Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

JENS BURGSCHWEIGER, BERND GNÄDIG, AND MARC C. STEINBACH

Optimization Models for Operative Planning in Drinking Water Networks

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OPTIMIZATION MODELS FOR OPERATIVE PLANNING IN DRINKING WATER NETWORKS

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ABSTRACT. The topic of this paper is minimum cost operative planning of pressurized water supply networks over a finite horizon and under reliable demand forecast. Since this is a very hard problem, it is desirable to employ sophisticated mathematical algorithms, which in turn calls for carefully designed models with suitable properties. The paper develops a nonlinear mixed integer model and a nonlinear programming model with favorable properties for gradient-based optimization methods, based on smooth component models for the network elements. In combination with further nonlinear programming techniques (to be reported elsewhere), practically satisfactory near-optimum solutions even for large networks can be generated in acceptable time using standard optimization software on a PC workstation. Such an optimization system is in operation at Berliner Wasserbetriebe.

0. INTRODUCTION

Municipal water supply systems constitute a central part of the public infrastructure and cause substantial costs, both in monetary and energetic terms. Avoiding unnecessary consumption of resources is therefore desirable for economical as well as ecological reasons. To achieve this goal, model-based decision support tools become increasingly important. The principal planning tasks include optimal network design to reduce investment costs and optimal network operation to minimize running costs. The subject of this paper is network operation. The mathematical problem of operative planning is hard because it involves both discrete and continuous decisions, in addition to the complexity caused by close-meshed networks and temporal coupling over the entire planning horizon. From a practical viewpoint, this makes it difficult to generate sufficiently accurate and reliable solutions in acceptable time. To achieve a reasonable compromise between model accuracy and computation times, the development of appropriate models for advanced optimization methods is important.

Because of the enormous complexity of the operative planning task, early mathematical approaches typically rely on substantially simplified network hydraulics (by dropping all nonlinearities, for instance) [13, 14, 18, 32, 39], which is often unacceptable in practice. Other authors employ discrete dynamic programming [8, 9, 11, 29, 31, 41], which is mathematically sound but only applicable to small networks unless specific properties can be exploited to increase efficiency. Optimization methods based on nonlinear models (mostly for the pumps only) are reported in [3, 10, 12, 24, 36]. These approaches employ computationally expensive meta-heuristics or suffer from inefficient coupling of gradient-based optimization with non-smooth simulation by existing network hydraulics software, such as EPANET [35]. More recent related work addresses modeling and optimization for networks of irrigation and sewage canals or for gas networks, see, e.g., [19, 23, 28, 38]. Note finally that the hydraulic equations already have an intrinsic optimization structure [1]; quite general mathematical formulations together with existence, uniqueness, and sensitivity results can be found in [16, 17].

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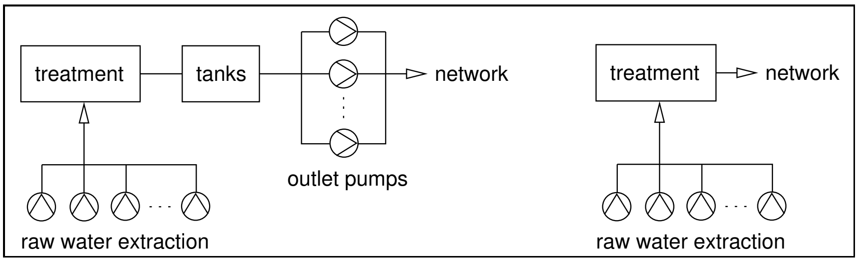

FIGURE 1. Schematic diagrams of waterworks with tanks (type I; left) and without tanks (type II; right)

The current paper addresses detailed and comprehensive models that are suited for gradient-based nonlinear optimization of daily network operations under reliable demand forecasts. These models possess certain favorable smoothness and regularity properties. Together with further work by the authors [4, 5, 7, 20, 21], they provide the basis for an optimization module that is in operation at Berliner Wasserbetriebe (BWB) for minimum cost operative planning of integrated raw water and pure water management in the municipal drinking water network. Our models are generic in that they are applicable to any pressurized water supply network consisting of similar elements. To add concreteness, however, we will often refer to the BWB network.

The paper is organized as follows. Section 1 outlines the overall structure and operation of urban water supply networks. Section 2 then develops a detailed and comprehensive physical model for dynamic network operation in continuous time, with emphasis on smooth approximations of the hydraulic pressure loss in pipes and of the aggregate efficiency of pumping stations. This model covers basic water hydraulics [15, 26, 30]. In Section 3 we consider the full operative planning problem with binary and continuous decisions in discrete time, providing both a generalized disjunctive programming (GDP) and a mixed integer nonlinear programming (MINLP) formulation. In developing the latter we avoid introducing additional nonlinearities or undesirable big-M terms and ensure that the relaxations are nondegenerate. Since a full mixed-integer optimization is impractical for large networks as in Berlin, we finally present a basic nonlinear programming (NLP) model in Section 4 which, in combination with special techniques addressing the binary decisions [7], is actually suited for practical computations. At BWB, a hierarchical solution strategy employs this model for the overall network optimization, followed by local mixed-integer optimizations for each waterworks or pumping station.

1. APPLICATION BACKGROUND

The technical process under consideration consists of the four main steps: raw water extraction, water treatment, intermediate storage, and distribution.

Raw water is extracted from reservoirs, either groundwater wells or sources of surface water such as lakes or rivers. It is treated in waterworks by various chemical and physical processes to guarantee the required quality standards. After the treatment, the clean water is either stored in pure water tanks or directly injected into the pressurized distribution network. In the fist case, pure water pumps have to be used in order to inject the water into the network. In the latter case, water treatment takes place under pressure so that increase of pressure is only produced by the raw water pumps; see Fig. 1.

There are additional pure water tanks and pumping stations within the distribution network. Generally, the tanks are filled during periods of low demand and deflated during the peak demand periods.

The filling of the tanks is controlled by valves. Such valves are also located at various other places within the network in order to control pressure and flow between adjacent

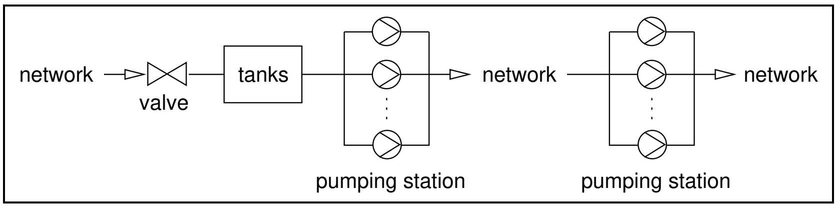

FIGURE 2. Schematic diagrams of pumping station with tanks (left) and without tanks (booster; right)

TABLE 1. Basic notation

Symbol	Explanation	Value	Unit
Q	Volumetric flow rate in arcs		m^3/s
D	Demand flow rate at junctions		m^3/s
H	Pressure at nodes (head)		m
$\bar{H}$	Constant pressure at reservoirs		m
ΔH	Pressure increase at pumps, decrease at valves		m
ρ	Water density	1000	kg/m^3
g	Gravity constant	9.81	m/s^2

regions. Pumping stations are required for emptying the tanks, but there are also pumping stations without a tank on the suction side. These stations (boosters) just increase the pressure from the suction side to the pressure side; see Fig. 2. The outlet pressure at household connections must always be kept in a certain range. This is ensured by continuously monitoring the network pressure at certain pressure measurement points.

Although there is plenty of surface water in the Berlin area, BWB produces the drinking water exclusively from ground water. This is because of the excellent water quality caused by the sandy ground: no sterilization or treatment of the raw water with additional chemicals are required; it is only aerated and filtered in the waterworks.

2. PHYSICAL MODEL

First we model the physical and technical network behavior in continuous time and discuss approximations of some components. The basic notation is given in Table 1. As usual, the physical model is macroscopic in space and time. Spatial model components are the physical network elements (pipes, tanks, and armatures, each described by a few dynamic variables). Short-period control actions such as starting up a pump or shutting a valve are considered to happen instantaneously. The dynamic variables may therefore have finitely many jump discontinuities but are assumed to be bounded and piecewise smooth (L^∞). Dynamic variables of which derivatives are taken will be considered to be bounded, continuous, and piecewise differentiable with bounded derivatives ($H^{1,\infty}$).

2.1. **Optimization Horizon.** The planning period is denoted as $I = [0, T]$. At BWB this time interval represents the following day, $T = 24\,\mathrm{h}$.

2.2. **Network Topology.** The mathematical model of the water network is based on a directed graph $G = (\mathcal{N}, \mathcal{A})$ whose node set represents junctions, reservoirs, and tanks,

$$\mathcal{N} = \mathcal{N}_{\mathrm{jc}} \cup \mathcal{N}_{\mathrm{rs}} \cup \mathcal{N}_{\mathrm{tk}},$$

and whose arc set represents pipes, pumps, and gate valves,

$$\mathcal{A} = \mathcal{A}_{\mathrm{pi}} \cup \mathcal{A}_{\mathrm{pu}} \cup \mathcal{A}_{\mathrm{vl}}.$$

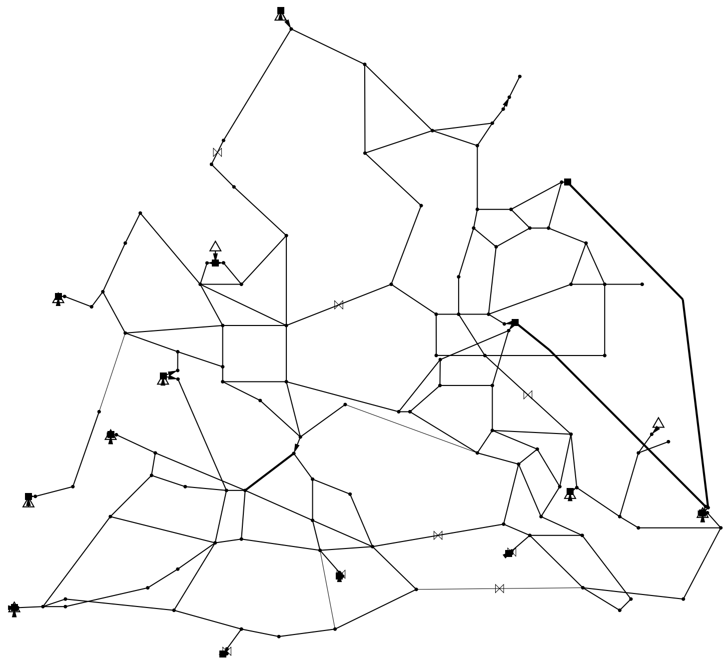

FIGURE 3. Test configuration for optimization at BWB

The set of pumps consists of raw water pumps (at the reservoirs) and pure water pumps (at the outlets of type I waterworks and pumping stations), $\mathcal{A}_{pu} = \mathcal{A}_{pr} \cup \mathcal{A}_{pp}$.

We denote individual arcs as $a \in \mathcal{A}$ or, using the tail and head $i, j \in \mathcal{N}$, as $ij \in \mathcal{A}$. The flow in an arc ij is defined to be positive when it is directed from i to j; otherwise it is negative (or possibly zero). The flow is always nonnegative in the pumps and in some other arcs where only one direction is possible.

Initial investigations toward optimization at BWB were conducted with a small network graph having 144 nodes and 192 arcs, the *test configuration* shown in Fig. 3. This heavily reduced graph, which contains mostly major pipes of 0.5 m or more in diameter, turned out to be too coarse for practical purposes. The operational model is therefore based on a larger graph with 1481 nodes and 1935 arcs, the *main network* illustrated in Fig. 4.

2.3. **Pressure and Flow.** The principal dynamic variables in the network model are node pressures and arc flows. Due to the incompressibility of water, pressure p can equivalently be expressed as an elevation difference Δh,

$$\Delta h = \frac{p}{g\rho},$$

where g is the gravity constant and ρ is the constant water density. In water management, pressure is therefore often measured by the fictitious elevation above sea level, the *head* H, which is the sum of the actual geodetic height and of the elevation difference corresponding to the hydraulic pressure. Thus, for instance, a network pressure of 3 bar at 60 m above sea level corresponds to the head $H = 90.6\,\text{m}$, a typical value in the Berlin area.

Dynamic pressure variables $H_j(t)$ with upper and lower bounds $H_j^{\pm}(t)$ are associated with every node $j \in \mathcal{N}$. The bounds are typically static, $H_j^{\pm}(t) = H_j^{\pm}$ for all $t \in [0, T]$, except at the outlets of waterworks and pumping stations. At BWB we impose network-wide default bounds, $H^- = 20\,\text{m}$ and $H^+ = 125\,\text{m}$, to keep iterates within reasonable physical limits during computations. Tighter bounds will be specified where appropriate.

Volumetric flow rates $Q_a(t)$ with bounds $Q_a^{\pm}(t)$ are associated with every arc $a \in \mathcal{A}$. Here we use default bounds $Q^{\pm} = \pm 10\,\text{m}^3/\text{s}$, and again tighter values where appropriate.

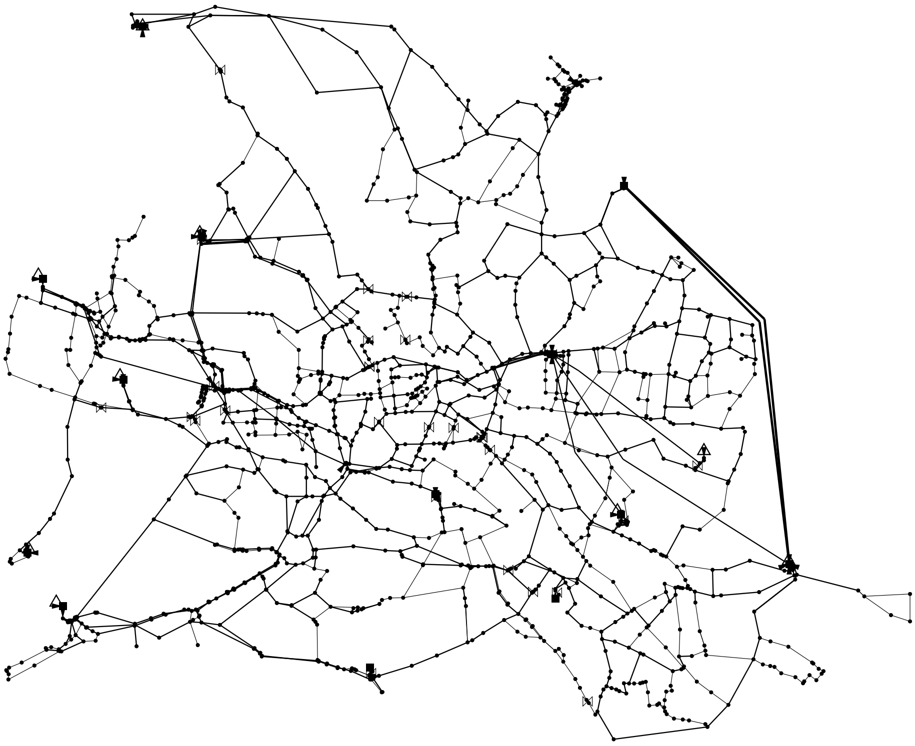

FIGURE 4. Main distribution network of BWB

Further degrees of freedom are the controlled pressure increase in pumps and pressure decrease in valves, $\Delta H_a(t)$, $a \in \mathcal{A}_{pu} \cup \mathcal{A}_{vl}$, with bounds $\Delta H_a^{\pm}(t)$. Here the default values are $\Delta H_a^- = 0$ in pumps, $\Delta H_a^- = -125\,\mathrm{m}$ in valves, and $\Delta H_a^+ = 125\,\mathrm{m}$ in both cases. Depending on the chosen degree of detail, additional variables may later be introduced in selected model components.

2.4. **Junction Model.** Every junction node $j \in \mathcal{N}_{jc}$ has an externally given demand profile $D_j(t)$ so that the continuity equation (conservation of mass) yields flow balance equations of Kirchhoff type,

$$\sum_{i:\, ij \in \mathcal{A}} Q_{ij}(t) - \sum_{k:\, jk \in \mathcal{A}} Q_{jk}(t) - D_j(t) = 0. \tag{1}$$

The default pressure bounds are replaced with local values $H_j^{\pm}(t)$ at the outlet junctions of waterworks and pumping stations, and at predefined *pressure measurement points* where sensors are installed that monitor the network state permanently to ensure safe operation.

2.5. **Reservoir Model.** Reservoirs $j \in \mathcal{N}_{rs}$ behave like unlimited sources of raw water where the pressure has a known constant value $\bar{H}_j$,

$$H_j(t) - \bar{H}_j = 0.$$

No further constraints need to be satisfied since the hourly as well as daily amounts of water extracted from the reservoirs are bounded by limits associated with the raw water pumps; see Section 2.8.

2.6. **Tank Model.** At the tanks $j \in \mathcal{N}_{tk}$, the flow balance equations are similar to (1) but involve the volumetric tank inflow $E_j(t)$ instead of a demand,

$$\sum_{i:\, ij\in\mathcal{A}} Q_{ij}(t) - \sum_{k:\, jk\in\mathcal{A}} Q_{jk}(t) - E_j(t) = 0. \tag{2}$$

The (possibly negative) tank inflow $E_j(t)$ depends on the head $H_j(t)$ through the effective tank filling volume $V_j(t)$. The latter is related to the filling level $h_j(t)$ by a characteristic function f_j,

$$h_j(t) = f_j(V_j(t), t),$$

whose inverse with respect to the first argument is given by the vertical profile of the cross-sectional tank area A_j,

$$V_j(t) = f_j^{-1}(h_j(t), t) = \int_0^{h_j(t)} A_j(h, t)\, dh. \tag{3}$$

Here the explicit time-dependence of A_j (and hence f_j) reflects the fact that one or more physical tanks constituting the conceptual tank $j \in \mathcal{N}_{tk}$ may be temporarily unavailable,

$$A_j(h, t) = \sum_{\nu=1}^{N_j} Y_{j\nu}(t) A_{j\nu}(h), \tag{4}$$

where the functions $A_{j\nu}(h)$ model the individual tank profiles, and the binary availability profiles $Y_{j\nu}(t)$ are externally given, representing maintenance schedules or the like. Substituting the sum (4) into the volume integral (3) and differentiating yields

$$\begin{aligned}
\dot V_j(t) &= \sum_{\nu=1}^{N_j} \dot Y_{j\nu}(t) \int_0^{h_j(t)} A_{j\nu}(h)\, dh + \sum_{\nu=1}^{N_j} Y_{j\nu}(t) A_{j\nu}(h_j(t)) \dot h_j(t) \\
&= \sum_{\nu=1}^{N_j} \dot Y_{j\nu}(t) V_{j\nu}(t) + A_j(h_j(t), t) \dot h_j(t) = \sum_{\nu=1}^{N_j} \dot Y_{j\nu}(t) V_{j\nu}(t) + E_j(t).
\end{aligned}$$

Here the first term (with Dirac measures $\dot Y_{j\nu}$) models abrupt effective volume changes due to adding or removing individual tanks while the second term is precisely the tank inflow.

The pressure variables H_j simply represent the tank filling above sea level,

$$H_j(t) = z_j + h_j(t), \tag{5}$$

where z_j is the elevation of the tank floor. Static bounds $H_j^{\pm}$ are naturally given by the elevation of the pumps (dry run) and the tank geometry (overflow). (The filling level and head as well as the corresponding bounds are always identical for connected individual tanks.) With (5) we can finally express the tank inflow as

$$E_j(t) = A_j(H_j(t) - z_j, t) \dot H_j(t). \tag{6}$$

2.7. **Pipe Model.** In every pipe, $a = ij \in \mathcal{A}_{pi}$, hydraulic friction causes a pressure loss. This friction loss is given by the formula of Darcy–Weisbach (cf. [15, §8.1]),

$$H_i(t) - H_j(t) = \lambda_a(v_a(t)) \frac{L_a}{d_a} \frac{v_a(t)|v_a(t)|}{2g}, \tag{7}$$

where L_a and d_a are the length and bore (inner diameter) of the pipe, respectively, and the average water velocity v_a is the flow rate divided by the cross-sectional pipe area A_a,

$$v_a(t) = \frac{Q_a(t)}{A_a} = \frac{4}{\pi d_a^2} Q_a(t).$$

In our model (using flow variables Q_a), the pressure loss equation thus reads

$$H_j(t) - H_i(t) + r_a(Q_a(t))\, Q_a(t)|Q_a(t)| = 0, \tag{8}$$

TABLE 2. Notation for the pipe model

Symbol	Explanation	Value	Unit
L	Pipe length		m
d	Pipe diameter (bore)		m
k	Pipe roughness		m
A	Pipe cross-sectional area		m^2
λ	Pipe friction coefficient		–
r	Pipe hydraulic loss coefficient		s^2/m^5
υ	Kinematic viscosity of water; at 10° C:	1.31e−6	m^2/s

where the hydraulic loss coefficient $r_a(Q_a)$ can be written

$$r_a(Q_a) = \frac{L_a}{2gd_aA_a^2}\lambda_a(Q_a/A_a) = \frac{8L_a}{\pi^2 g d_a^5}\lambda_a(v_a). \tag{9}$$

In the following subsections, we discuss empirical laws and suitable approximations for the friction coefficient $\lambda_a(v_a)$ and the associated friction loss $\Delta H_a(Q_a) = r_a(Q_a)\, Q_a|Q_a|$. Note that the pressure always decreases in the direction of flow; hence ΔH is an odd function of Q_a, and the positive coefficients λ_a, r_a depend only on $|v_a|$ and $|Q_a|$, respectively. This also applies to other approximations of the pressure loss ΔH_a. We just mention the most common one, the formula of Hazen–Williams,

$$\Delta H_a(Q_a) = r_a^{\mathrm{HW}}|Q_a|^{1.85}\operatorname{sign}(Q_a), \tag{10}$$

where the loss coefficient r_a^{HW} depends only on the pipe parameters.

2.7.1. *Friction Coefficient.* The friction coefficient $\lambda_a(v_a)$ in (7) and (9) is determined by the nature of the flow as characterized by the value of the non-dimensional Reynolds number,

$$\mathrm{Re}_a(t) = \frac{d_a}{\upsilon}|v_a(t)| = \frac{4}{\pi\upsilon d_a}|Q_a(t)|,$$

where υ denotes the kinematic viscosity of water.

Two cases have to be distinguished. (We drop the pipe subscript and time argument for simplicity). In the usual case of *turbulent* or *vortical* flow (Re > 2320), the friction coefficient depends not only on the Reynolds number (and hence the flow rate) but also on the roughness of the inner pipe surface, k, according to the law of Prandtl–Colebrook,

$$\frac{1}{\sqrt{\lambda^{\mathrm{PC}}}} = -2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{\lambda^{\mathrm{PC}}}} + \frac{k}{3.71\,d}\right). \tag{11}$$

This formula is also known as law of Colebrook–White.

For *laminar* flow (Re < 2320), the friction coefficient depends on the Reynolds number only, according to the law of Hagen–Poisseulle,

$$\frac{1}{\sqrt{\lambda^{\mathrm{HP}}}} = \frac{\mathrm{Re}\sqrt{\lambda^{\mathrm{HP}}}}{64} \iff \lambda^{\mathrm{HP}} = \frac{64}{\mathrm{Re}}. \tag{12}$$

Note that the pressure loss in this case grows *linearly* with the flow rate,

$$\Delta H_a(Q) = \frac{64}{4|Q|}\pi\upsilon d_a \frac{8L_a}{\pi^2 g d_a^5}Q|Q| = \frac{128\upsilon L_a}{\pi g d_a^4}Q,$$

whereas in the turbulent case it grows roughly quadratically.

Finally, two simplifications of the formula of Prandtl–Colebrook are commonly used: the law of Prandtl–Kármán for hydraulically smooth pipes,

$$\frac{1}{\sqrt{\lambda^{\mathrm{PKs}}}} = -2\log\frac{2.51}{\mathrm{Re}\sqrt{\lambda^{\mathrm{PKs}}}} = 2\log\frac{\mathrm{Re}\sqrt{\lambda^{\mathrm{PKs}}}}{2.51}, \tag{13}$$

and the law of Prandtl–Kármán for hydraulically rough pipes,

$$\frac{1}{\sqrt{\lambda^{\mathrm{PKr}}}} = -2\log\frac{k}{3.71\,d} \iff \lambda^{\mathrm{PKr}} = \left(2\log\frac{k}{3.71\,d}\right)^{-2}. \tag{14}$$

These expressions are obtained as the respective limit cases for $k = 0$ (hydraulically smooth pipe) and for $\mathrm{Re} \to \infty$ (hydraulically rough pipe).

Based on the above empirical laws, (11), (12), and (14), we consider two models for the friction loss $\Delta H_a(Q)$ along a pipe $a \in \mathcal{A}_{\mathrm{pi}}$: the *HP-PC friction model* as reference,

$$\Delta H_a^{\text{HP-PC}}(Q) = r_a^{\text{HP-PC}}(Q)\,Q|Q| \quad \text{with} \quad r_a^{\text{HP-PC}}(Q) = \begin{cases} r_a^{\mathrm{HP}}(Q), & \mathrm{Re} \le 2320, \\ r_a^{\mathrm{PC}}(Q), & \mathrm{Re} > 2320, \end{cases} \tag{15}$$

and the *PKr friction model* as simplification,

$$\Delta H_a^{\mathrm{PKr}}(Q) = r_a^{\mathrm{PKr}}\,Q|Q|, \tag{16}$$

where $r_a^{\mathrm{PC}}(Q)$, $r_a^{\mathrm{HP}}(Q)$, and r_a^{PKr} are given by (9) with (11), (12), or (14), respectively.

2.7.2. *Approximation of Friction Coefficient.* For various reasons, the two friction models just defined are not well suited for application in derivative-based optimization methods. The HP-PC model is highly accurate but has jump discontinuities at the transitions between laminar and turbulent flow. It is also implicit and therefore computationally expensive. The PKr model (with constant coefficient r^{PKr}) is simple and inexpensive but has a second order jump discontinuity at $Q = 0$ (where the HP-PC model is linear and hence smooth). Moreover, it is not very accurate for small $|Q|$, and one can show that asymptotically for large $|Q|$ only the second order derivative $(\Delta H^{\mathrm{PKr}})'' = \pm 2r^{\mathrm{PKr}}$ agrees with the HP-PC reference model, but not ΔH^{PKr} and $(\Delta H^{\mathrm{PKr}})'$.

Here we develop a globally smooth explicit approximation of ΔH that is asymptotically correct for the usual large flow rates. As an additional benefit, it does not underestimate ΔH for small flow rates, thus leading to conservative solutions.

To investigate the asymptotic behavior of the pressure loss, consider for fixed relative roughness k/d the function that implicitly defines the Prandtl–Colebrook coefficient,

$$F^{\mathrm{PC}}(\mathrm{Re},\lambda) = \frac{1}{\sqrt{\lambda}} + 2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{\lambda}} + \frac{k/d}{3.71}\right).$$

This function is defined on the positive orthant of $\mathbf{R}^2$ and has the following properties: it is smooth and convex jointly in both arguments, it is strictly decreasing in Re or λ when the other argument is fixed, and in these cases it has limit values

$$\lim_{\lambda\downarrow 0} F^{\mathrm{PC}}(\mathrm{Re},\lambda) = +\infty \qquad \lim_{\lambda\to+\infty} F^{\mathrm{PC}}(\mathrm{Re},\lambda) = 2\log\frac{k/d}{3.71},$$
$$\lim_{\mathrm{Re}\downarrow 0} F^{\mathrm{PC}}(\mathrm{Re},\lambda) = +\infty, \qquad \lim_{\mathrm{Re}\to+\infty} F^{\mathrm{PC}}(\mathrm{Re},\lambda) = 2\log\frac{k/d}{3.71} + \frac{1}{\sqrt{\lambda}}.$$

For every $\mathrm{Re} > 0$, these properties immediately imply the existence and uniqueness of the friction coefficient λ^{PC} such that $F^{\mathrm{PC}}(\mathrm{Re},\lambda^{\mathrm{PC}}) = 0$, since $0 \le k < d/2$ and

$$\sup_{\lambda} F^{\mathrm{PC}} = +\infty, \quad \inf_{\lambda} F^{\mathrm{PC}} \le 2\log\frac{k/d}{3.71} < 2\log\frac{1}{7.42} < -0.87 < 0.$$

This yields a smooth dependence $\lambda^{\mathrm{PC}} = f^{\mathrm{PC}}(\mathrm{Re})$ for fixed relative roughness k/d. It can further be shown that above we actually have equality $\inf_\lambda F^{\mathrm{PC}} = 2\log[k/(3.71d)]$, and that f^{PC} is convex, strictly decreasing, and bounded away from zero with

$$\inf f^{\mathrm{PC}} = \lim_{\mathrm{Re}\to\infty} f^{\mathrm{PC}}(\mathrm{Re}) = \left(2\log\frac{k/d}{3.71}\right)^{2}.$$

This infimum is precisely the friction coefficient λ^{PKr} (14).

Next, we derive a quadratic polynomial in Q that approximates the friction coefficient λ^{PC} up to second order for $Q \to +\infty$. To this end, define the positive constants

$$\alpha = \frac{2.51}{4/(\pi \upsilon d)}, \quad \beta = \frac{k/d}{3.71}, \quad \gamma = \frac{8L}{\pi^2 g d^5},$$

and let

$$x = Q, \quad y = \frac{1}{\sqrt{\lambda}}, \quad z = \frac{\alpha}{\beta}\frac{y}{x}.$$

For $Q > 0$, the pressure loss (8) and the law of Prandtl–Colebrook (11) then read

$$\Delta H = rx^2 = \gamma\lambda x^2, \quad y = -2\log\left(\alpha\frac{y}{x} + \beta\right) = -2\log\bigl[\beta(1+z)\bigr].$$

Since $\log w = \ln w / \ln 10$, the last equation can be rewritten as

$$\frac{\ln 10}{2} y = -\ln\beta - \ln(1+z),$$

yielding for $|z| < 1$

$$\ln\beta + \frac{\ln 10}{2}\frac{\beta}{\alpha} xz = -z + \frac{z^2}{2} - \frac{z^3}{3} \pm \cdots . \tag{17}$$

It is now easily seen that $\lambda^{PKr} = 1/(2\log\beta)^2$ is obtained with the series expansion of order zero, $\ln(1+z) \approx 0$, for small z (large x). Starting from the expansion of order one instead, $\ln(1+z) \approx z$, one obtains after some algebraic manipulations the coefficient

$$\lambda = \left(-\frac{\alpha}{(\beta\ln\beta)x} - \frac{1}{2\log\beta}\right)^2,$$

and the resulting pressure loss for large x,

$$\Delta H = \gamma\lambda x^2 = \gamma\left(\frac{\alpha}{\beta\ln\beta} + \frac{x}{2\log\beta}\right)^2 = r^{PKr}\left(\frac{2\alpha}{\beta\ln 10} + x\right)^2,$$

where $r^{PKr} = \gamma\lambda^{PKr} = \gamma/(2\log\beta)^2$. Letting

$$\delta = 2\alpha/(\beta\ln 10),$$

this formula can be shown to be asymptotically correct up to the constant $\lambda^{PKr}(\ln\beta)\delta^2 < 0$, so that we finally obtain the desired asymptotic approximation for $x \to +\infty$:

$$\Delta H \approx \gamma\lambda x^2 = r^{PKr}\left(x^2 + 2\delta x + (\ln\beta + 1)\delta^2\right). \tag{18}$$

As smooth global approximation for $\Delta H = H_i - H_j$ we now suggest a function of the general form

$$\Delta H^{PKrs}(x) = r^{PKr}\left(\sqrt{x^2 + a^2} + b + \frac{c}{\sqrt{x^2 + d^2}}\right)x, \tag{19}$$

with first and second order derivatives

$$(\Delta H^{PKrs})'(x) = r^{PKr}\left(\frac{2x^2 + a^2}{\sqrt{x^2 + a^2}} + b + \frac{cd^2}{\sqrt{x^2 + d^2}^3}\right),$$

$$(\Delta H^{PKrs})''(x) = r^{PKr}\left(\frac{2x^3 + 3a^2 x}{\sqrt{x^2 + a^2}^3} - \frac{3cd^2 x}{\sqrt{x^2 + d^2}^5}\right).$$

We call ΔH^{PKrs} the *smoothed PKr friction model* since it differs from the PKr model mainly in that it approximates the absolute value function $|x| = \sqrt{x^2}$ with $\sqrt{x^2 + a^2}$, thus smoothing out the kink at $x = 0$; see Fig. 5 (bottom left, solid line vs. dashed line).

Finally we have to select the parameters a, b, c, d in such a way that asymptotic correctness is guaranteed. To compare (18) with (19), let us define the asymptotic error

$$E(x) = \left(\sqrt{x^2 + a^2} + b + \frac{c}{\sqrt{x^2 + d^2}}\right)x - \left(x^2 + 2\delta x + (\ln\beta + 1)\delta^2\right). \tag{20}$$

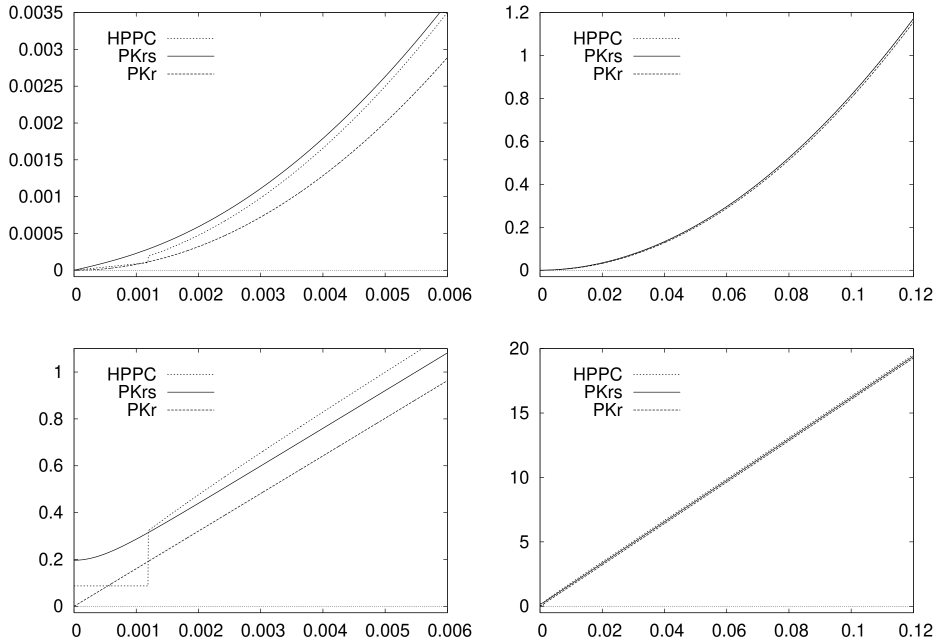

FIGURE 5. Comparison of friction loss models (top) and their first order derivatives (bottom) for small flow rates (left) and large flow rates (right); flow rates in m^3/s, friction loss in m/km, derivatives in $(\mathrm{m/km})/(\mathrm{m}^3/\mathrm{s})$

Then one obtains for $x \to \infty$

$$E(x) \to (b - 2\delta)x + \frac{a^2}{2} + c - (\ln\beta + 1)\delta^2, \quad E'(x) \to b - 2\delta, \quad E''(x) \to 0.$$

Asymptotic correctness thus determines the parameters

$$b = 2\delta, \quad c = (\ln\beta + 1)\delta^2 - \frac{a^2}{2},$$

satisfying $b > 0$ and (irrespective of a) $c < 0$. The parameters $a > 0$ and $d > 0$ influence the approximation only close to $x = 0$, where

$$\Delta H^{\mathrm{PKrs}}(0) = 0, \quad (\Delta H^{\mathrm{PKrs}})'(0) = r^{\mathrm{PKr}}(a + b + c/d), \quad (\Delta H^{\mathrm{PKrs}})''(0) = 0.$$

Thus a and d can be chosen to match any desired slope at $x = 0$ while the value and curvature remain zero. In addition, the relative contributions of the two square root terms at small flow values can be balanced.

2.8. **Pump Model.** Every pump, $a = ij \in \mathcal{A}_{\mathrm{pu}}$, increases the pressure by some controlled nonnegative amount $\Delta H_a(t)$,

$$H_j(t) - H_i(t) - \Delta H_a(t) = 0. \tag{21}$$

As with the tanks, such a conceptual pump usually consists of several physical pumps operated in parallel; see Figures 1 and 2. Aggregating pumps allows a largely simplified modeling of their combined behavior, based on a good approximation of the combined power consumption (or efficiency) which is a key factor in the cost. In what follows, we first study the model for an individual pump and then present suitable aggregate models for collections of raw water pumps and pure water pumps, respectively.

TABLE 3. Notation for the model of a single electric pump

Symbol	Explanation	Value	Unit
$\omega_{a\nu}(t)$	Relative speed of the pump		–
$n_{a\nu}(t)$	Absolute speed of the pump		1/min
$n_{\text{nom},a\nu}$	Nominal speed of the pump		1/min
$c_{H,0,a\nu}$	Coeff. of the pump characteristic		m
$c_{H,1,a\nu}$	Coeff. of the pump characteristic		s/m^2
$c_{H,2,a\nu}$	Coeff. of the pump characteristic		–
$c_{P,0,a\nu}$	Coeff. of the power characteristic		W
$c_{P,1,a\nu}$	Coeff. of the power characteristic		J/m^3
$c_{N,0,a\nu}$	Coeff. of the NPSH value characteristic		m
$c_{N,1,a\nu}$	Coeff. of the NPSH value characteristic		s/m^2
$c_{N,2,a\nu}$	Coeff. of the NPSH value characteristic		–
$v_{\text{in},a\nu}(t)$	Pump inflow velocity		m/s
$z_{\text{in},a\nu}$	Elevation of pump inlet		m
$z_{\text{NPSH},a\nu}$	Elevation of pump reference plane		m
$p_{\text{in},a\nu}(t)$	Pressure at pump inlet		bar
p_v	Vapor pressure of water; at 10° C:	0.0123	bar

TABLE 4. Model parameters of a waterworks outlet with four pumps, three of which are identical (BWB)

Pump	H_{suct}	H_{press}	n^-	n_{nom}	n^+	Q^-	Q_{nom}	Q^+
1–3	29.5	29.5	1180	1495	1540	0.2222	0.6944	0.8333
4	29.5	29.5	1180	1450	1500	0.1944	0.5000	0.5833
Pump	$c_{H,0}$	$c_{H,1}$	$c_{H,2}$	$c_{P,0}$	$c_{P,1}$	$c_{N,0}$	$c_{N,1}$	$c_{N,2}$
1–3	99.02	57.74	2.015	29572	106	6.997	1.408	1.796
4	99.14	81.31	2.356	25806	134	7.283	80.173	5.554

2.8.1. *Single Pump Model.* The individual pumps constituting an arc $a \in \mathcal{A}_{\text{pu}}$ have a common pressure increase $\Delta H_a(t)$ but different flow rates $Q_{a\nu}(t)$ that add up to the total arc flow,

$$Q_a(t) = \sum_{\nu=1}^{N_a} Q_{a\nu}(t). \tag{22}$$

Common pump types include electric centrifugal pumps and Diesel centrifugal pumps. The latter are usually used as backup units only and are therefore not addressed here. The technical model of each electric centrifugal pump is given by several characteristic diagrams involving the pump flow rate $Q_{a\nu}(t)$ and the non-dimensional relative speed,

$$\omega_{a\nu}(t) = \frac{n_{a\nu}(t)}{n_{\text{nom},a\nu}}, \tag{23}$$

where $n_{\text{nom},a\nu}$ denotes the nominal speed of the pump. The additional dynamic variables have respective bounds $Q^{\pm}_{a\nu}(t)$ and $\omega^{\pm}_{a\nu}(t)$ whose static default values $Q^{\pm}_{a\nu}$ and $\omega^{\pm}_{a\nu}$ are technical parameters of the pumps, whereas the total flow bounds $Q^{\pm}_a(t)$ are defined by the network operator depending on contractual and other requirements.

The characteristic diagrams are given in closed form with two or three empirical coefficients measured at the nominal speed; see Table 3. Actual values for a real waterworks at BWB are given in Table 4.

The common pressure increase $\Delta H_a(t)$ of all pumps is related to their respective flow rates and relative speeds by the *pump characteristic*,

$$\Delta H_a(t) = \omega_{av}(t)^2 \left(c_{H,0,av} - c_{H,1,av} \left(\frac{Q_{av}(t)}{\omega_{av}(t)} \right)^{c_{H,2,av}} \right). \tag{24}$$

The power consumption of each pump is given by the *power characteristic*,

$$P_{av}(t) = \omega_{av}(t)^3 \left(c_{P,0,av} - c_{P,1,av} \frac{Q_{av}(t)}{\omega_{av}(t)} \right). \tag{25}$$

Finally, the *required NPSH value (net positive suction head)* is given as

$$\mathrm{NPSH}_{av}(t) = \omega_{av}(t)^2 \left(c_{N,0,av} - c_{N,1,av} \left(\frac{Q_{av}(t)}{\omega_{av}(t)} \right)^{c_{N,2,av}} \right). \tag{26}$$

This value is bounded above by the *available NPSH value* (with a safety margin of $0.5\,\mathrm{m}$) to prevent cavitation inside the pump,

$$\mathrm{NPSH}_{av}(t) \le \mathrm{NPSHA}_{av}(t) - 0.5\,\mathrm{m}. \tag{27}$$

Here the available NPSH value of the pump is given as

$$\mathrm{NPSHA}_{av}(t) = z_{\mathrm{in},av} - z_{\mathrm{NPSH},av} + \frac{p_{\mathrm{in},av}(t) - p_v}{g\rho} + \frac{v_{\mathrm{in},av}(t)^2}{2g}, \tag{28}$$

where $z_{\mathrm{in},av}$ and $z_{\mathrm{NPSH},av}$ are the respective elevations of the pump inlet and of its NPSH reference plane. (For horizontally installed pumps the latter coincides with the elevation of the pump axis.) Further, $p_{\mathrm{in},av}(t) = g\rho(H_i(t) - z_{\mathrm{in},av})$ and p_v are the pressure at the pump inlet and the vapor pressure of water, respectively, and $v_{\mathrm{in},av}(t) = Q_{av}(t)/A_{\mathrm{in},av}$ denotes the pump inflow velocity of the water.

Certain types of pumps can only operate at fixed speed, $n_{av}(t) \equiv n_{\mathrm{nom},av}$. In this case we have $\omega_{av}(t) \equiv 1$, and the characteristics (24)–(26) simplify to

$$\Delta H_a(t) = c_{H,0,av} - c_{H,1,av} Q_{av}(t)^{c_{H,2,av}}, \tag{29}$$

$$P_{av}(t) = c_{P,0,av} - c_{P,1,av} Q_{av}(t), \tag{30}$$

$$\mathrm{NPSH}_{av}(t) = c_{N,0,av} - c_{N,1,av} Q_{av}(t)^{c_{N,2,av}}. \tag{31}$$

Inactive pumps of either type are disconnected from the network (by shutting valves) and behave like absent arcs: the flow rate, speed, and power are zero,

$$Q_{av}(t) = 0, \quad \omega_{av}(t) = 0, \quad P_{av}(t) = 0,$$

and the constraints (24), (25), (27) can be dropped. The binary activity status of a pump will be denoted as $Y_{av}(t)$. In contrast to the tank availability status, it is subject to optimization. One of the major difficulties in the mixed integer model below stems from the fact that a pump cannot be shut down continuously, in the sense that the admissible sets of flow rate and speed become disconnected, $Q_{av} \in \{0\} \cup [Q^-_{av}, Q^+_{av}]$ and $\omega_{av} \in \{0\} \cup [\omega^-_{av}, \omega^+_{av}]$. This is a consequence of the macroscopic time model: in practice, starting up a pump or shutting it down is actually a continuous procedure but requires a more complex sequence of actions, involving the opening or closing of additional valves on the suction and pressure sides of the pump in the proper order.

2.8.2. *Aggregated Raw Water Pump Model.* An arc $a \in \mathcal{A}_{\mathrm{pr}}$ typically represents a large collection of almost identical raw water pumps, which can be assumed to run at top efficiency for all flow values. (The smallest waterworks at BWB has 14 raw water pumps, the largest has 170, and the total number in all nine waterworks is 620.) Raw water pumping is thus modeled with constant specific energy demand per m^3, $w_{\mathrm{raw},a}$, yielding the total power consumption

$$P_a(t) = w_{\mathrm{raw},a} Q_a(t).$$

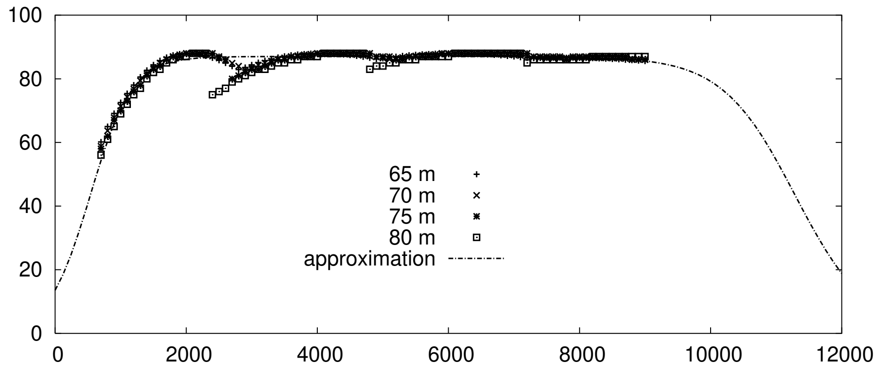

FIGURE 6. Efficiency curves of aggregated pure water pumps under optimal configuration

2.8.3. *Aggregated Pure Water Pump Model.* Pure water pumps appear in much smaller collections (three to six per outlet at BWB) that require a more detailed model based on an approximation of their combined efficiency. The exact combined efficiency is

$$\eta_a(t) = \frac{\rho g \Delta H_a(t) Q_a(t)}{P_a(t)},$$

where the numerator measures the total power output transferred to the water, and $P_a(t)$ denotes the total power consumption of the pumps,

$$P_a(t) = \sum_{\nu=1}^{N_a} P_{a\nu}(t). \tag{32}$$

Assuming that the optimal configuration of available pumps is selected under all operating conditions (yielding minimum power consumption P_a, or maximum efficiency η_a, for given Q_a and ΔH_a), we can neglect the moderate pressure dependence and approximate the flow dependence of the efficiency as

$$\eta_a(Q_a(t)) = \eta_a^{\max}\left(\frac{1}{\phi_a^-(Q_a(t))} - \frac{1}{\phi_a^+(Q_a(t))}\right) + 0.001,$$

where

$$\phi_a^\pm(Q) = 1 + \alpha_a^\pm \exp\left(\beta_a^\pm \frac{Q - q_a^\pm}{q_a^\pm}\right). \tag{33}$$

Here the parameters $\alpha_a^\pm$ and $\beta_a^\pm$ are fitted to the reference data, and the values $q_a^\pm$ are defined as

$$q_a^- = \min\left\{Q_{a\nu}^-\right\}_{\nu=1}^{N_a}, \quad q_a^+ = \sum_{\nu=1}^{N_a} Q_{a\nu}^+.$$

The latter determine roughly where the left and right slopes are located. The efficiency model is designed such that operation in the lower infeasible range $0 < Q_a < q_a^-$ is strongly discouraged through small values of the efficiency (see Fig. 6) whereas Q_a^+ is explicitly specified as an upper bound. However, q_a^+ may become relevant as a soft limit when one or more pumps are out of service.

2.8.4. *Restrictions in Pump Models.* Further constraints usually differ among certain subsets of the pumps. Let $\mathcal{W}$ denote the set of waterworks and pumping stations. For $w \in \mathcal{W}$, denote by $\mathcal{A}_{\text{pr}}(w)$ the associated set of raw water pumps (empty for pumping stations), by $\mathcal{A}_{\text{pp}}(w)$ the associated set of pure water pumps (empty for type II waterworks), and by $\mathcal{A}_{\text{pu}}(w) = \mathcal{A}_{\text{pr}}(w) \cup \mathcal{A}_{\text{pp}}(w)$ their union. Finally, let $\mathcal{A}_{\text{po}}$ denote the set of pumps at all waterworks outlets (not including the pumping stations).

For the raw water pumps, $a \in \mathcal{A}_{\text{pr}}$, there are additional bounds on the derivative of the flow rate,

$$\dot{Q}_a(t) \in [\dot{Q}_a^-(t), \dot{Q}_a^+(t)]. \tag{34}$$

This type of constraint is imposed to ensure the quality of the filtering process.

At the waterworks outlets, $a \in \mathcal{A}_{\text{po}}$, there may be additional bounds on the total daily discharge of the associated pumps,

$$\int_{t=0}^{T} Q_a(t)\, dt \in [\Sigma Q_a^-, \Sigma Q_a^+]. \tag{35}$$

This type of constraint models the availability of raw water; at BWB it is caused by contractual limits on the yearly groundwater extraction.

Finally, since electricity prices often include a component depending on the peak power, there is an upper bound on the combined power consumption of all pumps in a waterworks or pumping station, $w \in \mathcal{W}$. The disaggregated version of this constraint reads

$$\sum_{a \in \mathcal{A}_{\text{pu}}(w)} \sum_{\nu=1}^{N_a} P_{a\nu}(t) \le P_w^+, \tag{36}$$

and the (nonlinear) aggregated version reads

$$\sum_{a \in \mathcal{A}_{\text{pr}}(w)} w_{\text{raw},a} Q_a(t) + \sum_{a \in \mathcal{A}_{\text{pp}}(w)} \frac{\rho g \Delta H_a(t) Q_a(t)}{\eta_a(Q_a(t))} \le P_w^+. \tag{37}$$

Here the first sum is empty in pumping stations and the second sum is empty in waterworks of type II.

2.9. **Valve Model.** The pressure in a valve $a = ij \in \mathcal{A}_{\text{vl}}$ is decreased by some controlled amount $\Delta H_a(t)$,

$$H_j(t) - H_i(t) + \Delta H_a(t) = 0. \tag{38}$$

To ensure consistency of the pressure decrease with the generally unknown direction of flow, we impose a nonnegativity constraint

$$\Delta H_a(t) Q_a(t) \ge 0. \tag{39}$$

This model allows for the valve to be fully open ($\Delta H_a(t) = 0$) or fully closed ($Q_a(t) = 0$). During periods where the valve position is prescribed externally, the respective condition is introduced as an additional constraint,

$$\Delta H_a(t) = 0 \text{ (open)} \quad \text{or} \quad Q_a(t) = 0 \text{ (closed)},$$

and the other variable (Q_a or ΔH_a) remains free for optimization. The nonnegativity constraint is dropped in these cases, and the valve behaves either like a zero length pipe (without pressure loss) or like an absent link. A ternary valve mode $m_a(t)$ with the values *open, closed, controlled* designates which situation applies.

2.10. **Demand Model.** The demand forecast at BWB is generated by separate neural networks for the total daily consumption and for the hourly fractions of the total consumption. Input data are selected parameters from the weather forecast for the prediction horizon, the category of the day (weekday, holiday, vacation, etc.), and the actual consumption of the previous day. The neural networks have been trained on data collected over a period of four years. In addition, a long-term negative trend is adequately incorporated both in training and prediction. The basic approach is reported in [37]; details are confidential [6].

The BWB forecast method yields highly reliable predictions of the cumulative (network-wide) hourly demand but cannot generate predictions for individual junctions. Data on individual junctions are only available from the yearly read-out of the water meters. Although our optimization model permits arbitrary demand profiles D_{jt} at all junctions (see Section 2.4), we therefore assume a static spatial demand distribution. Thus we obtain a

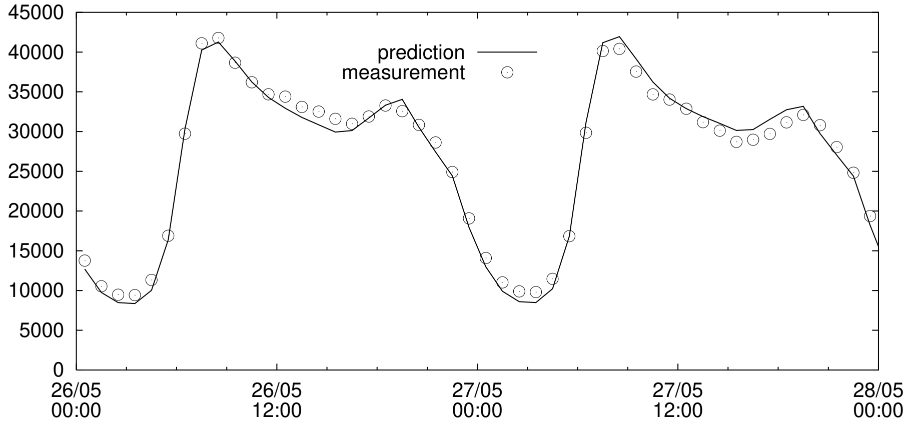

FIGURE 7. Typical profile of hourly network-wide demand in Berlin (m^3)

product structure with respect to space and time, where the demand at every junction is a constant fraction of the time-dependent cumulative demand. A typical hourly profile of the latter is given in Fig. 7, taken from two days in late May 2004.

2.11. **Initial and Terminal Conditions.** Initial values are only required for the tank filling levels and the flow rates of raw water pumps (whose derivatives appear in the constraints, (6) and (34)),

$$\begin{aligned} H_j(0) &= H_{j0}, \quad j \in \mathcal{N}_{\text{tk}}, \\ Q_a(0) &= Q_{a0}, \quad a \in \mathcal{A}_{\text{pr}}. \end{aligned}$$

All other dynamic variables are discontinuous in general and need only to be consistent between jumps.

Suitably tightened lower bounds are imposed on the tank filling levels at $t = T$ to prevent undesired finite horizon effects,

$$H_j(T) \geq H_{jT}^-, \quad j \in \mathcal{N}_{\text{tk}}.$$

Without such terminal constraints, operation costs during the current planning period could be reduced by deflating the tanks. On the following day, however, this would usually generate disproportionately increased costs or, even worse, render the network inoperable.

2.12. **Objective Function.** The goal is to minimize the variable operating costs, that is, the costs of raw water production and pure water production during the planning period,

$$K = K_{\text{raw}} + K_{\text{pure}} = \int_0^T \Bigg(\sum_{a \in \mathcal{A}_{\text{pr}}} \dot{K}_{\text{raw},a}(t) + \sum_{a \in \mathcal{A}_{\text{pp}}} \dot{K}_{\text{pure},a}(t) \Bigg)\, dt. \tag{40}$$

The variable costs include ground water extraction fees, costs for operational supplements, energy costs for raw water pumps, energy costs for raw water treatment, and energy costs for pure water pumps. This yields differential costs

$$\begin{aligned} \dot{K}_{\text{raw},a}(t) &= \dot{K}_{\text{fee},a}(t) + \dot{K}_{\text{suppl},a}(t) + \dot{K}_{\text{el,pump},a}(t) + \dot{K}_{\text{el,treat},a}(t), \\ \dot{K}_{\text{pure},a}(t) &= \dot{K}_{\text{el,pump},a}(t). \end{aligned}$$

In terms of the specific cost per m^3, $k_{\text{raw},a} = k_{\text{fee},a} + k_{\text{suppl},a}$, and the specific work per m^3, $w_{\text{raw},a} = w_{\text{pump},a} + w_{\text{treat},a}$, the contribution from raw water production can be written

$$\dot{K}_{\text{raw},a}(t) = \big[k_{\text{raw},a} + w_{\text{raw},a} k_{\text{el},a}(t)\big]\, Q_a(t), \tag{41}$$

TABLE 5. Notation for the cost function

Symbol	Explanation	Unit
K	Total daily operating cost	€
$k_{\text{el},a}(t)$	Price for electric energy at pump a	€/J
$k_{\text{raw},a}$	Specific price for raw water and treatment materials at pump a	€/m^3
$w_{\text{raw},a}$	Specific work for raw water pumping and treatment at pump a	J/m^3

and the contribution from pure water production reads

$$\dot{K}_{\text{pure},a}(t) = k_{\text{el},a}(t)\frac{\rho g \Delta H_a(t) Q_a(t)}{\eta_a(Q_a(t))}, \tag{42}$$

where $k_{\text{el},a}$ denotes the price for electric energy. Using the total power consumption (32), the corresponding expressions for the disaggregated pump model are

$$\dot{K}_{\text{raw},a}(t) = k_{\text{raw},a} Q_a(t) + k_{\text{el},a}(t) P_a(t), \tag{43}$$

$$\dot{K}_{\text{pure},a}(t) = k_{\text{el},a}(t) P_a(t). \tag{44}$$

The constants and coefficients are listed in Table 5.

2.13. **Summary.** The component models of all network elements, objective, and boundary conditions are now complete. Altogether, they form a continuous time optimization problem that might be supplemented by minimum up and down time constraints for the pumps to prevent solutions with excessive switching activity. Theoretically one could now invoke the maximum principle [33, 25] to formulate continuous time necessary optimality conditions, even including the discrete decisions [2]. However, due to the problem size and its combinatorial complexity, this can be considered an insurmountable task in practice, and the resulting multi-stage boundary value problem with switch and jump conditions would be intractable anyway. We therefore proceed with a discrete time nonlinear mixed integer formulation of the planning problem. Another reason to use discrete time is the fact that this is already common practice: demand forecasts, electricity prices, and operating schedules are typically specified in hourly intervals.

3. GDP AND MINLP MODELS

Relevant combinatorial aspects in our model include switching of pumps or (in the aggregated case) of waterworks and pumping stations, and the direction of flow in valves. After formulating all component models in discrete time, we will discuss the combinatorial issues and provide both a generalized disjunctive programming (GDP) formulation [22, 34] and a suitable MINLP formulation.

3.1. **Discrete Time Setting.** We consider a planning period of length T in discrete time, $t = 1, 2, \dots, T$, with initial conditions at $t = 0$. Subinterval t is denoted $I_t = (t-1, t)$ and has physical length Δt. At BWB, the planning period represents the following day, partitioned into 24 one-hour time-steps.

The discrete pressure variables in node $j \in \mathcal{N}$ are denoted H_{jt}, with upper and lower bounds $H^{\pm}_{jt}$, $t = 1, \dots, T$. Arc flow rates are similarly denoted Q_{at}, with bounds $Q^{\pm}_{at}$, and the pressure differences in pumps and valves are ΔH_{at}, with bounds $\Delta H^{\pm}_{at}$. The flows are assumed to be quasi-stationary and right-continuous with possible jumps at the grid points. That is, Q_{at} will be interpreted as constant value during $(t-1, t]$, whereas H_{jt} and ΔH_{at} will be interpreted as values at time t. Additional speed and flow variables ω_{avt} and Q_{avt} in the disaggregated pump model as well as powers P_{avt} and binary variables Y_{at}, Y_{avt} (including the tank availability profiles) are interpreted like Q_{at}.

3.2. **Model Formulation.** We begin with the full discrete time model for individual pure water pumps. (Raw water pumps are always aggregated from now on.) The objective is

$$\Delta t \sum_{t=1}^{T} \Bigg(\sum_{a \in \mathcal{A}_{\text{pr}}} (w_{\text{raw},a} k_{\text{el},at} + k_{\text{raw},a}) Q_{at} + \sum_{a \in \mathcal{A}_{\text{pp}}} k_{\text{el},at} P_{at} \Bigg) \to \min . \tag{45}$$

Basic equality constraints include the overall flow balances and pressure relations:

$$\sum_{i:\, ij \in \mathcal{A}} Q_{ijt} - \sum_{k:\, jk \in \mathcal{A}} Q_{jkt} - D_{jt} = 0, \qquad j \in \mathcal{N}_{\text{jc}}, \tag{46}$$

$$\sum_{i:\, ij \in \mathcal{A}} Q_{ijt} - \sum_{k:\, jk \in \mathcal{A}} Q_{jkt} - E_{jt}(H_{j,t-1}, H_{jt}) = 0, \qquad j \in \mathcal{N}_{\text{tk}}, \tag{47}$$

$$H_{jt} - \bar{H}_j = 0, \qquad j \in \mathcal{N}_{\text{rs}}, \tag{48}$$

$$H_{jt} - H_{it} + \varphi_a(Q_{at}) = 0, \qquad a \in \mathcal{A}_{\text{pi}}, \tag{49}$$

$$H_{jt} - H_{it} - \Delta H_{at} = 0, \qquad a \in \mathcal{A}_{\text{pu}}, \tag{50}$$

$$H_{jt} - H_{it} + \Delta H_{at} = 0, \qquad a \in \mathcal{A}_{\text{vl}}. \tag{51}$$

Here D_{jt} denotes the predicted consumption demand, E_{jt} the tank inflow (to be defined below), $\bar{H}_j$ the constant reservoir pressure, and $\varphi_a = \Delta H_a^{\text{PKrs}}$ the PKrs approximation (19) of the hydraulic pressure loss. Like Q_{at}, the flows D_{jt}, E_{jt} are interpreted as constant values during period t. The tank inflows are given as

$$E_{jt}(H_{j,t-1}, H_{jt}) = \frac{1}{\Delta t} \sum_{\nu=1}^{N_a} Y_{j\nu t} \Delta V_{j\nu}(H_{j,t-1}, H_{jt}),$$

where $\Delta V_{j\nu}$ denotes the change of the filling volume of tank $j\nu$ during period t,

$$\Delta V_{j\nu}(H_{j,t-1}, H_{jt}) = \int_{H_{j,t-1}}^{H_{jt}} A_{j\nu}(h - z_j)\, dh;$$

cf. Section 2.6. Here we require that the availability profiles are constant in each period, $Y_{j\nu}(\tau) \equiv Y_{j\nu t}$ for $\tau \in I_t$.

The nontrivial inequalities include discrete analogs of the restrictions (34)–(36) at the raw water pumps, outlets, and waterworks and pumping stations, respectively:

$$Q_{at} - Q_{a,t-1} \in [\Delta Q_{at}^-, \Delta Q_{at}^+], \qquad a \in \mathcal{A}_{\text{pr}}, \tag{52}$$

$$\Delta t \sum_{t=1}^{T} Q_{at} \in [\Sigma Q_a^-, \Sigma Q_a^+], \qquad a \in \mathcal{A}_{\text{po}}, \tag{53}$$

$$\sum_{a \in \mathcal{A}_{\text{pr}}(w)} w_{\text{raw},a} Q_{at} + \sum_{a \in \mathcal{A}_{\text{pp}}(w)} \sum_{\nu=1}^{N_a} P_{a\nu t} \le P_w^+, \qquad w \in \mathcal{W}, \tag{54}$$

where $\Delta Q_{at}^{\pm} = \dot{Q}_{at}^{\pm} \Delta t$.

Turning to the individual pure water pumps $\nu = 1, \dots, N_a$, $a \in \mathcal{A}_{\text{pp}}$, we define binary variables $Y_{a\nu t} \in \{0, 1\}$ designating the activity status during period t. The flow balance for an entire pump collection reads

$$Q_{at} - \sum_{\nu=1}^{N_a} Q_{a\nu t} = 0, \qquad a \in \mathcal{A}_{\text{pp}}. \tag{55}$$

Letting $z^0_{a\nu} = z_{\text{NPSH},a\nu} + p_\nu/(\rho g) + 0.5\,\text{m}$, cf. (28), the additional constraints for every active pump ($Y_{a\nu t} = 1$) can be written

$$\Delta H_{at} - \omega^2_{a\nu t}\left(c_{H,0,a\nu} - c_{H,1,a\nu}\left(\frac{Q_{a\nu t}}{\omega_{a\nu t}}\right)^{c_{H,2,a\nu}}\right) = 0, \tag{56}$$

$$P_{a\nu t} - \omega^3_{a\nu t}\left(c_{P,0,a\nu} - c_{P,1,a\nu}\frac{Q_{a\nu t}}{\omega_{a\nu t}}\right) = 0, \tag{57}$$

$$H_{it} + \frac{Q^2_{a\nu t}}{2gA^2_{\text{in},a\nu}} - z^0_{a\nu} - \omega^2_{a\nu t}\left(c_{N,0,a\nu} - c_{N,1,a\nu}\left(\frac{Q_{a\nu t}}{\omega_{a\nu t}}\right)^{c_{N,2,a\nu}}\right) \ge 0, \tag{58}$$

whereas every inactive pump ($Y_{a\nu t} = 0$) must satisfy

$$Q_{a\nu t} = 0, \tag{59}$$

$$P_{a\nu t} = 0, \tag{60}$$

$$\omega_{a\nu t} = 0. \tag{61}$$

For pumps with fixed speed the variables $\omega_{a\nu t}$ are dropped. Extra constraints during inactive periods are (59), (60), and during active periods we obtain

$$\Delta H_{at} - (c_{H,0,a\nu} - c_{H,1,a\nu} Q^{c_{H,2,a\nu}}_{a\nu t}) = 0, \tag{62}$$

$$P_{a\nu t} - (c_{P,0,a\nu} - c_{P,1,a\nu} Q_{a\nu t}) = 0, \tag{63}$$

$$H_{it} + \frac{Q^2_{a\nu t}}{2gA^2_{\text{in},a\nu}} - z^0_{a\nu} - (c_{N,0,a\nu} - c_{N,1,a\nu} Q^{c_{N,2,a\nu}}_{a\nu t}) \ge 0. \tag{64}$$

At the valves, the discrete version of the sign condition reads

$$\Delta H_{at} Q_{at} \ge 0, \quad a \in \mathcal{A}_{\text{vl}}. \tag{65}$$

However, this condition produces as feasible set the union of the positive and negative quadrant in Q_{at}-ΔH_{at}-space, which has disconnected interior. To capture the alternatives explicitly, we introduce binary decision variables $Y_{at} \in \{0, 1\}$ designating the direction of flow. The respective constraints are

$$Q_{at} \ge 0, \qquad \Delta H_{at} \ge 0 \qquad \text{(positive flow, } Y_{at} = 1\text{)}, \tag{66}$$

$$Q_{at} \le 0, \qquad \Delta H_{at} \le 0 \qquad \text{(negative flow, } Y_{at} = 0\text{)}. \tag{67}$$

The case of stagnant flow ($Q_{at} = 0$) is covered by both alternatives.

There are no initial conditions as in the continuous-time model, but initial values appear as parameters in the tank flow balances (47) and in the raw water gradient constraints (52). The relevant initial values are

$$H_{j0}, \quad j \in \mathcal{N}_{\text{tk}}, \tag{68}$$

$$Q_{a0}, \quad a \in \mathcal{A}_{\text{pr}}, \tag{69}$$

and the tightened terminal constraints read

$$H_{jT} \ge H^-_{jT}, \quad j \in \mathcal{N}_{\text{tk}}. \tag{70}$$

This completes the model formulation with disaggregated pumps.

With aggregated pumps we get the following modifications. The objective (45) has to be replaced with

$$\Delta t \sum_{t=1}^{T}\left(\sum_{a\in\mathcal{A}_{\text{pr}}} (w_{\text{raw},a} k_{\text{el},at} + k_{\text{raw},a}) Q_{at} + \sum_{a\in\mathcal{A}_{\text{pp}}} k_{\text{el},at}\frac{\rho g \Delta H_{at} Q_{at}}{\eta_a(Q_{at})}\right) \to \min. \tag{71}$$

Constraints (46)–(53) remain unchanged. The aggregated version of (54) corresponds to (37) and reads

$$\sum_{a\in\mathcal{A}_{\text{pr}}(w)} w_{\text{raw},a} Q_{at} + \sum_{a\in\mathcal{A}_{\text{pp}}(w)} \frac{\rho g \Delta H_{at} Q_{at}}{\eta_a(Q_{at})} \le P^+_w, \quad w \in \mathcal{W}. \tag{72}$$

The physical model for individual pumps, (55)–(61), is replaced as follows in the aggregated model. We define a single binary variable $Y_{at} \in \{0,1\}$ designating the activity status of the entire collection of pure water pumps $a \in \mathcal{A}_{\mathrm{pp}}$ during period t. With $\phi_a^{\pm}$ from (33), the aggregate efficiency in (71) and (72) is defined as

$$\eta_a(Q_{at}) = \eta_a^{\max}\left(\frac{1}{\phi_a^-(Q_{at})} - \frac{1}{\phi_a^+(Q_{at})}\right) + 0.001. \tag{73}$$

During active periods ($Y_{at} = 1$) there are no additional constraints, whereas an inactive waterworks outlet or pumping station ($Y_{at} = 0$) has to satisfy

$$Q_{at} = 0. \tag{74}$$

Constraints (62)–(64) are dropped since fixed speed pumps cannot be handled by aggregation. Finally, the valve constraints and boundary conditions (66)–(70) remain unchanged.

3.3. **GDP Formulation.** In generalized disjunctive programming (GDP), the selection among two or more alternative operating states characterized by different sets of constraints is formulated as a logical disjunction, with one set of constraints corresponding to each value of the discrete decision variable $Y \in \{Y_1, \dots, Y_k\}$:

$$\begin{bmatrix} Y = Y_1 \\ \text{constraint set } 1 \end{bmatrix} \vee \cdots \vee \begin{bmatrix} Y = Y_k \\ \text{constraint set } k \end{bmatrix}. \tag{75}$$

Special algorithms exist for GDP problems; they use branching on the discrete variables and exploit typical forms of the constraints [27, 40]. We use the notation "Y" for "$Y = 1$" and "$\neg Y$" for "$Y = 0$", which is common in the case of binary choices. This yields the GDP formulation for the disaggregated pump model:

$$\begin{aligned}
&\text{Minimize} && (45) \\
&\text{subject to} && \text{bounds on all variables,} \\
& && (46)\text{–}(51), \quad (54), (55), \quad (68)\text{–}(70), \\
& && \begin{bmatrix} Y_{avt} \\ (56)\text{–}(58) \end{bmatrix} \vee \begin{bmatrix} \neg Y_{avt} \\ (59)\text{–}(61) \end{bmatrix}, && a \in \mathcal{A}_{\mathrm{pp}} \text{ (variable speed)}, \\
& && \begin{bmatrix} Y_{avt} \\ (62)\text{–}(64) \end{bmatrix} \vee \begin{bmatrix} \neg Y_{avt} \\ (59), (60) \end{bmatrix}, && a \in \mathcal{A}_{\mathrm{pp}} \text{ (fixed speed)}, \\
& && \begin{bmatrix} Y_{at} \\ (66) \end{bmatrix} \vee \begin{bmatrix} \neg Y_{at} \\ (67) \end{bmatrix}, && a \in \mathcal{A}_{\mathrm{vl}}.
\end{aligned}$$

With the aggregated pump model we get instead:

$$\begin{aligned}
&\text{Minimize} && (71) \\
&\text{subject to} && \text{bounds on all variables,} \\
& && (46)\text{–}(51), \quad (72), (73), \quad (68)\text{–}(70), \\
& && Y_{at} \vee \begin{bmatrix} \neg Y_{at} \\ (74) \end{bmatrix}, && a \in \mathcal{A}_{\mathrm{pp}}, \\
& && \begin{bmatrix} Y_{at} \\ (66) \end{bmatrix} \vee \begin{bmatrix} \neg Y_{at} \\ (67) \end{bmatrix}, && a \in \mathcal{A}_{\mathrm{vl}}.
\end{aligned}$$

Notice that we have empty constraints in the disjunction $Y_{at} \vee [\neg Y_{at}\colon (74)]$ for $Y_{at} = 1$, as already mentioned above.

3.4. **MINLP Formulation.** For the MINLP formulation we wish to have suitable NLP relaxations so that branching on fractional values of a binary variable Y is possible. Moreover, the NLP relaxation should not introduce avoidable numerical difficulties (such as big-M terms or additional nonlinearities), and it should satisfy a constraint qualification at all feasible points, in the sense that the feasible sets for the cases $Y = 0$ and $Y = 1$ are connected by a nondegenerate polyhedron (having nonempty interior). Formulations with these properties will now be constructed.

3.4.1. *Relaxations.* Consider the individual pumps first. The straightforward relaxation approach would simply scale the flow and speed bounds,

$$Q_{avt} \in [Y_{avt} Q^-_{avt}, Y_{avt} Q^+_{avt}],$$
$$\omega_{avt} \in [Y_{avt} \omega^-_{avt}, Y_{avt} \omega^+_{avt}].$$

For $Y_{avt} \in \{0, 1\}$ this gives precisely the desired (disconnected) feasible sets, and the relaxation $Y_{avt} \in [0, 1]$ yields a 3-simplex in the positive orthant,

$$(Q_{avt}, \omega_{avt}, Y_{avt}) \in \big\{(Q, \omega, Y) \colon YQ^-_{avt} \le Q \le YQ^+_{avt}, \\ Y\omega^-_{avt} \le \omega \le Y\omega^+_{avt}, 0 \le Y \le 1\big\}. \tag{76}$$

For fixed speed pumps we drop ω_{avt} and the relaxation yields the triangle (2-simplex)

$$(Q_{avt}, Y_{avt}) \in \{(Q, Y) \colon YQ^-_{avt} \le Q \le YQ^+_{avt}, 0 \le Y \le 1\}. \tag{77}$$

Observe that for $Y_{avt} > 0$ the strictly positive lower and upper bounds on Q_{avt} and ω_{avt} guarantee that the quotient Q_{avt}/ω_{avt} remains bounded (and bounded away from zero):

$$0 < \frac{Q^-_{avt}}{\omega^+_{avt}} < \frac{Q_{avt}}{\omega_{avt}} < \frac{Q^+_{avt}}{\omega^-_{avt}} < \infty.$$

Moreover, since $Y_{avt} = 0$ implies $Q_{avt} = 0$ and $\omega_{avt} = 0$ (as required), the pump, power, and NPSH characteristics posses smooth extensions for $Y_{avt} \to 0$. The difficulty is that some of the resulting constraints will be incompatible with the pump inactivity, as we now show. From (56)–(58) we obtain in the limit $Y_{avt} \to 0$:

$$\Delta H_{at} = 0, \tag{78}$$
$$P_{avt} = 0, \tag{79}$$
$$H_{it} - z^0_{av} = \text{NPSHA}_{avt} - 0.5\,\text{m} \ge 0. \tag{80}$$

Constraint (79) is fine (and actually required), but the other two should be dropped. In fact, (78) is certainly wrong since $\Delta H_{at} > 0$, and (80) is potentially wrong, depending on the actual values of H_{it} and z^0_{av}.

For fixed speed pumps, we obtain from (62)–(64) in the limit $Y_{avt} \to 0$:

$$\Delta H_{at} - c_{H,0,av} = 0, \tag{81}$$
$$P_{avt} - c_{P,0,av} = 0, \tag{82}$$
$$H_{it} - z^0_{av} - c_{N,0,av} = \text{NPSHA}_{avt} - c_{N,0,av} - 0.5\,\text{m} \ge 0. \tag{83}$$

Here the constraints (81) and (82) are certainly wrong (the correct power is $P_{avt} = 0$), and (83) is potentially wrong.

Since the conflicting constraints in the relaxed MINLP formulation must become irrelevant for $Y_{avt} = 0$, we formulate them as inequality constraints that become inactive but avoid the usual big-M formulation. Thus (56) is replaced with

$$\Delta H_{at} - \omega^2_{avt} \left(c_{H,0,av} - c_{H,1,av} \left(\frac{Q_{avt}}{\omega_{avt}} \right)^{c_{H,2,av}} \right) \in (1 - Y_{avt})[\Delta H^-_{at}, \Delta H^+_{at}], \tag{84}$$

and (58) is replaced with

$$H_{it} + \frac{Q_{avt}^2}{2gA_{\text{in},av}^2} - Y_{avt}z_{av}^0 - \omega_{avt}^2\left(c_{N,0,av} - c_{N,1,av}\left(\frac{Q_{avt}}{\omega_{avt}}\right)^{c_{N,2,av}}\right) \geq 0. \quad (85)$$

For fixed speed pumps we also have to blend out the constant terms, obtaining

$$\Delta H_{at} - Y_{avt}c_{H,0,av} + c_{H,1,av}Q_{avt}^{c_{H,2,av}} \in (1 - Y_{avt})[\Delta H_{at}^-, \Delta H_{at}^+], \quad (86)$$

$$P_{avt} - Y_{avt}c_{P,0,av} + c_{P,1,av}Q_{avt} = 0, \quad (87)$$

$$H_{it} + \frac{Q_{avt}^2}{2gA_{\text{in},av}^2} - Y_{avt}z_{av}^0 - (Y_{avt}c_{N,0,av} - c_{N,1,av}Q_{avt}^{c_{N,2,av}}) \geq 0. \quad (88)$$

The relaxed formulation for aggregated pumps simply replaces the flow bounds with

$$Q_{at} \in [Y_{at}Q_{at}^-, Y_{at}Q_{at}^+]. \quad (89)$$

No conflicts arise in this relaxation since the approximate efficiency model is well-defined for all flow values $Q_{at} \in [0, Q_{at}^+]$ (in fact even for $Q_{at} \in \mathbf{R}$).

At the valves $a \in \mathcal{A}_{\text{vl}}$, we use a formulation similar to (84) and (86),

$$\Delta H_{at} \in [(1 - Y_{at})\Delta H_{at}^-, Y_{at}\Delta H_{at}^+], \quad (90)$$

$$Q_{at} \in [(1 - Y_{at})Q_{at}^-, Y_{at}Q_{at}^+], \quad (91)$$

forcing both variables to the negative or positive range if $Y_{at} = 0$ or $Y_{at} = 1$, respectively.

3.4.2. *Complete MINLP Formulation.* With the disaggregated pump model we obtain:

Minimize	(45)	
subject to	bounds on all variables,	
	(46)–(51), (54), (55), (68)–(70),	
	(76), (84), (57), (85),	$a \in \mathcal{A}_{\text{pp}}$ (variable speed),
	(77), (86), (87), (88),	$a \in \mathcal{A}_{\text{pp}}$ (fixed speed),
	(90), (91),	$a \in \mathcal{A}_{\text{vl}}$.

With the aggregated pump model we obtain:

Minimize	(71)	
subject to	bounds on all variables except pump flows,	
	(46)–(51), (72), (73), (68)–(70),	
	(89),	$a \in \mathcal{A}_{\text{pp}}$,
	(90), (91),	$a \in \mathcal{A}_{\text{vl}}$.

4. NLP Model

The discrete-time models in GDP and MINLP formulation are in principle solvable by global optimization methods based on nonlinear branch and bound type strategies. For small networks this may actually be practical. For routine application in daily operations planning of large networks, however, nonlinear mixed-integer models are by far too complex. At BWB it was therefore decided to employ a nonlinear programming (NLP) model. Under this provision, the main challenge consists in developing suitable techniques to incorporate combinatorial aspects as well as possible. One key ingredient in this respect is the aggregate efficiency model for collections of pure water pumps, which is particularly effective in combination with special constraints ensuring minimum up and down times for the pumps, as developed in [7].

The NLP model is formulated in discrete time and can be seen as a simplification of the MINLP model, from which it differs only as far as combinatorial aspects are concerned.

Here we formulate a basic NLP model that

- employs the aggregated pump model;
- addresses pump switching approximately via the efficiency model (73);
- may be refined by additional constraints.

This basic NLP model can be posed as follows:

$$\begin{array}{ll} \text{Minimize} & (71) \\ \text{subject to} & \text{bounds on all variables,} \\ & (46)\text{–}(51), \quad (72), (73), \quad (68)\text{–}(70), \\ & (65), \qquad a \in \mathcal{A}_{\mathrm{vl}}. \end{array}$$

5. SUMMARY

We have developed mathematical models for operative planning in drinking water networks by gradient-based optimization methods. Main elements from the nonlinear programming perspective include an inexpensive, globally smooth and asymptotically correct approximation of the hydraulic pressure loss in pipes, and a smooth approximation for the combined efficiency of collections of outlet pumps in waterworks and pumping stations. For pumps and valves we have provided nonlinear mixed integer formulations whose relaxations preserve desirable properties (smoothness, non-degeneracy) without introducing undesirable big-M terms or additional nonlinearities.

The approximation of aggregated pump efficiency not only reduces the computational cost significantly, in combination with special (smooth) minimum up and down time constraints developed in [7] it also allows to determine near-optimum network operation schedules by nonlinear programming methods. Operation schedules of individual pumps are then determined with the MINLP model separately for each waterworks and pumping station, with the total flow and common pressure increase given by the upper level optimization. Such a hierarchical approach is pursued in the optimization module for the large network of Berliner Wasserbetriebe, which has been in operation since June 2004.

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JENS BURGSCHWEIGER, BERLINER WASSERBETRIEBE, ABT. NA-G/W, 10864 BERLIN, GERMANY
E-mail address: `jens.burgschweiger@bwb.de`
URL: `http://www.bwb.de`

BERND GNÄDIG, BERLIN, GERMANY
E-mail address: `bernd.gnaedig@gmx.de`

MARC C. STEINBACH, ZUSE INSTITUTE BERLIN (ZIB), DEPARTMENT OPTIMIZATION, TAKUSTR. 7, 14195 BERLIN, GERMANY
E-mail address: steinbach@zib.de
URL: http://www.zib.de/steinbach