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**Polyhedral 3D Models for
compressors in gas networks**

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Polyhedral 3D Models for compressors in gas networks *

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Abstract

Compressor machines are crucial elements in a gas transmission network, required to compensate for the pressure loss caused by friction in the pipes. Modelling all physical and technical details of a compressor machine involves a large amount of nonlinearity, which makes it hard to use such models in the optimization of large-scale gas networks. In this paper, we are going to describe a modelling approach for the operating range of a compressor machine, starting from a physical reference model and resulting in a polyhedral representation in the 3D space of mass flow throughput as well as in- and outlet pressure.

In gas networks, compressors are used to increase the pressure of the incoming gas to a higher outflow pressure, thus counteracting the pressure loss caused by friction in pipes. This allows for the gas to be transported over long distances. A *compressor machine* (or compressor, for short) is powered by an associated *compressor drive*. The technical models for compressors and drives are highly nonlinear [1, 6]. As optimization models for gas networks usually involve switching compressors, this leads to hard-to-solve MINLPs. It is thus desirable to use simpler (i.e. polyhedral) yet accurate models for a compressor.

In this paper, we construct a polyhedral model for the operating range of a compressor machine in the three-dimensional space $(q, p^{\text{in}}, p^{\text{out}})$ of mass flow rate, inlet and outlet pressure. We will closely follow the steps as in [2] and analyse some of the assumptions that are made therein. In contrast to the construction in [2], we are considering technical restrictions from the drive and a non-constant compressibility factor. Moreover, we suggest a complexity-reducing postprocessing algorithm and provide computational results based on publicly available compressor data from the GASLIB [3].

For an overview on gas network optimization problems and the modeling of compressors we refer to [5, 4] and the references therein.

1 Physical compressor model

For every compressor machine $m \in \mathcal{M}$, our starting point of interest is a flow and pressure tuple $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$. A reasonably accurate modelling of a compressor

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Physical and model constants		
Temperature	T	[K]
Ambient temperature	T_{amb}	[K]
Isentropic exponent	κ	[−]
Specific gas constant	R_s	[kJ/(kg · K)]
Gas- and compressor-specific physical variables		
Pressure	p	[bar]
Mass flow rate	q	[kg/s]
Compressibility factor	z	[−]
Volumetric flow rate	Q	[m ³ /s]
Adiabatic head	H_{ad}	[kJ/kg]
Compressor speed	n	[rot./min]
Adiabatic efficiency	η_{ad}	[−]
Power	P	[kW]

Table 1: General physical quantities and constants.

machine involves many nonlinear and nonconvex constraints as well as several physical variables and quantities. An overview of the quantities that we consider constant or a variable together with their units is given in Table 1.

The physical and technical capabilities of m are given in a so-called characteristic diagram \mathcal{D}_m in the space of (Q, H_{ad}) . The physical model of a turbo compressor machine that we are using as a reference is given as follows [1, 6]:

$$z = z(p_m^{\text{in}}; T) \quad (1a)$$

$$Q = q_m R_s T z (p_m^{\text{in}})^{-1} \quad (1b)$$

$$H_{\text{ad}} = R_s T z \frac{\kappa}{\kappa - 1} \left[\left(\frac{p_m^{\text{out}}}{p_m^{\text{in}}} \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right] \quad (1c)$$

$$(Q, H_{\text{ad}}) \in \mathcal{D}_m \quad (1d)$$

$$n \in [n^{\text{min}}, n^{\text{max}}] \quad (1e)$$

$$H_{\text{ad}} = \chi(Q, n; A^{\text{speed}}) \quad (1f)$$

$$\eta_{\text{ad}} = \chi(Q, n; A^{\text{eff}}) \quad (1g)$$

$$P = q H_{\text{ad}} \eta_{\text{ad}}^{-1} \quad (1h)$$

$$P \leq \chi(n, T_{\text{amb}}; A^{\text{power}}) \quad (1i)$$

In constraint (1a), the compressibility factor z can be computed according to different formulas, see Section 2.2. The constraints (1a), (1b) and (1c) relate the problem variables $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$ to the characteristic diagram variables (Q, H_{ad}) . The constraints (1d) typically comprise a set of quadratic and possibly nonconvex inequalities. A restriction on the available power for compression as induced by the compressor drive is given by (1h) and (1i). In (1f), (1g) and (1i), $\chi(\cdot, \cdot; A)$ denotes a biquadratic function with some coefficient matrix $A \in \mathbb{R}^{3 \times 3}$.

2 Reformulated compressor model in $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$

The physical model is highly nonlinear, making it hard to find globally optimal solutions for large-scale gas network optimization problems. Moreover, in most cases, we are not directly interested in the values of most compressor-specific quantities of the physical model, which motivates the construction of a less complex compressor model as in Chapter 7.3.4 in [2]. Starting with a compressor machine $m \in \mathcal{M}$, every point (Q, H_{ad}) of its characteristic diagram can be transformed into a (curved) ray in the space of $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$ by inverting the equations (1b) and (1c). There is one degree of freedom in this transformation, denoted by p :

$$\begin{pmatrix} q \\ p^{\text{in}} \\ p^{\text{out}} \end{pmatrix} = g(Q, H_{\text{ad}}; p) = \begin{pmatrix} \frac{Qp}{R_s T z} \\ p \\ \left(\frac{H_{\text{ad}}}{R_s T z^{\frac{\kappa}{\kappa-1}}} + 1 \right)^{\frac{\kappa}{\kappa-1}} p \end{pmatrix} \in \mathbb{R}^3 \quad \text{for } p \geq 0. \quad (2)$$

Practically, in order to obtain a polyhedral approximation of the operating range in $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$, we apply (2) to a set of sample points within the characteristic diagram and on its boundary for a set of different values of p . This yields a set of points $\{q_m^k, p_m^{\text{in},k}, p_m^{\text{out},k}\}$. From this set, we remove all points that violate some of the technical bounds $p_m^{\text{in}} \geq p_m^{\text{in},\text{min}}$, $p_m^{\text{out}} \leq p_m^{\text{out},\text{max}}$, or $q_m^{\text{min}} \leq q_m \leq q_m^{\text{max}}$.

2.1 Impact of restricted compressor power

As mentioned before, the power for the compression process is provided by a compressor drive. This power is limited by an upper bound on the power that depends on the compressor speed n and the ambient air temperature T_{amb} [1]. The lower the ambient temperatures, the more power can be provided. In order to account for this power bound, we compute the required power P for every point of our set $\{q_m^k, p_m^{\text{in},k}, p_m^{\text{out},k}\}$ according to (1h) and discard all points that violate (1i). The convex hull \mathcal{P}_m of the remaining points yields the desired representation.

In Table 2, we show the impact of the power bound on our operating ranges in the space of $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$, based on a set of 5000 sampled instances on compressor data taken from the GASLIB [3]. It can be seen that roughly 2%-10% of the feasible instances for a compressor machine are rendered infeasible by the compressor drive restrictions. Typically, these instances are characterized by high throughput q_m and high inlet pressure p_m^{in} . Figure 1a shows a set of feasible and infeasible sample points.

2.2 Impact of variable compressibility factor

There exist several formulas for computing the compressibility factor z . Due to its accuracy for the pressure range that we are considering, we take the formula of PAPAY [1] as our reference:

$$z(p, T) = 1 - 3.52 \frac{p}{p_c} e^{-2.26 \frac{T}{T_c}} + 0.247 \left(\frac{p}{p_c} \right)^2 e^{-1.878 \frac{T}{T_c}}. \quad (3)$$

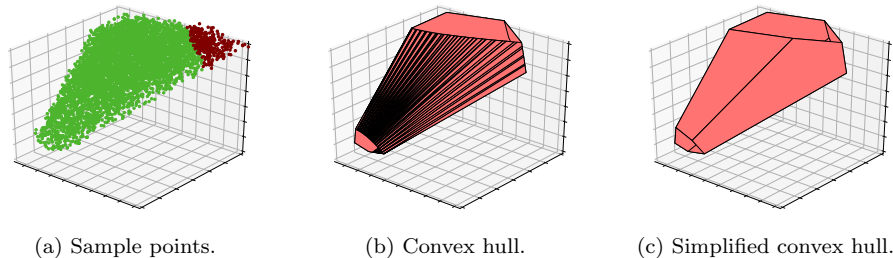


Figure 1: Construction of the operating range of a compressor machine.

compressor	cold	warm	hot
m_1	2.71%	5.66%	9.77%
m_2	1.85%	3.97%	5.83%
m_3	2.50%	5.55%	9.43%
m_4	1.86%	3.98%	5.90%
m_5	2.43%	5.37%	9.33%
m_6	1.85%	3.97%	5.83%

Table 2: Percentage of instances feasible for the compressor machine but infeasible due to the power bound of the compressor drive for different air temperatures.

p_c and T_c denote the pseudocritical pressure and temperature of the gas, respectively, that are constant in our case. Since we also assume T to be constant, (3) reduces to a quadratic equation in the pressure p .

Another formula, which is linear in p and suitable for pressure values up to 70 bar, has been proposed by the AMERICAN GAS ASSOCIATION (AGA) [1]:

$$z(p, T) = 1 + 0.257 \frac{p}{p_c} - 0.533 \frac{p}{p_c} \frac{T_c}{T}. \quad (4)$$

As a third and most simple alternative, used also in [2], the compressibility factor can also be considered constant with a value around $z = 0.9$.

Using the different z -factor formulas for the construction of our operating ranges leads to different polytopes in $(q_m, p_m^{\text{in}}, p_m^{\text{out}})$. There are errors in two directions: the polytopes may contain points that are infeasible for the physical model (false-positive; due to working with convex hulls), but they may also exclude feasible points (false-negative; due to using (4) or a constant value for z as compared to our reference formula (3)). Again, we have sampled 5000 instances and determined the percentage of false-positive and false-negative feasibility outcomes for the different z -factor computation variants. The results are shown in Table 3. It can be seen that the polytope obtained from using a constant compressibility factor contains the least percentage of instances that are technically infeasible. On the other hand, it also excludes some amount of technically feasible instances, which is not the case for AGA and PAPAY. Moreover, it turns out that the polyhedral approximation of the more exact PAPAY formula is better than the one for the AGA formula.

compressor	PAPAY		AGA		const $z = 0.9$	
	false-pos	false-neg	false-pos	false-neg	false-pos	false-neg
m_1	8.62%	0.00%	12.22%	0.03%	5.66%	5.52%
m_2	10.30%	0.00%	11.56%	0.00%	6.30%	3.29%
m_3	9.02%	0.00%	11.52%	0.00%	5.64%	6.64%
m_4	9.24%	0.00%	11.46%	0.00%	5.92%	3.42%
m_5	8.72%	0.00%	12.62%	0.03%	5.52%	5.51%
m_6	9.18%	0.00%	11.62%	0.02%	6.26%	3.13%

Table 3: Percentage of false-positive/-negative instances for different z formulas.

2.3 Reduction of polytope facets

The resulting polytope becomes more precise for a large set of sampling points in (Q, H_{ad}) and many values of p . As a downside, the convex hull representation gets more complex, i.e., the number of its vertices and facets sharply increases, blowing up the model formulation. Therefore, we propose Algorithm 1 to reduce the number of polytope facets until a given volume error tolerance is reached.

Tentatively trying different halfspaces is the most computationally expensive part of the algorithm. Hence, if the number of facets of the input polytope \mathcal{P} is large, it may be advantageous to only consider a (random) subset of all facets in every iteration. Our experience has shown that usually very few facets suffice to approximate any given polytope \mathcal{P} with a volume error of only 1% in reasonable time. Some results and computation times for our GASLIB compressors are shown in the following Table 4.

3 Conclusion

We have presented a method to construct a small yet quite accurate 3D polyhedral model for a compressor machine that approximates the true operating range as given by a nonlinear physical reference model. We computationally quantified the impact of neglecting the drives power restrictions as well as the

Algorithm 1: Polytope Facet Reduction(\mathcal{P}, τ)

Input: A polytope \mathcal{P} and a volume error tolerance $\tau \geq 0$.

Output: A polytope $\mathcal{Q} \supseteq \mathcal{P}$ with $\frac{\text{vol}(\mathcal{Q})}{\text{vol}(\mathcal{P})} - 1 \leq \tau$ and less facets than \mathcal{P} .

Compute $\text{vol}(\mathcal{P})$.

Let \mathcal{F} be the set of facets of \mathcal{P} (given as halfspace inequalities).

Initialize \mathcal{Q} to be some box around \mathcal{P} .

while $\frac{\text{vol}(\mathcal{Q})}{\text{vol}(\mathcal{P})} - 1 > \tau$ **do**

$f^* := \arg \min_{f \in \mathcal{F}} \text{vol}(\mathcal{Q} \cap f)$
 $\mathcal{Q} := \mathcal{Q} \cap f^*$

end

Return \mathcal{Q} .

compressor	(original)	$\tau = 0.1$		$\tau = 0.01$		$\tau = 0.001$	
	facets	facets	time[s]	facets	time[s]	facets	time[s]
m_1	256	7	5.20	9	9.71	23	128.56
m_2	286	6	2.75	8	6.51	23	102.48
m_3	259	6	3.37	10	12.71	16	50.96
m_4	284	6	4.39	9	9.67	23	109.44
m_5	261	6	3.80	9	9.60	18	74.32
m_6	284	6	3.40	9	10.59	21	81.96

Table 4: Computational results of facet reduction algorithm.

impact of the choice of the formula for computing the compressibility factor. It turns out that the polyhedral model approximates the true nonlinear operating range within an error of roughly 10%, which is acceptable for many applications.

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