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Non-Linear Battery Behavior in Electric Vehicle Scheduling Problems

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Abstract


The currently most popular approach to handle non-linear battery behavior for electric vehicle scheduling is to use a linear spline interpolation of the charge curve. We show that this can lead to approximate models that underestimate the charge duration and overestimate the state of charge, which is not desirable. While the error is of second order with respect to the interpolation step size, the associated mixed-integer linear programs do not scale well with the number of spline segments. It is therefore recommendable to use coarse interpolation grids adapted to the curvature of the charge curve, and to include sufficient safety margins to ensure solutions of approximate models remain feasible subjected to the exact charge curve.


1 Introduction and Related Literature

Shadowing the increasing interest in electrifying logistic systems, operations researchers are developing new optimization methods to handle the additional challenges introduced by deploying battery-powered vehicles. Logistics companies utilizing fully or partially electrified fleets must take battery capacities and the resulting range limitations into account by scheduling recharge events during service, which result in detours and vehicle downtime. Therefore, the *Electric Vehicle Scheduling Problem (EVSP)* is to assign a set of tasks or duties as well as recharge events to a fleet of electric vehicles such that no battery is ever fully depleted and the costs are minimized.

In Europe, operators prefer to recharge their vehicles with slow chargers at depots and fast chargers at selected external locations to minimize infrastructure acquisition costs. The amount of replenished driving range depends non-linearly on the charging time and the initial *state of charge (soc)*. The majority of EVSP papers (cf. surveys [EC19, PLL22]) consider simplified battery characteristics in such a way that either parts of the total battery capacity are ignored or that solutions can become infeasible in practice [OK20].

We have identified three approaches in the literature to incorporate non-linear battery behavior into EVSP models. *Energy expansion*, a method analogous to the well-known time expansion, is proposed in [vKNvdAH17] and [LLX19]. The non-linear behavior can be fully encoded in terms of the connections between discretized energy states. Naturally, this comes at a cost of significantly increased

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problem size, especially if one has to consider an additional time discretization to handle, for example, charger slot occupation.

Approaches that compute charge states exactly have been proposed by [Lee21], where a branch-and-price method exploits that charge duration is minimized in the objective, and by [DEG23], where a branch-and-check procedure produces cuts from schedules that can not be made energy-feasible by inserting recharge events.

Originally suggested in [MGMV17], the most popular approach is to use a piecewise linear spline interpolation $\hat{\xi}$ of a function ξ that maps the time spent charging an empty battery to the resulting *soc*. To the best of our knowledge, the implications of this particular modeling choice have hitherto only been examined in [ZMO22].

2 Piecewise Linear Charge Curve

Let $\xi : [0, t_{full}] \rightarrow [0, 1]$ be the *charge curve* giving the *soc* $y \in [0, 1]$ that results from charging an initially empty battery for t time units. According to the *Constant Current – Constant Voltage (CC-CV)* charging scheme commonly used for vehicle batteries [PJLV17], this function is linear until $\xi(t_V) = y_V$, then it grows monotonically and concavely towards the maximum *soc*, see Figure 1.

The charge curve is the solution to an autonomous non-linear ordinary differential equation $\xi'(t) = f(\xi(t))$ where f describes the charge rate as a function of *soc* [PJLV17]. f is constant on $[0, y_V]$ and monotonically non-increasing on $[y_V, 1]$, such that ξ is concave. f need not be differentiable in y_V . In general, there is no closed form for ξ , so it has to be approximated in some way.

The approximation that is most popular in the EVSP literature is a piecewise linear spline interpolation $\hat{\xi}$ of ξ . It is exact on the CC part, and it is straightforward to incorporate into mixed integer programming formulations as introduced in [MGMV17]. Namely, let $b_i = (t_i, y_i)$, $i = 0, \dots, n$, be the interpolation points of $\hat{\xi}$ such that $b_0 = (0, 0)$, $b_1 = (t_V, y_V)$ and $b_n = (t_{full}, 1)$. Let y_s and y_e be the *soc* at the beginning and the end of a recharge event, respectively, and θ the corresponding recharge duration. We can relate the duration and the charge state as follows. We first express (t_s, y_s) and (t_e, y_e) as convex combinations of their respective adjacent interpolation points, and then compute θ from the resulting time values, i.e., if $i_s = \arg \max_i \{y_i \leq y_s\}$ and $i_e =$

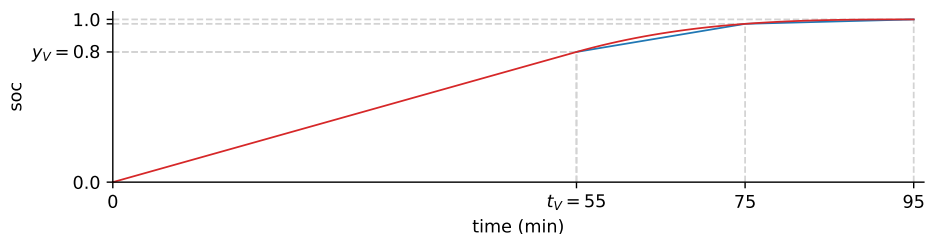


Figure 1: Example charge curve and linear spline interpolation based on real bus fast charging data. As is common in the literature, there are two CV phase segments.

$\arg \max_i \{y_i \leq y_e\}$, then

$$1 = \mu_{i_s} + \mu_{i_s+1} = \lambda_{i_e} + \lambda_{i_e+1} \quad (1)$$

$$y_s = \mu_{i_s} y_{i_s} + \mu_{i_s+1} y_{i_s+1} \quad (2)$$

$$y_e = \lambda_{i_e} y_{i_e} + \lambda_{i_e+1} y_{i_e+1} \quad (3)$$

$$\theta = \lambda_{i_e} t_{i_e} + \lambda_{i_e+1} t_{i_e+1} - \mu_{i_s} t_{i_s} - \mu_{i_s+1} t_{i_s+1} \quad (4)$$

Constraint (1) and $\mu_i = \lambda_i = 0$ for the remaining indices can be enforced in a MILP formulation via SOS type 2 constraints as in [MGMV17] or the papers building upon it. This commonly requires at least $2n |\mathcal{T}| |\mathcal{S}|$ additional binary decision variables where \mathcal{T} is the set of vehicle tasks and \mathcal{S} the set of charger slots.

3 Error Assessment

Computing θ in this way amounts to evaluating an approximation of a *charge duration function* $\Theta\xi : \{(y, z) \in [0, 1]^2 \mid y \leq z\} \rightarrow [0, t_{full}]$ that gives the duration required to reach some final *soc* z from an initial one y . Equivalently, if we were to compute z from y and θ , we execute a computation scheme for an approximation of a *charge increment function* $\Delta\xi : [0, 1] \times [0, t_{full}] \rightarrow [0, 1]$ that gives the amount of replenished *soc* if a battery is charged for θ time units from some initial charge state. Θ and Δ are function operators defined as

$$\Theta\xi(y, z) = \xi^{-1}(z) - \xi^{-1}(y) \quad \text{and} \quad \Delta\xi(y, \theta) = \xi(\xi^{-1}(y) + \theta) - y, \quad (5)$$

where we extend $\xi(t) = 1$ for $t > t_{full}$. To guarantee that solutions produced by approximate (linear) models using linear splines are also feasible for (non-linear) models using the exact curve, we want the approximate model to overestimate charge durations and to underestimate charge states, i.e., we need $\Theta\hat{\xi} \geq \Theta\xi$ and $\Delta\hat{\xi} \leq \Delta\xi$ to hold. Unfortunately, however, the natural looking implication

$$\hat{\xi}(t) \leq \xi(t) \quad \forall t \in [0, t_{full}] \implies \Theta\hat{\xi}(y, z) \geq \Theta\xi(y, z) \quad \forall y \leq z \in [0, 1] \quad (6)$$

does not hold in general, i.e., in the computation scheme (1)–(4) it is not true that underestimating the charge curve via a linear spline guarantees an overestimation of charging times, as we will show now. The analysis in [ZMO22], if we are not mistaken, takes these assumptions.

Proposition 1. *Given a linear spline interpolation $\hat{\xi}$ of a charge curve ξ , there exist input values $y < z$ such that $\Theta\hat{\xi}(y, z) < \Theta\xi(y, z)$.*

Proof. We have $\hat{\xi}(t_i) = \xi(t_i)$ for the interpolation points and $\hat{\xi}(t) < \xi(t)$ on the interior of CV segments (see Figure 1). Consequently, $\hat{\xi}^{-1}(y_i) = \xi^{-1}(y_i)$ and $\hat{\xi}^{-1}(y) > \xi^{-1}(y)$ on the CV interior. Hence, choosing $z = y_i$ for $i > 1$, we see

$$\Theta\hat{\xi}(y, z) = \Theta\hat{\xi}(y, y_i) = \hat{\xi}^{-1}(y_i) - \hat{\xi}^{-1}(y) < \xi^{-1}(y_i) - \xi^{-1}(y) = \Theta\xi(y, z) \quad (7)$$

for all $y_V < y < z$ and $y \neq y_j$ for all j . \square

Proposition 2. *There are y and θ such that $\Delta\hat{\xi}(y, \theta) > \Delta\xi(y, \theta)$.*

Proof. For $i > 1$ and $\theta > 0$ choose $y = \hat{\xi}(t_i - \theta)$, thus $t_i = \hat{\xi}^{-1}(y) + \theta$, and then

$$\Delta\hat{\xi}(y, \theta) + y = \xi(t_i) = \xi(\hat{\xi}^{-1}(y) + \theta) > \xi(\xi^{-1}(y) + \theta) = \Delta\xi(y, \theta) + y \quad (8)$$

by ξ being strictly monotonically increasing. \square

The charge duration underestimation of $\Theta\hat{\xi}$ is illustrated in Figure 2 and the charge state overestimation of $\Delta\hat{\xi}$ in Figure 3. Note that by Inequality (7), the charge duration is in fact always (not necessarily strictly) underestimated if the target charge state happens to be an interpolation point. Charge duration underestimation even occurs if we suppose only two interpolation points $(0, 0)$ and $(t_{full}, 1)$, as has been observed computationally by [ZLT⁺21], and is due to the derivative of the charge curve being overestimated near interpolation points. Since there are arguments for which $\Theta\hat{\xi}(y, z) > \Theta\xi(y, z)$ and $\Delta\hat{\xi}(y, \theta) < \Delta\xi(y, \theta)$ does hold, the functions $\Theta\hat{\xi}$ and $\Delta\hat{\xi}$ fluctuate around their exact counterparts.

Lemma 1. *The approximation error of $\Theta\hat{\xi}$ is*

$$\|\Theta\xi - \Theta\hat{\xi}\| = \|\xi^{-1} - \hat{\xi}^{-1}\| \leq \frac{h_{soc}^2}{8} \frac{\|f'\|}{f(y_V)^2}, \quad (9)$$

where $\|\cdot\|$ is the maximum norm and $h_{soc} = \max_{i=2, \dots, n} (y_i - y_{i-1})$.

Proof. Since the linear spline is accurate for the CC phase, any error occurs during the CV phase. There we have $(\xi^{-1})''(y) = -f'(y)/f(y)^2$ and since the charge rate is maximal during the CC phase, $\|f\| = f(y_V)$. Plugging this into a well-known error bound for linear spline approximation we obtain

$$\|\xi^{-1} - \hat{\xi}^{-1}\| \leq \frac{h_{soc}^2}{8} \|(\xi^{-1})''\| \leq \frac{h_{soc}^2}{8} \frac{\|f'\|}{\|f\|^2} = \frac{h_{soc}^2}{8} \frac{\|f'\|}{f(y_V)^2}. \quad (10)$$

To complete the proof, note that

$$\begin{aligned} \|\Theta\xi - \Theta\hat{\xi}\| &= \max_{0 \leq y \leq z \leq 1} \left| [\hat{\xi}^{-1}(z) - \xi^{-1}(z)] - [\hat{\xi}^{-1}(y) - \xi^{-1}(y)] \right| \\ &= \max_{y \in [0,1]} \left| \hat{\xi}^{-1}(y) - \xi^{-1}(y) \right| = \|\xi^{-1} - \hat{\xi}^{-1}\| \end{aligned} \quad (11)$$

since $\hat{\xi}^{-1} \geq \xi^{-1}$ and $\hat{\xi}^{-1}(y_i) = \xi^{-1}(y_i)$. \square

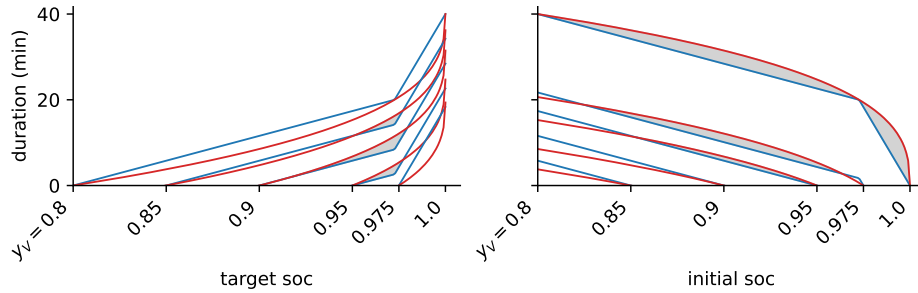


Figure 2: One-dimensional slices of $\Theta\xi$ and $\Theta\hat{\xi}$ computed from our example charge curve for $y = 0.8, 0.85, 0.9, 0.95, 0.975, 1$. We have $\Theta\xi(y, \cdot)$ and $\Theta\hat{\xi}(y, \cdot)$ on the left, and $\Theta\xi(\cdot, y)$ and $\Theta\hat{\xi}(\cdot, y)$ on the right. Critical areas where the approximation underestimates the charge duration are shaded.

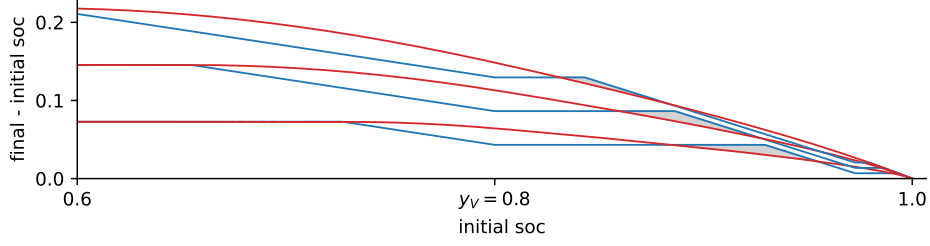


Figure 3: One-dimensional slices of $\Delta\xi$ and $\Delta\hat{\xi}$, computed from our example by $\Delta\xi(\cdot, \theta)$ and $\Delta\hat{\xi}(\cdot, \theta)$ for duration $\theta = 5, 10, 15$ minutes. Critical areas where the approximation overestimates the charge state are shaded.

Lemma 2. *The approximation error of $\Delta\hat{\xi}$ is*

$$\|\Delta\xi - \Delta\hat{\xi}\| = \|\xi - \hat{\xi}\| \leq \frac{h_{time}^2}{8} \|f'\| f(y_V), \quad (12)$$

where $\|\cdot\|$ is the maximum norm and $h_{time} = \max_{i=2, \dots, n} (t_i - t_{i-1})$.

Proof. Observe that $\xi''(t) = f'(\xi(t))f(\xi(t))$ and by the same error approximation as used for (10),

$$\begin{aligned} \|\Delta\xi - \Delta\hat{\xi}\| &= \max_{\substack{y \in [0,1] \\ \theta \in [0, t_{full}]}} \left| \xi(\xi^{-1}(y) + \theta) - \hat{\xi}(\hat{\xi}^{-1}(y) + \theta) \right| \\ &= \max_{\substack{y \in [0,1] \\ \theta \in [0, t_{full}]}} \left| [\xi(\hat{\xi}^{-1}(y) + \theta) - \hat{\xi}(\hat{\xi}^{-1}(y) + \theta)] - [\xi(\hat{\xi}^{-1}(y) + \theta) - \xi(\xi^{-1}(y) + \theta)] \right| \\ &= \max_{t \in [0, t_{full}]} \left| \xi(t) - \hat{\xi}(t) \right| = \|\xi - \hat{\xi}\| \leq \frac{h_{time}^2}{8} \|f'\| f(y_V) \quad (13) \end{aligned}$$

since $\hat{\xi}^{-1} \geq \xi^{-1}$ and $\hat{\xi}^{-1}(y_i) = \xi^{-1}(y_i)$. \square

In particular, the maximum possible underestimation of the charge duration respectively overestimation of the charge state is exactly the approximation error of the linear spline interpolation of the charge curve. The error becomes larger the faster the charge rate dissipates with growing *soc*, but decreases with the square of the interpolation step size. However, each additional linear segment introduces $2|\mathcal{T}||\mathcal{S}|$ new binary decision variables. We conclude that one should choose coarse interpolation grids adapted to the curvature of ξ and then include safety margins based on the error bound into the model to ensure solutions are feasible under exact ξ .

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