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The Price of Spite in Spot-checking games

(Brief Announcement)

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We introduce the class of spot-checking games (SC games). These games can be seen as the graphical counterpart of *security games*, and can be used to model problems where the goal is to distribute fare inspectors over a toll network. In an SC game, the pure strategies of network users correspond to paths in a graph, and those of the fare inspectors are subset of edges to be controlled. Mixed strategies of the network users entail a non-atomic traffic model without congestion, and can be represented using multicommodity flows. Similarly, mixed strategies of the inspector can be represented by flows in a time-extended duty graph and yield Markovian patrolling policies. With this model, best responses of the network users to a given inspector’s strategy correspond to shortest paths for some weights that depends on the control intensities.

Although SC games are not zero-sum, we show that a mixed Nash equilibrium can be computed by linear programming. However, the computation of a strong Stackelberg equilibrium is more relevant – because the inspector can credibly commit to a strategy– and we give a mixed integer programming (MIP) formulation for this problem. We show that the computation of such an equilibrium is NP-hard, even in the simplest case, in which the game has a “zero-sum plus costs” structure; Consequently, it is NP-hard to compute a strong Stackelberg equilibrium in a polymatrix game, *even if the game is pairwise zero-sum*.

Then, we study the *price of spite*, which measures how the payoff of the inspector degrades when committing to a Nash equilibrium, that is, when he loses his ability to credibly commit. In fact, the Nash equilibrium is easy to compute and we regard it as an efficient heuristic for the inspector, but this is also the most harmful strategy for the network users (thus the name *spite*). We give two upper bounds for this measure. The first one depends on the detour done by the flow of users’ best responses to the Nash controlling strategy, with respect to a particular metric. The second one is valid for networks with a distance-based toll: in this situation the price of spite is bounded from above by a constant which depends only on the ratio of the toll rate per kilometer to the average penalty to pay per evaded kilometer for a uniform control.

Finally, we report computational experiments on instances constructed from real data, for an application to the enforcement of a truck toll in Germany. These numerical results show the efficiency of the proposed methods, as well as the quality of the bounds derived in this article.

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