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Multi-Period Line Planning with Resource Transfers

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Abstract

Urban transportation systems are subject to a high level of variation and fluctuation in demand over the day. When this variation and fluctuation are observed in both time and space, it is crucial to develop line plans that are responsive to demand. A multi-period line planning approach that considers a changing demand during the planning horizon is proposed. If such systems are also subject to limitations of resources, a dynamic transfer of resources from one line to another throughout the planning horizon should also be considered. A mathematical modelling framework is developed to solve the line planning problem with transfer of resources during a finite length planning horizon of multiple periods. We analyze whether or not multi-period solutions outperform single period solutions in terms of feasibility and relevant costs. The importance of demand variation on multi-period solutions is investigated. We evaluate the impact of resource transfer constraints on the effectiveness of solutions. We also study the effect of line type designs and question the choice of period lengths along with the problem parameters that are significant for and sensitive to the optimality of solutions.

Keywords: line planning; multi-period planning; public transportation; urban transportation; mathematical programming; integer programming

1 Introduction

Along with the rapid growth in demand for urban transportation systems, researchers introduce various mathematical models for planning purposes. Public transportation

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planning is composed of five distinct stages including network design for infrastructure, line planning, timetabling, vehicle scheduling (rolling stock planning), and crew scheduling [9].

The *line planning problem* (LPP) is a long-established problem in the context of public transportation systems (see Schöbel [20] for a survey). LPP is solved on a *public transportation network* (PTN) which is composed of several stations and direct connections between stations. A line is a designated service by a vehicle on a path on the PTN. The line services are provided to transport passengers who demand to travel between a pair of stations (an origin and a destination). Based on a given PTN and a travel demand, the LPP seeks to find a set of lines together with their frequencies.

Demand for transportation is the *sine qua non* of the line planning problem; simply put, there is no necessity for transport services when there is no demand. The common models for line planning consider a finite length planning horizon, e.g., a day, a certain part of the day, an hour. Accordingly, the demand during that planning horizon is considered irrespective of its timing; the problem is to find the line services and their frequencies to satisfy the demand in a steady state manner during the planning horizon. In other words, a static demand rate is assumed for the complete planning horizon. A comprehensive review on the line planning problem by Schöbel [20] concludes by questioning the appropriateness of using the same line plan all over the day. One could easily unfold this question to assess the degree of fluctuation and variation in demand that requires the use of a nonsteady handling. Borndörfer et al. [5] note that the demand of the Istanbul Metrobüs system is extremely unsteady and asymmetric. They show that traditional line planning models are not convincing for these public transportation networks and discuss the lack of a modelling approach which adapts to the demand fluctuation during the planning horizon.

For a decision-making problem, when the problem environment or an aspect of the problem is not necessarily steady in time, multi-period planning and optimization arises as a remedy in the operations research literature. The use of multi-period modelling approaches has extended from production-and-inventory planning to unit commitment problems in electricity generation and from forestry planning to problems of routing and scheduling over time as thoroughly reviewed by Schrage [23]. In a multi-period planning problem, the finite-length planning horizon is dissected into (usually equal-length) periods as opposed to considering the planning horizon as a single-period during which parameters are stable and the problem environment is steady. Depending on the type and context of the problem, the planning horizon may be a day or tens of years while the period length may vary from minutes to a year (or a couple of years).

While a multi-period planning approach is a remedy for the non-steady aspects of the problem, it convolutes the problem by bringing in essential inter-period constraints that couple the underlying single-period problems with each other. If such constraints did not exist, the problem would be decomposable and each single-period problem would be solved separately. For example, the inventory balance constraints in a multi-period production planning problem relate the ending inventory level of one period to the beginning inventory

level of the next period. A multi-period facility location problem has to account for the existence of facilities throughout the periods according to closing and opening decisions. In a similar but more complicated fashion, daily unit commitment problems of electricity generators are ruled by the start-up and shut-down decisions as operations associated with either of these decisions may require a couple of hours to take effect during the day. As such, the interperiod constraints may be necessary to ensure the availability of resources throughout the periods and their allocation among the many activities and operations associated with the process. The best-known example to this would be simultaneous multi-period workforce and production planning problems.

In response to shortcomings of the sequential planning approach with the five stages of urban transportation system planning, various attempts, both exact and heuristic, have been made to integrate, in particular, two subsequent stages of the intermediate three (line planning, timetabling, and vehicle scheduling). Schöbel [21] is the first study to discuss an integration of these three stages by providing a general framework for integration and also summarizes the studies on integration of two subsequent stages in the literature. It is widely accepted that sequential planning leads to suboptimal solutions. As a matter of fact, not considering the availability of resources at later stages of planning in the earlier stages may even lead to infeasible solutions due to insufficiency of resources. This would be even more critical when line plans are not bounded by the (vehicle) fleet resources and optimal timetables cannot be realized due to insufficient vehicles ([8]).

Recognizing the connection between inter-period resource constraints in multi-period planning and integration of line planning with planning of resources associated with later stages of planning, we take on both challenges in the context of the line planning problem in urban transportation systems by

- proposing a multi-period planning approach in response to high levels of fluctuation and variation in demand, and
- integrating the multi-period strategic-level line planning decision with resource constraints that ensure the availability and allocation of operational resources throughout the periods.

In the case of line planning problems, an explicit consideration of changing demand in a multi-period setting is attempted for the first time. In the sequential planning approach, resources are considered in the last two stages. The consideration of resource availabilities in multi-period version of the problems in the first three stages calls for an integrated solution of these problems, which is a major open challenge in this area of research.

We lay the foundations of a multi-period line planning approach by introducing a multi-period line planning model in the form of an integer linear programming problem formulation. We extend this formulation with a consideration of resource allocation and resource transfer constraints in a multi-period setting and exemplify it with rotations of vehicles among lines. We present computational results to exhibit the value of and

understand conditions that call for a multi-period approach in line planning. We also investigate the sensitivity of system characteristics and pose the period length determination as an inherent optimization problem.

In Section 2, we provide a review of the LPP and its integration with other planning stages. Mathematical models for multi period line planning with resource allocation and transfers are presented in Section 3. In Section 4, we provide a comprehensive computational study of a multi-period line planning model for the case of the Istanbul Metrobüs system and exemplify resource transfer constraints with rotation of vehicles.

2 Literature Review

The LPP has been studied extensively in the literature with a variety of approaches. Schöbel [20] classifies the LPP models as passenger-oriented and cost-oriented according to their modeling approaches:

- In the passenger-oriented approach, lines along with frequencies maximize the number of direct travelers or minimize the traveling time. Bussieck et al. [7] introduce a first model to maximize the direct travels subject to a frequency constraint. For minimizing the traveling time, Schöbel and Scholl [22] present a mathematical model including restrictions on the budget. For a more detailed overview of line planning passenger-oriented mathematical models, see [20, 17, 11].
- In the cost-oriented approach, lines are selected to minimize the total cost which is composed of operational and fixed costs. Claessens et al. [10], with a cost minimizing objective function, consider passenger demand in the constraints along with line frequencies to find the optimal lines and frequencies. Later, Goossens et al. [13], following the cost-oriented approach, develop a model called multi-line planning problem which considers various halt patterns.

Following the modelling approach in Goossens et al.[13], new mathematical models have been extended for the LPP. Torres et al. [24] present a cost-oriented model on a tree network and implement it on the PTN in Quito. They minimize the total cost subject to capacity constraints and upper bounds on the line frequencies. Torres et al. [24] consider only closed lines which turned the Quito Trolébus system into a polynomially solvable case. In contrast, Borndörfer et al. [4] suggest that closed lines and open lines may lead to savings around 50% in total cost. Borndörfer et al. [5] consider an integer programming model for the Metrobüs in Istanbul which has a line topology as Trolébus in Quito.

In the 2000s, the movement towards the integration of several planning phases has accelerated in the LPP literature [18, 19]. Since the infrastructure of the PTN, as the subject of the first stage planning (network design), is fixed once implemented, line planning has received more attention in terms of integration efforts as this stage bears potential to improve service and reduce costs simultaneously when considered together with the

later stages of planning. Since timetabling comes next, it is the immediate stage to be integrated with line planning.

Kaspi et al. [16] integrate line planning and timetabling to minimize the operational costs and user inconvenience. Burggraefe et al. [6] present a heuristic approach where line planning and timetabling are coupled with each other iteratively. Yan and Goverde [25] also discuss an integration of line planning and timetabling with a more demand responsive approach where they introduce “multi-periodic” timetables to refer to irregular services running on the same section of a railway with different stop patterns and even different cycle times. In essence, “multi-periodic” timetables offer neither periodic nor aperiodic services in the traditional sense. Schöbel and Scholl [22] develop a novel modelling approach to integrate line planning with passenger routes. In the same vein, Borndörfer and Karbstein [2] discuss an alternative approach that distinguishes direct travels and routes including transfers by means of certain inequalities.

Distinct with its approach, Schöbel [21] integrates the three planning phases: line planning, timetabling and vehicle scheduling; a mathematical formulation of an integrated model is presented. In addition, the shortcomings of a sequential process and the reasons for not leading to a desirable transportation system in practice is discussed in detail. Accordingly, in the sequential approach, a feasible solution of the previous planning stage is considered as an input of the next stage. In case one of the planning stages is not necessarily solved to optimality, the planner has to solve the previous stage to find a better input.

For integration of the last two planning stages of vehicle and crew scheduling, Borndörfer et al. [3], Freling et al. [12] and Haase et al. [14] present exact approaches to optimally integrate and solve the vehicle scheduling and crew scheduling. The integration between these two stages may be considered as accomplished mostly although later works study various different aspects of both stages in an integrated manner.

According to Schrage [23], Holt et al. [15] is the first study to report a multi-period planning approach; their motivation is the fluctuation in demand orders in a manufacturing environment. Holt et al. [15] determine production quantities and also workforce levels for each month in a multi-year planning horizon. In the context of line planning problems, demand is obviously the most dynamic component of the problem environment. Demand information is usually accompanied with temporal information. In practice, however, the traditional LPPs do not necessarily recognize the fluctuation in demand and demand patterns. A single-period line planning solution supposes a simplified base demand which may be the maximum or the average demand observed in an hour during a day. However, there are some shortcomings of these simplifying approaches:

- Considering the maximum demand for an O-D pair, undertaking a worst-case approach, may lead to unnecessary costs and low utilization of the system during the times of the day when demand is really low.

- With average demand, one is highly likely to find solutions with unnecessary high frequencies for periods with low demand. Moreover, it is possible that the demand of some O-D pairs during some periods may not be satisfied due to insufficient frequencies.

Despite these well known shortcomings, traditional approaches rely on using static single-period approaches as the problem is already challenging from a computational point of view; LPPs are NP-Hard in general.

Our main motivation is to propose a multi-period line planning approach; to the best of our knowledge, this is a first in the corresponding literature. While our work has not been initiated with the purpose of integrating any of the planning stages, consideration of resource availabilities and allocation of resources to processes comes naturally and inherently with a multi-period planning. In the case of line planning problems, whether it is rolling stock or personnel, resources may be rotated from one line to another from the earlier periods to the later ones. The challenge is not only to determine how to allocate resources throughout periods and among lines, but also to account for unavailabilities during transfers and rotation of resources. As a matter of fact, our multi-period line planning modelling approach is novel for not only taking care of a nonsteady travel demand but also provides a new understanding of integration by considering the feasibility of operational plans with respect to resource availabilities at the strategic level line planning stage.

3 Mathematical Models

We develop mathematical models in the form of integer programming problems to formulate a multi-period line planning problem (MPLPP) which is then enriched by resource transfer and allocation constraints. Our starting point is a new LPP formulation that considers travel demand during a continuous-time planning horizon. This idealized continuous-time problem formulation provides a foundation to derive a multi-period line planning problem formulation through discretization of time.

3.1 A continuous-time LPP

A LPP includes

- a directed $PTN = (V, E)$ where V is the set of stations and E is the set of direct transport links/connections between stations;
- a predefined set of lines L while a line $l \in L$ is a simple path on PTN and defined by a sequence of stations;
- a planning horizon $[0, \tau]$ where $\mathcal{D}_e(t) \in \mathbb{N}$ is the number of passengers to travel on edge $e \in E$ until time $t \in [0, \tau]$.

A line (service) is said to serve all the transport links (corresponding to edges in E) between consecutive stations on the path from its starting station to the terminal station and covers part of the demand over those edges.

A feasible solution for a LPP involves a subset of lines in L and passenger capacities of services on lines such that the total service capacity of all lines serving an edge is sufficient to cover the edge's travel demand. For an edge $e \in E$, let $L_e \subseteq L$ be the set of lines that cover its demand. In the scope of a cost-oriented optimized line plan, two types of costs are considered: a fixed cost associated with the usage of a line service, c_l^f , and a variable cost rate associated with the service capacity in time, c_l^s , usually proportional to the length of a line $l \in L$. u_l for each line $l \in L$ denotes an upper bound on the service capacity due to physical limitations or safety regulations.

If we define

- $\mathcal{V}_l(t)$ to represent the passenger capacity of services provided on line $l \in L$ until time $t \in [0, \tau]$, and
- $\gamma(\mathcal{V}_l, \tau) = \begin{cases} 1, & \text{if } \exists t \in [0, \tau] : \mathcal{V}_l(t) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$

a continuous-time line planning problem can be formulated as

$$\text{minimize} \quad \sum_{l \in L} \left[c_l^f \gamma(\mathcal{V}_l, \tau) + c_l^s \int_0^\tau \mathcal{V}_l(t) dt \right] \quad (1)$$

$$\text{subject to} \quad \sum_{l \in L_e} [\mathcal{V}_l(t) - \mathcal{V}_l(s)] \geq \mathcal{D}_e(t) - \mathcal{D}_e(s) \quad \forall e \in E, \forall s, t \in [0, \tau], s < t, \quad (2)$$

$$0 \leq \mathcal{V}_l(t) \leq u_l \quad \forall l \in L, \forall t \in [0, \tau]. \quad (3)$$

The objective function (1) minimizes the total cost of line usage due to fixed costs and variable costs. Constraint (2) ensures that travel demand on an edge is satisfied at any time during the planning horizon while constraint (3) puts an upper bound on the service capacity of a line.

3.2 Discrete-time LPP

In order to formulate a more practical problem and develop the corresponding model, time discretization is a viable choice. In this respect, the planning horizon $[0, \tau]$ is divided into periods of a predetermined length Δt . If $\mathcal{T} = \lceil \tau / \Delta t \rceil$, then $T = \{t_1, t_2, t_3, \dots, t_{\mathcal{T}}\}$, where discrete time period $t_i \in T$ corresponds to $[\Delta t(i-1), \Delta t i] \in [0, \tau]$ and $t_{\mathcal{T}} \in T$ corresponds to $[\Delta t(\mathcal{T}-1), \tau] \in [0, \tau]$. Accordingly,

$$\mathbf{d}_e^{t_i} = \mathcal{D}_e(t_i) - \mathcal{D}_e(t_{i-1}), \quad \forall t_i \in T, \quad (4)$$

denotes the travel demand on edge $e \in E$ in time period $t_i \in T$ and

$$v_l^{t_i} = \mathcal{V}_l(t_i) - \mathcal{V}_l(t_{i-1}), \quad \forall t_i \in T, \quad (5)$$

denotes the service capacity on line $l \in L$ in time period $t_i \in T$. Consequently, for an edge $e \in E$,

$$\sum_{l \in L_e} v_l^{t_i} \geq \mathbf{d}_e^{t_i}, \quad \forall t_i \in T \quad (6)$$

becomes the feasibility condition for each edge $e \in E$ as discrete-time version of expression (2) ensuring that the demand in each discrete time period is covered by the capacity of the associated line services.

In an integer programming problem formulation for a discrete-time MPLPP, $y_l \in \{0, 1\}$ is a binary variable that takes value 1 if line $l \in L$ is selected and v_l^t is a non-negative integer variable that denotes the service capacity provided on line l in period $t \in T$, as defined earlier. The resulting integer programming problem formulation for *MPLPP* is

$$\text{minimize} \quad C^f \mathbf{y} + C^s \mathbf{v}^t \quad (7)$$

$$\text{subject to} \quad \mathbb{1}_e \mathbf{v}^t \geq \mathbf{d}_e^t \quad \forall e \in E, \forall t \in T, \quad (8)$$

$$\mathbf{A} \mathbf{y} - \mathbf{B} \mathbf{v}^t \leq \mathbf{0} \quad \forall t \in T, \quad (9)$$

$$\mathbf{v}^t \leq \mathbf{u} \quad \forall t \in T, \quad (10)$$

$$\mathbf{y} \in \mathbb{Z}_2^L = \{0, 1\}^L \quad (11)$$

$$\mathbf{v}^t \in \mathbb{N}^L \quad \forall t \in T. \quad (12)$$

where $\mathbb{1}_e \in \{0, 1\}^L$ and $[\mathbb{1}_e]_l = 1$ if $l \in L_e$. The objective function (7) minimizes the total cost which is composed of the variable cost and the fixed cost while the variable cost is multiplied by the level of service in each period. Constraints (8) in a closed form of expression (6) make sure that travel demand is satisfied in all periods. Coupling constraints (9) ensure that service is provided only on selected lines. Constraints (10) are upper-bounding the service capacity. Constraints (11) and (12) are domain constraints for all decision variables.

We may observe that if $\mathcal{T} = \lceil \tau / \Delta t \rceil = 1$, i.e., $T = \{t_1\}$, the above formulation solves a SPLPP; therefore, MPLPP generalizes SPLPP. We also note that MPLPP shall be decomposed with respect to time constraints if only the line selection variables, y_l , were also defined separately for each period.

3.3 Multi-period LPP with resource transfers

As mentioned earlier, a multi-period planning approach comes inherently with resource constraints and their allocation throughout periods. In this respect, we study a generalized version of MPLPP as MPLPP-RT where RT stands for resource transfers.

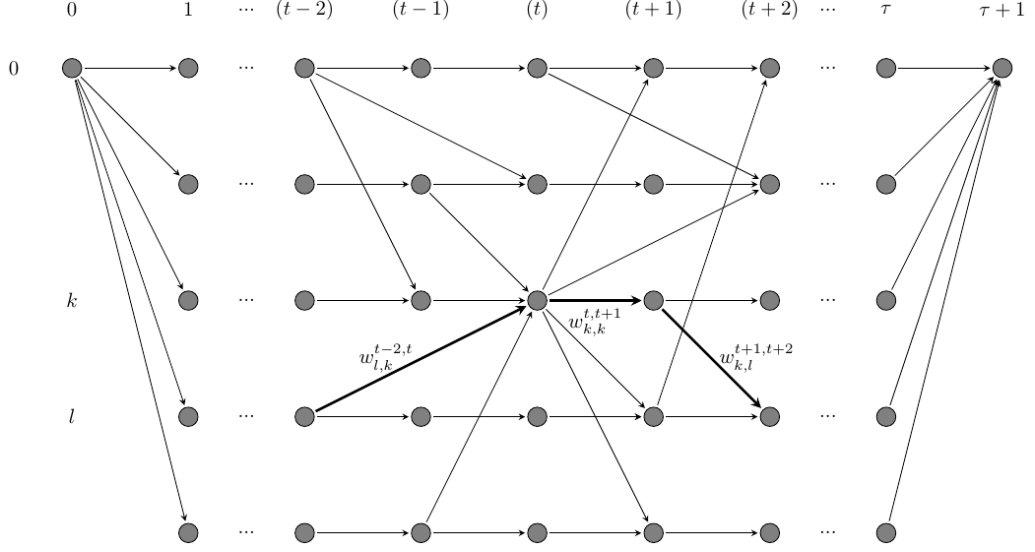


Figure 1: Resource transition between periods and lines

In order to integrate the assignment and allocation of resources to the MPLPP model, we may consider a single type of resource for the sake of simplicity. As the resource units are to be allocated among the lines throughout the periods, a network flow representation (based on the discretized planning horizon) for which the commodity corresponds to resource units is considered. In this network representation, $G = (N, A)$ with N denoting the set of nodes and A denoting the set of arcs, as demonstrated by a schematic representation in Figure 1,

- node $(l, i) \in N$ represents line $l \in L$ during the period $t_i \in T$,
- a source node represents the state of resources at the beginning of the planning horizon and is identified $(0, 0) \in N$,
- a sink node represents the state of resources at the end of the planning horizon and is identified as $(0, \mathcal{T} + 1)$, and
- an arc from node (l, i) to node (k, j) represents the flow of resource units from line l at the end of period t_i to line k at the beginning of a subsequent period t_j , i.e., $j > i$,

where line “0” represents the unused resource units.

If $w_{lk}^{t_i t_j}$ denotes the flow from node (l, i) to node (k, j) representing the number of resource units to be transferred from line l at period t_i to line k at period t_j when $j > i$ and r denotes the amount of available resource units, a closed form of network flow balance constraints is expressed as

$$\mathbf{G}\mathbf{w} = \mathbf{b} = (b_n) \begin{cases} r, & n = (0, 0) \\ -r, & n = (0, \mathcal{T} + 1) \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

where each row corresponds to a node $n = (l, i) \in N$.

The resource requirement of a line $l \in L$ is represented by $\mathbf{v}_l^{t_i}$ in formulation (7)-(12). While (13) ensures the feasibility of flow, constraint

$$\mathbf{F}\mathbf{w} = \mathbf{v} \quad (14)$$

somehow couples the network flow variables with the service capacity of line plans ensuring that levels of resources required by the lines to provide the associated service capacity are sufficient.

In this respect, an integer programming problem formulation for MPLPP-RT becomes

$$\text{minimize} \quad C^f \mathbf{y} + C^s \mathbf{v}^t \quad (15)$$

$$\text{subject to} \quad \mathbf{v}^t \geq \mathbf{d}_e^t \quad \forall e \in E, \forall t \in T, \quad (16)$$

$$\mathbf{A}\mathbf{y} - \mathbf{B}\mathbf{v}^t \leq \mathbf{0} \quad \forall t \in T, \quad (17)$$

$$\mathbf{v}^t \leq \mathbf{u} \quad \forall t \in T, \quad (18)$$

$$\mathbf{F}\mathbf{w} = \mathbf{v} \quad (19)$$

$$\mathbf{G}\mathbf{w} = \mathbf{b} \quad (20)$$

$$\mathbf{y} \in \mathbb{Z}_2^L \quad (21)$$

$$\mathbf{v}^t \in \mathbb{N}^L \quad \forall t \in T, \quad (22)$$

$$\mathbf{w}^{st} \in \mathbb{N}^{L \times L} \quad \forall s, t \in T, s < t, \quad (23)$$

where \mathbf{b} corresponds to the right-hand-side of (13).

3.4 Multi-period LPP with vehicle rotations

For the purpose of computations, we exemplify the system-wide resource with the rolling stock, i.e., vehicles, which can be considered as the most critical among such resources. Indeed, an integration of the LPP with usage of vehicles over time and allocation among lines is a realistic setting, if not the most crucial, in urban transportation systems. Therefore, we define a special case of MPLPP-RT as MPLPP with vehicle rotations (MPLPP-VR).

The vehicle allocation decision shall be directly associated with the service variable v_l^t . For the sake of simplicity, let us assume that v_l^t denotes the number of services provided on line l in period t and further suppose that a vehicle is assigned to a line for each service and is used only once during a period t . Accordingly, we define the following parameters:

- if the passenger capacity of a vehicle is \mathcal{K} passengers, the passenger capacity of a line service l in period t is $\mathcal{K}v_l^t$;
- an upper-bounding service capacity constraints shall be associated with safety regulations and limits the number of services on a line in a period with \mathcal{W} ;
- there are at most \mathcal{U} vehicles in the system that can be used simultaneously; and

- the transfer time (in number of discrete time periods) from line l to line k is ρ_{lk} .

Then, an integer programming formulation for MPLPP-VR as a special case of (15)-(22) is

$$\text{minimize} \quad \sum_{l \in L} c_l^f y_l + \sum_{l \in L} \sum_{t \in T} c_l^s v_l^t \quad (24)$$

$$\text{subject to} \quad \sum_{l \in L_e} \mathcal{K} v_l^t \geq d_e^t \quad \forall e \in E, \forall t \in T, \quad (25)$$

$$\mathcal{W} y_l - v_l^t \geq 0 \quad \forall l \in L, \forall t \in T, \quad (26)$$

$$\sum_{\substack{k \in L_0 \\ t - \rho_{kl} \geq 0}} w_{kl}^{t - \rho_{kl}, t} = v_l^t \quad \forall l \in L, \forall t \in T, \quad (27)$$

$$\sum_{\substack{k \in L_0 \\ t - \rho_{kl} \geq 0}} w_{kl}^{t - \rho_{kl}, t} - \sum_{\substack{k \in L_0 \\ t + \rho_{lk} \leq \mathcal{T}}} w_{lk}^{t, t + \rho_{lk}} = 0 \quad \forall l \in L_0, \forall t \in T, \quad (28)$$

$$\sum_{l \in L_0} \sum_{t \in T} w_{l_0 l}^{0t} = \mathcal{U}, \quad (29)$$

$$\sum_{l \in L_0} \sum_{t \in T} w_{ll_0}^{t|T|+1} = \mathcal{U}, \quad (30)$$

$$y_l \in \{0, 1\} \quad \forall l \in L, \quad (31)$$

$$v_l^t \in N \quad \forall l \in L, \forall t \in T, \quad (32)$$

$$w_{lk}^{st} \in N \quad \forall l, k \in L_0, \forall s \in \{0\} \cup T, \quad (33)$$

$$\forall t \in T \cup \{\mathcal{T} + 1\}, s < t.$$

where $L_0 = L \cup \{0\}$. In comparison to the closed form MPLPP-RT formulation, the set of constraints of MPLPP-VR can be described as follows:

- constraint (16) is replaced by (25),
- (26) is an explicit form of (17) and (18),
- constraint (27) establishes the direct relationship between \mathbf{w} and \mathbf{v} as in constraint (19) ensuring that sufficient numbers of vehicles, i.e., resource units, are assigned to a line in each period,
- constraints (28)-(30) are the flow balance constraints of the resource network representation as in (20).

In our computational study, we employ the MPLPP-VR formulation.

4 Computational Results

The aim of our computations is to demonstrate how the solutions change with the use of a multi-period planning approach and understand which parameters of the problem have significant impact. For this purpose, we consider the case of the Istanbul Metrobüs system

that experiences a high level of fluctuation in demand both temporally and spatially. The since network topology of the Metrobüs system being a simple line strips off inherent computational difficulties in LPPs and allows us to focus on multiperiodicity aspects.

4.1 Features of the problem instance

The Metrobüs is a bus rapid transit system that provides a backbone for the public transportation system of Istanbul with connections to underground rail, bus, and light rail. It has 44 stations from Beylikduzu on the far-west of the European land of Istanbul and Sogutlucemesme on the Asian land; the map in Figure 2 shows the geographical positioning of the system.

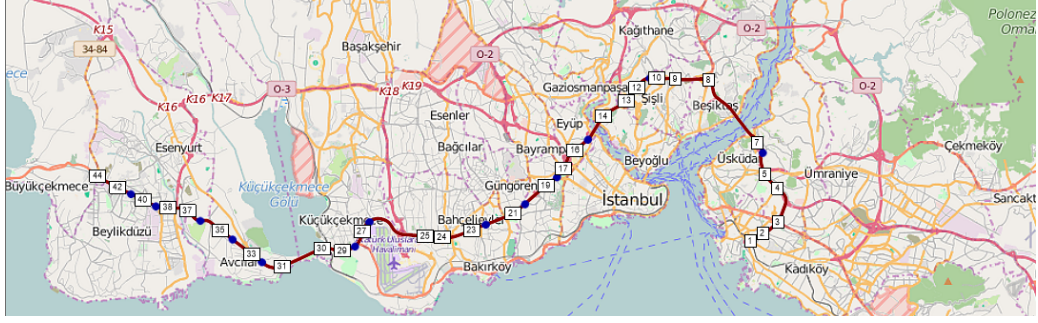


Figure 2: Istanbul map showing the Metrobüs system

BRT systems are known and popular for providing fast service. According to Basso et al. [1], 170 cities around the world have BRT systems covering 376 corridors and a distance of 5,046 km. It is also well-known that such systems usually suffer from excess demand as in the Istanbul case. Currently, the Metrobüs system works with 9 closed lines (34, 34T, 34BZ, 34U, 34Z, 34C, 34A, 34AS, 34G) as shown in Figure 3. The shaded area between Zincirlikuyu and Boğaziçi Koprusu shows the inland water Bosphorus while the other one between Ayvansaray and Halıcıoğlu is the Golden Horn, a primary inlet of the Bosphorus, which hosts the ancient harbor of Istanbul. Table 1 presents basic information on lines including starting and ending stations along with the length of each line.

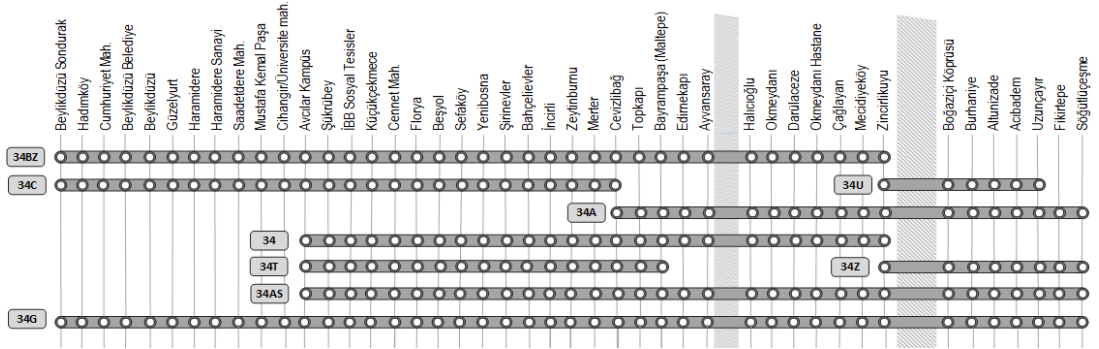
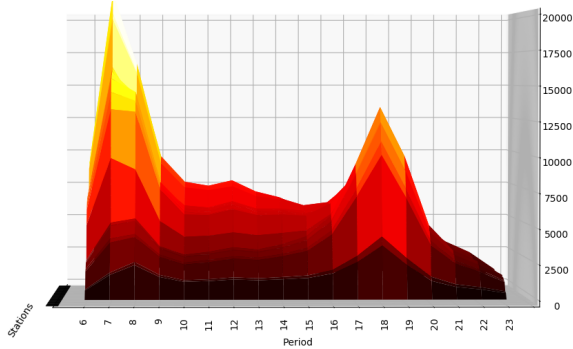


Figure 3: Network map of Metrobüs system with terminals and lines.

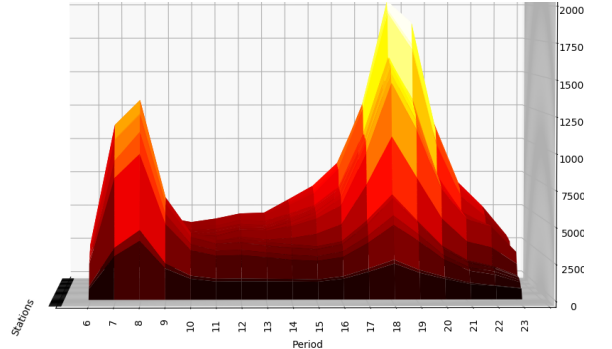
Table 1: Information on the lines used in Metrobüs system.

Line	Starting station	Ending station	Length (in meters)
34	12.Avcilar Kampus	37.Zincirlikuyu	29900
34A	26.Cevizlibag	44.Sogutluceme	22600
34C	1.Beylikduzu Sondurak	26.Cevizlibag	28600
34G	1.Beylikduzu Sondurak	44.Sogutluceme	51200
34U	37.Zincirlikuyu	42.Uzuncayir	9400
34T	12.Avcilar Kampus	28.Bayrampasa	19200
34Z	37.Zincirlikuyu	44.Sogutluceme	11300
34AS	12.Avcilar Kampus	44.Sogutluceme	41200
34BZ	1.Beylikduzu Sondurak	37.Zincirlikuyu	39900

Demand data for all O-D (station) pairs covering a planning horizon of one day is provided for periods of 1-hour length from 6 am to midnight (corresponding to 18 time periods). It is known that the number of passengers who travel on the network changes drastically depending on the time of the day and day of the week. We have three different daily demand data: an average weekday (denoted as Weekday hereafter), Saturday, and Sunday. Each daily data exhibits a high level of variation and asymmetry in time. In order to show the load on the network, we first convert O-D demand into edge demand, and show it both spatially and temporally. Figure 4 displays two charts showing the amount of edge demand in both forward (from west to east) and backward direction. The peak and the off-peak periods are easily observed: between 7 am to 9 am traffic demand is high in the direction from east to west and 5 pm to 8 pm from west to east.



(a) Forward (from west to east) direction



(b) Backward (from east to west) direction

Figure 4: Average weekday demand in Metrobüs system

In our models, the objective function includes a fixed cost for selecting a line and a variable cost for operating it. We suppose that fixed costs are mostly related to the cost of terminal stations. At a terminal station, additional space is needed for concourse or vehicle transfers, and terminals are usually facilitated with extra equipment. We assume that when a line is selected, the fixed cost should be charged once for the complete planning

horizon. We also suppose that variable cost is proportional to the line length. In the computations, we consider a unit cost of 1 as the rate per 1 meter of a line while we consider a cost of 180 as the fixed cost per line (by considering haphazardly 10 times of the unit operational cost per meter of a vehicle trip and multiplying it with the number of periods (18) in the planning horizon).

4.2 Baseline computational results

We consider the Metrob s system with a planning horizon of one day. Since the demand data is provided for 1-hour periods; the planning horizon is divided into 18 periods of 1-hour length since the planning horizon considers the day from 6 am to midnight.

In the demand data, the largest demand figure is 2158.8 in the Weekday data set while it is 688 and 390, respectively, for Saturday and Sunday. Besides, the average demands are 18.03, 15.14, and 11.05 respectively, for Weekday, Saturday and Sunday. For the baseline computations, parameters of the systems are set as follows: the capacity of a vehicle is 250 passengers ($\mathcal{K} = 250$) with 200 vehicles in the fleet ($\mathcal{U} = 200$) and a maximum number of 36 vehicles to be assigned to a line ($\mathcal{W} = 36$). The set of candidate lines among which the lines to be operated are selected is limited to the existing 9 lines.

All computational experiments are carried out on a computer with Intel Core(TM)i5-6200 CPU v2 2.30 GHz CPU and 4 GB RAM, using Gurobi Optimizer 7.5.2 as the integer programming solver with Python 3.6.2. All reported solutions are optimal.

4.2.1 Single-period approach vs. multi-period approach

To begin with, we should test the value of a dynamic multi-period planning approach against a static single-period approach. For this comparison, we solve a 1-day problem with the MPLPP formulation for once. Alternatively, we solve the problem of each 1-hour period separately with the SPLPP formulation and combine the solutions of 18 periods to make up a 1-day solution. While combining, we recalculate the fixed cost component by charging the fixed cost for each line only once in case a line is selected in more than one period.

Table 2 shows the results for the three daily instances (Weekday, Saturday, and Sunday) in terms of five solution metrics:

- Total cost is the sum of the fixed costs and operating costs; it is directly the value of the optimal objective function of MPLPP while it is recalculated for SPLPP to avoid multiple charges of the fixed cost for the same line.
- Total frequency is the total number of services/trips to run during the planning horizon.
- Distinct lines corresponds to the number of lines selected for the complete planning horizon.

- Line usage shows the sum of the number of times each line is used in 18 periods.
- Distance traveled shows the total distance traversed by all vehicles on all lines.

According to the results in Table 2, the total cost of the multi-period approach is significantly lower than that of the single-period approach in all three instances although total frequency and line usage are always higher with MPLPP. With MPLPP, line usage is almost always 18 (number of periods) times distinct lines as every selected line is used in all periods. Single-period solutions need to travel longer distances and operate more lines to satisfy the same demand. This is, indeed, the underlying reason for the sub-optimality of combined single-period solutions when compared to multi-period solutions. In addition, we also observe that most of the services are run on lines with shorter lengths in multi-period approach.

Table 2: A comparison between SPLPP and MPLPP ($\mathcal{K} = 250$, $\mathcal{U} = 200$, $\mathcal{W} = 36$)

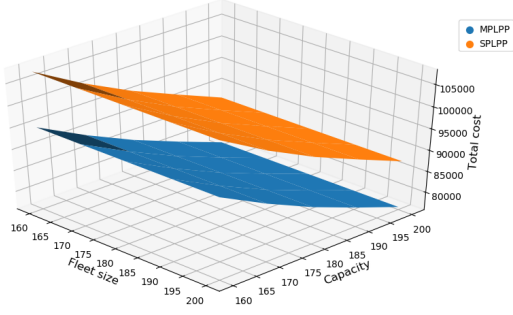
	Weekday		Saturday		Sunday	
	SPLPP	MPLPP	SPLPP	MPLPP	SPLPP	MPLPP
Total cost	64,058.80	61,334.60	51,731.40	48,807.00	37,636.80	35,377.80
Total frequency	875	1,070	635	822	457	569
Distinct lines	7	5	6	5	6	5
Line usage	49	90	43	90	38	90
Distance traveled	62798.80	60434.60	50651.40	47907.00	36556.80	34477.80

To complete our comparison, we expand this setting for various values of the system parameters. The baseline computations presented in Table 2 are repeated for $\mathcal{K} = 250$, $\mathcal{U} = 200$ and $\mathcal{W} = 36$. Figure 5 shows the total cost for settings where $\mathcal{K} \in [160, 200]$ and $\mathcal{U} \in [160, 200]$ for $\mathcal{W} = 30$ and $\mathcal{W} = 45$. Apparently for every possible setting with a feasible solution, SPLPP solutions are costlier than MPLPP solutions. At the same time, we observe that the total cost is higher when resources are more limited, i.e., when the vehicle capacity is small and the number of available vehicles is low. We therefore postulate that MPLPP provides substantially better solutions in terms of cost when compared to combined solutions of SPLPP. In both charts with different \mathcal{W} values, the results are the same.

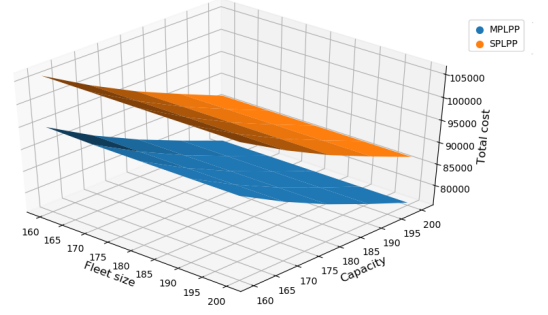
4.2.2 Size of the line set

In our computational experiments, the set of candidate lines is limited to the existing lines. However, this set can be expanded by enumerating all possible lines based on the current terminal stations used by the existing lines. This would make 45 lines, adding 36 to the existing 9 lines. This will affect the size of the problem and the solution time of the solver. The results for MPLPP showing the solution time along with other metrics are in Table 3.

We observe that an increase in the size of the set of candidate lines from 9 to 45 has a limited effect on the total cost (around 1% for Weekday data) while it yields a significant inflation in computational effort (around 1000 times of the original solution



(a) $\mathcal{W} = 30$



(b) $\mathcal{W} = 45$

Figure 5: Total cost with SPLPP and MPLPP for combinations of \mathcal{K} and \mathcal{U} .

Table 3: Comparison with respect to size of the candidate line set ($\mathcal{K} = 250$, $\mathcal{U} = 200$, $\mathcal{W} = 36$)

	Weekday		Saturday		Sunday	
	9 lines	45 lines	9 lines	45 lines	9 lines	45 lines
Total cost	61,334.60	61004	48,807.00	48417.00	35,377.80	35124.60
Total frequency	1,070	1037	822	628	569	531
Distinct lines	5	6	5	5	5	5
Line usage	90	108	90	89	90	89
Distance traveled	60434.60	59924.0	47907.00	47517.6	34477.80	34224.6
Solution time	3.56	5753.48	7.40	5068.34	5.21	5060.20

time). Accordingly, it is reasonable to conduct the analysis with the current set of lines for the sake of computational effort. On the other hand, even this limited experiment shows that computational complexity is a potential issue for further research on multi-period planning approach.

4.2.3 Level of variation in demand

In order to understand how sensitive the multi-period approach is to fluctuations in demand, we shall investigate the changes in optimal cost in response to changes in the level of the variation in demand. Based on a reference O-D demand data, new demand data are generated by either increasing or decreasing the variation with respect to time.

For a given O-D pair, the average demand over all periods is calculated first. Then, for any period with a demand above the average, the demand is increased by a fraction of the difference from the average while it is decreased by the same fraction of the difference from the average for periods with a demand below the average. With this modification, a demand data with more fluctuation and variation in time is generated for a given modification fraction. When the opposite is done, i.e., an increase for a period below the average and a decrease for a period above the average, a demand data with less variation is obtained. We use 25% and 50% as modification fractions, and obtain four new data set by changing the variation in both directions.

Table 4 shows two variation-related statistics for all three daily instances. Range,

denoted by δ , shows the difference between maximum demand and minimum demand among all demand figures (over all O-D pairs and periods). Both range and standard deviation, denoted by σ , decrease (increase) as the variation of demand decreases (increases). It should also be noted that with this modification, neither total demand over all O-D pairs nor total demand for a particular O-D pair changes. The modification alters only the distribution of passengers in time for a given O-D pair.

Table 4: Statistics of demand data for different levels of variation.

Modification fraction	Weekday		Saturday		Sunday	
	δ	σ	δ	σ	δ	σ
50% (decrease)	1079.40	23.27	344.00	14.94	195.00	10.77
25% (decrease)	1619.10	34.92	516.00	21.74	292.50	16.16
- (reference)	2158.80	46.56	688.00	28.99	390.00	21.55
25% (increase)	2698.50	58.19	860.00	36.23	487.50	26.94
50% (increase)	3238.20	69.83	1032.00	43.48	585.00	32.32

For all five demand data sets of all three daily instances, the change in optimal total cost is demonstrated in Figure 6. It is easily and clearly observed that the optimal objective function value increases when the level of demand variation increases. It should again be noted that for a given daily instance, the total demand for each O-D pair is the same for all five demand data sets. Therefore, the cost of an optimal line plan is clearly sensitive to the temporal variation/distribution in demand for a given total demand over a fixed length planning horizon. These results manifest the need and significance of multi-period approaches in the scope of line planning problems in response to high levels of demand variation and fluctuation. In addition, we also observe that the effect of variation is more evident when the total demand is higher as for the Weekday instance.

While investigating the effect of demand variation, we shall also avoid the potential bias due to system parameters such as vehicle capacity, fleet size, and the limit on the number of assigned vehicles to a line.

4.3 Vehicle rotations

It is not trivial to analyze the effect of considering vehicle transfers on either optimal solutions or feasibility. In the case of our baseline setting ($\mathcal{K} = 250$, $\mathcal{U} = 200$, $\mathcal{W} = 36$), the solutions for all three daily problems are the same in MPLPP and MPLPP-VR models with respect to the total cost even when vehicle transfer constraints are considered properly as in the formulation for MPLPP-VR. In other words, the optimal solution for MPLPP is feasible for the MPLPP-VR version of the problem for the baseline setting. It is clear that the assumption of vehicle transfers in more than one period may not alter the solution and the cost when the system has ample capacity. To highlight the potential effects of considering the actual time needed to transfer vehicles, we solve problems with more limited resources when compared to the baseline setting.

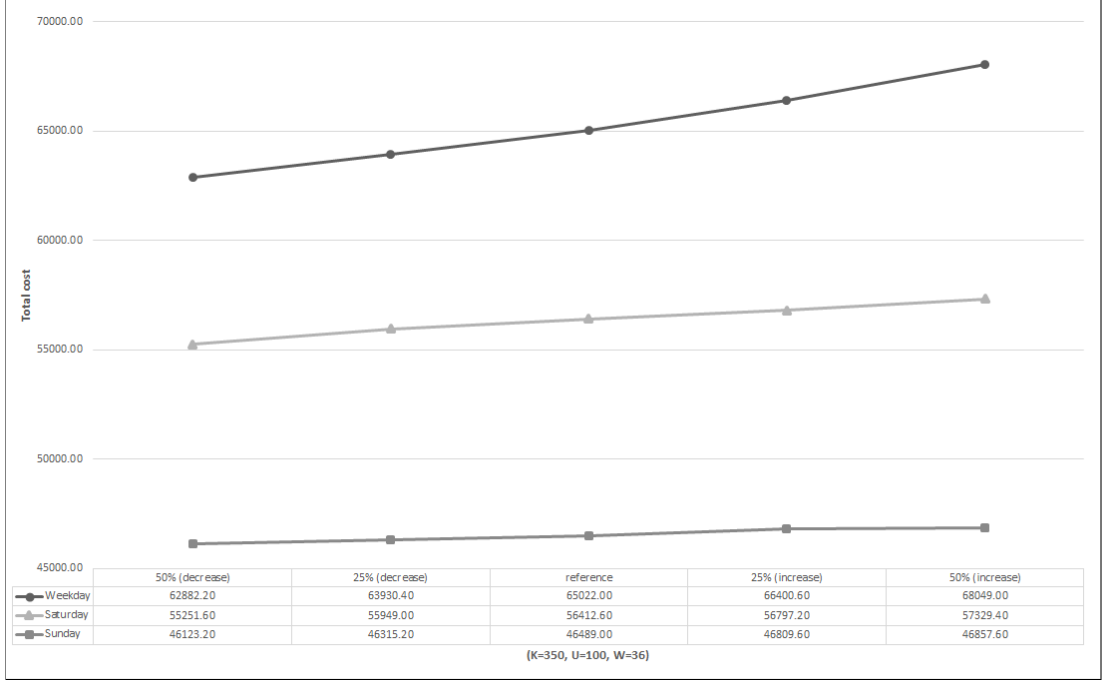


Figure 6: Optimal total cost values for demand data with different levels of variation for all daily instances.

In addition to the data used in MPLPP, MPLPP-VR also requires transfer times for vehicles. In order to calculate the time required for vehicle transfers, i.e., ρ_l^k for a pair of lines l and k , we consider the trip time of a vehicle on a line and the travel time of a vehicle from the terminal station of a line to the starting station of the other:

- The trip time of a vehicle is calculated by considering preparation time and station time in addition to the actual driving time. The preparation time is concerned with additional set-up and terminating operations, respectively, before and after each trip. Station time is the duration a vehicle spends at a station stop; the difference between arrival and departure time at each station is called station time. For the sake of simplicity, we suppose that all stations have the same station time. The driving speed of a vehicle is constant for the planning horizon.
- The sum of the trip time and travel time from the ending station of a line to the starting station of the other makes up the transfer time. Then, the transfer time is divided by the length of the period to calculate the transfer time in number of periods. For a pair of lines l and k , the transfer time from l to k is, then, denoted as ρ_{lk} . For instance, if the actual travel time is 100 minutes and the period length is 60 minutes, then $\rho_{lk} = 2$ since it would take longer than one period but shorter than two periods for a vehicle to transfer from l to k . By definition, for a pair of lines l and k , $\rho_{lk} \geq 1$.

Table 5 shows the comparison between MPLPP and MPLPP-VR with $\mathcal{K} = 220$, $\mathcal{U} = 100$, $\mathcal{W} = 36$. The results for Weekday demand show that the total cost of MPLPP-

VR is greater than that of MPLPP. This result demonstrates that all vehicles may not necessarily be available in all periods when transfers are considered. Even with the new parameter set, MPLPP and MPLPP-VR provide the same solutions for Saturday and Sunday. In order to observe the effect of MPLPP-VR with Saturday and Sunday demands, we further change the vehicle capacity, fleet size, and the maximum number of vehicles to be assigned to a line. Table 6 shows the results. The effect of vehicle transfer constraints is observed with Saturday demand as the total cost increases in the MPLPP-VR due to unavailability of transfers of vehicles from one period to the subsequent ones. With this new setting ($\mathcal{K} = 160$ and $\mathcal{U} = 80$), the Weekday solution is infeasible while the solutions of MPLPP and MPLPP-VR are still the same for Sunday due to ample resources in the system. The effect of vehicle transfer constraints are observable for Sunday demand only when the system resources are even more limited as $\mathcal{K} = 120$ and $\mathcal{U} = 70$ as seen in Table 6 again.

Table 5: A comparison between MPLPP and MPLPP-VR for all demand sets ($\mathcal{K} = 220$, $\mathcal{U} = 100$, $\mathcal{W} = 36$)

	Weekday		Saturday		Sunday	
	MPLPP	MPLPP-VR	MPLPP	MPLPP-VR	MPLPP	MPLPP-VR
Total cost	69790.60	70571.00	55296.80	55296.80	39790.60	39790.60
Total frequency	1103	969	937	1066	643	643
Distinct lines	6	6	5	5	5	5
Line usage	97	93	90	89	90	90
Distance traveled	68710.60	69491.00	54396.80	54396.80	38890.60	38890.60
Solution time	3.92	6.52	4.56	7.83	5.22	9.17

Table 6: A comparison between MPLPP and MPLPP-VR for Saturday and Sunday demand sets ($\mathcal{W} = 32$)

	Saturday ($\mathcal{K} = 160$, $\mathcal{U} = 80$)		Sunday ($\mathcal{K} = 120$, $\mathcal{U} = 70$)	
	MPLPP	MPLPP-VR	MPLPP	MPLPP-VR
Total cost	75835	76316.2	72155.80	72573.60
Total frequency	1203	1090	1067	990
Distinct lines	6	6	6	6
Line usage	91	91	99	97
Distance traveled	74755	75236.2	71075.80	71493.60
Solution time	4.21	7.28	3.31	7.26

4.4 Line types: Closed vs. open

The concept of open lines and closed lines are well-known in public transportation planning. Line type selection may be a key design issue in line planning. In the more common version, i.e., with closed lines, the service is provided in both directions on the path from the starting station to the ending station; for each service executed in one of the directions, a service is executed in the other direction. In essence, the service frequencies are the same in both directions, and mostly vehicles go back and forth on the same path. In an open

line, the service is provided only from the starting station to the ending station. A closed line corresponds to two open lines which operate on the same path in opposite directions with identical frequencies. In a line plan with closed lines only, the vehicles travel in both directions; consequently, some of the services are executed only for the sake of delivering the opposite direction rather than covering demand. Therefore, the traveled distance as well as the associated costs may increase unnecessarily. On the other hand, operating open lines may require more vehicle transfers and increase travel time for transfers resulting in a potentially more difficult-to-operate line plan. In this respect, it may be worthwhile to analyze the effect of allowable line types when the time needed to transfer vehicles between pairs of lines is an issue in the mathematical model.

We again assume that the resource related system parameters are sufficiently tight to observe the effect of transfers on MPLPP-VR solutions and set $\mathcal{K} = 220$, $\mathcal{U} = 200$, $\mathcal{W} = 36$. Table 7 reports the comparison among the two line types on all three daily instances. While solutions with “Closed Lines” correspond to the settings in the original baseline experiments, solutions with “Open Lines” consider the option of providing the service in only one of the directions of the original lines or running with different frequencies in opposite directions. From the cost perspective, we find out that the cost is larger when only closed lines are considered. It should also be noted that solution time is clearly larger when open lines are considered as the number of lines in the candidate set is twice as much.

Table 7: A comparison between line types for all demand sets ($\mathcal{K} = 220$, $\mathcal{U} = 200$, $\mathcal{W} = 36$)

Type of line	Weekdays		Saturdays		Sundays	
	Open Lines	Closed Lines	Open Lines	Closed Lines	Open Lines	Closed Lines
Total cost	55992.50	69253.40	48376.30	55296.80	37123.90	39790.60
Total frequency	2070	1197	1664	937	1132	643
Distinct lines	11	5	9	5	8	5
Line usage	180	90	161	90	141	90
Distance traveled	54012.50	68353.40	46756.30	54396.80	35683.90	38890.60
Solution time	18.70	6.05	22.46	9.86	25.41	8.66

In the case of the Istanbul Metrobüs, the demand is highly asymmetric in both time and space; this bears a huge advantage when open lines are allowed along with a multi-period planning approach. We observe this clearly in the results. Even when more lines are operated (leading to an increase in fixed costs), the optimal cost goes down by almost 20% for the Weekday which has the highest demand. This also shows that the time needed to transfer the vehicles among the starting stations of various lines may be easily compensated even when the resources are tight enough and operating open lines help to decrease the operational costs substantially.

4.5 Choice of period length

The period length in a multi-period problem is related to the temporal dimension of the O-D demand data; it specifies the time unit of the decision variables to determine the frequency of lines (through the number of vehicles of assigned to a line service). The length of the period also reveals the degree of discretization of time. And, since time is indeed a continuous phenomenon, it also determines the degree of approximation. In general, the degree of approximation is higher when time is discretized in larger units. Correspondingly, shorter period length is expected to lead to more accurate and less approximate solutions in practice. Although some degree of discretization is quite necessary so that the problems can be formulated in a discrete space and solutions shall be interpreted easily, its effect on the solution of the problem in terms of resource usage may not be as trivial as the accuracy. In this respect, we aim to investigate how both the accuracy and effectiveness of the solution change when alternative period lengths are used.

For the original Metrobüs demand data, the period length is one hour, i.e., 60 minutes. We now consider three scenarios for the length of the period: 60 minutes, 30 minutes and 15 minutes. The original demand data is transferred to shorter periods by allocating the demand of a longer period to a set of shorter ones by interpolating and smoothing out the demand according to the demand amount in previous and subsequent periods of the original longer period version. The transfer matrix for vehicle rotation is also updated for alternative period lengths.

Tables 8 - 10 report the results with MPLPP-VR formulation. Looking at the solution metrics closely, we observe that the total cost for the Weekday (see Table 8) first decreases when the period length goes from 60 to 30 minutes; but, then it increases again when the period length goes from 30 to 15 minutes. With the same parameter setting ($\mathcal{K} = 220$, $\mathcal{U} = 100$), however, for Saturday and Sunday, the total cost increases as the period length is shortened. We again check the results when system resources are tighter at a level for which the Weekday solution is already infeasible. Table 9 shows solutions for Saturday. Changing the period length from 60 to 30 minutes leads to a decrease in the total cost with $\mathcal{K} = 160$ and $\mathcal{U} = 80$; cost increases when the period length goes further down to 15 minutes. When $\mathcal{K} = 120$ and $\mathcal{U} = 70$, the total cost for Sunday (see Table 10) follows the same trend as that for Weekday with $\mathcal{K} = 160$ and $\mathcal{U} = 100$ and Saturday with $\mathcal{K} = 160$ and $\mathcal{U} = 80$. As a matter of fact, we make the following observations:

- It is probable that shortening the period length may decrease the total cost and provide even a better solution while increasing the accuracy.
- While shorter period lengths contribute to accuracy of the solutions by satisfying the demand in a timely manner since frequencies are arranged for shorter time periods, it may lead to inefficient use of resources due to discrete nature of resource capacities and shorter periods (leading to an increase in the number of periods) can lead to more slack in resource capacities.

Table 8: Comparison among alternative period lengths with Weekday demand ($\mathcal{K} = 220, \mathcal{U} = 100$)

Period length (minutes)	60	30	15
\mathcal{W}	36	18	9
Total cost	70,571.00	70,386.40	71,916.00
Total frequency	969	1,210	1,230
Distinct lines	6	5	5
Line usage	93	180	359
Distance traveled	69,491.00	69,486.40	71,016.00
Solution time	7.36	22.83	77.63

Table 9: Comparison among alternative period lengths with Saturday demand

Period length (minutes)	$(\mathcal{K} = 220, \mathcal{U} = 100)$			$(\mathcal{K} = 160, \mathcal{U} = 80)$		
	60	30	15	60	30	15
\mathcal{W}	36	18	9	32	16	8
Total cost	55296.80	56239.00	58134.00	76316.20	76196.40	78245.60
Total frequency	1066	1085	1,122	1090	1289	1321
Distinct lines	5	5	5	6	5	5
Line usage	89	179	359	91	179	356
Distance traveled	54396.80	55339.00	57234.00	75236.20	75296.40	77345.60
Solution time	8.50	26.58	78.68	9.74	18.96	72.08

Table 10: Comparison among alternative period lengths with Sunday demand

Period length (minutes)	$(\mathcal{K} = 220, \mathcal{U} = 100)$			$(\mathcal{K} = 120, \mathcal{U} = 70)$		
	60	30	15	60	30	15
\mathcal{W}	36	18	9	32	16	8
Total cost	39790.60	40809.80	42521.00	72573.60	72567.80	74210.60
Total frequency	643	654	672	990	1375	1404
Distinct lines	5	5	5	6	5	5
Line usage	90	176	337	97	180	360
Distance traveled	38890.60	39909.80	41621.00	71493.60	71667.80	73311.00
Solution time	10.36	38.74	121.24	7.80	21.19	108.01

The overall effect manifests itself as a trade-off between accuracy and effectiveness. It turns out that the length of the period may be an intricate choice to make. Owing to the trade-off between the accuracy and the effectiveness of the solution clearly makes this choice a subject of optimization, not to forget the dimension regarding the computational effort.

The effect of period length is not only observed as an increase or a decrease in the total cost. It may also alter the feasibility of the problem setting. In Table 11, we show the results with a different set of system parameters where $\mathcal{K} = 150$ and $\mathcal{U} = 100$ for the Weekday demand data set. When the period length is 60 minutes, the problem does not even have a feasible solution while the solutions for shorter period lengths are feasible.

We also observe that the solution time for the solver also increases significantly when the period length is shortened.

Table 11: Effect of length of period on the solution for Weekday instance with $\mathcal{K} = 150$ and $\mathcal{U} = 100$

	Period length (minutes)		
	60	30	15
\mathcal{W}	36	18	9
Total cost		102430.60	104287.60
Total frequency		1699	1709
Distinct lines		7	7
Line usage	inf	197	381
Distance traveled		101170.60	103027.60
Solution time		17.94	68.29

5 Discussion and conclusion

We present a multi-period planning approach for the well-known line planning problem; our approach is motivated by the drawbacks of traditional static line planning approaches for not being able to consider the dynamism of the demand. In practice, the traditional approaches may still work for systems with moderate demand load and where target service levels are already achieved with more than sufficient resources. For such systems, it is usually trivial to identify peak loads with respect to time and space and mostly as well as directions on the network. However, for overly-crowded systems for which many examples can be found as BRTs in different cities, unwanted passenger waiting times at stations resulting in longer travel times and lower service levels shall be handled if the changes in travel demand in time are considered explicitly. The new approach proposes consideration of longer planning horizons which are divided into periods of manageable length in terms of planning and coordination of both services and resources throughout the periods in the planning horizon.

We characterize the demand as a function of time, first; this helps us develop a continuous-time line planning problem for the first time in the literature. Then, for practical purposes, we develop an integer programming problem formulation for the multi-period line planning problem through discretization of the continuous planning horizon. In our computational study, we first work with this problem and show that

- both optimal solutions and resulting costs (represented by the objective functions) are improved significantly when a multi-period approach is employed as an alternative to combined solutions of traditional single-period line plan solutions;
- higher variation in demand benefits even more from a multi-period approach as higher fluctuations of demand in time leads to higher system costs even when the total demand does not change.

As a matter of fact, we are able to experimentally show that a multi-period approach shall outperform a traditional single-period approach under various circumstances. We also observe that the computational challenges of well-known line planning problem formulations are trivially inherited by the multi-period approach.

As easily and clearly observed from many examples in the literature and practice, decision-making and optimization with multi-period planning approaches naturally involve resource planning. Indeed, planning of resources is mostly what couples the time periods in a typical problem formulation. In this respect, we develop a generalization of the first multi-period planning problem formulation by integrating resource allocation and transfer constraints. Computations with this problem show that solutions may change significantly when resource constraints are involved and tight. Therefore, it shall be necessary to employ an approach where resource transfers are also included in order to obtain realistic and implementable solutions. We also observe that, the choice of line types as either open or closed is likely to contribute to solutions when multi-period planning approaches are employed. Especially, when the travel demand is highly asymmetric in space and experiences fluctuations in time, the flexibility with open lines contribute to the effect of a multi-period planning approach.

Last but not the least, our computations show that choosing the period length may be an intricate decision that is justified by a trade-off between accuracy of the solutions and efficiency of resource planning. It requires further analysis and understanding both the problem characteristics and the particular system on-hand as the parameters may play a significant role in the magnitude of this trade-off.

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