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The Maximum Diversity Assortment Selection Problem

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Abstract. In this paper, we introduce the Maximum Diversity Assortment Selection Problem (MADASS), which is a generalization of the 2-dimensional Cutting Stock Problem (2CSP). Given a set of rectangles and a rectangular container, the goal of 2CSP is to determine a subset of rectangles that can be placed in the container without overlapping, i.e., a feasible assortment, such that a maximum area is covered. In MADASS, we need to determine a set of feasible assortments, each of them covering a certain minimum threshold of the container, such that the diversity among them is maximized. Thereby, diversity is defined as minimum or average normalized Hamming-Distance of all assortment pairs. The MADASS Problem was used in the 11th AIMMS-MOPTA Competition in 2019. The methods we describe in this article and the computational results won the contest. In the following, we give a definition of the problem, introduce a mathematical model and solution approaches, determine upper bounds on the diversity, and conclude with computational experiments conducted on test instances derived from the 2CSP literature.

1 Introduction

The problem of packing rectangles into rectangular containers or to cut them from rectangular stock sheets arises in a variety of industrial operations. Thereby, one typically aims at determining a feasible solution where the wasted material or space is minimized. Consider for example the paper, glass, wood, textile, or metal industry. Here, rectangular pieces are needed for the production of certain goods, which are typically cut from given stock pieces [43]. Another common application arising in logistics is to load pallets or containers [49]. Furthermore, similar problems appear in the production and operation of microchips, namely

in the layout of processor chips [77] and the dynamic allocation of memory as well as in multiprocessor scheduling [43]. Finally, the editing and lay-outing of newspapers [76] or the arrangement of products on supermarket shelves [2] belong to this class of applications, too. These problems are typically modeled as some special variant of the 2-dim Cutting Stock Problem (2CSP), which we introduce in Section 2 and discuss in Section 3.

However, for practitioners it is often useful when they are presented not only one optimal but a set of diverse “near-optimal” solutions from which she or he can choose. This holds especially for those tasks where it is hard to formalize or model important side constraints. Consider, for example, the last two examples from above. If a supermarket wants to investigate the buying behavior of its customers, an arrangement of the products minimizing the empty space is certainly desirable. Nevertheless, for this particular task the result is not very meaningful. Instead, the company needs to conduct tests with a variety of arrangements to assess whether they increase the purchasing rates [52]. Similarly, when it comes to the layout of texts, pictures, or ads on a newspaper page, the result does not necessarily have to be minimal w.r.t. the resulting empty space, but has to come in some aesthetic appeal. Thus, presenting the user with a selection of assortments that cover some minimum threshold of the available area can be advantageous in all areas where experiences and subjective perceptions of humans have an impact on the chosen solution.

Problems of this kind are the motivation for the Maximum Diversity Assortment Selection Problem (MADASS), which we introduce in Section 2. We review the literature on two inherent subproblems in Section 3: the above mentioned 2CSP and the Maximum Diversity Problem (MDP), where a predefined number of elements has to be selected from a given set such that the diversity among them is maximized. Before discussing an MIQP model for MADASS in Section 5, we introduce MIQP formulations to determine upper bounds on the maximum diversity in Section 4. Next, we present a generic two-stage heuristic in Section 6. We present extensive computational experiments in Section 7. Finally, we conclude and give an outlook on future research in Section 8.

2 Definitions and Problem Setup

In MADASS, we are given a rectangle \mathcal{C} with width $w \in \mathbb{Z}_{\geq 0}$ and height $h \in \mathbb{Z}_{\geq 0}$, which we call *container*. Furthermore, we are given a set of rectangles $\mathcal{R} := \{R_1, \dots, R_n\}$ and each of them is associated with its width $w_i \in \mathbb{Z}_{\geq 0}$ and its height $h_i \in \mathbb{Z}_{\geq 0}$. Next, an *assortment* is a subset $\mathcal{A} \subseteq \mathcal{R}$ of rectangles, i.e., $\mathcal{A} \in \mathcal{P}(\mathcal{R})$ where $\mathcal{P}(\mathcal{R})$ denotes the powerset of \mathcal{R} . We call an assortment \mathcal{A} *feasible* if it can be placed in the container \mathcal{C} without overlapping, i.e., if we can assign a bottom-left corner coordinate $(x_i, y_i) \in \mathbb{R}^2$ to each rectangle $R_i \in \mathcal{A}$ such that $[x_i, x_i + w_i) \times [y_i, y_i + h_i) \subseteq [0, w) \times [0, h)$ and for all $R_i, R_j \in \mathcal{A}$ with $i \neq j$ we have $[x_i, x_i + w_i) \times [y_i, y_i + h_i) \cap [x_j, x_j + w_j) \times [y_j, y_j + h_j) = \emptyset$. In the sequel, we denote the set of all feasible assortments by \mathcal{F} . Next, each assortment \mathcal{A} has an associated value $v(\mathcal{A}) = \sum_{R_i \in \mathcal{A}} w_i h_i$, which is the sum of the areas of

the rectangles it contains. We call an assortment \mathcal{A}^* optimal if it is feasible and if $v(\mathcal{A}^*) \geq v(\mathcal{A})$ holds for every $\mathcal{A} \in \mathcal{F}$. Determining an optimal assortment is called the 2-dimensional Cutting Stock Problem (2CSP). Note that we otherwise allow an arbitrary placement of the rectangles inside the container, i.e., we do not impose any further conditions, and we do not allow the rotation of rectangles. An example can be found in Figure 1.

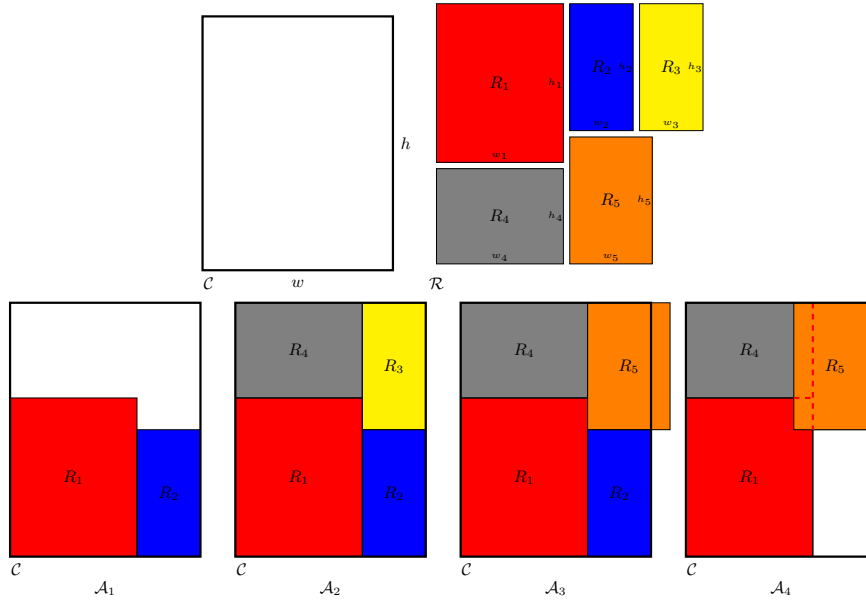


Fig. 1: Container \mathcal{C} with rectangle set \mathcal{R} . While assortment \mathcal{A}_1 is feasible, \mathcal{A}_2 is even optimal. On the other hand, assortments \mathcal{A}_3 and \mathcal{A}_4 are not feasible.

In MADASS we are furthermore given a threshold value $\underline{v} \in [0, v(\mathcal{C})]$, with $v(\mathcal{C}) := wh$ denoting the area of the container, as well as a natural number $m \in \mathbb{N}$ with $m \geq 2$. We call an assortment \mathcal{A} \underline{v} -good if it is feasible and if $v(\mathcal{A}) \geq \underline{v}$ and we denote the set of \underline{v} -good assortments by $\mathcal{F}_{\underline{v}} \subseteq \mathcal{F}$. Furthermore, a selection is a multi-subset $\mathcal{S} \subseteq \mathcal{P}(\mathcal{R})$ of assortments of cardinality $|\mathcal{S}| = m$, i.e., we allow that an assortment is contained more than once in \mathcal{S} . We call \mathcal{S} feasible if all of its assortments are \underline{v} -good, i.e., if $\mathcal{S} \subseteq \mathcal{F}_{\underline{v}}$. Additionally, we are given a diversity function δ for selections and call $\delta(\mathcal{S})$ the diversity of selection \mathcal{S} . The two diversity functions that we consider in this paper are based on the Hamming-Distances between assortment pairs contained in \mathcal{S} and are discussed in Subsection 2.1. Finally, a selection \mathcal{S}^* is called optimal if it is feasible and if $\delta(\mathcal{S}^*) \geq \delta(\mathcal{S})$ holds for all feasible selections \mathcal{S} . In the following, we denote a MADASS instance \mathcal{I} as quintuple $\mathcal{I} := (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta)$.

Lemma 1. *MADASS is NP-hard.*

Proof. We prove this by reduction from 2CSP, which is NP-hard, see Fekete and Schepers [29] and Garey and Johnson [32]. Given an instance of 2CSP with container \mathcal{C} and set of rectangles \mathcal{R} , for arbitrary m and δ , there exists a feasible assortment of value at least \underline{v} if and only if there exists a feasible selection for MADASS instance $\mathcal{I} := (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta)$.

2.1 Diversity of Selections

A common distance measure applied to the subsets of a common superset is the *Hamming-Distance* [41]. Given an assortment \mathcal{A} , we associate the vector $v_{\mathcal{A}} \in \{0, 1\}^n$ with it, where the i -th entry $v_{\mathcal{A}}^i$ is equal to 1 if rectangle R_i is contained in \mathcal{A} and 0 otherwise. For two assortments \mathcal{A} and \mathcal{A}' , the (normalized) Hamming-Distance is

$$d_H(\mathcal{A}, \mathcal{A}') := \frac{\sum_{i=1}^n |v_{\mathcal{A}}^i - v_{\mathcal{A}'}^i|}{|\mathcal{A}| + |\mathcal{A}'|}.$$

Furthermore, we set $d_H(\mathcal{A}, \mathcal{A}') = 0$ in the case $\mathcal{A} = \mathcal{A}' = \emptyset$. Note that we consider the normalized Hamming-Distance, i.e., we divide by $|\mathcal{A}| + |\mathcal{A}'|$, to ensure that the number of rectangles forming the assortments has no impact on the distance. An example demonstrating this rationale is given in Figure 2.

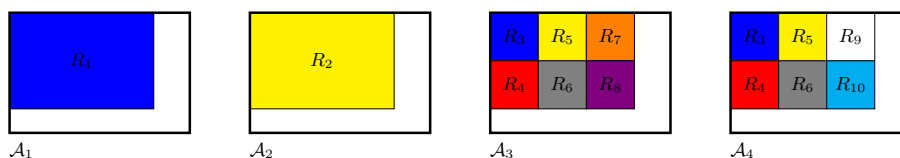


Fig. 2: The Hamming-Distance without normalization between \mathcal{A}_1 and \mathcal{A}_2 is 2, while the distance between \mathcal{A}_3 and \mathcal{A}_4 is 4. Using normalization, \mathcal{A}_1 and \mathcal{A}_2 have maximum distance 1, while the distance between \mathcal{A}_3 and \mathcal{A}_4 is $\frac{1}{3}$.

Lemma 2. *Let $\mathcal{I} := (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta)$ be a MADASS instance. For any two assortments $\mathcal{A} \subseteq \mathcal{R}$ and $\mathcal{A}' \subseteq \mathcal{R}$ we have $d_H(\mathcal{A}, \mathcal{A}') \in [0, 1]$.*

Based on the Hamming-Distance, we next define two diversity functions for selections. First, the *Minimum-Distance-Diversity*

$$\delta_{min}(\mathcal{S}) := \min_{\substack{i, j \in M \\ i < j}} \{d_H(\mathcal{A}_i, \mathcal{A}_j)\},$$

and second, the *Average-Distance-Diversity*

$$\delta_{avg}(\mathcal{S}) := \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m d_H(\mathcal{A}_i, \mathcal{A}_j).$$

In the remainder of this paper, we denote by $M := \{1, \dots, m\}$ the index set of the assortments in a selection.

Lemma 3. *Let $\mathcal{I} := (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta)$ be a MADASS instance and let $\mathcal{S} \subseteq \mathcal{P}(\mathcal{R})$ be a selection. Then $0 \leq \delta_{min}(\mathcal{S}) \leq \delta_{avg}(\mathcal{S}) \leq 1$.*

Proof. This can be seen from

$$\begin{aligned} \delta_{min}(\mathcal{S}) &= \min_{\substack{i, j \in M \\ i < j}} \{d_H(\mathcal{A}_i, \mathcal{A}_j)\} \\ &= \frac{2}{m(m-1)} \sum_{k=1}^{m-1} \sum_{l=k+1}^m \min_{\substack{i, j \in M \\ i < j}} \{d_H(\mathcal{A}_i, \mathcal{A}_j)\} \\ &\leq \frac{2}{m(m-1)} \sum_{k=1}^{m-1} \sum_{l=k+1}^m d_H(\mathcal{A}_k, \mathcal{A}_l) = \delta_{avg}(\mathcal{S}). \end{aligned}$$

3 Related Work and Subproblems

MADASS has first been introduced in the 11th AIMMS-MOPTA Optimization Modeling Competition [1,2], which is part of the MOPTA conference series held annually at Lehigh University. To the best of our knowledge, there exists no previous work regarding it. Therefore, we give an overview of the literature concerning its two inherent subproblems instead: The 2-dimensional Cutting Stock Problem (2CSP) and the Maximum Diversity Problem (MDP).

As discussed in Section 1, the packing and cutting of rectangular items into or from rectangular containers arises in the context of many different applications. Thus, the 2CSP is a topic that has been under investigation for a long time. According to Dowsland and Dowsland [24], the first mathematical formulation of the problem was given by Kantorovich [51] in 1939. Similar problem formulations were given by other authors during the 1950's as the work of Kantorovich was not translated until 1960.

For a general overview over 2CSP, we refer to the survey papers [20,23,26,46,48]. However, as there exist a variety of different variants of the 2CSP, we want to emphasize the work of Wäscher et al. [79] and Lodi et al. [57] regarding their classification. The former is based on the typology used in Dyckhoff [26]. Here, the different rectangular packing problems are identified as, e.g., Bin Packing, Knapsack and Cutting Stock Problems, and additionally classified according to their dimensionality, objective, as well as size, shape, and characteristics of the rectangles. On the other hand, the classification by Lodi et al. is based on the side constraints that need to be satisfied, e.g., if the rectangles can be placed freely in the container or are allowed to be rotated. Further, there exist weighted and an unweighted variants of the 2CSP. In the weighted case each rectangle is assigned a certain value, and the objective of the problem is to maximize the sum of the values of all placed rectangles. In the unweighted case, the value of each rectangle corresponds to its area. Thus, the objective can either be seen as maximizing the area of all placed rectangles or as minimizing the empty or wasted

space in the container. As mentioned, in this paper we consider the unweighted variant of 2CSP where the rectangles can be placed freely in the container and are not allowed to be rotated.

In the literature, many *Mixed Integer Programming (MIP)* models exist for 2CSP. For further details regarding MIP in general, we refer to Achterberg [3]. Hadjiconstantinou and Christofides [39] introduced a formulation using the straightforward technique of discretizing the container. To each integer tuple in the container is associated a binary variable indicating if the corresponding point is already covered by a placed rectangle. Other MIP models featuring less variables and constraints using the relative positions of pairs of rectangles were introduced by Belov et al. [12], and Egeblad and Pisinger [27] in 2009. The former approach is based on variables and corresponding constraints indicating whether two rectangles overlap when projected onto the x- or y-axis, from which at most one is allowed in a feasible assortment. The idea of the latter model is to use binary variables for each pair of rectangles to ensure that one of them is placed either over, under, left, or right of the other. Several other MIP formulations can be found in [10,35,42,44].

Furthermore, a variety of problem-specific *Branch-and-Bound* approaches has been developed [21,22,45]. To improve their performance, Boschetti et al. [18] present an upper bound, which can be used to significantly reduce the number of feasibility checks that have to be performed. Additionally, Fekete et al. [30] introduce an approach to reduce the time that is needed for checking the feasibility of subsets of rectangles, i.e., whether it can be placed into the container and therefore forms a feasible assortment. They use graph structures to model equivalence classes of assortments and can determine if a certain rectangle subset is feasible or not by checking it for cycles and cliques.

Next, we give a brief overview of the broad variety of heuristics and meta-heuristics that have been applied to 2CSP. We start with deterministic algorithms, which are typically embedded in an iterative procedure applying different randomized orderings of the rectangles. The first type of heuristic used for 2CSP are quasi-human algorithms that are inspired by the behavior of humans when solving a given problem. Consider, for example, the Least-Flexible-First algorithms of Wu et al. [78], Wu and Chan [77], and Huang and Chen [49]. Their basic idea is to pack rectangles that are less flexible due to their size in the beginning, in order to have more flexibility when finishing up the packing. On the other hand, Wei et al. [76] presented a Least-Waste-First heuristic where the rectangles are placed such that empty areas where no further rectangles can be placed are avoided. The third kind of placing procedures are so-called Best-Fit algorithms. In general, these heuristics are based on an evaluation function. Here, the rectangles are not only chosen and placed in a way such that the resulting empty space is as small as possible, but they also have to fit “well” with respect to the already placed ones. Examples for this type of heuristic can be found in the work of He et al. [43], de Armas et al. [7], and in particular in the IBHP heuristic of Shiangjen et al. [69]. The last deterministic approach we mention here is the Dynamic Decomposition algorithm of Wang [74]. His idea

is to sequentially decompose the container into smaller parts, pack them with rectangles, and rearrange them afterwards.

As mentioned above, several meta-heuristics have also been applied to 2CSP. The approaches can roughly be partitioned into Genetic Algorithms [11,17], Greedy Randomized Adaptive Search Procedures (GRASP) [5,62,63], TABU Search [6], and Simulated Annealing [27,55]. Further, there exist hybrid heuristics that combine deterministic algorithms and meta-heuristics [36,37,40].

The second problem, which is implicitly contained in MADASS, is to select a predefined number of elements from a given set such that the diversity among them is maximized. In our case this is the set of \underline{v} -good solutions $\mathcal{F}_{\underline{v}}$. This problem is known as the Maximum Diversity Problem or as Maximum Dispersion Problem (MDP) and is usually subdivided into the MAX-SUM and the MAX-MIN case. In the first case, the sum of the distances between the selected elements is maximized, while in the second case, one aims at maximizing the minimum distance between the chosen elements. This directly corresponds to our diversity measures δ_{avg} and δ_{min} .

Surveys on MDP have been published by Martí et al. [61] and Sandoya et al. [67]. As for 2CSP, there exists a variety of exact approaches to model and solve the MDP, including MIP and IQP formulations [34,53] as well as special Branch-and-Bound approaches [60]. Furthermore, different meta-heuristics have been used to tackle the problem, including for example the GRASP heuristic [66,70,71], TABU Search [25], and the Iterated Greedy Approach [58]. Finally, there also exist hybrid algorithms [31,68] and greedy heuristics [65].

4 Upper Bounds on the Diversity

In this section, we introduce two MIQP formulations to derive upper bounds on the maximum diversity of MADASS instances with respect to δ_{min} and δ_{avg} . The basic idea is to relax the problem by including assortments for which there may not exist a feasible placement in the container but that satisfy the \underline{v} -criterion, i.e., assortments contained in $\mathcal{G}_{\underline{v}} := \{\mathcal{A} \in \mathcal{P}(\mathcal{R}) \mid \underline{v} \leq v(\mathcal{A}) \leq v(\mathcal{C})\}$.

To simplify notation, we use an alternative expression for the Hamming-Distance in the sequel, namely

$$d_H(\mathcal{A}, \mathcal{A}') = \frac{\sum_{i=1}^n |v_{\mathcal{A}}^i - v_{\mathcal{A}'}^i|}{|\mathcal{A}| + |\mathcal{A}'|} = \frac{|\mathcal{A}| + |\mathcal{A}'| - 2|\mathcal{A} \cap \mathcal{A}'|}{|\mathcal{A}| + |\mathcal{A}'|} = 1 - \frac{2|\mathcal{A} \cap \mathcal{A}'|}{|\mathcal{A}| + |\mathcal{A}'|}.$$

4.1 Bounding the Minimum-Distance-Diversity δ_{min}

Lemma 4. *Let $\mathcal{I} := (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{min})$ be an instance of MADASS and let S denote a feasible selection for \mathcal{I} . Then*

$$\delta_{min}(S) \leq 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{G}_{\underline{v}}} \max_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \left\{ \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\}.$$

Proof. From the definition of δ_{min} and since $\mathcal{F}_v \subseteq \mathcal{G}_v$ it follows that

$$\begin{aligned}
\delta_{min}(\mathcal{S}) &\leq \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_v} \min_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \{d_H(\mathcal{A}_i, \mathcal{A}_j)\} \\
&= \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_v} \min_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \left\{ 1 - \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\} \\
&= \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_v} \left\{ 1 - \max_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \left\{ \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\} \right\} \\
&= 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_v} \max_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \left\{ \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\} \\
&\leq 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{G}_v} \max_{\substack{i, j \in \{1, \dots, m\} \\ i < j}} \left\{ \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\}. \tag{1}
\end{aligned}$$

The following MIQP formulation UB_{min} determines expression (1), i.e., an optimal selection with respect to δ_{min} in \mathcal{G}_v . Its variables and their meanings are listed in Table 1.

$$\max 1 - z \tag{2}$$

$$\text{s.t.} \quad \sum_{R_i \in \mathcal{R}} w_i h_i c_{ia} \geq v \quad \forall a \in M \tag{3}$$

$$\sum_{R_i \in \mathcal{R}} w_i h_i c_{ia} \leq v(\mathcal{C}) \quad \forall a \in M \tag{4}$$

$$\sum_{R_i \in \mathcal{R}} (c_{ia} + c_{ib}) = t_{ab} \quad \forall a, b \in M \text{ with } a < b \tag{5}$$

$$c_{ia} + c_{ib} \leq 1 + s_{iab} \quad \forall R_i \in \mathcal{R}, \forall a, b \in M \text{ with } a < b \tag{6}$$

$$\sum_{R_i \in \mathcal{R}} s_{iab} \leq \frac{1}{2} z t_{ab} \quad \forall a, b \in M \text{ with } a < b \tag{7}$$

$$c_{ia} \in \{0, 1\} \quad \forall R_i \in \mathcal{R}, \forall a \in M \tag{8}$$

$$s_{iab} \in \{0, 1\} \quad \forall R_i \in \mathcal{R}, \forall a, b \in M \text{ with } a < b \tag{9}$$

$$t_{ab} \in [0, \dots, 2|\mathcal{R}|] \quad \forall a, b \in M \text{ with } a < b \tag{10}$$

$$z \in [0, 1] \tag{11}$$

Constraints (3) and (4) ensure that the generated assortments are contained in \mathcal{G}_v . Additionally, t_{ab} is equal to $|\mathcal{A}_a| + |\mathcal{A}_b|$ due to constraint (5). Further, we have $s_{iab} = 1$ if and only if assortments \mathcal{A}_a and \mathcal{A}_b share rectangle R_i due to constraint (6), because we minimize expression (2). Therefore and analogously, the variable z is equal to the maximum value of $\frac{2|\mathcal{A}_a \cap \mathcal{A}_b|}{|\mathcal{A}_a| + |\mathcal{A}_b|}$ among all pairs $a, b \in M$ with $a < b$ due to constraint (7) and the objective function (2), too.

Variable	Description
c_{ia}	Binary variable indicating whether R_i is contained in \mathcal{A}_a
s_{iab}	Binary variable indicating whether R_i is contained in \mathcal{A}_a and \mathcal{A}_b
t_{ab}	Continuous variable equal to $ \mathcal{A}_a + \mathcal{A}_b $
z	Continuous variable equal to the non-constant objective function value

 Table 1: Variables for MIQP model UB_{min} .

4.2 Bounding the Average-Distance-Diversity δ_{avg}

The results from the previous subsection can also be adapted to δ_{avg} .

Lemma 5. *Let $(\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{avg})$ be an instance of MADASS and let \mathcal{S} denote a feasible selection. It holds that*

$$\delta_{avg}(\mathcal{S}) \leq 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{G}_{\underline{v}}} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|}.$$

Proof. From the definition of δ_{avg} and since $\mathcal{F}_{\underline{v}} \subseteq \mathcal{G}_{\underline{v}}$ it follows that

$$\begin{aligned} \delta_{avg}(\mathcal{S}) &\leq \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_{\underline{v}}} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m d_H(\mathcal{A}_i, \mathcal{A}_j) \\ &= \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_{\underline{v}}} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \left(1 - \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|}\right) \\ &= \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_{\underline{v}}} \frac{2}{m(m-1)} \left(\frac{m(m-1)}{2} - \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right) \\ &= \max_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_{\underline{v}}} \left\{ 1 - \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \right\} \\ &= 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{F}_{\underline{v}}} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|} \\ &\leq 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{G}_{\underline{v}}} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|}. \end{aligned} \tag{12}$$

The following MIQP formulation UB_{avg} determines expression (12), i.e., an optimal selection w.r.t. δ_{avg} in \mathcal{G}_v :

$$\max 1 - \frac{2}{m(m-1)} \sum_{a=1}^{m-1} \sum_{b=a+1}^m z_{ab} \quad (13)$$

$$\text{s.t.} \quad (3) - (6)$$

$$\sum_{R_i \in \mathcal{R}} y_{iab} = \frac{1}{2} z_{ab} l_{ab} \quad \text{for all } a, b \in M \text{ with } a < b \quad (14)$$

$$(8) - (10)$$

$$z_{ab} \in [0, 1] \quad \text{for all } a, b \in M \text{ with } a < b. \quad (15)$$

Most of the variables and constraints here are identical to the ones used in UB_{min} . However, here we introduce individual continuous variables (15) for each assortment pair \mathcal{A}_a and \mathcal{A}_b in order to determine their individual contributions, see (14), to the objective function (13).

4.3 Relation between Diversity Functions and its Bounds

In MADASS, if we consider two instances that only differ by the diversity function, we can make the following observations.

Lemma 6. *Let $\mathcal{I}_1 = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{min})$ and $\mathcal{I}_2 = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{avg})$ be MADASS instances with optimal selections \mathcal{S}_1 and \mathcal{S}_2 , respectively. Then it holds that $\delta_{min}(\mathcal{S}_1) \leq \delta_{avg}(\mathcal{S}_2)$.*

Proof. Applying Lemma 3 yields $\delta_{min}(\mathcal{S}_1) \leq \delta_{avg}(\mathcal{S}_1) \leq \delta_{avg}(\mathcal{S}_2)$.

Corollary 1. *Let $\mathcal{I}_1 = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{min})$ and $\mathcal{I}_2 = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{avg})$ be two instances of MADASS. Then any upper bound on the diversity of \mathcal{I}_2 is an upper bound on the diversity of \mathcal{I}_1 as well.*

This result can directly be applied to our MIQP model UB_{avg} .

Corollary 2. *Let $\mathcal{I} = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{min})$ be an instance of MADASS and let \mathcal{S} denote a feasible selection.*

$$\delta_{min}(\mathcal{S}) \leq 1 - \min_{\{\mathcal{A}_1, \dots, \mathcal{A}_m\} \subseteq \mathcal{G}_v} \frac{2}{m(m-1)} \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{2|\mathcal{A}_i \cap \mathcal{A}_j|}{|\mathcal{A}_i| + |\mathcal{A}_j|},$$

i.e., the optimal value of UB_{avg} is an upper bound on \mathcal{I} 's diversity.

However, this bound cannot be tighter than the one we derive using UB_{min} .

Corollary 3. *Let $\mathcal{I} = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta_{min})$. We have $UB_{min}^*(\mathcal{I}) \leq UB_{avg}^*(\mathcal{I})$, where $UB_{min}^*(\mathcal{I})$ and $UB_{avg}^*(\mathcal{I})$ denote the optimal solution values for the corresponding MIQP models.*

Proof. Let $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathcal{G}_v$ be optimal selections for $UB_{min}(\mathcal{I})$ and $UB_{avg}(\mathcal{I})$, respectively. By Lemma 3, it follows that

$$UB_{min}^*(\mathcal{I}) = \delta_{min}(\mathcal{S}_1) \leq \delta_{avg}(\mathcal{S}_1) \leq \delta_{avg}(\mathcal{S}_2) = UB_{avg}^*(\mathcal{I}).$$

5 An MIQP Model for MADASS

The rationale behind the following MIQP model for MADASS is the following. We construct a selection within $\mathcal{G}_{\underline{v}}$ by using either formulation UB_{min} or UB_{avg} , depending on the diversity function of the instance. See Section 4 for the definitions and more details. However, for each assortment we additionally add the constraints of a MIP formulation for 2CSP in order to ensure their feasibility, i.e., we guarantee that the selection is actually a subset of $\mathcal{F}_{\underline{v}}$ and therefore feasible itself. In the following example formulation (P) for the δ_{min} case, we use the inequalities of the MIP model of Egeblad and Pisinger [27]. It features the variables listed in Table 2.

$$(P) \quad \max \quad 1 - z \quad (16)$$

$$\text{s.t.} \quad \sum_{R_i \in \mathcal{R}} w_i h_i c_{ia} \geq \underline{v} \quad \forall a \in M \quad (17)$$

$$\sum_{R_i \in \mathcal{R}} w_i h_i c_{ia} \leq wh \quad \forall a \in M \quad (18)$$

$$\sum_{R_i \in \mathcal{R}} (c_{ia} + c_{ib}) = t_{ab} \quad \forall a, b \in M \text{ with } a < b \quad (19)$$

$$c_{ia} + c_{ib} \leq 1 + s_{iab} \quad \forall R_i \in \mathcal{R}, \forall a, b \in M \text{ with } a < b \quad (20)$$

$$\sum_{R_i \in \mathcal{R}} s_{iab} \leq \frac{1}{2} z t_{ab} \quad \forall a, b \in M \text{ with } a < b \quad (21)$$

$$l_{ija} + r_{ija} + u_{ija} + o_{ija} \geq c_{ia} + c_{ja} - 1 \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (22)$$

$$x_{ia} - x_{ja} + w l_{ija} \leq w - w_i \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (23)$$

$$x_{ja} - x_{ia} + w r_{ija} \leq w - w_j \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (24)$$

$$y_{ia} - y_{ja} + h u_{ija} \leq h - h_i \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (25)$$

$$y_{ja} - y_{ia} + h o_{ija} \leq h - h_j \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (26)$$

$$0 \leq x_{ia} \leq w - w_i \quad \forall R_i \in \mathcal{R}, \forall a \in M \quad (27)$$

$$0 \leq y_{ia} \leq h - h_i \quad \forall R_i \in \mathcal{R}, \forall a \in M \quad (28)$$

$$l_{ija}, r_{ija}, u_{ija}, o_{ija} \in \{0, 1\} \quad \forall R_i, R_j \in \mathcal{R} \text{ with } i < j, \forall a \in M \quad (29)$$

$$c_{ia} \in \{0, 1\} \quad \forall R_i \in \mathcal{R}, \forall a \in M \quad (30)$$

$$s_{iab} \in \{0, 1\} \quad \forall R_i \in \mathcal{R}, \forall a, b \in M \text{ with } a < b \quad (31)$$

$$t_{ab} \in [0, \dots, 2|\mathcal{R}|] \quad \forall a, b \in M \text{ with } a < b \quad (32)$$

$$z \in [0, 1]. \quad (33)$$

Variable	Description
c_{ia}	Binary variable indicating whether R_i is contained in \mathcal{A}_a
s_{iab}	Binary variable indicating whether R_i is contained in \mathcal{A}_a and \mathcal{A}_b
t_{ab}	Continuous variable equal to $ \mathcal{A}_a + \mathcal{A}_b $
z	Continuous variable equal to the non-constant objective function value
x_{ia}	Continuous variable determining the x-position of R_i in \mathcal{A}_a
y_{ia}	Continuous variable determining the y-position of R_i in \mathcal{A}_a
l_{ija}	Binary variable whether R_i is located left of R_j in \mathcal{A}_a
r_{ija}	Binary variable whether R_i is located right of R_j in \mathcal{A}_a
u_{ija}	Binary variable whether R_i is located under R_j in \mathcal{A}_a
o_{ija}	Binary variable whether R_i is located over R_j in \mathcal{A}_a

Table 2: Variables for MIQP model (P) for MADASS.

The objective function (16), constraints (17)–(21) and variables (30)–(33) correspond to Model UB_{min} and construct a selection with maximum Minimum-Distance-Diversity in \mathcal{G}_v . On the other hand, the constraints (22)–(28) and variables (27)–(29) originate from the MIP formulation of Egeblad and Pisinger [27] for 2CSP and ensure that the assortments are actually feasible. Thereby, the constraint (22) ensures that if rectangles R_i and R_j are used in assortment \mathcal{A}_a , i.e., $c_{ia} = c_{ja} = 1$, then at least one of the four variables l_{ija} , r_{ija} , u_{ija} , or o_{ija} has to be equal to 1. This implies that R_i has to be placed to the left of R_j (23), right of R_j (24), under R_j (25), or over R_j (26), which guarantees that the two rectangles do not overlap. Furthermore, by the definition of the positioning variables x_{ia} and y_{ia} , see (27) and (28), each rectangle is placed within the container.

5.1 A Benders Decomposition Algorithm

Next, we describe a Benders decomposition algorithm for the introduced MIQP formulation for MADASS. For details regarding Benders decompositions, we refer to Benders [13] and Geoffrion [33]. To derive it, we subdivide the model into its two subproblems. The higher-level problem consists in constructing a diverse selection of assortments in \mathcal{G}_v . The lower-level problems ensure the feasibility of the contained assortments. Thus, in our case the higher-level problem is UB_{min} or UB_{avg} , depending on the diversity function, and the lower-level problem is any MIP formulation or exact approach to 2CSP to check the feasibility of the single assortments, e.g., the variables and constraints from Egeblad and Pisinger [27] in example (P). If a solution for the higher-level problem has been found, but an assortment A is identified as infeasible by the lower-level problem, corresponding no-good-cuts

$$\sum_{R_i \in A} x_{ia} \leq |\mathcal{A}| - 1, \forall a \in M$$

Algorithm 1 Generic Two-Stage Algorithm**Input:** MADASS instance $\mathcal{I} = (\mathcal{C}, \mathcal{R}, m, \underline{v}, \delta)$ **Output:** Feasible Selection \mathcal{S} or UNSUCCESSFUL

-
- 1: $\mathcal{F}_s \leftarrow$ Sample feasible assortments A for 2CSP instance \mathcal{C}, \mathcal{R} with $v(A) \geq \underline{v}$
 - 2: **if** $\mathcal{F}_s = \emptyset$ **then**
 - 3: **return** UNSUCCESSFUL
 - 4: **else**
 - 5: **return** Solution of MDP algorithm for instance \mathcal{F}_s, m, δ
-

are added to the higher-level problem, which is solved again. Note that this separation problem, i.e., the lower-level problem, is NP-hard since we need solve an instance of 2CSP.

6 A Generic Two-Stage Heuristic

Next, we present a generic two-stage heuristic for MADASS, see Algorithm 1. In its first stage, we use any heuristic or exact solution approach for 2CSP to sample the space of \underline{v} -good assortments. We denote this sample set by $\mathcal{F}_s \subseteq \mathcal{F}$ in the sequel. Subsequently, we consider this subset in any exact or heuristic solution approach for MDP in order to determine a feasible selection of size m with respect to the diversity measure δ . Note that unless we are able to sample the complete set of \underline{v} -good assortments and do apply an exact MDP approach, the algorithm does not necessarily determine an optimal solution.

For many MDP approaches from the literature the distances between the assortments have to be known prior to their execution. However, depending on the size of \mathcal{F}_s , determining them can be quite time-consuming. Hence, we introduce a new heuristic for MDP that does not rely on the availability of these distances, see Algorithm 2.

The algorithm is based on the idea of a random exchange, i.e., we start with a selection \mathcal{S}_b of m randomly chosen assortments and then iteratively check if a complete or partial exchange with another k assortments increases the diversity of the selection. A similar idea was suggested by Ghosh [34], but in his approach, the exchange is based on an evaluation of all assortments. We avoid this by selecting the assortments completely at random and only determine the distances between the considered $m + k$ assortments in \mathcal{S}_c . Afterwards, we use an exact MIP formulation, depending on the diversity function that should be maximized, to choose m assortments from \mathcal{S}_c with maximum diversity, see Kuo et al. [53] for example. Note that the diversity cannot decrease. The number k of considered assortments for exchange increases with every 100 iterations that did not lead to an increase of the diversity, see line 5 of the algorithm. The count is reset whenever a more diverse selection was found. The idea here is, in particular when considering δ_{min} , that the diversity of the selection may depend on distances between multiple assortments. In this case, the exchange of only one assortment does not lead to an increase in the diversity. Thus, it is necessary to consider

Algorithm 2 MDP Random Exchange Heuristic

Input: $\mathcal{F}_s, m, \delta, \Delta$
Output: Selection \mathcal{S}_b

- 1: $\mathcal{S}_b \leftarrow$ Randomly choose m assortments $\mathcal{A}_1, \dots, \mathcal{A}_m \in \mathcal{F}_s$
- 2: $\delta_b \leftarrow \delta(\mathcal{S}_b)$
- 3: $k \leftarrow 1$
- 4: **While** $k \leq \Delta$
- 5: Choose $\mathcal{S}_r \subseteq \mathcal{F}_s \setminus \mathcal{S}_b$ with $|\mathcal{S}_r| = \lceil \frac{k}{100} \rceil$ at random
- 6: $\mathcal{S}_c \leftarrow \mathcal{S}_b \cup \mathcal{S}_r$
- 7: Determine $d_H(\mathcal{A}_i, \mathcal{A}_j)$ for all $\mathcal{A}_i, \mathcal{A}_j \in \mathcal{S}_c$ with $i < j$
- 8: $\mathcal{S}_b \leftarrow$ Solution of exact MDP approach for \mathcal{S}_c, m, δ
- 9: **If** $\delta(\mathcal{S}_b) > \delta_b$ **then**
- 10: $\delta_b \leftarrow \delta(\mathcal{S}_b)$
- 11: $k \leftarrow 1$
- 12: **Else**
- 13: $k \leftarrow k + 1$
- 14: **EndIf**
- 15: **EndWhile**
- 16:
- 17: **Return** \mathcal{S}_b

the replacement of more than one assortment at once. The algorithm terminates after Δ unsuccessful iterations.

7 Computational Experiments

In this section, we report on the results of our computational experiments that we conducted on the two instances from the MOPTA competition [2] as well as on modified 2CSP instances, which are widely used in the literature. We evaluate the results from solving an instantiation of the MIQP formulation directly, from applying the Benders approach presented in Section 5, and from an instantiation of our two-stage heuristic as introduced in Section 6. We compare these three approaches with respect to the diversity of the best solution they determined. Additionally, we present the results of our upper bound computations described in Section 4.

Before doing this, we investigate different exact and heuristic solution approaches for 2CSP with respect to their best generated solution value and the number of generated assortments. This is necessary in order to decide which MIP formulation to use within the MIQP model and which heuristic to employ in the first stage of the heuristic. For the latter we are particularly interested in the number of assortments satisfying the \underline{v} -criterion.

For our experiments, we considered $\underline{v} = (1 - \varepsilon)v^*$, for $\varepsilon = 5\%$ as threshold. Here, v^* denotes the best solution value which we determined during the corresponding 2CSP instance runs.

7.1 Computational Setup

All heuristic algorithms for 2CSP were implemented in Ada 2012 using the GNAT Pro 19 compiler [4] and run on an Intel(R) Xeon(TM) E5-2690 v4 CPU with 2.60GHz, four cores, and 32 GB RAM. The Benders approach was coded with Python v3.6, and for the MIP, and MIQP models, Gurobi v9.0 [38] was used as a solver. For all computations, we set a time limit of 3,600 seconds. Additionally, for the computation of the upper bounds the focus of Gurobi was set to improve primarily the bounds.

7.2 Test Instances

Since MADASS is a novel optimization problem, an important task was to come up with test instances. Before explaining how we derived test instances using 2CSP instances, we first of all explain how the two test instances for the MOPTA competition were created.

Generation of MOPTA Instances For the AIMMS-MOPTA competition, data generation procedures were devised to produce problems of any size, that could exhibit some variety in the shape of the rectangles, as measured by the aspect ratio (height-width ratio), and in the size of the rectangles, as measured by the surface.

The generation procedure for the first data set has 7 parameters $(n, w_{\min}, w_{\max}, \theta_1, h_{\min}, h_{\max}, \theta_2)$. It first generates samples w_i and h_i of real-valued random variables \mathcal{W}_i and \mathcal{H}_i representing the width and height of rectangle i for $i = 1, \dots, n$. \mathcal{W}_i and \mathcal{H}_i follow independent bounded power law distributions with shape parameters θ_1 and θ_2 respectively, restricted to the domain $[w_{\min}, w_{\max}] \times [h_{\min}, h_{\max}]$:

$$\mathbb{P}(w \leq \mathcal{W}_i \leq w + dw, h \leq \mathcal{H}_i \leq h + dh) \propto w^{\theta_1} h^{\theta_2} dw dh + o(dw dh) \quad (34)$$

for $w \in [w_{\min}, w_{\max}), h \in [h_{\min}, h_{\max})$.

This can be done by drawing some u_i, v_i uniformly in $[0, 1]$ and setting

$$w_i = \left((w_{\max}^{\theta_1+1} - w_{\min}^{\theta_1+1})u_i + w_{\min}^{\theta_1+1} \right)^{1/(\theta_1+1)}, \quad (35)$$

$$h_i = \left((h_{\max}^{\theta_2+1} - h_{\min}^{\theta_2+1})v_i + h_{\min}^{\theta_2+1} \right)^{1/(\theta_2+1)}. \quad (36)$$

The real-valued samples are then rounded up to the next integer. The data set was generated using $n = 200, w_{\min} = 40, w_{\max} = 200, \theta_1 = 1.8, h_{\min} = 40, h_{\max} = 200, \theta_2 = 0.8$. The container had width 300 and height 400. Having $\theta_3, \theta_4 > 0$ means that large rectangles are favored over small rectangles in the generation process.

The generation procedure for the second data set has 7 parameters $(n, s_{\min}, s_{\max}, \theta_3, \ell_{\min}, \ell_{\max}, \theta_4)$. It first generates samples s_i and ℓ_i of real-valued random

ratios \mathcal{S}_i and lengths \mathcal{L}_i , $i = 1, \dots, n$, following independent bounded power law distributions with shape parameters θ_3 and θ_4 respectively, restricted to the domain $[s_{\min}, s_{\max}] \times [\ell_{\min}, \ell_{\max}]$, defined similarly to 34. It then derives width and height samples w_i, h_i using a conditional rule: if $s_i \geq 1$, set $w_i = \ell_i/s_i$ and $h_i = \ell_i$ (tall rectangles); if $r_i < 1$, set $w_i = \ell_i$ and $h_i = s_i\ell_i$ (flat rectangles). The values are then rounded up to the next integer. The data set was generated using $n = 40$, $s_{\min} = 0.25$, $s_{\max} = 2$, $\theta_3 = 0.5$, $\ell_{\min} = 20$, $\ell_{\max} = 200$, $\theta_4 = 1$. The container was a square of sides of length 500.

The two data generation procedures are not equivalent. It can be checked that when $\mathcal{W}_i, \mathcal{H}_i$ follow truncated power laws, the distribution of the products $\mathcal{W}_i\mathcal{H}_i$ or the ratios $\mathcal{H}_i/\mathcal{W}_i$ do not themselves follow power laws. Thus, the two generation procedures control the distribution of the aspect ratios and surfaces in two different ways.

For both data sets, the parameters were determined after some tuning to make sure the problems were sufficiently challenging. This was done by estimating the computational time needed to obtain a pool of ϵ -optimal solutions to the two-dimensional knapsack problem formulated following [39]. To estimate the times, simplified instances were solved, obtained by scaling down by a factor 50 the dimensions of the container and rectangles and rounding them up to the next integer. Scale-and-round was used to reduce the number of binary variables needed to formulate the problems while hopefully preserving the relative degree of difficulty among the generated problems. The generation of a solution pool takes more time than the generation of a single solution but was deemed useful to measure the complexity of describing the set of good solutions from which a maximally-diverse solution is subsequently selected.

Derivation of Instances from 2CSP As mentioned, we additionally created new MADASS instances based on 2CSP instances, of which plentiful exist in the literature. In particular, we used the test instance packages listed in Table 3 to generate test instances for MADASS. Most of them can be found on the website of Wei and Wenbin [75], while the remaining ones were directly taken from the corresponding sources.

As mentioned above, except for the two instances from the competition, all other instances originate from 2CSP. Thus, the sum of the areas over all rectangles is in many cases equal to the area of the given container, since the focus is on the placement of the rectangles within the container. Hence, if we simply declare them to be MADASS instances, the number of \underline{v} -good assortments, for $\underline{v} = (1 - \epsilon)v^*$ with $\epsilon = 5\%$, and the maximum diversities would often be rather small. On the other hand, if we consider an instance with a set of rectangles \mathcal{R} whose sum of areas is much bigger than the area of the container, $\mathcal{F}_{\underline{v}}$ may contain many assortments having Hamming-Distance 1 to each other. Thus, we modify the instances in order to ensure that $\mathcal{F}_{\underline{v}}$ has suitable size by scaling down the container and thereby to guarantee that at least one rectangle has to be contained in at least two assortments of each feasible selection.

Package	# Instances	$ \mathcal{R} _{min}$	$ \mathcal{R} _{max}$	Source
<i>2dcsp</i>	60	20	50	Imahori [50]
<i>AH</i>	360	1,000	1,000	Bordfeldt [16]
<i>area</i>	6	33	500	Imahori [50]
<i>beng</i>	10	20	200	Bengtsson [14]
<i>Burke</i>	13	10	3,152	Burke et al. [19]
<i>bwmv</i>	500	20	100	Berkey and Wang [15] and Martello and Vigo [59]
<i>C</i>	21	16	197	Hopper and Turton [48]
<i>cgcut</i>	3	16	62	Christofides and Whitlock [21]
<i>CX</i>	7	50	15,000	Pinto and Oliveira [64]
<i>gcut</i>	13	10	50	Beasley [9]
<i>HC</i>	6	7	21	Hadjiconstantinou and Christofides [39]
<i>leung</i>	10	10	50	Leung et al. [56]
<i>N</i>	35	17	197	Hopper [47]
<i>ngcut</i>	12	7	22	Beasley [10]
<i>Nice</i>	6	25	1,000	Wang and Valenzela [73]
<i>Nice36</i>	33	100	5,000	Wang and Valenzela [73]
<i>OKP</i>	5	30	97	Fekete and Schepers [28]
<i>others</i>	4	40	200	AIMMS-MOPTA Competition [2], Babu and Babu [8] and Wang [72]
<i>Path</i>	6	25	1,000	Wang and Valenzela [73]
<i>Path36</i>	33	25	5,000	Wang and Valenzela [73]
<i>PB</i>	5	10	20	Lai and Chan [54]
<i>T</i>	35	17	199	Hopper [47]
<i>ZDF</i>	16	580	75,032	Shiangjen et al. [69]

Table 3: Packages of test instances given in the literature and by the AIMMS-MOPTA Competition. $|\mathcal{R}|_{min}$ is the smallest number of rectangles among all instances of the considered package, while $|\mathcal{R}|_{max}$ is the maximum cardinality of \mathcal{R} among all instances of the package.

Lemma 7. *Let \mathcal{I} be an instance of MADASS. Furthermore, let the total area of the given rectangles in \mathcal{R} be $A_{\mathcal{R}} := \sum_{R_i \in \mathcal{R}} w_i h_i$. If*

$$m\underline{v} > A_{\mathcal{R}}$$

holds, then at least one rectangle is contained in at least two assortments of each feasible selection.

Proof. Assume there exist m feasible assortments $\mathcal{A}_1, \dots, \mathcal{A}_m$ such that the intersection of each pair of differing assortments is empty, i.e., $\mathcal{A}_i \cap \mathcal{A}_j = \emptyset$ for all $i, j \in \{1, \dots, m\}$ with $i \neq j$. Further, let $\mathcal{A}_U := \bigcup_{i \in \{1, \dots, m\}} \mathcal{A}_i$ be the union of the considered assortments. Since all assortments are feasible, we have $v(\mathcal{A}_i) \geq \underline{v}$ for all $i \in \{1, \dots, m\}$. Then it follows that

$$A_{\mathcal{R}} \geq \sum_{R_i \in \mathcal{A}_U} w_i h_i = \sum_{R_i \in \mathcal{R}} w_i h_i = \sum_{i=1}^m v(\mathcal{A}_i) \geq m\underline{v},$$

which is a contradiction. □

Since we assume $\underline{v} = (1 - \varepsilon)v^*$ in the following and do not want to rely on the determination of v^* for every instance we consider the area of the container $v(\mathcal{C})$ instead. Hence, we want to scale down the container such that

$$m \cdot v(\mathcal{C}) > A_{\mathcal{R}}$$

still holds. On the other hand, as mentioned we additionally want to avoid instances with $A_{\mathcal{R}} \leq v(\mathcal{C})$ since this could imply that $\mathcal{F}_{\underline{v}}$ consists only of few elements if ε is small. Thus, we additionally request that

$$v(\mathcal{C}) < A_{\mathcal{R}}$$

and therefore, the goal is to scale the container of the 2CSP instance such that

$$1 < p := \frac{m \cdot v(\mathcal{C})}{A_{\mathcal{R}}} < m$$

holds. Thus, the scaling procedure works as follows. If $p \geq m$, let $s_r := \frac{h}{w}$ be the side-ratio of \mathcal{C} , and let $w_{max} := \max_{R_i \in \mathcal{R}} \{w_i\}$ and $h_{max} := \max_{R_i \in \mathcal{R}} \{h_i\}$ be the maximal width and height among all rectangles in \mathcal{R} . Then, we determine the minimum value of $p \in \mathbb{N}$ with $p \geq 2$ such that \mathcal{C} is scaled, the side-ratio s_r is preserved, and each rectangle of \mathcal{R} still fits into the resulting container. If $p \leq 5$, we determine the corresponding w and h and round them up to the next integer. Otherwise, we use the original container.

Note that if a certain instance is not equipped with a container, we proceed in the same way but set s_r to the average side-ratio of the rectangles in \mathcal{R} , which was done for the instances of the *area* package. Finally, we removed instances that occur twice after the scaling procedure and ended up with a test set consisting of 1,199 instances.

7.3 Evaluating Solution Approaches to 2CSP

In the remainder of this manuscript, we use the following abbreviations for 2CSP algorithms: with respect to heuristics we use *LWF*, *GRASP*, and *IBHP* for the Least-Waste-First heuristic of Wei et al. [76], the GRASP algorithm of Alvarez-Valdes et al. [5] and the IBHP heuristic of Shiangjen et al. [69]. Regarding MIP formulations, by *HC95* we refer to the IP model of Hadjiconstantinou and Christofides [39], *BKRS09* abbreviates the IP model of Belov et al. [12], and *EP09* corresponds to the MIP formulation of Egeblad and Pisinger [27].

Evaluation with respect to the Best Generated Assortment First of all, we compare the above mentioned 2CSP algorithms with respect to the value of their best generated assortment for each instance in the test set. The exact results can be found in Table 9 in Appendix A. In Table 4, we present the summarized results for the different instance packages. Note that the number of instances on which the different approaches obtained the best assortment do not have to sum

Package	$ \mathcal{R} _{min}$	$ \mathcal{R} _{max}$	# Instances	HC95	BKRS09	EP09	LWF	GRASP	IBHP
<i>2dcsp</i>	20	50	60	0	0	17	0	12	47
<i>AH</i>	1,000	1,000	360	0	0	0	95	1	264
<i>area</i>	33	500	6	0	0	0	1	0	6
<i>beng</i>	20	200	10	4	1	3	9	9	9
<i>Burke</i>	10	3,152	13	2	1	2	8	1	12
<i>bmwv</i>	20	100	500	39	12	112	93	96	463
<i>C</i>	16	197	21	9	6	9	21	18	20
<i>cgcut</i>	16	62	3	1	1	1	0	1	3
<i>CX</i>	50	15,000	7	0	0	1	6	6	7
<i>geut</i>	10	50	13	0	0	5	1	4	13
<i>HC</i>	7	21	6	6	4	6	3	5	4
<i>leung</i>	10	50	10	3	3	8	5	8	7
<i>N</i>	17	197	35	0	0	9	12	5	29
<i>ngcut</i>	7	22	12	10	9	12	7	11	9
<i>Nice</i>	25	1,000	6	0	0	0	3	0	3
<i>Nice36</i>	100	5,000	33	0	0	0	2	0	31
<i>OKP</i>	30	97	5	0	0	3	0	5	0
<i>others</i>	40	200	4	1	0	1	1	2	4
<i>Path</i>	25	1,000	6	0	0	1	1	0	6
<i>Path36</i>	25	5,000	33	0	0	0	16	0	33
<i>PB</i>	10	20	5	1	3	5	2	5	4
<i>T</i>	17	199	35	0	1	10	12	2	28
<i>ZDF</i>	580	75,032	16	0	0	0	12	9	16
Σ			1,199	76	41	205	310	200	1,018

Table 4: Number of instances on which the algorithms obtained an assortment with biggest value.

up to 1,199 as we counted instances on which two or more algorithms achieved the best solution multiple times. Additionally, one should keep in mind that we compare three exact approaches and three heuristics here.

The IBHP heuristic outperformed all other approaches with respect to the number of instances on which it determined an assortment with biggest value. This is the case for 1,018 of 1,199 instances, i.e., on 84.9% of the test set. The second-best approach in this context is the LWF heuristic with 310 of the 1,199 instances (25.9%), followed by approach EP09 (17.1%), and the GRASP heuristic (16.7%). Thus, when comparing all instances at once, it seems that IBHP is best suited for obtaining the best generated assortment. However, if we look at the different instance packages in more detail, we can observe that on packages with $|\mathcal{R}|_{max} > 197$ the best generated assortments were obtained by IBHP, but EP09 delivered the best results on packages with $|\mathcal{R}|_{max} \leq 22$.

If we only compare the MIP approaches with each other, we observe that EP09 leads to the best results, although HC95 could solve bigger instances in terms of the cardinality of \mathcal{R} . This is remarkable, as it is of larger size than the other two approaches with respect to the number of variables and constraints.

Evaluation with respect to the Number of Generated Assortments

Next, we compare the three heuristic approaches for 2CSP with respect to the number of assortments that were generated on the set of test instances. We do this, as in the first stage of the generic two-stage heuristic, the space of feasible \underline{v} -good assortments $\mathcal{F}_{\underline{v}}$ has to be sampled and the number of generated assortments is therefore an important factor. Note that we do not consider the

MIP approaches in this context, as they did not generate many feasible solutions during the solving process, even for small instances. The total number of generated assortments and the number of \underline{v} -good assortments, for $\underline{v} = (1 - \varepsilon)v^*$ with $\varepsilon = 5\%$, can be found in Table 10, in Appendix B. Note that for the number of \underline{v} -good solutions we only counted undominated assortments, i.e., assortments which were not contained in any other.

First, we compare the different algorithms with respect to the total number of generated assortments. In this case, IBHP obtained the most solutions on 921 of the 1,199 instances, i.e., on 76.8%, while the GRASP algorithm obtained the most on the remaining 278 ones (23.2%), see Table 5. If we take a closer look on the properties of the different instances, we can observe that GRASP obtained the most assortments on packages with $|\mathcal{R}|_{max} \leq 197$ and *Burke*.

Package	$ \mathcal{R} _{min}$	$ \mathcal{R} _{max}$	# Instances	total			\underline{v} -good		
				LWF	GRASP	IBHP	LWF	GRASP	IBHP
<i>2dcsp</i>	20	50	60	0	14	46	0	4	56
<i>AH</i>	1,000	1,000	360	0	0	360	0	0	360
<i>area</i>	33	500	6	0	0	6	0	0	6
<i>beng</i>	20	200	10	0	2	8	0	1	9
<i>Burke</i>	10	3,152	13	0	7	6	1	5	9
<i>bumv</i>	20	100	500	0	170	330	0	157	360
<i>C</i>	16	197	21	0	9	12	0	9	12
<i>cqcut</i>	16	62	3	0	2	1	0	2	1
<i>CX</i>	50	15,000	7	0	1	6	1	1	7
<i>gcut</i>	10	50	13	0	5	8	0	3	13
<i>HC</i>	7	21	6	0	6	0	1	6	3
<i>leung</i>	10	50	10	0	8	2	0	9	2
<i>N</i>	17	197	35	0	12	23	0	13	32
<i>ngcut</i>	7	22	12	0	12	0	6	12	7
<i>Nice</i>	25	1,000	6	0	1	5	0	0	6
<i>Nice36</i>	100	5,000	33	0	0	33	0	0	33
<i>OKP</i>	30	97	5	0	5	0	0	5	0
<i>others</i>	40	200	4	0	2	2	0	2	2
<i>Path</i>	25	1,000	6	0	1	5	0	1	5
<i>Path36</i>	25	5,000	33	0	1	32	0	0	33
<i>PB</i>	10	20	5	0	5	0	1	5	2
<i>T</i>	17	199	35	0	14	21	1	7	29
<i>ZDF</i>	580	75,032	16	0	1	15	0	0	16
Σ			1,199	0	278	921	11	232	1,003

Table 5: Number of instances on which the heuristics generated the most assortments in total and the most undominated \underline{v} -good assortments, for $\underline{v} = (1 - \varepsilon)v^*$ with $\varepsilon = 5\%$

After removing the dominated assortments from the set of solutions and only considering the assortments that are \underline{v} -good, the described situation intensifies, see Table 5. In this case, IBHP obtained the most assortments on 1,003 of the 1,199 instances (83.7%). The biggest instance on which GRASP delivers results superior to IBHP consists of 500 rectangles and the percentage of instances on that it is best decreases to 19.4%. Interestingly, LWF was able to generate the biggest set of assortments on 11 instances.

However, IBHP may also benefit here from its dominance with respect to the superior results regarding the best value found for the 2CSP instances. This is

due to the fact that we consider $\underline{v} = (1 - \varepsilon)v^*$, where v^* is the best solution value found by any of the approaches.

7.4 Evaluation with respect to Diversity

Based on the results from the two previous subsections, we instantiated the generic two-stage heuristic (GF) with IBHP and the random exchange heuristic presented in Subsection 6 and use MIP formulation EP09 for the instantiation of the MIQP model.

Next, we are going to compare the results of the solution approaches, discussed in Sections 5 and 6. Therefore, we generated instances of MADASS from the scaled 2CSP instances by setting $m = 6$, $\delta \in \{\delta_{min}, \delta_{avg}\}$, and $\underline{v} = (1 - \varepsilon)v^*$ for $\varepsilon = 5\%$, i.e., we created 2 test instances for each container and set of rectangles. Thus, we run every solution approach on 2,398 instances of MADASS. Recall that we always consider v^* to be equal to the value of the best generated assortment of the corresponding 2CSP instance, see Subsection 7.3. Thus, we run the MIQP formulation, the corresponding Benders decomposition approach (BD), and the two-stage heuristic for δ_{min} as well as for δ_{avg} . The detailed results we obtained can be found in Table 11, see Appendix C.

Evaluation with respect to δ_{min} First, we evaluate the results with respect to δ_{min} . Here, the heuristic delivered the best results on 1,124 of the 1,199 instances, i.e., on 93.7% of the instances and thus was the best performing approach among all considered algorithms, see Table 6. The second-best performing one in this context was the Benders approach, solving 60 instances best, i.e., 5.0%. However, the MIQP formulation was able to solve 10 instances to optimality, while the heuristic obtained only 4 optimal selections, see Table 7.

When we consider the maximum number of rectangles contained in the instances and investigate the single packages in more detail, one can observe that the heuristic performs well on any type of test instance, and outperforms the other approaches on 20 of the 23 packages. However, it performs less well on packages where the instances consist only of few rectangles, see, for example, *HC*, *ngcut*, and *PB*. Meanwhile, the Benders approach BD is especially strong in solving instances of this kind and led to the best results on small instances, even if the biggest instance on which it could obtain the best diversity value consisted of 29 rectangles only. Furthermore, it was able to generate the most diverse selections on all instance packages with 22 rectangles or less. Finally, the biggest instance, with respect to the number of contained rectangles, for which the most diverse solution was found by the MIQP formulation, consisted of 50 rectangles.

Evaluation with respect to δ_{avg} For δ_{avg} , we arrive at similar observations. Here, the heuristic obtained the best result on 1,092 instances, while the Benders approach could solve slightly more instances better than it did in the case of δ_{min} , i.e., 8.6%. Thus this approach seems to perform better with respect to the

Package	$ \mathcal{R} _{min}$	$ \mathcal{R} _{max}$	# Instances	δ_{min}			δ_{avg}		
				MIQP	BD	GF	MIQP	BD	GF
<i>2dcspl</i>	20	50	60	0	1	59	0	2	58
<i>AH</i>	1,000	1,000	360	0	0	360	0	0	360
<i>area</i>	33	500	6	0	0	6	0	0	6
<i>bergl</i>	20	200	10	0	1	9	0	2	8
<i>Burke</i>	10	3,152	13	1	1	10	1	1	10
<i>bwmv</i>	20	100	500	3	24	471	2	49	446
<i>C</i>	16	197	21	0	3	19	0	3	18
<i>cglcut</i>	16	62	3	0	0	3	0	0	3
<i>CX</i>	50	15,000	7	1	0	6	1	0	6
<i>glcut</i>	10	50	13	1	1	11	1	3	13
<i>HC</i>	7	21	6	1	4	0	2	6	0
<i>leung</i>	10	50	10	1	3	8	1	3	7
<i>N</i>	17	197	35	0	6	29	0	9	26
<i>ngcut</i>	7	22	12	4	8	2	7	12	2
<i>Nice</i>	25	1,000	6	0	0	6	0	0	6
<i>Nice36</i>	100	5,000	33	0	0	33	0	0	33
<i>OKP</i>	30	97	5	0	0	5	0	0	5
<i>others</i>	40	200	4	0	0	4	0	0	4
<i>Path</i>	25	1,000	6	0	0	6	0	1	5
<i>Path36</i>	25	5,000	33	0	0	33	0	1	32
<i>PB</i>	10	20	5	1	3	1	2	4	1
<i>T</i>	17	199	35	2	5	29	1	7	27
<i>ZDF</i>	580	75,032	16	0	0	14	0	0	16
Σ			1,199	15	60	1,124	18	103	1,092

Table 6: Number of instances on which the solution approaches obtained the best diversity with respect to δ_{min} and δ_{avg}

Average-Distance-Diversity, and its dominance on small instances with respect to the cardinality of \mathcal{R} increases. Meanwhile, the MIQP formulation could solve nearly the same amount of instances, see Table 6. Additionally, the Benders approach is now able to obtain the best result on instances with a size of up to 120 rectangles.

Thus, we can conclude again that the heuristic using IBHP and the random exchange heuristic is the best approach for deriving good solutions for MADASS instances, but when considering the Average-Distance-Diversity and an instance consisting of only a few rectangles, i.e., of less than 22, then the Benders approach is the solution approach of choice.

	δ_{min}			δ_{avg}		
	MIQP	BD	GF	MIQP	BD	GF
$ \mathcal{R} _{max}$	50	29	25,032	50	120	75,032
$ \mathcal{R} _{min}$	10	10	10	7	7	7
# Optimal	10	0	4	6	15	0

Table 7: Minimum and maximum cardinality of \mathcal{R} of the instances that are solved best by the approaches with respect to δ_{min} and δ_{avg}

Package	$ \mathcal{R} _{min}$	$ \mathcal{R} _{max}$	# Instances	δ_{min}		δ_{avg}
				UB _{min}	UB _{avg}	UB _{avg}
<i>2dcsp</i>	20	50	60	0.174	0.188	0.127
<i>AH</i>	1,000	1,000	360	0.307	0.274	0.225
<i>area</i>	33	500	6	0.256	0.253	0.190
<i>beng</i>	20	200	10	0.224	0.232	0.174
<i>Burke</i>	10	3,152	13	0.310	0.303	0.261
<i>bwmv</i>	20	100	500	0.265	0.267	0.227
<i>C</i>	16	197	21	0.215	0.233	0.164
<i>cgcut</i>	16	62	3	0.457	0.525	0.472
<i>CX</i>	50	15,000	7	0.422	0.419	0.371
<i>gcut</i>	10	50	13	0.332	0.362	0.289
<i>HC</i>	7	21	6	0.261	0.336	0.210
<i>leung</i>	10	50	10	0.231	0.287	0.197
<i>N</i>	17	197	35	0.272	0.311	0.229
<i>ngcut</i>	7	22	12	0.112	0.154	0.064
<i>Nice</i>	25	1,000	6	0.165	0.187	0.155
<i>Nice36</i>	100	5,000	33	0.446	0.439	0.396
<i>OKP</i>	30	97	5	0.687	0.689	0.578
<i>others</i>	40	200	4	0.180	0.201	0.115
<i>Path</i>	25	1,000	6	0.254	0.240	0.200
<i>Path36</i>	25	5,000	33	0.365	0.366	0.328
<i>PB</i>	10	20	5	0.256	0.367	0.306
<i>T</i>	17	199	35	0.281	0.308	0.255
<i>ZDF</i>	580	75,032	16	0.481	0.481	0.415
\emptyset			1,199	0.285	0.281	0.232

Table 8: Average distance between the bounds and the best obtained diversity

7.5 Evaluation of Upper Bounds

Finally, we look at the bounds obtained by MIQPs UB_{min} and UB_{avg} from Section 4. Recall that UB_{min} is only valid for δ_{min} , while UB_{avg} determines a bound valid for both diversity functions. It is important to note that any upper bound during the solving process is also valid. The results can be found in Table 11, see Appendix C.

Evaluation with respect to δ_{min} We start our evaluation with a comparison of both bounds on the instances w.r.t. δ_{min} . The minimum distance between any bound and the best result of any solution approach to MADASS is in both cases equal to 0, due to instance *ngcut01* since the maximum diversity that could be obtained coincides with the value of the bounds. Furthermore, the maximum distance between the bounds and best obtained diversity is 1, as for the biggest instances with respect to the number of contained rectangles, only selections with diversity equal to 0 are obtained, while the bounds are equal to 1.

Concerning the average distance between the bounds and the best obtained diversity, we can conclude that both bounds are nearly equally strong, with a slight advantage for UB_{avg} . But UB_{min} leads to better results on instances with less than 500 rectangles, making it better suited for smaller instances, while UB_{avg} performs best on instances with $|\mathcal{R}| \geq 500$, see Table 8. Furthermore, UB_{min} was the best bound for 552 instances, and UB_{avg} for 730 instances.

Evaluation with respect to δ_{avg} Finally, we present the results obtained by UB_{avg} for the instances with respect to δ_{avg} . The minimum distance between UB_{avg} and the value of a most diverse selection is again equal to 0, due to instance *ngcut01*. We also obtain a maximum distance of 1 between UB_{avg} and the best diversity due to instance *zdf16*.

The average distances to the best obtained diversities generated by the solution approaches to MADASS are depicted in the right column of Table 8. Clearly, the distances between UB_{avg} and the value of the most diverse selection are smaller than in the case of δ_{min} since the diversity of a selection with respect to δ_{avg} is bigger than or equal to its diversity with respect to δ_{min} , see Lemma 3. Furthermore, UB_{avg} seems to work best on packages with instances consisting of 1,000 rectangles or less. Nevertheless, it also obtains its biggest distance on a package with instances consisting of less than 100 rectangles, i.e., on package *OKP*. This suggests that either the solution approaches fail on determining selections with great diversity or that the relaxation of the conditions that the considered assortments have to fulfill is too weak. But this issue occurs for the bounds on δ_{min} as well. Nevertheless, with an average distance of 0.232 to all considered instances, UB_{avg} achieves better results for δ_{avg} than any bound on δ_{min} .

8 Conclusion and Outlook

In this article, we introduced the Maximum Diversity Assortment Selection Problem (MADASS), which is a novel generalization of the 2-dimensional Cutting Stock Problem (2CSP). First, we gave a mathematical definition of MADASS and introduced two diversity-functions for selections based on the Hamming-distance. Afterwards, we presented an overview of the literature focusing on its two inherent subproblems 2CSP and MDP. Next, we introduced two MIQPs that can be used to determine upper bounds on the diversity of MADASS instances. Based on them, we presented an exact MIQP formulation for MADASS, a Benders decomposition approach for it, as well as a generic two-stage heuristic. Furthermore, we compared different solution approaches for 2CSP with respect to the best assortment value and the number of generated assortments. And finally, we investigated the presented solution approaches for MADASS with respect to the maximum diverse selections they determined.

As a main result, the generic two-stage heuristic instantiated with IBHP and the random exchange for MDP delivered the best results with respect to the diversity among the presented solution approaches for MADASS. However, the Benders approaches led to more diverse selections on instances consisting of only few rectangles, especially with respect to δ_{avg} . Again, it is important to note that it is an exact algorithm, in contrast to the heuristic.

There are many directions for further research concerning MADASS. First of all, we are currently working on a possible improvement of the Benders decomposition approach by using the algorithm presented by Fekete et al. [30] for determining the maximality of an assortment since a lot of time is currently

spent in the lower-level problem. Additionally, it would be beneficial to experiment with other cuts than the no-good-cuts in order to improve the performance of the overall Benders approach.

Furthermore, we are investigating the structures of the solutions created by the different heuristics to try to combine them in order to further broaden the variety in the set of generated \underline{v} -good solutions. In this context, a modification of the shift strategy of IBHP, adapting the strength of the GRASP and LWF heuristics for small instances, would be of great advantage.

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A Appendix

In this appendix, we present the results of our computational experiments with respect to the value of the best generated assortment for each solution approach to 2CSP. In the two left columns of the following table, the name of the instance is shown together with the number of rectangles belonging to the instance. The remaining columns contain the results of the different solution approaches in percent, i.e., the value of the best assortment, generated by each approach, divided by the area of the container. Note that the best result for each of the instances is marked in blue and * indicates that the obtained value is optimal. Finally, – indicates that no solution could be obtained using the marked approach.

Table 9: Best obtained values for the 2CSP instances by the solution approaches for 2CSP

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
ALX	20	-	-	0.959*	0.939	0.951	0.958
ALY	20	-	-	0.979	0.934	0.971	0.973
ALZ	20	-	-	0.962	0.950	0.954	0.962
ALZZ	20	-	-	0.955	0.909	0.953	0.955
ALZZZ	20	-	-	0.955	0.909	0.953	0.955
ASX	20	-	-	0.955*	0.922	0.946	0.952
ASY	20	-	-	0.966	0.946	0.959	0.967
ASZ	20	-	-	0.977	0.956	0.972	0.973
ASZZ	20	-	-	0.968*	0.955	0.966	0.968*
ASZZZ	20	-	-	0.968*	0.955	0.966	0.968*
AVX	20	-	-	0.957*	0.947	0.957*	0.957*
AVY	20	-	-	0.976	0.964	0.976	0.976
AVZ	20	-	-	0.964	0.958	0.957	0.966
AVZZ	20	-	-	0.980*	0.945	0.980*	0.980*
AVZZZ	20	-	-	0.980*	0.945	0.980*	0.980*
BLX	30	-	-	0.975	0.951	0.973	0.975
BLY	30	-	-	0.980	0.969	0.980	0.980
BLZ	30	-	-	0.976	0.961	0.982	0.985
BLZZ	30	-	-	0.966	0.958	0.976	0.979
BLZZZ	30	-	-	0.972	0.958	0.980	0.981
BSX	30	-	-	0.975	0.955	0.975	0.968
BSY	30	-	-	0.980	0.960	0.977	0.983
BSZ	30	-	-	0.966	0.964	0.982	0.982
BSZZ	30	-	-	0.955	0.944	0.966	0.969
BSZZZ	30	-	-	0.965	0.961	0.981	0.981
BVX	30	-	-	0.981	0.941	0.970	0.969
BVY	30	-	-	0.975	0.969	0.984	0.983
BVZ	30	-	-	0.968	0.960	0.969	0.977
BVZZ	30	-	-	0.968	0.964	0.968	0.968
BVZZZ	30	-	-	0.971	0.968	0.969	0.985
CLX	40	-	-	0.964	0.956	0.974	0.973
CLY	40	-	-	0.967	0.962	0.977	0.986
CLZ	40	-	-	0.966	0.963	0.980	0.984
CLZZ	40	-	-	0.951	0.965	0.972	0.978
CLZZZ	40	-	-	0.970	0.968	0.982	0.985
CSX	40	-	-	0.958	0.969	0.976	0.992
CSY	40	-	-	0.952	0.953	0.985	0.984
CSZ	40	-	-	0.950	0.976	0.977	0.982
CSZZ	40	-	-	0.959	0.964	0.978	0.979
CSZZZ	40	-	-	0.963	0.964	0.973	0.986
CVX	40	-	-	0.970	0.975	0.980	0.984
CVY	40	-	-	0.962	0.972	0.979	0.982
CVZ	40	-	-	0.964	0.966	0.979	0.985
CVZZ	40	-	-	0.964	0.962	0.979	0.981
CVZZZ	40	-	-	0.963	0.973	0.984	0.983
DLX	50	-	-	0.966	0.977	0.982	0.990
DLY	50	-	-	0.947	0.973	0.983	0.988
DLZ	50	-	-	0.944	0.973	0.985	0.988
DLZZ	50	-	-	0.933	0.973	0.977	0.976
DLZZZ	50	-	-	0.963	0.961	0.974	0.985
DSX	50	-	-	0.941	0.983	0.981	0.985
DSY	50	-	-	0.945	0.976	0.977	0.988
DSZ	50	-	-	0.938	0.979	0.978	0.982
DSZZ	50	-	-	0.926	0.963	0.974	0.983
DSZZZ	50	-	-	0.961	0.979	0.982	0.984
DVX	50	-	-	0.978	0.969	0.982	0.981
DVY	50	-	-	0.975	0.981	0.984	0.991

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
DVZ	50	-	-	0.947	0.979	0.982	0.985
DVZZ	50	-	-	0.950	0.971	0.977	0.981
DVZZZ	50	-	-	0.928	0.957	0.967	0.968
AH1	1,000	-	-	0.006	0.997	0.929	0.989
AH2	1,000	-	-	0.006	0.976	0.898	0.989
AH3	1,000	-	-	0.005	0.975	0.894	0.993
AH4	1,000	-	-	0.005	0.981	0.956	0.978
AH5	1,000	-	-	0.005	0.982	0.971	0.993
AH6	1,000	-	-	0.005	0.994	0.951	0.993
AH7	1,000	-	-	0.006	0.988	0.962	0.990
AH8	1,000	-	-	0.005	0.956	0.880	0.970
AH9	1,000	-	-	0.005	0.961	0.952	0.998
AH10	1,000	-	-	0.006	0.994	0.883	0.992
AH11	1,000	-	-	0.006	0.945	0.952	0.990
AH12	1,000	-	-	0.005	0.997	0.939	0.998
AH13	1,000	-	-	0.005	0.981	0.893	0.980
AH14	1,000	-	-	0.005	0.954	0.948	0.967
AH15	1,000	-	-	0.005	0.995	0.929	0.986
AH16	1,000	-	-	0.004	0.975	0.910	0.990
AH17	1,000	-	-	0.005	0.960	0.940	0.981
AH18	1,000	-	-	0.005	0.767	0.941	0.986
AH19	1,000	-	-	0.005	0.996	0.959	0.994
AH20	1,000	-	-	0.005	0.968	0.895	0.987
AH21	1,000	-	-	0.004	0.980	0.898	0.980
AH22	1,000	-	-	0.005	0.909	0.841	0.993
AH23	1,000	-	-	0.005	0.916	0.874	0.980
AH24	1,000	-	-	0.005	0.997	0.920	0.996
AH25	1,000	-	-	0.004	0.987	0.894	0.985
AH26	1,000	-	-	0.006	0.951	0.903	0.991
AH27	1,000	-	-	0.005	0.914	0.863	0.969
AH28	1,000	-	-	0.005	0.890	0.902	0.976
AH29	1,000	-	-	0.006	0.961	0.974	0.984
AH30	1,000	-	-	0.005	0.984	0.951	0.985
AH31	1,000	-	-	0.006	0.793	0.933	0.986
AH32	1,000	-	-	0.010	0.972	0.971	0.990
AH33	1,000	-	-	0.007	0.994	0.981	0.993
AH34	1,000	-	-	0.006	0.946	0.950	0.990
AH35	1,000	-	-	0.006	0.916	0.965	0.987
AH36	1,000	-	-	0.006	0.928	0.946	0.972
AH37	1,000	-	-	0.007	0.990	0.952	0.989
AH38	1,000	-	-	0.009	0.947	0.974	0.997
AH39	1,000	-	-	0.006	0.878	0.929	0.986
AH40	1,000	-	-	0.008	0.991	0.972	0.996
AH41	1,000	-	-	0.006	0.862	0.862	0.984
AH42	1,000	-	-	0.008	0.960	0.956	0.993
AH43	1,000	-	-	0.005	0.971	0.941	0.989
AH44	1,000	-	-	0.007	0.930	0.956	0.986
AH45	1,000	-	-	0.008	0.966	0.973	0.994
AH46	1,000	-	-	0.008	0.998	0.943	0.995
AH47	1,000	-	-	0.007	0.999	0.973	0.998
AH48	1,000	-	-	0.006	0.995	0.969	0.996
AH49	1,000	-	-	0.007	0.998	0.934	0.999
AH50	1,000	-	-	0.006	0.957	0.970	0.998
AH51	1,000	-	-	0.010	0.992	0.973	0.998
AH52	1,000	-	-	0.006	0.937	0.930	0.930
AH53	1,000	-	-	0.006	0.945	0.967	0.976
AH54	1,000	-	-	0.005	0.973	0.970	0.995
AH55	1,000	-	-	0.006	0.824	0.968	0.980
AH56	1,000	-	-	0.005	0.990	0.954	0.989
AH57	1,000	-	-	0.008	0.898	0.911	0.978
AH58	1,000	-	-	0.008	0.950	0.993	0.993
AH59	1,000	-	-	0.006	0.962	0.931	0.997
AH60	1,000	-	-	0.005	0.947	0.941	0.998
AH61	1,000	-	-	0.005	0.993	0.972	0.991
AH62	1,000	-	-	0.005	0.995	0.954	0.992
AH63	1,000	-	-	0.004	0.998	0.943	0.995
AH64	1,000	-	-	0.005	0.988	0.962	0.990
AH65	1,000	-	-	0.005	0.995	0.946	0.991
AH66	1,000	-	-	0.005	0.983	0.959	0.990
AH67	1,000	-	-	0.005	0.992	0.956	0.987
AH68	1,000	-	-	0.006	0.993	0.967	0.990
AH69	1,000	-	-	0.006	0.997	0.992	0.995
AH70	1,000	-	-	0.006	0.994	0.958	0.992
AH71	1,000	-	-	0.005	0.993	0.950	0.989
AH72	1,000	-	-	0.006	0.996	0.981	0.988
AH73	1,000	-	-	0.004	0.999	0.957	0.989
AH74	1,000	-	-	0.005	0.999	0.958	0.988
AH75	1,000	-	-	0.005	0.994	0.952	0.989
AH76	1,000	-	-	0.005	0.994	0.968	0.993
AH77	1,000	-	-	0.005	0.996	0.982	0.994
AH78	1,000	-	-	0.004	0.994	0.963	0.991
AH79	1,000	-	-	0.004	0.996	0.948	0.993
AH80	1,000	-	-	0.005	0.985	0.955	0.989
AH81	1,000	-	-	0.005	0.990	0.970	0.994
AH82	1,000	-	-	0.005	0.996	0.965	0.996
AH83	1,000	-	-	0.005	0.996	0.964	0.993

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
AH84	1,000	-	-	0.007	0.993	0.988	0.995
AH85	1,000	-	-	0.005	0.998	0.968	0.994
AH86	1,000	-	-	0.005	0.993	0.961	0.993
AH87	1,000	-	-	0.004	0.995	0.939	0.995
AH88	1,000	-	-	0.004	0.998	0.936	0.993
AH89	1,000	-	-	0.006	0.993	0.967	0.993
AH90	1,000	-	-	0.005	0.996	0.964	0.994
AH91	1,000	-	-	0.007	0.997	0.987	0.994
AH92	1,000	-	-	0.011	0.999	0.988	0.998
AH93	1,000	-	-	0.006	0.998	0.980	0.993
AH94	1,000	-	-	0.007	0.994	0.984	0.995
AH95	1,000	-	-	0.008	0.998	0.987	0.997
AH96	1,000	-	-	0.008	0.997	0.991	0.996
AH97	1,000	-	-	0.006	0.996	0.975	0.994
AH98	1,000	-	-	0.007	0.997	0.984	0.995
AH99	1,000	-	-	0.006	0.996	0.959	0.992
AH100	1,000	-	-	0.008	0.999	0.979	0.995
AH101	1,000	-	-	0.009	0.997	0.978	0.994
AH102	1,000	-	-	0.007	0.976	0.987	0.994
AH103	1,000	-	-	0.006	0.992	0.974	0.993
AH104	1,000	-	-	0.006	0.991	0.954	0.992
AH105	1,000	-	-	0.011	0.998	0.989	0.999
AH106	1,000	-	-	0.013	0.998	0.994	0.997
AH107	1,000	-	-	0.008	0.987	0.983	0.991
AH108	1,000	-	-	0.008	0.997	0.981	0.998
AH109	1,000	-	-	0.005	0.996	0.953	0.989
AH110	1,000	-	-	0.009	0.987	0.980	0.995
AH111	1,000	-	-	0.006	0.989	0.963	0.994
AH112	1,000	-	-	0.007	0.997	0.984	0.993
AH113	1,000	-	-	0.006	0.991	0.965	0.996
AH114	1,000	-	-	0.007	0.992	0.979	0.997
AH115	1,000	-	-	0.007	0.983	0.984	0.995
AH116	1,000	-	-	0.009	0.993	0.988	0.995
AH117	1,000	-	-	0.008	0.990	0.982	0.993
AH118	1,000	-	-	0.009	0.995	0.982	0.996
AH119	1,000	-	-	0.008	0.995	0.989	0.993
AH120	1,000	-	-	0.006	0.989	0.982	0.994
AH121	1,000	-	-	0.006	0.995	0.912	0.993
AH122	1,000	-	-	0.006	0.990	0.935	0.995
AH123	1,000	-	-	0.006	0.994	0.929	0.995
AH124	1,000	-	-	0.006	0.995	0.938	0.995
AH125	1,000	-	-	0.006	0.985	0.940	0.995
AH126	1,000	-	-	0.006	0.996	0.944	0.994
AH127	1,000	-	-	0.007	0.992	0.945	0.995
AH128	1,000	-	-	0.006	0.994	0.936	0.994
AH129	1,000	-	-	0.007	0.993	0.953	0.994
AH130	1,000	-	-	0.006	0.993	0.933	0.996
AH131	1,000	-	-	0.007	0.994	0.920	0.994
AH132	1,000	-	-	0.006	0.995	0.936	0.995
AH133	1,000	-	-	0.006	0.995	0.926	0.995
AH134	1,000	-	-	0.006	0.991	0.940	0.997
AH135	1,000	-	-	0.006	0.995	0.940	0.995
AH136	1,000	-	-	0.006	0.994	0.928	0.995
AH137	1,000	-	-	0.006	0.994	0.932	0.995
AH138	1,000	-	-	0.007	0.996	0.917	0.994
AH139	1,000	-	-	0.007	0.991	0.934	0.994
AH140	1,000	-	-	0.007	0.984	0.938	0.994
AH141	1,000	-	-	0.006	0.995	0.922	0.994
AH142	1,000	-	-	0.007	0.994	0.926	0.995
AH143	1,000	-	-	0.006	0.993	0.911	0.993
AH144	1,000	-	-	0.006	0.995	0.917	0.994
AH145	1,000	-	-	0.006	0.994	0.925	0.994
AH146	1,000	-	-	0.006	0.989	0.931	0.996
AH147	1,000	-	-	0.006	0.996	0.948	0.994
AH148	1,000	-	-	0.007	0.993	0.934	0.995
AH149	1,000	-	-	0.006	0.994	0.938	0.994
AH150	1,000	-	-	0.006	0.995	0.943	0.995
AH151	1,000	-	-	0.009	0.956	0.966	0.994
AH152	1,000	-	-	0.009	0.988	0.977	0.994
AH153	1,000	-	-	0.009	0.954	0.970	0.996
AH154	1,000	-	-	0.010	0.937	0.976	0.996
AH155	1,000	-	-	0.009	0.931	0.969	0.995
AH156	1,000	-	-	0.009	0.984	0.977	0.996
AH157	1,000	-	-	0.010	0.962	0.979	0.992
AH158	1,000	-	-	0.010	0.988	0.976	0.996
AH159	1,000	-	-	0.009	0.990	0.976	0.996
AH160	1,000	-	-	0.010	0.979	0.975	0.994
AH161	1,000	-	-	0.008	0.989	0.976	0.994
AH162	1,000	-	-	0.010	0.993	0.975	0.996
AH163	1,000	-	-	0.009	0.932	0.979	0.995
AH164	1,000	-	-	0.010	0.981	0.971	0.995
AH165	1,000	-	-	0.009	0.961	0.970	0.994
AH166	1,000	-	-	0.009	0.981	0.973	0.996
AH167	1,000	-	-	0.010	0.987	0.975	0.995
AH168	1,000	-	-	0.011	0.967	0.973	0.994
AH169	1,000	-	-	0.010	0.986	0.979	0.996

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
AH170	1,000	-	-	0.009	0.961	0.973	0.997
AH171	1,000	-	-	0.009	0.969	0.974	0.994
AH172	1,000	-	-	0.010	0.962	0.976	0.995
AH173	1,000	-	-	0.008	0.985	0.974	0.994
AH174	1,000	-	-	0.010	0.981	0.978	0.993
AH175	1,000	-	-	0.008	0.983	0.973	0.995
AH176	1,000	-	-	0.009	0.986	0.977	0.994
AH177	1,000	-	-	0.010	0.961	0.970	0.994
AH178	1,000	-	-	0.010	0.960	0.972	0.995
AH179	1,000	-	-	0.008	0.982	0.974	0.995
AH180	1,000	-	-	0.009	0.922	0.969	0.994
AH181	1,000	-	-	0.006	0.998	0.953	0.998
AH182	1,000	-	-	0.006	0.997	0.946	0.998
AH183	1,000	-	-	0.006	0.999	0.960	0.998
AH184	1,000	-	-	0.006	0.998	0.965	0.997
AH185	1,000	-	-	0.006	0.997	0.956	0.997
AH186	1,000	-	-	0.006	0.997	0.962	0.998
AH187	1,000	-	-	0.007	0.997	0.963	0.997
AH188	1,000	-	-	0.006	0.997	0.956	0.997
AH189	1,000	-	-	0.006	0.999	0.967	0.998
AH190	1,000	-	-	0.006	0.997	0.956	0.998
AH191	1,000	-	-	0.006	0.997	0.953	0.998
AH192	1,000	-	-	0.006	0.997	0.961	0.998
AH193	1,000	-	-	0.006	0.998	0.954	0.998
AH194	1,000	-	-	0.006	0.998	0.954	0.998
AH195	1,000	-	-	0.007	0.998	0.963	0.998
AH196	1,000	-	-	0.007	0.997	0.955	0.997
AH197	1,000	-	-	0.007	0.998	0.954	0.998
AH198	1,000	-	-	0.006	0.998	0.960	0.998
AH199	1,000	-	-	0.006	0.997	0.962	0.998
AH200	1,000	-	-	0.006	0.998	0.960	0.998
AH201	1,000	-	-	0.006	0.998	0.959	0.997
AH202	1,000	-	-	0.006	0.998	0.959	0.997
AH203	1,000	-	-	0.006	0.997	0.963	0.997
AH204	1,000	-	-	0.006	0.998	0.955	0.998
AH205	1,000	-	-	0.006	0.996	0.960	0.998
AH206	1,000	-	-	0.007	0.997	0.958	0.997
AH207	1,000	-	-	0.006	0.998	0.958	0.997
AH208	1,000	-	-	0.006	0.998	0.959	0.997
AH209	1,000	-	-	0.007	0.998	0.954	0.998
AH210	1,000	-	-	0.006	0.998	0.955	0.999
AH211	1,000	-	-	0.010	0.994	0.985	0.999
AH212	1,000	-	-	0.010	0.995	0.980	0.999
AH213	1,000	-	-	0.010	0.997	0.983	1.000
AH214	1,000	-	-	0.010	0.994	0.982	1.000
AH215	1,000	-	-	0.008	0.996	0.979	1.000
AH216	1,000	-	-	0.010	0.999	0.982	0.999
AH217	1,000	-	-	0.009	0.995	0.988	1.000
AH218	1,000	-	-	0.011	0.995	0.983	1.000
AH219	1,000	-	-	0.011	0.996	0.976	0.999
AH220	1,000	-	-	0.008	0.989	0.982	0.999
AH221	1,000	-	-	0.011	0.997	0.988	1.000
AH222	1,000	-	-	0.009	0.996	0.986	0.999
AH223	1,000	-	-	0.011	0.998	0.984	0.999
AH224	1,000	-	-	0.011	0.998	0.986	0.999
AH225	1,000	-	-	0.010	0.996	0.985	0.999
AH226	1,000	-	-	0.008	0.999	0.978	0.999
AH227	1,000	-	-	0.009	0.998	0.982	1.000
AH228	1,000	-	-	0.010	0.994	0.980	1.000
AH229	1,000	-	-	0.009	0.997	0.984	0.999
AH230	1,000	-	-	0.010	0.989	0.982	0.999
AH231	1,000	-	-	0.009	0.996	0.980	0.999
AH232	1,000	-	-	0.009	0.997	0.981	0.999
AH233	1,000	-	-	0.010	0.999	0.984	0.999
AH234	1,000	-	-	0.009	0.989	0.982	0.999
AH235	1,000	-	-	0.010	0.997	0.987	1.000
AH236	1,000	-	-	0.009	0.998	0.985	0.999
AH237	1,000	-	-	0.009	0.994	0.986	0.999
AH238	1,000	-	-	0.008	0.997	0.980	0.999
AH239	1,000	-	-	0.010	0.997	0.979	0.999
AH240	1,000	-	-	0.010	0.998	0.983	0.999
AH241	1,000	-	-	0.007	0.987	0.926	0.994
AH242	1,000	-	-	0.007	0.997	0.932	0.994
AH243	1,000	-	-	0.007	0.997	0.938	0.995
AH244	1,000	-	-	0.007	0.994	0.927	0.995
AH245	1,000	-	-	0.006	0.997	0.925	0.994
AH246	1,000	-	-	0.007	0.997	0.930	0.994
AH247	1,000	-	-	0.007	0.988	0.928	0.995
AH248	1,000	-	-	0.006	0.994	0.934	0.994
AH249	1,000	-	-	0.006	0.997	0.941	0.995
AH250	1,000	-	-	0.006	0.997	0.939	0.993
AH251	1,000	-	-	0.007	0.987	0.928	0.994
AH252	1,000	-	-	0.007	0.994	0.929	0.995
AH253	1,000	-	-	0.006	0.986	0.935	0.994
AH254	1,000	-	-	0.006	0.996	0.927	0.994
AH255	1,000	-	-	0.007	0.925	0.928	0.995

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
AH256	1,000	-	-	0.006	0.996	0.930	0.994
AH257	1,000	-	-	0.007	0.997	0.931	0.994
AH258	1,000	-	-	0.007	0.997	0.940	0.995
AH259	1,000	-	-	0.007	0.997	0.940	0.994
AH260	1,000	-	-	0.007	0.994	0.923	0.994
AH261	1,000	-	-	0.007	0.997	0.926	0.995
AH262	1,000	-	-	0.007	0.997	0.942	0.994
AH263	1,000	-	-	0.007	0.994	0.935	0.994
AH264	1,000	-	-	0.007	0.998	0.928	0.995
AH265	1,000	-	-	0.007	0.995	0.943	0.994
AH266	1,000	-	-	0.007	0.997	0.943	0.994
AH267	1,000	-	-	0.007	0.990	0.935	0.994
AH268	1,000	-	-	0.007	0.978	0.929	0.995
AH269	1,000	-	-	0.007	0.995	0.935	0.994
AH270	1,000	-	-	0.007	0.973	0.936	0.995
AH271	1,000	-	-	0.010	0.922	0.978	0.999
AH272	1,000	-	-	0.010	0.988	0.980	0.999
AH273	1,000	-	-	0.011	0.990	0.982	0.999
AH274	1,000	-	-	0.010	0.990	0.982	0.999
AH275	1,000	-	-	0.010	0.891	0.981	0.998
AH276	1,000	-	-	0.010	0.993	0.982	0.998
AH277	1,000	-	-	0.010	0.904	0.982	0.999
AH278	1,000	-	-	0.011	0.987	0.980	0.999
AH279	1,000	-	-	0.011	0.925	0.980	0.998
AH280	1,000	-	-	0.010	0.991	0.982	0.998
AH281	1,000	-	-	0.010	0.990	0.982	0.999
AH282	1,000	-	-	0.010	0.944	0.981	0.999
AH283	1,000	-	-	0.010	0.880	0.982	0.998
AH284	1,000	-	-	0.010	0.972	0.980	0.999
AH285	1,000	-	-	0.010	0.963	0.979	0.999
AH286	1,000	-	-	0.010	0.950	0.979	0.998
AH287	1,000	-	-	0.010	0.985	0.979	0.998
AH288	1,000	-	-	0.010	0.991	0.981	0.999
AH289	1,000	-	-	0.010	0.965	0.982	0.998
AH290	1,000	-	-	0.010	0.974	0.980	0.998
AH291	1,000	-	-	0.010	0.993	0.983	0.998
AH292	1,000	-	-	0.011	0.992	0.981	0.998
AH293	1,000	-	-	0.010	0.973	0.981	0.999
AH294	1,000	-	-	0.010	0.992	0.982	0.998
AH295	1,000	-	-	0.010	0.965	0.979	0.999
AH296	1,000	-	-	0.010	0.885	0.980	0.999
AH297	1,000	-	-	0.010	0.975	0.980	0.998
AH298	1,000	-	-	0.010	0.987	0.982	0.999
AH299	1,000	-	-	0.010	0.976	0.979	0.999
AH300	1,000	-	-	0.010	0.990	0.979	0.999
AH301	1,000	-	-	0.007	1.000	0.956	1.000
AH302	1,000	-	-	0.007	1.000	0.953	1.000*
AH303	1,000	-	-	0.007	1.000	0.954	1.000
AH304	1,000	-	-	0.007	1.000	0.958	1.000*
AH305	1,000	-	-	0.007	1.000	0.963	1.000*
AH306	1,000	-	-	0.007	1.000	0.954	1.000
AH307	1,000	-	-	0.007	1.000	0.956	1.000*
AH308	1,000	-	-	0.007	1.000	0.955	1.000
AH309	1,000	-	-	0.007	1.000	0.965	1.000
AH310	1,000	-	-	0.007	1.000	0.958	1.000
AH311	1,000	-	-	0.007	1.000	0.955	1.000
AH312	1,000	-	-	0.007	1.000	0.961	1.000
AH313	1,000	-	-	0.007	1.000	0.956	1.000
AH314	1,000	-	-	0.007	1.000	0.955	1.000
AH315	1,000	-	-	0.007	1.000	0.960	1.000*
AH316	1,000	-	-	0.007	1.000	0.958	1.000
AH317	1,000	-	-	0.007	1.000	0.960	1.000*
AH318	1,000	-	-	0.007	1.000	0.960	1.000*
AH319	1,000	-	-	0.007	1.000	0.960	1.000*
AH320	1,000	-	-	0.007	1.000	0.959	1.000
AH321	1,000	-	-	0.007	1.000	0.956	1.000*
AH322	1,000	-	-	0.007	1.000	0.963	1.000
AH323	1,000	-	-	0.007	1.000	0.960	1.000
AH324	1,000	-	-	0.007	1.000	0.958	1.000*
AH325	1,000	-	-	0.007	1.000	0.960	1.000*
AH326	1,000	-	-	0.007	1.000	0.958	1.000
AH327	1,000	-	-	0.007	1.000	0.957	1.000
AH328	1,000	-	-	0.007	1.000	0.959	1.000
AH329	1,000	-	-	0.007	1.000	0.955	1.000
AH330	1,000	-	-	0.007	1.000	0.955	1.000*
AH331	1,000	-	-	0.010	1.000	0.988	1.000*
AH332	1,000	-	-	0.010	1.000	0.987	1.000*
AH333	1,000	-	-	0.010	1.000	0.989	1.000*
AH334	1,000	-	-	0.010	1.000	0.989	1.000*
AH335	1,000	-	-	0.011	1.000	0.988	1.000*
AH336	1,000	-	-	0.010	1.000	0.988	1.000*
AH337	1,000	-	-	0.010	1.000	0.988	1.000*
AH338	1,000	-	-	0.010	1.000	0.989	1.000*
AH339	1,000	-	-	0.010	1.000	0.988	1.000*
AH340	1,000	-	-	0.010	1.000	0.987	1.000*
AH341	1,000	-	-	0.010	1.000	0.987	1.000*

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
AH342	1,000	-	-	0.011	1.000	0.989	1.000*
AH343	1,000	-	-	0.010	1.000	0.987	1.000*
AH344	1,000	-	-	0.011	1.000	0.989	1.000*
AH345	1,000	-	-	0.011	1.000	0.987	1.000*
AH346	1,000	-	-	0.010	1.000	0.988	1.000*
AH347	1,000	-	-	0.010	1.000	0.988	1.000*
AH348	1,000	-	-	0.011	1.000	0.989	1.000*
AH349	1,000	-	-	0.010	1.000	0.988	1.000*
AH350	1,000	-	-	0.010	1.000	0.988	1.000*
AH351	1,000	-	-	0.011	1.000	0.988	1.000*
AH352	1,000	-	-	0.011	1.000	0.987	1.000*
AH353	1,000	-	-	0.011	1.000	0.987	1.000*
AH354	1,000	-	-	0.010	1.000	0.988	1.000*
AH355	1,000	-	-	0.010	1.000	0.989	1.000*
AH356	1,000	-	-	0.010	1.000	0.988	1.000*
AH357	1,000	-	-	0.010	1.000	0.987	1.000*
AH358	1,000	-	-	0.010	1.000	0.987	1.000*
AH359	1,000	-	-	0.010	1.000	0.988	1.000*
AH360	1,000	-	-	0.010	1.000	0.988	1.000*
ami33	33	-	-	0.960	0.981	0.970	0.990
ami49	49	-	-	0.942	0.981	0.970	0.984
pcb146	146	-	-	0.832	0.967	0.980	0.987
pcb500	500	-	-	0.309	1.000	0.996	1.000*
rp100	100	-	-	0.883	1.000*	0.993	1.000*
rp200	200	-	-	0.867	0.989	0.988	0.999
beng01	20	0.996*	0.996*	0.996*	0.981	0.989	0.981
beng02	40	1.000*	0.956	1.000*	1.000*	1.000*	1.000*
beng03	60	0.985	0.909	0.993	1.000*	1.000*	1.000*
beng04	80	1.000*	0.834	0.966	1.000*	1.000*	1.000*
beng05	100	0.993	-	0.928	1.000*	1.000*	1.000*
beng06	40	1.000*	0.992	1.000*	1.000*	1.000*	1.000*
beng07	80	0.998	0.678	0.975	1.000*	1.000*	1.000*
beng08	120	0.980	-	0.925	1.000*	1.000*	1.000*
beng09	160	0.972	-	0.924	1.000*	1.000*	1.000*
beng10	200	-	-	0.912	1.000*	1.000*	1.000*
n1	10	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
n2	20	0.926	0.954	0.973	0.977	0.965	0.984
n3	30	1.000*	0.966	1.000*	1.000*	0.995	1.000*
n4	40	0.144	0.817	0.975	0.983	0.987	0.991
n5	50	-	0.800	0.968	0.998	0.982	1.000*
n6	60	-	0.785	0.976	1.000*	0.993	1.000*
n7	70	-	-	0.958	0.996	0.989	1.000
n8	80	-	-	0.934	1.000*	0.998	1.000*
n9	100	-	-	0.883	1.000*	0.985	0.999
n10	200	-	-	0.788	0.999	0.996	1.000*
n11	300	0.747	-	0.773	1.000*	0.989	1.000*
n12	500	-	-	0.552	1.000*	0.998	1.000*
n13	3,152	-	-	0.005	1.000*	0.999	1.000*
C_1_1	20	0.948*	0.938	0.948*	0.900	0.948*	0.948*
C_1_2	20	0.984*	0.984*	0.984*	0.943	0.957	0.943
C_1_3	20	0.905	0.895	0.908	0.891	0.908	0.908
C_1_4	20	0.998*	0.949	0.998*	0.998*	0.988	0.998*
C_1_5	20	0.989*	0.939	0.989*	0.974	0.974	0.989*
C_1_6	20	0.941*	0.909	0.941*	0.934	0.941*	0.938
C_1_7	20	0.977*	0.975	0.977*	0.964	0.977*	0.977*
C_1_8	20	0.976*	0.971	0.976*	0.975	0.975	0.975
C_1_9	20	0.915*	0.913	0.915*	0.890	0.915*	0.915*
C_1_10	20	0.964*	0.964*	0.964*	0.946	0.964*	0.964*
C_1_11	40	0.973	0.908	0.964	0.986	0.986	0.987
C_1_12	40	0.972	0.856	0.972	0.978	0.983	0.993
C_1_13	40	0.937	0.878	0.965	0.965	0.960	0.971
C_1_14	40	0.964	0.902	0.969	0.962	0.972	0.975
C_1_15	40	0.974	0.836	0.972	0.980	0.980	0.987
C_1_16	40	0.907*	0.858	0.906	0.907*	0.907*	0.907*
C_1_17	40	0.989*	0.915	0.983	0.989*	0.989*	0.989*
C_1_18	40	0.906	0.886	0.905	0.907	0.907	0.907
C_1_19	40	0.925	0.881	0.968	0.986	0.974	0.986
C_1_20	40	0.982	0.847	0.975	0.996	0.986	0.996
C_1_21	60	0.694	-	0.921	0.935	0.930	0.935
C_1_22	60	0.967	-	0.949	0.975	0.974	0.980
C_1_23	60	0.857	0.826	0.925	0.954	0.953	0.957
C_1_24	60	0.625	-	0.890	0.874	0.903	0.911
C_1_25	60	0.979	-	0.948	0.999	0.987	0.999
C_1_26	60	0.957	0.830	0.956	0.990	0.978	0.991
C_1_27	60	0.960	-	0.956	0.986	0.981	0.989
C_1_28	60	0.965	0.837	0.959	0.970	0.969	0.970
C_1_29	60	0.959	-	0.963	0.993	0.986	0.995
C_1_30	60	0.698	-	0.929	0.938	0.938	0.944
C_1_31	80	0.735	-	0.875	0.966	0.957	0.966
C_1_32	80	0.781	-	0.872	0.961	0.955	0.962
C_1_33	80	0.698	-	0.842	0.893	0.900	0.908
C_1_34	80	0.752	-	0.870	0.950	0.952	0.957
C_1_35	80	0.773	-	0.883	0.967	0.968	0.971
C_1_36	80	0.791	-	0.887	0.969	0.965	0.969
C_1_37	80	0.824	-	0.886	0.938	0.946	0.948
C_1_38	80	0.817	-	0.872	0.946	0.949	0.954

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_1_39	80	0.854	-	0.909	0.986	0.978	0.989
C_1_40	80	0.848	-	0.901	0.991	0.987	0.992
C_1_41	100	0.782	-	0.882	0.997	0.988	0.997
C_1_42	100	0.838	-	0.852	0.992	0.982	0.993
C_1_43	100	0.815	-	0.855	0.999	0.993	0.999
C_1_44	100	0.797	-	0.883	0.987	0.983	0.987
C_1_45	100	0.815	-	0.874	0.976	0.966	0.980
C_1_46	100	0.711	-	0.874	0.955	0.956	0.962
C_1_47	100	0.789	-	0.880	0.993	0.983	0.994
C_1_48	100	0.886	-	0.868	0.991	0.985	0.992
C_1_49	100	0.748	-	0.893	0.995	0.991	0.998
C_1_50	100	0.773	-	0.865	0.970	0.970	0.975
C_2_51	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_52	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_53	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_54	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_55	20	1.000*	1.000*	1.000*	0.995	0.995	0.995
C_2_56	20	1.000*	0.996	1.000*	0.993	1.000*	0.996
C_2_57	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_58	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_59	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_60	20	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_61	40	1.000*	0.994	1.000*	1.000*	1.000*	1.000*
C_2_62	40	1.000*	0.995	1.000*	1.000*	1.000*	1.000*
C_2_63	40	1.000*	0.990	1.000*	1.000*	1.000*	1.000*
C_2_64	40	1.000*	0.973	1.000*	1.000*	1.000*	1.000*
C_2_65	40	1.000*	0.998	1.000*	1.000*	1.000*	1.000*
C_2_66	40	1.000*	0.987	1.000*	1.000*	1.000*	1.000*
C_2_67	40	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
C_2_68	40	1.000*	0.985	1.000*	1.000*	1.000*	1.000*
C_2_69	40	1.000*	0.994	1.000*	1.000*	1.000*	1.000*
C_2_70	40	1.000*	0.989	1.000*	1.000*	1.000*	1.000*
C_2_71	60	1.000*	0.916	0.996	1.000*	1.000*	1.000*
C_2_72	60	0.978	0.900	0.995	1.000*	1.000*	1.000*
C_2_73	60	0.966	0.884	0.991	1.000*	1.000*	1.000*
C_2_74	60	0.997	0.849	0.997	1.000*	1.000*	1.000*
C_2_75	60	0.986	0.908	0.997	1.000*	1.000*	1.000*
C_2_76	60	0.993	0.911	0.998	1.000*	1.000*	1.000*
C_2_77	60	1.000*	0.958	0.998	1.000*	1.000*	1.000*
C_2_78	60	0.985	0.884	0.997	1.000*	1.000*	1.000*
C_2_79	60	0.984	0.940	0.995	1.000*	1.000*	1.000*
C_2_80	60	0.949	0.867	0.984	1.000*	1.000*	1.000*
C_2_81	80	0.980	0.860	0.973	1.000*	1.000*	1.000*
C_2_82	80	0.965	0.861	0.982	1.000*	1.000*	1.000*
C_2_83	80	1.000*	0.819	0.986	1.000*	1.000*	1.000*
C_2_84	80	0.992	0.764	0.939	1.000*	1.000*	1.000*
C_2_85	80	0.987	0.857	0.953	1.000*	1.000*	1.000*
C_2_86	80	0.978	0.810	0.951	1.000*	1.000*	1.000*
C_2_87	80	0.979	0.767	0.963	1.000*	1.000*	1.000*
C_2_88	80	0.962	0.849	0.944	1.000*	1.000*	1.000*
C_2_89	80	0.943	0.867	0.957	1.000*	1.000*	1.000*
C_2_90	80	0.991	0.803	0.983	1.000*	1.000*	1.000*
C_2_91	100	0.986	-	0.965	1.000*	1.000*	1.000*
C_2_92	100	0.984	-	0.945	1.000*	1.000*	1.000*
C_2_93	100	0.995	-	0.978	1.000*	1.000*	1.000*
C_2_94	100	0.986	-	0.936	1.000*	1.000*	1.000*
C_2_95	100	0.847	-	0.945	1.000*	1.000*	1.000*
C_2_96	100	0.976	-	0.922	1.000*	1.000*	1.000*
C_2_97	100	1.000*	-	0.917	1.000*	1.000*	1.000*
C_2_98	100	0.976	-	0.907	1.000*	1.000*	1.000*
C_2_99	100	0.988	-	0.940	1.000*	1.000*	1.000*
C_2_100	100	0.881	-	0.942	1.000*	1.000*	1.000*
C_3_101	20	0.360	0.862	0.953	0.925	0.951	0.953
C_3_102	20	0.901	0.890	0.974*	0.932	0.957	0.973
C_3_103	20	0.323	0.845	0.908	0.892	0.890	0.908
C_3_104	20	0.797	0.927	0.989	0.942	0.973	0.978
C_3_105	20	0.700	0.925	0.976	0.961	0.976	0.976
C_3_106	20	0.179	0.902	0.964	0.938	0.964	0.961
C_3_107	20	0.866	0.912	0.972*	0.917	0.969	0.972*
C_3_108	20	0.282	0.899	0.972*	0.959	0.968	0.972*
C_3_109	20	0.422	0.869	0.943	0.933	0.943	0.943
C_3_110	20	0.031	0.907	0.955	0.911	0.950	0.950
C_3_111	40	-	0.856	0.976	0.963	0.964	0.992
C_3_112	40	0.079	-	0.963	0.953	0.968	0.994
C_3_113	40	-	-	0.957	0.952	0.959	0.972
C_3_114	40	-	-	0.943	0.939	0.944	0.969
C_3_115	40	-	-	0.953	0.951	0.957	0.975
C_3_116	40	0.006	0.776	0.911	0.920	0.915	0.921
C_3_117	40	-	-	0.969	0.967	0.965	0.985
C_3_118	40	-	-	0.894	0.892	0.895	0.906
C_3_119	40	-	-	0.956	0.970	0.953	0.982
C_3_120	40	-	0.775	0.968	0.966	0.961	0.990
C_3_121	60	-	-	0.893	0.937	0.934	0.954
C_3_122	60	-	-	0.941	0.952	0.966	0.981
C_3_123	60	-	-	0.940	0.972	0.964	0.987
C_3_124	60	-	-	0.924	0.943	0.941	0.959

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_3_125	60	-	-	0.957	0.960	0.974	0.991
C_3_126	60	-	-	0.919	0.957	0.951	0.981
C_3_127	60	-	-	0.949	0.983	0.982	0.995
C_3_128	60	-	-	0.913	0.955	0.962	0.982
C_3_129	60	-	-	0.932	0.961	0.960	0.980
C_3_130	60	-	-	0.882	0.943	0.946	0.961
C_3_131	80	-	-	0.886	0.977	0.959	0.988
C_3_132	80	-	-	0.862	0.938	0.917	0.951
C_3_133	80	-	-	0.897	0.958	0.945	0.965
C_3_134	80	-	-	0.866	0.969	0.959	0.986
C_3_135	80	-	-	0.893	0.982	0.957	0.994
C_3_136	80	-	-	0.847	0.961	0.959	0.980
C_3_137	80	-	-	0.873	0.962	0.955	0.979
C_3_138	80	-	-	0.835	0.940	0.932	0.956
C_3_139	80	-	-	0.844	0.957	0.945	0.972
C_3_140	80	-	-	0.836	0.970	0.975	0.993
C_3_141	100	-	-	0.853	0.984	0.975	0.998
C_3_142	100	-	-	0.828	0.972	0.964	0.984
C_3_143	100	-	-	0.837	0.956	0.972	0.998
C_3_144	100	-	-	0.807	0.974	0.971	0.996
C_3_145	100	-	-	0.822	0.975	0.963	0.985
C_3_146	100	-	-	0.844	0.961	0.953	0.973
C_3_147	100	-	-	0.840	0.955	0.966	0.983
C_3_148	100	-	-	0.851	0.951	0.969	0.990
C_3_149	100	-	-	0.811	0.981	0.978	0.997
C_3_150	100	-	-	0.835	0.955	0.943	0.972
C_4_151	20	0.987*	0.933	0.987*	0.967	0.978	0.978
C_4_152	20	0.969	0.936	0.987*	0.968	0.980	0.987*
C_4_153	20	0.965	0.949	0.984*	0.942	0.984*	0.975
C_4_154	20	0.974	0.970	0.987*	0.984	0.971	0.987*
C_4_155	20	0.969	0.952	0.973*	0.963	0.963	0.969
C_4_156	20	0.989*	0.961	0.989*	0.953	0.984	0.984
C_4_157	20	0.979	0.965	0.991*	0.970	0.975	0.970
C_4_158	20	0.967	0.923	0.989*	0.962	0.989*	0.983
C_4_159	20	0.989*	0.980	0.989*	0.989*	0.989*	0.989*
C_4_160	20	0.990*	0.955	0.990*	0.931	0.976	0.973
C_4_161	40	0.242	0.853	0.990	0.992	0.997	1.000*
C_4_162	40	0.263	0.810	0.995	0.995	0.998	0.999
C_4_163	40	-	0.910	0.983	0.998	0.986	1.000*
C_4_164	40	-	0.864	0.982	0.998	0.988	1.000
C_4_165	40	-	0.825	0.986	0.994	0.994	0.998
C_4_166	40	0.325	0.926	0.985	0.999	0.999	1.000*
C_4_167	40	-	0.844	0.991	0.999	0.999	0.999
C_4_168	40	0.244	0.903	0.997	0.998	0.999	0.999
C_4_169	40	0.423	0.877	0.989	0.995	0.993	0.999
C_4_170	40	0.472	0.878	0.987	0.998	0.993	0.998
C_4_171	60	-	-	0.968	1.000*	0.996	1.000*
C_4_172	60	-	-	0.973	1.000*	0.995	1.000*
C_4_173	60	0.134	-	0.983	1.000*	0.995	1.000*
C_4_174	60	-	-	0.973	1.000*	0.997	1.000*
C_4_175	60	-	-	0.982	1.000	0.998	1.000*
C_4_176	60	-	-	0.969	1.000*	0.997	1.000*
C_4_177	60	-	0.803	0.974	1.000*	0.999	1.000*
C_4_178	60	-	-	0.982	1.000*	0.997	1.000*
C_4_179	60	-	-	0.981	1.000*	0.997	1.000*
C_4_180	60	-	-	0.969	1.000	0.994	1.000*
C_4_181	80	-	-	0.941	1.000*	0.999	1.000*
C_4_182	80	-	-	0.954	0.999	0.994	1.000*
C_4_183	80	-	-	0.942	1.000*	0.998	1.000*
C_4_184	80	-	-	0.925	1.000*	0.999	1.000*
C_4_185	80	-	-	0.945	1.000*	0.996	1.000*
C_4_186	80	-	-	0.941	1.000*	0.999	1.000*
C_4_187	80	-	-	0.928	1.000	0.995	1.000*
C_4_188	80	-	-	0.935	1.000*	0.994	1.000*
C_4_189	80	-	-	0.912	1.000	0.996	1.000*
C_4_190	80	-	-	0.975	1.000*	0.998	1.000*
C_4_191	100	-	-	0.924	1.000*	0.998	1.000*
C_4_192	100	-	-	0.948	1.000*	0.998	1.000*
C_4_193	100	-	-	0.945	1.000*	0.998	1.000*
C_4_194	100	-	-	0.929	1.000*	0.998	1.000*
C_4_195	100	-	-	0.908	1.000*	0.998	1.000*
C_4_196	100	-	-	0.914	1.000*	0.998	1.000*
C_4_197	100	-	-	0.963	1.000*	0.999	1.000*
C_4_198	100	-	-	0.927	1.000*	0.999	1.000*
C_4_199	100	-	-	0.962	1.000*	0.999	1.000*
C_4_200	100	-	-	0.942	1.000*	0.999	1.000*
C_5_201	20	-	0.792	0.895	0.884	0.895	0.895
C_5_202	20	-	0.781	0.896*	0.861	0.896*	0.896*
C_5_203	20	-	0.842	0.896	0.879	0.896	0.896
C_5_204	20	-	0.852	0.972	0.950	0.963	0.959
C_5_205	20	-	0.847	0.954	0.943	0.947	0.947
C_5_206	20	-	0.791	0.920	0.920	0.919	0.920
C_5_207	20	-	0.885	0.960	0.916	0.960	0.960
C_5_208	20	-	0.808	0.942	0.930	0.940	0.941
C_5_209	20	-	0.768	0.923	0.912	0.923	0.923
C_5_210	20	-	-	0.935	0.909	0.935	0.935

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_5_211	40	-	-	0.945	0.917	0.935	0.949
C_5_212	40	-	-	0.939	0.942	0.945	0.951
C_5_213	40	-	-	0.925	0.924	0.924	0.925
C_5_214	40	-	-	0.936	0.934	0.934	0.948
C_5_215	40	-	-	0.945	0.941	0.940	0.966
C_5_216	40	-	-	0.845	0.844	0.848	0.850
C_5_217	40	-	-	0.951	0.926	0.942	0.951
C_5_218	40	-	-	0.882	0.878	0.879	0.888
C_5_219	40	-	-	0.920	0.928	0.928	0.944
C_5_220	40	-	-	0.946	0.936	0.932	0.948
C_5_221	60	-	-	0.897	0.922	0.916	0.929
C_5_222	60	-	-	0.930	0.952	0.940	0.960
C_5_223	60	-	-	0.912	0.947	0.945	0.954
C_5_224	60	-	-	0.855	0.886	0.882	0.890
C_5_225	60	-	-	0.937	0.949	0.958	0.976
C_5_226	60	-	-	0.912	0.938	0.926	0.946
C_5_227	60	-	-	0.927	0.937	0.937	0.956
C_5_228	60	-	-	0.932	0.944	0.938	0.964
C_5_229	60	-	-	0.933	0.944	0.953	0.969
C_5_230	60	-	-	0.876	0.902	0.896	0.910
C_5_231	80	-	-	0.828	0.908	0.916	0.933
C_5_232	80	-	-	0.870	0.915	0.909	0.923
C_5_233	80	-	-	0.816	0.884	0.872	0.890
C_5_234	80	-	-	0.857	0.922	0.916	0.935
C_5_235	80	-	-	0.878	0.951	0.932	0.961
C_5_236	80	-	-	0.887	0.939	0.931	0.947
C_5_237	80	-	-	0.848	0.933	0.926	0.941
C_5_238	80	-	-	0.861	0.933	0.929	0.944
C_5_239	80	-	-	0.895	0.934	0.931	0.952
C_5_240	80	-	-	0.907	0.948	0.953	0.968
C_5_241	100	-	-	0.847	0.936	0.946	0.976
C_5_242	100	-	-	0.819	0.944	0.945	0.954
C_5_243	100	-	-	0.845	0.952	0.957	0.984
C_5_244	100	-	-	0.801	0.954	0.944	0.967
C_5_245	100	-	-	0.844	0.949	0.928	0.958
C_5_246	100	-	-	0.824	0.922	0.925	0.947
C_5_247	100	-	-	0.850	0.933	0.939	0.962
C_5_248	100	-	-	0.802	0.945	0.948	0.967
C_5_249	100	-	-	0.848	0.950	0.958	0.972
C_5_250	100	-	-	0.860	0.933	0.924	0.942
C_6_251	20	-	0.924	0.971*	0.952	0.971*	0.966
C_6_252	20	-	0.836	0.977*	0.932	0.960	0.965
C_6_253	20	-	0.904	0.954*	0.916	0.951	0.948
C_6_254	20	-	0.880	0.966*	0.955	0.938	0.955
C_6_255	20	-	0.923	0.956*	0.944	0.955	0.955
C_6_256	20	-	0.853	0.934*	0.865	0.934*	0.934*
C_6_257	20	-	0.944	0.987*	0.960	0.975	0.987*
C_6_258	20	-	0.878	0.972*	0.954	0.961	0.956
C_6_259	20	-	0.901	0.976*	0.976*	0.976*	0.976*
C_6_260	20	-	0.891	0.968*	0.929	0.965	0.968*
C_6_261	40	-	-	0.986	0.985	0.992	0.992
C_6_262	40	-	-	0.978	0.978	0.989	0.992
C_6_263	40	-	-	0.983	0.980	0.990	0.992
C_6_264	40	-	-	0.976	0.989	0.983	0.993
C_6_265	40	-	-	0.969	0.982	0.987	0.994
C_6_266	40	-	-	0.985	0.988	0.991	0.995
C_6_267	40	-	-	0.983	0.985	0.994	0.994
C_6_268	40	-	-	0.982	0.982	0.987	0.995
C_6_269	40	-	-	0.984	0.979	0.983	0.990
C_6_270	40	-	-	0.980	0.980	0.987	0.990
C_6_271	60	-	-	0.977	0.987	0.990	0.996
C_6_272	60	-	-	0.974	0.991	0.989	0.997
C_6_273	60	-	-	0.934	0.979	0.981	0.995
C_6_274	60	-	-	0.963	0.982	0.993	0.995
C_6_275	60	-	-	0.976	0.989	0.988	0.998
C_6_276	60	-	-	0.977	0.983	0.989	0.994
C_6_277	60	-	-	0.985	0.987	0.993	0.996
C_6_278	60	-	-	0.963	0.985	0.989	0.996
C_6_279	60	-	-	0.962	0.981	0.989	0.995
C_6_280	60	-	-	0.967	0.984	0.988	0.995
C_6_281	80	-	-	0.941	0.990	0.992	0.998
C_6_282	80	-	-	0.966	0.991	0.989	0.999
C_6_283	80	-	-	0.967	0.980	0.994	0.999
C_6_284	80	-	-	0.926	0.991	0.991	0.998
C_6_285	80	-	-	0.933	0.985	0.987	0.995
C_6_286	80	-	-	0.940	0.987	0.991	0.997
C_6_287	80	-	-	0.926	0.977	0.989	0.995
C_6_288	80	-	-	0.939	0.982	0.991	0.997
C_6_289	80	-	-	0.956	0.981	0.990	0.999
C_6_290	80	-	-	0.936	0.991	0.994	0.997
C_6_291	100	-	-	0.935	0.981	0.992	0.998
C_6_292	100	-	-	0.921	0.982	0.989	0.997
C_6_293	100	-	-	0.922	0.992	0.992	0.998
C_6_294	100	-	-	0.927	0.994	0.993	0.999
C_6_295	100	-	-	0.924	0.987	0.992	0.999
C_6_296	100	-	-	0.918	0.984	0.991	0.998

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_6_297	100	-	-	0.921	0.985	0.994	0.998
C_6_298	100	-	-	0.942	0.989	0.994	0.999
C_6_299	100	-	-	0.938	0.994	0.996	0.999
C_6_300	100	-	-	0.938	0.993	0.994	0.997
C_7_301	20	-	0.873	0.928	0.917	0.925	0.928
C_7_302	20	-	0.817	0.859	0.826	0.859	0.859
C_7_303	20	-	0.845	0.917	0.907	0.917	0.917
C_7_304	20	-	0.803	0.841	0.788	0.841	0.841
C_7_305	20	-	0.844	0.876	0.853	0.876	0.876
C_7_306	20	-	0.859	0.888	0.864	0.888	0.888
C_7_307	20	-	0.808	0.915	0.900	0.915	0.915
C_7_308	20	-	0.833	0.852	0.775	0.852	0.852
C_7_309	20	-	0.851	0.871	0.851	0.871	0.871
C_7_310	20	-	0.779	0.914	0.913	0.914	0.914
C_7_311	40	-	-	0.920	0.904	0.916	0.920
C_7_312	40	-	-	0.880	0.887	0.879	0.894
C_7_313	40	-	-	0.937	0.936	0.932	0.939
C_7_314	40	-	-	0.858	0.827	0.860	0.862
C_7_315	40	-	-	0.884	0.889	0.887	0.893
C_7_316	40	-	-	0.904	0.913	0.910	0.918
C_7_317	40	-	-	0.863	0.852	0.859	0.869
C_7_318	40	-	-	0.909	0.907	0.905	0.913
C_7_319	40	-	-	0.894	0.886	0.893	0.898
C_7_320	40	-	-	0.855	0.851	0.844	0.855
C_7_321	60	-	-	0.880	0.897	0.900	0.915
C_7_322	60	-	-	0.907	0.940	0.922	0.945
C_7_323	60	-	-	0.872	0.877	0.871	0.886
C_7_324	60	-	-	0.875	0.924	0.918	0.926
C_7_325	60	-	-	0.878	0.896	0.885	0.899
C_7_326	60	-	-	0.843	0.850	0.855	0.858
C_7_327	60	-	-	0.878	0.896	0.885	0.900
C_7_328	60	-	-	0.858	0.891	0.880	0.896
C_7_329	60	-	-	0.905	0.931	0.928	0.936
C_7_330	60	-	-	0.880	0.929	0.918	0.932
C_7_331	80	-	-	0.868	0.914	0.883	0.918
C_7_332	80	-	-	0.848	0.885	0.872	0.889
C_7_333	80	-	-	0.855	0.915	0.901	0.916
C_7_334	80	-	-	0.873	0.906	0.895	0.908
C_7_335	80	-	-	0.868	0.885	0.881	0.893
C_7_336	80	-	-	0.848	0.885	0.877	0.894
C_7_337	80	-	-	0.831	0.902	0.879	0.908
C_7_338	80	-	-	0.854	0.894	0.894	0.906
C_7_339	80	-	-	0.827	0.877	0.873	0.890
C_7_340	80	-	-	0.855	0.891	0.879	0.894
C_7_341	100	-	-	0.843	0.905	0.897	0.910
C_7_342	100	-	-	0.842	0.902	0.883	0.910
C_7_343	100	-	-	0.828	0.889	0.885	0.894
C_7_344	100	-	-	0.854	0.897	0.889	0.904
C_7_345	100	-	-	0.845	0.907	0.899	0.916
C_7_346	100	-	-	0.816	0.918	0.896	0.925
C_7_347	100	-	-	0.841	0.895	0.881	0.899
C_7_348	100	-	-	0.823	0.888	0.877	0.892
C_7_349	100	-	-	0.857	0.923	0.918	0.938
C_7_350	100	-	-	0.833	0.902	0.897	0.914
C_8_351	20	-	0.893	0.963*	0.925	0.953	0.963*
C_8_352	20	-	0.790	0.958	0.934	0.951	0.953
C_8_353	20	-	0.891	0.976	0.951	0.964	0.976
C_8_354	20	-	-	0.962	0.945	0.949	0.948
C_8_355	20	-	0.871	0.969	0.924	0.955	0.955
C_8_356	20	-	0.839	0.978	0.957	0.972	0.978
C_8_357	20	-	0.787	0.965	0.923	0.947	0.951
C_8_358	20	-	0.769	0.954*	0.933	0.936	0.944
C_8_359	20	-	0.750	0.962	0.932	0.958	0.968
C_8_360	20	-	0.933	0.978*	0.944	0.957	0.978*
C_8_361	40	-	-	0.946	0.947	0.950	0.965
C_8_362	40	-	-	0.953	0.937	0.956	0.966
C_8_363	40	-	-	0.955	0.959	0.959	0.975
C_8_364	40	-	-	0.960	0.941	0.963	0.967
C_8_365	40	-	-	0.969	0.979	0.986	0.986
C_8_366	40	-	-	0.949	0.959	0.958	0.971
C_8_367	40	-	-	0.949	0.938	0.947	0.968
C_8_368	40	-	-	0.952	0.954	0.942	0.967
C_8_369	40	-	-	0.944	0.956	0.941	0.971
C_8_370	40	-	-	0.951	0.958	0.946	0.974
C_8_371	60	-	-	0.931	0.969	0.975	0.983
C_8_372	60	-	-	0.929	0.962	0.957	0.977
C_8_373	60	-	-	0.911	0.949	0.958	0.973
C_8_374	60	-	-	0.932	0.946	0.964	0.975
C_8_375	60	-	-	0.915	0.946	0.948	0.977
C_8_376	60	-	-	0.929	0.954	0.948	0.971
C_8_377	60	-	-	0.931	0.945	0.952	0.967
C_8_378	60	-	-	0.928	0.943	0.957	0.972
C_8_379	60	-	-	0.942	0.941	0.959	0.979
C_8_380	60	-	-	0.939	0.956	0.958	0.973
C_8_381	80	-	-	0.892	0.951	0.957	0.975
C_8_382	80	-	-	0.899	0.956	0.965	0.979

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_8_383	80	-	-	0.900	0.945	0.967	0.975
C_8_384	80	-	-	0.871	0.956	0.951	0.977
C_8_385	80	-	-	0.876	0.945	0.945	0.977
C_8_386	80	-	-	0.840	0.949	0.959	0.977
C_8_387	80	-	-	0.873	0.960	0.966	0.977
C_8_388	80	-	-	0.867	0.968	0.950	0.973
C_8_389	80	-	-	0.881	0.951	0.961	0.972
C_8_390	80	-	-	0.862	0.957	0.957	0.980
C_8_391	100	-	-	0.843	0.951	0.963	0.979
C_8_392	100	-	-	0.856	0.963	0.965	0.979
C_8_393	100	-	-	0.856	0.946	0.953	0.979
C_8_394	100	-	-	0.846	0.955	0.955	0.981
C_8_395	100	-	-	0.831	0.955	0.960	0.977
C_8_396	100	-	-	0.865	0.958	0.954	0.981
C_8_397	100	-	-	0.815	0.944	0.954	0.982
C_8_398	100	-	-	0.833	0.945	0.958	0.978
C_8_399	100	-	-	0.823	0.948	0.954	0.978
C_8_400	100	-	-	0.840	0.951	0.956	0.975
C_9_401	20	-	-	0.826	0.826	0.826	0.826
C_9_402	20	-	-	0.844	0.829	0.844	0.844
C_9_403	20	-	-	0.878	0.877	0.878	0.878
C_9_404	20	-	-	0.791	0.791	0.791	0.791
C_9_405	20	-	-	0.833	0.812	0.833	0.833
C_9_406	20	-	-	0.867	0.866	0.867	0.867
C_9_407	20	-	0.793	0.914	0.907	0.914	0.914
C_9_408	20	-	-	0.851	0.841	0.851	0.851
C_9_409	20	-	-	0.859	0.843	0.859	0.859
C_9_410	20	-	-	0.850	0.850	0.850	0.850
C_9_411	40	-	-	0.869	0.876	0.869	0.884
C_9_412	40	-	-	0.819	0.823	0.823	0.831
C_9_413	40	-	-	0.860	0.866	0.871	0.871
C_9_414	40	-	-	0.827	0.831	0.833	0.834
C_9_415	40	-	-	0.838	0.846	0.846	0.851
C_9_416	40	-	-	0.820	0.844	0.845	0.848
C_9_417	40	-	-	0.871	0.875	0.875	0.878
C_9_418	40	-	-	0.858	0.860	0.854	0.862
C_9_419	40	-	-	0.888	0.888	0.888	0.895
C_9_420	40	-	-	0.806	0.807	0.802	0.813
C_9_421	60	-	-	0.823	0.854	0.850	0.858
C_9_422	60	-	-	0.840	0.872	0.869	0.875
C_9_423	60	-	-	0.831	0.840	0.831	0.845
C_9_424	60	-	-	0.797	0.821	0.822	0.828
C_9_425	60	-	-	0.815	0.847	0.840	0.849
C_9_426	60	-	-	0.846	0.872	0.864	0.874
C_9_427	60	-	-	0.790	0.829	0.824	0.833
C_9_428	60	-	-	0.799	0.822	0.816	0.827
C_9_429	60	-	-	0.842	0.865	0.857	0.870
C_9_430	60	-	-	0.824	0.859	0.853	0.861
C_9_431	80	-	-	0.787	0.842	0.834	0.846
C_9_432	80	-	-	0.794	0.854	0.842	0.854
C_9_433	80	-	-	0.796	0.840	0.836	0.845
C_9_434	80	-	-	0.835	0.861	0.854	0.864
C_9_435	80	-	-	0.834	0.854	0.845	0.856
C_9_436	80	-	-	0.795	0.830	0.823	0.835
C_9_437	80	-	-	0.803	0.848	0.838	0.852
C_9_438	80	-	-	0.810	0.833	0.823	0.838
C_9_439	80	-	-	0.819	0.864	0.847	0.866
C_9_440	80	-	-	0.829	0.864	0.856	0.871
C_9_441	100	-	-	0.826	0.858	0.851	0.864
C_9_442	100	-	-	0.818	0.890	0.876	0.894
C_9_443	100	-	-	0.804	0.856	0.843	0.861
C_9_444	100	-	-	0.804	0.858	0.850	0.861
C_9_445	100	-	-	0.847	0.884	0.878	0.886
C_9_446	100	-	-	0.809	0.850	0.837	0.854
C_9_447	100	-	-	0.788	0.855	0.850	0.859
C_9_448	100	-	-	0.794	0.854	0.841	0.859
C_9_449	100	-	-	0.826	0.890	0.878	0.894
C_9_450	100	-	-	0.836	0.892	0.882	0.895
C_10_451	20	-	0.848	0.896*	0.873	0.896*	0.896*
C_10_452	20	-	0.897	0.965*	0.811	0.961	0.965*
C_10_453	20	-	0.793	0.930*	0.900	0.930*	0.930*
C_10_454	20	-	0.805	0.897*	0.880	0.897*	0.897*
C_10_455	20	-	0.902	0.943*	0.935	0.943*	0.943*
C_10_456	20	-	0.851	0.926	0.899	0.924	0.925
C_10_457	20	-	0.837	0.942	0.920	0.940	0.942
C_10_458	20	-	0.756	0.982*	0.922	0.971	0.956
C_10_459	20	-	0.775	0.953	0.926	0.943	0.953
C_10_460	20	-	0.940	0.971*	0.940	0.966	0.966
C_10_461	40	-	-	0.940	0.913	0.946	0.954
C_10_462	40	-	-	0.963	0.947	0.946	0.964
C_10_463	40	-	-	0.948	0.917	0.941	0.949
C_10_464	40	-	-	0.970	0.939	0.966	0.974
C_10_465	40	-	-	0.972	0.961	0.974	0.980
C_10_466	40	-	-	0.967	0.921	0.963	0.974
C_10_467	40	-	-	0.963	0.941	0.967	0.969
C_10_468	40	-	-	0.942	0.924	0.938	0.950

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
C_10_469	40	-	-	0.931	0.925	0.931	0.936
C_10_470	40	-	-	0.945	0.924	0.944	0.955
C_10_471	60	-	-	0.945	0.932	0.961	0.971
C_10_472	60	-	-	0.931	0.932	0.937	0.956
C_10_473	60	-	-	0.940	0.950	0.962	0.971
C_10_474	60	-	-	0.966	0.940	0.973	0.975
C_10_475	60	-	-	0.943	0.934	0.964	0.972
C_10_476	60	-	-	0.926	0.910	0.936	0.945
C_10_477	60	-	-	0.967	0.944	0.971	0.976
C_10_478	60	-	-	0.950	0.950	0.965	0.972
C_10_479	60	-	-	0.947	0.931	0.958	0.977
C_10_480	60	-	-	0.951	0.945	0.965	0.972
C_10_481	80	-	-	0.901	0.935	0.952	0.973
C_10_482	80	-	-	0.862	0.932	0.965	0.974
C_10_483	80	-	-	0.895	0.936	0.968	0.978
C_10_484	80	-	-	0.895	0.943	0.959	0.974
C_10_485	80	-	-	0.916	0.939	0.951	0.961
C_10_486	80	-	-	0.910	0.932	0.966	0.978
C_10_487	80	-	-	0.930	0.934	0.969	0.979
C_10_488	80	-	-	0.926	0.955	0.969	0.985
C_10_489	80	-	-	0.892	0.930	0.959	0.974
C_10_490	80	-	-	0.908	0.933	0.956	0.965
C_10_491	100	-	-	0.814	0.943	0.968	0.982
C_10_492	100	-	-	0.831	0.949	0.966	0.981
C_10_493	100	-	-	0.836	0.940	0.968	0.976
C_10_494	100	-	-	0.843	0.930	0.957	0.967
C_10_495	100	-	-	0.851	0.938	0.959	0.976
C_10_496	100	-	-	0.820	0.951	0.962	0.981
C_10_497	100	-	-	0.836	0.936	0.961	0.983
C_10_498	100	-	-	0.823	0.943	0.966	0.973
C_10_499	100	-	-	0.835	0.932	0.960	0.983
C_10_500	100	-	-	0.845	0.917	0.956	0.975
cgcut1	16	0.942*	0.942*	0.942*	0.930	0.942*	0.942*
cgcut2	23	0.931	0.858	0.929	0.931	0.931	0.966
cgcut3	62	-	-	0.880	0.968	0.933	0.971
lw161	16	0.991*	0.991*	0.991*	0.991*	0.991*	0.991*
lw163	16	1.000*	1.000*	1.000*	1.000*	1.000*	0.996
lw172	17	1.000*	0.997	1.000*	1.000*	1.000*	1.000*
lw251	25	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
lw252	25	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
lw253	25	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
lw281	28	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
lw283	28	1.000*	0.994	1.000*	1.000*	1.000*	1.000*
lw292	29	1.000*	0.983	1.000*	1.000*	1.000*	1.000*
lw491	49	0.950	0.883	0.992	1.000*	1.000*	1.000*
lw492	49	0.978	0.953	0.994	1.000*	1.000*	1.000*
lw493	49	0.978	0.927	0.991	1.000*	1.000*	1.000*
lw731	73	0.973	0.910	0.987	1.000*	1.000*	1.000*
lw732	73	0.996	0.872	0.964	1.000*	1.000*	1.000*
lw733	73	0.978	0.838	0.974	1.000*	1.000*	1.000*
lw971	97	-	-	0.951	1.000*	1.000*	1.000*
lw972	97	-	-	0.927	1.000*	1.000*	1.000*
lw973	97	-	-	0.932	1.000*	0.998	1.000*
lw1961	196	-	-	0.885	1.000*	0.998	1.000*
lw1963	196	-	-	0.938	1.000*	1.000	1.000*
lw1972	197	-	-	0.886	1.000*	1.000*	1.000*
50	50	-	-	0.947*	0.947*	0.947*	0.947*
100	100	-	-	0.940	0.972	0.993	0.995
500	500	-	-	0.377	1.000*	1.000*	1.000*
1000	1,000	-	-	0.012	1.000*	1.000*	1.000*
5000	5,000	-	-	-	1.000*	1.000*	1.000*
10000	10,000	-	-	-	1.000*	1.000*	1.000*
15000	15,000	-	-	-	1.000*	1.000*	1.000*
gcut01	10	-	0.761	0.804*	0.760	0.804*	0.804*
gcut02	20	-	-	0.908	0.906	0.927	0.929
gcut03	30	-	-	0.858	0.891	0.873	0.903
gcut04	50	-	-	0.866	0.939	0.913	0.952
gcut05	10	-	0.923	0.937*	0.937*	0.937*	0.937*
gcut06	20	-	-	0.916	0.913	0.911	0.916
gcut07	30	-	-	0.876	0.879	0.890	0.896
gcut08	50	-	-	0.864	0.930	0.905	0.954
gcut09	10	-	-	0.908*	0.864	0.908*	0.908*
gcut10	20	-	-	0.868	0.852	0.868	0.868
gcut11	30	-	-	0.901	0.909	0.903	0.927
gcut12	50	-	-	0.850	0.935	0.894	0.940
gcut13	32	-	-	0.971	0.971	0.983	0.985
hc2	8	0.438*	0.438*	0.438*	0.438*	0.438*	0.438*
hc3	7	0.846*	0.846*	0.846*	0.782	0.846*	0.846*
hc5	15	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
hc6	19	0.975*	0.968	0.975*	0.950	0.968	0.935
hc7	21	0.998*	0.993	0.998*	0.998*	0.998*	0.980
hc11	15	0.897*	0.897*	0.897*	0.890	0.897*	0.897*
leung1	10	-	0.856*	0.856*	0.846	0.856*	0.846
leung2	15	-	0.800	0.988	0.988	0.988	0.988
leung3	20	-	0.860	0.931*	0.871	0.931*	0.883
leung4	20	0.975*	0.932	0.975*	0.975*	0.975*	0.964

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
leung5	25	0.983	0.948	0.983	0.983	0.983	0.983
leung6	25	0.883	0.883	0.926	0.926	0.926	0.926
leung7	30	0.852	0.905	0.905	0.892	0.905	0.905
leung8	30	0.923*	0.923*	0.923*	0.923*	0.923*	0.923*
leung9	40	-	0.817	0.968	0.987	0.985	0.989
leung10	50	-	-	0.932	0.991	0.976	0.994
ngcut01	10	0.829*	0.829*	0.829*	0.819	0.829*	0.819
ngcut02	17	0.923*	0.923*	0.923*	0.923*	0.923*	0.923*
ngcut03	21	0.989*	0.961	0.989*	0.989*	0.989*	0.989*
ngcut04	7	0.843*	0.843*	0.843*	0.745	0.843*	0.843*
ngcut05	14	0.981*	0.981*	0.981*	0.981*	0.981*	0.981*
ngcut06	15	0.963*	0.956	0.963*	0.963*	0.963*	0.963*
ngcut07	8	0.438*	0.438*	0.438*	0.438*	0.438*	0.438*
ngcut08	13	0.925*	0.925*	0.925*	0.829	0.925*	0.904
ngcut09	18	0.983*	0.966	0.983*	0.926	0.981	0.973
ngcut10	13	0.692	0.717*	0.717*	0.717*	0.717*	0.717*
ngcut11	15	0.978*	0.978*	0.978*	0.914	0.978*	0.978*
ngcut12	22	0.852	0.880*	0.880*	0.880*	0.880*	0.880*
nice1	25	-	-	0.966	0.963	0.973	0.974
nice2	50	-	-	0.944	0.990	0.978	0.988
nice3	100	-	-	0.846	0.971	0.937	0.968
nice4	200	-	-	0.781	0.981	0.933	0.976
nice5	500	-	-	0.130	0.999	0.965	0.999
nice6	1,000	-	-	0.012	0.999	0.948	1.000*
nice3t	100	-	-	0.860	0.994	0.974	0.992
nice4t	200	-	-	0.879	0.998	0.969	0.996
nice6t	1,000	-	-	0.012	1.000	0.944	1.000*
nice11t	1,000	-	-	0.012	1.000	0.940	1.000*
nice12t	2,000	-	-	0.005	1.000	0.948	1.000*
nice15t	5,000	-	-	-	-	0.928	1.000*
nice21t	1,000	-	-	0.009	1.000	0.952	1.000*
nice22t	2,000	-	-	0.006	1.000	0.936	1.000*
nice25t	5,000	-	-	-	0.993	0.934	1.000*
nice31t	1,000	-	-	0.011	0.999	0.941	1.000*
nice32t	2,000	-	-	0.007	0.999	0.918	1.000*
nice35t	5,000	-	-	-	0.998	0.935	1.000*
nice41t	1,000	-	-	0.011	1.000	0.949	1.000*
nice42t	2,000	-	-	0.005	1.000	0.941	1.000*
nice45t	5,000	-	-	-	-	0.929	1.000*
nice51t	1,000	-	-	0.011	1.000	0.945	1.000*
nice52t	2,000	-	-	0.005	1.000	0.945	1.000*
nice55t	5,000	-	-	-	1.000	0.921	1.000*
nice61t	1,000	-	-	0.008	1.000	0.956	1.000*
nice62t	2,000	-	-	0.006	1.000	0.939	1.000*
nice65t	5,000	-	-	-	1.000	0.941	1.000*
nice71t	1,000	-	-	0.012	1.000	0.948	1.000*
nice72t	2,000	-	-	0.005	1.000	0.927	1.000*
nice75t	5,000	-	-	-	1.000	0.928	1.000*
nice81t	1,000	-	-	0.012	0.999	0.943	1.000*
nice82t	2,000	-	-	0.004	1.000	0.954	1.000*
nice85t	5,000	-	-	-	-	0.922	1.000*
nice91t	1,000	-	-	0.012	1.000	0.948	1.000*
nice92t	2,000	-	-	0.007	1.000	0.927	1.000*
nice95t	5,000	-	-	-	1.000	0.928	1.000*
nice101t	1,000	-	-	0.012	0.999	0.939	1.000*
nice102t	2,000	-	-	0.006	1.000	0.947	1.000*
nice105t	5,000	-	-	-	1.000	0.926	1.000*
N1a	17	-	0.900	0.954*	0.925	0.946	0.954*
N1b	17	-	0.846	0.960*	0.952	0.952	0.953
N1c	17	-	0.918	0.956*	0.933	0.949	0.949
N1d	17	-	0.840	0.961*	0.925	0.954	0.961*
N1e	17	-	0.912	0.965*	0.925	0.948	0.965*
N2a	25	-	0.853	0.973	0.955	0.961	0.971
N2b	25	-	0.852	0.981	0.957	0.971	0.981
N2c	25	-	0.927	0.983	0.952	0.974	0.983
N2d	25	-	0.891	0.977	0.950	0.978	0.975
N2e	25	-	0.878	0.978	0.962	0.978	0.977
N3a	29	-	0.830	0.976	0.968	0.972	0.984
N3b	29	-	0.833	0.983	0.965	0.972	0.982
N3c	29	-	0.913	0.981	0.971	0.974	0.982
N3d	29	-	0.722	0.967	0.962	0.973	0.982
N3e	29	-	0.865	0.983	0.986	0.981	0.986
N4a	49	-	-	0.977	0.989	0.991	0.997
N4b	49	-	-	0.984	0.994	0.987	0.997
N4c	49	-	-	0.962	0.991	0.984	0.998
N4d	49	-	-	0.978	0.992	0.994	0.998
N4e	49	-	-	0.972	0.996	0.992	0.997
N5a	73	-	-	0.971	0.999	0.996	1.000*
N5b	73	-	-	0.980	1.000	0.999	1.000*
N5c	73	-	-	0.927	1.000*	0.995	1.000*
N5d	73	-	-	0.981	1.000*	0.992	1.000*
N5e	73	-	-	0.971	0.993	0.990	0.998
N6a	97	-	-	0.920	1.000*	1.000	1.000*
N6b	97	-	-	0.933	1.000*	0.997	1.000*
N6c	97	-	-	0.917	1.000	0.995	1.000*
N6d	97	-	-	0.918	1.000*	0.992	1.000*

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Table 9 – continued from previous page

Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
N6e	97	-	-	0.938	1.000*	0.996	1.000*
N7a	197	-	-	0.879	1.000*	1.000*	1.000*
N7b	197	-	-	0.935	1.000*	1.000*	1.000*
N7c	197	-	-	0.929	1.000*	1.000*	1.000*
N7d	197	-	-	0.881	1.000*	1.000*	1.000*
N7e	197	-	-	0.937	1.000*	1.000	1.000*
okp1	50	-	0.851	0.997	0.988	0.997	0.974
okp2	30	-	0.901	0.988	0.968	0.988	0.956
okp3	30	-	0.944	0.988	0.953	0.988	0.968
okp4	61	-	0.841	0.989	0.994	0.997	0.982
okp5	97	-	-	0.940	0.979	0.994	0.986
babu	50	-	-	0.917	0.917	0.917	0.917
mopta1	200	-	-	0.936	0.993	0.992	0.997
mopta2	40	-	-	0.963	0.964	0.978	0.983
w83	42	0.974	0.879	0.972	0.972	0.974	0.974
path1	25	-	-	0.998	0.998	0.955	0.998
path2	50	-	-	0.986	0.993	0.993	0.996
path3	100	-	-	0.821	0.900	0.970	0.981
path4	200	-	-	0.789	0.957	0.963	0.986
path5	500	-	-	0.334	0.997	0.970	0.998
path6	1,000	-	-	0.025	0.976	0.915	0.994
path1t	25	-	-	0.993	0.997	0.957	0.997
path4t	200	-	-	0.946	0.996	0.995	0.998
path6t	1,000	-	-	0.075	1.000	0.969	1.000*
path11t	1,000	-	-	0.081	1.000	0.992	1.000*
path12t	2,000	-	-	0.053	1.000*	0.985	1.000*
path15t	5,000	-	-	-	1.000*	0.980	1.000*
path21t	1,000	-	-	0.103	1.000*	0.994	1.000*
path22t	2,000	-	-	0.054	1.000*	0.988	1.000*
path25t	5,000	-	-	-	1.000*	0.973	1.000*
path31t	1,000	-	-	0.075	1.000	0.983	1.000
path32t	2,000	-	-	0.042	1.000	0.968	1.000*
path35t	5,000	-	-	-	1.000	0.957	1.000*
path41t	1,000	-	-	0.050	1.000	0.985	1.000*
path42t	2,000	-	-	0.037	1.000*	0.990	1.000*
path45t	5,000	-	-	-	1.000	0.980	1.000*
path51t	1,000	-	-	0.156	1.000*	0.987	1.000*
path52t	2,000	-	-	0.064	1.000*	0.983	1.000*
path55t	5,000	-	-	-	1.000	0.951	1.000*
path61t	1,000	-	-	0.084	1.000	0.994	1.000*
path62t	2,000	-	-	0.052	1.000*	0.985	1.000*
path65t	5,000	-	-	-	1.000*	0.986	1.000*
path71t	1,000	-	-	0.042	0.983	0.961	0.999
path72t	2,000	-	-	0.056	1.000*	0.987	1.000*
path75t	5,000	-	-	-	1.000*	0.981	1.000*
path81t	1,000	-	-	0.123	1.000	0.991	1.000*
path82t	2,000	-	-	0.078	1.000*	0.974	1.000*
path85t	5,000	-	-	-	1.000	0.950	1.000*
path91t	1,000	-	-	0.120	1.000	0.983	1.000*
path92t	2,000	-	-	0.047	1.000	0.978	1.000*
path95t	5,000	-	-	-	1.000*	0.973	1.000*
path101t	1,000	-	-	0.039	0.998	0.955	1.000
path102t	2,000	-	-	0.035	1.000	0.978	1.000*
path105t	5,000	-	-	-	1.000*	0.960	1.000*
pb1	10	0.389	0.858*	0.858*	0.848	0.858*	0.858*
pb2	10	0.902*	0.902*	0.902*	0.902*	0.902*	0.902*
pb3	15	-	0.872	0.944*	0.901	0.944*	0.906
pb4	15	-	0.844	0.919	0.919	0.919	0.919
pb5	20	0.643	0.873	0.873	0.841	0.873	0.873
T1a	17	-	0.848	0.966*	0.939	0.958	0.964
T1b	17	-	0.917	0.961*	0.925	0.954	0.961*
T1c	17	-	0.855	0.916*	0.879	0.881	0.916*
T1d	17	-	0.899*	0.899*	0.899*	0.899*	0.899*
T1e	17	-	0.836	0.936*	0.919	0.934	0.932
T2a	25	-	0.814	0.983	0.945	0.956	0.967
T2b	25	-	0.792	0.983	0.934	0.975	0.978
T2c	25	-	0.898	0.983	0.947	0.980	0.983
T2d	25	-	0.878	0.978	0.962	0.978	0.977
T2e	25	-	0.852	0.981	0.957	0.971	0.981
T3a	29	-	0.739	0.967	0.962	0.973	0.982
T3b	29	-	0.824	0.981	0.951	0.976	0.985
T3c	29	-	0.859	0.980	0.963	0.973	0.986
T3d	29	-	0.875	0.978	0.967	0.978	0.980
T3e	29	-	0.833	0.983	0.965	0.972	0.982
T4a	49	-	-	0.982	0.990	0.988	0.997
T4b	49	-	-	0.962	0.991	0.984	0.998
T4c	49	-	-	0.978	0.992	0.994	0.998
T4d	49	-	-	0.977	0.989	0.991	0.997
T4e	49	-	-	0.984	0.994	0.987	0.997
T5a	73	-	-	0.927	1.000*	0.995	1.000*
T5b	73	-	-	0.971	0.999	0.997	1.000*
T5c	73	-	-	0.971	0.999	0.996	1.000*
T5d	73	-	-	0.981	1.000*	0.992	1.000*
T5e	73	-	-	0.980	1.000	0.999	1.000*
T6a	97	-	-	0.937	1.000*	0.998	1.000*
T6b	97	-	-	0.917	1.000	0.995	1.000*

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Instance	\mathcal{R}	v_{max}					
		HC95	BKRS09	EP09	LWF	GRASP	IBHP
T6c	97	-	-	0.938	1.000*	0.996	1.000*
T6d	97	-	-	0.933	1.000*	0.997	1.000*
T6e	97	-	-	0.904	1.000*	0.992	1.000*
T7a	199	-	-	0.907	1.000*	1.000	1.000*
T7b	199	-	-	0.913	1.000*	0.998	1.000*
T7c	199	-	-	0.916	1.000*	0.998	1.000*
T7d	199	-	-	0.929	1.000*	1.000*	1.000*
T7e	199	-	-	0.922	1.000*	0.998	1.000*
zdf1	580	-	-	0.611	1.000*	1.000	1.000*
zdf2	660	-	-	0.405	1.000*	1.000*	1.000*
zdf3	740	-	-	0.363	1.000*	1.000*	1.000*
zdf4	820	-	-	0.004	1.000*	1.000*	1.000*
zdf5	900	-	-	0.003	1.000*	1.000*	1.000*
zdf6	1,532	-	-	0.014	0.957	0.997	1.000
zdf7	2,432	-	-	0.014	-	0.999	1.000
zdf8	2,532	-	-	0.013	1.000*	1.000	1.000*
zdf9	5,032	-	-	-	1.000*	1.000*	1.000*
zdf10	5,064	-	-	-	1.000*	1.000	1.000*
zdf11	10,064	-	-	-	1.000*	1.000*	1.000*
zdf12	7,564	-	-	-	1.000*	1.000*	1.000*
zdf13	15,096	-	-	-	1.000*	1.000*	1.000*
zdf14	25,032	-	-	-	1.000*	1.000*	1.000*
zdf15	50,032	-	-	-	-	-	1.000*
zdf16	75,032	-	-	-	-	-	1.000*

B Appendix

In this appendix, we present the results of our computational experiments with respect to the number of assortments generated by the heuristics for 2CSP. In the two left columns of the following table, the name of the instance is shown together with the number of rectangles belonging to the instance. The remaining columns contain the number of generated \underline{v} -good assortments, for $\underline{v} = (1 - \varepsilon)v^*$ with $\varepsilon = 5\%$, and the number of all generated assortment. Here, the number of \underline{v} -good assortments consists only of undominated assortments, i.e., no assortment is contained within any other. If an instance is marked with ^d, then we could not guarantee that all dominated assortments were removed within one week of computation time. The columns for all generated assortments instead can contain dominated assortments since the dominated ones could not be removed within one week of computation time, for most of the instances. Note that the best result for each of the instances is marked in blue, and – indicates that no solution could be obtained using this approach.

Table 10: Number of \underline{v} -good assortments, and the total number of assortments, generated by the heuristics for 2CSP

Instance	\mathcal{R}	\underline{v} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
ALX	20	9	254	508	183	23,118	14,631
ALY	20	2	461	1,566	434	59,504	71,305
ALZ	20	28	2,006	3,085	831	64,692	48,932
ALZZ	20	1	190	323	51	17,888	10,841
ALZZZ	20	1	189	323	51	17,839	10,838
ASX	20	3	83	142	39	12,926	7,812
ASY	20	15	328	1,135	713	51,501	47,499
ASZ	20	24	816	2,745	1,133	65,531	82,530
ASZZ	20	9	112	159	79	15,467	9,934
ASZZZ	20	9	112	159	79	15,452	9,935
AVX	20	14	159	148	182	10,875	4,647
AVY	20	34	446	992	1,057	72,538	51,671

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Instance	\mathcal{R}	\mathcal{G} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
AVZ	20	145	806	6,472	3,297	112,153	161,370
AVZZ	20	9	92	74	288	18,797	7,445
AVZZZ	20	9	92	74	288	18,803	7,445
BLX	30	25	894	1,643	1,134	106,115	106,887
BLY	30	61	2,978	17,216	2,271	189,857	573,126
BLZ	30	232	10,079	46,185	9,880	313,091	1,222,685
BLZZ	30	115	17,247	72,307	4,136	216,984	676,781
BLZZZ	30	11	7,748	40,814	512	439,596	1,150,172
BSX	30	24	1,603	4,940	472	96,588	207,845
BSY	30	5	4,727	23,124	62	213,538	711,085
BSZ	30	103	11,636	39,352	4,916	286,398	1,073,842
BSZZ	30	44	11,714	5,437	2,276	240,902	107,677
BSZZZ	30	49	6,891	27,489	4,014	191,316	753,738
BVX	30	7	318	462	715	49,908	32,156
BVY	30	28	1,576	4,308	4,487	218,112	367,284
BVZ	30	198	1,599	27,569	7,621	211,735	1,444,057
BVZZ	30	563	12,857	65,727	7,585	203,440	1,420,871
BVZZZ	30	79	1,084	12,571	7,549	229,896	1,071,763
CLX	40	90	1,922	11,217	3,201	112,912	485,165
CLY	40	196	7,619	38,853	12,592	329,552	2,124,071
CLZ	40	557	13,770	117,854	15,710	260,061	3,442,902
CLZZ	40	621	34,789	221,467	11,014	304,389	3,733,708
CLZZZ	40	630	13,224	109,889	17,501	302,901	3,714,647
CSX	40	24	599	4,376	449	99,974	924,576
CSY	40	13	2,160	26,291	73	209,687	3,165,462
CSZ	40	1,706	14,005	97,418	13,571	230,239	3,733,327
CSZZ	40	913	53,792	132,403	11,378	356,965	2,581,219
CSZZZ	40	783	5,401	52,183	11,216	250,754	3,706,998
CVX	40	87	1,110	6,926	1,509	159,425	585,548
CVY	40	648	11,211	57,542	10,679	453,315	2,189,559
CVZ	40	127	11,128	70,222	1,304	389,965	3,218,404
CVZZ	40	920	62,973	168,529	12,998	281,457	3,070,213
CVZZZ	40	1,164	25,852	150,657	10,956	170,284	3,451,288
DLX	50	167	1,900	13,770	7,768	184,685	1,289,663
DLY	50	474	7,526	60,613	8,830	287,702	2,697,604
DLZ	50	599	8,934	73,148	8,864	240,221	2,999,576
DLZZ	50	821	152,320	256,034	8,477	358,923	2,961,760
DLZZZ	50	520	17,943	122,554	13,668	210,303	3,052,224
DSX	50	309	5,831	30,401	2,599	592,375	2,009,567
DSY	50	1,829	11,686	51,925	17,427	436,808	3,066,822
DSZ	50	4,762	14,194	89,140	15,811	286,513	3,121,696
DSZZ	50	1,070	78,443	103,255	8,499	358,565	2,765,103
DSZZZ	50	3,465	22,912	98,971	12,884	228,968	3,094,066
DVX	50	144	2,844	15,273	1,653	183,358	1,126,511
DVY	50	817	6,881	35,887	13,570	374,725	2,693,410
DVZ	50	2,276	17,984	89,200	14,029	402,178	3,005,911
DVZZ	50	2,660	75,826	203,956	13,557	298,796	2,919,244
DVZZZ	50	605	65,107	138,136	4,832	146,597	1,424,340
AH1	1,000	41	-	6,784	49	159	9,216
AH2	1,000	38	-	8,179	38	186	9,238
AH3	1,000	21	-	8,541	21	143	8,910
AH4	1,000	32	20	1,545	32	177	8,445
AH5	1,000	31	39	8,933	34	165	9,126
AH6	1,000	31	11	8,693	31	157	9,521
AH7	1,000	44	55	8,983	44	210	9,088
AH8	1,000	18	-	5,247	19	162	7,930
AH9	1,000	11	5	8,365	15	161	8,377
AH10	1,000	36	-	8,878	36	167	9,205
AH11	1,000	19	40	7,183	19	158	9,741
AH12	1,000	67	-	7,380	69	164	7,517
AH13	1,000	46	-	5,563	46	176	7,360
AH14	1,000	16	26	6,287	16	119	7,821
AH15	1,000	61	-	8,165	62	141	9,188
AH16	1,000	27	-	5,617	28	150	6,934
AH17	1,000	32	9	9,392	32	167	9,797
AH18	1,000	-	5	7,655	32	153	8,688
AH19	1,000	37	73	8,407	37	210	8,521
AH20	1,000	29	-	5,976	32	167	7,437
AH21	1,000	19	-	9,274	19	141	9,666
AH22	1,000	-	-	7,872	13	146	9,564
AH23	1,000	-	-	6,843	26	187	9,020
AH24	1,000	30	-	8,533	31	200	8,803
AH25	1,000	22	-	10,299	27	143	10,703
AH26	1,000	12	-	9,050	17	163	9,226
AH27	1,000	-	-	5,722	21	146	7,776
AH28	1,000	-	-	4,799	19	145	7,578
AH29	1,000	30	26	10,546	30	162	10,752
AH30	1,000	19	18	9,778	22	163	9,861
AH31	1,000	-	-	4,430	91	180	9,779
AH32	1,000	25	101	7,075	37	152	8,897
AH33	1,000	19	39	9,231	43	200	9,284
AH34	1,000	38	2	9,969	52	137	10,056
AH35	1,000	-	25	8,721	34	127	10,283
AH36	1,000	13	86	9,142	22	193	9,296
AH37	1,000	48	68	6,193	51	177	8,471
AH38	1,000	-	109	10,239	22	181	10,334

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Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
AH39	1,000	-	-	7,084	39	205	9,355
AH40	1,000	33	24	8,775	35	120	9,102
AH41	1,000	-	-	506	59	195	9,021
AH42	1,000	26	63	8,491	28	143	8,772
AH43	1,000	45	1	9,727	48	170	10,523
AH44	1,000	-	31	7,512	71	152	9,641
AH45	1,000	40	65	9,369	42	157	9,753
AH46	1,000	78	-	8,064	89	189	8,886
AH47	1,000	78	367	10,227	81	542	10,551
AH48	1,000	38	104	10,004	41	173	10,122
AH49	1,000	37	-	7,565	39	121	7,733
AH50	1,000	21	5	7,923	27	135	8,997
AH51	1,000	43	111	7,852	47	167	8,389
AH52	1,000	19	111	3,396	46	184	6,968
AH53	1,000	20	31	4,023	23	90	10,451
AH54	1,000	30	15	8,356	36	107	9,637
AH55	1,000	-	29	7,144	24	103	8,538
AH56	1,000	29	12	7,985	31	133	8,581
AH57	1,000	-	-	6,561	97	180	9,169
AH58	1,000	24	137	9,218	32	227	9,622
AH59	1,000	14	-	9,423	18	166	9,738
AH60	1,000	-	-	5,815	76	123	7,684
AH61	1,000	26	28	8,051	26	185	8,051
AH62	1,000	48	23	8,863	48	221	8,865
AH63	1,000	63	-	8,709	63	159	8,721
AH64	1,000	24	24	8,692	24	176	8,696
AH65	1,000	38	1	7,655	38	177	7,655
AH66	1,000	22	17	7,858	22	177	7,859
AH67	1,000	34	16	8,195	34	187	8,199
AH68	1,000	43	39	8,382	43	182	8,384
AH69	1,000	56	93	8,112	56	221	8,115
AH70	1,000	64	15	8,124	64	167	8,128
AH71	1,000	51	18	8,084	51	187	8,086
AH72	1,000	58	21	7,808	58	177	7,809
AH73	1,000	28	9	8,135	28	200	8,136
AH74	1,000	71	16	7,944	71	199	7,947
AH75	1,000	36	14	7,903	36	171	7,904
AH76	1,000	48	11	8,065	48	166	8,068
AH77	1,000	43	32	8,571	43	178	8,573
AH78	1,000	25	18	7,957	25	187	7,957
AH79	1,000	35	2	7,553	35	170	7,559
AH80	1,000	31	22	7,640	31	176	7,645
AH81	1,000	32	17	8,568	32	169	8,573
AH82	1,000	43	17	7,715	43	182	7,715
AH83	1,000	14	19	8,755	14	145	8,759
AH84	1,000	41	245	8,196	41	380	8,196
AH85	1,000	33	22	8,494	33	148	8,496
AH86	1,000	28	24	7,349	28	204	7,354
AH87	1,000	37	-	8,501	37	181	8,502
AH88	1,000	41	-	9,137	41	151	9,139
AH89	1,000	30	45	8,183	30	195	8,186
AH90	1,000	41	40	8,436	41	184	8,436
AH91	1,000	61	94	8,442	61	204	8,460
AH92	1,000	43	168	7,563	43	238	7,618
AH93	1,000	98	40	9,151	100	136	9,303
AH94	1,000	49	66	8,247	49	159	8,254
AH95	1,000	35	97	8,842	35	205	8,864
AH96	1,000	58	87	8,683	58	202	8,696
AH97	1,000	46	55	8,486	46	179	8,495
AH98	1,000	58	108	8,927	58	229	8,964
AH99	1,000	66	62	8,902	66	213	9,154
AH100	1,000	60	100	8,165	60	205	8,165
AH101	1,000	23	148	8,598	23	217	8,603
AH102	1,000	27	68	7,633	29	173	7,647
AH103	1,000	25	55	9,542	25	159	9,562
AH104	1,000	22	16	8,207	22	184	8,209
AH105	1,000	29	116	9,240	29	172	9,246
AH106	1,000	95	736	7,450	95	817	7,459
AH107	1,000	52	135	9,346	52	224	9,349
AH108	1,000	23	147	7,893	23	232	7,945
AH109	1,000	67	2	8,891	67	192	8,892
AH110	1,000	46	825	8,860	46	881	8,861
AH111	1,000	17	27	8,808	18	138	8,812
AH112	1,000	102	343	9,621	102	488	9,631
AH113	1,000	19	42	8,315	19	161	8,350
AH114	1,000	23	78	7,283	24	136	7,312
AH115	1,000	39	75	8,744	40	148	8,749
AH116	1,000	65	142	9,064	65	204	9,072
AH117	1,000	36	83	8,907	36	176	8,924
AH118	1,000	60	138	7,902	60	197	7,918
AH119	1,000	58	70	8,796	58	204	8,809
AH120	1,000	39	47	7,859	39	154	7,864
AH121	1,000	11	-	7,026	14	109	7,035
AH122	1,000	8	-	7,130	9	132	7,137
AH123	1,000	13	-	6,270	13	136	6,270
AH124	1,000	14	-	6,536	15	133	6,542

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Instance	\mathcal{R}	\mathcal{P} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
AH125	1,000	10	-	6,240	10	125	6,240
AH126	1,000	15	-	6,548	15	144	6,559
AH127	1,000	7	-	6,959	8	111	6,960
AH128	1,000	8	-	6,998	8	104	6,998
AH129	1,000	14	12	7,464	15	119	7,464
AH130	1,000	10	-	7,115	12	129	7,118
AH131	1,000	10	-	7,147	11	135	7,149
AH132	1,000	12	-	7,192	15	107	7,205
AH133	1,000	18	-	6,976	18	109	6,991
AH134	1,000	5	-	6,506	6	110	6,509
AH135	1,000	9	-	7,065	12	135	7,065
AH136	1,000	16	-	7,309	17	148	7,314
AH137	1,000	12	-	6,795	14	118	6,795
AH138	1,000	18	-	7,155	19	111	7,164
AH139	1,000	11	-	6,955	11	111	6,957
AH140	1,000	9	-	6,734	9	118	6,734
AH141	1,000	14	-	7,108	14	129	7,118
AH142	1,000	13	-	7,147	13	128	7,163
AH143	1,000	1	-	6,845	10	120	6,854
AH144	1,000	13	-	7,667	14	113	7,723
AH145	1,000	13	-	7,147	13	119	7,171
AH146	1,000	12	-	7,006	14	106	7,024
AH147	1,000	12	1	6,672	12	106	6,672
AH148	1,000	14	-	6,893	15	135	6,895
AH149	1,000	8	-	6,809	12	107	6,874
AH150	1,000	16	-	6,594	16	111	6,595
AH151	1,000	2	71	9,421	7	105	9,422
AH152	1,000	4	79	9,142	11	101	9,147
AH153	1,000	5	65	9,731	18	97	9,735
AH154	1,000	-	74	9,853	14	94	9,857
AH155	1,000	-	51	8,904	13	90	8,905
AH156	1,000	5	57	9,373	9	87	9,374
AH157	1,000	3	72	10,106	13	93	10,108
AH158	1,000	8	79	9,605	11	116	9,606
AH159	1,000	9	67	10,199	17	98	10,202
AH160	1,000	8	70	9,805	18	96	9,805
AH161	1,000	4	56	9,181	14	90	9,183
AH162	1,000	14	52	9,531	20	96	9,532
AH163	1,000	-	74	9,458	12	102	9,462
AH164	1,000	2	49	8,821	9	99	8,822
AH165	1,000	5	63	9,134	20	98	9,137
AH166	1,000	2	63	9,282	13	95	9,282
AH167	1,000	7	70	9,979	13	109	9,984
AH168	1,000	5	48	9,992	13	89	9,998
AH169	1,000	9	39	10,203	13	74	10,204
AH170	1,000	5	91	9,068	16	114	9,073
AH171	1,000	7	55	9,299	18	96	9,304
AH172	1,000	3	76	9,460	10	108	9,463
AH173	1,000	5	63	9,882	13	100	9,882
AH174	1,000	3	50	9,511	15	84	9,512
AH175	1,000	8	51	9,164	16	95	9,168
AH176	1,000	6	66	9,913	15	101	9,915
AH177	1,000	7	57	9,067	15	96	9,068
AH178	1,000	3	67	9,742	15	95	9,746
AH179	1,000	4	78	9,645	12	103	9,646
AH180	1,000	-	65	9,755	18	90	9,757
AH181	1,000	26	6	7,641	26	149	7,642
AH182	1,000	22	-	7,529	22	139	7,533
AH183	1,000	41	11	7,731	41	157	7,731
AH184	1,000	24	19	7,575	24	149	7,576
AH185	1,000	28	11	7,718	28	159	7,718
AH186	1,000	22	7	7,698	22	148	7,699
AH187	1,000	23	18	7,534	23	161	7,534
AH188	1,000	23	9	7,807	23	139	7,807
AH189	1,000	33	29	7,625	33	147	7,625
AH190	1,000	27	9	7,641	27	143	7,641
AH191	1,000	18	5	7,322	18	164	7,322
AH192	1,000	33	18	7,786	33	153	7,787
AH193	1,000	26	5	7,491	26	166	7,492
AH194	1,000	27	5	7,540	27	138	7,541
AH195	1,000	24	22	7,579	24	162	7,580
AH196	1,000	25	9	7,637	25	163	7,639
AH197	1,000	25	16	7,647	25	156	7,649
AH198	1,000	32	4	7,548	32	157	7,549
AH199	1,000	23	17	7,585	23	151	7,586
AH200	1,000	29	19	7,933	29	160	7,933
AH201	1,000	23	16	7,651	23	151	7,651
AH202	1,000	28	20	7,652	28	154	7,652
AH203	1,000	21	23	7,624	21	153	7,624
AH204	1,000	23	7	7,750	23	163	7,750
AH205	1,000	18	22	7,637	18	157	7,638
AH206	1,000	23	11	7,717	23	159	7,718
AH207	1,000	37	14	7,095	37	159	7,095
AH208	1,000	30	18	7,326	30	162	7,326
AH209	1,000	33	12	7,687	33	172	7,687
AH210	1,000	26	15	7,601	26	161	7,601

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Instance	\mathcal{R}	\mathcal{V} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
AH211	1,000	13	82	6,820	13	139	6,841
AH212	1,000	13	97	7,007	13	144	7,009
AH213	1,000	19	99	6,451	19	137	6,453
AH214	1,000	11	56	6,956	11	110	6,969
AH215	1,000	18	85	7,129	18	135	7,132
AH216	1,000	40	82	6,936	40	137	6,945
AH217	1,000	20	95	7,033	20	134	7,040
AH218	1,000	10	79	6,799	10	127	6,802
AH219	1,000	19	59	6,946	19	130	6,947
AH220	1,000	12	85	7,209	12	126	7,212
AH221	1,000	25	124	7,176	25	167	7,188
AH222	1,000	18	73	6,877	18	119	6,881
AH223	1,000	36	123	6,702	36	169	6,708
AH224	1,000	34	102	7,029	34	151	7,035
AH225	1,000	16	107	7,029	16	145	7,032
AH226	1,000	38	66	7,396	38	118	7,400
AH227	1,000	34	102	7,093	34	143	7,102
AH228	1,000	9	75	6,645	9	116	6,651
AH229	1,000	16	84	6,721	16	128	6,730
AH230	1,000	8	90	6,929	8	143	6,935
AH231	1,000	19	72	7,007	19	117	7,009
AH232	1,000	32	85	7,486	32	138	7,486
AH233	1,000	50	81	7,374	50	130	7,379
AH234	1,000	6	64	6,824	6	123	6,831
AH235	1,000	25	76	7,458	25	122	7,462
AH236	1,000	20	82	7,348	20	128	7,357
AH237	1,000	17	74	7,002	17	132	7,016
AH238	1,000	24	110	7,445	24	146	7,451
AH239	1,000	22	88	6,958	22	135	6,961
AH240	1,000	37	95	6,765	37	151	6,766
AH241	1,000	3	-	6,113	4	103	6,114
AH242	1,000	6	-	6,220	8	102	6,220
AH243	1,000	4	-	6,017	4	110	6,017
AH244	1,000	7	-	6,081	7	110	6,082
AH245	1,000	5	-	6,197	5	109	6,197
AH246	1,000	4	-	6,062	4	119	6,062
AH247	1,000	4	-	5,952	4	112	5,954
AH248	1,000	4	-	6,191	5	118	6,191
AH249	1,000	5	-	5,803	5	119	5,804
AH250	1,000	6	-	6,259	6	113	6,259
AH251	1,000	6	-	6,153	6	102	6,153
AH252	1,000	4	-	5,862	5	121	5,862
AH253	1,000	2	-	6,058	4	96	6,058
AH254	1,000	6	-	6,063	7	113	6,063
AH255	1,000	-	-	6,203	3	111	6,203
AH256	1,000	5	-	6,215	6	107	6,215
AH257	1,000	7	-	6,170	8	104	6,171
AH258	1,000	6	-	6,062	6	125	6,063
AH259	1,000	6	-	6,136	8	113	6,136
AH260	1,000	4	-	6,292	5	121	6,292
AH261	1,000	4	-	6,241	4	124	6,242
AH262	1,000	3	-	5,916	4	119	5,916
AH263	1,000	4	-	6,108	5	127	6,108
AH264	1,000	5	-	5,909	6	110	5,909
AH265	1,000	4	-	6,226	4	127	6,226
AH266	1,000	5	-	6,070	5	131	6,070
AH267	1,000	4	-	5,984	4	116	5,984
AH268	1,000	5	-	6,166	6	130	6,166
AH269	1,000	3	-	6,146	4	128	6,146
AH270	1,000	2	-	5,978	3	114	5,978
AH271	1,000	-	55	6,431	5	84	6,433
AH272	1,000	2	59	6,640	6	87	6,643
AH273	1,000	3	44	6,888	6	76	6,889
AH274	1,000	4	66	6,687	6	92	6,688
AH275	1,000	-	60	6,661	5	84	6,661
AH276	1,000	5	60	6,667	6	96	6,669
AH277	1,000	-	72	6,665	4	94	6,665
AH278	1,000	3	64	6,763	3	94	6,766
AH279	1,000	-	73	6,715	4	96	6,718
AH280	1,000	4	97	6,826	5	120	6,830
AH281	1,000	5	63	6,517	6	88	6,519
AH282	1,000	-	64	6,697	6	90	6,699
AH283	1,000	-	74	6,658	4	102	6,663
AH284	1,000	1	77	6,687	3	103	6,688
AH285	1,000	2	77	6,677	4	96	6,677
AH286	1,000	1	80	6,726	5	99	6,728
AH287	1,000	1	63	6,508	3	91	6,510
AH288	1,000	2	63	6,649	4	86	6,652
AH289	1,000	2	60	6,688	3	87	6,688
AH290	1,000	4	63	6,539	7	93	6,542
AH291	1,000	7	54	6,515	8	77	6,515
AH292	1,000	3	59	6,744	6	87	6,746
AH293	1,000	2	112	6,510	4	133	6,510
AH294	1,000	4	76	6,764	5	95	6,768
AH295	1,000	1	71	6,338	5	97	6,342
AH296	1,000	-	86	6,483	6	110	6,486

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Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
AH297	1,000	1	79	6,382	4	103	6,385
AH298	1,000	2	98	6,630	5	120	6,632
AH299	1,000	2	109	6,466	4	128	6,468
AH300	1,000	1	57	6,604	6	83	6,604
AH301	1,000	22	13	6,887	22	155	6,887
AH302	1,000	36	7	6,972	36	173	6,972
AH303	1,000	37	10	6,979	37	170	6,979
AH304	1,000	41	15	6,791	41	167	6,791
AH305	1,000	23	13	6,860	23	170	6,860
AH306	1,000	46	9	6,917	46	161	6,917
AH307	1,000	29	8	6,932	29	153	6,932
AH308	1,000	40	11	6,977	40	158	6,977
AH309	1,000	51	24	6,852	51	167	6,852
AH310	1,000	31	15	6,828	31	150	6,828
AH311	1,000	35	12	6,868	35	168	6,868
AH312	1,000	32	19	6,938	32	159	6,938
AH313	1,000	29	8	6,894	29	162	6,895
AH314	1,000	30	6	6,888	30	161	6,888
AH315	1,000	50	16	6,935	50	180	6,935
AH316	1,000	25	11	6,938	25	178	6,938
AH317	1,000	43	22	6,892	43	166	6,893
AH318	1,000	34	11	6,811	34	163	6,811
AH319	1,000	38	5	7,003	38	172	7,003
AH320	1,000	44	24	6,986	44	166	6,987
AH321	1,000	34	13	6,954	34	170	6,954
AH322	1,000	29	16	6,864	29	155	6,864
AH323	1,000	31	25	6,918	31	162	6,919
AH324	1,000	35	9	6,922	35	179	6,922
AH325	1,000	42	12	6,908	42	162	6,908
AH326	1,000	47	13	6,878	47	163	6,878
AH327	1,000	45	9	6,838	45	167	6,838
AH328	1,000	37	17	6,840	37	160	6,840
AH329	1,000	39	13	6,796	39	173	6,796
AH330	1,000	26	7	6,933	26	150	6,934
AH331	1,000	50	134	5,802	50	161	5,802
AH332	1,000	57	86	5,518	57	128	5,518
AH333	1,000	42	116	5,770	42	150	5,770
AH334	1,000	33	118	5,597	33	154	5,597
AH335	1,000	60	115	5,682	60	149	5,682
AH336	1,000	47	114	5,489	47	151	5,489
AH337	1,000	40	135	5,700	40	166	5,701
AH338	1,000	62	120	5,581	62	156	5,581
AH339	1,000	31	83	5,737	31	121	5,738
AH340	1,000	58	118	5,787	58	150	5,787
AH341	1,000	56	92	5,756	56	135	5,756
AH342	1,000	32	109	5,720	32	148	5,720
AH343	1,000	37	124	5,806	37	161	5,807
AH344	1,000	47	112	5,787	47	151	5,788
AH345	1,000	52	89	5,851	52	131	5,851
AH346	1,000	52	107	5,722	52	144	5,722
AH347	1,000	39	117	5,675	39	153	5,675
AH348	1,000	50	129	5,636	50	158	5,637
AH349	1,000	51	94	5,742	51	140	5,743
AH350	1,000	58	127	5,707	58	153	5,707
AH351	1,000	36	100	5,462	36	145	5,463
AH352	1,000	37	117	5,815	37	147	5,815
AH353	1,000	40	105	5,670	40	138	5,670
AH354	1,000	71	113	5,888	71	147	5,888
AH355	1,000	57	144	5,658	57	172	5,658
AH356	1,000	48	96	5,739	48	128	5,739
AH357	1,000	40	113	5,799	40	148	5,801
AH358	1,000	48	137	5,551	48	161	5,552
AH359	1,000	49	103	5,661	49	141	5,661
AH360	1,000	48	107	5,714	48	147	5,714
ami33	33	1,706	744	51,364	17,125	469,298	1,915,604
ami49	49	4,106	10,301	169,050	10,233	190,401	2,897,967
pcb146	146	34	15,848	51,944	199	16,606	99,510
pcb500	500	195	3,536	28,507	195	3,722	28,520
rp100	100	7,721	65,776	527,753	9,538	165,874	640,893
rp200	200	609	19,239	181,164	886	26,309	185,054
beng01	20	27	462	67	107	20,012	465
beng02	40	1,264	213,787	638,034	1,349	691,848	732,418
beng03	60	10,104	221,372	d ₁ 533,075	10,280	413,412	1,534,151
beng04	80	12,981	371,469	1,012,218	13,086	541,620	1,012,423
beng05	100	276	389,926	707,382	276	504,716	707,440
beng06	40	71	125,772	239,439	73	596,287	331,515
beng07	80	20,247	506,368	946,968	20,283	656,048	949,085
beng08	120	738	294,434	516,591	738	327,349	516,643
beng09	160	1,696	190,874	315,252	1,696	196,833	315,252
beng10	200	2,106	91,403	208,354	2,106	103,518	208,354
n1	10	1	1	1	1	133	4
n2	20	5	194	747	113	38,721	12,266
n3	30	5,295	30,690	322,168	6,923	429,863	689,714
n4	40	85	53,228	96,419	930	360,788	151,166
n5	50	3,276	157,529	608,017	9,514	470,473	1,030,953
n6	60	1	5,013	1	911	112,346	43,309

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Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
n7	70	808	268	10,074	2,002	79,835	60,651
n8	80	27,410	302,814	803,649	27,504	347,518	875,021
n9	100	1	501	186	818	40,417	14,634
n10	200	32	29,003	44,791	251	32,557	53,320
n11	300	1	2,164	1	196	5,204	19,445
n12	500	1,870	33,630	25,616	1,870	34,237	25,620
n13	3,152	77	9	794	77	11	794
C_1_1	20	-	20	14	9	25,475	167
C_1_2	20	1	4	2	11	7,815	66
C_1_3	20	4	146	77	20	19,332	420
C_1_4	20	27	84	50	82	35,520	715
C_1_5	20	8	9	1	64	16,072	154
C_1_6	20	8	16	8	79	17,470	256
C_1_7	20	4	38	19	30	17,579	119
C_1_8	20	8	10	9	127	20,269	209
C_1_9	20	4	24	17	19	21,751	232
C_1_10	20	4	5	5	34	10,731	70
C_1_11	40	38	46	37	2,012	139,582	1,874
C_1_12	40	6	157	19	70	95,534	20,157
C_1_13	40	26	212	33	818	118,080	6,555
C_1_14	40	4	341	114	22	150,563	4,707
C_1_15	40	54	237	23	1,317	98,041	6,782
C_1_16	40	267	4,592	4,029	2,542	160,632	83,936
C_1_17	40	12	68	12	1,384	90,603	4,343
C_1_18	40	37	8,164	3,645	122	134,898	31,020
C_1_19	40	24	153	26	2,764	130,881	6,162
C_1_20	40	1	32	1	1,013	137,800	3,452
C_1_21	60	2,146	23,096	36,785	8,442	167,549	103,613
C_1_22	60	254	6,427	430	2,879	196,676	20,255
C_1_23	60	398	6,284	12,936	1,096	223,674	64,176
C_1_24	60	6	12,248	242,204	36	122,647	553,873
C_1_25	60	1	335	1	3,335	173,296	12,635
C_1_26	60	54	1,833	68	1,567	163,028	16,416
C_1_27	60	46	2,149	163	540	156,098	11,403
C_1_28	60	239	3,492	1,392	5,961	156,542	53,288
C_1_29	60	53	955	47	7,639	199,087	13,529
C_1_30	60	441	11,080	17,741	3,403	114,639	251,368
C_1_31	80	100	6,784	16,134	1,165	108,821	132,180
C_1_32	80	542	14,468	36,315	2,714	147,233	80,368
C_1_33	80	283	12,657	d _{523,968}	923	114,481	664,192
C_1_34	80	389	15,034	31,131	2,203	100,301	168,660
C_1_35	80	231	6,787	6,689	723	110,012	44,495
C_1_36	80	600	8,587	14,186	6,521	130,984	83,646
C_1_37	80	282	8,813	87,398	1,117	106,893	233,821
C_1_38	80	76	9,022	82,838	718	112,433	181,808
C_1_39	80	224	8,208	451	1,525	134,006	19,047
C_1_40	80	220	4,480	524	5,470	167,067	15,432
C_1_41	100	1	2,172	1	935	105,824	6,893
C_1_42	100	64	2,838	667	1,948	75,462	34,614
C_1_43	100	1	1,330	1	1,515	97,569	10,126
C_1_44	100	78	5,733	542	745	89,321	20,990
C_1_45	100	523	1,326	9,850	2,637	78,990	53,580
C_1_46	100	945	14,667	52,653	2,580	97,206	181,314
C_1_47	100	52	13,886	307	398	91,786	10,715
C_1_48	100	135	6,453	761	682	100,052	18,963
C_1_49	100	73	1,206	77	947	90,077	13,467
C_1_50	100	138	10,885	13,150	460	92,250	45,032
C_2_51	20	13	1,321	207	20	40,440	758
C_2_52	20	257	571	202	638	17,180	982
C_2_53	20	94	534	137	348	32,757	1,195
C_2_54	20	410	1,104	214	1,593	50,021	1,052
C_2_55	20	35	187	42	215	17,320	847
C_2_56	20	8	682	60	17	32,875	121
C_2_57	20	81	948	214	119	24,839	574
C_2_58	20	7	390	169	19	11,848	635
C_2_59	20	19	944	88	32	29,056	209
C_2_60	20	57	813	124	264	22,494	390
C_2_61	40	51	321,024	430,929	51	730,383	448,559
C_2_62	40	2,650	220,673	592,644	2,689	726,678	620,443
C_2_63	40	372	158,705	551,150	406	671,255	603,122
C_2_64	40	43	140,654	588,779	45	514,340	619,599
C_2_65	40	16,403	246,365	553,344	17,025	846,175	606,322
C_2_66	40	13,214	212,964	480,516	13,968	601,545	519,233
C_2_67	40	6,342	414,099	371,939	6,451	953,549	396,670
C_2_68	40	45	274,967	832,713	46	770,736	967,163
C_2_69	40	9,359	252,263	315,147	9,456	772,421	323,606
C_2_70	40	52	276,344	357,621	55	721,792	405,581
C_2_71	60	109	321,017	d _{1,519,067}	109	528,326	1,519,801
C_2_72	60	1,494	224,123	d _{1,435,977}	1,514	400,039	1,436,344
C_2_73	60	8,852	252,796	d _{1,421,266}	9,102	518,716	1,421,663
C_2_74	60	112	235,467	d _{1,441,838}	115	454,886	1,444,245
C_2_75	60	3,344	425,785	d _{1,225,928}	3,354	664,140	1,229,550
C_2_76	60	120	206,049	d _{1,489,836}	121	459,585	1,490,118
C_2_77	60	77	285,491	1,085,982	77	459,722	1,086,298

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Table 10 – continued from previous page

Instance	\mathcal{R}	\mathcal{V} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
C_2_78	60	69	544,270	^d 1,228,330	69	695,628	1,229,646
C_2_79	60	136	261,793	^d 1,447,226	136	465,193	1,447,665
C_2_80	60	3,618	311,303	^d 1,573,693	3,722	638,789	1,574,174
C_2_81	80	8,277	367,137	^d 1,060,386	8,305	534,541	1,060,428
C_2_82	80	157	313,230	1,017,118	157	408,321	1,017,145
C_2_83	80	238	319,174	1,006,006	238	467,139	1,006,106
C_2_84	80	195	744,085	1,066,946	195	885,146	1,067,112
C_2_85	80	25,562	393,235	1,055,676	25,838	568,631	1,055,818
C_2_86	80	235	597,040	^d 1,001,754	235	683,726	1,001,888
C_2_87	80	3,692	394,495	^d 1,056,958	3,743	580,272	1,057,055
C_2_88	80	52	348,328	1,027,627	53	450,944	1,027,702
C_2_89	80	18,946	386,707	1,064,307	19,238	524,949	1,064,435
C_2_90	80	189	285,543	1,020,250	189	379,430	1,020,360
C_2_91	100	199	540,520	693,676	201	586,660	693,686
C_2_92	100	154	302,220	708,175	155	431,727	708,189
C_2_93	100	400	616,821	726,988	400	667,091	727,012
C_2_94	100	295	502,656	718,358	295	591,024	718,371
C_2_95	100	191	539,827	724,655	191	607,065	724,663
C_2_96	100	204	443,422	711,061	205	557,848	711,690
C_2_97	100	328	301,518	718,862	328	375,879	718,862
C_2_98	100	11,910	345,994	702,139	11,918	400,544	702,193
C_2_99	100	202	248,581	697,169	202	321,686	697,178
C_2_100	100	186	154,163	718,848	186	241,859	718,862
C_3_101	20	4	77	99	211	37,383	6,623
C_3_102	20	2	26	19	144	14,038	1,146
C_3_103	20	6	78	282	630	34,600	10,399
C_3_104	20	1	215	184	40	44,497	5,809
C_3_105	20	5	152	114	23	23,708	1,998
C_3_106	20	17	100	100	825	33,314	3,049
C_3_107	20	-	53	41	241	20,452	1,176
C_3_108	20	10	92	75	372	24,984	2,512
C_3_109	20	15	64	85	527	34,760	8,136
C_3_110	20	1	91	73	118	22,425	1,533
C_3_111	40	41	236	12,947	2,400	262,207	314,326
C_3_112	40	1	757	24,103	238	172,589	765,151
C_3_113	40	91	1,628	28,676	6,726	161,936	490,445
C_3_114	40	12	997	42,404	3,597	229,807	825,230
C_3_115	40	8	2,966	29,805	420	131,447	567,229
C_3_116	40	326	3,263	30,675	6,976	157,568	1,003,754
C_3_117	40	14	1,629	13,312	8,279	126,902	390,223
C_3_118	40	40	5,669	118,743	4,105	175,857	1,533,883
C_3_119	40	60	240	47,811	3,429	212,621	890,391
C_3_120	40	91	293	20,847	2,802	187,308	486,489
C_3_121	60	173	9,321	440,701	3,542	180,795	1,576,132
C_3_122	60	60	27,682	274,527	4,559	246,632	1,418,213
C_3_123	60	414	5,620	388,570	4,917	222,552	1,234,762
C_3_124	60	323	1,244	302,862	5,104	128,029	1,609,676
C_3_125	60	53	31,317	117,530	4,759	221,646	1,170,317
C_3_126	60	85	185	336,998	5,283	141,537	1,463,694
C_3_127	60	104	7,453	144,914	4,591	178,707	985,649
C_3_128	60	93	7,163	137,448	5,742	155,098	1,584,445
C_3_129	60	247	30,367	245,933	7,654	232,647	1,347,242
C_3_130	60	194	5,565	330,068	3,503	113,233	1,707,906
C_3_131	80	303	152	303,921	4,115	100,275	1,018,593
C_3_132	80	102	2,835	556,645	2,643	120,498	1,047,975
C_3_133	80	160	1,534	357,202	3,401	111,675	1,041,316
C_3_134	80	282	1,006	405,669	3,689	114,878	986,539
C_3_135	80	338	252	345,634	4,356	94,090	984,274
C_3_136	80	271	1,389	203,928	4,180	97,624	1,076,193
C_3_137	80	546	1,316	402,018	3,572	107,793	1,013,576
C_3_138	80	254	4,931	410,661	3,738	105,184	1,038,259
C_3_139	80	410	2,110	421,891	3,413	111,565	1,002,974
C_3_140	80	115	34,351	348,358	4,006	122,440	1,014,336
C_3_141	100	315	8,547	415,022	2,532	70,493	659,187
C_3_142	100	184	5,275	329,028	2,303	65,296	701,085
C_3_143	100	13	3,121	330,965	2,872	68,971	687,650
C_3_144	100	197	1,729	382,965	2,535	63,244	649,123
C_3_145	100	195	1,054	335,610	1,825	66,748	698,336
C_3_146	100	405	2,941	393,150	1,931	66,022	673,600
C_3_147	100	147	11,838	429,082	2,905	83,029	703,217
C_3_148	100	27	9,663	334,355	2,266	64,575	693,204
C_3_149	100	95	9,968	373,866	2,709	65,255	680,683
C_3_150	100	183	606	316,934	2,588	66,118	671,435
C_4_151	20	9	53	39	277	14,064	1,842
C_4_152	20	1	48	113	51	12,444	4,022
C_4_153	20	1	47	25	17	18,530	2,833
C_4_154	20	19	78	167	398	26,450	13,488
C_4_155	20	40	84	101	785	20,172	6,975
C_4_156	20	10	54	14	215	22,115	2,136
C_4_157	20	5	51	46	84	12,007	2,166
C_4_158	20	10	107	127	367	13,189	3,589
C_4_159	20	14	106	72	268	20,860	2,856
C_4_160	20	-	12	9	121	5,446	803
C_4_161	40	72	20,429	99,828	110	251,836	421,474
C_4_162	40	443	38,760	124,807	1,123	354,274	611,250

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Table 10 – continued from previous page

Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
C_4_163	40	4,118	19,132	261,062	12,802	301,939	1,053,017
C_4_164	40	2,819	21,093	347,556	12,244	268,005	1,132,693
C_4_165	40	3,216	22,979	256,468	13,425	316,450	1,122,072
C_4_166	40	1,700	31,961	147,672	3,972	257,133	658,449
C_4_167	40	5,457	58,108	307,766	11,685	342,024	1,175,004
C_4_168	40	1,360	36,583	90,893	7,050	353,920	575,259
C_4_169	40	3,747	13,797	110,127	10,423	338,897	623,865
C_4_170	40	1,936	10,801	92,862	13,058	305,625	813,063
C_4_171	60	13,483	86,034	^d 931,948	24,101	298,013	1,241,431
C_4_172	60	19,470	66,934	^d 1,059,303	26,451	234,733	1,161,162
C_4_173	60	2,364	70,557	688,933	3,557	305,288	785,534
C_4_174	60	14,384	52,088	775,760	21,448	234,347	1,062,987
C_4_175	60	4,564	133,943	675,822	5,367	372,089	897,907
C_4_176	60	16,373	59,241	757,359	23,283	259,940	986,788
C_4_177	60	10,870	110,799	688,162	12,843	278,137	770,309
C_4_178	60	13,806	118,832	691,569	17,319	337,184	888,885
C_4_179	60	12,117	98,009	812,166	15,358	240,554	985,466
C_4_180	60	5,615	115,789	^d 940,956	9,487	395,397	1,242,244
C_4_181	80	13,517	178,085	754,460	15,643	314,773	773,170
C_4_182	80	5,989	196,347	^d 869,595	9,011	322,430	881,747
C_4_183	80	14,626	97,828	760,427	20,598	230,277	805,924
C_4_184	80	12,278	260,700	742,820	14,844	406,639	765,119
C_4_185	80	11,478	157,966	727,626	13,576	318,613	744,244
C_4_186	80	17,801	197,027	729,446	21,222	332,485	752,001
C_4_187	80	12,081	171,291	722,713	15,628	318,654	743,583
C_4_188	80	8,085	117,085	810,094	13,504	259,005	833,353
C_4_189	80	8,828	147,096	744,131	13,055	274,255	780,706
C_4_190	80	15,090	175,651	728,220	16,414	301,060	744,870
C_4_191	100	10,082	184,903	506,760	10,358	246,152	508,915
C_4_192	100	12,577	179,789	497,352	13,436	265,460	500,629
C_4_193	100	11,032	164,933	513,466	11,593	225,814	518,068
C_4_194	100	10,970	166,188	494,498	11,779	238,249	497,195
C_4_195	100	9,442	193,849	512,084	10,526	241,695	514,394
C_4_196	100	10,435	173,233	516,439	12,243	251,061	519,573
C_4_197	100	9,481	184,727	546,100	10,476	240,627	549,578
C_4_198	100	13,356	144,935	560,094	14,280	219,379	563,852
C_4_199	100	15,843	129,210	560,271	16,420	200,853	563,683
C_4_200	100	11,341	126,480	562,098	13,970	210,743	566,991
C_5_201	20	13	27	27	806	23,552	2,422
C_5_202	20	4	6	7	173	8,177	624
C_5_203	20	12	167	136	693	19,064	2,419
C_5_204	20	23	67	103	708	23,287	4,428
C_5_205	20	5	35	23	249	13,114	867
C_5_206	20	6	10	6	405	17,072	1,282
C_5_207	20	1	19	14	145	12,428	1,002
C_5_208	20	2	52	28	270	15,734	1,082
C_5_209	20	5	16	12	405	18,255	3,362
C_5_210	20	3	6	6	120	10,579	706
C_5_211	40	5	781	386	677	111,314	70,253
C_5_212	40	22	637	342	2,033	59,042	303,154
C_5_213	40	90	783	387	1,984	78,654	180,208
C_5_214	40	17	2,127	1,662	678	135,152	132,872
C_5_215	40	14	226	249	1,374	73,385	136,777
C_5_216	40	149	3,548	16,372	3,429	98,544	593,852
C_5_217	40	19	636	126	1,501	57,387	120,706
C_5_218	40	63	2,603	15,832	1,379	100,105	670,824
C_5_219	40	27	451	1,453	1,710	94,459	215,383
C_5_220	40	67	385	273	1,351	90,020	77,997
C_5_221	60	49	11,093	34,436	1,375	116,205	655,947
C_5_222	60	106	2,345	1,186	2,992	125,981	771,850
C_5_223	60	87	2,035	5,400	2,107	108,607	756,564
C_5_224	60	315	1,844	98,202	2,977	63,496	1,334,073
C_5_225	60	56	2,893	890	2,025	104,828	321,516
C_5_226	60	96	491	20,196	1,485	79,419	855,101
C_5_227	60	22	3,236	4,429	914	85,710	425,768
C_5_228	60	99	213	2,128	2,308	92,074	1,047,777
C_5_229	60	51	3,208	2,917	1,457	110,472	819,448
C_5_230	60	179	3,868	135,008	1,832	51,694	1,478,874
C_5_231	80	129	704	31,894	2,034	52,992	1,048,271
C_5_232	80	39	11,043	141,736	1,071	69,628	814,721
C_5_233	80	183	855	126,412	1,783	51,062	978,721
C_5_234	80	180	444	58,697	1,942	41,512	965,075
C_5_235	80	128	276	10,785	1,895	54,535	984,613
C_5_236	80	240	688	34,947	2,334	51,803	1,007,231
C_5_237	80	388	1,481	68,409	2,278	54,510	971,050
C_5_238	80	61	1,564	69,067	1,454	56,711	835,249
C_5_239	80	160	4,728	44,603	1,576	53,993	826,473
C_5_240	80	85	2,735	4,151	1,271	49,536	730,541
C_5_241	100	3	616	12,511	1,460	32,321	679,869
C_5_242	100	20	2,001	25,112	1,179	32,543	699,752
C_5_243	100	23	523	3,595	1,212	31,310	631,569
C_5_244	100	76	871	11,111	918	31,703	705,560
C_5_245	100	108	248	43,147	1,228	30,638	662,305
C_5_246	100	60	2,075	86,856	1,104	32,733	646,058
C_5_247	100	43	1,672	22,187	1,097	35,347	628,310

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Table 10 – continued from previous page

Instance	\mathcal{R}	\mathcal{V} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
C_5_248	100	68	1,188	11,363	1,227	31,259	667,043
C_5_249	100	43	2,643	9,904	876	30,005	608,672
C_5_250	100	135	1,433	54,646	1,444	28,315	664,325
C_6_251	20	9	56	46	318	14,068	2,771
C_6_252	20	1	13	29	211	10,205	3,242
C_6_253	20	1	93	135	104	16,207	4,654
C_6_254	20	11	38	223	1,304	26,283	23,731
C_6_255	20	20	168	220	674	16,294	9,926
C_6_256	20	-	55	45	16	13,481	2,391
C_6_257	20	10	99	127	350	11,829	4,626
C_6_258	20	5	28	22	218	11,534	3,147
C_6_259	20	2	38	31	28	16,189	3,647
C_6_260	20	1	30	38	248	5,911	1,292
C_6_261	40	39	13,737	49,325	368	183,994	819,680
C_6_262	40	153	15,368	143,123	1,063	242,100	1,868,166
C_6_263	40	571	15,606	146,013	6,787	222,667	1,799,204
C_6_264	40	575	14,059	200,457	8,862	219,648	1,738,941
C_6_265	40	1,072	8,680	163,660	14,754	202,050	2,128,877
C_6_266	40	579	10,230	83,084	10,180	168,251	1,425,625
C_6_267	40	2,039	25,883	162,706	14,269	202,831	1,386,655
C_6_268	40	162	11,127	93,768	4,934	203,319	1,475,514
C_6_269	40	1,343	8,460	116,803	15,467	206,608	2,121,926
C_6_270	40	991	7,649	58,380	12,744	204,991	1,534,478
C_6_271	60	857	62,271	442,295	8,291	239,825	1,787,968
C_6_272	60	5,855	64,394	609,191	11,280	247,713	1,687,696
C_6_273	60	1,799	17,021	428,373	11,815	192,125	1,692,460
C_6_274	60	583	34,491	404,704	7,420	204,177	1,742,126
C_6_275	60	7,515	44,539	318,931	16,845	206,788	1,711,834
C_6_276	60	2,651	38,177	373,527	10,871	203,412	1,832,237
C_6_277	60	4,558	52,304	582,969	8,880	157,742	1,695,891
C_6_278	60	2,020	53,762	325,275	7,319	201,297	1,717,071
C_6_279	60	2,110	35,210	491,517	11,659	141,991	1,778,773
C_6_280	60	620	72,960	409,844	7,748	288,992	1,844,492
C_6_281	80	4,080	111,918	488,216	7,067	193,894	1,059,525
C_6_282	80	2,889	79,718	670,664	5,185	153,857	1,029,264
C_6_283	80	595	60,590	436,384	4,635	167,701	1,054,181
C_6_284	80	2,634	126,841	472,606	7,280	214,657	1,104,206
C_6_285	80	1,309	91,007	519,105	5,105	200,779	1,087,009
C_6_286	80	1,456	110,745	454,798	5,510	196,204	1,085,504
C_6_287	80	634	84,227	593,997	4,087	206,053	1,034,983
C_6_288	80	1,048	75,074	609,248	5,336	169,098	1,069,232
C_6_289	80	499	71,438	390,242	2,330	180,835	1,122,364
C_6_290	80	3,997	90,772	504,068	7,968	169,401	1,066,923
C_6_291	100	921	93,826	547,935	2,501	144,080	695,484
C_6_292	100	1,197	101,330	516,685	2,580	139,807	694,043
C_6_293	100	2,411	86,463	467,739	3,907	122,372	714,537
C_6_294	100	1,655	94,667	515,789	3,339	135,907	712,857
C_6_295	100	829	87,487	480,753	3,357	126,467	701,561
C_6_296	100	586	83,943	513,460	2,610	135,297	700,211
C_6_297	100	628	90,704	499,662	2,702	127,033	692,616
C_6_298	100	2,819	95,655	485,809	4,948	142,231	689,445
C_6_299	100	2,993	85,427	456,881	3,678	136,357	727,760
C_6_300	100	1,518	93,884	507,987	3,443	141,072	755,333
C_7_301	20	6	44	42	86	20,654	2,267
C_7_302	20	3	313	257	14	38,890	930
C_7_303	20	31	98	62	480	18,687	1,005
C_7_304	20	-	161	351	207	12,711	3,345
C_7_305	20	1	215	215	3	13,710	675
C_7_306	20	5	157	157	24	29,223	2,410
C_7_307	20	1	95	92	48	44,011	3,105
C_7_308	20	-	123	124	4	13,291	2,118
C_7_309	20	1	29	29	6	36,956	1,615
C_7_310	20	6	25	25	341	25,219	2,300
C_7_311	40	76	8,194	46,604	339	80,643	551,901
C_7_312	40	48	1,069	60,916	1,029	110,191	1,387,585
C_7_313	40	43	2,484	4,668	323	115,201	737,758
C_7_314	40	9	25,755	327,483	35	146,299	1,071,004
C_7_315	40	93	2,883	18,122	728	85,692	1,168,751
C_7_316	40	101	3,195	27,934	1,138	118,520	1,046,979
C_7_317	40	13	1,923	177,158	957	116,448	874,912
C_7_318	40	92	2,000	16,384	1,082	95,084	635,754
C_7_319	40	28	6,225	45,621	297	51,042	475,205
C_7_320	40	112	3,180	546,996	504	63,632	805,098
C_7_321	60	66	8,448	162,344	769	108,679	1,029,337
C_7_322	60	25	2,014	91,599	227	78,465	946,521
C_7_323	60	25	11,081	381,317	367	51,502	953,508
C_7_324	60	205	941	75,378	1,536	87,436	1,228,721
C_7_325	60	90	5,463	238,226	450	41,581	1,031,081
C_7_326	60	12	19,445	669,402	145	59,190	851,286
C_7_327	60	102	15,010	238,561	642	64,547	1,142,265
C_7_328	60	56	4,964	316,871	754	78,782	1,202,603
C_7_329	60	31	10,354	47,460	691	104,165	1,094,224
C_7_330	60	170	1,717	63,792	1,129	89,250	1,223,806
C_7_331	80	116	66	73,122	1,169	49,425	717,760
C_7_332	80	124	6,470	259,535	913	37,289	630,751
C_7_333	80	201	4,156	166,622	947	46,198	773,689

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Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
C_7_334	80	168	3,208	213,740	931	49,457	689,320
C_7_335	80	49	4,197	320,984	493	30,695	478,177
C_7_336	80	88	3,993	305,387	668	47,285	728,813
C_7_337	80	108	616	167,620	595	47,866	651,141
C_7_338	80	120	4,468	234,035	1,118	42,034	653,160
C_7_339	80	70	3,796	353,479	606	42,708	627,813
C_7_340	80	63	6,227	343,070	646	52,402	600,079
C_7_341	100	38	5,203	173,677	592	28,054	426,995
C_7_342	100	120	576	128,308	856	32,045	515,894
C_7_343	100	61	10,072	189,531	382	25,724	404,059
C_7_344	100	131	8,105	211,694	569	22,574	347,505
C_7_345	100	63	1,989	164,983	743	26,481	417,304
C_7_346	100	182	165	75,183	933	30,873	500,964
C_7_347	100	140	3,829	153,766	835	31,587	428,399
C_7_348	100	71	5,502	240,899	699	35,576	421,021
C_7_349	100	27	627	121,337	811	22,939	441,593
C_7_350	100	80	3,484	262,878	726	34,508	406,004
C_8_351	20	4	131	60	64	24,689	5,178
C_8_352	20	12	101	109	645	53,282	6,890
C_8_353	20	24	244	518	982	46,265	10,370
C_8_354	20	2	18	18	166	20,831	1,073
C_8_355	20	1	108	61	163	38,677	5,054
C_8_356	20	28	138	247	773	54,505	12,683
C_8_357	20	1	15	10	236	28,390	1,054
C_8_358	20	2	16	5	126	23,971	492
C_8_359	20	7	20	13	424	22,224	1,757
C_8_360	20	1	64	49	38	16,776	2,697
C_8_361	40	51	422	189	1,776	136,748	117,208
C_8_362	40	16	910	444	990	163,779	40,937
C_8_363	40	27	507	102	2,368	172,762	32,899
C_8_364	40	15	1,575	90	1,109	146,901	5,867
C_8_365	40	592	70,454	134,863	8,213	403,448	1,120,375
C_8_366	40	9	1,021	232	2,312	191,535	28,580
C_8_367	40	8	537	164	968	220,579	22,086
C_8_368	40	26	224	287	2,600	148,944	102,986
C_8_369	40	37	78	153	3,310	111,988	93,186
C_8_370	40	18	59	181	3,039	131,560	46,177
C_8_371	60	95	60,850	150,703	4,642	179,702	740,242
C_8_372	60	64	1,201	1,057	3,169	122,261	170,358
C_8_373	60	40	2,212	2,320	3,069	106,642	155,956
C_8_374	60	12	5,425	951	2,721	135,971	70,398
C_8_375	60	18	501	1,255	3,472	114,222	114,365
C_8_376	60	48	1,192	927	2,833	118,816	438,927
C_8_377	60	18	2,538	843	2,158	118,480	110,796
C_8_378	60	35	1,063	2,072	1,669	128,737	283,421
C_8_379	60	8	2,957	716	1,836	120,269	108,421
C_8_380	60	21	2,521	1,434	1,663	115,058	57,547
C_8_381	80	27	1,873	3,732	1,238	65,889	159,633
C_8_382	80	21	4,272	1,433	1,428	66,701	198,865
C_8_383	80	7	8,140	3,104	1,632	68,597	182,030
C_8_384	80	25	557	5,739	1,682	59,765	324,480
C_8_385	80	19	109	3,719	1,478	57,370	558,070
C_8_386	80	26	2,428	3,678	1,271	64,153	320,830
C_8_387	80	54	2,261	4,452	2,357	58,690	222,180
C_8_388	80	34	1,432	5,260	1,503	58,897	248,868
C_8_389	80	24	1,316	3,024	1,261	53,023	301,318
C_8_390	80	21	537	5,891	1,753	68,077	481,959
C_8_391	100	8	987	4,847	963	30,140	204,602
C_8_392	100	22	3,139	7,247	1,205	36,288	375,388
C_8_393	100	6	909	4,772	1,457	36,623	203,329
C_8_394	100	11	922	5,650	1,233	35,184	501,429
C_8_395	100	11	4,569	7,648	979	35,995	181,385
C_8_396	100	19	1,668	5,387	1,087	33,629	160,768
C_8_397	100	2	1,116	7,044	1,397	38,090	433,438
C_8_398	100	4	1,975	8,685	1,421	38,944	385,854
C_8_399	100	18	695	7,829	1,264	36,383	258,104
C_8_400	100	8	1,654	11,869	916	34,894	323,965
C_9_401	20	1	912	912	5	32,837	5,485
C_9_402	20	2	213	213	3	37,881	3,196
C_9_403	20	5	173	192	14	26,524	4,571
C_9_404	20	3	731	743	39	51,178	11,714
C_9_405	20	1	103	103	102	7,374	1,516
C_9_406	20	10	155	150	33	15,003	2,126
C_9_407	20	5	30	30	109	21,410	1,581
C_9_408	20	10	87	89	31	23,190	5,390
C_9_409	20	1	192	192	4	17,429	808
C_9_410	20	31	63	63	455	22,268	2,705
C_9_411	40	30	721	72,434	159	98,644	863,936
C_9_412	40	38	2,347	279,135	74	107,521	1,889,814
C_9_413	40	61	6,746	120,741	351	124,951	942,311
C_9_414	40	32	7,392	248,034	427	127,730	1,754,527
C_9_415	40	104	3,513	90,253	1,146	107,293	1,069,351
C_9_416	40	11	3,530	169,132	484	108,364	1,204,642
C_9_417	40	99	1,991	24,049	807	96,275	628,658
C_9_418	40	15	5,519	158,897	313	99,939	544,580
C_9_419	40	54	1,985	34,643	287	80,496	447,272

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Instance	\mathcal{R}	ν -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
C_9_420	40	1	3,201	224,125	2	103,992	1,656,256
C_9_421	60	141	4,982	328,700	1,134	97,648	1,051,648
C_9_422	60	231	9,425	374,860	579	78,608	903,993
C_9_423	60	73	5,566	391,782	213	62,183	914,100
C_9_424	60	9	12,279	121,116	835	99,259	954,696
C_9_425	60	291	10,289	196,186	1,466	66,890	1,046,624
C_9_426	60	214	6,973	306,264	1,000	79,312	1,170,454
C_9_427	60	372	11,374	271,969	1,662	89,429	1,098,848
C_9_428	60	31	9,272	243,617	622	83,141	885,493
C_9_429	60	459	5,539	581,807	1,497	106,891	1,107,959
C_9_430	60	570	12,656	354,606	1,705	83,918	1,150,166
C_9_431	80	385	4,807	142,380	1,337	50,028	653,172
C_9_432	80	294	9,231	94,588	1,304	52,687	677,480
C_9_433	80	81	9,560	102,574	753	57,612	613,981
C_9_434	80	189	10,540	178,188	808	39,037	610,728
C_9_435	80	175	4,142	226,654	431	55,791	494,099
C_9_436	80	278	6,047	162,190	1,069	54,309	572,515
C_9_437	80	273	2,884	169,620	1,087	53,213	640,405
C_9_438	80	62	10,443	266,380	527	48,057	559,797
C_9_439	80	383	1,652	160,944	1,292	44,127	711,061
C_9_440	80	166	1,167	261,460	462	57,016	548,543
C_9_441	100	180	3,704	118,376	807	32,460	437,457
C_9_442	100	222	865	213,753	967	35,591	531,978
C_9_443	100	298	2,367	123,223	1,231	40,525	460,457
C_9_444	100	253	3,178	148,132	558	30,170	341,746
C_9_445	100	362	4,874	190,888	1,204	30,586	439,351
C_9_446	100	286	4,413	156,956	1,021	30,305	482,892
C_9_447	100	448	4,091	177,178	1,218	34,603	446,970
C_9_448	100	379	3,041	131,857	1,113	35,644	392,199
C_9_449	100	407	1,007	161,619	987	33,969	474,715
C_9_450	100	578	1,938	186,423	1,223	32,923	426,188
C_10_451	20	3	4	4	47	6,924	148
C_10_452	20	-	158	93	41	20,695	2,392
C_10_453	20	2	29	18	92	11,124	499
C_10_454	20	5	6	6	87	7,310	296
C_10_455	20	2	10	7	52	11,383	195
C_10_456	20	2	398	349	276	27,844	7,265
C_10_457	20	1	24	19	68	13,808	834
C_10_458	20	-	8	6	80	19,361	398
C_10_459	20	1	97	88	29	14,236	775
C_10_460	20	9	77	60	177	9,085	1,403
C_10_461	40	3	999	235	462	78,951	34,750
C_10_462	40	13	263	177	494	66,776	19,651
C_10_463	40	4	793	299	292	88,739	30,972
C_10_464	40	4	693	127	301	118,761	10,558
C_10_465	40	68	22,049	96,321	9,135	257,615	574,278
C_10_466	40	-	556	155	770	86,164	33,282
C_10_467	40	6	807	116	502	117,268	26,516
C_10_468	40	9	707	1,127	974	103,618	96,188
C_10_469	40	5	2,375	2,163	1,840	112,678	224,662
C_10_470	40	8	1,125	641	495	54,918	33,173
C_10_471	60	8	12,689	968	1,167	115,652	57,419
C_10_472	60	17	4,018	5,408	724	97,060	189,234
C_10_473	60	10	25,590	1,857	997	134,142	104,980
C_10_474	60	5	3,765	1,225	1,700	107,043	103,940
C_10_475	60	3	2,337	764	950	85,653	76,615
C_10_476	60	5	7,491	10,049	791	97,047	300,660
C_10_477	60	9	1,297	176	743	79,958	63,945
C_10_478	60	58	22,504	144,958	1,371	143,981	811,074
C_10_479	60	2	6,229	1,156	430	124,574	86,817
C_10_480	60	5	6,024	701	1,578	113,755	125,189
C_10_481	80	8	5,371	3,815	687	60,613	131,710
C_10_482	80	2	2,258	1,892	701	59,423	193,630
C_10_483	80	3	9,943	4,621	1,156	68,746	159,829
C_10_484	80	10	5,354	5,244	1,106	66,567	263,394
C_10_485	80	14	10,534	12,764	833	68,818	215,232
C_10_486	80	4	16,307	3,027	1,097	76,309	136,027
C_10_487	80	3	9,149	2,243	559	73,786	88,590
C_10_488	80	19	10,904	1,942	789	68,546	127,450
C_10_489	80	2	749	2,887	680	49,689	233,443
C_10_490	80	22	14,622	6,923	1,106	68,911	359,881
C_10_491	100	4	6,975	3,700	474	37,532	173,149
C_10_492	100	5	10,138	2,577	1,098	39,371	173,755
C_10_493	100	17	4,687	2,287	715	36,912	160,080
C_10_494	100	6	13,245	11,774	621	40,581	181,512
C_10_495	100	4	10,928	6,471	744	36,505	128,241
C_10_496	100	8	2,784	2,501	618	31,783	198,735
C_10_497	100	1	1,319	2,879	680	34,243	231,190
C_10_498	100	24	4,752	2,809	755	38,980	310,680
C_10_499	100	-	2,088	6,700	1,121	40,898	192,359
C_10_500	100	-	2,095	13,639	678	41,306	293,524
cgcut1	16	11	350	156	44	8,745	1,431
cgcut2	23	6	56	10	539	143,084	20,536
cgcut3	62	1,094	10	d614,341	2,128	184,433	1,386,538
lw161	16	34	186	51	131	10,890	320
lw163	16	52	227	123	187	10,872	686

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Instance	\mathcal{R}	\mathcal{V} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
lw172	17	20	199	92	98	18,363	664
lw251	25	17	28,459	8,327	20	348,507	14,669
lw252	25	1,151	44,795	15,575	1,471	331,362	28,546
lw253	25	15	52,427	8,610	16	348,645	14,752
lw281	28	90	44,393	11,397	152	476,676	41,769
lw283	28	752	12,596	7,632	1,809	195,374	25,610
lw292	29	810	8,503	4,876	1,840	249,241	18,226
lw491	49	18,457	98,901	784,682	19,571	377,026	894,865
lw492	49	12,068	142,453	481,729	13,614	411,555	619,935
lw493	49	29,652	188,964	d_1 1,178,284	32,788	420,615	1,533,877
lw731	73	19,930	211,746	d_1 1,093,226	20,007	354,210	1,095,408
lw732	73	22,929	199,044	923,552	23,419	400,849	926,561
lw733	73	38,592	128,193	d_1 878,116	38,925	271,239	881,842
lw971	97	13,305	326,642	600,444	13,325	367,144	601,637
lw972	97	238	203,192	597,128	238	258,943	597,718
lw973	97	24,169	124,540	642,940	24,327	213,304	643,495
lw1961	196	1,030	61,140	174,484	1,031	62,730	174,824
lw1963	196	5,874	69,286	168,421	5,874	74,719	168,568
lw1972	197	8,936	74,267	167,756	8,940	78,413	167,909
50	50	2	2	2	87	26,252	720
100	100	621	20,175	31,166	1,835	34,051	185,927
500	500	1,276	4,577	27,451	1,276	4,795	27,451
1000	1,000	484	831	5,925	484	858	5,925
5000	5,000	17	24	301	17	24	301
10000	10,000	2	8	74	2	8	74
15000	15,000	3	9	68	3	9	68
gcut01	10	-	5	5	1	217	72
gcut02	20	2	579	1,608	20	132,973	85,505
gcut03	30	5	15	26,543	10	389,356	2,294,680
gcut04	50	451	36	106,307	2,149	178,200	2,414,529
gcut05	10	3	6	6	16	389	145
gcut06	20	16	197	454	132	122,445	67,149
gcut07	30	4	717	39,157	11	157,299	3,421,260
gcut08	50	7	-	70,647	246	202,541	1,787,950
gcut09	10	1	4	4	5	129	72
gcut10	20	2	348	503	6	67,037	69,553
gcut11	30	8	26	5,025	43	553,648	1,091,224
gcut12	50	1,999	2	82,893	2,874	183,553	2,666,679
gcut13	32	383	54,657	424,768	1,514	269,671	2,744,332
hc2	8	1	1	1	1	24	1
hc3	7	-	1	1	3	38	7
hc5	15	30	37	37	125	5,628	189
hc6	19	41	366	6	128	12,828	42
hc7	21	27	337	15	45	25,857	138
hc11	15	9	27	24	24	882	132
leung1	10	1	10	2	6	139	38
leung2	15	6	12	12	11	3,663	179
leung3	20	-	410	-	23	16,065	3,902
leung4	20	31	287	57	131	22,909	2,497
leung5	25	189	2,211	2,082	1,115	207,468	41,111
leung6	25	1,195	14,771	4,055	3,670	346,886	35,267
leung7	30	32	67,555	10,418	40	590,286	47,043
leung8	30	7	20,310	13,824	18	497,873	69,031
leung9	40	1,821	27,302	397,712	3,181	477,719	1,295,605
leung10	50	743	10,865	4,101	10,633	243,555	375,349
ngcut01	10	6	10	8	11	306	14
ngcut02	17	1	1	1	25	7,938	19
ngcut03	21	1	1	1	47	29,018	381
ngcut04	7	-	4	4	1	44	11
ngcut05	14	1	1	1	16	2,367	67
ngcut06	15	22	29	22	80	9,064	170
ngcut07	8	1	1	1	1	26	1
ngcut08	13	-	12	6	22	412	33
ngcut09	18	-	160	21	20	26,265	1,033
ngcut10	13	1	1	1	3	1,198	11
ngcut11	15	-	48	9	61	7,917	342
ngcut12	22	1	1	1	34	14,861	102
nice1	25	57	1,892	3,507	360	173,141	126,677
nice2	50	12,342	3,879	59,196	18,014	225,058	2,354,543
nice3	100	740	288	154,341	1,503	18,906	596,923
nice4	200	303	1	105,429	358	2,662	170,478
nice5	500	230	415	26,298	230	2,224	26,308
nice6	1,000	39	-	6,325	39	223	6,326
nice3t	100	5,413	6,832	201,569	5,537	253,929	665,583
nice4t	200	1,188	1,806	150,417	1,188	36,498	158,755
nice6t	1,000	39	-	6,338	39	210	6,338
nice11t	1,000	44	-	6,162	44	173	6,162
nice12t	2,000	7	-	1,677	7	28	1,677
nice15t	5,000	-	-	332	-	3	332
nice21t	1,000	48	2	6,640	48	162	6,640
nice22t	2,000	5	-	1,678	5	23	1,678
nice25t	5,000	2	-	322	2	3	322
nice31t	1,000	23	-	6,462	23	183	6,462
nice32t	2,000	5	-	1,753	5	19	1,753
nice35t	5,000	1	-	327	1	4	327

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Table 10 – continued from previous page

Instance	\mathcal{R}	χ -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
nice41t	1,000	35	-	6,256	35	190	6,256
nice42t	2,000	6	-	1,660	6	26	1,660
nice45t	5,000	-	-	307	-	3	307
nice51t	1,000	29	-	6,337	29	210	6,337
nice52t	2,000	10	-	1,744	10	28	1,744
nice55t	5,000	1	-	308	1	2	308
nice61t	1,000	34	9	6,015	34	171	6,015
nice62t	2,000	8	-	1,691	8	27	1,691
nice65t	5,000	1	-	328	1	3	328
nice71t	1,000	33	-	6,336	33	188	6,337
nice72t	2,000	6	-	1,680	6	23	1,680
nice75t	5,000	1	-	321	1	3	321
nice81t	1,000	33	-	6,183	33	171	6,183
nice82t	2,000	11	5	1,670	11	26	1,670
nice85t	5,000	-	-	291	-	3	291
nice91t	1,000	38	-	6,210	38	188	6,211
nice92t	2,000	6	-	1,715	6	27	1,716
nice95t	5,000	2	-	332	2	2	332
nice101t	1,000	28	-	6,262	28	188	6,262
nice102t	2,000	8	-	1,708	8	34	1,708
nice105t	5,000	1	-	331	1	3	331
N1a	17	6	73	62	308	7,975	1,479
N1b	17	5	44	112	163	11,364	3,432
N1c	17	3	41	14	121	5,124	607
N1d	17	5	120	80	108	10,906	1,703
N1e	17	4	50	110	191	10,984	4,229
N2a	25	116	2,259	7,007	2,024	116,086	98,512
N2b	25	57	1,497	3,509	1,639	107,938	61,439
N2c	25	24	340	714	1,006	62,477	23,250
N2d	25	60	2,446	3,872	1,644	116,737	75,282
N2e	25	43	691	1,724	2,157	114,575	77,703
N3a	29	109	1,889	13,941	3,041	142,945	222,039
N3b	29	75	1,627	6,020	4,434	154,851	136,411
N3c	29	100	950	5,747	3,067	106,354	107,597
N3d	29	139	6,788	12,549	1,810	229,556	148,244
N3e	29	205	8,187	19,756	2,406	201,397	267,950
N4a	49	1,668	20,596	480,271	11,199	172,958	1,827,858
N4b	49	3,054	44,757	321,245	18,989	192,347	1,457,283
N4c	49	4,336	27,870	712,631	21,242	191,385	1,992,485
N4d	49	2,053	27,017	319,435	13,310	226,020	1,346,895
N4e	49	3,332	75,921	323,934	16,196	297,828	1,522,975
N5a	73	12,424	117,372	720,013	17,477	264,928	954,519
N5b	73	11,986	150,164	763,379	14,406	263,514	936,578
N5c	73	11,890	141,589	690,759	15,661	236,516	1,019,800
N5d	73	10,464	49,146	663,187	14,199	204,116	904,359
N5e	73	6,115	154,196	506,658	10,110	212,909	1,048,688
N6a	97	22,128	41,817	487,675	22,745	96,197	525,001
N6b	97	9,785	113,214	447,667	11,007	126,803	581,048
N6c	97	7,250	95,959	496,062	8,258	119,796	584,222
N6d	97	11,357	125,460	506,837	11,993	185,381	524,774
N6e	97	13,751	156,337	545,435	14,153	192,893	572,009
N7a	197	1,245	68,586	163,161	1,245	70,813	163,298
N7b	197	1,233	51,434	164,039	1,240	56,037	164,151
N7c	197	7,212	40,656	167,412	7,217	45,119	167,684
N7d	197	9,551	49,661	160,085	9,561	53,384	160,488
N7e	197	7,413	55,572	163,423	7,422	60,766	163,610
okp1	50	848	4,249	90	1,082	265,882	2,517
okp2	30	17	240	10	306	94,775	1,151
okp3	30	4	401	166	490	116,133	17,711
okp4	61	32	4,630	480	41	312,790	999
okp5	97	40	9,539	1,961	48	245,506	2,186
babu	50	2,303	103,916	150,035	3,063	300,583	1,420,216
mopta1	200	880	23,354	23,222	1,859	373,231	102,774
mopta2	40	478	14,714	97,792	9,980	284,255	2,996,608
w83	42	958	7,627	1,022	1,841	411,297	6,327
path1	25	3	7	14	237	28,568	2,250
path2	50	1,863	81,768	94,455	9,212	176,569	619,246
path3	100	-	4,873	2,997	283	25,505	130,781
path4	200	23	1,302	5,157	154	3,538	60,227
path5	500	68	993	28,895	68	1,213	28,942
path6	1,000	2	-	2,876	2	28	6,440
path1t	25	3	7	16	230	29,539	2,272
path4t	200	979	56,918	68,363	981	57,334	234,243
path6t	1,000	66	256	5,492	66	449	5,492
path11t	1,000	71	1,888	5,543	71	1,918	5,543
path12t	2,000	36	66	1,280	36	79	1,280
path15t	5,000	2	10	325	2	11	325
path21t	1,000	191	2,526	5,849	191	2,556	5,849
path22t	2,000	48	83	1,380	48	100	1,380
path25t	5,000	2	5	291	2	7	291
path31t	1,000	61	265	5,391	61	300	5,392
path32t	2,000	10	9	1,350	10	19	1,350
path35t	5,000	2	2	303	2	5	303
path41t	1,000	46	172	5,645	46	214	5,645
path42t	2,000	32	59	1,339	32	72	1,339
path45t	5,000	3	13	294	3	13	294

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Table 10 – continued from previous page

Instance	\mathcal{R}	\mathcal{V} -good			total		
		LWF	GRASP	IBHP	LWF	GRASP	IBHP
path51t	1,000	128	2,107	5,528	128	2,142	5,528
path52t	2,000	25	81	1,344	25	92	1,344
path55t	5,000	2	1	322	2	3	322
path61t	1,000	223	2,698	5,504	223	2,728	5,504
path62t	2,000	27	63	1,297	27	75	1,297
path65t	5,000	3	13	296	3	13	296
path71t	1,000	3	33	4,581	4	53	6,315
path72t	2,000	37	105	1,349	37	113	1,349
path75t	5,000	4	11	331	4	11	331
path81t	1,000	138	2,737	5,491	138	2,780	5,491
path82t	2,000	19	85	1,280	19	100	1,280
path85t	5,000	2	-	321	2	3	321
path91t	1,000	73	837	5,463	73	915	5,464
path92t	2,000	11	57	1,314	11	76	1,314
path95t	5,000	3	6	286	3	7	286
path101t	1,000	7	16	6,336	7	51	6,336
path102t	2,000	12	37	1,401	12	39	1,401
path105t	5,000	2	3	288	2	3	288
pb1	10	3	5	5	9	352	44
pb2	10	1	1	1	7	116	10
pb3	15	1	154	49	22	12,529	1,813
pb4	15	105	644	345	135	12,675	3,426
pb5	20	90	1,461	852	939	37,700	8,219
T1a	17	2	148	58	114	9,720	1,593
T1b	17	5	120	80	108	10,910	1,703
T1c	17	2	3	1	42	3,083	205
T1d	17	1	1	1	11	1,215	53
T1e	17	10	28	20	235	6,310	781
T2a	25	7	43	159	788	35,752	6,056
T2b	25	1	1,100	452	306	74,513	7,650
T2c	25	10	980	2,380	539	79,824	43,960
T2d	25	43	680	1,727	2,170	113,138	77,739
T2e	25	57	1,501	3,509	1,649	108,070	61,430
T3a	29	139	6,791	12,410	1,810	229,679	146,545
T3b	29	48	10,881	3,917	1,292	137,545	37,360
T3c	29	102	2,367	22,926	3,445	153,391	300,282
T3d	29	80	5,244	6,874	1,944	122,129	72,860
T3e	29	77	1,627	6,013	4,444	154,843	136,246
T4a	49	4,800	50,996	438,725	13,677	197,040	1,586,524
T4b	49	4,380	27,768	715,491	21,486	190,669	2,000,458
T4c	49	2,104	27,351	317,125	13,700	229,888	1,336,905
T4d	49	1,663	20,737	478,509	11,150	174,332	1,820,765
T4e	49	3,046	44,667	321,903	18,886	191,894	1,460,071
T5a	73	12,088	141,645	685,624	15,911	236,620	1,011,925
T5b	73	11,880	219,119	721,193	17,172	313,481	1,025,473
T5c	73	12,369	117,379	731,163	17,391	264,946	969,584
T5d	73	10,444	49,111	664,287	14,169	203,975	905,834
T5e	73	11,865	149,100	755,617	14,253	261,450	926,973
T6a	97	17,080	132,438	511,296	17,940	166,841	535,383
T6b	97	7,223	96,359	499,548	8,214	120,302	588,449
T6c	97	13,717	156,148	545,478	14,119	192,657	572,062
T6d	97	9,913	113,400	450,718	11,152	127,007	584,999
T6e	97	11,851	114,573	540,923	12,548	168,162	570,198
T7a	199	4,123	69,718	163,420	4,138	75,513	163,569
T7b	199	7,266	60,150	160,778	7,271	65,118	160,871
T7c	199	1,448	49,667	160,800	1,448	55,418	160,945
T7d	199	8,055	74,060	167,026	8,057	81,385	167,111
T7e	199	7,908	57,455	169,332	7,908	63,371	169,459
zdf1	580	977	20,298	20,526	977	20,929	20,529
zdf2	660	557	12,893	16,138	557	13,375	16,139
zdf3	740	406	10,632	13,357	406	11,330	13,357
zdf4	820	518	7,813	10,513	518	8,314	10,513
zdf5	900	424	6,278	8,775	424	6,711	8,775
zdf6	1,532	1	9	774	1	13	1,753
zdf7	2,432	-	4	236	-	5	473
zdf8	2,532	89	9	645	89	9	890
zdf9	5,032	72	5	320	72	5	321
zdf10	5,064	12	2	399	12	2	399
zdf11	10,064	14	1	109	14	1	109
zdf12	7,564	17	10	172	17	10	172
zdf13	15,096	8	1	47	8	1	47
zdf14	25,032	1	2	15	1	2	15
zdf15	50,032	-	-	7	-	-	7
zdf16	75,032	-	-	10	-	-	10

C Appendix

In this appendix, we present the results of our computational experiments with respect to the diversity. Therefore, the following tables contain the results for

every instance of MADASS instantiated with $\delta \in \{\delta_{min}, \delta_{avg}\}$, $m = 6$, and $\underline{v} = (1-\varepsilon)v^*$ for $\varepsilon = 5\%$. In the two left columns of the following tables, the name of the instance is shown together with the number of rectangles belonging to the instance. The remaining columns contain the bounds on the current instance, presented in Section 4 and the results of the different solution approaches, i.e., the MIQP formulation, the Benders approach, and the instantiation of the generic two-stage heuristic with IBHP and the random exchange heuristic. Note that the best result for each of the instances is marked in blue, * indicates that the instance was solved to optimality, and – indicates that no selection with a diversity value greater than 0 could be obtained.

Table 11: Best obtained diversity by the solution approaches to MADASS

Instance	R	δ_{min}			δ_{min}			δ_{min}			
		UB	UB _{avg}	MIQP	BD	BD _{min}	GF	MIQP	BD	BD _{min}	GF
ALX	20	0.849	0.935	-	-	-	0.667	-	-	-	0.782
ALY	20	0.693	0.818	-	-	-	0.579	-	-	-	0.637
ALZ	20	0.629	0.623	-	-	-	0.417	-	-	-	0.442
ALZZ	20	0.908	0.959	-	-	0.600	0.692	-	-	0.781	0.806
ALZZZ	20	0.908	0.959	-	-	0.600	0.692	-	-	0.770	0.806
ASX	20	0.952	0.964	-	-	0.600	0.625	-	-	0.689	0.756
ASY	20	0.835	0.865	-	-	0.500	0.647	-	-	0.704	0.756
ASZ	20	0.702	0.704	-	-	0.500	0.500	-	-	0.692	0.704
ASZZ	20	0.889	0.958	-	-	0.529	0.625	-	-	0.618	0.713
ASZZZ	20	0.888	0.958	-	-	0.529	0.625	-	-	0.618	0.713
AVX	20	0.936	0.959	-	-	0.800	0.778	-	-	0.851	0.851
AVY	20	0.843	0.862	-	-	0.556	0.647	-	-	0.676	0.705
AVZ	20	0.689	0.708	-	-	0.538	0.692	-	-	0.364	0.536
AVZZ	20	0.924	0.935	-	-	0.538	0.692	-	-	0.681	0.796
AVZZZ	20	0.925	0.935	-	-	0.538	0.692	-	-	0.776	0.796
BLX	30	1.000	1.000	-	-	0.444	0.846	-	-	0.738	0.892
BLY	30	0.986	0.988	-	-	-	0.846	-	-	0.877	0.877
BLZ	30	0.948	0.938	-	-	-	0.714	-	-	0.802	0.802
BLZZ	30	0.676	0.673	-	-	-	0.438	-	-	0.469	0.469
BLZZZ	30	0.961	0.960	-	-	0.636	0.778	-	-	0.839	0.839
BSX	30	0.999	0.998	-	-	-	0.846	-	-	0.917	0.917
BSY	30	0.950	0.957	-	-	-	0.778	-	-	0.828	0.828
BSZ	30	0.864	0.880	-	-	-	0.667	-	-	0.828	0.828
BSZZ	30	0.421	0.421	-	-	-	0.244	-	-	0.279	0.279
BSZZZ	30	0.950	0.962	-	-	-	0.831	-	-	0.831	0.831
BVX	30	1.000	1.000	-	-	0.357	0.778	-	-	0.678	0.678
BVY	30	0.984	0.985	-	-	0.579	0.833	-	-	0.926	0.926
BVZ	30	0.876	0.944	-	-	0.360	0.818	-	-	0.874	0.874
BVZZ	30	0.661	0.661	-	-	-	0.714	-	-	0.794	0.794
BVZZZ	30	0.954	0.954	-	-	0.394	0.412	-	-	0.447	0.447
CLX	40	1.000	1.000	-	-	0.474	0.765	-	-	0.826	0.826
CLY	40	1.000	1.000	-	-	-	0.875	-	-	0.988	0.988
CLZ	40	0.977	0.975	-	-	-	0.875	-	-	0.939	0.939
CLZZ	40	0.778	0.776	-	-	-	0.810	-	-	0.846	0.846
CLZZZ	40	0.956	0.958	-	-	-	0.514	-	-	0.560	0.560
CSX	40	1.000	1.000	-	-	-	0.760	-	-	0.828	0.828
CSY	40	0.999	0.998	-	-	-	0.875	-	-	0.971	0.971
CSZ	40	0.970	0.966	-	-	-	0.882	-	-	0.925	0.925
CSZZ	40	0.751	0.744	-	-	-	0.769	-	-	0.837	0.837
CSZZZ	40	0.961	0.965	-	-	-	0.500	-	-	0.532	0.532
CVX	40	1.000	1.000	-	-	0.391	0.750	-	-	0.835	0.835
CVY	40	1.000	1.000	-	-	-	0.875	-	-	0.981	0.981
CVZ	40	0.985	0.987	-	-	-	0.875	-	-	0.957	0.957
CVZZ	40	0.834	0.821	-	-	0.212	0.810	-	-	0.879	0.879
CVZZZ	40	0.959	0.964	-	-	-	0.581	-	-	0.618	0.618
DLX	50	1.000	1.000	-	-	0.556	0.760	-	-	0.834	0.834
DLY	50	1.000	1.000	-	-	-	1.000*	-	-	1.000	1.000
DLZ	50	0.991	0.993	-	-	-	0.889	-	-	0.967	0.967
DLZZ	50	0.731	0.837	-	-	-	0.833	-	-	0.894	0.894
DLZZZ	50	0.960	0.959	-	-	-	0.600	-	-	0.641	0.641
							0.778			0.822	0.822

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD $_{min}$	GF	
DSX	50	1.000	1.000	-	-	1.000*	1.000
DSY	50	1.000	1.000	-	-	0.895	0.945
DSZ	50	0.988	0.986	-	-	0.820	0.875
DSZZ	50	0.820	0.816	-	-	0.565	0.602
DSZZZ	50	0.963	0.960	-	-	0.769	0.818
DVX	50	1.000	1.000	-	-	1.000*	1.000
DVY	50	1.000	1.000	-	-	0.895	0.985
DVZ	50	1.000	1.000	-	-	0.895	0.918
DVZZ	50	0.819	0.894	-	-	0.667	0.718
DVZZZ	50	0.433	0.829	-	-	0.241	0.263
AH1	1.000	0.943	0.882	-	-	0.690	0.752
AH2	1.000	0.940	0.879	-	-	0.689	0.724
AH3	1.000	0.943	0.856	-	-	0.686	0.715
AH4	1.000	0.941	0.896	-	-	0.698	0.730
AH5	1.000	0.937	0.844	-	-	0.694	0.748
AH6	1.000	0.845	0.807	-	-	0.711	0.744
AH7	1.000	0.942	0.870	-	-	0.694	0.748
AH8	1.000	0.944	0.815	-	-	0.585	0.693
AH9	1.000	0.931	0.848	-	-	0.667	0.701
AH10	1.000	0.943	0.861	-	-	0.682	0.722
AH11	1.000	0.871	0.870	-	-	0.699	0.743
AH12	1.000	0.902	0.901	-	-	0.682	0.718
AH13	1.000	0.941	0.832	-	-	0.699	0.724
AH14	1.000	0.940	0.808	-	-	0.694	0.730
AH15	1.000	0.886	0.875	-	-	0.699	0.721
AH16	1.000	0.936	0.877	-	-	0.695	0.729
AH17	1.000	0.939	0.826	-	-	0.689	0.721
AH18	1.000	0.850	0.803	-	-	0.684	0.738
AH19	1.000	0.848	0.821	-	-	0.666	0.766
AH20	1.000	0.946	0.843	-	-	0.657	0.682
AH21	1.000	0.836	0.821	-	-	0.689	0.710
AH22	1.000	0.941	0.850	-	-	0.599	0.631
AH23	1.000	0.937	0.850	-	-	0.703	0.730
AH24	1.000	0.938	0.840	-	-	0.690	0.738
AH25	1.000	0.939	0.870	-	-	0.702	0.726
AH26	1.000	0.940	0.850	-	-	0.688	0.721
AH27	1.000	0.943	0.798	-	-	0.571	0.612
AH28	1.000	0.937	0.804	-	-	0.596	0.664
AH29	1.000	0.941	0.875	-	-	0.704	0.742
AH30	1.000	0.935	0.882	-	-	0.698	0.717
AH31	1.000	0.862	0.854	-	-	0.420	0.440
AH32	1.000	0.915	0.877	-	-	0.393	0.497
AH33	1.000	0.889	0.870	-	-	0.643	0.733
AH34	1.000	0.945	0.859	-	-	0.543	0.685
AH35	1.000	0.953	0.864	-	-	0.432	0.631
AH36	1.000	0.857	0.842	-	-	0.623	0.695
AH37	1.000	0.870	0.829	-	-	0.440	0.657
AH38	1.000	0.907	0.877	-	-	0.554	0.698
AH39	1.000	0.942	0.827	-	-	0.493	0.516
AH40	1.000	0.901	0.892	-	-	0.493	0.584
AH41	1.000	0.942	0.874	-	-	0.388	0.472
AH42	1.000	0.890	0.892	-	-	0.555	0.702

Continued on next page

Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
AH43	1,000	0.949	0.870	-	-	0.612	0.733
AH44	1,000	0.946	0.880	-	-	0.595	0.629
AH45	1,000	0.940	0.883	-	-	0.620	0.715
AH46	1,000	0.878	0.876	-	-	0.563	0.735
AH47	1,000	0.887	0.868	-	-	0.713	0.783
AH48	1,000	0.949	0.955	-	-	0.546	0.667
AH49	1,000	0.948	0.845	-	-	0.495	0.594
AH50	1,000	0.840	0.821	-	-	0.622	0.728
AH51	1,000	0.910	0.900	-	-	0.514	0.725
AH52	1,000	0.958	0.864	-	-	0.619	0.533
AH53	1,000	0.941	0.844	-	-	0.336	0.359
AH54	1,000	0.955	0.939	-	-	0.496	0.608
AH55	1,000	0.854	0.821	-	-	0.364	0.486
AH56	1,000	0.854	0.820	-	-	0.590	0.695
AH57	1,000	0.898	0.866	-	-	0.490	0.516
AH58	1,000	0.902	0.892	-	-	0.427	0.614
AH59	1,000	0.864	0.843	-	-	0.649	0.747
AH60	1,000	0.940	0.843	-	-	0.543	0.657
AH61	1,000	0.978	0.977	-	-	0.694	0.727
AH62	1,000	0.977	0.970	-	-	0.694	0.736
AH63	1,000	0.978	0.970	-	-	0.677	0.710
AH64	1,000	0.978	0.969	-	-	0.699	0.735
AH65	1,000	0.976	0.970	-	-	0.703	0.731
AH66	1,000	0.981	0.978	-	-	0.685	0.720
AH67	1,000	0.981	0.972	-	-	0.701	0.728
AH68	1,000	0.982	0.963	-	-	0.699	0.729
AH69	1,000	0.981	0.974	-	-	0.706	0.738
AH70	1,000	0.979	0.980	-	-	0.712	0.735
AH71	1,000	0.982	0.957	-	-	0.698	0.729
AH72	1,000	0.979	0.978	-	-	0.695	0.726
AH73	1,000	0.981	0.975	-	-	0.694	0.730
AH74	1,000	0.983	0.954	-	-	0.702	0.731
AH75	1,000	0.978	0.977	-	-	0.696	0.724
AH76	1,000	0.986	0.953	-	-	0.693	0.723
AH77	1,000	0.980	0.978	-	-	0.692	0.731
AH78	1,000	0.981	0.977	-	-	0.687	0.715
AH79	1,000	0.991	0.948	-	-	0.705	0.720
AH80	1,000	0.978	0.979	-	-	0.712	0.731
AH81	1,000	0.979	0.975	-	-	0.705	0.745
AH82	1,000	0.983	0.979	-	-	0.698	0.732
AH83	1,000	0.990	0.968	-	-	0.681	0.731
AH84	1,000	0.990	0.976	-	-	0.700	0.732
AH85	1,000	0.990	0.962	-	-	0.686	0.716
AH86	1,000	0.981	0.978	-	-	0.695	0.724
AH87	1,000	0.983	0.974	-	-	0.696	0.738
AH88	1,000	0.985	0.952	-	-	0.688	0.730
AH89	1,000	0.980	0.979	-	-	0.700	0.731
AH90	1,000	0.977	0.950	-	-	0.706	0.738
AH91	1,000	0.966	0.974	-	-	0.489	0.642
AH92	1,000	0.990	0.987	-	-	0.615	0.725
AH93	1,000	0.983	0.978	-	-	0.416	0.552
AH94	1,000	0.979	0.971	-	-	0.537	0.675

Continued on next page

Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			GF	
				MIQP	BD _{min}	MIQP		BD _{min}
AH95	1,000	0.987	0.986	-	-	-	0.651	0.720
AH96	1,000	0.992	0.978	-	-	-	0.648	0.733
AH97	1,000	0.981	0.952	-	-	-	0.689	0.671
AH98	1,000	0.982	0.982	-	-	-	0.578	0.675
AH99	1,000	0.982	0.983	-	-	-	0.640	0.732
AH100	1,000	0.984	0.985	-	-	-	0.681	0.712
AH101	1,000	0.982	0.973	-	-	-	0.620	0.604
AH102	1,000	0.981	0.983	-	-	-	0.514	0.675
AH103	1,000	0.968	0.973	-	-	-	0.614	0.739
AH104	1,000	0.980	0.975	-	-	-	0.708	0.745
AH105	1,000	0.985	0.981	-	-	-	0.597	0.742
AH106	1,000	0.990	0.984	-	-	-	0.680	0.755
AH107	1,000	0.979	0.979	-	-	-	0.706	0.623
AH108	1,000	0.980	0.990	-	-	-	0.554	0.734
AH109	1,000	0.990	0.977	-	-	-	0.711	0.762
AH110	1,000	0.979	0.973	-	-	-	0.704	0.634
AH111	1,000	0.976	0.976	-	-	-	0.522	0.742
AH112	1,000	0.985	0.976	-	-	-	0.667	0.624
AH113	1,000	0.981	0.971	-	-	-	0.484	0.569
AH114	1,000	0.988	0.973	-	-	-	0.467	0.641
AH115	1,000	0.981	0.983	-	-	-	0.547	0.639
AH116	1,000	0.985	0.987	-	-	-	0.453	0.699
AH117	1,000	0.967	0.975	-	-	-	0.637	0.692
AH118	1,000	0.981	0.979	-	-	-	0.604	0.616
AH119	1,000	0.970	0.969	-	-	-	0.595	0.738
AH120	1,000	0.982	0.959	-	-	-	0.535	0.757
AH121	1,000	0.939	0.855	-	-	-	0.693	0.751
AH122	1,000	0.942	0.861	-	-	-	0.707	0.745
AH123	1,000	0.939	0.863	-	-	-	0.705	0.755
AH124	1,000	0.940	0.851	-	-	-	0.696	0.754
AH125	1,000	0.939	0.861	-	-	-	0.709	0.740
AH126	1,000	0.943	0.864	-	-	-	0.695	0.750
AH127	1,000	0.940	0.861	-	-	-	0.704	0.742
AH128	1,000	0.941	0.854	-	-	-	0.691	0.744
AH129	1,000	0.941	0.933	-	-	-	0.701	0.745
AH130	1,000	0.873	0.868	-	-	-	0.705	0.753
AH131	1,000	0.943	0.845	-	-	-	0.696	0.734
AH132	1,000	0.939	0.857	-	-	-	0.687	0.744
AH133	1,000	0.941	0.855	-	-	-	0.701	0.745
AH134	1,000	0.942	0.854	-	-	-	0.691	0.757
AH135	1,000	0.944	0.860	-	-	-	0.703	0.735
AH136	1,000	0.940	0.849	-	-	-	0.696	0.756
AH137	1,000	0.943	0.865	-	-	-	0.700	0.753
AH138	1,000	0.941	0.874	-	-	-	0.706	0.750
AH139	1,000	0.941	0.858	-	-	-	0.697	0.746
AH140	1,000	0.943	0.866	-	-	-	0.703	0.747
AH141	1,000	0.940	0.861	-	-	-	0.706	0.751
AH142	1,000	0.941	0.868	-	-	-	0.710	0.722
AH143	1,000	0.942	0.920	-	-	-	0.695	0.751
AH144	1,000	0.942	0.850	-	-	-	0.680	0.722
AH145	1,000	0.939	0.848	-	-	-	0.687	0.753
AH146	1,000	0.942	0.863	-	-	-	0.687	0.753

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD _{min}	MIQP	
AH147	1,000	0.943	0.833	-	-	0.691	0.741
AH148	1,000	0.944	0.867	-	-	0.710	0.755
AH149	1,000	0.942	0.867	-	-	0.690	0.740
AH150	1,000	0.943	0.866	-	-	0.696	0.740
AH151	1,000	0.900	0.865	-	-	0.530	0.607
AH152	1,000	0.879	0.833	-	-	0.477	0.551
AH153	1,000	0.888	0.838	-	-	0.582	0.655
AH154	1,000	0.901	0.886	-	-	0.503	0.627
AH155	1,000	0.892	0.841	-	-	0.564	0.607
AH156	1,000	0.894	0.852	-	-	0.496	0.607
AH157	1,000	0.961	0.961	-	-	0.418	0.483
AH158	1,000	0.960	0.962	-	-	0.583	0.652
AH159	1,000	0.946	0.948	-	-	0.536	0.623
AH160	1,000	0.946	0.854	-	-	0.492	0.574
AH161	1,000	0.951	0.834	-	-	0.494	0.570
AH162	1,000	0.906	0.851	-	-	0.559	0.643
AH163	1,000	0.874	0.832	-	-	0.477	0.536
AH164	1,000	0.948	0.848	-	-	0.580	0.660
AH165	1,000	0.896	0.836	-	-	0.561	0.615
AH166	1,000	0.898	0.848	-	-	0.508	0.594
AH167	1,000	0.903	0.829	-	-	0.522	0.592
AH168	1,000	0.963	0.968	-	-	0.472	0.540
AH169	1,000	0.906	0.850	-	-	0.447	0.558
AH170	1,000	0.892	0.869	-	-	0.519	0.561
AH171	1,000	0.890	0.850	-	-	0.553	0.602
AH172	1,000	0.952	0.871	-	-	0.521	0.581
AH173	1,000	0.880	0.854	-	-	0.521	0.597
AH174	1,000	0.960	0.959	-	-	0.464	0.526
AH175	1,000	0.891	0.839	-	-	0.596	0.665
AH176	1,000	0.885	0.829	-	-	0.474	0.520
AH177	1,000	0.948	0.833	-	-	0.522	0.589
AH178	1,000	0.959	0.956	-	-	0.517	0.588
AH179	1,000	0.889	0.850	-	-	0.499	0.597
AH180	1,000	0.900	0.866	-	-	0.494	0.562
AH181	1,000	0.976	0.962	-	-	0.685	0.710
AH182	1,000	0.980	0.951	-	-	0.680	0.705
AH183	1,000	0.988	0.966	-	-	0.681	0.716
AH184	1,000	0.978	0.953	-	-	0.687	0.719
AH185	1,000	0.988	0.953	-	-	0.687	0.723
AH186	1,000	0.980	0.951	-	-	0.687	0.711
AH187	1,000	0.978	0.951	-	-	0.690	0.719
AH188	1,000	0.982	0.958	-	-	0.670	0.705
AH189	1,000	0.975	0.970	-	-	0.685	0.719
AH190	1,000	0.987	0.951	-	-	0.684	0.717
AH191	1,000	0.980	0.972	-	-	0.681	0.713
AH192	1,000	0.979	0.965	-	-	0.687	0.716
AH193	1,000	0.986	0.950	-	-	0.677	0.715
AH194	1,000	0.991	0.973	-	-	0.682	0.711
AH195	1,000	0.980	0.976	-	-	0.689	0.726
AH196	1,000	0.979	0.953	-	-	0.693	0.717
AH197	1,000	0.978	0.951	-	-	0.691	0.716
AH198	1,000	0.978	0.953	-	-	0.688	0.723

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Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
AH109	1,000	0.988	0.950	-	-	0.689	-	-	0.716
AH200	1,000	0.989	0.949	-	-	0.690	-	-	0.721
AH201	1,000	0.979	0.952	-	-	0.694	-	-	0.726
AH202	1,000	0.976	0.952	-	-	0.681	-	-	0.713
AH203	1,000	0.978	0.953	-	-	0.690	-	-	0.725
AH204	1,000	0.988	0.953	-	-	0.680	-	-	0.712
AH205	1,000	0.977	0.962	-	-	0.677	-	-	0.715
AH206	1,000	0.987	0.962	-	-	0.696	-	-	0.720
AH207	1,000	0.980	0.953	-	-	0.680	-	-	0.713
AH208	1,000	0.979	0.969	-	-	0.671	-	-	0.703
AH209	1,000	0.988	0.951	-	-	0.696	-	-	0.728
AH210	1,000	0.987	0.952	-	-	0.686	-	-	0.717
AH211	1,000	0.975	0.977	-	-	0.665	-	-	0.692
AH212	1,000	0.982	0.970	-	-	0.658	-	-	0.693
AH213	1,000	0.988	0.968	-	-	0.635	-	-	0.701
AH214	1,000	0.972	0.970	-	-	0.671	-	-	0.702
AH215	1,000	0.989	0.981	-	-	0.630	-	-	0.679
AH216	1,000	0.986	0.964	-	-	0.668	-	-	0.702
AH217	1,000	0.976	0.975	-	-	0.633	-	-	0.671
AH218	1,000	0.986	0.970	-	-	0.629	-	-	0.684
AH219	1,000	0.982	0.979	-	-	0.657	-	-	0.694
AH220	1,000	0.972	0.977	-	-	0.631	-	-	0.666
AH221	1,000	0.975	0.986	-	-	0.661	-	-	0.703
AH222	1,000	0.976	0.977	-	-	0.645	-	-	0.684
AH223	1,000	0.980	0.979	-	-	0.665	-	-	0.714
AH224	1,000	0.980	0.978	-	-	0.666	-	-	0.712
AH225	1,000	0.988	0.977	-	-	0.662	-	-	0.702
AH226	1,000	0.981	0.965	-	-	0.609	-	-	0.646
AH227	1,000	0.984	0.970	-	-	0.642	-	-	0.692
AH228	1,000	0.989	0.985	-	-	0.639	-	-	0.681
AH229	1,000	0.975	0.975	-	-	0.615	-	-	0.650
AH230	1,000	0.979	0.964	-	-	0.657	-	-	0.693
AH231	1,000	0.972	0.976	-	-	0.626	-	-	0.657
AH232	1,000	0.978	0.972	-	-	0.671	-	-	0.706
AH233	1,000	0.984	0.972	-	-	0.631	-	-	0.673
AH234	1,000	0.979	0.964	-	-	0.648	-	-	0.679
AH235	1,000	0.986	0.975	-	-	0.647	-	-	0.697
AH236	1,000	0.979	0.964	-	-	0.654	-	-	0.693
AH237	1,000	0.975	0.967	-	-	0.656	-	-	0.695
AH238	1,000	0.975	0.965	-	-	0.664	-	-	0.705
AH239	1,000	0.976	0.964	-	-	0.655	-	-	0.698
AH240	1,000	0.978	0.972	-	-	0.678	-	-	0.717
AH241	1,000	0.941	0.867	-	-	0.694	-	-	0.725
AH242	1,000	0.941	0.873	-	-	0.691	-	-	0.724
AH243	1,000	0.940	0.874	-	-	0.700	-	-	0.724
AH244	1,000	0.941	0.866	-	-	0.685	-	-	0.717
AH245	1,000	0.942	0.860	-	-	0.687	-	-	0.721
AH246	1,000	0.942	0.855	-	-	0.695	-	-	0.717
AH247	1,000	0.940	0.864	-	-	0.688	-	-	0.725
AH248	1,000	0.940	0.862	-	-	0.688	-	-	0.720
AH249	1,000	0.938	0.861	-	-	0.694	-	-	0.724
AH250	1,000	0.940	0.874	-	-	0.690	-	-	0.721

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD $_{min}$	GF	
AH251	1.000	0.940	0.856	-	-	0.692	0.720
AH252	1.000	0.940	0.857	-	-	0.687	0.728
AH253	1.000	0.874	0.867	-	-	0.690	0.722
AH254	1.000	0.942	0.849	-	-	0.690	0.719
AH255	1.000	0.942	0.858	-	-	0.691	0.721
AH256	1.000	0.940	0.849	-	-	0.686	0.711
AH257	1.000	0.941	0.863	-	-	0.692	0.717
AH258	1.000	0.937	0.870	-	-	0.691	0.713
AH259	1.000	0.941	0.856	-	-	0.690	0.717
AH260	1.000	0.930	0.860	-	-	0.691	0.725
AH261	1.000	0.941	0.862	-	-	0.701	0.725
AH262	1.000	0.941	0.855	-	-	0.690	0.719
AH263	1.000	0.940	0.872	-	-	0.685	0.721
AH264	1.000	0.943	0.863	-	-	0.687	0.722
AH265	1.000	0.943	0.862	-	-	0.687	0.722
AH266	1.000	0.943	0.863	-	-	0.690	0.724
AH267	1.000	0.940	0.855	-	-	0.696	0.722
AH268	1.000	0.940	0.863	-	-	0.694	0.726
AH269	1.000	0.940	0.863	-	-	0.689	0.722
AH270	1.000	0.946	0.863	-	-	0.689	0.722
AH271	1.000	0.962	0.958	-	-	0.630	0.668
AH272	1.000	0.962	0.958	-	-	0.633	0.672
AH273	1.000	0.962	0.956	-	-	0.634	0.678
AH274	1.000	0.959	0.958	-	-	0.641	0.665
AH275	1.000	0.963	0.963	-	-	0.646	0.692
AH276	1.000	0.963	0.958	-	-	0.620	0.659
AH277	1.000	0.962	0.955	-	-	0.621	0.660
AH278	1.000	0.962	0.960	-	-	0.646	0.686
AH279	1.000	0.961	0.966	-	-	0.636	0.677
AH280	1.000	0.962	0.957	-	-	0.618	0.666
AH281	1.000	0.960	0.963	-	-	0.642	0.680
AH282	1.000	0.961	0.962	-	-	0.624	0.665
AH283	1.000	0.961	0.960	-	-	0.622	0.666
AH284	1.000	0.962	0.961	-	-	0.620	0.664
AH285	1.000	0.960	0.956	-	-	0.633	0.677
AH286	1.000	0.961	0.956	-	-	0.635	0.673
AH287	1.000	0.961	0.958	-	-	0.611	0.665
AH288	1.000	0.963	0.957	-	-	0.633	0.665
AH289	1.000	0.960	0.941	-	-	0.611	0.654
AH290	1.000	0.960	0.965	-	-	0.643	0.671
AH291	1.000	0.961	0.961	-	-	0.645	0.676
AH292	1.000	0.962	0.955	-	-	0.627	0.665
AH293	1.000	0.959	0.955	-	-	0.634	0.668
AH294	1.000	0.963	0.955	-	-	0.635	0.675
AH295	1.000	0.946	0.864	-	-	0.641	0.674
AH296	1.000	0.960	0.950	-	-	0.632	0.673
AH297	1.000	0.963	0.958	-	-	0.633	0.664
AH298	1.000	0.961	0.961	-	-	0.632	0.663
AH299	1.000	0.881	0.823	-	-	0.640	0.685
AH300	1.000	0.961	0.955	-	-	0.637	0.674
AH301	1.000	0.979	0.944	-	-	0.698	0.726
AH302	1.000	0.981	0.973	-	-	0.710	0.733

Continued on next page

Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
AH303	1,000	0.980	0.957	-	-	0.700	-	-	0.737
AH304	1,000	0.991	0.973	-	-	0.699	-	-	0.732
AH305	1,000	0.986	0.970	-	-	0.701	-	-	0.728
AH306	1,000	0.987	0.954	-	-	0.703	-	-	0.733
AH307	1,000	0.984	0.954	-	-	0.700	-	-	0.732
AH308	1,000	0.987	0.956	-	-	0.705	-	-	0.722
AH309	1,000	0.988	0.953	-	-	0.701	-	-	0.733
AH310	1,000	0.986	0.963	-	-	0.703	-	-	0.730
AH311	1,000	0.981	0.955	-	-	0.703	-	-	0.732
AH312	1,000	0.992	0.953	-	-	0.702	-	-	0.731
AH313	1,000	0.987	0.954	-	-	0.691	-	-	0.727
AH314	1,000	0.980	0.970	-	-	0.704	-	-	0.733
AH315	1,000	0.981	0.970	-	-	0.703	-	-	0.727
AH316	1,000	0.990	0.970	-	-	0.706	-	-	0.744
AH317	1,000	0.986	0.940	-	-	0.702	-	-	0.736
AH318	1,000	0.989	0.953	-	-	0.704	-	-	0.732
AH319	1,000	0.990	0.954	-	-	0.704	-	-	0.739
AH320	1,000	0.981	0.967	-	-	0.703	-	-	0.731
AH321	1,000	0.987	0.979	-	-	0.699	-	-	0.731
AH322	1,000	0.981	0.942	-	-	0.704	-	-	0.740
AH323	1,000	0.986	0.954	-	-	0.696	-	-	0.730
AH324	1,000	0.988	0.954	-	-	0.703	-	-	0.737
AH325	1,000	0.986	0.954	-	-	0.702	-	-	0.737
AH326	1,000	0.981	0.958	-	-	0.698	-	-	0.725
AH327	1,000	0.988	0.968	-	-	0.698	-	-	0.734
AH328	1,000	0.983	0.954	-	-	0.706	-	-	0.732
AH329	1,000	0.982	0.955	-	-	0.705	-	-	0.736
AH330	1,000	0.982	0.959	-	-	0.698	-	-	0.739
AH331	1,000	0.977	0.973	-	-	0.714	-	-	0.752
AH332	1,000	0.974	0.985	-	-	0.708	-	-	0.744
AH333	1,000	0.981	0.987	-	-	0.709	-	-	0.744
AH334	1,000	0.974	0.972	-	-	0.718	-	-	0.750
AH335	1,000	0.981	0.972	-	-	0.710	-	-	0.747
AH336	1,000	0.974	0.980	-	-	0.720	-	-	0.747
AH337	1,000	0.986	0.975	-	-	0.710	-	-	0.743
AH338	1,000	0.975	0.978	-	-	0.709	-	-	0.749
AH339	1,000	0.987	0.979	-	-	0.718	-	-	0.745
AH340	1,000	0.976	0.987	-	-	0.712	-	-	0.745
AH341	1,000	0.973	0.976	-	-	0.712	-	-	0.740
AH342	1,000	0.973	0.979	-	-	0.705	-	-	0.745
AH343	1,000	0.977	0.981	-	-	0.709	-	-	0.744
AH344	1,000	0.971	0.981	-	-	0.705	-	-	0.745
AH345	1,000	0.971	0.987	-	-	0.703	-	-	0.744
AH346	1,000	0.973	0.988	-	-	0.711	-	-	0.743
AH347	1,000	0.978	0.980	-	-	0.711	-	-	0.744
AH348	1,000	0.979	0.983	-	-	0.719	-	-	0.748
AH349	1,000	0.978	0.980	-	-	0.703	-	-	0.738
AH350	1,000	0.981	0.971	-	-	0.715	-	-	0.746
AH351	1,000	0.985	0.985	-	-	0.715	-	-	0.741
AH352	1,000	0.982	0.982	-	-	0.714	-	-	0.740
AH353	1,000	0.976	0.979	-	-	0.712	-	-	0.741
AH354	1,000	0.975	0.982	-	-	0.710	-	-	0.741

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Table 11 – continued from previous page

Instance	\mathcal{R}	δ_{min}			δ_{avg}			δ_{min}			δ_{min}		
		UB	MIQP	GF	UB	MIQP	GF	UB	MIQP	GF	UB	MIQP	GF
AH355	1,000	0.987	-	0.980	-	0.706	-	-	-	-	-	-	0.744
AH356	1,000	0.982	-	0.977	-	0.709	-	-	-	-	-	-	0.746
AH357	1,000	0.983	-	0.973	-	0.703	-	-	-	-	-	-	0.746
AH358	1,000	0.972	-	0.983	-	0.701	-	-	-	-	-	-	0.746
AH359	1,000	0.976	-	0.986	-	0.707	-	-	-	-	-	-	0.746
AH360	1,000	0.984	-	0.986	-	0.715	-	-	-	-	-	-	0.746
ami33	33	0.941	-	0.947	-	0.723	-	-	-	-	-	-	0.811
ami39	49	0.946	-	0.979	-	0.733	-	-	-	-	-	-	0.832
ami146	146	0.816	-	0.766	-	0.421	-	-	-	-	-	-	0.356
pc1500	500	0.997	-	0.993	-	0.818	-	-	-	-	-	-	0.867
pc100	100	0.971	-	0.970	-	0.703	-	-	-	-	-	-	0.767
tp200	200	0.971	-	0.972	-	0.703	-	-	-	-	-	-	0.752
beng01	20	0.886	-	0.926	-	0.474	-	0.600	-	0.709	-	-	0.633
beng02	40	0.927	-	0.952	-	0.714	-	-	-	-	-	-	0.798
beng03	60	0.950	-	0.952	-	0.744	-	-	-	-	-	-	0.790
beng04	80	0.957	-	0.957	-	0.750	-	-	-	-	-	-	0.790
beng05	100	0.955	-	0.959	-	0.735	-	-	-	-	-	-	0.782
beng06	40	0.941	-	0.956	-	0.704	-	-	-	-	-	-	0.780
beng07	80	0.957	-	0.958	-	0.745	-	-	-	-	-	-	0.790
beng08	120	0.956	-	0.958	-	0.744	-	-	-	0.814	-	-	0.778
beng09	160	0.961	-	0.961	-	0.737	-	-	-	-	-	-	0.775
beng10	200	0.963	-	0.963	-	0.746	-	-	-	-	-	-	0.773
n1	10	0.143	-	0.218	-	0.143	-	-	-	0.218	-	-	0.218*
n2	20	0.690	-	0.698	-	0.385	-	-	-	-	-	-	0.433
n3	30	0.894	-	0.867	-	0.600	-	-	-	-	-	-	0.679
n4	40	0.770	-	0.733	-	0.471	-	-	-	-	-	-	0.512
n5	50	0.837	-	0.807	-	0.500	-	-	-	-	-	-	0.550
n6	60	0.351	-	0.339	-	-	-	-	-	-	-	-	-
n7	70	0.815	-	0.761	-	-	-	-	-	-	-	-	0.341
n8	80	0.824	-	0.928	-	0.298	-	-	-	-	-	-	0.341
n9	100	0.388	-	0.360	-	0.673	-	-	-	-	-	-	0.717
n10	200	0.914	-	0.883	-	0.053	-	-	-	-	-	-	0.064
n11	300	0.437	-	0.389	-	0.471	-	-	-	-	-	-	0.064
n12	500	0.997	-	0.995	-	-	-	-	-	-	-	-	0.518
n13	3,152	0.996	-	0.987	-	-	-	-	-	-	-	-	-
C_1_1	20	0.192	-	0.227	-	0.806	-	-	-	-	-	-	0.846
C_1_2	20	0.165	-	0.204	-	0.627	-	-	-	-	-	-	0.688
C_1_3	20	0.292	-	0.346	-	0.032	-	-	-	0.172	-	-	0.083
C_1_4	20	0.283	-	0.324	-	0.056	-	-	-	0.231	-	-	0.083
C_1_5	20	0.118	-	0.154	-	0.152	-	-	-	-	-	-	0.192
C_1_6	20	0.206	-	0.237	-	-	-	-	-	-	-	-	-
C_1_7	20	0.213	-	0.243	-	0.118	-	-	-	0.141	-	-	0.053
C_1_8	20	0.208	-	0.233	-	0.053	-	-	-	-	-	-	0.053
C_1_9	20	0.281	-	0.297	-	0.059	-	-	-	-	-	-	0.053
C_1_10	20	0.220	-	0.248	-	-	-	-	-	-	-	-	0.111
C_1_11	40	0.248	-	0.239	-	-	-	-	-	0.196	-	-	0.049
C_1_12	40	0.180	-	0.208	-	0.026	-	-	-	0.187	-	-	0.042
C_1_13	40	0.242	-	0.258	-	0.026	-	-	-	-	-	-	0.042
C_1_14	40	0.265	-	0.261	-	0.054	-	-	-	-	-	-	0.074
C_1_15	40	0.221	-	0.225	-	0.026	-	-	-	-	-	-	0.033
C_1_16	40	0.377	-	0.408	-	0.121	-	-	-	-	-	-	0.140
C_1_17	40	0.217	-	0.231	-	0.026	-	-	-	-	-	-	0.026

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
C_1_18	40	0.305	0.397	-	-	0.118	-	-	0.134
C_1_19	40	0.163	0.186	-	-	0.026	-	-	0.026
C_1_20	40	0.185	0.186	-	-	-	-	-	-
C_1_21	60	0.359	0.361	-	-	0.113	-	-	0.124
C_1_22	60	0.211	0.250	-	-	0.063	-	-	0.075
C_1_23	60	0.202	0.290	-	-	0.093	-	-	0.106
C_1_24	60	0.434	0.405	-	-	0.126	-	-	0.143
C_1_25	60	0.195	0.199	-	-	-	-	-	-
C_1_26	60	0.197	0.210	-	-	0.026	-	-	0.032
C_1_27	60	0.280	0.273	-	-	0.035	-	-	0.045
C_1_28	60	0.273	0.271	-	-	0.071	-	-	0.074
C_1_29	60	0.166	0.206	-	-	0.017	-	-	0.017
C_1_30	60	0.305	0.303	-	-	0.094	-	-	0.113
C_1_31	80	0.255	0.270	-	-	0.064	-	-	0.071
C_1_32	80	0.303	0.301	-	-	0.082	-	-	0.095
C_1_33	80	0.447	0.429	-	-	0.118	-	-	0.135
C_1_34	80	0.320	0.315	-	-	0.082	-	-	0.092
C_1_35	80	0.259	0.258	-	-	0.054	-	-	0.064
C_1_36	80	0.264	0.272	-	-	0.068	-	-	0.080
C_1_37	80	0.314	0.307	-	-	0.082	-	-	0.089
C_1_38	80	0.323	0.323	-	-	0.091	-	-	0.104
C_1_39	80	0.233	0.227	-	-	0.046	-	-	0.052
C_1_40	80	0.238	0.240	-	-	0.040	-	-	0.051
C_1_41	100	0.210	0.206	-	-	-	-	-	-
C_1_42	100	0.243	0.229	-	-	0.036	-	-	0.041
C_1_43	100	0.196	0.221	-	-	-	-	-	-
C_1_44	100	0.230	0.226	-	-	0.031	-	-	0.038
C_1_45	100	0.224	0.242	-	-	0.065	-	-	0.073
C_1_46	100	0.298	0.290	-	-	0.076	-	-	0.085
C_1_47	100	0.252	0.254	-	-	0.026	-	-	0.034
C_1_48	100	0.242	0.230	-	-	0.037	-	-	0.042
C_1_49	100	0.215	0.221	-	-	0.010	-	-	0.010
C_1_50	100	0.296	0.283	-	-	0.065	-	-	0.070
C_2_51	20	0.892	0.920	-	0.600	0.667	-	0.721	0.784
C_2_52	20	0.836	0.840	-	-	0.417	-	-	0.530
C_2_53	20	0.909	0.924	-	0.600	0.600	-	0.673	0.713
C_2_54	20	0.797	0.806	-	0.308	0.500	-	0.522	0.580
C_2_55	20	0.826	0.930	-	0.167	0.400	-	-	0.528
C_2_56	20	0.882	0.919	-	0.600	0.500	0.642	0.677	0.630
C_2_57	20	0.824	0.926	-	0.364	0.556	-	0.632	0.665
C_2_58	20	0.845	0.939	-	0.579	0.500	-	0.646	0.626
C_2_59	20	0.904	0.938	-	0.667	0.667	-	0.746	0.761
C_2_60	20	0.843	0.936	-	0.571	0.556	-	0.679	0.649
C_2_61	40	0.858	0.941	-	0.600	0.724	-	0.784	0.793
C_2_62	40	0.939	0.952	-	-	0.739	-	0.496	0.790
C_2_63	40	0.878	0.949	-	-	0.739	-	0.815	0.794
C_2_64	40	0.943	0.943	-	0.200	0.727	-	-	0.798
C_2_65	40	0.855	0.942	-	-	0.714	-	-	0.789
C_2_66	40	0.902	0.946	-	0.600	0.714	-	0.729	0.798
C_2_67	40	0.921	0.950	-	0.625	0.750	-	-	0.809
C_2_68	40	0.931	0.938	-	-	0.714	-	-	0.784
C_2_69	40	0.937	0.935	-	0.667	0.739	-	0.793	0.788

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
C-2-70	40	0.846	0.938	-	-	0.714	-	-	0.702
C-2-71	60	0.949	0.958	-	-	0.765	-	0.798	0.785
C-2-72	60	0.947	0.945	-	-	0.757	-	0.784	0.784
C-2-73	60	0.946	0.945	-	-	0.737	-	0.792	0.792
C-2-74	60	0.889	0.956	-	-	0.737	-	0.802	0.802
C-2-75	60	0.800	0.939	-	-	0.737	-	0.807	0.807
C-2-76	60	0.942	0.956	-	-	0.758	-	0.818	0.818
C-2-77	60	0.909	0.948	-	0.292	0.737	-	0.786	0.786
C-2-78	60	0.879	0.948	-	-	0.750	-	0.793	0.793
C-2-79	60	0.950	0.952	-	-	0.750	-	0.789	0.789
C-2-80	60	0.945	0.955	-	-	0.733	-	0.789	0.789
C-2-81	80	0.954	0.951	-	-	0.756	-	0.795	0.795
C-2-82	80	0.866	0.961	-	-	0.745	-	0.795	0.795
C-2-83	80	0.954	0.958	-	-	0.733	-	0.785	0.785
C-2-84	80	0.957	0.957	-	-	0.741	-	0.780	0.780
C-2-85	80	0.953	0.943	-	-	0.741	-	0.787	0.787
C-2-86	80	0.957	0.957	-	-	0.751	-	0.779	0.779
C-2-87	80	0.953	0.953	-	-	0.731	-	0.779	0.779
C-2-88	80	0.889	0.947	-	-	0.755	-	0.783	0.783
C-2-89	80	0.953	0.957	-	-	0.727	-	0.783	0.783
C-2-90	80	0.959	0.961	-	-	0.750	-	0.798	0.798
C-2-91	100	0.956	0.960	-	-	0.750	-	0.784	0.784
C-2-92	100	0.960	0.961	-	-	0.750	-	0.790	0.790
C-2-93	100	0.962	0.952	-	-	0.746	-	0.792	0.792
C-2-94	100	0.956	0.956	-	-	0.741	-	0.786	0.786
C-2-95	100	0.952	0.954	-	-	0.741	-	0.778	0.778
C-2-96	100	0.957	0.958	-	-	0.733	-	0.775	0.775
C-2-97	100	0.965	0.965	-	-	0.750	-	0.799	0.799
C-2-98	100	0.963	0.951	-	-	0.754	-	0.788	0.788
C-2-99	100	0.880	0.962	-	-	0.745	-	0.789	0.789
C-2-100	100	0.959	0.962	-	-	0.741	-	0.784	0.784
C-3-101	20	0.515	0.510	-	-	0.200	-	0.255	0.255
C-3-102	20	0.569	0.571	-	-	0.118	-	0.164	0.164
C-3-103	20	0.555	0.567	-	-	0.286	-	0.315	0.315
C-3-104	20	0.694	0.698	-	-	0.310	-	0.388	0.388
C-3-105	20	0.537	0.562	-	-	0.226	-	0.268	0.268
C-3-106	20	0.504	0.500	-	0.062	0.200	-	0.244	0.244
C-3-107	20	0.560	0.570	-	-	0.172	-	0.243	0.243
C-3-108	20	0.572	0.573	-	-	0.200	-	0.246	0.246
C-3-109	20	0.525	0.516	-	-	0.200	-	0.245	0.245
C-3-110	20	0.534	0.565	-	-	0.200	-	0.248	0.248
C-3-111	40	0.557	0.553	-	-	0.226	-	0.255	0.255
C-3-112	40	0.508	0.520	-	-	0.220	-	0.252	0.252
C-3-113	40	0.538	0.550	-	-	0.241	-	0.274	0.274
C-3-114	40	0.579	0.574	-	-	0.226	-	0.285	0.285
C-3-115	40	0.543	0.544	-	-	0.250	-	0.292	0.292
C-3-116	40	0.632	0.642	-	-	0.214	-	0.241	0.241
C-3-117	40	0.590	0.582	-	-	0.213	-	0.241	0.241
C-3-118	40	0.627	0.619	-	-	0.250	-	0.279	0.279
C-3-119	40	0.516	0.510	-	-	0.219	-	0.233	0.233
C-3-120	40	0.501	0.497	-	-	0.194	-	0.217	0.217
C-3-121	60	0.595	0.576	-	-	0.234	-	0.255	0.255

Continued on next page

Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
C_3_122	60	0.572	0.559	-	-	0.317	-	-	0.369
C_3_123	60	0.591	0.514	-	-	0.222	-	-	0.251
C_3_124	60	0.588	0.588	-	-	0.209	-	-	0.231
C_3_125	60	0.581	0.528	-	-	0.200	-	-	0.224
C_3_126	60	0.591	0.533	-	-	0.213	-	-	0.230
C_3_127	60	0.592	0.571	-	-	0.191	-	-	0.237
C_3_128	60	0.557	0.538	-	-	0.191	-	-	0.214
C_3_129	60	0.577	0.566	-	-	0.238	-	-	0.272
C_3_130	60	0.545	0.531	-	-	0.208	-	-	0.234
C_3_131	80	0.555	0.528	-	-	0.187	-	-	0.203
C_3_132	80	0.620	0.594	-	-	0.235	-	-	0.263
C_3_133	80	0.627	0.592	-	-	0.205	-	-	0.229
C_3_134	80	0.570	0.541	-	-	0.206	-	-	0.227
C_3_135	80	0.474	0.518	-	-	0.187	-	-	0.217
C_3_136	80	0.581	0.546	-	-	0.194	-	-	0.208
C_3_137	80	0.547	0.524	-	-	0.194	-	-	0.218
C_3_138	80	0.618	0.588	-	-	0.221	-	-	0.249
C_3_139	80	0.579	0.550	-	-	0.220	-	-	0.239
C_3_140	80	0.579	0.556	-	-	0.210	-	-	0.230
C_3_141	100	0.565	0.530	-	-	0.205	-	-	0.228
C_3_142	100	0.584	0.548	-	-	0.200	-	-	0.222
C_3_143	100	0.602	0.545	-	-	0.187	-	-	0.201
C_3_144	100	0.560	0.523	-	-	0.192	-	-	0.210
C_3_145	100	0.573	0.527	-	-	0.188	-	-	0.203
C_3_146	100	0.579	0.540	-	-	0.200	-	-	0.220
C_3_147	100	0.652	0.583	-	-	0.195	-	-	0.224
C_3_148	100	0.581	0.554	-	-	0.210	-	-	0.228
C_3_149	100	0.591	0.556	-	-	0.200	-	-	0.222
C_3_150	100	0.602	0.567	-	-	0.192	-	-	0.223
C_4_151	20	0.888	0.946	-	0.538	0.500	-	0.645	0.652
C_4_152	20	0.712	0.735	-	-	0.286	-	-	0.381
C_4_153	20	0.867	0.944	-	-	0.556	-	0.742	0.660
C_4_154	20	0.751	0.837	-	-	0.364	-	0.506	0.507
C_4_155	20	0.785	0.864	-	-	0.185	-	-	0.453
C_4_156	20	0.896	0.945	-	-	0.375	-	0.548	0.497
C_4_157	20	0.797	0.868	-	-	0.231	-	0.398	0.363
C_4_158	20	0.772	0.865	-	-	0.103	-	0.558	0.474
C_4_159	20	0.898	0.945	-	-	0.600	-	0.750	0.750
C_4_160	20	0.865	0.954	-	-	0.200	-	0.373	0.373
C_4_161	40	0.964	0.969	-	-	0.692	-	-	0.717
C_4_162	40	0.952	0.965	-	-	0.667	-	-	0.751
C_4_163	40	0.948	0.962	-	-	0.704	-	-	0.779
C_4_164	40	0.954	0.966	-	-	0.667	-	0.467	0.738
C_4_165	40	0.960	0.965	-	-	0.655	-	-	0.758
C_4_166	40	0.952	0.966	-	-	0.643	-	-	0.747
C_4_167	40	0.953	0.969	-	-	0.667	-	-	0.769
C_4_168	40	0.949	0.959	-	-	0.643	-	-	0.742
C_4_169	40	0.944	0.955	-	-	0.692	-	-	0.789
C_4_170	40	0.949	0.957	-	-	0.655	-	-	0.740
C_4_171	60	0.967	0.969	-	-	0.659	-	-	0.758
C_4_172	60	0.960	0.970	-	-	0.700	-	-	0.779
C_4_173	60	0.963	0.965	-	-	0.700	-	-	0.793

Continued on next page

Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}			UB_{avg}			δ_{min}			δ_{min}		
		UB_{min}	MIQP	GF	UB_{avg}	MIQP	GF	BD_{min}	MIQP	GF	BD_{min}	MIQP	GF
C_4_174	60	0.966	-	0.968	-	-	-	-	-	-	-	-	0.773
C_4_175	60	0.963	-	0.969	-	-	-	-	-	-	-	-	0.784
C_4_176	60	0.965	-	0.965	-	-	-	-	-	-	-	-	0.776
C_4_177	60	0.965	-	0.967	-	-	-	-	-	-	-	-	0.786
C_4_178	60	0.966	-	0.967	-	-	-	-	-	-	-	-	0.787
C_4_180	60	0.964	-	0.972	-	-	-	-	-	-	-	-	0.781
C_4_181	80	0.964	-	0.965	-	-	-	-	-	-	-	-	0.786
C_4_182	80	0.971	-	0.974	-	-	-	-	-	-	-	-	0.771
C_4_183	80	0.969	-	0.972	-	-	-	-	-	-	-	-	0.785
C_4_184	80	0.968	-	0.970	-	-	-	-	-	-	-	-	0.793
C_4_185	80	0.963	-	0.969	-	-	-	-	-	-	-	-	0.797
C_4_186	80	0.971	-	0.972	-	-	-	-	-	-	-	-	0.779
C_4_187	80	0.961	-	0.966	-	-	-	-	-	-	-	-	0.765
C_4_188	80	0.970	-	0.973	-	-	-	-	-	-	-	-	0.759
C_4_189	80	0.965	-	0.971	-	-	-	-	-	-	-	-	0.792
C_4_190	80	0.971	-	0.974	-	-	-	-	-	-	-	-	0.792
C_4_191	100	0.969	-	0.974	-	-	-	-	-	-	-	-	0.793
C_4_192	100	0.973	-	0.975	-	-	-	-	-	-	-	-	0.790
C_4_193	100	0.974	-	0.973	-	-	-	-	-	-	-	-	0.793
C_4_194	100	0.968	-	0.971	-	-	-	-	-	-	-	-	0.794
C_4_195	100	0.969	-	0.970	-	-	-	-	-	-	-	-	0.771
C_4_196	100	0.968	-	0.972	-	-	-	-	-	-	-	-	0.795
C_4_197	100	0.978	-	0.977	-	-	-	-	-	-	-	-	0.792
C_4_198	100	0.972	-	0.975	-	-	-	-	-	-	-	-	0.782
C_4_199	100	0.976	-	0.976	-	-	-	-	-	-	-	-	0.795
C_4_200	100	0.970	-	0.974	-	-	-	-	-	-	-	-	0.134
C_5_201	20	0.349	-	0.381	-	-	-	-	-	-	-	-	0.098
C_5_202	20	0.483	-	0.482	-	-	-	-	-	-	-	-	0.205
C_5_203	20	0.394	-	0.394	-	-	-	-	-	-	-	-	0.235
C_5_204	20	0.510	-	0.508	-	-	-	-	-	-	-	-	0.110
C_5_205	20	0.273	-	0.313	-	-	-	-	-	-	-	-	0.086
C_5_206	20	0.329	-	0.337	-	-	-	-	-	-	-	-	0.124
C_5_207	20	0.345	-	0.345	-	-	-	-	-	-	-	-	0.160
C_5_208	20	0.360	-	0.373	-	-	-	-	-	-	-	-	0.103
C_5_209	20	0.295	-	0.327	-	-	-	-	-	-	-	-	0.070
C_5_210	20	0.348	-	0.358	-	-	-	-	-	-	-	-	0.133
C_5_211	40	0.402	-	0.400	-	-	-	-	-	-	-	-	0.137
C_5_212	40	0.349	-	0.344	-	-	-	-	-	-	-	-	0.100
C_5_213	40	0.400	-	0.394	-	-	-	-	-	-	-	-	0.160
C_5_214	40	0.372	-	0.373	-	-	-	-	-	-	-	-	0.119
C_5_215	40	0.317	-	0.315	-	-	-	-	-	-	-	-	0.196
C_5_216	40	0.544	-	0.562	-	-	-	-	-	-	-	-	0.097
C_5_217	40	0.399	-	0.395	-	-	-	-	-	-	-	-	0.180
C_5_218	40	0.466	-	0.457	-	-	-	-	-	-	-	-	0.139
C_5_219	40	0.337	-	0.331	-	-	-	-	-	-	-	-	0.095
C_5_220	40	0.336	-	0.330	-	-	-	-	-	-	-	-	0.162
C_5_221	60	0.416	-	0.401	-	-	-	-	-	-	-	-	0.124
C_5_222	60	0.358	-	0.348	-	-	-	-	-	-	-	-	0.137
C_5_223	60	0.336	-	0.327	-	-	-	-	-	-	-	-	0.170
C_5_224	60	0.503	-	0.482	-	-	-	-	-	-	-	-	0.112
C_5_225	60	0.303	-	0.298	-	-	-	-	-	-	-	-	0.094

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Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
C_5_226	60	0.351	0.345	-	-	0.126	0.141
C_5_227	60	0.364	0.304	-	-	0.129	0.144
C_5_228	60	0.338	0.324	-	-	0.105	0.124
C_5_229	60	0.342	0.328	-	-	0.129	0.132
C_5_230	60	0.408	0.385	-	-	0.146	0.158
C_5_231	80	0.409	0.380	-	-	0.124	0.136
C_5_232	80	0.418	0.418	-	-	0.165	0.182
C_5_233	80	0.523	0.496	-	-	0.141	0.151
C_5_234	80	0.428	0.395	-	-	0.132	0.117
C_5_235	80	0.352	0.319	-	-	0.102	0.113
C_5_236	80	0.381	0.358	-	-	0.118	0.133
C_5_237	80	0.360	0.342	-	-	0.114	0.124
C_5_238	80	0.404	0.382	-	-	0.121	0.150
C_5_239	80	0.363	0.345	-	-	0.121	0.138
C_5_240	80	0.358	0.346	-	-	0.116	0.134
C_5_241	100	0.314	0.294	-	-	0.105	0.117
C_5_242	100	0.381	0.350	-	-	0.106	0.118
C_5_243	100	0.329	0.308	-	-	0.089	0.098
C_5_244	100	0.340	0.316	-	-	0.103	0.113
C_5_245	100	0.358	0.325	-	-	0.107	0.122
C_5_246	100	0.377	0.348	-	-	0.119	0.136
C_5_247	100	0.405	0.373	-	-	0.119	0.134
C_5_248	100	0.371	0.338	-	-	0.117	0.129
C_5_249	100	0.363	0.331	-	-	0.101	0.113
C_5_250	100	0.418	0.385	-	-	0.111	0.130
C_6_251	20	0.878	0.956	-	0.600	0.529	0.699
C_6_252	20	0.593	0.598	-	-	0.152	0.222
C_6_253	20	0.879	0.957	-	0.600	0.529	0.634
C_6_254	20	0.727	0.728	-	-	0.385	0.440
C_6_255	20	0.941	0.876	-	0.400	0.500	0.603
C_6_256	20	0.777	0.962	-	0.333	0.333	0.443
C_6_257	20	0.805	0.878	-	-	0.400	0.486
C_6_258	20	0.752	0.876	-	0.154	0.167	0.267
C_6_259	20	0.945	0.951	-	0.375	0.444	0.583
C_6_260	20	0.922	0.958	-	0.143	0.333	0.454
C_6_261	40	0.970	0.971	-	0.407	0.613	0.672
C_6_262	40	0.967	0.968	-	-	0.667	0.759
C_6_263	40	0.954	0.967	-	-	0.643	0.755
C_6_264	40	0.966	0.970	-	-	0.667	0.800
C_6_265	40	0.966	0.968	-	-	0.643	0.742
C_6_266	40	0.960	0.970	-	-	0.600	0.692
C_6_267	40	0.962	0.973	-	-	0.692	0.780
C_6_268	40	0.957	0.965	-	-	0.630	0.693
C_6_269	40	0.959	0.962	-	-	0.655	0.762
C_6_270	40	0.962	0.962	-	-	0.600	0.696
C_6_271	60	0.968	0.971	-	-	0.591	0.677
C_6_272	60	0.963	0.973	-	-	0.667	0.635
C_6_273	60	0.965	0.968	-	-	0.619	0.689
C_6_274	60	0.969	0.972	-	-	0.556	0.645
C_6_275	60	0.970	0.972	-	-	0.674	0.718
C_6_276	60	0.963	0.970	-	-	0.610	0.681
C_6_277	60	0.975	0.977	-	-	0.622	0.714

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD $_{min}$	GF	
C-6-278	60	0.970	0.969	-	-	0.571	0.635
C-6-279	60	0.969	0.976	-	-	0.650	0.717
C-6-280	60	0.965	0.965	-	-	0.706	0.757
C-6-281	80	0.970	0.971	-	-	0.606	0.648
C-6-282	80	0.973	0.975	-	-	0.698	0.763
C-6-283	80	0.974	0.975	-	-	0.552	0.626
C-6-284	80	0.973	0.974	-	-	0.607	0.703
C-6-285	80	0.968	0.974	-	-	0.593	0.640
C-6-286	80	0.973	0.972	-	-	0.545	0.605
C-6-287	80	0.966	0.967	-	-	0.633	0.674
C-6-288	80	0.974	0.977	-	-	0.603	0.653
C-6-289	80	0.969	0.972	-	-	0.593	0.647
C-6-290	80	0.974	0.976	-	-	0.608	0.705
C-6-291	100	0.974	0.976	-	-	0.627	0.681
C-6-292	100	0.973	0.974	-	-	0.612	0.655
C-6-293	100	0.977	0.976	-	-	0.635	0.674
C-6-294	100	0.970	0.973	-	-	0.600	0.666
C-6-295	100	0.972	0.971	-	-	0.600	0.658
C-6-296	100	0.971	0.973	-	-	0.600	0.680
C-6-297	100	0.978	0.979	-	-	0.577	0.636
C-6-298	100	0.976	0.975	-	-	0.634	0.691
C-6-299	100	0.977	0.978	-	-	0.619	0.678
C-6-300	100	0.973	0.975	-	-	0.564	0.650
C-7-301	20	0.267	0.312	-	-	0.111	0.149
C-7-302	20	0.464	0.469	-	-	0.250	0.278
C-7-303	20	0.287	0.327	-	-	0.118	0.164
C-7-304	20	0.445	0.458	-	-	0.241	0.269
C-7-305	20	0.402	0.427	0.259	-	0.200	0.222
C-7-306	20	0.374	0.375	0.161	-	0.152	0.181
C-7-307	20	0.250	0.317	-	-	0.152	0.176
C-7-308	20	0.457	0.458	-	-	0.161	0.188
C-7-309	20	0.408	0.407	-	-	0.118	0.149
C-7-310	20	0.341	0.384	-	-	0.097	0.148
C-7-311	40	0.375	0.372	-	-	0.161	0.183
C-7-312	40	0.435	0.431	-	-	0.228	0.250
C-7-313	40	0.329	0.327	-	-	0.156	0.171
C-7-314	40	0.468	0.464	-	-	0.241	0.264
C-7-315	40	0.431	0.426	-	-	0.182	0.194
C-7-316	40	0.358	0.353	-	-	0.182	0.194
C-7-317	40	0.470	0.464	-	-	0.238	0.253
C-7-318	40	0.375	0.370	-	-	0.164	0.184
C-7-319	40	0.412	0.409	-	-	0.169	0.186
C-7-320	40	0.491	0.486	-	-	0.276	0.292
C-7-321	60	0.372	0.359	-	-	0.179	0.188
C-7-322	60	0.341	0.332	-	-	0.174	0.187
C-7-323	60	0.418	0.406	-	-	0.208	0.225
C-7-324	60	0.364	0.356	-	-	0.158	0.169
C-7-325	60	0.425	0.416	-	-	0.200	0.219
C-7-326	60	0.493	0.481	-	-	0.217	0.237
C-7-327	60	0.410	0.400	-	-	0.184	0.204
C-7-328	60	0.369	0.411	-	-	0.200	0.211
C-7-329	60	0.361	0.349	-	-	0.182	0.196

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Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}		UB_{avg}		δ_{min}		δ_{min}	
		MIQP	GF	MIQP	GF	MIQP	GF	MIQP	GF
C-7-330	60	0.332	0.325	-	0.157	-	-	-	0.176
C-7-331	80	0.405	0.375	-	0.177	-	-	-	0.190
C-7-332	80	0.402	0.375	-	0.193	-	-	-	0.216
C-7-333	80	0.402	0.383	-	0.163	-	-	-	0.176
C-7-334	80	0.406	0.383	-	0.171	-	-	-	0.183
C-7-335	80	0.406	0.408	-	0.216	-	-	-	0.229
C-7-336	80	0.436	0.408	-	0.185	-	-	-	0.205
C-7-337	80	0.390	0.368	-	0.165	-	-	-	0.180
C-7-338	80	0.414	0.392	-	0.169	-	-	-	0.187
C-7-339	80	0.460	0.432	-	0.176	-	-	-	0.195
C-7-340	80	0.379	0.399	-	0.200	-	-	-	0.218
C-7-341	100	0.435	0.396	-	0.179	-	-	-	0.194
C-7-342	100	0.448	0.405	-	0.157	-	-	-	0.175
C-7-343	100	0.470	0.424	-	0.188	-	-	-	0.207
C-7-344	100	0.460	0.420	-	0.203	-	-	-	0.224
C-7-345	100	0.442	0.416	-	0.174	-	-	-	0.187
C-7-346	100	0.360	0.341	-	0.156	-	-	-	0.170
C-7-347	100	0.462	0.427	-	0.192	-	-	-	0.209
C-7-348	100	0.464	0.416	-	0.193	-	-	-	0.217
C-7-349	100	0.385	0.359	-	0.160	-	-	-	0.174
C-7-350	100	0.420	0.378	-	0.179	-	-	-	0.197
C-8-351	20	0.893	0.945	0.375	0.467	0.583	0.601	-	0.601
C-8-352	20	0.497	0.504	-	0.241	-	-	-	0.295
C-8-353	20	0.477	0.471	-	0.241	-	-	-	0.264
C-8-354	20	0.176	0.209	0.152	0.056	0.301	0.093	-	0.093
C-8-355	20	0.523	0.530	-	0.212	-	-	-	0.282
C-8-356	20	0.471	0.475	-	0.241	-	-	-	0.272
C-8-357	20	0.232	0.255	-	0.056	-	-	-	0.080
C-8-358	20	0.290	0.300	-	-	-	-	-	0.064
C-8-359	20	0.163	0.190	0.059	0.053	0.159	0.070	-	0.070
C-8-360	20	0.925	0.952	0.667	0.500	0.639	0.639	-	0.639
C-8-361	40	0.271	0.266	-	0.083	-	-	-	0.102
C-8-362	40	0.268	0.267	-	0.096	-	-	-	0.108
C-8-363	40	0.289	0.232	-	0.067	-	-	-	0.083
C-8-364	40	0.311	0.305	-	0.083	-	-	-	0.103
C-8-365	40	0.701	0.712	-	0.378	-	-	-	0.426
C-8-366	40	0.261	0.255	-	0.083	-	-	-	0.094
C-8-367	40	0.277	0.273	-	0.083	-	-	-	0.103
C-8-368	40	0.262	0.265	-	0.099	-	-	-	0.120
C-8-369	40	0.262	0.258	-	0.096	-	-	-	0.107
C-8-370	40	0.240	0.238	-	0.083	-	-	-	0.104
C-8-371	60	0.539	0.520	-	0.218	-	-	-	0.244
C-8-372	60	0.221	0.226	-	0.091	-	-	-	0.097
C-8-373	60	0.212	0.231	-	0.093	-	-	-	0.100
C-8-374	60	0.289	0.280	-	0.083	-	-	-	0.094
C-8-375	60	0.260	0.258	-	0.091	-	-	-	0.098
C-8-376	60	0.272	0.263	-	0.084	-	-	-	0.098
C-8-377	60	0.290	0.282	-	0.091	-	-	-	0.103
C-8-378	60	0.262	0.256	-	0.093	-	-	-	0.105
C-8-379	60	0.250	0.246	-	0.084	-	-	-	0.098
C-8-380	60	0.237	0.256	-	0.091	-	-	-	0.101
C-8-381	80	0.240	0.250	-	0.082	-	-	-	0.092

Continued on next page

Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
C_8_382	80	0.300	0.286	-	-	0.076	0.085
C_8_383	80	0.294	0.281	-	-	0.082	0.083
C_8_384	80	0.269	0.254	-	-	0.085	0.090
C_8_385	80	0.242	0.231	-	-	0.075	0.084
C_8_386	80	0.240	0.241	-	-	0.081	0.090
C_8_387	80	0.263	0.249	-	-	0.081	0.097
C_8_388	80	0.211	0.237	-	-	0.088	0.088
C_8_389	80	0.311	0.290	-	-	0.075	0.092
C_8_390	80	0.245	0.230	-	-	0.082	0.075
C_8_391	100	0.200	0.263	-	-	0.075	0.085
C_8_392	100	0.298	0.270	-	-	0.071	0.084
C_8_393	100	0.282	0.261	-	-	0.075	0.086
C_8_394	100	0.259	0.234	-	-	0.083	0.093
C_8_395	100	0.288	0.263	-	-	0.076	0.085
C_8_396	100	0.219	0.240	-	-	0.076	0.093
C_8_397	100	0.285	0.264	-	-	0.077	0.084
C_8_398	100	0.267	0.251	-	-	0.075	0.089
C_8_399	100	0.284	0.268	-	-	0.077	0.094
C_8_400	100	0.269	0.245	-	-	0.087	0.257
C_9_401	20	0.379	0.432	0.241	-	0.226	0.267
C_9_402	20	0.456	0.453	-	-	0.241	0.217
C_9_403	20	0.306	0.363	-	-	0.188	0.297
C_9_404	20	0.484	0.481	-	-	0.267	0.196
C_9_405	20	0.438	0.470	-	-	0.188	0.180
C_9_406	20	0.333	0.398	-	-	0.152	0.128
C_9_407	20	0.362	0.361	-	-	0.086	0.192
C_9_408	20	0.313	0.408	-	-	0.152	0.217
C_9_409	20	0.403	0.404	-	-	0.188	0.172
C_9_410	20	0.446	0.444	-	-	0.125	0.211
C_9_411	40	0.392	0.391	-	-	0.188	0.248
C_9_412	40	0.456	0.453	-	-	0.233	0.198
C_9_413	40	0.409	0.409	-	-	0.182	0.233
C_9_414	40	0.459	0.456	-	-	0.226	0.207
C_9_415	40	0.460	0.456	-	-	0.182	0.217
C_9_416	40	0.433	0.430	-	-	0.206	0.172
C_9_417	40	0.430	0.429	-	-	0.152	0.215
C_9_418	40	0.465	0.457	-	-	0.200	0.192
C_9_419	40	0.409	0.418	-	-	0.169	0.281
C_9_420	40	0.413	0.462	-	-	0.258	0.213
C_9_421	60	0.414	0.413	-	-	0.196	0.193
C_9_422	60	0.410	0.400	-	-	0.184	0.236
C_9_423	60	0.449	0.444	-	-	0.217	0.234
C_9_424	60	0.494	0.486	-	-	0.217	0.216
C_9_425	60	0.476	0.463	-	-	0.196	0.196
C_9_426	60	0.386	0.432	-	-	0.175	0.224
C_9_427	60	0.493	0.479	-	-	0.213	0.243
C_9_428	60	0.489	0.482	-	-	0.226	0.192
C_9_429	60	0.403	0.402	-	-	0.172	0.188
C_9_430	60	0.386	0.428	-	-	0.180	0.198
C_9_431	80	0.480	0.455	-	-	0.188	0.203
C_9_432	80	0.479	0.467	-	-	0.185	0.208
C_9_433	80	0.490	0.468	-	-	0.194	-

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Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			GF	
				MIQP	BD _{min}	MIQP		BD _{min}
C ₉ _434	80	0.480	0.450	-	-	-	0.182	0.197
C ₉ _435	80	0.482	0.439	-	-	-	0.203	0.218
C ₉ _436	80	0.485	0.471	-	-	-	0.200	0.199
C ₉ _437	80	0.478	0.451	-	-	-	0.185	0.220
C ₉ _438	80	0.506	0.478	-	-	-	0.203	0.181
C ₉ _439	80	0.480	0.458	-	-	-	0.164	0.213
C ₉ _440	80	0.389	0.399	-	-	-	0.194	0.190
C ₉ _441	100	0.476	0.339	-	-	-	0.181	0.155
C ₉ _442	100	0.435	0.398	-	-	-	0.143	0.188
C ₉ _443	100	0.481	0.453	-	-	-	0.176	0.212
C ₉ _444	100	0.441	0.419	-	-	-	0.196	0.156
C ₉ _445	100	0.445	0.409	-	-	-	0.146	0.196
C ₉ _446	100	0.474	0.480	-	-	-	0.181	0.200
C ₉ _447	100	0.477	0.452	-	-	-	0.183	0.209
C ₉ _448	100	0.450	0.458	-	-	-	0.192	0.209
C ₉ _449	100	0.449	0.395	-	-	-	0.150	0.179
C ₉ _450	100	0.388	0.400	-	-	-	0.162	0.175
C ₁₀ _451	20	0.452	0.428	-	0.214	-	-	0.046
C ₁₀ _452	20	0.631	0.639	-	-	-	0.250	0.314
C ₁₀ _453	20	0.464	0.461	-	-	-	0.086	0.116
C ₁₀ _454	20	0.521	0.538	-	-	-	0.070	0.088
C ₁₀ _455	20	0.473	0.476	-	-	-	0.056	0.325
C ₁₀ _456	20	0.638	0.656	-	-	-	0.267	0.110
C ₁₀ _457	20	0.317	0.352	-	0.077	-	0.056	0.256
C ₁₀ _458	20	0.310	0.311	-	0.231	-	0.053	0.242
C ₁₀ _459	20	0.365	0.364	-	-	-	0.161	0.183
C ₁₀ _460	20	0.904	0.955	-	0.529	-	0.444	0.568
C ₁₀ _461	40	0.455	0.448	-	-	-	0.143	0.167
C ₁₀ _462	40	0.416	0.406	-	-	-	0.114	0.134
C ₁₀ _463	40	0.462	0.454	-	-	-	0.118	0.145
C ₁₀ _464	40	0.386	0.379	-	-	-	0.111	0.124
C ₁₀ _465	40	0.779	0.781	-	-	-	0.388	0.444
C ₁₀ _466	40	0.401	0.396	-	-	-	0.081	0.102
C ₁₀ _467	40	0.377	0.368	-	-	-	0.114	0.141
C ₁₀ _468	40	0.410	0.402	-	-	-	0.134	0.155
C ₁₀ _469	40	0.435	0.430	-	-	-	0.138	0.160
C ₁₀ _470	40	0.431	0.421	-	-	-	0.143	0.160
C ₁₀ _471	60	0.390	0.378	-	-	-	0.111	0.130
C ₁₀ _472	60	0.443	0.431	-	-	-	0.129	0.143
C ₁₀ _473	60	0.360	0.350	-	-	-	0.115	0.128
C ₁₀ _474	60	0.444	0.430	-	-	-	0.091	0.103
C ₁₀ _475	60	0.396	0.384	-	-	-	0.115	0.145
C ₁₀ _476	60	0.468	0.460	-	-	-	0.129	0.108
C ₁₀ _477	60	0.418	0.411	-	-	-	0.091	0.145
C ₁₀ _478	60	0.693	0.663	-	-	-	0.282	0.316
C ₁₀ _479	60	0.351	0.344	-	-	-	0.120	0.138
C ₁₀ _480	60	0.338	0.330	-	-	-	0.093	0.105
C ₁₀ _481	80	0.400	0.378	-	-	-	0.132	0.145
C ₁₀ _482	80	0.388	0.376	-	-	-	0.101	0.114
C ₁₀ _483	80	0.428	0.404	-	-	-	0.100	0.116
C ₁₀ _484	80	0.402	0.380	-	-	-	0.126	0.141
C ₁₀ _485	80	0.450	0.428	-	-	-	0.137	0.153

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
C_10_486	80	0.391	0.369	-	-	0.121	0.139
C_10_487	80	0.407	0.384	-	-	0.125	0.123
C_10_488	80	0.292	0.387	-	-	0.091	0.103
C_10_489	80	0.365	0.352	-	-	0.097	0.111
C_10_490	80	0.360	0.385	-	-	0.113	0.128
C_10_491	100	0.373	0.335	-	-	0.093	0.109
C_10_492	100	0.366	0.326	-	-	0.083	0.093
C_10_493	100	0.437	0.398	-	-	0.112	0.124
C_10_494	100	0.475	0.436	-	-	0.133	0.150
C_10_495	100	0.452	0.408	-	-	0.113	0.133
C_10_496	100	0.337	0.312	-	-	0.082	0.094
C_10_497	100	0.383	0.357	-	-	0.090	0.107
C_10_498	100	0.417	0.384	-	-	0.091	0.103
C_10_499	100	0.373	0.346	-	-	0.092	0.101
C_10_500	100	0.334	0.317	-	-	0.107	0.115
gcut1	16	0.563	0.595	-	-	0.250	0.316
gcut2	23	0.915	0.932	-	-	0.083	0.133
gcut3	62	0.680	0.836	-	-	0.455	0.496
lw161	23	0.779	0.779	-	-	0.474	0.590
lw163	16	0.759	0.767	-	0.500	0.474	0.592
lw172	17	0.584	0.613	-	0.474	0.474	0.582
lw251	25	0.897	0.924	-	0.368	0.333	0.398
lw252	25	0.854	0.845	-	0.571	0.714	0.796
lw253	25	0.844	0.845	-	0.500	0.583	0.638
lw281	28	0.927	0.944	-	0.667	0.684	0.604
lw283	28	0.933	0.952	-	-	0.750	0.818
lw292	29	0.931	0.954	-	0.524	0.652	0.800
lw491	49	0.952	0.951	-	-	0.652	0.760
lw492	49	0.961	0.962	-	-	0.778	0.816
lw493	49	0.961	0.966	-	-	0.724	0.803
lw731	73	0.968	0.896	-	-	0.600	0.659
lw732	73	0.975	0.976	-	-	0.784	0.833
lw733	73	0.968	0.976	-	-	0.765	0.832
lw971	97	0.972	0.973	-	-	0.778	0.829
lw972	97	0.975	0.977	-	-	0.769	0.822
lw973	97	0.970	0.980	-	-	0.844	0.822
lw1961	196	0.978	0.976	-	-	0.774	0.837
lw1963	196	0.984	0.985	-	-	0.760	0.816
lw1972	197	0.988	0.989	-	-	0.780	0.792
50	50	0.935	0.941	-	-	0.786	0.825
100	100	0.925	0.941	0.600	0.588	-	0.834
500	500	0.997	0.992	-	-	0.197	0.021
1000	1000	0.998	0.995	-	-	0.829	0.230
5000	5000	0.996	0.998	-	-	0.833	0.861
10000	10000	0.997	0.996	-	-	0.615	0.864
15000	15000	0.998	0.997	-	-	0.474	0.683
gcut01	10	0.250	0.996	-	-	0.346	0.528
gcut02	20	0.680	0.333	-	-	-	0.403
gcut03	30	0.712	0.684	-	-	-	0.257*
gcut04	50	0.704	0.712	-	-	-	0.257
gcut05	10	0.500	0.623	0.167	-	-	0.487
gcut06	20	0.678	0.687	-	0.167	0.167	0.414
						0.391	0.373
						-	0.233
						-	0.431

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{avg}		UB _{min}		UB _{avg}		UB _{min}		GF		δ_{min}		GF	
		MIQP	BD _{min}	MIQP	BD _{min}	MIQP	BD _{min}	MIQP	BD _{min}	MIQP	BD _{min}	MIQP	BD _{min}	MIQP	BD _{min}
gcut07	30	0.731	0.727	0.727	0.727	0.727	0.727	0.727	0.727	0.438	0.438	-	-	0.463	0.463
gcut08	50	0.695	0.692	0.692	0.692	0.692	0.692	0.692	0.692	0.370	0.370	-	-	0.419	0.419
gcut09	10	0.757	0.807	0.807	0.807	0.807	0.807	0.807	0.807	0.400	0.400	-	-	0.173*	0.173*
gcut10	20	0.729	0.735	0.735	0.735	0.735	0.735	0.735	0.735	0.429	0.429	-	-	0.463	0.463
gcut11	30	0.755	0.840	0.840	0.840	0.840	0.840	0.840	0.840	0.379	0.379	-	-	0.478	0.478
gcut12	50	0.699	0.697	0.697	0.697	0.697	0.697	0.697	0.697	0.778	0.778	-	-	0.420	0.420
gcut13	32	0.953	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.086*	0.086*	-	-	0.831	0.831
hcs	8	0.000	0.086	0.086	0.086	0.086	0.086	0.086	0.086	0.000*	0.000*	-	-	0.086*	0.086*
hcs	7	0.250	0.355	0.355	0.355	0.355	0.355	0.355	0.355	0.000*	0.000*	-	-	0.150*	0.150*
hcs	15	0.640	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.333	0.333	-	-	0.425	0.425
hcs	19	0.897	0.909	0.909	0.909	0.909	0.909	0.909	0.909	0.600	0.600	-	-	0.318	0.318
hcs	21	0.886	0.908	0.908	0.908	0.908	0.908	0.908	0.908	0.111	0.111	-	-	0.193	0.193
hcs11	15	0.896	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.571	0.571	-	-	0.697	0.697
leung1	10	0.600	0.831	0.831	0.831	0.831	0.831	0.831	0.831	0.439	0.439	-	-	0.556	0.556
leung2	15	0.077	0.114	0.114	0.114	0.114	0.114	0.114	0.114	0.200*	0.200*	-	-	0.164	0.164
leung3	20	0.854	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.077	0.077	-	-	0.325*	0.325*
leung4	20	0.923	0.909	0.909	0.909	0.909	0.909	0.909	0.909	0.412	0.412	-	-	0.537	0.537
leung5	25	0.927	0.940	0.940	0.940	0.940	0.940	0.940	0.940	0.538	0.538	-	-	0.686	0.686
leung6	25	0.790	0.858	0.858	0.858	0.858	0.858	0.858	0.858	0.667	0.667	-	-	0.791	0.791
leung7	30	0.893	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.565	0.565	-	-	0.558	0.558
leung8	30	0.916	0.952	0.952	0.952	0.952	0.952	0.952	0.952	0.684	0.684	-	-	0.362	0.362
leung9	40	0.939	0.953	0.953	0.953	0.953	0.953	0.953	0.953	0.778	0.778	-	-	0.824	0.824
leung10	50	0.190	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.760	0.760	-	-	0.833	0.833
ngcut01	10	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.114	0.114	-	-	0.128	0.128
ngcut02	17	0.067	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.111	0.111	-	-	0.111	0.111
ngcut03	21	0.053	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.067	0.067	-	-	0.091*	0.091*
ngcut04	7	0.200	0.237	0.237	0.237	0.237	0.237	0.237	0.237	0.053	0.053	-	-	0.173*	0.173*
ngcut05	14	0.000	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.000*	0.000*	-	-	0.062*	0.062*
ngcut06	15	0.167	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.000*	0.000*	-	-	0.183	0.183
ngcut07	8	0.000	0.086	0.086	0.086	0.086	0.086	0.086	0.086	0.111	0.111	-	-	0.086*	0.086*
ngcut08	13	0.800	0.911	0.911	0.911	0.911	0.911	0.911	0.911	0.333	0.333	-	-	0.149	0.149
ngcut09	18	0.795	0.806	0.806	0.806	0.806	0.806	0.806	0.806	0.250	0.250	-	-	0.602	0.602
ngcut10	13	0.083	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.444	0.444	-	-	0.713	0.713
ngcut11	15	0.596	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.083	0.083	-	-	0.555	0.555
ngcut12	22	0.119	0.154	0.154	0.154	0.154	0.154	0.154	0.154	0.083*	0.083*	-	-	0.113*	0.113*
nicee1	25	0.945	0.957	0.957	0.957	0.957	0.957	0.957	0.957	0.333	0.333	-	-	0.399	0.399
nicee2	50	0.942	0.949	0.949	0.949	0.949	0.949	0.949	0.949	0.111	0.111	-	-	0.145	0.145
nicee3	100	0.123	0.172	0.172	0.172	0.172	0.172	0.172	0.172	-	-	-	-	-	-
nicee4	200	0.108	0.173	0.173	0.173	0.173	0.173	0.173	0.173	-	-	-	-	-	-
nicee5	500	0.980	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.708	0.708	-	-	0.084	0.084
nicee6	1,000	0.964	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.708	0.708	-	-	0.739	0.739
nicee3t	100	0.968	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.750	0.750	-	-	0.792	0.792
nicee4t	200	0.968	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.742	0.742	-	-	0.719	0.719
nicee6t	1,000	0.992	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.683	0.683	-	-	0.776	0.776
nicee11t	1,000	0.990	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.692	0.692	-	-	0.721	0.721
nicee12t	2,000	0.955	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.571	0.571	-	-	0.635	0.635
nicee15t	5,000	0.998	0.978	0.978	0.978	0.978	0.978	0.978	0.978	0.315	0.315	-	-	0.354	0.354
nicee21t	1,000	0.977	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.684	0.684	-	-	0.718	0.718
nicee22t	2,000	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.580	0.580	-	-	0.631	0.631
nicee25t	5,000	0.999	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.315	0.315	-	-	0.361	0.361
nicee31t	1,000	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.689	0.689	-	-	0.722	0.722
nicee32t	2,000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.575	0.575	-	-	0.643	0.643

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
nice35t	5,000	0.998	0.998	-	-	0.311	0.518
nice41t	1,000	0.990	0.980	-	-	0.588	0.721
nice42t	2,000	0.999	0.977	-	-	0.589	0.643
nice45t	5,000	0.998	0.997	-	-	0.329	0.958
nice51t	1,000	0.979	0.980	-	-	0.692	0.718
nice52t	1,000	0.951	0.951	-	-	0.591	0.648
nice55t	5,000	0.999	1.001	-	-	0.304	0.943
nice61t	1,000	0.978	0.979	-	-	0.684	0.718
nice62t	5,000	0.951	0.951	-	-	0.635	0.635
nice65t	1,000	0.998	0.996	-	-	0.310	0.362
nice71t	1,000	0.979	0.979	-	-	0.690	0.720
nice72t	1,000	0.992	0.992	-	-	0.568	0.636
nice75t	5,000	1.000	0.999	-	-	0.296	0.935
nice81t	1,000	0.979	0.979	-	-	0.677	0.710
nice82t	2,000	0.999	0.952	-	-	0.592	0.653
nice85t	5,000	0.997	0.998	-	-	0.290	0.333
nice91t	1,000	0.978	0.980	-	-	0.684	0.714
nice92t	2,000	0.999	1.000	-	-	0.592	0.634
nice95t	5,000	0.997	0.997	-	-	0.304	0.350
nice101t	1,000	0.990	0.977	-	-	0.683	0.714
nice102t	2,000	0.995	0.952	-	-	0.595	0.653
nice105t	5,000	1.000	0.997	-	-	0.309	0.366
N1a	17	0.817	0.855	-	0.500	0.467	0.667
N1b	17	0.721	0.736	-	0.364	0.304	0.381
N1c	17	0.761	0.868	-	0.444	0.222	0.658
N1d	17	0.723	0.737	-	0.429	0.333	0.412
N1e	17	0.774	0.866	-	0.412	0.368	0.477
N2a	25	0.752	0.756	-	-	0.462	0.507
N2b	25	0.802	0.871	-	0.379	0.520	0.570
N2c	25	0.908	0.958	-	-	0.600	0.606
N2d	25	0.840	0.874	-	-	0.500	0.720
N2e	25	0.906	0.956	-	0.462	0.500	0.747
N3a	29	0.749	0.877	-	-	0.733	0.579
N3b	29	0.938	0.970	-	0.538	0.520	0.606
N3c	29	0.544	0.543	-	-	0.405	0.811
N3e	29	0.791	0.870	-	-	0.565	0.610
N4a	49	0.963	0.971	-	0.556	0.317	0.456
N4b	49	0.962	0.971	-	-	0.370	0.645
N4c	49	0.962	0.976	-	-	0.650	0.370
N4d	49	0.970	0.977	-	-	0.667	0.657
N4e	49	0.755	0.894	-	-	0.625	0.788
N5a	73	0.974	0.976	-	-	0.667	0.753
N5b	73	0.972	0.980	-	-	0.625	0.744
N5c	73	0.968	0.977	-	-	0.698	0.669
N5d	73	0.974	0.974	-	-	0.733	0.585
N5e	73	0.762	0.974	-	-	0.674	0.765
N6a	97	0.979	0.900	-	-	0.684	0.809
N6b	97	0.784	0.984	-	-	0.729	0.742
N6c	97	0.768	0.924	-	-	0.500	0.650
N6d	97	0.975	0.917	-	-	0.729	0.569
N6e	97	0.980	0.982	-	-	0.560	0.609
				-	-	0.714	0.613
				-	-	0.731	0.752
				-	-		0.781

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB_{min}	UB_{avg}	δ_{min}			GF
				MIQP	BD _{min}	GF	
N7a	197	0.986	0.988	-	-	0.783	0.829
N7b	197	0.986	0.987	-	-	0.789	0.819
N7c	197	0.985	0.986	-	-	0.789	0.815
N7d	197	0.984	0.986	-	-	0.773	0.815
N7e	197	0.984	0.985	-	-	0.782	0.812
ok41	50	0.979	0.987	-	-	0.115	0.173
ok42	30	0.962	0.962	-	-	0.222	0.163
ok43	30	0.980	0.983	-	-	0.474	0.164
ok44	61	1.000	1.000	0.333	-	0.446	0.146
ok45	97	1.000	1.000	-	-	0.308	0.399
babu	50	0.955	0.959	-	-	0.724	0.839
mopta1	200	1.000	1.000	-	-	1.000*	1.000
mopta2	40	0.850	0.928	-	-	0.667	0.729
w83	42	1.000	1.000	0.200	-	0.692	0.861
path1	25	0.200	0.197	-	-	0.043	0.072
path2	50	0.886	0.863	-	-	0.043	0.633
path3	100	0.407	0.366	-	-	0.103	0.118
path4	200	0.391	0.375	-	-	0.103	0.110
path5	500	0.916	0.914	-	-	0.574	0.611
path6	1,000	0.174	0.175	-	-	0.052	0.057
path1t	25	0.205	0.209	-	-	0.043	0.075
path4t	200	0.874	0.960	-	-	0.672	0.715
path6t	1,000	0.995	0.992	-	-	0.727	0.760
path11t	1,000	0.989	0.987	-	-	0.756	0.782
path12t	2,000	0.999	0.999	-	-	0.671	0.710
path15t	5,000	1.000	1.000	-	-	0.544	0.579
path21t	1,000	0.997	0.996	-	-	0.771	0.807
path22t	2,000	0.989	0.999	-	-	0.683	0.727
path25t	5,000	0.999	0.999	-	-	0.552	0.585
path31t	1,000	0.954	0.952	-	-	0.617	0.642
path32t	2,000	0.933	0.925	-	-	0.497	0.540
path35t	5,000	1.000	1.000	-	-	0.478	0.520
path41t	1,000	0.952	0.947	-	-	0.621	0.651
path42t	2,000	0.999	0.995	-	-	0.691	0.729
path45t	5,000	0.997	0.997	-	-	0.536	0.574
path51t	1,000	0.998	0.998	-	-	0.781	0.809
path52t	2,000	0.999	0.999	-	-	0.670	0.700
path55t	5,000	1.000	1.000	-	-	0.402	0.450
path61t	1,000	0.997	0.997	-	-	0.758	0.794
path62t	2,000	0.999	0.999	-	-	0.654	0.708
path65t	5,000	0.997	0.997	-	-	0.552	0.598
path71t	1,000	0.691	0.683	-	-	0.238	0.254
path72t	2,000	0.999	0.999	-	-	0.706	0.743
path75t	5,000	0.999	1.000	-	-	0.582	0.609
path81t	1,000	0.997	0.995	-	-	0.769	0.806
path82t	2,000	0.998	0.997	-	-	0.685	0.703
path85t	5,000	1.000	1.000	-	-	0.438	0.473
path91t	1,000	0.997	0.996	-	-	0.728	0.770
path92t	2,000	0.999	0.997	-	-	0.648	0.699
path95t	5,000	1.000	1.000	-	-	0.518	0.565
path101t	1,000	0.637	0.634	-	-	0.252	0.264
path102t	2,000	0.944	0.931	-	-	0.514	0.553

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Table 11 – continued from previous page

Instance	\mathcal{R}	UB _{min}	UB _{avg}	δ_{min}			GF
				MIQP	BD _{min}	BD _{min}	
path105t	5,000	0.965	0.951	-	-	0.287	0.313
p01	10	0.565	0.519	0.143	-	-	0.210
p02	10	0.566	0.708	0.143*	0.182	-	0.146*
p03	15	0.500	0.490	0.000*	0.182	-	0.353
p04	15	0.273	0.309	-	0.300	-	0.267
p05	20	0.860	0.936	-	0.600	0.429	0.309
T1a	17	0.554	0.549	-	0.333	0.231	0.176
T1b	17	0.721	0.738	-	0.444	0.333	0.282
T1c	17	0.621	0.688	0.182	-	-	0.477
T1d	17	0.622	0.701	0.182	-	-	-
T1e	17	0.719	0.742	0.182	-	-	0.230
T2a	25	0.810	0.708	0.400	-	-	0.296
T2b	25	0.586	0.578	-	-	0.257	0.309
T2c	25	0.751	0.869	-	-	0.538	0.603
T2d	25	0.905	0.956	-	-	0.733	0.811
T2e	25	0.802	0.871	-	0.379	0.520	0.606
T3a	29	0.544	0.543	-	-	0.316	0.356
T3b	29	0.785	0.777	-	0.333	0.419	0.474
T3c	29	0.890	0.963	-	0.439	0.667	0.763
T3d	29	0.788	0.890	-	0.571	0.533	0.611
T3e	29	0.729	0.735	-	-	0.389	0.457
T4a	49	0.962	0.976	-	-	0.657	0.744
T4b	49	0.962	0.976	-	-	0.657	0.731
T4c	49	0.970	0.977	-	-	0.600	0.698
T4d	49	0.963	0.971	-	-	0.697	0.772
T4e	49	0.960	0.971	-	-	0.692	0.749
T5a	73	0.971	0.976	-	-	0.652	0.743
T5b	73	0.972	0.980	-	-	0.667	0.736
T5c	73	0.973	0.976	-	-	0.714	0.761
T5d	73	0.968	0.974	-	-	0.674	0.743
T5e	97	0.973	0.980	-	-	0.744	0.801
T6a	97	0.978	0.982	-	-	0.714	0.774
T6b	97	0.768	0.917	-	-	0.562	0.598
T6c	97	0.980	0.982	-	-	0.714	0.771
T6d	97	0.785	0.924	-	-	0.568	0.614
T6e	97	0.980	0.982	-	-	0.719	0.757
T7a	199	0.985	0.986	-	-	0.786	0.816
T7b	199	0.982	0.985	-	-	0.769	0.810
T7c	199	0.978	0.980	-	-	0.758	0.800
T7d	199	0.986	0.989	-	-	0.787	0.828
T7e	199	0.983	0.984	-	-	0.763	0.800
zdf1	580	0.997	0.995	-	-	0.796	0.833
zdf2	660	0.997	0.997	-	-	0.780	0.820
zdf3	740	0.997	0.995	-	-	0.776	0.802
zdf4	820	0.998	0.998	-	-	0.769	0.800
zdf5	900	0.998	0.998	-	-	0.758	0.793
zdf6	1,532	0.999	0.996	-	-	0.320	0.366
zdf7	2,432	0.999	0.998	-	-	0.047	0.095
zdf8	2,532	0.999	0.999	-	-	0.667	0.741
zdf9	5,032	0.999	0.999	-	-	0.725	0.790
zdf10	5,064	0.998	0.999	-	-	0.660	0.711
zdf11	10,064	0.998	0.999	-	-	0.682	0.741

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Table 11 – continued from previous page

Instance	$ \mathcal{R} $	UB_{min}	UB_{avg}	δ_{min}			δ_{min}		
				MIQP	BD _{min}	GF	MIQP	BD _{min}	GF
zd12	7,464	0.998	0.999	-	-	0.688	-	-	0.715
zd13	12,006	0.998	0.999	-	-	0.593	-	-	0.701
zd14	23,032	1.000	1.000	-	-	0.008	-	-	0.428
zd15	55,032	1.000	1.000	-	-	-	-	-	0.001
zd16	75,032	1.000	1.000	-	-	-	-	-	0.001