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**Railway Track Allocation**

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**Simulation, Aggregation, and  
Optimization**

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# Railway Track Allocation

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## Simulation, Aggregation, and Optimization

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### Abstract

Today the railway timetabling process and the track allocation is one of the most challenging problems to solve by a railway company. Especially due to the deregulation of the transport market in the recent years several suppliers of railway traffic have entered the market in Europe. This leads to more potential conflicts between trains caused by an increasing demand of train paths.

Planning and operating railway transportation systems is extremely hard due to the combinatorial complexity of the underlying discrete optimization problems, the technical intricacies, and the immense size of the problem instances. In order to make best use of the infrastructure and to ensure economic operation, efficient planning of the railway operation is indispensable.

Mathematical optimization models and algorithms can help to automatize and tackle these challenges. Our contribution in this paper is to present a renewed planning process due to the liberalization in Europe and an associated concept for track allocation, that consists of three important parts, simulation, aggregation, and optimization. Furthermore, we present results of our general framework for real world data.

## 1 Introduction

This paper is about a framework for the allocation of available railway capacity. We address the simulation and the optimal allocation of the available railway track capacity. This problem is a major challenge for a railway company, independent of whether a free market, a private monopoly, or a public monopoly is given. Especially due to the deregulation of the transport market in the recent years several railway operators have entered the market. This leads to an increase of train path requests and consequently there are many conflicts between them. Furthermore, railway infrastructure networks

are very expensive assets and rather rigid due to a long-term down- and up-grade processes. Therefore, it must be the aim of all railway infrastructure providers to utilize the railway capacity in an efficient way.

Planning and operating railway transportation systems is extremely hard due to the combinatorial complexity of the underlying discrete optimization problems, the technical intricacies, and the immense size of the problem instances. Standardized processes, accurate simulation models, and mathematical optimization techniques can result in huge gains for both railway customers and operators, e.g., in terms of cost reductions or service quality improvements. We tackle this challenge by developing a framework using novel mathematical models and associated innovative algorithmic solution methods for large scale instances. This allows us to produce for the first time reliable solutions for a real world instance, i.e., the *Simplon* corridor in Switzerland.

From a mathematical point of view the optimization problem can be stated as a multi-commodity flow problem through an extremely large network in space and time with certain additional constraints. The problem is well known in the literature and in the seminal paper of [Caprara et al. \[2002\]](#) it was shown that it is  $\mathcal{NP}$ -hard. In [Section 4.1](#) we will describe the used optimization models and algorithms in more detail.

Only recently practical problem sizes are tractable due to the development of improved models and algorithms. Nevertheless a decomposition of the problem remains necessary. On the one hand for networks, or at least for long railway corridors, only simplified macroscopic models with a simplified routing through the railway infrastructure are considered, as in [Cai & Goh \[1994\]](#); [Brännlund et al. \[1998\]](#); [Caprara et al. \[2002\]](#); [Borndörfer et al. \[2006\]](#); [Cacchiani \[2007\]](#); [Borndörfer & Schlechte \[2007\]](#); [Cacchiani et al. \[2008\]](#); [Fischer et al. \[2008\]](#); [Borndörfer et al. \[2009\]](#). On the other hand, routing or re-routing through complex stations can be considered on a more detailed, but of course only on a local level, see [Zwaneveld et al. \[1996\]](#); [Caimi et al. \[2004\]](#); [Lusby et al. \[2006\]](#); [Caprara et al. \[2007\]](#); [Klabes \[2010\]](#); [Harrod \[2010\]](#). The only recent reference, to the best knowledge of the authors, describing the interaction of both levels is [Caimi et al. \[2009\]](#), in which a top-down approach is used.

We present a fully integrated bottom-up approach that is based on microscopic simulation, automatic network aggregation (and disaggregation), and discrete optimization. The contribution of this paper is to put several pieces and software tools together, i.e., `OpenTrack`, `NETCAST`, and `TS-OPT`, in order to release the optimization potential of the railway track allocation process. One challenging part is to establish interfaces between railway simulation software and state of the art mathematical optimization tools that allow an algorithmic transformation between the detailed level of simulation and a more aggregated level used in optimization. Hence, to tackle large size instances of the track allocation problem sophisticated mathematical tech-

niques are needed.

We will focus in that paper on the motivation and origin of our approach. Main contribution will be to discuss track allocation as part of a greater whole and to establish an interaction between microscopic simulation and macroscopic optimization. Section 2 will give a detailed description of the planning process of a railway system after the liberalization in Europe. The term railway capacity is discussed in Section 3. Finally, we present our framework of simulation based optimization of railway track allocations in Section 4.

## 2 Planning in Railway Transportation

The planning process in public transport can be divided into three major levels - strategic, tactical and operational planning, see Table 1. Public transport, especially railway transportation, is such a technically complex and large-scale system, that it is impossible to consider it at once to its full extend. In addition, the different planning horizon of certain decisions enforces a decomposition. Therefore a sequence of hierarchical planning steps has emerged over the years. In addition several important parties are involved in railway transportation planning process, i.e., train operating companies and railway infrastructure providers. We will use analogously the term railway undertaking (RU) and infrastructure manager (IM) to stick to the terminology of the European committee. Furthermore, several national and international institutions have a huge political influence on railway transportation, which is therefore somehow in the borderline between a social or public good and a product that can be traded on a free liberalized market. The special case of the changing European environment is discussed in detail in Klages [2010].

In contrast to that, fully private aviation companies or independent urban public transport companies can perform the complete planning more internally. In the airline industry the needed capacity, that are the slots at the airports, are given by grandfather rights in forehand, see Barnier et al. [2001] and Castelli et al. [2010]. Borndörfer et al. [2008, 2009] and Hanne & Dornberger [2010] give a recent survey about the potential of optimization for transportation systems and the differences between the planning process in the airline industry, the urban public transport, and the railway traffic. In case of urban public transport the planning process is discussed in Weider [2007] and Borndörfer et al. [2007]. A detailed description of the process in the airline industry can be found in Grönkvist [2005] and Barnhart & Laporte [2007]. Bussieck [1997] describes the use of discrete optimization in the planning process of public rail transport in case of an integrated system. Analogous considerations can be found in Liebchen [2006] and Lusby et al. [2009]. There the planning steps are classified with respect to the time

horizon and their general purposes.

level	time horizon	goal
strategic	5-15 years	resource acquisition
tactical	1-5 years	resource allocation
operational	24h - 1 year	resource consumption

**Table 1:** Planning steps in railroad traffic, source: [Bussieck et al. \[1997\]](#).

The strategic or long-term part (resource acquisition) concerns network design and line planning. On the tactical level (resource allocation) the offer, usually a timetable or a schedule has to be created, as well as the schedules for the needed resources. After that the resources, e.g., rolling stock, vehicles, aircrafts and crews, are considered on the operational stage (resource consumption).

Finally, on the day of operation re-scheduling and dispatching problems have to be faced. These kind of problems have a different flavor. On the one hand decisions and therefore solutions have to be made very fast because of the changing real-time setting. On the other hand only partial information about the "scenario" is given. Usually data has to be taken into consideration with respect to its appearance, i.e., in a so called on-line fashion. More details on these kind of problems in general can be found in [Grötschel et al. \[2001\]](#); [Albers \[2003\]](#). Recent approaches are to establish fast methods which bring the "real" situation back to the "planned" one, if possible, see [Potthoff et al. \[2008\]](#); [Rezanova & Ryan \[2010\]](#); [Jespersen-Groth et al. \[2009\]](#).

In [Klabes \[2010\]](#) the planning process is newly considered for the case of the segregated European railway system. The responsibilities of the planning steps refer directly to either the railway undertakings or the infrastructure manager. Nevertheless the long-term decisions in up- or downgrading the network is highly influenced either by the railway undertakings and their demands as well as by political conditions.

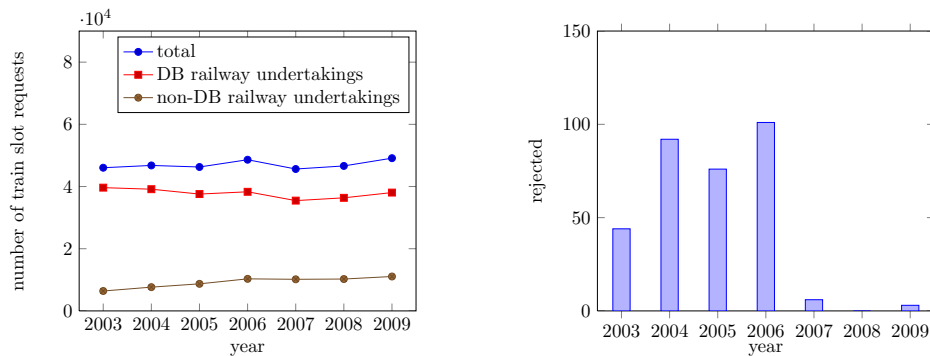
In case of a passenger railway undertaking the desired timetable to implement a given line plan induces train path requests, which is one input for the track allocation problem. The requirements of train path requests for cargo or freight railway operators differ a lot from the case of passenger trains because they usually have more flexibility, i.e., arrival and departures are only important at stations where loading has to be performed. In general freight railway operators need a mixture of annual and ad hoc train paths. This is of course highly influenced by the resident industry customers and freight concept of the operating railway undertakings. We have collected such data for the German subnetwork *hakafu\_simple* to estimate the demand of the railway freight transportation, see [Schlechte & Tanner \[2010\]](#).

The essential connection between (RU) and (IM) is the step to determine the complete track allocation, which is the focus of this paper. However,

we take mainly the point of view of a railway infrastructure provider, which is interested in optimizing the utilization of the network. That is to determine optimal track allocations with respect to utilization of infrastructure resources. In contrast to passenger timetabling where one asks for the ideal arrival and departure times to realize a timetable concept or a line plan. The result of that can be seen as a set of train path requests without flexibility. Railway optimization methods from the railway undertaking point of view for passenger traffic can be found in [Caprara et al. \[2007\]](#). State of the art modeling and optimization approaches to periodic timetabling, which is the usual type of schedule for passenger railway traffic, is widely studied by [Liebchen \[2006\]](#), [Caimi et al. \[2009\]](#) and others.

The induced competition for railway capacity allocation in public railway systems in Europe severely impacts the allocation procedure. In the past a single integrated railway company performed the complete planning. Its segregation reduces the competences of the railway infrastructure manager to perform network planning, capacity allocation and re-scheduling with respect to the infrastructure aspects only. Thus the infrastructure manager has just limited information during the planning process and needs to respect the confidential information of the railway undertakings. Moreover, new railway undertakings enter the established market for railway operation, which increases the complexity of the planning process. On the left hand of [Figure 1](#) the changing environment is illustrated by the growing number of train path requests from railway undertakings independent from the former integrated railway company "Deutsche Bahn". However, it can be observed that it is a long-term process to release the liberalization potential in the railway market. On the right hand of [Figure 1](#) the number of rejected train path requests for the same periods are shown. It can be seen that at the start of the segregation from 2003 until 2006 a lot of requests had to be rejected by DB Netz. Efforts to decrease these numbers by providing alternative paths were apparently successful in the following years. This leads to the suspicion that the complexity of the novel process was underestimated and was badly managed in the beginning, see also [Klabes \[2010\]](#). However, more transparency and flexibility of the allocation can be verified and documented. The business report for the year 2009 of the [Trasse Schweiz AG \[2009\]](#) documents this as well. In the Swiss network naturally a lot of different railway undertakings are operating, e.g., in 2009 they were 29 train operating companies which submitted train path requests. The geographical position in central Europe and the limited transportation possibilities through the Alps causes that. The future challenge for Switzerland will be to handle the complex track allocation process as the following extract already highlights:

The regulation of the conflicts arising train path orders of the annual timetable 2010 was despite or even because of the financial or economic crisis in comparison to the last years exten-



**Figure 1:** Requested train paths at DB, source: Klabes [2010]

sive and time-consuming. Indeed the number of submitted train path requests by cargo operators for the annual timetable 2010 decreased up to 10 percent in comparison to the last year. However, railway undertakings (RM) concentrated her orders due to the cost pressure and competitive market conditions on the most attractive time windows and stick much longer to their original requests. Nevertheless, we managed together with all infrastructure providers to find for all conflicts alternative train paths, which were accepted by the railway undertakings. There are three different railway infrastructure providers in Switzerland, i.e., BLS, SBB, and SOB. No train path request had to be rejected.

The competing railway undertakings should interact on a transparent and free market. The creation of such a market for railway capacity is a key target of the European Commission, hoping that it will lead to a more economic utilization of the railway infrastructure. Even more liberalization of the railway system should lead to a growing market and allow for innovation trends like in other old-established industries, e.g., aviation industry, telecommunication or energy market.

After the acceptance of train paths each railway undertaking determine the operating timetable, which acts as input for the planning of the needed resources. In case of the railway system the rolling stock rotations have to be constructed, which is very complex problem due to several regularities and maintenance requirements, see Fioole et al. [2006]; Anderegg et al. [2003]; Eidenbenz et al. [2003]; Peeters & Kroon [2008].

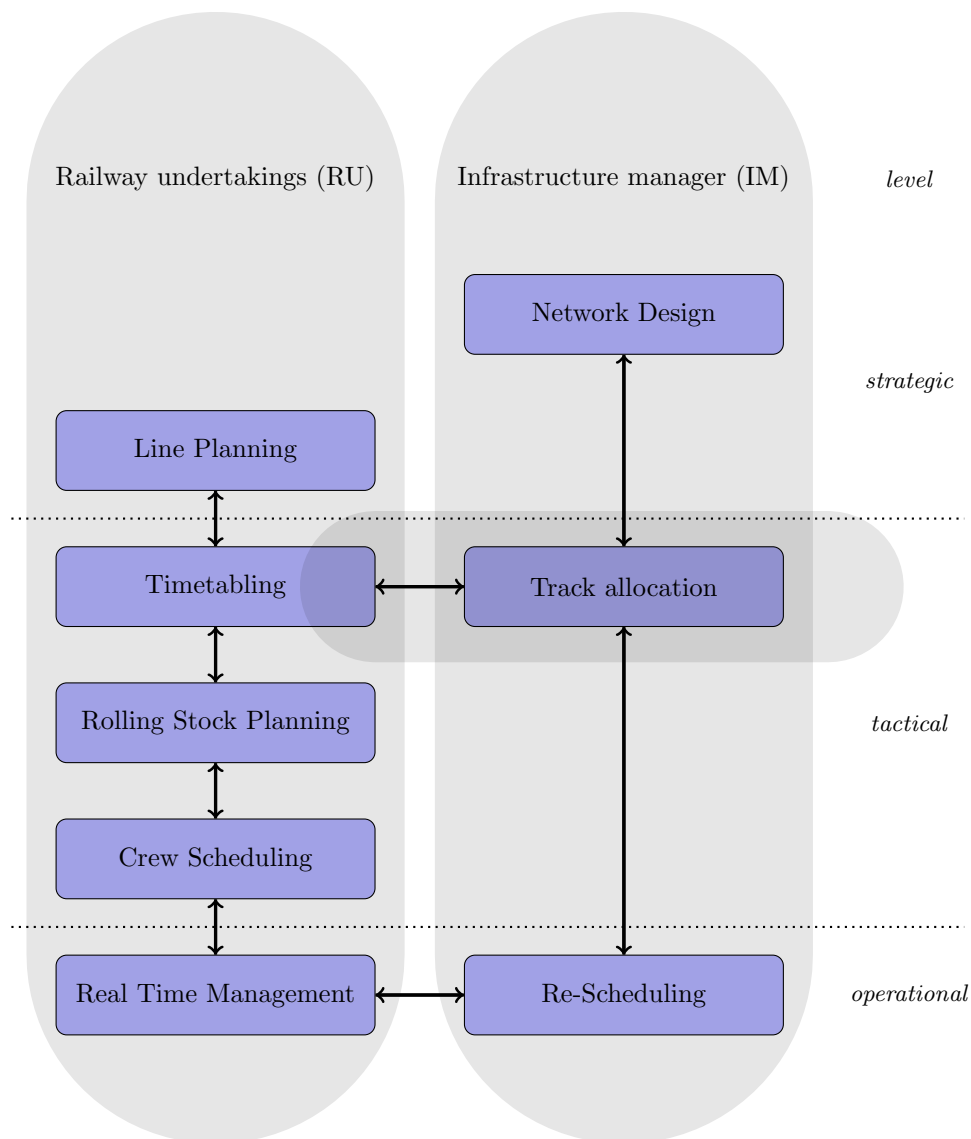
In public transport vehicle scheduling and in airline industry aircraft rotation planning are the analogous tasks, see Löbel [1997] and Grönkvist [2005]. Major objective is to reliably operate the timetable with a minimum cost, which is in general minimizing the number of engines, wagons,



vehicles, aircrafts etc. Another key requirement for planning railway rolling stock rotations is to provide regularity of the solutions. This means, e.g., that a train, that runs in the same way every day of the week, will also be composed in the same way every day of the week, always using the same cars from the same preceding trains in its rotation. Such a regime simplifies the operation of a railway significantly. However, the rule can not always be followed. Trains may run later on weekends, or not at all on certain days, e.g., in order to perform a maintenance operation or according to variations in the passenger demands. Although it is intuitively clear, it is even not easy to give a precise definition what regularity actually means.

The output of this planning step are passenger trips, that are trips of the published timetable, and deadhead trips, i.e., "empty" movements of the trains given by the constructed rolling stock rotation. These tasks need to be assigned to crews, which have to execute them. Recent work on railway crew scheduling can be found in [Abbink et al. \[2005\]](#) and [Bengtsson et al. \[2007\]](#). As already mentioned real time problems on the day of operation have quite different requirements, even if from a mathematical modeling point of view these problems can be formulated in a very similar way. In railway transportation disruption and delay management is very difficult because local decisions have a huge influence on the complete timetable system. Nevertheless, easy and fast rules of thumb are used to decide if trains have to be re-routed, have to wait, or even have to be canceled. [D'Ariano et al. \[2008\]](#), [Corman et al. \[2010b\]](#), and [Corman et al. \[2010a\]](#) presented a real-time traffic management system to support local dispatching in practice by operations research methods. On the basis of this renewed timetable, rolling stock rosters and crew schedules have to be adopted, see [Jespersen-Groth et al. \[2007\]](#); [Clausen et al. \[2010\]](#); [Potthoff et al. \[2008\]](#); [Rezanova & Ryan \[2010\]](#).

Every single step in this idealized sequential planning process is a difficult task by itself or even more has to be further divided and simplified into subproblems. In [Figure 2](#) the novel process is illustrated in case of the segregated railway industry in Europe. A main application of track allocation is to determine if a requested timetable is operational implementable or not, which is the main focus of this work. But, we want to mention that due to the segregation the track allocation process directly gives information about the infrastructure capacity. This feedback for the department concerning network design is very important and can support the evaluation of several alternatives. Even more long-term infrastructure decisions could be evaluated by applying automatically the track allocation process, e.g., without full details on a coarse macroscopic level but with different demand expectations. Even if we did not developed our models and tools for this purpose, it is clear that suitable extensions or simplifications the other way around of our models could support infrastructure decisions in a quantifiable way. For example major upgrades of the German railway system like



**Figure 2:** Idealized planning process for railway transportation in Europe.

the high-speed route from Erfurt to Nürnberg or the extension of the main station in Stuttgart can be evaluated from a reliable resource perspective. The billions of € for such large projects can then be justified by reasonable quantifications of the real capacity benefit with respect to the given expected demand.

An obvious, but unavoidable, disadvantage of the decomposition is that the in some sense an "optimal" solution for one step serves as fixed input for the subsequent problem. Therefore one cannot expect an overall "optimal"

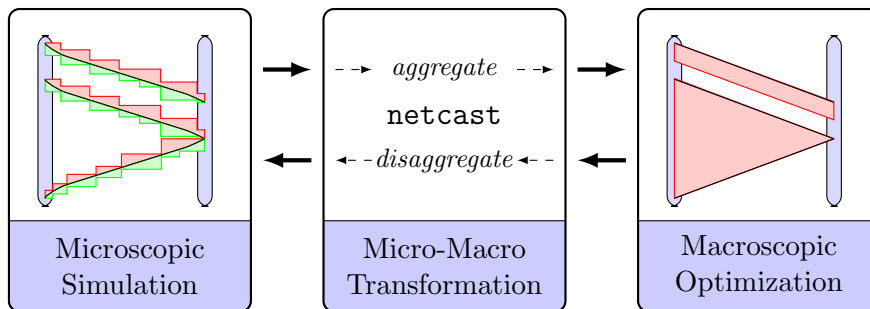
solution for the entire system in the end not even a feasible one. In that case former decisions have to be changed, and a part or the complete process has to be repeated. Prominent examples are regional scenarios for urban public transportation where traditional sequential approaches are not able to produce feasible schedules. Weider [2007] demonstrates in case of vehicle and duty scheduling how integrated models can cope with that and even more can increase the overall planning efficiency. Nevertheless, this hierarchic planning provides a partition of the traffic planning problem into manageable tasks. Tasks, which lead directly to quantifiable optimization problems and which nowadays can be solved by linear and integer programming to optimality or at least with proven optimality gaps. Problem standardization, automatization, organizing data, computational capabilities, mathematical modeling, and sophisticated algorithmic approaches on a problem specific but also on a general level afford this success story. As an prominent example for this we refer to the dutch railway timetable - the first railway timetable which was almost constructed from scratch. In fact the entire planning process was decomposed and each planning problem at Netherlands Railways (NS) was solved by the support of exact or heuristic mathematical approaches and sophisticated techniques, in particular linear, integer and constraint programming. More details can be found in the prizewinning work Kroon et al. [2009], which was honored with the *Franz Edelman Award* 2008. A prize which is rewarded to outstanding examples of management science and operations research practice in the world.

### 3 Modeling Railway Capacity

To establish an optimization process for the allocation of “railway capacity”, we first have to define capacity and derive a resource based model for a railway system in an appropriate way. Railway capacity has basically two dimensions, a space dimension which are the physical infrastructure elements as well as a time dimension that refers to the train movements, i.e., occupation or blocking times, on the physical infrastructure. A major challenge of both dimensions is the granularity, the potential size, and the arbitrary smooth variation of time. We will not describe microscopic aspects of the railway system. However, the definition of resources and capacity is inspired by the work of Landex et al. [2008]. In the literature several approaches work directly on a microscopic level with the disadvantage that only instances of small size can be handled, see Zwaneveld et al. [1996, 2001]; Lusby et al. [2009]; Fuchsberger [2007]; Delorme et al. [2009]; Klabes [2010]. This is why we restrict the use of microscopic and very detailed data to the Simulation case.

Nevertheless, on a planning stage it is not possible to consider all these details and also not necessary. Hence, the main application of a macroscopic

model is to evaluate different timetable concepts or infrastructure decisions on a coarse granularity. Only recently approaches were developed to tackle larger corridors or even network instances by exact optimization approaches. [Lusby et al. \[2009\]](#) give a recent survey on the track allocation problem and railway timetabling. [Liebchen \[2006\]](#) and [Caimi \[2009\]](#) classified the approaches in the literature according to used solution methods and scale of the models. With respect to that general classification, we are tackling the problem of non-periodic scheduling with state of the art linear and integer programming methods, see [Borndörfer & Schlechte \[2007\]](#), [Borndörfer et al. \[2009\]](#), and [Borndörfer et al. \[2010\]](#). This is called the level of optimization. The main advantage of the optimization approach is that the algorithm has a simultaneous view on the train path requests and on the entire railway network. Most of the heuristic approaches consider only a step-by-step solution by inserting new train paths without backtracking. This results only in small re-allocations of preassigned train paths, such that the new track allocation highly depends on the train paths that were assigned before. In [Caimi \[2009\]](#) a top-down approach is presented and used to handle the complete Swiss network by a priori decomposition of the network into different zones. In contrast to that, we use a bottom-up approach to define a macroscopic railway model, see [Schlechte et al. \[2011\]](#) and [Borndörfer et al. \[2010\]](#). This step is called aggregation. It means that a microscopic network with some given train routes is transformed by a set of algorithmic rules to a macroscopic level without losing important information with respect to the allocation of track capacity. This allows to get a tractable problem size for exact mathematical optimization and to exclude a lot of microscopic information that is useless for the purpose of track allocation.



**Figure 3:** Idealized closed loop for optimization of railway track allocation based on simulation.

## 4 Closing the Cycle - from Simulation to Optimization and back again

The concept of our framework is shown in Figure 3. Starting point is an accurate simulation tool that describes the railway system as "real" as it can be. In our studies, we are using the microscopic simulation tool `OpenTrack`, see [Hürlimann \[2001\]](#). This is a synchronous simulation tool. This means that the railway traffic is simulated synchronous, as it would be in reality. Hence, in a simulation tool the network, the signal system, and the train engines are modeled exact as they would behave in reality. The infrastructure is modeled by a graph and the movement of trains by differential equations. The input of a simulation tool is a timetable. Major aim is to analyze and prove if this single timetable is operable in reality, or not. It also simulates disturbances in operation to evaluates the robustness of a timetable. A recent and important example is the timetable verification test for the planned underground station in Stuttgart in Germany, where a desired timetable was tested by a simulation tool as a proof that the station is able to deal with the expected passenger demands.

The input for the *aggregation* is the microscopic network and a set of standard train routes representing the potential train paths in the network with accurate running and blocking times. It should be possible to export and extract this data by any simulation tool. Even if the set of potential train paths and simulation runs can be very large, this calculation of simulation data has only to be done once in the beginning. Based on that the aggregation method constructs a new network of station and tracks. Stations are either points where a train could potentially stop or wait or points of interactions of train routes like crossing or converging. Tracks are inserted between the stations with running times for each active train type and headway times between each pair of active train types. Those can easily be computed by the exported blocking times. Another important issue is the rounding of the exact running and headway times to a time discretization, e.g. six seconds. Such a discretization has either to be used to avoid an unsolvable size of the space-time network as well as it is not necessary and useful to plan a timetable on a level of seconds. According to that we developed a cumulative rounding technique that guarantees a small rounding error. The proof of that and the complete method is described and analyzed in [Schlechte et al. \[2011\]](#).

On the one hand the aggregated macroscopic models are precise enough to allow for valid allocations with respect to blocking times and on the other hand they are simplified and shrunk to a coarse level, which allows for solving large scale optimization track allocation problems. The main advantage of this approach is that we can estimate the quality of the macroscopic model, i.e., we developed bounds for the rounding error of running and headway

times in [Schlechte et al. \[2011\]](#).

We will explain the optimization in more detail in the next section. Major aim of optimization is to select from a set of feasible solutions the best one with respect to a given objective function. In our case this is to select the "best" timetable from all potential ones. The results of the optimization are macroscopic track allocations in an aggregated network. By trivial retransformation it is possible to lift them back into the microscopic infrastructure network and to refine the time discretization. Due to the construction and conservative definition of running and headway times all macroscopic train paths remain conflict-free after the disaggregation to the microscopic model. Our tool `NETCAST` manages the technical part by constructing and running the aggregation and disaggregation.

## 4.1 Optimization

As an input for the optimization model we have an aggregated description of the underlying network with stations and tracks that connect those stations. Furthermore, for each track running and headway times are given according to chosen train types and with respect to a fixed discretization. A train type is a collection of several trains with similar driving dynamics and train compositions. For the input we use a standardized format described in [Erol et al. \[2008\]](#). In addition train path requests are given as an input. A profit value is assigned to each request that is decreasing in case of deviations of the ideal arrival or departure times, i.e., feasible time windows have to be specified, see again the data format described in `TTPLIB`. Hence, passenger trains that are part of a network-wide periodic timetable have in most cases a very small interval that only permits its desired arrival and departure times. The optimization model tries to find a conflict-free track allocation such that each request respects its given time intervals. Moreover, it is possible to fix train paths in the model because they are already confirmed, but they still have to be considered because of the headway times. Obviously, train requests can be rejected in case there are no conflict-free train paths associated with this request during the time horizon.

Mathematically we model the problem by a space-time graph  $G = (V, A)$ . The nodes correspond to a pair of space and time. The arcs model a train traversing a track or a train waiting inside a station. For each train request an arc is defined, if the train runs on the corresponding track and if the corresponding points in time fit to its feasible time interval. A feasible solution is a set of train paths in  $G$ , at most one for each train path request, such that all headway conditions are respected.

We denote the set of train path requests by  $I$  and the set of all train paths in  $G$  by  $P = \cup_{i \in I} P_i$ . As mentioned before operational railway safety restrictions, i.e., minimum headway constraints, can be handled by conflict sets in  $G$ . This modeling approach was introduced by the pioneer works of

Brännlund et al. [1998] and Caprara et al. [2006] on railway track allocation. Each conflict  $\gamma \in \Gamma$  consists of a subset of arcs  $A_\gamma \subseteq A$  that are in conflict with respect to the headway times, in this case of conflict only one arc can be chosen from each conflict set. Introducing a decision variable  $x_p$  and utility value  $u_p$  for each path  $p \in P$ , the track allocation problem can be stated as the following path packing formulation (PPP):

$$\begin{aligned}
& \text{(PPP)} \\
& \max \quad \sum_{p \in P} u_p x_p & \text{(i)} \\
& \text{s.t.} \quad \sum_{p \in P_i} x_p \leq 1, \quad \forall i \in I & \text{(ii)} \\
& \quad \sum_{p \cap A_\gamma \neq \emptyset} x_p \leq 1, \quad \forall \gamma \in \Gamma & \text{(iii)} \\
& \quad x_p \in \{0, 1\}, \quad \forall p \in \mathcal{P} & \text{(iv)}
\end{aligned}$$

Constraints (PPP) (ii) ensure that each request is implemented by at most one path. Conflict constraints (PPP) (iii) make sure that no headway or station conflict is violated. (PPP) (iv) state that all path variables  $x_p$  are zero or one. Finally, objective (PPP) (i) is to maximize the profit of the schedule.

To tackle large scale instances and handle headway conflicts in an algorithmic efficient way we developed an extended model using so-called arc configuration variables, see Borndörfer & Schlechte [2007]. In Borndörfer et al. [2010] we focus on the ingredients of our optimization tool TS-OPT to produce high quality solutions in reasonable computation time; that are dynamic column generation, proximate bundle method and a rapid branching heuristic embedded in a branch and price scheme. Fischer & Helmberg [2010] propose a dynamic graph generation to solve the pricing problems for very large graphs, i.e., the original objective function has to fulfill the requirement that an earlier arrival is always beneficial. These are only some examples that documents that track allocation is a fruitful research area and that further algorithmic improvements are still possible and allow for the optimization of large-scale real world instances.

## 4.2 A Case Study

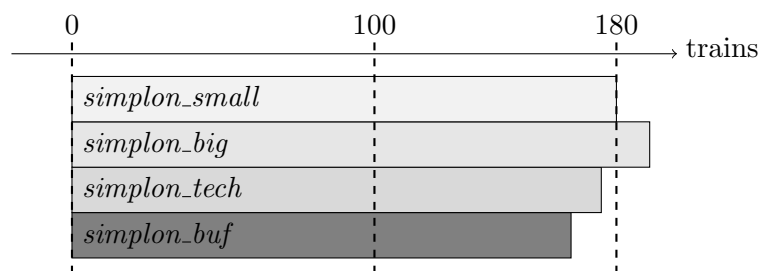
We applied our framework on real world data to analyze the technical and theoretical capacity of a railway corridor. As an example for the potential, Figure 4 shows simplified the output of an optimization for four different railway models of the considered corridor – the Simplon. There are only two north-south railway connections through the Alps in Switzerland, namely, the Gotthard corridor and the Lötschberg-Simplon corridor. The Simplon connects Switzerland and Italy and is therefore of strategic importance for the international railway freight traffic. The four network models *simplon\_small*, *simplon\_big*, *simplon\_tech*, and *simplon\_buf* differ in their

routing possibilities, buffer times, and hence in their theoretical ”maximum“ capacities, see Figure 4. The assumptions, details, i.e., the requested train mixture, and results of this case study on the Simplon corridor are presented in [Borndörfer et al. \[2010\]](#).

We tested our framework using the tools `OpenTrack`, `NETCAST`, and `TS-OPT`, to evaluate the Micro-Macro Transformation, to analyze macroscopic railway track allocations, and to provide a proof of concept on real world data. Finally, Figure 5 shows a track allocation for the Simplon produced by our approach and re-translated to `OpenTrack`, i.e., closing the loop between microscopic and macroscopic railway models.

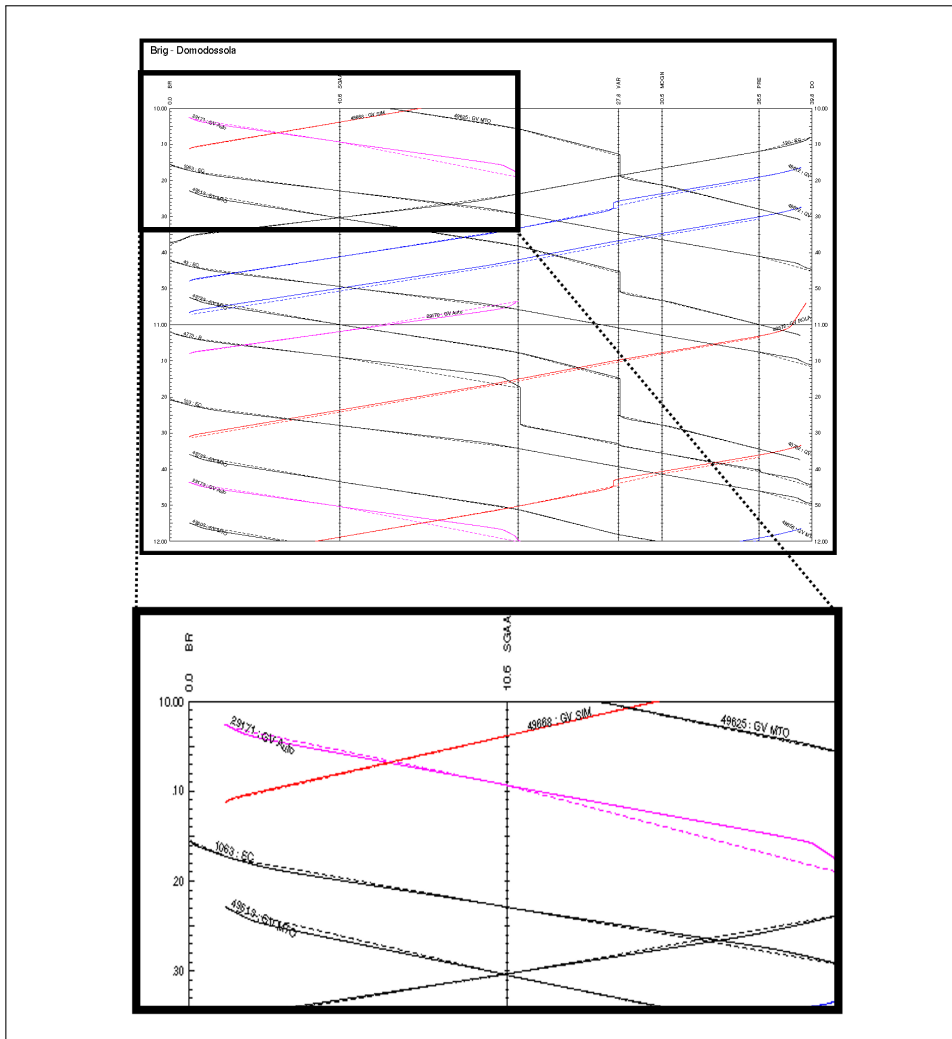
### 4.3 Conclusion

On the one hand the constructed macroscopic railway model is coarse enough to allow for complex optimization. On the other hand it is accurate enough to appropriately model the railway capacity and to produce timetables that can be re-translated into the microscopic model. Moreover, the core of the framework is to close the loop from simulation to optimization or from microscopic railway models to macroscopic ones. Establishing such interfaces between simulation and optimization is a very important point to allow for innovation. It activates the optimization potential of railway planning, e.g., in case of railway track allocation. In addition, it automatically integrates further developments of simulation tools. A fruitful and successful transfer of knowledge is only possible if bridges between theory and practice are built. Simulation tools are widespread and commonly used in almost all railway operating companies. However, innovative and state of the art discrete optimization has still potential to contribute to railway planning challenges. We believe that our framework and our solvers are a great example for that and will support railway planning, i.e., the process of railway track allocation, in the future.



**Figure 4:** Comparing the ”maximum“ number of scheduled trains for different networks (*simplon\_*) for instance 24h-tp-as using a 60s discretization.





**Figure 5:** OpenTrack traffic diagram of an optimized timetable, re-transformed to the (microscopic) simulation level using our Micro-Macro Transformation. All trains entering and leaving the corridor are shown from left (Brig) to right (Dommodossola) during 10:00 (top) and 12:00 (bottom). The dotted lines represent macroscopic train movements; they are linear. The “real” (simulated) timetable is plotted using solid lines; here, acceleration and braking phases are clearly perceivable.

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