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Technical Report

Linear Underestimators for bivariate functions with a fixed convexity behavior*

A documentation for the SCIP constraint handler `cons_bivariate`

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This is a technical report for the SCIP constraint handler `cons_bivariate`. We describe a cut-generation algorithm for a class of bivariate twice continuously differentiable functions with fixed convexity behavior over a box. Computational results comparing our cut-generation algorithms with state-of-the-art global optimization software on a series of randomly generated test instances are reported and discussed.

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1. Introduction

In this work we derive strong linear underestimators for bivariate functions $f : [l, u] \subseteq \mathbf{R}^2 \rightarrow \mathbf{R}$, $(x, y) \mapsto f(x, y)$, satisfying the following assumptions.

- (A1) $[l, u] := [l_x, u_x] \times [l_y, u_y] \subseteq \mathbf{R}^2$ is a box with $l_x < u_x$ and $l_y < u_y$,
- (A2) f is twice continuously differentiable on $[l, u]$,
- (A3) f has a fixed convexity behavior on $[l, u]$, i.e., the signs of the second partial derivatives and the determinant of the Hessian are the same for all points $(x, y) \in [l, u]$.

For a bivariate function $f(x, y)$ with a fixed convexity behavior, we distinguish five convexity patterns of f to derive tight underestimators: (i) convexity on the entire box, (ii) component-wise concavity in each variable, (iii) convexity in x and concavity in y , (iv) concavity in x and convexity in y , and (v) component-wise convexity and indefiniteness in the interior of the box.

Important classes of functions satisfying our assumptions (A1) – (A3) include the family of bivariate quadratic functions and the family of bivariate monomial functions of the form $x^p y^q$ restricted to the nonnegative orthant. Table 1 provides a complete characterization of the convexity behavior for these two families of functions.

Convexity behavior	$f(x, y) = a_{x,x}x^2 + a_{x,y}xy + a_{y,y}y^2 + b_x x + b_y y + c$	$f(x, y) = x^p y^q, (x, y) \in \mathbf{R}_{\geq 0}^2$
1. f is convex	$a_{x,x} \geq 0, a_{y,y} \geq 0$, and $a_{x,x}a_{y,y} - a_{x,y}^2 \geq 0$	$p^2 - p \geq 0, q^2 - q \geq 0$, and $pq(1 - p - q) \geq 0$
2. f is concave in x, y	$a_{x,x} \leq 0$ and $a_{y,y} \leq 0$	$p^2 - p \leq 0$ and $q^2 - q \leq 0$
3. f is non-linear convex in x and concave in y	$a_{x,x} > 0$ and $a_{y,y} \leq 0$	$p^2 - p > 0$ and $q^2 - q \leq 0$
4. f is concave in x and non-linear convex in y	$a_{x,x} \leq 0$ and $a_{y,y} > 0$	$p^2 - p < 0$ and $q^2 - q \leq 0$
5. f is indefinite and non-linear convex in x, y	$a_{x,x} > 0, a_{y,y} > 0$, and $a_{x,x}a_{y,y} - a_{x,y}^2 < 0$	$p^2 - p \geq 0, q^2 - q \geq 0$, and $pq(1 - p - q) < 0$

Table 1: Classes of fixed convexity behavior and criteria for bivariate quadratic and bivariate monomial functions. The latter functions are restricted to nonnegative domains $[l, u] \subseteq \mathbf{R}_{\geq 0}^2$.

Our goal is to provide a cut-generation algorithm for the construction and refinement of a linear relaxation of a possibly nonconvex set $\{(x, y, z) \in \mathbf{R}^3 : f(x, y) \leq cz\}$, where $f(x, y)$ satisfies our assumptions (A1) – (A3) and $c \in \mathbf{R}$. This is achieved by

constructing a maximally touching hyperplanes on the graph of the best possible convex underestimator of f on $[l, u]$, the so-called convex envelope which is denoted by $\text{vex}_{[l,u]}[f]$ (cf. [McC76]). The value of $\text{vex}_{[l,u]}[f]$ at $(x_0, y_0) \in [l, u]$ is given by (cf. [Roc70])

$$\text{vex}_{[l,u]}[f](x_0, y_0) = \min \left\{ \sum_{i=1}^3 \lambda_i f(x_i, y_i) \mid x_0 = \sum_{i=1}^3 \lambda_i x_i, y_0 = \sum_{i=1}^3 \lambda_i y_i, \right. \\ \left. 1 = \sum_{i=1}^3 \lambda_i, \lambda_i \geq 0, (x_i, y_i) \in [l, u] \right\}. \quad (1)$$

To determine a maximally touching hyperplane at the graph of $\text{vex}_{[l,u]}[f]$ at a given point $(x_0, y_0) \in [l, u]$, we compute suitable $\alpha, \beta, \delta \in \mathbf{R}$ and $\gamma \in \mathbf{R}_{\geq 0}$ such that $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, for all $(x, y) \in [l, u]$, and equality holds at least at (x_0, y_0) . Substituting $f(x, y)$ by a new variable $z \in \mathbf{R}$ yields the desired hyperplane.

When f is convex or component-wise concave on the box, an optimal solution of problem (1) can be easily computed which can be used to determine appropriate coefficients $\alpha, \beta, \gamma, \delta$. In the other cases, results from [TS01, JMW08] can be applied to find an optimal solution to the nonconvex problem (1). This is achieved by solving one or two univariate convex subproblems.

In general, these solutions do not lead to explicit formulas for the convex envelope and, hence, for the coefficients of the hyperplanes. However, based on these solutions we can derive the coefficients for the maximally touching hyperplanes using some elementary geometric arguments.

Locatelli and Schoen [LS09] have studied convex envelopes for twice continuously bivariate functions $f : P \rightarrow \mathbf{R}$ over a polytope $P \subseteq \mathbf{R}^2$, where f is indefinite in the interior of P , and either convex or concave over every facet of P . When choosing P to be a box, this class of functions include the indefinite functions studied in this report. Locatelli and Schoen provided a procedure returning, for a given point (x_0, y_0) , the value $\text{vex}_P[f](x_0, y_0)$ and a supporting hyperplane on the graph of $\text{vex}_P[f]$ at (x_0, y_0) . For this, they investigate an alternative representation for the convex envelope reading as

$$\text{vex}_{[l,u]}[f](x_0, y_0) = \max \quad c \\ \text{s. t.} \quad f(x, y) \geq a(x - x_0) + b(y - y_0) + c, \text{ for all } (x, y) \in P, \quad (2) \\ a, b, c \in \mathbf{R},$$

where a, b, c define the desired supporting hyperplane $z = ax + by + (c - ax_0 - by_0)$. The assumptions on f over P allow the authors to define a family of convex subproblems, where the optimum of problem (2) equals the overall optimum of the convex subproblems. The number of subproblems is bounded by 3^t , where t represents the number of facets of P over which f is strictly convex. We refer to [LS09], for more details on this analysis, and to [Loc10] for further extensions to multivariate quadratic polynomials and to bivariate polynomials. We also refer to [She97, BST09, TRX12] where connections between the two variants of optimization problems for computing convex envelopes are established for multilinear functions on boxes and for functions with polyhedral envelopes.

For our cut-generation algorithm, we follow [JMW08] and analyze the underlying problems as given in (1). We moreover point out that all the foundations of the presented analysis are already given in the literature. Nevertheless, we provide a detailed step-by-step derivation to provide a self-contained documentation for our code.

The cut-generation algorithm consists of two subroutines based on the evaluation of the convex envelope of f , and on a lifting technique. Let x_0, y_0 , and z_0 be the solution of the current relaxation, where z_0 corresponds to the value of $f(x_0, y_0)$ in the relaxation. If (x_0, y_0) is in the interior of the box $[l, u]$, we use [JMW08, TS01] to solve the optimization problem corresponding to the convex envelope at (x_0, y_0) . The solution of this problem can be used to construct a maximally touching, underestimating hyperplane, i.e., a hyperplane which is not dominated by another underestimating hyperplane. This subroutine is discussed in Section 2. If (x_0, y_0) is in the boundary of the box, the solution of the convex envelope problem may only provide an underestimator which is valid over a facet of the box. In Section 3 we apply a lifting technique to extend this locally valid underestimator to the entire box. Here, we use the results from [GKH⁺06, GHJ⁺08]. The presented formulas are implemented in the constraint handler “cons.bivariate” in the constraint integer programming framework SCIP [Ach07, Ach09]. A computational case study in Section 4 shows the good performance of the implemented constraint handler compared to state-of-the-art solvers.

2. Underestimators based on the convex envelope

In this section, we present explicit formulas for maximally touching hyperplanes for the graphs of the convex envelopes of our functions $f(x, y)$ at a given point (x_0, y_0) in the box $[l, u]$, i.e., we determine numbers $\alpha, \beta, \delta \in \mathbf{R}$ and $\gamma \in \mathbf{R}_{\geq 0}$ such that $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, for all $(x, y) \in [l, u]$ with equality at (x_0, y_0) .

2.1. The function $f(x, y)$ is convex in x and y

For a convex function $f : [l, u] \rightarrow \mathbf{R}$, the convex envelope over its domain is given by the function itself. Thus, the best possible linear underestimator of f at (x_0, y_0) is given by the tangent:

$$f(x, y) \geq \nabla f(x_0, y_0)^\top ((x, y) - (x_0, y_0)) + f(x_0, y_0).$$

2.2. The function $f(x, y)$ is concave in x and y

According to [McC76, Tar03, KS11] the convex envelope of a bivariate component-wise concave functions f is given as follows. If $f(l_x, l_y) + f(u_x, u_y) \leq f(l_x, u_y) + f(u_x, l_y)$, the convex envelope is given by

$$\text{vex}_{[l, u]}[f](x, y) = \begin{cases} \alpha_1 x + \beta_1 y - \delta_1, & \text{if } y_0 \leq \frac{u_y - l_y}{u_x - l_x} (x_0 - l_x) + l_y, \\ \alpha_2 x + \beta_2 y - \delta_2, & \text{if } y_0 > \frac{u_y - l_y}{u_x - l_x} (x_0 - l_x) + l_y, \end{cases}$$

where

$$\alpha_1 = \frac{f(u_x, l_y) - f(l_x, l_y)}{u_x - l_x}, \quad \beta_1 = \frac{f(u_x, u_y) - f(u_x, l_y)}{u_y - l_y}, \quad \delta_1 = -\frac{u_x(u_y - l_y)f(l_x, l_y) - (u_x - l_x)l_y f(u_x, u_y) + (u_x l_y - l_x u_y)f(u_x, l_y)}{(u_x - l_x)(u_y - l_y)},$$

$$\alpha_2 = \frac{f(u_x, u_y) - f(l_x, u_y)}{u_x - l_x}, \quad \beta_2 = \frac{f(l_x, u_y) - f(l_x, l_y)}{u_y - l_y}, \quad \delta_2 = -\frac{(u_x - l_x)u_y f(l_x, l_y) - l_x(u_y - l_y)f(u_x, u_y) + (l_x u_y - u_x l_y)f(l_x, u_y)}{(u_x - l_x)(u_y - l_y)}.$$

Otherwise, the convex envelope is given by

$$\text{vex}_{[l, u]}[f](x, y) = \begin{cases} \alpha_1 x + \beta_1 y - \delta_1, & \text{if } y_0 \leq \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y, \\ \alpha_2 x + \beta_2 y - \delta_2, & \text{if } y_0 > \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y, \end{cases}$$

where

$$\alpha_1 = \frac{f(u_x, l_y) - f(l_x, l_y)}{u_x - l_x}, \quad \beta_1 = \frac{f(l_x, u_y) - f(l_x, l_y)}{u_y - l_y}, \quad \delta_1 = -\frac{(u_x u_y - l_x l_y)f(l_x, l_y) - l_x(u_y - l_y)f(u_x, l_y) - (u_x - l_x)l_y f(l_x, u_y)}{(u_x - l_x)(u_y - l_y)},$$

$$\alpha_2 = \frac{f(u_x, u_y) - f(l_x, u_y)}{u_x - l_x}, \quad \beta_2 = \frac{f(u_x, u_y) - f(u_x, l_y)}{u_y - l_y}, \quad \delta_2 = -\frac{(l_x l_y - u_x u_y)f(u_x, u_y) + u_x(u_y - l_y)f(l_x, u_y) + (u_x - l_x)u_y f(u_x, l_y)}{(u_x - l_x)(u_y - l_y)}.$$

In the following we assume that the given point (x_0, y_0) is in the *interior* of $[l, u]$. The cases where (x_0, y_0) is in the boundary of $[l, u]$ are discussed in Section 3.

2.3. The function f is strictly convex in x and concave in y

We use results from [TS01, JMW08]. It follows that for a given point $(x_0, y_0) \in [l, u]$

$$\begin{aligned} \text{vex}_{[l, u]}[f](x_0, y_0) &= \min v(r, s, t) := t f(r, l_y) + (1 - t)f(s, u_y) \\ \text{s.t.} & \begin{cases} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = t \begin{pmatrix} r \\ l_y \end{pmatrix} + (1 - t) \begin{pmatrix} s \\ u_y \end{pmatrix}, \\ 0 \leq t \leq 1, \quad r, s \in [l_x, u_x]. \end{cases} \end{aligned} \quad (3)$$

Then, Problem (3) can be rewritten to the following univariate convex problem using the identities $t = \frac{y_0 - u_y}{l_y - u_y}$ and $r(s) = \frac{l_y - u_y}{y_0 - u_y}x_0 - \frac{l_y - y_0}{y_0 - u_y}s$ as $l_y < y_0 < u_y$ and $l_x < x_0 < u_x$

$$\min v_{\text{red}}(s) \quad \text{s.t.} \quad \max \left\{ l_x, \frac{y_0 - u_y}{l_y - y_0} \left[\frac{l_y - u_y}{y_0 - u_y} x_0 - u_x \right] \right\} \leq s \leq \min \left\{ \frac{y_0 - u_y}{l_y - y_0} \left[\frac{l_y - u_y}{y_0 - u_y} x_0 - l_x \right], u_x \right\}, \quad (4)$$

where $v_{\text{red}}(s)$ reads as

$$v_{\text{red}}(s) := v\left(r(s), s, \frac{y_0 - u_y}{l_y - u_y}\right) = \frac{y_0 - u_y}{l_y - u_y} f\left(\frac{l_y - u_y}{y_0 - u_y} x_0 - \frac{l_y - y_0}{y_0 - u_y} s, l_y\right) + \frac{l_y - y_0}{l_y - u_y} f(s, u_y).$$

Let s^* denote an optimal solution of the reduced Problem (4). Then, the point (s^*, r^*, t^*) , with $t^* = \frac{y_0 - u_y}{l_y - u_y}$ and $r^* = r(s^*) = \frac{l_y - u_y}{y_0 - u_y}x_0 - \frac{l_y - y_0}{y_0 - u_y}s^*$, is an optimal solution of Problem (3). Moreover, we have that $\text{vex}_{[l, u]}[f](x_0, y_0) = v_{\text{red}}(s^*)$.

By construction, $\text{vex}_{[l_x, u_x] \times [l_y, u_y]}[f](x, y)$ is linear over the segment connecting (r^*, l_y) and (s^*, u_y) which contains the point (x_0, y_0) . Moreover, f is convex in x for a fixed

$y \in \{l_y, u_y\}$. Thus, a maximally touching hyperplane on the graph of $\text{vex}_{[l,u]}[f]$ at (x_0, y_0) is defined by the point p_1 and the direction vectors q_1 and q_2 given by

$$p_1 := \begin{pmatrix} x_0 \\ y_0 \\ v_{\text{red}}(s^*) \end{pmatrix}, \quad q_1 := \begin{pmatrix} s^* - r^* \\ u_y - l_y \\ f(s^*, u_y) - f(r^*, l_y) \end{pmatrix}, \quad q_2 := \begin{pmatrix} 1 \\ 0 \\ \frac{\partial f}{\partial x}(\bar{x}, \bar{y}) \end{pmatrix},$$

where the point $(\bar{x}, \bar{y}) \in \{(r^*, l_y), (s^*, u_y)\}$ has to be chosen as follows. If $l_x < s^* < u_x$, set $(\bar{x}, \bar{y}) = (s^*, u_y)$. If $s^* \in \{l_x, u_x\}$ and $l_x < r^* < u_x$, set $(\bar{x}, \bar{y}) = (r^*, l_y)$. Otherwise, both points $(\bar{x}^1, \bar{y}^1) = (r^*, l_y)$ and $(\bar{x}^2, \bar{y}^2) = (s^*, u_y)$ yield valid inequalities. To understand the choice of (\bar{x}, \bar{y}) we consider the case where $l_x < s^* < u_x$ and $l_x < r^* < u_x$. Then, $\frac{\partial f}{\partial x}(s^*, u_y) = \frac{\partial f}{\partial x}(r^*, l_y)$. If, for instance, $s^* = l_x$ and $l_x < r^* < u_x$, the point (x', u_x) with $\frac{\partial f}{\partial x}(x', u_x) = \frac{\partial f}{\partial x}(r^*, l_y)$ satisfies $x' \leq l_x = s^*$. By convexity of f in x , it follows that $\frac{\partial f}{\partial x}$ is nondecreasing in x and thus $\frac{\partial f}{\partial x}(x', u_x) = \frac{\partial f}{\partial x}(r^*, l_y) \leq \frac{\partial f}{\partial x}(s^*, u_y)$. Then, $f(x, u_y) \geq \frac{\partial f}{\partial x}(s^*, u_y)(x - s^*) + f(s^*, u_y) \geq \frac{\partial f}{\partial x}(r^*, l_y)(x - s^*) + f(s^*, u_y)$ for all $x \in [l, u]$. As the resulting hyperplane underestimates $f(x, y)$ for fixed $y \in \{l_y, u_y\}$, and f is convex in x and concave in y , it follows that the hyperplane is a touching hyperplane on the graph of $\text{vex}_{[l,u]}[f]$ at (x_0, y_0) . This yields the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$ with

$$\begin{aligned} \alpha &= (u_y - l_y) \frac{\partial f}{\partial x}(\bar{x}, \bar{y}), \\ \beta &= f(s^*, u_y) - f(r^*, l_y) - (s^* - r^*) \frac{\partial f}{\partial x}(\bar{x}, \bar{y}), \\ \gamma &= (u_y - l_y), \\ \delta &= \alpha x_0 + \beta y_0 - \gamma v_{\text{red}}(s^*). \end{aligned}$$

2.4. The function f is concave in x and strictly convex in y

Switch the variables and apply Subsection 2.3.

2.5. The function is not convex, but strictly convex in x and y

By Theorem 3.1 in [JMW08], the value of the convex envelope of f on $[l, u]$ at (x_0, y_0) is given by

$$\begin{aligned} \min v(x_1, x_2, y_1, y_2, t) &:= tf(x_1, y_1) + (1-t)f(x_2, y_2) \\ \text{s.t.} \quad \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} &= t \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + (1-t) \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \\ 0 \leq t \leq 1, \quad &(x_1, y_1), (x_2, y_2) \in \mathcal{F}, \end{aligned} \tag{5}$$

where \mathcal{F} denotes the boundary of the box $[l, u]$, i.e., either $x_1 \in \{l_x, u_x\}$ or $y_1 \in \{l_y, u_y\}$, and $x_2 \in \{l_x, u_x\}$ or $y_2 \in \{l_y, u_y\}$. This can be used in a case distinction which simplifies Problem (5) by assigning (x_1, y_1) and (x_2, y_2) to different facets of the box $[l, u]$. Thus,

we get up to six simplified optimization problems because we have to consider two assignments to parallel facets and four assignments to adjacent facets. The minimum of all six cases yields the value of the convex envelope. According to [JMW08], this case distinction can be avoided by exploiting a geometrical property of indefinite and $(n-1)$ -convex functions: Their concave directions are contained in a pair of orthants of \mathbf{R}^2 . To determine this pair for a given function, we can compute the eigenvector to the negative eigenvalue of the Hessian $\mathcal{H}_f(\bar{x}, \bar{y})$ of f at the midpoint $(\bar{x}, \bar{y}) = (\frac{1}{2}(u_x + l_x), \frac{1}{2}(u_y + l_y))$ of the box. If the eigenvector has entries with different signs, then the concave directions of f at any point in $[l, u]$ are contained in the union $(\mathbf{R}_{\geq 0} \times \mathbf{R}_{\leq 0}) \cup (\mathbf{R}_{\leq 0} \times \mathbf{R}_{\geq 0})$ [**pattern A**], otherwise the concave directions are contained in the union $(\mathbf{R}_{\geq 0} \times \mathbf{R}_{\geq 0}) \cup (\mathbf{R}_{\leq 0} \times \mathbf{R}_{\leq 0})$ [**pattern B**]. Each pattern contains still three possible assignments of (x_1, y_1) and (x_x, y_2) to the facets of $[l, u]$.

2.5.1. Pattern A

Pattern A can lead to two possible types of structures for the subdivision of the box w.r.t. the description of the convex envelope. See Figure 1. The figure reflects that all lines in $A_i, i = 1, 2, 3$, connecting the facets of the box are contained in $(\mathbf{R}_{\geq 0} \times \mathbf{R}_{\leq 0}) \cup (\mathbf{R}_{\leq 0} \times \mathbf{R}_{\geq 0})$. The knowledge on the type of subdivision for the box further reduces the number of possible assignments to the endpoints of a minimizing segment.

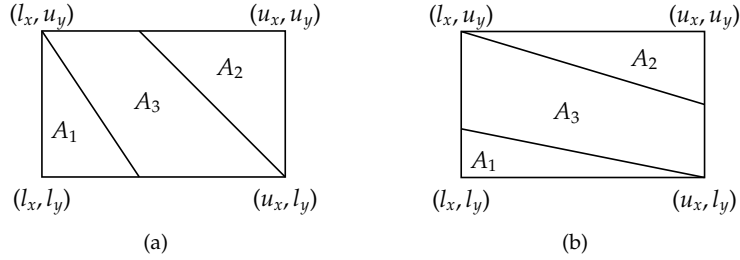


Figure 1: Pattern A: Possible subdivisions of the box w.r.t. the description of the convex envelope.

The type of subdivision can be determined as follows.

Lemma 2.1 (e.g., cf. Ex. 5 in [JMW08]). *If $f(l_x, u_y) + (l_y - u_y) \frac{\partial f}{\partial y}(l_x, u_y) \geq f(u_x, l_y) + (l_x - u_x) \frac{\partial f}{\partial x}(u_x, l_y)$, the structure for the subdivision of the convex envelope corresponds to Figure 1 (a). Otherwise, the structure corresponds to Figure 1 (b).*

Subsequently we discuss the convex envelope with a subdivision as in Figure 1 (a). The formulas for the other case can be derived analogously by interchanging x and y . This gives rise to a situation as in Figure 1 (a).

Note that Lemma 2.1 only provides information about the general shape of the concrete subdivision. To determine a minimizing segment for a given point, we have

to solve two auxiliary problems. The minimal value of the two problems is then equivalent to the value of the convex envelope. The first auxiliary problem corresponds to subdomain A_3 , where the endpoints of the possible minimizing segment are contained in the parallel facets given by $y = l_y$ and $y = u_y$. The second auxiliary problem corresponds to the subdomains A_1 and A_2 depending on the position of the point (x_0, y_0) . If the point (x_0, y_0) is below the diagonal of the box connecting (l_x, u_y) and (u_x, l_y) , the endpoints of the possible minimizing segment are contained in the orthogonal facets $x = l_x$ and $y = l_y$ (subdomain A_1). Otherwise, the endpoints of the possible minimizing segment are contained in the orthogonal facets $x = u_x$ and $y = u_y$ (subdomain A_2).

Auxiliary problem 1: Parallel facets In this case the optimization problem reduces to

$$\begin{aligned} \varrho(x_0, y_0) &:= \min \quad tf(r, l_y) + (1-t)f(s, u_y) \\ \text{s.t.} \quad &\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = t \begin{pmatrix} r \\ l_y \end{pmatrix} + (1-t) \begin{pmatrix} s \\ u_y \end{pmatrix}, \\ &0 \leq t \leq 1, \quad r, s \in [l_x, u_x]. \end{aligned} \quad (6)$$

This subproblem is identical to the case considered in Subsection 2.3.

Auxiliary problem 2: Orthogonal facets If the point (x_0, y_0) is below the diagonal, i.e., $y_0 \leq \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y$, we consider the subproblem

$$\begin{aligned} \omega_1(x_0, y_0) &:= \min \quad tf(l_x, r) + (1-t)f(s, l_y) \\ \text{s.t.} \quad &\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = t \begin{pmatrix} l_x \\ r \end{pmatrix} + (1-t) \begin{pmatrix} s \\ l_y \end{pmatrix}, \\ &0 < t < 1, \quad r \in [l_y, u_y], \quad t \in [l_x, u_x]. \end{aligned} \quad (7)$$

Following ([JMW08, Lem. 4.2]), we set $s(t) = (x_0 - l_x t)/(1-t)$ and $r(t) = (y_0 - (1-t)l_y)/t$. Then, Problem (7) can be transformed into the following univariate convex optimization problem

$$\min \quad v_{\text{red}}(t) := t f\left(l_x, \frac{y_0 - (1-t)l_y}{t}\right) + (1-t)f\left(\frac{x_0 - l_x t}{1-t}, l_y\right), \quad \text{s.t.} \quad t \in \left[\frac{y_0 - l_y}{u_y - l_y}, \frac{u_x - x_0}{u_x - l_x}\right]. \quad (8)$$

An optimal solution $t^* \in (0, 1)$ to Problem (8) yields the point (t^*, s^*, r^*) , with $s^* = s(t^*)$ and $r^* = r(t^*)$, i.e., $\omega_1(x_0, y_0) = v_{\text{red}}(t^*)$.

If the point (x_0, y_0) is above the diagonal, i.e., $y_0 > \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y$, we consider

the subproblem

$$\begin{aligned} \omega_2(x_0, y_0) &:= \min t f(u_x, r) + (1-t) f(s, u_y) \\ \text{s.t. } \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} &= t \begin{pmatrix} u_x \\ r \end{pmatrix} + (1-t) \begin{pmatrix} s \\ u_y \end{pmatrix}, \\ 0 < t < 1, & r \in [l_y, u_y], s \in [l_x, u_x]. \end{aligned} \quad (9)$$

Using $s(t) = (x_0 - u_x t)/(1-t)$ and $r(t) = (y_0 - (1-t)u_y)/t$, Problem (9) is equivalent to the following univariate convex problem

$$\min v_{\text{red}}(t) := t f\left(u_x, \frac{y_0 - (1-t)u_y}{t}\right) + (1-t) f\left(\frac{x_0 - u_x t}{1-t}, u_y\right), \quad \text{s.t. } t \in \left[\frac{u_y - y_0}{u_y - l_y}, \frac{x_0 - l_x}{u_x - l_x}\right], \quad (10)$$

that can be solved using standard numerical methods. Let $t^* \in (0, 1)$ be an optimal solution of Problem (10). Then, the point (t^*, s^*, r^*) with $s^* = s(t^*)$ and $r^* = r(t^*)$ is an optimal solution for Problem (9).

Thus, the value of the convex envelope is the minimum of the optimal value $\varrho(x_0, y_0)$ of Problem (6) corresponding to the parallel case and of either $\omega_1(x_0, y_0)$ or $\omega_2(x_0, y_0)$ of Problems (7) and (9), respectively, corresponding to the orthogonal case:

$$\text{vex}_{[l,u]}[f](x_0, y_0) = \begin{cases} \min\{\varrho(x_0, y_0), \omega_1(x_0, y_0)\}, & \text{if } y_0 \leq \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y, \\ \min\{\varrho(x_0, y_0), \omega_2(x_0, y_0)\}, & \text{if } y_0 > \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y. \end{cases}$$

To construct supporting hyperplanes, we thus need to consider three cases:

- (i) $\text{vex}_{[l,u]}[f](x_0, y_0) = \varrho(x_0, y_0)$,
- (ii) $y_0 \leq \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y$ and $\text{vex}_{[l,u]}[f](x_0, y_0) = \omega_1(x_0, y_0)$,
- (iii) $y_0 > \frac{l_y - u_y}{u_x - l_x}(x_0 - l_x) + u_y$ and $\text{vex}_{[l,u]}[f](x_0, y_0) = \omega_2(x_0, y_0)$.

Case (i): A minimizing segment is given by the optimal solution of Problem (6), and the formulas for a linear underestimator can be derived as in Subsection 2.3.

Case (ii): A minimizing segment is given by an optimal solution (t^*, s^*, r^*) to Problem (7). The desired hyperplane on the graph of the convex envelope at the point (x_0, y_0) is given by the two points $p_1 = (l_x, r^*, f(l_x, r^*))$ and $p_2 = (s^*, l_y, f(s^*, l_y))$ and a direction vector q . If $s^* \neq u_x$, then $q = (1, 0, \frac{\partial f}{\partial x}(s^*, l_y))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned} \alpha &= (r^* - l_y) \frac{\partial f}{\partial x}(s^*, l_y), \\ \beta &= (s^* - l_x) \frac{\partial f}{\partial x}(s^*, l_y) + f(l_x, r^*) - f(s^*, l_y), \\ \gamma &= r^* - l_y, \\ \delta &= \alpha l_x + \beta r^* - \gamma f(l_x, r^*). \end{aligned}$$

If $s^* = u_x$ and $r^* \neq u_y$, then $q = (0, 1, \frac{\partial f}{\partial y}(l_x, r^*))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (r^* - l_y) \frac{\partial f}{\partial y}(l_x, r^*) - f(l_x, r^*) + f(s^*, l_y), \\ \beta &= (s^* - l_x) \frac{\partial f}{\partial y}(l_x, r^*), \\ \gamma &= s^* - l_x, \\ \delta &= \alpha l_x + \beta r^* - \gamma f(l_x, r^*).\end{aligned}$$

If $s^* = u_x$ and $r^* = u_y$, then $q = (1, 0, \min\{\frac{\partial f}{\partial x}(s^*, l_y), \frac{\partial f}{\partial x}(l_x, r^*)\})$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (r^* - l_y) \min\{\frac{\partial f}{\partial x}(s^*, l_y), \frac{\partial f}{\partial x}(l_x, r^*)\}, \\ \beta &= (s^* - l_x) \min\{\frac{\partial f}{\partial x}(s^*, l_y), \frac{\partial f}{\partial x}(l_x, r^*)\} + f(l_x, r^*) - f(s^*, l_y), \\ \gamma &= r^* - l_y, \\ \delta &= \alpha l_x + \beta r^* - \gamma f(l_x, r^*).\end{aligned}$$

Case (iii): A minimizing segment is given by an optimal solution (t^*, s^*, r^*) to Problem (9). The desired hyperplane on the graph of the convex envelope at the point (x_0, y_0) is given by the two points $p_1 = (u_x, r^*, f(u_x, r^*))$ and $p_2 = (s^*, u_y, f(s^*, u_y))$ and a direction vector q . If $s^* \neq l_x$, then $q = (1, 0, \frac{\partial f}{\partial x}(s^*, u_y))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (u_y - r^*) \frac{\partial f}{\partial x}(s^*, u_y), \\ \beta &= (u_x - s^*) \frac{\partial f}{\partial x}(s^*, u_y) - f(u_x, r^*) + f(s^*, u_y), \\ \gamma &= u_y - r^*, \\ \delta &= \alpha s^* + \beta u_y - \gamma f(s^*, u_y).\end{aligned}$$

If $s^* = l_x$ and $r^* \neq l_y$, then $q = (0, 1, \frac{\partial f}{\partial y}(u_x, r^*))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (u_y - r^*) \frac{\partial f}{\partial y}(u_x, r^*) + f(u_x, r^*) - f(s^*, u_y), \quad \beta = (u_x - s^*) \frac{\partial f}{\partial y}(u_x, r^*), \quad \gamma = u_x - s^*, \\ \delta &= \alpha s^* + \beta u_y - \gamma f(s^*, u_y).\end{aligned}$$

If $s^* = l_x$ and $r^* = l_y$, then $q = (1, 0, \min\{\frac{\partial f}{\partial x}(s^*, u_y), \frac{\partial f}{\partial x}(u_x, r^*)\})$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (u_y - r^*) \min\{\frac{\partial f}{\partial x}(s^*, u_y), \frac{\partial f}{\partial x}(u_x, r^*)\}, \\ \beta &= (u_x - s^*) \min\{\frac{\partial f}{\partial x}(s^*, u_y), \frac{\partial f}{\partial x}(u_x, r^*)\} - f(u_x, r^*) + f(s^*, u_y), \\ \gamma &= u_y - s^*, \\ \delta &= \alpha s^* + \beta u_y - \gamma f(s^*, u_y).\end{aligned}$$

2.5.2. Pattern B

Figure 2 displays the two possible structures for the subdivision of the box w.r.t. the description of the convex envelope for pattern B. If $f(l_x, l_y) + (u_y - l_y) \frac{\partial f}{\partial y}(l_x, l_y) \geq f(u_x, u_y) + (l_x - u_x) \frac{\partial f}{\partial x}(u_x, u_y)$, the structure corresponds to Figure 2 (a). Otherwise, the structure of the subdivision corresponds to Figure 2 (b).

Subsequently, we discuss the convex envelope with a subdivision as in Figure 2 (a). The formulas for the other case can be derived analogously by interchanging x and y leading to a situation as in Figure 2 (a).

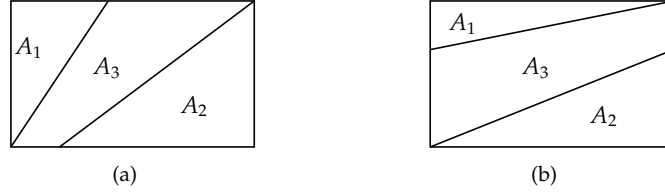


Figure 2: Pattern B: Possible subdivisions of the box w.r.t the description of the convex envelope.

Analogously to pattern A, we have to solve two auxiliary problems:

Auxiliary problem 1: Parallel facets In this case the optimization problem reduces to

$$\begin{aligned} \varrho(x_0, y_0) &:= \min t f(r, l_y) + (1-t) f(s, u_y) \\ \text{s.t. } &\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = t \begin{pmatrix} r \\ l_y \end{pmatrix} + (1-t) \begin{pmatrix} s \\ u_y \end{pmatrix}, \\ &0 \leq t \leq 1, \quad r, s \in [l_x, u_x]. \end{aligned} \quad (11)$$

This subproblem is identical to the case considered in Subsection 2.3.

Auxiliary problem 2: Orthogonal facets If the point (x_0, y_0) is above the diagonal connecting (l_x, l_y) and (u_x, u_y) , i.e., $y_0 \geq \frac{u_y - l_y}{u_x - l_x} (x_0 - l_x) + l_y$, we consider the subproblem

$$\begin{aligned} \omega_1(x_0, y_0) &:= \min t f(r, u_y) + (1-t) f(l_x, s) \\ \text{s.t. } &\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = t \begin{pmatrix} r \\ u_y \end{pmatrix} + (1-t) \begin{pmatrix} l_x \\ s \end{pmatrix}, \\ &0 < t < 1, \quad r \in [l_x, u_x], \quad s \in [l_y, u_y]. \end{aligned} \quad (12)$$

Using $r(t) = l_x + (x_0 - l_x)/t$ and $s(t) = (y_0 - tu_y)/(1-t)$, Problem (12) can be transformed into the following univariate convex optimization problem

$$\min v_{\text{red}}(t) := t f\left(l_x + \frac{x_0 - l_x}{t}, u_y\right) + (1-t)f\left(l_x, \frac{y_0 - tu_y}{1-t}\right), \quad \text{s.t. } t \in \left[\frac{x_0 - l_x}{u_x - l_x}, \frac{y_0 - l_y}{u_y - l_y}\right].$$

Let $t^* \in (0, 1)$ be the optimal solution. Then, the point (t^*, s^*, r^*) with $s^* = s(t^*)$ and $r^* = r(t^*)$ is an optimal solution for Problem (12).

If the point (x_0, y_0) is below the diagonal connecting (l_x, l_y) and (u_x, u_y) , i.e., $y_0 < \frac{u_y - l_y}{u_x - l_x}(x_0 - l_x) + l_y$, we consider the subproblem

$$\begin{aligned} \omega_2(x_0, y_0) &:= \min t f(r, l_y) + (1-t)f(u_x, s) \\ \text{s.t. } \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} &= t \begin{pmatrix} r \\ l_y \end{pmatrix} + (1-t) \begin{pmatrix} u_x \\ s \end{pmatrix}, \\ 0 < t < 1, & r \in [l_x, u_x], y \in [l_y, u_y]. \end{aligned} \quad (13)$$

Using $r(t) = u_x + (x_0 - u_x)/t$ and $s(t) = (y_0 - tl_y)/(1-t)$, Problem (13) can be transformed into the following univariate convex optimization problem

$$\min v_{\text{red}}(t) := t f\left(u_x + \frac{x_0 - u_x}{t}, l_y\right) + (1-t)f\left(u_x, \frac{y_0 - tl_y}{1-t}\right), \quad \text{s.t. } t \in \left[\frac{u_x - x_0}{u_x - l_x}, \frac{u_y - y_0}{u_y - l_y}\right].$$

Let $t^* \in (0, 1)$ be the optimal solution. Then, the point (t^*, s^*, r^*) with $s^* = s(t^*)$ and $r^* = r(t^*)$ is an optimal solution for Problem (13).

Thus, the value of the convex envelope is the minimum of the optimal value $\varrho(x_0, y_0)$ of Problem (11) corresponding to the parallel case and of either $\omega_1(x_0, y_0)$ or $\omega_2(x_0, y_0)$ of Problems (12) and (13), respectively, corresponding to the orthogonal case:

$$\text{vex}_{[l,u]}[f](x_0, y_0) = \begin{cases} \min\{\varrho(x_0, y_0), \omega_1(x_0, y_0)\}, & \text{if } y_0 \geq \frac{u_y - l_y}{u_x - l_x}(x_0 - l_x) + l_y, \\ \min\{\varrho(x_0, y_0), \omega_2(x_0, y_0)\}, & \text{if } y_0 < \frac{u_y - l_y}{u_x - l_x}(x_0 - l_x) + l_y. \end{cases}$$

To construct supporting hyperplanes, we thus need to consider three cases:

- (i) $\text{vex}_{[l,u]}[f](x_0, y_0) = \varrho(x_0, y_0)$,
- (ii) $y_0 \geq \frac{u_y - l_y}{u_x - l_x}(x_0 - l_x) + l_y$ and $\text{vex}_{[l,u]}[f](x_0, y_0) = \omega_1(x_0, y_0)$,
- (iii) $y_0 < \frac{u_y - l_y}{u_x - l_x}(x_0 - l_x) + l_y$ and $\text{vex}_{[l,u]}[f](x_0, y_0) = \omega_2(x_0, y_0)$.

Case (i): A minimizing segment is given by the optimal solution of Problem (11), and the formulas for a linear underestimators can be derived as in Subsection 2.3.

Case (ii): A minimizing segment is given by the optimal solution (t^*, s^*, r^*) to Problem (12). The desired hyperplane on the graph of the convex envelope at the point (x_0, y_0) is given by the two points $p_1 = (r^*, u_y, f(r^*, u_y))$ and $p_2 = (l_x, s^*, f(l_x, s^*))$

and a direction vector q . If $r^* \neq u_x$, then $q = (1, 0, \frac{\partial f}{\partial x}(r^*, u_y))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (u_y - s^*) \frac{\partial f}{\partial x}(r^*, u_y), \\ \beta &= (l_x - r^*) \frac{\partial f}{\partial x}(r^*, u_y) + f(r^*, u_y) - f(l_x, s^*), \\ \gamma &= u_y - s^*, \\ \delta &= \alpha l_x + \beta s^* - \gamma f(l_x, s^*).\end{aligned}$$

If $r^* = u_x$ and $s^* \neq l_y$, then $q = (0, 1, \frac{\partial f}{\partial y}(l_x, s^*))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (s^* - u_y) \frac{\partial f}{\partial y}(l_x, s^*) + f(r^*, u_y) - f(l_x, s^*), \\ \beta &= (r^* - l_x) \frac{\partial f}{\partial y}(l_x, s^*), \\ \gamma &= r^* - l_x, \\ \delta &= \alpha l_x + \beta s^* - \gamma f(l_x, s^*).\end{aligned}$$

If $r^* = u_x$ and $s^* = l_y$, then $q = (1, 0, \min\{\frac{\partial f}{\partial x}(r^*, u_y), \frac{\partial f}{\partial x}(l_x, s^*)\})$ yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (u_y - s^*) \min\{\frac{\partial f}{\partial x}(r^*, u_y), \frac{\partial f}{\partial x}(l_x, s^*)\}, \\ \beta &= (l_x - r^*) \min\{\frac{\partial f}{\partial x}(r^*, u_y), \frac{\partial f}{\partial x}(l_x, s^*)\} + f(r^*, u_y) - f(l_x, s^*), \\ \gamma &= (u_y - s^*), \\ \delta &= \alpha l_x + \beta s^* - \gamma f(l_x, s^*).\end{aligned}$$

Case (iii): A minimizing segment is given by the optimal solution (t^*, s^*, r^*) to Problem (13). The desired hyperplane on the graph of the convex envelope at the point (x_0, y_0) is now given by the two points $p_1 = (r^*, l_y, f(r^*, l_y))$ and $p_2 = (u_x, s^*, f(u_x, s^*))$ and a direction vector q . If $r^* \neq l_x$, then $q = (1, 0, \frac{\partial f}{\partial x}(r^*, l_y))$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (s^* - l_y) \frac{\partial f}{\partial x}(r^*, l_y), \\ \beta &= (r^* - u_x) \frac{\partial f}{\partial x}(r^*, l_y) - f(r^*, l_y) + f(u_x, s^*), \\ \gamma &= s^* - l_y, \\ \delta &= \alpha u_x + \beta s^* - \gamma f(u_x, s^*).\end{aligned}$$

If $r^* = l_x$ and $s^* \neq u_y$, then $q = (0, 1, \frac{\partial f}{\partial y}(u_x, s^*))$, yielding the linear underestimator

$\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (l_y - s^*) \frac{\partial f}{\partial y}(u_x, s^*) - f(r^*, l_y) + f(u_x, s^*), \\ \beta &= (u_x - r^*) \frac{\partial f}{\partial y}(u_x, s^*), \\ \gamma &= u_x - r^*, \\ \delta &= \alpha u_x + \beta s^* - \gamma f(u_x, s^*).\end{aligned}$$

If $r^* = u_x$ and $s^* = l_y$, then $q = (1, 0, \min\{\frac{\partial f}{\partial x}(r^*, l_y), \frac{\partial f}{\partial x}(u_x, s^*)\})$, yielding the linear underestimator $\gamma f(x, y) \geq \alpha x + \beta y - \delta$, where

$$\begin{aligned}\alpha &= (s^* - l_y) \min\{\frac{\partial f}{\partial x}(r^*, l_y), \frac{\partial f}{\partial x}(u_x, s^*)\} \\ \beta &= (r^* - u_x) \min\{\frac{\partial f}{\partial x}(r^*, l_y), \frac{\partial f}{\partial x}(u_x, s^*)\} - f(r^*, l_y) + f(u_x, s^*), \\ \gamma &= s^* - l_y, \\ \delta &= \alpha u_x + \beta s^* - \gamma f(u_x, s^*).\end{aligned}$$

3. Underestimators on the boundary by lifting

In this section we calculate linear underestimators for f when the given point (x_0, y_0) is in the boundary of the domain $[l, u]$, and f is either convex in one variable and concave in the other, or f is convex in both variables and indefinite. In these cases, an optimal solution of the underlying optimization problems in Equations (3) and (5) provides a lower-dimensional estimator that is, in general, only valid over the corresponding face of the box. In the following, we apply the *lifting technique* to the lower-dimensional estimators to compute estimators that are valid on the entire box.

The concept of the lifting was introduced by Padberg in [Pad75] to compute tight linear inequalities for linear zero-one problems. It has been adopted in [GKH⁺06, GHJ⁺08, BMSMW10] to derive linear and convex underestimators for concrete examples of low-dimensional non-linear functions by exploiting certain analytic and geometric properties. In a general setting, the lifting technique has been studied in [RT10].

Applied to our setting, the key idea of the lifting procedure is the following (see [GHJ⁺08]). Given a bivariate function $f : \mathbf{R}^2 \rightarrow \mathbf{R}$, $(x, y) \mapsto f(x, y)$ on a box $[l, u] := [l_x, u_x] \times [l_y, u_y] \subseteq \mathbf{R}^2$. We first fix one variable to one of its bounds. For the purpose of illustration assume that x is fixed to its lower bound l_x . Assume furthermore that we have a function $g : \mathbf{R} \rightarrow \mathbf{R}$ underestimating $f(l_x, y)$ over $[l_y, u_y]$ at hand. The functions which we consider in this work are either convex or concave when restricted to a facet of the box so that we can underestimate them by a tangent or a secant, respectively. Our aim is to determine a best possible *lifting coefficient* $\mu \in \mathbf{R}$ such that

$$f(x, y) \geq \mu(x - l_x) + g(y) \quad \text{holds for all } (x, y) \in [l, u].$$

This gives rise to the following non-linear optimization problem

$$\mu := \inf \left\{ \frac{f(x,y) - g(y)}{x - l_x} \mid x \in (l_x, u_x], y \in [l_y, u_y] \right\}. \quad (14)$$

If we fix x to its upper bound u_x and assume that $h : \mathbf{R} \rightarrow \mathbf{R}$ is an underestimating function for $f(u_x, y)$ on $[l_y, u_y]$, we determine a best possible number $\tau \in \mathbf{R}$ with

$$f(x, y) \geq \tau(x - u_x) + h(y) \quad \text{for all } (x, y) \in [l, u].$$

Using that $x - u_x \leq 0$ for $x \in [l_x, u_x]$, we arrive at the following optimization task

$$\tau := \sup \left\{ \frac{f(x,y) - h(y)}{x - u_x} \mid x \in (l_x, u_x], y \in [l_y, u_y] \right\}. \quad (15)$$

We remark that the optimization problems can be, in general, extremely difficult to solve. In the following we exploit the specific structure of our bivariate functions to determine appropriate lifting coefficients.

To complete our *cut-generation procedure* from the previous section, we have to investigate the lifting

1. from a facet over which the function is concave into a direction in which the function is convex,
2. from a facet over which the function is convex into a direction in which the function is concave,
3. from a facet over which the function is convex into a direction in which the function is convex.

For this, we use elementary arguments (or slightly different versions of them) that have been already used in [GKH⁺06, GHJ⁺08]. Cases 1 and 2 concern bivariate functions being convex in one variable and concave in the second one. Case 3 concerns 1-convex indefinite functions.

3.1. Lifting from a facet over which the function is concave into a direction in which the function is convex

Let $f : [l, u] \rightarrow \mathbf{R}$ be a bivariate function that is convex in x and concave in y . We assume that the given point (x_0, y_0) is contained in the boundary of the box where $x_0 \in \{l_x, u_x\}$ and $l_y \leq y_0 \leq u_y$. As f is concave in y , the best linear underestimator for $f(x_0, y)$ on $[l_y, u_y]$ is given by the secant $s : \mathbf{R} \rightarrow \mathbf{R}$ on the graph of $f(x_0, y)$ through the points $(l_y, f(x_0, l_y))$ and $(u_y, f(x_0, u_y))$, i.e., s is given by

$$s(y) := \frac{f(x_0, u_y) - f(x_0, l_y)}{u_y - l_y} (y - l_y) + f(x_0, l_y).$$

Next, we extend $s(y)$ to a globally valid underestimator of the form

$$f(x, y) \geq \alpha(x - x_0) + s(y).$$

case (a): $x_0 = l_x$. We will argue that

$$\alpha = \frac{\partial f}{\partial x}(l_x, \bar{y}), \quad \text{where } \bar{y} := \begin{cases} l_y, & \text{if } \frac{\partial f}{\partial x}(l_x, u_y) \geq \frac{\partial f}{\partial x}(l_x, l_y), \\ u_y, & \text{otherwise.} \end{cases}$$

We lift the underestimator into a direction in which the function is convex. As the underestimator and the function coincide at $y \in \{l_y, u_y\}$, it follows that $\alpha \leq \frac{\partial f}{\partial x}(l_x, \bar{y})$. Along the line $y = \bar{y}$, the lifting coefficient $\frac{\partial f}{\partial x}(l_x, \bar{y})$ is the best possible. The resulting linear underestimator is valid for f over $[l, u]$ because (i) it underestimates f along the lines $y = l_y$ and $y = u_y$ and (ii) it underestimates f along each segment from (x, l_y) to (x, u_y) for all $x \in [l_x, u_x]$ as it is linear in y while f is concave in y . Therefore, a *best possible* linear underestimator is given by

$$f(x, y) \geq \frac{\partial f}{\partial x}(l_x, \bar{y}) (x - l_x) + \frac{f(l_x, u_y) - f(l_x, l_y)}{u_y - l_y} (y - l_y) + f(l_x, l_y).$$

case (b): $x_0 = u_x$. Analogously to (a) the best possible lifting coefficient is given by

$$\alpha = \frac{\partial f}{\partial x}(u_x, \bar{y}), \quad \text{where } \bar{y} := \begin{cases} l_y, & \text{if } \frac{\partial f}{\partial x}(u_x, u_y) \leq \frac{\partial f}{\partial x}(u_x, l_y), \\ u_y, & \text{otherwise.} \end{cases}$$

This yields the following linear underestimator

$$f(x, y) \geq \frac{\partial f}{\partial x}(u_x, \bar{y}) (x - u_x) + \frac{f(u_x, u_y) - f(u_x, l_y)}{u_y - l_y} (y - l_y) + f(u_x, l_y).$$

3.2. Lifting from a facet over which the function is convex into a direction in which the function is concave

Let $f : [l, u] \rightarrow \mathbf{R}$ be a bivariate function that is convex in x and concave in y and consider the point (x_0, y_0) , where $l_x < x_0 < u_x$ and $y_0 \in \{l_y, u_y\}$. As f is convex when y is fixed, the best linear underestimator is given by the tangent $t : \mathbf{R} \rightarrow \mathbf{R}$ on the graph of $f(x, y_0)$ at $(x_0, f(x_0, y_0))$ with

$$t(x) := \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + f(x_0, y_0).$$

Next, we extend $t(x)$ to a globally valid underestimator of the form

$$f(x, y) \geq t(x) + \beta(y - y_0).$$

To determine an appropriate β , we use a geometric argument that has been also applied in [GKH⁺06].

case (a): $y_0 = l_y$. For every fixed $x \in [l_x, u_x]$ the segment connecting the points $(x, l_y, t(x))$ and $(x, u_y, f(x, u_y))$ underestimates $f(x, y)$ on $[l_y, u_y]$ as $t(x) \leq f(x, l_y)$ and f is concave for every fixed x . A valid lifting coefficient $\beta \in \mathbf{R}$ is given by the minimal slope $\gamma(x) = \frac{f(x, u_y) - t(x)}{u_y - l_y}$ over $x \in [l_x, u_x]$. Note that $\gamma(x)$ is convex because f is convex. This means that each critical point x satisfying the following first-order condition forms a global minimum of γ :

$$\frac{\partial \gamma}{\partial x}(x) = \frac{1}{u_y - l_y} \left(\frac{\partial f}{\partial x}(x, u_y) - t'(x) \right) \stackrel{!}{=} 0.$$

Therefore, $\beta = \frac{f(\bar{x}, u_y) - t(\bar{x})}{u_y - l_y}$, where \bar{x} is a critical point satisfying the first-order condition, provided such point exists and is contained in the relevant domain $[l_x, u_x]$. If such a point does not exist, set $\bar{x} = l_x$ if $\gamma(l_x) \leq \gamma(u_x)$, and $\bar{x} = u_x$ otherwise.

This leads to the following linear underestimator

$$f(x, y) \geq \frac{\partial f}{\partial x}(x_0, l_y) (x - x_0) + f(x_0, l_y) + \frac{f(\bar{x}, u_y) - t(\bar{x})}{u_y - l_y} (y - l_y).$$

case (b): $y_0 = u_y$. Similar to case (a), for every fixed $x' \in [l_x, u_x]$ the segment connecting the points $(x', l_y, f(x', l_y))$ and $(x', u_y, t(x'))$ underestimates $f(x', y)$ on $[l_y, u_y]$. A valid lifting coefficient $\beta \in \mathbf{R}$ is given by the maximal slope $\gamma(x) = \frac{f(x, l_y) - t(x)}{l_y - u_y}$ over $x \in [l_x, u_x]$. As $l_y - u_y < 0$ and f is convex in x , it follows that γ is concave. Thus, each point satisfying the following first-order condition provides a global maximum for γ :

$$\frac{\partial \gamma}{\partial x}(x) = \frac{1}{l_y - u_y} \left(\frac{\partial f}{\partial x}(x, l_y) - t'(x) \right) \stackrel{!}{=} 0.$$

Let \bar{x} be a critical point satisfying the first-order condition, provided such point exists and is contained in $[l_x, u_x]$. Otherwise we set $\bar{x} = l_x$, if $\gamma(l_x) \geq \gamma(u_x)$, and $\bar{x} = u_x$, else. Then, $\beta = \gamma(\bar{x})$ yields the following linear underestimator

$$f(x, y) \geq \frac{\partial f}{\partial x}(x_0, u_y) (x - x_0) + f(x_0, u_y) + \frac{f(\bar{x}, l_y) - t(\bar{x})}{l_y - u_y} (y - u_y).$$

3.3. Lifting from a facet over which the function is convex into a direction in which the function is convex.

Let $f : [l, u] \rightarrow \mathbf{R}$ be a bivariate function that is strictly convex in both x and y but indefinite. Note that the lifting technique has been already applied in a similar setting in [GH]⁺08] for a class of functions which are convex in each variable but not necessarily indefinite at each point of the box.

Consider a point $(x_0, y_0) \in [l, u]$ which is contained in the boundary of the box, i.e., $x_0 \in \{l_x, u_x\}$ or $y_0 \in \{l_y, u_y\}$. W.l.o.g., we assume $x_0 \in \{l_x, u_x\}$. Otherwise we can interchange the variables x and y and consider the function $f(y, x)$ on $[l_y, u_y] \times [l_x, u_x]$

and the point (y_0, x_0) . As f is convex in y , the best convex underestimator for $f(x_0, y)$ is the function $f(x_0, y)$ itself. A valid lifting coefficient $\alpha \in \mathbf{R}$ is required to satisfy

$$f(x, y) \geq \alpha(x - x_0) + f(x_0, y), \quad \text{for all } (x, y) \in [l, u].$$

For this, we define

$$\mu(x, y) := \frac{f(x, y) - f(x_0, y)}{x - x_0}.$$

The best possible lifting coefficient α is given by the infimum and the supremum of $\mu(x, y)$ over $[l, u]$ for $x_0 = l_x$ and $x_0 = u_x$, respectively.

case (a): $x_0 = l_x$. A valid lifting coefficient α corresponds to the infimum of $\mu(x, y)$. As already mentioned in [GH]⁺08], $\mu(x, y)$ is the differential quotient of f in x , for every fixed $y \in [l_y, u_y]$. By convexity of f in x , it follows that $\mu(x, y) \geq \frac{\partial f}{\partial x}(l_x, y)$ for all $(x, y) \in [l, u]$. We can exploit the assumptions on f to show monotonicity of $\frac{\partial f}{\partial x}(l_x, y)$ in y . Formally the assumptions on f mean

- $\frac{\partial^2 f}{\partial x^2}(x, y) > 0$, $\frac{\partial^2 f}{\partial y^2}(x, y) > 0$ for all (x, y) in the interior of $[l, u]$,
- $\frac{\partial^2 f}{\partial x^2}(x, y) \frac{\partial^2 f}{\partial y^2}(x, y) - [\frac{\partial^2 f}{\partial x \partial y}(x, y)]^2 < 0$ for all (x, y) in the interior of $[l, u]$.

Thus, $[\frac{\partial^2 f}{\partial x \partial y}(x, y)]^2 > \frac{\partial^2 f}{\partial x^2}(x, y) \frac{\partial^2 f}{\partial y^2}(x, y) > 0$ for all (x, y) in the interior of $[l, u]$. As we assume f to be twice continuously differentiable, it follows that $\frac{\partial^2 f}{\partial x \partial y}(x, y)$ is either non-positive or non-negative over $[l, u]$ which implies monotonicity of $\frac{\partial f}{\partial x}(l_x, y)$ in y . Thus, $\mu(x, y) \geq \frac{\partial f}{\partial x}(l_x, y) \geq \frac{\partial f}{\partial x}(l_x, y^*)$ for all $(x, y) \in [l, u]$, where

$$y^* := \begin{cases} l_y, & \text{if } \frac{\partial f}{\partial x}(l_x, l_y) \leq \frac{\partial f}{\partial x}(l_x, u_y), \\ u_y, & \text{otherwise.} \end{cases}$$

A linear underestimator for $f(x, y)$ w.r.t. the given point (x_0, y_0) can be obtained as follows:

$$\begin{aligned} f(x, y) &\geq \frac{\partial f}{\partial x}(l_x, y^*)(x - l_x) + f(l_x, y) \\ &\geq \frac{\partial f}{\partial x}(l_x, y^*)(x - l_x) + \frac{\partial f}{\partial y}(l_x, y_0)(y - y_0) + f(l_x, y_0). \end{aligned}$$

case (b): $x_0 = u_x$. With

$$y^* := \begin{cases} l_y, & \text{if } \frac{\partial f}{\partial x}(u_x, l_y) \geq \frac{\partial f}{\partial x}(u_x, u_y), \\ u_y, & \text{otherwise,} \end{cases}$$

we obtain the following linear underestimator

$$f(x, y) \geq \frac{\partial f}{\partial x}(u_x, y^*)(x - u_x) + \frac{\partial f}{\partial y}(u_x, y_0)(y - y_0) + f(u_x, y_0).$$

We remark that the assumptions on f are crucial to use the derived lifting coefficient. The next example deals with a convex function and shows that the derived lifting coefficients are not valid for this class of functions.

Example 3.1. Consider the bivariate function $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ given by $f(x, y) := x^2 + x(y^2 - 2y)$ restricted to the box $[1, 2] \times [0, 2]$ over which f is convex. We fix x to its lower bound 1 and consider the point $(x_0, y_0) = (1, 0.5)$. Assume we want to lift the tangent $t(y) = -y + 0.75$ on the graph of $f(1, y)$ at $y_0 = 0.5$ along the line $x = 1$ to a valid underestimator of the form $f(x, y) = t(y) + \alpha(x - 1)$. Note that the first partial derivative $\frac{\partial f}{\partial x}(1, y) = y^2 - 2y + 2$ attains its global minimum at $y = 1 \in [0, 2]$. Therefore, we can choose α to be $\frac{\partial f}{\partial x}(1, 1) = 1$ yielding $f(x, y) \geq x - y - 0.25$. We remark that for $y \in \{l_y, u_y\}$, $\frac{\partial f}{\partial x}(1, y) = 2$. Setting $\alpha = 2$ gives the following relation $f(x, y) \geq 2x - y - 1.25$ that does not hold for the point $(1.5, 0.5)$, for instance

$$f(1.5, 0.5) = 1.125 \quad \not\geq \quad 1.25 = 2 \cdot 1.5 - 0.5 - 1.25.$$

The following two examples show in detail that $\frac{\partial^2 f}{\partial x \partial y}$ is monotone if f is a bivariate quadratic or monomial function.

Example 3.2. For an arbitrary bivariate quadratic function given by $f(x, y) := a_{x,x}x^2 + a_{x,y}xy + a_{y,y}y^2 + b_x x + b_y y + c$, the second partial derivative $\frac{\partial^2 f}{\partial x \partial y}(l_x, y) = a_{x,y}$ is a constant, that allows us to consider the endpoints of the interval only. Note that if the constant $a_{x,y} = 0$, our bivariate quadratic function f is separable.

Example 3.3. For any monomial function of the form $f(x, y) = x^p y^q$, for some $p, q \neq 0$, restricted to a box $[l, u] \subseteq \mathbf{R}_{\geq}^2$, we have that

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = pqx^{p-1}y^{q-1}$$

is either non-negative for all $(x, y) \in \mathbf{R}_{\geq 0}^2$ or non-positive for all $(x, y) \in \mathbf{R}_{\geq 0}^2$. Thus, critical points can only occur, if one of the lower bounds are equal to zero and one variable is fixed to this lower bound for the lifting step. In such a case, the function f is the zero function over the corresponding facet of the box. It follows that it is again sufficient to consider the endpoints of the interval of the second variable, only.

4. Computations

This section presents a computational study of the linear underestimator technique as discussed in the previous sections.

4.1. Implementation

The underestimator techniques have been implemented as a new constraint handler in the constraint integer programming framework SCIP [Ach07, Ach09], which has

recently been extended to handle general MINLPs [BHV09, Vig12]. The new constraint handler `cons_bivariate` has been released the first time with SCIP 2.1.0 in October 2011 as a beta version.

SCIP solves MINLPs by a branch-and-bound algorithm. The problem is recursively split into smaller subproblems, thereby creating a search tree and implicitly enumerating all potential solutions. At each subproblem, domain propagation is performed to exclude further values from the variables' domains, and a linear relaxation is solved to achieve a local lower bound – assuming the problem is to minimize the objective function. The relaxation is strengthened by adding further linear inequalities, which cut off the optimal solution of the relaxation. Primal heuristics are used as supplementary methods to improve the upper bound.

A constraint handler in SCIP defines the semantics and the algorithms to process constraints of a certain class. Each constraint handler has to implement an enforcement method. In enforcement, the handler has to decide whether the optimal solution of the linear relaxation satisfies all of its constraints. If the solution violates one or more constraints, the handler may resolve the infeasibility by adding linear inequalities, performing a domain reduction, or a branching.

Our constraint handler handles bivariate constraints of the form $\ell \leq f(x, y) + cz \leq r$, where $f : [l_x, u_x] \times [l_y, u_y] \rightarrow \mathbf{R}$ is a bivariate function with fixed convexity behavior, $c \in \mathbf{R}$, $\ell \in \mathbf{R} \cup \{-\infty\}$, and $u \in \mathbf{R} \cup \{\infty\}$. The function $f(x, y)$ has to be passed to the constraint handler in form of an expression trees. Additionally, the convexity behavior of the function (recall Table 1) has to be specified. For enforcement and during separation rounds, the constraint handler generates a linear inequality from a linear under- or overestimators of $f(x, y)$ (as described in the previous sections). The univariate convex optimization problems are solved by Newton's method. If the generated inequality does not cut off the optimal solution of the linear relaxation, spatial branching is applied on either x or y . E.g., if $f(x, y)$ is convex in x and concave in y and the current relaxation optimum $(\hat{x}, \hat{y}, \hat{z})$ violates the inequality $f(\hat{x}, \hat{y}) + c\hat{z} \leq r$, then variable y is proposed as branching candidate to SCIP. From all branching candidates that are registered by all constraint handlers, SCIP selects a branching variable and branching point according to a pseudo-costs based variable selection rule, see [BLL⁺09, BHV09, Vig12] for details. Further, a feasibility-based bound tightening (FBBT) rule is applied to deduce tighter variable bounds for x , y , or z from the constraint and the bounds on these variables, see [Vig12] for details.

During presolve, SCIP reformulates a MINLP into a form where it can construct a linear relaxation. The reformulation mainly consists of introducing new auxiliary variables and nonlinear constraints for subexpressions of nonlinear functions. For example, a general monomial function $x^p y^q$ has so far been reformulated by SCIP into a product $w_1 w_2$ and two new constraints $w_1 = x^p$ and $w_2 = y^q$, because SCIP knows how to compute linear under- and overestimators for these functions. With the new constraint handler for bivariate functions, there is no more need for reformulating monomials $x^p y^q$ with $l_x \geq 0$ and $l_y \geq 0$. Additionally, bivariate quadratic functions $a_x x + a_y y + a_{xy} xy + a_{xx} x^2 + a_{yy} y^2$ are automatically recognized as bivariate terms and handled by the new constraint handler.

4.2. Test set

Initially we considered the problem libraries GLOBALLIB [GLO] and MINLPLIB [BDM03]. However, they contain only a few instances with bivariate quadratic terms or monomials, mainly of the form x/y . To investigate the computational benefit of having a convex underestimator for bivariate functions at hand, we created a set of nonlinear optimization problems where bivariate functions occur in form of quadratic functions and monomials, e.g., $3x_1^2 + x_1x_2 - x_x^2 + 2x_1^{0.3}x_2^{1.5} - 4x_2^{1.2}x_3^{2.5}$.

The random generation of problems with constraints can lead to infeasibility. As the proposed constraint handler aims at strong lower bounds on the problem, feasible problems are required in order to compare the quality of the bounds. This is achieved by the following problem class where we vary the number of variables $Nvars$ and constraints $Ncons$, and the maximum degree Deg over all constraints.

$$\begin{aligned} & \min \epsilon \\ \text{s. t. } & \sum_{i=1}^{Deg-1} \sum_{j=1}^{Deg-i} \sum_{k=1}^{Nvars} \sum_{l=k+1}^{Nvars} a_{c,i,j,k,l} x_k^{p_{c,i,j,k}} x_l^{q_{c,i,j,l}} + \sum_{k=1}^{Nvars} b_{c,k} x_k^2 \leq \epsilon, \quad \text{for all } c \in \{1, \dots, Ncons\}, \\ & x \in [l, u], \end{aligned}$$

where

- $p_{c,i,j,k}$: If $i = j = 1$ then $p_{c,i,j,k} = 1$. Otherwise, $p_{c,i,j,k}$ is uniformly random set to a value in $\{(i-1) + 0.2, (i-1) + 0.4, \dots, (i-1) + 1\}$.
- $q_{c,i,j,l}$: If $i = j = 1$ then $q_{c,i,j,l} = 1$. Otherwise, $q_{c,i,j,l}$ is uniformly random set to a value in $\{(j-1) + 0.2, (j-1) + 0.4, \dots, (j-1) + 1\}$.
- $a_{c,i,j,k,l}$: If $i = j = 1, k$ odd, and $l = k + 1$, then $a_{c,i,j,k,l}$ is uniformly random in $\{-4, -3, \dots, 3, 4\}$. If $i > 1$ or $j > 1$, then $a_{c,i,j,k,l}$ is with probability $2/Nvars$ in $\{-4, -3, \dots, 3, 4\}$. Otherwise, we set $a_{c,i,j,k,l} = 0$.
- $b_{c,k}$: The coefficient $b_{c,k}$ is chosen uniformly at random from $\{-4, -3, \dots, 3, 4\}$.
- $[l, u]$: For each $k \in \{1, \dots, Nvars\}$ the lower bound l_k is uniformly at random set to a value in $\{0, 1, 2, 3, 4\}$. The upper bound u_k is the sum of $l_k + 1$ and a value which is chosen uniformly at random from $\{0, 1, 2, 3, 4\}$. To avoid numerical inconsistencies we check that $u_k^{Deg} \leq 2000$.

The condition $i = j = 1$ deals with the quadratic case. It ensures that integer exponents leading to quadratic terms $x_i x_j$ are generated. The condition $(l = k + 1), l$ odd, leads to bivariate quadratic terms $x_1 x_2, x_3 x_4, \dots$ and thereby the univariate quadratic terms x_k^2 can be associated to a unique bivariate quadratic monomial.

The implemented methods are of particular interest if the optimal or intermediate solutions are attained in the interior of the underlying boxes $[l, u]$. Otherwise, only the lifting methods are executed. Thus, an ellipsoid constraint is optionally added to

the problems which cuts off the boundary of the box. It reads

$$\sum_{k=1}^{\text{Nvars}} \left(\frac{x_k - \text{midpoint}_k}{\text{intervallength}_k/2} \right)^2 = \sum_{k=1}^{\text{Nvars}} \left(\frac{x_k - (u_k + l_k)/2}{(u_k - l_k)/2} \right)^2 \leq 1 \quad (16)$$

The following settings are considered.

- Nvars $\in \{10, 20, 30\}$.
- Deg $\in \{2, 3, 4, 5\}$.
- Ncons $\in \{1, 2, 3, 5, 10\}$.
- Enable/Disable the ellipsoid constraint (16).

Hence, there are $3 \cdot 5 \cdot 4 \cdot 2 = 120$ different settings. For each setting we generate 10 random instances which leads to 1,200 instances in total.

4.3. Experimental Setup

We compared SCIP 3.0.1 (with the new constraint handler enabled or disabled) with BARON 11.8.0 [TS05] and COUENNE 0.4 [BLL⁺09]. SCIP and BARON use CPLEX 12.5 for solving LP relaxations, COUENNE uses CLP 1.14. SCIP and COUENNE use Ipopt for finding local optimal solutions to an NLP, BARON automatically chooses between Conopt, Ipopt, Minos, and Snopt.

We run all experiments with GAMS 24.0.1 under openSuSE Linux 12.1 64bit on a Dell PowerEdge M1000e blade with 48 GB RAM and two Intel Xeon X5672 CPUs running at 3.20 GHz. The timelimit is 30 minutes and the gap tolerance is 0.01%.

4.4. Results

Table 2 summarizes the results for SCIP, SCIP(bivar), which stands for SCIP with the new constraint handler enabled, BARON, and COUENNE when run on all 1,200 instances, excluding those 5 instances for which at least one of the 4 algorithms aborted due to numerical difficulties. The detailed results of the single instances can be found in Section A. First, we report the number of instances which are solved and solved fastest by an algorithm, and for which an algorithm computes the best dual bound. An algorithm is called fastest if it is within one second of the minimal solution time for an instance. A dual bound for a solver is called best dual bound, if the bound is within 0.01% of the best dual bound for that instance. Second, for each solver we calculated mean values of the solution time (in which unsolved instances are accounted for with the time limit), the number of processed nodes, and the dual gap at termination. The mean values are computed according to [Ach07, Section A.3], where the shifted geometric mean, defined as

$$\left(\prod_{i \in [n]} \max(\varepsilon, v_i + s) \right)^{1/n} - s,$$

	SCIP	SCIP(bivar)	BARON	COUENNE
#solved	375	977	678	760
#fastest	20	636	239	212
#best dual bound	377	1041	721	802
time (sh. geom. mean)	491.7	67.6	233.7	162.3
nodes (sh. geom. mean)	2233.0	385.9	1086.2	3741.5
dual gap (arith. mean)	29.56%	4.22%	20.22%	18.12%

Table 2: Computational results for 1,200 randomly generated polynomial instances.

is calculated with $\varepsilon = 1$ and $s = 10$ for solution times and with $\varepsilon = 1$ and $s = 100$ for node counts. The *dual gap* for a problem with dual bound \underline{v} and best known objective value v^* is defined as

$$\text{dual gap} := \min\left(1, \frac{|v^* - \underline{v}|}{\max(1, |v^*|)}\right),$$

see [ABH12]. A justification for the truncation at 1 is the claim that all bounds that are far away from the optimal value are equally useless. However, projecting the gaps onto $[0, 1]$ allows for meaningful arithmetic means.

The results in Table 2 allow an overall ranking of the four algorithms as the ranking for most of the individual performance parameters is the same. SCIP(bivar) clearly outperforms the other algorithms, followed by COUENNE, BARON and SCIP. SCIP(bivar) solves at least 200 instances more than the solvers BARON and COUENNE, which also results in much better average dual gaps and mean solution times when compared to BARON and COUENNE.

We further restrict our attention to those 569 instances which are solved by BARON, COUENNE, and SCIP(bivar), see Table 3. Also for these instances, SCIP(bivar) is more often the fastest algorithm than the other solvers. The mean of the computation times shows that COUENNE is 24% slower than SCIP(bivar) and that BARON needs approximately twice as much time as SCIP(bivar). The small solving time of SCIP(bivar) is likely related to the low number of processed branch-and-bound nodes, which indicates the strength of the relaxations used in SCIP(bivar). This claim is supported by the direct comparison of SCIP and SCIP(bivar) restricted to those 368 instances which are solved by both algorithms. SCIP needs a mean of 20.1 seconds and 1312.3 nodes while SCIP(bivar) uses only 9.6 seconds and 395.7 nodes. Thus, SCIP(bivar) can utilize the improved relaxations to avoid branching steps and prune nodes earlier, thus accelerating the solving process.

In Figure 3, we refine the analysis of the dual gaps for the 1,200 instances w.r.t. the number of variables N_{Vars} , the number of constraints N_{Cons} , the maximal degree Deg of the polynomials, and the presence of the ellipsoid constraint in Equation (16). First, for 10 Variables per instance we observe that SCIP(bivar), BARON, and COUENNE have about the same dual gap, which is close to zero. For instances with more variables, the dual gaps of BARON and COUENNE increase to more than 34%, while the dual gap of SCIP(bivar) increases modestly to only 13%.

	SCIP(bivar)	BARON	COUENNE
#fastest	254	214	104
time (sh. geom. mean)	15.7	29.8	19.5
nodes (sh. geom. mean)	381.9	421.7	555.1

Table 3: Summary of 569 instances solved by SCIP(bivar), BARON, as well as COUENNE.

Second, an increase of the number of constraints leads only to a modest increase of the dual gap for all solvers. SCIP(bivar)’s dual gap is between 3.8 and 7.2 times smaller than the one of second best algorithm, COUENNE.

Third, the maximal degree has a significant influence on the dual gap of all solvers. For a degree of two, BARON’s and COUENNE’s dual gaps are close to zero while SCIP(bivar)’s gap is about 6%. For degrees of three and four, the dual gaps of BARON, COUENNE, and SCIP increase heavily, whereas SCIP(bivar)’s gap even decreases. For degree two, we construct only uni- and bivariate quadratic monomials, but no monomials with fractional exponents like $x^{0.2}y^{1.4}$, which is allowed for larger degrees. As a consequence, the programs corresponding to degree two are quadratic programs for which solvers like BARON and COUENNE may be able to compute tighter relaxations than SCIP(bivar). For instances with higher degree, there are monomials like $x^{0.4}y^{1.4}$, which are concave in one direction and convex in the other direction, and monomials like $x^{1.4}y^{2.2}$, which are 1-convex and indefinite. These cases are handled by the presented constraint handler in SCIP(bivar).

Fourth, if the ellipsoid constraint is enabled the dual gaps of SCIP, SCIP(bivar), BARON, and COUENNE increase by factors of about 4, 8, 4, and 6, respectively. The activation of the ellipsoid constraint forces the optimal solution to be attained in the interior of the given domains, which obviously causes some problems for the algorithms due to weaker relaxations. Yet, SCIP(bivar) returns a dual gap which is at least three times better than the gap of the other algorithms. This shows that both the lifting technique used to cut-off points at the boundary and the linear underestimators based on the convex envelopes used to cut-off points in the interior have a significant influence on the performance of SCIP(bivar). This concludes the discussion corresponding to Figure 3.

Integral Exponents

In the tests above, we constructed monomials with fractional exponents like $x^{0.4}y^{1.8}$. A last comparison is devoted to instances with integral exponents only, i.e., we round up the exponents such that we obtain monomials like x^1y^2 . Table 4 compares the computational results of SCIP and SCIP(bivar) applied to a test set of 1,200 instances with integral exponents. In contrast to the instances with fractional exponents reported in Table 2, SCIP can solve about 300 instances more whereas SCIP(bivar) solves about 350 instances less.

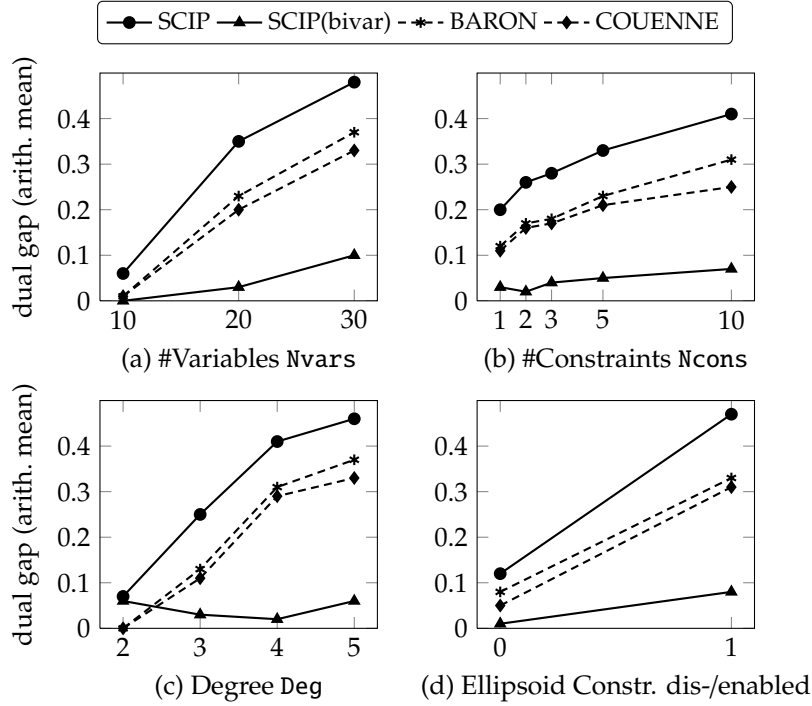


Figure 3: Dual gaps of the solvers w.r.t. to the number of variables N_{Vars} , constraints N_{Cons} , the maximal degree Deg , and the ellipsoid constraint dis- and enabled.

To understand the good performance of SCIP on this test set, consider the monomials $x^{0.4}y^{1.8}$ and $x^{2.6}y^{1.6}$. SCIP introduces new variables $v_{0.4}, v_{2.6}$, and $w_{2.6}, w_{1.6}$ for the univariate convex or concave monomials $x^{0.4}, x^{2.6}$, and $y^{1.8}, y^{1.6}$, respectively. Afterwards, it relaxes the univariate monomials and the bilinear product terms $v_{0.4}w_{1.8}$ and $v_{2.6}w_{1.6}$ by their convex and concave envelopes. If only integral exponents are allowed, the monomials change to x^1y^2 and x^3y^2 . Thus, less variables are introduced, namely v_1, v_3 , and w_2 , and the bilinear terms v_1w_2 and v_3w_2 have a common variable which is helpful in the process of relaxation. Table 5 depicts the average quotient of the number of variables or constraints in the reformulated problems between SCIP(bivar) and SCIP. It is seen that the number of constraints that have to be processed by SCIP(bivar) is much larger than those that are processed by SCIP, while for the instances with fractional exponents and degree > 2 , SCIP(bivar) has to process even less constraints than SCIP.

A possible explanation for SCIP(bivar)'s bad performance is the absence of monomials like $x^{0.4}y^{1.8}$, which are strictly concave in one variable and convex in the other one. We already indicated this in the discussion of Figure 3 (c), where no monomials like $x^{0.4}y^{1.8}$ occur for degree two, while this is the case for higher degrees. This conjecture

	SCIP	SCIP(bivar)
#solved	670	618
#fastest	478	220
#best dual bound	1073	739
time (sh. geom. mean)	204.5	249.0
nodes (sh. geom. mean)	1807.9	1054.9
dual gap (arith. mean)	22.58%	26.19%

Table 4: Computational results for 1,200 randomly generated polynomial instances with integral exponents.

Maximal degree		2	3	4	5
Nvars	Fractional exponents	2.76	0.80	0.76	0.81
	Integral exponents	2.76	4.25	2.70	3.92
Ncons	Fractional exponents	9.89	0.77	0.74	0.80
	Integral exponents	9.89	5.77	2.94	4.24

Table 5: Average growth of number of variables and constraints in reformulated problem from SCIP to SCIP(bivar) (only instances without ellipsoid constraint are considered).

is further affirmed by Table 6, which displays the dual gaps for SCIP and SCIP(bivar) w.r.t. the maximal degree of the programs with fractional and integral exponents. The numbers show an enormous increase of the dual gap of SCIP(bivar) for integral exponents compared to fractional exponents. (Note that the dual gaps are computed w.r.t. the best solutions found by SCIP and SCIP(bivar) and thus the gaps can be larger than the gaps in Figure 3 (c) where also the solutions of BARON and COUENNE are considered.)

Maximal degree		2	3	4	5
SCIP	Fractional exponents	7.03%	26.12%	46.33%	52.42%
	Integral exponents	7.04%	18.85%	31.58%	32.85%
SCIP(bivar)	Fractional exponents	6.26%	3.01%	6.10%	11.23%
	Integral exponents	6.26%	23.58%	35.74%	39.20%

Table 6: Dual gaps of SCIP and SCIP(bivar) for instances with fractional exponents and instances with integral exponents.

5. Conclusions

The new constraint handler used in SCIP(bivar) can reduce the solution time and improve the dual bounds of programs containing bivariate functions with a fixed convexity behavior. While the implemented methods of the constraint handler are applicable for general bivariate functions with a fixed convexity behavior, an automatic detection of the convexity behavior has been only implemented for the class of bivariate quadratic functions and for the class of monomial functions restricted to the nonnegative orthant (cf. Table 1). Using the callable library of SCIP, a user can manually provide the convexity type of a function to the constraint handler so that the advantages of this new tool can already be exploited for further functions.

Finally, we remark that for many specific bivariate functions of practical relevance, explicit formulas for the convex envelope are available (e.g., see [TS01, JMW08, KS11, KS12]). An incorporation of such explicit formulas could be helpful for our constraint handler to avoid the numerical solution of the nonlinear subproblems involved in the cut generation.

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A. Detailed computational results

In this section, the results of the solvers on the single instances are reported. We record the number of seconds for solving the problem, or, if a time limit was hit, the lower and upper bound on the optimal value at termination. Additionally, we collect the number of nodes that have been processed.

In the detailed result tables, each entry shows the number of seconds a certain solver needs to solve a problem and the number of branch-and-bound nodes it has processed. If the problem has not been solved within the given time limit and the solver did not abort, then we report either the *dual gap* and *primal gap* in parentheses, or the dual bound and primal bound in brackets. The gaps are reported if the optimal value of the instance is known, otherwise the bounds are given. Additional to the dual gap, which was defined in Section 4.4, we define the *primal gap* [ABH12] for a problem with primal bound \bar{v} and best known objective value v^* as

$$\text{primal gap} := \min\left(1, \frac{|\bar{v} - v^*|}{\max(1, |v^*|)}\right).$$

For each instance, the fastest solution time or – in case all solvers hit the time limit – the best primal and dual gaps (or bounds), are depicted in bold face. (A solution time is marked as best time, if it is within one second of the minimal solution time for that instance. A dual or primal gap/bound for a solver that hit the time limit is marked as best gap/bound, if no other solver solved the instance and if the gap/bound is within 0.01% of the best gap/bound for that instance.)

A.1. Results for instances with fractional exponents

instance				SCIP	SCIP(bivar)	BARON	COUENNE				
Nvars	Deg	Ncons (16)	#	time nodes (dgap) (pgap)	time nodes (dgap) (pgap)	time nodes (dgap) (pgap)	time nodes (dgap) (pgap)				
10	2	1	1	0.04	1	0.02	1	0.03	1	0.11	0
10	2	1	2	0.03	1	0.03	1	0.06	1	0.03	0
10	2	1	3	0.02	1	0.02	1	0.05	1	0.23	0
10	2	1	4	0.03	1	0.01	1	0.05	1	0.07	0
10	2	1	5	0.02	1	0.01	1	0.05	1	0.01	0
10	2	1	6	0.03	1	0.03	1	0.06	1	0.11	0
10	2	1	7	0.05	1	0.02	1	0.05	1	0.07	0
10	2	1	8	0.03	1	0.01	1	0.04	1	0.01	0
10	2	1	9	0.04	1	0.01	1	0.05	1	0.11	0
10	2	1	10	0.04	1	0.01	1	0.05	1	0.02	0
10	2	1	✓ 1	46.41	12.5k	1800.04	6053.3k	2.21	296	1.91	1841
10	2	1	✓ 2	34.30	16.2k	190.72	429.5k	1.86	389	1.83	684
10	2	1	✓ 3	8.26	2856	1031.95	2359.6k	1.06	83	0.39	50
10	2	1	✓ 4	35.14	10.9k	1800.01	4969.0k	2.08	316	1.12	718
10	2	1	✓ 5	32.62	8985	5.53	635	2.43	258	1.28	810
10	2	1	✓ 6	45.55	9160	21.13	2366	2.39	226	1.62	1450
10	2	1	✓ 7	51.12	12.5k	1800.03	4345.8k	2.48	495	1.28	939
10	2	1	✓ 8	5.69	1917	1800.01	5620.4k	1.03	99	1.74	60
10	2	1	✓ 9	68.76	19.4k	450.08	936.6k	3.74	533	2.68	3360

instance					SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
10	2	1	✓	10	34.72	10.1k	3.02	363	1.48	161	2.02	1036
10	2	2		1	0.04	1	0.01	1	0.05	1	0.05	0
10	2	2		2	0.05	1	0.02	1	0.06	1	0.04	0
10	2	2		3	0.29	59	0.32	39	0.12	5	0.80	22
10	2	2		4	0.49	217	0.44	165	0.16	19	0.22	26
10	2	2		5	0.07	11	0.05	1	0.05	1	0.12	0
10	2	2		6	0.03	1	0.02	1	0.05	1	0.02	0
10	2	2		7	0.02	1	0.01	1	0.05	1	0.04	0
10	2	2		8	0.21	29	0.28	19	0.07	1	0.49	0
10	2	2		9	0.11	19	0.11	8	0.07	3	0.12	4
10	2	2		10	0.26	17	0.13	7	0.05	1	0.03	0
10	2	2	✓	1	49.14	30.1k	1800.00	5260.7k	1.63	117	1.16	680
10	2	2	✓	2	20.34	4506	26.63	4302	1.82	191	1.80	412
10	2	2	✓	3	51.08	26.3k	28.90	4862	3.34	382	1.48	1338
10	2	2	✓	4	71.99	18.1k	88.13	13.3k	1.92	176	1.17	852
10	2	2	✓	5	30.30	21.9k	622.11	1300.1k	9.67	2495	1.06	640
10	2	2	✓	6	41.04	11.1k	32.85	5482	2.62	443	1.11	659
10	2	2	✓	7	60.67	13.4k	1800.00	5354.3k	4.43	678	1.38	644
10	2	2	✓	8	16.15	6324	2.41	279	2.98	675	1.19	897
10	2	2	✓	9	31.98	18.6k	8.91	2034	0.96	67	0.65	244
10	2	2	✓	10	11.63	6382	11.06	2273	0.96	34	0.70	204
10	2	3		1	0.28	47	0.36	41	0.04	1	0.02	0
10	2	3		2	0.28	47	0.31	29	0.05	1	0.06	6
10	2	3		3	0.11	17	0.25	17	0.06	1	0.04	0
10	2	3		4	0.86	652	0.79	220	0.14	3	1.05	21
10	2	3		5	0.26	55	0.16	43	0.06	1	0.09	0
10	2	3		6	0.30	55	0.11	5	0.07	1	0.26	0
10	2	3		7	0.36	121	0.42	79	0.07	1	0.06	0
10	2	3		8	0.12	19	0.09	17	0.06	1	0.13	2
10	2	3		9	0.02	1	0.10	1	0.05	1	0.04	0
10	2	3		10	0.30	85	0.36	82	0.08	1	0.19	0
10	2	3	✓	1	29.86	8821	16.13	2544	2.76	316	1.13	698
10	2	3	✓	2	28.63	11.7k	1800.01	2870.8k	1.87	190	1.08	294
10	2	3	✓	3	12.75	2213	30.27	4527	2.39	276	1.16	730
10	2	3	✓	4	53.43	24.2k	31.99	5100	4.47	604	1.53	1525
10	2	3	✓	5	28.39	11.2k	18.49	4007	3.08	401	0.99	613
10	2	3	✓	6	22.27	8600	4.70	622	2.13	197	1.08	847
10	2	3	✓	7	78.21	22.2k	89.55	14.6k	9.13	1631	2.58	2562
10	2	3	✓	8	87.04	29.2k	46.56	7852	3.66	434	1.13	702
10	2	3	✓	9	82.95	22.6k	137.16	21.3k	2.42	280	1.21	648
10	2	3	✓	10	143.63	85.1k	13.69	9140	25.13	7279	2.58	2736
10	2	5		1	0.31	33	0.43	44	0.03	1	0.09	2
10	2	5		2	0.18	25	0.35	27	0.04	1	0.12	4
10	2	5		3	0.27	33	0.34	45	0.08	1	0.24	8
10	2	5		4	0.33	61	0.53	54	0.10	1	0.77	12
10	2	5		5	0.18	19	0.14	22	0.06	1	0.09	0
10	2	5		6	2.19	5153	1.15	547	0.74	73	0.69	216
10	2	5		7	0.34	47	0.35	53	0.05	1	0.10	0
10	2	5		8	0.26	35	0.21	15	0.05	1	0.06	0
10	2	5		9	0.30	47	0.34	19	0.11	1	0.43	8

instance					SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	2	5	10		0.54	195	0.80	169	0.13	1	4.63	39
10	2	5	✓ 1		28.48	10.2k	41.45	7136	0.66	25	0.67	240
10	2	5	✓ 2		20.05	4029	48.27	7684	2.73	122	0.86	188
10	2	5	✓ 3		26.61	11.6k	20.70	3858	0.81	27	0.77	262
10	2	5	✓ 4		36.53	9984	34.03	8446	1.79	129	1.23	698
10	2	5	✓ 5		10.57	2313	8.77	2270	2.85	333	1.82	1439
10	2	5	✓ 6		95.25	56.8k	51.51	11.7k	2.70	345	1.15	739
10	2	5	✓ 7		21.79	6538	19.63	3694	1.82	167	1.23	730
10	2	5	✓ 8		9.70	2814	53.60	18.2k	0.99	35	0.94	298
10	2	5	✓ 9		33.45	16.9k	7.90	6109	2.76	323	1.46	818
10	2	5	✓ 10		41.12	11.9k	91.71	16.0k	1.72	195	1.15	918
10	2	10	1		1.14	1201	2.66	1229	0.23	22	0.43	40
10	2	10	2		0.32	59	0.45	17	0.08	1	0.74	6
10	2	10	3		0.35	47	0.61	59	0.08	1	0.14	2
10	2	10	4		0.35	77	0.61	121	0.08	1	0.21	4
10	2	10	5		0.25	27	0.46	19	0.03	1	0.05	0
10	2	10	6		0.32	43	0.61	31	0.07	1	0.27	0
10	2	10	7		0.32	39	0.43	65	0.05	1	0.11	0
10	2	10	8		0.31	37	0.47	27	0.07	1	0.14	4
10	2	10	9		0.21	23	0.49	23	0.07	1	0.06	0
10	2	10	10		0.48	243	0.93	154	0.19	15	0.89	122
10	2	10	✓ 1		68.13	15.6k	225.80	21.6k	2.38	221	1.39	626
10	2	10	✓ 2		16.45	3674	19.91	3452	2.44	217	1.56	800
10	2	10	✓ 3		19.64	6443	64.76	6994	1.31	87	1.35	447
10	2	10	✓ 4		95.18	34.8k	297.92	27.4k	3.53	376	1.75	1184
10	2	10	✓ 5		46.87	11.5k	94.93	11.7k	2.44	223	1.60	690
10	2	10	✓ 6		33.25	12.9k	114.99	14.6k	2.28	163	1.27	591
10	2	10	✓ 7		20.70	7186	36.73	5971	1.74	127	0.79	344
10	2	10	✓ 8		15.90	5175	19.33	2996	2.15	175	3.95	882
10	2	10	✓ 9		68.88	17.1k	159.69	17.9k	3.21	345	2.03	1198
10	2	10	✓ 10		36.52	12.6k	132.55	14.3k	2.79	218	1.45	790
10	3	1	1		0.12	1	0.05	1	0.09	1	0.25	4
10	3	1	2		4.58	102	1.43	2000	0.23	7	1.02	26
10	3	1	3		3.83	13	1.02	7	0.13	7	1.03	14
10	3	1	4		1.34	1715	0.69	299	0.84	75	1.33	194
10	3	1	5		0.69	1	1.03	1	0.08	1	0.74	0
10	3	1	6		5.39	21	(1.80%)	(0.00%)	0.08	1	(1.03%)	(100.00%)
10	3	1	7		0.22	1	0.08	1	0.08	1	0.03	0
10	3	1	8		2.01	3	0.56	3	0.07	1	0.42	2
10	3	1	9		0.40	33	0.41	41	0.22	17	0.47	25
10	3	1	10		0.18	1	0.02	1	0.03	1	0.02	0
10	3	1	✓ 1		(0.01%)	(0.00%)	60.92	5895	26.30	825	16.32	8881
10	3	1	✓ 2		(1.63%)	(0.01%)	1.35	27	28.36	1179	15.16	9142
10	3	1	✓ 3		(0.08%)	(0.00%)	166.42	19.6k	27.64	1352	13.28	7326
10	3	1	✓ 4		(0.13%)	(0.00%)	141.73	15.1k	17.71	788	11.96	6289
10	3	1	✓ 5		(0.08%)	(0.00%)	76.74	7396	9.97	483	12.57	6458
10	3	1	✓ 6		1800.03	868.5k	102.28	14.3k	21.88	1436	19.73	12.0k
10	3	1	✓ 7		669.79	93.0k	213.18	53.6k	18.29	1431	7.94	5555
10	3	1	✓ 8		959.97	450.9k	2.23	47	6.76	589	3.31	2212
10	3	1	✓ 9		1717.73	577.6k	1.45	49	18.56	923	10.50	6832

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	3	1	✓ 10	(0.01%)	(0.00%)	1800.02	1792.9k	43.00	2738	12.21	7538
10	3	2	1	0.43	15	0.56	25	0.37	7	0.79	4
10	3	2	2	2.43	27	0.86	1	0.23	1	1.56	22
10	3	2	3	3.06	121	1.02	1	0.28	3	(2.05%)	(100.00%)
10	3	2	4	1.23	797	0.69	212	0.78	29	1.34	112
10	3	2	5	5.96	56	1.17	17	0.11	1	(0.00%)	(100.00%)
10	3	2	6	(0.00%)	(0.40%)	(0.09%)	(0.14%)	0.31	7	1.06	19
10	3	2	7	0.59	83	0.79	108	0.34	7	0.85	26
10	3	2	8	(7.20%)	(0.00%)	(1.79%)	(0.09%)	1800.00	5629.3k	(7.15%)	(100.00%)
10	3	2	9	3.65	129	0.94	75	0.11	3	1.67	32
10	3	2	10	0.19	5	0.42	21	0.15	1	0.19	0
10	3	2	✓ 1	(0.01%)	(0.00%)	4.19	93	49.67	909	19.63	6828
10	3	2	✓ 2	(0.05%)	(0.00%)	1.24	11	192.88	4545	19.93	7403
10	3	2	✓ 3	(0.13%)	(0.00%)	1.79	35	60.41	1513	19.34	6390
10	3	2	✓ 4	(0.90%)	(0.00%)	813.92	393.8k	31.27	1395	12.97	4769
10	3	2	✓ 5	(0.15%)	(0.00%)	1.30	19	27.26	517	18.28	6968
10	3	2	✓ 6	(0.29%)	(0.00%)	1.32	23	158.45	4685	22.51	8092
10	3	2	✓ 7	1800.02	742.2k	109.90	22.9k	16.67	464	16.71	6723
10	3	2	✓ 8	(0.07%)	(0.00%)	0.70	5	43.68	1091	39.85	17.5k
10	3	2	✓ 9	(0.07%)	(0.00%)	0.50	5	36.58	1228	11.88	5635
10	3	2	✓ 10	(0.01%)	(0.00%)	1800.04	2144.7k	113.78	3651	19.15	7084
10	3	3	1	1800.09	2940.1k	1.20	3	(0.00%)	(1.54%)	5.23	330
10	3	3	2	(0.51%)	(0.05%)	(0.69%)	(0.05%)	(0.00%)	(0.05%)	1.62	80
10	3	3	3	6.72	32	1.10	1	0.16	1	1.69	37
10	3	3	4	1800.02	5017.2k	40.24	96.3k	0.62	1	2.07	108
10	3	3	5	0.94	103	0.93	79	0.57	1	1.71	30
10	3	3	6	0.80	41	0.63	88	0.30	1	1.87	11
10	3	3	7	0.86	145	0.63	41	0.99	7	1.90	42
10	3	3	8	0.31	5	0.11	3	0.22	1	0.37	2
10	3	3	9	1800.11	2243.1k	3.91	2000	7.36	121	10.96	545
10	3	3	10	0.88	41	0.35	1	0.14	5	(0.00%)	(100.00%)
10	3	3	✓ 1	(2.55%)	(0.00%)	1.37	3	135.74	1111	73.41	11.7k
10	3	3	✓ 2	(0.25%)	(0.00%)	619.01	41.1k	74.33	1075	58.20	13.6k
10	3	3	✓ 3	(0.36%)	(0.00%)	0.96	13	30.18	361	30.41	7405
10	3	3	✓ 4	(0.21%)	(0.00%)	2.57	39	47.65	697	13.78	3578
10	3	3	✓ 5	(4.42%)	(0.00%)	1058.19	41.3k	302.48	3532	61.61	12.5k
10	3	3	✓ 6	<i>abort</i>		300.97	30.3k	40.90	529	19.07	5088
10	3	3	✓ 7	(1.56%)	(0.00%)	1.11	7	86.39	972	51.12	8803
10	3	3	✓ 8	1800.04	192.9k	251.62	21.1k	55.55	1733	32.04	10.4k
10	3	3	✓ 9	(60.69%)	(0.00%)	1.07	3	1576.19	22.9k	302.01	61.6k
10	3	3	✓ 10	(0.02%)	(0.00%)	1.04	7	156.98	3974	41.85	11.9k
10	3	5	1	1.42	261	0.33	1	(0.00%)	(3.97%)	3.03	46
10	3	5	2	(0.00%)	(2.88%)	7.91	2000	0.80	1	3.35	40
10	3	5	3	(0.14%)	(1.52%)	1800.12	3909.6k	0.86	1	4.42	76
10	3	5	4	(0.00%)	(12.61%)	26.54	1220	1800.00	2208.5k	6.41	102
10	3	5	5	0.70	9	0.26	3	0.17	1	0.79	0
10	3	5	6	0.85	15	0.81	35	0.43	5	4.29	10
10	3	5	7	(0.68%)	(0.00%)	4.97	2000	(0.00%)	(29.19%)	5.47	306
10	3	5	8	1.12	19	2.42	232	0.31	1	3.01	18
10	3	5	9	(0.92%)	(1.65%)	1800.00	2724.1k	0.90	1	4.22	32

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	3	5	10	0.35	1	0.72	19	0.14	1	1.58	3
10	3	5	✓ 1	(0.55%)	(0.00%)	3.81	35	202.78	2396	620.26	131.5k
10	3	5	✓ 2	(0.15%)	(0.00%)	1.20	9	238.93	2630	89.24	17.2k
10	3	5	✓ 3	(0.23%)	(0.00%)	597.71	34.1k	75.27	849	49.65	8780
10	3	5	✓ 4	(2.73%)	(0.00%)	2.92	11	270.06	2609	189.24	30.9k
10	3	5	✓ 5	(3.01%)	(0.00%)	1800.05	202.6k	170.03	1450	145.51	23.9k
10	3	5	✓ 6	(0.43%)	(0.00%)	3.98	27	87.53	1095	60.06	11.4k
10	3	5	✓ 7	(0.57%)	(0.00%)	5.51	47	74.24	1529	46.27	7760
10	3	5	✓ 8	(0.44%)	(0.00%)	634.65	40.5k	68.95	494	44.96	6269
10	3	5	✓ 9	(0.03%)	(0.00%)	390.31	22.5k	41.46	1292	34.62	6181
10	3	5	✓ 10	(0.27%)	(0.00%)	569.75	25.1k	61.76	492	32.13	5443
10	3	10	1	(2.38%)	(0.00%)	0.95	5	14.97	65	23.48	827
10	3	10	2	1.44	230	1800.01	2538.4k	0.73	7	7.21	44
10	3	10	3	36.15	43.4k	3.17	38	1.54	11	6.88	70
10	3	10	4	(2.37%)	(3.42%)	38.51	2574	1800.00	444.5k	12.40	144
10	3	10	5	28.75	18.8k	9.15	240	3.92	39	13.79	169
10	3	10	6	1.34	119	2.00	79	1.75	1	6.57	52
10	3	10	7	0.81	26	2.38	220	0.30	1	0.90	0
10	3	10	8	(0.00%)	(0.95%)	25.38	2012	6.69	53	10.01	90
10	3	10	9	5.94	571	3.40	149	2.89	7	7.17	88
10	3	10	10	(2.15%)	(0.00%)	3.59	35	112.26	6410	28.99	1189
10	3	10	✓ 1	(3.98%)	(0.00%)	2.23	9	341.37	2130	182.23	15.3k
10	3	10	✓ 2	(1.14%)	(0.00%)	959.98	16.0k	254.66	2642	130.13	12.8k
10	3	10	✓ 3	(0.21%)	(0.00%)	2.20	3	196.17	2393	96.72	9400
10	3	10	✓ 4	(1.49%)	(0.00%)	13.97	55	374.23	2887	129.56	13.4k
10	3	10	✓ 5	(5.50%)	(0.00%)	2.69	3	465.62	2449	149.55	14.3k
10	3	10	✓ 6	(3.73%)	(0.00%)	(0.23%)	(0.00%)	314.67	2256	244.76	20.7k
10	3	10	✓ 7	(0.03%)	(0.00%)	2.76	3	50.02	183	113.52	11.4k
10	3	10	✓ 8	(2.45%)	(0.00%)	74.28	253	290.27	1566	227.69	19.1k
10	3	10	✓ 9	(3.19%)	(0.00%)	9.41	43	99.61	513	109.58	8588
10	3	10	✓ 10	(7.63%)	(0.00%)	3.79	9	935.09	9375	404.64	37.9k
10	4	1	1	1800.01	4659.0k	0.42	1	1.44	43	0.24	0
10	4	1	2	1.15	324	0.90	2	0.62	7	1.25	46
10	4	1	3	1.34	39	1.56	7	0.32	1	1.29	22
10	4	1	4	9.71	408	1.35	1	0.67	25	1.72	128
10	4	1	5	5.44	137	88.73	466.3k	0.61	3	1.63	6
10	4	1	6	1.27	100	0.36	1	0.77	9	2.07	24
10	4	1	7	(2.33%)	(0.04%)	3.23	1	0.80	21	2.02	151
10	4	1	8	0.16	1	0.09	1	0.06	1	0.04	0
10	4	1	9	0.61	27	1.30	2	0.17	1	0.66	4
10	4	1	10	0.28	1	0.16	1	0.06	1	0.05	0
10	4	1	✓ 1	(0.31%)	(0.00%)	2.24	2	57.04	1283	62.92	19.5k
10	4	1	✓ 2	(2.66%)	(0.00%)	2.16	2	179.76	4526	153.58	43.9k
10	4	1	✓ 3	1800.04	398.6k	2.13	2	18.94	680	50.44	21.3k
10	4	1	✓ 4	(16.27%)	(0.00%)	1.41	3	692.06	15.1k	617.68	192.4k
10	4	1	✓ 5	(0.02%)	(0.00%)	2.04	2	11.68	198	11.03	2924
10	4	1	✓ 6	(13.58%)	(0.00%)	1.88	2	476.97	7807	155.93	38.7k
10	4	1	✓ 7	(0.02%)	(0.00%)	1.78	2	18.65	773	45.80	20.9k
10	4	1	✓ 8	(0.05%)	(0.00%)	653.67	226.7k	112.20	6889	68.48	23.2k
10	4	1	✓ 9	(0.18%)	(0.00%)	1.91	2	88.42	1919	137.84	49.0k

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	4	1	✓ 10	(1.63%)	(0.00%)	2.42	2	122.35	2428	85.92	24.7k
10	4	2	1	1.68	107	2.37	2	1.35	5	2.85	14
10	4	2	2	18.15	12.5k	2.66	193	3.18	53	7.48	246
10	4	2	3	1800.11	1524.4k	5.43	27	6.97	57	11.83	274
10	4	2	4	(0.00%)	(1.30%)	1.92	221	1.03	1	2.09	20
10	4	2	5	3.33	301	2.69	2	1800.00	1197.3k	3.90	161
10	4	2	6	(0.00%)	(0.05%)	3.73	2	2.18	21	7.51	109
10	4	2	7	2.50	158	3.27	2	2.47	15	5.25	58
10	4	2	8	82.29	30.4k	2.38	2	2.98	33	8.81	144
10	4	2	9	2.87	297	0.84	11	1.64	5	3.34	52
10	4	2	10	1.11	260	1.42	2	0.59	1	2.13	26
10	4	2	✓ 1	(20.85%)	(0.00%)	3.97	2	1019.74	9985	918.50	162.4k
10	4	2	✓ 2	(22.43%)	(0.00%)	1744.10	51.9k	835.35	9381	355.52	58.2k
10	4	2	✓ 3	(6.42%)	(0.00%)	4.50	17	782.53	5351	470.27	54.2k
10	4	2	✓ 4	(14.17%)	(0.00%)	1.91	2	706.79	18.4k	256.70	38.6k
10	4	2	✓ 5	(6.51%)	(0.00%)	3.66	2	692.14	8002	386.52	71.1k
10	4	2	✓ 6	(4.19%)	(0.00%)	12.24	2	664.23	5389	389.91	53.5k
10	4	2	✓ 7	(6.14%)	(0.00%)	2.57	2	941.93	7494	459.71	57.6k
10	4	2	✓ 8	(2.82%)	(0.00%)	10.24	2	192.29	2591	126.42	17.3k
10	4	2	✓ 9	(42.17%)	(0.00%)	2.52	3	(0.03%)	(0.00%)	1221.71	171.7k
10	4	2	✓ 10	(15.84%)	(0.00%)	4.21	2	493.85	6991	118.23	25.3k
10	4	3	1	10.24	1902	3.49	2	1.65	1	11.44	117
10	4	3	2	(0.05%)	(0.03%)	3.86	3	1.36	1	2.96	18
10	4	3	3	2.24	247	1.65	100	3.15	21	6.28	89
10	4	3	4	1.72	469	0.86	42	1.13	1	3.60	25
10	4	3	5	1800.14	1018.5k	1.46	13	6.63	35	16.67	205
10	4	3	6	0.65	21	0.89	43	0.46	1	5.84	12
10	4	3	7	(0.00%)	(21.86%)	3.56	21	(0.00%)	(20.46%)	14.94	127
10	4	3	8	3.94	479	1.99	173	3.54	15	7.00	111
10	4	3	9	(0.00%)	(3.14%)	5.73	2	1.54	7	8.22	30
10	4	3	10	2.70	369	1.60	97	3.67	39	9.85	106
10	4	3	✓ 1	(100.00%)	(0.04%)	27.84	2	1112.90	5502	539.36	56.3k
10	4	3	✓ 2	(4.58%)	(0.00%)	4.56	2	1046.42	22.5k	1122.32	115.8k
10	4	3	✓ 3	(2.57%)	(0.00%)	8.00	2	219.78	4648	271.53	26.7k
10	4	3	✓ 4	(8.51%)	(0.00%)	3.27	2	318.43	3445	268.40	33.7k
10	4	3	✓ 5	(100.00%)	(38.09%)	1.12	3	(0.00%)	(0.00%)	(100.00%)	(0.64%)
10	4	3	✓ 6	(0.46%)	(0.00%)	1800.06	482.5k	150.64	1892	131.73	15.5k
10	4	3	✓ 7	(84.18%)	(0.00%)	4.21	3	(1.07%)	(0.00%)	1893.05	206.0k
10	4	3	✓ 8	(2.00%)	(0.00%)	1800.06	47.2k	297.58	4029	247.99	24.0k
10	4	3	✓ 9	(13.96%)	(0.00%)	5.39	2	977.53	20.0k	337.87	24.7k
10	4	3	✓ 10	(6.48%)	(0.00%)	72.71	2	429.39	6054	406.79	49.5k
10	4	5	1	7.44	513	5.79	142	8.16	37	21.12	208
10	4	5	2	479.48	125.7k	3.17	47	13.56	43	21.88	400
10	4	5	3	(1.41%)	(0.00%)	12.56	175	119.30	349	105.66	2456
10	4	5	4	1800.01	1733.2k	1.74	7	13.00	55	11.86	417
10	4	5	5	(0.81%)	(2.02%)	4.99	2	(0.00%)	(2.02%)	17.36	130
10	4	5	6	(0.93%)	(6.50%)	(0.00%)	(61.60%)	35.02	117	46.06	1319
10	4	5	7	27.74	1051	6.46	171	11.69	29	34.72	183
10	4	5	8	5.79	762	2.01	89	2.91	7	11.75	48
10	4	5	9	(0.12%)	(0.00%)	7.59	3	91.57	163	41.11	438

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	4	5	10	2.77	79	7.58	95	2.97	1	20.07	76
10	4	5	✓ 1	(5.63%)	(0.00%)	57.06	2	1079.95	9991	633.37	58.4k
10	4	5	✓ 2	(3.23%)	(0.00%)	8.47	2	576.89	12.3k	874.09	56.0k
10	4	5	✓ 3	(46.67%)	(0.00%)	4.97	3	(1.41%)	(0.00%)	(7.04%)	(0.00%)
10	4	5	✓ 4	(2.14%)	(0.00%)	15.49	2	267.48	10.2k	191.63	12.9k
10	4	5	✓ 5	(17.70%)	(0.00%)	9.91	2	(0.04%)	(0.00%)	1807.12	107.2k
10	4	5	✓ 6	(8.70%)	(0.00%)	7.49	3	948.21	5401	663.41	38.2k
10	4	5	✓ 7	(100.00%)	(0.00%)	(29.67%)	(0.00%)	(0.89%)	(0.00%)	(0.02%)	(0.00%)
10	4	5	✓ 8	(67.87%)	(0.01%)	6.62	2	1390.15	21.4k	1092.64	85.7k
10	4	5	✓ 9	(23.50%)	(0.00%)	7.76	3	(0.19%)	(0.00%)	(0.15%)	(0.00%)
10	4	5	✓ 10	(30.60%)	(0.00%)	8.66	2	1800.01	4435	(0.07%)	(0.00%)
10	4	10	1	(0.69%)	(1.09%)	2.94	2	1800.00	347.5k	54.99	1160
10	4	10	2	471.82	19.8k	85.48	662	89.78	107	90.45	1004
10	4	10	3	(28.44%)	(17.96%)	475.60	24.9k	(0.00%)	(17.96%)	238.68	3436
10	4	10	4	137.48	18.0k	25.18	257	36.02	73	79.11	194
10	4	10	5	(5.35%)	(100.00%)	17.66	3	(0.00%)	(2.22%)	66.87	1091
10	4	10	6	1572.36	19.6k	7.58	3	58.16	69	81.14	482
10	4	10	7	(0.04%)	(0.00%)	155.81	4831	252.22	583	197.38	5157
10	4	10	8	(0.00%)	(0.03%)	11.90	2	39.69	49	63.65	721
10	4	10	9	(49.16%)	(0.00%)	69.96	329	770.86	767	261.02	4296
10	4	10	10	(0.07%)	(0.00%)	11.08	37	29.28	33	61.41	349
10	4	10	✓ 1	(0.56%)	(0.00%)	30.28	2	473.37	13.0k	289.96	12.9k
10	4	10	✓ 2	(18.43%)	(0.00%)	2.68	3	1800.02	1514	1859.20	51.3k
10	4	10	✓ 3	(21.29%)	(0.00%)	7.48	2	1800.00	1452	1581.84	45.6k
10	4	10	✓ 4	(19.09%)	(0.00%)	(6.57%)	(0.00%)	(1.02%)	(0.00%)	(0.01%)	(0.00%)
10	4	10	✓ 5	(27.51%)	(0.00%)	10.60	3	1800.00	1393	(0.03%)	(0.00%)
10	4	10	✓ 6	(9.95%)	(0.00%)	73.36	3	1417.71	6682	922.30	31.8k
10	4	10	✓ 7	(45.66%)	(0.00%)	2.04	3	(0.15%)	(0.00%)	(1.35%)	(0.00%)
10	4	10	✓ 8	(8.98%)	(0.00%)	11.43	2	(0.02%)	(0.00%)	1751.86	65.0k
10	4	10	✓ 9	(83.00%)	(0.00%)	5.17	2	1270.50	8957	890.23	25.2k
10	4	10	✓ 10	(13.26%)	(0.00%)	34.11	19	1800.00	2309	971.05	34.5k
10	5	1	1	1.66	22	1.43	2	0.32	1	0.62	0
10	5	1	2	0.27	5	0.14	1	0.12	1	(0.00%)	(100.00%)
10	5	1	3	0.33	3	0.22	1	0.15	1	(0.00%)	(100.00%)
10	5	1	4	15.34	21.3k	1.14	188	1.74	25	4.29	139
10	5	1	5	0.87	45	2.13	3	0.45	1	(0.12%)	(100.00%)
10	5	1	6	(1.18%)	(0.00%)	(0.00%)	(0.78%)	(0.00%)	(0.78%)	4.10	59
10	5	1	7	3.70	63	1.78	20	1800.00	1225.3k	2.81	20
10	5	1	8	19.86	102	3.04	1	1.79	19	4.10	39
10	5	1	9	111.96	104.5k	(0.00%)	(0.09%)	1.05	1	3.02	38
10	5	1	10	(0.22%)	(0.63%)	2.18	2	1.64	15	3.29	79
10	5	1	✓ 1	(26.89%)	(0.01%)	2.50	2	1800.00	486.3k	387.66	70.6k
10	5	1	✓ 2	(0.04%)	(0.00%)	1.98	2	1394.44	777.3k	122.16	25.3k
10	5	1	✓ 3	(9.94%)	(0.00%)	4.24	2	1800.00	558.5k	362.44	67.6k
10	5	1	✓ 4	(0.02%)	(0.00%)	209.80	22.1k	1800.00	968.6k	68.73	13.8k
10	5	1	✓ 5	(0.72%)	(0.00%)	3.99	2	1800.00	640.4k	462.91	90.8k
10	5	1	✓ 6	(4.48%)	(0.00%)	2.47	3	1800.00	635.0k	202.99	39.8k
10	5	1	✓ 7	(3.89%)	(0.00%)	3.89	2	1800.00	164.0k	730.11	137.0k
10	5	1	✓ 8	(3.65%)	(0.00%)	2.95	2	565.06	4614	546.87	102.5k
10	5	1	✓ 9	(2.53%)	(0.00%)	2.99	2	1800.00	582.9k	401.08	74.5k

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	5	1	✓ 10	(0.21%)	(0.00%)	3.47	2	1800.00	516.5k	330.30	59.3k
10	5	2	1	(0.13%)	(0.00%)	(0.00%)	(47.29%)	1800.00	781.5k	19.76	351
10	5	2	2	8.25	206	(0.00%)	(6.27%)	4.67	21	16.62	71
10	5	2	3	25.89	7718	3.78	15	13.03	113	11.45	513
10	5	2	4	0.87	15	0.60	7	0.57	1	0.99	0
10	5	2	5	8.04	7092	3.12	2	1.20	1	5.99	20
10	5	2	6	0.76	15	0.90	63	0.62	1	4.17	17
10	5	2	7	1.61	249	2.28	524	2.19	3	7.58	68
10	5	2	8	1.47	27	3.06	2	2.40	3	8.68	24
10	5	2	9	1800.15	2044.1k	4.03	9	5.08	25	11.69	92
10	5	2	10	(0.06%)	(0.00%)	5.64	102	1800.00	813.7k	(0.32%)	(100.00%)
10	5	2	✓ 1	(14.58%)	(0.00%)	2.06	7	1800.00	114.8k	1452.22	141.0k
10	5	2	✓ 2	(98.49%)	(0.00%)	3.60	2	(2.95%)	(0.00%)	1886.20	178.7k
10	5	2	✓ 3	(2.47%)	(0.00%)	5.59	2	1800.00	84.0k	1050.11	84.0k
10	5	2	✓ 4	(100.00%)	(0.06%)	(0.00%)	(0.06%)	(0.00%)	(0.06%)	314.85	32.4k
10	5	2	✓ 5	(2.16%)	(0.00%)	50.02	2	1800.00	123.3k	1592.68	116.5k
10	5	2	✓ 6	(2.17%)	(0.00%)	1800.07	237.0k	1800.00	268.5k	276.87	30.4k
10	5	2	✓ 7	(86.86%)	(0.02%)	(2.13%)	(0.02%)	(0.00%)	(0.01%)	438.35	41.5k
10	5	2	✓ 8	(16.41%)	(0.00%)	4.87	2	(0.88%)	(0.00%)	(2.95%)	(0.00%)
10	5	2	✓ 9	(30.27%)	(0.00%)	8.35	2	(1.01%)	(0.00%)	1572.82	144.6k
10	5	2	✓ 10	(3.23%)	(0.00%)	5.87	2	(0.02%)	(0.00%)	(0.04%)	(0.00%)
10	5	3	1	1800.00	1388.9k	1800.01	1585.0k	6.01	13	14.88	101
10	5	3	2	(2.38%)	(0.41%)	7.65	27	1029.26	3271	119.62	3845
10	5	3	3	(0.06%)	(0.99%)	8.42	2	1800.00	403.6k	23.26	88
10	5	3	4	(0.51%)	(0.29%)	7.45	2	(0.00%)	(0.24%)	16.20	102
10	5	3	5	3.47	49	4.79	33	4.17	1	8.09	20
10	5	3	6	218.69	77.9k	9.07	3	17.29	27	20.66	128
10	5	3	7	4.26	209	2.37	101	1800.00	584.4k	20.80	80
10	5	3	8	40.60	6537	5.14	229	8.56	27	21.61	181
10	5	3	9	12.24	56	7.36	2	1.65	1	7.67	6
10	5	3	10	44.81	2928	1.72	3	20.23	53	24.47	351
10	5	3	✓ 1	(7.35%)	(0.00%)	8.01	2	1800.00	7572	1145.15	76.7k
10	5	3	✓ 2	(4.65%)	(0.00%)	5.13	3	1800.00	76.8k	1129.06	55.5k
10	5	3	✓ 3	(10.61%)	(0.00%)	11.93	3	(0.05%)	(0.00%)	(0.41%)	(0.00%)
10	5	3	✓ 4	(7.81%)	(0.00%)	8.18	2	1800.00	4412	1296.41	87.7k
10	5	3	✓ 5	(29.71%)	(0.00%)	2.20	3	(0.01%)	(0.00%)	1547.32	69.2k
10	5	3	✓ 6	(22.19%)	(0.00%)	10.30	3	(1.10%)	(0.00%)	(1.32%)	(0.00%)
10	5	3	✓ 7	(22.91%)	(0.00%)	(0.71%)	(0.00%)	1800.00	70.6k	925.15	63.8k
10	5	3	✓ 8	(2.92%)	(0.00%)	62.50	2	1800.00	222.3k	338.44	21.0k
10	5	3	✓ 9	(5.43%)	(0.00%)	7.10	2	1800.00	13.6k	973.95	63.4k
10	5	3	✓ 10	(87.11%)	(0.00%)	11.06	3	(2.61%)	(0.00%)	(0.22%)	(0.00%)
10	5	5	1	(0.00%)	(2.21%)	10.38	2	132.18	299	41.61	431
10	5	5	2	(17.34%)	(6.43%)	2.55	19	1155.52	5897	94.45	1291
10	5	5	3	(23.22%)	(100.00%)	10.46	2	(0.00%)	(1.43%)	150.49	1166
10	5	5	4	(0.12%)	(0.00%)	128.34	3	245.14	195	113.92	1463
10	5	5	5	821.27	69.1k	25.76	249	124.40	183	96.35	1476
10	5	5	6	1.27	15	1.51	27	1.79	9	21.78	12
10	5	5	7	(0.53%)	(0.00%)	22.66	2	19.08	17	41.44	376
10	5	5	8	1800.00	431.2k	9.63	2	12.70	55	33.09	150
10	5	5	9	59.75	689	15.46	2	61.25	75	63.84	476

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
10	5	5	10	(0.00%)	(76.65%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	31.57	64
10	5	5	✓ 1	(2.74%)	(0.00%)	40.72	2	(0.01%)	(0.00%)	1078.23	44.5k
10	5	5	✓ 2	(6.80%)	(0.00%)	41.89	2	(0.01%)	(0.00%)	1110.30	36.8k
10	5	5	✓ 3	(22.35%)	(0.00%)	21.71	2	(1.81%)	(0.00%)	(0.15%)	(0.00%)
10	5	5	✓ 4	(12.19%)	(0.00%)	257.82	3	(0.02%)	(0.00%)	(0.01%)	(0.00%)
10	5	5	✓ 5	(49.81%)	(0.00%)	16.71	3	(1.63%)	(0.00%)	(0.97%)	(0.00%)
10	5	5	✓ 6	(0.33%)	(0.00%)	(0.02%)	(0.00%)	1800.00	103.4k	297.37	12.1k
10	5	5	✓ 7	(21.72%)	(0.00%)	21.15	2	1800.00	19.0k	1097.39	37.4k
10	5	5	✓ 8	(3.51%)	(0.00%)	13.05	2	(0.04%)	(0.00%)	674.63	28.9k
10	5	5	✓ 9	(11.48%)	(0.00%)	13.13	2	(0.18%)	(0.00%)	1801.90	70.1k
10	5	5	✓ 10	(10.87%)	(0.00%)	15.19	3	(0.40%)	(0.00%)	1728.72	74.5k
10	5	10	1	(0.45%)	(0.00%)	97.57	467	150.50	71	163.76	612
10	5	10	2	4.32	121	17.89	1	18.21	17	37.12	97
10	5	10	3	(2.28%)	(6.42%)	(0.00%)	(6.88%)	15.90	61	62.20	28
10	5	10	4	347.40	9552	70.03	275	101.21	29	84.69	242
10	5	10	5	(0.75%)	(1.97%)	(0.56%)	(100.00%)	(0.00%)	(0.93%)	112.33	428
10	5	10	6	(55.44%)	(100.00%)	33.07	2	51.62	275	62.51	204
10	5	10	7	(55.51%)	(100.00%)	(0.00%)	(100.00%)	1153.70	445	295.99	2011
10	5	10	8	4.72	61	17.77	2	17.59	1	78.25	64
10	5	10	9	1.88	16	5.18	2	5.59	1	13.05	2
10	5	10	10	(15.41%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(2.65%)	225.18	1337
10	5	10	✓ 1	(10.29%)	(0.00%)	98.78	3	(0.44%)	(0.00%)	(0.27%)	(0.00%)
10	5	10	✓ 2	(6.72%)	(0.00%)	26.86	3	(0.84%)	(0.00%)	(0.32%)	(0.00%)
10	5	10	✓ 3	(11.44%)	(0.00%)	29.21	2	(1.48%)	(0.00%)	(0.26%)	(0.00%)
10	5	10	✓ 4	(2.34%)	(0.00%)	60.51	3	1800.00	36.4k	1428.87	24.9k
10	5	10	✓ 5	(3.15%)	(0.00%)	189.32	2	1800.00	35.7k	1484.44	35.8k
10	5	10	✓ 6	(11.59%)	(0.00%)	27.31	2	(0.01%)	(0.00%)	1859.13	36.5k
10	5	10	✓ 7	(100.00%)	(0.00%)	12.32	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
10	5	10	✓ 8	(7.00%)	(0.00%)	884.59	3	(0.03%)	(0.00%)	(0.66%)	(0.00%)
10	5	10	✓ 9	(5.16%)	(0.00%)	24.58	2	(0.32%)	(0.00%)	(0.05%)	(0.00%)
10	5	10	✓ 10	(100.00%)	(0.00%)	41.33	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	2	1	1	0.05	1	0.05	1	0.10	1	0.32	0
20	2	1	2	0.05	1	0.05	1	0.05	1	0.12	0
20	2	1	3	0.08	1	0.03	1	0.07	1	0.35	0
20	2	1	4	0.05	1	0.04	1	0.08	1	0.05	0
20	2	1	5	0.03	1	0.03	1	0.05	1	0.03	0
20	2	1	6	0.04	1	0.01	1	0.05	1	0.07	0
20	2	1	7	0.05	1	0.02	1	0.05	1	0.02	0
20	2	1	8	0.07	1	0.03	1	0.05	1	0.07	0
20	2	1	9	0.03	1	0.02	1	0.05	1	0.03	0
20	2	1	10	0.05	1	0.03	1	0.05	1	0.12	0
20	2	1	✓ 1	(0.25%)	(0.00%)	189.76	2611	1800.00	4483.9k	20.91	23.9k
20	2	1	✓ 2	(1.05%)	(0.00%)	(0.50%)	(0.00%)	56.97	5205	12.17	14.6k
20	2	1	✓ 3	(4.87%)	(0.01%)	(0.84%)	(0.01%)	119.08	10.7k	7.96	9326
20	2	1	✓ 4	(1.87%)	(0.00%)	1800.00	64.4k	222.37	22.7k	61.28	72.2k
20	2	1	✓ 5	(14.83%)	(0.08%)	(7.40%)	(0.08%)	674.79	78.8k	88.88	101.3k
20	2	1	✓ 6	(4.66%)	(0.01%)	(3.03%)	(0.01%)	768.26	82.1k	8.51	8358
20	2	1	✓ 7	(2.13%)	(0.03%)	(0.04%)	(0.03%)	77.13	7289	33.86	35.7k
20	2	1	✓ 8	(0.41%)	(0.00%)	(0.33%)	(0.00%)	73.32	6409	4.17	3252
20	2	1	✓ 9	(2.63%)	(0.01%)	(1.65%)	(0.01%)	351.92	39.6k	4.64	4142

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	2	1	✓ 10	(4.03%)	(0.01%)	(1.66%)	(0.01%)	461.73	44.1k	18.26	19.9k
20	2	2	1	0.14	17	0.24	15	0.06	1	0.12	0
20	2	2	2	0.05	1	0.08	1	0.05	1	0.13	0
20	2	2	3	0.28	59	0.45	51	0.08	1	1.14	1
20	2	2	4	0.11	1	0.03	1	0.16	3	0.08	0
20	2	2	5	0.13	11	0.09	1	0.05	1	0.37	2
20	2	2	6	0.13	15	0.30	7	0.06	1	0.19	0
20	2	2	7	0.46	140	0.54	209	0.12	1	0.72	12
20	2	2	8	0.38	107	0.30	102	0.10	1	0.33	6
20	2	2	9	7.96	6316	3.38	4134	0.59	41	0.63	147
20	2	2	10	0.08	1	0.02	1	0.06	1	0.07	0
20	2	2	✓ 1	(2.50%)	(0.01%)	(2.36%)	(0.01%)	169.25	12.3k	50.30	55.1k
20	2	2	✓ 2	(9.16%)	(0.02%)	(2.95%)	(0.02%)	176.58	17.2k	74.33	85.1k
20	2	2	✓ 3	(10.09%)	(0.02%)	(3.59%)	(0.02%)	608.61	70.3k	28.73	32.0k
20	2	2	✓ 4	(3.06%)	(0.01%)	(2.35%)	(0.01%)	1800.00	141.6k	144.71	166.8k
20	2	2	✓ 5	(0.69%)	(0.00%)	1800.00	891.4k	364.93	33.2k	20.57	20.1k
20	2	2	✓ 6	(4.98%)	(0.01%)	(14.35%)	(0.01%)	145.54	10.4k	3.79	2748
20	2	2	✓ 7	(42.61%)	(0.02%)	(39.61%)	(0.02%)	179.70	12.1k	77.03	84.8k
20	2	2	✓ 8	(6.13%)	(0.00%)	(5.36%)	(0.00%)	93.96	4236	57.09	68.8k
20	2	2	✓ 9	(0.53%)	(0.01%)	(0.02%)	(0.01%)	33.62	2325	11.07	10.5k
20	2	2	✓ 10	(2.00%)	(0.04%)	1800.02	701.3k	388.19	31.3k	21.34	22.9k
20	2	3	1	0.08	3	0.20	5	0.09	1	0.55	6
20	2	3	2	0.41	105	0.54	126	0.30	13	0.68	30
20	2	3	3	0.20	17	0.27	23	0.06	1	0.13	0
20	2	3	4	0.31	29	0.40	41	0.06	1	0.23	2
20	2	3	5	0.27	45	0.40	54	0.14	1	0.29	18
20	2	3	6	0.27	34	0.24	7	0.07	1	0.11	2
20	2	3	7	0.35	57	0.46	63	0.05	1	0.12	0
20	2	3	8	0.05	3	0.11	5	0.08	1	0.28	2
20	2	3	9	0.34	61	0.42	101	0.10	1	0.22	4
20	2	3	10	0.09	1	0.05	1	0.08	1	0.19	0
20	2	3	✓ 1	(11.74%)	(0.18%)	(0.37%)	(0.18%)	(0.00%)	(0.17%)	9.16	7568
20	2	3	✓ 2	(1.49%)	(0.00%)	1800.03	170.9k	1800.00	2292.9k	17.02	18.5k
20	2	3	✓ 3	(23.38%)	(0.02%)	(14.73%)	(0.02%)	617.38	37.6k	29.22	27.3k
20	2	3	✓ 4	(0.41%)	(0.01%)	(0.05%)	(0.01%)	113.53	8027	10.36	9094
20	2	3	✓ 5	(1.04%)	(0.01%)	1800.03	512.1k	309.20	20.3k	33.98	35.7k
20	2	3	✓ 6	(1.60%)	(0.00%)	(2.02%)	(0.00%)	88.31	4841	37.17	38.1k
20	2	3	✓ 7	(12.74%)	(0.01%)	(12.23%)	(0.01%)	404.48	26.3k	53.64	51.7k
20	2	3	✓ 8	(6.85%)	(0.01%)	(9.19%)	(0.01%)	(0.02%)	(0.00%)	275.26	325.5k
20	2	3	✓ 9	(60.07%)	(0.01%)	(42.24%)	(0.01%)	77.54	4517	56.34	60.3k
20	2	3	✓ 10	(5.48%)	(0.00%)	(0.27%)	(0.00%)	812.98	65.5k	84.94	101.4k
20	2	5	1	0.50	71	0.77	73	0.08	1	0.17	0
20	2	5	2	0.24	45	0.33	23	0.11	1	0.13	0
20	2	5	3	0.40	47	0.60	48	0.08	1	0.18	2
20	2	5	4	0.53	251	0.94	365	0.13	1	0.40	6
20	2	5	5	0.48	139	0.80	293	0.09	1	0.54	6
20	2	5	6	3.79	1585	5.94	1845	0.19	1	0.40	32
20	2	5	7	0.53	123	0.75	129	0.12	1	0.33	10
20	2	5	8	0.32	69	0.51	70	0.13	1	0.57	30
20	2	5	9	0.38	55	0.41	101	0.08	1	0.76	2

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	2	5	10	0.59	177	0.85	132	0.13	1	0.82	12
20	2	5	✓ 1	(100.00%)	(0.08%)	(100.00%)	(0.08%)	481.95	24.5k	36.46	32.9k
20	2	5	✓ 2	(2.28%)	(0.03%)	(6.77%)	(0.03%)	106.43	5444	19.29	16.1k
20	2	5	✓ 3	(5.85%)	(0.01%)	(4.86%)	(0.01%)	43.60	2481	13.33	11.0k
20	2	5	✓ 4	(3.34%)	(0.00%)	(3.47%)	(0.00%)	424.46	22.4k	66.99	62.8k
20	2	5	✓ 5	(2.23%)	(0.00%)	(0.10%)	(0.00%)	310.75	20.4k	98.11	100.3k
20	2	5	✓ 6	(25.69%)	(0.00%)	(45.45%)	(0.00%)	61.33	5814	15.20	14.3k
20	2	5	✓ 7	(1.32%)	(0.01%)	1800.05	297.2k	184.27	14.2k	11.94	10.4k
20	2	5	✓ 8	(1.27%)	(0.01%)	(1.62%)	(0.01%)	125.51	7065	34.31	31.6k
20	2	5	✓ 9	(3.18%)	(0.00%)	(1.30%)	(0.00%)	141.45	9677	53.59	49.3k
20	2	5	✓ 10	(12.67%)	(0.05%)	(19.84%)	(0.05%)	125.98	8822	15.40	11.5k
20	2	10	1	3.35	2254	13.73	3365	0.36	13	0.72	64
20	2	10	2	2.76	1659	5.86	1320	0.32	7	0.58	85
20	2	10	3	1.10	329	3.25	435	0.08	1	0.41	0
20	2	10	4	1.87	801	5.98	1109	0.22	3	0.69	18
20	2	10	5	1.62	493	3.05	511	0.18	1	0.58	32
20	2	10	6	0.86	691	2.67	1003	0.80	17	1.08	84
20	2	10	7	17.13	3278	24.60	2641	0.08	1	0.71	24
20	2	10	8	1.36	603	3.80	689	0.14	1	0.79	4
20	2	10	9	1.10	447	1.55	334	0.26	1	0.85	64
20	2	10	10	1.19	449	2.06	828	0.24	3	0.70	42
20	2	10	✓ 1	(4.91%)	(0.00%)	(6.52%)	(0.00%)	296.99	15.1k	48.05	39.5k
20	2	10	✓ 2	(0.46%)	(0.01%)	(0.21%)	(0.01%)	47.26	1989	25.45	20.3k
20	2	10	✓ 3	(2.06%)	(0.01%)	(2.32%)	(0.01%)	120.24	6708	16.57	14.5k
20	2	10	✓ 4	(3.16%)	(0.00%)	(6.48%)	(0.00%)	188.08	8272	33.94	26.3k
20	2	10	✓ 5	(6.83%)	(0.00%)	(2.53%)	(0.00%)	288.86	18.5k	120.81	112.5k
20	2	10	✓ 6	(3.93%)	(0.01%)	(4.65%)	(0.01%)	1606.63	85.7k	147.86	134.5k
20	2	10	✓ 7	(8.95%)	(0.00%)	(11.89%)	(0.00%)	189.56	8445	61.88	48.6k
20	2	10	✓ 8	(2.83%)	(0.00%)	(3.41%)	(0.00%)	42.62	1457	77.33	69.4k
20	2	10	✓ 9	(3.04%)	(0.02%)	(1.41%)	(0.02%)	1512.38	76.2k	169.18	139.2k
20	2	10	✓ 10	(2.91%)	(0.00%)	(2.45%)	(0.00%)	128.53	4923	54.71	44.2k
20	3	1	1	0.20	3	0.28	1	0.05	1	0.25	0
20	3	1	2	0.59	25	0.23	7	0.21	1	0.99	19
20	3	1	3	(0.47%)	(0.11%)	1.14	1	1800.00	5176.9k	2.06	106
20	3	1	4	0.73	1	0.02	1	0.04	1	0.11	0
20	3	1	5	(0.16%)	(0.81%)	(0.02%)	(0.81%)	1.32	51	4.38	113
20	3	1	6	7.04	15.6k	1.16	1	(0.00%)	(2.60%)	1.67	24
20	3	1	7	4.04	1	0.74	1	0.07	1	0.24	0
20	3	1	8	5.36	186	0.80	1	0.33	3	4.09	36
20	3	1	9	2.48	3	1.04	3	0.11	1	0.51	2
20	3	1	10	3.47	681	(0.40%)	(0.04%)	0.60	13	(0.55%)	(100.00%)
20	3	1	✓ 1	(66.47%)	(0.00%)	(30.94%)	(0.00%)	(0.02%)	(0.00%)	(1.75%)	(0.00%)
20	3	1	✓ 2	(100.00%)	(0.07%)	8.86	57	996.69	26.3k	(4.34%)	(0.00%)
20	3	1	✓ 3	(18.25%)	(0.00%)	55.39	247	1800.00	37.8k	(0.04%)	(0.00%)
20	3	1	✓ 4	(4.35%)	(0.00%)	(1.37%)	(0.00%)	1800.00	30.9k	1894.87	652.4k
20	3	1	✓ 5	(100.00%)	(100.00%)	1494.16	3333	(39.32%)	(0.00%)	(29.36%)	(0.00%)
20	3	1	✓ 6	(80.21%)	(100.00%)	28.58	127	(0.15%)	(0.00%)	(5.10%)	(0.00%)
20	3	1	✓ 7	(6.36%)	(0.00%)	20.71	65	1800.00	23.0k	(0.42%)	(0.00%)
20	3	1	✓ 8	(100.00%)	(0.00%)	261.53	823	(0.20%)	(0.00%)	(0.32%)	(0.00%)
20	3	1	✓ 9	(45.90%)	(0.00%)	(35.08%)	(0.00%)	1370.77	25.7k	(0.03%)	(0.00%)

instance					SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	3	1	✓	10	(39.11%)	(0.00%)	(21.45%)	(0.00%)	(1.18%)	(0.00%)	(0.96%)	(0.00%)
20	3	2		1	(4.67%)	(0.00%)	0.63	3	2.70	61	7.90	562
20	3	2		2	4.98	287	1800.04	4601.9k	0.93	1	4.07	61
20	3	2		3	(4.40%)	(2.34%)	2.94	2000	0.87	7	6.05	61
20	3	2		4	(0.96%)	(0.20%)	1.31	253	3.90	173	3.45	114
20	3	2		5	(0.74%)	(0.00%)	(0.53%)	(0.00%)	1.20	11	5.78	58
20	3	2		6	(0.21%)	(0.02%)	0.79	1	35.43	1079	16.08	1136
20	3	2		7	(0.00%)	(0.40%)	1.48	41	2.22	37	4.58	261
20	3	2		8	(0.00%)	(0.17%)	2.34	1	2.24	31	9.45	298
20	3	2		9	32.06	3228	6.07	600	4.42	43	11.24	282
20	3	2		10	6.22	2546	3.27	2000	0.44	7	4.91	137
20	3	2	✓	1	(38.78%)	(0.00%)	309.95	495	(5.40%)	(0.00%)	(6.75%)	(0.00%)
20	3	2	✓	2	(16.53%)	(0.00%)	4.73	11	(0.45%)	(0.00%)	(1.63%)	(0.00%)
20	3	2	✓	3	(17.70%)	(0.00%)	39.90	115	(2.97%)	(0.00%)	(3.81%)	(0.00%)
20	3	2	✓	4	(49.76%)	(0.00%)	172.90	341	(1.35%)	(0.00%)	(1.22%)	(0.00%)
20	3	2	✓	5	(100.00%)	(0.00%)	(41.15%)	(0.00%)	(27.04%)	(0.00%)	(21.56%)	(0.00%)
20	3	2	✓	6	(100.00%)	(0.00%)	3.66	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	3	2	✓	7	(12.34%)	(0.00%)	247.86	503	(0.06%)	(0.00%)	(0.12%)	(0.00%)
20	3	2	✓	8	(43.03%)	(0.00%)	4.03	3	(0.83%)	(0.00%)	(2.25%)	(0.00%)
20	3	2	✓	9	(10.38%)	(0.00%)	(6.73%)	(0.00%)	(0.02%)	(0.00%)	(1.33%)	(0.00%)
20	3	2	✓	10	(14.18%)	(0.00%)	10.30	33	(3.22%)	(0.00%)	(5.38%)	(0.00%)
20	3	3		1	(0.00%)	(0.05%)	41.55	2000	2.40	5	8.34	111
20	3	3		2	1800.13	1165.2k	17.07	824	2.86	13	17.71	248
20	3	3		3	387.52	6660	3.35	7	27.48	201	29.00	688
20	3	3		4	(0.00%)	(1.02%)	0.35	1	(0.00%)	(1.68%)	13.97	497
20	3	3		5	(0.04%)	(0.00%)	1.98	3	4.92	41	14.30	695
20	3	3		6	(0.00%)	(0.21%)	0.66	1	29.68	265	9.86	642
20	3	3		7	1.17	79	0.87	67	0.82	1	3.54	16
20	3	3		8	4.83	570	0.43	1	81.97	1697	14.80	1025
20	3	3		9	1.73	164	1.92	1013	1.02	3	(0.00%)	(100.00%)
20	3	3		10	(0.00%)	(3.64%)	8.20	312	(0.00%)	(3.96%)	7.54	138
20	3	3	✓	1	(100.00%)	(5.01%)	86.48	117	1543.42	7828	(36.16%)	(5.01%)
20	3	3	✓	2	(100.00%)	(0.00%)	7.02	15	(51.81%)	(0.00%)	(43.83%)	(0.00%)
20	3	3	✓	3	(100.00%)	(0.00%)	3.20	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	3	3	✓	4	(13.80%)	(0.00%)	9.67	19	(0.22%)	(0.00%)	(3.01%)	(0.00%)
20	3	3	✓	5	(20.51%)	(0.00%)	(18.41%)	(0.00%)	(0.26%)	(0.00%)	(7.78%)	(0.00%)
20	3	3	✓	6	(33.11%)	(0.00%)	193.53	371	(0.23%)	(0.00%)	(5.79%)	(0.00%)
20	3	3	✓	7	(32.91%)	(0.00%)	12.81	23	(1.57%)	(0.00%)	(5.48%)	(0.00%)
20	3	3	✓	8	(34.83%)	(0.00%)	2.67	3	(4.40%)	(0.00%)	(6.16%)	(0.00%)
20	3	3	✓	9	(14.05%)	(0.00%)	32.14	43	(0.33%)	(0.00%)	(0.64%)	(0.00%)
20	3	3	✓	10	(74.04%)	(0.00%)	6.49	23	(22.32%)	(0.00%)	(17.10%)	(0.00%)
20	3	5		1	(0.00%)	(0.72%)	1.31	9	10.94	33	16.43	185
20	3	5		2	(0.00%)	(0.72%)	1.22	3	30.83	67	26.80	449
20	3	5		3	(0.00%)	(0.66%)	25.86	43	5.49	37	14.10	376
20	3	5		4	(0.19%)	(0.83%)	1.68	9	22.63	71	30.00	775
20	3	5		5	(0.83%)	(1.95%)	43.18	1110	37.29	113	37.87	705
20	3	5		6	33.82	2255	11.06	580	5.45	23	14.97	170
20	3	5		7	33.68	2794	9.96	693	4.93	17	16.31	174
20	3	5		8	(0.23%)	(0.07%)	0.80	1	(0.00%)	(0.07%)	7.14	38
20	3	5		9	(0.19%)	(3.89%)	3.43	1	1800.00	309.2k	16.79	262

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	3	5	10	(0.20%)	(0.00%)	1.86	80	(0.03%)	(0.00%)	18.24	136
20	3	5	✓ 1	(100.00%)	(0.00%)	1132.04	653	(31.81%)	(0.00%)	(14.49%)	(0.00%)
20	3	5	✓ 2	(83.21%)	(0.00%)	8.22	7	(1.57%)	(0.00%)	(4.94%)	(0.00%)
20	3	5	✓ 3	(14.85%)	(0.00%)	4.94	3	(0.24%)	(0.00%)	(0.51%)	(0.00%)
20	3	5	✓ 4	(100.00%)	(0.00%)	1130.55	855	(17.39%)	(0.00%)	(13.16%)	(0.00%)
20	3	5	✓ 5	(72.34%)	(0.00%)	3.69	7	(44.72%)	(0.00%)	(40.74%)	(0.00%)
20	3	5	✓ 6	(48.36%)	(0.00%)	(29.11%)	(0.00%)	(10.87%)	(0.00%)	(12.60%)	(0.00%)
20	3	5	✓ 7	(34.40%)	(0.00%)	100.21	53	(6.76%)	(0.00%)	(5.04%)	(0.00%)
20	3	5	✓ 8	(40.03%)	(0.00%)	322.51	221	(1.27%)	(0.00%)	(5.75%)	(0.00%)
20	3	5	✓ 9	(100.00%)	(0.00%)	12.99	21	(21.25%)	(0.00%)	(44.27%)	(0.00%)
20	3	5	✓ 10	(8.68%)	(0.00%)	6.89	9	(0.09%)	(0.00%)	(1.37%)	(0.00%)
20	3	10	1	(27.76%)	(0.00%)	25.43	139	666.92	633	382.25	8589
20	3	10	2	(4.71%)	(5.12%)	112.03	2000	(0.00%)	(1.98%)	79.75	1118
20	3	10	3	(6.06%)	(0.00%)	6.24	35	326.02	341	172.75	4310
20	3	10	4	(0.33%)	(0.09%)	28.07	209	(0.00%)	(0.45%)	62.17	386
20	3	10	5	(100.00%)	(0.00%)	2.29	3	(100.00%)	(0.00%)	(70.82%)	(0.00%)
20	3	10	6	1800.27	594.0k	1.31	3	32.98	53	63.18	315
20	3	10	7	(100.00%)	(100.00%)	29.53	2000	(0.00%)	(0.63%)	167.17	3428
20	3	10	8	(51.77%)	(7.33%)	8.30	21	412.87	481	232.87	3961
20	3	10	9	1800.30	166.0k	513.78	5340	43.91	25	49.02	469
20	3	10	10	(0.44%)	(0.00%)	197.56	3080	27.89	71	64.34	714
20	3	10	✓ 1	(45.76%)	(0.00%)	5.24	3	(11.35%)	(0.00%)	(11.11%)	(0.00%)
20	3	10	✓ 2	(54.63%)	(0.00%)	13.30	3	(7.71%)	(0.00%)	(11.69%)	(0.00%)
20	3	10	✓ 3	(18.77%)	(0.00%)	1660.73	409	(1.60%)	(0.00%)	(4.23%)	(0.00%)
20	3	10	✓ 4	(44.57%)	(0.00%)	82.62	23	(17.72%)	(0.00%)	(18.94%)	(0.00%)
20	3	10	✓ 5	(34.64%)	(0.00%)	8.48	3	(12.65%)	(0.00%)	(12.74%)	(0.00%)
20	3	10	✓ 6	(31.08%)	(0.00%)	10.13	5	(18.62%)	(0.00%)	(13.16%)	(0.00%)
20	3	10	✓ 7	(87.07%)	(0.00%)	3.40	3	(44.37%)	(0.00%)	(37.14%)	(0.00%)
20	3	10	✓ 8	(39.87%)	(0.00%)	22.81	9	(19.44%)	(0.00%)	(12.78%)	(0.00%)
20	3	10	✓ 9	(20.34%)	(0.00%)	14.28	3	(1.64%)	(0.00%)	(6.71%)	(0.00%)
20	3	10	✓ 10	(100.00%)	(0.00%)	1163.32	335	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	1	1	(8.35%)	(0.00%)	128.93	1	0.95	7	3.42	40
20	4	1	2	7.62	130	2.08	12	1.12	1	5.91	32
20	4	1	3	(0.02%)	(0.13%)	1800.16	3631.5k	12.14	2107	10.09	431
20	4	1	4	1800.00	1461.6k	3.04	2	5.93	83	11.69	454
20	4	1	5	(0.01%)	(0.00%)	3.83	2	14.10	285	35.56	2835
20	4	1	6	(0.09%)	(0.00%)	8.23	151	2.21	33	6.07	274
20	4	1	7	16.80	29	1.65	2	0.15	1	1.24	2
20	4	1	8	1.29	180	2.18	2	1.10	1	3.54	37
20	4	1	9	(0.39%)	(0.10%)	2.84	2	1.36	11	5.93	19
20	4	1	10	(0.53%)	(0.26%)	2.79	3735	(0.00%)	(0.60%)	3.20	12
20	4	1	✓ 1	(100.00%)	(0.00%)	7.46	2	(39.48%)	(0.00%)	(58.33%)	(0.00%)
20	4	1	✓ 2	(20.42%)	(0.00%)	10.80	2	(0.49%)	(0.00%)	(10.98%)	(0.00%)
20	4	1	✓ 3	(100.00%)	(0.00%)	5.89	3	(87.42%)	(0.00%)	(82.98%)	(0.00%)
20	4	1	✓ 4	(22.81%)	(0.00%)	14.58	2	(3.28%)	(0.00%)	(11.19%)	(0.00%)
20	4	1	✓ 5	(19.84%)	(0.00%)	5.77	2	(13.01%)	(0.00%)	(9.11%)	(0.00%)
20	4	1	✓ 6	(100.00%)	(0.00%)	6.94	2	(43.68%)	(0.00%)	(29.88%)	(0.00%)
20	4	1	✓ 7	(15.08%)	(0.00%)	5.91	2	(6.36%)	(0.00%)	(6.57%)	(0.00%)
20	4	1	✓ 8	(92.47%)	(0.00%)	7.22	2	(28.78%)	(0.00%)	(25.58%)	(0.00%)
20	4	1	✓ 9	(100.00%)	(0.00%)	10.67	2	(95.21%)	(0.00%)	(100.00%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	4	1	✓ 10	(48.95%)	(0.00%)	6.67	2	(3.67%)	(0.00%)	(23.19%)	(0.00%)
20	4	2	1	7.82	1482	45.58	1	14.42	61	16.12	376
20	4	2	2	(0.30%)	(0.00%)	3.19	105	5.14	7	13.54	44
20	4	2	3	(0.00%)	(0.41%)	7.17	2	1800.00	392.1k	20.76	74
20	4	2	4	(0.48%)	(0.89%)	20.95	2	7.95	23	44.19	374
20	4	2	5	(0.11%)	(0.03%)	(0.00%)	(19.69%)	15.45	463	21.03	1211
20	4	2	6	(0.36%)	(0.28%)	9.15	3	25.95	149	21.73	225
20	4	2	7	(0.25%)	(0.58%)	5.14	2	1800.00	643.6k	40.81	142
20	4	2	8	(0.00%)	(0.20%)	6.22	2	1800.00	609.7k	61.15	923
20	4	2	9	1800.21	744.6k	5.30	2	1800.00	254.0k	21.13	81
20	4	2	10	(0.17%)	(0.00%)	3.00	2	(0.00%)	(1.18%)	18.45	150
20	4	2	✓ 1	(85.88%)	(0.00%)	0.38	1	(43.16%)	(0.00%)	(41.07%)	(0.00%)
20	4	2	✓ 2	(58.39%)	(0.00%)	11.60	2	(36.63%)	(0.00%)	(27.76%)	(0.00%)
20	4	2	✓ 3	(100.00%)	(0.00%)	15.56	2	(76.55%)	(0.00%)	(100.00%)	(0.00%)
20	4	2	✓ 4	(93.87%)	(0.00%)	9.43	2	(73.83%)	(0.00%)	(75.51%)	(0.00%)
20	4	2	✓ 5	(100.00%)	(0.00%)	16.96	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	2	✓ 6	(100.00%)	(0.00%)	15.96	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	2	✓ 7	(100.00%)	(0.00%)	28.71	2	(62.45%)	(0.00%)	(54.70%)	(0.00%)
20	4	2	✓ 8	(100.00%)	(0.00%)	6.69	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	2	✓ 9	(100.00%)	(0.00%)	23.89	2	(93.34%)	(0.00%)	(81.91%)	(0.00%)
20	4	2	✓ 10	(53.91%)	(0.00%)	30.19	2	(20.20%)	(0.00%)	(22.74%)	(0.00%)
20	4	3	1	296.75	48.6k	11.53	2	42.07	123	55.05	718
20	4	3	2	(2.25%)	(100.00%)	11.67	2	26.94	47	76.73	1114
20	4	3	3	1800.56	411.9k	19.07	45	13.35	17	45.67	216
20	4	3	4	(1.24%)	(5.76%)	9.40	2	112.05	235	66.17	378
20	4	3	5	(54.27%)	(100.00%)	927.51	72.8k	(0.00%)	(11.85%)	189.66	3923
20	4	3	6	(0.14%)	(0.05%)	39.86	2	17.11	23	47.79	228
20	4	3	7	(0.45%)	(1.74%)	(0.00%)	(100.00%)	(0.00%)	(0.07%)	92.56	2705
20	4	3	8	28.23	8165	0.80	2	12.58	15	23.36	388
20	4	3	9	(0.07%)	(0.19%)	1.47	3	(0.00%)	(0.46%)	48.04	624
20	4	3	10	1800.38	922.8k	8.38	2	31.17	83	46.56	610
20	4	3	✓ 1	(88.83%)	(0.00%)	87.58	2	(66.69%)	(0.00%)	(61.33%)	(0.00%)
20	4	3	✓ 2	(100.00%)	(0.00%)	47.35	2	(79.42%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 3	(86.11%)	(0.00%)	109.37	2	(60.02%)	(0.00%)	(69.39%)	(0.00%)
20	4	3	✓ 4	(100.00%)	(0.00%)	70.42	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 5	(59.84%)	(0.00%)	74.22	2	(31.15%)	(0.00%)	(38.86%)	(0.00%)
20	4	3	✓ 6	(100.00%)	(100.00%)	24.41	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 7	(100.00%)	(0.00%)	52.82	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 8	(100.00%)	(0.00%)	3.68	2	(70.89%)	(0.00%)	(66.47%)	(0.00%)
20	4	3	✓ 9	(100.00%)	(0.00%)	23.32	2	(31.92%)	(0.00%)	(72.23%)	(0.00%)
20	4	3	✓ 10	(52.02%)	(0.00%)	26.21	2	(33.97%)	(0.00%)	(30.27%)	(0.00%)
20	4	5	1	(6.31%)	(0.00%)	13.85	37	(0.25%)	(0.00%)	910.48	28.5k
20	4	5	2	(100.00%)	(100.00%)	(9.28%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(8.74%)
20	4	5	3	(87.30%)	(5.79%)	173.21	31.1k	(74.40%)	(2.44%)	1030.16	16.0k
20	4	5	4	(1.69%)	(100.00%)	21.98	3	(0.00%)	(4.48%)	82.82	88
20	4	5	5	(5.25%)	(6.12%)	38.24	2	(0.00%)	(4.26%)	105.00	3121
20	4	5	6	(58.68%)	(0.00%)	5.07	13	1588.58	3983	(32.17%)	(0.00%)
20	4	5	7	(2.46%)	(100.00%)	1800.63	339.1k	68.75	33	107.05	316
20	4	5	8	(36.71%)	(100.00%)	(0.00%)	(100.00%)	(22.47%)	(0.02%)	(0.10%)	(0.00%)
20	4	5	9	(9.99%)	(100.00%)	13.58	2	(0.00%)	(2.78%)	300.64	3669

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
20	4	5	10	(30.49%)	(100.00%)	<i>abort</i>		(13.22%)	(1.36%)	1650.50	32.1k
20	4	5	✓ 1	(100.00%)	(0.00%)	14.02	3	(100.00%)	(0.00%)	(89.26%)	(0.00%)
20	4	5	✓ 2	(100.00%)	(0.00%)	23.71	2	(81.80%)	(0.00%)	(89.43%)	(0.00%)
20	4	5	✓ 3	(60.59%)	(0.00%)	82.86	2	(46.50%)	(0.00%)	(29.77%)	(0.00%)
20	4	5	✓ 4	(100.00%)	(0.00%)	96.08	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 5	(32.50%)	(100.00%)	24.62	2	(7.59%)	(0.00%)	(6.48%)	(0.00%)
20	4	5	✓ 6	(100.00%)	(0.00%)	5.11	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 7	(100.00%)	(0.00%)	49.14	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 8	(100.00%)	(0.00%)	5.65	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 9	(100.00%)	(0.00%)	71.64	2	(91.19%)	(0.00%)	(63.74%)	(0.00%)
20	4	5	✓ 10	(100.00%)	(0.00%)	9.17	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	1	(100.00%)	(100.00%)	<i>abort</i>		(59.99%)	(0.61%)	1815.12	9672
20	4	10	2	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(71.43%)	(1.87%)	(5.58%)	(0.00%)
20	4	10	3	(76.26%)	(0.00%)	33.07	59	1799.86	391	627.95	3151
20	4	10	4	(84.13%)	(0.00%)	(9.28%)	(0.00%)	(87.84%)	(0.00%)	(33.01%)	(0.00%)
20	4	10	5	(100.00%)	(36.60%)	40.37	2	(100.00%)	(0.00%)	(100.00%)	(0.80%)
20	4	10	6	(36.46%)	(100.00%)	(0.00%)	(90.15%)	311.14	87	396.40	1416
20	4	10	7	(13.37%)	(100.00%)	(0.00%)	(100.00%)	(6.34%)	(0.95%)	1693.26	19.4k
20	4	10	8	(78.07%)	(0.00%)	6.85	2	(10.68%)	(0.00%)	1511.98	11.8k
20	4	10	9	(24.87%)	(1.17%)	770.40	685	(25.19%)	(0.00%)	(9.01%)	(0.00%)
20	4	10	10	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)
20	4	10	✓ 1	(100.00%)	(0.00%)	1325.51	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 2	(80.78%)	(0.00%)	142.26	2	(73.20%)	(0.00%)	(51.13%)	(0.00%)
20	4	10	✓ 3	(89.03%)	(0.00%)	62.71	2	(70.97%)	(0.00%)	(54.01%)	(0.00%)
20	4	10	✓ 4	(100.00%)	(0.00%)	247.89	2	(93.60%)	(0.00%)	(65.72%)	(0.00%)
20	4	10	✓ 5	(96.34%)	(0.00%)	193.35	3	(63.61%)	(0.00%)	(31.06%)	(0.00%)
20	4	10	✓ 6	(57.48%)	(0.00%)	638.21	3	(42.88%)	(0.00%)	(36.67%)	(0.00%)
20	4	10	✓ 7	(100.00%)	(0.00%)	251.86	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 8	(57.08%)	(0.00%)	403.47	2	(34.78%)	(0.00%)	(41.03%)	(0.00%)
20	4	10	✓ 9	(100.00%)	(0.00%)	209.07	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 10	(100.00%)	(0.00%)	17.50	3	(100.00%)	(0.00%)	(74.24%)	(0.00%)
20	5	1	1	(1.04%)	(0.38%)	4.74	1	(0.00%)	(0.42%)	15.98	252
20	5	1	2	(0.01%)	(0.00%)	3.37	2	5.09	39	(0.23%)	(100.00%)
20	5	1	3	5.82	2563	3.98	2	1.97	5	7.45	42
20	5	1	4	(0.00%)	(0.28%)	5.13	2	10.32	57	18.22	206
20	5	1	5	39.67	3130	22.33	2	7.84	85	20.41	315
20	5	1	6	(0.00%)	(0.90%)	1.54	2	1.63	1	(0.00%)	(100.00%)
20	5	1	7	23.95	409	7.32	1	(0.00%)	(0.23%)	15.02	38
20	5	1	8	(0.74%)	(0.22%)	8.45	105	1800.00	350.6k	35.34	763
20	5	1	9	(0.23%)	(0.03%)	262.21	156.4k	1800.00	761.0k	24.68	452
20	5	1	10	6.80	325	64.33	2	2.85	7	5.37	24
20	5	1	✓ 1	(100.00%)	(0.00%)	14.25	2	(67.23%)	(0.00%)	(53.63%)	(0.00%)
20	5	1	✓ 2	(71.51%)	(0.00%)	14.36	2	(3.91%)	(0.00%)	(14.59%)	(0.00%)
20	5	1	✓ 3	(83.64%)	(0.00%)	10.16	2	(36.29%)	(0.00%)	(29.17%)	(0.00%)
20	5	1	✓ 4	(48.90%)	(0.00%)	8.83	2	(21.20%)	(0.00%)	(20.87%)	(0.00%)
20	5	1	✓ 5	(100.00%)	(0.00%)	6.78	2	(77.46%)	(0.00%)	(74.93%)	(0.00%)
20	5	1	✓ 6	(50.86%)	(0.00%)	9.66	2	(0.30%)	(0.00%)	(12.80%)	(0.00%)
20	5	1	✓ 7	(43.74%)	(0.00%)	12.14	2	(33.13%)	(0.00%)	(25.88%)	(0.00%)
20	5	1	✓ 8	(100.00%)	(0.00%)	7.40	2	(100.00%)	(0.00%)	(76.09%)	(0.00%)
20	5	1	✓ 9	(100.00%)	(0.00%)	124.03	2	(100.00%)	(0.00%)	(64.74%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
20	5	1	✓ 10	(100.00%)	(0.02%)	13.51	2	(100.00%)	(0.02%)	(100.00%)	(0.00%)
20	5	2	1	(14.04%)	(100.00%)	226.10	2	(0.00%)	(0.20%)	187.09	1040
20	5	2	2	(0.10%)	(4.35%)	(5.02%)	(6.62%)	(0.00%)	(6.61%)	62.68	1551
20	5	2	3	(1.12%)	(0.00%)	56.93	2	160.25	173	80.16	888
20	5	2	4	(0.15%)	(0.10%)	24.47	2	23.07	13	57.45	206
20	5	2	5	(0.94%)	(0.00%)	22.95	2	(0.00%)	(9.17%)	53.19	417
20	5	2	6	(43.56%)	(100.00%)	(0.00%)	(55.50%)	(2.31%)	(0.74%)	537.88	8539
20	5	2	7	(1.10%)	(0.27%)	196.18	2	467.22	591	293.45	4296
20	5	2	8	(2.84%)	(0.71%)	55.66	2	1800.00	14.2k	105.77	1380
20	5	2	9	(3.05%)	(100.00%)	5.05	2	1800.00	194.2k	147.39	3058
20	5	2	10	(0.11%)	(0.00%)	(0.00%)	(100.00%)	1800.00	252.3k	48.22	658
20	5	2	✓ 1	(100.00%)	(0.00%)	46.75	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓ 2	(33.12%)	(0.00%)	15.80	2	(19.50%)	(0.00%)	(20.79%)	(0.00%)
20	5	2	✓ 3	(90.77%)	(0.00%)	36.25	2	(83.58%)	(0.00%)	(45.29%)	(0.00%)
20	5	2	✓ 4	(100.00%)	(0.00%)	152.16	2	(43.73%)	(0.00%)	(34.03%)	(0.00%)
20	5	2	✓ 5	(33.98%)	(0.00%)	61.31	2	(17.90%)	(0.00%)	(20.51%)	(0.00%)
20	5	2	✓ 6	(100.00%)	(0.00%)	3.31	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓ 7	(100.00%)	(0.00%)	174.37	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓ 8	(100.00%)	(0.00%)	29.49	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓ 9	(100.00%)	(0.00%)	83.80	2	(83.27%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓ 10	(100.00%)	(0.00%)	25.58	2	(90.92%)	(0.00%)	(100.00%)	(0.00%)
20	5	3	1	(33.61%)	(100.00%)	9.96	7	(8.76%)	(0.69%)	833.59	12.6k
20	5	3	2	2.90	126	22.17	2	1800.00	290.1k	74.71	14
20	5	3	3	(0.00%)	(0.07%)	16.62	2	53.49	53	124.14	262
20	5	3	4	(0.02%)	(1.24%)	3.70	2	(0.00%)	(1.24%)	176.90	846
20	5	3	5	(5.48%)	(100.00%)	17.19	2	(0.00%)	(6.14%)	219.72	5719
20	5	3	6	(9.55%)	(100.00%)	(0.00%)	(100.00%)	1272.83	1160	335.88	2874
20	5	3	7	175.98	3100	1.94	1	202.32	203	192.63	1970
20	5	3	8	(2.16%)	(100.00%)	(0.00%)	(17.69%)	(0.00%)	(0.80%)	311.49	3613
20	5	3	9	(44.28%)	(100.00%)	504.49	21.9k	(0.00%)	(1.36%)	363.41	4251
20	5	3	10	(17.79%)	(2.34%)	18.31	2	(15.94%)	(2.34%)	496.18	7815
20	5	3	✓ 1	(100.00%)	(0.00%)	1554.67	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	3	✓ 2	(18.30%)	(0.00%)	48.61	2	(5.25%)	(0.00%)	(3.69%)	(0.00%)
20	5	3	✓ 3	(35.68%)	(0.00%)	28.24	2	(21.99%)	(0.00%)	(15.76%)	(0.00%)
20	5	3	✓ 4	(100.00%)	(0.00%)	78.70	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	3	✓ 5	(45.21%)	(0.00%)	37.24	2	(42.05%)	(0.00%)	(28.91%)	(0.00%)
20	5	3	✓ 6	(100.00%)	(0.00%)	264.27	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	3	✓ 7	(77.47%)	(0.00%)	100.32	2	(49.92%)	(0.00%)	(53.06%)	(0.00%)
20	5	3	✓ 8	(55.96%)	(0.00%)	95.44	2	(48.73%)	(0.00%)	(33.02%)	(0.00%)
20	5	3	✓ 9	(60.65%)	(0.00%)	37.81	2	(43.15%)	(0.00%)	(37.01%)	(0.00%)
20	5	3	✓ 10	(66.74%)	(0.00%)	87.58	2	(61.94%)	(0.00%)	(40.51%)	(0.00%)
20	5	5	1	(100.00%)	(0.00%)	1563.64	57	(70.80%)	(0.00%)	(59.27%)	(0.00%)
20	5	5	2	(26.18%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(2.17%)	438.83	3377
20	5	5	3	(13.00%)	(18.41%)	59.44	3	(0.00%)	(9.62%)	254.32	318
20	5	5	4	(100.00%)	(100.00%)	17.39	13	(100.00%)	(0.70%)	(14.41%)	(0.00%)
20	5	5	5	(0.00%)	(0.06%)	62.43	221	(0.00%)	(0.18%)	137.82	28
20	5	5	6	(41.54%)	(100.00%)	387.42	17.6k	(7.28%)	(3.67%)	1499.44	15.6k
20	5	5	7	(15.67%)	(100.00%)	906.43	244	(8.40%)	(0.54%)	467.00	3123
20	5	5	8	(2.74%)	(0.00%)	598.88	2	1521.93	715	569.22	4445
20	5	5	9	(100.00%)	(100.00%)	(10.44%)	(100.00%)	(58.66%)	(0.00%)	(14.15%)	(0.11%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	5	5	10	(100.00%)	(69.39%)	39.36	2	(100.00%)	(1.64%)	(100.00%)	(0.00%)
20	5	5	✓ 1	(100.00%)	(0.00%)	57.21	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	5	✓ 2	(67.88%)	(0.00%)	1636.39	2	(53.23%)	(0.00%)	(42.66%)	(0.00%)
20	5	5	✓ 3	(54.89%)	(0.00%)	52.12	3	(38.79%)	(0.00%)	(26.82%)	(0.00%)
20	5	5	✓ 4	(100.00%)	(0.00%)	8.09	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	5	✓ 5	(44.29%)	(0.00%)	219.28	2	(30.03%)	(0.00%)	(24.11%)	(0.00%)
20	5	5	✓ 6	(100.00%)	(0.00%)	11.74	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	5	✓ 7	(100.00%)	(0.00%)	53.80	3	(100.00%)	(0.00%)	(83.18%)	(0.00%)
20	5	5	✓ 8	(100.00%)	(0.00%)	9.67	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	5	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(80.07%)	(0.00%)	(54.06%)	(0.00%)
20	5	5	✓ 10	(100.00%)	(0.00%)	173.30	2	(98.14%)	(0.00%)	(79.82%)	(0.00%)
20	5	10	1	(20.15%)	(4.96%)	(0.00%)	(4.96%)	(28.73%)	(5.54%)	1414.12	5571
20	5	10	2	(28.26%)	(0.00%)	19.74	2	(32.73%)	(0.00%)	(6.52%)	(0.00%)
20	5	10	3	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(26.15%)	(20.32%)	800.02	2783
20	5	10	4	(100.00%)	(48.93%)	20.07	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	10	5	(100.00%)	(0.00%)	37.31	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	10	6	(39.05%)	(0.00%)	69.55	33	(60.18%)	(0.00%)	(9.95%)	(0.00%)
20	5	10	7	(67.44%)	(0.00%)	14.87	2	(76.20%)	(0.00%)	(13.85%)	(0.00%)
20	5	10	8	(100.00%)	(100.00%)	(0.35%)	(100.00%)	(97.43%)	(0.00%)	(32.63%)	(1.52%)
20	5	10	9	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(5.28%)
20	5	10	10	(100.00%)	(100.00%)	67.43	2	1104.71	69	720.33	3529
20	5	10	✓ 1	(59.23%)	(0.00%)	1328.84	2	(58.84%)	(0.00%)	(34.76%)	(0.00%)
20	5	10	✓ 2	(47.17%)	(0.00%)	887.32	2	(44.38%)	(0.00%)	(22.63%)	(0.00%)
20	5	10	✓ 3	(88.78%)	(0.27%)	1174.45	3	(89.01%)	(0.00%)	(58.45%)	(0.00%)
20	5	10	✓ 4	(100.00%)	(0.00%)	378.40	3	(100.00%)	(0.00%)	(89.35%)	(0.00%)
20	5	10	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	10	✓ 6	(54.82%)	(0.00%)	(100.00%)	(0.00%)	(59.75%)	(0.00%)	(33.98%)	(0.00%)
20	5	10	✓ 7	(57.90%)	(0.00%)	(100.00%)	(0.00%)	(62.13%)	(0.00%)	(32.80%)	(0.00%)
20	5	10	✓ 8	(82.28%)	(0.00%)	614.53	3	(90.84%)	(0.00%)	(57.73%)	(0.33%)
20	5	10	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	10	✓ 10	(87.22%)	(0.00%)	133.14	2	(67.88%)	(0.00%)	(42.74%)	(0.00%)
30	2	1	1	0.08	3	0.01	1	0.08	1	0.27	2
30	2	1	2	0.07	3	0.03	1	0.08	1	(0.00%)	(100.00%)
30	2	1	3	0.04	1	0.04	1	0.07	1	0.52	0
30	2	1	4	8.66	23.3k	0.03	1	1800.00	3580.3k	2.97	1119
30	2	1	5	0.06	1	0.02	1	0.06	1	0.02	0
30	2	1	6	0.09	3	0.03	1	0.11	1	1.10	2
30	2	1	7	0.08	1	0.03	1	0.05	1	0.11	0
30	2	1	8	0.07	1	0.02	1	0.08	1	0.12	0
30	2	1	9	0.34	15	0.03	1	0.14	7	0.66	2
30	2	1	10	0.08	1	0.06	1	0.08	1	0.15	0
30	2	1	✓ 1	(13.82%)	(0.03%)	(8.85%)	(0.03%)	1800.00	85.6k	736.55	587.2k
30	2	1	✓ 2	(23.55%)	(0.02%)	(16.30%)	(0.02%)	(0.23%)	(0.02%)	(0.05%)	(0.00%)
30	2	1	✓ 3	(17.32%)	(0.04%)	(6.92%)	(0.04%)	1800.00	89.9k	447.52	355.1k
30	2	1	✓ 4	(100.00%)	(1.87%)	(100.00%)	(1.87%)	(0.00%)	(1.86%)	1380.10	1078.4k
30	2	1	✓ 5	(100.00%)	(0.33%)	(100.00%)	(0.33%)	(0.00%)	(0.32%)	190.04	154.2k
30	2	1	✓ 6	(13.54%)	(0.00%)	(2.91%)	(0.00%)	1800.00	74.1k	254.28	246.3k
30	2	1	✓ 7	(37.96%)	(0.08%)	(10.74%)	(0.08%)	(0.00%)	(0.08%)	344.53	226.7k
30	2	1	✓ 8	(17.09%)	(0.01%)	(10.25%)	(0.01%)	(0.01%)	(0.01%)	(0.15%)	(0.00%)
30	2	1	✓ 9	(9.72%)	(0.02%)	(4.01%)	(0.02%)	1800.00	75.7k	248.25	216.3k

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
30	2	1	✓ 10	(8.02%)	(0.01%)	(1.47%)	(0.01%)	1800.00	69.7k	1333.96	1078.4k
30	2	2	1	0.34	51	0.37	21	0.93	614	0.24	0
30	2	2	2	1.26	684	1.04	537	0.91	58	1.01	126
30	2	2	3	0.17	11	0.09	2	0.17	1	(0.00%)	(100.00%)
30	2	2	4	0.09	1	0.05	1	0.09	1	0.56	0
30	2	2	5	0.28	41	0.39	58	0.11	1	0.61	6
30	2	2	6	1.40	361	0.71	201	0.49	17	1.17	6
30	2	2	7	1.02	247	0.59	128	0.16	1	0.72	14
30	2	2	8	0.10	1	0.07	1	0.05	1	0.12	0
30	2	2	9	0.37	55	0.43	51	0.11	1	0.49	14
30	2	2	10	0.64	161	0.52	103	0.18	1	0.89	26
30	2	2	✓ 1	(95.46%)	(0.23%)	(66.23%)	(0.23%)	(0.00%)	(0.23%)	(0.07%)	(0.00%)
30	2	2	✓ 2	(25.15%)	(0.02%)	(21.35%)	(0.02%)	(0.17%)	(0.02%)	1834.81	1512.4k
30	2	2	✓ 3	(4.91%)	(0.04%)	(3.58%)	(0.04%)	(0.14%)	(0.04%)	926.04	626.1k
30	2	2	✓ 4	(39.16%)	(0.06%)	(37.57%)	(0.06%)	(17.72%)	(0.06%)	(0.86%)	(0.00%)
30	2	2	✓ 5	(7.67%)	(0.01%)	(5.78%)	(0.01%)	(0.84%)	(0.00%)	(0.02%)	(0.00%)
30	2	2	✓ 6	(21.21%)	(0.00%)	(17.64%)	(0.00%)	(2.81%)	(0.00%)	(0.53%)	(0.00%)
30	2	2	✓ 7	(35.69%)	(0.01%)	(25.57%)	(0.01%)	(0.04%)	(0.01%)	(0.14%)	(0.00%)
30	2	2	✓ 8	(10.42%)	(0.01%)	(3.57%)	(0.01%)	(0.02%)	(0.01%)	(0.04%)	(0.00%)
30	2	2	✓ 9	(26.34%)	(0.01%)	(27.94%)	(0.01%)	(0.10%)	(0.01%)	(0.15%)	(0.00%)
30	2	2	✓ 10	(6.27%)	(0.00%)	(4.13%)	(0.00%)	(0.01%)	(0.00%)	537.35	419.0k
30	2	3	1	0.36	81	0.60	23	0.17	1	0.64	29
30	2	3	2	0.42	169	0.49	195	0.18	1	0.47	4
30	2	3	3	0.99	371	0.97	315	0.19	1	0.78	30
30	2	3	4	1800.03	11.0M	0.88	165	0.41	9	0.99	71
30	2	3	5	0.83	305	1.11	481	0.10	1	0.84	5
30	2	3	6	0.92	343	1.35	423	0.09	1	0.18	0
30	2	3	7	0.47	123	0.52	125	0.15	3	1.13	8
30	2	3	8	0.74	161	0.70	167	0.07	1	0.18	0
30	2	3	9	1.04	365	0.54	93	0.34	3	1.33	22
30	2	3	10	0.35	51	0.52	24	0.10	1	0.11	0
30	2	3	✓ 1	(10.56%)	(0.00%)	(9.90%)	(0.00%)	1800.00	50.4k	(0.03%)	(0.00%)
30	2	3	✓ 2	(30.87%)	(0.03%)	(28.36%)	(0.03%)	(0.00%)	(0.03%)	490.66	348.2k
30	2	3	✓ 3	(100.00%)	(0.01%)	(100.00%)	(0.01%)	(3.44%)	(0.01%)	(0.46%)	(0.00%)
30	2	3	✓ 4	(5.07%)	(0.01%)	(1.63%)	(0.01%)	(0.03%)	(0.01%)	(0.02%)	(0.00%)
30	2	3	✓ 5	(21.71%)	(0.01%)	(19.26%)	(0.01%)	1800.00	48.3k	559.89	426.1k
30	2	3	✓ 6	(100.00%)	(0.05%)	(100.00%)	(0.05%)	(0.00%)	(0.04%)	(0.82%)	(0.00%)
30	2	3	✓ 7	(4.61%)	(0.01%)	(3.30%)	(0.01%)	1800.00	49.8k	597.23	401.9k
30	2	3	✓ 8	(18.05%)	(0.01%)	(20.44%)	(0.01%)	1800.00	41.4k	912.08	604.6k
30	2	3	✓ 9	(17.26%)	(0.03%)	(16.17%)	(0.03%)	(1.63%)	(0.03%)	(0.15%)	(0.00%)
30	2	3	✓ 10	(48.14%)	(0.04%)	(31.37%)	(0.04%)	1800.00	52.2k	496.09	358.7k
30	2	5	1	34.05	22.3k	4.02	1083	0.21	1	0.83	12
30	2	5	2	18.01	3011	10.22	1928	0.27	1	0.86	4
30	2	5	3	0.62	63	0.84	69	0.08	1	0.26	0
30	2	5	4	84.51	11.6k	39.75	14.8k	0.69	27	0.95	80
30	2	5	5	62.74	11.4k	29.37	6627	0.15	1	0.85	12
30	2	5	6	8.62	1589	3.99	768	0.54	9	1.21	43
30	2	5	7	2.38	713	2.94	695	0.11	1	0.37	4
30	2	5	8	246.26	72.7k	403.17	89.8k	1800.00	4570.6k	2.65	829
30	2	5	9	0.85	255	1.04	229	0.10	1	0.71	4

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	2	5	10	2.24	826	(0.00%)	(0.77%)	0.30	5	0.98	90
30	2	5	✓ 1	(26.27%)	(0.01%)	(25.37%)	(0.01%)	(0.03%)	(0.01%)	1647.12	1189.3k
30	2	5	✓ 2	(45.83%)	(0.00%)	(38.67%)	(0.00%)	(0.37%)	(0.00%)	(0.43%)	(0.00%)
30	2	5	✓ 3	(26.98%)	(0.00%)	(24.79%)	(0.00%)	(0.03%)	(0.00%)	(0.06%)	(0.00%)
30	2	5	✓ 4	(47.63%)	(0.00%)	(52.75%)	(0.00%)	(0.60%)	(0.00%)	(0.49%)	(0.00%)
30	2	5	✓ 5	(15.04%)	(0.01%)	(10.39%)	(0.01%)	1800.00	48.4k	(0.09%)	(0.00%)
30	2	5	✓ 6	(100.00%)	(0.00%)	(92.49%)	(0.00%)	(0.28%)	(0.00%)	(1.09%)	(0.00%)
30	2	5	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(4.20%)	(0.00%)	(0.49%)	(0.00%)
30	2	5	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(1.07%)	(0.00%)	(2.95%)	(0.00%)
30	2	5	✓ 9	(11.52%)	(0.01%)	(8.58%)	(0.01%)	(0.03%)	(0.01%)	(0.17%)	(0.00%)
30	2	5	✓ 10	(17.89%)	(0.01%)	(10.95%)	(0.01%)	(0.00%)	(0.01%)	(0.06%)	(0.00%)
30	2	10	1	2.14	1265	5.70	1501	0.27	3	0.85	14
30	2	10	2	117.60	14.8k	94.80	12.4k	1.11	33	1.49	154
30	2	10	3	1800.02	5619.9k	112.72	37.7k	1.16	31	1.97	120
30	2	10	4	60.71	6491	107.86	6990	0.74	9	1.43	90
30	2	10	5	38.68	10.4k	85.70	11.8k	0.49	13	1.07	36
30	2	10	6	4.33	945	10.42	2389	0.24	1	1.15	22
30	2	10	7	3.75	8412	5.58	2178	0.41	13	1.39	40
30	2	10	8	1800.01	3791.4k	186.17	25.2k	0.91	33	1.72	370
30	2	10	9	2.34	789	5.41	1448	0.23	1	0.71	4
30	2	10	10	108.25	22.2k	130.53	18.9k	0.99	23	1.66	82
30	2	10	✓ 1	(7.34%)	(0.00%)	(6.49%)	(0.00%)	(0.03%)	(0.00%)	(0.04%)	(0.00%)
30	2	10	✓ 2	(29.22%)	(0.00%)	(29.93%)	(0.00%)	1800.00	37.8k	(0.20%)	(0.00%)
30	2	10	✓ 3	(22.26%)	(0.00%)	(23.43%)	(0.00%)	1800.00	33.3k	(0.10%)	(0.00%)
30	2	10	✓ 4	(13.69%)	(0.00%)	(10.96%)	(0.00%)	(0.02%)	(0.00%)	(0.29%)	(0.00%)
30	2	10	✓ 5	(41.99%)	(0.01%)	(50.83%)	(0.01%)	(0.01%)	(0.01%)	831.72	492.5k
30	2	10	✓ 6	(13.63%)	(0.00%)	(14.53%)	(0.00%)	1800.00	31.3k	(0.23%)	(0.00%)
30	2	10	✓ 7	(5.76%)	(0.01%)	(3.75%)	(0.01%)	1800.00	39.1k	(0.11%)	(0.00%)
30	2	10	✓ 8	(13.29%)	(0.00%)	(13.52%)	(0.00%)	1800.00	37.4k	(0.01%)	(0.00%)
30	2	10	✓ 9	(13.25%)	(0.02%)	(12.04%)	(0.02%)	(0.00%)	(0.02%)	1762.27	1007.1k
30	2	10	✓ 10	(48.91%)	(0.00%)	(49.27%)	(0.00%)	(0.04%)	(0.00%)	(0.67%)	(0.00%)
30	3	1	1	6.02	277	12.79	2000	0.59	7	5.69	84
30	3	1	2	(0.00%)	(0.15%)	0.73	1	1800.00	2518.2k	3.79	92
30	3	1	3	6.41	29	1.13	16	0.25	1	(0.00%)	(100.00%)
30	3	1	4	1800.00	4435.6k	(0.02%)	(0.00%)	1100.54	631.5k	2.84	12
30	3	1	5	(0.15%)	(0.00%)	(0.31%)	(0.52%)	2.75	149	4.19	74
30	3	1	6	1.77	7	0.91	7	0.21	1	2.40	8
30	3	1	7	7.07	196	1.50	161	1.34	35	4.83	46
30	3	1	8	0.81	1	0.97	5	0.15	1	0.88	2
30	3	1	9	3.06	126	1.07	38	0.40	1	1.68	10
30	3	1	10	(0.17%)	(0.22%)	2.09	1000	1.82	31	5.51	481
30	3	1	✓ 1	(100.00%)	(0.00%)	448.37	297	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	1	✓ 2	(50.12%)	(0.00%)	275.24	239	(31.45%)	(0.00%)	(16.98%)	(0.00%)
30	3	1	✓ 3	(31.95%)	(0.00%)	619.66	367	(17.74%)	(0.00%)	(8.53%)	(0.00%)
30	3	1	✓ 4	(100.00%)	(0.00%)	(74.85%)	(0.00%)	(64.46%)	(0.00%)	(41.13%)	(0.00%)
30	3	1	✓ 5	(62.71%)	(0.00%)	(44.06%)	(0.00%)	(29.92%)	(0.00%)	(11.96%)	(0.00%)
30	3	1	✓ 6	(15.98%)	(0.00%)	(10.53%)	(0.00%)	(0.79%)	(0.00%)	(2.85%)	(0.00%)
30	3	1	✓ 7	(100.00%)	(0.00%)	(93.25%)	(0.00%)	(68.65%)	(0.00%)	(27.92%)	(0.00%)
30	3	1	✓ 8	(59.31%)	(0.00%)	(37.57%)	(0.00%)	(30.61%)	(0.00%)	(6.75%)	(0.00%)
30	3	1	✓ 9	(22.92%)	(0.00%)	14.82	13	(4.73%)	(0.00%)	(8.88%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	3	1	✓ 10	(15.72%)	(0.00%)	(11.31%)	(0.00%)	(5.69%)	(0.00%)	(3.60%)	(0.00%)
30	3	2	1	(0.88%)	(4.44%)	3.09	11	16.01	287	11.58	161
30	3	2	2	22.31	4608	(0.00%)	(1.28%)	1.88	11	8.58	143
30	3	2	3	(0.00%)	(5.21%)	(0.00%)	(4.56%)	(0.00%)	(4.56%)	14.49	338
30	3	2	4	17.02	2842	1.42	1	1800.00	876.8k	(0.00%)	(100.00%)
30	3	2	5	1800.29	2522.2k	1800.03	3454.5k	1.81	35	11.23	290
30	3	2	6	28.54	2525	7.27	1212	1800.00	226.5k	13.83	485
30	3	2	7	1800.29	5330.0k	1.55	1	0.89	1	5.59	60
30	3	2	8	(0.09%)	(0.13%)	(0.00%)	(0.10%)	1800.00	1876.5k	7.71	239
30	3	2	9	(0.00%)	(0.03%)	(0.01%)	(0.30%)	13.87	137	24.59	1397
30	3	2	10	1800.00	424.8k	4.82	141	310.11	4531	147.45	20.3k
30	3	2	✓ 1	(49.63%)	(0.00%)	82.02	67	(28.73%)	(0.00%)	(20.35%)	(0.00%)
30	3	2	✓ 2	(100.00%)	(0.00%)	809.04	361	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	2	✓ 3	(100.00%)	(0.00%)	1403.12	515	(54.98%)	(0.00%)	(41.43%)	(0.00%)
30	3	2	✓ 4	(35.94%)	(0.00%)	13.24	7	(20.58%)	(0.00%)	(11.59%)	(0.00%)
30	3	2	✓ 5	(100.00%)	(0.00%)	910.24	405	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	2	✓ 6	(100.00%)	(0.01%)	(100.00%)	(0.01%)	(100.00%)	(0.01%)	(100.00%)	(0.00%)
30	3	2	✓ 7	(71.20%)	(0.00%)	913.03	287	(23.77%)	(0.00%)	(12.36%)	(0.00%)
30	3	2	✓ 8	(100.00%)	(0.00%)	53.58	19	(65.51%)	(0.00%)	(53.25%)	(0.00%)
30	3	2	✓ 9	(48.89%)	(0.00%)	67.96	27	(29.24%)	(0.00%)	(16.25%)	(0.00%)
30	3	2	✓ 10	(100.00%)	(0.00%)	59.27	19	(75.16%)	(0.00%)	(67.69%)	(0.00%)
30	3	3	1	157.54	16.6k	5.27	77	8.75	55	39.83	787
30	3	3	2	(0.07%)	(0.45%)	3.94	1	11.79	67	28.92	633
30	3	3	3	(0.04%)	(0.00%)	2.35	27	(0.00%)	(1.20%)	20.11	604
30	3	3	4	(0.00%)	(0.61%)	(0.02%)	(0.00%)	4.76	35	13.22	117
30	3	3	5	105.42	8243	0.41	1	28.98	183	24.61	512
30	3	3	6	(0.09%)	(0.22%)	(0.00%)	(3.93%)	(0.00%)	(0.02%)	17.89	62
30	3	3	7	(0.35%)	(100.00%)	94.50	5340	85.23	533	21.13	573
30	3	3	8	2.96	755	0.65	7	2.22	7	14.39	58
30	3	3	9	(0.38%)	(0.81%)	1.88	1	8.08	45	22.41	723
30	3	3	10	1800.00	2167.0k	8.99	1168	19.66	243	18.96	534
30	3	3	✓ 1	(86.65%)	(0.00%)	544.82	191	(48.56%)	(0.00%)	(32.19%)	(0.00%)
30	3	3	✓ 2	(100.00%)	(0.00%)	62.66	23	(93.88%)	(0.00%)	(75.30%)	(0.00%)
30	3	3	✓ 3	(100.00%)	(0.00%)	149.02	35	(77.31%)	(0.00%)	(51.29%)	(0.00%)
30	3	3	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 5	(100.00%)	(0.00%)	20.46	7	(52.80%)	(0.00%)	(40.09%)	(0.00%)
30	3	3	✓ 6	(100.00%)	(0.00%)	(81.14%)	(0.00%)	(71.22%)	(0.00%)	(39.81%)	(0.00%)
30	3	3	✓ 7	(100.00%)	(0.00%)	22.43	19	(85.81%)	(0.00%)	(68.90%)	(0.00%)
30	3	3	✓ 8	(80.03%)	(0.00%)	(44.15%)	(0.00%)	(58.08%)	(0.00%)	(41.57%)	(0.00%)
30	3	3	✓ 9	(100.00%)	(0.00%)	551.05	147	(60.64%)	(0.00%)	(78.33%)	(0.00%)
30	3	3	✓ 10	(38.53%)	(0.00%)	(13.34%)	(0.00%)	(19.25%)	(0.00%)	(13.92%)	(0.00%)
30	3	5	1	1800.26	1173.3k	38.85	4580	4.82	7	23.75	94
30	3	5	2	1800.27	1822.8k	40.91	5340	19.16	87	23.55	135
30	3	5	3	1800.24	351.3k	12.81	582	23.09	41	34.59	312
30	3	5	4	(2.87%)	(0.12%)	5.54	45	(0.05%)	(0.12%)	479.81	20.4k
30	3	5	5	(6.26%)	(100.00%)	263.36	2000	(0.00%)	(2.61%)	(0.16%)	(0.00%)
30	3	5	6	(8.68%)	(100.00%)	8.11	1	828.02	1533	267.45	8063
30	3	5	7	(16.31%)	(100.00%)	543.77	2000	(0.00%)	(0.38%)	161.43	2683
30	3	5	8	(3.39%)	(0.00%)	35.71	245	61.21	47	72.66	582
30	3	5	9	(1.47%)	(2.12%)	8.67	11	52.72	153	88.88	1071

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
30	3	5	10	(1.97%)	(100.00%)	101.09	2000	(0.00%)	(0.99%)	256.55	11.2k
30	3	5	✓ 1	(33.45%)	(0.00%)	47.74	11	(19.06%)	(0.00%)	(13.24%)	(0.00%)
30	3	5	✓ 2	(60.04%)	(0.00%)	36.83	11	(39.07%)	(0.00%)	(28.67%)	(0.00%)
30	3	5	✓ 3	(80.80%)	(0.00%)	45.29	9	(49.76%)	(0.00%)	(33.91%)	(0.00%)
30	3	5	✓ 4	(69.63%)	(0.00%)	20.19	7	(43.62%)	(0.00%)	(37.15%)	(0.00%)
30	3	5	✓ 5	(38.60%)	(0.00%)	50.05	15	(21.36%)	(0.00%)	(32.35%)	(0.04%)
30	3	5	✓ 6	(100.00%)	(0.00%)	14.80	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 7	(100.00%)	(0.00%)	36.25	11	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 8	(96.92%)	(0.00%)	429.87	79	(58.04%)	(0.00%)	(32.73%)	(0.00%)
30	3	5	✓ 9	(43.22%)	(0.00%)	79.78	13	(24.60%)	(0.00%)	(21.05%)	(0.00%)
30	3	5	✓ 10	(100.00%)	(0.00%)	11.33	3	(91.76%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	1	(66.31%)	(4.34%)	1275.23	3060	(3.81%)	(4.34%)	737.41	10.7k
30	3	10	2	(0.59%)	(9.53%)	280.63	4327	(0.00%)	(0.32%)	170.68	676
30	3	10	3	(10.97%)	(100.00%)	223.94	1460	(6.62%)	(0.05%)	(0.55%)	(0.00%)
30	3	10	4	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(51.34%)	(8.91%)	(35.95%)	(0.00%)
30	3	10	5	(65.21%)	(100.00%)	374.30	2000	(0.00%)	(2.15%)	418.01	6492
30	3	10	6	(33.22%)	(100.00%)	318.34	2000	(0.00%)	(5.16%)	335.45	3149
30	3	10	7	(22.86%)	(100.00%)	1475.68	1300	(0.00%)	(0.43%)	410.20	4981
30	3	10	8	(100.00%)	(100.00%)	1574.88	3840	(12.51%)	(11.75%)	722.29	13.1k
30	3	10	9	(78.23%)	(100.00%)	740.45	2000	(20.34%)	(4.42%)	970.26	17.2k
30	3	10	10	(0.27%)	(84.78%)	5.47	7	(0.00%)	(12.89%)	103.93	333
30	3	10	✓ 1	(89.87%)	(0.00%)	233.94	25	(60.72%)	(0.00%)	(46.85%)	(0.00%)
30	3	10	✓ 2	(100.00%)	(0.00%)	26.18	5	(100.00%)	(0.00%)	(56.45%)	(0.00%)
30	3	10	✓ 3	(88.69%)	(0.00%)	31.47	5	(64.28%)	(0.00%)	(38.57%)	(0.00%)
30	3	10	✓ 4	(100.00%)	(0.00%)	37.86	5	(71.76%)	(0.00%)	(62.65%)	(0.00%)
30	3	10	✓ 5	(58.21%)	(0.00%)	35.27	5	(38.83%)	(0.00%)	(29.66%)	(0.00%)
30	3	10	✓ 6	(98.61%)	(0.00%)	45.80	7	(71.15%)	(0.00%)	(48.60%)	(0.00%)
30	3	10	✓ 7	(100.00%)	(0.00%)	171.47	23	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 8	(68.17%)	(0.00%)	30.99	3	(47.80%)	(0.00%)	(29.67%)	(0.00%)
30	3	10	✓ 9	(83.84%)	(0.00%)	17.64	3	(71.01%)	(0.00%)	(37.98%)	(0.00%)
30	3	10	✓ 10	(67.52%)	(0.00%)	19.06	3	(32.72%)	(0.00%)	(29.34%)	(0.00%)
30	4	1	1	(0.10%)	(0.00%)	4.15	2	7.06	58	23.55	208
30	4	1	2	(0.12%)	(0.39%)	4.38	9	13.97	183	18.86	176
30	4	1	3	133.13	92.5k	3.16	10	10.57	87	18.45	427
30	4	1	4	(1.07%)	(0.00%)	1.26	2	2.78	27	7.55	107
30	4	1	5	22.37	843	4.84	3	6.89	69	26.09	392
30	4	1	6	(0.29%)	(0.00%)	4.56	2	(0.00%)	(2.20%)	(0.00%)	(100.00%)
30	4	1	7	3.45	91	1.91	1	0.92	9	4.30	6
30	4	1	8	(0.07%)	(0.00%)	2.34	2	1.80	19	6.31	36
30	4	1	9	(0.05%)	(0.00%)	16.04	2	3.27	21	8.76	122
30	4	1	10	8.11	166	6.55	1	1.35	9	8.93	42
30	4	1	✓ 1	(36.64%)	(0.00%)	23.76	2	(29.34%)	(0.00%)	(24.33%)	(0.00%)
30	4	1	✓ 2	(100.00%)	(0.00%)	15.53	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	1	✓ 3	(100.00%)	(0.00%)	44.72	2	(68.06%)	(0.00%)	(70.59%)	(0.00%)
30	4	1	✓ 4	(75.73%)	(0.00%)	12.80	2	(54.93%)	(0.00%)	(42.26%)	(0.00%)
30	4	1	✓ 5	(100.00%)	(0.00%)	9.65	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	1	✓ 6	(100.00%)	(0.00%)	17.12	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	1	✓ 7	(71.86%)	(0.00%)	13.27	2	(39.97%)	(0.00%)	(33.26%)	(0.00%)
30	4	1	✓ 8	(33.32%)	(0.00%)	28.25	2	(11.64%)	(0.00%)	(37.08%)	(0.00%)
30	4	1	✓ 9	(100.00%)	(0.00%)	28.84	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
30	4	1	✓ 10	(45.12%)	(0.00%)	23.27	2	(36.32%)	(0.00%)	(39.27%)	(0.00%)
30	4	2	1	1800.00	752.4k	1800.22	1879.5k	1800.00	220.0k	25.15	42
30	4	2	2	(0.08%)	(0.02%)	6.42	2	8.77	273	16.59	74
30	4	2	3	(0.03%)	(0.00%)	48.98	2	(0.00%)	(0.41%)	60.03	1079
30	4	2	4	(0.05%)	(3.04%)	3.33	2	1800.00	96.1k	49.57	371
30	4	2	5	(0.03%)	(0.01%)	7.35	2	1800.00	533.0k	70.01	562
30	4	2	6	894.05	9111	14.92	51	70.30	151	84.61	1623
30	4	2	7	(1.56%)	(100.00%)	35.61	2	313.26	709	114.61	2363
30	4	2	8	(0.16%)	(100.00%)	241.35	2	(0.00%)	(0.33%)	45.84	1179
30	4	2	9	(0.13%)	(0.00%)	3.46	43	29.04	53	55.79	277
30	4	2	10	(100.00%)	(0.00%)	24.24	2	(98.57%)	(0.00%)	1004.24	31.2k
30	4	2	✓ 1	(100.00%)	(0.00%)	38.76	2	(97.91%)	(0.00%)	(100.00%)	(0.00%)
30	4	2	✓ 2	(100.00%)	(0.00%)	33.50	2	(79.20%)	(0.00%)	(95.99%)	(0.00%)
30	4	2	✓ 3	(100.00%)	(0.00%)	50.26	2	(84.68%)	(0.00%)	(96.41%)	(0.00%)
30	4	2	✓ 4	(93.01%)	(0.00%)	45.08	2	(56.65%)	(0.00%)	(66.88%)	(0.00%)
30	4	2	✓ 5	(62.56%)	(0.00%)	73.88	2	(40.79%)	(0.00%)	(36.74%)	(0.00%)
30	4	2	✓ 6	(100.00%)	(0.00%)	23.71	2	(77.63%)	(0.00%)	(74.83%)	(0.00%)
30	4	2	✓ 7	(100.00%)	(0.00%)	279.30	2	(100.00%)	(0.00%)	(95.56%)	(0.00%)
30	4	2	✓ 8	(100.00%)	(0.00%)	28.24	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	2	✓ 9	(100.00%)	(0.00%)	115.77	2	(91.18%)	(0.00%)	(82.42%)	(0.00%)
30	4	2	✓ 10	(66.89%)	(0.00%)	91.75	2	(37.02%)	(0.00%)	(47.97%)	(0.00%)
30	4	3	1	(1.92%)	(100.00%)	24.13	2	(0.00%)	(0.77%)	249.44	4491
30	4	3	2	(11.09%)	(100.00%)	(0.00%)	(100.00%)	1375.61	1121	632.89	8449
30	4	3	3	(7.24%)	(100.00%)	(0.00%)	(0.29%)	1800.00	252.7k	513.87	7526
30	4	3	4	(0.01%)	(0.05%)	15.67	2	220.34	447	106.92	1104
30	4	3	5	1800.40	511.4k	19.22	2	41.68	143	57.57	151
30	4	3	6	(2.35%)	(100.00%)	(0.28%)	(100.00%)	(0.00%)	(0.03%)	311.09	4181
30	4	3	7	(3.38%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(2.82%)	412.14	15.2k
30	4	3	8	(4.66%)	(100.00%)	4.66	1	(0.00%)	(1.13%)	236.35	1637
30	4	3	9	(7.52%)	(100.00%)	1100.55	4571	(6.32%)	(0.01%)	(0.18%)	(0.00%)
30	4	3	10	(37.09%)	(3.32%)	(0.00%)	(100.00%)	(17.68%)	(0.65%)	(12.86%)	(0.00%)
30	4	3	✓ 1	(100.00%)	(0.00%)	40.68	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 2	(100.00%)	(0.00%)	340.30	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 3	(100.00%)	(0.00%)	1122.05	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 4	(100.00%)	(0.00%)	37.71	1	(69.76%)	(0.00%)	(67.54%)	(0.00%)
30	4	3	✓ 5	(95.09%)	(0.00%)	106.66	1	(71.60%)	(0.00%)	(58.45%)	(0.00%)
30	4	3	✓ 6	(100.00%)	(0.00%)	1491.12	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 7	(100.00%)	(0.00%)	81.26	2	(92.94%)	(0.00%)	(78.96%)	(0.00%)
30	4	3	✓ 8	(100.00%)	(0.00%)	45.11	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 9	(100.00%)	(0.00%)	301.09	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	1	(8.56%)	(0.12%)	(0.00%)	(100.00%)	(4.36%)	(0.14%)	(0.41%)	(0.00%)
30	4	5	2	(9.04%)	(0.05%)	4.06	2	(4.60%)	(0.05%)	(0.20%)	(0.00%)
30	4	5	3	(14.65%)	(100.00%)	31.74	3	(2.95%)	(0.59%)	974.23	9548
30	4	5	4	(100.00%)	(100.00%)	<i>abort</i>		(88.40%)	(3.11%)	(41.40%)	(0.00%)
30	4	5	5	(27.90%)	(100.00%)	36.03	2	(0.00%)	(0.22%)	548.17	3318
30	4	5	6	(25.89%)	(0.98%)	(2.88%)	(0.94%)	(19.08%)	(0.10%)	(12.07%)	(0.00%)
30	4	5	7	(14.66%)	(100.00%)	(0.00%)	(100.00%)	1096.97	365	1018.96	4424
30	4	5	8	(35.84%)	(1.56%)	95.75	169	(26.70%)	(1.56%)	(9.29%)	(0.00%)
30	4	5	9	(25.63%)	(100.00%)	42.66	2	(15.95%)	(4.33%)	(2.26%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	4	5	10	(20.80%)	(0.23%)	3.45	3	(23.09%)	(0.23%)	(7.86%)	(0.00%)
30	4	5	✓ 1	(100.00%)	(0.00%)	80.88	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 2	(100.00%)	(0.01%)	549.72	2	(100.00%)	(0.00%)	(100.00%)	(0.01%)
30	4	5	✓ 3	(100.00%)	(0.00%)	57.45	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 4	(100.00%)	(0.00%)	86.14	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 5	(100.00%)	(0.00%)	352.92	2	(80.93%)	(0.00%)	(91.97%)	(0.00%)
30	4	5	✓ 6	(100.00%)	(0.00%)	129.02	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 7	(100.00%)	(0.00%)	833.05	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 8	(100.00%)	(0.00%)	20.11	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 9	(100.00%)	(0.00%)	49.60	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 10	(100.00%)	(0.00%)	940.74	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	1	(79.10%)	(100.00%)	44.40	11	(60.83%)	(0.00%)	(52.56%)	(0.71%)
30	4	10	2	(100.00%)	(100.00%)	98.56	2	(100.00%)	(6.37%)	1779.47	3236
30	4	10	3	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(1.97%)
30	4	10	4	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(5.12%)	(88.24%)	(0.00%)
30	4	10	5	(100.00%)	(0.00%)	10.46	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	6	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(10.24%)	(18.36%)	(0.00%)
30	4	10	7	(100.00%)	(1.50%)	(100.00%)	(2.44%)	(100.00%)	(0.00%)	(100.00%)	(2.44%)
30	4	10	8	(26.05%)	(0.00%)	(12.43%)	(0.00%)	(29.95%)	(0.00%)	(23.39%)	(0.00%)
30	4	10	9	(72.47%)	(2.54%)	(25.74%)	(100.00%)	(65.37%)	(0.00%)	(59.54%)	(0.44%)
30	4	10	10	(100.00%)	(100.00%)	145.02	209	(100.00%)	(0.00%)	(37.08%)	(0.00%)
30	4	10	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 2	(100.00%)	(0.00%)	732.13	2	(68.52%)	(0.00%)	(73.06%)	(0.00%)
30	4	10	✓ 3	(100.00%)	(0.00%)	19.58	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(92.14%)	(0.00%)	(78.14%)	(0.00%)
30	4	10	✓ 6	(100.00%)	(0.00%)	62.04	1	(78.04%)	(0.00%)	(75.12%)	(0.00%)
30	4	10	✓ 7	(100.00%)	(0.00%)	97.39	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 8	(100.00%)	(0.00%)	1216.55	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 9	(100.00%)	(0.00%)	602.71	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 10	(93.91%)	(0.00%)	(100.00%)	(0.00%)	(61.63%)	(0.00%)	(63.19%)	(0.00%)
30	5	1	1	(0.00%)	(0.11%)	8.82	1	1800.00	400.1k	44.51	221
30	5	1	2	1800.56	560.9k	4.90	2	48.75	147	80.83	1749
30	5	1	3	(0.59%)	(0.34%)	6.24	2	937.23	5581	416.40	37.1k
30	5	1	4	1646.75	8091	42.72	3715	235.70	955	208.57	6902
30	5	1	5	(0.31%)	(0.68%)	9.29	2	1800.00	400.0k	29.16	160
30	5	1	6	(2.07%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(0.38%)	199.00	10.4k
30	5	1	7	(0.13%)	(0.00%)	1800.21	2530.4k	1800.00	372.5k	22.36	40
30	5	1	8	1800.00	898.0k	33.73	931	42.14	149	108.48	1332
30	5	1	9	(0.06%)	(0.78%)	7.59	2	(0.00%)	(1.46%)	22.59	62
30	5	1	10	(0.05%)	(0.41%)	8.26	2	1800.00	821.2k	20.79	26
30	5	1	✓ 1	(100.00%)	(0.01%)	18.11	2	(100.00%)	(0.01%)	(100.00%)	(0.00%)
30	5	1	✓ 2	(100.00%)	(0.00%)	26.40	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 3	(100.00%)	(0.00%)	26.82	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 4	(100.00%)	(0.00%)	16.37	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 5	(100.00%)	(0.00%)	22.45	2	(99.20%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 6	(100.00%)	(0.00%)	42.61	2	(69.04%)	(0.00%)	(85.05%)	(0.00%)
30	5	1	✓ 7	(100.00%)	(0.00%)	24.65	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 8	(100.00%)	(0.00%)	31.00	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓ 9	(100.00%)	(0.00%)	40.73	2	(65.47%)	(0.00%)	(61.97%)	(0.00%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	5	1	✓ 10	(49.44%)	(0.00%)	28.34	2	(33.97%)	(0.00%)	(29.37%)	(0.00%)
30	5	2	1	(59.17%)	(5.14%)	(0.00%)	(100.00%)	(29.72%)	(0.00%)	(22.73%)	(0.50%)
30	5	2	2	(3.53%)	(100.00%)	170.99	1	1800.02	2194	317.26	3769
30	5	2	3	(10.03%)	(100.00%)	1174.37	2	1637.12	1471	501.53	4622
30	5	2	4	(14.48%)	(100.00%)	37.85	2	(0.00%)	(0.43%)	412.23	2618
30	5	2	5	(20.81%)	(0.07%)	174.88	2	(14.11%)	(0.08%)	(3.10%)	(0.00%)
30	5	2	6	(10.57%)	(4.06%)	28.58	23	1800.00	219.3k	212.38	1862
30	5	2	7	(0.23%)	(0.60%)	23.43	1	80.24	55	146.92	549
30	5	2	8	(21.35%)	(100.00%)	103.03	3	(10.91%)	(0.00%)	(2.17%)	(0.00%)
30	5	2	9	(41.84%)	(100.00%)	(0.00%)	(100.00%)	(18.52%)	(0.28%)	(4.96%)	(0.00%)
30	5	2	10	(15.80%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(0.92%)	679.44	7329
30	5	2	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 2	(100.00%)	(0.00%)	539.03	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 3	(100.00%)	(0.00%)	1312.74	2	(92.63%)	(0.00%)	(86.12%)	(0.00%)
30	5	2	✓ 4	(100.00%)	(0.00%)	89.70	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 5	(100.00%)	(0.00%)	181.72	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 6	(100.00%)	(0.00%)	41.68	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 7	(100.00%)	(0.01%)	63.84	2	(100.00%)	(0.00%)	(100.00%)	(0.01%)
30	5	2	✓ 8	(100.00%)	(0.00%)	253.66	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 9	(100.00%)	(0.00%)	164.14	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓ 10	(100.00%)	(0.00%)	48.10	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	1	(1.68%)	(1.00%)	66.05	2	1044.56	201	310.12	2714
30	5	3	2	(12.48%)	(100.00%)	(0.17%)	(100.00%)	(0.00%)	(0.19%)	689.80	4822
30	5	3	3	(20.18%)	(100.00%)	(0.00%)	(100.00%)	(17.24%)	(2.11%)	691.65	2303
30	5	3	4	(100.00%)	(1.03%)	(8.52%)	(1.03%)	(78.82%)	(0.00%)	(41.91%)	(0.88%)
30	5	3	5	(19.06%)	(14.84%)	37.50	67	1781.61	427	776.50	3524
30	5	3	6	(79.56%)	(100.00%)	1800.00	146.3k	(61.87%)	(0.07%)	(21.08%)	(0.00%)
30	5	3	7	(11.42%)	(0.00%)	(100.00%)	(0.00%)	(7.44%)	(0.00%)	1807.98	28.0k
30	5	3	8	(37.51%)	(100.00%)	44.16	2	(19.70%)	(0.09%)	(5.85%)	(0.00%)
30	5	3	9	(52.61%)	(100.00%)	949.47	2	(32.01%)	(1.92%)	1003.83	6041
30	5	3	10	(80.83%)	(100.00%)	9.42	3	(43.39%)	(0.13%)	(30.42%)	(3.98%)
30	5	3	✓ 1	(100.00%)	(0.00%)	827.59	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 3	(100.00%)	(0.00%)	682.63	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 4	(100.00%)	(0.00%)	1340.66	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 5	(100.00%)	(0.00%)	583.95	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 6	(100.00%)	(0.00%)	1295.24	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 7	(100.00%)	(0.00%)	1701.35	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 8	(100.00%)	(0.00%)	1115.35	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 9	(100.00%)	(0.00%)	233.09	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 10	(100.00%)	(0.00%)	75.73	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	1	(53.03%)	(100.00%)	(0.00%)	(100.00%)	(37.24%)	(0.00%)	(25.71%)	(0.36%)
30	5	5	2	(100.00%)	(17.32%)	(0.00%)	(17.32%)	(100.00%)	(17.32%)	(63.09%)	(0.00%)
30	5	5	3	(100.00%)	(100.00%)	(32.26%)	(0.00%)	(81.09%)	(0.00%)	(92.02%)	(1.92%)
30	5	5	4	(100.00%)	(100.00%)	56.38	2	(100.00%)	(0.45%)	(100.00%)	(0.00%)
30	5	5	5	(40.93%)	(0.63%)	(0.00%)	(100.00%)	(29.30%)	(0.00%)	(22.75%)	(0.34%)
30	5	5	6	(40.15%)	(100.00%)	(0.00%)	(100.00%)	(34.86%)	(0.92%)	(18.00%)	(0.00%)
30	5	5	7	(27.18%)	(0.00%)	(5.37%)	(0.00%)	(22.14%)	(0.00%)	(14.76%)	(0.00%)
30	5	5	8	(100.00%)	(0.00%)	(100.00%)	(0.24%)	(100.00%)	(0.00%)	(100.00%)	(13.39%)
30	5	5	9	(69.93%)	(100.00%)	(0.00%)	(100.00%)	(54.68%)	(0.00%)	(33.47%)	(1.44%)

instance				SCIP		SCIP(bivar)		BARON		COUENNE	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)	(dgap)	(pgap)
30	5	5	10	(54.58%)	(100.00%)	<i>abort</i>		(34.43%)	(0.00%)	(24.10%)	(1.27%)
30	5	5	✓ 1	(100.00%)	(0.00%)	111.47	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 2	(73.94%)	(0.00%)	1327.09	2	(54.32%)	(0.00%)	(48.95%)	(0.00%)
30	5	5	✓ 3	(100.00%)	(0.00%)	228.95	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 4	(100.00%)	(0.00%)	577.69	2	(87.80%)	(0.00%)	(75.98%)	(0.00%)
30	5	5	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 6	(100.00%)	(0.00%)	212.46	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 7	(100.00%)	(0.00%)	825.47	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 8	(100.00%)	(0.00%)	372.13	1	(97.91%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.12%)
30	5	5	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	1	(54.12%)	(100.00%)	429.27	3	(54.71%)	(0.00%)	(35.02%)	(0.30%)
30	5	10	2	(100.00%)	(11.46%)	351.74	161	(100.00%)	(0.29%)	(100.00%)	(1.93%)
30	5	10	3	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(5.04%)
30	5	10	4	(100.00%)	(0.00%)	101.37	2	(100.00%)	(0.00%)	(86.00%)	(0.00%)
30	5	10	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(18.91%)
30	5	10	6	(100.00%)	(0.00%)	109.79	39	(100.00%)	(0.12%)	(100.00%)	(0.00%)
30	5	10	7	(100.00%)	(0.01%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)
30	5	10	8	(47.04%)	(100.00%)	(0.00%)	(100.00%)	(61.64%)	(0.12%)	(13.79%)	(0.00%)
30	5	10	9	(100.00%)	(0.86%)	86.94	27	(100.00%)	(0.00%)	(100.00%)	(2.44%)
30	5	10	10	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(100.00%)	(0.00%)	(100.00%)	(44.27%)
30	5	10	✓ 1	(79.92%)	(0.00%)	348.07	2	(70.20%)	(0.00%)	(62.52%)	(0.00%)
30	5	10	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(92.91%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 4	(96.07%)	(0.00%)	702.60	2	(65.82%)	(0.00%)	(69.14%)	(0.00%)
30	5	10	✓ 5	(100.00%)	(0.00%)	166.84	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 6	(100.00%)	(0.00%)	517.04	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)	(91.67%)	(0.00%)	(93.30%)	(0.00%)
30	5	10	✓ 8	(100.00%)	(0.00%)	191.85	3	(100.00%)	(0.00%)	(98.32%)	(0.00%)
30	5	10	✓ 9	(100.00%)	(0.00%)	189.62	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 10	(100.00%)	(0.00%)	400.72	3	(100.00%)	(0.00%)	(100.00%)	(0.48%)

A.2. Results for instances with integral exponents

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	#	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)
10	2	1	1	0.04	1	0.02	1
10	2	1	2	0.03	1	0.02	1
10	2	1	3	0.01	1	0.02	1
10	2	1	4	0.04	1	0.01	1
10	2	1	5	0.01	1	0.01	1
10	2	1	6	0.04	1	0.02	1
10	2	1	7	0.06	1	0.02	1
10	2	1	8	0.02	1	0.00	1
10	2	1	9	0.04	1	0.01	1
10	2	1	10	0.04	1	0.01	1
10	2	1	✓ 1	46.38	12.5k	1800.04	5994.7k
10	2	1	✓ 2	34.64	16.2k	188.32	429.5k
10	2	1	✓ 3	8.30	2856	1029.00	2359.6k
10	2	1	✓ 4	35.22	10.9k	1800.01	4960.1k
10	2	1	✓ 5	32.66	8985	5.38	635
10	2	1	✓ 6	45.71	9160	20.97	2366
10	2	1	✓ 7	51.14	12.5k	1800.03	4358.9k
10	2	1	✓ 8	5.70	1917	1800.01	5637.0k
10	2	1	✓ 9	68.76	19.4k	446.16	936.6k
10	2	1	✓ 10	34.80	10.1k	3.13	363
10	2	2	1	0.03	1	0.00	1
10	2	2	2	0.05	1	0.02	1
10	2	2	3	0.28	59	0.30	39
10	2	2	4	0.49	217	0.50	165
10	2	2	5	0.07	11	0.10	1
10	2	2	6	0.03	1	0.02	1
10	2	2	7	0.02	1	0.01	1
10	2	2	8	0.22	29	0.26	19
10	2	2	9	0.13	19	0.12	8
10	2	2	10	0.27	17	0.14	7
10	2	2	✓ 1	49.11	30.1k	1800.00	5273.4k
10	2	2	✓ 2	20.44	4506	26.56	4302
10	2	2	✓ 3	51.33	26.3k	29.17	4862
10	2	2	✓ 4	73.01	18.1k	88.12	13.3k
10	2	2	✓ 5	30.35	21.9k	634.42	1300.1k
10	2	2	✓ 6	41.10	11.1k	32.83	5482
10	2	2	✓ 7	60.92	13.4k	1800.00	5430.2k
10	2	2	✓ 8	16.33	6324	2.47	279
10	2	2	✓ 9	31.98	18.6k	8.97	2034
10	2	2	✓ 10	11.73	6382	11.23	2273
10	2	3	1	0.27	47	0.27	41
10	2	3	2	0.28	47	0.31	29
10	2	3	3	0.11	17	0.26	17
10	2	3	4	0.88	652	0.58	220
10	2	3	5	0.26	55	0.34	43
10	2	3	6	0.32	55	0.12	5
10	2	3	7	0.35	121	0.42	79
10	2	3	8	0.11	19	0.10	17

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	2	3	9	0.03	1	0.10	1
10	2	3	10	0.29	85	0.21	82
10	2	3	✓ 1	29.89	8821	16.07	2544
10	2	3	✓ 2	28.64	11.7k	1800.02	2820.6k
10	2	3	✓ 3	12.81	2213	30.29	4527
10	2	3	✓ 4	53.42	24.2k	31.98	5100
10	2	3	✓ 5	28.45	11.2k	18.55	4007
10	2	3	✓ 6	22.04	8600	4.70	622
10	2	3	✓ 7	79.39	22.2k	89.65	14.6k
10	2	3	✓ 8	86.21	29.2k	46.34	7852
10	2	3	✓ 9	82.95	22.6k	136.26	21.3k
10	2	3	✓ 10	143.47	85.1k	13.63	9140
10	2	5	1	0.31	33	0.44	44
10	2	5	2	0.18	25	0.36	27
10	2	5	3	0.28	33	0.35	45
10	2	5	4	0.35	61	0.52	54
10	2	5	5	0.17	19	0.14	22
10	2	5	6	2.05	5153	1.14	547
10	2	5	7	0.32	47	0.36	53
10	2	5	8	0.26	35	0.22	15
10	2	5	9	0.30	47	0.35	19
10	2	5	10	0.54	195	0.80	169
10	2	5	✓ 1	28.59	10.2k	41.08	7136
10	2	5	✓ 2	19.92	4029	48.29	7684
10	2	5	✓ 3	26.60	11.6k	20.94	3858
10	2	5	✓ 4	36.50	9984	34.33	8446
10	2	5	✓ 5	10.70	2313	8.82	2270
10	2	5	✓ 6	93.71	56.8k	51.40	11.7k
10	2	5	✓ 7	21.84	6538	19.46	3694
10	2	5	✓ 8	9.80	2814	53.45	18.2k
10	2	5	✓ 9	33.49	16.9k	7.59	6109
10	2	5	✓ 10	40.99	11.9k	91.16	16.0k
10	2	10	1	1.08	1201	2.85	1229
10	2	10	2	0.25	59	0.38	17
10	2	10	3	0.33	47	0.60	59
10	2	10	4	0.35	77	0.71	121
10	2	10	5	0.24	27	0.51	19
10	2	10	6	0.33	43	0.48	31
10	2	10	7	0.31	39	0.50	65
10	2	10	8	0.31	37	0.47	27
10	2	10	9	0.20	23	0.33	23
10	2	10	10	0.48	243	0.94	154
10	2	10	✓ 1	68.60	15.6k	226.02	21.6k
10	2	10	✓ 2	16.38	3674	20.07	3452
10	2	10	✓ 3	19.61	6443	64.83	6994
10	2	10	✓ 4	96.84	34.8k	299.32	27.4k
10	2	10	✓ 5	46.94	11.5k	94.88	11.7k
10	2	10	✓ 6	33.19	12.9k	115.75	14.6k
10	2	10	✓ 7	20.79	7186	36.90	5971
10	2	10	✓ 8	15.98	5175	19.50	2996

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	2	10	✓	9	68.76	17.1k	156.46	17.9k
10	2	10	✓	10	36.52	12.6k	132.48	14.3k
10	3	1		1	0.11	1	0.05	1
10	3	1		2	0.14	1	0.10	1
10	3	1		3	0.09	1	0.01	1
10	3	1		4	0.16	1	0.24	23
10	3	1		5	0.13	1	0.20	11
10	3	1		6	0.10	1	0.01	1
10	3	1		7	0.15	1	0.08	1
10	3	1		8	0.12	3	0.05	1
10	3	1		9	0.24	9	0.18	1
10	3	1		10	0.13	1	0.01	1
10	3	1	✓	1	184.09	37.8k	172.74	12.0k
10	3	1	✓	2	1154.12	601.0k	1800.03	1000.2k
10	3	1	✓	3	110.73	20.4k	142.63	12.7k
10	3	1	✓	4	255.93	40.6k	307.56	35.2k
10	3	1	✓	5	156.42	57.2k	1800.07	3198.7k
10	3	1	✓	6	243.00	50.9k	1800.00	1172.5k
10	3	1	✓	7	192.65	32.5k	1800.03	2446.4k
10	3	1	✓	8	339.24	66.9k	1800.03	2649.5k
10	3	1	✓	9	930.63	279.5k	(0.01%)	(0.00%)
10	3	1	✓	10	357.94	94.2k	1800.00	965.8k
10	3	2		1	0.47	57	0.67	79
10	3	2		2	0.26	12	0.12	11
10	3	2		3	0.17	1	0.35	6
10	3	2		4	0.19	13	0.38	15
10	3	2		5	0.39	65	0.73	137
10	3	2		6	0.14	1	0.32	7
10	3	2		7	0.43	71	0.89	326
10	3	2		8	0.82	317	1.30	415
10	3	2		9	0.18	11	0.22	13
10	3	2		10	0.22	9	0.54	57
10	3	2	✓	1	707.91	219.5k	420.66	29.2k
10	3	2	✓	2	140.22	35.9k	278.95	28.0k
10	3	2	✓	3	1591.00	1182.6k	1800.03	1876.5k
10	3	2	✓	4	657.38	248.9k	516.53	51.5k
10	3	2	✓	5	624.62	177.3k	556.31	68.1k
10	3	2	✓	6	459.77	44.3k	925.53	194.9k
10	3	2	✓	7	854.96	437.2k	590.60	154.8k
10	3	2	✓	8	889.40	233.3k	1800.05	1903.4k
10	3	2	✓	9	232.22	36.5k	308.85	20.9k
10	3	2	✓	10	298.11	41.2k	1800.03	1381.7k
10	3	3		1	0.53	117	1.07	135
10	3	3		2	0.35	25	0.47	39
10	3	3		3	0.25	4	0.30	7
10	3	3		4	0.41	41	1.06	661
10	3	3		5	0.39	41	0.97	75
10	3	3		6	0.14	5	0.04	1
10	3	3		7	0.44	27	0.85	75
10	3	3		8	0.18	1	0.44	17

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	3	3		9	0.62	99	1.77	145
10	3	3		10	0.21	13	0.49	35
10	3	3	✓	1	1115.77	252.3k	(0.02%)	(0.00%)
10	3	3	✓	2	120.36	28.1k	1800.05	927.3k
10	3	3	✓	3	106.57	19.5k	1800.04	1002.5k
10	3	3	✓	4	495.63	84.5k	852.85	54.8k
10	3	3	✓	5	400.25	73.8k	1800.04	516.3k
10	3	3	✓	6	1290.91	904.8k	403.58	33.5k
10	3	3	✓	7	575.17	88.0k	(0.76%)	(0.00%)
10	3	3	✓	8	379.05	153.0k	356.20	23.2k
10	3	3	✓	9	522.71	46.1k	1800.04	453.8k
10	3	3	✓	10	578.72	447.3k	493.18	30.8k
10	3	5		1	0.61	89	1.17	101
10	3	5		2	0.66	131	1.19	161
10	3	5		3	0.14	11	0.85	25
10	3	5		4	0.39	39	0.83	47
10	3	5		5	0.46	46	0.70	41
10	3	5		6	0.52	63	1.14	115
10	3	5		7	0.20	1	0.69	22
10	3	5		8	0.36	16	0.77	29
10	3	5		9	0.54	129	1.23	312
10	3	5		10	0.23	13	0.79	31
10	3	5	✓	1	1800.02	294.7k	(0.77%)	(0.00%)
10	3	5	✓	2	444.84	50.4k	(0.04%)	(0.00%)
10	3	5	✓	3	485.77	288.2k	671.53	97.2k
10	3	5	✓	4	842.60	374.3k	1069.67	162.8k
10	3	5	✓	5	276.79	53.3k	895.43	30.2k
10	3	5	✓	6	1221.33	292.8k	(0.20%)	(0.00%)
10	3	5	✓	7	270.18	47.7k	827.56	23.9k
10	3	5	✓	8	169.45	28.4k	545.82	35.1k
10	3	5	✓	9	169.34	22.6k	663.02	19.7k
10	3	5	✓	10	592.74	334.1k	1800.04	765.1k
10	3	10		1	1800.04	4134.2k	4.24	170
10	3	10		2	0.56	59	2.08	63
10	3	10		3	1.00	245	7.04	305
10	3	10		4	0.84	165	4.24	229
10	3	10		5	0.92	219	11.18	446
10	3	10		6	1800.05	2702.9k	36.33	7611
10	3	10		7	1800.00	4217.3k	1.99	117
10	3	10		8	0.70	283	3.90	177
10	3	10		9	1.19	223	9.35	490
10	3	10		10	1.02	315	5.27	307
10	3	10	✓	1	257.31	23.3k	(0.49%)	(0.00%)
10	3	10	✓	2	406.65	56.6k	(1.10%)	(0.00%)
10	3	10	✓	3	741.58	170.8k	(0.29%)	(0.00%)
10	3	10	✓	4	1444.33	681.2k	(0.01%)	(0.00%)
10	3	10	✓	5	598.65	87.7k	(0.91%)	(0.00%)
10	3	10	✓	6	1800.03	471.7k	(2.18%)	(0.00%)
10	3	10	✓	7	360.80	165.5k	1800.10	49.9k
10	3	10	✓	8	1800.00	657.8k	(0.10%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
10	3	10	✓	9	627.42	108.1k	(0.17%)	(0.00%)
10	3	10	✓	10	830.00	66.0k	(0.94%)	(0.00%)
10	4	1		1	0.50	23	0.28	3
10	4	1		2	0.92	107	0.73	55
10	4	1		3	0.18	47	0.18	5
10	4	1		4	0.64	229	0.52	175
10	4	1		5	0.27	39	0.24	1
10	4	1		6	1.00	427	1.12	143
10	4	1		7	0.52	59	0.45	15
10	4	1		8	0.71	635	0.24	5
10	4	1		9	0.62	71	0.96	113
10	4	1		10	0.54	99	0.24	1
10	4	1	✓	1	471.62	80.5k	594.38	393.7k
10	4	1	✓	2	(0.08%)	(0.00%)	1800.05	201.3k
10	4	1	✓	3	1800.03	1084.7k	1800.05	969.8k
10	4	1	✓	4	(0.14%)	(0.00%)	1166.23	74.1k
10	4	1	✓	5	1129.70	710.9k	1800.07	1150.1k
10	4	1	✓	6	(0.30%)	(0.00%)	(0.07%)	(0.00%)
10	4	1	✓	7	360.87	209.0k	175.15	95.0k
10	4	1	✓	8	756.81	449.2k	1800.08	1365.8k
10	4	1	✓	9	1800.03	814.4k	1800.04	1543.7k
10	4	1	✓	10	(0.48%)	(0.00%)	(0.14%)	(0.00%)
10	4	2		1	0.50	7	0.94	67
10	4	2		2	1.21	467	1.16	109
10	4	2		3	2.60	1569	2.04	236
10	4	2		4	0.77	297	1.18	182
10	4	2		5	0.83	249	17.46	12.3k
10	4	2		6	1.09	493	2.25	492
10	4	2		7	0.59	97	0.93	77
10	4	2		8	1.63	229	1.73	145
10	4	2		9	0.53	175	0.89	63
10	4	2		10	0.66	433	0.88	115
10	4	2	✓	1	(0.02%)	(0.00%)	(0.23%)	(0.00%)
10	4	2	✓	2	(5.24%)	(0.00%)	(2.65%)	(0.00%)
10	4	2	✓	3	1800.00	202.3k	1800.07	203.4k
10	4	2	✓	4	(2.15%)	(0.00%)	(10.77%)	(0.00%)
10	4	2	✓	5	1800.00	593.2k	1458.74	39.7k
10	4	2	✓	6	(0.22%)	(0.00%)	(0.82%)	(0.00%)
10	4	2	✓	7	(0.25%)	(0.00%)	(1.85%)	(0.00%)
10	4	2	✓	8	(0.02%)	(0.00%)	(0.42%)	(0.00%)
10	4	2	✓	9	1800.04	456.9k	1409.14	55.3k
10	4	2	✓	10	780.73	58.8k	855.86	32.7k
10	4	3		1	4.41	1360	8.06	587
10	4	3		2	4.44	587	4.85	239
10	4	3		3	2.29	587	4.66	309
10	4	3		4	0.80	313	1.37	103
10	4	3		5	36.19	14.2k	53.81	32.2k
10	4	3		6	0.38	13	0.83	28
10	4	3		7	4.06	748	6.94	237
10	4	3		8	1.97	359	3.75	201

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	4	3	9	0.68	51	0.76	55
10	4	3	10	0.62	45	1.53	140
10	4	3	✓ 1	1800.04	596.3k	1800.09	329.3k
10	4	3	✓ 2	1800.05	784.2k	1800.17	413.2k
10	4	3	✓ 3	1800.04	559.4k	1800.09	49.6k
10	4	3	✓ 4	1725.45	112.5k	(0.21%)	(0.00%)
10	4	3	✓ 5	(15.34%) (0.00%)		(35.73%)	(0.00%)
10	4	3	✓ 6	416.39	28.7k	1004.82	23.8k
10	4	3	✓ 7	(70.97%) (0.00%)		(100.00%)	(0.00%)
10	4	3	✓ 8	1326.70	217.3k	(0.04%)	(0.00%)
10	4	3	✓ 9	(1.17%) (0.00%)		(4.00%)	(0.00%)
10	4	3	✓ 10	(0.38%) (0.00%)		(7.83%)	(0.00%)
10	4	5	1	0.90	227	2.55	111
10	4	5	2	5.53	813	5.10	194
10	4	5	3	11.87	1543	27.30	498
10	4	5	4	3.22	671	582.90	30.6k
10	4	5	5	9.79	8538	8.19	636
10	4	5	6	1.53	239	4.05	133
10	4	5	7	2.76	1942	8.78	773
10	4	5	8	7.93	4699	6.39	369
10	4	5	9	77.39	4035	95.91	1285
10	4	5	10	1.19	221	4.89	84
10	4	5	✓ 1	1661.58	118.2k	(0.42%)	(0.00%)
10	4	5	✓ 2	(0.56%) (0.00%)		(3.42%)	(0.00%)
10	4	5	✓ 3	(21.81%) (0.00%)		(52.30%)	(0.00%)
10	4	5	✓ 4	(0.01%)	(0.00%)	1800.12	42.0k
10	4	5	✓ 5	(0.97%) (0.00%)		(5.08%)	(0.00%)
10	4	5	✓ 6	1260.05	51.5k	(0.65%)	(0.00%)
10	4	5	✓ 7	1800.04	87.8k	(3.48%)	(0.00%)
10	4	5	✓ 8	(1.59%) (0.00%)		(7.51%)	(0.00%)
10	4	5	✓ 9	(7.51%) (0.00%)		(38.94%)	(0.00%)
10	4	5	✓ 10	(0.93%) (0.00%)		(30.14%)	(0.00%)
10	4	10	1	1.42	438	3.83	191
10	4	10	2	49.33	3726	181.23	1193
10	4	10	3	38.24	2118	165.08	1080
10	4	10	4	21.62	931	165.70	1007
10	4	10	5	99.09	8124	570.80	5125
10	4	10	6	26.92	5493	47.67	831
10	4	10	7	11.15	1368	92.00	685
10	4	10	8	40.06	6341	131.62	1539
10	4	10	9	22.51	2689	60.54	453
10	4	10	10	37.93	2973	127.71	959
10	4	10	✓ 1	1800.11	177.3k	(0.25%)	(0.00%)
10	4	10	✓ 2	(3.73%) (0.00%)		(19.28%)	(0.00%)
10	4	10	✓ 3	(2.01%) (0.00%)		(18.27%)	(0.00%)
10	4	10	✓ 4	(1.12%) (0.00%)		(19.66%)	(0.00%)
10	4	10	✓ 5	(0.31%) (0.00%)		(7.46%)	(0.00%)
10	4	10	✓ 6	(0.04%) (0.00%)		(9.37%)	(0.00%)
10	4	10	✓ 7	(3.75%) (0.00%)		(100.00%)	(0.00%)
10	4	10	✓ 8	(0.54%) (0.00%)		(6.25%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#		time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	4	10	✓	9	(0.69%)	(0.00%)	(30.53%)	(0.00%)
10	4	10	✓	10	(0.14%)	(0.00%)	(12.96%)	(0.00%)
10	5	1		1	0.60	43	0.76	1
10	5	1		2	0.18	3	0.17	1
10	5	1		3	0.69	123	0.95	63
10	5	1		4	0.56	87	1.14	185
10	5	1		5	0.61	85	0.58	41
10	5	1		6	0.62	69	0.56	71
10	5	1		7	0.70	699	0.81	38
10	5	1		8	1.01	55	0.77	71
10	5	1		9	0.68	109	0.96	105
10	5	1		10	0.69	71	0.64	89
10	5	1	✓	1	(2.43%)	(0.00%)	(1.05%)	(0.00%)
10	5	1	✓	2	1800.03	1485.2k	1800.10	650.2k
10	5	1	✓	3	(0.20%)	(0.00%)	1800.13	597.5k
10	5	1	✓	4	1800.03	1867.8k	318.78	24.4k
10	5	1	✓	5	1800.00	412.1k	1800.00	196.8k
10	5	1	✓	6	(0.01%)	(0.00%)	1050.98	70.8k
10	5	1	✓	7	1800.04	195.8k	1800.07	339.5k
10	5	1	✓	8	(1.10%)	(0.00%)	(1.29%)	(0.00%)
10	5	1	✓	9	1800.09	264.2k	1800.10	108.9k
10	5	1	✓	10	1800.05	366.4k	1800.12	439.5k
10	5	2		1	3.69	91	3.52	85
10	5	2		2	5.99	193	7.62	175
10	5	2		3	1.15	361	2.02	217
10	5	2		4	0.56	59	0.89	22
10	5	2		5	0.65	69	1.16	69
10	5	2		6	0.41	25	0.89	15
10	5	2		7	1.82	173	36.67	5470
10	5	2		8	1.86	281	1.56	39
10	5	2		9	0.85	125	1.76	103
10	5	2		10	0.50	9	1.43	49
10	5	2	✓	1	(10.77%)	(0.00%)	(20.69%)	(0.00%)
10	5	2	✓	2	(4.77%)	(0.00%)	(7.43%)	(0.00%)
10	5	2	✓	3	457.33	66.0k	(0.03%)	(0.00%)
10	5	2	✓	4	730.77	79.1k	1800.23	266.5k
10	5	2	✓	5	(0.01%)	(0.00%)	(0.25%)	(0.00%)
10	5	2	✓	6	549.47	54.5k	(0.03%)	(0.00%)
10	5	2	✓	7	353.81	27.1k	(0.07%)	(0.00%)
10	5	2	✓	8	(1.25%)	(0.00%)	(3.19%)	(0.00%)
10	5	2	✓	9	(1.06%)	(0.00%)	(1.63%)	(0.00%)
10	5	2	✓	10	1800.00	164.6k	(0.12%)	(0.00%)
10	5	3		1	12.71	2239	27.28	795
10	5	3		2	2.66	371	10.57	285
10	5	3		3	1.25	353	2.13	119
10	5	3		4	0.93	73	78.23	13.6k
10	5	3		5	1.55	73	3.69	67
10	5	3		6	12.99	2296	43.31	970
10	5	3		7	3.52	586	36.04	6695
10	5	3		8	0.86	269	1.99	51

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
10	5	3	9	0.62	45	1.20	15
10	5	3	10	6.98	1662	16.30	399
10	5	3	✓ 1	(5.46%) (0.00%)		(28.60%) (0.00%)	
10	5	3	✓ 2	974.08	368.8k	(0.09%)	(0.00%)
10	5	3	✓ 3	(0.20%) (0.00%)		(1.97%) (0.00%)	
10	5	3	✓ 4	(0.10%) (0.00%)		(2.05%) (0.00%)	
10	5	3	✓ 5	(0.16%) (0.00%)		(8.60%) (0.00%)	
10	5	3	✓ 6	(0.18%) (0.00%)		(5.65%) (0.00%)	
10	5	3	✓ 7	(6.86%) (0.00%)		(47.32%) (0.00%)	
10	5	3	✓ 8	1800.00	148.0k	(0.78%)	(0.00%)
10	5	3	✓ 9	1800.07	167.6k	(0.03%)	(0.00%)
10	5	3	✓ 10	(43.74%) (0.00%)		(100.00%) (1.54%)	
10	5	5	1	0.77	201	4.08	99
10	5	5	2	1.20	201	8.73	141
10	5	5	3	53.38	5691	313.20	2314
10	5	5	4	4.92	309	29.86	147
10	5	5	5	3.19	185	20.86	157
10	5	5	6	0.25	1	1.96	7
10	5	5	7	9.18	1507	39.35	552
10	5	5	8	1.15	109	125.13	4021
10	5	5	9	51.88	1179	688.77	9745
10	5	5	10	6.99	1148	33.80	755
10	5	5	✓ 1	1800.08	197.8k	(1.36%)	(0.00%)
10	5	5	✓ 2	1800.00	107.5k	(3.79%)	(0.00%)
10	5	5	✓ 3	(0.57%) (0.00%)		(19.82%) (0.00%)	
10	5	5	✓ 4	(0.53%) (0.00%)		(3.96%) (0.00%)	
10	5	5	✓ 5	(4.21%) (0.00%)		(25.56%) (0.00%)	
10	5	5	✓ 6	405.47	33.5k	(0.05%)	(0.00%)
10	5	5	✓ 7	1519.52	85.7k	(8.86%)	(0.00%)
10	5	5	✓ 8	1800.11	91.1k	(7.35%)	(0.00%)
10	5	5	✓ 9	(4.09%) (0.00%)		(22.38%) (0.00%)	
10	5	5	✓ 10	(2.86%) (0.00%)		(6.35%) (0.00%)	
10	5	10	1	14.70	553	278.01	659
10	5	10	2	6.27	2885	43.18	547
10	5	10	3	22.99	4613	135.60	1123
10	5	10	4	4.06	486	96.65	357
10	5	10	5	6.50	1755	38.32	412
10	5	10	6	19.01	2723	198.49	517
10	5	10	7	134.07	5297	(3.36%)	(0.10%)
10	5	10	8	6.60	1039	102.38	367
10	5	10	9	0.81	73	13.09	53
10	5	10	10	73.11	3570	962.08	2843
10	5	10	✓ 1	(11.76%) (0.00%)		(95.61%) (0.00%)	
10	5	10	✓ 2	1800.00	230.8k	(2.76%)	(0.00%)
10	5	10	✓ 3	(1.70%) (0.00%)		(13.31%) (0.00%)	
10	5	10	✓ 4	1361.38	52.0k	(9.44%)	(0.00%)
10	5	10	✓ 5	(0.03%) (0.00%)		(1.98%) (0.00%)	
10	5	10	✓ 6	(1.06%) (0.00%)		(19.17%) (0.00%)	
10	5	10	✓ 7	(2.75%) (0.00%)		(100.00%) (0.00%)	
10	5	10	✓ 8	(0.05%) (0.00%)		(8.44%) (0.00%)	

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
10	5	10	✓	9	1469.72	47.4k	(1.41%)	(0.00%)
10	5	10	✓	10	(6.70%) (0.00%)		(100.00%) (0.00%)	
20	2	1		1	0.05	1	0.06	1
20	2	1		2	0.04	1	0.05	1
20	2	1		3	0.07	1	0.03	1
20	2	1		4	0.06	1	0.05	1
20	2	1		5	0.03	1	0.02	1
20	2	1		6	0.03	1	0.01	1
20	2	1		7	0.04	1	0.02	1
20	2	1		8	0.07	1	0.03	1
20	2	1		9	0.03	1	0.02	1
20	2	1		10	0.05	1	0.02	1
20	2	1	✓	1	(0.26%) (0.00%)		191.57	2611
20	2	1	✓	2	(1.05%) (0.00%)		(0.50%) (0.00%)	
20	2	1	✓	3	(4.93%) (0.00%)		(0.85%) (0.00%)	
20	2	1	✓	4	(1.88%) (0.00%)		1800.00	67.5k
20	2	1	✓	5	(14.92%) (0.00%)		(7.47%) (0.00%)	
20	2	1	✓	6	(4.67%) (0.00%)		(3.04%) (0.00%)	
20	2	1	✓	7	(2.17%) (0.00%)		(0.07%) (0.00%)	
20	2	1	✓	8	(0.42%) (0.00%)		(0.33%) (0.00%)	
20	2	1	✓	9	(2.64%) (0.00%)		(1.66%) (0.00%)	
20	2	1	✓	10	(4.04%) (0.00%)		(1.66%) (0.00%)	
20	2	2		1	0.14	17	0.26	15
20	2	2		2	0.04	1	0.08	1
20	2	2		3	0.30	59	0.47	51
20	2	2		4	0.11	1	0.03	1
20	2	2		5	0.13	11	0.09	1
20	2	2		6	0.11	15	0.30	7
20	2	2		7	0.48	140	0.53	209
20	2	2		8	0.38	107	0.28	102
20	2	2		9	8.02	6316	3.41	4134
20	2	2		10	0.09	1	0.02	1
20	2	2	✓	1	(2.52%) (0.00%)		(2.39%) (0.00%)	
20	2	2	✓	2	(9.19%) (0.00%)		(2.95%) (0.00%)	
20	2	2	✓	3	(10.17%) (0.00%)		(3.61%) (0.00%)	
20	2	2	✓	4	(3.06%) (0.00%)		(2.36%) (0.00%)	
20	2	2	✓	5	(0.69%) (0.00%)		1800.00	894.2k
20	2	2	✓	6	(4.99%) (0.00%)		(14.36%) (0.00%)	
20	2	2	✓	7	(42.73%) (0.00%)		(39.63%) (0.00%)	
20	2	2	✓	8	(6.12%) (0.00%)		(5.36%) (0.00%)	
20	2	2	✓	9	(0.54%) (0.00%)		(0.03%) (0.00%)	
20	2	2	✓	10	(2.05%) (0.00%)		1800.01	705.6k
20	2	3		1	0.08	3	0.20	5
20	2	3		2	0.41	105	0.54	126
20	2	3		3	0.21	17	0.40	23
20	2	3		4	0.28	29	0.42	41
20	2	3		5	0.26	45	0.41	54
20	2	3		6	0.27	34	0.24	7
20	2	3		7	0.35	57	0.46	63
20	2	3		8	0.05	3	0.12	5

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	2	3	9	0.34	61	0.40	101
20	2	3	10	0.09	1	0.05	1
20	2	3	✓ 1	(11.94%) (0.00%)		(0.57%) (0.00%)	
20	2	3	✓ 2	(1.48%) (0.00%)		1800.02 161.7k	
20	2	3	✓ 3	(23.58%) (0.00%)		(14.64%) (0.00%)	
20	2	3	✓ 4	(0.42%) (0.00%)		(0.06%) (0.00%)	
20	2	3	✓ 5	(1.04%) (0.00%)		1800.03 468.6k	
20	2	3	✓ 6	(1.60%) (0.00%)		(2.02%) (0.00%)	
20	2	3	✓ 7	(12.74%) (0.00%)		(12.30%) (0.00%)	
20	2	3	✓ 8	(6.80%) (0.00%)		(9.07%) (0.00%)	
20	2	3	✓ 9	(60.09%) (0.00%)		(42.30%) (0.00%)	
20	2	3	✓ 10	(5.42%) (0.00%)		(0.28%) (0.00%)	
20	2	5	1	0.57	71	0.63	73
20	2	5	2	0.37	45	0.49	23
20	2	5	3	0.40	47	0.41	48
20	2	5	4	0.71	251	1.05	365
20	2	5	5	0.49	139	0.85	293
20	2	5	6	3.81	1585	5.95	1845
20	2	5	7	0.49	123	0.77	129
20	2	5	8	0.38	69	0.52	70
20	2	5	9	0.37	55	0.44	101
20	2	5	10	0.73	177	0.85	132
20	2	5	✓ 1	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	2	5	✓ 2	(2.31%) (0.00%)		(6.80%) (0.00%)	
20	2	5	✓ 3	(5.89%) (0.00%)		(4.87%) (0.00%)	
20	2	5	✓ 4	(3.34%) (0.00%)		(3.47%) (0.00%)	
20	2	5	✓ 5	(2.24%) (0.00%)		(0.10%) (0.00%)	
20	2	5	✓ 6	(25.69%) (0.00%)		(45.46%) (0.00%)	
20	2	5	✓ 7	(1.32%) (0.00%)		1800.04 295.3k	
20	2	5	✓ 8	(1.29%) (0.00%)		(1.62%) (0.00%)	
20	2	5	✓ 9	(3.18%) (0.00%)		(1.31%) (0.00%)	
20	2	5	✓ 10	(12.74%) (0.00%)		(19.90%) (0.00%)	
20	2	10	1	3.57	2254	13.83	3365
20	2	10	2	2.85	1659	5.84	1320
20	2	10	3	0.89	329	3.25	435
20	2	10	4	1.94	801	5.90	1109
20	2	10	5	1.68	493	3.13	507
20	2	10	6	0.89	691	2.82	1003
20	2	10	7	17.16	3278	24.71	2641
20	2	10	8	1.42	603	3.83	689
20	2	10	9	0.86	447	1.63	334
20	2	10	10	0.97	449	2.04	828
20	2	10	✓ 1	(4.92%) (0.00%)		(6.52%) (0.00%)	
20	2	10	✓ 2	(0.46%) (0.00%)		(0.21%) (0.00%)	
20	2	10	✓ 3	(2.06%) (0.00%)		(2.32%) (0.00%)	
20	2	10	✓ 4	(3.18%) (0.00%)		(6.48%) (0.00%)	
20	2	10	✓ 5	(6.83%) (0.00%)		(2.69%) (0.00%)	
20	2	10	✓ 6	(3.94%) (0.00%)		(4.81%) (0.00%)	
20	2	10	✓ 7	(8.95%) (0.00%)		(11.89%) (0.00%)	
20	2	10	✓ 8	(2.83%) (0.00%)		(3.41%) (0.00%)	

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	2	10	✓	9	(3.04%)	(0.00%)	(1.43%)	(0.00%)
20	2	10	✓	10	(2.93%)	(0.00%)	(2.45%)	(0.00%)
20	3	1		1	0.14	1	0.02	1
20	3	1		2	0.12	1	0.06	1
20	3	1		3	0.53	21	0.31	1
20	3	1		4	0.13	1	0.41	5
20	3	1		5	0.17	1	0.18	1
20	3	1		6	0.22	3	0.22	1
20	3	1		7	0.25	1	0.33	5
20	3	1		8	0.44	42	0.41	35
20	3	1		9	0.24	23	0.39	7
20	3	1		10	0.13	1	0.13	1
20	3	1	✓	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	3	1	✓	2	(58.91%)	(0.00%)	(100.00%)	(0.00%)
20	3	1	✓	3	(4.12%)	(0.00%)	(8.09%)	(0.00%)
20	3	1	✓	4	(3.17%)	(0.00%)	(2.93%)	(0.00%)
20	3	1	✓	5	(62.63%)	(0.00%)	(72.25%)	(0.00%)
20	3	1	✓	6	(49.15%)	(0.00%)	(54.25%)	(0.00%)
20	3	1	✓	7	(19.13%)	(0.00%)	(22.83%)	(0.00%)
20	3	1	✓	8	(9.91%)	(0.00%)	(16.53%)	(0.00%)
20	3	1	✓	9	(55.87%)	(0.00%)	(63.74%)	(0.00%)
20	3	1	✓	10	(9.74%)	(0.00%)	(11.63%)	(0.00%)
20	3	2		1	0.42	53	0.66	91
20	3	2		2	0.46	25	0.95	49
20	3	2		3	0.43	41	0.72	93
20	3	2		4	1.20	147	2.75	137
20	3	2		5	(0.00%)	(0.91%)	1.04	100
20	3	2		6	0.67	158	1.51	231
20	3	2		7	0.54	43	0.97	75
20	3	2		8	0.52	69	0.70	129
20	3	2		9	1.31	269	4.14	697
20	3	2		10	0.47	95	0.71	49
20	3	2	✓	1	(3.23%)	(0.00%)	(5.75%)	(0.00%)
20	3	2	✓	2	(30.38%)	(0.00%)	(50.21%)	(0.00%)
20	3	2	✓	3	(25.22%)	(0.00%)	(27.25%)	(0.00%)
20	3	2	✓	4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	3	2	✓	5	(50.67%)	(0.00%)	(72.93%)	(0.00%)
20	3	2	✓	6	(22.90%)	(0.00%)	(37.21%)	(0.00%)
20	3	2	✓	7	(8.58%)	(0.00%)	(14.98%)	(0.00%)
20	3	2	✓	8	(45.25%)	(0.00%)	(73.05%)	(0.00%)
20	3	2	✓	9	(24.64%)	(0.00%)	(38.27%)	(0.00%)
20	3	2	✓	10	(10.44%)	(0.00%)	(23.13%)	(0.00%)
20	3	3		1	0.96	625	1.71	417
20	3	3		2	1.81	1479	(0.00%)	(1.85%)
20	3	3		3	2.93	151	4.45	205
20	3	3		4	4.32	3945	24.11	8617
20	3	3		5	0.70	187	1.84	375
20	3	3		6	0.61	65	2.27	217
20	3	3		7	0.26	11	0.79	31
20	3	3		8	1.10	351	3.77	482

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	3	3	9	0.70	75	1.18	79
20	3	3	10	0.79	191	2.34	317
20	3	3	✓ 1	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	3	3	✓ 2	(62.23%) (0.00%)		(100.00%) (0.00%)	
20	3	3	✓ 3	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	3	3	✓ 4	(14.60%) (0.00%)		(22.27%)	(0.00%)
20	3	3	✓ 5	(5.53%) (0.00%)		(13.68%)	(0.00%)
20	3	3	✓ 6	(24.44%) (0.00%)		(37.05%)	(0.00%)
20	3	3	✓ 7	(13.25%) (0.00%)		(21.65%)	(0.00%)
20	3	3	✓ 8	(13.06%) (0.00%)		(32.87%)	(0.00%)
20	3	3	✓ 9	(15.39%) (0.00%)		(27.63%)	(0.00%)
20	3	3	✓ 10	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	3	5	1	1.45	133	10.84	2407
20	3	5	2	6.92	3182	121.85	27.8k
20	3	5	3	1800.08	1726.4k	164.00	8821
20	3	5	4	1.52	227	6.70	433
20	3	5	5	1.93	367	23.54	1111
20	3	5	6	2.95	413	(1.48%)	(0.00%)
20	3	5	7	1.71	525	17.46	914
20	3	5	8	0.90	287	2.37	217
20	3	5	9	0.65	81	3.00	129
20	3	5	10	0.49	19	0.92	64
20	3	5	✓ 1	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	3	5	✓ 2	(29.59%) (0.00%)		(45.58%)	(0.00%)
20	3	5	✓ 3	(17.87%) (0.00%)		(34.50%)	(0.00%)
20	3	5	✓ 4	(86.45%) (0.00%)		(100.00%) (0.00%)	
20	3	5	✓ 5	(48.86%) (0.00%)		(88.63%)	(0.00%)
20	3	5	✓ 6	(26.31%) (0.00%)		(44.36%)	(0.00%)
20	3	5	✓ 7	(100.00%) (0.00%)		(100.00%) (0.00%)	
20	3	5	✓ 8	(43.65%) (0.00%)		(96.05%)	(0.00%)
20	3	5	✓ 9	(33.24%) (0.00%)		(75.97%)	(0.00%)
20	3	5	✓ 10	(6.81%) (0.00%)		(20.38%)	(0.00%)
20	3	10	1	4.29	1130	116.97	3717
20	3	10	2	161.19	27.8k	1365.74	15.3k
20	3	10	3	3.29	425	118.56	1248
20	3	10	4	3.76	807	42.18	919
20	3	10	5	158.27	11.3k	(31.76%)	(6.73%)
20	3	10	6	0.66	61	5.76	256
20	3	10	7	72.60	1564	670.66	2980
20	3	10	8	56.57	2146	527.53	2819
20	3	10	9	6.18	405	129.89	1127
20	3	10	10	9.46	2217	157.99	4104
20	3	10	✓ 1	(6.30%) (0.00%)		(15.51%)	(0.00%)
20	3	10	✓ 2	(15.76%) (0.00%)		(28.88%)	(0.00%)
20	3	10	✓ 3	(5.90%) (0.00%)		(11.88%)	(0.00%)
20	3	10	✓ 4	(67.98%) (0.00%)		(100.00%) (0.00%)	
20	3	10	✓ 5	(36.46%) (0.00%)		(100.00%) (0.00%)	
20	3	10	✓ 6	(19.41%) (0.00%)		(38.49%)	(0.00%)
20	3	10	✓ 7	(68.01%) (0.00%)		(100.00%) (0.00%)	
20	3	10	✓ 8	(34.55%) (0.00%)		(66.56%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
20	3	10	✓	9	(11.21%) (0.00%)		(34.25%)	(0.00%)
20	3	10	✓	10	(52.23%) (0.00%)		(100.00%)	(0.00%)
20	4	1		1	1.89	525	1.38	195
20	4	1		2	8.97	6058	8.58	3111
20	4	1		3	27.61	902	5.11	894
20	4	1		4	3.48	1280	1.33	423
20	4	1		5	1.62	1139	0.91	31
20	4	1		6	0.68	37	0.77	3
20	4	1		7	1.24	916	0.99	215
20	4	1		8	4.31	6273	1.72	1004
20	4	1		9	3.73	733	2.27	195
20	4	1		10	0.97	249	1.18	111
20	4	1	✓	1	(57.54%) (0.00%)		(72.24%)	(0.00%)
20	4	1	✓	2	(26.45%)	(0.00%)	(22.75%)	(0.00%)
20	4	1	✓	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	1	✓	4	(11.57%) (0.00%)		(19.06%)	(0.00%)
20	4	1	✓	5	(21.25%)	(0.00%)	(20.43%)	(0.00%)
20	4	1	✓	6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	1	✓	7	(16.51%)	(0.00%)	(14.16%)	(0.00%)
20	4	1	✓	8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	1	✓	9	(96.91%)	(0.00%)	(87.69%)	(0.00%)
20	4	1	✓	10	(64.26%)	(0.00%)	(58.99%)	(0.00%)
20	4	2		1	48.86	31.2k	15.29	1341
20	4	2		2	5.84	1220	7.02	799
20	4	2		3	2.46	745	2.03	161
20	4	2		4	167.52	109.8k	1800.00	1693.8k
20	4	2		5	420.95	37.3k	1800.00	1194.4k
20	4	2		6	28.40	2847	5.30	233
20	4	2		7	6.59	412	2.66	131
20	4	2		8	14.75	2190	12.18	435
20	4	2		9	0.81	85	1.39	29
20	4	2		10	160.45	13.2k	53.74	1911
20	4	2	✓	1	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	4	2	✓	2	(19.43%) (0.00%)		(33.93%)	(0.00%)
20	4	2	✓	3	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	4	2	✓	4	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	4	2	✓	5	(17.47%) (0.00%)		(21.87%)	(0.00%)
20	4	2	✓	6	(55.33%) (0.00%)		(85.14%)	(0.00%)
20	4	2	✓	7	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	4	2	✓	8	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	4	2	✓	9	(42.00%) (0.00%)		(48.51%)	(0.00%)
20	4	2	✓	10	(79.73%) (0.00%)		(100.00%)	(0.00%)
20	4	3		1	1.99	804	2.65	241
20	4	3		2	71.02	29.7k	816.09	370.7k
20	4	3		3	54.53	12.2k	53.91	809
20	4	3		4	10.95	3683	5.24	219
20	4	3		5	569.26	33.5k	226.63	3432
20	4	3		6	1.49	71	2.17	65
20	4	3		7	44.36	15.3k	18.26	337
20	4	3		8	16.72	2055	9.93	388

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	4	3	9	3.01	205	4.20	334
20	4	3	10	1056.93	70.2k	299.14	4484
20	4	3	✓ 1	(21.69%)	(0.00%)	(36.53%)	(0.00%)
20	4	3	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 3	(36.73%)	(0.00%)	(48.67%)	(0.00%)
20	4	3	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 5	(41.24%)	(0.00%)	(33.09%)	(0.00%)
20	4	3	✓ 6	(37.00%)	(0.00%)	(44.07%)	(0.00%)
20	4	3	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 8	(70.20%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	3	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	1	101.95	4107	243.54	1401
20	4	5	2	430.16	4737	775.44	3533
20	4	5	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	4	1409.68	264.7k	783.30	7731
20	4	5	5	(7.90%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	6	394.24	66.7k	583.80	8212
20	4	5	7	116.98	23.6k	75.60	1496
20	4	5	8	1661.35	90.5k	(5.53%)	(0.00%)
20	4	5	9	34.72	3827	62.12	582
20	4	5	10	271.15	14.0k	178.45	3410
20	4	5	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 3	(77.62%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 5	(26.58%)	(0.00%)	(30.98%)	(0.00%)
20	4	5	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	5	✓ 10	(37.50%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	1	(6.07%)	(0.00%)	(10.30%)	(0.00%)
20	4	10	2	(60.86%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	3	(0.44%)	(0.00%)	(1.17%)	(0.00%)
20	4	10	4	(29.27%)	(0.00%)	(49.69%)	(0.01%)
20	4	10	5	353.73	19.8k	499.70	1205
20	4	10	6	(56.74%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	7	(10.45%)	(0.00%)	(22.56%)	(0.00%)
20	4	10	8	(49.03%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	9	(29.82%)	(0.00%)	(44.70%)	(0.00%)
20	4	10	10	(72.17%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 3	(38.90%)	(0.00%)	(59.17%)	(0.00%)
20	4	10	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 5	(73.67%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 6	(43.70%)	(0.00%)	(68.78%)	(0.00%)
20	4	10	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓ 8	(70.60%)	(0.00%)	(100.00%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
20	4	10	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	4	10	✓	10	(49.46%)	(0.00%)	(100.00%)	(0.00%)
20	5	1		1	1.68	221	1.95	77
20	5	1		2	3.43	344	9.44	475
20	5	1		3	1.82	151	3.33	181
20	5	1		4	3.44	318	6.38	283
20	5	1		5	14.53	2749	5.91	616
20	5	1		6	1.02	139	2.93	69
20	5	1		7	12.51	2123	5.09	235
20	5	1		8	1.77	203	1.95	174
20	5	1		9	33.69	2367	23.88	2430
20	5	1		10	1.73	288	174.50	297.8k
20	5	1	✓	1	(64.35%)	(0.00%)	(62.17%)	(0.00%)
20	5	1	✓	2	(100.00%)	(0.00%)	(95.32%)	(0.00%)
20	5	1	✓	3	(75.53%)	(0.00%)	(82.37%)	(0.00%)
20	5	1	✓	4	(42.45%)	(0.00%)	(42.17%)	(0.00%)
20	5	1	✓	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	1	✓	6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	1	✓	7	(39.45%)	(0.00%)	(44.70%)	(0.00%)
20	5	1	✓	8	(33.95%)	(0.00%)	(32.04%)	(0.00%)
20	5	1	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	1	✓	10	(69.91%)	(0.00%)	(81.52%)	(0.00%)
20	5	2		1	4.88	413	5.93	359
20	5	2		2	24.27	25.0k	9.72	425
20	5	2		3	297.27	226.9k	45.80	3423
20	5	2		4	4.06	1189	24.78	1879
20	5	2		5	10.78	3122	57.23	681
20	5	2		6	689.12	9659	444.47	3417
20	5	2		7	37.95	1705	83.79	1184
20	5	2		8	17.54	857	85.86	843
20	5	2		9	260.66	8465	117.57	2092
20	5	2		10	47.77	20.4k	113.45	43.8k
20	5	2	✓	1	(77.87%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓	2	(27.59%)	(0.00%)	(30.82%)	(0.00%)
20	5	2	✓	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓	4	(86.62%)	(0.00%)	(81.02%)	(0.00%)
20	5	2	✓	5	(17.00%)	(0.00%)	(33.50%)	(0.00%)
20	5	2	✓	6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓	8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	2	✓	9	(76.24%)	(0.00%)	(91.81%)	(0.00%)
20	5	2	✓	10	(17.77%)	(0.00%)	(36.54%)	(0.00%)
20	5	3		1	116.34	16.8k	1800.82	227.1k
20	5	3		2	0.80	129	2.31	69
20	5	3		3	4.78	2283	19.91	656
20	5	3		4	(8.20%)	(0.00%)	(15.97%)	(0.00%)
20	5	3		5	10.77	2099	30.27	497
20	5	3		6	(0.65%)	(0.69%)	(5.44%)	(0.00%)
20	5	3		7	25.38	7237	1800.52	351.9k
20	5	3		8	262.65	87.2k	1800.00	143.6k

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons (16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
20	5	3	9	1671.88	497.7k	218.59	1289
20	5	3	10	75.42	7206	290.88	2400
20	5	3	✓ 1	(86.30%) (0.00%)		(100.00%)	(0.00%)
20	5	3	✓ 2	(10.37%)	(0.00%)	(9.46%)	(0.00%)
20	5	3	✓ 3	(15.74%) (0.00%)		(18.11%)	(0.00%)
20	5	3	✓ 4	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	3	✓ 5	(25.66%) (0.00%)		(44.31%)	(0.00%)
20	5	3	✓ 6	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	3	✓ 7	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	3	✓ 8	(26.99%) (0.00%)		(56.40%)	(0.00%)
20	5	3	✓ 9	(36.01%) (0.00%)		(57.01%)	(0.00%)
20	5	3	✓ 10	(81.69%) (0.00%)		(100.00%)	(0.00%)
20	5	5	1	(32.60%) (0.00%)		(35.47%)	(0.00%)
20	5	5	2	27.79	4276	154.78	963
20	5	5	3	372.42	8522	(8.02%)	(1.22%)
20	5	5	4	345.37	6602	1399.52	8210
20	5	5	5	3.52	211	19.86	120
20	5	5	6	328.43	14.4k	641.45	849
20	5	5	7	169.72	7328	(1.31%)	(0.00%)
20	5	5	8	776.61	18.8k	(2.37%)	(0.00%)
20	5	5	9	1137.99	13.2k	(100.00%)	(0.00%)
20	5	5	10	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 1	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 2	(24.58%) (0.00%)		(32.86%)	(0.00%)
20	5	5	✓ 3	(78.19%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 4	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 5	(28.34%) (0.00%)		(36.38%)	(0.00%)
20	5	5	✓ 6	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 7	(57.71%) (0.00%)		(100.00%)	(0.00%)
20	5	5	✓ 8	(50.74%) (0.00%)		(97.88%)	(0.00%)
20	5	5	✓ 9	(66.82%) (0.00%)		(95.36%)	(0.00%)
20	5	5	✓ 10	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	10	1	374.63	18.4k	(8.82%)	(0.00%)
20	5	10	2	(0.02%) (0.00%)		(24.68%)	(0.00%)
20	5	10	3	(31.28%) (0.00%)		(67.63%)	(0.00%)
20	5	10	4	(8.11%) (0.00%)		(51.81%)	(0.70%)
20	5	10	5	(100.00%) (0.00%)		(100.00%)	(69.93%)
20	5	10	6	1800.00	60.0k	(18.54%)	(0.00%)
20	5	10	7	(70.54%) (0.00%)		(100.00%)	(0.00%)
20	5	10	8	175.03	2030	(93.92%)	(0.00%)
20	5	10	9	(100.00%) (0.00%)		(100.00%)	(6.94%)
20	5	10	10	95.36	3419	(3.27%)	(0.00%)
20	5	10	✓ 1	(27.26%) (0.00%)		(51.26%)	(0.00%)
20	5	10	✓ 2	(27.80%) (0.00%)		(73.00%)	(0.00%)
20	5	10	✓ 3	(100.00%) (0.00%)		(100.00%)	(0.00%)
20	5	10	✓ 4	(45.18%) (0.00%)		(95.98%)	(0.00%)
20	5	10	✓ 5	(100.00%) (0.00%)		(100.00%)	(0.43%)
20	5	10	✓ 6	(26.76%) (0.00%)		(47.82%)	(0.00%)
20	5	10	✓ 7	(70.45%) (0.00%)		(100.00%)	(0.00%)
20	5	10	✓ 8	(57.79%) (0.00%)		(100.00%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
20	5	10	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
20	5	10	✓	10	(50.92%)	(0.00%)	(95.90%)	(0.00%)
30	2	1		1	0.08	3	0.01	1
30	2	1		2	0.08	3	0.03	1
30	2	1		3	0.04	1	0.03	1
30	2	1		4	8.40	23.3k	0.03	1
30	2	1		5	0.05	1	0.02	1
30	2	1		6	0.09	3	0.03	1
30	2	1		7	0.07	1	0.03	1
30	2	1		8	0.07	1	0.03	1
30	2	1		9	0.32	15	0.03	1
30	2	1		10	0.08	1	0.06	1
30	2	1	✓	1	(13.78%)	(0.00%)	(8.89%)	(0.00%)
30	2	1	✓	2	(23.59%)	(0.00%)	(16.32%)	(0.00%)
30	2	1	✓	3	(17.35%)	(0.00%)	(6.96%)	(0.00%)
30	2	1	✓	4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	1	✓	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	1	✓	6	(13.54%)	(0.00%)	(2.91%)	(0.00%)
30	2	1	✓	7	(38.08%)	(0.00%)	(10.83%)	(0.00%)
30	2	1	✓	8	(17.08%)	(0.00%)	(10.25%)	(0.00%)
30	2	1	✓	9	(9.55%)	(0.00%)	(4.03%)	(0.00%)
30	2	1	✓	10	(7.90%)	(0.00%)	(1.48%)	(0.00%)
30	2	2		1	0.34	51	0.36	21
30	2	2		2	1.19	684	0.85	537
30	2	2		3	0.17	11	0.09	2
30	2	2		4	0.08	1	0.04	1
30	2	2		5	0.29	41	0.39	58
30	2	2		6	1.36	361	0.67	201
30	2	2		7	0.99	247	0.49	128
30	2	2		8	0.10	1	0.07	1
30	2	2		9	0.36	55	0.42	51
30	2	2		10	0.60	161	0.51	103
30	2	2	✓	1	(95.47%)	(0.00%)	(66.30%)	(0.00%)
30	2	2	✓	2	(25.23%)	(0.00%)	(21.37%)	(0.00%)
30	2	2	✓	3	(4.95%)	(0.00%)	(3.62%)	(0.00%)
30	2	2	✓	4	(39.25%)	(0.00%)	(37.66%)	(0.00%)
30	2	2	✓	5	(7.68%)	(0.00%)	(5.78%)	(0.00%)
30	2	2	✓	6	(21.21%)	(0.00%)	(17.64%)	(0.00%)
30	2	2	✓	7	(35.70%)	(0.00%)	(25.60%)	(0.00%)
30	2	2	✓	8	(10.43%)	(0.00%)	(3.58%)	(0.00%)
30	2	2	✓	9	(26.35%)	(0.00%)	(27.97%)	(0.00%)
30	2	2	✓	10	(6.27%)	(0.00%)	(4.13%)	(0.00%)
30	2	3		1	0.38	81	0.52	23
30	2	3		2	0.43	169	0.52	195
30	2	3		3	1.02	371	0.91	315
30	2	3		4	1800.04	11.0M	0.90	165
30	2	3		5	0.66	305	0.94	481
30	2	3		6	1.17	343	1.31	423
30	2	3		7	0.47	123	0.53	125
30	2	3		8	0.77	161	0.78	167

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	2	3	9	1.13	365	0.64	93
30	2	3	10	0.24	51	0.52	24
30	2	3	✓ 1	(10.56%)	(0.00%)	(9.90%)	(0.00%)
30	2	3	✓ 2	(30.92%)	(0.00%)	(28.40%)	(0.00%)
30	2	3	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	3	✓ 4	(5.12%)	(0.00%)	(1.64%)	(0.00%)
30	2	3	✓ 5	(21.71%)	(0.00%)	(19.26%)	(0.00%)
30	2	3	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	3	✓ 7	(4.62%)	(0.00%)	(3.31%)	(0.00%)
30	2	3	✓ 8	(18.06%)	(0.00%)	(20.46%)	(0.00%)
30	2	3	✓ 9	(17.30%)	(0.00%)	(16.20%)	(0.00%)
30	2	3	✓ 10	(48.20%)	(0.00%)	(31.42%)	(0.00%)
30	2	5	1	33.95	22.3k	4.00	1083
30	2	5	2	18.11	3011	10.20	1928
30	2	5	3	0.53	63	0.69	69
30	2	5	4	84.68	11.6k	39.73	14.8k
30	2	5	5	61.66	11.4k	29.45	6627
30	2	5	6	8.45	1589	4.01	768
30	2	5	7	2.53	713	2.86	695
30	2	5	8	242.23	72.7k	402.30	89.8k
30	2	5	9	0.80	255	1.05	229
30	2	5	10	2.27	826	(0.00%)	(0.77%)
30	2	5	✓ 1	(26.27%)	(0.00%)	(25.37%)	(0.00%)
30	2	5	✓ 2	(45.83%)	(0.00%)	(38.98%)	(0.00%)
30	2	5	✓ 3	(27.13%)	(0.00%)	(24.79%)	(0.00%)
30	2	5	✓ 4	(47.63%)	(0.00%)	(52.67%)	(0.00%)
30	2	5	✓ 5	(15.05%)	(0.00%)	(10.40%)	(0.00%)
30	2	5	✓ 6	(100.00%)	(0.00%)	(92.55%)	(0.00%)
30	2	5	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	5	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	2	5	✓ 9	(11.52%)	(0.00%)	(8.58%)	(0.00%)
30	2	5	✓ 10	(17.90%)	(0.00%)	(10.95%)	(0.00%)
30	2	10	1	2.09	1265	5.69	1501
30	2	10	2	118.09	14.8k	95.19	12.4k
30	2	10	3	1800.02	5609.9k	114.21	37.7k
30	2	10	4	60.24	6491	107.55	6990
30	2	10	5	38.79	10.4k	85.75	11.8k
30	2	10	6	4.43	945	10.38	2389
30	2	10	7	3.79	8412	5.58	2178
30	2	10	8	1800.01	3807.8k	184.07	25.2k
30	2	10	9	2.36	789	5.20	1448
30	2	10	10	108.16	22.2k	131.78	18.9k
30	2	10	✓ 1	(7.34%)	(0.00%)	(6.49%)	(0.00%)
30	2	10	✓ 2	(28.93%)	(0.00%)	(30.09%)	(0.00%)
30	2	10	✓ 3	(22.26%)	(0.00%)	(23.44%)	(0.00%)
30	2	10	✓ 4	(13.69%)	(0.00%)	(10.96%)	(0.00%)
30	2	10	✓ 5	(41.85%)	(0.00%)	(50.83%)	(0.00%)
30	2	10	✓ 6	(13.66%)	(0.00%)	(14.53%)	(0.00%)
30	2	10	✓ 7	(5.78%)	(0.00%)	(3.76%)	(0.00%)
30	2	10	✓ 8	(13.30%)	(0.00%)	(13.52%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	2	10	✓	9	(13.27%)	(0.00%)	(12.06%)	(0.00%)
30	2	10	✓	10	(48.91%)	(0.00%)	(49.27%)	(0.00%)
30	3	1		1	0.53	45	0.71	1
30	3	1		2	0.66	165	0.65	61
30	3	1		3	0.38	51	0.18	5
30	3	1		4	0.44	33	0.70	27
30	3	1		5	0.46	19	0.42	3
30	3	1		6	0.29	1	0.54	1
30	3	1		7	0.42	17	0.72	61
30	3	1		8	0.29	1	0.11	1
30	3	1		9	0.43	31	0.43	3
30	3	1		10	0.47	37	0.81	169
30	3	1	✓	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	1	✓	2	(46.54%)	(0.00%)	(48.97%)	(0.00%)
30	3	1	✓	3	(35.59%)	(0.00%)	(35.64%)	(0.00%)
30	3	1	✓	4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	1	✓	5	(73.18%)	(0.00%)	(85.62%)	(0.00%)
30	3	1	✓	6	(16.02%)	(0.00%)	(21.61%)	(0.00%)
30	3	1	✓	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	1	✓	8	(49.52%)	(0.00%)	(45.49%)	(0.00%)
30	3	1	✓	9	(38.26%)	(0.00%)	(56.22%)	(0.00%)
30	3	1	✓	10	(23.17%)	(0.00%)	(26.60%)	(0.00%)
30	3	2		1	0.45	19	0.76	39
30	3	2		2	1.18	283	(0.00%)	(1.74%)
30	3	2		3	1.42	719	3.15	1090
30	3	2		4	0.72	87	1.06	101
30	3	2		5	0.60	19	0.84	37
30	3	2		6	0.57	49	1.63	209
30	3	2		7	0.45	39	0.85	45
30	3	2		8	0.86	408	(0.00%)	(1.10%)
30	3	2		9	0.78	181	2.24	622
30	3	2		10	0.54	97	1.26	373
30	3	2	✓	1	(20.35%)	(0.00%)	(29.22%)	(0.00%)
30	3	2	✓	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	2	✓	3	(68.48%)	(0.00%)	(77.11%)	(0.00%)
30	3	2	✓	4	(61.85%)	(0.00%)	(75.13%)	(0.00%)
30	3	2	✓	5	(52.38%)	(0.00%)	(57.81%)	(0.00%)
30	3	2	✓	6	(39.61%)	(0.00%)	(79.36%)	(0.00%)
30	3	2	✓	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	2	✓	8	(63.77%)	(0.00%)	(82.80%)	(0.00%)
30	3	2	✓	9	(41.42%)	(0.00%)	(57.59%)	(0.00%)
30	3	2	✓	10	(96.17%)	(0.00%)	(100.00%)	(0.00%)
30	3	3		1	0.73	65	1800.28	3048.7k
30	3	3		2	1.44	305	4.54	366
30	3	3		3	3.08	603	17.14	1192
30	3	3		4	(0.16%)	(0.00%)	(0.16%)	(0.00%)
30	3	3		5	0.80	65	2.14	117
30	3	3		6	0.46	57	1.51	105
30	3	3		7	0.57	9	1.39	92
30	3	3		8	1.23	307	4.25	1219

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)
30	3	3	9	5.92	2879	56.27	15.2k
30	3	3	10	1.07	312	3.76	501
30	3	3	✓ 1	(62.33%)	(0.00%)	(76.47%)	(0.00%)
30	3	3	✓ 2	(88.75%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 5	(78.14%)	(0.00%)	(92.87%)	(0.00%)
30	3	3	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	3	✓ 9	(32.15%)	(0.00%)	(42.22%)	(0.00%)
30	3	3	✓ 10	(68.92%)	(0.00%)	(93.43%)	(0.00%)
30	3	5	1	4.08	206	32.86	555
30	3	5	2	13.22	3037	809.61	64.7k
30	3	5	3	169.69	18.7k	1800.50	68.6k
30	3	5	4	7.40	1713	100.40	5758
30	3	5	5	12.37	889	118.06	1810
30	3	5	6	12.37	3891	1800.51	321.8k
30	3	5	7	2.83	397	57.95	1234
30	3	5	8	709.87	327.3k	(0.00%)	(0.92%)
30	3	5	9	5.27	1317	69.22	2599
30	3	5	10	1.48	314	8.92	1216
30	3	5	✓ 1	(40.57%)	(0.00%)	(68.78%)	(0.00%)
30	3	5	✓ 2	(67.66%)	(0.00%)	(92.78%)	(0.00%)
30	3	5	✓ 3	(79.06%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 6	(84.14%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 7	(88.04%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 8	(51.10%)	(0.00%)	(64.54%)	(0.00%)
30	3	5	✓ 9	(64.67%)	(0.00%)	(100.00%)	(0.00%)
30	3	5	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	1	142.02	3519	(6.09%)	(0.00%)
30	3	10	2	5.71	493	86.85	839
30	3	10	3	585.47	13.4k	(9.81%)	(0.00%)
30	3	10	4	(88.53%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	5	1485.96	314.4k	(45.13%)	(0.00%)
30	3	10	6	1800.00	111.0k	(7.36%)	(0.29%)
30	3	10	7	19.00	1002	405.49	2371
30	3	10	8	136.41	2885	(27.29%)	(0.00%)
30	3	10	9	24.83	2901	668.74	2537
30	3	10	10	67.17	7643	(0.02%)	(0.00%)
30	3	10	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 5	(41.55%)	(0.00%)	(56.71%)	(0.00%)
30	3	10	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	3	10	✓ 8	(46.47%)	(0.00%)	(65.98%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
30	3	10	✓	9	(46.98%) (0.00%)		(70.65%)	(0.00%)
30	3	10	✓	10	(47.48%) (0.00%)		(82.13%)	(0.00%)
30	4	1		1	0.84	127	1.61	1
30	4	1		2	4.90	516	6.52	3309
30	4	1		3	0.59	59	0.88	9
30	4	1		4	0.83	235	2.23	101
30	4	1		5	34.67	9849	178.78	212.5k
30	4	1		6	18.99	1041	3.84	1572
30	4	1		7	0.99	285	1.02	1
30	4	1		8	2.48	916	1.43	199
30	4	1		9	10.77	658	2.64	111
30	4	1		10	0.69	161	0.87	3
30	4	1	✓	1	(35.35%) (0.00%)		(41.71%)	(0.00%)
30	4	1	✓	2	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	3	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	4	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	5	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	6	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	7	(69.25%)	(0.00%)	(68.84%)	(0.00%)
30	4	1	✓	8	(69.75%)	(0.00%)	(68.30%)	(0.00%)
30	4	1	✓	9	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	1	✓	10	(58.52%) (0.00%)		(58.88%)	(0.00%)
30	4	2		1	133.15	4195	40.01	1920
30	4	2		2	527.41	253.4k	(0.02%)	(0.00%)
30	4	2		3	9.69	1248	12.08	501
30	4	2		4	675.86	241.2k	51.75	4171
30	4	2		5	2.17	469	2.05	155
30	4	2		6	118.42	3516	28.75	912
30	4	2		7	221.28	2515	50.93	1051
30	4	2		8	142.28	9817	34.50	1449
30	4	2		9	1800.00	442.7k	88.84	16.5k
30	4	2		10	374.74	51.3k	73.39	1829
30	4	2	✓	1	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	2	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	3	(68.25%) (0.00%)		(81.23%)	(0.00%)
30	4	2	✓	4	(88.25%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	5	(54.15%) (0.00%)		(63.11%)	(0.00%)
30	4	2	✓	6	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	7	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	8	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	9	(100.00%) (0.00%)		(100.00%)	(0.00%)
30	4	2	✓	10	(92.54%) (0.00%)		(100.00%)	(0.00%)
30	4	3		1	14.02	1615	12.14	141
30	4	3		2	761.66	17.7k	(0.00%)	(0.26%)
30	4	3		3	1321.02	18.4k	249.11	2290
30	4	3		4	922.71	76.2k	105.66	1640
30	4	3		5	1800.00	316.8k	221.75	6589
30	4	3		6	340.93	29.9k	1800.00	493.2k
30	4	3		7	97.86	14.3k	52.74	3710
30	4	3		8	71.97	5310	676.58	62.1k

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16) #	time	nodes	time	nodes
				(dgap)	(pgap)	(dgap)	(pgap)
30	4	3	9	(6.39%)	(0.00%)	(2.20%)	(0.00%)
30	4	3	10	(4.22%)	(0.00%)	462.65	2479
30	4	3	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 4	(76.07%)	(0.00%)	(88.58%)	(0.00%)
30	4	3	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	3	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	1	(4.96%)	(0.00%)	(8.03%)	(0.00%)
30	4	5	2	(2.02%)	(0.03%)	(0.89%)	(0.00%)
30	4	5	3	925.34	10.2k	1800.00	130.8k
30	4	5	4	(4.71%)	(0.00%)	(13.87%)	(0.00%)
30	4	5	5	(19.30%)	(0.00%)	(13.93%)	(0.00%)
30	4	5	6	(10.47%)	(3.21%)	(6.36%)	(0.00%)
30	4	5	7	(9.82%)	(0.00%)	(3.46%)	(0.00%)
30	4	5	8	(14.85%)	(0.39%)	(11.38%)	(0.00%)
30	4	5	9	(2.75%)	(0.00%)	(0.78%)	(0.00%)
30	4	5	10	1800.17	274.5k	1801.47	63.0k
30	4	5	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 5	(84.59%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	5	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	1	(82.47%)	(0.00%)	(95.79%)	(0.00%)
30	4	10	2	(50.63%)	(0.00%)	(58.14%)	(0.47%)
30	4	10	3	(40.28%)	(0.00%)	(60.83%)	(0.00%)
30	4	10	4	(46.72%)	(0.00%)	(44.00%)	(0.00%)
30	4	10	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	6	(15.13%)	(0.00%)	(37.15%)	(0.00%)
30	4	10	7	(85.26%)	(0.00%)	(78.83%)	(0.00%)
30	4	10	8	(8.30%)	(0.00%)	(12.82%)	(0.00%)
30	4	10	9	(28.74%)	(0.00%)	(47.13%)	(5.43%)
30	4	10	10	(94.84%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 6	(61.28%)	(0.00%)	(83.68%)	(0.00%)
30	4	10	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time	nodes	time	nodes
					(dgap)	(pgap)	(dgap)	(pgap)
30	4	10	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	4	10	✓	10	(86.83%)	(0.00%)	(100.00%)	(0.00%)
30	5	1		1	1600.00	197.2k	176.84	20.8k
30	5	1		2	25.79	6496	17.62	2018
30	5	1		3	1800.00	321.9k	59.10	18.3k
30	5	1		4	1800.00	512.3k	117.33	2940
30	5	1		5	1.87	228	3.07	155
30	5	1		6	111.37	21.3k	1038.73	914.5k
30	5	1		7	15.32	4437	12.60	1078
30	5	1		8	173.34	25.4k	238.78	3639
30	5	1		9	2.03	129	2.09	95
30	5	1		10	1.13	101	1.97	17
30	5	1	✓	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	6	(98.34%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	1	✓	8	(80.63%)	(0.00%)	(91.99%)	(0.00%)
30	5	1	✓	9	(66.59%)	(0.00%)	(73.25%)	(0.00%)
30	5	1	✓	10	(47.74%)	(0.00%)	(53.11%)	(0.00%)
30	5	2		1	(34.20%)	(0.00%)	(10.99%)	(0.00%)
30	5	2		2	1014.90	86.6k	(2.52%)	(0.00%)
30	5	2		3	(1.09%)	(0.00%)	(1.53%)	(0.00%)
30	5	2		4	1017.84	378.4k	1800.00	451.7k
30	5	2		5	(2.02%)	(0.00%)	(1.28%)	(0.00%)
30	5	2		6	(1.35%)	(0.00%)	(2.99%)	(0.00%)
30	5	2		7	47.53	1798	124.11	1266
30	5	2		8	(9.67%)	(2.20%)	(5.55%)	(0.00%)
30	5	2		9	354.46	37.9k	429.77	5121
30	5	2		10	680.98	27.0k	990.32	36.3k
30	5	2	✓	1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	2	✓	10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3		1	1800.00	391.2k	1800.63	122.2k
30	5	3		2	702.24	172.3k	618.42	9617
30	5	3		3	1256.91	88.4k	(0.85%)	(0.00%)
30	5	3		4	(38.30%)	(1.37%)	(26.56%)	(0.00%)
30	5	3		5	(3.89%)	(0.00%)	(8.95%)	(1.09%)
30	5	3		6	(69.65%)	(0.00%)	(54.58%)	(0.00%)
30	5	3		7	1800.00	204.3k	(1.80%)	(0.00%)
30	5	3		8	(4.86%)	(0.00%)	(4.83%)	(0.00%)

instance				SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16) #	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	5	3	9	402.32	10.3k	(0.05%)	(0.00%)
30	5	3	10	(5.45%)	(0.00%)	(6.85%)	(0.00%)
30	5	3	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 2	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 3	(100.00%)	(0.00%)	(100.00%)	(100.00%)
30	5	3	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	3	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	1	(20.83%)	(0.00%)	(28.62%)	(0.75%)
30	5	5	2	156.30	17.9k	966.04	2637
30	5	5	3	(100.00%)	(19.01%)	(100.00%)	(0.00%)
30	5	5	4	(46.36%)	(0.00%)	(74.05%)	(0.00%)
30	5	5	5	(13.67%)	(0.00%)	(31.79%)	(0.32%)
30	5	5	6	(7.61%)	(0.00%)	(25.17%)	(0.00%)
30	5	5	7	(15.14%)	(0.00%)	(22.23%)	(0.00%)
30	5	5	8	(18.40%)	(0.00%)	(76.30%)	(2.59%)
30	5	5	9	(12.83%)	(0.00%)	(44.41%)	(0.00%)
30	5	5	10	(14.21%)	(0.00%)	(23.84%)	(0.00%)
30	5	5	✓ 1	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 2	(94.82%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 4	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 9	(90.14%)	(0.00%)	(100.00%)	(0.00%)
30	5	5	✓ 10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	1	1066.97	3555	(37.28%)	(0.00%)
30	5	10	2	(100.00%)	(5.53%)	(100.00%)	(0.00%)
30	5	10	3	(100.00%)	(0.00%)	(100.00%)	(59.69%)
30	5	10	4	(65.47%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	5	(100.00%)	(0.00%)	(100.00%)	(5.29%)
30	5	10	6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	7	(100.00%)	(0.00%)	(100.00%)	(8.10%)
30	5	10	8	(9.69%)	(0.00%)	(37.11%)	(0.00%)
30	5	10	9	(71.81%)	(0.00%)	(87.20%)	(0.00%)
30	5	10	10	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 1	(44.86%)	(0.00%)	(75.04%)	(0.00%)
30	5	10	✓ 2	(93.02%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 3	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 4	(82.18%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 5	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 6	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 7	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓ 8	(100.00%)	(0.00%)	(100.00%)	(0.00%)

instance					SCIP		SCIP(bivar)	
Nvars	Deg	Ncons	(16)	#	time (dgap)	nodes (pgap)	time (dgap)	nodes (pgap)
30	5	10	✓	9	(100.00%)	(0.00%)	(100.00%)	(0.00%)
30	5	10	✓	10	(100.00%)	(0.00%)	(100.00%)	(0.00%)