Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

KLAUS EHRHARDT AND MARC C. STEINBACH

Nonlinear Optimization in Gas Networks

ZIB-Report 03-46 (December 2003)

NONLINEAR OPTIMIZATION IN GAS NETWORKS

KLAUS EHRHARDT AND MARC C. STEINBACH

ABSTRACT. The operative planning problem in natural gas distribution networks is addressed. An optimization model focusing on the governing PDE and other nonlinear aspects is presented together with a suitable discretization for transient optimization in large networks by SQP methods. Computational results for a range of related dynamic test problems demonstrate the viability of the approach.

1. INTRODUCTION

Natural gas is a primary energy source of increasing relevance, mainly due to its favorable environmental properties. In Germany, gas is primarily used for heating, industries, and power generation. Our industry partner Ruhrgas operates the largest gas network in Germany, a combined transport and distribution network with roughly 11000 km of pipes and 26 compressor stations requiring up to 831 MW. The annual gas demand in 2002 was 612000 GWh, with daily demands varying between 773 and 3109 GWh.

The current paper is concerned with operative planning, or transient technical optimization (TTO), in gas networks. This planning level addresses the task of controlling the network load distribution over the next 24 to 48 hours to satisfy the actual demand subject to physical, technical, and contractual constraints as well as target values for gas production, storage, purchase, and sale determined by the mid-term planning. The objective is to minimize the variable operating costs, which are dominated by the cost for the gas transport, i.e., the fuel consumption of compressors.

Due to reliable temperature forecasts we can neglect demand uncertainty and hence use a deterministic model, but the operative planning problem involves PDE constraints (gas flow) as well as substantial combinatorial aspects (switching compressors on or off, opening or closing valves), leading to a currently intractable mixed-integer PDE boundary value problem. Here we focus on the nonlinear aspects, assuming that combinatorial decisions are externally given—ideally by an enclosing mixed-integer optimization framework.

Typical subjects of the earlier literature are *steady-state* optimization in tree-structured networks by dynamic programming [17], later surveyed in [2], sequential linearization for nonlinear mixed-integer models on general network topologies [11], optimization of single compressor stations by simulated annealing [18], or evaluation criteria for mixed-integer methods and lower bounds on the total energy consumption [1]. For *transient* network optimization with given binary decisions, a gradient method is proposed in [14] based on a highly detailed model [8] that is used in the commercial simulation tool SIMONE; an extended simplex method for a largely simplified model is developed in [16]. The related problem of controlling networks of open water channels is studied in [5]. First approaches for the *mixed-integer TTO* problem, with rather coarse approximations of nonlinearities, are developed in [13] and later in [6]. To address the full TTO problem, our own work aims at a future integration with related mixed-integer approaches that are currently being developed in a partner project [9]. A second partner project studies stochastic aspects [7].

2000 *Mathematics Subject Classification.* 93C20, 90C06, 90C30, 90C35, 93C95.

Key words and phrases. Gas networks, operative planning, large-scale optimization.

This work has been supported by the Federal Ministry of Education and Science (BMBF) under grant 03STM5B4.

2. Optimization Model

In the following we first describe physical and technical restrictions and the cost function of the TTO problem with prescribed binary decisions. Then we present appropriate space and time discretizations and add contractual restrictions to obtain a large-scale nonlinear optimization problem (NLP).

2.1. **Planning Horizon.** At Ruhrgas, the end of a *gas day* at 6:00 a.m. is a natural choice for the end of the horizon since the state of the network at this time is well predictable. A reoptimization is to be performed every hour. Therefore we operate with a *breathing horizon* of length 24 to 48 hours that starts at the next full hour and ends after the following gas day. In the current paper we are only concerned with a single optimization problem and denote time by $t \in [0, t_e]$.

2.2. **Pipes.** Consider a pipe segment with circular cross section. Let D denote the diameter and L the length. Independent variables are the distance $x \in [0, L]$ and time t; relevant dynamic variables are the gas density ρ, velocity v, flow rate $q = \rho v$, pressure p, and temperature T (all depending on x and t). The altitude of the pipe at x is denoted as $h(x)$. The following material is based on [12, 15].

The gas flow in the pipe is governed by thermodynamic conservation laws. Conservation of mass (continuity equation) and of momentum (pressure loss equation) form a hyperbolic PDE system that is coupled with the equation of state for a real gas. Under a given temperature field $T(x, t)$, the latter replaces an (impractical) PDE modeling heat exchange with the ground (cf. [15]),

$$
\begin{aligned}
\partial_t \rho + \partial_x q &= 0 , && (1)\\
\partial_t q + \partial_x p + \partial_x(\rho v^2) + g\rho\partial_x h &= -\frac{\lambda(q)}{2D}\rho v|v| , && (2)\\
p &= \gamma(T) z(p, T)\rho . && (3)
\end{aligned}
$$

Here γ is a constant field given by T, and the compressibility $z(p, T)$ describes the deviation of the real gas from ideal behavior ($z = 1$). For pressures up to 70 bar, the latter is empirically modeled according to the formula of the American Gas Association (AGA),

$$
z(p, T) = 1 + 0.257(p/p_c) - 0.533\frac{(p/p_c)}{(T/T_c)} , \tag{4}
$$

where p_c and T_c denote the pseudo-critical pressure and temperature of the gas mixture, respectively.

For the usual turbulent flow in gas pipes, the friction coefficient $\lambda(q)$ in (2) is empirically modeled by the implicit formula of Prandtl–Colebrook,

$$
\frac{1}{\sqrt{\lambda(q)}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}(q)\sqrt{\lambda(q)}} + \frac{k}{3.71D}\right) . \tag{5}
$$

Here k is the pipe rugosity (the roughness of its inner surface) and, with η denoting the dynamic viscosity of the gas, $\mathrm{Re}(q)$ is the Reynolds number,

$$
\mathrm{Re}(q) = \frac{D}{\eta}q \approx 10^6 . \tag{6}
$$

In what follows we scale (1)–(3) by the cross-sectional pipe area, $A = \frac{\pi}{4}D^2$, or by $1/A$. Without changing notation, we replace the density with the linear density, $\rho \leftarrow A\rho$ (kg/m), the flow rate with the mass flow, $q \leftarrow Aq$ (kg/s), and $\gamma \leftarrow \gamma/A$, leaving (1) and (3) unchanged. Equation (2) is then simplified by dropping the kinetic energy terms $\partial_t q$ and $\partial_x(\rho v^2)$ (which contribute less than one percent to the sum of all terms under normal

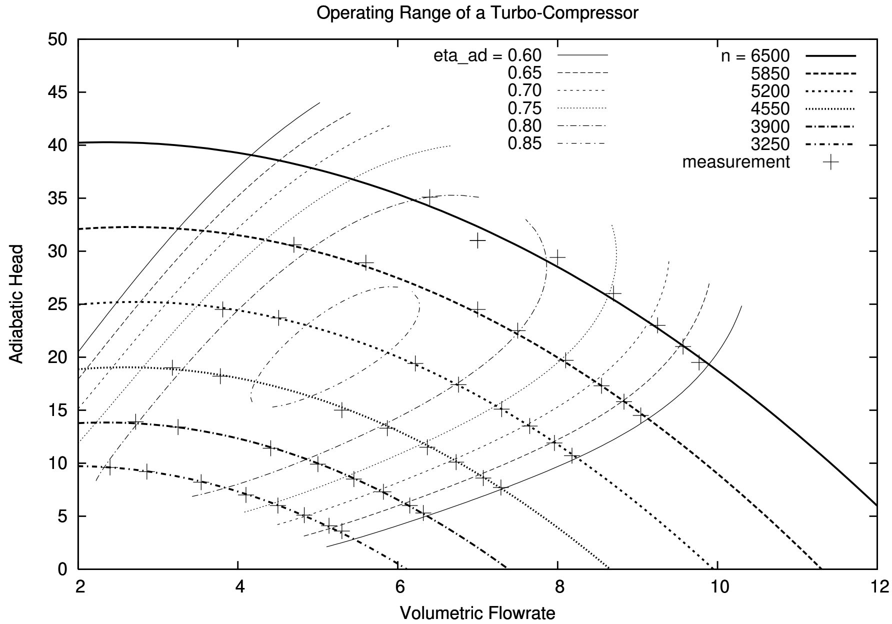

FIGURE 1. Characteristic diagram of a turbo-compressor with measurements (+) and isolines for speed (n, *thick lines*) and adiabatic efficiency (eta_ad, *thin lines*)

operating conditions; see [15]) and substituting $v = q/\rho$ in the hydraulic resistance term,

$$A\partial_x p + g\rho\partial_x h = -\frac{\lambda(q)}{2D}\frac{q|q|}{\rho} \,. \tag{7}$$

2.3. **Compressors.** The physical process of increasing the pressure of a gas flow q from p_{in} to p_{out} (under adiabatic conditions) requires the theoretical power

$$N_{th} = c_1 q H_{ad} = c_1 q \; c_2 z(p_{in}, T_{in}) T_{in} \frac{\kappa}{\kappa - 1}\left[\left(\frac{p_{out}}{p_{in}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] \,. \tag{8}$$

Here c_1, c_2 are constants, H_{ad} denotes the adiabatic head, and κ denotes the isentropic exponent of the gas.

In the following we concentrate on turbo-compressors (also called centrifugal compressors) driven by gas turbines. The technical behavior of this most common compressor type is described by a characteristic diagram as shown in Fig. 1. The speed n and the adiabatic efficiency η_{ad} of the compressor can both be modeled as bivariate quadratic polynomials in q and H_{ad} obtained as fits to discrete measurements. The fuel consumption per time unit then reads

$$B = c_3 N b(n, N) \tag{9}$$

where N is the actual power consumption of the compressor, $N = N_{th}/\eta_{ad}$, and b is its (empirical) specific fuel consumption. The fuel B is taken directly from the gas flow; it amounts to roughly 0.5% of the compressor throughput.

For our purposes a simplified model is sufficient. We consider entire compressor stations (typically consisting of about half a dozen compressor units) as a single *idealized* compressor satisfying (8) and having constant efficiency and specific fuel consumption, so that $B = \tilde{c}_3 N_{th}$ with $\tilde{c}_3 = c_3 b/\eta_{ad}$.

2.4. **Further Elements.** Real gas networks contain further active and passive elements of which the following are relevant in our context.

Connections: are short pipes with a constant relative pressure loss $c \in (0, 1]$ and with identical input and output flows,

$$p_{\text{out}} = c p_{\text{in}} \ , \quad q_{\text{out}} = q_{\text{in}} \ . \tag{10}$$

Valves: can be open or closed, with corresponding pressure and flow relations

$$p_{\text{out}} = p_{\text{in}} \ , \quad q_{\text{out}} = q_{\text{in}} \quad (\text{open}) \ , \quad q_{\text{in}} = q_{\text{out}} = 0 \quad (\text{closed}) \ . \tag{11}$$

Regulators (or **control valves**): are valves that reduce the gas pressure by a controlled amount $\Delta p \in [\Delta p_{\min}, \Delta p_{\max}]$ when open,

$$p_{\text{out}} = p_{\text{in}} - \Delta p \ , \quad q_{\text{out}} = q_{\text{in}} \quad (\text{open}) \ , \quad q_{\text{in}} = q_{\text{out}} = 0 \quad (\text{closed}) \ . \tag{12}$$

2.5. **Network.** The network topology is modeled by a directed graph $G = (\mathcal{N}, \mathcal{A})$ whose vertex set consists of provider nodes $\mathcal{N}_+$ (sources), customer nodes $\mathcal{N}_-$ (sinks), and interior nodes $\mathcal{N}_0$ (junctions),

$$\mathcal{N} = \mathcal{N}_+ \cup \mathcal{N}_- \cup \mathcal{N}_0 \ . \tag{13}$$

The set of arcs consists of the network elements described above: pipes $\mathcal{A}_{\text{pi}}$, connections $\mathcal{A}_{\text{cn}}$, compressors $\mathcal{A}_{\text{cs}}$, valves $\mathcal{A}_{\text{vl}}$, and regulators $\mathcal{A}_{\text{rg}}$,

$$\mathcal{A} = \underbrace{\mathcal{A}_{\text{pi}} \cup \mathcal{A}_{\text{cn}}}_{\text{passive}} \cup \underbrace{\mathcal{A}_{\text{cs}} \cup \mathcal{A}_{\text{vl}} \cup \mathcal{A}_{\text{rg}}}_{\text{active (controlled)}} \ . \tag{14}$$

Individual arcs will be denoted as $a \in \mathcal{A}$ or, using the tail and head $i, j \in \mathcal{N}$, as $ij \in \mathcal{A}$. Below we always require that the flow is directed from i to j.

2.6. **Objective.** The objective consists in minimizing the variable operating costs, that is, the total fuel costs of all compressors,

$$\text{cost}_{\text{cs}} = \sum_{a \in \mathcal{A}_{\text{cs}}} c_a \int_0^{t_e} B_a(t) \, dt \ . \tag{15}$$

2.7. **Terminal constraint.** Since the operative planning problem has a finite horizon, some terminal constraint is required to ensure reasonable operating conditions for $t > t_e$. Otherwise the optimization would yield abnormally low values of pressure and network gas content at $t = t_e$ to reduce compressor duties and thus fuel costs.

A straightforward approach to prevent such behavior consists in placing a lower bound on the total gas mass at the end of the horizon (where we neglect the small amount of gas in non-pipe elements),

$$m_{\min} \le \sum_{a \in \mathcal{A}_{\text{pi}}} \int_0^{L_a} \rho_a(x, t_e) \, dx \ . \tag{16}$$

2.8. **Discretization.** A suitable discretization for transient optimization of large networks should be coarse but reliable (i.e., at least mass-conserving), and stable. We use a full a priori discretization in space and time. Only pipes are relevant for the space discretization; all other arc types are assumed to have zero length. For each pipe we use the "grid" consisting of both end points only, $\Gamma_a = \{0, L_a\}$, $a \in \mathcal{A}_{\text{pi}}$. (A physical pipe may consist of arbitrarily many arcs $a \in \mathcal{A}_{\text{pi}}$.) The planning horizon $I = [0, t_e]$ is divided equidistantly to yield a time grid $\Gamma_I = \{0, 1, 2, \dots, t_e\}$. (With no loss of generality the length of subintervals is chosen as time unit here, $\Delta t = 1$.) The variable vector is denoted

$$x = (x_0, \dots, x_{t_e}) \quad \text{with} \quad x_t = (p_t, q_t, s_t, u_t) \ , \quad t = 0, \dots, t_e \ . \tag{17}$$

Here the states at time t consist of the node pressures $p_t = (p_{it})_{i \in \mathcal{N}}$, the arc flows $q_t = (q_{it}, q_{jt})_{ij \in \mathcal{A}}$, and further states in pipes and compressors, $s_t = (s_{at})_{a \in \mathcal{A}}$, while the (piecewise constant) controls on subinterval $(t-1,t)$ are the pressure changes $u_t = (u_{at})_{a\in\mathcal{A}}$ in compressors and regulators:

$$\begin{array}{llll} s_{ijt} = \rho_{jt}\ , & ij \in \mathcal{A}_{\text{pi}}\ , & u_{at} = \Delta p_{at}\ , & a \in \mathcal{A}_{\text{cs}}\ , \\ s_{at} = N_{at}\ , & a \in \mathcal{A}_{\text{cs}}\ , & u_{at} = \Delta p_{at}\ , & a \in \mathcal{A}_{\text{rg}}\ . \end{array} \tag{18}$$

In all other arc types, s_{at} and u_{at} are empty.

The PDE and equation of state are discretized with implicit Euler schemes in space and time, yielding for $a = ij \in \mathcal{A}_{\text{pi}}$, $t \in \{1,\dots,t_e\}$, and $t_- = t-1$:

$$\frac{\rho_{jt}-\rho_{jt_-}}{\Delta t} + \frac{q_{jt}-q_{it}}{L_a} = 0\ , \tag{19}$$

$$A_a\frac{p_{jt}-p_{it}}{L_a} + g\rho_{jt}\frac{h_j-h_i}{L_a} + \frac{\lambda(q_{jt})}{2D_a}\frac{q_{jt}^2}{\rho_{jt}} = 0\ , \tag{20}$$

$$p_{jt} - \gamma(T_{jt})z(p_{jt},T_{jt})\rho_{jt} = 0\ . \tag{21}$$

The simplified fuel consumption model for compressor $a = ij \in \mathcal{A}_{\text{cs}}$ reads

$$B_{at} = \tilde{c}_{3a}N_{at} = c_{1a}c_{2a}\tilde{c}_{3a}q_{jt}z(p_{it},T_{it})T_{it}\frac{\kappa}{\kappa-1}\left[\left(\frac{p_{jt}}{p_{it}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]\ . \tag{22}$$

The integrals in the objective and in the terminal constraint are approximated by the trapezoidal rule with respect to the grids Γ_a and Γ_I, respectively,

$$\text{cost}_{\text{cs}} = \sum_{a\in\mathcal{A}_{\text{cs}}} c_a \sum_{t=1}^{t_e}\frac{B_{at_-}+B_{at}}{2}\Delta t\ , \quad m_{\min} \le \sum_{ij\in\mathcal{A}_{\text{pi}}} L_{ij}\frac{\rho_{it_e}+\rho_{jt_e}}{2}\ . \tag{23}$$

This completes the nontrivial arc equations, which are coupled by flow balance equations in every internal node $j \in \mathcal{N}_0$ for $t \in \{1,\dots,t_e\}$,

$$\sum_{i:\, ij\in\mathcal{A}} q_{ijt} = \sum_{k:\, jk\in\mathcal{A}} q_{jkt}\ . \tag{24}$$

Finally there are initial conditions $x_0 = \hat{x}_0$ and simple bounds on all variables, where contractual constraints are specified as hourly profiles for the pressure bounds in terminal nodes ($i \in \mathcal{N}_+ \cup \mathcal{N}_-$) and for the flow bounds in provider and customer arcs ($ij \in \mathcal{A} : i \in \mathcal{N}_+$ or $j \in \mathcal{N}_-$).

3. Computational Results

All computations are performed on a single 2.4 GHz Pentium IV processor of a dual processor Linux PC workstation with 4 GB RAM. The optimization model is implemented in Fortran 77 and compiled with the GNU g77 compiler.

In the results that follow, the large-scale NLP resulting from the discretization are solved by the general-purpose nonlinear optimization code SNOPT [4] in combination with the automatic differentiation add-on SnadiOpt [3].

3.1. **Test Problem.** The test network in Fig. 2 has three compressors (Cs A–C), one valve (Vl 1), and one regulator (Rg 1) to supply gas from two providers (P 1–2) to three customers (C 1–3). Pipe segments vary from 0.5–1.1 m in diameter and 50–120 km in length, with a total length of 920 km. Provider P 1 supplies roughly 3–8 times as much gas as P 2, while the main customer C 3 demands roughly 30 times as much gas as each of the other two customers. We assume constant demand profiles for C 1, C 2, and a sinusoidal daily demand profile for C 3. The planning horizon is 48 hours, with subintervals of one hour (a natural choice because of contractual profiles). The length of each pipeline segment $a \in \mathcal{A}_{\text{pi}}$ is $L_a = 10$ km. The NLP has 21168 states, 192 controls, and 21170 constraints. Using stationary solutions or optimal transient solutions of close-by scenarios as initial estimates,

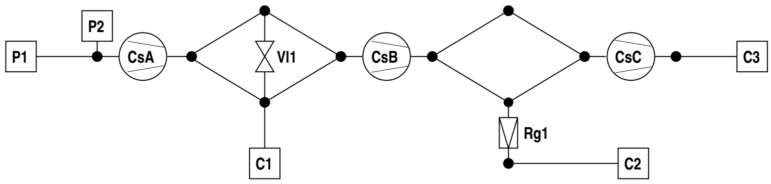

FIGURE 2. Flowchart of the test network

SNOPT takes 17–49 major iterations and typically 4–10 minutes (up to 25 minutes in rare cases) for a tolerance of 10^{-6}.

We consider three test cases, each with two scenarios differing in the binary decisions. In scenario one, all three compressors are working all the time and the valve is open all the time. Scenario two differs only in that the second compressor (Cs B) is switched off during 08:00–12:00. In case one, the lower bound on all compressor powers is zero, which leads to solutions where compressor Cs B occasionally operates below its technical limit, see Fig. 3a/b. This shows that the combinatorial nature of the on/off decision must not be ignored. Case two poses realistic positive lower limits on all compressor powers, yielding the results in Fig. 3c/d. In case three, finally, the altitude increases from sea level to 600 m between P 1 and C 3. Results are shown in Fig. 3e/f.

3.2. **Discussion.** In case one, all pressure profiles behave approximately periodically, following the biggest customer's demand with different phase shifts. The main burden lies on the second compressor (Cs B) which yields an average pressure increase of 11.6 bar, whereas Cs A and Cs C yield slightly less than 9.6 bar on average. The regulator produces a constant pressure drop of 10 bar (the upper bound). Typical end effects are observable during 42:00–48:00 where the control profiles deviate from the periodical behavior, and at the very beginning where the network leaves its initial stationary state.

When compressor Cs B is switched off during 08:00–12:00 in scenario two, substantial changes in all control profiles are observed roughly during 02:00–18:00 (six hours before and after the interruption), but otherwise the optimal solution is almost identical to scenario one. This shows that perturbations during limited periods of time yield control responses whose magnitude decays with the distance from the perturbation period, an effect which justifies to neglect end effects if the planning horizon is sufficiently long.

Altogether, optimal solutions in case one exhibit precisely the qualitative behavior that one would expect—under the counterfactual assumption, however, that a compressor can produce any nonnegative pressure increase.

Case two shows the effect of placing realistic lower limits on the power consumption of each compressor, resulting in approximate minimal pressure increases of 8 bar for compressor Cs A and 5 bar for Cs B, Cs C. The periodic behavior of the control profiles prevails, but their amplitudes become smaller and the duty is more evenly distributed among the three compressors. Moreover, the end effects become less accentuated. Switching off the second compressor leads again to an optimal solution that is almost identical except during the six hours before and after the interruption.

Increasing the altitude along the network in case three, finally, yields optimal solutions that are very similar to case two except that the peak pressures produced by each compressor are larger than before.

4. SUMMARY

We have presented a nonlinear optimization model and a suitable discretization for minimum-cost operative planning under prescribed binary decisions in natural gas distribution networks. Computational results obtained with an SQP method demonstrate the viability of this approach.

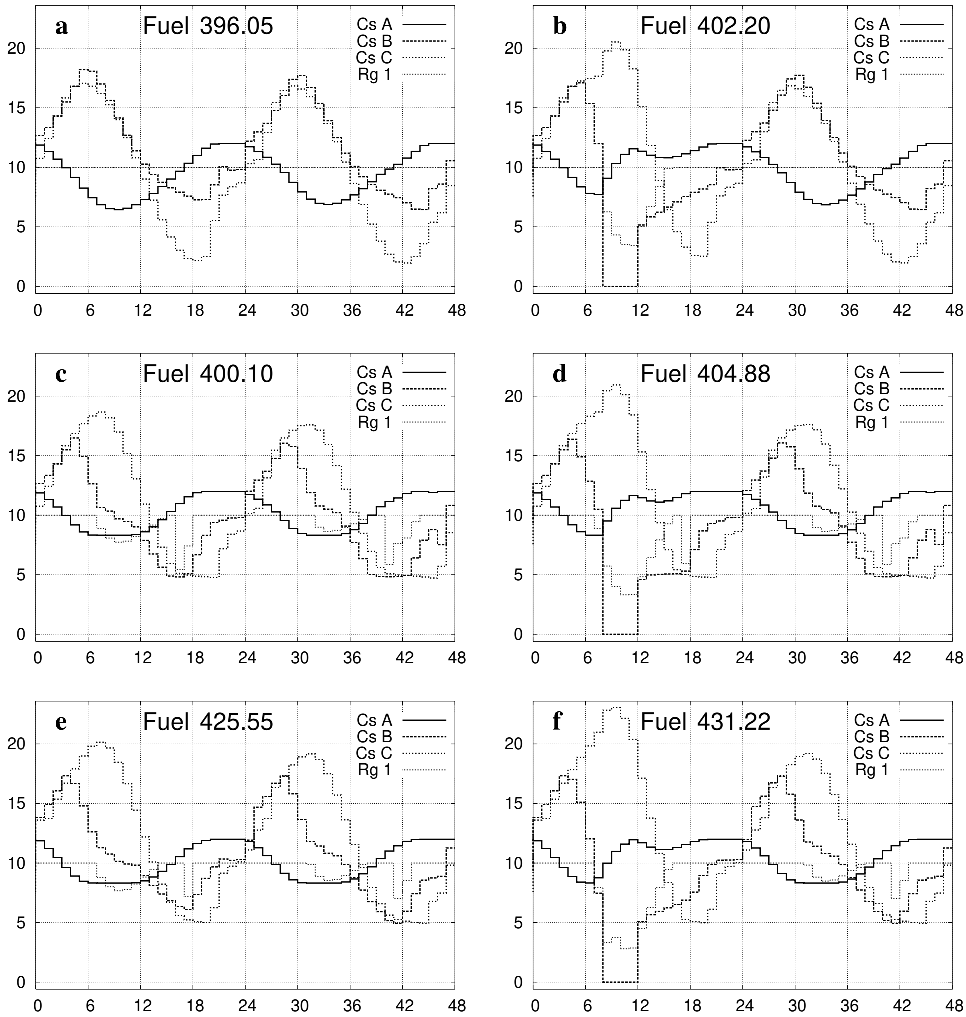

FIGURE 3. Optimal pressure changes (bar) versus time (h) and optimal cumulative fuel consumption (1000 kg) for six related test problems

ACKNOWLEDGMENTS

We would like to express our sincere thanks to Ruhrgas AG (Essen) and PSI AG (Berlin) for their close cooperation, especially on the practical aspects of the project. Ruhrgas also supplied the measurement data for Fig. 1. We are also indebted to P. E. Gill and E. M. Gertz for free academic licenses of their software packages SNOPT and SnadiOpt, respectively.

REFERENCES

[1] E. A. BOYD, L. R. SCOTT, AND S. WU, *Evaluating the quality of pipeline optimization algorithms*, in 29th Annual Meeting, Pipeline Simulation Interest Group, 1997. Paper 9709.

[2] R. G. CARTER, *Pipeline optimization: Dynamic Programming after 30 years*, in PSIG [10]. Paper 9803.

[3] E. M. GERTZ, P. E. GILL, AND J. MUETHERING, *Users guide for SnadiOpt: a package adding automatic differentiation to SNOPT*, Technical Memorandum ANL/MCS-TM-245, Argonne National Labs, Jan. 2001.

[4] P. E. GILL, W. MURRAY, AND M. S. SAUNDERS, *SNOPT: An SQP algorithm for large-scale constrained optimization*, SIAM J. Optim., 12 (2002), pp. 979–1006.

[5] M. GUGAT, G. LEUGERING, K. SCHITTKOWSKI, AND E. J. P. G. SCHMIDT, *Modelling, stabilization, and control of flow in networks of open channels*, in Online Optimization of Large Scale Systems, M. Grötschel, S. O. Krumke, and J. Rambau, eds., Springer, Berlin, 2001, pp. 251–270.

[6] P. HACKLÄNDER, *Integrierte Betriebsplanung von Gasversorgungssystemen*, Verlag Mainz, Wissenschaftsverlag, Aachen, 2002.
[7] T. HEIDENREICH, *Linearisierung in transienten Optimierungsmodellen des Gastransports*, Diploma thesis, Universität Duisburg, 2002.
[8] J. KRÁLIK, P. STIEGLER, Z. VOSTRÝ, AND J. ZÁVORKA, *A universal dynamic simulation model of gas pipeline networks*, IEEE Trans. Syst., Man, Cybern., SMC-14 (1984), pp. 597–606.
[9] A. MARTIN AND M. MÖLLER, *Cutting planes for the optimization of gas networks*, Preprint 2253, Fachbereich Mathematik, Technische Universität Darmstadt, 2002.
[10] PIPELINE SIMULATION INTEREST GROUP, *PSIG 30th Annual Meeting, Denver, Colorado*, 1998.
[11] K. F. PRATT AND J. G. WILSON, *Optimization of the operation of gas transmission systems*, Transactions of the Institute of Measurement and Control, 6 (1984), pp. 261–269.
[12] D. RIST, *Dynamik realer Gase*, Springer, Berlin, 1996.
[13] E. SEKIRNJAK, *Mixed integer optimization for gas transmission and distribution systems*. Vortragsmanuskript, INFORMS-Meeting, Seattle, Oct. 1998.
[14] Z. VOSTRÝ, *Transient optimization of gas transport and distribution*, in Proceedings of 2nd International Workshop SIMONE on Innovative Approaches to Modeling and Optimal Control of Large Scale Pipeline Networks, Prague, 1993, pp. 53–62.
[15] J. F. WILKINSON, D. V. HOLLIDAY, E. H. BATEY, AND K. W. HANNAH, *Transient Flow in Natural Gas Transmission Systems*, American Gas Association, 1964.
[16] D. D. WOLF AND Y. SMEERS, *The gas transmission problem solved by an extension of the simplex algorithm*, Management Sci., 46 (2000), pp. 1454–1465.
[17] P. J. WONG AND R. E. LARSON, *Optimization of tree-structured natural-gas transmission networks*, J. Math. Anal. Appl., 24 (1968), pp. 613–626.
[18] S. WRIGHT, M. SOMANI, AND C. DITZEL, *Compressor station optimization*, in PSIG [10]. Paper 9805.

KLAUS EHRHARDT, KONRAD-ZUSE-ZENTRUM FÜR INFORMATIONSTECHNIK BERLIN, DEPARTMENT OPTIMIZATION, TAKUSTR. 7, 14195 BERLIN-DAHLEM, GERMANY
E-mail address: `ehrhardt@zib.de`

MARC C. STEINBACH, KONRAD-ZUSE-ZENTRUM FÜR INFORMATIONSTECHNIK BERLIN, DEPARTMENT OPTIMIZATION, TAKUSTR. 7, 14195 BERLIN-DAHLEM, GERMANY
E-mail address: `steinbach@zib.de`
URL: `http://www.zib.de/steinbach`