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## **Re-optimization of Rolling Stock Rotations**

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# Re-optimization of Rolling Stock Rotations

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**Abstract** The Rolling Stock Rotation Problem is to schedule rail vehicles in order to cover timetabled trips by a cost optimal set of vehicle rotations. The problem integrates several facets of railway optimization, i.e., vehicle composition, maintenance constraints, and regularity aspects. In industrial applications existing schedules often have to be re-optimized to integrate timetable changes or construction sites. We present an integrated modeling and algorithmic approach for this task as well as computational results for industrial problem instances of DB Fernverkehr AG.

## 1 Introduction

The Rolling Stock Rotation Problem (RSRP) deals with the implementation of a railway timetable by constructing *rolling stock rotations* to operate passenger trips by rail vehicles. Rail vehicles or rolling stock is the most expensive and limited asset of a railway company. This problem integrates several major requirements like *vehicle composition rules*, *maintenance constraints*, *infrastructure capacities*, and *regularity stipulations*. A detailed problem description, a Mixed Integer Programming formulation, and an algorithm to solve this problem in an integrated manner is described in detail in [2].

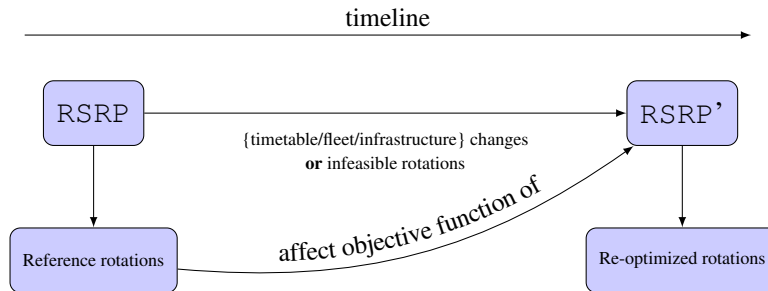
In this paper, we report on one of the most important and challenging industrial applications of the RSRP, i.e., *Re-optimization* or *Re-scheduling*.

A concept of Re-optimization for the RSRP is illustrated in Figure 1 and can be summarized as follows. At some point in time a railway undertaking has to tackle an instance of the RSRP and agrees on a solution, ideally, utilizing an optimiza-

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**Fig. 1** Concept of Re-optimization for the RSRP.

tion algorithm or by manual planning, see box reference rotations. At another point in time this problem changes that much, such that the reference rotation plan can no longer be viable. Thus, a new problem has to be solved, *i.e.*, RSRP'. The most important difference to the previous planning step is that much of the reference rotation plan was already implemented: Crew was scheduled for vehicle operations and maintenance tasks, capacity consumption of parking areas was reserved, and most important in a segregated railway system, e.g., in Europe and Germany: train paths were already allocated for the timed deadhead trips. Therefore a major goal is to change as less as possible in comparison to the original rotation plan. This re-optimization planning problem occurs very often at a railway company. There are various causes that can lead to a situation where the implemented rotation plan becomes infeasible in an unexpected manner. Predictable and unpredictable construction sites are one main reason which have to be integrated in a timetable and which result in technically not feasible rotation plans. In addition, fleet changes due to disruption of operations or technical constraints, e.g., more restrictive maintenance intervals, modified speed limits for rolling stock vehicles, or changed infrastructure capacity, ask for a modification of the rotation plans.

Depending on how large and how long the infeasibilities and its consequences are re-optimization or re-scheduling is required either by the dispatchers or in sufficiently lasting cases by the tactical and strategical divisions of the railway companies. In the latter case the problem is considered as cyclic planning problem as it is introduced in [2].

The paper contributes an adaption of the generic Mixed Integer Programming approach presented in [2] to re-optimize rolling stock rotations. We show how to incorporate re-optimization requirements into the hypergraph based formulation.

This paper is organized as follows. Section 2 defines the considered problem including an overview of the hypergraph based formulation. In Section 3 we introduce an objective function for the re-optimization case and provide relations to industrial use cases. Computational results in Section 4 show that our model and algorithm produces high quality and implementable results even for complicated re-optimization settings. Rotation planners of Deutsche Bahn validated the resulting rolling stock rotations from a detailed technical and operational point of view.

## 2 The Rolling Stock Rotation Problem

The set of timetabled passenger trips is denoted by  $T$ . A trip  $t \in T$  has a departure and arrival time in our standard week. A solution of the RSRP has to contain timed *maintenance services* on each vehicle. A *maintenance service* and the set of maintenance services is denoted by  $s \in S$ . Let  $V$  be a set of *nodes* representing departures and arrivals of vehicles operating passenger trips of  $T$  and let  $A \subseteq (V \cup S)^2$  be a set of directed standard arcs. We define a set  $H \subseteq 2^A$ , called *hyperarcs*. The RSRP *hypergraph* is denoted by  $G = (V \cup S, A, H)$ . The hyperarc  $h \in H$  covers  $t \in T$ , if each standard arc  $a \in h \subseteq A$  represents an arc between the departure and arrival of  $t$ . We define the set of all hyperarcs that cover  $t \in T$  by  $H(t)$ . Using a hyperarc formulation provides us the modeling power to directly integrate vehicle composition rules and daily regularity aspects, see [1].

A *maintenance constraint*  $l$  is represented by a resource function  $r_l : S \cup A \mapsto \mathbb{Q}_+$ , a *resource upper bound*  $U_l \in \mathbb{Q}_+$ , and a sub-set of maintenance services of  $S$ . Each service of this sub-set can be performed to *reset* the resource  $r_l$  to fulfill the bound  $U_l$ . A *feasible path*  $P \subseteq A$  in  $G$  w.r.t. a maintenance constraint is a simple path starting and ending at service nodes such that:  $\sum_{s \in P} r_l(s) + \sum_{a \in P} r_l(a) \leq U_l$ , *i.e.*, the sum of all consumed resources on  $P$  has to be smaller than or equal to the bound of a maintenance constraint. A *feasible rotation* is a cycle of feasible paths. The RSRP states as follows:

Let  $T$  be a set of timetabled passenger trips and let  $G = (V \cup S, A, H)$  be a RSRP hypergraph with a cost function  $\mathbf{c} : H \mapsto \mathbb{Q}_+$ . Furthermore let  $L$  be a set of maintenance constraints. The RSRP is to find a cost minimal set of hyperarcs  $H_0 \subseteq H$  such that:

- Each timetabled trip  $t \in T$  is covered by exactly one hyperarc  $a \in H_0$ .
- The set  $\bigcup_{a \in H_0} a$  is a set of feasible rotations w.r.t. all maintenance constraints of  $L$ .

We define sets of incoming and outgoing hyperarcs of  $v \in V$  in the RSRP hypergraph  $G$  as  $H(v)^{\text{in}} := \{h \in H \mid \exists a \in h : a = (u, v)\}$  and  $H(v)^{\text{out}} := \{h \in H \mid \exists a \in h : a = (v, w)\}$ , respectively. Let  $x_h$  be a binary decision variable for each hyperarc  $h$ . Finally, the RSRP without maintenance constraints can be stated as a mixed integer program as follows:

$$\min \sum_{h \in H} \mathbf{c}_h x_h, \quad (\text{MP})$$

$$\sum_{h \in H(t)} x_h = 1 \quad \forall t \in T, \quad (1)$$

$$\sum_{h \in H(v)^{\text{in}}} x_h = \sum_{h \in H(v)^{\text{out}}} x_h \quad \forall v \in V, \quad (2)$$

$$x \in \{0, 1\}^{|H|} \quad (3)$$

The linear objective function minimizes the total cost and is directly related to the cost of operating a timetable. For each trip  $t \in T$  the covering constraints (1) assign exactly one hyperarc of  $H(t)$  to  $t$ . The equalities (2) are flow conservation constraints for each node  $v \in V$  that imply the set of rotations in the arc set  $A$ . Finally, (3) states that  $x$  has to be binary. The formulation can be directly extended to also handle the maintenance constraints introduced in the problem description, see [2]. We present here only this relaxed version of the problem formulation to simplify notation. However, the extension of the model including maintenance constraints is straight forward and does not affect the content and contribution of the paper. Nevertheless, in our computational study we provide results for the case with maintenance constraints.

### 3 Re-optimization

As introduced in Section 1 the major requirement for the re-optimization case is to change as less as possible in the reference rotation plan. We handle this requirement by defining a detailed objective function based on the reference solution.

$$\mathbf{c} : H \mapsto \mathbb{Q}_+ : \mathbf{c}(h) := \left\langle \begin{pmatrix} \mathbf{c}_1(h) \\ \mathbf{c}_2(h) \\ \mathbf{c}_3(h) \\ \mathbf{c}_4(h) \\ \mathbf{c}_5(h) \\ \mathbf{c}_6(h) \\ \mathbf{c}_7(h) \\ \mathbf{c}_8(h) \\ \mathbf{c}_9(h) \\ \mathbf{c}_{10}(h) \\ \mathbf{c}_{11}(h) \end{pmatrix}, \begin{pmatrix} p_1(h) \\ p_2(h) \\ p_3(h) \\ p_4(h) \\ p_5(h) \\ p_6(h) \\ p_7(h) \\ p_8(h) \\ p_9(h) \\ p_{10}(h) \\ p_{11}(h) \end{pmatrix} \right\rangle \quad (4)$$

*... connection deviations*  
*... deadhead deviations*  
*... composition deviations*  
*... rotation deviations*  
*... service deviations*  
*... vehicles*  
*... services*  
*... deadhead distance*  
*... regularity*  
*... couplings*

Equation 4 illustrates our approach. Our objective function for the re-optimization case is the sum of the original objective function  $\sum_{i=7}^{11} \mathbf{c}_i p_i$  and re-optimization cost  $\sum_{i=1}^6 \mathbf{c}_i p_i$ . All parts of the re-optimization part are computed as follows. Let  $h \in H$  be a hyperarc. First we reinterpret  $h$  in the reference rotations, *i.e.*, we search the timetabled trips that are connected or covered by  $h$  in the reference rotation plan, if they still exist. In a second step we compute a property  $p_i$ ,  $i = 1, \dots, 5$  for  $h$  that states the number of differences that  $h$  has w.r.t. the reference rotations. Examples for this are:

- Let trips  $t_1, t_2$  exist in the reference rotations and both trips are not connected there, then if hyperarc  $h$  connects  $t_1$  and  $t_2$ , let  $p_1(h) = |h|$ , otherwise  $p_1(h) = 0$ .
- If  $h$  implies that an additional deadhead trip has to be allocated  $p_2(h) = 1$ , otherwise  $p_2(h) = 0$ .

- If  $h$  covers trip  $t$  that exists in the reference rotations and is operated by a different vehicle composition that  $h$  represents  $p_3(h) \geq 1$ , otherwise  $p_3(h) = 0$ . The exact numeric number depends on  $|h|$ , how these vehicles are oriented, which fleets are used etc..
- If  $h$  implies that  $t$  is operated in a different feasible rotation  $p_4(h) = |h|$ , otherwise  $p_4(h) = 0$ .
- If  $h$  implies a different maintenance service before or after a timetabled trip  $p_5(h) = 1$ , otherwise  $p_5(h) = 0$ .

Solutions of re-optimization scenarios often have the characteristic that major parts of the reference rotations are not changed but some more or less small parts have to be modified. In some cases, however, new timetabled trips have to be incorporated in the reference rotation plans. To handle this we also have to consider properties of the original objective function  $\sum_{i=7}^{11} \mathbf{c}_i p_i$  for re-optimization instances, *i.e.*, the number of vehicles consumed by an hyperarc, the number for maintenance services, the deadhead distance, irregularities, and number coupling activities. Finally all of this individual properties are multiplied by individual cost parameters  $\mathbf{c}_i$ ,  $i = 1, \dots, 11$  that can be adjusted to the requirements of industrial railway applications.

As introduced, all of the requirements for re-optimization can be incorporated into our model by penalizing local deviations w.r.t. the reference rotations implied by the corresponding hyperarc. Therefore, the general model and algorithm presented in [2] can be directly used to solve re-optimization instances.

## 4 Computational results

We implemented our re-optimization model and algorithm in a computer program, called ROTOR 2.0. This implementation takes use of the commercial mixed integer programming solver CPLEX 12.5. ROTOR 2.0 is integrated in the IT system of Deutsche Bahn and is evaluated and used by planners of Deutsche Bahn Fernverkehr. All our computations were performed on computers with an Intel(R) Xeon(R) CPU X5672 with 3.20 GHz, 12 MB cache, and 48 GB of RAM in multi thread mode.

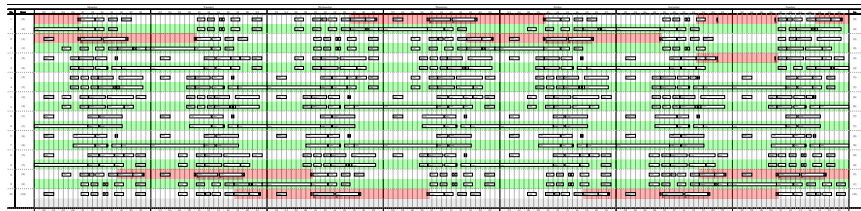


Fig. 2 Comparison of reference and re-optimized rolling stock rotations.

The considered instances include scenarios where vehicles got broken, where the timetable was changed due to the track sharing with other railway operators. And we also tackle instances where the fleet size increases, *i.e.*, for the case when new vehicles are available and have to be integrated in the current operations. All scenarios were given by Deutsche Bahn Fernverkehr AG. Figure 4 shows a difference view of the reference solution and the re-optimized provided by ROTOR solution in green. The rows alternate between the reference solution – or the rotation plan to be repaired – and the re-optimized solution. Table 1 lists the sizes of the instances, *i.e.*, the number of *trips*, *compositions*, *fleets*, and *maintenances*. In addition the total number of nodes ( $|V|$ ) and hyperarcs ( $|H|$ ) clearly show that this is large scale real world optimization.

instance	trips	compositions	fleets	maintenances	$ V $	$ H $	vehicles	slacks	dev. heads	dev. configurations	dev. fleets	dev. orientations	gap [%]	hh:mm:ss
RSRP_11	104	2	1	0	486	186,130	9	0	0	0	0	0	0.00	00:03:13
RSRP_12	104	2	1	1	486	192,612	9	0	0	0	0	0	0.09	00:04:05
RSRP_13	104	2	1	2	486	198,758	9	0	1	0	0	0	1.65	00:05:35
RSRP_21	805	2	2	0	9,810	15,770,498	55	0	3	1	1	0	0.00	00:09:27
RSRP_22	805	2	2	2	9,810	18,768,740	55	0	1	0	0	0	0.15	03:34:00
RSRP_31	788	2	2	0	7,776	11,727,856	55	0	29	2	2	0	0.00	00:07:59
RSRP_32	788	2	2	2	7,776	14,019,208	55	0	30	2	2	0	0.28	01:17:17
RSRP_41	789	10	4	0	16,790	42,764,116	61	0	39	7	45	23	0.32	00:49:36
RSRP_42	789	10	4	4	16,790	54,640,466	59	0	40	7	42	17	0.91	02:48:20

**Table 1** Key numbers of scenarios and re-optimization results with ROTOR 2.0 and CPLEX 12.5.

Furthermore, Table 1 provides results for the re-optimization instances arising at Deutsche Bahn. The first and second column after the vertical line report on the number of used vehicles and slacks (trips that are not covered) of our solution. The next four columns denote the number of deviations w.r.t. the reference solution. Finally, the last two columns show the proven worst case optimality gap and the total computation time.

Our computational study demonstrates that our re-optimization approach can be used to produce high quality solutions for large-scale real-world rolling stock rotation planning problems in reasonable computation time.

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