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# Analysis of operating modes of complex compressor stations\*

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## Abstract

We consider the modeling of operation modes for complex compressor stations (i.e., ones with several in- or outlets) in gas networks. In particular, we propose a refined model that allows to precompute tighter relaxations for each operation mode. These relaxations may be used to strengthen the compressor station submodels in gas network optimization problems. We provide a procedure to obtain the refined model from the input data for the original model. This procedure is based on a nontrivial reduction of the graph representing the gas flow through the compressor station in an operation mode.

## 1 Introduction

Gas transmission networks are a crucial part of the European energy supply infrastructure. The gas flow is driven by pressure potentials. To maintain the necessary pressure levels and control the routing of the gas in the network, compressor stations are used. In the German transmission network compressor stations usually interconnect two or more pipeline systems. They often have a complex internal structure, allowing them to realize different routing patterns and compression levels [8]. An example of such a complex compressor station is shown in Figure 1. In particular, some or all boundary nodes of the compressor station may serve as inlet or outlet, depending on the requirements of the surrounding network.

In this paper, we consider the compressor station modeling introduced in [8]. This model combines a network containing compressors and valves and a set of switching states for these elements to describe all feasible operation modes of a compressor station. The constraints describing the technical capability of a compressor may be nonlinear and nonconvex, leading to hard-to-solve MINLP models. We propose model refinements that aim to help the solution process to detect unsuitable operation modes early. These model refinements are otherwise redundant and are supposed to be used in addition to the compressor model employed. For instance, to improve the model a natural idea is to precompute, for each operation mode, bounds on the minimum and maximum flow and pressure that can be handled and to include this information in the model. However, the modeling of an operation mode from [8] and the data available

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\*This paper is the long version of the extended abstract [5].

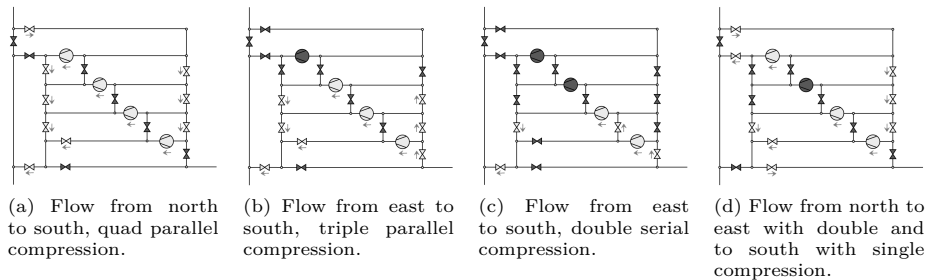


Figure 1: Four operation modes of a large compressor station. Elements colored dark are “closed”.

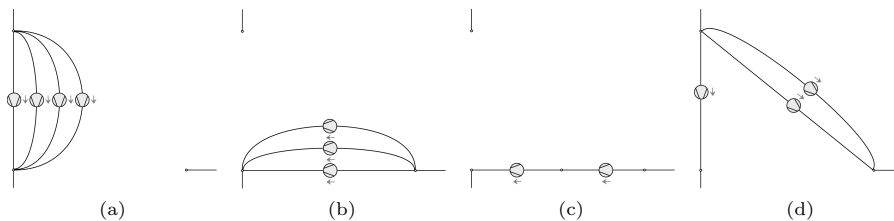


Figure 2: Reduced representation of the four operation modes shown in Figure 1. All compressor symbols correspond to active compressors, i.e., ones that are not working in bypass.

in the GasLib [7] do not specify whether a compressor is actively compressing or bypassed, which are very distinct behaviors. Thus no nontrivial flow bounds may be obtained for an operation mode.

We develop techniques for analyzing the original representation of operation modes to generate a more detailed representation where each operation mode prescribes for each compressor whether it is compressing or in bypass. This allows to compute tight bounds for the pressure/flow combinations that can be handled by each operation mode. The crucial ingredient is a method to obtain a reduced representation of an operation mode to cope with redundancies due to the original representation. Examples of such reduced representations are shown in Figure 2. Moreover, these reduced representations facilitate the classification of operation modes according to flow patterns, i.e., which boundary nodes are sources or sinks for the gas flow in the compressor station.

**Related work** We briefly mention some related papers and refer to [11] for a comprehensive overview. Most of the work on optimization of compressor stations has focused on simple compressor stations that are compressing in a predetermined direction from a single inlet to a single outlet. The fact that a compressor station usually features several (often distinct) compressors has been dealt with by using an aggregated model, like a range for the power required by the compression process [9, 2], box constraints for flows and pressures [1], or a polyhedral model [13, 12, 6]. Papers using a detailed model for the operation of a single compressor usually assume that a compressor station consists of several

identical compressors that are operated in parallel [13], the only discrete decision being the number of compressors switched on. A recent exception is the work of [10], considering the selection of a configuration of the compressors consisting of serial stages of compressors used in parallel. Complex multi-way compressor stations with multiple operating modes are only considered in [8] and related work.

The remaining paper is structured as follows. Section 2 recalls the model introduced in [8]. In Section 3, we propose a method that reduces the description of a single operation mode to a kind of “normal form”. This is used in Section 4 to detect redundancy and generate a set of redundancy-free operation modes that, as a set, are equivalent to the original operation modes. Section 5 explains the classification of operation modes according to flow patterns and how this can be incorporated in the model. Finally, we report on some computational results in Section 6.

## 2 Model for complex compressor stations

We use the following model for compressor stations that closely follows the modeling proposed in [8].

**Network** We represent a compressor station as a directed graph  $(V, A_{va} \cup A_{cg} \cup A_{cv} \cup A_{sc})$ , where the arc set consists of the set of valves  $A_{va}$ , the set of compressors  $A_{cg}$ , the set of control valves  $A_{cv}$ , and the set of shortcuts  $A_{sc}$ . Valves can be *open* or *closed* and are used to control the route of gas through the compressor station. In this paper, we treat compressors as abstract entities that facilitate pressure increase and that may operate in one of the states *closed* (no gas flow), *active* (compressing), and *bypass* (gas flow without compression). In fact, each compressor may actually be a *compressor group* [8] that consists of one or several compressors, possibly allowing several series-parallel configurations, each of which employs a subset of the compressors. A control valve may operate in the same set of three states, but the *active* state reduces the pressure of the gas flowing through the control valve. Finally, shortcuts are convenient modeling elements that allow gas flow between two nodes without pressure drop. Moreover, we partition the set of nodes into boundary nodes  $V_{\pm}$  and inner nodes  $V_0$ .

For each node  $u \in V$  we introduce a variable for the pressure  $p_u$  with non-negative lower and upper bounds  $\underline{p}_u$  and  $\bar{p}_u$ , i.e.,

$$0 \leq \underline{p}_u \leq p_u \leq \bar{p}_u \quad \text{for all } u \in V. \quad (1)$$

For each arc  $a \in A$  there is a variable for the mass flow  $q_a$  with lower and upper bounds  $\underline{q}_a$  and  $\bar{q}_a$ , i.e.,

$$\underline{q}_a \leq q_a \leq \bar{q}_a \quad \text{for all } a \in A. \quad (2)$$

Positive mass flow values indicate flow in the direction of the arc, whereas negative values represent flow in the opposite direction. The precise values of the bounds depend on the type and state of an element and are discussed below. For each node  $u \in V$  we denote the sets of incoming and outgoing arcs by

$\delta^-(u)$  and  $\delta^+(u)$ , respectively. We define  $b_u$ , the excess of mass flow at node  $u$ , whereby at inner nodes the mass flow is conserved, i.e.,

$$b_u := \sum_{a \in \delta^-(u)} q_a - \sum_{a \in \delta^+(u)} q_a \quad \text{for all } u \in V, \quad (3)$$

$$b_u = 0 \quad \text{for all } u \in V_0. \quad (4)$$

**Shortcuts** For shortcuts the following holds:

$$\underline{q}_a = -\infty, \bar{q}_a = \infty \quad \text{for all } a \in A_{\text{sc}}, \quad (5a)$$

$$p_u = p_v \quad \text{for all } (u, v) \in A_{\text{sc}}. \quad (5b)$$

**Valves** For each valve  $a \in A_{\text{va}}$ , a binary variable  $s_a$  distinguishes between the states *open* and *closed* where  $s_a = 1$  corresponds to *open*,  $s_a = 0$  corresponds to *closed*.

$$s_a = 0 \implies q_a = 0 \quad \text{for all } a \in A_{\text{va}}, \quad (6a)$$

$$s_a = 1 \implies \begin{cases} \underline{q}_a = -\infty, \bar{q}_a = \infty, \\ p_u = p_v \end{cases} \quad \text{for all } a = (u, v) \in A_{\text{va}}. \quad (6b)$$

**Compressors and control valves** For each compressor  $a \in A_{\text{cg}}$  or control valve  $a \in A_{\text{cv}}$ , binary variables  $s_a, s_a^{\text{ac}}, s_a^{\text{bp}}$  distinguish between the states *active*, *bypass* and *closed* where  $s_a^{\text{ac}} = 1$  corresponds to *active*,  $s_a^{\text{bp}} = 1$  corresponds to *bypass* and  $s_a = 0$  corresponds to *closed*. We model the capabilities of an active compressor or control valve  $a$  by an abstract set  $P_a \subseteq \mathbb{R}_{\geq 0}^3$  of feasible inlet pressure, outlet pressure, and mass flow. Thus our methods apply to all variants of compressor and control valve models. Thus compressors and control valves are modeled by the constraints

$$s_a = s_a^{\text{ac}} + s_a^{\text{bp}} \quad \text{for all } a \in A_{\text{cg}}, \quad (7a)$$

$$s_a = 0 \implies q_a = 0 \quad \text{for all } a \in A_{\text{cg}}, \quad (7b)$$

$$s_a^{\text{bp}} = 1 \implies \begin{cases} \underline{q}_a = -\infty, \bar{q}_a = \infty, \\ p_u = p_v \end{cases} \quad \text{for all } a = (u, v) \in A_{\text{cg}}. \quad (7c)$$

$$s_a^{\text{ac}} = 1 \implies (p_u, p_v, q_a) \in P_a \subseteq \mathbb{R}_{\geq 0}^3 \quad \text{for all } a = (u, v) \in A_{\text{cg}}. \quad (7d)$$

The constraints describing the capability set  $P_a$  of a compressor may be non-linear and nonconvex, leading to hard-to-solve MINLPs for large-scale networks [8, 10].

In the remainder of the paper, compressors and control valves will be handled exactly the same, unless mentioned otherwise. For simplicity, we will only talk about compressors with the understanding that the techniques also work for control valves.

**Operation modes** An operation mode specifies the switching state of each active element (valves, compressors) and thus the route of the gas flow through the compressor station. Operation modes are modeled in [3, Section 6.1.8] by a triple  $(A_{\text{active}}, \mathcal{M}, d)$ , where  $A_{\text{active}} = A_{\text{va}} \cup A_{\text{cg}} \cup A_{\text{cv}}$  is the set of active

elements. The set  $\mathcal{M} \subseteq \{0, 1\}^{A_{\text{active}}}$  describes each operation mode  $m \in \mathcal{M}$  by stating whether an active arc  $a$  is *open* ( $m_a = 1$ ) or *closed* ( $m_a = 0$ ). In the case of an *open* compressor it is not yet specified whether this compressor is *active* or in *bypass*. Finally, the function  $d : A_{\text{active}} \times \mathcal{M} \rightarrow \{-1, 0, 1\}$  describes whether the flow direction for an active arc  $a = (u, v)$  is restricted or not, using the convention

$$d(a, m) = \begin{cases} -1, & \text{if gas flows from } v \text{ to } u \text{ in operation mode } m, \\ 0, & \text{if the flow direction on } a \text{ is undefined in operation mode } m, \\ 1, & \text{if gas flows from } u \text{ to } v \text{ in operation mode } m. \end{cases}$$

We introduce binary decision variables  $s_m \in \{0, 1\}$  for all  $m \in \mathcal{M}$  to formulate that *exactly one* operation mode has to be chosen, i.e.,

$$\sum_{m \in \mathcal{M}} s_m = 1. \quad (8a)$$

Further, an element  $a \in A_{\text{active}}$  may only be used as specified by the elements of  $\mathcal{M}$ :

$$\sum_{m \in \mathcal{M}} m_a s_m \geq s_a \quad \text{for all } a \in A_{\text{active}}, \quad (8b)$$

$$\sum_{m \in \mathcal{M}} (1 - m_a) s_m \geq (1 - s_a) \quad \text{for all } a \in A_{\text{active}}. \quad (8c)$$

Finally, if an operation mode that specifies flow directions for some of the sub-network elements is chosen, we have to guarantee that these flow direction requirements are satisfied. This is achieved by adding the constraints:

$$\left( 1 + \sum_{(a,m) \in A_{\text{active}} \times \mathcal{M}} d(a, m) s_m \right) \bar{q}_a \geq q_a \quad \text{for all } a \in A_{\text{active}}, \quad (8d)$$

$$\left( 1 - \sum_{(a,m) \in A_{\text{active}} \times \mathcal{M}} d(a, m) s_m \right) \underline{q}_a \leq q_a \quad \text{for all } a \in A_{\text{active}}. \quad (8e)$$

As mentioned in the introduction, the fact that this representation does not specify whether an *open* compressor is *active* or running in *bypass* precludes us from obtaining tight bounds for flows and pressures obtainable by an operation mode. We thus propose a more detailed representation where each operation mode prescribes for each compressor whether it is *active* or in *bypass*. To obtain this representation from the original one we enumerate all *active/bypass* combinations for each operation mode. Since this leads to very many and largely redundant operation modes, we apply the methods described in Section 3 to obtain a smaller, yet complete set of fully specified operation modes. These are described by a tuple  $(A_{\text{active}}, \mathcal{M}', d')$ . The set  $\mathcal{M}' \subseteq \{0, 1\}^{A_{\text{active}}} \times \{0, 1\}^{A_{\text{active}}}$  specifies for each operation mode  $(m^{ac}, m^{bp}) \in \mathcal{M}'$  whether a valve  $a$  is *open* ( $m_a^{ac} = 0, m_a^{bp} = 1$ ) or *closed* ( $m_a^{ac} = 1, m_a^{bp} = 0$ ) and whether a compressor  $b$  is in *bypass* ( $m_b^{ac} = 0, m_b^{bp} = 1$ ), *active* ( $m_b^{ac} = 1, m_b^{bp} = 0$ ) or *closed* ( $m_b^{ac} = 0, m_b^{bp} = 0$ ). The function  $d' : A_{\text{active}} \times \mathcal{M}' \rightarrow \{-1, 0, 1\}$  describes restrictions on the flow direction for active arcs, using the same convention as

the function  $d$  in the original formulation. The new model can then be formulated analogously to (8a)–(8e) with  $\mathcal{M}'$  instead of  $\mathcal{M}$ ,  $d'$  instead of  $d$  and two additional constraints enforcing the full state for all compressors as specified by the elements of  $\mathcal{M}'$ :

$$\sum_{\substack{m \in \mathcal{M}' \\ m=(m^{ac}, m^{bp})}} m_a^{ac} s_m \geq s_a^{ac} \quad \text{for all } a \in A_{cg}, \quad (8f)$$

$$\sum_{\substack{m \in \mathcal{M}' \\ m=(m^{ac}, m^{bp})}} (1 - m_a^{ac}) s_m \geq (1 - s_a^{ac}) \quad \text{for all } a \in A_{cg}. \quad (8g)$$

For each of these operation modes, we can now compute tight pressure and inflow bounds by solving the optimization problem given by (9)–(14) together with respective objective functions. Then, with  $\underline{p}_u(m), \bar{p}_u(m)$  and  $\underline{b}_u(m), \bar{b}_u(m)$  denoting the pressure bounds and mass flow excess bounds, respectively, for node  $u$  in operation mode  $m$ , the following inequalities are valid:

$$\sum_{m \in \mathcal{M}} \underline{p}_u(m) s_m \leq p_u \leq \sum_{m \in \mathcal{M}} \bar{p}_u(m) s_m \quad \text{for all } u \in V, \quad (8h)$$

$$\sum_{m \in \mathcal{M}} \underline{b}_u(m) s_m \leq b_u \leq \sum_{m \in \mathcal{M}} \bar{b}_u(m) s_m \quad \text{for all } u \in V. \quad (8i)$$

From now on, we call the formulation using the original operation modes the *compact formulation* and the one using fully specified operation modes the *extended formulation*. Moreover, the extended formulation enriched with the above bounds (8h)–(8i) for each operation mode will be called *bounded extended formulation*.

### 3 Topology simplification for a single operation mode

Our goal in this section is to simplify the topology of a single operation mode of a compressor station to obtain a reduced representation suitable for comparing operation modes via graph isomorphism detection.

We consider the network  $N^m = (V, A^m, \underline{q}, \bar{q}, \underline{p}, \bar{p})$  corresponding to a fully specified operation mode  $m$  derived from the station network as follows. First, all closed elements are removed. Second, every shortcut, open valve and compressor in bypass is equivalently replaced by two opposing shortcuts with lower flow bound equal to zero. Third, all weakly connected components, which do not contain a boundary node, and all shortcuts with upper flow bound equal to zero are removed. This is also a valid transformation with respect to pressure since the constraints for open valves or bypassed compressors are equivalent to those of shortcuts. Hence, the arc set  $A^m$  consists only of shortcuts and active



compressors. Thus, the model for a single operation mode reduces to

$$0 \leq \underline{p}_u \leq p_u \leq \bar{p}_u \quad \text{for all } u \in V, \quad (9)$$

$$0 \leq \underline{q}_a \leq q_a \leq \bar{q}_a \quad \text{for all } a \in A, \quad (10)$$

$$b_u = 0 \quad \text{for all } u \in V_0, \quad (11)$$

$$\underline{q}_a = 0, \bar{q}_a = \infty \quad \text{for all } a \in A_{sc}, \quad (12)$$

$$p_u = p_v \quad \text{for all } (u, v) \in A_{sc}, \quad (13)$$

$$(p_u, p_v, q_a) \in P_a \subseteq \mathbb{R}_{\geq 0}^3 \quad \text{for all } a = (u, v) \in A_{cg}. \quad (14)$$

However, the network may be highly redundant, as a shortcut usually indicates that the incident nodes are identical. Thus we can reduce the size of the network by contracting a shortcut as follows. We identify the incident nodes of the shortcut and update the pressure bounds of the remaining node to be the intersection of the pressure intervals for the original two nodes. If there are any other arcs between the two nodes, we do keep them as self-loops. But we need to be careful when applying this contraction since shortcuts sometimes do carry important information on the topology of feasible flows. An example of when contracting a shortcut would lead to a loss of information is shown in Figure 3.

We now devise a criterion for safely removing shortcuts. Since it is clear from the context that we are dealing with a network corresponding to a specific operation mode, we omit the superscript from  $N^m$  in the following in order to keep the notation simple. We consider the shortcut subgraph of  $N$ ,  $G^{sc}$ , its set of sources  $V_+^{sc}$ , its set of sinks  $V_-^{sc}$  and for all sources  $w \in V_+^{sc}$  the set  $\vec{R}_N(w) \subseteq V_-^{sc}$  of sinks reachable using only shortcuts:

$$G^{sc} := (V, A_{sc}) \quad (15)$$

$$V_+^{sc} := V_{\pm} \cup \{w \in V \mid \exists u \in V : (u, w) \in A_{cg}\} \quad (16)$$

$$V_-^{sc} := V_{\pm} \cup \{w \in V \mid \exists u \in V : (w, u) \in A_{cg}\} \quad (17)$$

$$\vec{R}_N(w) := \{u \in V_-^{sc} : \exists w\text{-}u\text{-path in } G^{sc}\} \quad \text{for all } w \in V_+^{sc} \quad (18)$$

**Proposition 1** Consider a shortcut  $\tilde{a} = (u, v)$  with  $u \in V \setminus \{V_+^{sc}\}$  and the network  $N'$  arising from  $N$  when contracting  $\tilde{a}$  to  $v$ . If

$$\vec{R}_N(w) = \vec{R}_{N'}(w) \quad \text{for all } w \in V_+^{sc} \quad (19)$$

then for every admissible flow-pressure combination  $(p', q')$  for  $N'$  there exists



Figure 3: All arcs  $a$  in this network are shortcuts with  $\underline{q}_a = 0$  and  $\bar{q}_a = \infty$ . With exception of  $(u, v)$  they do not carry information and can be contracted. However,  $(u, v)$  may not be contracted as it carries the information that no flow is permitted from  $w$  to  $t$ , but only in the opposite direction.

an admissible flow-pressure combination  $(p, q)$  for  $N$  such that

$$q'_a = q_a \quad \text{for all } a \in A_{cg} \quad (20)$$

$$b'_w = b_w \quad \text{for all } w \in V_{\pm} \quad (21)$$

$$p'_w = p_w \quad \text{for all } w \in V_{\pm} \quad (22)$$

and vice versa. □

PROOF Consider a flow-pressure combination  $(p', q')$  for  $N'$ . Define the pressure vector  $p$  by

$$p_w := \begin{cases} p'_w & w \in V \setminus \{u\}, \\ p'_v & w = u. \end{cases} \quad (23)$$

Then,  $p, p'$  trivially satisfy (22). Also,  $p$  is admissible pressure: By definition, it satisfies (13) for  $\tilde{a}$ . It satisfies (9) for  $u$  and  $v$  since by our definition of contraction the pressure bounds on  $v$  in  $N'$  are the intersection of the pressure intervals for  $u$  and  $v$  in  $N$  and  $p'$  is admissible. Also, it satisfies (9), (13) and (14) (with respect to pressure) for all other nodes and arcs, because  $p'$  does.

We obtain  $q$  by constructing a flow decomposition of it based on a flow decomposition of  $q'$ . Consider a flow decomposition of  $q'$  into paths and cycles. Let  $P' = v_1, \dots, v, \dots, v_k$  be a path with  $v_1, v_k \in V_{\pm}$  and let  $\kappa$  be the flow value on  $P'$  in the flow decomposition of  $q'$ . We now construct a corresponding path  $P$  in  $N$  with identical endpoints and flow value, but as part of the flow decomposition of  $q$ . If  $P'$  does not contain  $v$ , we simply set  $P := P'$ . Otherwise, the idea is to partition  $P'$  into shortcut subpaths that go between compressors or boundary nodes. Exactly one of these subpaths contains  $v$  and is thus possibly not in  $N$ . But, by (19) we can replace this shortcut subpath in  $N'$  with a shortcut subpath in  $N$  that has the same endpoints.

Consider the partition of  $P'$  into shortcut subpaths  $P'_1, \dots, P'_n, \dots, P'_r$  by removal of all compressors and let  $P'_n = v_i, \dots, v, \dots, v_j$ . By (16) and (17) we have  $v_i \in V_+^{sc}$  and  $v_j \in V_-^{sc}$ , and by (18) we have  $v_j \in \vec{R}_{N'}(v_i)$ . Therefore, by (19) we have  $v_j \in \vec{R}_N(v_i)$ . Let  $P_n = v_i, \dots, v_j$  be a shortcut path in  $N$  and define  $P$  to be  $P'$  with  $P'_n$  replaced by  $P_n$ .

Similarly, let  $C'$  be a cycle in  $N'$  with flow value  $\kappa'$  in the flow decomposition of  $q'$ . If  $C'$  contains a compressor, we can employ the same reasoning as before to obtain a cycle  $C$  in  $N$  with identical flow value  $\kappa'$ , but as part of the flow decomposition of  $q$ . Otherwise, if  $C$  consists of shortcuts only, we let the flow value on it as part of the flow decomposition of  $q$  be zero.

Altogether, this gives a flow decomposition of  $q$ . Since by construction from  $q'$  to  $q$  only possibly the flow on shortcuts could have changed (to a non-negative value), (20), (10) and (14) (with respect to flow) hold. Also, for every path in the flow decomposition of  $q'$  there exists exactly one corresponding path in the flow decomposition of  $q$  and that path has identical endpoints and flow value. By this, (21) and (11) hold. Therefore,  $q$  is both admissible and satisfies the proposition.

Analogously, the same reasoning can be employed in the other direction to construct a flow-pressure combination  $(p', q')$  for  $N'$  from a flow-pressure combination  $(p, q)$  for  $N$ .

We remark that the proposition can be stated and proved similarly without the premise that  $u \in V \setminus \{V_+^{sc}\}$ , but it requires some technical refinements in the statement and the proof of the proposition in order to account for the case that  $u \notin N'$ .

## 4 Detection of redundant operation modes

We reduce the network representation of a single operation mode by iteratively contracting all shortcuts which are incident to at most one boundary node and which can be contracted in accordance with Proposition 1. On the reduced network we tighten all flow and pressure bounds by solving the optimization problem given by (9)-(14) with the respective objective functions. All operation modes turning out to be infeasible are discarded. Finally, we detect redundancy between the remaining feasible operation modes by an extended isomorphism test between their reduced network representations. More specifically, for operation modes  $m^1$  and  $m^2$  with reduced network representations  $G^1 = (V^1, A^1)$  and  $G^2 = (V^2, A^2)$ , respectively, we say that  $m^1$  is *dominated* by  $m^2$  if all of the following hold:

- There exists a graph isomorphism  $\phi : V_1 \rightarrow V_2$ .
- All boundary nodes are fixed points of  $\phi$ .
- All element types are conserved by  $\phi$ .
- All feasible flow and/or pressure values for elements of  $G^1$  are also feasible for the elements' images in  $G^2$ .

This can be formalized as follows:

$$\phi(u) = u \quad \text{for all } u \in V_{\pm}^1, \quad (24)$$

$$u \in V_0^1 \iff \phi(u) \in V_0^2 \quad \text{for all } u \in V^1, \quad (25)$$

$$(u, v) \in A_{cg}^1 \iff (\phi(u), \phi(v)) \in A_{cg}^2 \quad \text{for all } (u, v) \in A^1, \quad (26)$$

$$(u, v) \in A_{cv}^1 \iff (\phi(u), \phi(v)) \in A_{cv}^2 \quad \text{for all } (u, v) \in A^1, \quad (27)$$

$$(u, v) \in A_{sc}^1 \iff (\phi(u), \phi(v)) \in A_{sc}^2 \quad \text{for all } (u, v) \in A^1, \quad (28)$$

$$\underline{p}_u \geq \underline{p}_{\phi(u)}, \bar{p}_u \leq \bar{p}_{\phi(u)} \quad \text{for all } u \in V_1, \quad (29)$$

$$P_{(u,v)} \subseteq P_{(\phi(u), \phi(v))} \quad \text{for all } (u, v) \in A_{cg}^1. \quad (30)$$

Then,  $G^1$  and  $G^2$  are topologically equivalent by (24)-(28). In addition, by (29)-(30) all flow and pressure combinations, that are feasible in operation mode  $m^1$ , are also feasible in operation mode  $m^2$ . Therefore, we can iteratively discard *dominated* operation modes such that for our new, redundancy-free set of operation modes  $\mathcal{M}'$  no decision  $m^1 \in \mathcal{M}'$  is *dominated* by another decision  $m^2 \in \mathcal{M}'$ . We note that this extended isomorphism test can be performed on the basis of any general graph isomorphism test, which generates all isomorphisms between  $G^1$  and  $G^2$  (with  $V_{\pm}^1$  as fixed points), and by checking if any of these isomorphisms fulfils (24)-(30).

## 5 Classification of operation modes by compressor station flow patterns

Because arcs in our network model can admit flow in either direction, most boundary nodes cannot generally be classified as either source or sink of the compressor station. Instead, they may take on either role, depending on the general network situation and the selected operation mode of the station. Thus, compressor stations often admit multiple *flow patterns*. Here, a *flow pattern* describes a partition of the boundary nodes into sinks and sources. This corresponds to a mass flow excess bound of the form  $b_u \leq 0$  (if  $u$  is a sink) or  $b_u \geq 0$  (if  $u$  is a source) for every boundary node  $u$ . We say that a compressor station admits a flow pattern if it admits a flow which satisfies the flow pattern.

On the contrary, a compressor station usually admits very few or even just a single flow pattern if its operation mode is fixed. This is mostly due to the fixed working direction of active compressors. Therefore, we suggest as an additional modelling step the classification of operation modes by the flow patterns they admit, in addition to introducing binary variables for the different flow patterns. The idea is that, when solving a gas network optimization task, branching on the flow patterns should quickly identify the viable flow patterns and thereby reduce the set of operation modes that need to be considered.

We incorporate this into our model as follows. For every flow pattern  $F_i$ ,  $i \in I$ , that a compressor station admits, let  $\mathcal{F}_i$  be the set of operation modes of the station that admit this flow pattern. Then, every feasible operation mode is contained in at least one of the sets  $\mathcal{F}_i$  for some  $i \in I$ . For all  $i \in I$  let  $f_i \in \{0, 1\}$  be a binary variable which is 0 if flow pattern  $F_i$  is not chosen, i.e., the selected operation mode does not admit  $F_i$ . This is expressed by the constraints

$$f_i = \sum_{m \in \mathcal{F}_i} s_m \quad \text{for all } i \in I. \quad (31)$$

Note that due to the fact that a single operation mode can admit multiple flow patterns  $f_i = 1$  does not generally imply that flow pattern  $F_i$  is chosen. It just means that the selected operation mode also admits this flow pattern. This modelling choice was made to avoid splitting the operation modes further to achieve a one-to-one correspondence between operation modes and flow patterns, which would inflate the model. Hence, the model only requires that at least one flow pattern is chosen:

$$\sum_{i \in I} f_i \geq 1. \quad (32)$$

Finally, we couple the choice of the flow pattern with flow bounds for the boundary nodes according to the flow pattern:

$$\sum_{i \in I} \underline{b}_u(F_i) f_i \leq b_u \leq \sum_{i \in I} \bar{b}_u(F_i) f_i \quad \text{for all } u \in V_{\pm} \quad (33)$$

with

$$\begin{aligned} \underline{b}_u(F_i) &:= \begin{cases} \min_{m \in \mathcal{F}_i} \underline{b}_u(m) & \text{if } u \text{ is a sink for } F_i \\ 0 & \text{otherwise} \end{cases} & \text{for all } i \in I, u \in V_{\pm}, \\ \bar{b}_u(F_i) &:= \begin{cases} \max_{m \in \mathcal{F}_i} \bar{b}_u(m) & \text{if } u \text{ is a source for } F_i \\ 0 & \text{otherwise} \end{cases} & \text{for all } i \in I, u \in V_{\pm}. \end{aligned}$$

## 6 Computational results

When applied to a single compressor station with a linear model for the compressors, earlier computational results in [5] showed some benefit of the proposed model extensions for detecting infeasibility early. However, presumably due to the simplified nature of this setting, the overall impact was negligible.

We therefore evaluated the effectiveness of our approach on an entire gas network with several compressor stations. For this, we used the openly available gas network benchmark instance GasLib-582 [7], consisting of a total of 582 nodes and 609 arcs. It contains two compressor stations which have six active elements including one compressor group and 15 active elements including three compressor groups, respectively. In addition, the second compressor station contains two control valves. Since in the original data, the compressor stations are only specified by their active elements, we had to manually determine the corresponding station subnetworks. For this we expanded the subnetworks induced by the stations' active elements to a reasonable depth and without adding any additional active elements or long pipes. The resulting station subnetworks are both connected, and have three boundary nodes each.

For the first compressor station the original data describes four operation modes which, as explained in Section 2, for all active elements specify one of the states *open* or *closed* and possibly a flow direction. But they do not specify for *open* compressor groups (or control valves) whether they are *active* or in *bypass*. Enumerating all combinations of *active* and *bypass* leads to six fully specified operation modes of which three remain after filtering out those that are invalid with respect to computed flow and pressure bounds on the elements or detected as redundant by the method presented in Section 4. These three remaining operation modes are then used for the formulation of our extended and bounded extended models. Similarly for the second compressor station the original data contains 19 operation modes which correspond to 225 fully specified operation modes of which ultimately 52 remain that are used in the formulation of our extended and bounded extended models. The number of operation modes after the different processing steps are also listed in detail in Table 1.

The GasLib-582 benchmark data contains a set of nomination files which each describe an inflow-outflow-situation at the boundary nodes of the GasLib-582 gas network together with pressure bounds or fixations. For these 4227 nominations, we created the gas network optimization model from [3]. We chose this model as it is a standard MILP model and thus suitable for experiments with standard MIP solvers. Of the 4227 nominations, 35 nominations are recognized as infeasible during model setup (which involves certain preprocessing). Each basic MILP model was enhanced with our model extensions described in the preceding sections.

	original data	# of operation modes		
		after full specification	after removal of invalid ones	after removal of redundant ones
station 1	4	6	5	3
station 2	19	225	81	52

Table 1: Shown are the number of operation modes after the different processing steps. The operation modes in the left-most column are used in the *compact formulation*. The operation modes in the right-most column are used in the *extended formulation* and *extended bounded formulations*.

In total, we compared 13 different model variants: the *compact formulation* (C), the *extended formulation* (E) and the *bounded extended formulations* which consist of the *extended formulation* enriched with flow bounds (F) as in (8i), pressure bounds (P) as in (8h), flow pattern constraints (M) as in (31)-(33), flow pattern constraints with increased branching priority on the flow pattern decision variables (M+) and all combinations of the former (FP, FM, FM+, PM, PM+, FPM, FPM+). All computations were performed with Gurobi 7.5.1 [4] on a single thread and with a time limit of one hour. Because we observed that some of the problems are intrinsically numerically challenging, we performed all computations three times with the same three different random seeds and averaged the performance over these three runs. In addition, we excluded 60 of the 4192 instances from our analysis due to numerical instability.

The results demonstrate the success of our approach as the original *compact formulation* is outperformed by all other models. Moreover, they show that the model formulation benefits both from our selection of fully specified operation modes as well as from the addition of operation mode specific bounds as can be seen in Figure 4. For the comparison of two models, we only considered these test instances where they showed a noticeable performance difference which means runtime difference of at least one second and a ratio of runtimes of least 1.05. On the ca. 75% of instances which exhibited a noticeable performance difference, the *extended model* and the FPM+ model were faster than the *compact model* by median factors of 1.12 and 1.25, respectively. The mean runtimes ranged from 37.7 seconds for the FPM+ model to 44.6 seconds for the *compact formulation*. However, our results also show that the different types of bounds used in the *bounded extended formulations* vary greatly in effectiveness. While the addition of pressure bounds improves the model a lot, the addition of flow bounds only has a small positive effect, and the addition of flow pattern constraints even decreases the model’s performance in some cases.

## 7 Conclusion

We considered the optimization model for operation modes of compressor stations from [3, Section 6.1.8]. We developed improved formulations of the model and a data preprocessing technique based on network topology reduction in order to address two problems of the original formulation.

First, in the original model operation modes are not fully specified, and as a result each operation mode in the original model may correspond to expo-



Figure 4: The cells in the heatmap show the median ratios of runtimes (column model divided by row model) over the ca. 75% of test instances which exhibited noticeable performance differences.

nentially many fully specified operation modes. We solved this by developing a different formulation of the model which uses fully specified operation modes. We determined the set of fully specified operation modes for our new model by enumeration of fully specified operation modes contained in the original model, and used bound propagation to filter out those that are physically infeasible. In addition, we developed different versions of our model by incorporating additional bounds into it.

Second, operation modes can be highly redundant. Different operation modes might describe the exact same behavior of a compressor station within a gas network. But this is difficult to tell due to the complexity of the network. To solve this problem we developed a method to faithfully reduce the topology of the network that each operation mode enables. This reduced network representation for each operation mode allowed us to effectively compare them by means of graph isomorphisms, and detect redundancy between them.

Lastly, we evaluated our models on reference benchmark data for gas networks, and found that our methods yielded an improvement in the model's performance of up to 25%.

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