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Reoptimization Gaps versus Model Errors in Online-Dispatching of Service Units for ADAC

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Reoptimization Gaps versus Model Errors in Online-Dispatching of Service Units for ADAC [1]

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Abstract

Under high load, the automated dispatching of service vehicles for the German Automobile Association (ADAC) must reoptimize a dispatch for 100–150 vehicles and 400 requests in about ten seconds to near optimality. In the presence of service contractors, this can be achieved by the column generation algorithm ZIBDIP. In metropolitan areas, however, service contractors cannot be dispatched automatically because they may decline. The problem: a model without contractors yields larger optimality gaps within ten seconds. One way out are simplified reoptimization models. These compute a short-term dispatch containing only some of the requests: unknown future requests will influence future service anyway. The simpler the models the better the gaps, but also the larger the model error. What is more significant: reoptimization gap or reoptimization model error? We answer this question in simulations on real-world ADAC data: only the new models ShadowPrice and $\text{ZIBDIP}_{\text{dummy}}$ can keep up with ZIBDIP.

Key words: vehicle dispatching, real-time, integer linear programming, dynamic column generation, dummy contractor, shadow price model

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1 Issues and Motivation

Currently, the German Automobile Association (ADAC) evaluates an automated dispatching system for service vehicles (units) and service contractors (contractors) on the basis of exact cost-reoptimization. This means that a current dispatch is maintained, which contains all known yet unserved requests and which is near optimal on the basis of the current data; whenever a unit becomes idle its next request is read from the current dispatch; at each event (new request, finished service, etc.) the dispatch is updated by a reoptimization run.

A feasible current dispatch for all known requests and available service vehicles is a partition of the requests into tours for units and contractors such that each request is in exactly one tour and each unit drives exactly one tour (maybe directly to its home position) so that the cost function is minimized. Cost contributions come from driving costs for units, fixed service costs per requests for contractors, and a strictly convex lateness cost for the violation of soft time windows at each request (currently quadratic). The latter cost structure is chosen so as to avoid large individual waiting times for customers.

It is not a-priori clear that such a rigorous reoptimization yields the best, or even a good, long-term cost (the *online issue* of the dispatching problem). Indeed, at times in the literature it is claimed that exact reoptimization (i.e., with small optimality gap) does not pay in practice because of the unknown future requests [1, p. 5]. In the case of this particular application, however, the results of exact reoptimization are satisfying [2], in concordance with [3, Sec. 8.4].

Although the reoptimization problem, which is modeled as a set partitioning problem for tours, has an astronomical number of variables, it can be solved by a dynamic column generation procedure. An effective method to obtain provably good solutions in ten seconds (the *real-time aspect* of the dispatching problem) is *dynamic pricing control*, which is the main feature of our ZIBDIP algorithm (a thorough description of the algorithm and computational results can be found in [4]).

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As it turns out, the fixed costs for service by contractors bound the dual values of requests. Thus, contractors substantially contribute to the success of ZIBDIP. The contractor, however, may in practice decline to serve suggested requests, in which case this request has to be manually reentered into the system, with the additional constraint that it must not be assigned to this contractor again. This is a time consuming process. In metropolitan areas, contractors decline so often that the ADAC decided to remove contractors from the model.

In simulations on ADAC production data (three days in December 2002 with high load) without contractors, we encountered significant reoptimization gaps. For 2002/12/13, e.g., Fig. 1 shows the gap of the reoptimization result to the respective lower bound coming from the optimal solution of the LP relaxation (this lower bound was computed a-posteriori for each reoptimization). The reoptimization still works well in most cases, but under high load the solutions – delivered after ten seconds – exhibit optimality gaps around 3% on average but up to 10% in peak load situations.

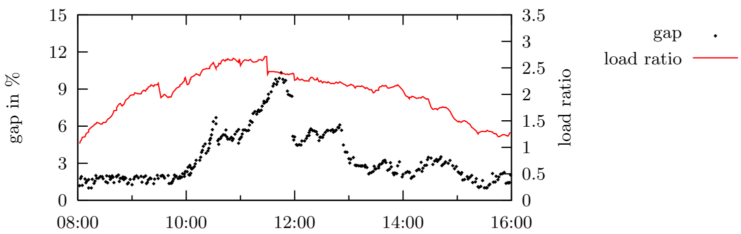


Fig. 1. Optimality gap over time of ZIBDIP (the load ratio is the number of requests per unit in a reoptimization problem).

One way to overcome this problem is to consider *simplified reoptimization models* that stem from the following considerations: In principle, for each unit we only have to determine the next request to work on. The complete dispatch is computed only to pick up future synergies by considering more than one request per unit. Synergies that are implemented only very far in the future will be disturbed by new requests anyway; therefore, an exact pre-calculation of the best decisions in, say, two hours may not really be necessary; consequently, one can try to cover only *a subset of requests* in a reoptimization step.

The issue of this experimental work is: should one stick to the complete model and accept occasional substantial reoptimization gaps, or is it better to simplify the reoptimization model so as to eliminate the reoptimization gap? This question is answered on the basis of simulation studies, performed on the aforementioned ADAC production data: we first compare the original ZIBDIP reoptimization to several methods to select subsets of requests that have to be covered by any solution of the reoptimization run. Then ZIBDIP competes with two simple online heuristics for the ZIBDIP model in order to estimate how even larger reoptimization gaps harm in the long run.

2 Simplified Models

We developed and evaluated the following strategies for the selection of requests to be covered in a reoptimization run. In the sequel, we describe the original and each simplified model in more detail.

We will use R and U to denote the set of requests and units, respectively. In all our models, there is a binary selection variable x_T for each feasible tour T. Such a tour is given by a unit u and a sequence of requests to be served by u in the given order. We call the set of all feasible tours $\mathcal{T}$ and the set of all feasible tours for unit u is written as $\mathcal{T}_u$.

We denote by c_T the cost coefficient of tour T. This is a weighted sum of strictly convex lateness costs, linear drive costs, and strictly convex overtime costs. Lateness costs in the reoptimization are incurred whenever a request is served after a waiting time of more than 15 min. The true target for the waiting time is higher. The 15 min deadline in the reoptimization problem was derived from the following consideration: the true waiting time for a request should lead to the same lateness costs as the fixed contractor costs for serving that request. This is motivated by the wish that requests that can not be served inside the true time window by a unit should be served by a contractor in order to reach the true target time. The exact formula including the numerical values of the coefficients of the cost function can not be disclosed here.

Let (a_{vT}) be the incidence matrix of requests and tours.

2.1 The Original Model ZIBDIP

The original reoptimization problem solved by ZIBDIP without contractors reads as follows.

$$\begin{aligned}
\min \sum_{T\in\mathcal{T}} c_T x_T \quad \text{s.t.} & & & \\
\sum_{T\in\mathcal{T}} a_{vT} x_T = 1 & \qquad \forall v \in R & & \text{(partitioning requests)} \\
\sum_{T\in\mathcal{T}_u} x_T = 1 & \qquad \forall u \in U & & \text{(partitioning units)} \\
x_T \in \{0,1\} & \qquad \forall T \in \mathcal{T} & &
\end{aligned}$$

In contrast to the following simplifications this model guarantees that, after every reoptimization, each request is assigned to exactly one unit because of the set partitioning constraint. Every unit has to drive exactly one tour, where the direct move to its home position is also a feasible tour, the *drive-home tour*.

2.2 The Simplified Model 4-ZIBDIP

Select those requests that are among the four closest to some unit. This can be generalized to k-ZIBDIP. In the following, *k-close requests* are requests that are among the k closest to some unit, denoted by $R_k \subseteq R$. In formulae, we obtain the following model:

$$\begin{aligned}
\min \sum_{T\in\mathcal{T}} c_T x_T \quad \text{s.t.} & & \\
\sum_{T\in\mathcal{T}} a_{vT} x_T = 1 & \quad \forall v \in R_k & \text{(partitioning } k\text{-close requests)}\\
\sum_{T\in\mathcal{T}_u} x_T = 1 & \quad \forall u \in U & \text{(partitioning units)}\\
x_T \in \{0,1\} & \quad \forall T \in \mathcal{T} &
\end{aligned}$$

2.3 The Simplified Model PTC (Prescribed Total Cover)

Relax the set partitioning condition to set packing, and require that a request set of cardinality twice the number of units is covered by tours of units. This leads to the following model, where n_T is the number of requests in tour T:

$$\begin{aligned}
\min \sum_{T\in\mathcal{T}} c_T x_T \quad \text{s.t.} & & \\
\sum_{T\in\mathcal{T}} a_{vT} x_T \leq 1 & \quad \forall v \in R & \text{(packing requests)}\\
\sum_{T\in\mathcal{T}} n_T x_T \geq 2|U| & & \text{(cardinality)}\\
\sum_{T\in\mathcal{T}_u} x_T = 1 & \quad \forall u \in U & \text{(partitioning units)}\\
x_T \in \{0,1\} & \quad \forall T \in \mathcal{T} &
\end{aligned}$$

Note that if we replace the cardinality constraint by

$$\sum_{T\in\mathcal{T}} n_T x_T \geq \min\{2|U|, |R|\}$$

the PTC model is equivalent to the original ZIBDIP model if there are at most two requests per unit on average.

Solve the LP relaxation of ZIBDIP. To find an integral solution, relax the set partitioning condition to set packing and change the cost of each tour to its reduced cost from the hopefully near optimal LP solution. In the following, the new cost coefficient $\tilde{c}^T$ of a tour T is the *reduced cost* of T w. r. t. the best LP solution that can be found in time. Because the LP solution algorithm works by dynamic column generation, this solution is an optimal solution to the last RLP that could be solved in time. The resulting model reads as follows:

$$
\begin{aligned}
\min \sum_{T \in \mathcal{T}} \tilde{c}_T x_T \quad \text{s.t.} & & & \\
\sum_{T \in \mathcal{T}} a_{vT} x_T \leq 1 & \qquad \forall v \in R & & \text{(packing requests)} \\
\sum_{T \in \mathcal{T}_u} x_T = 1 & \qquad \forall u \in U & & \text{(partitioning units)} \\
x_T \in \{0, 1\} & \qquad \forall T \in \mathcal{T} & &
\end{aligned}
$$

In this model, requests are assigned to units only if their LP dual prices together with the drive-home cost of a unit pay enough to weigh out the primal costs of their service. This requires that the LP relaxation can be solved fast, since the LP is not simplified at all.

This model is motivated by the fact that not only the column generation process is slowed down by the absence of contractors but also the IP-solution process. This can be explained as follows: In the presence of contractors, for each request there is an elementary column covering exactly that request. That way, each set packing solution using cheap tours through suitable requests can be augmented to a feasible set partitioning solution by adding such elementary columns, each at the fixed cost of the corresponding contractor. When there are no contractors, such elementary columns may become much more expensive than the price for a contractor, and for this reason they may even be overseen in the column generation process. From the remaining columns it may be difficult to augment a set of nice tours to a feasible set partitioning solution at reasonable costs. Relaxing the set partitioning condition to set packing on the model-level by-passes this problem completely and may lead to a faster IP-solution process.

2.5 The Simplified Model ZIBDIP$_{dummy}$

Introduce a dummy contractor. This contractor can be assigned arbitrarily many requests at the same time at no extra cost, i.e., in reality, these requests are unassigned for the moment. In order to enforce a cost for the assignment to the dummy contractor, its arrival time at any request is a fixed time, the *dummy contractor delay.* In our case, 135 min were chosen. In the following, d_v is the dummy contractor delay, i.e., the lateness cost for 135 min additional delay at v (on top of the current age of v). By using decision variables y_v to indicate whether request v should be served by the dummy contractor, we obtain the following model:

$$
\begin{aligned}
\min \sum_{T\in\mathcal{T}} c_T x_T + \sum_{v\in\text{requests}} d_v y_v \quad \text{s.t.} & & & \\
\sum_{T\in\mathcal{T}} a_{vT} x_T + \sum_{v\in R} y_v &= 1 & \forall v \in R & \quad \text{(partitioning requests)}\\
\sum_{T\in\mathcal{T}_u} x_T &= 1 & \forall u \in U & \quad \text{(partitioning units)}\\
x_T &\in \{0,1\} & \forall T \in \mathcal{T} & \\
y_v &\in \{0,1\} & \forall v \in R &
\end{aligned}
$$

This model implies that, in an optimal solution, for any request in a tour of a unit, service will start after at most 135 minutes after reoptimization; otherwise, the request would have been assigned to the dummy contractor.

We remark that all simplified models, including ZIBDIP$_{\text{dummy}}$, can be augmented to accommodate real contractors as soon as this might be reasonable again. (The re-introduction of real contractors requires, however, that acceptance or decline of real contractors can be predicted, in other words: reliable contracts are signed.)

3 Simplified Reoptimization Algorithms

We furthermore evaluated two heuristics for the original model, which were used in the reoptimization process as replacements for ZIBDIP. One should mention that in each reoptimization with either model, the solutions of the previous reoptimization are reused as start solutions – a simple but essential technique to stabilize the dispatching process in case of occasional suboptimal reoptimization.

3.1 The Simplified Algorithm BestInsert

A new dispatch is obtained by taking the dispatch of the previous reoptimization, removing all requests that have been served in the meantime, and inserting new requests at minimal additional cost w. r. t. to the original ZIB-DIP-model.

3.2 The Simplified Algorithm 2-Exchange

A first tentative dispatch is computed by BestInsert. This dispatch is then improved by successively exchanging two requests between distinct time slots in the dispatch if this decreases the cost. It has to be noted that the complicated cost function for tours leads to quite some computational effort for the calculation of the 2-Exchange solutions.

4 Computational Results

The simulation data stems from three days of production at ADAC in December 2002; instance sizes are given in Table 1. Depending on the instance, between 1700 and 2100 reoptimization runs were triggered.

instance	*requests*	*units*	*requests per unit*
2002/12/07	2123	125	16.98
2002/12/13	2537	146	17.38
2002/12/14	1731	131	13.21

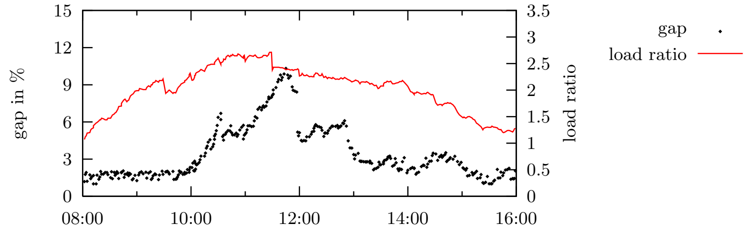
Table 1
Sizes of high load instances used for simulation.

The software ran on a standard Linux PC equipped with 2.4 GHz Pentium 4 CPU, 4 GB RAM, distribution SuSE 9.0 using kernel 2.4.21-202-smp. It was compiled with gcc 3.3.1 and used the LP / MIP solver CPLEX 8.0. Each reoptimization run was interrupted after 10 seconds run-time.

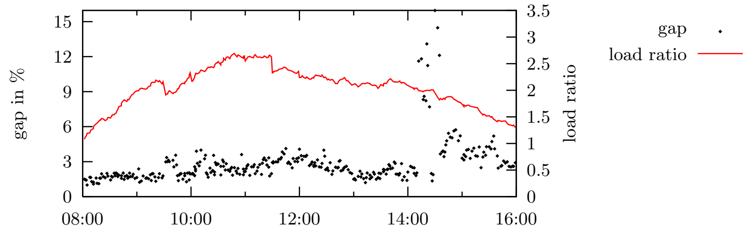
4.1 Simplified Models

Since all our simplified models by design do not guarantee service for all requests under low load, we evaluated them in the following way: If the load ratio was less or equal to 2.0, reoptimization was performed using the original ZIBDIP model. If the load ratio exceeded 2.0 we employed the respective simplified model (this is natural since these models were designed for high load situations).

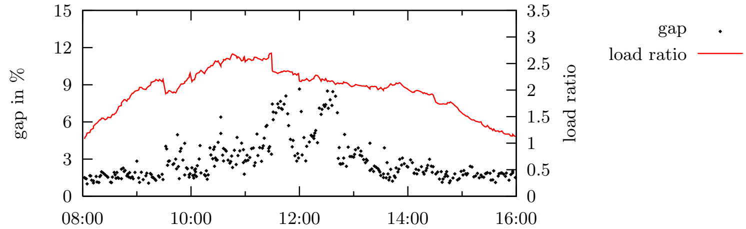
First of all, we checked whether the simplified models can reduce the optimality gaps of the reoptimization solutions that could be computed in 10 s (see Fig. 2). It can be seen that all models reduce the gap significantly, i.e., the corresponding optimization problems are easier to solve in 10 s.



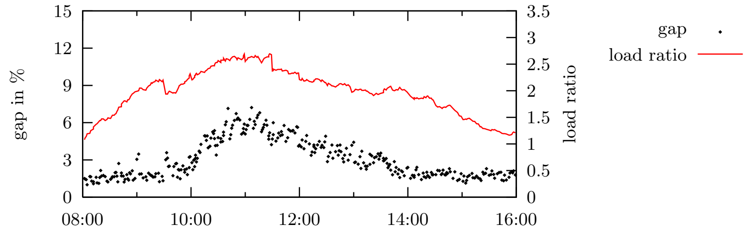
(a) ZIBDIP



(b) 4-ZIBDIP



(c) PTC

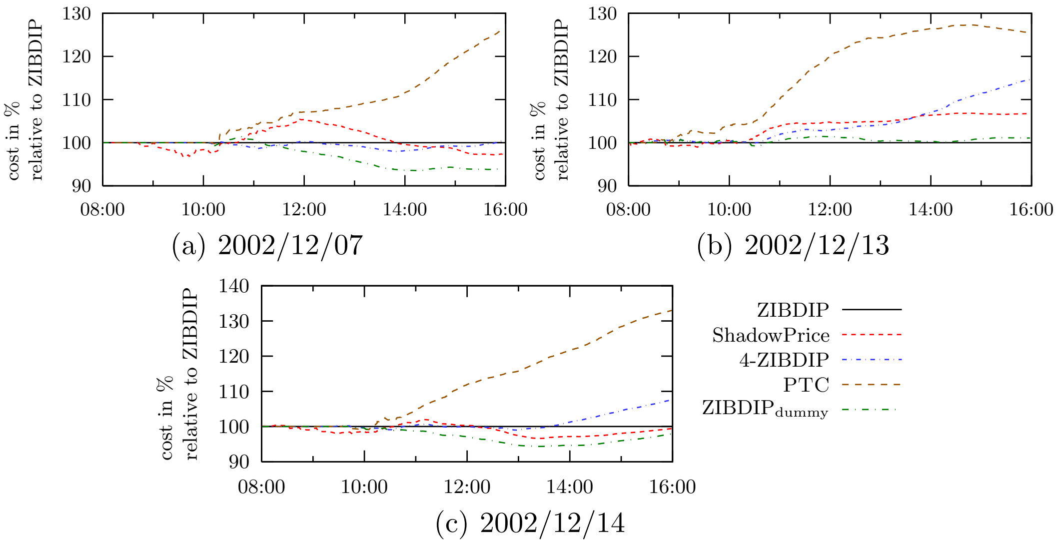


(d) $\text{ZIBDIP}_{\text{dummy}}$

Fig. 2. Optimality gaps and load ratios for simplified models and ZIBDIP. The optimality gap of ShadowPrice is inevitably infinite, since the lower bound the LP provides w. r. t. the modified cost (which is the reduced cost) is zero.

We think that some single large optimality gaps for 4-ZIBDIP and PTC stem from switching back to ZIBDIP if the load ratio drops temporarily below 2.0. The switches are particularly “unsmooth” for these two models, since ZIBDIP has to run essentially without a feasible start solution. This discontinuity in operation is certainly a drawback of 4-ZIBDIP and PTC.

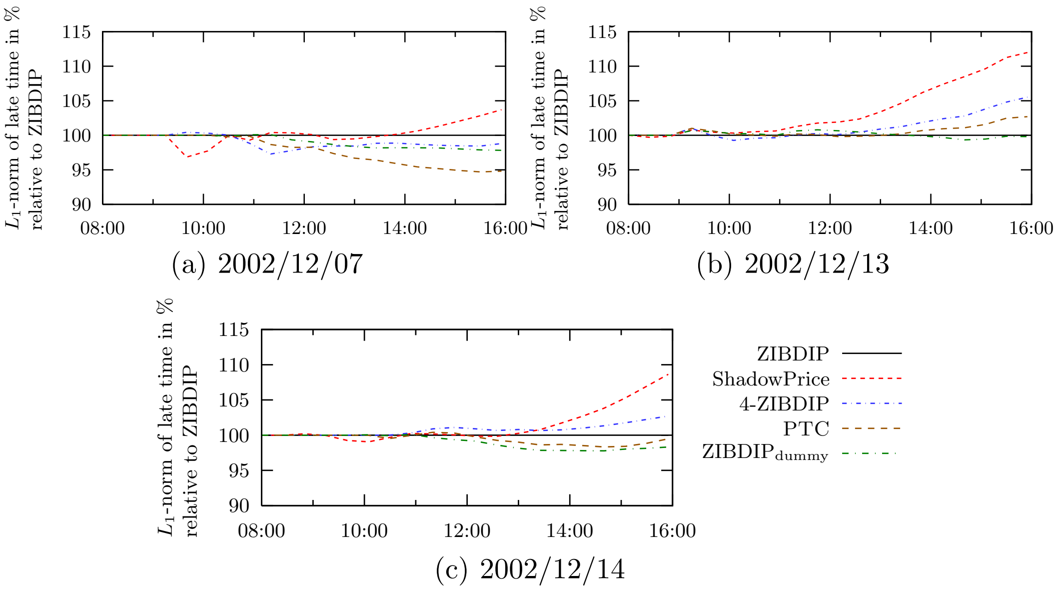
Next, we investigated the cost over time w. r. t. the reoptimization cost function, designed in cooperation with ADAC.



(a) 2002/12/07 (b) 2002/12/13 (c) 2002/12/14

Fig. 3. Comparison of ZIBDIP and simplified models w.r.t. the nonlinear cost function used by ADAC.

The results: only ShadowPrice and ZIBDIP$_{\text{dummy}}$ are competitive against ZIBDIP, although ShadowPrice seems to degrade in performance in the largest instance (b). In two out of three instances, ShadowPrice and ZIBDIP$_{\text{dummy}}$ have even slightly lower long-term cost than ZIBDIP, though by a small margin. In the largest instance with the most difficult reoptimization problems, however, the original ZIBDIP is superior. On average, however, the results are in favor of ZIBDIP$_{\text{dummy}}$.



(a) 2002/12/07 (b) 2002/12/13 (c) 2002/12/14

Fig. 4. ZIBDIP vs. simplified models: L_1-norm of lateness time.

Since the reoptimization cost function of ADAC is quite a complicated mixture of lateness, drive, and overtime costs, we decided to investigate two standard

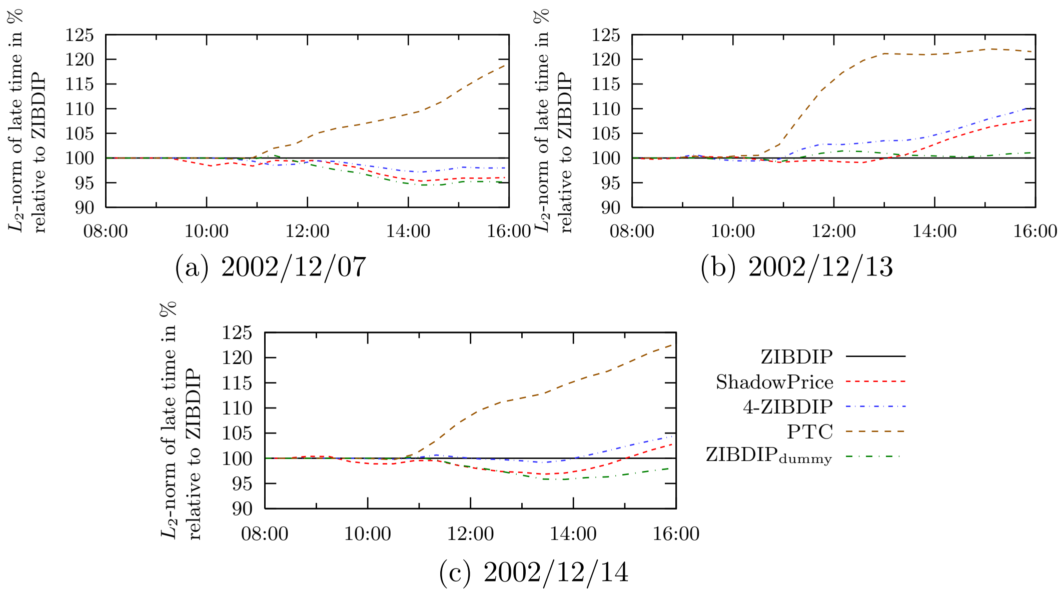


Fig. 5. ZIBDIP vs. simplified models: L_2-norm of lateness time.

measures on the so-called lateness time vectors (see Fig. 4 and 5). The lateness time of a request is its waiting time portion that exceeds the allowed waiting time, fixed by ADAC. We calculated the L_1 norms and the L_2 norms of the lateness time vectors (one entry for each request). The former norms measure the average waiting time, the latter norms penalize in particular large individual lateness times, which is desirable from a fairness point of view. One should mention that these two criteria are also of vital interest in the evaluation of the long-term behavior of online-algorithms. The ADAC reoptimization objective was chosen to contain more aspects since reoptimization of L_1 and L_2 norms alone, resp., did not lead to satisfactory overall results.

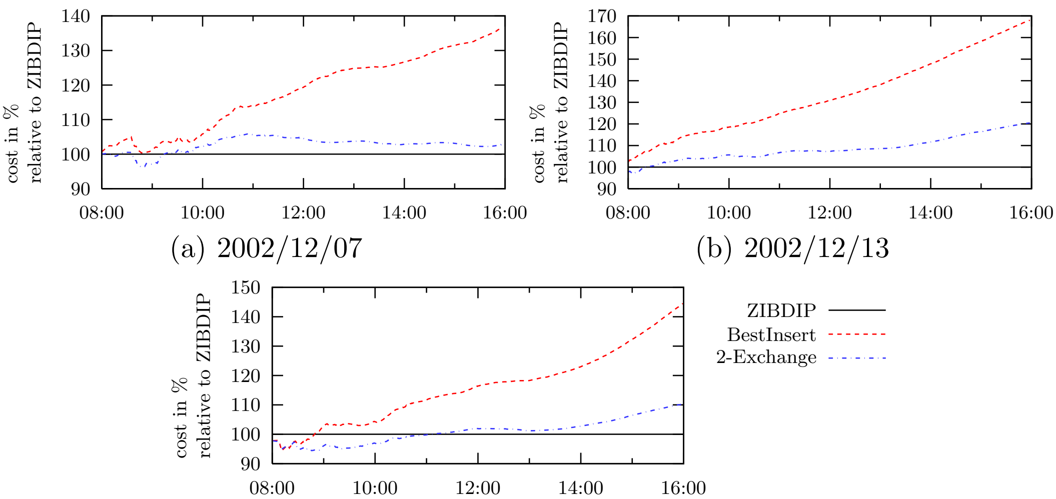
It is apparent, that w. r. t. these lateness time measures, ZIBDIP$_{\text{dummy}}$ is never worse than second best; moreover, it performs best in four out of six evaluations. ShadowPrice shows the worst L_1 norms, although the L_2 norms are good. We have no explanation for this.

The good L_1 norms of PTC are due to the fact that, obviously, individual requests are postponed in favor of new requests that can be served faster. This can be seen very clearly in the L_2 norm diagrams, in which PTC performs worst. Uncontrolled deferment of requests is a very undesired property of an online algorithm. Therefore, PTC can not be recommended for tasks in which fairness is an issue. In our application, fairness certainly is an issue, whence the ADAC cost function contains a strictly convex waiting time penalty.

The answer to our main question is that the model error of most of our high-load models leads to worse long-term behavior than the computational error that ZIBDIP produces (Fig. 3). Therefore, model simplifications have to be treated with great care. In our case, $\text{ZIBDIP}_{\text{dummy}}$ delivers the overall slightly best solution. One needs to be careful, though: a substantially smaller contractor delay of 45 min would lead to a tiny reoptimization gap; it, however, would at the same time produce unacceptable long-term costs because too many requests stay unassigned for too long. (This was, by the way, observed when we were looking for a good dummy contractor delay. Thus, $\text{ZIBDIP}_{\text{dummy}}$ involves some parameter tuning that the original ZIBDIP does not.)

4.2 Simplified Reoptimization Algorithms

The results so far could lead us to the conclusion to keep the original model but to use simplified reoptimization algorithms, since it seems that the optimality gap does not harm too much. After all, the implementation of a dynamic column generation procedure means a substantially larger effort, which is important especially in the industrial context.



(a) 2002/12/07 (b) 2002/12/13 (c) 2002/12/14

Fig. 6. Comparison of ZIBDIP and the heuristics w. r. t. the nonlinear cost function used by ADAC.

Since we hear quite frequently such arguments in order to promote the use of heuristics rather than exact mathematical programming methods, we followed also this line in our simulation experiments and found out the following: Larger computational errors in the reoptimization can increase the long-term costs even more significantly than the model errors above.

This is most incisively shown by the bad performance of BestInsert (Fig. 6, 7, and 8). Even 2-Exchange can not catch up with ZIBDIP and $\text{ZIBDIP}_{\text{dummy}}$ in the heavier instances. In the largest instance (b), 2-Exchange ends up at a long-term cost of 20 % above ZIBDIP and $\text{ZIBDIP}_{\text{dummy}}$. Especially striking

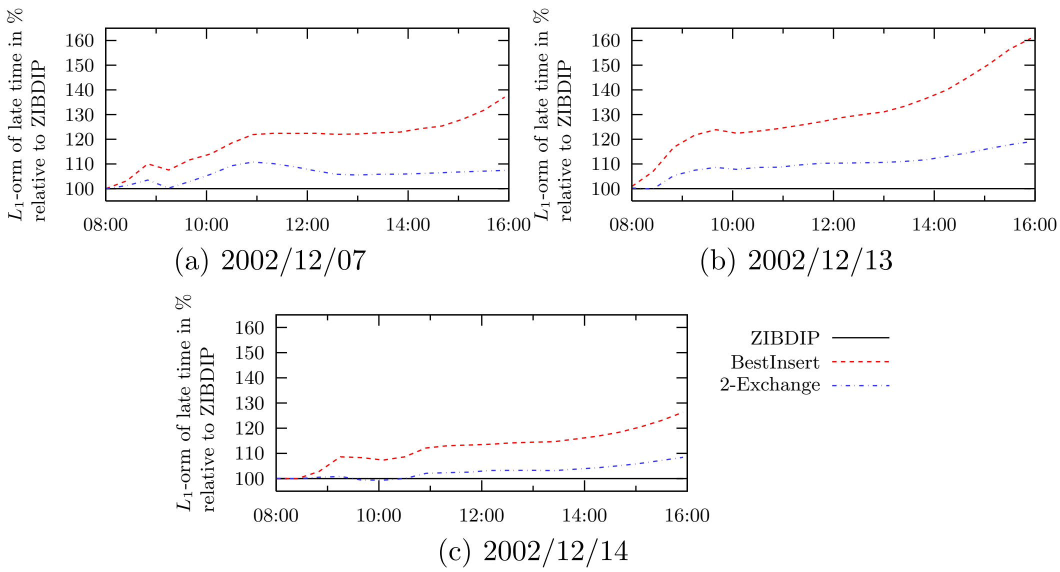


Fig. 7. ZIBDIP vs. heuristics: L_1-norm of lateness time.

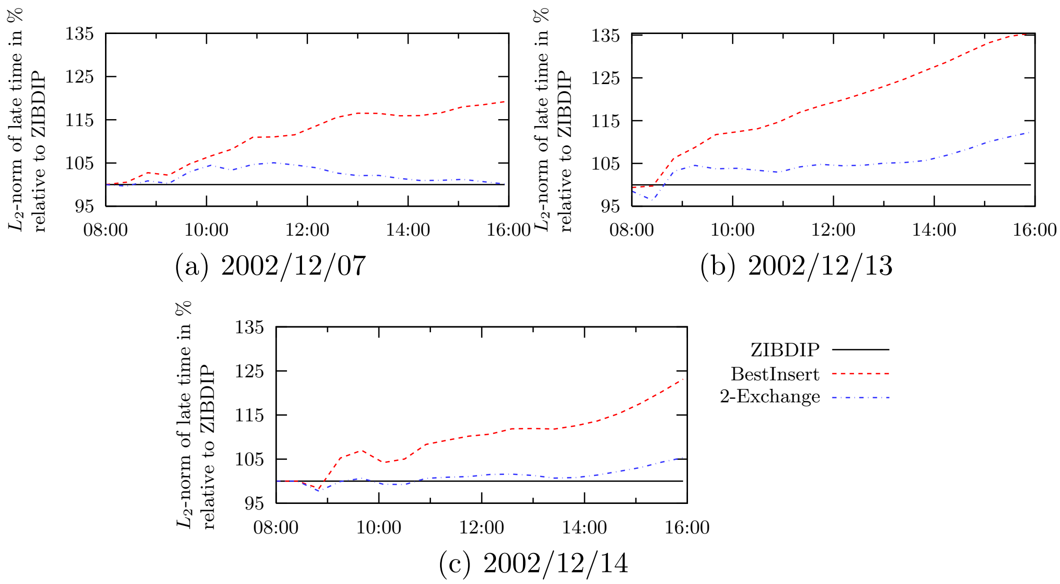


Fig. 8. ZIBDIP vs. heuristics: L_2-norm of lateness time.

is the fact that, in the largest instance, the cost of 2-Exchange is constantly increasing over time relative to ZIBDIP. That means: the reoptimization errors accumulate.

In particular: in our application it is certainly not true, that deliberately sticking to the suboptimal solutions of heuristics like BestInsert in order to leave space for future requests can yield superior long-term behavior (compare [1, p. 5]). We are not saying that reoptimization is the best possible policy, maybe not even in our application. We claim: if anything is wrong with the reoptimization policy then this defect is not cured by using suboptimal solutions to the reoptimization problems.

The good overall performance of $\text{ZIBDIP}_{\text{dummy}}$ may stem not only from closing the optimality gap in the reoptimization process; it seems, moreover, that the special model of $\text{ZIBDIP}_{\text{dummy}}$ makes perfectly sense in the dynamic environment: since requests that are assigned to the dummy contractor would otherwise be served quite far in the future, with a high probability their position in the dispatch will change anyway. These considerations led us to the conclusion to install $\text{ZIBDIP}_{\text{dummy}}$ as the default reoptimization model in the automatic dispatching software for ADAC.

5 Significance

The production software for automated dispatching of ADAC service vehicles is delivered by Intergraph Public Saftety (IPS), based on the ZIBDIP algorithm. In the view of the results presented in this work, ADAC has filed a change request for the production software: $\text{ZIBDIP}_{\text{dummy}}$ is now the standard reoptimization model because it has proven to be more robust against sudden load increase.

The key learning is that rigorous reoptimization on the basis of mathematical programming – though myopic w. r. t. unknown future requests – yields the best results in this particular application. Whether or not statistic information about future requests can be fruitfully integrated into the reoptimization framework, is work in progress, as is the investigation of randomized online-algorithms.

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