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## **Demand-Driven Line Planning with Selfish Routing**

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# Demand-Driven Line Planning with Selfish Routing

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**Abstract** Bus rapid transit systems in developing and newly industrialized countries are often operated at the limits of passenger capacity. In particular, demand during morning and afternoon peaks is hardly or even not covered with available line plans. In order to develop demand-driven line plans, we use two mathematical models in the form of integer programming problem formulations. While the actual demand data is specified with origin-destination pairs, the arc-based model considers the demand over the arcs derived from the origin-destination demand. In order to test the accuracy of the models in terms of demand satisfaction, we simulate the optimal solutions and compare number of transfers and travel times. We also question the effect of a selfish route choice behavior which in theory results in a Braess-like paradox by increasing the number of transfers when system capacity is increased with additional lines.

## 1 Introduction

A common characteristic of most urban transportation systems is the variation in demand during different times of the day and different days of the week. This characteristic is even more notable in bus rapid transit (BRT) systems as such systems typically serve the bulk of the demand, exerting a trunk function for the whole trans-

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portation system of a region/area. The demand varies heavily during the day, with morning and evening peaks if it is a weekday while timings of morning and evening peaks may change for the weekends. In addition to fluctuations with respect to time, the demand is typically highly asymmetric with respect to its distribution on the line. Such systems, again typically, cover a central line of a highly populated city. In particularly densely populated cities of such countries, (the population is young and the workforce moves from usually outer regions of the cities to the central parts, i.e.,) the movement of population is typically in reverse directions for morning peaks vs. evening peaks. As a result, the system-wide transportation demand exerts significant changes in both time and space.

Traditional line planning is mostly concerned with a static demand and addresses the case of fluctuating demand by constructing a base service, which is augmented in peak hours, or vice versa, i.e., constructing a peak service which is decremented during non-peak hours. In BRT systems with demand sensitivity in both space and time, a demand-driven approach in line planning would be more viable. As a result, line plans are more susceptible to infrastructure capacity and fleet capacity. In this respect, an analytical demand-driven approach should consider the effects of such limitations on the level of demand satisfaction in the system.

In traditional mathematical programming formulations of line planning problems, the relation between capacities and the demand are roughly reflected/considered. There is usually a predetermined frequency requirement rather a demand amount to be satisfied ([2]). We use mathematical formulations where origin-destination (OD) demand between pairs of stations to be satisfied in a finite length planning horizon is explicitly represented as in [3]. In order to investigate the accuracy of demand-driven line planning approaches, we consider a simplistic underlying network structure: a line network. On a line network, each pair of stations is connected via a single path. The alternative mathematical formulations differ from each other with respect to the way demand is represented: an arc-based model where the OD demand is transformed into arc demands without considering the passenger routes explicitly and an OD-based model where the demand is represented in its original form. First, we aim to test the accuracy of the solutions provided by alternative formulations. To this end, we simulate the optimal line plan solutions to observe the differences in basic matrices. In the simulation, we consider passengers with a selfish route choice behavior that is expected to result in a Braess-like paradox.

## 2 The Braess-like Paradox with Selfish Routing

The passenger load on lines may significantly change based on the route choice behavior of passengers. The route choice behavior is indeed a very complex problem when there are uncertainties involved; yet, it can be simplified in favor of selecting a certain criterion such as minimizing the number of connections or travel time.

When the transport system is sensitive to capacities, the route choice behavior of passengers may play an important role in service levels. We observe a Braess-like paradox with respect to the simple criterion of minimizing connections when the capacity of a system is increased by adding lines.

In order to illustrate our observation, we consider a network consisting of stations 1 to 5 along a single path with three lines denoted  $A$ ,  $B$  and  $C$  as shown in Figure 1a and OD demands as in Figure 1b. Each line has a passenger capacity of  $N$ .

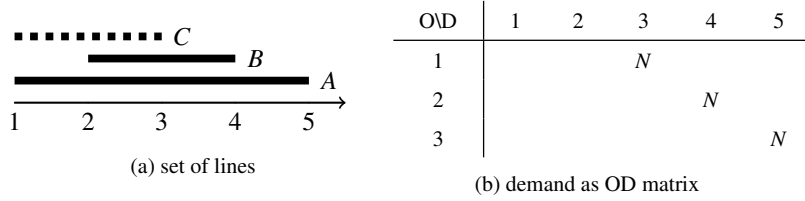


Fig. 1: Alternative line plans and corresponding OD demand matrix

We compare two different line plans:

- Line plan  $\{A, B\}$ , consists of line  $A$  and line  $B$ . Naturally,  $N$  passengers from 1 to 3 take line  $A$ . Therefore, all passengers from 2 to 4 must use line  $B$ . Finally,  $N$  passengers from 3 to 5 take line  $A$ .
- Line plan  $\{A, B, C\}$ , is likely to change the situation. Passengers from 1 to 3 now have the opportunity to choose between line  $A$  and line  $C$ . Under a selfish behavior assumption, each passenger takes the first non-full line to come. Supposing that both lines work with the same frequency, this leads to a split of  $N$  passengers between  $A$  and  $C$ : on average  $N/2$  passengers on each line. Hence, line  $A$  is not full at station 2. Again, passengers taking the first non-full line causes roughly  $N/2$  of the demand from 2 to 4 to take line  $A$  instead of  $B$ . All vehicles on line  $A$  are, then, full while traveling from station 2 to station 3 where  $N/2$  passengers hop off. In consequence, only about  $N/2$  seats are empty on line  $A$  at station 3, so only  $N/2$  of the passengers waiting at station 3 to go to station 5 can take line  $A$ . The rest  $N/2$  is forced to take line  $B$  from 3 to 4, and connect to line  $A$  at station 4 when the passengers traveling from 2 to 4 empty their seats.

We observe that introducing a new line to increase the passenger capacity may lead to a worse situation with respect to minimal number of connections due to selfish behavior of passengers. All passengers could be handled via direct connections with line plan  $\{A, B\}$ . On the contrary, line plan  $\{A, B, C\}$  leads to passengers transfers in station 4. Table 1 summarizes the comparison between the two line plans.

line plan	fixed cost	operational cost	direct travelers	travelers with transfer at station 4
$\{A, B\}$	$2 \cdot f$	$X$	$3 \cdot N$	0
$\{A, B, C\}$	$3 \cdot f$	$X + c_{1,2} + c_{2,3}$	$2 \cdot N + \frac{N}{2}$	$\frac{N}{2}$

Table 1: Comparison of line plan  $\{A, B\}$  and line plan  $\{A, B, C\}$ 

### 3 Alternative formulations for demand-driven line planning

We consider an underlying traffic network represented by a directed graph  $G = (N, A)$  where  $N$  denotes the stations and  $A$  denotes the set of traffic links between the stations. For a finite length planning horizon, OD demand is specified as  $d_{sq}$  for each  $(s, q)$  pair of stations with  $s, q \in N$ . When the paths along which passengers travel are known in advance (as is the case for a network with tree structure), OD demand can be transformed into arc demand as denoted by  $d_a, a \in A$ .

To formulate the line planning problem, we consider a given line set  $L$ ;  $c_l^f$  and  $c_l^o$  denote the fixed costs of line  $l$  and the operational costs of assigning a vehicle to line  $l$ , respectively. In a line plan, at most  $V$  vehicles can be used while at most  $M$  vehicles can be assigned to any line. A passenger route is defined not only by the physical path on the traffic network but also by each line used to traverse the traffic link along that path. For an OD-pair  $(s, q)$ ,  $R_{sq}$  denotes the set of routes from  $s$  to  $q$ . The set of all routes is denoted by  $R = \bigcup_{s,q} R_{sq}$ .

In an arc-based demand coverage model ( $DCM_A$ ) as described in Borndörfer et al. [1], passenger routes are not considered explicitly. Therefore, two decision variables are sufficient in the integer programming formulation of the problem:  $x_l \in \{0, 1\}$  taking the value of 1 if line  $l \in L$  is selected, and an integer variable  $v_l \in \mathbb{N}$  denoting the number of vehicles assigned to line  $l$ . However, in an OD-based model ( $DCM_{OD}$ ), an additional decision variable  $z_r \in \mathbb{N}$  is used to denote the number of passengers taking route  $r$ . We obtain the following integer programming problem formulation where  $g_{la}^r$  is an indicator denoting the traversal of arc  $a$  by line  $l$  on route  $r$  and  $\rho(r)$  is some penalty function that characterizes how attractive the route is for passengers.

$$\begin{aligned}
\min \quad & \sum_{l \in L} c_l^f x_l + \sum_{l \in L} c_l^o v_l + \sum_{r \in R} z_r \rho(r) && (DCM_{OD}) \\
\text{subject to} \quad & \sum_{r \in R_{sq}} z_r \geq d_{sq} && \forall (s, q) \in D \\
& \sum_{r \in R} z_r g_{la}^r \leq \kappa v_l && \forall l \in L, a \in A \\
& Mx_l - v_l \geq 0 && \forall l \in L
\end{aligned}$$

$$\begin{aligned} \sum_{l \in L} v_l &\leq V \\ x_l &\in \{0, 1\}, v_l \in \mathbb{N} \quad \forall l \in L \\ z_r &\in \mathbb{N} \quad \forall r \in R \end{aligned}$$

It is clear that  $DCM_{OD}$  is larger in terms of number of decision variables when compared to  $DCM_A$ . On the other hand,  $DCM_{OD}$ , when solved to optimality, provides a more accurate solution with respect to demand satisfaction and with respect to minimal number of connections/transfers. To illustrate how  $DCM_{OD}$  can be advantageous in comparison to  $DCM_A$ , we consider the example in Figure 2.

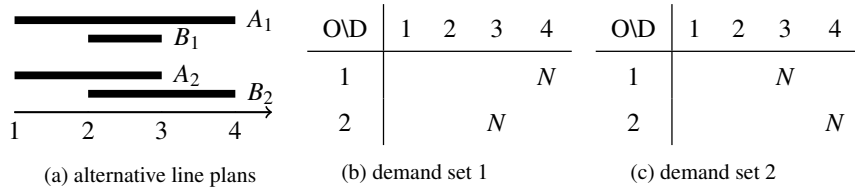


Fig. 2: Optimal solution examples for two demand sets

In the example, the arc demands are the same for demand set 1 and demand set 2; the arc capacities provided by line plan  $\{A_1, B_1\}$  and line plan  $\{A_2, B_2\}$  are equivalent. As a matter of fact, according to  $DCM_A$  either line plan is optimal for both demand sets. In contrast, only line plan  $\{A_1, B_1\}$  is optimal for demand set 1 with the OD-based demand model,  $DCM_{OD}$  and  $\{A_2, B_2\}$  for demand set 2 because they allow all passengers to arrive their destination by a direct connection. Using line plan  $\{A_1, B_1\}$  for demand set 2 forces the passenger of OD pair (2, 4) to take line  $B_1$  first (as  $A_1$  is full) and then to transfer to  $A_1$  at station 3. Using line plan  $\{A_2, B_2\}$  for demand set 1 does not provide a direct connection for the OD pair (1, 4).

## 4 Computational Results

When the simultaneous effect of ignoring passenger routes on demand satisfaction level and the selfish route choice behavior resulting in a Braess-like paradox is considered, the impact on the outcomes of seemingly optimal line plans might be complicated. In order to test both effects, we run simulations for a real-life system, the Istanbul Metrobüs, which is a BRT system with high fluctuations in demand during the day. We compare the performance of the optimal solutions from  $DCM_A$  model against two versions of the  $DCM_{OD}$  model. In version I,  $\rho$  is set to zero while in version II routes are penalized according to the number of necessary transfers. We

calculate both passenger travel times and number of transfers under the assumption that the schedule can be implemented. We select three different time periods of one hour length that are significantly different from each other in terms of the demand characteristics. In the simulation, each one-hour solution is tested for 10 consecutive periods to approximate a continuously run schedule. Most importantly, we assume a selfish route choice behavior and lines arriving in a random order, thus inducing unwanted transfers. The results are shown in Table 2.

Instance	Model	Operational costs (km)	Average travel time (min)	Transfers per passenger
08h - 09h	$DCM_A$	5597.3	37.1	0.19
	$DCM_{OD}$ I	5657.9	35.4	0.19
	$DCM_{OD}$ II	5732.0	36.4	0.08
14h - 15h	$DCM_A$	2263.9	37.5	0.18
	$DCM_{OD}$ I	2284.6	37.3	0.19
	$DCM_{OD}$ II	2309.3	37.9	0.04
18h - 19h	$DCM_A$	6868.2	39.5	0.24
	$DCM_{OD}$ I	6886.8	38.3	0.28
	$DCM_{OD}$ II	6983.7	40.6	0.08

Table 2: Results of computational experiments

Overall, the three models show similar results although the set of optimal lines may differ from each other. As expected, non-zero penalty affects the ratio of transfers to passengers with negligible increase in operational costs. In terms of average travel times, OD-based model results in small savings which are lost when transfer penalty is introduced. To conclude, however, the impact of model choice can only be observed in the number of transfers and when using an appropriate penalty function. Simulation results show that optimal line plan solutions may fall short in response to time-sensitive demand. This observation shall stimulate new ideas and approaches to improve the accuracy of mathematical models in demand-sensitive line planning.

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