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# Multistage Stochastic Programming in Strategic Telecommunication Network Planning<sup>3</sup>

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<sup>3</sup>This work is based on a diploma thesis in mathematics by Jonas Schweiger at Technische Universität Berlin, Germany.



# Multistage Stochastic Programming in Strategic Telecommunication Network Planning

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**Abstract**—Mobile communication is nowadays taken for granted. Having started primarily as a service for speech communication, data service and mobile Internet access are now driving the evolution of network infrastructure. Operators are facing the challenge to match the demand by continuously expanding and upgrading the network infrastructure. However, the evolution of the customer's demand is uncertain.

We introduce a novel (long-term) network planning approach based on multistage stochastic programming, where demand evolution is considered as a stochastic process and the network is extended as to maximize the expected profit. The approach proves capable of designing large-scale realistic UMTS networks with a time-horizon of several years. Our mathematical optimization model, the solution approach, and computational results are presented in this paper.

**Keywords**—UMTS, Network Evolution, Multistage, Stochastic Programming

## I. INTRODUCTION

Telecommunication is fundamental to the information society we live in. In the German market in the year 2009, for example, 19 million users generated over 33 million gigabyte of traffic using mobile data services. The 3rd generation networks (3G or UMTS) in Germany comprise over 39 000 sites with some 120 000 cells in total. From 1998 to 2008, the four network operators in Germany invested over 28 billion euros into their infrastructure (all figures taken from [1]).

A careful allocation of the resources is thus crucial for the profitability of a network operator: a network should be dimensioned to match customers' demand. As this demand evolves over time, the infrastructure has to evolve accordingly. The demand evolution is hard to

predict, however, and thus constitutes a strong source of uncertainty. Strategic network planning has to take this uncertainty into account, and the planned network evolution should be adaptable to changing market conditions. The application of advanced planning methods under the consideration of uncertainty can improve the profitability of the network and creates a competitive advantage.

Planning of wireless networks is a challenging task and automated planning procedures are required in current and future applications such as LTE. We contribute by proposing a planning framework which is suitable for several technologies and demonstrate the practical applicability by performing computational experiments using realistic, large-scale planning scenarios. We choose a setting that allows to perform experiments on realistic data without guessing too many parameters. This is nowadays the case for the introduction of UMTS, starting around the year 2003. Therefore, a network evolution from the year 2004 onwards is considered. Most relevant parameters have become publicly accessible and are no longer a trade secret of the industry.

A popular approach to evaluate investment opportunities in several domains are *real options* [14]. The real options approach treats investment projects as options on the resulting future cash-flows and uses the financial market for a risk-neutral monetary valuation of investment opportunities. Real options have been applied in several applications, including telecommunications (see [5], for example). In order to properly apply the theory of real options, the project has to be embedded in an appropriate market. This is hardly possible in our application.

Stochastic programming has been suggested to value real options in the absence of a market embedding, see de Neufville and Wang [16], for example. A risk-adjusted discount rate has to be used and the result is an implicit evaluation of 1a scenario tree. Since the evolution of the network can be divided into several stages, multi-

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stage stochastic programming is a suitable framework to model strategic telecommunication network planning. We refer to Dentcheva, Ruszczyński and Shapiro [2] for an introduction to stochastic programming.

This paper is organized as follows: In the Section II, we introduce the system model that describes UMTS networks. In Section III, we describe our model of the uncertain parameter and the construction of a scenario trees. We formulate the optimization model for the network planning problem under uncertainty in Section IV. Computational experiments verify the applicability of our approach and are presented in Section V. Section VI concludes the paper.

## II. A MODEL FOR SERVICE COVERAGE

Coverage and capacity are the main determinants of UMTS network performance. Coverage is primarily treated as a function of the received signal level, while (soft) capacity is a matter of interference and resources. The interplay of signal strength, signal quality, and cell capacity is highly complex for UMTS. For detailed descriptions of UMTS radio networks and their intricacies we refer to [3], [11], [10].

In this section, we first present a simplified model of UMTS radio network performance that allows for answering the question of interest in network planning, namely, which of the (anticipated) traffic demand can be served by a given network configuration. We then introduce the constraints to be faced during an network roll-out. We finally sketch how a spatial traffic demand profile can be obtained on the basis of a reference service and a population map.

### A. The Ability of a Network to Satisfy Service Demand

Simplified models can capture network coverage and capacity sufficiently well for the purposes of strategic network planning. The basic idea is to determine the area in which all of the traffic demand can be served. We derive such a model in two steps. First, the notion of a cell's coverage area is introduced. This is the area in which the cell's signal can be received with a strong enough level to allow for basic communication in the presence of no to little interference. Next, the notion of a cell's service area is defined. This is the area from which the traffic demand can be served in the presence of inter-cell interference assuming that surrounding cell's are equally loaded.

This work is based on Geerdes [3], and we adopt the notation and definitions from there.

1) *Cells*: For our purposes a radio network is described by a collection  $\mathcal{C}$  of *cells* installed at *sites*. Each of the cells is associated with radio parameters such as pilot transmission power, antenna height, antenna type, the dominant radiation direction as determined by the azimuth as well as mechanical and electrical tilt. A site typically hosts three cells.

2) *The Coverage Area of a Cell*: The pilot signal emitted by the transmission unit of a cell  $i \in \mathcal{C}$  is the basis for network access by the mobile devices. Mobile devices determine the signal strength of the surrounding cells by sensing the cell's *pilot channel*. A signal level of some threshold  $\pi_{\text{Pilot}}$  is required for successful receptions

$$\gamma_i(x)p_i^{(P)} \geq \pi_{\text{Pilot}} \quad (1)$$

where  $p_i^{(P)}$  denotes the transmission power on the pilot channel and  $\gamma_i(x)$  the signal attenuation of cell  $i$  to point  $x$  in the planning area  $\mathcal{A}$ .

We denote the area in which (1) is met for a cell as the cell's *coverage area*. That is, outside of this area coverage cannot be assumed. Inside of this area, radio service may or may not be available depending on the desired service and the level of interference.

3) *The Service Area of a Cell*: The *cell load* describes the amount by which the capacity of a cell is used and is an important performance indicator for wireless networks. In the downlink transmission direction (that is, from the cell to the mobile devices), the cell load is the fraction of the total available transmission power that is required to service the demand.

The definition of a cell's service area ultimately bases on the ability to compute cell load for a specific traffic demand scattered across the cell's coverage area in the presence of interference. In a simplified version, the cell load can be estimated using a generalized pole capacity equation, which is described in detail by Geerdes [3].

This simplified capacity model can still take many of the effects linked to UMTS into account such as power control on dedicated transmission links, intra- and inter-cell interference, noise, service specific carrier-to-interference ratios (CIR), a rich service mix, and spatially heterogeneous service demand.

For the purpose of our work here, the most important property is that this capacity model can be used to define the *service area* of a cell as the maximal subset of the coverage area from which all demand can be serviced without exceeding a given load threshold. We use the threshold of 60%, which is common in network dimensioning.

We now sketch how the service area is algorithmically determined. Further details can be found in [12]. In network planning, signal propagation path-losses are

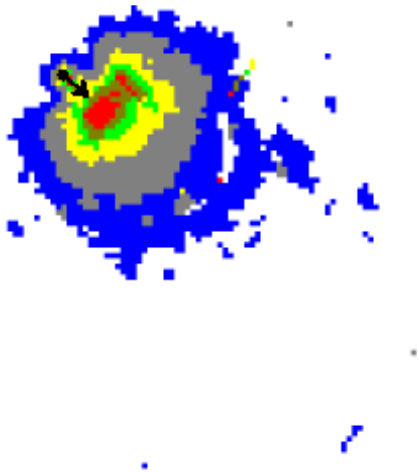


Fig. 1. Service area of a cell for different load intensities

typically predicted for pixels of a resolution between 20 m and 50 m with an accuracy of 1 dB. The coverage area can thus be arranged into a sequence of sets, where each set contains its predecessor. The first set comprises the pixel with minimum path-loss, the second set the pixel with the least two path-loss values, and so on. In a given spatial service demand map, each pixel is associated with its individual demand. By definition of the set sequence above, the cell load grows along the sequence. The service area is the last pixel set for which the cell load threshold is not exceeded.

In an empty network the coverage and service areas coincide. With increasing traffic the capacity eventually no longer suffices to service the demand at the cell borders and the service areas shrink.

4) *Technology Upgrades*: Several network features are available to increase the efficiency of a UMTS network; the activation of additional frequency block, enhanced modulation and coding schemes and alike. We refer to these measures as *technology upgrades*. Technology upgrades reduce the impact of a single user onto the cell load and, thus, add capacity to the system. We take account for this by scaling the user intensity. Figure 1 shows the service area of a cell for different upgrades, i. e., load intensities.

## B. Network Evolution

A key driver for the development of the network over time is the (anticipated) evolution of the traffic demand. A network design that is fit to service a specific traffic load intensity will eventually become overloaded when the traffic scales up. For the purpose of strategic network planning, the network shall be evolved in such a way that overload is penalized.

By construction of the cell service areas, we assume that the demand in a pixel can be satisfied if and only if it lies in the service area of some active cell. Consequently revenues are generated within the service areas of active cells and the operator wants to choose a set of cells that maximizes the overall (discounted) profit.

In the beginning of the life cycle of a service, the traffic first increases slowly. In case of a successful service, a steep increase is then observed after some time and the network gets populated and eventually overloaded. With increasing traffic, the service areas of the cells scale down and penalize overload. Pixel whose demand was initially satisfied, then may cease to be serviced. Since the operator then losses revenues, the construction of additional cells might be beneficial. Moreover, areas that are not profitable to serve in the beginning may become profitable with increasing traffic.

## C. Constraints on Network Evolution

Several constraints on network extension apply in practice. We focus on limitations in the number of cells that can be built in each period and on constraints on the coverage provided by the network.

1) *Limited Construction Activities*: The activation of new sites is assumed to involve construction activities; antennas have to be mounted, equipped with transmission units, and connected to the network. The same holds true if a new cell is to be opened on an existing site; technicians have to mount and configure the transmission unit. We assume that the capacity of constructing new sites and cells is limited. This is a reasonable assumption, since the deployment of a network involves the construction of ten-thousands of cells.

2) *Coverage Requirements*: A minimum coverage is expected and may be stipulated by license terms from network service providers in the mobile telecommunication mass market. In Section II-A.3 we introduced the cell service area, which is the part of the covered area from which all traffic can be serviced without exceeding some load threshold. The area where the cell provides coverage might be larger. The coverage constraints are not formulated on basis of the service area but on the basis of *coverage areas*. We denote the coverage area of cell  $i$  with  $\mathcal{A}_i^{(c)}$ . The total coverage area  $\mathcal{A}^{(c)}$  of a network configuration is the union of all coverage areas of cell installed:

$$\mathcal{A}^{(c)} = \bigcup_{i \text{ is installed}} \mathcal{A}_i^{(c)}$$

Let  $w : \mathcal{A} \rightarrow \mathbb{R}_{\geq 0}$  be a function weighting the planning area  $\mathcal{A}$ , such as some population density. The coverage requirement is fulfilled if the weight of the total coverage

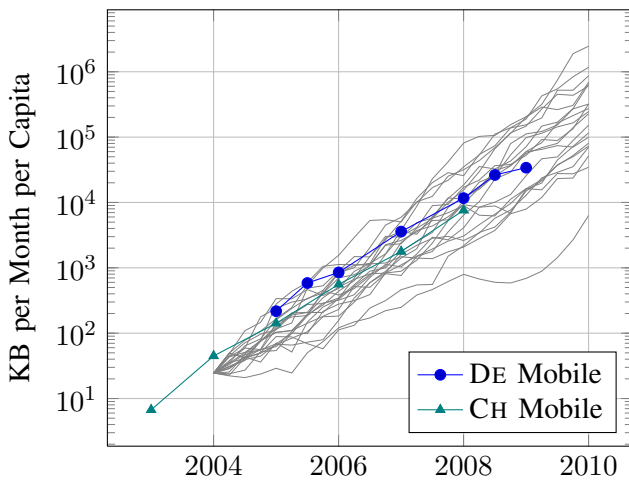


Fig. 2. Observed traffic and 20 sample paths of geometric Brownian motion (gray)

area exceeds the required percentage of the weight of the planning area. The required percentage is expressed by the parameter  $\alpha \in [0, 1]$ . The coverage requirement is satisfied if

$$\int_{\mathcal{A}^{(e)}} w(x) dx \geq \alpha \int_{\mathcal{A}} w(x) dx.$$

#### D. Traffic Per Capita Defines Spatial Service Demand

The spatial service demand can be derived from the monthly traffic per capita that is expected to be generated by mobile data services. In Section III, a stochastic model for the evolution of the demand is developed.

Given a fixed average monthly traffic per capita and a population map for the planning area, the monthly traffic volume in each pixel can be computed. Assuming that a fixed fraction of this traffic volume (25% is a common value in the literature) is spent in the busy hour of a day, the average traffic rate is computed for the pixel in the busy hour. This traffic rate is interpreted as the number of “reference users” of some reference service. These “users” generate the equivalence of the desired traffic volume. The evaluation of the network and the computation of cell service areas is then performed based on these “reference users” per pixel.

### III. STOCHASTIC MODEL OF TRAFFIC EVOLUTION

We assume future demand to be the main source of uncertainty. We model the demand in terms of the traffic volume that is generated by consumers using mobile services. The large dynamic range in the traffic observations, depicted in Figure 2, indicates that prediction errors can lead to poor planning decisions.

The power of stochastic optimization lies in the consideration of different future evolutions of key parameters in the optimization model. In multistage stochastic optimization, tree structured processes are most suitable to describe randomness. They have the property that scenarios share the first part of the random path until the paths split. Until the splitting point, these parts are indistinguishable and only the conditional distribution of future events is known at that time. Repeatedly splitting of bundles of scenarios makes the problem multistage.

In the following, a three step approach to compute scenario trees is described. In the first step, we model random influences as a stochastic process. Second, the time horizon is discretized and sample paths from the stochastic process are drawn. Third, a scenario tree is constructed. The tree approximates the information structure (i.e., the increase of information over time) inherent in the scenarios (and the probabilities) with as few nodes and scenarios as possible. The scenario tree is the input for a stochastic program that can be solved numerically. Figure 3 shows the scheme of the scenario tree construction.

*a) Traffic as Geometric Brownian Motion:* The traffic in the mobile networks has commonly been perceived as exponentially growing in the time horizon under consideration. The geometric Brownian motion is appropriate to model exponential growth with random disturbance and models the uncertain traffic evolution. The geometric Brownian motion is a stochastic process  $\mathbb{X}_t$  that solves the stochastic differential equation

$$d\mathbb{X}_t = \mu\mathbb{X}_t dt + \sigma\mathbb{X}_t dW_t.$$

The process and its properties are described, for example, by Shreve [13].

*b) Sampling Traffic Paths:* Paths are sampled from this distribution. The time horizon is discretized such that the sample points coincide with the points in time where decision have to be made. Each path represents a possible evolution of the demand. Historical data helps to get a reasonable choice of the parameters of the geometric Brownian motion. Figure 2 shows the observed traffic data in the mobile networks in Germany and Switzerland and 20 sample paths of the geometric Brownian motion. The sampled paths all have the same probability, so a large number of samples is needed to capture the characteristics of the distribution. We refer to Glasserman [4] for sampling techniques.

*c) Scenario Tree Construction:* The information structure described by the sample paths is approximated by a scenario tree. Scenario trees are constructed from sample paths using SCENRED, a tool for scenario reduction and scenario tree construction written by

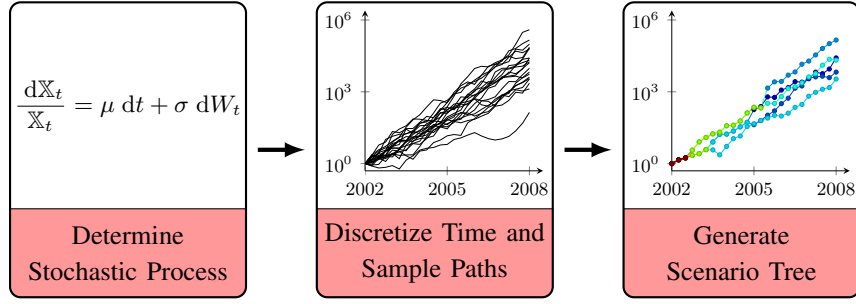


Fig. 3. Three stage approach for scenario tree construction

Heitsch [6]. SCENRED constructs a scenario tree from a given set of scenarios by minimizing an appropriate metric on the space of probability distributions, see Heitsch and Römisch [7] for details.

For our computational experiments, we constructed two different scenario trees. Both trees result from a tree construction on the basis of 300 sample paths of the geometric Brownian Motion with drift  $\mu = 1.6$  and volatility  $\sigma = 0.6$ . Different parameter settings for SCENRED were used to generate trees with different numbers of nodes. Due to the large range of the values of the sample path, we apply SCENRED to the logarithm of the sample path and transform the resulting tree back. Figure 4 visualizes the two trees. The colors of the nodes represent the access probabilities.

#### IV. OPTIMIZING NETWORK EVOLUTION

The main decision in the network evolution is where and when to build new cells. We are given a set  $\mathcal{C}$  of *candidate locations* for cells and a scenario tree with nodes  $n \in \mathcal{N}$ . The decision for the construction of a cell is represented by a binary *construction variable*  $y_{ni}$ . The model sets  $y_{ni} = 1$  if cell  $i$  is already built in node  $n$  and  $y_{ni} = 0$  otherwise. Activating the cells is only possible once the coverage requirement is fulfilled.

The planning area  $\mathcal{A}$  is discretized into two-dimensional pixel  $x \in \mathcal{A}$ . The service area  $\mathcal{A}_{ni}^{(s)}$  in node  $n$  is the subset of the planning area in which all demand for network traffic is serviced. As described in Section II-A.3, the shape of the service area depends on the demand intensity and might differ for different nodes in the scenario tree. Binary *service variables*  $x_{nx}^{(s)}$  indicate whether a pixel is serviced by the network. These are forced to 0 if the coverage requirement is not fulfilled and the constructed cells are not yet active. The set of potentially servicing cells (built and unbuilt) of a pixel  $x$  in a node  $n$  is denoted by

$$\mathcal{C}_{nx}^{(s)} = \left\{ i \in \mathcal{C} \mid x \in \mathcal{A}_{ni}^{(s)} \right\}$$

Analogously, binary *coverage variables*  $x_{nx}^{(c)}$  are set to 1 if the pixel  $x$  is within the coverage area of an active cell in node  $n$ . These variables allow the formulation of the coverage requirements.

##### A. Multistage Stochastic Program for Expected Profit

The aim of the optimization procedure is to determine an optimal network evolution plan, i. e., a plan when and where to construct cells. All technical and regulatory requirements described in Section II-C have to be modelled. We formulate a multistage stochastic integer program. Table I summarizes the variables used in the model.

$$\begin{aligned} \max \quad & \sum_{n \in \mathcal{N}} \mathbb{P}(n) \left[ \sum_{x \in \mathcal{A}} \widehat{C}_{nx} x_{nx}^{(s)} - \sum_{i \in \mathcal{C}} (\widehat{c}_{ni}^{(cap)} (y_{ni} - y_{p(n)i}) + \widehat{c}_{ni}^{(op)} y_{ni}) \right] \quad (2a) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nx}^{(s)} \quad \forall n \in \mathcal{N}, x \in \mathcal{A} \quad (2b) \\ & \sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \geq x_{nx}^{(c)} \quad \forall n \in \mathcal{N}, x \in \mathcal{A} \quad (2c) \\ & y_{ni} \geq y_{p(n)i} \quad \forall n \in \mathcal{N}, i \in \mathcal{C} \quad (2d) \\ & \sum_{i \in \mathcal{C}} (y_{ni} - y_{p(n)i}) \leq K_n \quad \forall n \in \mathcal{N} \quad (2e) \\ & \sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \geq \left( \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \right) c_n \quad \forall n \in \mathcal{N} \quad (2f) \\ & \sum_{x \in \mathcal{A}} x_{nx}^{(s)} \leq |\mathcal{A}| c_n \quad \forall n \in \mathcal{N} \quad (2g) \\ & y \in \{0, 1\}^{\mathcal{N} \times \mathcal{C}}, x^{(s)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}} \\ & x^{(c)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}}, c \in \{0, 1\}^{\mathcal{N}} \quad (2h) \end{aligned}$$

The objective is to maximize the expected net present value over an appropriate scenario tree. The net present value is the sum of all future discounted cash-flows, i. e., the discounted revenues and the discounted expenditures. The expectation is easily calculated for discrete probability distributions; we sum the discounted cash-flows of all nodes and weight them by their probability. Within the brackets the total discounted cash-flow in node  $n$  is

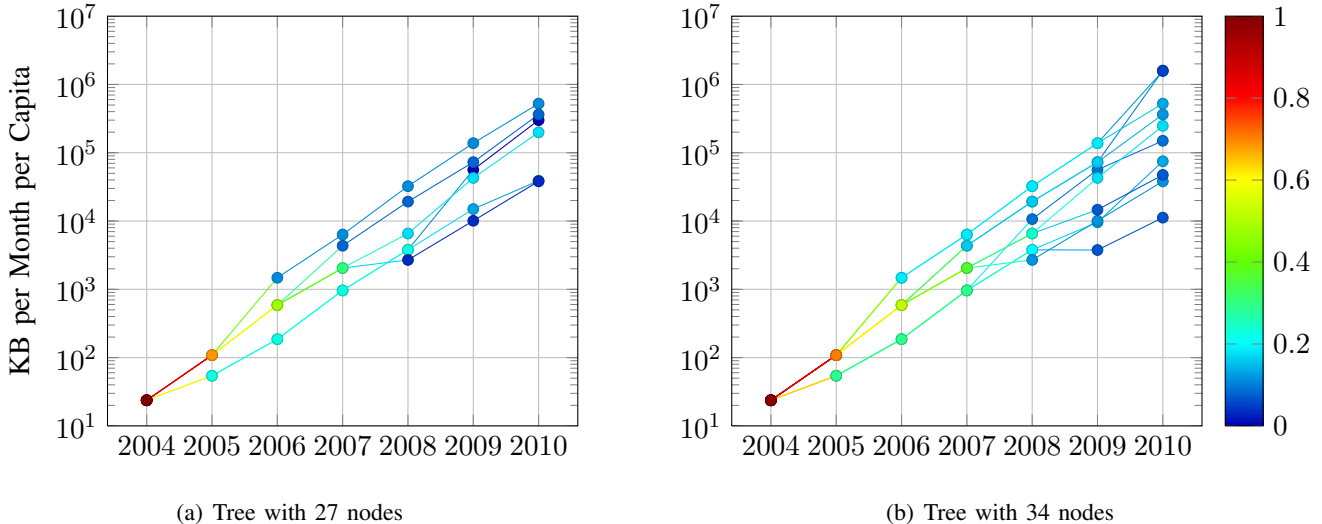


Fig. 4. The scenario trees for annual planning, colors visualize probabilities

computed. The first sum within the brackets computes the revenues while the second computes the capital and the operational expenses in the node. Constraints (2b) and (2c) ensure the correct setting of service and coverage variables, respectively. Destruction or deactivation of cells is not allowed, which is reflected by monotonicity of the variables  $y_{ni}$  and is ensured by constraint (2d). For the father of the root node  $r$  of the scenario tree, artificial variables  $y_{p(r)i} = 0$  are included. Constraint (2e) ensures that the construction limit is not violated. The difference  $y_{ni} - y_{p(n)i}$  is 1 if the cell is built in node  $n$ . The sum counts the number of cells built in node  $n$  which is limited by the construction limit  $K_n$ . The constraints (2f) and (2g) ensure that the coverage requirement is fulfilled at the moment the network starts operation. If enough coverage is provided,  $c_n = 1$  and constraint (2g) is void. If  $c_n = 0$ , constraint (2f) is always fulfilled and constraint (2g) forces all service variables  $x_{nx}^{(s)}$  to 0. The domains of the decision variables are specified in (2h).

## V. COMPUTATIONAL EXPERIMENTS

In this section, we describe our realistic planning scenario, present presolving steps to make the resulting optimization problem computationally tractable, and report optimization results with respect to two different scenario trees. Experiments on another planning scenario and with different time horizons can be found in [12].

### A. Planning Scenario

We study our methods in a realistic planning scenario comprising the area of the city of Hamburg, Germany, and surroundings. The dataset is provided by a German

TABLE I  
VARIABLES AND COEFFICIENTS IN OPTIMIZATION MODELS

Name	Domain	Interpretation
<i>Sets</i>		
$\mathcal{A}$	$\subseteq \mathbb{R}^3$	Planning area, usually divided into pixel
$\mathcal{C}$		Candidate set of cells
$\mathcal{A}_{ni}^{(s)}$	$\subseteq \mathcal{A}$	Service area of cell $i$ in node $n$
$\mathcal{A}_i^{(c)}$	$\subseteq \mathcal{A}$	Coverage area of cell $i$
$\mathcal{C}_{nx}^{(s)}$	$\subseteq \mathcal{C}$	Cells that can service pixel $x$ in node $n$
$\mathcal{C}_x^{(c)}$	$\subseteq \mathcal{C}$	Cells that can cover pixel $x$
<i>Scenario Tree</i>		
$\mathcal{N}$		Nodes in scenario tree
$p(n)$	$\in \mathcal{N}$	The father of node $n$ in the scenario tree
$\mathbb{P}(n)$	$\in (0, 1]$	Probability of node $n$
<i>Coefficients and Parameters</i>		
$K_n$	$\in \mathbb{N}$	Construction limit per timeperiod
$\alpha_n$	$\in [0, 1]$	Coverage requirement parameter
$w_{nx}$	$\in \mathbb{R}_{\geq 0}$	Weights on pixel
$\hat{C}_{nx}$	$\in \mathbb{R}_{\geq 0}$	Disc. revenue generated by pixel $x$ at node $n$
$\hat{c}_{ni}^{(cap)}$	$\in \mathbb{R}_{\geq 0}$	Disc. capital expenses for cell $i$ at node $n$
$\hat{c}_{ni}^{(op)}$	$\in \mathbb{R}_{\geq 0}$	Disc. operational expenses for cell $i$ at node $n$
<i>Decision Variables</i>		
$y_{ni}$	$\in \{0, 1\}$	Cell $i$ is built at node $n$
$x_{nx}^{(s)}$	$\in \{0, 1\}$	Pixel $x$ is serviced by an active cell at node $n$
$x_{nx}^{(c)}$	$\in \{0, 1\}$	Pixel $x$ is covered by an active cell at node $n$
$c_n$	$\in \{0, 1\}$	Coverage requirement is met at node $n$

network operator. Graphical representations of the network can unfortunately not be included into the paper.

The scenario contains data for 1476 cells at 477 sites. Due to the large number of cells, we plan on site basis, i. e., we decide which sites to build. Table II gives an overview over the scenario data and the planning parameters.



TABLE II  
SCENARIO PROPERTIES AND PLANNING PARAMETERS

Parameter	Value
<i>Scenario Properties</i>	
Area	4 682.34 km <sup>2</sup>
Pixel Size	50 m
Number of Pixel	1 872 936
Pixel with positive weight	296 538
Population Estimate	1 800 000
Candidates Locations	477 Sites with 1 476 Cells
<i>Planning Parameters</i>	
Base-line traffic	50 MB p. Cap. p. Mon.
Risk-adjusted interest rate (annual)	20 %
Construction limit	60 and 100
Max. number of technology upgrades	5
Traffic impact reduction per upgrade	50 %

### B. The Effect of Presolving

Removing redundant variables and constraints can speed up the process of solving mixed-integer programs. In the following, we present fast and exact presolving methods cause significant reductions in the problem size and thereby make the problem computationally tractable. The impact of the different presolving methods is summarized in Table III. The first row gives the problem size without presolving. In each of the following rows, one presolving method after the other is added.

1) *Removing variables without impact:* Service variables for pixel that do not generate cash-flow have no impact on the objective function. Fixing these variables to 0 is always feasible and does not change a solutions objective function value. This fixing should hence be done in a presolving step and the variables should not be included into the model. In this case, the corresponding service constraints (2b) are redundant and also deleted. The same applies to coverage variables corresponding to pixel with weight 0. Especially in sparsely populated areas, this presolving step can reduce the problem size significantly. The resulting problem size of the model if this presolving step is applied is given in the second row of Table III.

2) *Pixel Aggregation:* Two pixel that can be serviced by the same set of servers are either both covered or both uncovered. The pixel are not distinguishable by the model and can be aggregated. The aggregation of pixel results in a different discretization of the planning area. Since the shapes of the service areas depend on the traffic, the discretization of the planning area might be different in each node. Of course, an aggregated discretization is also used for the baseline traffic for the coverage requirement. The sizes of the models with aggregated pixels are given in the third row of Table III.

TABLE III  
IMPACT OF PRESOLVING ON THE MODEL SIZE FOR THE SCENARIO TREE WITH 32 NODES

	Constraints	Variables
Without presolving	127 375 491	127 375 900
Only pixel with weight > 0	20 180 427	20 180 836
Pixel aggregation	834 905	835 314
Variable reuse	619 944	620 353
Fix coverage requirement	430 737	430 295

3) *Reusing Service Variables as Coverage Variables:* Suppose a node  $n$  and two pixel  $x$  and  $y$ , where the cell of possible servers of  $x$  is equal to the set of possibly covering cells of  $y$ :

$$\mathcal{C}_{nx}^{(s)} = \mathcal{C}_y^{(c)}$$

If the coverage requirement is not fulfilled, the service variables are forced to 0. In this case, setting the coverage variables to 0 is always feasible. If the coverage requirement is fulfilled and one cell  $i \in \mathcal{C}_{nx}^{(s)}$  is built, then coverage and service variables can be set to one. In both cases they take the same value and aggregating them does neither affect the feasibility of the problem and nor change the objective function value. We can, however, eliminate some variables and constraints. In our experiments, this procedure can be applied to about half of the coverage variables. This allows a reduction of the model size of about 25 %. The figures are displayed in the fourth row of Table III.

4) *Presolving the Coverage Requirement:* If the coverage requirement cannot be fulfilled, the service and coverage variables for the respective node can be fixed to 0 and eliminated from the model. By solving a small optimization model that maximizes the coverage attainable in a node, nodes where this is the case can be identified efficiently. In our planning scenario, however, the coverage requirement can be fulfilled already in the root node and no savings are possible.

5) *Mandatory Coverage Requirement:* If the coverage requirement is mandatory in some stage, the coverage requirement variables for the respective stages are fixed to 1. If the coverage requirement is fulfilled once, it is fulfilled in all following stages. A monitoring is thus only needed in the first stage the coverage requirement is demanded. The respective constraints and coverage variables can be deleted from the model. The result is a further reduction of the number of variables and constraints as displayed in the fifth row of Table III.

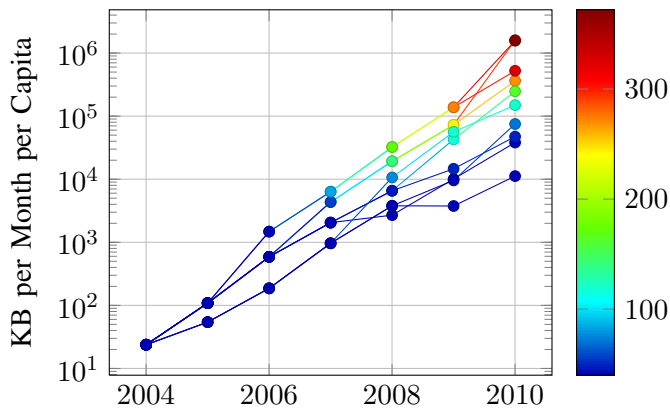


Fig. 5. Evolution of the number of installed sites (represented by the color of the nodes) in Hamburg in the scenario tree with 34 nodes and a construction limit of 100

### C. Computational Results

We conduct our computational experiments on quad-core PCs with 16GB main memory. All MIPs are generated with ZIMPL [9] and solved with CPLEX Version 12.2 [8] with a time limit of 10 hours and a relative gap limit of 0.5 %.

Generally, large differences in the network evolution across the different scenarios are observed. Consider, for example, the Hamburg annually planning scenario optimized on the scenario tree with 34 nodes and a maximum construction limit of 100. The evolution of the number of installed sites in each node is visualized in Figure 5 by the color of the node. At the root node, 40 sites are built to ensure the coverage requirement (57 % of the population are covered in the root). In the scenarios with poor traffic evolution, very few further sites are constructed after that; less than 10 with a probability of 26 %. In the scenarios with a high traffic increase, considerably more sites are built; 327 or more in 2010 with a probability of 22 %.

Negative cash-flows occur only in 3 nodes. The aggregated cash-flow is only negative in the scenario with the poorest traffic evolution (probability 7 %). In this case, the initial investment of 4 mill. is never recovered. Discounting changes this situation noticeably. Four scenarios with a total probability of 39 % have a negative final aggregated discounted cash-flow. In these scenarios, the investment in an alternative project with the same return is favorable. Taking all scenarios into account, however, the project is profitable with an expected discounted profit of 14 410 188 €. A summary of the optimization results in the different parameter settings is displayed in Table IV.

Demanding for coverage in the root node considerably decreases the performance in terms of expected profit.

TABLE V

COMPARISON OF STOCHASTIC OPTIMIZATION OVER SCENARIO TREE AND OPTIMIZATION WITH THE EXPECTED TRAFFIC EVOLUTION AND EVALUATED ON SCENARIO TREE.

	Scen. Tree	Exp. Traffic Evol.
	Expectation	Expectation
<i>Hamburg annually with mandatory coverage in root</i>		
27 Nodes, 100 Cap	12 972 933 €	11 130 202 € (-14.2 %)
27 Nodes, 60 Cap	12 906 688 €	11 147 457 € (-13.6 %)
34 Nodes, 100 Cap	14 410 188 €	11 681 093 € (-18.9 %)
34 Nodes, 60 Cap	13 969 286 €	11 592 296 € (-17.0 %)
<i>Hamburg annually</i>		
27 Nodes, 100 Cap	15 286 450 €	13 295 759 € (-13.0 %)
27 Nodes, 60 Cap	15 233 750 €	13 286 692 € (-12.8 %)
34 Nodes, 100 Cap	16 659 934 €	13 843 274 € (-16.9 %)
34 Nodes, 60 Cap	16 198 435 €	13 721 703 € (-15.3 %)

The figures for the trees differ. The expectation is about 10 % higher for the tree with 34 nodes. The reduction of the construction limit from 100 to 60 has only a minor effect in the scenario tree with 27 nodes. In the setting with the tree with 34 nodes, the expected profit suffers from the reduced construction limit. The limit is exhausted only in the scenarios with high increase in the traffic, which are the most profitable.

1) *Comparison to a Clairvoyant*: The solution of our stochastic program is a non-anticipative network evolution plan for each root-leaf path in the scenario tree. If the non-anticipativity constraints are relaxed, the model decomposes into independent subproblems for the leaves which can be optimized individually. This corresponds to the perspective of a clairvoyant. The expected profit of the clairvoyant's strategy provides an upper bound for the expected profit of the stochastic program and is displayed in the third column of Table IV.

The difference between the solution of the stochastic program and the clairvoyant's solutions is surprisingly small. In all settings, the clairvoyant outperforms us by at most 2.2 %. The absolute difference between the expectations of the clairvoyant's strategy and the non-anticipative strategy is called *expected value of perfect information* (EVPI). Since the clairvoyant's problem is a relaxation of the stochastic program, the EVPI is always non-negative. The EVPI in the different settings is stated in the last column of Table IV. We observe that the EVPI is considerable higher in the tree with 34 nodes than in the tree with 27 nodes. However, the EVPI always represents less than 2.2 % of the expected profit.

2) *Deterministic Optimization for the Expected Traffic Evolution*: Traditionally, uncertain data is treated by optimizing for the expected data evolution. The resulting

TABLE IV

COMPARISON OF OPTIMIZATION OVER THE SCENARIO TREE AND OVER THE INDIVIDUAL SCENARIOS, I. E., THE RELAXATION OF THE NON-ANTICIPATIVITY CONSTRAINTS

	Opt. over scen. tree	Relax. of non-anticipativity	
	Expectation	Expectation	EVPI
<i>Annually</i>			
27 Nodes, 100 Cap	15 286 450 €	15 291 819 € ( +0.0 %)	5 368 €
27 Nodes, 60 Cap	15 233 750 €	15 245 549 € ( +0.1 %)	11 798 €
34 Nodes, 100 Cap	16 659 934 €	16 895 924 € ( +1.4 %)	235 991 €
34 Nodes, 60 Cap	16 198 435 €	16 510 183 € ( +1.9 %)	311 749 €
<i>Annually with mandatory coverage in root</i>			
27 Nodes, 100 Cap	12 972 933 €	13 024 427 € ( +0.4 %)	51 494 €
27 Nodes, 60 Cap	12 906 688 €	12 969 340 € ( +0.5 %)	62 652 €
34 Nodes, 100 Cap	14 410 188 €	14 595 176 € ( +1.3 %)	184 988 €
34 Nodes, 60 Cap	13 969 286 €	14 279 881 € ( +2.2 %)	310 595 €

network evolution can then be evaluated in the different scenarios of a scenario tree and the expected profit of that evolution strategy can be calculated. The result of this experiment is displayed in Table V and compared to the solution of the stochastic optimization.

A significantly poorer performance is observed. The expected profit decreases between 13 % and 19 %. This comparison shows that the stochastic programming approach yields significantly better results than planning on the expected demand evolution. In practice, however, the network operator would probably not stick to the planning on the basis of the expected traffic evolution, but would re-optimize regularly to adjust the planning to the shifted expectation of the future demand evolution. The procedure in practice would be a rolling horizon approach and would probably yield better solutions.

## VI. CONCLUSIONS

In this paper, we propose, implement, and analyze the application of multistage stochastic programming in strategic cellular network planning. Network operators in the telecommunication industry face strong, hardly predictable demand evolution. This applies to the types of the services requested as well as to the demand intensity. Strategic planning is substantial in this highly uncertain market environment, since huge financial investments have to be mastered.

The classical approaches for strategic decision taking, such as net present value and decision tree analysis, have considerable shortcomings. The popular real options approach addresses most of them, but the theoretical barrier for the proper application is high and the conditions often not fulfilled.

Recently, stochastic programming has been proposed as an alternative to real options, which avoids the assumptions from financial mathematics [15]. The steady

advances in MIP technology as well as computer technology render the routine solution of large-scale mixed-integer programs possible. These two observations are at the starting point of our work. The goal is to analyze how stochastic programming can be used to tackle realistic strategic network planning problems.

We use a realistic system model of UMTS radio cells, which takes signal propagation and interferences into account. We devise an approach for strategic UMTS radio network planning that maps both, cell coverage as well as a cell's capability to serve demand, into the notion of a cell's service area. This also allows to model the increase of cell capacity through technology upgrades.

The demand evolution is modeled as a continuous stochastic process, which is approximated by a discrete scenario tree. We use a three-stage approach for the construction of non-uniform scenario trees that serve as input of the stochastic program.

We conduct computational experiments on a realistic planning scenario. We study eight different planning settings in total. The corresponding deterministic equivalent problems are solved using a commercial MIP solver to small optimality gaps within 10 hours running time. Our aggregation technique allows to substantially reduce the problem sizes of the corresponding deterministic equivalent programs; on realistic input data a reduction from about 120 000 000 to 600 000 variables and constraints is achieved in some instances (see Table III).

The optimization result provides a tree-like network deployment plan, where construction over time is tuned to the demand evolution. The evolution paths differ considerable across the tree. Compared to deterministic optimization for the expected demand evolution, the expected profit increases by at most 18.9 % if our approach

is applied. In conclusion, the acceptable solution times as well as the obtained results encourage the application of stochastic methods in this setting.

Relaxing the non-anticipativity of the network evolution yield results from a clairvoyant's perspective. We observe improvements of at most 2% in the expected profit. The value of perfect information is hence rather small in the planning scenarios under consideration.

The framework can be extended to use different risk-measures, such as some multi-period Average-Value-at-Risk, instead of the expected profit.

Our approach can be applied to plan the evolution of other cellular radio network technologies as well. Planning for LTE requires merely small changes in the system model and an update of model parameters. The market model should be updated to match the present market conditions with a high degree of penetration (approaching saturation) and flat rate contracts. An application to planning optical fiber access networks (FTTX) may also be feasible, but would require an in-depth analysis not conducted here.

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