



Exploiting Symmetry in Mixed-Integer Nonlinear Programming

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zur Erlangung des Grades
Master of Science

vorgelegt von
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8. Juli 2019

Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und eigenhändig sowie ohne unerlaubte fremde Hilfe und ausschließlich unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

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Zusammenfassung

Das modernste Verfahren zum Lösen von nicht-konvexen gemischt-ganzzahligen nichtlinearen Programmen ist spatial branch-and-bound. Probleme mit einer großen Anzahl an symmetrischen Lösungen können eine Herausforderung für diesen Algorithmus darstellen, da äquivalente Teilbäume erforscht werden ohne tatsächlichen Fortschritt zu machen. Solche Symmetrien zu erkennen und zu handhaben ist ein wichtiger Aspekt einer performanten Implementierung. Abgesehen von wenigen Ausnahmen konzentriert sich die meiste Literatur auf Permutations-Symmetrien in gemischt-ganzzahligen linearen Programmen, häufig mit der Einschränkung auf Probleme mit spezieller Struktur. Im allgemeinen Fall ist die einzige verfügbare Symmetrie Handhabungs-Technik, das Problem mit Hilfe von simplen Symmetrie Brechungs-Ungleichungen zu reformulieren.

Wir erweitern die exisierende Arbeit zu Permutations-basierter Symmetrie, indem wir zeigen, wie stärkere Ungleichungen für bestimmte Untergruppen der Formulierungsgruppe benutzt werden können, wenn sie eine bestimmte Struktur haben. Außerdem entwickeln wir drei dynamische Methoden zur Symmetriebrechung, die jene Ungleichungen während des branch-and-bound Prozesses statt vorher generieren. Diese Verfahren nutzen die LP Lösung und branching Informationen, um zu entscheiden, wie Symmetrie gebrochen wird. Darüber hinaus präsentieren wir einen neuen Algorithmus zur Erkennung von Komplementär-Symmetrie in quadratischen Problemen, was eine andere Art von Symmetrie ist, die bisher nicht für nichtlineare Probleme untersucht wurde. Die entscheidende Idee des Algorithmus ist, ein Hilfsproblem zu konstruieren und zu lösen, das einen Koeffizientenvergleich vor und nach der Komplementierung modelliert.

Eine umfangreiche computerbasierte Studie von öffentlich verfügbaren benchmark-Instanzen untersucht die Wirksamkeit von allen besprochenen Ansätzen und vergleicht sie mit Methoden aus der Literatur. Unsere Ergebnisse zeigen, dass eine Beschleunigung der Rechenzeit um ca. 25% erreicht werden kann, indem die verbesserten Ungleichungen für Untergruppen benutzt werden. Sie weisen auch darauf hin, dass das Hinzufügen von Symmetriebrechungs-Ungleichungen für allgemeine ganzzahlige Variablen ein entscheidender Faktor für das Lösen von gemischt-ganzzahligen Programmen sein kann, während andere etablierte Methoden in diesem Fall entweder nicht anwendbar oder schwer zu implementieren sind.

Abstract

The state-of-the-art method for solving nonconvex mixed-integer nonlinear programs is spatial branch-and-bound. Problems containing a large amount of symmetric solutions can pose a challenge for this algorithm, since equivalent sub-trees are explored without making actual progress. Detecting and Handling such symmetries is an important aspect of a high-performance implementation. With only a few exceptions, most literature focuses on permutation-based symmetries in mixed-integer linear programs, often restricting to problems with special structure. For the more general case, the only available symmetry handling technique is to reformulate the problem by means of simple symmetry breaking inequalities.

We extend the existing work for permutation-based symmetry by showing how stronger inequalities can be used if a subgroup of the formulation group has a certain structure. In addition to this, we develop three dynamic symmetry handling methods that generate those inequalities during the branch-and-bound process instead of beforehand. These techniques utilize the LP solution and branching information to decide how symmetry is broken. Besides that, we present a new algorithm for detecting complementary symmetry in quadratic problems, which is a different type of symmetry that has not been studied for nonlinear problems, yet. The key idea of the algorithm is to construct and solve an auxiliary problem that models a coefficient comparison before, and after complementation.

An exhaustive computational study of publicly available benchmark instances investigates the effectiveness of all discussed approaches and compares them to methods from the literature. Our results show that a speed-up of about 25% on affected instances can be achieved by using the improved inequalities for subgroups. They also indicate that adding symmetry breaking constraints for general integer variables can be a decisive factor for solving mixed-integer nonlinear programs, where other well-proven methods are either not applicable or difficult to implement in practice.

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Chapter 1

Introduction

A variety of mathematical optimization problems appearing in real-world applications contain some kind of symmetry. Prominent examples include packing, partitioning and graph coloring problems. It has been known for a while that this can heavily slow down branch-and-bound based solving processes, see e.g., Jeroslow [20]. The reason is that inherently equivalent sub-trees are explored without providing new information to the solver.

In the context of integer linear programming, this topic has been studied extensively and effective symmetry handling and detection techniques have been developed. Among the most successful ones are “Orbital Fixing” and “Isomorphism Pruning” by Margot [29, 30], “Orbital Branching” by Ostrowski et al. [35, 36] and the polyhedral methods for problems with special structure by Hojny et al. [17]. We also refer to Margot [31] for an overview and to Pfetsch et al. [38] for an extensive computational comparison of the different methods.

All of the techniques mentioned above are restricted to symmetries affecting integer variables. In practice, they are often only applied to binary variables. While this suffices for linear problems, solving mixed-integer non-linear programs (MINLPs) with spatial branch-and-bound also requires to branch on continuous variables. The only available method that is applicable to symmetries on continuous variables is to use simple linear symmetry breaking inequalities. While it was historically the first idea for handling symmetry, it did not prove to be competitive for mixed-integer linear programs (MIPs), see Pfetsch et al. [38].

Liberti [25] studied the approach of adding static symmetry breaking inequalities for general MINLPs. They also generalized the symmetry detection algorithms used for MIPs to MINLPs and performed some computational experiments on a selection of real-world problems. However, those experiments did not distinguish between symmetries on continuous, integer, and binary variables. They also did not compare the method of adding symmetry breaking constraints for binary variables with other techniques from the literature. It remains an open question whether breaking symmetry on continuous variables is actually beneficial for solving MINLPs. In the end, it might well be the case that restricting to integer and/or binary variables and using well-established techniques to handle the respective symmetries leads to better results in practice. Furthermore, no dynamic symmetry breaking methods have been developed for MINLPs so far.

This thesis considers general nonconvex MINLPs and aims to answer some of the open questions mentioned above. To this end, we first extend the existing concepts of symmetry breaking constraints to capture more types of symmetries appearing in practice. A usual approach is to test whether the formulation group of a problem is a symmetric group, in which case inequalities that enforce a total order on the affected variables are added. We show how such inequalities can also be used if only a subgroup is isomorphic to the symmetric group. Consider for example the problem given by

$$\begin{aligned} \max_{x \in \mathbb{R}^6, t \in \mathbb{R}} \quad & t \\ t - \sqrt{(x_1 - x_3)^2 + (x_2 - x_4)^2} & \leq 0 \\ t - \sqrt{(x_1 - x_5)^2 + (x_2 - x_6)^2} & \leq 0 \\ t - \sqrt{(x_3 - x_5)^2 + (x_4 - x_6)^2} & \leq 0 \\ x_1, \dots, x_6 & \in [0, 1] \end{aligned}$$

which models the problem of placing the three points $(x_i, x_{i+1})^T \in \mathbb{R}^2$, $i = 1, 2, 3$, within the unit square, while maximizing the minimum distance between any pair of points. The symmetry group of this problem is generated by the permutations $(1\ 2)(4\ 5)$, $(2\ 3)(5\ 6)$, and $(1\ 4)(2\ 5)(3\ 6)$, which is not the symmetric group. In such situations, it is possible to apply general-purpose symmetry breaking constraints to fix a maximum among the affected variables. However, stronger inequalities can be derived by considering that the subgroup generated by only the first two permutations is isomorphic to the symmetric group on three elements.

In addition to this, we develop dynamic variants which use information obtained during the solving process to decide how symmetry should be broken. Our proposed algorithms are based on the LP solution and branching decisions. Furthermore, we show how another type of symmetry, so called complementary symmetry, can be detected in mixed-integer quadratically constrained programs. This type was introduced by Christophel et al. [7] for binary linear programs, but has not been studied for general MINLPs so far. For an example, note that in any feasible solution of the problem described above, we can obtain another solution of the same value by replacing the values of x_1 , x_3 , and x_5 by $1 - x_1$, $1 - x_3$, and $1 - x_5$, respectively. This kind of symmetry is not captured by permutations.

Finally, we report on results of extensive computational experiments with the MINLPLib [33] that we performed. They include an analysis of the symmetries appearing in the testset as well as a comparison of the different techniques presented in this thesis and selected methods from the literature.

The rest of the thesis is structured as follows. Chapter 2 provides notation and covers basic concepts of mixed-integer nonlinear programming. In Chapter 3, we describe how different kinds of symmetry can be formally defined. Chapter 4 explains the detection of permutation-based symmetries. Our new algorithm for detecting complementary symmetry in quadratic

problems is presented in Chapter 5. Proceeding to symmetry handling, the theory of static techniques is discussed in Chapters 6, including new extensions to symmetry breaking inequalities from the literature. In Chapter 7, we then propose dynamic procedures for generating such inequalities. Finally, we report on the results of our computational experiments in Chapter 8 and give concluding remarks and an outlook in Chapter 9.

Chapter 2

Preliminaries

Throughout the thesis, the set of natural numbers \mathbb{N} includes 0 and n , m , and k are always natural numbers. We denote with $[n] := \{1, \dots, n\}$ the set of natural numbers from 1 to n and with $\mathbb{R}_\infty := \mathbb{R} \cup \{-\infty, \infty\}$ the set of real numbers including infinity. Furthermore, \oplus denotes the *XOR* operator.

2.1 Mixed-Integer Nonlinear Programming

We study optimization problems of the form

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ & g_k(x) \leq 0 \quad k \in [m] \\ & x \in [x^L, x^U] \\ & x_j \in \mathbb{Z} \quad j \in \mathcal{I}, \end{aligned} \tag{2.1}$$

where $c \in \mathbb{R}^n$ denotes the objective function, $g_k : \mathbb{R}^n \rightarrow \mathbb{R}$ are continuous functions, $x^L, x^U \in \mathbb{R}_\infty^n$ the lower and upper bounds of the variables, respectively, and $\mathcal{I} \subseteq [n]$ the index set of integer variables.

Problem (2.1) is called a *mixed-integer nonlinear program* (MINLP). For any MINLP P , let

$$\mathcal{F}_P := \{x \in [x^L, x^U] \mid g_k(x) \leq 0 \ \forall k \in [m], \ x_j \in \mathbb{Z} \ \forall j \in \mathcal{I}\}$$

be the *feasible set* and

$$\mathcal{G}_P := \{x \in \mathcal{F}_P \mid c^T x \leq c^T y \ \forall y \in \mathcal{F}_P\}$$

be the *optimal set* of P . Furthermore, a *relaxation* of P is a problem P' with the same objective function and $\mathcal{F}_P \subseteq \mathcal{F}_{P'}$. Consequently, the optimal value of P' is a lower bound on the optimal value of P (in the case of minimization).

Note that assuming a linear objective function is not a restriction of generality. If there was a nonlinear objective function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, we

could introduce an auxiliary variable t and reformulate the problem as

$$\begin{aligned} \min_{(x,t) \in \mathbb{R}^{n+1}} \quad & t \\ & f(x) - t \leq 0 \\ & g_k(x) \leq 0 \quad k \in [m] \\ & x \in [x^L, x^U] \\ & x_j \in \mathbb{Z} \quad j \in \mathcal{I}. \end{aligned}$$

The class of MINLPs includes some prominent special cases. P is called

- a) *convex*, if all g_k are convex, and *nonconvex* otherwise,
- b) *mixed-integer linear program (MIP)* if all g_k are linear functions, i.e., for all $k \in [m]$ there are $a_k \in \mathbb{R}^n$, and $b_k \in \mathbb{R}$ such that

$$g_k(x) = a_k^T x + b_k \quad \forall x \in \mathbb{R}^n,$$

- c) *mixed-integer quadratically constrained program (MIQCP)* if all g_k are quadratic functions, i.e., for all $k \in [m]$ there are $Q_k \in \mathbb{R}^{n \times n}$, $a_k \in \mathbb{R}^n$, and $b_k \in \mathbb{R}$ such that

$$g_k(x) = x^T Q_k x + a_k^T x + b_k \quad \forall x \in \mathbb{R}^n,$$

- d) *binary linear program (BLP)* if it is a MIP with $[x^L, x^U] = [0, 1]^n$ and $\mathcal{I} = [n]$, and
- e) *linear program (LP)* if it is a MIP with $\mathcal{I} = \emptyset$.

Our focus is on nonconvex MINLPs, since they are the most general and difficult to solve.

2.2 Spatial Branch-and-Bound

One of the widely-used frameworks for solving nonconvex MINLPs is *spatial branch-and-bound* (sB&B), see e.g. Wolsey [47]. The idea of branch-and-bound based algorithms is to build a tree in which each node represents a sub-problem of its parent. The feasible region of each node is equal to the union of the feasible regions of its children. The root of the tree corresponds to the original problem and the step of subdividing a node into (typically two) sub-problems is called *branching*.

In order to benefit from the sub-division, the algorithm tries to find valid lower and upper bounds on the objective values of the sub-problems. Lower bounds are obtained by solving a (typically convex) relaxation at each node. The most widely-used variant uses LP-relaxations, which are formed by dropping integrality conditions and – in the case of MINLPs – over- and underestimating nonlinear constraints. So-called *primal heuristics* try to find feasible solutions from which globally valid upper bounds can be deduced. If a node is proven to be infeasible or – in case of a minimization problem – its lower bound is higher than the best known global

upper bound, it can be *pruned*. The algorithm strongly depends on the efficient computation of such bounds, as without them, it degenerates to an exhaustive search of the solution space.

For integer variables, branching can be performed in the following way. After the LP-relaxation at the current node is solved, any integer variable with fractional value in the LP solution is chosen according to some *branching rule*. Then the variable is rounded once down and once up and the bounds of the variable are adjusted accordingly. For example, if $x \in \mathbb{Z}$ has the LP solution value $x^* = 3.7$, then the sub-problems are formed by setting $x^U = 3$ in one child node, and $x^L = 4$ in the other. Branching on a binary variable results in a fixing to 0 and 1, respectively. If the problem is a MIP, branchings on integer variables are suffice and the procedure always terminates with an optimal solution, see Wolsey [47].

In case of nonconvex MINLPs, however, the situation is different. Assume that enough branchings on the integer variables have been performed such that none of them has a fractional LP solution value at the current node. Then the LP solution is still not guaranteed to be feasible due to the nonconvex constraints. To resolve this infeasibility, branching has to be continued on continuous variables which is called spatial branching. The difference to the formerly described procedure is that bounds cannot be rounded. If the LP solution value of a continuous variable x is $x^* = 3.7$, the sub-problems are formed by setting $x^U = 3.7$ and $x^L = 3.7$, respectively, since we cannot use strict inequalities for practical purposes. Nonetheless, it is possible that spatial branching results in tighter convex under- and over-estimators on the new intervals and eventually leads to an LP solution that is feasible for the original problem. While sB&B is generally not guaranteed to terminate, it solves *bounded* MINLPs to ϵ -optimality, see Horst et al. [18]. Finitely terminating sB&B algorithms have also been developed for certain problems with special structure, see Parker et al. [37] and A. Al-Khayyal et al. [1].

To understand why symmetry can lead to difficulties in the sB&B process, consider a problem of the form

$$\begin{aligned} \min_{x \in \mathbb{R}^4} \quad & \alpha(x_1 + x_2) + \beta(x_3 + x_4) \\ & g_1(x_1, x_2) + g_1(x_2, x_1) \leq 0 \\ & g_2(x_3, x_4) + g_2(x_4, x_3) \leq 0 \\ & x_1, x_2 \in \{0, 1\} \\ & x_3, x_4 \in [0, 1], \end{aligned} \tag{2.2}$$

where $\alpha, \beta \in \mathbb{R}$ and $g_1, g_2 : \mathbb{R}^2 \rightarrow \mathbb{R}$. There is a symmetry between the binary variables x_1 and x_2 in the following sense. For any feasible solution, swapping the values of x_1 and x_2 leads to another feasible solution with the same objective value. The same holds for the continuous variables x_3 and x_4 . Figure 2.1 shows a possible sB&B tree for (2.2). The LP solution at each respective node is denoted by x^* . In the root node, we have $x_1^* = x_2^* = 0.5$ and x_1 is chosen as branching variable. So x_1 is fixed to 0 and 1, respectively. The LP solution of the sub-problems P_1 and P_2

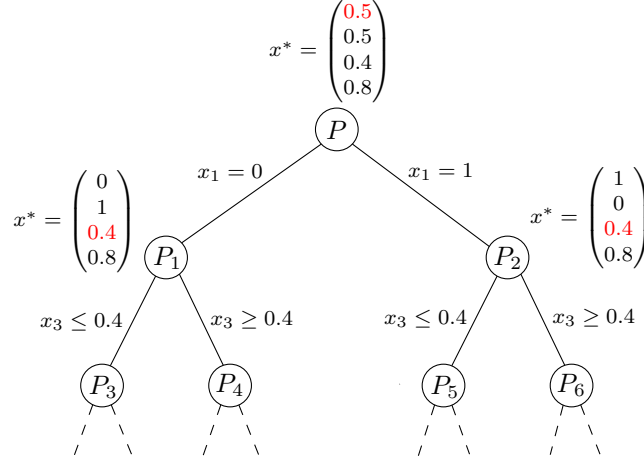


FIGURE 2.1: An example sB&B tree for (2.2). Depicted are the LP solutions x^* at each node and the branching decisions next to each edge. The first branching is performed on the binary variable x_1 . It is fixed to 0 and 1, respectively. Afterwards, no integral variables with fractional LP solution values remain. The process continues with spatial branchings to resolve infeasibilities due to nonconvex constraints. The two sub-trees rooted at P_1 and P_2 are equivalent up to permutations of the symmetric variables x_1 and x_2 .

now has $x_2^* = 1 - x_1^*$ (which could for example happen due to a constraint $x_1 + x_2 = 1$). Consequently, no further branchings on the binary variables are necessary. However, the LP solutions are still infeasible due to some nonconvexity and spatial branching is performed on x_3 . Since x_1 and x_2 are symmetric and the rest of the problem is independent of these variables, the whole sub-trees rooted at P_1 and P_2 are symmetric. Any optimal solution of P_1 corresponds to an optimal solution of P_2 which can be obtained by permuting x_1 and x_2 . Furthermore, there could be more symmetric sub-problems *within* each sub-tree, since x_3 and x_4 are also symmetric. Without symmetry handling, it is possible that all of these equivalent nodes are explored and solved, while it would suffice to consider only one of them.

Most symmetry breaking techniques only apply to symmetries on binary variables. Some extensions to integer variables are known, but did not prove to be successful in practice, see Pfetsch et al. [38]. Continuous variables are usually ignored when handling symmetries in MIPs, since they are not branched on and are therefore not considered harmful to the B&B process. For MINLPs, however, they can be relevant due to spatial branchings.

Chapter 3

Symmetry in MINLPs

In Chapter 2 we showed how the sB&B tree of a symmetric problem can contain equivalent optimal solutions. Ideally, we would like to partition the optimal set such that each class contains exactly those optima that can be mapped onto each other by *permuting* the variable indices. This partition could then be used to make sure that only one representative of each class is considered during sB&B. In the following, we introduce the basic concepts of group theory that we need to formalize this notion of symmetry. For more detailed information about groups see, e.g., Rotman [42] .

3.1 Group Theory

Throughout the thesis, we consider finite *permutation groups*, i.e., bijections from the set $[n]$ (for some $n \in \mathbb{N}$) to itself equipped with composition as group operation. The group of all such permutations for a given n is called the *symmetric group* of $[n]$ and is denoted by \mathcal{S}_n . Every permutation group G is a subgroup of \mathcal{S}_n for which we write $G \leq \mathcal{S}_n$.

Intuitively, we assume that the groups appearing in this thesis permute the set of variables of problem (2.1). In the language of group theory, we are considering two different actions of such a permutation group, though. In the first case, it acts on the set of variable indices $[n]$ by simply applying the bijection, i.e., $(\pi, i) \mapsto \pi(i)$, $\pi \in G$. In the second case, it acts on \mathbb{R}^n by permuting the entries of a vector x , i.e., $(\pi, x) \mapsto \pi(x) := (x_{\pi^{-1}(1)}, \dots, x_{\pi^{-1}(n)})$.

For a group G acting on a set X , the *orbit* of $x \in X$ refers to the set $orb(x, G) := \{\pi(x) \mid \pi \in G\}$. For example, if a permutation group G acts on $[n]$ as described above, the orbit $orb(i, G)$ of some $i \in [n]$ consists of all variable indices that i can be moved to by any permutation in G . In the case of the action on \mathbb{R}^n , the orbit $orb(x, G)$ of $x \in \mathbb{R}^n$ contains all vectors which can be the result of permuting the entries of x by an element of G .

Remark 3.1. In the literature, it is often not distinguished between the action on $[n]$ and the action on \mathbb{R}^n , since there is an intuitive connection between them. It is usually clear which action is meant, depending on whether the orbits contain variable indices or vectors. Therefore, we will only specify the considered action when necessary. Furthermore, whenever we speak of the orbits of a group action, we actually mean the *non-trivial* orbits, i.e., orbits containing more than one element.

Any permutation can be decomposed into disjoint cycles and we use standard *cyclic notation* to represent permutations. For example, $\pi \in \mathcal{S}_6$ defined by $\pi(1) = 2$, $\pi(2) = 5$, $\pi(3) = 3$, $\pi(4) = 6$, $\pi(5) = 2$ and $\pi(6) = 4$ is denoted by $\pi = (1\ 2\ 5)(4\ 6)$. Note that fixed points are left out.

For $I \subseteq [n]$, the groups

$$\begin{aligned} \text{stab}(G, I) &:= \{\pi \in G \mid \pi(I) = I\} \\ G^I &:= \{\pi \in G \mid \pi(j) = j \ \forall j \in I\} \end{aligned}$$

denote the *setwise* and *pointwise stabilizer* of I in G , respectively. We write $G = \langle \pi_1, \dots, \pi_k \rangle$ for the group G that is generated by the permutations π_1, \dots, π_k . Furthermore, for any $\pi \in G$ and an orbit ω of G , we denote by $\pi|_\omega$ the permutation defined by

$$\pi|_\omega(i) = \begin{cases} \pi(i) & i \in \omega, \\ i & i \notin \omega. \end{cases}$$

In some sense, $\pi|_\omega$ can be seen as the projection of π onto ω . Similarly, $G|_\omega := \{\pi|_\omega : \pi \in G\}$ which is also a permutation group. This follows from the fact that the orbits form a disjoint partition of $[n]$ and the cycles of each permutation in G lie completely in one of the orbits.

Certain kinds of actions of symmetric groups are of particular interest to us. Let $G \leq \mathcal{S}_n$ be a permutation group such that $G \cong \mathcal{S}_m$ for some $m \leq n$. If $m = n$, we call the action of G on $[n]$ a *coordinate action*. Otherwise, assume that there is a matrix A with m columns, such that A contains each number in $[n]$ exactly once and each $\pi \in G$ permutes the columns of A by acting on its entries. Then we call it a *matrix action*, denoted by $\mathcal{M}(\mathcal{S}_m, n)$. Such a matrix can only exist if $n = km$ for some $k \geq 2$. In all other cases, there is no obvious interpretation of the action of G . As we will see in Chapter 8, coordinate and matrix actions seem to appear frequently in practice.

Example 3.2. The groups

- $G_1 = \langle (1\ 2), (2\ 3), (3\ 4) \rangle \leq \mathcal{S}_4$,
- $G_2 = \langle (1\ 2)(5\ 6)(9\ 10), (2\ 3)(6\ 7)(10\ 11), (3\ 4)(7\ 8)(11\ 12) \rangle \leq \mathcal{S}_{12}$

are both isomorphic to \mathcal{S}_4 . G_1 acts as a coordinate action and G_2 as the matrix action $\mathcal{M}(\mathcal{S}_4, 12)$ permuting the columns of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}.$$

A group $G \leq \mathcal{S}_n$ can also be the direct product of different permutation groups, i.e., there are $H_1, \dots, H_k \leq \mathcal{S}_n$ such that $G = H_1 \times \dots \times H_k$, in which case H_1, \dots, H_k are called the *factors* of G .

3.2 Permutation-based Symmetries

Given a problem P of the form (2.1), we would, ideally, like to know the group of all permutations of variable indices mapping optimal solutions to optimal solutions by acting on \mathbb{R}^n , as defined by Liberti [25].

Definition 3.3. The *solution group* G_P^* of P is defined by

$$G_P^* := \text{stab}(\mathcal{G}_P, \mathcal{S}_n).$$

Knowing G_P^* would allow us to compute the orbits of all optimal points inducing a partition of \mathcal{G}_P . Then we could try to make sure that only one representative of each orbit is considered during sB&B.

However, computing the solution group would require *a-priori* knowledge about the optimal set \mathcal{G}_P which is generally not available. Instead, the usual approach is to use a (possibly large) subgroup of G_P^* that can be computed efficiently. One candidate is the following group, as defined by Margot [31].

Definition 3.4. The *symmetry group* \hat{G}_P of P is the group of all permutations of $[n]$ that, by acting on \mathbb{R}^n , maps feasible points to feasible points and preserves the objective value, i.e.,

$$\hat{G}_P := \{ \pi \in \mathcal{S}_n \mid c^T x = c^T \pi(x) \text{ and } \pi(x) \in \mathcal{F}_P \ \forall x \in \mathcal{F}_P \}.$$

It follows directly from the definition that \hat{G}_P is a subgroup of G_P^* . An example shows that, in general, $\hat{G}_P \neq G_P^*$.

Example 3.5. Let P be defined by

$$\begin{aligned} \min_{(x,y) \in \mathbb{R}^2} \quad & (x - y)^2 \\ & x \geq 1 \\ & y \geq 0. \end{aligned} \tag{3.1}$$

Then the optimal value is 0, so for every optimal solution, $x = y$ holds and we have $G_P^* = \mathcal{S}_2$. However, permutation $\pi = (1 \ 2)$ maps the feasible point $(1, 0)$ to the infeasible point $(0, 1)$ and thus $\pi \notin \hat{G}_P$. Since P has only 2 variables, \hat{G}_P consists of just the identity permutation.

Unfortunately, the situation does not become easier when considering the symmetry group since it is implicitly defined in terms of the feasible region. We cite the proof of the following theorem by Margot [31] for convenience of the reader.

Theorem 3.6. *Computing \hat{G}_P for MIP problems P is NP-hard.*

Proof. Consider an arbitrary MIP P over n variables. If $\mathcal{F}_P = \emptyset$, then $\hat{G}_P = \mathcal{S}_n$ according to definition 3.4. Now we add two variables y_1 and y_2 and the constraint $y_1 + y_2 = 1$ to P and call the modified problem P' . If the symmetry group of P' is \mathcal{S}_{n+2} , then P is infeasible, since y_1 and y_2 do

not appear in P . On the other hand, if P is infeasible, then P' is infeasible, too. Therefore we get

$$\mathcal{F}_{P'} = \mathcal{S}_{n+2} \iff \mathcal{F}_P = \emptyset.$$

However, it is widely known that deciding feasibility of MIPs is NP-hard. \square

Remark 3.7. Theorem 3.6 obviously holds for MINLPs since MIPs are a subclass of MINLPs.

Paving the way towards a procedure for computing symmetry algorithmically, we relax the considered group once again by considering *formulation* symmetry rather than *problem* symmetry.

Definition 3.8. The group \bar{G}_P that “fixes the formulation” of P is defined by

$$\begin{aligned} \bar{G}_P := \{ \pi \in \mathcal{S}_n \mid & c = \pi(c) \wedge \pi(j) \in \mathcal{I} \ \forall j \in \mathcal{I} \\ & \wedge x^L = \pi(x^L) \wedge x^U = \pi(x^U) \\ & \wedge \exists \sigma \in \mathcal{S}_m : g_{\sigma(k)}(\pi(x)) = g_k(x) \ \forall k \in [m], \forall x \in \mathcal{F}_P \}. \end{aligned}$$

In contrast to the previous groups, \bar{G}_P depends on the specific formulation of the problem. The step from problem to formulation symmetry is necessary if we want to develop an automated procedure to compute a subgroup of G_P^* .

Again, $\bar{G}_P \leq \hat{G}_P$ follows directly from the definition and we will use an example to show that $\bar{G}_P \neq \hat{G}_P$, in general.

Example 3.9. Let P be defined by

$$\begin{aligned} \min_{(x,y) \in \mathbb{R}^2} \quad & x + y \\ & x + y \geq 1 \\ & 2x + y \leq 2 \\ & x, y \in \{0, 1\}. \end{aligned}$$

The only feasible points of P are $(1, 0)$ and $(0, 1)$, so $\hat{G}_P = \mathcal{S}_2$. However, the term $2x + y$ is different for the two feasible points and therefore $\pi = \begin{pmatrix} 1 & 2 \end{pmatrix} \notin \bar{G}_P$.

If the problem is nonlinear, another relaxation step is necessary. In practice, the functions g_k are represented by mathematical expressions. As Liberti [25] pointed out, algorithmically deciding whether two such expressions are equal can be reduced to the problem of deciding whether a nonlinear equation has a solution, which is undecidable, see Zhu [48]. Consider the example of the mathematically equivalent expressions $\sin(x)$ and $\cos(x + \pi/2)$ to get an intuition for this fact.

Now assume that we have an oracle `equal`(f, g) which checks whether the representations of two functions $f, g : \mathbb{R}^n \rightarrow \mathbb{R}$ are equal such that we

have

$$\text{equal}(f, g) = \text{true} \implies \begin{cases} \text{dom}(f) = \text{dom}(g) \\ f(x) = g(x) \quad \forall x \in \text{dom}(f). \end{cases}$$

The important thing to note here is that $\text{equal}(f, g) = \text{false}$ does not imply $f \neq g$. The oracle only guarantees correctness if its result is positive. Writing $f \equiv g$ whenever $\text{equal}(f, g) = \text{true}$, we now replace the equality in Definition 3.8 by that notion of congruence.

Definition 3.10. The *formulation group* G_P of P is defined by

$$\begin{aligned} G_P := \{ \pi \in \mathcal{S}_n \mid & c = \pi(c), \wedge \pi(j) \in \mathcal{I} \quad \forall j \in \mathcal{I} \\ & \wedge x^L = \pi(x^L) \wedge x^U = \pi(x^U) \\ & \wedge \exists \sigma \in \mathcal{S}_m : g_{\sigma(k)} \circ \pi \equiv g_k \quad \forall k \in [m] \}. \end{aligned}$$

As the congruence induced by the oracle is weaker than the equality in Definition 3.8, G_P is a subgroup of \bar{G}_P . The size of the formulation group depends on the oracle that is used. In Section 4.2 we will see how such an oracle can look like in practice.

Remark 3.11. In the literature for MIPs, \bar{G}_P is usually called the formulation group, see e.g., Margot [31]. The reason is that for linear problems, \bar{G}_P is equal to the symmetry group of the LP-relaxation, as Liberti [25] showed. In that case, the last relaxation step is not necessary, since comparing the defining matrix and right-hand side vector of the MIP (i.e., the formulation) suffices to verify the conditions of \bar{G}_P . Whenever we consider a MIP and its formulation group we implicitly assume that the oracle just check if the constraint matrix and right-hand side vector are equal.

3.3 Complement-based Symmetries

While symmetries based on variable permutations are the most studied and cover the vast majority of practically relevant cases, there are other kinds of symmetries which are not captured by the formulation group. One of those is *complementary symmetry*. As a motivating example, we consider again the problem from Chapter 1.

Example 3.12. Let P be defined by

$$\begin{aligned} & \max_{x, y \in \mathbb{R}^3, t \in \mathbb{R}} t \\ & t - \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \leq 0 \\ & t - \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \leq 0 \\ & t - \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} \leq 0 \\ & x_1, x_2, x_3, y_1, y_2, y_3 \in [0, 1] \end{aligned} \tag{3.2}$$

which models the problem of placing three points within the unit square such that the minimum distance between any pair of points is maximized. Variables x_i and y_i represent the x - and y -coordinate of the i -th point. The constraints enforce that the objective value t is no larger than the Euclidean distance between each pair of points.

There are some obvious permutation-based symmetries in this problem. First of all, we can swap the coordinates of two points simultaneously, leading to a solution of the same value that corresponds to the same geometric constellation. Furthermore, swapping the x -coordinate of each point with its respective y -coordinate also leads to a new feasible solution with the same objective value. Geometrically, this corresponds to mirroring the solution along the line $x - y = 0$. Assuming the variables are ordered as $(x_1, x_2, x_3, y_1, y_2, y_3, t)$, the formulation group is given by

$$G_P = \langle \pi_1, \pi_2, \pi_3 \rangle \quad \text{with}$$

$$\pi_1 = (1\ 2)(4\ 5), \quad \pi_2 = (2\ 3)(5\ 6), \quad \pi_3 = (1\ 4)(2\ 5)(3\ 6).$$

Here, π_1 and π_2 each swap two points with each other, while π_3 permutes the respective coordinates of all points. Together they generate the whole group. Note that we assume that the oracle detects $(x_1 - x_2)^2 = (x_2 - x_1)^2$.

Now we consider the set of optimal solutions \mathcal{G}_P . Figure 3.1 shows the four different geometric constellations that appear in the optimal set. In each of these sets, the points can be permuted arbitrarily, leading to six distinct solutions. In total, \mathcal{G}_P contains 24 points. Our goal is to consider only one representative in each orbit under the action of G_P on \mathcal{G}_P .

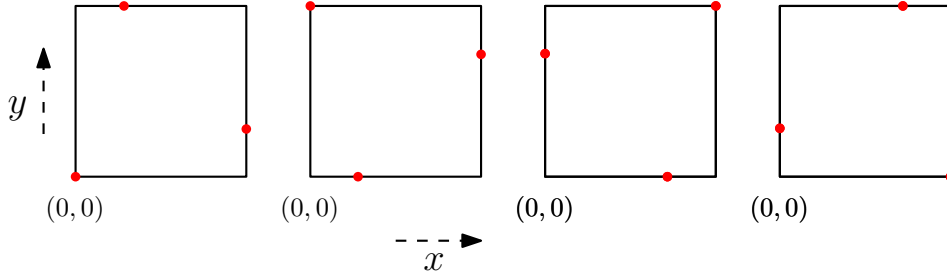


FIGURE 3.1: The four geometric constellations of optimal solutions in (3.2).

For the sake of simplicity, let us consider only the subgroup of G_P that is generated by π_1 and π_2 for now, so only permutations that swap the points with each other. This would leave us with four solutions corresponding exactly to the four geometric constellations in Figure 3.1, since each orbit contains all solutions which belong to the same constellation. However, geometrically speaking, even those four solutions are equivalent, in the sense that they are equal up to reflections. To be more precise, the solutions can be transformed into each other by replacing either all x_i with $(1 - x_i)$, all y_i with $(1 - y_i)$, or both at the same time. Now since π_3 corresponds to mirroring along $x - y = 0$, it maps the second and fourth solution in Figure 3.1 onto each other. This means that the corresponding orbits are

merged. Nonetheless, three distinct equivalence classes of optimal solutions remain, while ideally we would like to have only one.

Having seen that there can be symmetries which do not involve permuting variables, the question arises how such symmetries can be described without using geometric interpretations. In Example 3.12, variable x_i was replaced by its complement $(1 - x_i)$. Depending on the bounds of the variable, we might have to replace it by some other value, though. The following definition generalizes this notion of complementation.

Definition 3.13. Let P be a MINLP, $I \subseteq [n]$ and $d \in \mathbb{R}^n$ be given by

$$d_i = \begin{cases} x_i^U + x_i^L & \text{if } x_i^L, x_i^U \neq \pm\infty \\ 0 & \text{otherwise.} \end{cases}$$

The *complement point* $\bar{x}^{(I)}$ of $x \in \mathbb{R}^n$ w.r.t. I is defined by

$$\bar{x}_i^{(I)} := \begin{cases} d_i - x_i & i \in I, \\ x_i & i \notin I. \end{cases}$$

We say that P contains a *complementary symmetry along I* if

$$\forall x \in \mathcal{F}_P : \quad \bar{x}^{(I)} \in \mathcal{F}_P \quad \text{and} \quad c^T \bar{x}^{(I)} = c^T x.$$

Geometrically, the complement point is obtained by mirroring for each $i \in I$ the original point on the axis $x_i = d_i/2$, which is the midpoint of the bounding interval of variable x_i . The definition is therefore independent of the position of the feasible region in \mathbb{R}^n . For BLPs, this kind of symmetry was studied by Christophel et al. [7]. They brought the concepts of permutation-based and complement-based symmetry together by means of *signed permutations*. However, as we will see in Chapter 5, their approach is not applicable to nonlinear problems.

Naturally, there are even further types of symmetries based on the geometric properties of the feasible region. For example, if we replace the unit box in Example 3.12 by the unit circle, then each feasible solution can be transferred into an equivalent one by arbitrary rotations around the center. Such symmetries are still not captured by the above concept of complementariness.

Chapter 4

Classical Symmetry Detection

Computing the formulation group of a MINLP is usually reduced to computing the *automorphism group* of vertex-colored graphs, see Margot [31]. In Section 4.1, we explain how the reduction works for MIPs. While Salvagnin [43] seems to be the first one to describe the procedure, it has been rediscovered by other authors, such as Berthold et al. [6]. Section 4.2 introduces the concept of expression graphs which are needed to extend the graph automorphism idea to MINLPs. In Section 4.3, we will explain this generalization which is due to Liberti [25].

4.1 Reduction to Graph Automorphisms

A *colored graph* is a graph (V, E) together with two functions $r : V \rightarrow \mathbb{N}$ and $s : E \rightarrow \mathbb{N}$ describing the vertex and edge colors, respectively. An automorphism π of a colored graph is a bijection on the set of its vertices V such that, for any two vertices $v, w \in V$, $(\pi(v), \pi(w)) \in E$ if and only if $(v, w) \in E$, while vertex and edge colors are preserved. We will construct a colored bipartite graph such that each of its automorphisms corresponds to a permutation in the formulation group of the problem.

Let $P := \min\{c^T x \mid Ax \leq b, x_i \in \mathbb{Z} \forall i \in \mathcal{I}\}$ be a MIP. The two parts that the vertex set of our bipartite graph consists of are $V' = \{v_1, \dots, v_n\}$ and $V'' = \{w_1, \dots, w_m\}$, i.e., $V = V' \dot{\cup} V''$. The set V' contains a vertex for each variable and the set V'' a vertex for each linear constraint of P . Vertices in V' are colored according to objective coefficient, bounds, and type (i.e., integral or continuous) of the respective variable. This means that two vertices have the same color if and only if all of those values are the same. Similarly, vertices in V'' share a color if the right-hand side of their corresponding constraints are equal. Each color is only used for either constraint or variable vertices. Furthermore, two vertices $v_j \in V'$ and $w_i \in V''$ are connected by an edge if $A_{ij} \neq 0$ and the edge is then colored according to coefficient A_{ij} .

It is not hard to see that each automorphism of this graph corresponds to a permutation in G_P . Since there is a one-to-one correspondence of variables in P and vertices in V' , projecting the automorphism onto V' directly gives the element in G_P . Note that in the presence of redundant constraints, it is possible for two graph automorphisms to yield the same permutation. For example, automorphisms that fix each variable vertex

and only permute the constraint vertices all correspond to the identity permutation on $[n]$. On the other hand, each permutation in G_P can be extended to an automorphism of the graph. Consequently, we obtain the whole formulation group. The following example illustrates the idea.

Example 4.1. Let P be defined by

$$\begin{aligned} \min_{x \in \mathbb{R}^4} \quad & x_1 + x_2 + x_3 - x_4 \\ & x_1 + x_2 + 2x_3 + x_4 \leq 15 \\ & x_1 + x_2 \geq 3 \\ & 2x_3 - x_4 \geq 3 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned} \tag{4.1}$$

It consists of four non-negative variables and three linear constraints. There are no integral variables, i.e., $\mathcal{I} = \emptyset$. Figure 4.1 shows the corresponding graph with edge colors. It has one vertex for each constraint, numbered according to their order in (4.1), and one for each variable. Since the second and third constraint have the same right-hand side, they share a color. We need two different colors for the variable vertices, since x_1, x_2 and x_3 have an objective coefficient of 1 and a lower bound of 0. x_4 has the same bound but a different objective coefficient and therefore gets a different color. Furthermore, there appear four different non-zero coefficients in the constraints, so we need four edge colors. The only non-trivial automorphism of the graph swaps x_1 with x_2 and fixes all remaining vertices, so $G_P = \langle (1\ 2) \rangle \leq \mathcal{S}_4$.

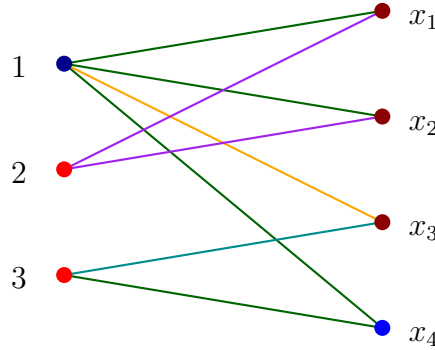


FIGURE 4.1: The colored graph corresponding to (4.1). The vertices on the left-hand side correspond to constraints. They are colored according to their right-hand side value, while the colors of variable vertices represent their bounds and the edge colors their corresponding coefficients.

It should be noted that the computational complexity of the graph automorphism problem is still unknown, i.e., no polynomial algorithm for the problem is known, but it has also not been shown to be NP-hard. For practical purposes, there are several software tools for computing graph automorphisms that work effectively in practice, e.g., **bliss** [21], **nauty** [32], and **saucy** [11]. However, they can only handle vertex colors. If the problem is a BLP, there are only two possible coefficients, so we do not need

any edge colors. For general MIPs, we have to transform the graph such that only vertices are colored, but automorphisms can still be translated to elements of the formulation group.

One way to achieve this, first presented by Salvagnin [43], is to replace each edge (v, w) by an intermediate vertex u and two edges (v, u) , (u, w) , and then giving u the color of the edge (v, w) . Using this transformation, no edge colors are needed anymore and the information of the constraint coefficients is encoded by the intermediate vertices. Note that this only works since the original graph is bipartite. In addition to this, one can use an idea by Puget [39] to reduce the number of intermediate vertices. If a variable appears with the same coefficient in different constraints, only one intermediate vertex is needed for this combination of variable and coefficient. In other words, we do not need one intermediate vertex for each non-zero entry of the matrix A , but only one per distinct value in each of its columns.

Example 4.1 (continued). Figure 4.2 shows the transformed graph for Example 4.1. For variable x_4 , only one intermediate vertex has to be inserted, since it appears with a coefficient of 1 in both the first and third constraint.

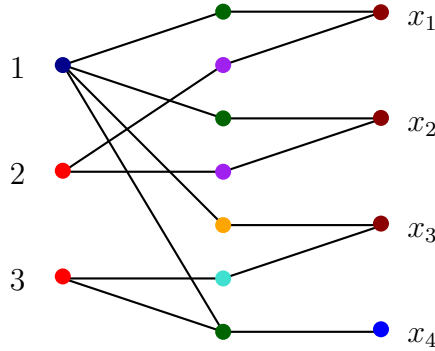


FIGURE 4.2: The transformed graph without edge colors for (4.1). The intermediate vertices in the middle are colored according to the respective coefficients.

Remark 4.2. The procedure to reduce the number of vertices described above is called *grouping by variables*. Alternatively, it is also possible to perform *grouping by constraints*, see Pfetsch et al. [38], because the original graph is bipartite. This means that one color for each unique coefficient within a constraint is used. Whether one of the two methods is better than the other depends on the problem.

Another fundamentally different method to remove edge colors can be found in the manual of `nauty` (version 2.4) [32]. We will not discuss this procedure here, since experiments by Pfetsch et al. [38] indicate that, on average, it is outperformed by the method described above.

4.2 Expression Graphs

In state-of-the-art sB&B based MINLP solvers, nonlinear functions are often encoded by means of directed acyclic graphs. These graphs will prove helpful for the detection of symmetries in MINLPs. To this end, we consider the class of factorable functions.

Definition 4.3. Given a set of univariate functions \mathcal{L} , the set of *factorable functions* is the smallest set that contains \mathcal{L} and that is closed under addition, multiplication and composition.

We consider the set \mathcal{L} that consists of the univariate functions \exp , \log , \sin , \cos , $(\cdot)^k$, and $|\cdot|$ as well as constant functions and the identity. So the class contains all functions that are given as a recursive sum and product of those operations. For example,

$$f(x, y) = |3\exp(x^3) - y + \log(\sin^2(y)\sqrt{x}) - 4.5|$$

is a factorable function.

An *expression graph* is a directed acyclic graph (DAG) $T = (V, A)$ such that for all $v \in V$,

$$\begin{aligned} \delta^+(v) = 0 & \iff v \text{ is a variable or constant,} \\ \delta^+(v) = 1 & \iff v \text{ is a univariate function in } \mathcal{L}, \\ \delta^+(v) > 1 & \iff v \text{ is a sum or a product,} \end{aligned}$$

where $\delta^+(v)$ denotes the *outdegree* of v . We can recursively define an expression graph $T_f = (V_f, A_f)$ for any factorable function f as follows: If $f \in \mathcal{L}$ and is not the identity or constant, then $V_f = \{x_f, s_f\}$ and $A_f = \{(s_f, x_f)\}$, where x_f is the variable and s_f the operator symbol of f . The identity and constant function are represented by a single vertex that is the respective variable or constant. If f is the sum or product of factorable functions g_1, \dots, g_k , then

$$V_f = \{s_f\} \cup \bigcup_{i=1}^k V_{g_i} \quad \text{and} \quad A_f = \bigcup_{i=1}^k \left(A_{g_i} \cup \{(s_f, r_i)\} \right),$$

where r_i is the root of T_{g_i} , i.e., the unique vertex v with $\delta^-(v) = 0$, and s_f is '+' or '*', respectively. Similarly, compositions are constructed by connecting a vertex for the operator symbol with the expression graph corresponding to the argument function. Figure 4.3 shows an example of an expression DAG.

While mathematically, the above definition of expression graphs is precise, in an implementation it is not sufficient to construct the DAGs in the described way. The issue is that two different expressions can represent the same factorable function. For example, the expressions $3(x + y)$ and $3x + 3y$ describe the same function but lead to different expression graphs. Thus, the representation of a MINLP depends on its formulation. It is possible, however, to reduce the number of DAGs corresponding to

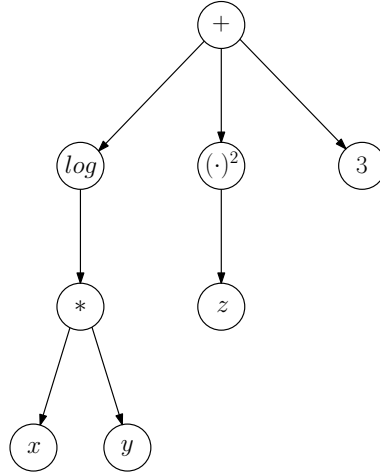


FIGURE 4.3: An expression DAG for the factorable function $f(x, y, z) = \log(xy) + z^2 + 3$. Each leaf represents either a variable or a constant. All other vertices correspond to operators.

an expression by defining a set of simplification rules, see Cohen [8]. This can be combined with scaling rules ($g(x) \leq 0$ and $2g(x) \leq 0$ should be the same constraints) and a fixed order on the arguments, e.g., constants first, followed by variables and operators sorted lexicographically by their names, see Liberti et al. [27]. While these rules reduce the number of possible DAGs significantly, there are still functions that are equal but lead to different expression graphs, e.g., $\sin(x)$ and $\cos(x + \pi/2)$.

It is also possible to merge vertices corresponding to the same variable and even identify whole common sub-expressions. For example, the function

$$f(x, y, z) = x + xy + |xy| + 2yz \quad (4.2)$$

can be represented by the expression graph depicted in Figure 4.4.

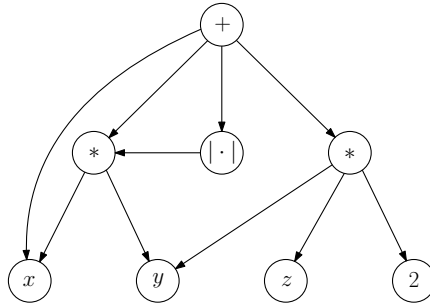


FIGURE 4.4: An expression DAG for (4.2) using common sub-expressions. The multiplication vertex on the left hand side has two incoming edges, since the product appears twice in the expression. Furthermore, there is only one shared vertex for each variable.

Canonicalizing the representation as much as possible can simplify and speed up many different components of MINLP solvers. For symmetry

detection, it is of particular importance, since we will compare expression graphs in order to decide whether they represent the same function.

4.3 Extension to MINLPs

In the case of general MINLPs, the formulation group can be computed in a similar fashion as in Section 4.1. The only ingredient we need is an oracle `equal` as described in Section 3.2. To this end, the expression graphs defined in the previous section can be used. In fact, we can simply compare two expression graphs and use this as an oracle to decide whether two functions are equal. The overall procedure was described by Liberti [25]. We connect the expression graphs of each constraint to one combined DAG, using the fact that the constraints operate on the same set of variables. Then we compute the automorphism group of this graph and project it onto the variable vertices. This will give us the formulation group with respect to the oracle that compares expression graphs.

Let P be of the form (2.1) and for each $k \in [m]$, let $T_k = (V_k, A_k)$ be an expression graph representing g_k . Then the oracle described above is given by

$$\text{equal}_{DAG}(g_i, g_j) = \text{true} \iff T_i = T_j.$$

The DAG $D_P = (V_P, A_P)$ is built by identifying vertices in $\bigcup_{k=1}^m V_k$ that represent the same variable and then adding each edge in $\bigcup_{k=1}^m A_k$. As in the case of MIPs, all variable nodes are colored according to their bounds, objective coefficient and integrality type. Additionally, we have vertices for constants and operators. Two vertices corresponding to constants get the same color if their values are equal. The color of operator vertices depends on their name (operator type).

Liberti [25] showed that the automorphism group of D_P , colored as described above, can be projected onto the set of variables (by restricting the permutations to variable vertices) to obtain G_P with respect to `equal`_{DAG}. It was also shown by Liberti [25] that for expression graphs corresponding to linear functions, the oracle `equal`_{DAG} is equivalent to equality of the coefficients if the graphs are built in a normalized way (e.g., sorting all children by their names). In other words, for MIPs the method based on expression graphs is consistent with the method described in the previous section.

In practice, linear constraints are usually not stored as DAGs. However, the two methods would create the same colored sub-graph for the linear part of the problem and all constraints use the same set of variables. Therefore, one can create the linear part first and then add the nonlinear part corresponding to the expression graphs while using the already created variable vertices. The only thing that has to be taken care of is that, in contrast to the linear case, we build a directed graph for MINLPs. Fortunately, there is a simple way to transform the DAG into an undirected graph. We add one “root” vertex for each constraint and connect it to the previous root operator vertex. Then we move constants to the right-hand

side and color the new vertices accordingly, similar to the previous section. The following example visualizes the whole construction.

Example 4.4. Let P be defined by

$$\begin{aligned} \min_{x \in \mathbb{R}^3} \quad & -x_1 - x_2 - x_3 \\ & -x_1 - x_2 \leq 2 \\ & \sqrt{x_1^2 + x_2^2 + x_3^2} \leq 5. \end{aligned} \tag{4.3}$$

It contains three unbounded variables, one linear, and one nonlinear constraint. Figure 4.5 depicts the corresponding undirected graph. The green vertices represent the coefficients in the first constraint, while the blue intermediate vertices are the square terms. Since the only non-trivial automorphism swaps x_1 and x_2 , as well as the respective intermediate vertices, we have $G_P = \langle (1\ 2) \rangle \leq \mathcal{S}_3$.

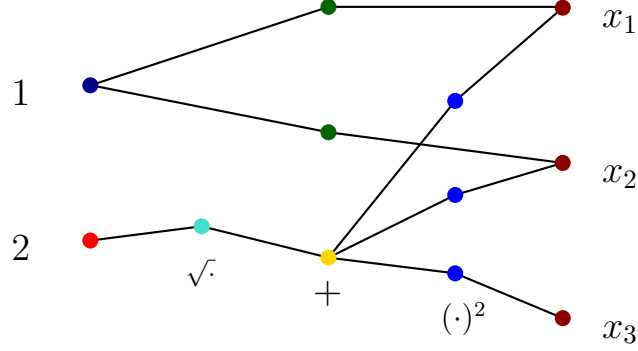


FIGURE 4.5: The undirected vertex-colored graph for (4.3). The vertices on the left-hand side correspond to the two constraints and the ones on the right-hand side to the three variables. Among the intermediate vertices in the middle, the green ones are the coefficients in the linear constraint, while the others represent all operators appearing in the nonlinear constraint.

Chapter 5

Detecting Complementary Symmetry in MIQCPs

Graph isomorphisms are useful for finding permutation based symmetries. As we have seen in Section 3.3, however, there exist other types of symmetries. Christophel et al. [7] showed how the problem formulation of BLPs can be extended to detect combinations of permutation and complement based symmetries, described by signed permutations where a negative index corresponds to the complement of the respective variable. The group of these generalized permutations can be obtained by using the graph automorphism method for the extended problem. Unfortunately, their construction relies on linearity of the constraints and cannot be transferred to MINLPs.

Though it is not clear how complementary symmetries can be detected for general nonlinear problems, in the following we will show how it can be done for quadratic problems. To this end, we consider MIQCPs as defined in Section 2.1. For ease of notation, we will use Q^k , a^k and b^k instead of Q_k , a_k , and b_k for the constraint defining matrices, vectors, and constants, i.e., the k -th constraint is given by

$$x^T Q^k x + a^{kT} x + b^k \leq 0.$$

5.1 Detection via Coefficient Comparison

According to Definition 3.13, a problem P has a complementary symmetry with respect to some $I \subseteq [n]$ if replacing x by $\bar{x}^{(I)}$ preserves the feasible set and the objective function. If we want to develop an algorithmic procedure for the detection of such symmetry, we have to relax the conditions by considering the *formulation* of the problem, similar to the case of permutation symmetry in Section 3.2.

The idea is the following. Using the notation of Definition 3.13, we consider affine transformations $\gamma_I : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $\gamma_I(x) = Rx + s$, where $R \in \mathbb{R}^{n \times n}$ and $s \in \mathbb{R}^n$ are defined by

$$R_{ij} = \begin{cases} 1 & \text{if } i = j \wedge i \notin I, \\ -1 & \text{if } i = j \wedge i \in I, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad s_i = \begin{cases} d_i & \text{if } i \in I, \\ 0 & \text{otherwise.} \end{cases}$$

If the transformation preserves the objective and constraint functions, i.e., $c^T x = c^T \gamma_I(x)$ and $g_k(x) = g_k(\gamma_I(x))$ for all $k \in [m]$ and $x \in \mathbb{R}^n$, then it also preserves the feasible set and therefore, we have a complementary symmetry along I .

As explained in Section 3.2, the hard part is checking the equality $g_k(x) = g_k(\gamma_I(x))$. If all occurring functions are polynomials, however, this is equivalent to comparing the coefficients of each monomial, which is illustrated in the following example.

Example 5.1. Let P be defined by

$$\begin{aligned} \max_{x, y \in \mathbb{R}^2, t \in \mathbb{R}} \quad & t \\ t - (x_1 - x_2)^2 - (y_1 - y_2)^2 \leq & 0 \\ x_1, x_2, y_1, y_2 \in & [0, 2], \end{aligned} \tag{5.1}$$

which models the problem of placing two points in the square $[0, 2] \times [0, 2]$ such that their distance is maximized. Note that in contrast to Example 3.12, we use the square of the norm in the first constraint in order to obtain a quadratic problem. While this affects the objective value, it does not change the feasible and optimal set.

The constraint can be reformulated as

$$t - x_1^2 - x_2^2 + 2x_1x_2 - y_1^2 - y_2^2 + 2y_1y_2 \leq 0.$$

We use (x_1, x_2, y_1, y_2, t) as order for the variables and consider the index set $I_1 = \{1, 2\}$ which corresponds to complementing x_1 and x_2 , i.e., replacing them with $2 - x_1$ and $2 - x_2$, respectively. Since

$$\begin{aligned} & -(2 - x_1)^2 - (2 - x_2)^2 + 2(2 - x_1)(2 - x_2) \\ = & -4 + 4x_1 - x_1^2 - 4 + 4x_2 - x_2^2 + 8 - 4x_1 - 4x_2 + 2x_1x_2 \\ = & -x_1^2 - x_2^2 + 2x_1x_2, \end{aligned}$$

we can see that all coefficients in the constraint are preserved by the complementation. Consequently, the constraint functions are equal before and after replacing x_1 and x_2 , so P has a complementary symmetry along I_1 . The same holds for $I_2 = \{3, 4\}$ and $I_3 = \{1, 2, 3, 4\}$.

The corresponding affine transformation γ_{I_1} as defined above is given by

$$R = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad s = (2, 2, 0, 0, 0)^T.$$

Replacing x_1 with $2 - x_1$ and x_2 with $2 - x_2$ is equivalent to applying γ_{I_1} to the feasible set.

For the special case of MIQCPs, there is a closed form for checking

equality of the constraints. We only have to apply the affine transformation and check whether the constraint defining matrices and vectors are preserved, as we did in Example 5.1. However, it is not tractable to do this for each possible index set $I \subseteq [n]$. Since we want to detect as much complementary symmetry as possible, a natural approach is to search for the largest such set. The following theorem shows how this can be achieved by constructing and solving an auxiliary problem which encapsulates the ideas described above.

Theorem 5.2. *Let P be an MIQCP, $d \in \mathbb{R}^n$ be defined as in Definition 3.13 and let z^* be a feasible solution of the quadratic problem*

$$\max_{z \in \mathbb{R}^n} \sum_{i=1}^n z_i \quad (5.2a)$$

$$z_i - z_j = 0 \quad i < j \in [n], \ k \in [m] : Q_{ij}^k \neq 0 \quad (5.2a)$$

$$z_i = 0 \quad i \in U \quad (5.2b)$$

$$g_i^k(z) = 0 \quad i \in [n], \ k \in [m] \quad (5.2c)$$

$$h^k(z) = 0 \quad k \in [m] \quad (5.2d)$$

$$z_i \in \{0, 1\} \quad i \in [n] \quad (5.2e)$$

where

$$U = \{i \in [n] : c_i \neq 0 \vee (x_i^L = -\infty \oplus x_i^U = \infty)\}$$

$$g_i^k(z) = -2a_i^k z_i + \sum_{j=1}^n (Q_{ij}^k + Q_{ji}^k) d_j z_j (1 - 2z_i)$$

$$h^k(z) = \sum_{i=1}^n a_i^k d_i z_i + \sum_{i,j=1}^n Q_{ij}^k d_i d_j z_i z_j.$$

Then P contains a complementary symmetry along $I^* := \{i \in [n] : z_i^* = 1\}$.

Proof. We first note that for any $I \subseteq [n]$ the entries of the complement point $\bar{x}^{(I)}$ can be written as

$$\bar{x}_i^{(I)} = (1 - z_i)x_i + z_i(d_i - x_i) \quad \text{with } z_i = \begin{cases} 1 & i \in I, \\ 0 & i \notin I. \end{cases} \quad (5.3)$$

If we now replace x by $\bar{x}^{(I)}$ and the problem does not change, i.e., all coefficients of quadratic, bilinear, and linear terms as well as the constant are the same as in the original problem, it follows directly that (3.13) is fulfilled for I . For ease of notation, let \bar{x} be the complement with respect

to some arbitrary, but fixed set $I \subseteq [n]$. Then

$$\begin{aligned}
\bar{x}^T Q^k \bar{x} &= \sum_{i,j=1}^n Q_{ij}^k \bar{x}_i \bar{x}_j \\
&= \sum_{i,j=1}^n Q_{ij}^k \left(((1 - z_i) x_i + z_i (d_i - x_i)) ((1 - z_j) x_j + z_j (d_j - x_j)) \right) \\
&= \sum_{i,j=1}^n Q_{ij}^k \left(x_i x_j ((1 - 2z_i) (1 - 2z_j)) + d_i d_j z_i z_j \right. \\
&\quad \left. + x_i (1 - 2z_i) d_j z_j + x_j (1 - 2z_j) d_i z_i \right), \\
a^{kT} \bar{x} &= \sum_{i=1}^n a_i^k \bar{x}_i \\
&= \sum_{i=1}^n a_i^k ((1 - z_i) x_i + z_i (d_i - x_i)) \\
&= \sum_{i=1}^n (x_i (1 - 2z_i) a_i^k + a_i^k d_i z_i)
\end{aligned}$$

So for the k -th constraint, the coefficient of the bilinear term $x_i x_j$ is given by

$$Q_{ij}^k (1 - 2z_i) (1 - 2z_j), \quad (5.4)$$

the coefficient of the linear term x_i by

$$a_i^k (1 - 2z_i) + \sum_{j=1}^n (Q_{ij}^k + Q_{ji}^k) d_j z_j (1 - 2z_i), \quad (5.5)$$

and the constant term by

$$b^k + \sum_{i=1}^n a_i^k d_i z_i + \sum_{i,j=1}^n Q_{ij}^k d_i d_j z_i z_j. \quad (5.6)$$

The bilinear coefficients are the same as the original ones if and only if (5.4) is equal to Q_{ij}^k . If $Q_{ij}^k = 0$, this is always true, and otherwise it is equivalent to constraint (5.2a), since $z_i \in \{0, 1\}$. Note that swapping i and j leads to the same constraint and for quadratic terms, i.e., $i = j$, the condition becomes $(1 - 2z_i)^2 = 1$, which always holds. For this reason, we need constraints (5.2a) only for $i < j$.

As for the linear coefficients, we need (5.5) equal to a_i^k which is equivalent to constraint (5.2c). Similarly, the condition that (5.6) is equal to b^k is equivalent to (5.2d).

Since we assume a linear objective function, the set I can only contain indices of variables that do not appear in the objective function. Also, variables which are unbounded in exactly one direction can never be complemented. These two conditions are encoded by constraints (5.2b).

Thus, for any feasible solution of (5.2), I^* fulfills (3.13) and so P contains a complementary symmetry along I^* . \square

Solving the auxiliary problem (5.2) to optimality gives us the largest set of variable indices that can be complemented simultaneously. The size of (5.2) depends strongly on the density pattern of the matrices in the original problem. If N is the total number of non-zeros in the quadratic matrices Q^k , then it has $\mathcal{O}(N + m \cdot n)$ non-redundant constraints containing $2N$ quadratic terms in total.

We return to Example 5.1 to demonstrate how Theorem 5.2 can be used.

Example 5.1 (continued). We again rewrite the constraint in (5.1) as

$$t - x_1^2 - x_2^2 - y_1^2 - y_2^2 + 2x_1x_2 + 2y_1y_2 \leq 0$$

and use (x_1, x_2, y_1, y_2, t) as order for the variables. Then the auxiliary problem as defined in (5.2) is given by

$$\begin{aligned} \max_{z \in \mathbb{R}^5} \quad & \sum_{i=1}^5 z_i \\ & z_1 - z_2 = 0 \\ & z_3 - z_4 = 0 \\ & z_5 = 0 \\ & -2z_1 + 4z_1^2 + 2z_2^2 - 4z_1z_2 = 0 \quad (g_1 \text{ and } g_2) \\ & -2z_3 + 4z_3^2 + 2z_4^2 - 4z_3z_4 = 0 \quad (g_3 \text{ and } g_4) \\ & -2z_5 = 0 \quad (g_5) \\ & -z_1^2 - z_2^2 + 2z_1z_2 - z_3^2 - z_4^2 + 2z_3z_4 = 0 \quad (h) \\ & z_i \in \{0, 1\} \quad i = 1, \dots, 5. \end{aligned}$$

Since the z -variables are binary, the constraints g_1, \dots, g_4 and h are all redundant. This is due to the absence of linear terms in the original problem. Constraint g_5 is also equivalent to $z_5 = 0$ and therefore redundant. Thus, the optimal solution is $(1, 1, 1, 1, 0)$. Going back to the original problem P , this means that it contains a complementary symmetry along the set $\{1, 2, 3, 4\}$. In other words, replacing simultaneously the values of all x - and y -variables in a feasible solution of P by their complements leads to another feasible solution with the same objective value.

5.2 Extending the Algorithm

Finding complementary symmetries with Theorem 5.2 has two downsides. The first one has to do with dependencies between the variables. Even though the solution of the auxiliary problem gives us the largest set of variables that can be complemented, it does not tell us which variables *have to* be complemented *simultaneously*. In fact, replacing an x -variable

by its complement in Example 5.1, forces us to do the same with the other x -variable, which corresponds to mirroring along the line $y = 0.5$. So the x -variables cannot be complemented independently of each other. While it is possible to do all complementations at the same time (geometrically a mirroring along $x - y = 0$), complementing the x -variables does not require us to do the same with the y -variables.

Fortunately, there is a small addition that can fix this. To this end, not that the coefficients of two variables in problem (5.2) depend on each other if and only if the variables appear in a bilinear term somewhere in the problem.

Lemma 5.3. *Let P be a quadratic problem and z be any feasible solution to the auxiliary problem P' of P , as defined in Theorem 5.2. If $z_i = 1$ for some $i \in [n]$, then for all $j \in [n] \setminus \{i\}$*

$$Q_{ij}^k = 0 \quad \forall k \in [m] \quad \Longleftrightarrow \quad \exists \tilde{z} \in \mathcal{F}_{P'} : \tilde{z}_j = 0 \wedge \tilde{z}_i = 1.$$

Proof. First, let $j \in [n] \setminus \{i\}$ such that $Q_{ij}^k = 0$ for all $k \in [m]$. It is clear that $\mathbf{0} \in \mathbb{R}^n$ is always a feasible solution of P' . We also know that there is at least one feasible solution with $z_i = 1$. So starting with $\mathbf{0}$ and setting all variables to 1 that depend on z_i leads to another feasible solution \tilde{z} . Since $Q_{ij}^k = 0$ for all $k \in [m]$, z_i and z_j do not appear together in any of the constraints (5.2a) - (5.2e). Thus, $\tilde{z}_i = 1$ and $\tilde{z}_j = 0$. The other implication follows directly from constraints (5.2a). \square

Lemma 5.3 tells us that we only have to partition the optimal solution of the auxiliary problem according to the bilinear coefficients in order to gain the desired information about dependencies.

The second downside is that the auxiliary problem compares coefficients for each constraint separately. However, it is possible that complementary symmetry involves permuting constraints in addition to complementing variables. To see this, we will consider a variant of the point-packing problem, where two points have to be placed inside a triangle instead of a square.

Example 5.4. Let P be defined by

$$\begin{aligned} \max_{x, y \in \mathbb{R}^2, t \in \mathbb{R}} \quad & t \\ t - (x_1 - x_2)^2 - (y_1 - y_2)^2 & \leq 0 \\ x_i + 2y_i & \leq 2 & i = 1, 2 \\ x_i - 2y_i & \leq 2 & i = 1, 2 \\ x_1, x_2 & \in [0, 2] \\ y_1, y_2 & \in [-1, 1] \end{aligned}$$

The first constraint implicitly models the objective function corresponding to maximizing (the square of) the distance between the points $(x_1, y_1)^T$ and $(x_2, y_2)^T$. Leaving the objective constraint and variable t aside and interpreting $(x_1, y_1)^T$ and $(x_2, y_2)^T$ as points in \mathbb{R}^2 , the feasible region can

be embedded into the two dimensional plane, as shown in Figure 5.1. We can see that P contains a complementary symmetry along the y -variables, but not along the x -variables.

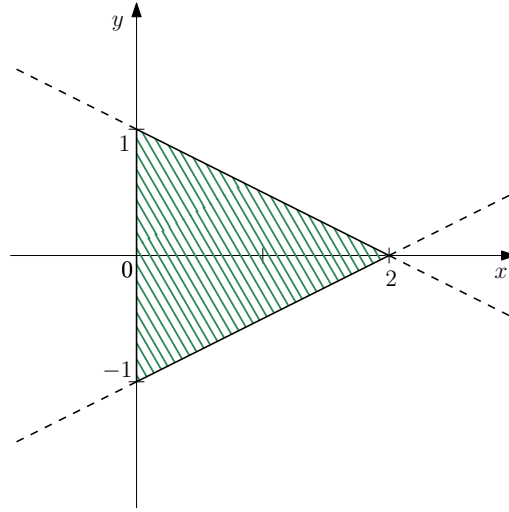


FIGURE 5.1: 2D-visualization of the feasible region in Example 5.4 The two dashed lines that bound the triangle correspond to the linear constraints.

Since the second and third set of constraints are linear, the corresponding constraints (5.2c) in the auxiliary problem are

$$\begin{aligned} -2x_i &= 0 \\ -4y_i &= 0 \\ 4y_i &= 0 \end{aligned}$$

for $i = 1, 2$. Thus, in contrast to our geometric expectation, $\mathbf{0} \in \mathbb{R}^5$ is the only solution. The reason is that each of the linear constraints by itself does not inherit complementary symmetry. It only emerges due to the combination of the two constraints that build the border of the triangle. When replacing y_1 by $-y_1$, the respective linear constraints become

$$\begin{aligned} x_1 - 2y_1 &\leq 2, \\ x_1 + 2y_1 &\leq 2, \end{aligned}$$

so each is turned into the other one. For quadratic constraints this seems unlikely to happen, but in the linear case, usually more than one constraint is necessary for complementary symmetry, since the coefficients of a single one are always changing after complementation. Ideally, we would like to detect such symmetries as well.

Considering that symmetries like the ones in Example 5.4 are most likely hidden in the linear part of a problem, we can make use of the method presented by Christophel et al. [7] and combine it with the auxiliary problem from Theorem 5.2. They showed the following.

Theorem 5.5. *There is a one-to-one correspondence between the complementary symmetries of the BLP*

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ & Ax - b \leq 0 \\ & x \in \{0, 1\}^n \end{aligned} \quad (5.7)$$

and the permutation-based symmetries of the extended problem

$$\begin{aligned} \min_{(x, \bar{x}) \in \mathbb{R}^{2n}} \quad & c^T x + c^T \bar{x} \\ & Ax - A\bar{x} - 2b + A\mathbf{1} \leq 0 \\ & x + \bar{x} - \mathbf{1} = 0 \\ & x, \bar{x} \in \{0, 1\}^n. \end{aligned} \quad (5.8)$$

Of course, we do not want to restrict ourselves to binary problems. It is possible to slightly modify Theorem 5.5 in order to generalize it to MIPs, though. For the sake of simplicity, we state the modified version only for bounded problems. The proof works analogously to the binary case.

Theorem 5.6. *There is a one-to-one correspondence between the complementary symmetries of the (bounded) MIP*

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ & Ax - b \leq 0 \\ & x \in [x^L, x^U] \\ & x_i \in \mathbb{Z} \quad i \in \mathcal{I}, \end{aligned} \quad (5.9)$$

and the permutation-based symmetries of the extended problem

$$\begin{aligned} \min_{(x, \bar{x}) \in \mathbb{R}^{2n}} \quad & c^T x + c^T \bar{x} \\ & Ax - A\bar{x} - 2b + Ad \leq 0 \\ & x + \bar{x} - d = 0 \\ & x, \bar{x} \in [x^L, x^U] \\ & x_i, \bar{x}_i \in \mathbb{Z} \quad i \in \mathcal{I}, \end{aligned} \quad (5.10)$$

where $\mathcal{I} \subseteq [n]$, $x^L, x^U \in \mathbb{R}^n$, and $d = x^L + x^U \in \mathbb{R}^n$.

In order to use Theorem 5.6 for our purpose, we subdivide an MIQCP P into its linear and its quadratic part such that it can be written as

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ & x^T Q^k x + a^{kT} x + b^k \leq 0 \quad k \in [m - l] \\ & Ax - b \leq 0 \\ & x \in [x^L, x^U] \end{aligned} \quad (5.11)$$

with $A \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$ and $x^L, x^U \neq \pm\infty$. The ingredients for extending the detection procedure are the following.

1. Build the modified problem of Theorem 5.6 for the linear part of (5.11) and compute the generators of its formulation group. Filter out the generators that only complement and do not permute variables, i.e., that contain only non-trivial cycles of the form (x_i, \bar{x}_i) for $i \in [n]$. Let the set of those generators be denoted by H .
2. Build the auxiliary problem of Theorem 5.2 for the quadratic part of (5.11). For each variable x_j that is not affected by any permutation in H , add the constraint $x_j = 0$ to the auxiliary problem. This ensures that the solution will not include any variables that cannot be complemented in the linear part.
3. Solve the modified auxiliary problem and partition the solution such that two variables x_i, x_j are in the same class if and only if $Q_{ij}^k \neq 0$ for any $k \in [m - l]$.

To demonstrate this, we get back to the point-packing problem in the triangle.

Example 5.3 (continued). As in Example 5.1, we use (x_1, x_2, y_1, y_2, t) as order for the variables. Firstly, the linear part of the original problem is given by

$$\begin{aligned}
 \max_{x, y \in \mathbb{R}^2, t \in \mathbb{R}} \quad & t \\
 x_i + 2y_i &\leq 2 & i = 1, 2 \\
 x_i - 2y_i &\leq 2 & i = 1, 2 \\
 x_1, x_2 &\in [0, 2] \\
 y_1, y_2 &\in [-1, 1] \\
 t &\in [-M, M].
 \end{aligned}$$

For the sake of simplicity, we added bounds for the objective variable t , where M is just a large enough number. As in Theorem 5.6, we extend the problem with complementary variables. Using $d = (2, 2, 0, 0, 0)^T$ and $Ad = (2, 2, 2, 2)^T$, we get

$$\begin{aligned}
 \max_{x, y, \bar{x}, \bar{y} \in \mathbb{R}^2, t, \bar{t} \in \mathbb{R}} \quad & t + \bar{t} \\
 x_i + 2y_i - \bar{x}_i - 2\bar{y}_i &\leq 0 & i = 1, 2 \\
 x_i - 2y_i - \bar{x}_i + 2\bar{y}_i &\leq 0 & i = 1, 2 \\
 x_i - \bar{x}_i &= 0 & i = 1, 2 \\
 y_i - \bar{y}_i &= 0 & i = 1, 2 \\
 t - \bar{t} &= 0 \\
 x_1, x_2, \bar{x}_1, \bar{x}_2 &\in [0, 2] \\
 y_1, y_2, \bar{y}_1, \bar{y}_2 &\in [-1, 1] \\
 t, \bar{t} &\in [-M, M].
 \end{aligned}$$

The formulation group of this extended problem is generated by the permutations

$$\begin{aligned}\pi_1 &= (y_1 \bar{y}_1), \quad \pi_2 = (y_2 \bar{y}_2), \quad \pi_3 = (t \bar{t}), \\ \text{and} \quad \pi_4 &= (x_1 x_2)(y_1 y_2)(\bar{x}_1 \bar{x}_2)(\bar{y}_1 \bar{y}_2).\end{aligned}$$

Filtering out the complementing permutations, we get $H = \{\pi_1, \pi_2, \pi_3\}$.

Secondly, we need the quadratic part of the original problem. Without the triangle-bounding linear constraints, the problem is exactly the same as the point-packing problem in Example 5.1. We only need to add the constraints $x_1 = 0$ and $x_2 = 0$, since x_1 and x_2 are not affected by any permutation in H . The optimal solution of the (modified) auxiliary problem is then $(0, 0, 1, 1, 0)$. Remember that this means that we can complement y_1 and y_2 , but no other variables.

As a last step, we want to check whether the y -variables can be complemented independently. Looking at the quadratic constraint again, we see that they do appear in a bilinear term which means they are not independent of each other. So we have found out that the original problem contains a complementary symmetry along $\{3, 4\}$ (the indices of the y -variables) and whenever one is replaced by its complement, the same has to be done with the other one. This is exactly what we expected from the geometric interpretation.

As explained in the beginning of this section, solving (5.2) basically performs a coefficient comparison of the quadratic and linear terms before and after replacing variables with their complements. Unfortunately, polynomials of higher degree cannot be represented by a matrix and a vector like in the quadratic case. In theory, comparing coefficients can also detect complementary symmetry for a specific index set I of a general polynomial problem, but it is not clear whether it is possible to construct an analogous auxiliary problem.

Another open question is whether the two types of symmetry can be combined in some way. One approach could be to extend the problem by variables \bar{x}_i for each $i \in [n]$ in such a way that the formulation group contains the permutation $(x_i \bar{x}_i)$ if and only if x_i can be complemented. It is possible, however, that the problems would become too large for practical purposes.

Christophel et al. [7], on the other hand, use a slightly more general notion of complement-based symmetry which combines classical permutations of variables and their complementation by means of signed permutations. Under that notion, permutations like $(x_1 \bar{x}_2)$ are also valid. In the linear context, Theorem 5.6 can be used to actually find all such permutations of the generalized form. For nonlinear problems, however, it is not clear how to detect them.

Chapter 6

Static Symmetry Breaking Constraints

We now turn to symmetry *handling* methods. Returning to permutation-based symmetry, once the formulation group of a MINLP has been computed, the question is how this information can be used to reduce the size of the sB&B-tree. As explained in Section 3.2, any subgroup of the solution group induces an equivalence relation on the set of optimal points. Two optima lie in the same equivalence class if they are mapped onto each other by some permutation in the subgroup. Symmetry methods try to reduce the number of optimal points while ensuring that at least one representative from each equivalence class stays feasible. In general, it is very hard to achieve that exactly one solution per class remains.

Symmetry breaking techniques for sB&B based solvers can be divided into two categories, namely *static* and *dynamic* ones. The idea of static methods is to change the problem formulation such that a preferably small subset of the optimal set remains feasible. Dynamic methods, on the other hand, try to use symmetry information during the sB&B process to identify equivalent subtrees and prune all but one of them.

The most common static approach is to add linear constraints to the problem formulation that break the symmetry. If only two variables x and y can be permuted, one can just add the constraint $x \leq y$ (or $x \geq y$). In the case of more than two affected variables, we have to make sure that there are no conflicts between the added constraints.

A different approach by Fischetti et al. [14] called *orbital shrinking*, which is based on projection onto the so-called *fixed space*. However, it only works for problems that are convex after dropping integrality conditions and is therefore not relevant for our more general setting.

We will give a definition of symmetry breaking constraints and cite some folklore results from the literature in Section 6.1. Then, in Section 6.2, we discuss ideas by other authors for avoiding conflicting constraints developed. New symmetry breaking inequalities for symmetric subgroups of the formulation group are presented in Section 6.3, which is our contribution to this topic.

6.1 Inequalities from the Literature

We first give a formal definition of symmetry breaking constraints in a similar fashion to Liberti [25].

Definition 6.1. Given a MINLP P and a permutation group $G \leq \mathcal{S}_n$, a set of inequalities $S = \{f_i(x) \leq 0\}_{i \in [k]}$ for $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i \in [k]$ is called a set of *symmetry breaking constraints* (SBCs) for P with respect to G if

$$\forall y \in \mathcal{G}_P \quad \exists \pi \in G : \quad f_i(\pi(y)) \leq 0 \quad \forall i \in [k].$$

To get an understanding of this definition, assume that G is a subgroup of the solution group G_P^* . In that case, we have $\pi(y) \in \mathcal{G}_P$ for all $\pi \in G$ and therefore at least one optimal solution is still feasible after adding the constraints in S to the problem. For practical purposes, G is always (a subgroup of) the formulation group.

Remark 6.2. Despite their name, SBCs are not *guaranteed* to break symmetry in any way. For example, redundant constraints are also SBCs for a problem. A more intuitive name would be solution preserving constraints with respect to G . We chose to stick to symmetry breaking constraints to be consistent with Liberti [25]. After all, the important property is that not *all* optimal solutions are cutoff. For the SBCs that we use, it will be clear that they also break symmetry in the sense that they shrink the formulation group.

For any set S of SBCs for P with respect to G , it follows immediately from the definition that

1. S is a set of SBCs for P w.r.t. any $H \geq G$,
2. $T \subseteq S$ is a set of SBCs for P w.r.t. G ,
3. S is a set of SBCs for P w.r.t. $G|_\omega$ for any orbit ω of G if all f_i involve only variables in ω .

The idea is to use SBCs to modify the feasible region such that it contains only one element from each orbit under the action of G on the feasible region. For BLPs, one could theoretically add the inequalities

$$\sum_{i=1}^n 2^{i-1} x_i \geq \sum_{i=1}^n 2^{i-1} x_{\pi(i)} \quad \forall \pi \in G,$$

to enforce a lexicographic ordering and produce a so-called *minimal fundamental domain*, which fulfills the desired property, see J. Friedman [19] and Sherali et al. [45]. However, aside from the fact that there is one inequality for each element of the group, they do not behave numerically stable in practice due to the size of the coefficients. Furthermore, they are not applicable if continuous variables are involved.

As Pfetsch et al. [38] observed, distinct factors of a product group can be considered independently for any symmetry handling purpose, since the

factors act on disjoint sets of elements. If not stated otherwise, we will always consider single factors.

Most literature about static symmetry breaking techniques focuses on SBCs based on orbits under action of a group G on the set of variable indices. For the remainder of this chapter, we denote the set of these orbits by Ω_G . The following two theorems are both taken from Liberti [25].

Theorem 6.3. *Let P be a MINLP, $G \leq G_P^*$ and $\omega \in \Omega_G$. For an arbitrary but fixed $k \in \omega$, the set of inequalities*

$$\{x_k \geq x_j \mid j \in \omega \setminus \{k\}\} \quad (6.1)$$

is a set of SBCs for P w.r.t. G .

We call the inequalities in (6.1) *weak SBCs*. They fix one variable of the orbit as maximum. By swapping the inequality signs, it could of course also be fixed as a minimum. For a single orbit, we can therefore always bring an element of our choice to the first position, regardless of which kind of group it is. If we know more about the structure of the group, though, we can use stronger inequalities.

Theorem 6.4. *Let P be a MINLP, $G \leq G_P^*$ and $\omega \in \Omega_G$. If $G|_\omega \cong \mathcal{S}_k$ and $\omega = \{j_1, j_2, \dots, j_k\}$ then the set of inequalities*

$$\{x_{j_i} \geq x_{j_{i+1}} \mid i \in \{1, \dots, k-1\}\} \quad (6.2)$$

is a set of SBCs for P w.r.t. G .

So whenever a permutation group acts like the symmetric group on one of its orbits, we can add the so-called *strong SBCs* (6.2). They enforce a total order (which can be chosen arbitrarily) on that orbit. We will give a few examples of different cases that this can apply to.

Example 6.5. Let P be defined by

$$\begin{aligned} \max_{x \in \mathbb{R}^5} \quad & x_1 + x_2 - x_3 + x_4 - x_5 \\ & x_1 + x_2 + x_3 + x_4 + x_5 \leq 1 \\ & x_1, \dots, x_5 \in [0, 1]. \end{aligned}$$

Then the formulation group is $G_P = \langle (1\ 2), (2\ 4), (3\ 5) \rangle$. It has two factors since the first two generators act on a different set of variables than the third one, i.e., it can be written as $G_P = \langle (1\ 2), (2\ 4) \rangle \times \langle (3\ 5) \rangle$. Those sets of variable indices are, in this case, also the orbits, namely $\omega_1 = \{1, 2, 4\}$ and $\omega_2 = \{3, 5\}$. Restricting to ω_1 , we can see that $G|_{\omega_1} \cong \mathcal{S}_3$. According to Theorem 6.4, we can choose an arbitrary order on ω_1 , say $(4, 1, 2)$, and add the strong SBCs $x_4 \geq x_1$, $x_1 \geq x_2$ to the problem. As for ω_2 , we also get $G|_{\omega_2} \cong \mathcal{S}_2$, but for orbits with two elements, weak and strong SBCs coincide and we can just add $x_3 \geq x_5$.

Example 6.6. Let P be defined by

$$\begin{aligned}
 \max_{x \in \mathbb{R}^3} \quad & x_1 + x_2 + x_3 \\
 & x_1 + x_2 + x_3 \leq 6 \\
 & x_1 + 2x_2 \geq 5 \\
 & x_2 + 2x_3 \geq 5 \\
 & x_3 + 2x_1 \geq 5 \\
 & (x_1 + x_2)^2 \geq 1 \\
 & (x_2 + x_3)^2 \geq 1 \\
 & (x_3 + x_1)^2 \geq 1 \\
 & x_1, x_2, x_3 \in \mathbb{Z}.
 \end{aligned}$$

There is a symmetry between all three variables, but we cannot swap them arbitrarily. The formulation group is the cyclic group $G_P = \langle (1\ 2\ 3) \rangle$ and set of optimal solutions is $\mathcal{G}_P = \{(1, 2, 3)^T, (3, 1, 2)^T, (2, 3, 1)^T\}$. There is only one orbit, but $G_P \not\cong \mathcal{S}_3$, so we cannot use Theorem 6.4 to enforce an arbitrary order. In fact, adding the constraints $x_1 \geq x_2$ and $x_2 \geq x_3$ would cut off every optimal solution. Nonetheless, Theorem 6.3 still allows us to fix a maximum, e.g., by adding $x_1 \geq x_2$ and $x_1 \geq x_3$.

Example 6.7. Let P be defined by

$$\begin{aligned}
 \max_{x \in \mathbb{R}^6} \quad & \sum_{i=1}^6 x_i \\
 & x_1 + x_2 + x_3 \leq 2 \\
 & x_4 + x_5 + x_6 \leq 1 \\
 & x_1 + x_4 \geq 1 \\
 & x_2 + x_5 \geq 1 \\
 & x_3 + x_6 \geq 1 \\
 & x_1, \dots, x_6 \in \{0, 1\}.
 \end{aligned}$$

In this problem, we can arbitrarily swap x_1, x_2 , and x_3 , but due to the last three constraints, we always have to permute x_4, x_5 , and x_6 accordingly. So the formulation group is $G_P = \langle (1\ 2)(4\ 5), (2\ 3)(5\ 6) \rangle$. Like in the previous example, we have two different orbits, namely $\omega_1 = \{1, 2, 3\}$ and $\omega_2 = \{4, 5, 6\}$. The difference is that this time there is only a single factor, so the orbits belong to the same component of the group. Still, restricting to either of the orbits, we get $G_P|_{\omega_1} \cong \mathcal{S}_3$ and $G_P|_{\omega_2} \cong \mathcal{S}_3$, respectively. Therefore, we can add strong SBCs for ω_1 again. However, Theorem 6.4 does not tell us whether we can add strong SBCs for ω_2 on top of it. In fact, the set of optimal solutions is

$$\mathcal{G}_P = \left\{ (0, 1, 1, 1, 0, 0)^T, (1, 0, 1, 0, 1, 0)^T, (1, 1, 0, 0, 0, 1)^T \right\}$$

and adding strong SBCs for both orbits simultaneously, i.e.,

$$\begin{array}{ll} x_1 \geq x_2, & x_4 \geq x_5, \\ x_2 \geq x_3, & x_5 \geq x_6, \end{array}$$

cuts off all of the optimal solutions.

As we have seen, orbits can depend on each other in which case we can only add SBCs for one of them. Considering Example 6.7, one might think that in such cases it is not necessary to handle more than one orbit, since the order of the second one is implied by the first one. In general, that is not true. For example, if the values of variables corresponding to the first orbit are equal in all optimal solutions, then SBCs for that orbit do not actually cut off any optimal solutions, while SBCs for the second one could do so. Consequently, the choice of the orbit can have a significant impact on the solving process in practice.

The group in Example 6.7 acts as a matrix action (see Section 3.1). These kind of groups play an important role in practice, as they frequently appear in real-world problems. If all involved variables are binary, Hojny et al. [17] showed how such symmetries can be handled using *orbitopes*, the convex hulls of binary matrices with lexicographically non-increasing columns w.r.t. the group. While the technique was developed for BLPs, it can directly be transferred to binary MINLPs, since it relies only on the structure of the group and not on the linearity of the constraints. For groups acting on continuous variables, however, the method is not applicable, since the generated inequalities make use of the integrality of the variables.

Theorems 6.3 and 6.4 give rise to a simple framework for generating SBCs which is described in Algorithm 1. Its ingredients are the following subroutines. `computeFactors` subdivides a given group into its distinct factors. For each factor, `isSymmetricGroup` then checks whether a given group is isomorphic to the symmetric group of some degree. In that case, the algorithm determines whether the group has only a single orbit in Line 5, i.e., whether it is a coordinate action. In Example 6.5, the two factors are such coordinate actions, while in Example 6.7 we have a matrix action. In the latter case or if no symmetric group was found, `chooseOrbit` computes the orbits of a group and selects one to break symmetry on (according to some rule) in Line 8 or 12, respectively. As for the first two subroutines, efficient algorithms and implementations thereof exist, see e.g., `PermLib` [41]. In Section 8.1, we will explain how orbits are chosen in our implementation.

The different situations in the examples above also give rise to the question under which circumstances orbits are independent of each other, i.e., whether there is something else that can be done Lines 12 – 13 of Algorithm 1. In the next section, we will discuss criteria for independent orbits from the literature.

Algorithmus 1 : Simple framework for generating SBCs**Input** : G_P : formulation group of MINLP P **Output** : S : set of SBCs for P w.r.t. G_P

```

1  $S \leftarrow \emptyset$ 
2  $\mathcal{R} \leftarrow \text{computeFactors}(G_P)$ 
3 for  $G \in \mathcal{R}$  do
4   if  $\text{isSymmetricGroup}(G)$  then
5     if  $\text{numberOfOrbits}(G) = 1$  then
6        $\text{add inequalities (6.2) for } G \text{ to } S$ 
7     else
8        $\omega \leftarrow \text{chooseOrbit}(G)$ 
9        $\text{add inequalities (6.2) for } \omega \text{ to } S$ 
10    end
11  else
12     $\omega \leftarrow \text{chooseOrbit}(G)$ 
13     $\text{add inequalities (6.1) for } \omega \text{ to } S$ 
14  end
15 end
16 return  $S$ 

```

6.2 Orbital Independence

The following theorem gives the first set of conditions for independent orbits and is due to Liberti [25]:

Theorem 6.8. *Let P be a MINLP, $G \leq G_P^*$ and $\omega, \theta \in \Omega_G$ and let S_ω, S_θ be sets of SBCs for P w.r.t. $G|_\omega$ and $G|_\theta$, respectively. If the two conditions*

1. $\exists H \leq G|_{\omega \cup \theta} : H|_\omega \cong \mathcal{C}_{|\omega|} \text{ and } H|_\theta \cong \mathcal{C}_{|\theta|}$
2. $\gcd(|\omega|, |\theta|) = 1$

hold, then $S_\omega \cup S_\theta$ is a set of SBCs for P w.r.t. G .

Two orbits fulfilling the conditions of Theorem 6.8 are called *coprime*. In practice, coprime orbits appear rarely within the same factor of the formulation group (see Section 8.2). Therefore, the result is more of theoretical interest.

In an attempt to give more practically relevant conditions, another approach based on stabilizers of orbits was proposed by Liberti et al. [26]. The idea is to extend SBCs for an orbit ω by (re-)computing orbits with respect to the pointwise stabilizer of ω and is based on the following observation.

Theorem 6.9. *Let P be a MINLP, $G \leq G_P^*$, $\omega \in \Omega_G$ and $\theta \in \Omega_{G^\omega}$. Then the union of constraints (6.1) for ω and θ are a set of SBCs for P w.r.t. G . If additionally $G|_\omega \cong \mathcal{S}_{|\omega|}$ or $G|_\theta \cong \mathcal{S}_{|\theta|}$, the respective constraints can be replaced by (6.2).*

Liberti et al. [26] used this to build an iterative procedure, at each step choosing the largest orbit under the pointwise stabilizer of the union of the previously used orbits.

The idea was extended by Dias et al. [13] in the following way. Instead of iteratively building a set of independent orbits, find a possibly large subset Ω_I of Ω_G such that the elements are pairwise independent, i.e. for each pair $\omega, \theta \in \Omega_I$ we have $\omega \in \Omega_{G^\theta}$ and $\theta \in \Omega_{G^\omega}$. The largest such subset can be found by computing a maximal clique of a graph that encodes the independence relation between the orbits. However, in an extension of the that article, Dias et al. [12] also showed that a set of pairwise independent orbits is not necessarily independent as a whole. In other words, the tested conditions are not sufficient for adding SBCs for each orbit in Ω_I simultaneously. Thus, further heuristic methods have to be applied to extract a set of (fully) independent orbits from Ω_I . The problem of finding the largest such set is computationally very hard, since a naive approach would require a lot of stabilizer computations.

Both approaches suffer from another drawback: The fact that they rely on *pointwise* stabilizers is a strong restriction. In most practical cases, this will, only apply to distinct factors of the formulation group. Most real-world problems have formulation groups with well-understood structure (i.e., isomorphisms of symmetric groups), where computing orbits and stabilizers does not provide any benefit at all. Furthermore, other formulation groups appearing in practice often have only a single orbit, in which case the approach is not productive either. Instead, we will consider a different idea based on subgroups of the formulation group.

6.3 SBCs from Subgroups

As mentioned in Section 6.1, SBCs with respect to a group G are also valid for any overgroup of G (i.e., a group H with $G \leq H$). Consequently, we could consider using only a subgroup of the formulation group to generate SBCs, whenever that seems fitting. While it might be counter-intuitive at first, we have already come across examples where this approach can be beneficial.

Example 6.10. Consider again the point-packing problem with 3 points from Example 3.12. We saw that the formulation group is given by

$$G_P = \langle \pi_1, \pi_2, \pi_3 \rangle \quad \text{with}$$

$$\pi_1 = (1\ 2)(4\ 5), \quad \pi_2 = (2\ 3)(5\ 6), \quad \pi_3 = (1\ 4)(2\ 5)(3\ 6).$$

Algorithm 1 detects that G_P consists of a single factor which is not isomorphic to a symmetric group. Thus, it adds the weak SBCs $x_1 \geq x_2$, $x_1 \geq x_3$ and $x_1 \geq y_i$ for $i = 1, 2, 3$, since all of these variables lie in the same orbit. We have also seen that there are 24 optimal solutions corresponding to four geometric constellations. For each of these constellations, the weak SBCs

determine which point corresponds to (x_1, y_1) , while two possibilities are left for the other two points. So eight distinct optimal solutions remain.

If instead of the whole formulation group, we only consider the subgroup $H = \langle \pi_1, \pi_2 \rangle$, then the algorithm will detect that $H \cong \mathcal{S}_3$. Since there are two orbits $\omega_1 = \{1, 2, 3\}$ and $\omega_2 = \{4, 5, 6\}$ under H , it will proceed to choose one of the orbits, say ω_1 , and add the strong SBCs $x_1 \geq x_2$ and $x_2 \geq x_3$. These constraints determine the total order of the points in each geometric constellation, leaving only four optimal solutions feasible.

So in certain cases where the formulation group contains a “hidden” symmetric group, more symmetry can be broken by restricting to the respective subgroup. Such groups appear for example in the `pointpack*` instances of the MINLPLib [33]. A problem, however, is that there is no clear way to decide which subgroup to use. Testing all possible subgroups is not computationally tractable and even restricting to subgroups which are generated by subsets of the set of generators would be exponential in the number of generators. In Section 8.1 we propose a heuristic for choosing a suited subgroup purposely built to detect symmetric groups.

The observation made above does not imply that restricting to a symmetric subgroup is always advantageous. The following problem illustrates this.

Example 6.11. Let P be defined by

$$\begin{aligned} \max_{x \in \mathbb{R}^6} \quad & \sum_{i=1}^6 x_i \\ & x_1 + x_2 + x_3 \leq 6 \\ & x_4 + x_5 + x_6 \leq 6 \\ & (x_1 - x_2)^2 + (x_4 - x_5)^2 = 1 \\ & (x_2 - x_3)^2 + (x_5 - x_6)^2 = 1 \\ & (x_3 - x_1)^2 + (x_6 - x_4)^2 = 1 \\ & x_i \leq 3 \quad i = 1, \dots, 6 \\ & x_i \in \mathbb{Z} \quad i = 1, \dots, 6. \end{aligned}$$

The formulation group is the same as in the previous example, i.e.,

$$G_P = \langle (1\ 2)(4\ 5), (2\ 3)(5\ 6), (1\ 4)(2\ 5)(3\ 6) \rangle.$$

In every optimal solution, the first three and last three variables sum up to 6, respectively. The quadratic constraints enforce that one of the two sets contains the values 1, 2, and 3, while in the other one all values are 2. So the set of optimal solutions contains six points with $x_1 = x_2 = x_3 = 2$ and six points with $x_4 = x_5 = x_6 = 2$. Now adding the weak SBCs $x_1 \geq x_j$ for $j = 2, \dots, 6$ cuts off all but two optimal solutions, namely $(3, 2, 1, 2, 2, 2)^T$ and $(3, 1, 2, 2, 2, 2)^T$. On the other hand, adding strong SBCs $x_1 \leq x_2 \leq x_3$ – based on the same subgroup as in the previous example – leaves us with the six solutions where $x_1 = x_2 = x_3 = 2$ in addition to $(3, 2, 1, 2, 2, 2)^T$.

So in this example, using the standard weak SBCs actually breaks more symmetry than using strong SBCs to enforce a total order on a subset of the variables.

Fortunately, we can still do better than this. In fact, one can extend the set of strong SBCs for the subgroup with some weak SBCs that take the whole group into account, as the following theorem shows. The idea is that the maximum element of the subgroup can also be fixed to be the maximum element of its orbit in the whole group.

Theorem 6.12. *Let P be a MINLP, $G \leq G_P^*$ and $H \leq G$ such that $H|_\omega \cong S_k$ for an orbit $\omega = \{x_{j_1}, \dots, x_{j_k}\}$ of H . Also, let θ be the unique orbit of G such that $\omega \subseteq \theta$. Then*

$$x_{j_i} \geq x_{j_{i+1}} \quad i \in [k-1] \quad (6.3)$$

$$x_{j_1} \geq x_\ell \quad \ell \in \theta \setminus \omega \quad (6.4)$$

is a system of SBCs for P w.r.t. G .

Proof. Let y be an optimal solution of P . We have to show that there exists a permutation $\pi \in G$, s.t. $\pi(y)$ fulfills (6.3) and (6.4). Of course, G acts transitively on θ , so there is a $\pi_1 \in G$ such that inequalities (6.4) hold for $\pi_1(y)$. Since $H|_\omega \cong S_k$, there is also a $\pi_2 \in H$, such that $\pi_2(\pi_1(y))$ fulfills inequalities (6.3). Now π_2 might swap variables within $\theta \setminus \omega$, but we know that it is setwise fixed by π_2 , i.e., $\pi_2(\theta \setminus \omega) = \theta \setminus \omega$. Together with the fact that $\pi_2(y_{j_1}) \geq y_{j_1}$, we get that inequalities (6.4) also hold for $\pi_2(\pi_1(y))$. \square

Theorem 6.12 allows us to combine strong and weak SBCs under certain conditions. We will demonstrate how this can be applied to the previous example.

Example 6.11 (continued). After having detected the symmetric subgroup and adding the strong SBCs $x_1 \geq x_2 \geq x_3$, we have to find the orbit of G_P that contains the used variables. In this case, there is only one orbit that contains all variables. Therefore, we take the maximum in the total order induced by the strong SBCs, which is x_1 , and add the constraints $x_1 \geq x_4$, $x_1 \geq x_5$, and $x_1 \geq x_6$. So to summarize, we have added the constraints

$$\begin{aligned} x_2 &\geq x_3 & \text{and} \\ x_1 &\geq x_j & j \in \{2, 4, 5, 6\}. \end{aligned}$$

These SBCs combined cut off all optima apart from $(3, 2, 1, 2, 2, 2)^T$.

Note that in all of the above example problems, the algorithms for orbital independence from the previous section would have no effect, since they all have only a single orbit anyway.

Algorithmus 2 : Extended framework for generating SBCs

Input : G_P : formulation group of MINLP P **Output** : S : set of SBCs for P w.r.t. G_P

```

1   $S \leftarrow \emptyset$ 
2   $\mathcal{R} \leftarrow \text{computeFactors}(G_P)$ 
3  for  $G \in \mathcal{R}$  do
4      if  $\text{isSymmetricGroup}(G)$  then
5          if  $\text{numberOfOrbits}(G) = 1$  then
6              add inequalities (6.2) for  $G$  to  $S$ 
7          else
8               $\omega \leftarrow \text{chooseOrbit}(G)$ 
9              add inequalities (6.2) for  $\omega$  to  $S$ 
10         end
11     else
12          $\mathcal{H} \leftarrow \text{chooseSubgroups}(G)$ 
13          $f \leftarrow \text{false}$ 
14         for  $H \in \mathcal{H}$  do
15             if  $\text{isSymmetricGroup}(H)$  then
16                  $\omega \leftarrow \text{chooseOrbit}(H)$ 
17                  $\theta \leftarrow \text{computeOverorbit}(G, \omega)$ 
18                 add inequalities (6.3) and (6.4) for  $(\omega, \theta)$  to  $S$ 
19                  $f \leftarrow \text{true}$ 
20                 break
21             end
22         end
23         if  $f = \text{false}$  then
24              $\omega \leftarrow \text{chooseOrbit}(G)$ 
25             add inequalities (6.1) for  $\omega$  to  $S$ 
26         end
27     end
28 end
29 return  $S$ 

```

Theorem 6.12 can be used to extend Algorithm 1. Instead of only testing whether an entire factor is isomorphic to the symmetric group, we try to find a subgroup of the factor that fulfills this property. The extended framework is described in Algorithm 2. Lines 4 – 11 are the same as in Algorithm 1. If the current factor is not a symmetric group, however, a set of subgroups is chosen by a subroutine `chooseSubgroups` in Line 12. Afterwards, the symmetric group test is performed for all of the subgroups in Lines 14 – 22. If such a subgroup is found, the corresponding inequalities (6.3) and (6.4) are added to the problem and the algorithm leaves the loop. To this end, `computeOverorbit` finds the unique orbit of the action of G which encloses the chosen orbit ω of the action of subgroup H , as described in Theorem 6.12. Only if neither the whole factor nor any of the subgroups

are isomorphic to a symmetric groups, the general purpose weak SBCs are added in Lines 23 – 26.

Remark 6.13. Theorem 6.12 does not require the whole subgroup to be a coordinate or matrix action (as in Example 6.10) since we can use only one of the orbits anyway. It suffices that the subgroup is isomorphic to a symmetric group when restricted to one of its orbits. The `hadamard*` and `netmod*` instances of the MINLPLib are examples where such groups (that do not act as matrix actions) appear.

Chapter 7

Breaking Symmetry Dynamically

In contrast to the methods from the previous chapter, dynamic symmetry breaking techniques do not reformulate the problem before the solving process. Instead, they try to reduce the tree size during the sB&B process. The most widely-known dynamic methods for BLPs are *Orbital Fixing* and *Isomorphism Pruning*, both introduced by Margot [29, 30], as well as *Orbital Branching* by Ostrowski et al. [35, 36]. For binary problems with special structure, the separation and propagation routines for orbitopes by Hojny et al. [17] also fall into this category. An extensive computational comparison by Pfetsch et al. [38] revealed that – in the context of linear programming – dynamic methods prove to be more successful than static methods, with Orbital Fixing and Isomorphism Pruning being the most effective ones in general.

Despite the fact that none of these methods can directly be transferred to continuous variables, it is still worth understanding why dynamic approaches can be superior, possibly paving the way towards a similar technique for MINLPs. Isomorphism Pruning and Orbital Fixing work differently, but often result in the same prunings, see Pfetsch et al. [38]. We will focus on the latter.

7.1 Orbital Fixing

There is another way of looking at SBCs when the underlying orbit consists of binary variables. For example, the weak SBCs (6.1) for an orbit ω with maximum index k can be translated to the implication

$$x_k = 0 \implies x_j = 0 \quad \forall j \in \omega.$$

The key idea of Orbital Fixing – and most other dynamic approaches – is to delay the decision about which element of an orbit should be the maximum until one of them is branched on. With static SBCs, we have to pick an element beforehand. Since the first branching decisions are usually the most impactful, it can be a good idea to *adjust* the order accordingly. The main idea of Orbital Fixing is the following. As soon as an orbit is branched on for the first time, consider the sub-tree in which the respective variable is set to 0. In this branch, all other variables in that orbit are set to 0 as well.

In order to be able to apply this propagation to more than one orbit, we have to use the setwise stabilizer of all variables that have been branched to 1 at the respective node. The reason is the possibility of dependent orbits as explained in Section 6.1. Formally, Orbital Fixing is based on the following result, which was firstly obtained by Margot [29] for a specific branching rule and later proven to be independent of the branching rule by Ostrowski [34]. It extends the idea explained above to variables that are fixed for other reasons than branching. To this end, denote by $B \subseteq [n]$ the index set of all binary variables, by $B_0, B_1 \subseteq B$ the indices of variables branched to 0 and 1, respectively, at some node, and by $F_0, F_1 \subseteq B$ the indices of variables that have been fixed to 0 and 1, respectively, by some other symmetry independent method.

Theorem 7.1. *Let P be a MINLP, $G \leq G_P^*$ and ω an orbit of $\text{stab}(G, B_1)$.*

1. *If $\omega \cap (B_0 \cup F_0) \neq \emptyset$ then all variables in ω can be fixed to 0.*
2. *If $\omega \cap F_1 \neq \emptyset$ then all variables in ω can be fixed to 1.*

Algorithm 3 describes the whole procedure which can be applied to any node of the sB&B tree. Again, efficient algorithms and implementations for the subroutines `computeSetwiseStabilizer` and `computeOrbits` are available. Note that, since we compute the orbits with respect to $\text{stab}(G_P, B_1)$, any orbit containing a variable in B_1 only contains variables in B_1 . Therefore we do not have to consider this in Lines 5 and 7.

Algorithmus 3 : Orbital Fixing

Input : G_P : formulation group of MINLP P ,
 B_0, B_1 : binary var. indices branched on 0/1 resp.,
 F_0, F_1 : binary var. indices fixed to 0/1 resp.
Output : L_0, L_1 : binary var. indices that can be fixed to 0, 1 resp.

```

1  $L_0, L_1 \leftarrow \emptyset$ 
2  $H \leftarrow \text{computeSetwiseStabilizer}(G_P, B_1)$ 
3  $\Omega \leftarrow \text{computeOrbits}(H)$ 
4 foreach  $\omega \in \Omega$  with  $\omega \cap B \neq \emptyset$  do
5   if  $\omega \cap (B_0 \cup F_0) \neq \emptyset$  then
6      $L_0 \leftarrow L_0 \cup (\omega \setminus (B_0 \cup F_0))$ 
7   else if  $\omega \cap F_1 \neq \emptyset$  then
8      $L_1 \leftarrow L_1 \cup (\omega \setminus F_1)$ 
9   end
10 end
11 return  $L_0, L_1$ 
```

By using the stabilizer, two things are achieved implicitly. If there are two or more dependent orbits and one has been branched on first, once all variables in that orbit are set to 1, the other orbits “reappear“ under the stabilizer. On the other hand, the algorithm implicitly distinguishes between different types of groups. If we have a symmetric group, for example, it will have generators which are also in the stabilizer and therefore

the respective orbit can be used to apply Theorem 7.1 even further. Both ideas are best understood by considering a matrix action.

Example 7.2. Let P be a MINLP with formulation group $G_P = \langle \pi_1, \pi_2 \rangle$, where

$$\pi_1 = (1\ 2)(4\ 5)(7\ 8) \quad \text{and} \quad \pi_2 = (2\ 3)(5\ 6)(8\ 9).$$

This is a matrix action with orbits $\omega_1 = \{1, 2, 3\}$, $\omega_2 = \{4, 5, 6\}$, and $\omega_3 = \{7, 8, 9\}$. Now say the first branching is performed on x_1 . Then in the sub-tree where $x_0 = 0$, we have $B_1 = \emptyset$ and $\text{stab}(G_P, \emptyset) = G_P$, so we can fix $x_2 = 0$ and $x_3 = 0$. In the other sub-tree, where $x_1 = 1$, we have $B_1 = \{1\}$ and $\text{stab}(G_P, \{1\}) = \langle \pi_2 \rangle$. The orbits of this stabilizer are $\{2, 3\}$, $\{5, 6\}$, and $\{8, 9\}$. If the next branching decision is $x_2 = 0$, the stabilizer and thus the orbits stay the same, so we could still fix $x_3 = 0$, as well. This corresponds to enforcing a lexicographical order on orbit ω_1 . If we had a cyclic group on that orbit instead, we would not have been able to do the second fixing, since the only generator would not be in the stabilizer after the $x_1 = 1$ branching.

Furthermore, if we arrive at a node with branchings $x_1 = 1$, $x_2 = 1$, and $x_3 = 1$, we have $B_1 = \{1, 2, 3\}$ and thus again $\text{stab}(G_P, \{1, 2, 3\}) = G_P$. So for any further branchings on one of the orbits ω_2 or ω_3 , we could use Orbital Fixing to find further fixings. This could also happen for other more complicated groups than the symmetric group.

There are two aspects of Orbital Fixing that contribute to its outstanding performance. Firstly, the fact that symmetry breaking decisions are adjusted to branching decisions, not only in terms of orbit choices, but also regarding the order that is enforced in an orbit. Secondly, the structure of the formulation group is implicitly taken into account without the need for explicit group type computations. One disadvantage is the possibly large amount of stabilizer and orbit computations that is required. However, the experiments by Pfetsch et al. [38], indicate that this does generally to have a big impact in practice.

Even though Theorem 7.1 was originally shown for MIPs, it does not rely on the linearity of constraints. In MINLPs, the handling of symmetries in non-binary variables is of greater interest, since branchings are also performed on continuous variables. While the basic concept of Orbital Fixing could in principle be transferred to non-binary variables, there is a major obstacle involved: Branchings on non-binary variables do not result in fixings, but merely in bound changes. Consequently, it would not suffice to compute the *setwise* stabilizers of certain branchings, since they do not ensure that the respective variables have the same value. In other words, permutations mapping non-binary branching variables onto each other could potentially destroy the lexicographic order that we want to enforce. Instead, the use of *pointwise* stabilizers would be necessary, which is a much stronger restriction resulting in a loss of some of the advantages of Orbital Fixing.

Nonetheless, it is possible to develop a dynamic procedure that is based on the SBCs from Chapter 6 but makes decisions depending on the course of the solving process.

7.2 Generating global SBCs Dynamically

The main question we have to answer for developing a dynamic procedure is when and in which order symmetry should be broken. As we have seen, in certain cases one out of several orbits has to be chosen for which SBCs should be applied. If symmetry is handled during the solving process, the order which is enforced on the variables within that orbit is also relevant. We propose two different approaches.

Decisions based on branchings. One way to decide how to break symmetry is to use branching information. Since the objective of breaking symmetry is to avoid equivalent sub-trees in the sB&B tree, it seems reasonable that the branching decisions contain information about which symmetries could be more important to break than others.

Based on this idea, Algorithm 4 describes a procedure for dynamically generating SBCs which can be called at each node of the sB&B tree. In doing so, we have to remember which orbit was chosen and in which order the variables in the orbit were used to avoid adding conflicting inequalities. Additionally, we pass a symmetric subgroup H of the formulation group (if it exists), in order to be able to take advantage of Theorem 6.12. The idea is to choose an orbit of H according to some rule and then dynamically bring the variables in that orbit into an order. At each call of the algorithm at most one new variable is added to the order.

Lines 1 – 7 are initialization steps. Depending on whether there is a symmetric (sub-)group (encoded by $H \neq \emptyset$) or not, we consider the orbit of the current branching variable with respect to H or G_P . The first branching variable with a non-trivial orbit determines the orbit that is used for breaking symmetry, which is stored in Line 9. Whenever a new variable in the chosen orbit is branched on, Lines 11 – 21 are entered and the branching variable is implicitly given its place in the order by adding weak SBCs for all variables in the orbit that have not been used, i.e., that have not been branched on thus far. To this end, the symmetry breaking direction is determined by some yet to be defined procedure `determineBreakingDirection`. So depending on the direction, the newly found variable is either inserted as the next largest or the next smallest element in the order of the orbit. If $H = \emptyset$, we can only add those weak SBCs with respect to the first found variable. For that reason, all remaining variables are added to the set of used variables in Line 16. Otherwise, SBCs for the “enclosing” orbit (see Theorem 6.12) are added in Lines 17 – 19, when the first variable is added to the order. Note that, although only weak SBCs are added in each iteration, together they correspond to inequalities (6.3) and (6.4).

Algorithmus 4 : Dynamic generation of global SBCs

Input : G_P : formulation group of MINLP P ,
 H : symmetric subgroup of G (or \emptyset),
 ω : orbit that is used (or \emptyset),
 U : list of variables in ω that have been used,
 b : branching decision of this node

Output : S : set of SBCs that should be added to P

```

1  $S \leftarrow \emptyset$ 
2  $x \leftarrow b.var$ 
3 if  $H = \emptyset$  then
4    $G \leftarrow G_P$ 
5 else
6    $G \leftarrow H$ 
7 end
8 if  $\omega = \emptyset$  and  $|orb(x, G)| > 1$  then
9    $\omega \leftarrow orb(x, G)$ 
10 end
11 if  $orb(x, G) = \omega$  and  $x \notin U$  then
12    $d \leftarrow \text{determineBreakingDirection}(b)$ 
13    $U.add(x)$ 
14   add inequalities (6.1) for  $x$  and  $\omega \setminus U$  with direction  $d$  to  $S$ 
15   if  $H = \emptyset$  then
16      $U.add(\omega)$ 
17   else if  $|U| = 1$  then
18      $\theta \leftarrow orb(x, G_P)$ 
19     add inequalities (6.1) for  $x$  and  $\theta \setminus \omega$  with direction  $d$  to  $S$ 
20   end
21 end
22 return  $S$ 

```

The only question remaining is how the symmetry breaking direction is determined in Line 12. Since the involved variables are symmetric in the problem formulation, one might think that the direction does not matter. Once one of those variables is branched on, however, the situation is not symmetric anymore. After a branching, we would like the sub-problems to be equally hard to solve. One common indicator for this is the volume of polyhedra associated with relaxations of the sub-problems, see for example Lee et al. [24]. Based on this idea, we propose the following procedure.

Assuming for the sake of simplicity that all variables are continuous and bounded, we take a look at the box induced by the bounds of two symmetric variables x and y . A symmetry breaking inequality between x and y cuts off a triangle in the box, as shown in Figure 7.1. Depending on the breaking direction, either the upper or the lower triangle remains feasible. Now consider a branching $\{x \mid x \leq \alpha\} \cup \{x \mid x \geq \alpha\}$, where x is a variable with lower bound $\ell(x)$ and upper bound $u(x)$ and $\alpha \in [\ell(x), u(x)]$ is the branching point. It divides the triangle into two parts which is

depicted by the dashed vertical line in Figure 7.1. In case we have a non-symmetric relaxation when the branching is performed, α does not need to be the midpoint of the interval of x . If it is greater than $(\ell(x) + u(x)) / 2$, the pieces of the lower triangle are volume-wise more balanced than the ones of the upper triangle. The opposite is the case if α is smaller than $(\ell(x) + u(x)) / 2$. Thus, letting the symmetry breaking direction depend on the first relevant branching decision could potentially have an impact on the difficulty of the sub-problems. In the example of Figure 7.1 we would add the inequality $x \geq y$. Of course, the relaxation used in the sB&B process is probably smaller than the box, but the above consideration could still be used to heuristically decide how to break symmetry.

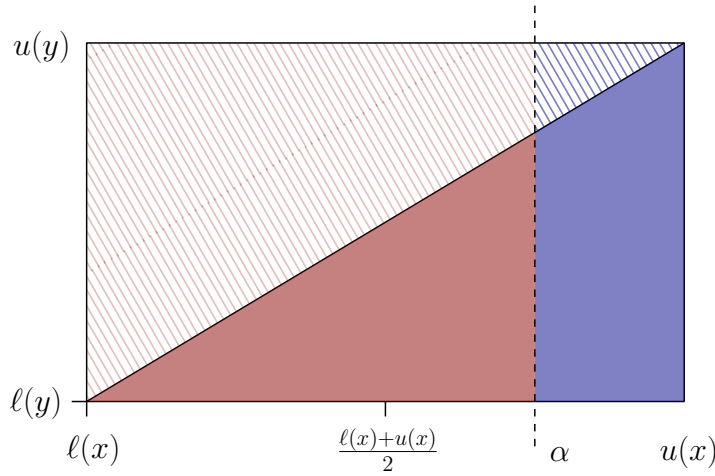


FIGURE 7.1: The direction of SBCs determine the size of the sub-problems. Here, α is greater than the midpoint of the interval of x . Using the inequality $x \geq y$ cuts off the upper triangle and results in more balanced sub-problems, depicted by the filled areas.

In the case of binary variables, branching means fixing a variable, so the above argument is not relevant. For general integer variables, however, the same principle can be applied, since the number of integral points in the sub-problems also depends on the volume of the triangle pieces.

Decisions based on LP solution. Another possibility is to let symmetry breaking decisions depend on the LP solutions. The idea is that symmetric variables might have different values in an LP relaxation, and so the choice about the order of the variables can actually make a difference. One approach could be to use the inequality which cuts off the LP solution, while the other possibility is to “follow” the suggested order. It is not clear which approach is more promising. In general, cutting off LP solutions can lead to progress in the solution process. On the other hand, we know that due to the symmetry there is at least one symmetric solution fulfilling any SBC we could add, so in the end we might be better off just

Algorithmus 5 : Dynamic generation of global SBCs based on LP solutions

Input : G_P : formulation group of MINLP P ,
 H : symmetric subgroup of G (or \emptyset),
 ω : orbit that is used (or \emptyset),
 U : list of variables in ω that have been used,
 x^* : current LP solution,
 cut : whether SBCs should try to cut off the LP solution

Output : S : set of SBCs that should be added to P

```

1  $S \leftarrow \emptyset$ 
2  $x \leftarrow b.var$ 
3 if  $H = \emptyset$  then
4    $G \leftarrow G_P$ 
5 else
6    $G \leftarrow H$ 
7 end
8 if  $\omega = \emptyset$  then
9    $\omega \leftarrow \text{chooseOrbit}(G)$ 
10 end
11  $X \leftarrow \omega \setminus U$ 
12 sort  $X$  non-increasingly by LP solution value
13  $x_{prev} \leftarrow X.first$ 
14  $U.add(x_{prev})$ 
15 for  $x_i \in X$  with  $x_i \neq x_{prev}$  do
16   if  $H = \emptyset$  then
17     add  $x_i \leq x_{prev}$  to  $S$ 
18      $U.add(x_i)$ 
19   else if  $x_i^* < x_{prev}^*$  then
20     if  $cut$  then
21       add  $x_i \geq x_{prev}$  to  $S$ 
22     else
23       add  $x_i \leq x_{prev}$  to  $S$ 
24     end
25      $U.add(x_i)$ 
26      $x_{prev} \leftarrow x_i$ 
27   end
28 end
29 return  $S$ 

```

using the order of the LP solution and hoping that subsequent nodes will be invalid because of the SBCs.

Algorithm 5 describes the procedure which works similarly to Algorithm 4 at each node, but uses the LP relaxation to make decisions instead of the branchings. In Line 9, the orbit on which to break symmetry is chosen according to some rule. We could, for example, take a random orbit on which not all variables have an equal LP solution value if we want to make sure that the LP solution is indeed asymmetric on the orbit. Then, in Line 11, a set X is built which contains all variables in the chosen orbit which have not been used thus far. It is sorted non-increasingly by the LP solution values in Line 12. The main loop (Lines 15 – 28) goes through the sorted list X and adds SBCs whenever distinct values are found. To this end, the algorithm provides a flag '*cut*' which encodes whether SBCs should be used to cutoff the LP solution whenever possible or whether they should preserve the order in the LP-relaxations.

Remark 7.3. In each iteration of Lines 15 – 28, only one variable for each set of equal solution values is used for the SBCs. The place of the other variables in the order is left open until later iterations. The disadvantage of this is that there is no connection between the variables used in subsequent iterations. One could just add all “undecided” variables to the end of the order and then still fix their relative order later on in the same way. We simply omitted this in the description of the algorithm to keep it more simple.

7.3 Separation Routine for SBC Inequalities

One disadvantage of Algorithm 4 is that the decisions from one sub-tree can determine the decisions made in another one. The reason is that *global* symmetry breaking constraints are added to the problem formulation. To see this, consider the following example.

Example 7.4. Assume that we have a MINLP P with 5 variables and formulation group $G_P = \langle (2\ 3)(4\ 5) \rangle$. Let the top part of the sB&B tree be as depicted in Figure 7.2. The first branching is performed on x_1 . In the left-hand sub-tree, the second branching variable is x_2 , while in the right-hand sub-tree, it is x_4 . Since x_1 does not appear in a non-trivial orbit of G_P , Algorithm 4 does not add any SBCs in the nodes corresponding to P_1 or P_2 . Assuming that P_3 is processed next, it finds that x_2 is in orbit $\omega_1 = \{2, 3\}$ and adds $x_2 \geq x_3$ (or $x_2 \leq x_3$) as a global constraint to the problem. Since the two orbits of G_P are dependent on each other, no further SBCs can be added. So despite x_4 being in the non-trivial orbit $\omega_2 = \{4, 5\}$, we cannot break the symmetry between x_4 and x_5 anymore. However, considering the arguments from the previous chapter, it might be more beneficial to use orbit ω_2 to break symmetry in the right-hand sub-tree.

In order to mitigate the described problem, we could use local symmetry breaking inequalities at each node instead of adding global constraints.

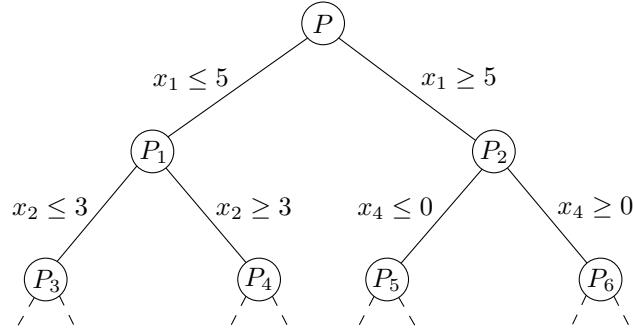


FIGURE 7.2: Example sB&B tree with branchings on continuous variables.

Algorithm 6 describes a corresponding separation routine. The orbit and order of the variables in the orbit are chosen in the same way as in Algorithm 4, based on the branching decisions. In Lines 9 – 25, two ordered sets U and L are built to represent that order. U contains the largest and L the smallest variables in the orbit. All variables in the chosen orbit that have not been branched on are not contained in either of the sets. Therefore, they lie somewhere between the last element of U and the last element of L (which is the largest element in the order of L). Afterwards, the order encoded by U and L is enforced by separation of symmetry breaking inequalities (including the ones corresponding to the enclosing orbit) in Lines 26 – 35. The difference to Algorithm 4 is that the choices in one node do not affect the choices made in any node that is on the same level but in a different sub-tree. Within one sub-tree, however, the decisions are still consistent, since the set of branchings at each node is a subset of the one at its parent.

Algorithmus 6 : Dynamic separation of local SBCs

Input : G_P : formulation group of MINLP P ,
 H : symmetric subgroup of G (or \emptyset),
 B : list of branching decisions of this node

Output : –

```

1   $\omega \leftarrow \emptyset$ 
2   $U \leftarrow \emptyset$ 
3   $L \leftarrow \emptyset$ 
4  if  $H = \emptyset$  then
5     $G \leftarrow G_P$ 
6  else
7     $G \leftarrow H$ 
8  end
9  for  $b \in B$  do
10    $x \leftarrow b.var$ 
11   if  $\omega = \emptyset$  and  $|orb(x, G)| > 1$  then
12      $\omega \leftarrow orb(x, G)$ 
13   end
14   if  $\omega = orb(x, G)$  then
15      $d \leftarrow \text{determineBreakingDirection}(b)$ 
16     if  $d = \geq$  then
17        $U.add(x)$ 
18     else
19        $L.add(x)$ 
20     end
21     if  $H = \emptyset$  then
22       break
23     end
24   end
25 end
26 if  $\omega \neq \emptyset$  then
27   separate ineq. (6.2) for  $U$  with dir.  $\geq$ 
28   separate ineq. (6.2) for  $L$  with dir.  $\leq$ 
29   separate ineq. (6.1) for  $U.last$  and  $\omega \setminus (L \cup U)$  with dir.  $\geq$ 
30   separate ineq. (6.1) for  $L.last$  and  $\omega \setminus (L \cup U)$  with dir.  $\leq$ 
31   if  $U \neq \emptyset$  then
32      $\theta \leftarrow orb(U.first, G_P)$ 
33     separate ineq. (6.1) for  $U.first$  and  $\theta \setminus \omega$  with dir.  $\geq$ 
34   end
35 end
36 return

```

Chapter 8

Computational Experiments

We performed several computational experiments in order to analyze symmetries in the MINLPLib [33] and evaluate the performance of the different techniques that were discussed in this thesis. The goal is to answer the following questions:

- Which instances in the MINLPLib contain permutation symmetry and how long does it take to compute them? What kind of symmetries do appear?
- Which are the best static and dynamic SBC variants? Does it pay off to break symmetry on continuous variables at all?
- For binary instances, can any of the methods keep up with Orbital Fixing?
- Can we find any (quadratic) instances in the MINLPLib with complementary symmetry?
- Does the performance improve if these are handled additionally?

To this end, we implemented the described algorithms in C as plug-ins for the constraint integer programming solver SCIP [44], originally developed by Achterberg [3]. In Section 8.1, we provide more detailed information about the implementation and software that was used. Afterwards, Section 8.2 covers the analysis of symmetries in the MINLPLib and Section 8.3 reports on the performance of the different methods. In these two sections we will only deal with permutation symmetries and not mention this at each point again. Section 8.4 then deals with experiments for complementary symmetry.

The computations were performed on a Linux cluster of 64bit Intel(R) Xeon(R) CPU E5-2690 v4 2.60GHz with 35MB cache and 128GB main memory. On each computer, only a single process was run at a time to avoid a mutual slow due to parallel processing which makes results incomparable. The code was compiled with gcc 7.4.0 using -O3 optimization.

All reported running times are in seconds and the time limit for each computation was 1 hour. Whenever an instance ran into the time limit, it was evaluated as 3600 seconds. When reporting aggregated results, we use

the *shifted geometric mean*. For values a_1, \dots, a_k and shift s it is given by

$$\left(\prod_{i=1}^k (a_i + s) \right)^{1/k} - s.$$

The purpose of the shift is to reduce the otherwise strong impact of easy instances on the mean values, see Achterberg [2]. We use a shift of $s = 1$ for time and $s = 100$ for nodes.

8.1 Implementation Details

The implementation is integrated into a development version of SCIP 6.0.1 which uses expression grahs as described in Section 4.2. It already contains a plug-in for symmetry detection in MIPs, which has been developed by Pfetsch et al. [38] and integrated into SCIP since version 5.0 [15]. We extended this plug-in to also detect permutation symmetries in MINLPs, as described in Section 4.3. The static symmetry breaking methods from Chapter 6 are implemented in a separate presolver while the dynamic variants are realized in a constraint handler. For all computations, the *component* constraint handler (`cons_components`) was turned off to avoid interactions with the symmetry handling code that could distort the results. We used SCIP with SoPlex 4.0.1 [46] as LP solver, Ipopt 3.12.11 [10] as NLP solver, as well as CppAD 20180000.0 [9] and bliss 0.73 [21].

In the following, we give more details about the individual modules of the implementation.

Symmetry Detection. There are two possibilities for the timing of detecting symmetry: before or after presolving. In theory, presolving could both eliminate or introduce symmetries. For this reason, we performed both experiment to compare the results. It should be noted that we cannot detect symmetries of MINLPs directly on the formulation level, since we need the expression graphs for nonlinear constraints. In SCIP, they are constructed during the first presolving step. When we say “before” presolving, we actually mean as soon as the expression graphs are available.

The graph automorphisms are computed using bliss 0.73 [21]. The graph is constructed in the following order: First, all variable nodes are created, then it is extended according to the linear constraints as described in Section 4.1, and lastly, the structure of the expression graphs is included to capture all nonlinear constraints. Following the experiments done by Pfetsch et al. [38], we use either grouping by variables or by constraints for the intermediate vertices, depending on which method produces the smaller graph. Also, the number of generators is limited to 1500 to avoid exploding computation times.

Group Type Analysis. We used the C++ library `PermLib` [41] to compute the factors of all formulation groups and determine the type of group action they represent, whenever possible.

Heuristic for Symmetric Groups. Hojny [16] suggested a heuristic which can detect coordinate and matrix actions if the generators of a group are given in a particular “nice” form. It searches for sets of generators that are compositions of disjoint 2-cycles and checks whether they fulfill certain properties. We use and extend this heuristic in two ways.

Firstly, we take the inclusion-wise largest subset of generators of a component that fulfills the above criteria. If several such sets exist, we stick to the one with the largest number of 2-cycles. We then apply the heuristic, possibly detecting subgroups that are coordinate or matrix actions. This allows us to use the results from Section 6.3.

Secondly, whenever the previous approach failed, we go one step further and detect whether the subset generates a subgroup which is isomorphic to a symmetric group when *projected to a single orbit*, see Remark 6.13. So if $\{\pi_1, \dots, \pi_k\}$ is a subset of generators which are compositions of disjoint 2-cycles, we try to find an orbit ω of the subgroup $H = \langle \pi_1, \dots, \pi_k \rangle$ such that $H|_{\omega} \cong \mathcal{S}_{|\omega|}$. The original heuristic also goes over all orbits and checks the required properties and whether they fit together. Therefore, just a slight modification is needed to achieve this.

Orbitopes. Matrix actions on binary variables can be handled by orbitopes, see Kaibel et al. [22]. Since SCIP version 5.0 [15], it includes a constraint handler which implements propagation and separation algorithms for orbitopes by Kaibel et al. [23], Bendotti et al. [4], and Hojny et al. [17]. The situations in which we use this plug-in are described below.

Symmetry Breaking Constraints. In all versions, the factors of the formulation group are computed and the respective algorithm is executed for each factor separately. For the static variants, the implementation provides the possibility to decide whether binary or continuous variables should be prioritized, whenever the algorithm has to choose between several orbits. In both cases, integer variables come after the binary ones in the priority order. If the choice has to be made between orbits of different length, the largest one is used in order to maximize the number of SBCs generated. Furthermore, probing is applied to the largest and smallest variable, whenever we add SBCs for an orbit containing binary variables.

For the implementation of Algorithm 6, we handle matrix actions in a special way that depends on the affected variables. If it acts solely on binary variables, it can be handled completely by the orbitope constraint handler (see above). We expect the propagation and separation routines for orbitopes to be stronger in that case, since our general purpose SBCs do not exploit the special structure of such actions. If a matrix action moves binary and non-binary variables simultaneously, we first order the rows of the corresponding matrix. In Section 3.1, we saw that each row of the

matrix contains exactly the variables of one of the orbits, so either binary or non-binary variables exclusively. Now the sub-matrix of all binary rows is handled as if it was an orbitope. Algorithm 6 is then used for the non-binary rows, but only if all variables have been (locally) fixed to the same value. Otherwise we would use dependent orbits at the same time which is not feasible. Consequently, binary variables are always prioritized and Algorithm 6 will mainly be active on factors with no binary symmetry.

Continuous variables that appear exclusively in linear terms (possibly within nonlinear constraints) are ignored in all of the variants. The reason is that SCIP never branches on such variables, so there is nothing to gain by breaking symmetry on them.

Orbital Fixing. For Orbital Fixing, we use the propagation plug-in already included in SCIP, implemented by M. Pfetsch [15].

8.2 Results for Computing Symmetry

Since the effectiveness of any symmetry handling method is dependent on the amount of symmetry that is detected, we firstly analyzed the formulation groups of all 1632 instances in the MINLPLib before and after presolving, taking only permutation-based symmetry into account.

Table 8.1 shows the total number of symmetric instances, mean number of generators and mean running times. There are 352 instances initially containing formulation symmetry. After presolving, the number reduces to 261. Thus, presolving alone already eliminates a significant percentage of formulation symmetries. This can also be seen in the reduced number of generators after presolving. In 35 occasions, however, there are more generators after presolving. For 17 of those cases, it even introduced symmetry to previously non-symmetric instances (e.g., `kissing2`). The computation times for symmetry detection are in general very low with a shifted geometric mean of less than 0.2 seconds. This is not unexpected considering that MINLPs are usually not very large compared to MIPs, for which not much time was needed during the experiments by Pfetsch et al. [38], either. For instances without formulation symmetry, the same observation can be made. The only instances that ran into the time limit during group type analysis were `cont6-qq`, `watercontamination0202`, and `watercontamination0303`. This is surprising, since the time needed during this step for all other instances never exceeded 4 seconds. When comparing the results with the groups found by Liberti [25], note that the formulation group depends on the way the expression DAGs are constructed, so different implementations can lead to different results.

The following instances are not included in Table 8.1. On `hadamard_9`, the time limit has already been reached during construction and simplification of expression graphs and was therefore left out of both runs. As for the second row of the table, `pedigree_sim2000` and `truck` hit the time limit during presolving, while `st_test1`, `st_test2`, `st_test3`, `st_test5`,

TABLE 8.1: Results for symmetry detection in MINLPLib (1632 instances), split into instances with, and without symmetry. Shown are the shifted geometric means of the time in seconds to compute graph automorphisms (graph time) and to compute the group types with permlib (group time), the geometric mean of the number of generators (#gens), and the number of instances that ran into the time limit during symmetry computation (#limits)

timing	with symmetry				without symmetry		
	#	graph time	group time	#gens	#	graph time	#limits
before presol	350	0.12	0.04	6.2	1,278	0.04	3
after presol	261	0.09	0.01	3.6	1,357	0.10	0

`st_test6`, `ex9_2_8`, `nemhaus`, `hybriddynamic_fixed`, `st_miqp1`, `st_bp1`, and `st_bp2` were all solved before symmetry computation was started.

For all affected instances we then tried to identify the types of the group actions using `PermLib`. The individual factors together with the order of the group as well as the number of affected variables are depicted in Table A.1 (before presolving) and Table A.2 (after presolving). The order of the groups ranges from 2 to $10^{3416.2}$ and the percentage of affected variables from less than 0.1% to 100%.

Before presolving, there are 92 instances for which at least one factor could not be identified by `PermLib`. Furthermore, 8 instances contain a so-called *wreath product*, denoted by the symbol ' \wr '. An explanation of wreath products as well as inequalities handling them in the case of binary variables can be found in Hojny [16]. Those inequalities rely on the linearity of the problem and cannot be extended to MINLPs. Since we are not aware of any method to handle wreath products in the general setting, We did not cover this type of group. All other factors are coordinate and matrix actions, often of small size.

After presolving, the situation is as follows: 47 instances with unknown factors remain, 3 groups contain a wreath product and there are two instances, namely `color_lab3_3x0` and `torsion100`, with a *Klein four-group*, which we also did not cover. Again, all other factors are coordinate and matrix actions.

We then tested the heuristic for detecting symmetric groups that was described in the previous section. Interestingly enough, it was able to find every single coordinate and matrix action. An explanation for this performance is that `bliss` seems to represent such groups by exactly the simple form that the heuristic expects. Based on this insight, all further experiments used the heuristic instead of `PermLib`. This also allowed us to detect the “hidden“ symmetric groups (see Sec. 6.3) by using the extended heuristic.

8.3 Results for Handling Symmetry

Having analyzed which symmetries are present in the MINLPLib, we now report on the performance of the symmetry handling methods that were discussed. For all of the following experiments, we computed the formulation group *after* presolving. The reason is that doing so beforehand could result in constraints which become redundant or even infeasible due to

presolving. For static variants, SBCs are therefore added after the group computation and directly before the sB&B process begins.

From a theoretical point of view, we expect that breaking symmetry speeds up the solving process since the size of the feasible region is reduced. However, in practice it is possible that the inequalities have no effect due to decisions made in other parts of the solver, but still increase the size of LP-relaxations, thereby slowing down the computation times.

The Testset. On 28 out of the 261 symmetric instances, SCIP crashed or returned a wrong solution/bound for at least one of the settings due to symmetry unrelated procedures. These are

arki0024	chp_shorttermplan2d	ex8_3_12	ex8_3_14
ex8_6_2	gasnet_al1	gasnet_al2	gasnet_al3
gasnet_al4	gasnet_al5	gastrans135	iswath2
jbearing100	jbearing25	jbearing50	jbearing75
kport40	nd_netgen-2000-3-4-b-a-ns_7	nuclear104	oil
routingdelay_bigm	routingdelay_proj	steenbrf	torsion75
transswitch2383wpp	watercontamination0303	waterno2_01	waterno2_02

Our testset consists of the remaining 233 instances and is denoted by **M-sym-all**. We also report on results for the following subsets:

- **M-sym-bin**: 121 instances in **M-sym-all** for which at least one binary variable is affected by symmetry
- **M-sym-int**: 135 instances in **M-sym-all** for which at least one integer *or* binary variable is affected by symmetry
- **M-sym-onlycont**: 98 instances in **M-sym-all** for which *no* integer or binary variable is affected by symmetry
- **M-sym-unknown**: 49 instances in **M-sym-all** with at least one group factor that is not a coordinate or matrix action.

Note that **M-sym-unknown** also contains the instances with wreath and Klein groups. The reason is that they do not allow for strong SBC and we do not have any other structure specific method to handle them.

As our basis of comparison we use SCIP with default settings where no symmetry is handled. It is denoted by **default**. For all runs, we used a gap limit of 10^{-4} .

Experiment with Static Algorithms. The first performance experiment compares three different static SBC variants, namely

- **S**: Algorithm 2 with priority in binary variables
- **S-cont**: Algorithm 2 with priority in continuous variables
- **S-orbmax**: only adding weak SBCs (6.1) for each orbit with priority in binary variables.

TABLE 8.2: Comparison of different static variants. Depicted are the shifted geometric means of the number of B&B nodes (`#nodes`) and CPU time in seconds (`time`), the number of instances solved to optimality (`#opt`), the shifted geometric mean of the remaining gap in % (`gap`), the number of instances in which SBCs were added (`#act`), the number of full symmetric groups (`#full`), symmetric subgroups (`#sub`), and symmetric groups on single orbits (`#orb`) found, the total number of SBCs added (`#sbcs`), and the shifted geometric mean of the time used for detecting and handling symmetry (`sym-time`).

setting	#nodes	time	#opt	#act	#full	#sub	#orb	#sbcs	sym-time
M-sym-all(233):									
default	2,062	253.4	109	-	-	-	-	-	-
S	1,691	231.6	114	218	349	19	19	8,372	0.15
S-cont	1,745	236.3	112	218	349	19	19	8,372	0.12
S-orbmax	2,114	251.7	111	218	-	-	-	8,559	0.12
M'sym-bin(121):									
default	3,503	464.6	47	-	-	-	-	-	-
S	2,974	410.2	51	120	102	6	8	3,859	0.19
S-cont	3,154	426.2	49	120	102	6	8	3,859	0.13
S-orbmax	3,622	450.0	49	120	-	-	-	3,882	0.12
M-sym-int(135):									
default	3,359	318.1	58	-	-	-	-	-	-
S	2,576	263.4	63	134	105	6	8	3,932	0.17
S-cont	2,716	272.5	61	134	105	6	8	3,932	0.12
S-orbmax	3,333	303.2	60	134	-	-	-	3,955	0.11
M-sym-onlycont(98):									
default	1,032	185.1	51	-	-	-	-	-	-
S	930	194.1	51	84	244	13	11	4,440	0.13
S-cont	930	194.1	51	84	244	13	11	4,440	0.13
S-orbmax	1,110	194.8	51	84	-	-	-	4,604	0.13
M-sym-unknown(49):									
default	9,187	1588.2	8	-	-	-	-	-	-
S	5,519	1431.7	9	49	-	19	19	2,729	0.20
S-cont	5,515	1432.2	9	49	-	19	19	2,729	0.20
S-orbmax	7,757	1502.8	9	49	-	-	-	2,953	0.17

TABLE 8.3: Comparison of static variants on instances solved to optimality by all settings.

setting	M-sym-all(109)		M-sym-bin(47)		M-sym-int(58)		M-sym-onlycont(51)	
	#nodes	time	#nodes	time	#nodes	time	#nodes	time
default	368	11.5	1,180	17.6	947	11.8	87	11.1
S	306	10.6	831	14.0	645	9.3	104	12.3
S-cont	312	10.8	864	14.8	666	9.7	104	12.3
S-orbmin	351	11.3	1,077	16.2	799	10.5	105	12.4

Table 8.2 shows the results. The first thing to note is that **S** solves 5 more instances of **M-sym-all** than the default setting. The computation time is also reduced by about 9%. A Wilcoxon signed rank test, see Berthold [5], confirmed this reduction to be statistically significant with a p -value of less than 0.005. The time needed for all symmetry related computations is very low compared to the total solving time. On 15 instances, the algorithm does not add any constraints. One of them is **ball_mk3_10** which is detected to be infeasible during presolving. Due to the timing structure of SCIP, the symmetry detection is still called, but the solving process terminates, before the SBCs are added. The other 14 instances contain only symmetries affecting continuous variables that appear solely in linear constraints.

Comparing **S** with **S-cont**, we can see that they show similar computation times and node numbers, but the first variant is able to solve 2 more instances. This is an indication that breaking symmetry on binary variables could be more important than on continuous ones. Variant **S-orbmax**

TABLE 8.4: Comparison of strong and weak SBCs on 9 unknown group instances solved to optimality by both settings, run for 10 permutations. Depicted are the shifted geometric means of the number of B&B nodes (`#nodes`) and CPU time in seconds (`time`) and the number of instances solved to optimality on all permutations (`#solved`).

setting	#nodes	time	#solved
S	1,294	20.3	9
S-orbmax	2,050	27.0	8

solves 3 instances less than **S** and does not achieve a significant decrease in mean computation time or number of nodes over **default**, while still solving 2 instances more. We conclude that detecting symmetric groups and handling them by strong SBCs is an important factor for the performance of symmetry breaking methods for MINLPs. Interestingly, the experiments by Pfetsch et al. [38] found the opposite to be the case for MIPs.

The results for **M-sym-bin** and **M-sym-int** do not change the picture much. One interesting thing to point out here is that out of the 5 instances that can only be solved when using SBCs, 4 contain symmetric binary variables while the last one, namely **ball_mk2_30**, has only non-binary integer variables. For that instance, the default setting ran into the time limit, while variant **S** was able to solve it in less than a second. Testing this again on 10 permutations showed that the behaviour is consistent.

Now turning to **M-sym-onlycont**, we observe that the computation time of all static symmetry breaking variants was larger than for **default**, while the mean number of nodes decreased for **S** and **S-cont**. The latter is not necessarily a sign of improvement, though, since it could be the case that simply less nodes were processed for instances running into the time limit. Table 8.3 presents a comparison of the mean number of nodes and time for the first four testsets, restricted to instances that were solved to optimality by all settings. Here we can see that the number of nodes for **M-sym-onlycont** actually increases significantly for all SBC variants and the computation times are also larger by about 10%. These results, however, are not statistically significant. It seems that, in general, adding static SBCs for continuous variables does more harm to the solving process than good by reducing the tree size.

Table 8.3 also confirms the observations made for integer and binary symmetries. Variant **S** significantly reduces the number of nodes and time, being slightly faster than **S-cont**, while **S-orbmax** shows little improvement over the default setting.

Finally, we examine **M-sym-unknown**. Here we are mostly interested in comparing **S** and **S-orbmax**, since we want to evaluate the effectiveness of the inequalities in Theorem 6.12. The heuristic found 19 symmetric subgroups and 19 subgroups that were isomorphic to a symmetric group when projected onto a single orbit. Each instance contains at most one of these, so for 11 instances in **M-sym-unknown**, the behaviour of **S** and **S-orbmax** is the same. Table 8.2 shows that variant **S** uses less time and nodes overall. However, the testset seems to be quite hard overall, with only 9 of the 49 instances being solved. To get a more meaningful comparison, we repeated the experiment for all 9 solved instances with 5 different permutations.

The results are depicted in Table 8.4. Here, variant **S** needs about 25% less time and about 37% less nodes on average. For one of the permutations, **S-orbmax** also fails to solve one of the instances. So despite the relatively small size of the sample, there is an indication that detecting symmetric sub-groups and using the refined SBCs from Theorem 6.12 improves the performance of such symmetry breaking techniques.

Experiment with Dynamic Algorithms. In a second performance experiment, we compared the following dynamic variants:

- **D**: Algorithm 4
- **D-lp**: Algorithm 5 with `cut = true`
- **D-lpf**: Algorithm 5 with `cut = false`
- **D-sepa**: Algorithm 6
- **D-sepa***: Algorithm 6, but breaking symmetry in the opposite direction **D-sepa**

The results are presented in Tables 8.5 and 8.6. All variants manage to solve more instances than the default setting. It is 1 more for the algorithms that make decision based on branchings (**D**, **D-sepa**, and **D-sepa***) and 2 for the LP-based variants (**D-lp**, **D-lpf**). It should be noted that there are instances which are only solved by the first type and not by the second, and vice versa (see Appendix B). So it seems that, depending on the structure of the problem, either approach can be more beneficial.

We observe that there is no noticeable difference between **D-sepa** and **D-sepa***. While the latter variant detect more cutoffs, they do not result in shorter computation times or less number of nodes. This indicates that the approach behind the branching-based procedure described in Section 7.2 does not have an impact on the strength of the inequalities. Both methods are not able to improve on the default setting for **M-sym-bin**. It seems that for the three instances which are only solved by **D**, **D-lp**, and **D-lpf**, having the SBCs as global constraints and therefore being able to use that information in other places during the solving process is an important aspect.

Considering only the instances solved by all settings in Table 8.6, we see that all dynamic variants reduce the time and number of nodes in comparison to **default**. Interestingly, the numbers for the different dynamic algorithms are all similar. Looking at the different subsets of **M-sym-all**, we again observe that breaking symmetry on binary and integer variables clearly improves the performance while doing so on continuous variables is not beneficial. The computation times of all variants were about 34% smaller for **M-sym-bin** and about 32% smaller for **M-sym-int** compared to the default setting. According to the Wilcoxon signed rank test, these results are statistically significant with a *p-value* of less than 0.0005. For the testset **M-sym-onlycont** on the other hand, the time and number of nodes are larger than for **default**, just as we observed in the static case. The

TABLE 8.5: Comparison of different dynamic variants. Depicted are the shifted geometric means of the number of B&B nodes (`#nodes`) and CPU time in seconds (`time`), the number of instances solved to optimality (`#opt`), the shifted geometric mean of the remaining gap in % (`gap`), the geometric means of the number of domain reductions performed (`#red`) and the number of node cutoffs detected (`#cutoff`), the total number of global SBCs added (`#sbc`), and the shifted geometric mean of the time used for the method including symmetry computation (`sym-time`).

setting	#nodes	time	#opt	#red	#cutoff	#sbc	sym-time
M-sym-all(233):							
default	2,062	253.4	109	-	-	-	-
D	1,868	248.3	110	-	-	7,757	0.10
D-lp	1,898	247.1	111	-	-	5,765	0.10
D-lpf	1,858	243.7	111	-	-	6,310	0.10
D-sepa	1,512	248.5	110	0.4	0.1	-	0.10
D-sepa*	1,505	247.4	110	0.4	0.3	-	0.10
M-sym-bin(121):							
default	3,503	464.6	47	-	-	-	-
D	2,749	394.0	50	-	-	55	0.09
D-lp	2,778	391.2	50	-	-	84	0.09
D-lpf	2,792	390.7	50	-	-	83	0.09
D-sepa	2,031	438.4	47	0.9	0.1	-	0.09
D-sepa*	2,037	439.4	47	0.9	0.1	-	0.09
M-sym-int(135):							
default	3,359	318.1	58	-	-	-	-
D	2,647	271.4	61	-	-	113	0.08
D-lp	2,679	269.4	61	-	-	101	0.08
D-lpf	2,689	269.5	61	-	-	100	0.08
D-sepa	1,854	281.1	59	0.7	0.1	-	0.08
D-sepa*	1,892	282.8	59	0.7	0.4	-	0.08
M-sym-onlycont(98):							
default	1,032	185.1	51	-	-	-	-
D	1,143	219.6	49	-	-	7,644	0.12
D-lp	1,168	219.4	50	-	-	5,664	0.13
D-lpf	1,103	212.2	50	-	-	6,210	0.13
D-sepa	1,137	209.8	51	-	0.1	-	0.12
D-sepa*	1,091	205.7	51	-	0.3	-	0.13

TABLE 8.6: Comparison of dynamic variants on instances solved to optimality by all settings.

setting	M-sym-all(103)		M-sym-bin(43)		M-sym-int(54)		M-sym-onlycont(49)	
	#nodes	time	#nodes	time	#nodes	time	#nodes	time
default	317	10.5	903	15.4	749	10.3	90	10.7
D-sepa	232	9.4	435	10.1	385	7.0	119	12.8
D	234	9.3	439	10.2	394	7.1	117	12.5
D-lp	243	9.4	443	10.0	400	7.0	127	13.0
D-lpf	227	9.1	439	10.0	394	7.0	107	12.1
D-sepa*	228	9.3	438	10.2	401	7.1	106	12.3

differences are again not statistically significant. We still conclude that, despite their ability to adapt to the solving process, dynamic variants also do not benefit from breaking symmetry on continuous variables.

Overall, we find that the variants which add global constraints (D, D-lp, and D-lpf) seem to be superior. On the one hand, they solve less instances and are slower for M-sym-onlycont, but on the other hand, this is the testset where default showed the best performance and the global variants solved more instances on the integer testset. Apart from the few (un-)solved instances, the differences are not very big, as Table 8.6 showed. So again, a more refined implementation could change this picture.

The times for the symmetry computations are relatively low compared to the total time and are only noticeable for very large instances that are

TABLE 8.7: Comparison of the best static and dynamic variants and orbital fixing. Depicted are the shifted geometric means of the number of B&B nodes (`#nodes`) and CPU time in seconds (`time`), the number of instances solved to optimality (`#opt`), the shifted geometric mean of the remaining gap in % (`gap`), and the shifted geometric mean of the time used for detecting and handling symmetry (`method-time`).

setting	#nodes	time	#opt	method-time
M-sym-all(233):				
default	2,062	253.4	109	-
S	1,691	231.6	114	0.15
D-lpf	1,858	243.7	111	0.10
orbfix	1,903	231.5	113	0.23
M-sym-bin(121):				
default	3,503	464.6	47	-
S	2,974	410.2	51	0.19
D-lpf	2,792	390.7	50	0.09
orbfix	2,822	387.9	51	0.31

TABLE 8.8: Comparison of different variants on instances solved to optimality by all settings.

setting	M-sym-all(108)		M-sym-bin(47)	
	#nodes	time	#nodes	time
default	373	11.4	1,180	17.6
S	311	10.5	831	14.0
D-lpf	293	10.3	626	11.9
orbfix	280	9.7	644	12.1

not solved anyway. Putting all results together, the LP-based variants **D-lp** and **D-lpf** are the best on average.

Comparison of Best Variants. For the last experiment of this section, we took the best performing variants from the two previous experiment and compared them with each other. In addition to this, we wanted to see how they perform on binary instances in comparison to Orbital Fixing (**orbfix**), see Section 7.1. The results can be found in Tables 8.7 and 8.8. On the whole testset, **S** is able to solve the most instances, with 1 more than Orbital Fixing. However, this instance is **ball_mk2_30**, which does not have binary variables, so **orbfix** is not active anyway. **D-lpf** solves 3 instances less than **S**.

Turning to the results on all solved instances in Table 8.8, we see that **orbfix** clearly outperforms **S**, in particular when restricting to binary symmetries. **D-lpf**, on the other hand, can keep up with Orbital Fixing and even shows a slightly lower mean time for **M-sym-bin**. The least we can say is that it is competitive on this testset.

We conclude that none of the examined symmetry breaking methods is able to use information about symmetries on continuous variables to reduce computation times. In all other cases, it seems that a combination of Orbital Fixing for binary variables and a variant of symmetry breaking constraints for general integer variables is the best approach overall. For the latter one, adding static symmetry breaking constraints is the best option, albeit there are some hints that further refinements in the LP-based dynamic procedure could lead to an overall better algorithm.

Detailed results for each instance in all experiments of this section can be found in Appendix B.

8.4 Results for Complementary Symmetry

For the last part of the computational study, we tested the algorithm for detecting complementary symmetry in MIQCPs described in Chapter 5. The testset consisted of the 861 quadratic instances in the MINLPLib that do not have more than 10,000 variables or constraints. For each of them, we used PySCIPOpt [28, 40] to construct and solve the auxiliary problem, and to compute the components of the solution, i.e., which variable complementations are dependent of each other. The results are presented in Table 8.9. The algorithm detected 18 instances with complementary symmetry. The proportion of variables that are affected ranges from 6.5% to 100% and the number of components from 2 to 53, although in all except one case, there are not more than 8. We can also see that the computation times for solving the auxiliary problem are quite low. Note that the mean CPU time in the last line of the table includes all instances without complementary symmetry. The meaning of the remaining columns will be explained below.

TABLE 8.9: List of instances with complementary symmetry found in the MINLPLib. Also depicted are the total (#) and relative (%) number of variables affected by this symmetry, the number of independent components that the variables are partitioned in (#comps), the number of generators of the formulation group before (orig) and after (mod) modification of the problem, and the CPU time for solving the auxiliary problem (time). The last line shows the shifted geometric mean of the detection time, including instances without complementary symmetry.

name	affected vars		#comps	#perm-gens		time
	#	%		orig	mod	
eigena2	2500	100.0	53	49	43	0.40
ex14_1_6	3	33.3	2	0	0	0.00
himmell6	12	66.7	2	0	0	0.00
knp3-12	36	97.3	4	13	10	0.00
knp4-24	96	99.0	6	26	21	0.00
knp5-40	200	99.5	8	43	38	0.00
knp5-41	205	99.5	8	44	39	0.00
knp5-42	210	99.5	8	45	40	0.00
knp5-43	215	99.5	8	46	41	0.10
knp5-44	220	99.5	8	47	42	0.00
ringpack_10_1	10	14.3	2	0	0	0.00
ringpack_10_2	10	12.5	2	1	1	0.00
ringpack_20_1	20	9.3	3	0	0	0.30
ringpack_20_2	20	8.5	3	1	1	0.40
ringpack_20_3	40	15.8	4	18	14	0.40
ringpack_30_1	30	6.9	4	0	0	1.10
ringpack_30_2	30	6.5	4	1	1	1.30
st_robot	3	37.5	2	0	0	0.00
total average:						0.14

As a second step, we modified each problem by taking one variable per component and setting its upper bound to the midpoint of its interval. As explained, in Chapter 5, a complementary symmetry on a variable x can be viewed as a classical symmetry through the permutation $(x \bar{x})$. Restricting the variable to the lower half of its interval corresponds to adding the inequality $x \leq \bar{x}$. Thus, it is a valid symmetry breaking inequality as long as we use not more than one variable per component. Since complementary symmetry and permutation symmetry can interact with each other, in the sense that they affect the same variables, changing the bounds in the

TABLE 8.10: Comparison of variant **S** on the original and modified versions of instances containing complementary symmetry. The experiment was performed for 10 permutations. Depicted are the number of solved instances across all permutations and the arithmetic means of the number of nodes (#nodes), the CPU time in seconds (time), and the remaining gap at the time limit (gap) for each instance. The last line shows the shifted geometric means of the number of nodes and the CPU time over all instances.

name	original				modified			
	#opt	#nodes	time	gap	#opt	#nodes	time	gap
eigena2	0	1	3,600.0	>10 ⁴	1	1	3,503.0	>10 ⁴
ex14_1_6	10	1	0.5	0.0	10	1	0.5	0.0
himmel16	10	2,272	7.1	0.0	10	702	3.1	0.0
knp3-12	0	86,374	3,600.0	137.2	0	98,018	3,600.0	130.3
knp4-24	0	4,774	3,600.0	405.6	0	7,611	3,600.0	327.8
knp5-40	0	495	3,600.0	706.9	0	464	3,600.0	624.6
knp5-41	0	515	3,600.0	729.8	0	304	3,600.0	683.3
knp5-42	0	257	3,600.0	782.7	0	334	3,600.0	683.7
knp5-43	0	392	3,600.0	787.0	0	529	3,600.0	668.2
knp5-44	0	313	3,600.0	805.5	0	393	3,600.0	697.0
ringpack_10_1	0	115,130	3,600.0	15.9	0	116,390	3,600.0	27.4
ringpack_10_2	0	114,498	3,600.0	7.3	0	116,414	3,600.0	8.5
ringpack_20_1	0	11,984	3,600.0	214.6	0	14,688	3,600.0	195.5
ringpack_20_2	0	9,930	3,600.0	450.0	0	15,242	3,600.0	422.9
ringpack_20_3	0	23,513	3,600.0	74.9	0	28,714	3,600.0	69.9
ringpack_30_1	0	8,225	3,600.0	>10 ⁴	0	10,840	3,600.0	>10 ⁴
ringpack_30_2	0	5,668	3,600.0	>10 ⁴	0	8,575	3,600.0	405.2
st_robot	10	1	0.5	0.0	10	1	0.5	0.0
total		2,577	1,080.0			2,759	1,037.5	

described way can change the formulation group of the problem. For this reason, we computed the formulation group for the original and modified problem. Table 8.9 shows the number of generators in each case. We observe that 12 of the instances contain permutation symmetry. Despite the fact that the number of generators decreases with the modification for 9 of them, it is always a relatively small reduction.

We then solved 10 permutations of the original and modified problems with SBC variant **S**. Table 8.10 compares the results. The testset is overall quite hard, with 14 of the 18 instances not being solved in either case. For one permutation of **eigena2**, however, the modified problem could be solved, while the original one could not. The table also shows the remaining gaps for instances that ran into the time limit. Here we see that in all but two cases, the gap consistently decreases for the modified version, even though not by much.

The results are neither consistent nor strong enough to prove the effectiveness of the applied method. Nonetheless, they serve as a hint that handling complementary symmetry can be beneficial in certain cases, particularly when considering the small amount of affected variables in the present instances. Our experiment should be viewed as a small motivating step towards more research on this topic. It is entirely unclear, for example, how one should handle the case in which a problem contains both types of symmetries on the same variables. We simply chose an obvious and easy to implement approach.

Chapter 9

Conclusion and Outlook

In this thesis, we studied nonconvex mixed-integer nonlinear programs. Solving such problems with spatial branch-and-bound can be difficult if they contain a large amount of symmetric solutions. In order to avoid the exploration of equivalent sub-trees, it is possible to use automatic procedures to detect and break symmetries.

Recap and Results We first described the different types of symmetries that can be found in MINLPs and explained how permutation-based symmetry can be detected by computing the formulation group. Then we developed a new procedure for detecting complementary symmetry in MIQCPs (Chapter 5). Its key component is an auxiliary problem which models a coefficient comparison before, and after replacing variables with their complements. Solving it gives the largest set of possible complementations.

In terms of symmetry handling, our focus was on classical symmetries defined by means of permutations. Section 6.3 presented new improved symmetry breaking inequalities. If the formulation group does not act in a well-understood way, but one of its subgroups is isomorphic to a symmetric group, we can use these inequalities to enforce a total order on a subset of the affected variables. They can be used in any static method in addition to previously existing symmetry breaking constraints.

In Chapter 7, we developed several dynamic variants of the SBC routines. Inspired by the most successful symmetry breaking methods for MIPs, they try to improve their performance by adapting to the solving process. Since symmetry cannot be broken simultaneously for different orbits that are dependent on each other, the idea behind dynamic methods is to postpone the respective decisions until more information is available. The proposed algorithms use either the LP solution or branching information at each node of the sB&B tree to decide how to break symmetry.

We performed a variety of computational experiments that gave several insights. Firstly, SBCs are a competitive symmetry breaking method for MINLPs, even though they do not perform as good as Orbital Fixing for binary variables. Furthermore, breaking symmetry on integer variables is an important factor for certain problems arising in practice. Since extending Orbital Fixing or other binary methods to the general integer case is not an easy task, SBCs can serve as a good instrument to handle this. We have found no indication that breaking continuous symmetries speeds up

the sB&B process. It is not clear what the reason is, since such symmetries can lead to significantly larger trees for MINLPs in exactly the same way as integer symmetries.

The subgroup approach proved to be an improvement to the existing inequalities. In comparison to the general-purpose weak SBCs, it was able to reduce the computation time on solved instances by about 25% on average for a suitable sub-testset of the MINLPLib. On the other hand, none of the dynamic variants was able to keep up with their static counterpart in terms of overall performance. Although the LP-based dynamic variants were faster than Orbital Fixing on the all-optimal binary testset, they did not solve as many instances in total. One possible explanation is that, due to the generality of MINLPs, these algorithms are not able to make use of the sB&B information in the same way that similar methods for MIPs are.

We also analyzed the formulation groups of all instances in our testset, which was previously only done for subsets of the MINLPLib. In terms of complementary symmetry, our experiments showed that the new detection algorithm is effective in practice, as the auxiliary problems can be solved very fast. We found 18 quadratic instances in the MINLPLib that contain complementary symmetry. For those, changing the bounds to break the symmetry slightly improved the performance.

Possible Future Work It remains an open question, how symmetries on continuous variables can be exploited to improve the performance of ssB&B solvers. While we have found no indication that using SBCs for these variables is beneficial, the testset we used is quite diverse. It is possible that breaking continuous symmetries is only important for MINLPs with special structure. Identifying such cases could be the first step towards a better understanding of the underwhelming performance of the testset methods.

Even though the dynamic variants did not show convincing results, they were mostly due to a small number of instances where the dynamic approach seems to fail entirely. Apart from these, some variants performed very well on the rest of the testset. This can be seen as a hint that more refined versions and implementations could make dynamic procedures viable for MINLPs. To this end, analyzing the cases of failure would be key.

Another possible subject of further research lies in our work on complementary symmetry. In Section 5.1, we showed how symmetry of this type can be described by means of affine transformations of a certain form. In a similar way, permutation-based symmetries can be described by permutation matrices, which are also affine transformations. This gives rise to a more general notion of symmetry. It is possible that the auxiliary problem from Theorem 5.2 can be modified to detect symmetries with respect to *arbitrary* affine transformations. The usefulness of such a generalization depends on the quantity of other symmetry types that can actually be found in practice.

Bibliography

- [1] Faiz A. Al-Khayyal and Hanif Sherahli. “On Finitely Terminating Branch-and-Bound Algorithms for Some Global Optimization Problems”. *SIAM Journal on Optimization* 10.4 (2000), pp. 1049–1057. DOI: 10.1137/S105262349935178X.
- [2] Tobias Achterberg. “Constraint Integer Programming”. PhD thesis. TU Berlin, 2007.
- [3] Tobias Achterberg. “SCIP: solving constraint integer programs”. *Mathematical Programming Computation* 1.1 (2009), pp. 1–41. DOI: 10.1007/s12532-008-0001-1.
- [4] Pascale Bendotti, Pierre Fouilhoux, and Cécile Rottner. *Orbitopal fixing for the full (sub)-orbitope and application to the Unit Commitment Problem*. Tech. rep. Optimization Online. URL: http://www.optimization-online.org/DB_HTML/2017/10/6301.html.
- [5] Timo Berthold. “Heuristic algorithms in global MINLP solvers”. PhD thesis. 2014.
- [6] Timo Berthold and Marc E. Pfetsch. “Detecting Orbitopal Symmetries”. *Operations Research Proceedings 2008*. Ed. by Bernhard Fleischmann, Karl Heinz Borgwardt, Robert Klein, and Alex Tuma. Springer-Verlag, 2009, pp. 433–438. DOI: 10.1007/978-3-642-00142-0_70.
- [7] Philipp M. Christophel, Menal Güzelsoy, and Imre Pólik. *New symmetries in mixed-integer linear optimization: Symmetry heuristics and complement-based symmetries*. Tech. rep. Optimization Online, 2014. URL: http://www.optimization-online.org/DB_HTML/2014/07/4466.html.
- [8] Joel Cohen. “Computer Algebra and Symbolic Computation: Mathematical Methods”. A K Peters/CRC Press, 2003.
- [9] “COIN-OR: CppAD, a package for differentiation of CPP algorithms”. <http://www.coin-or.org/CppAD>.
- [10] “COIN-OR: Ipopt, Interior point optimizer”. <http://www.coin-or.org/Ipopt>.
- [11] Paul T. Darga, Hadi Katebi, Matk Liffiton, Igor L. Markov, and Karem Sakallah. “Saucy”. <http://vlsicad.eecs.umich.edu/BK/SAUCY/>.
- [12] Gustavo Dias and Leo Liberti. “Exploiting symmetries in mathematical programming via orbital independence”. *Annals of Operations Research* 277 (2019), pp. 1–34. DOI: 10.1007/s10479-019-03145-x.

- [13] Gustavo Dias and Leo Liberti. “Orbital Independence in Symmetric Mathematical Programs”. *COCOA 2015 Proceedings*. Ed. by Zaixin Lu, Donghyun Kim, Weili Wu, Wei Li, and Ding-Zhu Du. Vol. 9486. LNCS. Springer, 2015, pp. 467–480. DOI: 10.1007/978-3-319-26626-8_34.
- [14] Matteo Fischetti and Leo Liberti. “Orbital Shrinking”. *Combinatorial Optimization*. Ed. by Ali Ridha Mahjoub, Vangelis Markakis, Ioannis Milis, and Vangelis Paschos. Vol. 7422. LNCS. Springer Berlin Heidelberg, 2012, pp. 48–58. DOI: 10.1007/978-3-642-32147-4_6.
- [15] Ambros Gleixner, Leon Eifler, Tristan Gally, Gerald Gamrath, Patrick Gemander, Robert Lion Gottwald, Gregor Hendel, Christopher Hojny, Thorsten Koch, Matthias Miltenberger, Benjamin Müller, Marc E. Pfetsch, Christian Puchert, Daniel Rehfeldt, Franziska Schlösser, Felipe Serrano, Yuji Shinano, Jan Merlin Viernickel, Stefan Vigerske, Dieter Weninger, Jonas T. Witt, and Jakob Witzig. *The SCIP Optimization Suite 5.0*. eng. Tech. rep. 17–61. Takustr. 7, 14195 Berlin: ZIB, 2017.
- [16] Chrisopther Hojny. “Symmetries in Binary Programs – A Polyhedral Perspective”. sierke Verlag, 2018.
- [17] Christopher Hojny and Marc E. Pfetsch. “Polytopes associated with symmetry handling”. *Mathematical Programming* 175.1 (2019), pp. 197–240. DOI: 10.1007/s10107-018-1239-7.
- [18] R. Horst and H. Tuy. “Global Optimization: Deterministic Approaches”. Springer, Berlin, 1990.
- [19] Eric J. Friedman. “Fundamental Domains for Integer Programs with Symmetries”. *COCOA 2007 proceedings*. Ed. by Andreas Dress, Yin-feng Xu, and binhai Zhu. Vol. 4616. LNCS. Springer, 2007, pp. 146–153. DOI: 10.1007/978-3-540-73556-4_17.
- [20] R. G. Jeroslow. “Trivial integer programs unsolvable by branch-and-bound”. *Mathematical Programming* 6.1 (1974), pp. 105–109. DOI: 10.1007/BF01580225.
- [21] Tommi Juntilla and Petteri Kaski. “bliss: A tool for computing automorphism groups and canonical labelings of graphs”. <http://www.tcs.hut.fi/Software/bliss/>.
- [22] Volker Kaibel and Marc E. Pfetsch. “Packing and Partitioning Orbitopes”. *Mathematical Programming* 114.1 (2008), pp. 1–36. DOI: 10.1007/s10107-006-0081-5.
- [23] Volker Kaibel, Matthias Peinhardt, and Marc E. Pfetsch. “Orbitopal Fixing”. *Integer Programming and Combinatorial Optimization*. Ed. by Matteo Fischetti and David P. Williamson. Springer Berlin Heidelberg, 2007, pp. 74–88. DOI: 10.1007/978-3-540-72792-7_7.

- [24] Jon Lee, Daphne Skipper, and Emily Speakman. “Algorithmic and modeling insights via volumetric comparison of polyhedral relaxations”. *Mathematical Programming* 170.1 (2018), pp. 121–140. DOI: 10.1007/s10107-018-1272-6.
- [25] Leo Liberti. “Reformulations in mathematical programming: Automatic symmetry detection and exploitation”. *Mathematical Programming* 131 (2010), pp. 273–304. DOI: 10.1007/s10107-010-0351-0.
- [26] Leo Liberti and James Ostrowski. “Stabilizer-based symmetry breaking constraints for mathematical programs”. *Journal of Global Optimization* 60.2 (2014), pp. 183–194. DOI: 10.1007/s10898-013-0106-6.
- [27] Leo Liberti and Alexander Zaslavski. “Writing Global Optimization Software”. 2006, pp. 211–262. DOI: 10.1007/0-387-30528-9_8.
- [28] Stephan Maher, Matthias Miltenberger, Joao Pedro Pedroso, Daniel Rehfeldt, Robert Schwarz, and Felipe Serrano. “PySCIPOpt: Mathematical Programming in Python with the SCIP Optimization Suite”. *Mathematical Software - ICMS 2016*. Vol. 9725. 2016, pp. 301–307. DOI: 10.1007/978-3-319-42432-3_37.
- [29] François Margot. “Exploiting orbits in symmetric ILP”. *Mathematical Programming* 98.1 (2003), pp. 3–21. DOI: 10.1007/s10107-003-0394-6.
- [30] François Margot. “Pruning by isomorphism in branch-and-cut”. *Mathematical Programming* 94.1 (2002), pp. 71–90. DOI: 10.1007/s10107-002-0358-2.
- [31] François Margot. “Symmetry in Integer Linear Programming”. *50 Years of Integer Programming 1985 - 2008*. Ed. by M. Jünger, T. Liebling, D. Naddef, G. L. Nemhauser, G. Pulleyblank, G. Reinelt, G. Rinaldi, and L. Wolsey. Springer-Verlag, 2010. Chap. 17, pp. 647–681. DOI: 10.1007/978-3-540-68279-0_17.
- [32] Breandan D. McKay. “The nauty program”. <http://users.cecs.anu.edu.au/~bdm/nauty>.
- [33] “MINLP Lib, MINLP Library”. <http://www.minlplib.org/>.
- [34] James Ostrowski. “Symmetry in Integer Programming”. PhD thesis. Lehigh University, 2009.
- [35] James Ostrowski, Jeff Linderoth, Fabrizio Rossi, and Stefano Smriglio. “Orbital Branching”. *Integer Programming and Combinatorial Optimization*. Ed. by Matteo Fischetti and David P. Williamson. Vol. 4513. LNCS. Berlin, Heidelberg: Springer Berlin Heidelberg, 2007, pp. 104–118.
- [36] James Ostrowski, Jeff Linderoth, Fabrizio Rossi, and Stefano Smriglio. “Orbital branching”. *Mathematical Programming* 126.1 (2011), pp. 147–178. DOI: 10.1007/s10107-009-0273-x.

- [37] Shectman J. Parker and Nikolaos Sahinidis. “A Finite Algorithm for Global Minimization of Separable Concave Programs”. *Journal of Global Optimization* 12 (1998), pp. 1–36.
- [38] Marc E. Pfetsch and Thomas Rehn. “A computational comparison of symmetry handling methods for mixed integer programs”. *Math. Program. Comput.* 11.1 (2019), pp. 37–93. DOI: 10.1007/s12532-018-0140-y.
- [39] Jean-Francois Puget. “Automatic Detection of Variable and Value Symmetries”. *Principles and Practice of Constraint Programming - CP 2005*. Ed. by Peter van Beek. Vol. 3709. 2005, pp. 475–489. DOI: 10.1007/11564751_36.
- [40] “PySCIPOpt”. <https://github.com/SCIP-Interface/PySCIPOpt>.
- [41] Thomas Rehn. “PermLib: Permutation computation library”. <http://www.math.uni-rostock.de/~rehn/software/permlib.html>.
- [42] Joseph J. Rotman. “An Introduction to the Theory of Groups”. Springer, 1994.
- [43] Domenico Salvagnin. “A dominance procedure for integer programming”. MA thesis. University of Padova, 2005.
- [44] “SCIP–Solving Constraint Integer Programs”. <https://scip.zib.de>.
- [45] Hanif D. Sherali and J. Cole Smith. “Improving Discrete Model Representations via Symmetry Considerations”. *Management Science* 47.10 (2001), pp. 1311–1440. DOI: 10.1287/mnsc.47.10.1396.10265.
- [46] “SoPlex–Sequential object-oriented simPlex”. <https://soplex.zib.de>.
- [47] Laurence A Wolsey. “Integer Programming”. New York: Wiley, 1998.
- [48] Wenxing Zhu. “Unsolvability of some optimization problems”. *Applied Mathematics and Computation* 174 (2006), pp. 921–926. DOI: 10.1016/j.amc.2005.05.025.

Appendices

Appendix A

Symmetry in MINLPLib

TABLE A.1: List of symmetric instances in MINLPLib before presolving. Depicted are the proportion of affected variables in %, the number of binary/integer/continuous variables that are affected (B/I/C), the $\log_{10}|G|$ of the size, and the type of each individual factor of the formulation group (factors).

name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
arki0002	93.7	0/0/2304	5.7	$(\mathcal{M}(\mathcal{S}_6, 1152))^2$
arki0003	0.1	0/0/2	0.3	\mathcal{S}_2
arki0005	0.8	0/0/18	179.2	$(\mathcal{S}_2)^9$
arki0006	0.8	0/0/18	179.2	$(\mathcal{S}_2)^9$
arki0008	1.0	0/0/50	64.5	\mathcal{S}_{50}
arki0009	1.0	0/0/81	35.0	$(\mathcal{S}_5)^{10}, \mathcal{S}_{11}, \mathcal{M}(\mathcal{S}_{10}, 20)$
arki0010	1.4	0/0/56	24.6	$(\mathcal{S}_5)^5, \mathcal{S}_{11}, \mathcal{M}(\mathcal{S}_{10}, 20)$
arki0011	1.1	0/0/217	285.6	$\mathcal{M}(\mathcal{S}_3, 6), (\mathcal{S}_9)^3,$ $\mathcal{M}(\mathcal{S}_{20}, 40), \mathcal{S}_{144}$
arki0012	1.0	0/0/199	274.8	$\mathcal{M}(\mathcal{S}_3, 6), (\mathcal{S}_9)^3, \mathcal{S}_{144},$ $\mathcal{M}(\mathcal{S}_{11}, 22)$
arki0013	1.1	0/0/217	285.6	$\mathcal{M}(\mathcal{S}_3, 6), (\mathcal{S}_9)^3,$ $\mathcal{M}(\mathcal{S}_{20}, 40), \mathcal{S}_{144}$
arki0014	0.6	0/0/113	83.8	$\mathcal{M}(\mathcal{S}_3, 6), \mathcal{S}_{40}, (\mathcal{S}_9)^3,$ $\mathcal{M}(\mathcal{S}_{20}, 40)$
arki0016	0.1	0/0/5	2.1	\mathcal{S}_5
arki0018	22.0	0/0/2159	763.4	$\mathcal{S}_{18}, (\mathcal{S}_2)^{366}, (\mathcal{S}_3)^{129},$ $(\mathcal{S}_4)^{59}, (\mathcal{S}_5)^{27}, (\mathcal{S}_{10})^4,$ $(\mathcal{S}_6)^{24}, (\mathcal{S}_{11})^7, (\mathcal{S}_7)^{15},$ $(\mathcal{S}_{12})^2, (\mathcal{S}_8)^9, (\mathcal{S}_{13})^4,$ $(\mathcal{S}_9)^7, (\mathcal{S}_{14})^2, (\mathcal{S}_{15})^2, \mathcal{S}_{16}$
autocorr_bern20-03	95.2	20/0/0	0.9	$\mathcal{M}(\mathcal{S}_2 \wr \mathcal{S}_2, 20)$
autocorr_bern20-05	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern20-10	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern20-15	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern25-03	92.3	24/0/0	0.6	$(\mathcal{M}(\mathcal{S}_2, 12))^2$
autocorr_bern25-06	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-13	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-19	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-25	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern30-04	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-08	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-15	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-23	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-30	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern35-04	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-09	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-18	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-26	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-35fix	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern40-05	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-10	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-20	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-30	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-40	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
autocorr_bern45-05	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-11	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-23	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-34	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-45	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern50-06	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern50-13	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern50-25	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern55-06	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern55-14	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern55-28	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern60-08	98.4	60/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 60)$
autocorr_bern60-15	98.4	60/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 60)$
ball_mk2_10	100.0	0/10/0	6.6	\mathcal{S}_{10}
ball_mk2_30	100.0	0/30/0	32.4	\mathcal{S}_{30}
bchoco05	2.2	0/0/2	0.3	\mathcal{S}_2
bchoco06	1.7	0/0/2	0.3	\mathcal{S}_2
bchoco07	1.5	0/0/2	0.3	\mathcal{S}_2
bchoco08	1.2	0/0/2	0.3	\mathcal{S}_2
carton7	67.4	137/35/49	91.7	$\mathcal{M}(\mathcal{S}_2, 16)$, 2 unknown
carton9	100.0	216/72/72	136.8	$\mathcal{M}(\mathcal{S}_3, 135)$, 2 unknown
cecil_13	14.3	60/0/60	9.0	$(\mathcal{M}(\mathcal{S}_2, 4))^{30}$
celar6-sub0	13.7	88/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 88)$
chp_partload	13.3	10/0/290	15.7	$(\mathcal{S}_2)^4$, $\mathcal{M}(\mathcal{S}_2, 156)$, 2 unknown
chp_shorttermplan1a	4.8	0/0/48	39.0	1 unknown
chp_shorttermplan1b	31.4	96/0/432	91.1	$(\mathcal{S}_2)^{48}$, $\mathcal{M}(\mathcal{S}_2, 384)$, 1 unknown
chp_shorttermplan2a	66.7	192/0/864	151.7	$(\mathcal{S}_3)^{48}$, $\mathcal{M}(\mathcal{S}_2, 480)$, $\mathcal{M}(\mathcal{S}_2, 384)$, 1 unknown
chp_shorttermplan2b	3.4	0/0/48	16.1	1 unknown
chp_shorttermplan2c	88.2	384/0/1776	364.9	$\mathcal{M}(\mathcal{S}_4, 960)$, $\mathcal{M}(\mathcal{S}_2, 480)$, $(\mathcal{S}_6)^{48}$, $\mathcal{M}(\mathcal{S}_2, 384)$, 1 unknown
chp_shorttermplan2d	81.5	480/0/2064	128.0	$\mathcal{M}(\mathcal{S}_4, 768)$, $(\mathcal{S}_4)^{48}$, $(\mathcal{M}(\mathcal{S}_2, 480))^2$, $\mathcal{M}(\mathcal{S}_2, 576)$, 1 unknown
color_lab3_3x0	57.4	182/0/0	0.9	$\mathcal{M}(\mathcal{S}_2, 132)$, 1 unknown
color_lab3_4x0	82.8	328/0/0	0.9	$\mathcal{M}(\mathcal{S}_2, 264)$, 1 unknown
crossdock_15x7	85.3	180/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 180)$
crossdock_15x8	99.6	240/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 240)$
crudeoil_lee1_05	0.7	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee1_06	0.6	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee1_07	0.5	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee1_08	0.5	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee1_09	0.4	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee1_10	0.4	0/0/4	1.7	\mathcal{S}_4
crudeoil_lee2_05	1.2	0/0/14	6.6	1 unknown
crudeoil_lee2_06	1.0	0/0/14	6.6	1 unknown
crudeoil_lee2_07	0.9	0/0/14	6.6	1 unknown
crudeoil_lee2_08	0.8	0/0/14	6.6	1 unknown
crudeoil_lee2_09	0.7	0/0/14	6.6	1 unknown
crudeoil_lee2_10	0.6	0/0/14	6.6	1 unknown
crudeoil_lee3_05	1.2	0/0/16	8.3	1 unknown
crudeoil_lee3_06	1.0	0/0/16	8.3	1 unknown
crudeoil_lee3_07	0.9	0/0/16	8.3	1 unknown
crudeoil_lee3_08	0.8	0/0/16	8.3	1 unknown
crudeoil_lee3_09	0.7	0/0/16	8.3	1 unknown
crudeoil_lee3_10	0.6	0/0/16	8.3	1 unknown
crudeoil_lee4_05	1.4	0/0/27	15.5	1 unknown
crudeoil_lee4_06	1.2	0/0/27	15.5	1 unknown
crudeoil_lee4_07	1.0	0/0/27	15.5	1 unknown
crudeoil_lee4_08	0.9	0/0/27	15.5	1 unknown
crudeoil_lee4_09	0.8	0/0/27	15.5	1 unknown
crudeoil_lee4_10	0.7	0/0/27	15.5	1 unknown
crudeoil_li02	31.0	90/0/312	0.9	$(\mathcal{M}(\mathcal{S}_2, 134))^3$
crudeoil_li05	21.1	30/0/168	0.3	$\mathcal{M}(\mathcal{S}_2, 198)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
crudeoil_li06	20.5	30/0/168	0.3	$\mathcal{M}(S_2, 198)$
crudeoil_li11	20.9	42/0/204	0.3	$\mathcal{M}(S_2, 246)$
crudeoil_li21	21.8	54/0/240	0.3	$\mathcal{M}(S_2, 294)$
cvxnonsep_normcon30	13.3	0/4/0	0.6	$(S_2)^2$
cvxnonsep_normcon30r	13.3	0/4/4	0.6	$(\mathcal{M}(S_2, 4))^2$
cvxnonsep_normcon40	10.0	0/2/2	0.6	$(S_2)^2$
cvxnonsep_normcon40r	10.0	0/2/6	0.6	$(\mathcal{M}(S_2, 4))^2$
cvxnonsep_psig20	14.3	0/0/3	0.8	S_3
cvxnonsep_psig20r	14.3	0/0/6	0.8	$\mathcal{M}(S_3, 6)$
cvxnonsep_psig30	6.5	0/0/2	0.3	S_2
cvxnonsep_psig30r	6.5	0/0/4	0.3	$\mathcal{M}(S_2, 4)$
cvxnonsep_psig40	19.5	0/4/4	1.2	$(S_2)^4$
cvxnonsep_psig40r	24.4	0/4/16	1.5	$(\mathcal{M}(S_2, 4))^5$
deb7	2.5	10/0/10	6.6	$\mathcal{M}(S_{10}, 20)$
deb8	2.4	10/0/10	6.6	$\mathcal{M}(S_{10}, 20)$
deb9	2.5	10/0/10	6.6	$\mathcal{M}(S_{10}, 20)$
densitymod	97.9	23040/0/0	73.3	$\mathcal{M}(S_7, 2688),$ $\mathcal{M}(S_{53}, 20352)$
eigena2	100.0	0/0/2500	64.5	$\mathcal{M}(S_{50}, 2500)$
elec100	99.7	0/0/300	158.7	1 unknown
elec200	99.8	0/0/600	375.7	1 unknown
elec25	98.7	0/0/75	26.0	1 unknown
elec50	99.3	0/0/150	65.3	1 unknown
elf	100.0	24/0/30	0.8	$\mathcal{M}(S_3, 54)$
emfl050_3_3	1.7	0/0/27	8.3	1 unknown
emfl050_5_5	1.3	0/0/75	32.7	1 unknown
emfl100_3_3	0.9	0/0/27	8.3	1 unknown
emfl100_5_5	0.8	0/0/75	32.7	1 unknown
ex14_1_5	66.7	0/0/4	1.4	S_4
ex2_1_3	57.1	0/0/8	0.3	$\mathcal{M}(S_2, 8)$
ex5_2_5	81.8	0/0/27	0.8	$\mathcal{M}(S_3, 27)$
ex6_1_1	88.9	0/0/8	0.3	$\mathcal{M}(S_2, 8)$
ex6_1_3	92.3	0/0/12	0.3	$\mathcal{M}(S_2, 12)$
ex6_2_10	85.7	0/0/6	0.3	$\mathcal{M}(S_2, 6)$
ex6_2_12	80.0	0/0/4	0.3	$\mathcal{M}(S_2, 4)$
ex6_2_13	85.7	0/0/6	0.3	$\mathcal{M}(S_2, 6)$
ex6_2_14	80.0	0/0/4	0.3	$\mathcal{M}(S_2, 4)$
ex6_2_9	80.0	0/0/4	0.3	$\mathcal{M}(S_2, 4)$
ex8_1_6	66.7	0/0/2	0.3	S_2
ex8_3_1	95.7	0/0/110	2.1	1 unknown
ex8_3_11	95.7	0/0/110	2.1	1 unknown
ex8_3_12	95.8	0/0/115	2.1	1 unknown
ex8_3_13	95.7	0/0/110	2.1	1 unknown
ex8_3_14	95.5	0/0/105	2.1	1 unknown
ex8_3_2	95.5	0/0/105	2.1	1 unknown
ex8_3_3	95.5	0/0/105	2.1	1 unknown
ex8_3_4	95.5	0/0/105	2.1	1 unknown
ex8_3_5	95.5	0/0/105	2.1	1 unknown
ex8_3_7	94.5	0/0/120	2.1	1 unknown
ex8_3_8	95.2	0/0/120	2.1	1 unknown
ex8_3_9	96.2	0/0/75	2.1	1 unknown
ex8_4_6	40.0	0/0/6	0.8	$\mathcal{M}(S_3, 6)$
ex8_6_1	82.9	0/0/63	3.7	1 unknown
ex8_6_2	65.6	0/0/21	3.7	$\mathcal{M}(S_7, 21)$
ex9_1_8	14.3	0/0/2	0.3	S_2
ex9_2_2	16.7	0/0/2	0.3	S_2
ex9_2_6	94.1	0/0/16	1.2	$S_2 \wr S_2, \mathcal{M}(S_2, 12)$
gabriel04	17.7	0/0/64	7.8	$(\mathcal{M}(S_2, 32))^2$
gams02	10.7	0/0/1363	3076.7	$S_{403}, (S_{480})^2$
gams03	95.0	380/1900/0	4.8	$\mathcal{M}(S_5, 600),$ $(\mathcal{M}(S_2, 240))^4,$ $(\mathcal{M}(S_3, 360))^2$
gancns	5.0	0/0/18	3.4	$(S_2)^6, (S_3)^2$
gasnet_al1	6.6	12/0/36	1.5	$(\mathcal{M}(S_2, 22))^2, S_2 \wr S_2$
gasnet_al2	6.6	12/0/36	1.5	$(\mathcal{M}(S_2, 22))^2, S_2 \wr S_2$
gasnet_al3	6.6	12/0/36	1.5	$(\mathcal{M}(S_2, 22))^2, S_2 \wr S_2$
gasnet_al4	6.6	12/0/36	1.5	$(\mathcal{M}(S_2, 22))^2, S_2 \wr S_2$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
gasnet_al5	6.6	12/0/36	1.5	$(\mathcal{M}(S_2, 22))^2, S_2 \wr S_2$
gastrans	11.3	4/0/8	0.6	$(\mathcal{M}(S_2, 6))^2$
gastrans135	5.9	0/0/70	76.4	$(S_2)^6, \mathcal{M}(S_{29}, 58)$
gastrans582_cold13	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cold13_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cold17	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cold17_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cool12	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cool12_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cool14	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_cool14_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_freezing27	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_freezing27_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_freezing30	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_freezing30_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_mild10	4.1	0/0/89	115.0	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^7, 1 \text{ unknown}$
gastrans582_mild10_95	4.1	0/0/89	115.0	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^7, 1 \text{ unknown}$
gastrans582_mild11	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_mild11_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_warm15	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_warm15_95	3.9	0/0/85	114.7	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^6, 1 \text{ unknown}$
gastrans582_warm31	4.1	0/0/89	115.0	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^7, 1 \text{ unknown}$
gastrans582_warm31_95	4.1	0/0/89	115.0	$\mathcal{M}(S_3, 6), (S_2)^{19}, S_3, S_5,$ $(\mathcal{M}(S_2, 4))^7, 1 \text{ unknown}$
gear	80.0	0/4/0	0.6	$(S_2)^2$
gear2	96.6	24/0/4	0.6	$(\mathcal{M}(S_2, 14))^2$
gear3	88.9	0/4/4	0.6	$(\mathcal{M}(S_2, 4))^2$
gear4	66.7	0/4/0	0.6	$(S_2)^2$
graphpart_2g-0044-1601	98.0	48/0/0	0.8	$\mathcal{M}(S_3, 48)$
graphpart_2g-0055-0062	98.7	75/0/0	0.8	$\mathcal{M}(S_3, 75)$
graphpart_2g-0066-0066	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_2g-0077-0077	99.3	147/0/0	0.8	$\mathcal{M}(S_3, 147)$
graphpart_2g-0088-0088	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_2g-0099-9211	99.6	243/0/0	0.8	$\mathcal{M}(S_3, 243)$
graphpart_2g-1010-0824	99.7	300/0/0	0.8	$\mathcal{M}(S_3, 300)$
graphpart_2pm-0044-0044	98.0	48/0/0	0.8	$\mathcal{M}(S_3, 48)$
graphpart_2pm-0055-0055	98.7	75/0/0	0.8	$\mathcal{M}(S_3, 75)$
graphpart_2pm-0066-0066	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_2pm-0077-0777	99.3	147/0/0	0.8	$\mathcal{M}(S_3, 147)$
graphpart_2pm-0088-0888	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_2pm-0099-0999	99.6	243/0/0	0.8	$\mathcal{M}(S_3, 243)$
graphpart_3g-0234-0234	98.6	72/0/0	0.8	$\mathcal{M}(S_3, 72)$
graphpart_3g-0244-0244	99.0	96/0/0	0.8	$\mathcal{M}(S_3, 96)$
graphpart_3g-0333-0333	98.8	81/0/0	0.8	$\mathcal{M}(S_3, 81)$
graphpart_3g-0334-0334	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_3g-0344-0344	99.3	144/0/0	0.8	$\mathcal{M}(S_3, 144)$
graphpart_3g-0444-0444	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_3pm-0234-0234	98.6	72/0/0	0.8	$\mathcal{M}(S_3, 72)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
graphpart_3pm-0244-0244	99.0	96/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 96)$
graphpart_3pm-0333-0333	98.8	81/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 81)$
graphpart_3pm-0334-0334	99.1	108/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 108)$
graphpart_3pm-0344-0344	99.3	144/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 144)$
graphpart_3pm-0444-0444	99.5	192/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 192)$
graphpart_clique-20	98.4	60/0/0	1.1	1 unknown
graphpart_clique-30	98.9	90/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 90)$
graphpart_clique-40	99.2	120/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 120)$
graphpart_clique-50	99.3	150/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 150)$
graphpart_clique-60	99.4	180/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 180)$
graphpart_clique-70	99.5	210/0/0	0.8	$\mathcal{M}(\mathcal{S}_3, 210)$
hadamard_4	94.1	16/0/0	2.8	1 unknown
hadamard_5	96.2	25/0/0	4.2	1 unknown
hadamard_6	97.3	36/0/0	5.7	1 unknown
hadamard_7	98.0	49/0/0	7.4	1 unknown
hadamard_8	98.5	64/0/0	9.2	1 unknown
heatexch_gen1	3.6	0/0/4	1.4	\mathcal{S}_4
heatexch_gen2	3.4	0/0/5	2.1	\mathcal{S}_5
heatexch_gen3	4.3	0/0/25	25.2	\mathcal{S}_{25}
heatexch_spec1	7.0	0/0/4	1.4	\mathcal{S}_4
heatexch_spec2	6.5	0/0/5	2.1	\mathcal{S}_5
heatexch_spec3	9.6	0/0/25	25.2	\mathcal{S}_{25}
heatexch_trigen	2.7	8/0/0	4.6	\mathcal{S}_8
hmittelman	35.3	6/0/0	0.9	$(\mathcal{S}_2)^3$
hvb11	8.5	625/0/210	938.4	$(\mathcal{S}_2)^{32}, \mathcal{S}_{64}, \mathcal{S}_{30}, \mathcal{S}_{116}, 1$ unknown
iswath2	2.1	0/0/134	99.0	$\mathcal{S}_{54}, (\mathcal{S}_4)^{20}$
ivalues	49.3	0/100/0	0.3	$\mathcal{M}(\mathcal{S}_2, 100)$
knp3-12	97.3	0/0/36	9.5	1 unknown
knp4-24	99.0	0/0/96	25.2	1 unknown
knp5-40	99.5	0/0/200	50.0	1 unknown
knp5-41	99.5	0/0/205	51.6	1 unknown
knp5-42	99.5	0/0/210	53.2	1 unknown
knp5-43	99.5	0/0/215	54.9	1 unknown
knp5-44	99.5	0/0/220	56.5	1 unknown
korcns	4.2	0/0/4	0.6	$(\mathcal{S}_2)^2$
kport20	54.5	5/17/33	2.0	$\mathcal{M}(\mathcal{S}_3, 15), (\mathcal{M}(\mathcal{S}_2, 10))^4$
kport40	73.4	29/55/112	10.1	$(\mathcal{M}(\mathcal{S}_2, 14))^2, \mathcal{M}(\mathcal{S}_9, 63),$ $(\mathcal{M}(\mathcal{S}_3, 21))^5$
lop97ic	0.2	0/0/4	0.6	$(\mathcal{S}_2)^2$
lop97icx	78.7	0/763/14	1871.9	$(\mathcal{S}_2)^7, \mathcal{S}_{763}$
maxcsp-ehi-85-297-12	2.3	48/0/0	1.2	$(\mathcal{M}(\mathcal{S}_2, 12))^4$
maxcsp-ehi-85-297-36	2.0	41/0/0	1.4	$\mathcal{M}(\mathcal{S}_2, 14), \mathcal{M}(\mathcal{S}_3, 15),$ $\mathcal{M}(\mathcal{S}_2, 12)$
maxcsp-ehi-85-297-71	1.2	24/0/0	0.6	$(\mathcal{M}(\mathcal{S}_2, 12))^2$
maxcsp-ehi-90-315-70	0.5	12/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 12)$
maxcsp-langford-3-11	98.1	616/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 616)$
maxmin	96.3	0/0/26	9.0	1 unknown
mbtd	100.0	200/0/10	1.3	$\mathcal{M}(\mathcal{S}_2, 100), 1$ unknown
meanvar-orl400_05_e_8	25.0	0/0/400	868.8	\mathcal{S}_{400}
minsurf100	100.0	0/0/5304	0.3	$\mathcal{M}(\mathcal{S}_2, 5304)$
minsurf25	99.9	0/0/1404	0.3	$\mathcal{M}(\mathcal{S}_2, 1404)$
minsurf50	100.0	0/0/2704	0.3	$\mathcal{M}(\mathcal{S}_2, 2704)$
minsurf75	100.0	0/0/4004	0.3	$\mathcal{M}(\mathcal{S}_2, 4004)$
netmod_dol1	99.9	462/0/1536	15.4	1 unknown
netmod_dol2	17.9	0/0/357	521.9	$\mathcal{S}_{36}, \mathcal{S}_{74}, \mathcal{S}_{101}, \mathcal{S}_{57}, \mathcal{S}_{89}$
netmod_kar1	28.9	44/0/88	2.7	$\mathcal{M}(\mathcal{S}_2, 24), \mathcal{M}(\mathcal{S}_5, 60),$ $\mathcal{M}(\mathcal{S}_2, 48)$
netmod_kar2	28.9	44/0/88	2.7	$\mathcal{M}(\mathcal{S}_2, 24), \mathcal{M}(\mathcal{S}_5, 60),$ $\mathcal{M}(\mathcal{S}_2, 48)$
nuclear104	91.2	10816/0/10907	9.8	$\mathcal{M}(\mathcal{S}_{13}, 21723)$
nuclear14	74.9	576/0/594	2.9	$\mathcal{M}(\mathcal{S}_6, 1170)$
nuclear25	75.7	625/0/645	2.1	$\mathcal{M}(\mathcal{S}_5, 1270)$
nuclear49	84.5	2401/0/2443	3.7	$\mathcal{M}(\mathcal{S}_7, 4844)$
nuclearva	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$
nuclearvb	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$
nuclearvc	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
nuclearvd	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$
nuclearve	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$
nuclearvf	50.4	168/0/9	0.8	$\mathcal{M}(\mathcal{S}_3, 177)$
nvs09	90.9	0/10/0	6.6	\mathcal{S}_{10}
oil	16.7	4/0/253	42.2	$\mathcal{M}(\mathcal{S}_4, 88), \mathcal{M}(\mathcal{S}_3, 12), \mathcal{S}_{28}, \mathcal{S}_3, \mathcal{M}(\mathcal{S}_4, 16), (\mathcal{M}(\mathcal{S}_2, 4))^8, (\mathcal{M}(\mathcal{S}_2, 6))^2, (\mathcal{M}(\mathcal{S}_2, 8))^2, (\mathcal{M}(\mathcal{S}_2, 10))^3, \mathcal{M}(\mathcal{S}_5, 20)$
oil2	3.0	0/0/28	2.1	$\mathcal{S}_2, (\mathcal{M}(\mathcal{S}_2, 4))^2, \mathcal{M}(\mathcal{S}_2, 8), \mathcal{M}(\mathcal{S}_2, 10)$
orth_d4m6_pl	90.5	0/0/38	0.3	$\mathcal{M}(\mathcal{S}_2, 38)$
pedigree_sp_top4_250	11.8	0/0/100	158.0	\mathcal{S}_{100}
pedigree_sp_top4_300	19.7	0/0/119	196.7	\mathcal{S}_{119}
pedigree_sp_top5_200	5.5	0/0/76	111.3	\mathcal{S}_{76}
pedigree_sp_top5_250	10.3	0/0/100	158.0	\mathcal{S}_{100}
pooling_bental5pq	94.6	0/0/87	0.8	$\mathcal{M}(\mathcal{S}_3, 87)$
pooling_bental5stp	95.8	0/0/114	0.8	$\mathcal{M}(\mathcal{S}_3, 114)$
pooling_bental5tp	94.6	0/0/87	0.8	$\mathcal{M}(\mathcal{S}_3, 87)$
powerflow2383wpr	0.1	0/0/24	0.6	$(\mathcal{M}(\mathcal{S}_2, 12))^2$
procurement1large	2.0	20/0/120	27.6	$(\mathcal{S}_2)^{40}, (\mathcal{S}_3)^{20}$
procurement1mot	3.1	2/0/22	4.9	$(\mathcal{S}_2)^6, (\mathcal{S}_3)^4$
procurement2mot	3.0	2/0/22	4.9	$(\mathcal{S}_2)^6, (\mathcal{S}_3)^4$
product	11.1	0/0/172	180.3	$\mathcal{S}_{90}, (\mathcal{S}_2)^{11}, (\mathcal{S}_3)^2, \mathcal{S}_4, (\mathcal{S}_5)^2, \mathcal{S}_6, \mathcal{S}_{14}, 1 \text{ unknown}$
product2	64.8	128/0/1714	488.4	$(\mathcal{M}(\mathcal{S}_4, 88))^2, \mathcal{S}_{90}, (\mathcal{M}(\mathcal{S}_4, 72))^2, (\mathcal{S}_3)^2, (\mathcal{S}_4)^5, (\mathcal{M}(\mathcal{S}_4, 80))^2, (\mathcal{M}(\mathcal{S}_4, 48))^{11}, (\mathcal{S}_{20})^5, (\mathcal{S}_{21})^2, (\mathcal{S}_{12})^2, (\mathcal{S}_8)^{21}, \mathcal{S}_{13}, (\mathcal{M}(\mathcal{S}_4, 56))^2, (\mathcal{M}(\mathcal{S}_4, 64))^3, \mathcal{S}_{17}, 1 \text{ unknown}$
radar-2000-10-a-6_lat_7	76.4	1529/0/6116	2638.3	$\mathcal{M}(\mathcal{S}_{12}, 60), \mathcal{M}(\mathcal{S}_{51}, 255), \mathcal{M}(\mathcal{S}_{11}, 55), \mathcal{M}(\mathcal{S}_{635}, 3175), \mathcal{M}(\mathcal{S}_8, 40), \mathcal{M}(\mathcal{S}_{21}, 105), \mathcal{M}(\mathcal{S}_5, 25), (\mathcal{M}(\mathcal{S}_{30}, 150))^3, (\mathcal{M}(\mathcal{S}_9, 45))^2, (\mathcal{M}(\mathcal{S}_{16}, 80))^2, (\mathcal{M}(\mathcal{S}_{26}, 130))^2, \mathcal{M}(\mathcal{S}_{220}, 1100), (\mathcal{M}(\mathcal{S}_{33}, 165))^2, \mathcal{M}(\mathcal{S}_{18}, 90), \mathcal{M}(\mathcal{S}_{23}, 115), \mathcal{M}(\mathcal{S}_{37}, 185), \mathcal{M}(\mathcal{S}_{44}, 220), (\mathcal{M}(\mathcal{S}_{34}, 170))^2, \mathcal{M}(\mathcal{S}_{43}, 215), \mathcal{M}(\mathcal{S}_{19}, 95), (\mathcal{M}(\mathcal{S}_{28}, 140))^2$
radar-3000-10-a-8_lat_7	50.1	1504/0/6016	3416.2	$\mathcal{M}(\mathcal{S}_{861}, 4305), \mathcal{M}(\mathcal{S}_{71}, 355), \mathcal{M}(\mathcal{S}_{315}, 1575), \mathcal{M}(\mathcal{S}_{257}, 1285)$
ringpack_10_2	25.0	20/0/0	57.5	$\mathcal{M}(\mathcal{S}_2, 20)$
ringpack_20_2	17.0	40/0/0	385.9	$\mathcal{M}(\mathcal{S}_2, 40)$
ringpack_20_3	100.0	213/0/40	445.5	1 unknown
risk2bpb	21.3	0/0/99	48.2	$(\mathcal{S}_2)^3, (\mathcal{S}_3)^{12}, (\mathcal{S}_6)^3, (\mathcal{S}_{13})^3$
routingdelay_bigm	3.2	8/0/28	3.6	$(\mathcal{S}_2)^{10}, (\mathcal{M}(\mathcal{S}_2, 8))^2$
routingdelay_proj	3.2	8/0/28	3.6	$(\mathcal{S}_2)^{10}, (\mathcal{M}(\mathcal{S}_2, 8))^2$
rsyn0815h	1.3	0/0/5	1.7	$\mathcal{S}_2, \mathcal{S}_3$
rsyn0815m	2.4	0/0/5	1.4	$\mathcal{S}_2, \mathcal{S}_3$
rsyn0815m02h	1.1	0/0/10	6.7	$(\mathcal{S}_2)^2, (\mathcal{S}_3)^2$
rsyn0815m02m	2.1	0/0/10	6.7	$(\mathcal{S}_2)^2, (\mathcal{S}_3)^2$
rsyn0815m03h	1.1	0/0/15	17.4	$(\mathcal{S}_2)^3, (\mathcal{S}_3)^3$
rsyn0815m03m	2.1	0/0/15	17.1	$(\mathcal{S}_2)^3, (\mathcal{S}_3)^3$
rsyn0815m04h	1.1	0/0/20	33.2	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^4$
rsyn0815m04m	2.1	0/0/20	32.3	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^4$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
sepasequ_complex	5.4	0/0/27	10.7	$\mathcal{S}_2, (\mathcal{S}_5)^3, \mathcal{S}_2 \wr \mathcal{S}_5$
sporttournament10	4.3	2/0/0	0.3	\mathcal{S}_2
st_e09	66.7	0/0/2	0.3	\mathcal{S}_2
st_e18	100.0	0/0/2	0.3	\mathcal{S}_2
st_rv9	39.2	0/0/20	3.0	$(\mathcal{S}_2)^{10}$
steenbrf	46.1	0/0/216	0.9	$(\mathcal{M}(\mathcal{S}_2, 72))^3$
super1	2.8	12/0/24	5.8	$\mathcal{M}(\mathcal{S}_3, 12), (\mathcal{S}_2)^9, (\mathcal{S}_3)^2$
super2	2.4	8/0/24	4.9	$(\mathcal{S}_2)^9, (\mathcal{S}_3)^2, \mathcal{M}(\mathcal{S}_2, 8)$
super3	1.8	0/0/24	4.3	$(\mathcal{S}_2)^9, (\mathcal{S}_3)^2$
super3t	2.3	0/0/24	4.3	$(\mathcal{S}_2)^9, (\mathcal{S}_3)^2$
syn15h	4.1	0/0/5	1.1	$\mathcal{S}_2, \mathcal{S}_3$
syn15m	9.1	0/0/5	1.1	$\mathcal{S}_2, \mathcal{S}_3$
syn15m02h	3.3	0/0/10	6.7	$(\mathcal{S}_2)^2, (\mathcal{S}_3)^2$
syn15m02m	5.9	0/0/10	6.7	$(\mathcal{S}_2)^2, (\mathcal{S}_3)^2$
syn15m03h	3.3	0/0/15	16.8	$(\mathcal{S}_2)^3, (\mathcal{S}_3)^3$
syn15m03m	5.9	0/0/15	16.8	$(\mathcal{S}_2)^3, (\mathcal{S}_3)^3$
syn15m04h	3.3	0/0/20	31.4	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^4$
syn15m04m	5.9	0/0/20	31.4	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^4$
synheat	6.9	0/0/4	1.4	\mathcal{S}_4
t1000	95.2	0/0/954	1934.7	$(\mathcal{S}_2)^5, \mathcal{S}_{274}, \mathcal{S}_{381},$ $\mathcal{M}(\mathcal{S}_2, 10), \mathcal{S}_{279}$
topopt-cantilever_60x40_50	8.9	0/0/3000	451.2	$(\mathcal{S}_2)^{1500}$
topopt-mbb_60x40_50	8.9	0/0/3000	451.2	$(\mathcal{S}_2)^{1500}$
topopt-zhou-rozvany_75	28.6	0/0/400	60.2	$(\mathcal{S}_2)^{200}$
torsion100	100.0	0/0/5308	0.3	$\mathcal{M}(\mathcal{S}_2, 5308)$
torsion25	100.0	0/0/1408	0.3	$\mathcal{M}(\mathcal{S}_2, 1408)$
torsion50	100.0	0/0/2708	0.6	1 unknown
torsion75	100.0	0/0/4008	0.3	$\mathcal{M}(\mathcal{S}_2, 4008)$
transswitch2383wpr	0.1	4/0/24	0.6	$(\mathcal{M}(\mathcal{S}_2, 14))^2$
turkey	1.5	0/0/8	1.2	$(\mathcal{S}_2)^4$
unitcommit_200_0_5_mod_7	2.3	0/0/647	350.9	$(\mathcal{S}_2)^{19}, (\mathcal{S}_3)^4, (\mathcal{S}_4)^6, (\mathcal{S}_5)^2,$ $\mathcal{S}_{10}, (\mathcal{S}_6)^5, \mathcal{M}(\mathcal{S}_{154}, 462),$ $\mathcal{S}_7, (\mathcal{S}_{11})^2, (\mathcal{S}_8)^4$
unitcommit_200_100_1_mod_8	0.6	0/0/144	72.6	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^5, (\mathcal{S}_4)^4, \mathcal{S}_5,$ $(\mathcal{S}_6)^2, \mathcal{S}_{10}, (\mathcal{S}_7)^3, (\mathcal{S}_{11})^2,$ $\mathcal{S}_8, (\mathcal{S}_9)^3$
unitcommit_200_100_2_mod_7	1.8	0/0/628	360.2	$(\mathcal{S}_2)^4, \mathcal{M}(\mathcal{S}_{161}, 483),$ $(\mathcal{S}_3)^4, (\mathcal{S}_4)^3, (\mathcal{S}_5)^5, (\mathcal{S}_6)^2,$ $(\mathcal{S}_{10})^4, \mathcal{S}_7, \mathcal{S}_8, \mathcal{S}_{12}, \mathcal{S}_9$
unitcommit_200_100_2_mod_8	0.6	0/0/145	73.4	$(\mathcal{S}_2)^4, (\mathcal{S}_3)^4, (\mathcal{S}_4)^3, (\mathcal{S}_5)^5,$ $(\mathcal{S}_6)^2, (\mathcal{S}_{10})^4, \mathcal{S}_7, \mathcal{S}_8, \mathcal{S}_{12},$ \mathcal{S}_9
unitcommit_50_20_2_mod_8	0.6	0/0/34	14.9	$\mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_5, (\mathcal{S}_6)^2, \mathcal{S}_8$
wager	7.7	0/0/12	4.1	$(\mathcal{S}_4)^3$
watercontamination0202r	4.1	0/0/8	0.3	$\mathcal{M}(\mathcal{S}_2, 8)$
watercontamination0303r	2.1	0/0/8	0.3	$\mathcal{M}(\mathcal{S}_2, 8)$
watertreatnd_flow	8.6	0/0/36	5.4	$(\mathcal{S}_2)^{18}$
waterund14	2.4	0/0/3	0.8	\mathcal{S}_3
waterund22	2.1	0/0/3	0.8	\mathcal{S}_3
waterund25	1.7	0/0/2	0.3	\mathcal{S}_2
waterund27	1.9	0/0/8	4.6	\mathcal{S}_8
waterund28	28.4	0/0/216	0.6	1 unknown
waterund36	1.2	0/0/4	1.4	\mathcal{S}_4

TABLE A.2: List of symmetric instances in MINLPLib after presolving. Depicted are the proportion of affected variables in %, the number of binary/integer/continuous variables that are affected (B/I/C), the $\log_{10}|G|$ of the size, and the type of each individual factor of the formulation group (factors).

name	affected vars		$\log_{10} G $	formulation group
	%	B/I/C		factors
arki0002	90.1	0/0/1392	5.7	$(\mathcal{M}(\mathcal{S}_6, 696))^2$
arki0005	0.2	0/0/2	0.3	\mathcal{S}_2
arki0006	0.2	0/0/2	0.3	\mathcal{S}_2
arki0008	1.8	0/0/50	64.5	\mathcal{S}_{50}
arki0016	0.1	0/0/5	2.1	\mathcal{S}_5
arki0017	1.0	0/0/42	2.1	$(\mathcal{M}(\mathcal{S}_2, 6))^7$
arki0018	19.8	0/0/1946	1056.8	$\mathcal{S}_{19}, (\mathcal{S}_2)^{226}, \mathcal{S}_{29}, (\mathcal{S}_3)^{72}, (\mathcal{S}_4)^{47}, (\mathcal{S}_5)^{25}, (\mathcal{S}_{10})^2, (\mathcal{S}_6)^{14}, \mathcal{S}_{20}, (\mathcal{S}_7)^{13}, (\mathcal{S}_{11})^6, \mathcal{S}_{30}, (\mathcal{S}_8)^6, (\mathcal{S}_{12})^3, \mathcal{S}_{31}, (\mathcal{S}_9)^{10}, (\mathcal{S}_{13})^3, \mathcal{S}_{50}, \mathcal{S}_{23}, (\mathcal{S}_{14})^3, \mathcal{S}_{24}, \mathcal{S}_{51}, (\mathcal{S}_{15})^3, \mathcal{S}_{25}, \mathcal{S}_{70}, (\mathcal{S}_{16})^2$
arki0024	9.1	0/0/176	4.1	$(\mathcal{S}_2)^{10}, \mathcal{M}(\mathcal{S}_2, 66), \mathcal{M}(\mathcal{S}_3, 90)$
autocorr_bern20-03	95.2	20/0/0	0.9	$\mathcal{M}(\mathcal{S}_2 \wr \mathcal{S}_2, 20)$
autocorr_bern20-05	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern20-10	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern20-15	95.2	20/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 20)$
autocorr_bern25-03	92.3	24/0/0	0.6	$(\mathcal{M}(\mathcal{S}_2, 12))^2$
autocorr_bern25-06	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-13	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-19	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern25-25	92.3	24/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 24)$
autocorr_bern30-04	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-08	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-15	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-23	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern30-30	96.8	30/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 30)$
autocorr_bern35-04	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-09	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-18	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-26	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern35-35fix	94.4	34/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 34)$
autocorr_bern40-05	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-10	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-20	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-30	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern40-40	97.6	40/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 40)$
autocorr_bern45-05	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-11	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-23	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-34	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern45-45	95.7	44/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 44)$
autocorr_bern50-06	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern50-13	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern50-25	98.0	50/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
autocorr_bern55-06	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern55-14	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern55-28	96.4	54/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 54)$
autocorr_bern60-08	98.4	60/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 60)$
autocorr_bern60-15	98.4	60/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 60)$
ball_mk2_10	100.0	0/10/0	6.6	\mathcal{S}_{10}
ball_mk2_30	100.0	0/30/0	32.4	\mathcal{S}_{30}
ball_mk3_10	100.0	3/0/0	0.8	\mathcal{S}_3
carton7	18.0	24/0/12	0.9	$(\mathcal{M}(\mathcal{S}_2, 12))^3$
carton9	6.5	12/0/4	0.3	$\mathcal{M}(\mathcal{S}_2, 16)$
celar6-sub0	100.0	640/0/0	0.6	1 unknown
chp_partload	7.2	10/0/40	0.3	$\mathcal{M}(\mathcal{S}_2, 50)$
chp_shorttermplan1b	31.2	96/0/288	0.3	$\mathcal{M}(\mathcal{S}_2, 384)$
chp_shorttermplan2a	74.0	100/0/580	0.6	$\mathcal{M}(\mathcal{S}_2, 384), \mathcal{M}(\mathcal{S}_2, 296)$
chp_shorttermplan2c	93.1	108/0/1164	2.0	$\mathcal{M}(\mathcal{S}_4, 592), \mathcal{M}(\mathcal{S}_2, 384), \mathcal{M}(\mathcal{S}_2, 296)$
chp_shorttermplan2d	85.2	296/0/1640	2.3	$\mathcal{M}(\mathcal{S}_4, 768), \mathcal{M}(\mathcal{S}_2, 576), (\mathcal{M}(\mathcal{S}_2, 296))^2$
color_lab3_3x0	71.4	170/0/0	0.9	$\mathcal{M}(\mathcal{S}_2, 132), \mathcal{M}(\mathcal{S}_2 \times \mathcal{S}_2, 38)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
color_lab3_4x0	99.7	316/0/0	0.9	$\mathcal{M}(\mathcal{S}_2, 264)$, 1 unknown
crossdock_15x7	85.3	180/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 180)$
crossdock_15x8	99.6	240/0/0	0.3	$\mathcal{M}(\mathcal{S}_2, 240)$
crudeoil_li02	31.5	36/0/132	1.2	\mathcal{S}_2 , $\mathcal{M}(\mathcal{S}_2, 54)$, $(\mathcal{M}(\mathcal{S}_2, 56))^2$
crudeoil_li06	21.5	28/0/152	0.3	$\mathcal{M}(\mathcal{S}_2, 180)$
crudeoil_li21	22.2	40/0/186	0.3	$\mathcal{M}(\mathcal{S}_2, 226)$
crudeoil_pooling_dt4	0.0	0/0/2	0.3	\mathcal{S}_2
cvxnonsep_normcon30	13.3	0/4/0	0.6	$(\mathcal{S}_2)^2$
cvxnonsep_normcon30r	13.3	0/4/4	0.6	$(\mathcal{M}(\mathcal{S}_2, 4))^2$
cvxnonsep_normcon40	10.0	0/2/2	0.6	$(\mathcal{S}_2)^2$
cvxnonsep_normcon40r	10.0	0/2/6	0.6	$(\mathcal{M}(\mathcal{S}_2, 4))^2$
cvxnonsep_psig20	14.3	0/0/3	0.8	\mathcal{S}_3
cvxnonsep_psig20r	14.6	0/0/6	0.8	$\mathcal{M}(\mathcal{S}_3, 6)$
cvxnonsep_psig30	6.5	0/0/2	0.3	\mathcal{S}_2
cvxnonsep_psig30r	6.6	0/0/4	0.3	$\mathcal{M}(\mathcal{S}_2, 4)$
cvxnonsep_psig40	19.5	0/4/4	1.2	$(\mathcal{S}_2)^4$
cvxnonsep_psig40r	24.4	0/4/16	1.5	$(\mathcal{M}(\mathcal{S}_2, 4))^5$
densitymod	97.9	23040/0/0	73.3	$\mathcal{M}(\mathcal{S}_7, 2688)$, $\mathcal{M}(\mathcal{S}_{53}, 20352)$
eigena2	100.0	0/0/2500	64.5	$\mathcal{M}(\mathcal{S}_{50}, 2500)$
elec100	99.7	0/0/300	158.7	1 unknown
elec200	99.8	0/0/600	375.7	1 unknown
elec25	98.7	0/0/75	26.0	1 unknown
elec50	99.3	0/0/150	65.3	1 unknown
elf	101.9	24/0/30	0.8	$\mathcal{M}(\mathcal{S}_3, 54)$
emfl100_5_5	0.5	0/0/23	20.0	\mathcal{S}_2 , \mathcal{S}_{21}
ex14_1_5	66.7	0/0/4	1.4	\mathcal{S}_4
ex2_1_3	50.0	0/0/4	0.3	$\mathcal{M}(\mathcal{S}_2, 4)$
ex5_2_5	81.8	0/0/27	0.8	$\mathcal{M}(\mathcal{S}_3, 27)$
ex8_1_6	66.7	0/0/2	0.3	\mathcal{S}_2
ex8_3_1	95.5	0/0/105	2.1	1 unknown
ex8_3_11	95.5	0/0/105	2.1	1 unknown
ex8_3_12	95.7	0/0/110	2.1	1 unknown
ex8_3_13	95.5	0/0/105	2.1	1 unknown
ex8_3_14	95.2	0/0/100	2.1	1 unknown
ex8_3_2	94.7	0/0/90	2.1	1 unknown
ex8_3_3	94.7	0/0/90	2.1	1 unknown
ex8_3_4	94.7	0/0/90	2.1	1 unknown
ex8_3_5	94.7	0/0/90	2.1	1 unknown
ex8_3_7	93.8	0/0/105	2.1	1 unknown
ex8_3_8	95.5	0/0/105	2.1	1 unknown
ex8_3_9	95.9	0/0/70	2.1	1 unknown
ex8_4_6	40.0	0/0/6	0.8	$\mathcal{M}(\mathcal{S}_3, 6)$
ex8_6_1	90.0	0/0/63	3.7	1 unknown
ex8_6_2	84.0	0/0/21	3.7	$\mathcal{M}(\mathcal{S}_7, 21)$
ex9_1_8	33.3	0/0/2	0.3	\mathcal{S}_2
ex9_2_2	28.6	0/0/2	0.3	\mathcal{S}_2
ex9_2_6	92.3	0/0/12	1.2	$\mathcal{S}_2 \wr \mathcal{S}_2$, $\mathcal{M}(\mathcal{S}_2, 8)$
gabriel04	18.6	0/0/64	0.6	$(\mathcal{M}(\mathcal{S}_2, 32))^2$
gams03	95.0	380/1881/0	4.8	$\mathcal{M}(\mathcal{S}_5, 595)$, $(\mathcal{M}(\mathcal{S}_2, 238))^4$, $(\mathcal{M}(\mathcal{S}_3, 357))^2$
gasnet_al1	6.7	4/0/20	0.6	$\mathcal{M}(\mathcal{S}_2, 14)$, $\mathcal{M}(\mathcal{S}_2, 10)$
gasnet_al2	6.6	4/0/20	0.6	$\mathcal{M}(\mathcal{S}_2, 14)$, $\mathcal{M}(\mathcal{S}_2, 10)$
gasnet_al3	6.7	4/0/20	0.6	$\mathcal{M}(\mathcal{S}_2, 14)$, $\mathcal{M}(\mathcal{S}_2, 10)$
gasnet_al4	6.7	4/0/20	0.6	$\mathcal{M}(\mathcal{S}_2, 14)$, $\mathcal{M}(\mathcal{S}_2, 10)$
gasnet_al5	6.7	4/0/20	0.6	$\mathcal{M}(\mathcal{S}_2, 14)$, $\mathcal{M}(\mathcal{S}_2, 10)$
gastrans135	0.9	0/0/4	0.6	$(\mathcal{S}_2)^2$
gastrans582_cold13	3.2	0/0/20	2.1	$(\mathcal{S}_2)^4$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cold13_95	3.2	0/0/20	2.1	$(\mathcal{S}_2)^4$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cold17	3.2	0/0/20	2.1	$(\mathcal{S}_2)^4$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cold17_95	3.2	0/0/20	2.1	$(\mathcal{S}_2)^4$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cool12	3.5	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cool12_95	3.5	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cool14	3.5	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_cool14_95	3.5	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_freezing27	3.6	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_freezing27_95	3.6	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$
gastrans582_freezing30	3.6	0/0/22	2.4	$(\mathcal{S}_2)^5$, $(\mathcal{M}(\mathcal{S}_2, 4))^3$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
gastrans582_freezing30_95	3.6	0/0/22	2.4	$(S_2)^5, (\mathcal{M}(S_2, 4))^3$
gastrans582_mild10	3.2	0/0/20	2.1	$(S_2)^4, (\mathcal{M}(S_2, 4))^3$
gastrans582_mild10_95	3.2	0/0/20	2.1	$(S_2)^4, (\mathcal{M}(S_2, 4))^3$
gastrans582_mild11	3.5	0/0/22	2.4	$(S_2)^5, (\mathcal{M}(S_2, 4))^3$
gastrans582_mild11_95	3.5	0/0/22	2.4	$(S_2)^5, (\mathcal{M}(S_2, 4))^3$
gastrans582_warm15	2.9	0/0/18	1.8	$(S_2)^3, (\mathcal{M}(S_2, 4))^3$
gastrans582_warm15_95	2.9	0/0/18	1.8	$(S_2)^3, (\mathcal{M}(S_2, 4))^3$
gastrans582_warm31	2.9	0/0/18	1.8	$(S_2)^3, (\mathcal{M}(S_2, 4))^3$
gastrans582_warm31_95	2.9	0/0/18	1.8	$(S_2)^3, (\mathcal{M}(S_2, 4))^3$
gear	80.0	0/4/0	0.6	$(S_2)^2$
gear2	96.6	24/0/4	0.6	$(\mathcal{M}(S_2, 14))^2$
gear3	80.0	0/4/0	0.6	$(S_2)^2$
gear4	66.7	0/4/0	0.6	$(S_2)^2$
graphpart_2g-0044-1601	104.3	48/0/0	0.8	$\mathcal{M}(S_3, 48)$
graphpart_2g-0055-0062	98.7	75/0/0	0.8	$\mathcal{M}(S_3, 75)$
graphpart_2g-0066-0066	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_2g-0077-0077	99.3	147/0/0	0.8	$\mathcal{M}(S_3, 147)$
graphpart_2g-0088-0088	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_2g-0099-9211	99.6	243/0/0	0.8	$\mathcal{M}(S_3, 243)$
graphpart_2g-1010-0824	99.7	300/0/0	0.8	$\mathcal{M}(S_3, 300)$
graphpart_2pm-0044-0044	104.3	48/0/0	0.8	$\mathcal{M}(S_3, 48)$
graphpart_2pm-0055-0055	98.7	75/0/0	0.8	$\mathcal{M}(S_3, 75)$
graphpart_2pm-0066-0066	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_2pm-0077-0777	99.3	147/0/0	0.8	$\mathcal{M}(S_3, 147)$
graphpart_2pm-0088-0888	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_2pm-0099-0999	99.6	243/0/0	0.8	$\mathcal{M}(S_3, 243)$
graphpart_3g-0234-0234	98.6	72/0/0	0.8	$\mathcal{M}(S_3, 72)$
graphpart_3g-0244-0244	99.0	96/0/0	0.8	$\mathcal{M}(S_3, 96)$
graphpart_3g-0333-0333	98.8	81/0/0	0.8	$\mathcal{M}(S_3, 81)$
graphpart_3g-0334-0334	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_3g-0344-0344	99.3	144/0/0	0.8	$\mathcal{M}(S_3, 144)$
graphpart_3g-0444-0444	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_3pm-0234-0234	98.6	72/0/0	0.8	$\mathcal{M}(S_3, 72)$
graphpart_3pm-0244-0244	99.0	96/0/0	0.8	$\mathcal{M}(S_3, 96)$
graphpart_3pm-0333-0333	98.8	81/0/0	0.8	$\mathcal{M}(S_3, 81)$
graphpart_3pm-0334-0334	99.1	108/0/0	0.8	$\mathcal{M}(S_3, 108)$
graphpart_3pm-0344-0344	99.3	144/0/0	0.8	$\mathcal{M}(S_3, 144)$
graphpart_3pm-0444-0444	99.5	192/0/0	0.8	$\mathcal{M}(S_3, 192)$
graphpart_clique-20	98.4	60/0/0	1.1	1 unknown
graphpart_clique-30	98.9	90/0/0	1.1	1 unknown
graphpart_clique-40	99.2	120/0/0	1.1	1 unknown
graphpart_clique-50	99.3	150/0/0	1.1	1 unknown
graphpart_clique-60	99.4	180/0/0	1.1	1 unknown
graphpart_clique-70	99.5	210/0/0	1.1	1 unknown
hadamard_4	94.1	16/0/0	2.8	1 unknown
hadamard_5	96.2	25/0/0	4.2	1 unknown
hadamard_6	97.3	36/0/0	5.7	1 unknown
hadamard_7	98.0	49/0/0	7.4	1 unknown
hadamard_8	98.5	64/0/0	9.2	1 unknown
hmittelman	46.2	6/0/0	0.9	$(S_2)^3$
iswath2	2.1	0/0/134	99.0	$S_{54}, (S_4)^{20}$
ivalues	98.5	0/200/0	0.6	$(\mathcal{M}(S_2, 100))^2$
jbearing100	100.0	0/0/5000	0.3	$\mathcal{M}(S_2, 5000)$
jbearing25	95.9	0/0/1200	0.3	$\mathcal{M}(S_2, 1200)$
jbearing50	100.0	0/0/2500	0.3	$\mathcal{M}(S_2, 2500)$
jbearing75	98.6	0/0/3700	0.3	$\mathcal{M}(S_2, 3700)$
kissing2	96.2	0/0/744	144.1	$\mathcal{M}(S_{93}, 744)$
knp3-12	97.3	0/0/36	9.5	1 unknown
knp4-24	99.0	0/0/96	25.2	1 unknown
knp5-40	99.5	0/0/200	50.0	1 unknown
knp5-41	99.5	0/0/205	51.6	1 unknown
knp5-42	99.5	0/0/210	53.2	1 unknown
knp5-43	99.5	0/0/215	54.9	1 unknown
knp5-44	99.5	0/0/220	56.5	1 unknown
kport20	56.1	5/17/33	2.0	$\mathcal{M}(S_3, 15), (\mathcal{M}(S_2, 10))^4$
kport40	69.1	15/46/89	10.1	$\mathcal{M}(S_2, 14), \mathcal{M}(S_3, 15),$ $(\mathcal{M}(S_3, 21))^4, \mathcal{M}(S_2, 10), \mathcal{M}(S_9, 27)$

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
lop97icx	3.4	0/0/6	0.9	$(S_2)^3$
maxcsp-ehi-85-297-12	2.3	48/0/0	1.2	$(M(S_2, 12))^4$
maxcsp-ehi-85-297-36	2.0	41/0/0	1.4	$M(S_2, 14), M(S_3, 15), M(S_2, 12)$
maxcsp-ehi-85-297-71	1.2	24/0/0	0.6	$(M(S_2, 12))^4$
maxcsp-ehi-90-315-70	0.5	12/0/0	0.3	$(M(S_2, 12))^2$
maxcsp-langford-3-11	98.1	616/0/0	0.3	$M(S_2, 616)$
maxmin	96.0	0/0/24	9.0	1 unknown
mbtd	101.0	200/0/10	1.3	$M(S_2, 100), 1 \text{ unknown}$
nd_netgen-2000-3-4-b-a-ns_7	0.1	2/0/5	1.1	$S_3, M(S_2, 4)$
netmod_doll	99.9	462/0/1524	15.4	1 unknown
netmod_kar1	99.8	136/0/312	4.1	1 unknown
netmod_kar2	99.8	136/0/312	4.1	1 unknown
nuclear104	83.9	10816/0/91	9.8	$M(S_{13}, 10907)$
nuclear14	60.2	576/0/18	2.9	$M(S_6, 594)$
nuclear25	61.3	625/0/20	2.1	$M(S_5, 645)$
nuclear49	73.3	2401/0/42	3.7	$M(S_7, 2443)$
nuclearva	46.8	144/0/9	0.8	$M(S_3, 153)$
nuclearvb	46.8	144/0/9	0.8	$M(S_3, 153)$
nuclearvc	46.8	144/0/9	0.8	$M(S_3, 153)$
nuclearvd	46.8	144/0/9	0.8	$M(S_3, 153)$
nuclearve	46.8	144/0/9	0.8	$M(S_3, 153)$
nuclearvf	46.8	144/0/9	0.8	$M(S_3, 153)$
nvs09	90.9	0/10/0	6.6	S_{10}
oil	16.1	0/0/102	13.2	$(M(S_3, 6))^2, (S_2)^{16}, S_4,$ $(M(S_2, 4))^2, M(S_5, 10), M(S_4, 8),$ $M(S_4, 28)$
oil2	3.6	0/0/14	1.2	$S_2, (M(S_2, 4))^3$
orth_d4m6_pl	90.5	0/0/38	0.3	$M(S_2, 38)$
pooling_bental5pq	94.6	0/0/87	0.8	$M(S_3, 87)$
pooling_bental5stp	95.8	0/0/114	0.8	$M(S_3, 114)$
pooling_bental5tp	94.6	0/0/87	0.8	$M(S_3, 87)$
powerflow2383wpp	0.1	0/0/16	0.6	$(M(S_2, 8))^2$
powerflow2383wpr	0.1	0/0/16	0.6	$(M(S_2, 8))^2$
procurement1large	3.1	0/0/94	18.7	$(S_2)^{26}, (S_3)^{14}$
procurement1mot	4.1	0/0/10	1.5	$(S_2)^5$
procurement2mot	3.8	0/0/10	1.5	$(S_2)^5$
product	1.9	0/0/9	2.7	$(S_2)^2, S_5$
product2	87.2	128/0/528	262.6	$(M(S_4, 24))^3, (M(S_4, 48))^2, S_{20},$ $(S_8)^3, S_{12}, (M(S_4, 60))^2, (S_{24})^7,$ $(M(S_4, 12))^2, (S_{16})^3, (M(S_4, 36))^2$
radar-2000-10-a-6_lat_7	76.41529	0/3058	2638.3	$M(S_{43}, 129), (M(S_{33}, 99))^2,$ $M(S_{44}, 132), M(S_{635}, 1905),$ $(M(S_{34}, 102))^2, M(S_{11}, 33),$ $M(S_{12}, 36), M(S_{18}, 54),$ $M(S_{51}, 153), M(S_{19}, 57),$ $M(S_{220}, 660), (M(S_{16}, 48))^2,$ $M(S_{21}, 63), M(S_{37}, 111),$ $(M(S_{28}, 84))^2, M(S_{23}, 69),$ $M(S_8, 24), (M(S_{30}, 90))^3,$ $(M(S_9, 27))^2, (M(S_{26}, 78))^2,$ $M(S_5, 15)$
radar-3000-10-a-8_lat_7	50.11504	0/3008	3416.2	$M(S_{71}, 213), M(S_{315}, 945),$ $M(S_{861}, 2583), M(S_{257}, 771)$
ringpack_10_2	25.3	20/0/0	0.3	$M(S_2, 20)$
ringpack_20_2	17.1	40/0/0	0.3	$M(S_2, 40)$
ringpack_20_3	100.4	213/0/40	8.7	1 unknown
ringpack_30_2	13.0	60/0/0	0.3	$M(S_2, 60)$
routingdelay_bigm	2.0	4/0/4	1.2	$(S_2)^4$
routingdelay_proj	4.6	4/0/20	3.6	$(S_2)^{12}$
rsyn0815m	5.3	0/0/5	1.1	S_2, S_3
rsyn0815m02m	3.9	0/0/10	2.2	$(S_2)^2, (S_3)^2$
rsyn0815m03m	3.8	0/0/15	3.2	$(S_2)^3, (S_3)^3$
rsyn0815m04m	3.7	0/0/20	4.3	$(S_2)^4, (S_3)^4$
sepasequ_complex	3.3	0/0/12	3.9	$S_2, S_2 \wr S_5$
sporttournament10	4.3	2/0/0	0.3	S_2
st_e09	66.7	0/0/2	0.3	S_2
st_e18	100.0	0/0/2	0.3	S_2

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name	affected vars		formulation group	
	%	B/I/C	$\log_{10} G $	factors
st_qpc-m3c	90.9	0/0/10	1.5	$(\mathcal{S}_2)^5$
st_rv9	39.2	0/0/20	3.0	$(\mathcal{S}_2)^{10}$
steenbrf	46.1	0/0/216	0.9	$(\mathcal{M}(\mathcal{S}_2, 72))^3$
syn15m	11.9	0/0/5	1.1	$\mathcal{S}_2, \mathcal{S}_3$
t1000	95.2	0/0/954	1934.7	$(\mathcal{S}_2)^5, \mathcal{S}_{274}, \mathcal{S}_{381}, \mathcal{M}(\mathcal{S}_2, 10), \mathcal{S}_{279}$
tln2	100.0	0/6/0	0.3	$\mathcal{M}(\mathcal{S}_2, 6)$
topopt-zhou-rozvany_75	28.6	0/0/400	60.2	$(\mathcal{S}_2)^{200}$
torsion100	100.0	0/0/5002	0.9	$\mathcal{S}_2, \mathcal{M}(\mathcal{S}_2 \times \mathcal{S}_2, 5000)$
torsion25	100.0	0/0/1252	0.9	$\mathcal{S}_2, 1 \text{ unknown}$
torsion50	100.0	0/0/2502	1.2	$\mathcal{S}_2, 1 \text{ unknown}$
torsion75	100.0	0/0/3752	0.9	$\mathcal{S}_2, 1 \text{ unknown}$
transswitch2383wpp	0.1	4/0/16	0.6	$(\mathcal{M}(\mathcal{S}_2, 10))^2$
turkey	2.0	0/0/8	1.2	$(\mathcal{S}_2)^4$
util	5.9	2/0/0	0.3	\mathcal{S}_2
waste	3.1	0/0/38	8.3	$(\mathcal{S}_2)^7, (\mathcal{S}_3)^8$
watercontamination0202	1.5	0/0/4	0.3	$\mathcal{M}(\mathcal{S}_2, 4)$
watercontamination0303	1.0	0/0/4	0.3	$\mathcal{M}(\mathcal{S}_2, 4)$
waterno2_01	45.9	0/0/28	0.6	$(\mathcal{M}(\mathcal{S}_2, 14))^2$
waterno2_02	25.7	0/0/28	0.6	$(\mathcal{M}(\mathcal{S}_2, 14))^2$
waterund28	28.6	0/0/212	0.6	1 unknown

Appendix B

Detailed Computational Results

TABLE B.1: Detailed results for variant **default** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of added SBCs (#sbc), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#sbc	sym-time
arki0002	24	limit	-	-	-
arki0005	439	limit	-	-	-
arki0006	97	limit	-	-	-
arki0008	1	32.87	-	-	-
arki0016	8,950	limit	-	-	-
arki0017	10,327	limit	995.85	-	-
arki0018	2	limit	-	-	-
autocorr_bern20-03	1	0.02	-	-	-
autocorr_bern20-05	35,724	60.21	-	-	-
autocorr_bern20-10	189,870	1,224.42	-	-	-
autocorr_bern20-15	204,468	2,638.87	-	-	-
autocorr_bern25-03	1	0.02	-	-	-
autocorr_bern25-06	1,251,262	limit	21.67	-	-
autocorr_bern25-13	96,256	limit	103.70	-	-
autocorr_bern25-19	36,317	limit	267.08	-	-
autocorr_bern25-25	20,061	limit	417.52	-	-
autocorr_bern30-04	1,914,925	2,897.03	-	-	-
autocorr_bern30-08	234,553	limit	189.53	-	-
autocorr_bern30-15	28,016	limit	305.74	-	-
autocorr_bern30-23	5,427	limit	527.73	-	-
autocorr_bern30-30	3,166	limit	782.57	-	-
autocorr_bern35-04	859,563	limit	58.07	-	-
autocorr_bern35-09	78,039	limit	371.35	-	-
autocorr_bern35-18	4,383	limit	495.41	-	-
autocorr_bern35-26	2,091	limit	805.50	-	-
autocorr_bern35-35fix	1,791	limit	981.77	-	-
autocorr_bern40-05	356,042	limit	195.02	-	-
autocorr_bern40-10	38,276	limit	530.93	-	-
autocorr_bern40-20	1,985	limit	725.06	-	-
autocorr_bern40-30	1,075	limit	924.50	-	-
autocorr_bern40-40	1,295	limit	1,077.95	-	-
autocorr_bern45-05	249,290	limit	234.12	-	-
autocorr_bern45-11	17,443	limit	686.73	-	-
autocorr_bern45-23	1,388	limit	881.57	-	-
autocorr_bern45-34	827	limit	1,103.40	-	-
autocorr_bern45-45	795	limit	1,212.30	-	-
autocorr_bern50-06	126,523	limit	430.15	-	-
autocorr_bern50-13	6,979	limit	745.06	-	-
autocorr_bern50-25	889	limit	1,031.93	-	-
autocorr_bern55-06	112,692	limit	441.52	-	-
autocorr_bern55-14	2,963	limit	932.52	-	-
autocorr_bern55-28	664	limit	1,158.56	-	-
autocorr_bern60-08	28,194	limit	714.08	-	-
autocorr_bern60-15	2,350	limit	1,010.90	-	-
ball_mk2_10	1	0.00	-	-	-
ball_mk2_30	3,884,971	3,319.32	-	-	-
ball_mk3_10	0	0.01	-	-	-

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name	#nodes	total-time	gap	#sbc	sym-time
carton7	345,039	903.75	-	-	-
carton9	739,244	limit	61.40	-	-
celar6-sub0	1,531	limit	-	-	-
chp_partload	683	limit	-	-	-
chp_shorttermplan1b	28,571	limit	5.72	-	-
chp_shorttermplan2a	541	149.30	-	-	-
chp_shorttermplan2c	18,529	limit	-	-	-
color_lab3_3x0	57,584	limit	694.88	-	-
color_lab3_4x0	44,923	limit	1,284.98	-	-
crossdock_15x7	61,895	limit	595.15	-	-
crossdock_15x8	52,112	limit	1,701.34	-	-
crudeoil_li02	1,690,305	limit	1.13	-	-
crudeoil_li06	20,619	392.00	-	-	-
crudeoil_li21	182,940	limit	2.04	-	-
crudeoil_pooling_dt4	20,874	limit	8.16	-	-
cvxnonsep_normcon30	1	0.11	-	-	-
cvxnonsep_normcon30r	1	0.10	-	-	-
cvxnonsep_normcon40	1	0.14	-	-	-
cvxnonsep_normcon40r	10	0.43	-	-	-
cvxnonsep_psig20	8,165,216	limit	231.12	-	-
cvxnonsep_psig20r	1	0.15	-	-	-
cvxnonsep_psig30	7,384,054	limit	155.14	-	-
cvxnonsep_psig30r	68	1.00	-	-	-
cvxnonsep_psig40	6,800,441	limit	113.51	-	-
cvxnonsep_psig40r	51	0.96	-	-	-
densitymod	2,443	limit	-	-	-
eigena2	1	limit	-	-	-
elec100	1,387	limit	-	-	-
elec200	1	limit	-	-	-
elec25	55,730	limit	-	-	-
elec50	15,684	limit	-	-	-
elf	254	1.96	-	-	-
emfl100_5_5	3,151	limit	-	-	-
ex14_1_5	1	0.01	-	-	-
ex2_1_3	1	0.17	-	-	-
ex5_2_5	1	1.00	-	-	-
ex8_1_6	17	0.15	-	-	-
ex8_3_1	718,916	limit	23.00	-	-
ex8_3_11	565,310	limit	30.77	-	-
ex8_3_13	383,219	limit	16.86	-	-
ex8_3_2	658,346	limit	42.73	-	-
ex8_3_3	681,244	limit	39.22	-	-
ex8_3_4	606,268	limit	62.01	-	-
ex8_3_5	652,718	limit	1,496.29	-	-
ex8_3_7	59	limit	-	-	-
ex8_3_8	127,806	limit	207.11	-	-
ex8_3_9	795,578	limit	31.06	-	-
ex8_4_6	3	limit	-	-	-
ex8_6_1	1	limit	-	-	-
ex9_1_8	1	0.00	-	-	-
ex9_2_2	1	0.14	-	-	-
ex9_2_6	1	0.11	-	-	-
gabriel04	8,108	329.06	-	-	-
gams03	1	limit	-	-	-
gastrans582_cold13	22	33.83	-	-	-
gastrans582_cold13_95	35	25.47	-	-	-
gastrans582_cold17	47	37.88	-	-	-
gastrans582_cold17_95	115	42.50	-	-	-
gastrans582_cool12	38	30.21	-	-	-
gastrans582_cool12_95	30	41.69	-	-	-
gastrans582_cool14	34	35.30	-	-	-
gastrans582_cool14_95	26	29.95	-	-	-
gastrans582_freezing27	0	22.42	-	-	-
gastrans582_freezing27_95	1	36.80	-	-	-
gastrans582_freezing30	22	44.60	-	-	-
gastrans582_freezing30_95	5	32.23	-	-	-
gastrans582_mild10	17	29.91	-	-	-
gastrans582_mild10_95	47	29.43	-	-	-

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name	#nodes	total-time	gap	#sbcs	sym-time
gastrans582_mild11	63	37.16	-	-	-
gastrans582_mild11_95	31	37.12	-	-	-
gastrans582_warm15	1	17.80	-	-	-
gastrans582_warm15_95	20	28.08	-	-	-
gastrans582_warm31	35	38.94	-	-	-
gastrans582_warm31_95	65	30.87	-	-	-
gear	26,844	18.68	-	-	-
gear2	37,733	21.81	-	-	-
gear3	27,844	19.48	-	-	-
gear4	1,268	0.45	-	-	-
graphpart_2g-0044-1601	1	0.33	-	-	-
graphpart_2g-0055-0062	15	1.46	-	-	-
graphpart_2g-0066-0066	23	2.93	-	-	-
graphpart_2g-0077-0077	305	7.04	-	-	-
graphpart_2g-0088-0088	71	9.52	-	-	-
graphpart_2g-0099-9211	332	17.46	-	-	-
graphpart_2g-1010-0824	45	13.69	-	-	-
graphpart_2pm-0044-0044	10	1.25	-	-	-
graphpart_2pm-0055-0055	39	1.97	-	-	-
graphpart_2pm-0066-0066	186	3.72	-	-	-
graphpart_2pm-0077-0777	195	5.35	-	-	-
graphpart_2pm-0088-0888	113	7.82	-	-	-
graphpart_2pm-0099-0999	6,712	85.56	-	-	-
graphpart_3g-0234-0234	29	2.36	-	-	-
graphpart_3g-0244-0244	36	5.13	-	-	-
graphpart_3g-0333-0333	20	3.60	-	-	-
graphpart_3g-0334-0334	287	6.44	-	-	-
graphpart_3g-0344-0344	261	9.57	-	-	-
graphpart_3g-0444-0444	7,694	120.45	-	-	-
graphpart_3pm-0234-0234	204	2.62	-	-	-
graphpart_3pm-0244-0244	752	5.93	-	-	-
graphpart_3pm-0333-0333	1,044	6.97	-	-	-
graphpart_3pm-0334-0334	3,675	22.85	-	-	-
graphpart_3pm-0344-0344	21,755	142.66	-	-	-
graphpart_3pm-0444-0444	514,592	limit	6.15	-	-
graphpart_clique-20	2,106	18.03	-	-	-
graphpart_clique-30	51,613	655.53	-	-	-
graphpart_clique-40	173,325	limit	31.74	-	-
graphpart_clique-50	31,179	limit	229.29	-	-
graphpart_clique-60	21,557	limit	705.30	-	-
graphpart_clique-70	12,286	limit	597.77	-	-
hadamard_4	173	0.56	-	-	-
hadamard_5	121,260	176.12	-	-	-
hadamard_6	105,001	limit	965.33	-	-
hadamard_7	2,571	limit	136,825.00	-	-
hadamard_8	13	limit	-	-	-
hmittelman	1	0.03	-	-	-
ivalues	72,272	limit	409.38	-	-
kissing2	1	909.72	-	-	-
knp3-12	123,860	limit	197.28	-	-
knp4-24	5,991	limit	485.79	-	-
knp5-40	1,885	limit	770.96	-	-
knp5-41	2,185	limit	811.31	-	-
knp5-42	2,139	limit	807.71	-	-
knp5-43	2,367	limit	765.37	-	-
knp5-44	841	limit	763.42	-	-
kport20	153,400	717.51	-	-	-
lop97icx	2,083	24.25	-	-	-
maxcsp-ehi-85-297-12	264	limit	-	-	-
maxcsp-ehi-85-297-36	294	limit	-	-	-
maxcsp-ehi-85-297-71	256	limit	-	-	-
maxcsp-ehi-90-315-70	201	limit	-	-	-
maxcsp-langford-3-11	26,758	limit	-	-	-
maxmin	107,240	limit	73.33	-	-
mbtd	1,971	limit	165.63	-	-
netmod_doll	4,151	limit	64.95	-	-
netmod_kar1	3,757	117.01	-	-	-
netmod_kar2	3,757	117.14	-	-	-

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name	#nodes	total-time	gap	#sbc	sym-time
nuclear14	30	limit	-	-	-
nuclear25	15	limit	-	-	-
nuclear49	21	limit	-	-	-
nuclearva	22	limit	-	-	-
nuclearvb	111	limit	-	-	-
nuclearvc	212	limit	-	-	-
nuclearvd	287	limit	-	-	-
nuclearve	59	limit	-	-	-
nuclearvf	66	limit	-	-	-
nvs09	614	1.47	-	-	-
oil2	118	27.58	-	-	-
orth_d4m6_pl	430,653	limit	168.62	-	-
pooling_bental5pq	1	0.17	-	-	-
pooling_bental5tp	40	2.10	-	-	-
pooling_bental5tp	1	0.13	-	-	-
powerflow2383wpp	1	limit	-	-	-
powerflow2383wpr	1	limit	-	-	-
procurement1large	96,632	limit	375.14	-	-
procurement1mot	838,722	limit	491.08	-	-
procurement2mot	167	4.45	-	-	-
product	990	87.65	-	-	-
product2	1	14.16	-	-	-
radar-2000-10-a-6_lat_7	15,896	limit	138.30	-	-
radar-3000-10-a-8_lat_7	12,097	limit	10,110.83	-	-
ringpack_10_2	128,530	limit	3.95	-	-
ringpack_20_2	8,185	limit	233.33	-	-
ringpack_20_3	26,531	limit	82.91	-	-
ringpack_30_2	61	limit	1,400.00	-	-
rsyn0815m	151	1.06	-	-	-
rsyn0815m02m	1,397	13.55	-	-	-
rsyn0815m03m	1,949	22.54	-	-	-
rsyn0815m04m	6,390	77.33	-	-	-
sepasequ_complex	62,343	limit	62.94	-	-
sporttournament10	1	0.70	-	-	-
st_e09	1	0.03	-	-	-
st_e18	1	0.00	-	-	-
st_qpc-m3c	1	0.02	-	-	-
st_rv9	1,791	3.22	-	-	-
syn15m	1	0.33	-	-	-
t1000	0	0.18	-	-	-
tln2	1	0.01	-	-	-
topopt-zhou-rozvany_75	464	limit	-	-	-
torsion100	20	limit	380.39	-	-
torsion25	1	limit	319.02	-	-
torsion50	16	limit	373.99	-	-
turkey	1	19.78	-	-	-
util	15	0.23	-	-	-
waste	101	limit	118.63	-	-
watercontamination0202	19	107.23	-	-	-
waterund28	10,038	limit	7.76	-	-

TABLE B.2: Detailed results for variant **S** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of added SBCs (#sbc), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#sbc	sym-time
arki0002	1	limit	-	10	0.02
arki0005	439	limit	-	-	0.05
arki0006	97	limit	-	-	0.05
arki0008	1	56.94	-	49	0.02
arki0016	8,100	limit	-	4	0.02
arki0017	9,880	limit	989.38	7	0.02
arki0018	1	limit	-	1,500	11.26

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name	#nodes	total-time	gap	#sbcs	sym-time
autocorr_bern20-03	1	0.03	-	3	0.00
autocorr_bern20-05	51,135	78.52	-	1	0.00
autocorr_bern20-10	178,610	1,165.82	-	1	0.00
autocorr_bern20-15	199,585	2,654.88	-	1	0.01
autocorr_bern25-03	1	0.03	-	2	0.00
autocorr_bern25-06	1,299,056	limit	27.50	1	0.00
autocorr_bern25-13	81,901	limit	123.16	1	0.01
autocorr_bern25-19	34,835	limit	248.45	1	0.02
autocorr_bern25-25	14,128	limit	395.34	1	0.01
autocorr_bern30-04	1,576,889	2,369.11	-	1	0.01
autocorr_bern30-08	187,993	limit	202.81	1	0.01
autocorr_bern30-15	31,798	limit	337.93	1	0.02
autocorr_bern30-23	5,075	limit	577.67	1	0.03
autocorr_bern30-30	3,331	limit	715.09	1	0.03
autocorr_bern35-04	723,324	limit	73.44	1	0.00
autocorr_bern35-09	89,577	limit	362.86	1	0.00
autocorr_bern35-18	5,020	limit	523.06	1	0.03
autocorr_bern35-26	2,316	limit	753.41	1	0.05
autocorr_bern35-35fix	2,152	limit	1,023.68	1	0.05
autocorr_bern40-05	382,288	limit	184.05	1	0.00
autocorr_bern40-10	36,234	limit	539.29	1	0.01
autocorr_bern40-20	2,281	limit	730.04	1	0.02
autocorr_bern40-30	1,030	limit	961.25	1	0.08
autocorr_bern40-40	1,029	limit	1,096.93	1	0.11
autocorr_bern45-05	264,890	limit	232.04	1	0.00
autocorr_bern45-11	16,016	limit	688.66	1	0.02
autocorr_bern45-23	1,407	limit	870.30	1	0.05
autocorr_bern45-34	1,067	limit	1,052.60	1	0.11
autocorr_bern45-45	726	limit	1,238.79	1	0.13
autocorr_bern50-06	129,307	limit	417.01	1	0.00
autocorr_bern50-13	6,709	limit	862.03	1	0.02
autocorr_bern50-25	844	limit	1,028.12	1	0.10
autocorr_bern55-06	105,194	limit	446.02	1	0.01
autocorr_bern55-14	2,970	limit	929.77	1	0.03
autocorr_bern55-28	737	limit	1,122.54	1	0.13
autocorr_bern60-08	31,553	limit	708.34	1	0.01
autocorr_bern60-15	2,738	limit	1,115.49	1	0.04
ball_mk2_10	1	0.00	-	9	0.00
ball_mk2_30	1	0.02	-	29	0.00
ball_mk3_10	0	0.01	-	-	0.00
carton7	231,293	583.91	-	3	0.00
carton9	827,919	limit	60.55	1	0.01
celar6-sub0	1,393	limit	-	3	0.21
chp_partload	594	limit	-	1	0.03
chp_shorttermplan1b	50,326	limit	6.80	1	0.02
chp_shorttermplan2a	441	129.43	-	2	0.03
chp_shorttermplan2c	23,771	limit	-	5	0.35
color_lab3_3x0	52,768	limit	555.55	4	0.01
color_lab3_4x0	39,319	limit	1,257.62	4	0.01
crossdock_15x7	66,311	limit	463.06	1	0.01
crossdock_15x8	53,833	limit	1,759.14	1	0.01
crudeoil_li02	1,546,195	limit	1.13	3	0.00
crudeoil_li06	108,420	1,522.29	-	1	0.00
crudeoil_li21	166,926	limit	1.93	1	0.00
crudeoil_pooling_dt4	39,600	limit	-	-	0.00
cvxnonsep_normcon30	1	0.10	-	2	0.00
cvxnonsep_normcon30r	1	0.12	-	2	0.00
cvxnonsep_normcon40	1	0.05	-	2	0.00
cvxnonsep_normcon40r	29	0.67	-	2	0.00
cvxnonsep_psig20	8,320,026	limit	228.77	2	0.00
cvxnonsep_psig20r	1	0.16	-	2	0.00
cvxnonsep_psig30	7,296,791	limit	155.01	1	0.00
cvxnonsep_psig30r	87	0.96	-	1	0.00
cvxnonsep_psig40	6,278,508	limit	113.45	4	0.00
cvxnonsep_psig40r	22	0.86	-	5	0.00
densitymod	4,241	limit	-	58	4.38
eigena2	1	limit	-	49	1.15
elec100	1	limit	-	299	1.79

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name	#nodes	total-time	gap	#sbc	sym-time
elec200	1	limit	-	599	26.18
elec25	4,407	limit	-	74	0.02
elec50	1	limit	-	149	0.20
elf	47	1.58	-	2	0.00
emfl100_5_5	41	limit	32,331.87	21	0.04
ex14_1_5	1	0.02	-	3	0.00
ex2_1_3	1	0.20	-	1	0.00
ex5_2_5	1	1.17	-	2	0.00
ex8_1_6	17	0.16	-	1	0.00
ex8_3_1	372,917	limit	23.00	4	0.00
ex8_3_11	436,514	limit	25.07	4	0.00
ex8_3_13	305,588	limit	15.67	4	0.00
ex8_3_2	488,518	limit	40.66	4	0.00
ex8_3_3	554,861	limit	39.22	4	0.00
ex8_3_4	482,282	limit	62.01	4	0.00
ex8_3_5	504,187	limit	1,496.29	4	0.00
ex8_3_7	26	limit	-	4	0.00
ex8_3_8	123,120	limit	207.11	4	0.00
ex8_3_9	698,909	limit	31.06	4	0.00
ex8_4_6	5	limit	-	2	0.00
ex8_6_1	1,391	limit	-	6	0.00
ex9_1_8	1	0.00	-	1	0.00
ex9_2_2	1	0.14	-	1	0.00
ex9_2_6	1	0.11	-	4	0.00
gabriel04	4,837	251.45	-	2	0.00
gams03	1	limit	-	12	1.57
gastrans582_cold13	1	14.98	-	2	0.00
gastrans582_cold13_95	16	25.53	-	2	0.00
gastrans582_cold17	41	30.73	-	1	0.01
gastrans582_cold17_95	32	27.78	-	1	0.01
gastrans582_cool12	1	16.64	-	2	0.00
gastrans582_cool12_95	31	47.71	-	2	0.01
gastrans582_cool14	77	55.02	-	2	0.00
gastrans582_cool14_95	22	39.45	-	2	0.01
gastrans582_freezing27	1	28.62	-	1	0.00
gastrans582_freezing27_95	1	33.68	-	1	0.01
gastrans582_freezing30	21	43.96	-	1	0.00
gastrans582_freezing30_95	25	29.32	-	1	0.00
gastrans582_mild10	29	37.09	-	2	0.01
gastrans582_mild10_95	18	27.11	-	2	0.00
gastrans582_mild11	5,014	286.29	-	2	0.00
gastrans582_mild11_95	13	37.66	-	2	0.00
gastrans582_warm15	7	23.89	-	2	0.01
gastrans582_warm15_95	27	25.08	-	2	0.00
gastrans582_warm31	12	36.71	-	2	0.00
gastrans582_warm31_95	22	22.93	-	2	0.00
gear	7,756	5.65	-	2	0.00
gear2	32,993	22.30	-	2	0.00
gear3	7,832	5.91	-	2	0.00
gear4	822	0.40	-	2	0.00
graphpart_2g-0044-1601	1	0.31	-	2	0.01
graphpart_2g-0055-0062	167	2.20	-	2	0.00
graphpart_2g-0066-0066	9	2.44	-	2	0.00
graphpart_2g-0077-0077	19	3.26	-	2	0.00
graphpart_2g-0088-0088	27	6.50	-	2	0.00
graphpart_2g-0099-9211	201	14.75	-	2	0.00
graphpart_2g-1010-0824	66	17.53	-	2	0.00
graphpart_2pm-0044-0044	6	0.68	-	2	0.00
graphpart_2pm-0055-0055	11	1.48	-	2	0.01
graphpart_2pm-0066-0066	118	2.92	-	2	0.00
graphpart_2pm-0077-0777	102	4.75	-	2	0.00
graphpart_2pm-0088-0888	266	8.40	-	2	0.00
graphpart_2pm-0099-0999	3,731	54.28	-	2	0.00
graphpart_3g-0234-0234	71	2.44	-	2	0.00
graphpart_3g-0244-0244	28	3.44	-	2	0.00
graphpart_3g-0333-0333	23	3.35	-	2	0.00
graphpart_3g-0334-0334	217	6.12	-	2	0.00
graphpart_3g-0344-0344	28	8.00	-	2	0.01

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name	#nodes	total-time	gap	#sbcs	sym-time
graphpart_3g-0444-0444	3,256	63.58	-	2	0.01
graphpart_3pm-0234-0234	41	2.48	-	2	0.00
graphpart_3pm-0244-0244	606	5.72	-	2	0.00
graphpart_3pm-0333-0333	335	3.81	-	2	0.01
graphpart_3pm-0334-0334	930	9.67	-	2	0.00
graphpart_3pm-0344-0344	7,691	68.87	-	2	0.00
graphpart_3pm-0444-0444	197,619	1,302.55	-	2	0.00
graphpart_clique-20	235	5.31	-	5	0.01
graphpart_clique-30	3,897	76.65	-	5	0.01
graphpart_clique-40	23,007	672.29	-	5	0.01
graphpart_clique-50	39,276	limit	99.22	5	0.02
graphpart_clique-60	18,375	limit	147.04	5	0.02
graphpart_clique-70	11,393	limit	248.74	5	0.04
hadamard_4	28	0.48	-	15	0.00
hadamard_5	8,899	15.49	-	24	0.01
hadamard_6	162,449	limit	561.11	35	0.03
hadamard_7	5,229	limit	24,783.33	48	0.30
hadamard_8	12	limit	-	63	3.18
hmittelman	1	0.02	-	3	0.00
ivalues	72,670	limit	448.77	2	0.01
kissing2	1	1,848.14	-	92	5.47
knnp3-12	88,748	limit	137.81	35	0.00
knnp4-24	7,631	limit	410.79	95	0.04
knnp5-40	140	limit	765.25	199	0.19
knnp5-41	1,238	limit	652.19	204	0.21
knnp5-42	51	limit	791.33	209	0.22
knnp5-43	393	limit	825.54	214	0.24
knnp5-44	526	limit	835.42	219	0.25
kport20	25,409	122.71	-	6	0.00
lop97icx	2,083	24.14	-	-	0.01
maxcsp-ehi-85-297-12	207	limit	-	4	2.05
maxcsp-ehi-85-297-36	230	limit	-	4	1.99
maxcsp-ehi-85-297-71	239	limit	-	2	1.18
maxcsp-ehi-90-315-70	147	limit	-	1	0.74
maxcsp-langford-3-11	28,408	limit	-	1	0.09
maxmin	106,089	limit	46.83	23	0.00
mbtd	1,834	limit	173.33	10	0.47
netmod_doll	3,929	limit	56.98	41	0.48
netmod_kar1	12,784	366.66	-	19	0.02
netmod_kar2	12,784	367.12	-	19	0.04
nuclear14	57	limit	-	5	0.03
nuclear25	47	limit	-	4	0.03
nuclear49	21	limit	-	6	0.13
nuclearva	24	limit	-	2	0.00
nuclearvb	265	limit	-	2	0.01
nuclearvc	48	limit	-	2	0.01
nuclearvd	396	limit	-	2	0.01
nuclearve	90	limit	-	2	0.01
nuclearvf	220	limit	-	2	0.00
nvs09	15	0.18	-	9	0.00
oil2	111	24.29	-	4	0.01
orth_d4m6_pl	449,178	limit	263.43	1	0.00
pooling_bental5pq	1	0.16	-	2	0.00
pooling_bental5stp	6,076	33.82	-	2	0.00
pooling_bental5tp	1	0.13	-	2	0.00
powerflow2383wpp	1	limit	-	2	0.22
powerflow2383wpr	1	limit	-	2	0.26
procurement1large	88,537	limit	375.13	-	0.01
procurement1mot	844,579	limit	522.61	-	0.00
procurement2mot	167	4.37	-	-	0.00
product	4,293	315.27	-	5	0.00
product2	1	12.39	-	290	0.21
radar-2000-10-a-6_lat_7	177	876.41	-	1,500	128.65
radar-3000-10-a-8_lat_7	881	1,351.73	-	1,500	81.11
ringpack_10_2	115,984	limit	3.95	1	0.00
ringpack_20_2	7,483	limit	400.00	1	0.02
ringpack_20_3	17,334	limit	566.67	17	0.16
ringpack_30_2	3,536	limit	-	1	0.04

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name	#nodes	total-time	gap	#sbc	sym-time
rsyn0815m	151	1.03	-	-	0.00
rsyn0815m02m	1,422	11.57	-	-	0.00
rsyn0815m03m	1,347	18.47	-	-	0.00
rsyn0815m04m	4,006	49.69	-	-	0.00
sepasequ_complex	56,576	limit	53.37	9	0.00
sporttournament10	1	0.51	-	1	0.00
st_e09	1	0.03	-	1	0.00
st_e18	1	0.02	-	1	0.00
st_qpc-m3c	1	0.02	-	5	0.01
st_rv9	237	2.09	-	10	0.00
syn15m	1	0.31	-	-	0.00
t1000	0	0.88	-	-	0.00
tln2	1	0.01	-	1	0.01
topopt-zhou-rozvany_75	448	limit	-	200	0.09
torsion100	13	limit	380.15	3	0.10
torsion25	1	limit	319.68	3	0.02
torsion50	17	limit	467.20	7	0.07
turkey	1	19.98	-	-	0.01
util	29	0.39	-	1	0.00
waste	201,968	limit	85.82	23	0.01
watercontamination0202	19	99.20	-	1	0.00
waterund28	12,562	limit	21.74	3	0.00

TABLE B.3: Detailed results for variant **S-cont** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of added SBCs (#sbc), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#sbc	sym-time
arki0002	1	limit	-	10	0.02
arki0005	439	limit	-	-	0.05
arki0006	97	limit	-	-	0.06
arki0008	1	56.52	-	49	0.03
arki0016	8,100	limit	-	4	0.02
arki0017	9,980	limit	989.38	7	0.02
arki0018	1	limit	-	1,500	11.28
autocorr_bern20-03	1	0.04	-	3	0.00
autocorr_bern20-05	51,135	77.93	-	1	0.00
autocorr_bern20-10	178,610	1,168.96	-	1	0.00
autocorr_bern20-15	199,585	2,656.35	-	1	0.01
autocorr_bern25-03	1	0.04	-	2	0.00
autocorr_bern25-06	1,302,552	limit	27.50	1	0.01
autocorr_bern25-13	81,634	limit	123.47	1	0.01
autocorr_bern25-19	34,719	limit	248.60	1	0.02
autocorr_bern25-25	14,021	limit	395.34	1	0.02
autocorr_bern30-04	1,576,889	2,367.09	-	1	0.00
autocorr_bern30-08	188,041	limit	202.81	1	0.01
autocorr_bern30-15	31,932	limit	337.93	1	0.01
autocorr_bern30-23	5,075	limit	577.67	1	0.03
autocorr_bern30-30	3,360	limit	714.86	1	0.03
autocorr_bern35-04	720,896	limit	73.53	1	0.00
autocorr_bern35-09	89,523	limit	362.86	1	0.01
autocorr_bern35-18	5,005	limit	523.06	1	0.03
autocorr_bern35-26	2,294	limit	755.01	1	0.03
autocorr_bern35-35fix	2,185	limit	1,021.97	1	0.07
autocorr_bern40-05	382,904	limit	184.05	1	0.00
autocorr_bern40-10	36,567	limit	539.04	1	0.01
autocorr_bern40-20	2,278	limit	730.04	1	0.04
autocorr_bern40-30	1,024	limit	961.25	1	0.05
autocorr_bern40-40	1,029	limit	1,096.93	1	0.08
autocorr_bern45-05	263,873	limit	232.07	1	0.00
autocorr_bern45-11	16,195	limit	687.32	1	0.01
autocorr_bern45-23	1,404	limit	870.30	1	0.07
autocorr_bern45-34	1,066	limit	1,052.60	1	0.10

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name	#nodes	total-time	gap	#sbcs	sym-time
autocorr_bern45-45	726	limit	1,238.79	1	0.13
autocorr_bern50-06	129,704	limit	416.95	1	0.01
autocorr_bern50-13	6,584	limit	862.03	1	0.03
autocorr_bern50-25	842	limit	1,028.12	1	0.08
autocorr_bern55-06	104,744	limit	446.14	1	0.00
autocorr_bern55-14	2,934	limit	930.85	1	0.04
autocorr_bern55-28	742	limit	1,122.54	1	0.12
autocorr_bern60-08	32,085	limit	707.80	1	0.01
autocorr_bern60-15	2,740	limit	1,115.49	1	0.04
ball_mk2_10	1	0.01	-	9	0.00
ball_mk2_30	1	0.02	-	29	0.00
ball_mk3_10	0	0.00	-	-	0.00
carton7	231,293	586.67	-	3	0.01
carton9	827,606	limit	60.55	1	0.00
celar6-sub0	1,358	limit	-	3	0.19
chp_partload	614	limit	-	1	0.01
chp_shorttermplan1b	37,379	limit	4.90	1	0.00
chp_shorttermplan2a	1,546	280.43	-	2	0.01
chp_shorttermplan2c	21,553	limit	-	5	0.02
color_lab3_3x0	52,523	limit	555.58	4	0.00
color_lab3_4x0	39,337	limit	1,257.62	4	0.00
crossdock_15x7	66,007	limit	463.20	1	0.00
crossdock_15x8	53,797	limit	1,759.14	1	0.01
crudeoil_li02	1,541,687	limit	1.13	3	0.00
crudeoil_li06	26,990	596.63	-	1	0.00
crudeoil_li21	168,678	limit	1.74	1	0.00
crudeoil_pooling_dt4	39,647	limit	-	-	0.01
cvxnonsep_normcon30	1	0.11	-	2	0.00
cvxnonsep_normcon30r	1	0.13	-	2	0.00
cvxnonsep_normcon40	1	0.06	-	2	0.00
cvxnonsep_normcon40r	29	0.68	-	2	0.00
cvxnonsep_psig20	8,302,421	limit	228.78	2	0.00
cvxnonsep_psig20r	1	0.17	-	2	0.00
cvxnonsep_psig30	7,199,280	limit	155.01	1	0.00
cvxnonsep_psig30r	87	0.96	-	1	0.00
cvxnonsep_psig40	6,261,646	limit	113.45	4	0.00
cvxnonsep_psig40r	22	0.80	-	5	0.00
densitymod	4,241	limit	-	58	4.36
eigena2	1	limit	-	49	1.15
elec100	1	limit	-	299	1.84
elec200	1	limit	-	599	26.58
elec25	4,409	limit	-	74	0.02
elec50	1	limit	-	149	0.22
elf	149	1.89	-	2	0.00
emfl100_5_5	41	limit	32,331.87	21	0.03
ex14_1_5	1	0.02	-	3	0.00
ex2_1_3	1	0.21	-	1	0.00
ex5_2_5	1	1.17	-	2	0.00
ex8_1_6	17	0.16	-	1	0.00
ex8_3_1	373,811	limit	23.00	4	0.00
ex8_3_11	434,832	limit	25.07	4	0.01
ex8_3_13	304,184	limit	15.67	4	0.00
ex8_3_2	490,250	limit	40.66	4	0.00
ex8_3_3	554,397	limit	39.22	4	0.00
ex8_3_4	483,888	limit	62.01	4	0.00
ex8_3_5	502,844	limit	1,496.29	4	0.00
ex8_3_7	26	limit	-	4	0.00
ex8_3_8	123,227	limit	207.11	4	0.00
ex8_3_9	696,943	limit	31.06	4	0.00
ex8_4_6	5	limit	-	2	0.00
ex8_6_1	1,391	limit	-	6	0.00
ex9_1_8	1	0.00	-	1	0.00
ex9_2_2	1	0.14	-	1	0.00
ex9_2_6	1	0.11	-	4	0.00
gabriel04	4,837	251.55	-	2	0.00
gams03	1	limit	-	12	1.51
gastrans582_cold13	1	14.97	-	2	0.01
gastrans582_cold13_95	16	25.48	-	2	0.01

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name	#nodes	total-time	gap	#sbc	sym-time
gastrans582_cold17	41	30.77	-	1	0.00
gastrans582_cold17_95	32	28.28	-	1	0.00
gastrans582_cool12	1	16.86	-	2	0.01
gastrans582_cool12_95	31	47.79	-	2	0.01
gastrans582_cool14	77	55.17	-	2	0.00
gastrans582_cool14_95	22	39.78	-	2	0.00
gastrans582_freezing27	1	28.64	-	1	0.00
gastrans582_freezing27_95	1	33.46	-	1	0.00
gastrans582_freezing30	21	43.66	-	1	0.00
gastrans582_freezing30_95	25	29.65	-	1	0.01
gastrans582_mild10	29	37.58	-	2	0.00
gastrans582_mild10_95	18	27.03	-	2	0.01
gastrans582_mild11	5,014	286.23	-	2	0.01
gastrans582_mild11_95	13	38.05	-	2	0.00
gastrans582_warm15	7	24.12	-	2	0.01
gastrans582_warm15_95	27	25.31	-	2	0.01
gastrans582_warm31	12	36.73	-	2	0.00
gastrans582_warm31_95	22	22.89	-	2	0.00
gear	7,756	5.69	-	2	0.00
gear2	32,993	22.50	-	2	0.00
gear3	7,832	6.03	-	2	0.00
gear4	822	0.40	-	2	0.01
graphpart_2g-0044-1601	1	0.30	-	2	0.00
graphpart_2g-0055-0062	167	2.20	-	2	0.00
graphpart_2g-0066-0066	9	2.39	-	2	0.00
graphpart_2g-0077-0077	19	3.23	-	2	0.00
graphpart_2g-0088-0088	27	6.50	-	2	0.00
graphpart_2g-0099-9211	201	14.86	-	2	0.01
graphpart_2g-1010-0824	66	17.34	-	2	0.00
graphpart_2pm-0044-0044	6	0.71	-	2	0.00
graphpart_2pm-0055-0055	11	1.54	-	2	0.00
graphpart_2pm-0066-0066	118	2.91	-	2	0.00
graphpart_2pm-0077-0777	102	4.53	-	2	0.00
graphpart_2pm-0088-0888	266	8.56	-	2	0.00
graphpart_2pm-0099-0999	3,731	54.04	-	2	0.00
graphpart_3g-0234-0234	71	2.43	-	2	0.01
graphpart_3g-0244-0244	28	3.37	-	2	0.00
graphpart_3g-0333-0333	23	3.30	-	2	0.00
graphpart_3g-0334-0334	217	6.05	-	2	0.00
graphpart_3g-0344-0344	28	7.82	-	2	0.00
graphpart_3g-0444-0444	3,256	63.42	-	2	0.01
graphpart_3pm-0234-0234	41	2.56	-	2	0.00
graphpart_3pm-0244-0244	606	5.53	-	2	0.00
graphpart_3pm-0333-0333	335	4.02	-	2	0.00
graphpart_3pm-0334-0334	930	9.58	-	2	0.00
graphpart_3pm-0344-0344	7,691	69.18	-	2	0.00
graphpart_3pm-0444-0444	197,619	1,303.71	-	2	0.01
graphpart_clique-20	235	5.39	-	5	0.00
graphpart_clique-30	3,897	76.86	-	5	0.01
graphpart_clique-40	23,007	671.09	-	5	0.01
graphpart_clique-50	39,107	limit	99.38	5	0.02
graphpart_clique-60	18,248	limit	147.34	5	0.03
graphpart_clique-70	11,430	limit	248.41	5	0.04
hadamard_4	28	0.49	-	15	0.00
hadamard_5	8,899	15.47	-	24	0.00
hadamard_6	161,512	limit	561.11	35	0.03
hadamard_7	5,216	limit	24,783.33	48	0.30
hadamard_8	12	limit	-	63	3.19
hmittelman	1	0.03	-	3	0.00
ivalues	72,993	limit	448.66	2	0.01
kissing2	1	1,835.72	-	92	5.44
kn3-12	88,801	limit	137.80	35	0.00
kn4-24	7,631	limit	410.79	95	0.04
kn5-40	140	limit	765.25	199	0.20
kn5-41	1,238	limit	652.19	204	0.20
kn5-42	51	limit	791.33	209	0.24
kn5-43	393	limit	825.54	214	0.23
kn5-44	526	limit	835.42	219	0.25

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name	#nodes	total-time	gap	#sbc	sym-time
kport20	102,932	516.76	-	6	0.00
lop97icx	2,083	24.24	-	-	0.00
maxcsp-ehi-85-297-12	209	limit	-	4	2.08
maxcsp-ehi-85-297-36	232	limit	-	4	1.92
maxcsp-ehi-85-297-71	238	limit	-	2	1.15
maxcsp-ehi-90-315-70	147	limit	-	1	0.65
maxcsp-langford-3-11	28,498	limit	-	1	0.11
maxmin	106,325	limit	46.80	23	0.00
mbtd	1,834	limit	173.33	10	0.46
netmod_doll	3,954	limit	56.98	41	0.46
netmod_kar1	12,784	368.67	-	19	0.05
netmod_kar2	12,784	365.88	-	19	0.04
nuclear14	18	limit	-	5	0.01
nuclear25	65	limit	-	4	0.00
nuclear49	16	limit	-	6	0.03
nuclearva	22	limit	-	2	0.00
nuclearvb	90	limit	-	2	0.00
nuclearvc	974	limit	-	2	0.00
nuclearvd	217	limit	-	2	0.00
nuclearve	53	limit	-	2	0.00
nuclearvf	49	limit	-	2	0.00
nvs09	15	0.16	-	9	0.00
oil2	111	24.22	-	4	0.00
orth_d4m6_pl	447,537	limit	263.71	1	0.00
pooling_bental5pq	1	0.17	-	2	0.01
pooling_bental5stp	6,076	33.51	-	2	0.00
pooling_bental5tp	1	0.14	-	2	0.00
powerflow2383wpp	1	limit	-	2	0.23
powerflow2383wpr	1	limit	-	2	0.26
procurement1large	88,537	limit	375.13	-	0.01
procurement1mot	844,307	limit	522.63	-	0.01
procurement2mot	167	4.35	-	-	0.00
product	4,293	314.15	-	5	0.01
product2	1	31.81	-	290	0.07
radar-2000-10-a-6_lat_7	9,661	limit	138.29	1,500	6.55
radar-3000-10-a-8_lat_7	10,855	limit	11,690.49	1,500	8.87
ringpack_10_2	116,180	limit	3.95	1	0.00
ringpack_20_2	7,517	limit	400.00	1	0.02
ringpack_20_3	17,353	limit	566.67	17	0.16
ringpack_30_2	3,483	limit	-	1	0.06
rsyn0815m	151	1.03	-	-	0.00
rsyn0815m02m	1,422	11.57	-	-	0.00
rsyn0815m03m	1,347	18.47	-	-	0.00
rsyn0815m04m	4,006	49.69	-	-	0.00
sepasequ_complex	56,621	limit	53.31	9	0.01
sporttournament10	1	0.53	-	1	0.00
st_e09	1	0.02	-	1	0.00
st_e18	1	0.02	-	1	0.00
st_qpc-m3c	1	0.01	-	5	0.00
st_rv9	237	2.13	-	10	0.00
syn15m	1	0.31	-	-	0.00
t1000	0	0.89	-	-	0.00
tln2	1	0.01	-	1	0.00
topopt-zhou-rozvany_75	448	limit	-	200	0.09
torsion100	13	limit	380.15	3	0.10
torsion25	1	limit	319.68	3	0.03
torsion50	17	limit	467.20	7	0.06
turkey	1	20.01	-	-	0.00
util	29	0.36	-	1	0.00
waste	201,900	limit	85.82	23	0.01
watercontamination0202	19	99.28	-	1	0.00
waterund28	12,564	limit	21.74	3	0.00

TABLE B.4: Detailed results for variant **S-orbmax** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of added SBCs (#sbc), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#sbc	sym-time
arki0002	10	limit	-	10	0.02
arki0005	439	limit	-	-	0.05
arki0006	97	limit	-	-	0.06
arki0008	1	64.25	-	49	0.02
arki0016	8,270	limit	-	4	0.03
arki0017	9,971	limit	989.38	7	0.02
arki0018	1	limit	-	1,500	13.11
autocorr_bern20-03	1	0.03	-	3	0.00
autocorr_bern20-05	51,135	77.41	-	1	0.00
autocorr_bern20-10	178,610	1,166.54	-	1	0.00
autocorr_bern20-15	199,585	2,651.33	-	1	0.01
autocorr_bern25-03	1	0.04	-	2	0.00
autocorr_bern25-06	1,295,777	limit	27.50	1	0.01
autocorr_bern25-13	81,211	limit	123.93	1	0.01
autocorr_bern25-19	34,968	limit	248.35	1	0.02
autocorr_bern25-25	14,034	limit	395.34	1	0.02
autocorr_bern30-04	1,576,889	2,371.47	-	1	0.00
autocorr_bern30-08	186,505	limit	203.08	1	0.01
autocorr_bern30-15	31,935	limit	337.93	1	0.02
autocorr_bern30-23	5,065	limit	578.44	1	0.04
autocorr_bern30-30	3,327	limit	715.09	1	0.04
autocorr_bern35-04	717,327	limit	73.61	1	0.00
autocorr_bern35-09	89,888	limit	362.79	1	0.00
autocorr_bern35-18	5,028	limit	523.06	1	0.02
autocorr_bern35-26	2,291	limit	755.01	1	0.04
autocorr_bern35-35fix	2,178	limit	1,023.68	1	0.06
autocorr_bern40-05	382,472	limit	184.05	1	0.00
autocorr_bern40-10	35,913	limit	539.77	1	0.01
autocorr_bern40-20	2,274	limit	730.04	1	0.05
autocorr_bern40-30	1,024	limit	961.25	1	0.06
autocorr_bern40-40	1,035	limit	1,096.93	1	0.10
autocorr_bern45-05	265,934	limit	231.91	1	0.00
autocorr_bern45-11	16,244	limit	686.65	1	0.00
autocorr_bern45-23	1,406	limit	870.30	1	0.06
autocorr_bern45-34	1,071	limit	1,052.60	1	0.10
autocorr_bern45-45	727	limit	1,238.79	1	0.11
autocorr_bern50-06	128,874	limit	417.14	1	0.01
autocorr_bern50-13	6,673	limit	862.03	1	0.02
autocorr_bern50-25	842	limit	1,028.12	1	0.08
autocorr_bern55-06	102,638	limit	446.83	1	0.00
autocorr_bern55-14	2,963	limit	929.77	1	0.03
autocorr_bern55-28	739	limit	1,122.54	1	0.13
autocorr_bern60-08	32,070	limit	707.80	1	0.01
autocorr_bern60-15	2,744	limit	1,115.49	1	0.05
ball_mk2_10	0	0.03	-	9	0.00
ball_mk2_30	4,129,026	2,819.40	-	29	0.00
ball_mk3_10	0	0.00	-	-	0.00
carton7	231,293	583.11	-	3	0.00
carton9	827,707	limit	60.55	1	0.00
celar6-sub0	1,464	limit	-	3	0.20
chp_partload	594	limit	-	1	0.03
chp_shorttermplan1b	50,440	limit	6.80	1	0.02
chp_shorttermplan2a	441	129.73	-	2	0.01
chp_shorttermplan2c	20,998	limit	-	5	0.07
color_lab3_3x0	55,234	limit	555.26	4	0.01
color_lab3_4x0	41,674	limit	1,252.25	4	0.01
crossdock_15x7	66,125	limit	463.14	1	0.01
crossdock_15x8	53,667	limit	1,759.14	1	0.01
crudeoil_li02	1,539,569	limit	1.13	3	0.01
crudeoil_li06	108,420	1,523.62	-	1	0.00
crudeoil_li21	166,633	limit	1.93	1	0.01
crudeoil_pooling_dt4	39,609	limit	-	-	0.01
cvxnonsep_normcon30	1	0.11	-	2	0.00

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name	#nodes	total-time	gap	#sbcs	sym-time
cvxnonsep_normcon30r	1	0.12	-	2	0.00
cvxnonsep_normcon40	1	0.07	-	2	0.00
cvxnonsep_normcon40r	29	0.68	-	2	0.00
cvxnonsep_psig20	8,257,990	limit	230.67	2	0.00
cvxnonsep_psig20r	1	0.14	-	2	0.00
cvxnonsep_psig30	7,290,134	limit	155.01	1	0.00
cvxnonsep_psig30r	87	0.92	-	1	0.00
cvxnonsep_psig40	6,213,802	limit	113.45	4	0.00
cvxnonsep_psig40r	22	0.80	-	5	0.00
densitymod	3,276	limit	-	58	3.87
eigena2	1	limit	-	49	1.16
elec100	1,767	limit	-	299	1.79
elec200	1	limit	-	599	26.21
elec25	73,520	limit	-	74	0.02
elec50	8,356	limit	-	149	0.21
elf	252	1.88	-	2	0.00
emfl100_5_5	4,635	limit	17,404.38	21	0.03
ex14_1_5	1	0.03	-	3	0.00
ex2_1_3	1	0.22	-	1	0.00
ex5_2_5	1	1.17	-	2	0.00
ex8_1_6	17	0.15	-	1	0.00
ex8_3_1	653,674	limit	22.01	19	0.00
ex8_3_11	530,645	limit	25.07	19	0.00
ex8_3_13	411,081	limit	18.91	19	0.00
ex8_3_2	624,687	limit	40.66	19	0.00
ex8_3_3	655,453	limit	39.22	19	0.00
ex8_3_4	521,961	limit	62.01	19	0.00
ex8_3_5	599,281	limit	1,496.29	19	0.00
ex8_3_7	110	limit	-	19	0.00
ex8_3_8	103,775	limit	207.11	19	0.00
ex8_3_9	702,711	limit	31.30	19	0.00
ex8_4_6	26	limit	-	2	0.00
ex8_6_1	1	limit	-	20	0.00
ex9_1_8	1	0.01	-	1	0.00
ex9_2_2	1	0.14	-	1	0.00
ex9_2_6	1	0.09	-	4	0.00
gabriel04	4,837	251.76	-	2	0.00
gams03	1	limit	-	12	0.33
gastrans582_cold13	1	15.11	-	2	0.00
gastrans582_cold13_95	16	25.06	-	2	0.01
gastrans582_cold17	41	30.85	-	1	0.00
gastrans582_cold17_95	32	27.90	-	1	0.00
gastrans582_cool12	1	16.39	-	2	0.00
gastrans582_cool12_95	31	47.51	-	2	0.01
gastrans582_cool14	77	54.90	-	2	0.01
gastrans582_cool14_95	22	39.21	-	2	0.01
gastrans582_freezing27	1	28.48	-	1	0.00
gastrans582_freezing27_95	1	33.18	-	1	0.00
gastrans582_freezing30	21	43.23	-	1	0.00
gastrans582_freezing30_95	25	29.53	-	1	0.00
gastrans582_mild10	29	37.41	-	2	0.00
gastrans582_mild10_95	18	26.99	-	2	0.00
gastrans582_mild11	5,014	285.04	-	2	0.01
gastrans582_mild11_95	13	37.67	-	2	0.00
gastrans582_warm15	7	24.38	-	2	0.01
gastrans582_warm15_95	27	24.98	-	2	0.01
gastrans582_warm31	12	36.69	-	2	0.00
gastrans582_warm31_95	22	22.94	-	2	0.01
gear	7,756	5.84	-	2	0.00
gear2	32,993	22.24	-	2	0.01
gear3	7,832	5.94	-	2	0.00
gear4	822	0.38	-	2	0.00
graphpart_2g-0044-1601	3	1.00	-	2	0.00
graphpart_2g-0055-0062	11	1.46	-	2	0.00
graphpart_2g-0066-0066	23	4.01	-	2	0.00
graphpart_2g-0077-0077	479	9.19	-	2	0.00
graphpart_2g-0088-0088	37	8.44	-	2	0.00
graphpart_2g-0099-9211	313	16.73	-	2	0.00

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name	#nodes	total-time	gap	#sbc	sym-time
graphpart_2g-1010-0824	66	15.09	-	2	0.00
graphpart_2pm-0044-0044	5	1.37	-	2	0.00
graphpart_2pm-0055-0055	76	1.76	-	2	0.00
graphpart_2pm-0066-0066	197	3.72	-	2	0.00
graphpart_2pm-0077-0777	317	5.69	-	2	0.01
graphpart_2pm-0088-0888	272	7.54	-	2	0.00
graphpart_2pm-0099-0999	3,164	40.49	-	2	0.00
graphpart_3g-0234-0234	17	2.34	-	2	0.00
graphpart_3g-0244-0244	22	3.43	-	2	0.00
graphpart_3g-0333-0333	58	3.06	-	2	0.00
graphpart_3g-0334-0334	351	6.56	-	2	0.00
graphpart_3g-0344-0344	202	11.90	-	2	0.00
graphpart_3g-0444-0444	7,543	104.74	-	2	0.00
graphpart_3pm-0234-0234	250	2.64	-	2	0.00
graphpart_3pm-0244-0244	997	7.83	-	2	0.00
graphpart_3pm-0333-0333	421	4.40	-	2	0.00
graphpart_3pm-0334-0334	2,470	20.71	-	2	0.00
graphpart_3pm-0344-0344	16,264	135.75	-	2	0.00
graphpart_3pm-0444-0444	473,433	3,366.39	-	2	0.01
graphpart_clique-20	1,448	11.58	-	5	0.00
graphpart_clique-30	8,607	116.76	-	5	0.00
graphpart_clique-40	163,089	2,924.52	-	5	0.00
graphpart_clique-50	41,139	limit	105.38	5	0.00
graphpart_clique-60	22,702	limit	281.98	5	0.01
graphpart_clique-70	15,566	limit	455.66	5	0.02
hadamard_4	28	0.50	-	15	0.01
hadamard_5	26,593	34.60	-	24	0.00
hadamard_6	141,255	limit	683.33	35	0.02
hadamard_7	3,869	limit	38,388.89	48	0.09
hadamard_8	12	limit	-	63	1.14
hmittelman	1	0.02	-	3	0.00
ivalues	72,267	limit	449.04	2	0.01
kissing2	1	1,621.09	-	92	5.61
kn3-12	107,477	limit	168.89	35	0.00
kn4-24	2,656	limit	415.41	95	0.04
kn5-40	1,417	limit	697.12	199	0.19
kn5-41	1,089	limit	776.21	204	0.21
kn5-42	477	limit	860.03	209	0.22
kn5-43	465	limit	833.27	214	0.23
kn5-44	366	limit	837.32	219	0.24
kport20	37,564	197.67	-	6	0.00
lop97icx	2,083	24.26	-	-	0.00
maxcsp-ehi-85-297-12	202	limit	-	4	2.03
maxcsp-ehi-85-297-36	232	limit	-	4	1.59
maxcsp-ehi-85-297-71	238	limit	-	2	1.11
maxcsp-ehi-90-315-70	146	limit	-	1	0.73
maxcsp-langford-3-11	28,371	limit	-	1	0.07
maxmin	94,273	limit	66.11	23	0.00
mbtd	1,948	limit	160.00	10	0.45
netmod_doll	4,348	limit	44.79	89	0.13
netmod_kar1	6,656	225.50	-	19	0.01
netmod_kar2	6,656	224.59	-	19	0.01
nuclear14	4	limit	-	5	0.01
nuclear25	41	limit	-	4	0.01
nuclear49	18	limit	-	6	0.03
nuclearva	194	limit	-	2	0.00
nuclearvb	365	limit	-	2	0.00
nuclearvc	382	limit	-	2	0.00
nuclearvd	94	limit	-	2	0.00
nuclearve	167	limit	-	2	0.00
nuclearvf	361	limit	-	2	0.00
nvs09	3	0.09	-	9	0.00
oil2	111	24.42	-	4	0.00
orth_d4m6_pl	447,716	limit	263.69	1	0.00
pooling_bental5pq	1	0.15	-	2	0.00
pooling_bental5tp	9,794	55.31	-	2	0.00
pooling_bental5tp	1	0.13	-	2	0.00
powerflow2383wpp	1	limit	-	2	0.24

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name	#nodes	total-time	gap	#sbc	sym-time
powerflow2383wpr	1	limit	-	2	0.25
procurement1large	88,537	limit	375.13	-	0.01
procurement1mot	843,897	limit	522.63	-	0.00
procurement2mot	167	4.44	-	-	0.00
product	4,293	315.94	-	5	0.00
product2	1	18.22	-	253	0.09
radar-2000-10-a-6_lat_7	28,806	limit	630.17	1,500	14.11
radar-3000-10-a-8_lat_7	14,107	limit	37,261.75	1,500	10.37
ringpack_10_2	115,257	limit	3.95	1	0.00
ringpack_20_2	7,481	limit	400.00	1	0.02
ringpack_20_3	27,656	limit	12.35	29	0.13
ringpack_30_2	3,483	limit	-	1	0.06
rsyn0815m	151	1.03	-	-	0.00
rsyn0815m02m	1,422	11.57	-	-	0.00
rsyn0815m03m	1,347	18.47	-	-	0.00
rsyn0815m04m	4,006	49.69	-	-	0.00
sepasequ_complex	56,461	limit	82.62	9	0.00
sporttournament10	1	0.51	-	1	0.00
st_e09	1	0.01	-	1	0.00
st_e18	1	0.02	-	1	0.00
st_qpc-m3c	1	0.03	-	5	0.00
st_rv9	237	2.12	-	10	0.00
syn15m	1	0.31	-	-	0.00
t1000	0	0.90	-	-	0.00
tlm2	1	0.01	-	1	0.00
topopt-zhou-rozvany_75	450	limit	-	200	0.09
torsion100	13	limit	380.15	3	0.11
torsion25	1	limit	319.68	3	0.02
torsion50	17	limit	467.20	7	0.06
turkey	1	19.95	-	-	0.00
util	29	0.39	-	1	0.00
waste	206,454	limit	82.26	23	0.00
watercontamination0202	19	98.69	-	1	0.00
waterund28	12,653	limit	21.74	3	0.00

TABLE B.5: Detailed results for variant D on M-sym-all. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of orbitopes handled (#orbitope), the number of sbcs added (#sbc), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#orbitope	#sbc	sym-time
arki0002	4	limit	-	-	-	0.03
arki0005	439	limit	-	-	-	0.06
arki0006	97	limit	-	-	-	0.05
arki0008	1	33.17	-	-	-	0.02
arki0016	8,718	limit	-	-	-	0.02
arki0017	9,214	limit	996.25	-	-	0.02
arki0018	2	limit	-	-	-	8.85
autocorr_bern20-03	1	0.02	-	1	-	0.00
autocorr_bern20-05	23,260	43.16	-	1	-	0.00
autocorr_bern20-10	92,832	608.86	-	1	-	0.01
autocorr_bern20-15	89,307	1,169.29	-	1	-	0.00
autocorr_bern25-03	1	0.04	-	2	-	0.00
autocorr_bern25-06	723,859	1,923.60	-	1	-	0.00
autocorr_bern25-13	99,523	limit	78.02	1	-	0.01
autocorr_bern25-19	38,603	limit	218.25	1	-	0.01
autocorr_bern25-25	18,867	limit	387.21	1	-	0.01
autocorr_bern30-04	783,673	1,177.71	-	1	-	0.00
autocorr_bern30-08	201,616	limit	171.14	1	-	0.00
autocorr_bern30-15	33,143	limit	238.26	1	-	0.01
autocorr_bern30-23	8,121	limit	460.03	1	-	0.01
autocorr_bern30-30	3,650	limit	675.90	1	-	0.01
autocorr_bern35-04	759,042	limit	59.03	1	-	0.00
autocorr_bern35-09	84,780	limit	322.03	1	-	0.00

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name	#nodes	total-time	gap	#orbitope	#sbscs	sym-time
autocorr_bern35-18	7,463	limit	449.75	1	-	0.01
autocorr_bern35-26	2,538	limit	671.49	1	-	0.02
autocorr_bern35-35fix	1,946	limit	901.51	1	-	0.00
autocorr_bern40-05	344,302	limit	157.00	1	-	0.01
autocorr_bern40-10	40,320	limit	489.50	1	-	0.00
autocorr_bern40-20	2,118	limit	644.42	1	-	0.02
autocorr_bern40-30	1,257	limit	857.07	1	-	0.03
autocorr_bern40-40	1,276	limit	1,048.21	1	-	0.04
autocorr_bern45-05	291,158	limit	213.23	1	-	0.00
autocorr_bern45-11	16,410	limit	615.10	1	-	0.00
autocorr_bern45-23	1,340	limit	782.92	1	-	0.03
autocorr_bern45-34	790	limit	1,102.14	1	-	0.05
autocorr_bern45-45	603	limit	1,193.35	1	-	0.06
autocorr_bern50-06	111,581	limit	407.69	1	-	0.00
autocorr_bern50-13	6,741	limit	714.84	1	-	0.01
autocorr_bern50-25	834	limit	918.77	1	-	0.04
autocorr_bern55-06	111,155	limit	425.80	1	-	0.00
autocorr_bern55-14	2,770	limit	961.58	1	-	0.01
autocorr_bern55-28	702	limit	1,088.26	1	-	0.06
autocorr_bern60-08	32,686	limit	704.38	1	-	0.00
autocorr_bern60-15	2,464	limit	989.17	1	-	0.02
ball_mk2_10	1	0.02	-	-	-	0.00
ball_mk2_30	57	0.11	-	-	-	0.00
ball_mk3_10	0	0.00	-	-	-	0.00
carton7	1,371	limit	127.53	-	-	0.00
carton9	338,042	limit	75.28	-	-	0.01
celar6-sub0	2,071	limit	-	1	-	0.21
chp_partload	69	limit	-	-	-	0.01
chp_shorttermplan1b	40,633	limit	5.79	-	-	0.00
chp_shorttermplan2a	134	141.68	-	-	-	0.00
chp_shorttermplan2c	21,310	limit	-	-	-	0.00
color_lab3_3x0	45,494	limit	495.78	2	-	0.00
color_lab3_4x0	38,682	limit	1,066.23	2	-	0.01
crossdock_15x7	54,510	limit	316.21	1	-	0.01
crossdock_15x8	45,192	limit	1,448.01	1	-	0.01
crudeoil_li02	119	limit	-	-	-	0.00
crudeoil_li06	7,160	limit	0.90	-	-	0.01
crudeoil_li21	2,392	limit	-	-	-	0.00
crudeoil_pooling_dt4	19,071	limit	8.17	-	-	0.00
cvxnonsep_normcon30	1	0.10	-	-	-	0.00
cvxnonsep_normcon30r	1	0.12	-	-	-	0.00
cvxnonsep_normcon40	1	0.13	-	-	-	0.00
cvxnonsep_normcon40r	10	0.44	-	-	-	0.00
cvxnonsep_psig20	7,767,553	limit	227.18	-	-	0.00
cvxnonsep_psig20r	1	0.15	-	-	-	0.00
cvxnonsep_psig30	7,073,121	limit	155.00	-	-	0.00
cvxnonsep_psig30r	68	1.04	-	-	-	0.00
cvxnonsep_psig40	6,240,835	limit	113.24	-	-	0.00
cvxnonsep_psig40r	36	0.87	-	-	-	0.00
densitymod	511	limit	-	2	-	3.91
eigena2	1	limit	-	-	-	1.13
elec100	404	limit	-	-	-	1.80
elec200	1	limit	-	-	-	26.42
elec25	3,673	limit	-	-	-	0.03
elec50	3,564	limit	-	-	-	0.21
elf	55	limit	6,663.44	-	-	0.00
emf100_5_5	3,224	limit	29,267.53	1	-	0.03
ex14_1_5	1	0.01	-	-	-	0.00
ex2_1_3	1	0.17	-	-	-	0.00
ex5_2_5	1	1.04	-	-	-	0.00
ex8_1_6	19	0.13	-	-	-	0.00
ex8_3_1	374,403	limit	23.00	-	-	0.00
ex8_3_11	436,330	limit	25.07	-	-	0.00
ex8_3_13	305,811	limit	15.67	-	-	0.00
ex8_3_2	489,163	limit	40.66	-	-	0.00
ex8_3_3	555,099	limit	39.22	-	-	0.00
ex8_3_4	479,337	limit	62.01	-	-	0.00
ex8_3_5	502,872	limit	1,496.29	-	-	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
ex8_3_7	26	limit	-	-	-	0.00
ex8_3_8	123,141	limit	207.11	-	-	0.00
ex8_3_9	697,576	limit	31.06	-	-	0.01
ex8_4_6	12	limit	-	-	-	0.00
ex8_6_1	654	limit	-	-	-	0.00
ex9_1_8	1	0.01	-	-	-	0.00
ex9_2_2	1	0.15	-	-	-	0.00
ex9_2_6	1	0.09	-	-	-	0.00
gabriel04	7,047	321.73	-	-	-	0.00
gams03	1	limit	-	-	-	0.33
gastrans582_cold13	41,516	1,680.14	-	-	-	0.00
gastrans582_cold13_95	23	26.49	-	-	-	0.00
gastrans582_cold17	1	19.07	-	-	-	0.00
gastrans582_cold17_95	37	33.26	-	-	-	0.00
gastrans582_cool12	25	27.44	-	-	-	0.01
gastrans582_cool12_95	24	43.14	-	-	-	0.00
gastrans582_cool14	2,560	229.80	-	-	-	0.00
gastrans582_cool14_95	11	35.31	-	-	-	0.00
gastrans582_freezing27	1	25.33	-	-	-	0.00
gastrans582_freezing27_95	1	36.61	-	-	-	0.00
gastrans582_freezing30	21	44.28	-	-	-	0.00
gastrans582_freezing30_95	21	41.15	-	-	-	0.00
gastrans582_mild10	24	42.59	-	-	-	0.00
gastrans582_mild10_95	20	36.72	-	-	-	0.00
gastrans582_mild11	42	33.44	-	-	-	0.01
gastrans582_mild11_95	1	18.97	-	-	-	0.00
gastrans582_warm15	1,043	303.93	-	-	-	0.01
gastrans582_warm15_95	21	33.01	-	-	-	0.00
gastrans582_warm31	29	37.95	-	-	-	0.01
gastrans582_warm31_95	18	30.48	-	-	-	0.00
gear	14,488	9.11	-	-	-	0.00
gear2	99	limit	-	-	-	0.00
gear3	14,076	9.71	-	-	-	0.00
gear4	1,060	0.37	-	-	-	0.00
graphpart_2g-0044-1601	1	0.35	-	1	-	0.00
graphpart_2g-0055-0062	5	1.17	-	1	-	0.00
graphpart_2g-0066-0066	11	2.35	-	1	-	0.00
graphpart_2g-0077-0077	54	4.71	-	1	-	0.00
graphpart_2g-0088-0088	23	6.79	-	1	-	0.00
graphpart_2g-0099-9211	70	13.72	-	1	-	0.00
graphpart_2g-1010-0824	24	12.75	-	1	-	0.00
graphpart_2pm-0044-0044	5	1.16	-	1	-	0.00
graphpart_2pm-0055-0055	21	1.84	-	1	-	0.00
graphpart_2pm-0066-0066	30	3.23	-	1	-	0.00
graphpart_2pm-0077-0777	34	4.20	-	1	-	0.00
graphpart_2pm-0088-0888	79	8.28	-	1	-	0.00
graphpart_2pm-0099-0999	1,569	28.43	-	1	-	0.00
graphpart_3g-0234-0234	17	2.02	-	1	-	0.00
graphpart_3g-0244-0244	17	4.64	-	1	-	0.00
graphpart_3g-0333-0333	10	3.32	-	1	-	0.00
graphpart_3g-0334-0334	94	5.80	-	1	-	0.00
graphpart_3g-0344-0344	125	8.35	-	1	-	0.00
graphpart_3g-0444-0444	1,117	32.22	-	1	-	0.00
graphpart_3pm-0234-0234	30	2.26	-	1	-	0.00
graphpart_3pm-0244-0244	175	3.96	-	1	-	0.00
graphpart_3pm-0333-0333	439	4.72	-	1	-	0.00
graphpart_3pm-0334-0334	876	9.42	-	1	-	0.00
graphpart_3pm-0344-0344	2,300	30.58	-	1	-	0.00
graphpart_3pm-0444-0444	90,112	699.09	-	1	-	0.00
graphpart_clique-20	341	5.76	-	1	-	0.00
graphpart_clique-30	2,682	54.74	-	1	-	0.00
graphpart_clique-40	43,499	849.54	-	1	-	0.00
graphpart_clique-50	39,565	limit	105.83	1	-	0.00
graphpart_clique-60	20,965	limit	276.40	1	-	0.00
graphpart_clique-70	13,118	limit	249.43	1	-	0.00
hadamard_4	173	0.56	-	-	-	0.00
hadamard_5	21,849	34.48	-	-	-	0.00
hadamard_6	123,327	limit	766.67	-	-	0.03

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
hadamard_7	3,513	limit	125,300.00	-	-	0.28
hadamard_8	18	limit	-	-	-	3.13
hmittelman	1	0.02	-	3	-	0.00
ivalues	63,645	limit	486.95	-	-	0.00
kissing2	1	914.55	-	-	-	5.44
knp3-12	74,763	limit	142.33	-	-	0.00
knp4-24	6,853	limit	391.48	-	-	0.03
knp5-40	327	limit	820.08	-	-	0.19
knp5-41	1,243	limit	727.74	-	-	0.21
knp5-42	520	limit	830.93	-	-	0.21
knp5-43	890	limit	807.30	-	-	0.24
knp5-44	609	limit	823.92	-	-	0.25
kport20	49,856	257.36	-	-	-	0.00
lop97icx	2,083	25.45	-	-	-	0.00
maxcsp-ehi-85-297-12	245	limit	-	4	-	0.00
maxcsp-ehi-85-297-36	263	limit	-	3	-	0.00
maxcsp-ehi-85-297-71	195	limit	-	2	-	0.00
maxcsp-ehi-90-315-70	139	limit	-	1	-	0.00
maxcsp-langford-3-11	31,278	limit	-	1	-	0.00
maxmin	172,863	limit	9.03	-	-	0.00
mbtd	1,570	limit	151.09	1	-	0.47
netmod_doll	4,799	limit	47.82	-	-	0.47
netmod_kar1	3,372	117.70	-	-	-	0.05
netmod_kar2	3,372	118.05	-	-	-	0.06
nuclear14	30	limit	-	-	-	0.00
nuclear25	15	limit	-	-	-	0.01
nuclear49	21	limit	-	-	-	0.03
nuclearva	22	limit	-	-	-	0.00
nuclearvb	111	limit	-	-	-	0.00
nuclearvc	212	limit	-	-	-	0.00
nuclearvd	287	limit	-	-	-	0.00
nuclearve	59	limit	-	-	-	0.01
nuclearvf	66	limit	-	-	-	0.00
nvs09	39	0.20	-	-	-	0.00
oil2	168	81.68	-	-	-	0.00
orth_d4m6_pl	426,404	limit	166.56	-	-	0.00
pooling_bental5pq	1	0.16	-	-	-	0.00
pooling_bental5stp	7,198	37.57	-	-	-	0.00
pooling_bental5tp	1	0.14	-	-	-	0.00
powerflow2383wpp	1	limit	-	-	-	0.23
powerflow2383wpr	1	limit	-	-	-	0.26
procurement1large	87,925	limit	375.13	-	-	0.01
procurement1mot	842,483	limit	522.71	-	-	0.00
procurement2mot	167	4.48	-	-	-	0.00
product	4,293	316.28	-	-	-	0.01
product2	1	9.48	-	-	-	0.07
radar-2000-10-a-6_lat_7	341	361.31	-	-	-	0.00
radar-3000-10-a-8_lat_7	4,924	limit	292.58	-	-	0.00
ringpack_10_2	110,478	limit	8.22	1	-	0.00
ringpack_20_2	8,833	limit	233.33	1	-	0.02
ringpack_20_3	17,380	limit	566.67	-	-	0.16
ringpack_30_2	61	limit	1,400.00	1	-	0.06
rsyn0815m	151	1.01	-	-	-	0.00
rsyn0815m02m	1,422	11.68	-	-	-	0.00
rsyn0815m03m	1,347	18.43	-	-	-	0.00
rsyn0815m04m	4,006	50.04	-	-	-	0.00
sepasequ_complex	54,234	limit	62.58	-	-	0.00
sporttournament10	1	0.69	-	1	-	0.00
st_e09	1	0.01	-	-	-	0.00
st_e18	1	0.02	-	-	-	0.00
st_qpc-m3c	1	0.03	-	-	-	0.00
st_rv9	331	2.78	-	-	-	0.00
syn15m	1	0.32	-	-	-	0.00
t1000	0	0.88	-	-	-	0.00
tlm2	1	0.00	-	-	-	0.01
topopt-zhou-rozvany_75	478	limit	-	-	-	0.00
torsion100	20	limit	380.39	-	-	0.11
torsion25	1	limit	319.02	-	-	0.03

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name	#nodes	total-time	gap	#orbitope	#sbc	sym-time
torsion50	16	limit	373.99	-	-	0.06
turkey	1	20.01	-	-	-	0.00
util	14	0.28	-	1	-	0.00
waste	203,212	limit	84.75	-	-	0.00
watercontamination0202	33	98.04	-	-	-	0.00
waterund28	10,033	limit	7.76	-	-	0.00

TABLE B.6: Detailed results for variant **D-1p** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of orbitopes handled (#orbitope), the number of sbcs added (#sbc), and the time for symmetry computating and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#orbitope	#sbc	sym-time
arki0002	11	limit	-	-	10	0.03
arki0005	922	limit	-	-	1	0.06
arki0006	97	limit	-	-	-	0.06
arki0008	1	32.86	-	-	-	0.03
arki0016	9,317	limit	-	-	5	0.03
arki0017	9,226	limit	995.08	-	6	0.01
arki0018	2	limit	-	-	77	8.99
autocorr_bern20-03	1	0.03	-	1	-	0.00
autocorr_bern20-05	23,260	40.98	-	1	-	0.00
autocorr_bern20-10	92,832	584.71	-	1	-	0.00
autocorr_bern20-15	89,307	1,126.27	-	1	-	0.00
autocorr_bern25-03	1	0.03	-	2	-	0.00
autocorr_bern25-06	723,859	1,857.02	-	1	-	0.00
autocorr_bern25-13	104,248	limit	76.35	1	-	0.01
autocorr_bern25-19	40,459	limit	216.25	1	-	0.00
autocorr_bern25-25	19,296	limit	385.40	1	-	0.01
autocorr_bern30-04	783,673	1,141.84	-	1	-	0.00
autocorr_bern30-08	210,547	limit	169.33	1	-	0.01
autocorr_bern30-15	34,741	limit	233.29	1	-	0.01
autocorr_bern30-23	8,285	limit	457.26	1	-	0.02
autocorr_bern30-30	3,707	limit	672.47	1	-	0.01
autocorr_bern35-04	765,720	limit	58.85	1	-	0.00
autocorr_bern35-09	88,078	limit	320.01	1	-	0.00
autocorr_bern35-18	7,661	limit	447.49	1	-	0.00
autocorr_bern35-26	2,551	limit	668.02	1	-	0.03
autocorr_bern35-35fix	1,962	limit	900.76	1	-	0.00
autocorr_bern40-05	349,434	limit	156.47	1	-	0.00
autocorr_bern40-10	42,094	limit	486.92	1	-	0.01
autocorr_bern40-20	2,153	limit	643.85	1	-	0.02
autocorr_bern40-30	1,266	limit	854.80	1	-	0.03
autocorr_bern40-40	1,295	limit	1,048.21	1	-	0.05
autocorr_bern45-05	294,467	limit	212.93	1	-	0.00
autocorr_bern45-11	16,671	limit	615.10	1	-	0.01
autocorr_bern45-23	1,365	limit	782.92	1	-	0.02
autocorr_bern45-34	810	limit	1,092.85	1	-	0.05
autocorr_bern45-45	609	limit	1,193.35	1	-	0.07
autocorr_bern50-06	115,133	limit	406.22	1	-	0.00
autocorr_bern50-13	6,884	limit	714.84	1	-	0.01
autocorr_bern50-25	833	limit	918.77	1	-	0.03
autocorr_bern55-06	112,931	limit	425.29	1	-	0.01
autocorr_bern55-14	2,790	limit	959.75	1	-	0.02
autocorr_bern55-28	705	limit	1,088.26	1	-	0.06
autocorr_bern60-08	33,570	limit	703.49	1	-	0.01
autocorr_bern60-15	2,535	limit	988.55	1	-	0.02
ball_mk2_10	1	0.00	-	-	-	0.00
ball_mk2_30	3,884,971	3,298.63	-	-	-	0.00
ball_mk3_10	0	0.01	-	-	-	0.00
carton7	260,530	686.28	-	-	2	0.00
carton9	887,030	limit	60.22	-	1	0.01
celar6-sub0	2,072	limit	-	1	-	0.21
chp_partload	778	limit	-	-	1	0.01

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name	#nodes	total-time	gap	#orbitope	#sbc	sym-time
chp_shorttermplan1b	41,240	limit	6.06	-	-	0.00
chp_shorttermplan2a	135	135.91	-	-	-	0.00
chp_shorttermplan2c	21,346	limit	-	-	-	0.00
color_lab3_3x0	46,659	limit	495.24	2	-	0.00
color_lab3_4x0	38,542	limit	1,066.23	2	-	0.01
crossdock_15x7	56,942	limit	312.25	1	-	0.01
crossdock_15x8	47,047	limit	1,426.76	1	-	0.01
crudeoil_li02	1,527,328	limit	1.13	-	3	0.00
crudeoil_li06	58,816	926.55	-	-	1	0.00
crudeoil_li21	153,002	limit	2.53	-	1	0.01
crudeoil_pooling_dt4	32,663	limit	-	-	1	0.00
cvxnonsep_normcon30	1	0.10	-	-	-	0.00
cvxnonsep_normcon30r	1	0.11	-	-	-	0.00
cvxnonsep_normcon40	1	0.14	-	-	-	0.00
cvxnonsep_normcon40r	10	0.39	-	-	1	0.00
cvxnonsep_psig20	8,095,612	limit	229.64	-	3	0.00
cvxnonsep_psig20r	1	0.14	-	-	-	0.00
cvxnonsep_psig30	7,155,868	limit	154.98	-	1	0.00
cvxnonsep_psig30r	68	1.02	-	-	1	0.00
cvxnonsep_psig40	6,257,927	limit	113.43	-	4	0.00
cvxnonsep_psig40r	48	0.93	-	-	4	0.00
densitymod	511	limit	-	2	-	3.91
eigena2	1	limit	-	-	-	1.17
elec100	159	limit	-	-	1,663	1.83
elec200	1	limit	-	-	-	25.93
elec25	2,389	limit	-	-	269	0.02
elec50	589	limit	-	-	639	0.22
elf	187	1.76	-	-	2	0.00
emfl100_5_5	3,225	limit	29,267.53	1	-	0.03
ex14_1_5	1	0.02	-	-	-	0.00
ex2_1_3	1	0.16	-	-	-	0.00
ex5_2_5	1	1.03	-	-	-	0.00
ex8_1_6	10	0.12	-	-	1	0.00
ex8_3_1	373,703	limit	23.00	-	-	0.00
ex8_3_11	429,617	limit	25.07	-	-	0.00
ex8_3_13	303,666	limit	15.67	-	-	0.00
ex8_3_2	485,541	limit	40.66	-	-	0.00
ex8_3_3	554,310	limit	39.22	-	-	0.01
ex8_3_4	479,764	limit	62.01	-	-	0.00
ex8_3_5	499,196	limit	1,496.29	-	-	0.00
ex8_3_7	26	limit	-	-	-	0.00
ex8_3_8	123,077	limit	207.11	-	-	0.00
ex8_3_9	697,103	limit	31.06	-	-	0.00
ex8_4_6	9	limit	-	-	3	0.00
ex8_6_1	654	limit	-	-	-	0.00
ex9_1_8	1	0.01	-	-	-	0.01
ex9_2_2	1	0.15	-	-	-	0.00
ex9_2_6	1	0.12	-	-	-	0.00
gabriel04	7,563	332.88	-	-	2	0.01
gams03	1	limit	-	-	-	0.33
gastrans582_cold13	14,262	593.88	-	-	1	0.00
gastrans582_cold13_95	99,388	limit	-	-	-	0.00
gastrans582_cold17	1	18.61	-	-	-	0.00
gastrans582_cold17_95	27	32.13	-	-	-	0.00
gastrans582_cool12	25	25.81	-	-	-	0.00
gastrans582_cool12_95	24	42.80	-	-	-	0.00
gastrans582_cool14	124	47.17	-	-	-	0.00
gastrans582_cool14_95	20	36.16	-	-	-	0.00
gastrans582_freezing27	1	25.32	-	-	-	0.00
gastrans582_freezing27_95	1	36.22	-	-	-	0.00
gastrans582_freezing30	21	43.14	-	-	-	0.00
gastrans582_freezing30_95	1	29.92	-	-	-	0.00
gastrans582_mild10	24	41.96	-	-	-	0.01
gastrans582_mild10_95	20	36.15	-	-	-	0.00
gastrans582_mild11	18	25.96	-	-	-	0.00
gastrans582_mild11_95	1	18.49	-	-	-	0.00
gastrans582_warm15	28,888	2,446.33	-	-	-	0.00
gastrans582_warm15_95	17	31.51	-	-	-	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
gastrans582_warm31	35	36.38	-	-	-	0.01
gastrans582_warm31_95	65	30.97	-	-	-	0.00
gear	10,834	8.42	-	-	2	0.00
gear2	24,051	15.19	-	-	2	0.00
gear3	10,885	8.34	-	-	2	0.00
gear4	803	0.27	-	-	2	0.00
graphpart_2g-0044-1601	1	0.32	-	1	-	0.00
graphpart_2g-0055-0062	5	1.15	-	1	-	0.00
graphpart_2g-0066-0066	11	2.28	-	1	-	0.00
graphpart_2g-0077-0077	54	4.54	-	1	-	0.00
graphpart_2g-0088-0088	23	6.64	-	1	-	0.00
graphpart_2g-0099-9211	70	12.93	-	1	-	0.00
graphpart_2g-1010-0824	24	12.23	-	1	-	0.00
graphpart_2pm-0044-0044	5	1.16	-	1	-	0.00
graphpart_2pm-0055-0055	21	1.86	-	1	-	0.00
graphpart_2pm-0066-0066	30	3.22	-	1	-	0.00
graphpart_2pm-0077-0777	34	4.18	-	1	-	0.00
graphpart_2pm-0088-0888	79	8.09	-	1	-	0.00
graphpart_2pm-0099-0999	1,569	27.80	-	1	-	0.00
graphpart_3g-0234-0234	17	1.96	-	1	-	0.00
graphpart_3g-0244-0244	17	4.53	-	1	-	0.00
graphpart_3g-0333-0333	10	3.28	-	1	-	0.00
graphpart_3g-0334-0334	94	5.54	-	1	-	0.00
graphpart_3g-0344-0344	125	8.27	-	1	-	0.00
graphpart_3g-0444-0444	1,117	31.68	-	1	-	0.00
graphpart_3pm-0234-0234	30	2.15	-	1	-	0.00
graphpart_3pm-0244-0244	175	3.74	-	1	-	0.00
graphpart_3pm-0333-0333	439	4.62	-	1	-	0.00
graphpart_3pm-0334-0334	876	9.22	-	1	-	0.00
graphpart_3pm-0344-0344	2,300	29.82	-	1	-	0.00
graphpart_3pm-0444-0444	90,112	682.69	-	1	-	0.00
graphpart_clique-20	341	5.95	-	1	-	0.00
graphpart_clique-30	2,682	55.77	-	1	-	0.00
graphpart_clique-40	43,499	848.37	-	1	-	0.00
graphpart_clique-50	39,277	limit	105.91	1	-	0.00
graphpart_clique-60	21,067	limit	276.26	1	-	0.00
graphpart_clique-70	13,065	limit	249.43	1	-	0.00
hadamard_4	173	0.57	-	-	-	0.00
hadamard_5	21,849	33.82	-	-	-	0.00
hadamard_6	123,138	limit	766.67	-	-	0.03
hadamard_7	3,481	limit	125,400.00	-	-	0.29
hadamard_8	18	limit	-	-	-	3.16
hmittelman	1	0.03	-	3	-	0.00
ivalues	73,868	limit	356.43	-	2	0.01
kissing2	1	913.97	-	-	-	5.46
knp3-12	109,798	limit	142.58	-	70	0.00
knp4-24	5,475	limit	391.00	-	227	0.03
knp5-40	393	limit	797.54	-	397	0.19
knp5-41	222	limit	782.18	-	553	0.20
knp5-42	457	limit	759.73	-	452	0.21
knp5-43	306	limit	842.26	-	453	0.24
knp5-44	235	limit	840.02	-	504	0.25
kport20	97,294	332.65	-	-	7	0.00
lop97icx	2,083	24.07	-	-	-	0.00
maxcsp-ehi-85-297-12	274	limit	-	4	-	0.00
maxcsp-ehi-85-297-36	295	limit	-	3	-	0.00
maxcsp-ehi-85-297-71	224	limit	-	2	-	0.00
maxcsp-ehi-90-315-70	159	limit	-	1	-	0.00
maxcsp-langford-3-11	33,283	limit	-	1	-	0.00
maxmin	79,691	limit	46.83	-	36	0.00
mbtd	1,569	limit	151.09	1	-	0.46
netmod_doll	4,794	limit	47.82	-	-	0.48
netmod_kar1	3,372	117.93	-	-	-	0.05
netmod_kar2	3,372	120.72	-	-	-	0.04
nuclear14	52	limit	-	-	15	0.01
nuclear25	48	limit	-	-	10	0.00
nuclear49	26	limit	-	-	21	0.02
nuclearva	22	limit	-	-	3	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
nuclearvb	480	limit	-	-	3	0.00
nuclearvc	97	limit	-	-	3	0.00
nuclearvd	227	limit	-	-	3	0.00
nuclearve	98	limit	-	-	3	0.00
nuclearvf	250	limit	-	-	3	0.00
nvs09	614	1.39	-	-	-	0.00
oil2	232	53.92	-	-	-	0.00
orth_d4m6_pl	433,055	limit	210.64	-	1	0.00
pooling_bental5pq	1	0.17	-	-	-	0.00
pooling_bental5stp	25,450	152.27	-	-	3	0.00
pooling_bental5tp	1	0.12	-	-	-	0.00
powerflow2383wpp	1	limit	-	-	-	0.24
powerflow2383wpr	1	limit	-	-	-	0.26
procurement1large	92,443	limit	378.29	-	36	0.01
procurement1mot	851,934	limit	505.27	-	5	0.00
procurement2mot	167	4.41	-	-	-	0.00
product	4,293	314.52	-	-	-	0.00
product2	1	9.30	-	-	-	0.07
radar-2000-10-a-6_lat_7	15,884	limit	138.30	-	-	0.00
radar-3000-10-a-8_lat_7	11,945	limit	10,110.83	-	-	0.00
ringpack_10_2	111,638	limit	8.22	1	-	0.00
ringpack_20_2	8,977	limit	233.33	1	-	0.02
ringpack_20_3	17,358	limit	566.67	-	-	0.15
ringpack_30_2	61	limit	1,400.00	1	-	0.06
rsyn0815m	149	1.07	-	-	4	0.00
rsyn0815m02m	1,296	10.65	-	-	7	0.00
rsyn0815m03m	1,464	19.06	-	-	11	0.00
rsyn0815m04m	3,450	48.06	-	-	15	0.00
sepasequ_complex	54,200	limit	62.59	-	-	0.01
sporttournament10	1	0.69	-	1	-	0.00
st_e09	1	0.02	-	-	-	0.00
st_e18	1	0.01	-	-	-	0.00
st_qpc-m3c	1	0.02	-	-	-	0.00
st_rv9	261	2.73	-	-	7	0.00
syn15m	1	0.31	-	-	-	0.00
t1000	0	0.89	-	-	-	0.00
tlm2	1	0.01	-	-	-	0.00
topopt-zhou-rozvany_75	451	limit	-	-	200	0.00
torsion100	20	limit	380.39	-	-	0.10
torsion25	1	limit	319.02	-	-	0.02
torsion50	16	limit	373.99	-	-	0.06
turkey	1	19.94	-	-	-	0.00
util	14	0.28	-	1	-	0.00
waste	205,277	limit	83.05	-	-	0.01
watercontamination0202	33	96.51	-	-	-	0.00
waterund28	10,026	limit	7.76	-	-	0.01

TABLE B.7: Detailed results for variant **D-1pf** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of orbitopes handled (#orbitope), the number of sbcs added (#sbcs), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
arki0002	23	limit	-	-	10	0.02
arki0005	816	limit	-	-	1	0.06
arki0006	97	limit	-	-	-	0.08
arki0008	1	32.76	-	-	-	0.02
arki0016	8,980	limit	-	-	5	0.02
arki0017	9,372	limit	996.18	-	6	0.01
arki0018	2	limit	-	-	71	9.10
autocorr_bern20-03	1	0.03	-	1	-	0.00
autocorr_bern20-05	23,260	41.40	-	1	-	0.00
autocorr_bern20-10	92,832	587.68	-	1	-	0.00
autocorr_bern20-15	89,307	1,124.71	-	1	-	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
autocorr_bern25-03	1	0.03	-	2	-	0.00
autocorr_bern25-06	723,859	1,852.49	-	1	-	0.00
autocorr_bern25-13	105,918	limit	75.74	1	-	0.01
autocorr_bern25-19	40,716	limit	215.60	1	-	0.01
autocorr_bern25-25	19,241	limit	385.42	1	-	0.01
autocorr_bern30-04	783,673	1,141.83	-	1	-	0.00
autocorr_bern30-08	210,552	limit	169.33	1	-	0.00
autocorr_bern30-15	34,660	limit	233.45	1	-	0.00
autocorr_bern30-23	8,262	limit	457.26	1	-	0.01
autocorr_bern30-30	3,670	limit	675.01	1	-	0.02
autocorr_bern35-04	776,239	limit	58.59	1	-	0.00
autocorr_bern35-09	87,644	limit	320.31	1	-	0.00
autocorr_bern35-18	7,695	limit	447.31	1	-	0.01
autocorr_bern35-26	2,555	limit	668.02	1	-	0.02
autocorr_bern35-35fix	1,957	limit	901.51	1	-	0.00
autocorr_bern40-05	352,890	limit	156.14	1	-	0.00
autocorr_bern40-10	42,352	limit	486.67	1	-	0.01
autocorr_bern40-20	2,137	limit	643.85	1	-	0.01
autocorr_bern40-30	1,268	limit	854.80	1	-	0.03
autocorr_bern40-40	1,276	limit	1,048.21	1	-	0.03
autocorr_bern45-05	296,053	limit	212.83	1	-	0.00
autocorr_bern45-11	16,706	limit	615.10	1	-	0.00
autocorr_bern45-23	1,352	limit	782.92	1	-	0.03
autocorr_bern45-34	805	limit	1,092.85	1	-	0.04
autocorr_bern45-45	612	limit	1,193.08	1	-	0.07
autocorr_bern50-06	115,543	limit	406.04	1	-	0.00
autocorr_bern50-13	6,913	limit	714.84	1	-	0.01
autocorr_bern50-25	840	limit	914.33	1	-	0.04
autocorr_bern55-06	115,104	limit	424.60	1	-	0.00
autocorr_bern55-14	2,812	limit	958.49	1	-	0.01
autocorr_bern55-28	705	limit	1,088.26	1	-	0.05
autocorr_bern60-08	33,674	limit	703.32	1	-	0.00
autocorr_bern60-15	2,532	limit	988.55	1	-	0.01
ball_mk2_10	1	0.01	-	-	-	0.00
ball_mk2_30	3,884,971	3,336.53	-	-	-	0.00
ball_mk3_10	0	0.00	-	-	-	0.00
carton7	271,560	686.34	-	-	3	0.00
carton9	849,860	limit	60.22	-	1	0.00
celar6-sub0	2,071	limit	-	1	-	0.21
chp_partload	786	limit	-	-	1	0.00
chp_shorttermplan1b	41,280	limit	6.06	-	-	0.00
chp_shorttermplan2a	135	136.59	-	-	-	0.00
chp_shorttermplan2c	21,299	limit	-	-	-	0.00
color_lab3_3x0	46,820	limit	494.85	2	-	0.00
color_lab3_4x0	38,687	limit	1,066.23	2	-	0.01
crossdock_15x7	56,784	limit	312.38	1	-	0.00
crossdock_15x8	47,283	limit	1,425.00	1	-	0.01
crudeoil_li02	1,484,820	limit	1.14	-	3	0.00
crudeoil_li06	45,205	763.71	-	-	1	0.01
crudeoil_li21	158,853	limit	1.72	-	1	0.01
crudeoil_pooling_dt4	19,960	limit	8.18	-	1	0.00
cvxnonsep_normcon30	1	0.11	-	-	-	0.00
cvxnonsep_normcon30r	1	0.12	-	-	-	0.00
cvxnonsep_normcon40	1	0.12	-	-	-	0.00
cvxnonsep_normcon40r	10	0.44	-	-	1	0.00
cvxnonsep_psig20	8,151,067	limit	227.98	-	3	0.00
cvxnonsep_psig20r	1	0.18	-	-	-	0.01
cvxnonsep_psig30	7,204,990	limit	155.02	-	1	0.00
cvxnonsep_psig30r	68	1.04	-	-	1	0.00
cvxnonsep_psig40	6,442,387	limit	112.65	-	4	0.00
cvxnonsep_psig40r	50	0.98	-	-	4	0.00
densitymod	511	limit	-	2	-	3.96
eigena2	1	limit	-	-	-	1.14
elec100	44	limit	-	-	2,024	1.80
elec200	1	limit	-	-	-	26.31
elec25	1,874	limit	-	-	284	0.03
elec50	511	limit	-	-	519	0.22
elf	176	1.76	-	-	2	0.00

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name	#nodes	total-time	gap	#orbitope	#sbc	sym-time
emfl100_5_5	3,226	limit	29,267.53	1	-	0.03
ex14_1_5	1	0.01	-	-	-	0.00
ex2_1_3	1	0.17	-	-	-	0.00
ex5_2_5	1	1.01	-	-	-	0.00
ex8_1_6	15	0.12	-	-	1	0.00
ex8_3_1	373,777	limit	23.00	-	-	0.00
ex8_3_11	434,839	limit	25.07	-	-	0.00
ex8_3_13	304,880	limit	15.67	-	-	0.00
ex8_3_2	489,622	limit	40.66	-	-	0.00
ex8_3_3	554,680	limit	39.22	-	-	0.00
ex8_3_4	483,344	limit	62.01	-	-	0.00
ex8_3_5	503,868	limit	1,496.29	-	-	0.00
ex8_3_7	26	limit	-	-	-	0.00
ex8_3_8	122,890	limit	207.11	-	-	0.00
ex8_3_9	696,756	limit	31.06	-	-	0.00
ex8_4_6	5	limit	-	-	3	0.00
ex8_6_1	654	limit	-	-	-	0.00
ex9_1_8	1	0.00	-	-	-	0.00
ex9_2_2	1	0.16	-	-	-	0.00
ex9_2_6	1	0.11	-	-	-	0.00
gabriel04	7,529	353.65	-	-	2	0.00
gams03	1	limit	-	-	-	0.35
gastrans582_cold13	14,275	589.00	-	-	1	0.00
gastrans582_cold13_95	98,709	limit	-	-	-	0.00
gastrans582_cold17	1	18.35	-	-	-	0.00
gastrans582_cold17_95	27	32.24	-	-	-	0.00
gastrans582_cool12	25	25.88	-	-	-	0.01
gastrans582_cool12_95	24	42.38	-	-	-	0.00
gastrans582_cool14	124	47.52	-	-	-	0.01
gastrans582_cool14_95	20	36.09	-	-	-	0.00
gastrans582_freezing27	1	25.02	-	-	-	0.00
gastrans582_freezing27_95	1	36.01	-	-	-	0.00
gastrans582_freezing30	21	42.98	-	-	-	0.00
gastrans582_freezing30_95	1	29.85	-	-	-	0.00
gastrans582_mild10	24	41.87	-	-	-	0.00
gastrans582_mild10_95	20	35.80	-	-	-	0.00
gastrans582_mild11	18	25.98	-	-	-	0.01
gastrans582_mild11_95	1	18.66	-	-	-	0.00
gastrans582_warm15	28,888	2,429.33	-	-	-	0.01
gastrans582_warm15_95	17	31.99	-	-	-	0.00
gastrans582_warm31	35	36.80	-	-	-	0.01
gastrans582_warm31_95	65	30.62	-	-	-	0.00
gear	10,972	8.70	-	-	2	0.00
gear2	30,935	18.46	-	-	2	0.00
gear3	11,771	9.45	-	-	2	0.00
gear4	563	0.21	-	-	2	0.00
graphpart_2g-0044-1601	1	0.34	-	1	-	0.00
graphpart_2g-0055-0062	5	1.12	-	1	-	0.00
graphpart_2g-0066-0066	11	2.34	-	1	-	0.00
graphpart_2g-0077-0077	54	4.48	-	1	-	0.00
graphpart_2g-0088-0088	23	6.54	-	1	-	0.00
graphpart_2g-0099-9211	70	13.23	-	1	-	0.00
graphpart_2g-1010-0824	24	12.33	-	1	-	0.00
graphpart_2pm-0044-0044	5	1.11	-	1	-	0.00
graphpart_2pm-0055-0055	21	1.86	-	1	-	0.00
graphpart_2pm-0066-0066	30	3.05	-	1	-	0.00
graphpart_2pm-0077-0777	34	3.99	-	1	-	0.00
graphpart_2pm-0088-0888	79	8.09	-	1	-	0.00
graphpart_2pm-0099-0999	1,569	27.44	-	1	-	0.00
graphpart_3g-0234-0234	17	1.99	-	1	-	0.00
graphpart_3g-0244-0244	17	4.56	-	1	-	0.00
graphpart_3g-0333-0333	10	3.13	-	1	-	0.00
graphpart_3g-0334-0334	94	5.53	-	1	-	0.00
graphpart_3g-0344-0344	125	8.39	-	1	-	0.00
graphpart_3g-0444-0444	1,117	31.76	-	1	-	0.00
graphpart_3pm-0234-0234	30	2.16	-	1	-	0.00
graphpart_3pm-0244-0244	175	3.68	-	1	-	0.00
graphpart_3pm-0333-0333	439	4.71	-	1	-	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
graphpart_3pm-0334-0334	876	9.24	-	1	-	0.00
graphpart_3pm-0344-0344	2,300	30.03	-	1	-	0.00
graphpart_3pm-0444-0444	90,112	680.71	-	1	-	0.00
graphpart_clique-20	341	5.87	-	1	-	0.00
graphpart_clique-30	2,682	55.03	-	1	-	0.00
graphpart_clique-40	43,499	848.07	-	1	-	0.00
graphpart_clique-50	39,471	limit	105.83	1	-	0.00
graphpart_clique-60	21,117	limit	276.26	1	-	0.00
graphpart_clique-70	13,065	limit	249.43	1	-	0.00
hadamard_4	173	0.55	-	-	-	0.00
hadamard_5	21,849	33.94	-	-	-	0.00
hadamard_6	121,998	limit	770.37	-	-	0.03
hadamard_7	3,494	limit	125,375.00	-	-	0.29
hadamard_8	18	limit	-	-	-	3.17
hmittelman	1	0.01	-	3	-	0.00
ivalues	77,279	limit	365.50	-	2	0.01
kissing2	1	915.56	-	-	-	5.47
knp3-12	79,044	limit	145.68	-	79	0.00
knp4-24	4,446	limit	447.37	-	303	0.03
knp5-40	178	limit	785.11	-	469	0.18
knp5-41	1,271	limit	661.17	-	491	0.21
knp5-42	367	limit	797.11	-	436	0.21
knp5-43	334	limit	788.54	-	593	0.23
knp5-44	203	limit	779.83	-	591	0.25
kport20	67,556	333.46	-	-	7	0.00
lop97icx	2,083	24.35	-	-	-	0.00
maxcsp-ehi-85-297-12	276	limit	-	4	-	0.00
maxcsp-ehi-85-297-36	293	limit	-	3	-	0.00
maxcsp-ehi-85-297-71	221	limit	-	2	-	0.00
maxcsp-ehi-90-315-70	165	limit	-	1	-	0.00
maxcsp-langford-3-11	32,612	limit	-	1	-	0.00
maxmin	135,137	limit	43.85	-	35	0.00
mbtd	1,568	limit	151.09	1	-	0.47
netmod_doll	4,789	limit	47.82	-	-	0.48
netmod_kar1	3,372	117.94	-	-	-	0.05
netmod_kar2	3,372	118.36	-	-	-	0.04
nuclear14	19	limit	-	-	15	0.00
nuclear25	66	limit	-	-	10	0.01
nuclear49	19	limit	-	-	20	0.03
nuclearva	22	limit	-	-	2	0.00
nuclearvb	338	limit	-	-	3	0.01
nuclearvc	424	limit	-	-	3	0.00
nuclearvd	57	limit	-	-	3	0.01
nuclearve	82	limit	-	-	3	0.00
nuclearvf	1,098	limit	-	-	3	0.00
nvs09	614	1.45	-	-	-	0.00
oil2	232	54.91	-	-	-	0.00
orth_d4m6_pl	409,157	limit	166.41	-	1	0.00
pooling_bental5pq	1	0.17	-	-	-	0.00
pooling_bental5tp	179	4.08	-	-	3	0.00
pooling_bental5tp	1	0.12	-	-	-	0.00
powerflow2383wpp	1	limit	-	-	-	0.23
powerflow2383wpr	1	limit	-	-	-	0.26
procurement1large	99,233	limit	375.13	-	35	0.01
procurement1mot	927,261	limit	552.12	-	5	0.00
procurement2mot	167	4.46	-	-	-	0.00
product	4,293	314.87	-	-	-	0.00
product2	1	9.22	-	-	-	0.07
radar-2000-10-a-6_lat_7	15,917	limit	138.30	-	-	0.00
radar-3000-10-a-8_lat_7	12,097	limit	10,110.83	-	-	0.00
ringpack_10_2	111,943	limit	8.22	1	-	0.01
ringpack_20_2	8,974	limit	233.33	1	-	0.02
ringpack_20_3	17,340	limit	566.67	-	-	0.15
ringpack_30_2	61	limit	1,400.00	1	-	0.06
rsyn0815m	151	1.02	-	-	3	0.00
rsyn0815m02m	1,422	11.94	-	-	6	0.00
rsyn0815m03m	1,347	18.32	-	-	9	0.01
rsyn0815m04m	3,857	52.02	-	-	12	0.00

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name	#nodes	total-time	gap	#orbitope	#sbcs	sym-time
sepasequ_complex	54,365	limit	62.58	-	-	0.01
sporttournament10	1	0.67	-	1	-	0.00
st_e09	1	0.02	-	-	-	0.00
st_e18	1	0.01	-	-	-	0.00
st_qpc-m3c	1	0.03	-	-	-	0.00
st_rv9	240	2.70	-	-	6	0.00
syn15m	1	0.32	-	-	-	0.00
t1000	0	0.87	-	-	-	0.00
tln2	1	0.01	-	-	-	0.00
topopt-zhou-rozvany_75	442	limit	-	-	200	0.00
torsion100	20	limit	380.39	-	-	0.10
torsion25	1	limit	319.02	-	-	0.02
torsion50	16	limit	373.99	-	-	0.06
turkey	1	19.97	-	-	-	0.00
util	14	0.26	-	1	-	0.00
waste	204,774	limit	83.06	-	-	0.01
watercontamination0202	33	96.14	-	-	-	0.00
waterund28	10,025	limit	7.76	-	-	0.00

TABLE B.8: Detailed results for variant **D-sepa** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of domain reductions performed (#red), the number of node cutoffs detected (#cutoff), and the time for symmetry computating and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#red	#cutoff	sym-time
arki0002	4	limit	-	-	-	0.02
arki0005	439	limit	-	-	-	0.06
arki0006	97	limit	-	-	-	0.05
arki0008	1	33.17	-	-	-	0.02
arki0016	8,718	limit	-	-	-	0.02
arki0017	9,214	limit	996.25	-	-	0.02
arki0018	2	limit	-	-	-	8.92
autocorr_bern20-03	1	0.02	-	-	-	0.00
autocorr_bern20-05	23,260	43.16	-	403	-	0.00
autocorr_bern20-10	92,832	608.86	-	351	-	0.01
autocorr_bern20-15	89,307	1,169.29	-	376	-	0.00
autocorr_bern25-03	1	0.04	-	-	-	0.00
autocorr_bern25-06	723,859	1,923.60	-	3,561	3	0.00
autocorr_bern25-13	99,523	limit	78.02	836	-	0.00
autocorr_bern25-19	38,603	limit	218.25	641	-	0.00
autocorr_bern25-25	18,867	limit	387.21	463	-	0.01
autocorr_bern30-04	783,673	1,177.71	-	15,862	-	0.00
autocorr_bern30-08	201,616	limit	171.14	2,634	-	0.00
autocorr_bern30-15	33,143	limit	238.26	285	-	0.01
autocorr_bern30-23	8,121	limit	460.03	479	-	0.01
autocorr_bern30-30	3,650	limit	675.90	141	-	0.02
autocorr_bern35-04	759,042	limit	59.03	5,994	-	0.00
autocorr_bern35-09	84,780	limit	322.03	7,392	-	0.01
autocorr_bern35-18	7,463	limit	449.75	520	-	0.01
autocorr_bern35-26	2,538	limit	671.49	102	-	0.03
autocorr_bern35-35fx	1,946	limit	901.51	152	-	0.00
autocorr_bern40-05	344,302	limit	157.00	5,281	-	0.01
autocorr_bern40-10	40,320	limit	489.50	678	-	0.01
autocorr_bern40-20	2,118	limit	644.42	89	-	0.02
autocorr_bern40-30	1,257	limit	857.07	101	-	0.03
autocorr_bern40-40	1,276	limit	1,048.21	45	-	0.03
autocorr_bern45-05	291,158	limit	213.23	7,357	-	0.00
autocorr_bern45-11	16,410	limit	615.10	1,467	-	0.00
autocorr_bern45-23	1,340	limit	782.92	79	-	0.03
autocorr_bern45-34	790	limit	1,102.14	70	-	0.05
autocorr_bern45-45	603	limit	1,193.35	42	-	0.05
autocorr_bern50-06	111,581	limit	407.69	1,105	-	0.00
autocorr_bern50-13	6,741	limit	714.84	321	-	0.01

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name	#nodes	total-time	gap	#red	#cut off	sym-time
autocorr_bern50-25	834	limit	918.77	22	-	0.03
autocorr_bern55-06	111,155	limit	425.80	585	-	0.00
autocorr_bern55-14	2,770	limit	961.58	249	-	0.01
autocorr_bern55-28	702	limit	1,088.26	86	-	0.06
autocorr_bern60-08	32,686	limit	704.38	1,090	-	0.00
autocorr_bern60-15	2,464	limit	989.17	1	-	0.02
ball_mk2_10	1	0.02	-	-	-	0.00
ball_mk2_30	57	0.11	-	-	1	0.00
ball_mk3_10	0	0.00	-	-	-	0.00
carton7	1,371	limit	127.53	12	-	0.00
carton9	338,042	limit	75.28	5,579	25	0.00
celar6-sub0	2,071	limit	-	133	143	0.20
chp_partload	69	limit	-	8	-	0.01
chp_shorttermplan1b	40,633	limit	5.79	578	-	0.00
chp_shorttermplan2a	134	141.68	-	18	1	0.00
chp_shorttermplan2c	21,310	limit	-	389	-	0.00
color_lab3_3x0	45,494	limit	495.78	472	20	0.00
color_lab3_4x0	38,682	limit	1,066.23	827	22	0.01
crossdock_15x7	54,510	limit	316.21	5,633	102	0.01
crossdock_15x8	45,192	limit	1,448.01	3,875	32	0.01
crudeoil_li02	119	limit	-	9	-	0.00
crudeoil_li06	7,160	limit	0.90	284	4	0.00
crudeoil_li21	2,392	limit	-	30	1	0.01
crudeoil_pooling_dt4	19,071	limit	8.17	-	-	0.00
cvxnonsep_normcon30	1	0.10	-	-	-	0.00
cvxnonsep_normcon30r	1	0.12	-	-	-	0.00
cvxnonsep_normcon40	1	0.13	-	-	-	0.00
cvxnonsep_normcon40r	10	0.44	-	-	-	0.00
cvxnonsep_psig20	7,767,553	limit	227.18	-	-	0.00
cvxnonsep_psig20r	1	0.15	-	-	-	0.00
cvxnonsep_psig30	7,073,121	limit	155.00	-	-	0.00
cvxnonsep_psig30r	68	1.04	-	-	-	0.00
cvxnonsep_psig40	6,240,835	limit	113.24	-	2	0.00
cvxnonsep_psig40r	36	0.87	-	-	-	0.00
densitymod	511	limit	-	170	237	3.94
eigena2	1	limit	-	-	-	1.16
elec100	404	limit	-	-	-	1.79
elec200	1	limit	-	-	-	26.28
elec25	3,673	limit	-	-	-	0.03
elec50	3,564	limit	-	-	-	0.21
elf	55	limit	6,663.44	7	-	0.00
emfl100_5_5	3,224	limit	29,267.53	-	-	0.02
ex14_1_5	1	0.01	-	-	-	0.00
ex2_1_3	1	0.17	-	-	-	0.00
ex5_2_5	1	1.04	-	-	-	0.00
ex8_1_6	19	0.13	-	-	-	0.00
ex8_3_1	374,403	limit	23.00	-	-	0.00
ex8_3_11	436,330	limit	25.07	-	-	0.00
ex8_3_13	305,811	limit	15.67	-	-	0.00
ex8_3_2	489,163	limit	40.66	-	-	0.00
ex8_3_3	555,099	limit	39.22	-	-	0.00
ex8_3_4	479,337	limit	62.01	-	-	0.00
ex8_3_5	502,872	limit	1,496.29	-	-	0.00
ex8_3_7	26	limit	-	-	-	0.00
ex8_3_8	123,141	limit	207.11	-	-	0.00
ex8_3_9	697,576	limit	31.06	-	-	0.00
ex8_4_6	12	limit	-	-	-	0.00
ex8_6_1	654	limit	-	-	-	0.00
ex9_1_8	1	0.01	-	-	-	0.00
ex9_2_2	1	0.15	-	-	-	0.00
ex9_2_6	1	0.09	-	-	-	0.00
gabriel04	7,047	321.73	-	-	-	0.00
gams03	1	limit	-	-	-	0.34
gastrans582_cold13	41,516	1,680.14	-	-	-	0.00
gastrans582_cold13_95	23	26.49	-	-	-	0.00
gastrans582_cold17	1	19.07	-	-	-	0.00
gastrans582_cold17_95	37	33.26	-	-	-	0.00
gastrans582_cool12	25	27.44	-	-	-	0.00

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name	#nodes	total-time	gap	#red	#cutoff	sym-time
gastrans582_cool12_95	24	43.14	-	-	-	0.00
gastrans582_cool14	2,560	229.80	-	-	-	0.01
gastrans582_cool14_95	11	35.31	-	-	-	0.00
gastrans582_freezing27	1	25.33	-	-	-	0.00
gastrans582_freezing27_95	1	36.61	-	-	-	0.00
gastrans582_freezing30	21	44.28	-	-	-	0.00
gastrans582_freezing30_95	21	41.15	-	-	-	0.00
gastrans582_mild10	24	42.59	-	-	-	0.00
gastrans582_mild10_95	20	36.72	-	-	-	0.00
gastrans582_mild11	42	33.44	-	-	-	0.01
gastrans582_mild11_95	1	18.97	-	-	-	0.00
gastrans582_warm15	1,043	303.93	-	-	-	0.00
gastrans582_warm15_95	21	33.01	-	-	-	0.00
gastrans582_warm31	29	37.95	-	-	-	0.00
gastrans582_warm31_95	18	30.48	-	-	-	0.00
gear	14,488	9.11	-	-	4	0.00
gear2	99	limit	-	25	-	0.00
gear3	14,076	9.71	-	-	3	0.00
gear4	1,060	0.37	-	-	-	0.00
graphpart_2g-0044-1601	1	0.35	-	-	-	0.00
graphpart_2g-0055-0062	5	1.17	-	2	1	0.00
graphpart_2g-0066-0066	11	2.35	-	3	2	0.00
graphpart_2g-0077-0077	54	4.71	-	2	2	0.00
graphpart_2g-0088-0088	23	6.79	-	8	1	0.00
graphpart_2g-0099-9211	70	13.72	-	11	2	0.00
graphpart_2g-1010-0824	24	12.75	-	4	3	0.00
graphpart_2pm-0044-0044	5	1.16	-	3	-	0.00
graphpart_2pm-0055-0055	21	1.84	-	4	2	0.00
graphpart_2pm-0066-0066	30	3.23	-	3	1	0.00
graphpart_2pm-0077-0777	34	4.20	-	3	1	0.00
graphpart_2pm-0088-0888	79	8.28	-	15	2	0.00
graphpart_2pm-0099-0999	1,569	28.43	-	12	1	0.00
graphpart_3g-0234-0234	17	2.02	-	4	2	0.00
graphpart_3g-0244-0244	17	4.64	-	5	3	0.00
graphpart_3g-0333-0333	10	3.32	-	3	-	0.00
graphpart_3g-0334-0334	94	5.80	-	10	5	0.00
graphpart_3g-0344-0344	125	8.35	-	12	3	0.00
graphpart_3g-0444-0444	1,117	32.22	-	24	44	0.00
graphpart_3pm-0234-0234	30	2.26	-	3	1	0.00
graphpart_3pm-0244-0244	175	3.96	-	13	1	0.00
graphpart_3pm-0333-0333	439	4.72	-	10	5	0.00
graphpart_3pm-0334-0334	876	9.42	-	58	1	0.00
graphpart_3pm-0344-0344	2,300	30.58	-	19	100	0.00
graphpart_3pm-0444-0444	90,112	699.09	-	32	47	0.00
graphpart_clique-20	341	5.76	-	16	5	0.00
graphpart_clique-30	2,682	54.74	-	142	25	0.00
graphpart_clique-40	43,499	849.54	-	226	19	0.00
graphpart_clique-50	39,565	limit	105.83	81	2	0.00
graphpart_clique-60	20,965	limit	276.40	409	19	0.00
graphpart_clique-70	13,118	limit	249.43	27	20	0.00
hadamard_4	173	0.56	-	-	-	0.00
hadamard_5	21,849	34.48	-	-	-	0.01
hadamard_6	123,327	limit	766.67	-	-	0.03
hadamard_7	3,513	limit	125,300.00	-	-	0.31
hadamard_8	18	limit	-	-	-	3.19
hmittelman	1	0.02	-	-	-	0.00
ivalues	63,645	limit	486.95	-	6	0.00
kissing2	1	914.55	-	-	-	5.45
knp3-12	74,763	limit	142.33	-	328	0.00
knp4-24	6,853	limit	391.48	-	3	0.03
knp5-40	327	limit	820.08	-	-	0.19
knp5-41	1,243	limit	727.74	-	-	0.20
knp5-42	520	limit	830.93	-	-	0.21
knp5-43	890	limit	807.30	-	-	0.23
knp5-44	609	limit	823.92	-	-	0.24
kport20	49,856	257.36	-	1,322	-	0.00
lop97icx	2,083	25.45	-	-	-	0.00
maxcsp-ehi-85-297-12	245	limit	-	3	-	0.00

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name	#nodes	total-time	gap	#red	#cut off	sym-time
maxcsp-ehi-85-297-36	263	limit	-	5	-	0.00
maxcsp-ehi-85-297-71	195	limit	-	-	-	0.00
maxcsp-ehi-90-315-70	139	limit	-	-	-	0.00
maxcsp-langford-3-11	31,278	limit	-	196	242	0.00
maxmin	172,863	limit	9.03	-	-	0.00
mbtd	1,570	limit	151.09	4	1	0.46
netmod_doll	4,799	limit	47.82	-	-	0.46
netmod_kar1	3,372	117.70	-	-	-	0.05
netmod_kar2	3,372	118.05	-	-	-	0.05
nuclear14	30	limit	-	-	-	0.01
nuclear25	15	limit	-	-	-	0.00
nuclear49	21	limit	-	-	-	0.02
nuclearva	22	limit	-	-	-	0.01
nuclearvb	111	limit	-	-	-	0.00
nuclearvc	212	limit	-	-	-	0.00
nuclearvd	287	limit	-	-	-	0.00
nuclearve	59	limit	-	-	-	0.00
nuclearvf	66	limit	-	-	-	0.00
nvs09	39	0.20	-	-	1	0.00
oil2	168	81.68	-	-	-	0.00
orth_d4m6_pl	426,404	limit	166.56	-	43	0.00
pooling_bental5pq	1	0.16	-	-	-	0.00
pooling_bental5stp	7,198	37.57	-	-	-	0.00
pooling_bental5tp	1	0.14	-	-	-	0.00
powerflow2383wpp	1	limit	-	-	-	0.23
powerflow2383wpr	1	limit	-	-	-	0.27
procurement1large	87,925	limit	375.13	-	-	0.01
procurement1mot	842,483	limit	522.71	-	-	0.00
procurement2mot	167	4.48	-	-	-	0.00
product	4,293	316.28	-	-	-	0.00
product2	1	9.48	-	-	-	0.07
radar-2000-10-a-6_lat_7	341	361.31	-	26,206	-	0.00
radar-3000-10-a-8_lat_7	4,924	limit	292.58	144,643	-	0.00
ringpack_10_2	110,478	limit	8.22	1,428	37	0.00
ringpack_20_2	8,833	limit	233.33	9	3	0.01
ringpack_20_3	17,380	limit	566.67	-	-	0.15
ringpack_30_2	61	limit	1,400.00	-	-	0.05
rsyn0815m	151	1.01	-	-	-	0.00
rsyn0815m02m	1,422	11.68	-	-	-	0.00
rsyn0815m03m	1,347	18.43	-	-	-	0.00
rsyn0815m04m	4,006	50.04	-	-	-	0.01
sepasequ_complex	54,234	limit	62.58	-	-	0.01
sporttournament10	1	0.69	-	-	-	0.00
st_e09	1	0.01	-	-	-	0.00
st_e18	1	0.02	-	-	-	0.00
st_qpc-m3c	1	0.03	-	-	-	0.00
st_rv9	331	2.78	-	-	4	0.00
syn15m	1	0.32	-	-	-	0.00
t1000	0	0.88	-	-	-	0.00
tln2	1	0.00	-	-	-	0.00
topopt-zhou-rozvany_75	478	limit	-	-	-	0.00
torsion100	20	limit	380.39	-	-	0.11
torsion25	1	limit	319.02	-	-	0.02
torsion50	16	limit	373.99	-	-	0.06
turkey	1	20.01	-	-	-	0.00
util	14	0.28	-	1	-	0.00
waste	203,212	limit	84.75	-	-	0.01
watercontamination0202	33	98.04	-	-	-	0.00
waterund28	10,033	limit	7.76	-	-	0.01

TABLE B.9: Detailed results for variant **D-sepa*** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of domain reductions performed (#red), the number of node cutoffs detected (#cutoff), and the time for symmetry computating and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#red	#cutoff	sym-time
arki0002	7	limit	-	-	-	0.02
arki0005	439	limit	-	-	-	0.06
arki0006	97	limit	-	-	-	0.05
arki0008	1	33.28	-	-	-	0.02
arki0016	8,690	limit	-	-	-	0.02
arki0017	9,218	limit	996.25	-	-	0.01
arki0018	3	limit	-	-	-	9.13
autocorr_bern20-03	1	0.03	-	-	-	0.00
autocorr_bern20-05	23,260	42.80	-	403	-	0.00
autocorr_bern20-10	92,832	611.47	-	351	-	0.01
autocorr_bern20-15	89,307	1,169.16	-	376	-	0.00
autocorr_bern25-03	1	0.04	-	-	-	0.00
autocorr_bern25-06	723,859	1,918.40	-	3,561	3	0.00
autocorr_bern25-13	99,959	limit	77.82	836	-	0.01
autocorr_bern25-19	38,909	limit	218.19	641	-	0.00
autocorr_bern25-25	18,846	limit	387.63	463	-	0.01
autocorr_bern30-04	783,673	1,180.86	-	15,862	-	0.00
autocorr_bern30-08	200,308	limit	171.36	2,621	-	0.00
autocorr_bern30-15	33,125	limit	238.26	285	-	0.01
autocorr_bern30-23	8,117	limit	460.03	479	-	0.01
autocorr_bern30-30	3,661	limit	675.01	141	-	0.02
autocorr_bern35-04	757,819	limit	59.03	5,987	-	0.00
autocorr_bern35-09	83,542	limit	323.06	7,322	-	0.00
autocorr_bern35-18	7,459	limit	449.75	520	-	0.01
autocorr_bern35-26	2,510	limit	673.97	102	-	0.02
autocorr_bern35-35fix	1,947	limit	901.51	152	-	0.00
autocorr_bern40-05	345,337	limit	156.90	5,290	-	0.01
autocorr_bern40-10	40,561	limit	489.12	680	-	0.00
autocorr_bern40-20	2,121	limit	643.85	89	-	0.02
autocorr_bern40-30	1,259	limit	857.07	101	-	0.03
autocorr_bern40-40	1,258	limit	1,048.21	45	-	0.04
autocorr_bern45-05	288,957	limit	213.42	7,318	-	0.00
autocorr_bern45-11	16,341	limit	615.10	1,458	-	0.01
autocorr_bern45-23	1,351	limit	782.92	79	-	0.02
autocorr_bern45-34	792	limit	1,102.14	70	-	0.05
autocorr_bern45-45	609	limit	1,193.35	42	-	0.04
autocorr_bern50-06	110,062	limit	408.33	1,056	-	0.00
autocorr_bern50-13	6,737	limit	714.84	321	-	0.00
autocorr_bern50-25	832	limit	918.77	22	-	0.03
autocorr_bern55-06	112,664	limit	425.34	599	-	0.00
autocorr_bern55-14	2,775	limit	961.58	250	-	0.02
autocorr_bern55-28	702	limit	1,088.26	86	-	0.06
autocorr_bern60-08	32,822	limit	704.24	1,091	-	0.01
autocorr_bern60-15	2,487	limit	989.09	1	-	0.02
ball_mk2_10	1	0.00	-	-	-	0.00
ball_mk2_30	227	0.28	-	-	84	0.00
ball_mk3_10	0	0.00	-	-	-	0.00
carton7	1,371	limit	127.53	12	-	0.00
carton9	338,042	limit	75.28	5,579	25	0.00
celar6-sub0	2,072	limit	-	133	143	0.21
chp_partload	69	limit	-	8	-	0.01
chp_shorttermplan1b	40,593	limit	5.79	578	-	0.00
chp_shorttermplan2a	134	142.71	-	18	1	0.00
chp_shorttermplan2c	21,405	limit	-	390	-	0.00
color_lab3_3x0	46,725	limit	495.24	474	20	0.00
color_lab3_4x0	38,873	limit	1,066.23	827	23	0.01
crossdock_15x7	54,490	limit	316.24	5,628	102	0.00
crossdock_15x8	45,328	limit	1,443.87	3,878	32	0.01
crudeoil_li02	119	limit	-	9	-	0.01
crudeoil_li06	7,160	limit	0.90	284	4	0.00
crudeoil_li21	2,392	limit	-	30	1	0.01
crudeoil_pooling_dt4	18,944	limit	8.17	-	-	0.00

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name	#nodes	total-time	gap	#red	#cut off	sym-time
cvxnonsep_normcon30	1	0.12	-	-	-	0.00
cvxnonsep_normcon30r	1	0.13	-	-	-	0.00
cvxnonsep_normcon40	1	0.12	-	-	-	0.00
cvxnonsep_normcon40r	10	0.40	-	-	-	0.00
cvxnonsep_psig20	7,939,826	limit	228.54	-	11	0.00
cvxnonsep_psig20r	1	0.16	-	-	-	0.00
cvxnonsep_psig30	7,096,224	limit	155.00	-	1	0.00
cvxnonsep_psig30r	68	1.03	-	-	-	0.00
cvxnonsep_psig40	6,412,437	limit	113.25	-	23851	0.00
cvxnonsep_psig40r	41	0.95	-	-	-	0.00
densitymod	511	limit	-	170	237	3.92
eigena2	1	limit	-	-	-	1.14
elec100	287	limit	-	-	-	1.81
elec200	1	limit	-	-	-	26.50
elec25	6,859	limit	-	-	11	0.03
elec50	2,055	limit	-	-	-	0.20
elf	55	limit	6,663.44	7	-	0.00
emfl100_5_5	3,224	limit	29,267.53	-	-	0.02
ex14_1_5	1	0.01	-	-	-	0.00
ex2_1_3	1	0.15	-	-	-	0.00
ex5_2_5	1	0.99	-	-	-	0.00
ex8_1_6	17	0.14	-	-	-	0.00
ex8_3_1	373,853	limit	23.00	-	-	0.00
ex8_3_11	436,021	limit	25.07	-	-	0.00
ex8_3_13	305,827	limit	15.67	-	-	0.00
ex8_3_2	489,793	limit	40.66	-	-	0.00
ex8_3_3	552,618	limit	39.22	-	-	0.00
ex8_3_4	481,679	limit	62.01	-	-	0.00
ex8_3_5	505,614	limit	1,496.29	-	-	0.00
ex8_3_7	26	limit	-	-	-	0.00
ex8_3_8	123,325	limit	207.11	-	-	0.00
ex8_3_9	697,524	limit	31.06	-	-	0.00
ex8_4_6	14	limit	-	-	1	0.00
ex8_6_1	654	limit	-	-	-	0.00
ex9_1_8	1	0.00	-	-	-	0.00
ex9_2_2	1	0.15	-	-	-	0.00
ex9_2_6	1	0.11	-	-	-	0.00
gabriel04	6,831	324.49	-	-	-	0.00
gams03	1	limit	-	-	-	0.35
gastrans582_cold13	41,516	1,676.32	-	-	-	0.00
gastrans582_cold13_95	23	26.72	-	-	-	0.00
gastrans582_cold17	1	19.00	-	-	-	0.00
gastrans582_cold17_95	37	33.32	-	-	-	0.00
gastrans582_cool12	25	26.35	-	-	-	0.00
gastrans582_cool12_95	24	43.36	-	-	-	0.00
gastrans582_cool14	2,560	228.85	-	-	-	0.01
gastrans582_cool14_95	11	35.38	-	-	-	0.00
gastrans582_freezing27	1	25.53	-	-	-	0.00
gastrans582_freezing27_95	1	37.07	-	-	-	0.00
gastrans582_freezing30	21	44.26	-	-	-	0.00
gastrans582_freezing30_95	21	41.10	-	-	-	0.00
gastrans582_mild10	24	42.82	-	-	-	0.01
gastrans582_mild10_95	20	36.88	-	-	-	0.00
gastrans582_mild11	42	33.02	-	-	-	0.00
gastrans582_mild11_95	1	19.24	-	-	-	0.00
gastrans582_warm15	1,043	302.74	-	-	-	0.01
gastrans582_warm15_95	21	33.15	-	-	-	0.00
gastrans582_warm31	29	37.46	-	-	-	0.01
gastrans582_warm31_95	18	29.87	-	-	-	0.00
gear	18,615	11.92	-	-	146	0.00
gear2	99	limit	-	25	-	0.01
gear3	18,455	11.80	-	-	56	0.00
gear4	912	0.34	-	-	60	0.00
graphpart_2g-0044-1601	1	0.37	-	-	-	0.00
graphpart_2g-0055-0062	5	1.23	-	2	1	0.00
graphpart_2g-0066-0066	11	2.33	-	3	2	0.00
graphpart_2g-0077-0077	54	4.63	-	2	2	0.00
graphpart_2g-0088-0088	23	6.82	-	8	1	0.00

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name	#nodes	total-time	gap	#red	#cutoff	sym-time
graphpart_2g-0099-9211	70	13.64	-	11	2	0.00
graphpart_2g-1010-0824	24	12.71	-	4	3	0.00
graphpart_2pm-0044-0044	5	1.20	-	3	-	0.00
graphpart_2pm-0055-0055	21	1.91	-	4	2	0.00
graphpart_2pm-0066-0066	30	3.25	-	3	1	0.00
graphpart_2pm-0077-0777	34	4.21	-	3	1	0.00
graphpart_2pm-0088-0888	79	8.42	-	15	2	0.00
graphpart_2pm-0099-0999	1,569	28.37	-	12	1	0.00
graphpart_3g-0234-0234	17	2.08	-	4	2	0.00
graphpart_3g-0244-0244	17	4.61	-	5	3	0.00
graphpart_3g-0333-0333	10	3.23	-	3	-	0.00
graphpart_3g-0334-0334	94	5.67	-	10	5	0.00
graphpart_3g-0344-0344	125	8.71	-	12	3	0.00
graphpart_3g-0444-0444	1,117	32.07	-	24	44	0.00
graphpart_3pm-0234-0234	30	2.28	-	3	1	0.00
graphpart_3pm-0244-0244	175	3.79	-	13	1	0.00
graphpart_3pm-0333-0333	439	4.78	-	10	5	0.00
graphpart_3pm-0334-0334	876	9.26	-	58	1	0.00
graphpart_3pm-0344-0344	2,300	30.76	-	19	100	0.00
graphpart_3pm-0444-0444	90,112	706.99	-	32	47	0.00
graphpart_clique-20	341	5.68	-	16	5	0.00
graphpart_clique-30	2,682	55.21	-	142	25	0.00
graphpart_clique-40	43,499	847.70	-	226	19	0.00
graphpart_clique-50	39,284	limit	105.91	81	2	0.00
graphpart_clique-60	21,199	limit	275.97	427	19	0.00
graphpart_clique-70	13,136	limit	249.43	27	20	0.00
hadamard_4	173	0.59	-	-	-	0.00
hadamard_5	21,849	34.57	-	-	-	0.00
hadamard_6	123,144	limit	766.67	-	-	0.03
hadamard_7	3,507	limit	125,375.00	-	-	0.30
hadamard_8	18	limit	-	-	-	3.17
hmittelman	1	0.03	-	-	-	0.00
ivalues	69,513	limit	444.53	-	107	0.00
kissing2	1	913.34	-	-	-	5.56
kn3-12	100,093	limit	123.12	-	246	0.00
kn4-24	9,153	limit	389.97	-	1	0.03
kn5-40	1,131	limit	782.40	-	-	0.19
kn5-41	452	limit	798.99	-	-	0.20
kn5-42	192	limit	871.81	-	-	0.22
kn5-43	265	limit	878.66	-	-	0.23
kn5-44	799	limit	784.37	-	-	0.25
kport20	66,735	342.70	-	1,761	10	0.00
lop97icx	2,083	25.31	-	-	-	0.01
maxcsp-ehi-85-297-12	245	limit	-	3	-	0.00
maxcsp-ehi-85-297-36	264	limit	-	5	-	0.00
maxcsp-ehi-85-297-71	195	limit	-	-	-	0.00
maxcsp-ehi-90-315-70	143	limit	-	-	-	0.00
maxcsp-langford-3-11	31,227	limit	-	192	239	0.00
maxmin	165,677	limit	51.13	-	75	0.00
mbtd	1,568	limit	151.09	4	1	0.46
netmod_doll	4,789	limit	47.82	-	-	0.52
netmod_kar1	3,372	117.88	-	-	-	0.04
netmod_kar2	3,372	118.22	-	-	-	0.03
nuclear14	30	limit	-	-	-	0.00
nuclear25	15	limit	-	-	-	0.01
nuclear49	21	limit	-	-	-	0.03
nuclearva	22	limit	-	-	-	0.00
nuclearvb	111	limit	-	-	-	0.00
nuclearvc	212	limit	-	-	-	0.00
nuclearvd	287	limit	-	-	-	0.00
nuclearve	59	limit	-	-	-	0.01
nuclearvf	66	limit	-	-	-	0.00
nvs09	285	0.66	-	-	3	0.00
oil2	168	81.26	-	-	-	0.00
orth_d4m6_pl	443,898	limit	246.89	-	330	0.00
pooling_bental5pq	1	0.15	-	-	-	0.00
pooling_bental5stp	436	5.20	-	-	-	0.00
pooling_bental5tp	1	0.14	-	-	-	0.00

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name	#nodes	total-time	gap	#red	#cut off	sym-time
powerflow2383wpp	1	limit	-	-	-	0.24
powerflow2383wpr	1	limit	-	-	-	0.26
procurement1large	87,979	limit	375.13	-	-	0.02
procurement1mot	839,224	limit	522.87	-	-	0.00
procurement2mot	167	4.39	-	-	-	0.00
product	4,293	315.91	-	-	-	0.00
product2	1	9.42	-	-	-	0.07
radar-2000-10-a-6_lat_7	341	362.62	-	26,206	-	0.00
radar-3000-10-a-8_lat_7	4,924	limit	292.58	144,585	-	0.00
ringpack_10_2	110,024	limit	8.22	1,420	37	0.00
ringpack_20_2	8,831	limit	233.33	9	3	0.02
ringpack_20_3	17,360	limit	566.67	-	-	0.15
ringpack_30_2	61	limit	1,400.00	-	-	0.06
rsyn0815m	151	1.01	-	-	-	0.00
rsyn0815m02m	1,422	11.56	-	-	-	0.01
rsyn0815m03m	1,347	18.31	-	-	-	0.00
rsyn0815m04m	4,006	49.83	-	-	-	0.01
sepasequ_complex	54,290	limit	62.58	-	-	0.01
sporttournament10	1	0.72	-	-	-	0.00
st_e09	1	0.03	-	-	-	0.00
st_e18	1	0.01	-	-	-	0.00
st_qpc-m3c	1	0.02	-	-	-	0.00
st_rv9	274	2.68	-	-	-	0.00
syn15m	1	0.28	-	-	-	0.00
t1000	0	0.88	-	-	-	0.00
tln2	1	0.00	-	-	-	0.00
topopt-zhou-rozvany_75	480	limit	-	-	-	0.00
torsion100	20	limit	380.39	-	-	0.12
torsion25	1	limit	319.02	-	-	0.02
torsion50	16	limit	373.99	-	-	0.06
turkey	1	19.82	-	-	-	0.00
util	14	0.29	-	1	-	0.00
waste	203,402	limit	84.75	-	-	0.01
watercontamination0202	33	98.89	-	-	-	0.00
waterund28	10,016	limit	7.76	-	-	0.01

TABLE B.10: Detailed results for variant **orbfix** on **M-sym-all**. Depicted are the number of nodes (#nodes), the total CPU time in seconds (total-time), the remaining gap at the time limit as reported by SCIP (gap), the number of domain reductions performed (#red), the number of node cutoffs detected (#cut off), and the time for symmetry computing and handling in seconds (sym-time). If the gap is not displayed despite the problem hitting the time limit, then SCIP reported either "Large" or "infinite" as gap.

name	#nodes	total-time	gap	#red	#cut off	sym-time
arki0002	24	limit	-	-	-	0.00
arki0005	439	limit	-	-	-	0.00
arki0006	97	limit	-	-	-	0.00
arki0008	1	32.90	-	-	-	0.00
arki0016	8,947	limit	-	-	-	0.00
arki0017	10,303	limit	995.85	-	-	0.00
arki0018	2	limit	-	-	-	0.00
autocorr_bern20-03	1	0.02	-	-	-	0.00
autocorr_bern20-05	44,669	70.63	-	153	-	0.07
autocorr_bern20-10	118,016	715.60	-	317	-	0.25
autocorr_bern20-15	103,785	1,291.37	-	260	-	0.25
autocorr_bern25-03	1	0.03	-	-	-	0.00
autocorr_bern25-06	755,891	1,950.32	-	726	-	2.32
autocorr_bern25-13	106,084	limit	82.71	431	-	0.75
autocorr_bern25-19	38,204	limit	219.21	141	-	0.20
autocorr_bern25-25	19,353	limit	356.71	84	-	0.15
autocorr_bern30-04	1,395,483	1,905.29	-	598	2	3.61
autocorr_bern30-08	217,344	limit	164.48	134	-	0.75
autocorr_bern30-15	40,833	limit	204.19	216	-	0.31
autocorr_bern30-23	5,775	limit	490.40	23	-	0.03
autocorr_bern30-30	5,066	limit	658.05	9	-	0.06

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name	#nodes	total-time	gap	#red	#cutoff	sym-time
autocorr_bern35-04	1,152,819	limit	60.59	140	-	3.37
autocorr_bern35-09	108,443	limit	333.73	325	-	0.50
autocorr_bern35-18	5,815	limit	471.82	65	-	0.05
autocorr_bern35-26	1,913	limit	742.39	19	-	0.06
autocorr_bern35-35fix	1,957	limit	971.43	5	-	0.00
autocorr_bern40-05	361,809	limit	166.52	179	-	1.57
autocorr_bern40-10	43,101	limit	507.54	19	-	0.31
autocorr_bern40-20	2,156	limit	641.79	32	-	0.03
autocorr_bern40-30	1,304	limit	908.86	5	-	0.06
autocorr_bern40-40	1,459	limit	1,095.03	8	-	0.09
autocorr_bern45-05	253,717	limit	232.33	38	-	1.05
autocorr_bern45-11	16,708	limit	613.20	5	-	0.17
autocorr_bern45-23	1,119	limit	799.77	22	-	0.06
autocorr_bern45-34	984	limit	1,082.72	13	-	0.09
autocorr_bern45-45	665	limit	1,173.04	5	-	0.09
autocorr_bern50-06	118,615	limit	416.67	31	-	0.58
autocorr_bern50-13	7,654	limit	729.62	5	-	0.05
autocorr_bern50-25	825	limit	937.44	18	-	0.06
autocorr_bern55-06	106,392	limit	436.54	23	-	0.54
autocorr_bern55-14	3,162	limit	947.20	8	-	0.04
autocorr_bern55-28	636	limit	1,070.55	5	-	0.05
autocorr_bern60-08	32,680	limit	706.80	12	-	0.20
autocorr_bern60-15	2,459	limit	986.81	5	-	0.07
ball_mk2_10	1	0.00	-	-	-	0.00
ball_mk2_30	3,884,971	3,333.13	-	-	-	2.50
ball_mk3_10	0	0.00	-	-	-	0.00
carton7	265,440	710.44	-	55,092	34	1.37
carton9	819,699	limit	54.54	166,261	89	4.74
celar6-sub0	1,545	limit	-	95	-	0.28
chp_partload	688	limit	-	20	8	0.03
chp_shorttermplan1b	28,599	limit	5.72	-	-	0.00
chp_shorttermplan2a	808	166.12	-	6	-	0.00
chp_shorttermplan2c	19,663	limit	-	6	-	0.00
color_lab3_3x0	56,660	limit	535.23	670	2	0.66
color_lab3_4x0	38,975	limit	1,249.71	1,007	47	0.39
crossdock_15x7	70,060	limit	480.32	14	1	0.51
crossdock_15x8	52,001	limit	1,701.34	-	-	0.20
crudeoil_li02	1,573,805	limit	1.15	14,862	37	11.92
crudeoil_li06	20,619	393.46	-	-	-	0.05
crudeoil_li21	182,708	limit	2.04	-	-	0.67
crudeoil_pooling_dt4	18,099	limit	8.16	-	-	0.00
cvxnonsep_normcon30	1	0.12	-	-	-	0.00
cvxnonsep_normcon30r	1	0.11	-	-	-	0.00
cvxnonsep_normcon40	1	0.11	-	-	-	0.00
cvxnonsep_normcon40r	10	0.40	-	-	-	0.00
cvxnonsep_psig20	8,155,273	limit	231.12	-	-	6.65
cvxnonsep_psig20r	1	0.16	-	-	-	0.00
cvxnonsep_psig30	7,382,522	limit	155.14	-	-	5.75
cvxnonsep_psig30r	68	1.02	-	-	-	0.00
cvxnonsep_psig40	6,723,979	limit	113.51	-	-	5.50
cvxnonsep_psig40r	51	1.00	-	-	-	0.00
densitymod	3,028	limit	-	903	-	6.54
eigena2	1	limit	-	-	-	0.00
elec100	1,385	limit	-	-	-	0.00
elec200	1	limit	-	-	-	0.00
elec25	55,810	limit	-	-	-	0.09
elec50	15,723	limit	-	-	-	0.00
elf	134	1.66	-	2	-	0.00
emfl100_5_5	3,147	limit	-	-	-	0.00
ex14_1_5	1	0.01	-	-	-	0.00
ex2_1_3	1	0.15	-	-	-	0.00
ex5_2_5	1	1.03	-	-	-	0.00
ex8_1_6	17	0.13	-	-	-	0.00
ex8_3_1	718,415	limit	23.00	-	-	0.63
ex8_3_11	564,934	limit	30.77	-	-	0.55
ex8_3_13	381,171	limit	16.86	-	-	0.37
ex8_3_2	658,798	limit	42.73	-	-	0.69
ex8_3_3	680,778	limit	39.22	-	-	0.82

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name	#nodes	total-time	gap	#red	#cut off	sym-time
ex8_3_4	606,003	limit	62.01	-	-	0.52
ex8_3_5	650,956	limit	1,496.29	-	-	0.66
ex8_3_7	59	limit	-	-	-	0.00
ex8_3_8	127,806	limit	207.11	-	-	0.09
ex8_3_9	795,980	limit	31.06	-	-	0.80
ex8_4_6	3	limit	-	-	-	0.00
ex8_6_1	1	limit	-	-	-	0.00
ex9_1_8	1	0.00	-	-	-	0.00
ex9_2_2	1	0.14	-	-	-	0.00
ex9_2_6	1	0.11	-	-	-	0.00
gabriel04	6,181	278.60	-	-	-	0.01
gams03	1	limit	-	-	-	0.00
gastrans582_cold13	22	33.63	-	-	-	0.00
gastrans582_cold13_95	35	25.70	-	-	-	0.00
gastrans582_cold17	47	37.60	-	-	-	0.01
gastrans582_cold17_95	115	42.39	-	-	-	0.00
gastrans582_cool12	38	30.20	-	-	-	0.00
gastrans582_cool12_95	30	41.12	-	-	-	0.00
gastrans582_cool14	34	35.40	-	-	-	0.00
gastrans582_cool14_95	26	30.06	-	-	-	0.00
gastrans582_freezing27	0	22.64	-	-	-	0.00
gastrans582_freezing27_95	1	36.87	-	-	-	0.00
gastrans582_freezing30	22	44.22	-	-	-	0.00
gastrans582_freezing30_95	5	31.69	-	-	-	0.00
gastrans582_mild10	17	30.48	-	-	-	0.00
gastrans582_mild10_95	47	29.25	-	-	-	0.00
gastrans582_mild11	63	36.64	-	-	-	0.01
gastrans582_mild11_95	31	37.01	-	-	-	0.00
gastrans582_warm15	1	17.66	-	-	-	0.00
gastrans582_warm15_95	20	28.47	-	-	-	0.00
gastrans582_warm31	35	38.93	-	-	-	0.00
gastrans582_warm31_95	65	31.10	-	-	-	0.00
gear	26,844	18.77	-	-	-	0.01
gear2	25,556	15.62	-	283	-	0.01
gear3	27,844	19.28	-	-	-	0.01
gear4	1,268	0.45	-	-	-	0.00
graphpart_2g-0044-1601	1	0.33	-	-	-	0.00
graphpart_2g-0055-0062	7	1.20	-	3	-	0.00
graphpart_2g-0066-0066	11	2.27	-	3	-	0.00
graphpart_2g-0077-0077	81	5.19	-	2	-	0.00
graphpart_2g-0088-0088	16	6.64	-	4	-	0.00
graphpart_2g-0099-9211	63	11.33	-	4	-	0.00
graphpart_2g-1010-0824	16	10.98	-	5	-	0.00
graphpart_2pm-0044-0044	9	1.18	-	2	-	0.00
graphpart_2pm-0055-0055	13	1.69	-	5	-	0.00
graphpart_2pm-0066-0066	21	2.86	-	4	-	0.00
graphpart_2pm-0077-0777	77	4.62	-	2	-	0.00
graphpart_2pm-0088-0888	30	5.66	-	4	-	0.00
graphpart_2pm-0099-0999	584	16.21	-	10	-	0.00
graphpart_3g-0234-0234	15	1.88	-	2	-	0.00
graphpart_3g-0244-0244	12	4.29	-	4	-	0.00
graphpart_3g-0333-0333	10	3.26	-	3	-	0.00
graphpart_3g-0334-0334	45	5.22	-	2	-	0.00
graphpart_3g-0344-0344	25	5.80	-	3	1	0.00
graphpart_3g-0444-0444	1,661	35.46	-	16	18	0.00
graphpart_3pm-0234-0234	30	2.12	-	6	-	0.00
graphpart_3pm-0244-0244	140	3.52	-	6	-	0.00
graphpart_3pm-0333-0333	415	4.31	-	2	-	0.00
graphpart_3pm-0334-0334	1,531	12.24	-	2	-	0.00
graphpart_3pm-0344-0344	7,016	53.00	-	48	21	0.00
graphpart_3pm-0444-0444	182,761	1,405.42	-	2	-	0.00
graphpart_clique-20	539	6.87	-	5	-	0.00
graphpart_clique-30	3,557	61.92	-	13	-	0.00
graphpart_clique-40	20,809	441.39	-	16	1	0.00
graphpart_clique-50	38,394	limit	102.13	19	-	0.00
graphpart_clique-60	20,962	limit	345.64	5	-	0.00
graphpart_clique-70	13,878	limit	216.05	20	-	0.00
hadamard_4	79	0.46	-	21	-	0.00

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name	#nodes	total-time	gap	#red	#cutoff	sym-time
hadamard_5	35,843	55.78	-	995	-	0.08
hadamard_6	127,575	limit	811.11	504	-	0.66
hadamard_7	3,601	limit	63,833.33	82	-	0.15
hadamard_8	14	limit	-	63	-	0.89
hmittelman	1	0.02	-	-	-	0.00
ivalues	72,237	limit	409.52	-	-	0.10
kissing2	1	908.01	-	-	-	0.00
kn3-12	123,815	limit	197.28	-	-	0.08
kn4-24	5,991	limit	485.79	-	-	0.00
kn5-40	1,885	limit	770.96	-	-	0.00
kn5-41	2,185	limit	811.31	-	-	0.00
kn5-42	2,139	limit	807.71	-	-	0.00
kn5-43	2,367	limit	765.37	-	-	0.00
kn5-44	841	limit	763.42	-	-	0.00
kport20	153,400	714.60	-	-	-	0.37
lop97icx	2,083	24.23	-	-	-	0.01
maxcsp-ehi-85-297-12	261	limit	-	-	-	0.00
maxcsp-ehi-85-297-36	296	limit	-	-	-	0.00
maxcsp-ehi-85-297-71	257	limit	-	-	-	0.00
maxcsp-ehi-90-315-70	200	limit	-	-	-	0.00
maxcsp-langford-3-11	27,640	limit	-	4	-	0.00
maxmin	106,817	limit	73.36	-	-	0.13
mbtd	1,647	limit	156.25	24	-	0.52
netmod_doll	4,197	limit	44.61	1,541	2	0.79
netmod_karl	1,143	54.60	-	147	11	0.01
netmod_karl2	1,143	54.96	-	147	11	0.02
nuclear14	30	limit	-	-	-	0.01
nuclear25	15	limit	-	-	-	0.00
nuclear49	21	limit	-	-	-	0.03
nuclearva	22	limit	-	-	-	0.00
nuclearvb	111	limit	-	-	-	0.01
nuclearvc	269	limit	-	4	-	0.01
nuclearvd	159	limit	-	7	-	0.00
nuclearve	59	limit	-	-	-	0.00
nuclearvf	66	limit	-	-	-	0.00
nvs09	614	1.48	-	-	-	0.00
oil2	1,697	94.88	-	-	-	0.00
orth_d4m6_pl	433,001	limit	168.52	-	-	0.48
pooling_bental5pq	1	0.15	-	-	-	0.00
pooling_bental5tp	40	2.13	-	-	-	0.00
pooling_bental5tp	1	0.13	-	-	-	0.00
powerflow2383wpp	1	limit	-	-	-	0.00
powerflow2383wpr	1	limit	-	-	-	0.00
procurement1large	93,263	limit	375.15	-	-	1.21
procurement1mot	850,260	limit	473.05	-	-	3.03
procurement2mot	167	4.43	-	-	-	0.02
product	990	88.44	-	-	-	0.01
product2	1	14.23	-	-	-	0.00
radar-2000-10-a-6_lat_7	707	721.64	-	8,259	-	0.00
radar-3000-10-a-8_lat_7	7,351	limit	1,621.70	275,171	-	0.00
ringpack_10_2	116,925	limit	3.95	4	863	0.32
ringpack_20_2	7,490	limit	233.33	1	1	0.03
ringpack_20_3	37,878	limit	82.91	6,433	-	0.49
ringpack_30_2	9,906	limit	1,400.00	1	2	0.17
rsyn0815m	151	1.02	-	-	-	0.02
rsyn0815m02m	1,190	11.65	-	-	-	0.01
rsyn0815m03m	1,949	22.50	-	-	-	0.01
rsyn0815m04m	6,634	76.82	-	-	-	0.10
sepasequ_complex	61,795	limit	63.92	-	-	0.36
sporttournament10	1	0.72	-	-	-	0.00
st_e09	1	0.02	-	-	-	0.00
st_e18	1	0.02	-	-	-	0.00
st_qpc-m3c	1	0.02	-	-	-	0.00
st_rv9	1,791	3.32	-	-	-	0.01
syn15m	1	0.32	-	-	-	0.00
t1000	0	0.20	-	-	-	0.00
tln2	1	0.01	-	-	-	0.00
topopt-zhou-rozvany_75	464	limit	-	-	-	0.00

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name	#nodes	total-time	gap	#red	#cutoff	sym-time
torsion100	20	limit	380.39	-	-	0.00
torsion25	1	limit	319.02	-	-	0.00
torsion50	16	limit	373.99	-	-	0.00
turkey	1	19.85	-	-	-	0.00
util	14	0.23	-	1	-	0.00
waste	81,999	limit	91.24	-	-	0.30
watercontamination0202	19	107.19	-	-	-	0.00
waterund28	10,017	limit	7.76	-	-	0.00