Zuse Institute Berlin



Takustr. 7 14195 Berlin Germany

Ralf Lenz¹

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 1 (D) 0000-0001-6949-3737

Zuse Institute Berlin Takustr. 7 14195 Berlin Germany

Telephone: $+49\,30\,84185-0$ Telefax: $+49\,30\,84185-125$

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Pipe Merging for Transient Gas Network Problems

Ralf Lenz 10*

Abstract

In practice, transient gas transport problems frequently have to be solved for large-scale gas networks. Gas network optimization problems typically belong to the class of Mixed-Integer Nonlinear Programming Problems (MINLP). However current state-of-the-art MINLP solvers are not yet mature enough to solve large-scale real-world instances. Therefore, an established approach in practice is to solve the problems with respect to a coarser representation of the network and thereby reducing the size of the underlying model. Two well-known aggregation methods that effectively reduce the network size are parallel and serial pipe merges. However, these methods have only been studied in stationary gas transport problems so far. This paper closes this gap and presents parallel and serial pipe merging methods in the context of transient gas transport. To this end, we introduce the concept of equivalent and heuristic subnetwork replacements. For the heuristic methods, we conduct a huge empirical evaluation based on real-world data taken from one of the largest gas networks in Europe. It turns out that both, parallel and serial pipe merging can be considered as appropriate aggregation methods for real-world transient gas flow problems.

1 Introduction

The optimization of transient large-scale gas transport problems is a challenging task and comes in two shades: First, flows in pipelines arise from nonlinear potential differences between adjacent nodes in meshed pipeline networks. Accounting also for the combinatorial complexity arising from network control decisions, such as routing gas flows in compressor stations and satisfying feasible operating ranges, gas transport problems are genuinely located in the field of Mixed-Integer Nonlinear Programming (MINLP). Second, practically relevant gas networks frequently contain up to several thousands of elements, see e.g., Carvalho et al. (2009), Geißler et al. (2015), and Schmidt et al. (2017). As a consequence, high-dimensional optimization problems arise as MINLPs. Such problems have not been successfully optimized so far, mainly for two reasons: On the one hand this is due to the limited capability of current state-of-the-art MINLP solvers and on the other hand research about transient gas problems is still in its early stages. While a large body of the literature on gas network optimization is concerned with the stationary case (for stationary optimization problems and state-of-the-art approaches we refer to Koch et al. (2015); Pfetsch et al. (2015) and the references therein), controlling and optimizing transient gas networks has become more prominent across different countries and continents in recent years, see for example, the computational studies conducted on parts of the German gas network Moritz (2007), Finnish gas network Aalto (2015), along the East Coast of the United States Zlotnik et al. (2015), and between West and East China Liu et al. (2019). While these studies consider rather small networks, there is already the need of transmission system operators (TSO) to solve transient gas flow

^{*}Zuse Institute Berlin, Takustr. 7, 14195 Berlin, Germany, lenz@zib.de

problems in practice on large-scale networks that contain thousands of elements, see Lenz (2021).

In fact, also TSOs typically deal with an aggregated representation of their networks, when making decisions in practice. This applies, for example, to the daily network control, evaluation of potential new contracts with clients, validation of worst-case scenarios, and network expansion planning. The decision-making of these applications is usually acquired by running simulation or optimization tools on networks, where certain parts are coarsely represented that are considered to be of minor importance for the respective decision process.

For the operation of stationary gas transport, a few approaches already exist that reduce the network size, see Ríos-Mercado et al. (2002), Clees et al. (2018) and Groß et al. (2019). Ríos-Mercado et al. (2002) present a reduction technique, which shrinks passive subnetworks between compressing arcs to single nodes. The authors show that if the reduced graph is a tree, then the flows along the compressing arcs are fixed and the resulting flows on the original passive subnetworks are uniquely determined. In the case that the reduced graph is cyclic however, the authors suggest using a numerical approximation for the flow on the reduced graph. Clees et al. (2018) and Groß et al. (2019) apply the concept of generalized series-parallel graphs, see Korneyenko (1994), by successively merging pipes in parallel and series and shrinking leaf nodes, which reduces the size of the considered networks in both papers about roughly 70%. Indeed, generalized series-parallel graphs frequently occur in network applications, either directly or as substructures. Through the successive application of serial and parallel merges and leaf node reductions, they are reducible to a single arc, i.e., to the K_2 . Moreover, generalized series-parallel graphs can be detected in linear time Takamizawa et al. (1982). From the computational studies in the literature, it can be seen that parallel and serial pipe merges have a high impact on reducing the size of gas networks, however, these methods have only been studied in the stationary case (Lenz and Schwarz (2016); Groß et al. (2019)).

In this paper, we close this gap and present parallel and serial pipe merging techniques for the transient case. Thereby, this paper provides necessary merging methods that can be used to acquire solutions for transient gas flow problems on aggregated networks. As basis, we introduce the concept of *equivalent subnetwork replacements*. For a transient gas flow problem, equivalent replacements allow to recover a solution for the original subnetwork from the solution of the aggregated subnetwork. This concept is generic in that it includes pipe merging, but is not necessarily restricted to it. Hence, it can be applied in a broader setting within a gas network aggregation scheme that also comprises other aggregation methods, as done in Lenz (2021).

The remainder of this paper is structured as follows: In Section 2 we present modeling preliminaries. In particular we introduce the necessary modeling of transient gas flows in pipelines, on which the merging procedures studied in this paper base upon. Afterwards, in Section 3 we introduce a general concept of equivalent subnetwork replacements for transient gas networks. In Section 4, we highlight properties that pipe merges should generally satisfy in the transient case. In Sections 5 and 6, we apply the concept of equivalent replacements to parallel and serial pipe merges. In contrast to the serial merge, we show that the parallel merge is an equivalent procedure. Therefore, we present a heuristic approach to serial pipe merging that bases on the proposed properties from Section 4. Then, we conduct an empirical study and evaluate this heuristic method on several pairs of serial pipelines using fine-grained real-world state data over an entire year. To connect the presented merging methods with the literature, we highlight their analogy to the stationary case. The paper ends with a conclusion in Section 7.

2 Modeling Prerequisites

We model the topology of a gas network as directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ with node set \mathcal{V} and arc set $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$, where we allow for multiple (anti-)parallel arcs between any pair of nodes.¹ The nodes consist of sources $\mathcal{V}^+ \subseteq \mathcal{V}$, sinks $\mathcal{V}^- \subseteq \mathcal{V}$, and transshipment nodes $\mathcal{V}^* := \mathcal{V} \setminus (\mathcal{V}^+ \cup \mathcal{V}^-)$. Moreover, we assume that $\mathcal{V}^+ \cap \mathcal{V}^- = \emptyset$ holds. Most importantly, we focus on pipes $\mathcal{A}_{pi} \subseteq \mathcal{A}$ in this paper, whereas the arc set \mathcal{A} typically contains additional arcs, such as valves, regulators and compressors.

Furthermore, we represent the time interval [0, T] by a sequence of discretized points in time 0 < 1 < ... < k, where 0 corresponds to the initial time point and 1, ..., k to the future time points. The difference between two successive points in time is given by τ . We define the following time sets $\mathcal{T}_0 := \{0, ..., k\}$ and $\mathcal{T} := \mathcal{T}_0 \setminus \{0\}$, where $|\mathcal{T}| = k$ denotes the cardinality of \mathcal{T} . A demand vector $b \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{T}|}$ is given for all nodes $v \in \mathcal{V}$ and for all time points $t \in \mathcal{T}$, where all sources $v \in \mathcal{V}^+$ hold $b_{v,t} \geq 0$, all sinks $v \in \mathcal{V}^+$ hold $b_{v,t} \leq 0$, and all transshipment nodes $v \in \mathcal{V}^*$ hold $b_{v,t} = 0$.

We model the physical state of the gas network using pressure in [bar] and mass flow in [kg/s]. More precisely, we associate a non-negative pressure variable $p_{v,t} \in$ $[\underline{p}_v, \overline{p}_v]$ with every node $v \in \mathcal{V}$ and time point $t \in \mathcal{T}$, where the initial pressure value at t = 0 is given input data. For each pipe $a = (\ell, r) \in \mathcal{A}_{pi}$ and for each $t \in \mathcal{T}$, we use two flow variables $q_{\ell,t}$ and $q_{r,t}$ describing the arcs' in and outflow at nodes ℓ and r. Moreover, we assume all variable bounds to be time-independent.

2.1 Modeling pipelines

Gas flows in pipelines are frequently modeled as one-dimensional in space, where pipes have cylindrical shapes. These assumptions enable to represent gas flows by a hyperbolic system of nonlinear partial differential equations, the so-called *Euler* equations. In this paper, we focus on constant gas temperature, which reduces this system for a single pipe $a = (\ell, r) \in \mathcal{A}_{pi}$ to the conservation of mass and momentum, and reads

$$\partial_t \rho + \partial_x (\rho v) = 0,$$
 (1a)

$$\partial_t(\rho v) + \partial_x(p + \rho v^2) + g\rho s_a + \frac{\lambda_a}{2D_a}\rho v|v| = 0,$$
(1b)

see, for example, LeVeque (2002) and Brouwer et al. (2011). Here, the symbols x and t denote the spatial and temporal coordinate of the system. This means that for pipe $a = (\ell, r)$ the term x describes the distance to node ℓ , and similarly the term t describes the distance to t_0 . The parameters $D_a \in \mathbb{R}^{\geq 0}$ and $s_a \in \mathbb{R}$ denote the diameter and the slope of the pipe, whereas g represents the gravitational constant. Moreover, λ_a represents the friction coefficient of the pipe, which we calculate by means of the formula of *Nikuradse* Nikuradse (1950), and ρ describes the density and v the velocity of the gas.

¹Actually, we deal with multigraphs, but to keep the notation simple, we restrict to the notation of simple graphs. Whenever needed, it will be evident from the context, to which of the different (anti-)parallel arcs is referred to.

In addition, we consider the equation of state for real gases. It links the gas pressure p, density ρ , and temperature T and is given by

$$p = R_s \rho T z(p, T). \tag{2}$$

The equation of state for real gases introduces two new terms, the specific gas constant R_s and the compressibility factor z(p,T), where the latter corresponds to a correction factor from the equation of state for ideal gases.

In the following, we further simplify System (1) in a similar fashion as done, for example, in Burlacu et al. (2019). Firstly, by using the speed of sound $c_s = \sqrt{p/\rho}$ the spatial derivative term in Equation (1b) can be reformulated to

$$p + \rho v^2 = p \left(1 + \frac{v^2}{c_s^2} \right). \tag{3}$$

Assuming that the velocity of gas flow typically holds in practice $v \ll c_s$, then (3) enables us to neglect the term $\partial_x(\rho v^2)$ in Equation (1b), see also the section about semi-linear equations in Domschke et al. (2017), Burlacu et al. (2019), and Hennings et al. (2019).

Secondly, we assume a constant compressibility factor z := z(p, T). Then, by virtue of the Equation of state (2), the speed of sound $c_s = \sqrt{p/\rho}$ transforms to $c_s^2 = R_s T z$, since

$$p = R_s \rho T z \quad \iff \quad \frac{p}{\rho} = R_s T z \quad \iff \quad c_s^2 = R_s T z,$$
 (4)

cf. also the nonlinear models in Bales et al. (2009) and Domschke et al. (2015). Given that we treat R_s, T and z as constants, then c_s is also constant. In line with different approaches in the literature see, e.g., Hahn et al. (2017) and Burlacu et al. (2019), we set the speed of sound to a constant value, here $c_s := 340 \text{ [m/s]}$ for all pipes. In natural gas, the speed of sound is approximately given by this value, see Domschke et al. (2017).

Thirdly, we drop the term $\partial_t(\rho v)$ in Equation (1b), as it contributes insignificantly to the momentum Equation under normal operating conditions, see, for example, Wilkinson et al. (1964) and Ehrhardt and Steinbach (2005). Finally, we reformulate the system in terms of the physical quantities that we use as state variables, namely pressure p and mass flow q. Provided that the pipelines have cylindrical shapes, the mass flow in pipe $a \in \mathcal{A}_{pi}$ is given by $q = A_a \rho v$ with cross-sectional area $A_a = D_a^2 \pi/4$, leading to

$$\frac{A_a}{c_s^2}\partial_t p + \partial_x q = 0, (5a)$$

$$\partial_x p + \frac{gs}{c_s^2} p + \frac{\lambda_a \ c_s^2}{2D_a A_a^2} \frac{q|q|}{p} = 0.$$
(5b)

Note that System (5) is often referred to as *friction-dominated* model, see model variant (FD1) in Brouwer et al. (2011) and (ISO3) in Domschke et al. (2017). Following the discretization scheme presented in Lenz (2021), the discretized equations then read for a pipe $a = (\ell, r) \in \mathcal{A}_{pi}$ and two consecutive time steps $t - 1, t \in \mathcal{T}$

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + \alpha_a \tau \left(q_{r,t} - q_{\ell,t} \right) = 0, \tag{6}$$

,

$$p_{r,t}^2 - p_{\ell,t}^2 + \beta_a \left(p_{r,t}^2 + p_{\ell,t}^2 \right) + \gamma_a \left(q_{r,t} + q_{\ell,t} \right) \left| q_{r,t} + q_{\ell,t} \right| = 0, \tag{7}$$

with

$$\alpha := \frac{c_{\mathrm{s}}^2}{A_a} \frac{2}{L_a} \qquad \text{and} \qquad \gamma_a := \frac{\lambda_a \ c_{\mathrm{s}}^2 \ L_a}{4 \ D_a \ A_a^2} \qquad \text{and} \qquad \beta_a := \frac{g \ (h_r - h_\ell)}{c_{\mathrm{s}}^2}$$

2.2 Modeling line-pack

Pipelines primarily serve to transport gas between sources and sinks. However, they can also be used as storages due to the compressible nature of gas. This process is referred to as line-pack and plays a crucial role for the operation of gas networks, especially in periods of volatile demands. For example, in the case of sudden high withdrawals, line-pack can be used as safety stock.

Line-pack describes the amount of gas in a system at a given point in time, which we specify as mass denoted in [kg]. For a single pipeline $a = (\ell, r)$ at time step $t \in \mathcal{T}_0$, the line-pack $LP_{a,t}$ can be determined by $LP_{a,t} = \rho_{a,t}^{mean} vol_a$, where $\rho_{a,t}^{mean}$ denotes the mean density and $vol_a = L_a A_a$ the pipe volume. Using the equation of state $p^{mean} = c_s^2 \rho^{mean}$ allows to represent the line-pack $LP_{a,t}$ of a single pipe as

$$LP_{a,t} = \frac{p_{a,t}^{mean}}{c_{\rm s}^{2}} vol_a, \tag{8}$$

where the mean pressure $p_{a,t}^{mean}$ along the pipeline *a* can be approximated by using a closed-form expression from the stationary case, cf. Koch et al. (2015) or Saleh (2002):

$$p_{a,t}^{mean} = \frac{2}{3} \left(p_{\ell,t} + p_{r,t} - \frac{p_{\ell,t} \, p_{r,t}}{p_{\ell,t} + p_{r,t}} \right) \tag{9}$$

for given pressure values $p_{\ell,t}$ and $p_{r,t}$.

3 Equivalent subnetwork replacements

When solving transient gas flow problems on aggregated networks, the resulting pressure and flow solutions should "appropriately" approximate the ones obtained by solving the problem on the network before aggregation. To this end, the problems should ideally be feasible on both the original and the aggregated network at the same time. In the following, we formalize this requirement most generally for a method that replaces a subnetwork $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{A}_C)$ with $\mathcal{V}_C \subseteq \mathcal{V}, \mathcal{A}_C \subseteq \mathcal{A}$, by an aggregated structure $\mathcal{G}_C^{agg} = (\mathcal{V}_C^{agg}, \mathcal{A}_C^{agg})$. In our context, \mathcal{G}_C represents two parallel or serial pipes and \mathcal{G}_C^{agg} the merged pipe, in both cases with their incident nodes.

Further, let the entire network after aggregating \mathcal{G}_C be given by

$$\mathcal{G}^{\mathrm{agg}} = (\mathcal{V}^{\mathrm{agg}}, \mathcal{A}^{\mathrm{agg}}) := \left((\mathcal{V} \setminus \mathcal{V}_C) \cup \mathcal{V}_C^{\mathrm{agg}}, (\mathcal{A} \setminus \mathcal{A}_C) \cup \mathcal{A}_C^{\mathrm{agg}} \right).$$

and let the part of the original network \mathcal{G} that coincides with the aggregated network \mathcal{G}^{agg} be given by

$$\mathcal{G} \cap \mathcal{G}^{\mathrm{agg}} = \Big(\mathcal{V} \cap \mathcal{V}^{\mathrm{agg}}, \mathcal{A} \cap \mathcal{A}^{\mathrm{agg}} \Big).$$

Note that $\mathcal{G} \cap \mathcal{G}^{\text{agg}} = \mathcal{G} \setminus \mathcal{G}_C = \mathcal{G}^{\text{agg}} \setminus \mathcal{G}_C^{\text{agg}}$ holds. In the following, let us consider an arbitrary *Transient Control Problem*, where we only require the pipes to be modeled as in Equations (6) and (7), and let

 $\mathcal{X}_{\mathcal{G}}$ be a solution of the Transient Control Problem before aggregating \mathcal{G}_{C} ,

- $\mathcal{X}_{\mathcal{G}^{\mathrm{agg}}}$ be the solution of a Transient Control Problem after aggregating \mathcal{G}_C ,
- $\mathcal{X}_{\mathcal{G}_C}$ the restriction of $\mathcal{X}_{\mathcal{G}}$ to the variables induced by \mathcal{G}_C , and
- $\mathcal{X}_{\mathcal{G}_{C}^{\mathrm{agg}}}$ the restriction of $\mathcal{X}_{\mathcal{G}^{\mathrm{agg}}}$ to the variables induced by $\mathcal{G}_{C}^{\mathrm{agg}}$.

We understand equivalent replacements of subnetworks in such a way that it is possible to extend a solution for the aggregated subnetwork to a solution for the nonaggregated subnetwork. The following definition formalizes the concept of equivalent replacements.

Definition 3.1. Let $\mathcal{G}_{C}^{agg} \cap \mathcal{G}_{C} \neq \emptyset$. We say that an aggregated subnetwork \mathcal{G}_{C}^{agg} equivalently replaces a non-aggregated subnetwork \mathcal{G}_{C} , if

- i) a solution $\mathcal{X}_{\mathcal{G}_{C}^{agg}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}_{C}}$, and
- ii) a solution $\mathcal{X}_{\mathcal{G}_C}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}_C^{agg}}$.

In other words, the first condition of the definition says that after solving the *Transient Control Problem* on the aggregated network and fixing the solution values of those variables that are induced by common elements of $\mathcal{G}_C^{\text{agg}}$ and \mathcal{G}_C , values can be found for the variables that are induced by the remaining elements in $\mathcal{G}_C \setminus (\mathcal{G}_C \cap \mathcal{G}_C^{\text{agg}})$ such that the corresponding constraints of the *Transient Control Problem* are satisfied. Together, both conditions guarantee that there is a solution to the *Transient Control Problem* for the network before and after aggregation of \mathcal{G}_C , if it exists for either of them.

The following proposition states that a solution on the aggregated network \mathcal{G}^{agg} can be extended to a solution on the non-aggregated network \mathcal{G} and vice versa.

Proposition 3.2. Assume that \mathcal{G}_C^{agg} equivalently replaces a subnetwork \mathcal{G}_C according to Definition 3.1. Then,

- i) a solution $\mathcal{X}_{\mathcal{G}^{agg}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}}$, and
- ii) a solution $\mathcal{X}_{\mathcal{G}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}^{agg}}$.

Proof. We show that (i) a solution $\mathcal{X}_{\mathcal{G}^{agg}}$ can be extended to a solution $\mathcal{X}_{\mathcal{G}}$. The second claim follows analogously.

Let $\mathcal{X}_{\mathcal{G}}$ be defined as follows: It is going to be the solution $\mathcal{X}_{\mathcal{G}^{agg}}$ for all elements in $\mathcal{G} \setminus \mathcal{G}_C$ and all elements in $\mathcal{G}_C \cap \mathcal{G}_C^{agg}$. It remains to define $\mathcal{X}_{\mathcal{G}}$ for the elements in $\mathcal{G}_C \setminus (\mathcal{G}_C \cap \mathcal{G}_C^{agg})$, but from Definition 3.1 follows that there exists an assignment that makes it feasible for \mathcal{G}_C .

When restricting to the possible (multiple) application of aggregation procedures that equivalently replace subnetworks, then, from Proposition 3.2 follows that a solution for the non-aggregated elements can be extended to a solution for all original network elements. Consequently, one would like to apply aggregation methods that establish equivalent replacements.

4 Preserving properties in the merged structures.

In the case that an aggregation method does not equivalently replace a subnetwork, we call the procedure *heuristic*. To reduce possible effects of heuristic methods on the physical behavior in the remaining network, we aim at developing these methods such that for every solution $\mathcal{X}_{\mathcal{G}_{C}^{agg}}$, there is ideally a point close to $\mathcal{X}_{\mathcal{G}_{C}^{agg}}$ that can be

extended to a solution $\mathcal{X}_{\mathcal{G}_C}$. To achieve this for the heuristic serial pipe merge, we figured out that preserving specific properties of the network and its physical state plays a crucial role:

- i) maintain the volume of the serial pipes,
- ii) maintain the difference in height induced by the serial pipes, and
- iii) minimize the error in terms of pressure and flow realizations between the subnetwork before and after the aggregation, i.e., between the serial and merged pipes.

The rationale for these properties is the following. Firstly, to generate substitute structures that maintain the pipe volume enables to store a similar amount of gas (line-pack) over time as before the aggregation, which can be used to buffer peaks in gas flows. Secondly, to maintain the difference in height induced by the serial pipes, keeps the height and slope of the remaining non-aggregated network elements constant. Thirdly, we minimize the error between the pressure and flow realizations at common elements of the aggregated and non-aggregated structures. To this end, we develop a special-tailored method for the heuristic serial pipe merging procedure in Section 6, whereas in the case of the equivalent parallel pipe merge procedure, this error is zero by construction.

Finally, for the serial pipe merge, we evaluate the physical impact on the remaining network by using simulations. Our intensive computational experiments suggest that following the described criteria extremely well features $\mathcal{X}_{\mathcal{G}} \approx \mathcal{X}_{\mathcal{G}^{agg}}$ and thereby keeps the impact of the aggregation on the remaining network small.

5 Merging Parallel Pipes

In this section, we investigate parallel pipe merging under transient conditions. We present a merging approach that equivalently replaces parallel pipes according to Definition 3.1. Even though not being a heuristic method, we will see that the merging procedure satisfies all of the deployed aspects from Section 4. We conclude this section by highlighting the analogy to the parallel merge from the stationary setting.

System of equations for the parallel and merged pipes. In the following, we consider two parallel pipes $a = (\ell, r)$ and $b = (\ell, r)$ as a stand-alone subnetwork. For each pipe a and b, a system of equations \mathcal{F}_a and \mathcal{F}_b represents the corresponding discretized continuity and momentum equations. Besides, system \mathcal{F}_{fc} represents the flow conservation at nodes ℓ and r for all $t \in \mathcal{T}$ and is given by

$$\mathcal{F}_{fc} = \begin{cases} q_{\ell,t} = b_{\ell,t} \tag{10} \\ \end{cases}$$

$$f^c = \left\{ q_{r,t} = b_{r,t}. \right. \tag{11}$$

The parallel merge substitutes both pipes for a new pipe $c = (\ell, r)$. Here, the merge replaces $\mathcal{F}_{orig} := \mathcal{F}_a \cup \mathcal{F}_b \cup \mathcal{F}_{fc}$ by $\mathcal{F}_{agg} := \mathcal{F}_c \cup \mathcal{F}_{fc}$, where system \mathcal{F}_c represents the discretized continuity and momentum equation of the merged pipe c.

The system of equations \mathcal{F}_a of pipe a is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r_a,t} - q_{\ell_a,t}) \alpha_a \tau = 0,$$
(12)

$$p_{r,t}^{2} \left(1 + \beta_{a}\right) - p_{\ell,t}^{2} \left(1 - \beta_{a}\right) + \left(q_{r_{a},t} + q_{\ell_{a},t}\right) |q_{r_{a},t} + q_{\ell_{a},t}| \gamma_{a} = 0,$$
(13)

$$p_{\ell,t} \in [\underline{p}_{\ell}, \overline{p}_{\ell}], \ p_{r,t} \in [\underline{p}_{r}, \overline{p}_{r}], \ q_{\ell_{a},t} \in [\underline{q}_{\ell_{a}}, \overline{q}_{\ell_{a}}], \ q_{r_{a},t} \in [\underline{q}_{r_{a}}, \overline{q}_{r_{a}}].$$
(14)

The system of equations \mathcal{F}_b of pipe b is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r_b,t} - q_{\ell_b,t}) \alpha_b \tau = 0, \qquad (15)$$

$$p_{r,t}^2 \left(1+\beta_b\right) - p_{\ell,t}^2 \left(1-\beta_b\right) + \left(q_{r_b,t}+q_{\ell_b,t}\right) |q_{r_b,t}+q_{\ell_b,t}| \gamma_b = 0, \tag{16}$$

$$p_{\ell,t} \in [\underline{p}_{\ell}, \overline{p}_{\ell}], \ p_{r,t} \in [\underline{p}_{r}, \overline{p}_{r}], \ q_{\ell_{b},t} \in [\underline{q}_{\ell_{t}}, \overline{q}_{\ell_{b}}], \ q_{r_{b},t} \in [\underline{q}_{r_{t}}, \overline{q}_{r_{b}}]. \tag{17}$$

The system of equations \mathcal{F}_c of the merged pipe c is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r,t} - q_{\ell,t}) \alpha_c \tau = 0,$$
(18)

$$p_{r,t}^2 \left(1 + \beta_c\right) - p_{\ell,t}^2 \left(1 - \beta_c\right) + \left(q_{r_c,t} + q_{\ell_c,t}\right) |q_{r_c,t} + q_{\ell_c,t}| \gamma_c = 0, \tag{19}$$

$$p_{\ell,t} \in [\underline{p}_{\ell}, \overline{p}_{\ell}], \ p_{r,t} \in [\underline{p}_{r}, \overline{p}_{r}], \tag{20}$$

$$q_{\ell_c,t} \in \left[\underline{q}_{\ell_a} + \underline{q}_{\ell_b}, \overline{q}_{\ell_a} + \overline{q}_{\ell_b}\right], \ q_{r_c,t} \in \left[\underline{q}_{r_a} + \underline{q}_{r_b}, \overline{q}_{r_a} + \overline{q}_{r_b}\right]. \tag{21}$$

We recall that the parameters of pipe a and pipe b are given by

$$\alpha_i := \frac{2 c_s^2}{A_i L_i}, \qquad \gamma_i := \frac{\lambda_a c_s^2 L_i}{4D_i A_i^2}, \qquad \beta_i := \frac{g (h_r - h_\ell)}{R_s T z}$$

with $\alpha_i, \gamma_i > 0$ and $\beta_i \in \mathbb{R}$ for $i \in \{a, b\}$.

An illustration of the parallel pipes a, b and the merged pipe c with corresponding variables for a single time step $t \in \mathcal{T}$ is provided in Figure 1. Considering multiple time steps, then k-many parallel merges reduce the size of the model by $k \cdot |\mathcal{T}| \cdot 2$ variables, since only two flow variables are needed per time step instead of four, and $k \cdot |\mathcal{T}| \cdot 8$ constraints (the discretized continuity and momentum equations and lower and upper bounds of two flow variables), cf. Equations (12) - (21).



Figure 1: Two parallel pipes (left) and the merged pipe (right) for time step t with associated pressure variables $p_{\ell,t}$ and $p_{r,t}$ as well as in and outflow variables $q_{\ell_a,t}, q_{r_a,t}$ and $q_{\ell_b,t}, q_{r_b,t}$ and $q_{\ell_c,t}, q_{r_c,t}$.

5.1 Merging approach

In this subsection, we determine the parameters α_c , β_c and γ_c in system \mathcal{F}_c such that the aggregation equivalently replaces pipes a and b by the merged pipe c. According to Definition 3.1, equivalent replacement means that a solution of the *Transient Control Problem* for the merged pipe c can be extended to a solution for the non-aggregated pipes a and b. Here, this translates to finding appropriate parameters α_c , β_c , γ_c such that a solution of system \mathcal{F}_{agg} can be extended to a solution of system \mathcal{F}_{orig} .

Please note that in the case of an iterative aggregation scheme, as done in Lenz (2021), the parallel - and also the serial - merging procedure might be applied to pipelines that already have been merged in the course of the aggregation and thus

differ for example in length. For this reason, it is advantageous that we derive aggregated parameters α_c , β_c , γ_c and do not require to determine disaggregated values for each particular physical entity, like the diameter, length, and friction factor.

Firstly, the requirement to keep the difference in height along pipe $c = (\ell, r)$ as induced from pipes a and b leads to $\beta_c := \beta_a = \beta_b = g (h_r - h_\ell) / c_s^2$. On this basis, we determine the remaining parameters α_c and γ_c such that the same pressure and flow realizations are obtained for the merged pipe as for the parallel pipes. Theorem 5.1 formalizes the concept of the parallel pipe merge.

Theorem 5.1. Given two parallel pipes a and b between nodes ℓ and r, and pipe parameters $\alpha_a, \gamma_a > 0$ and $\alpha_b, \gamma_b > 0$. Under the assumption $\beta_c := \beta_a = \beta_b$, pipelines a and b can be equivalently replaced by a single pipe $c = (\ell, r)$ with

$$\alpha_c = \frac{\alpha_a \cdot \alpha_b}{\alpha_a + \alpha_b}, \qquad and \qquad \gamma_c = \frac{\gamma_b \cdot \gamma_a}{\left(\sqrt{\gamma_b} + \sqrt{\gamma_a}\right)^2}$$

We restrict the proof to one time step. Applying the arguments of the proof sequentially to all points in time then completes the proof. Moreover, we assume that the in and outflow of the merged pipe c equals the one of both parallel pipes, i.e.,

$$q_{\ell_c,t} = q_{\ell_a,t} + q_{\ell_b,t} \qquad \forall t \in \mathcal{T}, \quad \text{and}$$
(22)

$$q_{r_c,t} = q_{r_a,t} + q_{r_b,t}, \qquad \forall t \in \mathcal{T}.$$
(23)

Proof. The proof contains two steps. At first we deduce parameter α_c from the continuity Equations (12), (15) and (18) and secondly we derive parameter γ_c from the momentum Equations (13), (16) and (19).

Determine parameter α_c . Using Equations (22) and (23), we get

$$(q_{\ell_c,t} - q_{r_c,t}) \alpha_c = (q_{\ell_a,t} + q_{\ell_b,t} - q_{r_a,t} - q_{r_b,t}) \alpha_c.$$
(24)

All Equations (12), (15), (18) contain the expression $p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1}$, hence we deduce from (12) and (15):

$$(q_{\ell_a,t} - q_{r_a,t}) \,\alpha_a = (q_{\ell_b,t} - q_{r_b,t}) \,\alpha_b \tag{25}$$

and from Equations (12) and (18):

$$(q_{\ell_c,t} - q_{r_c,t}) \alpha_c = (q_{\ell_a,t} - q_{r_a,t}) \alpha_a$$

$$\stackrel{(24)}{\longleftrightarrow} (q_{\ell_a,t} - q_{r_a,t}) \alpha_c + (q_{\ell_b,t} - q_{r_b,t}) \alpha_c = (q_{\ell_a,t} - q_{r_a,t}) \alpha_a$$

$$\stackrel{(24)}{\longleftrightarrow} (q_{\ell_a,t} - q_{r_a,t}) (\alpha_a - \alpha_c) = (q_{\ell_b,t} - q_{r_b,t}) \alpha_c.$$
(26)

Finally, we conclude from Equations (25) and (26):

$$\Rightarrow \quad \frac{\alpha_a - \alpha_c}{\alpha_c} = \frac{\alpha_a}{\alpha_b}$$
$$\Rightarrow \quad \alpha_c = \frac{\alpha_a \alpha_b}{\alpha_a + \alpha_b}$$

Determine parameter γ_c . Given that $\beta_a = \beta_b$ holds, it follows from Equations (13) and (16):

$$|q_{r_a,t} + q_{\ell_a,t}||q_{r_a,t} + q_{\ell_a,t}||\gamma_a = (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}||\gamma_b.$$
(27)

From Equation (27) follows together with $\gamma_a, \gamma_b > 0$ that

(

$$sgn(q_{r_a,t} + q_{\ell_a,t}) = sgn(q_{r_b,t} + q_{\ell_b,t})$$
(28)

holds for all $t \in \mathcal{T}$. Using (28), then Equation (27) transforms to

$$q_{r_a,t} + q_{\ell_a,t} = \sqrt{\frac{\gamma_b}{\gamma_a}} (q_{r_b,t} + q_{\ell_b,t}).$$
(29)

Provided that $\beta_c = \beta_b$ holds, we further deduce from Equations (16) and (19):

$$\begin{aligned} (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}|\gamma_b \\ &= (q_{r_c,t} + q_{\ell_c,t})|q_{r_c,t} + q_{\ell_c,t}|\gamma_c \\ &= (q_{r_a,t} + q_{r_b,t} + q_{\ell_a,t} + q_{\ell_b,t})|q_{r_a,t} + q_{r_b,t} + q_{\ell_a,t} + q_{\ell_b,t}|\gamma_c \\ \stackrel{(29)}{=} \left(\sqrt{\frac{\gamma_b}{\gamma_a}}(q_{r_b,t} + q_{\ell_b,t}) + q_{r_b,t} + q_{\ell_b,t}\right) \\ &\cdot \left| \left(\sqrt{\frac{\gamma_b}{\gamma_a}}(q_{r_b,t} + q_{\ell_b,t}) + q_{r_b,t} + q_{\ell_b,t}\right) \right|\gamma_c \\ \\ \stackrel{\gamma_a,\gamma_b>0}{=} (q_{r_b,t} + q_{\ell_b,t})|q_{r_b,t} + q_{\ell_b,t}| \left(1 + \sqrt{\frac{\gamma_b}{\gamma_a}}\right)^2 \gamma_c \\ \Rightarrow \gamma_c &= \frac{\gamma_b}{\left(1 + \sqrt{\frac{\gamma_b}{\gamma_a}}\right)^2} = \frac{\gamma_a \gamma_b}{\gamma_a + \gamma_b + 2\sqrt{\gamma_a \gamma_b}} \\ \Rightarrow \left[\gamma_c = \frac{\gamma_a \gamma_b}{\left(\sqrt{\gamma_a} + \sqrt{\gamma_b}\right)^2} \right]. \end{aligned}$$

This merging procedure indeed preserves the requirements from Section 4 on the accumulated volume and line-pack, as stated by the following corollary.

Corollary 5.2. Given two parallel pipes a and b between nodes ℓ and r. Then, the parallel pipe merge from Theorem 5.1 yields for the merged pipe $c = (\ell, r) vol_c = vol_a + vol_b$, and $LP_{c,t} = LP_{a,t} + LP_{b,t}$.

Proof. (i) The volume of both pipes a and b is given by $vol_a = L_a A_a$ and $vol_b = L_b A_b$. By virtue of parameter $\alpha_c = 2 c_s^2 / (vol_c)$, we deduce from Theorem 5.1:

$$\begin{aligned} \alpha_c &= \frac{\alpha_a \cdot \alpha_b}{\alpha_a + \alpha_b} \quad \Rightarrow \quad vol_c^{-1} = \frac{vol_a^{-1} \cdot vol_b^{-1}}{vol_a^{-1} + vol_b^{-1}} \\ &\Rightarrow \quad vol_c \; = \; \frac{vol_a^{-1} + vol_b^{-1}}{vol_a^{-1} \cdot vol_b^{-1}} \; = \; \frac{vol_b + vol_a}{vol_a \cdot vol_b} \cdot vol_a \cdot vol_b \\ &\Rightarrow \quad vol_c \; = \; vol_a + vol_b. \end{aligned}$$

(ii) Using $vol_c = vol_a + vol_b$ and the line-pack formula (8), where the mean pressure is approximated by (9), then $p_{c,t}^{mean} = p_{a,t}^{mean} = p_{b,t}^{mean}$ holds, and consequently

$$LP_{c,t} = \frac{p_{c,t}^{mean}}{c_s^2} vol_c = \frac{p_{a,t}^{mean}}{c_s^2} vol_a + \frac{p_{b,t}^{mean}}{c_s^2} vol_b = LP_{a,t} + LP_{b,t}.$$

5.2 Relation of transient and stationary parallel pipe merging

We conclude this section by showing the analogy between the transient and stationary parallel pipe merges. A common approximation for the interdependency of pressure and flow along a pipeline $a = (\ell, r)$ is governed by the stationary Weymouth equation

$$p_r^2 - \beta_a^{st} \, p_\ell^2 + \gamma^{st} \, q_a |q_a| = 0. \tag{30}$$

For the sake of readability, we distinguish between the stationary parameters $\gamma_a^{st}, \beta_a^{st}$ and the transient parameters here. Without explicitly stating the parameters $\gamma_a^{st}, \beta_a^{st} > 0$, we mention that $\beta_a^{st} = 1$ represents a horizontal pipe in the stationary case and refer to Fügenschuh et al. (2015) for further information.

Merging parallel pipelines in the stationary setting has been independently investigated in Lenz and Schwarz (2016) and Groß et al. (2019), see the following proposition.

Proposition 5.3 (Lenz and Schwarz (2016) and Groß et al. (2019)). Given two parallel pipes a and b between nodes ℓ and r, and pipe characteristics $\gamma_a^{st}, \gamma_b^{st}, \beta_a^{st}, \beta_b^{st} > 0$. Then, both pipes can be equivalently replaced by an artificial pipe $c = (\ell, r)$ with

$$\beta_c^{st} = \beta_a^{st} = \beta_b^{st}, \quad and \quad \gamma_c^{st} = \frac{\gamma_b^{st} \gamma_a^{st}}{\left(\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}\right)^2}.$$

In sum, we can state that the parallel pipe merge in the transient and stationary case are similarly calculated by

$$\gamma_c^{tr} = \frac{\gamma_b^{tr} \, \gamma_a^{tr}}{\left(\sqrt{\gamma_b^{tr}} + \sqrt{\gamma_a^{tr}}\right)^2} \quad \text{and} \quad \gamma_c^{st} = \frac{\gamma_b^{st} \, \gamma_a^{st}}{\left(\sqrt{\gamma_b^{st}} + \sqrt{\gamma_a^{st}}\right)^2}$$

6 Merging Serial Pipes

In this section, we investigate serial pipe merging under transient conditions. But other than the parallel merge, we show that it is not possible to equivalently replace two serial pipes by one pipe that throughout admits the same pressure and flow realizations. However, solving transient gas network optimization problems benefits from reducing the network size by applying serial merges. For this reason, we introduce a new method that approximates the feasible region of the original system of equations by using a sampling-based regression approach. More precisely, the approach samples the feasible region of the original system and determines a parameter of a new reduced, but structurally equivalent system that minimizes the error incurred through sampling. Afterwards, we evaluate this method on different pairs of serial pipes using real-world data. It turns out that this method performs tremendously well in the sense that it approximates the solution space of the original system "very adequately".

Remark 6.1. We point out that serial pipe merges are not in the spirit of refining the spatial discretization which would be required in order to converge towards a solution of System (1). Instead, this aggregation procedure enlarges the step size of the spatial discretization by removing the intermediate node and thus dealing with an entire pipe instead of two pipe segments. Hence, we apply the merge from the viewpoint of optimization with the intension to reduce the number of variables and constraints present in the underlying transient gas flow model. In the remainder of this section, we consider two serial pipes $a = (\ell, m)$ and b = (m, r) as a stand-alone subnetwork, where node $m \in \mathcal{V}^*$ is a transshipment node and has deg(m) = 2. Each pipeline induces two flow variables at time step $t \in \mathcal{T}$, which indicate the amount of flow entering and leaving the pipeline, see Figure 2.



Figure 2: Illustration of the variables present at modeling serial pipes a and b at time step t.

Given that we model flow variables towards the orientation of the arc with positive sign, the flow conservation constraints at these three nodes read

$$q_{\ell,t} - b_{\ell,t} = 0,$$

$$q_{m_b,t} - q_{m_a,t} - b_{m,t} = 0,$$

$$-q_{r,t} + b_{r,t} = 0,$$

where demand values $b_{v,t} \geq 0$ correspond to inflow and $b_{v,t} \leq 0$ to outflow values from the remaining network at $v \in \{\ell, r\}$. Since deg(m) = 2 and $m \in \mathcal{V}^*$, i.e., $b_{m,t} = 0$ holds for all $t \in \mathcal{T}$, it is possible to eliminate one flow variable by introducing $q_{m,t}$ and substituting $q_{m,t} := q_{m_a,t} = -q_{m_b,t}$. Considering multiple time steps, then, after eliminating the flow conservation constraint at node m and one flow variable, k-many serial merges reduce the size of the model by $k \cdot |\mathcal{T}| \cdot 2$ variables (where the two variables are given by $p_{m,t}$ and $q_{m,t}$) and $k \cdot |\mathcal{T}| \cdot 6$ constraints (the discretized continuity and momentum equations, the lower and upper pressure and flow bounds), cf. Equations (33) - (41).

System of equations for the serial and merged pipes. For each pipe $a = (\ell, m)$ and b = (m, r), a system of equations \mathcal{F}_a and \mathcal{F}_b represents the corresponding discretized continuity and momentum equations and variable bounds. Besides, system \mathcal{F}_{fc} represents the flow conservation at nodes ℓ and r for all $t \in \mathcal{T}$ and is given by

$$\int q_{\ell,t} = b_{\ell,t},\tag{31}$$

$$\mathcal{F}_{fc} = \left\{ q_{r,t} = b_{r,t}. \right.$$

$$(32)$$

Note that the flow conservation at node m is already embedded in \mathcal{F}_a and \mathcal{F}_b by using $q_{m,t}$ from above. The serial merge substitutes both pipes a and b for a new pipe $c = (\ell, r)$. Here, the merge replaces $\mathcal{F}_{orig} := \mathcal{F}_a \cup \mathcal{F}_b \cup \mathcal{F}_{fc}$ by $\mathcal{F}_{agg} := \mathcal{F}_c \cup \mathcal{F}_{fc}$, where system \mathcal{F}_c represents the discretized continuity and momentum equation of the merged pipe c.

The system of equations \mathcal{F}_a of pipe a is given for all $t \in \mathcal{T}$ by

$$p_{m,t} - p_{m,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{m,t} - q_{\ell,t}) \alpha_a \tau = 0,$$
(33)

$$p_{m,t}^2(1+\beta_a) - p_{\ell,t}^2(1-\beta_a) + (q_{m,t}+q_{\ell,t}) |q_{m,t}+q_{\ell,t}|\gamma_a = 0,$$
(34)

$$p_{\ell,t} \in [p_{\ell}, \overline{p}_{\ell}], \ p_{m,t} \in [p_{m}, \overline{p}_{m}], \ q_{\ell,t} \in [q_{\ell}, \overline{q}_{\ell}], \ q_{m,t} \in [q_{m}, \overline{q}_{m}].$$
(35)

The system of equations \mathcal{F}_b of pipe b is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{m,t} - p_{m,t-1} + (q_{r,t} - q_{m,t}) \alpha_b \tau = 0, \qquad (36)$$

$$p_{r,t}^2(1+\beta_b) - p_{m,t}^2(1-\beta_b) + (q_{r,t}+q_{m,t}) |q_{r,t}+q_{m,t}| \gamma_b = 0,$$
(37)

$$p_{m,t} \in [\underline{p}_m, \overline{p}_m], \ p_{r,t} \in [\underline{p}_r, \overline{p}_r], \ q_{m,t} \in [\underline{q}_m, \overline{q}_m], \ q_{r,t} \in [\underline{q}_r, \overline{q}_r].$$
(38)

The system of equations \mathcal{F}_c of the merged pipe $c = (\ell, r)$ is given for all $t \in \mathcal{T}$ by

$$p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r,t} - q_{\ell,t}) \alpha_c \tau = 0,$$
(39)

$$p_{r,t}^2(1+\beta_c) - p_{\ell,t}^2(1-\beta_c) + (q_{r,t}+q_{\ell,t}) |q_{r,t}+q_{\ell,t}| \gamma_c = 0,$$
(40)

$$p_{\ell,t} \in [\underline{p}_{\ell}, \overline{p}_{\ell}], \ p_{r,t} \in [\underline{p}_{r}, \overline{p}_{r}], \ q_{\ell,t} \in [\underline{q}_{\ell}, \overline{q}_{\ell}], \ q_{r,t} \in [\underline{q}_{r}, \overline{q}_{r}].$$
(41)

We recall that the parameters are given by

$$\alpha_i := \frac{2 \operatorname{c_s}^2}{A_i L_i}, \quad \gamma_i := \frac{\lambda_a \operatorname{c_s}^2 L_i}{4D_i A_i^2}, \quad \beta_a := \frac{g \left(h_m - h_\ell\right)}{\operatorname{c_s}^2}, \quad \beta_b := \frac{g \left(h_r - h_m\right)}{\operatorname{c_s}^2},$$

with $\alpha_i, \gamma_i > 0$ for $i \in \{a, b\}$ and $\beta_a, \beta_b \in \mathbb{R}$.

As for the parallel merge, the serial merge requires determining the parameters $\alpha_c, \gamma_c > 0$ and $\beta_c \in \mathbb{R}$ in system \mathcal{F}_c . While the requirements from Section 4 to (i) maintain the volume of the serial pipes, and (ii) retain the difference in height along the serial pipes, naturally allow determining the parameters α_c, β_c , the calculation of the parameter γ_c is more sophisticated and is part of the regression-based merging method in Subsection 6.2.

Concerning α_c : The property to keep the volume of both serial pipes reads $vol_c = vol_a + vol_b$. Given that the volume can be extracted from the parameters

$$\alpha_a = \frac{2 c_s^2}{vol_a}$$
 and $\alpha_b = \frac{2 c_s^2}{vol_b}$,

it is possible to derive α_c from α_a and α_b :

$$vol_{c} = vol_{a} + vol_{b} \quad \Rightarrow \quad \alpha_{c} = \frac{2 c_{s}^{2}}{vol_{a} + vol_{b}} = \left(\frac{vol_{a}}{2 c_{s}^{2}} + \frac{vol_{b}}{2 c_{s}^{2}}\right)^{-1}$$
$$\Rightarrow \quad \alpha_{c} = \left(\frac{1}{\alpha_{a}} + \frac{1}{\alpha_{b}}\right)^{-1}.$$
(42)

Remark 6.2 (Serial pipes - line pack). Other than in parallel pipe merging, the serial pipe merge might not necessarily retain the line-pack of the serial pipes, i.e., it might hold $LP_{c,t} \neq LP_{a,t} + LP_{b,t}$. Here, the line-pack is only retained, if $p_{c,t}^{mean} = p_{a,t}^{mean} + p_{b,t}^{mean}$ holds. This can be derived from the representation of the line-pack in Equation (8) together with the condition $vol_c = vol_a + vol_b$, similarly as done in Corollary 5.2 for parallel pipe merging.

Concerning β_c : The difference in height $h_r - h_\ell$ along the serial pipes is governed by the parameters β_a and β_b . Keeping this difference along the merged pipe c follows naturally by assuming that the parameters β_a and β_b are additive, i.e.,

$$(h_r - h_m) + (h_m - h_\ell) = h_r - h_\ell$$

$$\Rightarrow \quad \frac{g(h_r - h_m)}{c_s^2} + \frac{g(h_m - h_\ell)}{c_s^2} = \frac{g(h_r - h_\ell)}{c_s^2}.$$

By defining $\beta_c := g (h_r - h_\ell) / c_s^2$, then β_c fulfills

=

$$\beta_c = \beta_a + \beta_b. \tag{43}$$

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However, we remark that the geographic height at specific spatial coordinates might be different between the serial pipes and the merged pipe. This happens, for example, if there is a local hill or valley at node m, i.e., when $h_m \notin h_\ell + \lambda(h_r - h_\ell)$ holds for all $\lambda \in [0, 1]$.

6.1 Counterexample to equivalent transient serial pipe merging

In the previous subsection, we have seen that α_c and β_c are uniquely determined by retaining the volume and the difference in height of the serial pipes. Ideally and in addition to both requirements, the systems \mathcal{F}_{orig} and \mathcal{F}_{agg} are both satisfied at once. However, in the following we provide a counterexample to satisfying all these requirements simultaneously. To this end, we throughout consider horizontal pipes, which implies β_a , β_b , $\beta_c = 0$. For the counterexample, we first introduce some preliminary results about transient gas flows in a single and in serial pipes. We start by proving that the solution of system \mathcal{F}_{agg} for a single horizontal pipe is unique, given an initial state and a future demand. This fact is not self-evident, for example, Weltsch (2018) shows that the solution of the frequently used *implicit Box* discretization scheme is not unique for a single pipe.

Lemma 6.3. Let pipe $a = (\ell, r)$ with parameters $\alpha_a, \gamma_a > 0$, initial pressure values $p_{\ell,t_0}, p_{r,t_0} \geq 0$, and demand values $b_{\ell,t}$ and $b_{r,t}$ for all $t \in \mathcal{T}$ be given. Then, the following nonlinear system

$$\begin{aligned} p_{r,t} - p_{r,t-1} + p_{\ell,t} - p_{\ell,t-1} + (q_{r,t} - q_{\ell,t}) \, \alpha_a &= 0 & \forall \ t \in \mathcal{T} \\ p_{r,t}^2 - p_{\ell,t}^2 + (q_{r,t} + q_{\ell,t}) \, |q_{r,t} + q_{\ell,t}| \gamma_a &= 0 & \forall \ t \in \mathcal{T} \\ q_{\ell,t} &= b_{\ell,t} & \forall \ t \in \mathcal{T} \\ q_{r,t} &= b_{r,t} & \forall \ t \in \mathcal{T} \end{aligned}$$

has a unique solution $(p_{\ell,t}, p_{r,t}, q_{\ell,t}, q_{r,t})_{t \in \mathcal{T}}$ with non-negative values $p_{\ell,t} \in [\underline{p}_{\ell}, \overline{p}_{\ell}]$ and $p_{r,t} \in [p_r, \overline{p}_r]$, or it is infeasible.

Proof. We show that the pressure values $p_{\ell,t}, p_{r,t}$ are uniquely determined for given values $p_{\ell,t-1}, p_{r,t-1}$ and $b_{\ell,t}, b_{r,t}$ at an arbitrary time step $t \in \mathcal{T} \setminus \{|\mathcal{T}|\}$. Then, starting with the first time step t = 1 and applying this argument iteratively to two consecutive time steps via induction, completes the proof.

For a single step from t - 1 to t, the discretized continuity and momentum equations reduce to

$$p_{r,t} = s_1 - p_{\ell,t} \tag{44}$$

$$p_{r,t}^2 - p_{\ell,t}^2 + s_2 = 0, (45)$$

where the parameters s_1, s_2 are given by

$$s_1 := p_{r,t-1} + p_{\ell,t-1} - (q_{r,t} - q_{\ell,t}) \alpha_a$$

$$s_2 := (q_{r,t} + q_{\ell,t}) |q_{r,t} + q_{\ell,t}| \gamma_a.$$

Combining Equations (44) and (45) yields

$$\Rightarrow \quad (s_1 - p_{\ell,t})^2 - p_{\ell,t}^2 + s_2 = 0 \Rightarrow \quad p_{\ell,t} = \frac{s_1^2 + s_2}{2 s_1}, \quad \text{if } s_1 \neq 0.$$

Hence, $p_{\ell,t}$ is uniquely defined only depending on the parameters $s_1 \neq 0$ and s_2 . Then, $p_{r,t}$ is also uniquely defined by virtue of Equation (45). On the other hand, if $s_1 = 0$, then (44) reduces to $p_{r,t} = -p_{\ell,t}$, which, according to the non-negativity requirement of the pressure variables, is only the case, if $p_{r,t} = p_{\ell,t} = 0$ holds. Finally, all uniquely determined values $p_{r,t}, p_{\ell,t}$ are either non-negative and within the bounds, or the system is infeasible. Let us now come back to two serial pipes. The following lemma characterizes stationary behavior in a time-dependent setting. More precisely, given a constant and balanced demand over time and initial pressure values in steady-state, then there exist a solution in steady-state.

Lemma 6.4. Let two serial pipes $a = (\ell, m)$ and b = (m, r) be given, with deg(m) = 2 and $b_{m,t} = 0$ for all $t \in \mathcal{T}$. Further, let

- i) a constant and balanced demand $b_{\ell,t}, b_{r,t} := \hat{q}$ for all $t \in \mathcal{T}$, with $q_{\ell,t}, q_{r,t} := \hat{q}$ and $\underline{q}_{\ell} \leq q_{\ell} \leq \overline{q}_{\ell}$ and $\underline{q}_{r} \leq q_{r} \leq \overline{q}_{r}$, and
- *ii)* initial pressure values $p_{\ell,t_0} \in [\underline{p}_{\ell}, \overline{p}_{\ell}]$ and $p_{m,t_0} \in [\underline{p}_m, \overline{p}_m]$ and $p_{r,t_0} \in [\underline{p}_r, \overline{p}_r]$ in steady-state, *i.e.*,

$$p_{m,t_0} = \left(p_{\ell,t_0}^2 - \left(q_{m,t_0} + q_{\ell,t_0}\right) \left|q_{m,t_0} + q_{\ell,t_0}\right| \gamma_a\right)^{1/2} \tag{46}$$

$$p_{r,t_0} = \left(p_{m,t_0}^2 - \left(q_{r,t_0} + q_{m,t_0}\right) \left|q_{r,t_0} + q_{m,t_0}\right| \gamma_b\right)^{1/2} \tag{47}$$

be given. Then, $(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{\ell,t}, q_{m,t}, q_{r,t})_{t \in \mathcal{T}}$ with $q_{m,t} := q_{\ell,t}, q_{r,t}$ and $p_{\ell,t} := p_{\ell,t_0}$ and $p_{m,t} := p_{m,t_0}$ and $p_{r,t} := p_{r,t_0}$ is a solution of system \mathcal{F}_{orig} .

Proof. We have to show that Equations (31) - (38) are satisfied. Firstly, setting $q_{m,t} = \hat{q}$ and $p_{\ell,t} = p_{\ell,t_0}, p_{m,t} = p_{m,t_{t_0}}$ and $p_{r,t} = p_{r,t_0}$ for all $t \in \mathcal{T}$ together with $q_{\ell,t}, q_{r,t} = \hat{q}$ satisfy Equations (33) and (36). Secondly, from the values $p_{m,t} = p_{m,t_0}$ and $p_{r,t} = p_{r,t_0}$, as stated in (46) and (47), follow that (34) and (37) are satisfied. Finally, all variables hold the bounds and thus $(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{\ell,t}, q_{m,t}, q_{r,t})_{t\in\mathcal{T}}$ is a solution of system (31) - (38).

Under the premise of Lemma 6.4, it is possible to derive γ_c such that the merged pipe and serial pipes entail the same pressure and flow realizations. The following corollary formalizes this claim and explicitly states a formula for γ_c .

Corollary 6.5. Consider a horizontal pipe $c = (\ell, r)$ that is split into two pipe segments $a = (\ell, m)$ and b = (m, r) with $\beta_a = \beta_b = 0$ and $\gamma_a, \gamma_b > 0$ and let deg(m) = 2 and $b_{m,t} = 0$ for all $t \in \mathcal{T}_0$. Further, let

- i) a constant and balanced demand $b_{\ell,t}, b_{r,t} := \hat{q}$ for all $t \in \mathcal{T}_0$, with $q_{\ell,t}, q_{r,t} := \hat{q}$ and $\underline{q}_{\ell} \leq q_{\ell} \leq \overline{q}_{\ell}$ and $\underline{q}_r \leq q_r \leq \overline{q}_r$, and
- ii) initial pressure values $p_{\ell,t_0} \in [\underline{p}_{\ell}, \overline{p}_{\ell}]$ and $p_{m,t_0} \in [\underline{p}_m, \overline{p}_m]$ and $p_{r,t_0} \in [\underline{p}_r, \overline{p}_r]$ in steady-state, i.e.,

$$p_{m,t_0} = \left(p_{\ell,t_0}^2 - (q_{m,t_0} + q_{\ell,t_0}) | q_{m,t_0} + q_{\ell,t_0} | \gamma_a\right)^{1/2}$$
$$p_{r,t_0} = \left(p_{m,t_0}^2 - (q_{r,t_0} + q_{m,t_0}) | q_{r,t_0} + q_{m,t_0} | \gamma_b\right)^{1/2}$$

be given. Then, it follows $\gamma_c = \gamma_a + \gamma_b$.

Proof. From Lemma 6.4 follows that $(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{\ell,t}, q_{m,t}, q_{r,t})_{t \in \mathcal{T}}$ is feasible for \mathcal{F}_{orig} with given solution values in the lemma. Then, the momentum equations for pipes a, b and c reduce to

$$\begin{cases} p_{m,t}^2 - p_{\ell,t}^2 + 4\,\hat{q}^2\,\gamma_a = 0 \quad \forall t \in \mathcal{T}, \\ p_{r,t}^2 - p_{m,t}^2 + 4\,\hat{q}^2\,\gamma_b = 0 \quad \forall t \in \mathcal{T}, \end{cases}$$

$$\Rightarrow \quad p_{r,t}^2 - p_{\ell,t}^2 + 4\,\hat{q}^2(\gamma_a + \gamma_b) = 0 \quad \forall t \in \mathcal{T},$$

$$\Rightarrow \quad \gamma_c = \gamma_a + \gamma_b.$$

In Subsection 6.4, we show that merging serial pipes in the stationary case also yields $\gamma_c = \gamma_a + \gamma_b$, cf. Equation (60). Nevertheless, it is not possible to establish the relation $\gamma_c = \gamma_a + \gamma_b$ in general. It even suffices to slightly distort a balanced demand of a system in steady-state in order to generate a counterexample, as we show in the following. The counterexample works as follows: We consider two equal horizontal serial pipes, i.e., $\alpha_a = \alpha_b$, $\gamma_a = \gamma_b$ and $\beta_a = \beta_b = 0$, and assume that the stationary assumptions from Corollary 6.5 (and Lemma 6.4) hold, i.e., (i) a constant and balanced demand over time, and (ii) initial pressure values in steady-state. As we know from Corollary 6.5, these assumptions allow us to derive a parameter γ_c that establishes the same pressure and flow realizations for a merged pipe c as for the serial pipes a and b. Afterwards, we slightly change the demand, which finally requires a different parameter $\tilde{\gamma_c} \neq \gamma_c$ in order to obtain the same pressure profiles for the merged pipe as for the serial pipes.

Example 6.6 (Counterexample to equivalent transient serial pipe merging). Consider a stand-alone network consisting of two horizontal pipes $a = (\ell, m)$ and b = (m, r) in serial, with $\alpha_a, \alpha_b = 1.0, \gamma_a, \gamma_b = 1.0$ and $\beta_a, \beta_b = 0$ and $\tau = 1$ for a single time step from t_0 to t_1 . Let a demand be given by $b_{\ell,t} = 5.0, b_{r,t} = -5.0$ and $b_{m,t} = 0.0$, which, for this network translates to $\hat{q}_{\ell,t_1}, \hat{q}_{r,t_1} = 5.0$. Further, let an initial state be given that fulfills stationary conditions

$$\hat{p}_{\ell,t_0} = 50.00, \quad \hat{p}_{m,t_0} = 48.99, \quad \hat{p}_{r,t_0} = 47.96,$$
(48)

(rounded to two decimal places) with respect to flow values $\hat{q}_{\ell,t_0} = \hat{q}_{m,t_0} = \hat{q}_{r,t_0} = 5.0$. Then, from Lemma 6.4 follows that a solution of system \mathcal{F}_{orig} for both serial pipes a and b is given by

$$\hat{p}_{\ell,t_1} = \hat{p}_{\ell,t_0}, \quad \hat{p}_{m,t_1} = \hat{p}_{m,t_0}, \quad \hat{p}_{r,t_1} = \hat{p}_{r,t_0}, \quad and \quad \hat{q}_{m,t_1} = 5.0.$$

Besides, the assumptions of Corollary 6.5 are satisfied, and hence the parameter γ_c for the merged pipe c holds

$$\gamma_c = \gamma_a + \gamma_b = 2.0. \tag{49}$$

Then, for γ_c as given in (49) and any choice of $\alpha_c \geq 0$, $(\hat{p}_{\ell,t_1}, \hat{p}_{r,t_1}, \hat{q}_{\ell,t_1}, \hat{q}_{r,t_1})$ is the unique solution of system \mathcal{F}_{agg} for the merged pipe c according to Lemma 6.3.

However, when slightly changing the demand to

$$\tilde{q}_{\ell,t_1} = 10.0, \quad \tilde{q}_{r,t_1} = 5.0,$$

while keeping the initial conditions of both serial pipes from (48), the solution of system \mathcal{F}_{orig} (for the serial pipes) changes to

$$\tilde{p}_{\ell,t_1} = 52.57, \quad \tilde{p}_{m,t_1} = 49.91, \quad \tilde{p}_{r,t_1} = 48.56, \quad \tilde{q}_{m,t_1} = 6.52.$$

But the solution $(\tilde{p}_{\ell,t_1}, \tilde{p}_{r,t_1}\tilde{q}_{\ell,t_1}, \tilde{q}_{r,t_1})$ cannot be realized with γ_c as determined in Equation (49), since Equation (40) together with given values $\tilde{p}_{\ell,t_1}, \tilde{p}_{r,t_1}, \tilde{q}_{\ell,t_1}$ and \tilde{q}_{r,t_1} uniquely determines $\tilde{\gamma}_c = 1.80 \neq 2.0$.

6.2 Sampling based regression approach

Motivated by the counterexample above, which shows that it is not possible to equivalently replace two serial pipes by one pipe, we develop a sampling-based merging approach that minimizes the error incurred. At first, we sample the feasible region of system \mathcal{F}_{orig} . Then, we use a least-squares regression to acquire a linear model. The model returns a parameter γ_c that minimizes the violation of the samples with respect to system \mathcal{F}_{aqq} .

Sampling procedure. We are given two serial pipes $a = (\ell, m)$ and b = (m, r) with corresponding parameters $\alpha_a, \alpha_b, \beta_a, \beta_b, \gamma_a, \gamma_b$ and a time horizon \mathcal{T}_0 . In the following, we generate a sampling set $S := \{s_k | k \in \mathcal{K}\}$ that approximates the feasible region of the serial pipes' system \mathcal{F}_{orig} . Here, we use index $k \in \mathcal{K}$ to denote a particular sample s_k . A single sample $s_k \in S$ reads

$$\left(p_{\ell,t_0}^k, \dots, p_{\ell,t_n}^k, p_{m,t_0}^k, \dots, p_{m,t_n}^k, p_{r,t_0}^k, \dots, p_{r,t_n}^k, q_{\ell,t_1}^k, \dots, q_{\ell,t_n}^k, q_{m,t_1}^k, \dots, q_{m,t_n}^k, q_{r,t_1}^k, \dots, q_{r,t_n}^k\right)$$

where its domain is given by

$$s_k \in [\underline{p}_{\ell}, \overline{p}_{\ell}]^{|\mathcal{T}_0|} \times [\underline{p}_m, \overline{p}_m]^{|\mathcal{T}_0|} \times [\underline{p}_r, \overline{p}_r]^{|\mathcal{T}_0|} \times [\underline{q}_{\ell}, \overline{q}_{\ell}]^{|\mathcal{T}|} \times [\underline{q}_m, \overline{q}_m]^{|\mathcal{T}|} \times [\underline{q}_r, \overline{q}_r]^{|\mathcal{T}|}.$$

The generation of a particular sample s_k consists of the following three steps, as sketched in Algorithm 1:

- i) generate initial pressure values $p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k$ by GENERATEINITIALSTATE,
- ii) generate demand values $\left(q_{\ell,t}^k, q_{r,t}^k\right)_{t \in \mathcal{T}}$ by GENERATEDEMAND,
- iii) generate the remaining values $\left(p_{\ell,t}^k, p_{m,t}^k, p_{r,t}^k, q_{m,t}^k\right)_{t \in \mathcal{T}}$ by RUNSAMPLECOMPLETION.

In order to imitate a "more realistic" physical behavior than sampling arbitrary values within the bounds, we build each sample upon an initial state that fulfills stationary conditions. To this end, we select a random flow value $\hat{q}_{t_0}^k \in [\underline{q}, \overline{q}]$ and set $q_{\ell,t_0}^k, q_{m,t_0}^k, q_{r,t_0}^k := \hat{q}_{t_0}^k$ in GENERATEINITIALSTATE. In this way, a flow direction along both pipes is predetermined at t_0 , and thus, it either holds $p_{\ell,t_0}^k > p_{m,t_0}^k > p_{r,t_0}^k$ or $p_{r,t_0}^k > p_{m,t_0}^k > p_{\ell,t_0}^k$. To initialize these pressure values, we first generate a random value for the pressure variable that is supposed to have the highest value, i.e., p_{ℓ,t_0}^k in the case of sgn $(\hat{q}_{t_0}^k) > 0$, or p_{r,t_0}^k in the case of sgn $(\hat{q}_{t_0}^k) < 0$, and then use the given flow value $\hat{q}_{t_0}^k$ together with the discretized momentum equation to successively determine the remaining initial pressure values, see Algorithm 1. This flow value also serves for the generation of the demand. Based on q_{ℓ,t_0}^k and q_{r,t_0}^k , we derive random flow values $q_{\ell,t}^k$ and $q_{r,t}^k$ at each time step $t \in \mathcal{T}$ that are only allowed to vary within a predefined range from their respective predecessors, i.e., $q_{\ell,t}^k \in [q_{\ell,t-1}^k - q_{\epsilon}, q_{\ell,t-1}^k + q_{\epsilon}]$ and $q_{r,t}^k \in [q_{r,t-1}^k - q_{\epsilon}, q_{r,t-1}^k + q_{\epsilon}]$, as described by the subroutine GENERATEDEMAND in Algorithm 1. This assumption seems reasonable in order to avoid a completely arbitrary demand profile. In this context, we also refer to the upper Sub-figures 4 - 5, which illustrate historical in and outflows of different pairs of serial pipes taken from real-world data. These figures show that sudden peaks in the difference between the in and outflow of two serial pipes occur rarely. However, in general it is possible that flows are more volatile than being generated here.

Algorithm 1 Sampling generation in regression-based serial pipe merge

Generation of sampling set S

1: $\mathcal{S} \leftarrow \emptyset$ 2: while $k \leftarrow 1 < |\mathcal{K}|$ do \triangleright the number of generated samples $|\mathcal{K}|$ is predefined $\left(p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k, q_{\ell,t_0}^k, q_{r,t_0}^k\right) \leftarrow \text{GenerateInitialState}$ 3: $\left(q_{\ell,t}^{k}, q_{r,t}^{k}\right)_{t \in \mathcal{T}} \leftarrow \text{GENERATEDEMAND}\left(q_{\ell,t_{0}}^{k}, q_{r,t_{0}}^{k}\right)$ 4: isSampleValid \leftarrow RUNSAMPLECOMPLETION $(p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k, q_{\ell,t}^k, q_{r,t}^k)_{t \in \mathcal{T}}$ 5:if isSampleValid then 6: $\mathcal{S} \leftarrow \left(p_{\ell,\tilde{t}}^{k}, p_{m,\tilde{t}}^{k}, p_{r,\tilde{t}}^{k}, q_{\ell,t}^{k}, q_{m,t}^{k}, q_{r,t}^{k} \right)_{\tilde{t} \in \mathcal{T}_{0}, t \in \mathcal{T}}$ \triangleright add sample to sampling set 7: $k \leftarrow k + 1$ 8: end if 9:

10: end while

Generation of an (i) initial state, (ii) demand and (iii) sample completion

function GENERATEINITIALSTATE 11: 12:Select $\hat{q}_{t_0}^k \in \left[\underline{q}, \overline{q}\right]$ and set $q_{\ell, t_0}^k, q_{r, t_0}^k := \hat{q}_{t_0}^k \quad \triangleright$ select initial random flow in in $\left[\underline{q}, \overline{q}\right]$ if $q_{\ell,t_0}^k, q_{r,t_0}^k < 0$ then 13: \triangleright flow from node r to node ℓ Select $p_{\ell,t_0}^k \in [p,\overline{p}]$ 14: $p_{m,t_0}^k \leftarrow \left(\left((p_{\ell,t_0}^k)^2 (1-\beta_a) + (2 \cdot \hat{q}_{t_0}^k)^2 \gamma_a \right) / (1+\beta_a) \right)^{1/2}$ 15: $p_{r,t_0}^k \leftarrow \left(\left((p_{m,t_0}^k)^2 (1-\beta_b) + (2 \cdot \hat{q}_{t_0}^k)^2 \gamma_b \right) / (1+\beta_b) \right)^{1/2}$ where ℓ is the product of the product o 16:17: elseSelect $p_{r,t_0}^k \in \left[p, \overline{p}\right]$ 18: $p_{m,t_0}^k \leftarrow \left(\left((p_{r,t_0}^k)^2 (1+\beta_b) + (2 \cdot \hat{q}_{t_0}^k)^2 \gamma_b \right) / (1-\beta_b) \right)^{1/2}$ 19: $p_{\ell,t_0}^k \leftarrow \left(\left((p_{m,t_0}^k)^2 (1+\beta_a) + (2 \cdot \hat{q}_{t_0}^k)^2 \gamma_a \right) / (1-\beta_a) \right)^{1/2}$ 20: 21:end if **return** $(p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k, q_{\ell,t_0}^k, q_{r,t_0}^k)$ 22:end function 23:24: function GENERATEDEMAND $(q_{\ell,t_0}^k, q_{r,t_0}^k)$ for $t \leftarrow 1$ to $|\mathcal{T}|$ do 25:Select $q_{\ell,t}^k \in \left[q_{\ell,t-1}^k - q_\epsilon, q_{\ell,t-1}^k + q_\epsilon\right]$ Select $q_{r,t}^k \in \left[q_{r,t-1}^k - q_\epsilon, q_{r,t-1}^k + q_\epsilon\right]$ 26:27: 28: end for return $\left(q_{\ell,t}^k, q_{r,t}^k\right)_{t\in\mathcal{T}}$ 29: 30: end function 31: **function** RUNSAMPLECOMPLETION $((p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k, q_{\ell,t}^k, q_{r,t}^k)_{t \in \mathcal{T}})$ 32: solve system of Equations (31) - (38) \triangleright where $(p_{\ell,t_0}^k, p_{m,t_0}^k, p_{r,t_0}^k, q_{\ell,t}^k, q_{r,t}^k)_{t \in \mathcal{T}}$ is given return Feasibility of system (31) - (38) 33: 34: end function

So far, after the generation of (i) an initial state and (ii) a demand scenario at nodes ℓ and r, the values of the bold parameters are fixed for a particular sample in $s_k \in S$

$$\begin{pmatrix} \boldsymbol{p}_{\ell,t_0}^{k}, ..., p_{\ell,t_n}^{k}, \boldsymbol{p}_{m,t_0}^{k}, ..., p_{m,t_n}^{k}, \boldsymbol{p}_{r,t_0}^{k}, ..., p_{r,t_n}^{k}, \boldsymbol{q}_{\ell,t_1}^{k}, ..., \boldsymbol{q}_{\ell,t_n}^{k}, q_{m,t_1}^{k}, ..., q_{m,t_n}^{k}, \\ \boldsymbol{q}_{r,t_1}^{k}, ..., \boldsymbol{q}_{r,t_n}^{k} \end{pmatrix}.$$

To complete the sample, we apply the method RUNSAMPLECOMPLETION. The underlying system \mathcal{F}_{orig} of the sample completion either admits a feasible solution $(p_{\ell,t}, p_{m,t}, p_{r,t}, q_{m,t})_{t \in \mathcal{T}}$ or is infeasible. Our preliminary computational experiments on the sample completion even pose the conjecture that the solution of this sample completion is unique. To check the feasibility of this system for a particular sample s_k , we use SCIP version 7.0.0 Gamrath et al. (2020). The sample completion is an NLP-Problem. For a reasonable amount of time steps $|\mathcal{T}|$, it is solved in significantly less than one second for a single sample. Nevertheless we set a time limit in order to compute thousands of samples on short notice that are needed for the merging procedure. In the case that sample s_k is feasible for \mathcal{F}_{orig} , we add it to the sampling set \mathcal{S} , indicated by the Boolean isSampleValid.

From the counterexample above, we already know that a sample is most likely not feasible for \mathcal{F}_{agg} . Therefore, after generating $|\mathcal{K}|$ -many samples, we determine γ_c by using linear regression. The regression minimizes the error of the samples in \mathcal{S} with respect to their corresponding realizations in system \mathcal{F}_{agg} . In general, different forms of linear regression exist that mostly vary in the measure of the error by using different norms. Based on preliminary computational tests, we decided to use an ordinary least-squares regression.

Regression. For a particular sample $s_k \in S$, we introduce slack (residuum) values $r_t^k, \tilde{r}_t^k \in \mathbb{R}$ for all $t \in \mathcal{T}$ with respect to the following equations present in \mathcal{F}_{orig} :

$$p_{r,t}^{k} - p_{r,t-1}^{k} + p_{\ell,t}^{k} - p_{\ell,t-1}^{k} + \left(q_{r,t}^{k} - q_{\ell,t}^{k}\right) \alpha_{c} \tau + r_{t}^{k} = 0,$$
(50)

$$(p_{r,t}^k)^2 (1+\beta_c) - (p_{\ell,t}^k)^2 (1-\beta_c) + \left(q_{r,t}^k + q_{\ell,t}^k\right) |q_{r,t}^k + q_{\ell,t}^k| \gamma_c + \tilde{r}_t^k = 0.$$
(51)

However, since the parameters α_c and β_c are uniquely determined by virtue of Equations (42) - (43) before sampling, and since each sample together with α_c uniquely determines the residuum r_t^k in (50), the only remaining degree of freedom is to determine γ_c in (51). That is, we minimize the slack values \tilde{r}_t^k for all $t \in \mathcal{T}$ across all samples. Thereby, the regression allows to calculate γ_c in such a way that it might compensate a possible inaccurate impact of the beforehand determined parameter $\beta_c = \beta_a + \beta_b$.

In the following, we determine γ_c analytically, given that a linear ordinary leastsquares regression admits a closed-form solution, see also Hastie et al. (2009). To this end, we define a function $f \colon \mathbb{R} \to \mathbb{R}$ that describes the sum of squared slack values for all samples, which is to be minimized:

$$f(\gamma_c) := \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \left((p_{r,t}^k)^2 \left(1 + \beta_c \right) - (p_{\ell,t}^k)^2 (1 - \beta_c) + \left(q_{r,t}^k + q_{\ell,t}^k \right) |q_{r,t}^k + q_{\ell,t}^k |\gamma_c \right)^2.$$

The minimum of f is obtained at $\partial_{\gamma_c} f = 0$ with

$$\partial_{\gamma_c} f = \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} 2\Big((p_{r,t}^k)^2 \left(1 + \beta_c\right) - (p_{\ell,t}^k)^2 (1 - \beta_c) \Big)$$

+
$$(q_{r,t}^k + q_{\ell,t}^k) |q_{r,t}^k + q_{\ell,t}^k|\gamma_c) (q_{r,t}^k + q_{\ell,t}^k) |q_{r,t}^k + q_{\ell,t}^k|,$$

finally yielding

$$\arg\min_{\gamma_{c}} f(\gamma_{c}) = (52)$$

$$-\frac{\sum_{k} \sum_{t} \left(\left((p_{r,t}^{k})^{2} (1+\beta_{c}) - (p_{\ell,t}^{k})^{2} (1-\beta_{c}) \right) \left(q_{r,t}^{k} + q_{\ell,t}^{k} \right) |q_{r,t}^{k} + q_{\ell,t}^{k}| \right)}{\left(\sum_{k} \sum_{t} \left(q_{r,t}^{k} + q_{\ell,t}^{k} \right) |q_{r,t}^{k} + q_{\ell,t}^{k}| \right)^{2}}.$$

Note that (52) is indeed the point where the minimum of f is achieved, since $\partial_{\gamma_c}^2 f(\gamma_c) = \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} 2\left(\left(q_{r,t}^k + q_{\ell,t}^k\right) |q_{r,t}^k + q_{\ell,t}^k|\right)^2 \ge 0$ holds.

6.3 Evaluation of serial merging approach on real-world data

To validate whether the regression-based merging approach leads to pressure-flow relationships that are reasonable close to the ones of the serial pipes, we use realworld data. For the evaluation, we use six pairs of serial pipes $S_1, S_2, S_3, S_4, S_5, S_6$ taken from a real-world network operated by our industrial cooperation partner Open Grid Europe GmbH². All examples are located in different network parts and cover a wide range of possible pipe characteristics. For example, the serial pipelines S_1, S_3, S_4 , and S_5 have pairwise similar lengths, while S_2 and S_6 consist of one long and one short pipeline. Besides, the pipelines in S_2, S_4 , and S_6 have notably different slopes (more than 1%), while the pipes in S_5 have even opposite slopes of mild magnitude. Table 1 provides an overview of the pipe characteristics.

Cases	Pipe a				Pipe b			
	L_a	D_a	s_a	λ_a	L_b	D_b	s_b	λ_b
	[km]	[m]	%	-	[km]	[m]	%	-
S_1	13.003	0.889	0.56	6.46e-03	12.607	0.889	0.55	6.46e-03
S_2	9.408	0.694	1.31	7.88e-03	0.487	0.694	0.04	7.88e-03
S_3	14.388	1.086	0.91	6.29e-03	15.386	1.086	0.61	6.29e-03
S_4	15.065	0.996	1.41	7.46e-03	14.853	0.996	0.07	7.46e-03
S_5	16.090	1.036	0.04	5.77e-03	16.183	1.036	-0.04	5.77e-03
S_6	17.500	0.982	0.37	6.38e-03	0.100	0.982	2.40	6.38e-03

Table 1: Characteristics of the pairs of serial pipes a and b used in the evaluation of the sampling-based merging approach, with length L, diameter D, slope s, and friction factor λ .

For each example, we generate a sampling set S that consists of 2,000 samples, where a particular sample comprises four time steps. The bounds of all flow variables are given by $\underline{q} = -500$ and $\overline{q} = 500$ in all six cases, and the volatility of the demand is determined by the parameter $q_{\epsilon} = 50$.

For each of these six pairs of serial pipes $a = (\ell, m)$ and b = (m, r), we test the merging approach by running two different kind of simulations, (i) for the serial pipes,

²https://oge.net, accessed in April 2021.

and (ii), for the merged pipe $c = (\ell, r)$. For the used demand values $(b_{\ell,t}, b_{r,t})_{t\in\mathcal{T}}$ and the initial pressure values required for the simulation, we use historical data that covers an entire year, i.e., 365 days, split into 15-minute intervals resulting in 35040 time steps. Since operative planning of gas networks typically comprises a time horizon of about an entire day Ehrhardt and Steinbach (2005); Steinbach (2007), we run in total 365 simulations, where the time horizon of a particular simulation is one day and consists of 96 time steps. Due to the considered large and fine-grained time range, we can assume that the simulation results provide us with a representative picture for the impact of the presented merging procedure on the gas transport for real-world scenarios. Thereby, our study accounts for seasonal demand fluctuations, as winters typically entail higher gas consumptions than summers. For a visualization of the applied demand scenarios $(b_{\ell,t}, b_{r,t})_{t\in\mathcal{T}}$ with $\mathcal{T} = \{1, \dots, 35040\}$, see the upper Sub-figures 3 - 5. Here, the demand values $b_{\ell,t} := q_{\ell,t}$ and $b_{r,t} := q_{r,t}$ determine the in and outflow values at the left ℓ and right node r of the pipes $a = (\ell, m)$ and b = (m, r).

Afterwards, we compare the simulated pressure values at common nodes of the original (serial) and aggregated (merged) pipes with respect to Measures (53) - (55). We decided to use *absolute deviation* (AD) and *mean absolute deviation* (MAD) Leys et al. (2013), where we take averages with respect to the number of considered days and time steps. Taking averages in fact means that we consider the mean AD and mean MAD. Besides, we report the real absolute deviation AD. These measures allow interpreting the results in terms of their physical units, here pressure values in [bar] and mass flow values in [kg/s].

AD:
$$\max_{d \in \{1,...,365\}} \max_{t \in \{1,...,96\}} \left| x_{d_t}^{\text{orig}} - x_{d_t}^{\text{agg}} \right|,$$
(53)

mean AD:
$$\frac{1}{365} \max_{t \in \{1,...,96\}} \left| x_t^{\text{orig}} - x_t^{\text{agg}} \right|$$
, and (54)

mean MAD:
$$\frac{1}{365} \sum_{d=1}^{305} \left(\frac{1}{96} \cdot \sum_{t=1}^{96} \left| x_t^{\text{orig}} - x_t^{\text{agg}} \right| \right).$$
 (55)

Results. Table 2 summarizes the differences of simulated pressure values at the common nodes ℓ and r of the serial and merged pipes. It can be seen that the deviations are very small for all examples but S_3 , varying from 0.0006 to 0.04 [bar] in the absolute deviation (AD). Only for S_3 there is a difference of 0.5 [bar] (AD). However, when taking the average over all 365 maximal daily differences, it reduces to a value of 0.05 [bar] (mean AD) and likewise to 0.02 [bar] (mean MAD), which implies that a value of 0.5 in the absolute deviation is rather an exception. These small differences suggest that $\mathcal{X}_{\mathcal{G}_C} \approx \mathcal{X}_{\mathcal{G}_C^{agg}}$ holds and that the presented regression-based merging procedure suitably approximates the serial pipes in the spirit of equivalent subnetwork replacements. However, we remark that more severe demand situations could stress the pipes more.

Finally, the middle and lower Sub-figures 3- 5 visualize the simulated pressure profiles of the serial pipes (red) and the merged pipe (green) at node ℓ (middle sub-figures) and node r (lower sub-figures). These figures additionally depict the historical pressure profiles at both nodes in cyan.

6.4 Relation of transient and stationary serial pipe merging

We know from Section 5 that the parallel merge allows us to determine pipe parameters for the merged pipe that preserve the flow and pressure profiles of the parallel



Figure 3: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_1 and S_2 . The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.



Figure 4: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_3 and S_4 . The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.



Figure 5: Comparison of simulated pressure profiles of serial pipes (red) and merged pipe (green), and the historical pressure values (cyan) at node ℓ (middle sub-figures) and node r (lower sub-figures) for S_5 and S_6 . The upper sub-figures illustrate the historical demand scenarios $(b_{\ell,t})_{t\in\mathcal{T}}$ (blue) and $(b_{r,t})_{t\in\mathcal{T}}$ (orange) at nodes ℓ and r of both serial pipes.

		S_1	S_2	S_3	S_4	S_5	S_6
node ℓ	AD mean AD mean MAD	$\begin{array}{c} 4.07e{-}02\\ 8.10e{-}03\\ 6.01e{-}03\end{array}$	2.75e-03 7.53e-04 5.92e-04	5.02e-01 5.10e-02 2.00e-02	3.73e-02 2.34e-02 2.16e-02	$\begin{array}{c} 4.20e{-}02\\ 6.03e{-}03\\ 3.73e{-}03 \end{array}$	6.22e - 04 2.60e - 04 2.23e - 04
node r	AD mean AD mean MAD	$\begin{array}{c} 4.88e{-02} \\ 8.30e{-03} \\ 6.16e{-03} \end{array}$	$\begin{array}{c} 2.72e{-}03\\ 7.44e{-}04\\ 5.83e{-}04\end{array}$	$5.13e - 01 \\ 4.04e - 02 \\ 2.10e - 02$	$\begin{array}{c} 3.73e{-}02\\ 2.29e{-}02\\ 2.06e{-}02\end{array}$	$\begin{array}{c} 3.78e{-02} \\ 6.12e{-03} \\ 3.81e{-03} \end{array}$	8.36e-04 3.53e-04 3.00e-04

Table 2: Difference of simulated pressure values between the serial pipes and the merged pipe for all six examples $S_1, S_2, S_3, S_4, S_5, S_6$ stated at nodes ℓ and r in [bar].

pipelines in the stationary and transient case. However, when merging serial pipes in the transient case, this property is in general not true, as we have seen in Example 6.6. Nevertheless Corollary 6.5 reveals that "stationary" assumptions enable us to determine such a pipe parameter. While this corollary restricts to horizontal pipes, we show in the remainder of this section that this property also holds for non-horizontal pipelines in the stationary case.

Again, we distinguish between stationary parameters γ^{st} , β^{st} and transient parameters γ^{tr} , β^{tr} , as done for parallel pipe merging in Section 5. The following proposition formalizes the concept of serial merging in the stationary setting. Previous versions for serial merging of horizontal pipes can be found in Lenz and Schwarz (2016) and Groß et al. (2019). Here, we generalize these results by additionally accounting for the difference in height along the pipes.

Proposition 6.7. Given two serial pipes $a = (\ell, r), b = (\ell, r)$ with pipe characteristics $\gamma_a^{st}, \gamma_b^{st} > 0$ and $\beta_a^{st}, \beta_b^{st} \ge 0$ respectively and $b_m = 0$, deg(m) = 2. Then, both pipes can equivalently be replaced by an artificial pipe $c = (\ell, r)$ with

$$\beta_c^{st} = \beta_a^{st} \, \beta_b^{st} \quad and \quad \gamma_c^{st} = \gamma_a^{st} + \beta_a^{st} \, \gamma_b^{st}$$

Proof. From $b_m = 0$ and $\deg(m) = 2$ follows $q_c := q_a = q_b$. For two pipes in serial, each one is represented by a single Equation (30):

$$p_{\ell}^{2} - \beta_{a}^{st} p_{m}^{2} + \gamma_{a}^{st} q_{c} |q_{c}| = 0$$
(56)

$$p_m^2 - \beta_b^{st} p_r^2 + \gamma_b^{st} q_c |q_c| = 0.$$
(57)

Then, it follows

$$\stackrel{(57)}{\Rightarrow} \quad \beta_a^{st} p_m^2 - \beta_a^{st} \beta_b^{st} p_r^2 + \beta_a^{st} \gamma_b q_c |q_c| = 0 \tag{58}$$

$$\stackrel{(56),(58)}{\Rightarrow} p_{\ell}^{2} - \beta_{a}^{st} p_{m}^{2} + \beta_{a}^{st} p_{m}^{2} - \beta_{a}^{st} \beta_{b}^{st} p_{r}^{2} + (\gamma_{a}^{st} + \beta_{a}^{st} \gamma_{b}^{st}) q_{c} |q_{c}| = 0$$

$$\Rightarrow \qquad p_{\ell}^{2} - \beta_{a}^{st} \beta_{b}^{st} p_{r}^{2} + (\gamma_{a}^{st} + \beta_{a}^{st} \gamma_{b}^{st}) q_{c} |q_{c}| = 0$$

$$\Rightarrow \qquad \beta_{c}^{st} = \beta_{a}^{st} \beta_{b}^{st} \qquad \text{and} \qquad \gamma_{c}^{st} = \gamma_{a}^{st} + \beta_{a}^{st} \gamma_{b}^{st}.$$

$$(59)$$

In the stationary case, horizontal serial pipes hold
$$\beta_a^{st} = \beta_b^{st} = 1$$
, see Fügenschuh et al. (2015), then from (59) directly follows

$$\beta_c^{st} = 1 \quad \text{and} \quad \gamma_c^{st} = \gamma_a^{st} + \gamma_b^{st},$$
(60)

which coincides with the formula provided in Corollary 6.5 for transient serial merges under "stationary" conditions.

7 Conclusion

In this paper, we studied the merge of parallel and serial pipelines in transient gas networks. First, we introduced the concept of *equivalent* and *heuristic* subnetwork replacements. Equivalent replacements allow to recover a solution for the original subnetwork from the solution of the aggregated (merged) subnetwork. This is a generic concept and can thus be applied to other aggregation methods than pipe merging.

It turned out that the parallel merge is an equivalent replacement, whereas the serial merge is a heuristic method. Therefore, we evaluated the serial pipe merge in a huge empirical study based on real-world fine-grained state data covering an entire year taken from the German gas network. Besides the equivalent parallel merge, we found that the serial merge only leads to small errors. We thus conclude that the presented serial merge can be considered as appropriate aggregation methods for real-world transient gas flow problems.

Finally, we remark that the used pipe modeling from Section 2 forms the basis of the presented merging methods. Hence, an interesting avenue for future research would be to investigate whether it is possible to derive an appropriate discretization of the Euler system (1) that leads to equivalent replacements for both merging methods simultaneously.

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