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Rounding and Propagation Heuristics for Mixed Integer Programming

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Rounding and Propagation Heuristics for Mixed Integer Programming

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Abstract

Primal heuristics are an important component of state-of-the-art codes for mixed integer programming. In this paper, we focus on primal heuristics that only employ computationally inexpensive procedures such as rounding and logical deductions (propagation). We give an overview of eight different approaches. To assess the impact of these primal heuristics on the ability to find feasible solutions, in particular early during search, we introduce a new performance measure, the *primal integral*. Computational experiments evaluate this and other measures on MIPLIB 2010 benchmark instances.

1 Introduction: primal heuristics for MIP

Mixed integer programming (MIP) is to solve the optimization problem

$$\tilde{x}_{\text{opt}} = \operatorname{argmin}\{c^T x \mid Ax \leq b, l \leq x \leq u, x_j \in \mathbb{Z} \text{ for all } j \in J\}, \quad (1)$$

with $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $l, u \in \hat{\mathbb{R}}^n$ (with $\hat{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$), and $J \subseteq N = \{1, \dots, n\}$.

In state-of-the-art MIP solvers, primal heuristics play a major role in finding and improving integer feasible solutions at an early stage of the solution process. Knowing good solutions early during optimization helps to prune the search tree and to simplify the problem via dual reductions. Further, it proves the feasibility of a problem and a practitioner might be satisfied with a solution that is proven to be within a certain gap to optimality.

This article gives an overview about rounding and propagation heuristics for MIP that are integrated into SCIP [1], which is a state-of-the-art non-commercial solver and framework for mixed integer programming.

The ZI Round heuristic has been introduced by Wallace [5], more details on the other primal heuristics can be found in [2, 1, 3].

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2 Rounding heuristics

The goal of rounding heuristics is to convert a fractional solution \bar{x} of the system $Ax \leq b, l \leq x \leq u$ into an integral solution, i.e., $x_j \in \mathbb{Z} \forall j \in J$. All rounding heuristics described in this section use the notion of *up-* and *down-locks*. For a MIP (1), we call the number of positive coefficients $\xi_j^+ := |\{i: a_{ij} > 0\}|$ the *up-locks* of the variable x_j ; the number of negative coefficients is called the *down-locks* ξ_j^- of x_j .

Simple Rounding is a very cheap heuristic that iterates over the set of fractional variables of some LP-feasible point. It only performs roundings, which guarantee to keep all linear constraints satisfied. Consider an integer variable $x_j, j \in J$, with fractional LP solution \bar{x}_j . If $\xi_j^- = 0$, we can safely set $\tilde{x}_j := \lfloor \bar{x}_j \rfloor$ without violating any linear constraint. Analogously, if $\xi_j^+ = 0$, we can set $\tilde{x}_j := \lceil \bar{x}_j \rceil$. If all integer variables with fractional \bar{x}_j can be rounded that way, then \tilde{x} will be a feasible MIP solution.

In contrast to *Simple Rounding*, *Rounding* also performs roundings which potentially lead to a violation of some linear constraints, trying to recover from this infeasibility by further roundings later on. The solutions that can be found by *Rounding* are a superset of the ones that can be found by *Simple Rounding*. Like *Simple Rounding*, the *Rounding* heuristic takes up- and down-locks of an integer variable with fractional LP value \bar{x}_j into account. As long as no linear constraint is violated, the algorithm iterates over the fractional variables and applies a rounding into the direction of fewer locks, updating the *activities* $A_i \tilde{x}$ of the LP rows after each step, A_i being the i -th row of A . If there is a violated linear constraint, hence $A_i \tilde{x} > b_i$ for some i , the heuristic will try to find a fractional variable that can be rounded in a direction such that the violation of the constraint is decreased, using the number of up- and down-locks as a tie breaker. If no rounding can decrease the violation of the constraint, the procedure is aborted.

ZI Round[5] reduces the *integer infeasibility* of an LP solution step-by-step by shifting fractional values towards integrality, but not necessarily rounding them. For each integer variable x_j with fractional solution value \bar{x}_j , the heuristic calculates bounds for both possible rounding directions of \bar{x}_j such that the obtained solution stays LP-feasible. \bar{x}_j is shifted by the corresponding bound into the direction which reduces the *fractionality* $\min\{\bar{x}_j - \lfloor \bar{x}_j \rfloor, \lceil \bar{x}_j \rceil - \bar{x}_j\}$ most. The set of fractional variables might be processed several times by *ZI Round*. It either terminates with a MIP solution \tilde{x} or aborts if the integer infeasibility could not be decreased anymore or if a predefined iteration limit has been reached.

3 Propagation heuristics

The goal of propagation heuristics is to construct a feasible MIP solution \tilde{x} from scratch or from a start solution, while applying domain propagation to exclude variable values that would lead to an infeasibility or to a solution that is inferior to the incumbent. In contrast to diving or LNS heuristics (see, e.g., [2]), propagation heuristics do not solve any LP during search.

The *Shifting* heuristic is similar to *Rounding*, but it tries to continue in the case that no rounding can decrease the violation of a linear constraint. In this case, the value of a continuous variable or an integer variable with integral

value will be shifted in order to decrease the violation of the constraint. To avoid cycling, the procedure terminates after a certain number of *non-improving shifts*. A shift is called *non-improving*, if it neither reduces the number of fractional variables nor the number of violated rows.

Shift-and-Propagate tries to find a MIP solution by alternately fixing variables and propagating these fixings. Starting with an initial, typically infeasible, solution in which each variable is assumed to be at one of its bounds, it iterates over all integer variables $x_j, j \in J$ in nondecreasing order of their impact on the activity of the linear constraints. In each step, the heuristic fixes a variable x_j to a value \tilde{x}_j such that the overall infeasibility gets maximally reduced. The fixing $x_j = \tilde{x}_j$ is then propagated to reduce further variable domains. In case that the domain propagation detects the infeasibility of the current partial solution, the domain of x_j is reset to its previous state and the variable is postponed. The procedure is aborted when a predefined iteration limit is exceeded.

4 Improvement heuristics

Improvement heuristics consider the incumbent solution as a starting point and try to construct an improved solution with better objective value.

Oneopt is a straightforward improvement heuristic: given a feasible MIP solution \tilde{x} , the value of an integer variable $x_j, j \in J$, can be decreased for $c_j > 0$ or increased for $c_j < 0$ if the resulting solution is still feasible. If more than one variable can be shifted, they are sorted by non-decreasing impact $|c_j \delta_j|$ on the objective and sequentially shifted until no more improvements can be obtained. Here, $\delta_j \in \mathbb{Z}$ denotes how far the variable can be shifted into the desired direction without losing feasibility. *Oneopt* often succeeds in improving solutions which were found by the rounding heuristics described in Section 2, since their defensive approach to round into the direction of fewer locks tends to over-fulfill linear constraints, sacrificing solution quality.

The *Twoopt* heuristic attempts to improve a feasible MIP solution \tilde{x} by altering the solution values of pairs of variables. Only variables which share a pre-defined ratio of LP rows are considered as pairs. Each step of the heuristic consists of improving the objective value by shifting one variable, and then compensating the resulting infeasibilities by shifting a second variable, without completely losing the objective improvement. Similarly to *Oneopt*, pairs are processed in non-decreasing order of their impact on the objective.

5 Computational experiments

In MIP solving, the running time to optimality and the number of branch-and-bound nodes are typical measures for comparison. For primal heuristics, the time needed to find a first feasible solution, an optimal solution, or a solution within a certain gap to optimality are favorable measures that concentrate on the primal part of the solution process. Nevertheless, the trade-off between speed and solution quality is not well covered by any of them.

We suggest a new performance measure that takes into account the overall solution process. The goal is to measure the progress of convergence towards the optimal solution over the entire solving time. Let \tilde{x} be a solution for a MIP,

and \tilde{x}_{opt} be an optimal (or best known) solution for that MIP. We define the *primal gap* $\gamma \in [0, 1]$ of \tilde{x} as:

$$\gamma(\tilde{x}) := \begin{cases} 0, & \text{if } |c^T \tilde{x}_{\text{opt}}| = |c^T \tilde{x}| = 0, \\ 1, & \text{if } c^T \tilde{x}_{\text{opt}} \cdot c^T \tilde{x} < 0, \\ \frac{|c^T \tilde{x}_{\text{opt}} - c^T \tilde{x}|}{\max\{|c^T \tilde{x}_{\text{opt}}|, |c^T \tilde{x}|\}}, & \text{else.} \end{cases} \quad (2)$$

Let $t_{\text{max}} \in \mathbb{R}_{\geq 0}$ be a limit on the solution time of a MIP solver. Considering a log file of a MIP solver for a certain problem instance within a fixed computational environment, we define its *primal gap function* $p: [0, t_{\text{max}}] \mapsto [0, 1]$:

$$p(t) := \begin{cases} 1, & \text{if no incumbent found until point } t, \\ \gamma(\tilde{x}(t)), & \text{with } \tilde{x}(t) \text{ being the incumbent solution at point } t. \end{cases} \quad (3)$$

$p(t)$ is a step function that changes whenever a new incumbent is found. It is zero from the point on at which the optimal solution is found. We define the *primal integral* $P(T)$ of a run until a point in time $T \in [0, t_{\text{max}}]$ as:

$$P(T) := \int_{t=0}^T p(t) dt = \sum_{i=1}^I p(t_{i-1}) \cdot (t_i - t_{i-1}), \quad (4)$$

where $t_i \in [0, T]$ for $i \in 1, \dots, I-1$ are the points in time when a new incumbent solution is found, $t_0 = 0$, $t_I = T$.

We suggest to use $P(t_{\text{max}})$ for measuring the quality of primal heuristics. It favors finding good solutions early. The fraction $P(t_{\text{max}})/t_{\text{max}}$ can be seen as the average solution quality during the search process. Spoken differently, the smaller $P(t_{\text{max}})$ is, the better is the expected quality of the incumbent solution if we stop the solver at an arbitrary point in time.

We used the benchmark set of the MIPLIB 2010 [4] as test set for our experiments. Since we are interested in the primal part of the solution process, we excluded the four infeasible instances **triptim1**, **enlight14**, **ns1766074**, and **ash608gpia-3col**; further, **mspp16** was excluded since it terminated for memory reasons during presolving for all our tests. Thus, 82 test instances remained. We performed four different runs: SCIP without any primal heuristics (Heur.OFF), SCIP using only the rounding and propagation heuristics which are described in this paper (R&P), SCIP using all default heuristics except the ones described in this paper (NoR&P), and SCIP with default settings (DEF). All experiments were conducted with a time limit of one hour, a memory limit of 4 GB on a 3.00 GHz Intel® Core™2 Extreme CPU X9650 with 6144KB Cache and 8 GB RAM.

Figure 1 exemplarily shows the primal gap function $p(t)$ for the four settings applied to the instance **n3seq24**. A square shows when a new primal solution is found and its quality. It can be seen that the two settings that use rounding and propagation heuristics find solutions earlier and hence have a smaller primal integral $P(t_{\text{max}})$. For all four settings, SCIP found the optimal solution within an hour, but timed out without proving optimality. Interestingly, with disabled heuristics (Heur.OFF), the optimal solution was found in the smallest amount of time. Nevertheless, the behavior of the default settings (DEF) seems favorable since primal solutions of reasonable quality are found much earlier.

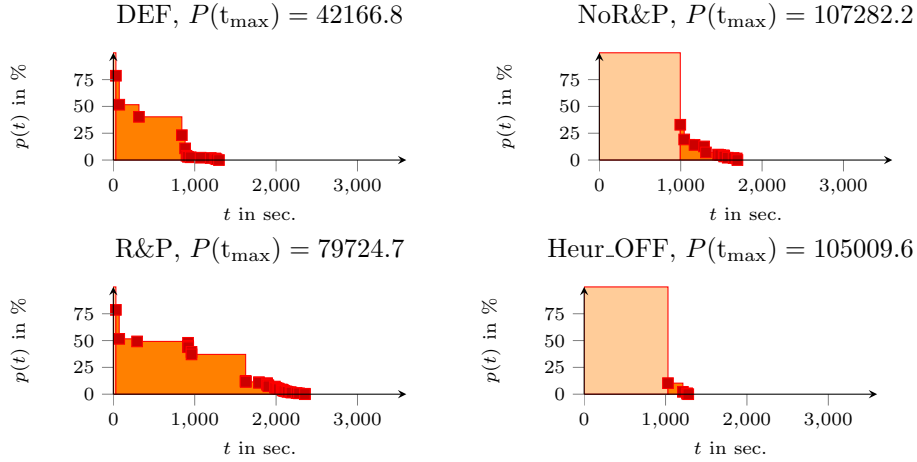


Figure 1: The solving process depicted for the instance `n3seq24`

	DEF	NoR&P	R&P	Heur_OFF
$\phi(P)$	0.44	0.47	0.61	1.0
$\phi(t_1)$	8.54	17.48	10.88	57.66
$\phi(t_{\text{opt}})$	215.63	218.18	236.92	263.46
$\phi(t_{\text{solved}})$	712.90	676.10	746.48	838.60

Table 1: Computed mean values for all four settings

Table 1 shows aggregated results of our experiments. The first row shows the evaluation of the *normalized* primal integral over the entire testset. For each instance, the corresponding integral $P(t_{\text{max}})$ is divided by the integral obtained with the setting `Heur_OFF`. This reference value is then used to compute the geometric mean $\phi(P)$ for each setting. The remainder of the table shows the geometric means of the time until the first (t_1) and the best (t_{opt}) primal solution were found and the mean solving time (t_{solved}).

The advantage of rounding and propagation heuristics can be best seen in the time to first solution t_1 : they are valuable for finding start solutions. The impact on the time to the best solution and the overall solving time is much smaller, the latter even showing a degradation. The primal integral $P(t_{\text{max}})$ implies the following ranking: To get good primal solutions, using cheap and expensive heuristics together (`DEF`) is slightly better than using only expensive heuristics (`noR&P`), which is considerably better than using only cheap ones (`R&P`), which is much better than using no heuristics (`Heur_OFF`). This behavior is only partially expressed by the other measures.

Detailed computational results including charts for the development of the primal gap function $p(t)$ can be found in the Appendix.

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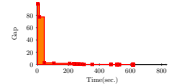
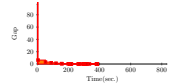
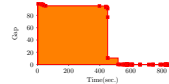
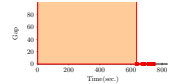
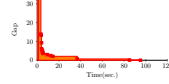
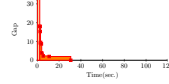
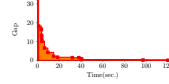
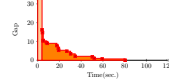
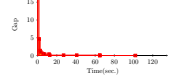
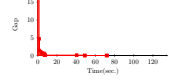
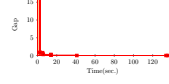
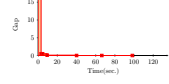
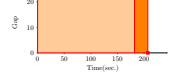
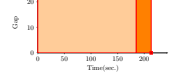
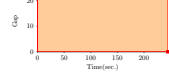
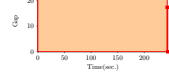
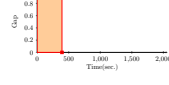
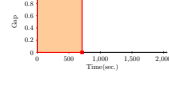
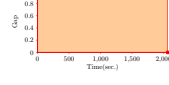
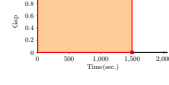
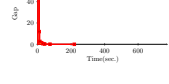
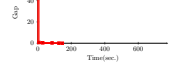
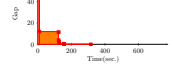
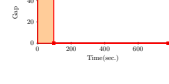
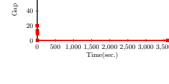
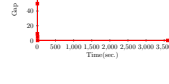
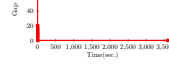
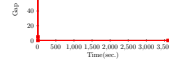
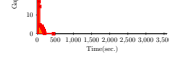
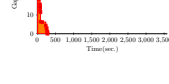
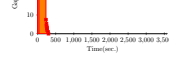
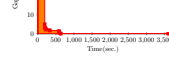
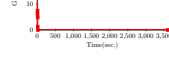
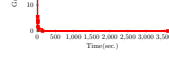
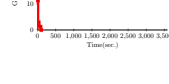
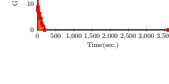
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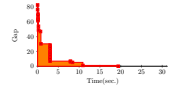
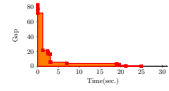
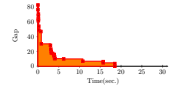
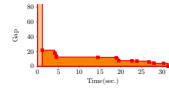
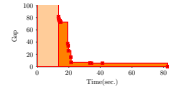
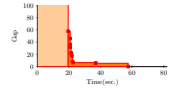
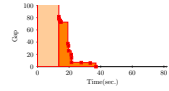
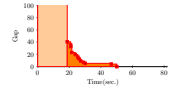
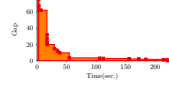
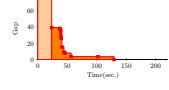
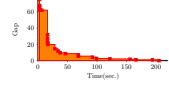
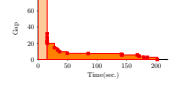
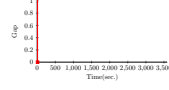
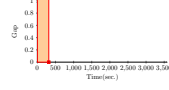
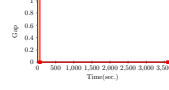
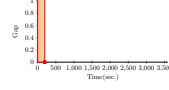
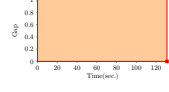
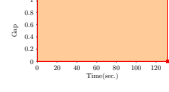
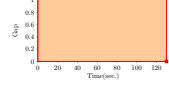
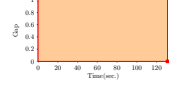
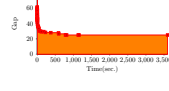
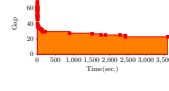
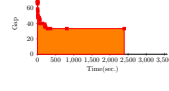
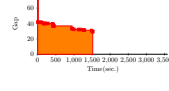
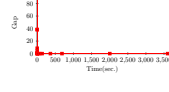
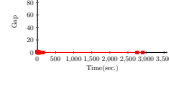
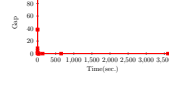
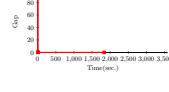
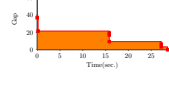
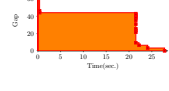
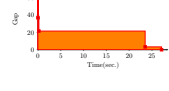
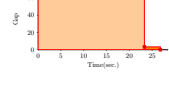
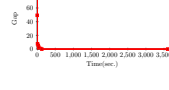
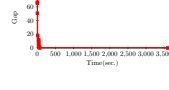
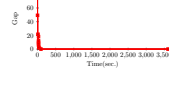
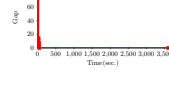
Appendix

In Table 2, the results of the experiment are explicitly shown for each of the 82 instances in our test set. Different performance measures are evaluated for the four settings Heur.OFF, R&P, NoR&P, and DEF, see above. For each instance, the first row shows the time until the first (t_1), and the best (t_{opt}) solution were found, the overall solving time (t_{solved}) and the normalized integral value ($P(t_{max})$). The second row shows the development of $p(t)$ over time. Note that according to the definition of the normalized integral, $P(t_{max})$ will always be 1.00 for the setting without primal heuristics.

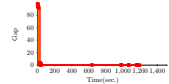
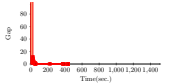
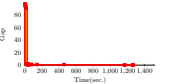
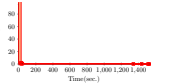
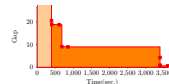
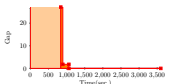
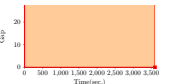
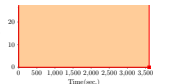
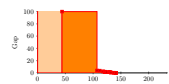
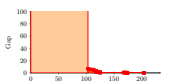
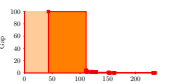
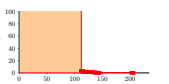
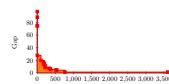
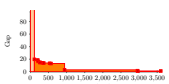
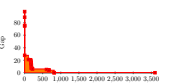
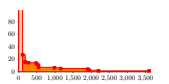
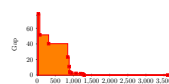
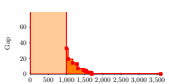
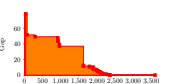
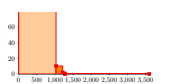
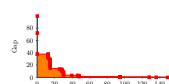
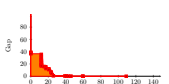
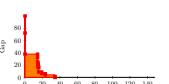
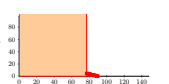
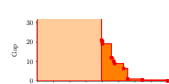

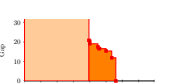
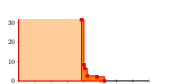




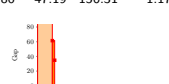

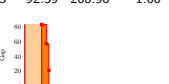

For the figures, a square at 3600 seconds indicates that the solver hit the time limit, because the incumbent solution was not proven to be optimal. Note that the light shaded areas, which indicate the period without an incumbent solution, are cut off at the top.

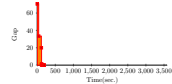
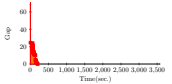
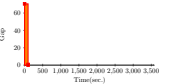
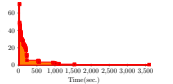
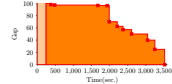
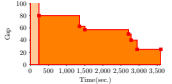
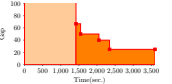
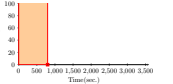
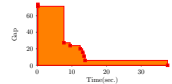
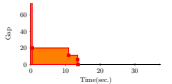
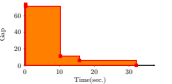
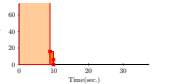
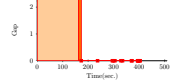
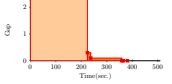
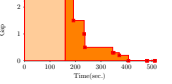
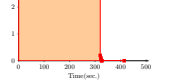
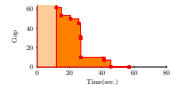
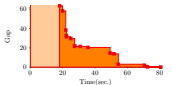
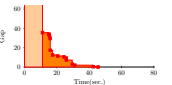
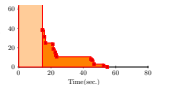
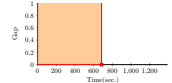
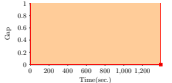
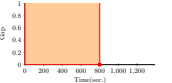
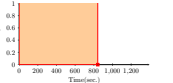
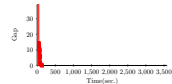
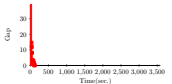
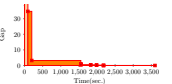
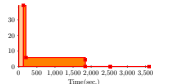
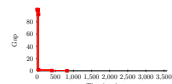
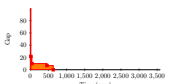
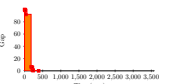
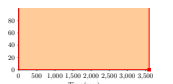
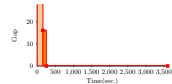
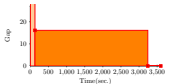
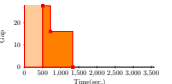
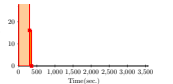
Problem Name	DEF				NoR&P				R&P				Heur.OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
30n20b8	256.63	3287.83	3600.0	0.45	533.64	3582.95	3600.0	0.67	3600.0	3600.0	3600.0	1.14	2680.8	3594.77	3600.0	1.00
acc-tight5	185.25	185.25	185.25	1.03	226.27	226.27	226.27	1.26	536.83	536.83	536.84	2.98	180.29	180.29	180.29	1.00
aflow40b	0.77	353.51	1274.95	0.33	14.12	592.35	3516.76	0.45	0.76	986.43	1886.23	2.10	32.64	630.78	3600.0	1.00
air04	41.0	45.63	48.99	1.24	41.34	56.39	58.69	1.27	35.13	38.71	42.33	1.06	33.05	36.82	36.93	1.00
app1-2	3600.0	3600.0	3600.0	1.34	137.01	3485.08	3485.09	0.60	3600.0	3600.0	3600.0	1.34	2685.16	2685.16	2685.17	1.00
bab5	500.97	688.34	3600.0	0.72	571.07	1905.29	3600.0	0.82	758.96	794.2	3600.0	1.11	613.21	1980.27	3600.0	1.00
beasleyC3	0.07	917.53	3600.0	0.05	0.03	2690.61	3600.0	0.07	0.07	0.07	3600.0	0.21	3600.0	3600.0	3600.0	1.00

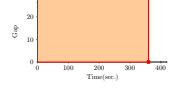
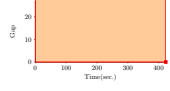
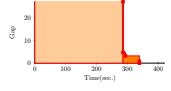
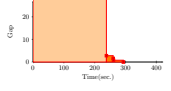
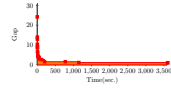
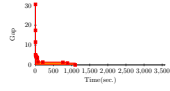
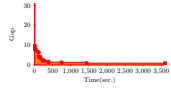
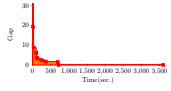
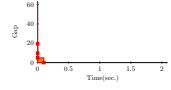
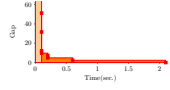
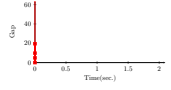
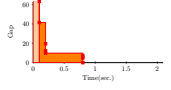
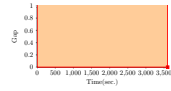
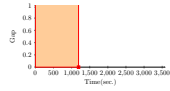
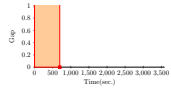
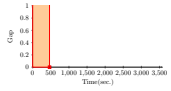
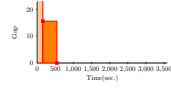
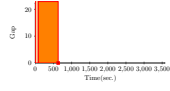
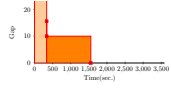
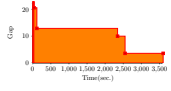
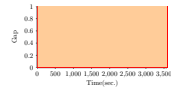
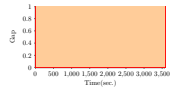
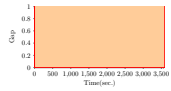
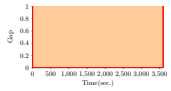
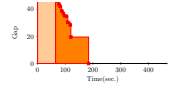
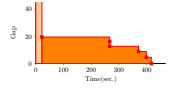
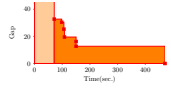
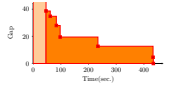
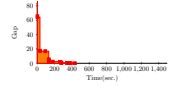
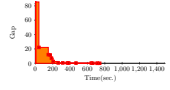
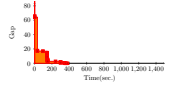
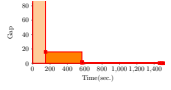
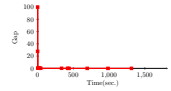
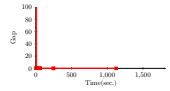
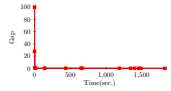
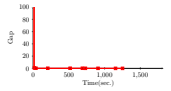
Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
biella1	2.51	619.04	701.01	0.07	4.52	386.06	419.45	0.02	2.5	838.8	897.29	0.69	635.66	745.69	944.05	1.00
																
bienst2	1.1	95.03	169.6	0.46	2.25	30.69	223.84	0.44	0.97	120.42	125.46	0.42	4.05	80.44	149.69	1.00
																
binkar10.1	1.36	102.26	129.01	0.49	1.32	105.5	131.83	0.44	2.55	136.23	179.03	0.82	3.15	98.17	171.45	1.00
																
bley_x1	181.68	206.85	206.85	0.77	184.61	212.53	212.55	0.79	245.34	245.34	245.34	1.01	243.42	244.21	244.22	1.00
																
bnatt350	393.34	393.34	393.34	0.26	709.23	709.23	709.24	0.47	2067.97	2067.97	2067.97	1.38	1495.95	1495.95	1495.96	1.00
																
core2536-691	8.43	219.18	219.19	0.10	7.21	145.7	145.7	0.08	8.34	316.47	316.48	0.24	95.84	778.98	778.99	1.00
																
cov1075	0.01	234.81	3600.0	0.21	0.09	6.57	3600.0	0.10	0.0	7.36	3600.0	0.21	6.91	7.28	3600.0	1.00
																
csched010	31.48	449.62	3189.21	0.60	10.14	374.06	2818.98	0.50	40.03	301.69	3544.72	1.17	13.75	636.55	3600.0	1.00
																
danoint	8.58	20.74	3600.0	0.61	4.95	144.23	3600.0	0.49	10.43	102.04	3596.58	0.96	7.27	186.98	3600.0	1.00
																

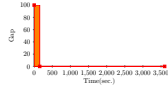
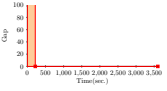
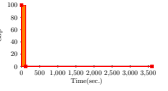
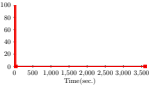
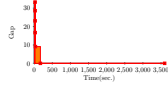
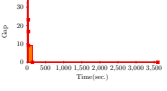
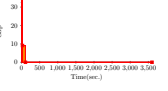
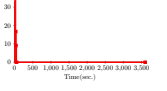
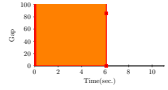
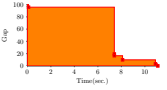
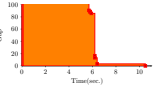
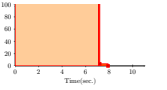
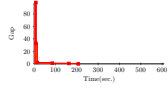
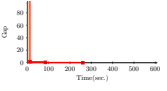
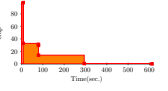
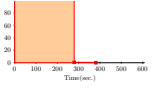
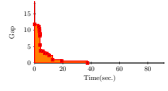
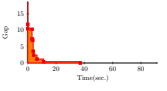
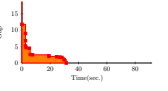
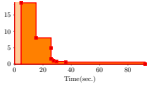
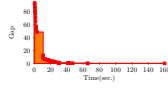
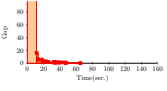
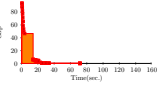
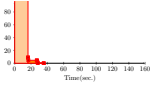
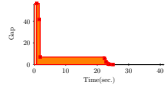
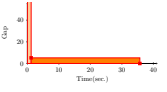
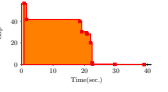
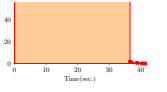
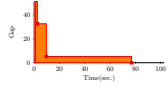
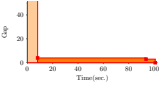
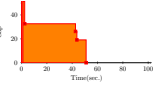
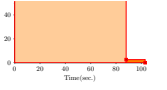
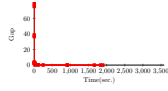
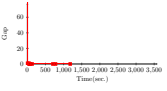
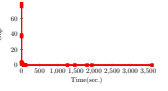
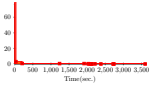
Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
dfn-gwin-UUM	0.02	19.44	104.69	0.39	0.01	24.89	108.87	0.45	0.02	18.47	98.04	0.55	1.14	31.43	125.07	1.00
																
eil33-2	13.51	82.59	94.24	0.99	19.53	57.52	80.3	1.02	13.48	37.05	76.56	0.87	18.71	50.11	81.94	1.00
																
eilB101	2.47	220.59	241.0	0.70	23.55	128.95	158.15	1.15	2.58	205.23	235.44	0.71	15.26	201.53	248.37	1.00
																
enlight13	10.62	10.62	1938.46	0.05	315.02	315.02	452.68	1.62	66.63	66.63	3600.0	0.34	193.77	193.77	2564.5	1.00
																
ex9	131.17	131.17	131.17	1.00	131.96	131.96	131.96	1.01	130.36	130.36	130.36	0.99	131.23	131.23	131.23	1.00
																
glass4	0.58	1679.56	3600.0	0.95	0.43	2429.37	3600.0	0.96	0.38	2392.02	2489.41	0.87	0.94	2866.82	2964.95	1.00
																
gmu-35-40	0.0	2004.04	3600.0	0.14	0.36	2908.79	3193.88	0.18	0.0	650.65	3600.0	0.15	3.22	1832.67	3276.45	1.00
																
iis-100-0-cov	0.0	28.62	1988.89	0.19	0.12	27.81	1863.8	0.42	0.0	27.13	1545.83	0.22	23.28	26.77	1478.99	1.00
																
iis-bupa-cov	0.01	127.69	3600.0	0.33	0.3	65.23	3600.0	0.39	0.0	94.46	3600.0	0.40	27.08	43.43	3600.0	1.00
																

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
iis-pima-cov	0.01	245.66	641.41	0.23	0.35	224.63	657.33	0.37	0.02	177.67	583.23	0.19	42.08	406.79	666.64	1.00
lectsched-4-obj	14.8	79.06	79.07	0.31	14.97	83.62	83.63	0.32	15.55	93.52	93.53	0.36	101.17	239.19	239.2	1.00
m100n500k4r1	0.0	18.4	3600.0	1.00	0.03	12.3	3600.0	1.00	0.01	40.73	3600.0	1.01	0.68	2.88	3600.0	1.00
macrophage	0.01	205.4	3600.0	0.22	10.32	1588.24	3600.0	0.14	0.0	3403.14	3600.0	2.08	35.91	3049.12	3600.0	1.00
map18	4.4	246.77	317.26	0.22	54.51	424.76	500.01	0.46	4.33	354.17	376.22	0.33	196.31	265.93	318.38	1.00
map20	4.62	316.41	419.39	0.10	52.88	372.95	472.83	0.23	4.56	316.8	405.42	0.16	289.64	369.28	403.33	1.00
mcsched	0.11	157.72	159.86	0.21	0.38	35.74	184.27	0.13	0.12	37.37	148.26	0.21	6.5	29.95	178.24	1.00
mik-250-1-100-1	0.0	0.13	131.97	0.01	0.01	0.11	130.49	0.01	0.0	5.16	122.18	0.01	1.01	432.24	3193.47	1.00
mine-166-5	0.01	44.99	46.64	0.69	36.8	50.29	53.43	0.95	0.01	45.64	46.78	0.69	39.03	50.58	53.46	1.00

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
mine-90-10	0.01	1657.66	1911.35	0.84	24.65	438.38	603.12	0.71	0.01	1265.59	1514.87	1.02	28.06	1525.87	1647.92	1.00
																
msc98-ip	394.86	3481.54	3600.0	0.19	840.17	1064.75	3600.0	0.24	3600.0	3600.0	3600.0	1.00	3600.0	3600.0	3600.0	1.00
																
mzzv11	0.04	141.49	288.18	0.96	103.0	203.92	220.85	0.93	0.05	234.6	285.85	0.99	111.95	205.9	277.44	1.00
																
n3div36	2.41	818.48	3600.0	0.41	110.48	2962.52	3600.0	1.05	2.42	818.07	3600.0	0.31	106.9	2202.45	3600.0	1.00
																
n3seq24	31.46	1297.75	3600.0	0.40	994.89	1697.2	3600.0	1.02	31.25	3590.62	3600.0	0.76	1030.03	1278.57	3600.0	1.00
																
n4-3	0.07	148.95	631.79	0.12	0.09	109.28	1032.29	0.08	0.06	34.61	523.35	0.09	77.12	89.77	799.61	1.00
																
neos-1109824	7.91	15.99	235.22	1.06	1.62	15.11	163.08	0.42	7.86	11.17	279.51	1.07	7.74	10.52	304.19	1.00
																
neos-1337307	26.84	2372.34	3600.0	1.02	25.18	39.38	3600.0	0.96	24.28	181.57	3600.0	0.93	26.23	29.55	3600.0	1.00
																
neos-1396125	31.86	47.19	150.31	1.17	28.59	166.45	392.23	0.98	36.45	92.39	260.96	1.60	24.99	282.79	447.85	1.00
																

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
neos13	2.17	193.92	206.58	0.42	2.55	202.86	238.96	0.22	2.1	98.44	124.33	0.53	22.12	1538.69	3600.0	1.00
																
neos-1601936	246.59	3517.45	3517.46	3.38	237.45	2946.48	3600.0	2.73	1419.96	2345.89	3600.0	2.74	797.54	797.54	797.55	1.00
																
neos18	0.16	37.64	149.83	0.92	0.37	13.56	87.4	0.31	0.16	32.1	124.88	0.98	8.86	9.95	101.2	1.00
																
neos-476283	162.55	403.84	421.71	0.51	223.77	403.65	428.65	0.70	160.06	511.8	556.23	0.51	319.77	414.48	456.84	1.00
																
neos-686190	11.88	56.85	65.37	1.11	18.08	80.7	93.09	1.43	11.12	45.51	55.88	0.75	14.7	54.83	66.26	1.00
																
neos-849702	684.53	684.53	684.53	0.81	1396.76	1396.76	1396.76	1.66	801.06	801.06	801.07	0.95	842.15	842.15	842.16	1.00
																
neos-916792	43.97	132.83	563.83	0.19	33.92	115.29	528.25	0.15	92.69	2179.78	3600.0	0.70	127.81	2527.55	3600.0	1.00
																
neos-934278	0.11	823.18	823.19	0.01	13.86	631.21	631.21	0.02	0.11	389.54	389.54	0.05	3600.0	3600.0	3600.0	1.00
																
net12	158.93	252.71	3600.0	0.57	127.13	3241.0	3600.0	2.04	504.37	1331.99	3004.87	2.15	298.99	354.52	3045.95	1.00
																

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
netdiversion	360.34	360.34	431.38	1.51	422.65	422.65	554.26	1.77	285.64	338.6	401.51	1.20	237.76	293.44	328.04	1.00
																
newdano	1.78	3598.52	3600.0	1.76	3.57	1099.34	3459.53	0.65	1.56	1437.89	3600.0	1.68	6.88	714.09	3600.0	1.00
																
noswot	0.01	15.41	381.75	0.02	0.09	2.14	302.55	0.83	0.01	75.28	632.39	0.00	0.12	0.84	219.48	1.00
																
ns1208400	3600.0	3600.0	3600.0	7.76	1191.82	1191.82	1903.62	2.57	695.84	695.84	695.85	1.50	463.8	463.8	585.43	1.00
																
ns1688347	155.61	541.74	541.75	0.55	81.71	629.33	629.34	0.53	334.67	1552.74	1552.82	1.16	30.58	2548.98	3600.0	1.00
																
ns1758913	3600.0	3600.0	3600.0	1.00	3600.0	3600.0	3600.0	1.00	3600.0	3600.0	3600.0	1.00	3600.0	3600.0	3600.0	1.00
																
ns1830653	65.03	183.69	682.52	0.85	22.07	416.86	746.31	0.75	71.14	470.8	874.02	1.14	46.61	433.98	784.68	1.00
																
opm2-z7-s2	0.03	436.38	442.66	0.19	7.29	740.62	743.26	0.31	0.03	380.38	382.35	0.20	148.18	1502.84	1504.68	1.00
																
pg5_34	0.01	1313.26	1417.1	0.10	3.6	1123.98	1272.91	0.21	0.0	1824.41	1909.51	0.26	17.46	1250.58	1473.57	1.00
																

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
pigeon-10	0.0	152.42	3600.0	4.84	218.41	218.41	3600.0	6.94	0.0	113.37	3600.0	3.60	31.44	31.44	3600.0	1.00
																
pw-myciel4	23.65	171.43	3600.0	1.22	26.69	138.89	3600.0	1.20	26.38	108.41	3600.0	1.08	28.73	52.25	3600.0	1.00
																
qiu	0.06	6.09	61.76	0.84	0.04	11.06	81.57	1.04	0.06	10.52	54.15	0.87	7.13	7.9	63.14	1.00
																
rail507	7.43	207.1	303.18	0.05	12.31	260.18	361.65	0.05	7.38	611.5	679.32	0.23	278.95	380.74	453.91	1.00
																
rani16x16	0.01	37.42	213.09	0.11	0.0	37.12	251.08	0.08	0.02	31.31	210.66	0.13	4.63	92.13	223.25	1.00
																
reblock67	0.0	160.41	245.61	0.43	11.19	65.31	283.58	0.74	0.01	71.61	209.67	0.48	16.27	35.81	206.25	1.00
																
rmatr100-p10	0.77	24.98	59.2	0.08	1.2	35.66	66.71	0.08	0.78	38.9	56.33	0.26	36.61	41.35	67.8	1.00
																
rmatr100-p5	1.06	76.76	115.69	0.09	8.05	101.26	113.47	0.13	1.06	50.96	92.94	0.19	87.99	102.99	113.03	1.00
																
rmine6	0.01	1893.13	2808.4	0.07	5.87	1182.11	1925.61	0.13	0.0	1942.44	3600.0	0.08	33.42	2738.63	3600.0	1.00
																

Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
rocII-4-11	22.77	117.16	139.87	0.55	28.92	96.07	108.68	0.57	29.43	132.75	140.52	0.67	30.15	255.2	259.82	1.00
rococoC10-001000	0.09	485.0	1626.56	0.24	3.37	646.23	1704.96	0.20	0.1	313.1	1415.92	0.34	13.84	384.62	981.9	1.00
roll3000	3.63	479.02	3600.0	0.48	3.61	245.47	3600.0	0.74	16.59	572.78	3600.0	2.34	12.64	484.77	3600.0	1.00
satellites1-25	303.35	1417.32	1419.37	0.44	285.96	625.36	1244.68	0.66	277.48	1086.96	1660.38	0.88	267.51	950.24	2374.85	1.00
sp98ic	2.04	1153.66	3600.0	0.46	10.69	2626.06	3600.0	0.29	2.02	2871.37	3600.0	0.56	209.69	2312.39	3600.0	1.00
sp98ir	7.2	231.91	286.09	0.52	7.18	60.19	163.48	0.47	10.09	30.85	108.43	0.61	16.79	40.38	140.4	1.00
tanglegram1	0.16	1.28	3600.0	0.44	276.61	702.24	1596.85	0.08	0.16	3600.0	3600.0	0.44	3600.0	3600.0	3600.0	1.00
tanglegram2	0.01	20.01	48.35	0.79	3.24	3.24	3.77	0.59	0.02	13.02	43.76	0.79	5.42	11.74	12.16	1.00
timtab1	1.16	212.69	591.87	0.44	1.26	102.86	529.93	0.37	11.09	194.58	515.85	1.01	14.09	26.16	439.05	1.00

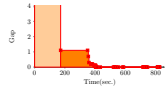
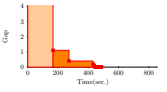
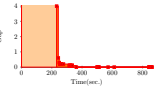
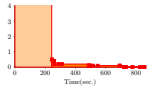
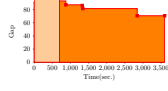
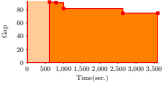
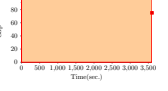
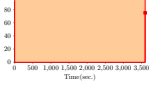
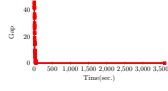
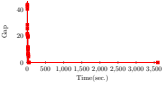
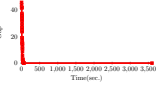
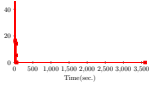
Problem Name	DEF				NoR&P				R&P				Heur_OFF			
	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$	t_1	t_{opt}	t_{solved}	$P(t_{max})$
unitcal.7	172.12	825.49	1695.51	0.72	166.6	805.37	1051.57	0.69	231.51	935.35	1333.29	0.96	242.79	1159.38	1438.29	1.00
																
vpphard	698.86	2842.75	3600.0	0.84	609.93	2635.11	3600.0	0.84	3600.0	3600.0	3600.0	1.00	3600.0	3600.0	3600.0	1.00
																
zib54-UUE	0.34	49.47	3600.0	0.42	0.1	43.81	3600.0	0.24	0.33	62.66	3600.0	0.45	15.05	54.43	3600.0	1.00
																

Table 2: The results for every instance