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# The Rolling Stock Rotation Planning Problem under Revenue Considerations

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## 1 Introduction

Planning rolling stock movements in industrial passenger railway applications is a long-term process based on timetables which are also often valid for long periods of time. For these timetables rotation plans, i.e., plans of railway vehicle movements are constructed as templates for these periods. These rotation plans will gain accuracy the closer the day of operation comes. During operation the rotation plans are affected by all kinds of unplanned events such as natural disasters, technical problems, or man-made impediments (strikes). In such events the rolling stock rotations, maintenance plans, and crew schedules have to be adapted. In this paper we focus on such cases of events where it is known for sure that vehicle, driver, or crew, capacities are not sufficient to cover all trips of the timetable or where it is not possible to transport all expected passenger of the timetabled trips. Thus the rotation plans planned in advance are not feasible anymore. This leads to a second level of optimization decisions. First a decision which timetabled trip is canceled or changed in the way it is operated. Afterwards an optimal rotation plan for the chosen set of timetabled trips and their vehicle configurations has to be computed. For each trip there is an estimation on the revenue depending on the number of available seats of the vehicle configuration that is selected to operate the trip. Thus a non-optimal selection of trips or, respectively, of vehicle configuration leads to decreased revenues of the railway company.

Finding new or revised tours of rolling stock vehicles through the timetable after disruptions is a well studied topic in the literature, see [Cacchiani et al.(2014)] for an overview. Usually, a rescheduling based on a timetable update is done, followed by the construction of new rotations that reward the recovery of parts of the obsolete rotations. We consider

a different, novel, and more integrated approach. The idea is to guide the cancellation of the trips or reconfiguration of the vehicle composition used to operate a trip of the timetable by the rotation planning process, which is based on the mixed integer programming approach presented in [Reuther(2017)]. The goal is to minimize the operating costs while cancelling or operating trips with insufficient vehicle configurations in the sense of passenger capacities inflicts opportunity costs and loss of revenue.

## 2 A MIP Model for Rolling Stock Rotation Problem under Revenue Considerations

In this section we consider the *Rolling Stock Rotation Problem* (RSRP) and extend a hypergraph-based integer programming formulation to suit our setting. We focus on the main modeling ideas and refer the reader to the paper [Reuther et al.(2012)] for technical details.

A cyclic planning horizon of one *standard week* is considered. Let  $T$  denote the set of timetabled passenger trips. Let  $V$  be a set of *nodes* representing timetabled departures and arrivals of vehicles operating passenger trips of  $T$ . Trips that could be operated with two or more vehicles have the appropriate number of arrival and departure nodes. Let further  $A \subseteq V \times V$  be a set of directed standard arcs, and  $H \subseteq 2^A$  a set of *hyperarcs*. Thus, a hyperarc  $h \in H$  is a set of standard arcs and includes always an equal number of tail and head nodes, i.e., arrival and departure nodes. A hyperarc  $h \in H$  *covers*  $t \in T$  if each standard arc  $a \in h$  represents an arc between the departure and arrival of  $t$ . Each of the standard arcs  $a$  represents a vehicle that is required to operate  $t$ . We define the set of all hyperarcs that cover  $t \in T$  by  $H(t) \subseteq H$ . By defining hyperarcs appropriately, vehicle composition rules and regularity aspects can be directly handled by the model. Hyperarcs that contain arrival and departure nodes of different trips are used to model deadhead trips between the operation of two (or more if couplings are involved) trips. The RSRP *hypergraph* is denoted by  $G = (V, A, H)$ . We define sets of hyperarcs coming into and going out of  $v \in V$  in the RSRP hypergraph  $G$  as  $H(v)^{\text{in}} := \{h \in H \mid \exists a \in h : a = (u, v)\}$  and  $H(v)^{\text{out}} := \{h \in H \mid \exists a \in h : a = (v, w)\}$ , respectively. Let finally  $R$  be a set of resources,  $k_r \in \mathbb{N}, \forall r \in R$  denote a capacity of resource  $r$  and  $\delta_{r,t}$  the respective resource consumption of a trip  $t \in T$ . Examples resource are the number of vehicles available to operate the timetable, the length of a trip in kilometres or hours, or the working time of the crew that operates the trip. For each timetabled trip  $t \in T$  a revenue  $p_t \in \mathbb{Q}^+$  is given to reflect amount of passengers that are expected to take this trip. Let  $c_h \in \mathbb{Q}^+ \forall t_h$  denote the costs that are associated with  $h$  respectively the vehicle movements behind it. Though in  $c_h$  all costs for vehicle usage, deadhead trip costs, energy consumption are combined including a penalty for choosing a configuration that offers less seat capacity

than passengers expected for this trip. The *Rolling Stock Rotation Problem under Revenue Considerations* (RSRPRC) is to find a cost minimal set of hyperarcs  $H_0 \subseteq H$  such that the capacities  $k_r, \forall r \in R$  are not exceeded by the trips  $t \in T$  covered by a hyperarc  $h \in H_0$  and  $\bigcup_{h \in H_0} h \subseteq A$  is a set of *rotations*, i.e., a packing of cycles (each node is covered at most once).

Using a binary decision variable for each hyperarc and a slack variable for each trip, the RSRPRC can be stated as an integer program as follows:

$$\min \sum_{h \in H} c_h x_h - \sum_{t \in T} p_t s_t, \quad (1)$$

$$\sum_{t \in T} \sum_{h \in H(t)} \delta_{r,t} x_h \leq k_r, \quad \forall r \in R \quad (2)$$

$$\sum_{h \in H(t)} x_h = 1 - s_t \quad \forall t \in H, \quad (3)$$

$$\sum_{h \in H(v)^{\text{in}}} x_h - \sum_{h \in H(v)^{\text{out}}} x_h = 0 \quad \forall v \in V, \quad (4)$$

$$x_h \in \{0, 1\} \quad \forall h \in H, \quad (5)$$

$$s_t \in \mathbb{Q}_+ \quad \forall t \in T. \quad (6)$$

The RSRP, and therefore also the RSRPRC, is  $\mathcal{NP}$ -hard, even if constraints (3) are trivially fulfilled, i.e.,  $|H(t)| = 1$  for all trips  $t \in T$ , see [Heismann(2014)].

### 3 Computational Results

The proposed model was implemented in our algorithmic framework ROTOR, that is integrated in the IT environment of DB Fernverkehr AG [Reuther et al.(2012)], as well as in ROPT [LBW(2017)] developed by the LBW Optimization GmbH. In the first case the implementation makes use of the commercial mixed integer programming solver **Gurobi** as an internal LP solver

Our implementation is tested on two real-world scenarios provided by our industrial partners. The first set of instances is related to strike periods in Germany. Each scenario has different fleet sizes, vehicle characteristics, and different underlying networks which cover wide parts of Germany. To compare our solution approach we run ROTOR without the revenue optimization approach on instances that contain a limited number of trips of the normal DBF timetable. This list of trips was created by planners of DBF as a alternative timetable offer during the strike period. It depends on a rough guess which train drivers were on strike and which not. Although this list is the result of the planning at DBF there were some changes made before really operating the trips during that period. Nevertheless, these rotations are very close to the operated ones and therefore a most

Table 1:  $ice^{DB}$ : Instance with  $\approx 50\%$  manually cancelled trips by planners of DBF.

Name	$ T $	$ H (\times 10^6)$	$\sum \delta_t$ (km)	Dh (km)	Cost( $\times 10^x$ )	CPU(s)	$\sum p_t(\times 10^x)$
$ice1^{DB}$	379	0.9	296094	8777	1.74	70	1.08
$ice2^{DB}$	456	4.8	165906	13506	1.00	622	0.61
$ice3^{DB}$	335	1.6	186653	6279	1.42	489	0.94
$iceT^{DB}$	232	1.9	131899	9370	0.69	441	0.27

Table 2: Results for the RSRPRC on DBF instances.

Name	$ T $	$ H (\times 10^6)$	$\sum \delta_t$ (km)	Dh (km)	Cost( $\times 10^x$ )	CPU(s)	$\sum p_t(\times 10^y)$
$ice1$	700	0.9	295737	3587	1.70	229	1.24
$ice2$	973	5.9	166311	4807	0.98	5458	0.66
$ice3$	922	4.0	187142	5265	1.37	5507	1.10
$iceT$	915	3.2	132073	6391	0.62	882	0.37

appropriate candidate to compare to. The revenues of a trip are dependent on an estimated number of passengers and their distance travelled, i.e., a longer train ride of a passenger increases the revenue. Table 1 shows the main characteristics of the solution process and its outcome. The first two columns show the instance name, respectively fleet, and the number of trips included. Columns three and four give the sum of the trip and deadhead trip distance of all used vehicles of the solution. Since the costs are confidential column *Cost* shows only a factor of the operational cost of the computed solution. Column *CPU* presents the run time of the optimization process. The last column gives the sum of the revenues  $p_t$  for all trips included in the solution.

Table 2 shows the results of the optimization runs with revenues considered. We applied a capacity limit for each instance, respectively fleet, depending on the aggregated trip length of all trips included in the corresponding instance with manually canceled trips. Hence, optimized rotations of both approaches have an amount of comparable working hours of the train drivers. The aggregated deadhead trip length of the optimized solutions save between  $\approx 32\%$  and  $\approx 64\%$  of the aggregated deadhead km. Also the operational costs of the optimized solutions decrease which is a consequence of the decreased number of deadhead kilometres. Comparing the last columns of the two tables shows that while keeping a comparable amount of offered trains and costs to operate them the revenues could be increased significantly.

The second case where we applied our approach is a scenario from a railway company in Canada. In this instance 288 passenger trips have to be covered with 17 different vehicle

Table 3: Canada instance.

Name	$ T $	$ H $	vehicle types	compositions
<i>can</i>	288	1386826	3	17

Table 4: Results for Canada instance.

Name	Rev.	$ T $	$\sum \delta_t$ (km)	Dh (km)	Cost( $\times 10^x$ )	CPU(s)	$\sum p_t$	$ A $	$ B $
<i>can</i>	no	288	50084	7112	1.797	8664	3.854	25	65
<i>can</i>	yes	288	51258	8286	1.920	9003	4.528	85	25

compositions containing a locomotive and up to eight coaches of either type A (68 seats) or B (56 seats). This leads to 1.4 million hyperarcs in the model (Table 3). These scenarios were solved with ROPT. In contrast to the DBF instance in this case trip cancellations should be avoided, but configurations with less seats than passengers expected on the respective trip are allowed. Additionally a capacity limit on the total number coaches of each type types is applied. Table 4 shows the results of the optimization runs for instance *can* with and without revenues considered. In the case without considering the revenues each trip had to be covered by a vehicle configuration minimizing the overall operational costs. It shows that changing the vehicle composition and increasing the deadhead kilometres leads to an higher revenue when they are considered in the model.

## 4 Conclusion

We presented the integration of a revenue guided decision making into a mixed integer programming approach to solve the RSRPRC. The proposed approach leads to promising results for situations with an heavily decreased offer of passenger railway trips, like strike periods. We could show that it might be efficient to increase the offered seats due to changed vehicle configurations and increase the number of operated vehicles to gain higher revenues.

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