



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Scaling and Rounding Periodic Event Scheduling Instances To Different Period Times

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Abstract. The Periodic Event Scheduling Problem (PESP) is a notoriously hard combinatorial optimization problem, essential for the design of periodic timetables in public transportation. The coefficients of the integer variables in the standard mixed integer linear programming formulations of PESP are the period time, e.g., 60 for a horizon of one hour with a resolution of one minute. In many application scenarios, lines with different frequencies have to be scheduled, leading to period times with many divisors. It then seems natural to consider derived instances, where the period time is a divisor of the original one, thereby smaller, and bounds are scaled and rounded accordingly. To this end, we identify two rounding schemes: wide and tight. We then discuss the approximation performance of both strategies, in theory and practice.

Keywords: Timetabling · Mixed-Integer Programming · Public Transport.

1 Introduction

When planning public transport networks, particularly periodically operated networks, one of the many challenges is the definition of a timetable. Not only it is directly impactful for passengers' satisfaction, but it also affects further operational planning tasks, such as managing rolling stock and personnel assignments. Therefore, being able to compute periodic timetables with efficiency as well as efficacy is of practical importance, and as it turns out, also of high technical and computational difficulty. The Periodic Event Scheduling Problem (PESP) [10] is the standard model used to design and optimize periodic timetables. It is a well-known NP-complete problem, even in restrictively simpler instances [7]. Although a potpourri of solution methods has been designed (e.g., [1, 2, 8]), so far none of them has managed total success on the infamous PESPLib benchmarking library [3], where all instances currently sit unresolved.

In this work we propose a simple new strategy. We will employ previously known methods implemented in the `ConcurrentPESP` solver [1], but on slightly modified instances. The modification will consist in dividing the period time by some of its integer factors, and with it also all lower and upper bounds on activity durations. As this means scaling down the instance to a de facto equivalent copy, we further proceed as follows: To maintain the useful property of integral arc bounds [9], we round all scaled bounds, which could otherwise have non-trivial fractional part. Rather than rounding to the closest integer, we devise two main strategies: *tightening* and *widening*. Tightening means making activities bounds tighter by rounding lower bounds upwards and upper bounds downwards. Widening, on the contrary, does the opposite. Tightened instances can be used for primal searches, as they will always produce solutions that are feasible for the original instance, whereas widened instances can be used for dual searches instead, as they are a relaxation of the original problem.

Our main motivation for this scaling and rounding procedure is that decreasing the period time while maintaining integral bounds reduces the magnitude of the entries of the constraint matrix in the standard MILP formulations for PESP, and thus allows for tighter LP relaxations.

In Section 2 we will recapitulate the most basic PESP knowledge, in Section 3 we will describe our novel approach, and in Section 4 we will present our computational experiments.

2 The Periodic Event Scheduling Problem

The Periodic Event Scheduling Problem (PESP) captures several aspects of periodic timetabling in public transport [5], and is defined as follows:

Definition 1 ([10]). *Given an instance $\mathcal{I} = (G, T, \ell, u, w)$, where G is a directed graph of events $V(G)$ and activities $A(G)$, $T \in \mathbb{N}$ is a period time, vectors $\ell, u \in \mathbb{Z}^{A(G)}$ are lower and upper bounds on the activities, and $w \in \mathbb{R}_{\geq 0}^{A(G)}$ is a weight vector, the Periodic Event Scheduling Problem (PESP) is to find a periodic timetable $\pi \in [0, T)^{V(G)}$ and a periodic tension $x \in \mathbb{R}^{A(G)}$ such that*

$$\pi_j - \pi_i \equiv x_a \pmod{T} \quad \forall a = (i, j) \in A(G), \quad (1)$$

$$\ell \leq x \leq u, \quad (2)$$

$$w^\top x \text{ is minimum}, \quad (3)$$

or to decide that no such π and x exist.

A simple machinery to formulate PESP as a mixed-integer linear program (MILP) is to use auxiliary integer variables $p \in \mathbb{Z}^{A(G)}$, so called *periodic offsets*, such that (1) can be rewritten as

$$\pi_j - \pi_i + p_a T = x_a \quad \forall a = (i, j) \in A(G). \quad (4)$$

Another common mixed-integer program formulation for PESP is the cycle formulation, which given an integral cycle basis \mathcal{B} and a corresponding cycle matrix $\Gamma \in \{-1, 0, 1\}^{\mathcal{B} \times A(G)}$, then substitutes (1) with

$$\Gamma x = Tz, \quad (5)$$

where we again have auxiliary integer variables $z \in \mathbb{Z}^{\mathcal{B}}$, so called *cycle offsets* [6]. In either case, the integer variables are multiplied with the comparably large factor T , which deteriorates the quality of linear programming relaxations.

3 Scaling and Rounding of Instances

As discussed, we now propose instance-alteration strategies to hopefully achieve speed improvements in solution discovery. The basic concept of our strategies is that of scaling instances by some factor of their period time and then carefully rounding any non-integral bounds. Explicitly, given $k \in \mathbb{N}$ such that k divides T , and arc bounds $[\ell, u]$, we define two scaled and rounded bounding strategies as:

$$\textit{tight rounded bounds:} \quad \left[\left\lfloor \frac{\ell}{k} \right\rfloor, \left\lfloor \frac{u}{k} \right\rfloor \right] \quad (6)$$

$$\textit{wide rounded bounds:} \quad \left[\left\lceil \frac{\ell}{k} \right\rceil, \left\lceil \frac{u}{k} \right\rceil \right]. \quad (7)$$

Then, given a PESP instance \mathcal{I} , we indicate as $\mathcal{I}_{k,t}$ the corresponding instance where all arc bounds are scaled by k and tightly rounded, and the period time is set to T/k . Similarly, $\mathcal{I}_{k,w}$ indicates the case of wide rounding. Of course $\mathcal{I}_{1,t} = \mathcal{I}_{1,w} = \mathcal{I}$. We quickly summarize some immediate observations.

Lemma 1. *Let \mathcal{I} be a PESP instance with period time T , and let k be a divisor of T .*

1. *If (π, x) is a feasible solution to $\mathcal{I}_{k,t}$, then $(k\pi, kx)$ is feasible for \mathcal{I} .*
2. *If (π, x) is a feasible solution to \mathcal{I} , then $(\pi/k, x/k)$ is feasible for $\mathcal{I}_{k,w}$.*

For the optimal objective values $\text{opt}(\cdot)$ in terms of weighted periodic tension $w^\top x$, Lemma 1 has the following consequences:

Theorem 1. *Let $\mathcal{I} = (G, T, \ell, u, w)$ be a PESP instance, and let k be a divisor of T .*

1. *If $\mathcal{I}_{k,t}$ is feasible, then \mathcal{I} is feasible, and $\text{opt}(\mathcal{I}) \leq k \cdot \text{opt}(\mathcal{I}_{k,t})$.*
2. *If \mathcal{I} is feasible, then $\mathcal{I}_{k,w}$ is feasible, and $k \cdot \text{opt}(\mathcal{I}_{k,w}) \leq \text{opt}(\mathcal{I})$.*

Proof. Let (π, x) be an optimal solution to $\mathcal{I}_{k,t}$. Then $(k\pi, kx)$ is feasible for \mathcal{I} by Lemma 1, and so $\text{opt}(\mathcal{I}) \leq w^\top(kx) = k \cdot \text{opt}(\mathcal{I}_{k,t})$. The proof for wide rounding is analogous.

All of the above bounds are immediately verified to be tight, for example when all activity bounds are divisible by k , because then $\mathcal{I}_{k,t} = \mathcal{I}_{k,w}$.

However, for realistic instances, e.g., the PESPLib railway instances [3], the tight rounding procedure reveals an excess of naïveté, as two issues arise. The span of an activity $a \in A(G)$ is defined to be the width of its bound interval, i.e., $u_a - \ell_a$. There are activities a which span small enough that for some divisors k we have

$$\left\lfloor \frac{u_a}{k} \right\rfloor < \left\lceil \frac{\ell_a}{k} \right\rceil, \quad (8)$$

and of course this then renders $\mathcal{I}_{k,t}$ a priori infeasible. Moreover, there are often many free activities, i.e., activities with span equal to $T - 1$. These arcs intend to model transfer activities, and do not affect the feasibility of \mathcal{I} , but once scaled and rounded in $\mathcal{I}_{k,t}$ they are suddenly quite active instead.

Therefore, to overcome both issues, we also constructed instances $\tilde{\mathcal{I}}_{k,t}$, where any activity that is either free or satisfying (8) is left unchanged, and the bounds on the remaining *unproblematic* activities a are instead set to

$$\left[k \left\lceil \frac{\ell_a}{k} \right\rceil, k \left\lfloor \frac{u_a}{k} \right\rfloor \right], \quad (9)$$

and the period time remains T . Similarly for $\tilde{\mathcal{I}}_{k,w}$. This strategy basically tightens the unproblematic activities, while leaving the others as in the original instance \mathcal{I} . As a final attempt we also tested a further modification of $\tilde{\mathcal{I}}_{k,t}$, where only the unproblematic upper bounds were rounded downwards, leaving the lower bounds unchanged. The rationale of this one-sided alteration is that the periodic tensions tend to be particularly frequently at the lower bound in quality solutions. We denote such instances by $\tilde{\mathcal{I}}_{k,ot}$, and define $\tilde{\mathcal{I}}_{k,ow}$ analogously.

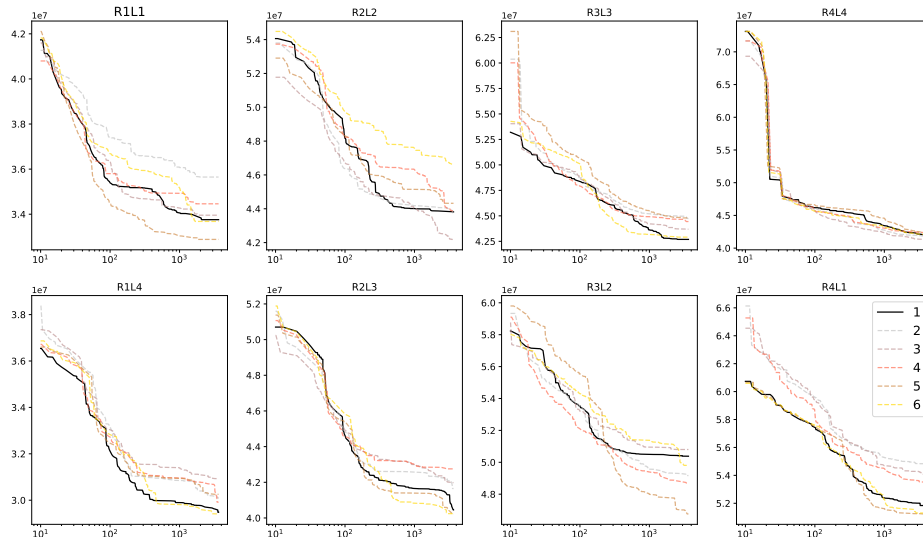


Fig. 1. Tests of the $\tilde{\mathcal{I}}_{k,ot}$ instances. The time axis is in logarithmic scale, the y -axis is the weighted periodic slack $w^\top(x - \ell)$.

4 Computational Results

All our experiments were conducted on an Intel i7-9700K CPU with 32 GB RAM, using the latest version of `ConcurrentPESP` [1], and using Gurobi 10 [4] as underlying MILP solver.

Initially we tested the $\mathcal{I}_{k,t}$ and $\mathcal{I}_{k,w}$ formulations for all positive integral divisors of $T = 60$, i.e., $k \in \{2, 3, 4, 5, 6, 10, 12, 15, 20, 30\}$, on very easy to solve sub-instances of R1L1, the smallest PESPLib instance [3]. As expected, many tightened instances were outright infeasible, if not for the smallest divisors. In any case, the new formulations showed no advantage, in none of these tests, which were all quite quickly solved to optimality. We went on and conducted the same tests on R1L1 and R4L4, with a time limit of one hour. The exact same patterns repeated.

Further, we repeated the tests with all the same parameters, but now on the instances adjusted so as to deal with the infeasibility issues explained above, namely those denoted by $\tilde{\mathcal{I}}_{k,t}$ and $\tilde{\mathcal{I}}_{k,w}$. Almost everything was now feasible, but the non-scaled instance performed best at all times.

To conclude, we tested $\tilde{\mathcal{I}}_{k,ot}$. As divisors, we only used the smaller ones, i.e., $k \in \{2, 3, 4, 5, 6\}$, which from the previous tests were the only ones possibly close to the non-scaled instance's performance. As instances, instead, we took R1L1, R2L2, R3L3, R4L4, R1L4, R2L3, R3L2, R4L1. The

time limit was one hour. A picture of these test runs is shown in Figure 1. In all of these tests there were satisfactory speed improvements in the primal search. For example, for R3L2, the test on the original instance finds a primal $\sim 11.4\%$ above the current PESPlib incumbent in terms of weighted slack $w^\top(x - \ell)$, compared to only $\sim 3.4\%$ above on $\tilde{I}_{5,ot}$. Sometimes the scaled instances outright beat the non-scaled instance, on the whole time window, and sometimes it only was better for certain time intervals. This suggests it could be beneficial to implement these strategies in a concurrent fashion. Regardless, it still seems unpredictable which scaling factors k perform well or not.

Concerning dual bounds, our tests with instances of the form $\tilde{I}_{k,ow}$ remained inconclusive, with no clear winner.

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