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An IP approach to toll enforcement optimization on German motorways

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Abstract

This paper proposes the first model for toll enforcement optimization on German motorways. The enforcement is done by mobile control teams and our goal is to produce a schedule achieving network-wide control, proportional to spatial and time-dependent traffic distributions. Our model consists of two parts. The first plans control tours using a vehicle routing approach with profits and some side constraints. The second plans feasible rosters for the control teams. Both problems can be modeled as Multi-Commodity Flow Problems. Adding additional coupling constraints produces a large-scale integrated integer programming formulation. We show that this model can be solved to optimality for real world instances associated with a control area in East Germany.

1 Introduction

In 2005 Germany introduced a distance-based toll for commercial trucks weighing twelve tonnes or more in order to fund growing investments for maintenance and extensions of motorways. The enforcement of the toll is the responsibility of the German Federal Office for Goods Transport (BAG). It is implemented by a combination of 300 automatic stationary gantry bridges and by tours of 300 control vehicles on the entire highway network. The control tours are operated by teams, composed of one or two inspectors. The vehicles and crews are based at a number of depots and are responsible for certain control areas. The goal of our approach is to construct a set of tours that guarantee a network-wide control whose intensity is proportional to spatial and time dependent traffic distributions. The tours must fit within a feasible crew roster, respecting all legal rules, over a time horizon of several weeks. An important restriction is that each team and vehicle can only control highway sections in their associated control area, close to their base depots. Because of this restriction we can not use sequential and partly anonymous planning approaches for duty scheduling and rostering like in public transport (see [8] ch. 1), since those would lead to infeasible staff rosters. Hence, personalized duty roster planning must be used in our model. These two components are combined into an integrated model using suitable constraints.

To the best knowledge of the authors no optimization approach for toll enforcement has appeared in the literature. Related publications deal with problems such as tax evasion or ticket evasion in public transport; they mainly discuss

the expected behaviour of evaders or payers from a theoretical point of view, e.g. [1], or optimal levels of inspection, see [2]. A recent approach to inspector scheduling in public transport was proposed DSB S-tog in Denmark [5], but in contrast to our problem they focus on temporal scheduling of the inspectors and not on their routes through the network.

The paper is structured as follows: In Section 2 we present models for tour and roster planning, and build from these components an integrated toll enforcement optimization model. This model is used in Section 3 for a computational study. We present results for two instances of our *Toll Enforcement (Optimization) Problem (TEP)* which are part of the MIPLIB 2010[4]. Our computations show that real-world instances of the TEP can be solved to proven optimality for an entire control region over a time horizon of four weeks.

2 Optimal Toll Enforcement

Our toll enforcement optimization model consists of two parts. Section 2.1 presents a model for tour planning, and Section 2.2 a model for the assignment of feasible staff rosters to all inspectors. These two components are combined into an integrated model using suitable constraints.

2.1 A Graph and IP-Model for the Planning of Inspector Tours

The TEP can be described in terms of a *section graph* $G = (S, N)$, in which the nodes $s \in S$ represent *control sections*, which are sub-parts of the motorway network with a length of approximate 25-50 km. An edge $n \in N$ connects two sections, if they have at least one motorway junction or motorway exit in common. Furthermore, there is a given planning horizon T , e.g., four weeks, and some given time discretization Δ . Since it is required to define both the spatial routing and the temporal sequence of the tours, we extend G to a space-time digraph $D = (V, A)$, the *tour planning graph*. Its nodes $v \in V$ are either defined as a pair of a section and a point in time, i.e., $v = (s, t) \in S \times Z := \{0, \Delta, \dots, T - \Delta, T\} \subset [0, T]$, or they represent artificial start and end nodes d_s and d_t for the vehicle paths (depot nodes). Directed arcs connect either adjacent time intervals of the same section, or they connect adjacent sections, i.e., $\forall s \in S$ there is $((s, t_1), (s, t_2)) \in A$ with $t_2 = t_1 + \Delta$, starting at $t_1 = 0$ until $t_2 = T$, and if $(s_1, s_2) \in N$ it holds that $((s_1, t), (s_2, t + \Delta)) \in A \forall t \in Z \setminus \{T\}$. Figure 1 illustrates this construction. In addition, arcs are inserted from the start depot to all other non-depot nodes and from all non-depot nodes to the end depot node. Finally, a profit value is associated with each node $v = (s, t)$.

We consider the problem of finding a feasible (d_s, d_t) -path in D for each vehicle f on each day, that respects a tour length restriction of 8h30. Each control tour corresponds to such a *control path*. The profit w_p of a control path p is the sum of the profit of its visited nodes. This approach could be seen as a vehicle routing problem with profits under some additional constraints. Vehicle routing is an well-established research area, see [6] for an overview. For the case of dealing with profits Feillet et al. [3] give a literature survey.

There are restrictions on the feasible starting and ending times of the control tours; the feasible times are defined by the (*Working*) *Time Windows*. Let P be

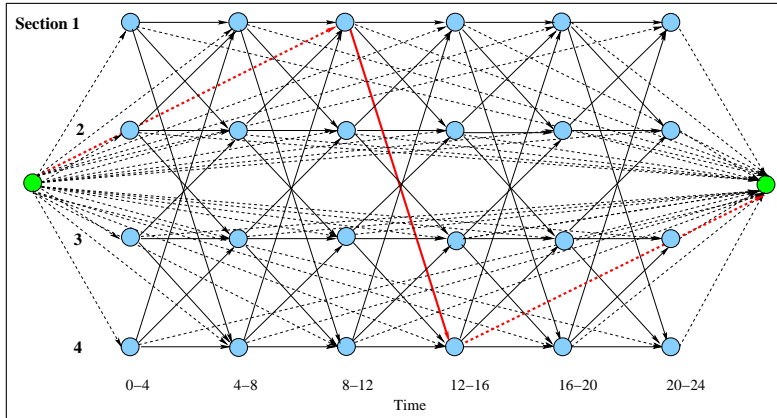


Figure 1: Construction of the tour planning graph.

set of all control paths in D and $P_{f,j} \subset P$ the set of all paths that are feasible for vehicle $f \in F$ and start at day $j \in J$. Furthermore for a given section $s \in S$, the set of all paths $p \in P$ that visit a node $v = (s, t_i) \in V$ is denoted by P_s and the minimum control quota is named by κ_s . The *Tour Planning Problem (TPP)* is then formulated as a 0/1 multi-commodity flow problem in D , where vehicles f represent the commodities. We introduce 0/1-variables z_p , $p \in P$, that indicate if tour p is chosen or not. Then the TPP can be modeled by the following integer program:

$$\max \sum_{p \in P} w_p z_p \quad (1)$$

$$\sum_{p \in P_{f,j}} z_p \leq 1, \quad \forall (f, j) \in F \times J \quad (2)$$

$$\sum_{p \in P_s} z_p \geq \kappa_s, \quad \forall s \in S \quad (3)$$

$$z_p \in \{0, 1\}, \quad \forall p \in P. \quad (4)$$

In this formulation the objective function (1) maximizes the profit of all selected tours. The requirement that each vehicle f can do at most one tour per day is assured by Constraints (2). Constraints (3) guarantee that at least κ_s paths, that traverse section s , are chosen in any feasible control schedule. The last constraints (4) are the integrality constraints.

2.2 Integration of Duty Roster Planning

The second task in the TEP is the planning of the rosters, called the *Inspector Rostering Problem (IRP)*. There, the objective is to minimize the costs, which can be real costs of a duty or artificial costs that penalize some feasible but inappropriate sequence of duties. An example for this is if a duty on the next day respects the minimum rest times, but starts earlier than the day before, e.g., Mo 8-17, Tu 6-15.

We formulate the IRP as a multi-commodity flow problem in a directed graph $\tilde{D} = (\tilde{V} := (\hat{V} \cup \{s, t\}), \tilde{A})$ with two artificial start and end nodes s, t . The nodes $v \in \hat{V}$ represent duties as a pair of day and time-window. The arcs $(u, v) \in \tilde{A} \subseteq \tilde{V} \times \tilde{V}$ model a feasible sequence of two duties according to legal rules. Again the depot nodes s and t are connected with all non-depot nodes. Furthermore, let M be the set of all inspectors and W the set of all weeks. Then $\hat{V}_w \subseteq \hat{V}$ is defined as the set of all duties in week $w \in W$. In addition, let t_v be the duration of duty $v \in \hat{V}$, while a_{min}^m and a_{max}^m indicate the minimum or maximum weekly labor time of inspector $m \in M$. Introducing flow variables $x_{u,v}^m$ for each arc (u, v) and inspector m , we propose the following integer programming formulation for the IRP:

$$\min \sum_{m \in M} \sum_{(u,v) \in \tilde{A}} c_{u,v} x_{u,v}^m \quad (5)$$

$$\sum_v x_{s,v}^m = 1, \quad \forall m \in M \quad (6)$$

$$\sum_k x_{v,k}^m - \sum_u x_{u,v}^m = 0, \quad \forall v \in \hat{V}, m \in M \quad (7)$$

$$\sum_{u \in \hat{V}_w} \sum_v t_u x_{u,v}^m \leq a_{max}^m, \quad \forall w \in W, m \in M \quad (8)$$

$$\sum_{u \in \hat{V}_w} \sum_v t_u x_{u,v}^m \geq a_{min}^m, \quad \forall w \in W, m \in M \quad (9)$$

$$x_{u,v}^m \in \{0, 1\}, \quad \forall (u, v) \in \tilde{A}, m \in M. \quad (10)$$

The objective function (5) minimizes the cost. Constraints (6) guarantee that exactly one roster path is chosen for each inspector. Such a path is called *Inspector Roster Path*. By (7) we model the flow conservation in each non-depot node. Maximum and minimum weekly working times are enforced by inequalities (8,9). Finally, in (10) we have the integrality constraints for the flow variables.

The TPP and the IRP are connected by coupling constraints into an integrated formulation for the TEP. To this purpose, we denote by $P_{f,u}$ the set of all control paths feasible for vehicle f and duty $u \in \hat{V}$. The parameter n_f gives the number of inspectors in vehicle f and by $m \in f$ it is meant, that the inspector m uses vehicle f (which is a fixed assignment). This leads to the following coupling constraint.

$$\sum_{p \in P_{f,u}} n_f z_p - \sum_{m \in f} \sum_v x_{u,v}^m = 0 \quad \forall f \in F, u \in \hat{V} \quad (11)$$

The constraints (11) ensure that for each control path p in D all inspectors in the corresponding team have a feasible inspector roster path with a duty in the time horizon of the planned tour. The objective function is a combination of collecting the profit (1) and minimizing the cost (5).

Table 1: IP-Solution analysis of some instances of the control region Brandenburg. The parameter “dc” indicates the use of duty costs, “dm” a pre-given duty mix according to the time windows and $\Delta(h)$ the time discretization. The value of the root LP is denoted by $v(\text{lp})$ and the best integer solution by v^* . The solution time limit equals $6h = 21600\text{sec}$.

inst.	$\Delta(h)$	dc	dm	columns	rows	$v(\text{lp})$	v^*	gap(%)	time(sec.)
T1	4	x	x	136264	17245	368997.56	350288.76	0.06	12283.0 ¹
T2	4	x	-	136264	17237	449883.49	435343.00	opt.	181.62
T3	4	-	x	128808	17257	533597.42	499561.80	1.84	21600.00
T4	2	x	x	376328	22877	677133.58	644458.66	0.05	21600.00
T5	2	x	-	376328	22869	732316.22	709276.99	0.04	3554.2 ¹
T6	2	-	x	368872	22889	846722.87	796495.47	1.52	21600.00
bab5	4	x	x	21600	4964	117062.16	106411.84	opt.	640.34
bab3	2	x	x	393800	23069	686288.87	655559.30	0.45	21600.00

3 Computational Results and Conclusion

We tested our model on instances associated with the control region Brandenburg that are based on real-world data. Six instances (T1, . . . , T6) model four week planning periods and basic legal rules like minimum rest times, but they differ in some parameter settings, see Table 1. The others belong to the MIPLIB 2010, see below. In the table, columns “dc” and “dm” characterize the instances as follows: “dc” stands for using direct duty costs while “dm” demands a duty mix regarding to the time windows in the roster, e.g., approx. 40% of all duties must begin at 6am and end at 15pm. All computations were done on a PC with an Intel i7 Quad-Core processor with 2.97 GHz and 16 GB RAM. CPLEX 12.2 [7] was applied as an IP solver using four threads. Furthermore, we used a time limit of 6h and a limit of 10 GB for the Branch&Bound tree.

Table 1 shows the results of our computations. The term $v(\text{lp})$ denotes the optimal value of the root LP and v^* the best integer value. An important result is that all duty-cost instances could be solved to near-optimality within the given time limit of six hours. An interesting observation is that a larger number of rows and columns does not always lead to an increase of solution times or higher integrality gaps. Especially in the 4h-case (T1, T2), a major part of the complexity originates from the duty mix constraints. In fact, the solution time is reduced drastically if we omit them. Another important observation is that the problem becomes more difficult if we replace duty costs by coefficients that penalize inappropriate rosters (T3, T6).

The MIPLIB 2010 problem library [4] contains two instances from the TEP. Both model the TPP in a slightly different way than described in Section 2.1, which, however, produces equivalent results. The first instance is bab5²: It is similar to T1, but models a plan for 8 days, see Table 1. It can be solved optimally within 15 minutes by CPLEX 12.2.

The second instance bab3 has the same parameters as T4 (see Table 1), except for a small difference with respect to the length of a duty. It is more difficult to solve compared to T4 since it has a gap of 0.45% after 6 hours of running

¹Memory-Limit reached

²“BAB” is the official abbreviation of the German motorways

CPLEX. Even after several days of computation this instance could not be solved to optimality.

We conclude that we are able to model the TEP by two graph models and then to formulate it by an IP that integrates all important legal rules. Solving this IP results in high quality solutions for real-world instances with time discretizations of two or four hours. In the future we want to integrate additional rules and implement advanced algorithms to solve more complex instances.

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