

Master's Thesis

Optimization of a Master Surgery Schedule

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Overview

During the past years hospitals saw themselves confronted with increasing economical pressure (WB06, p. V). Therefore, optimizing the general operational procedures has gained in importance. The revenue of a hospital depends on the kinds and quantity of treatments performed and on the efficient use and utilization of the corresponding resources. About 25 – 50% of the treatment costs of a patient needing surgery incurs in the operating rooms (WB06, p. 58). Hence skillful management of the operating rooms can have a large impact on the overall revenue of a hospital. Beliën and Demeulemeester (BD07) describe the planning of operating room (OR) schedules as a multi-stage process. In the first stage OR time is allocated to the hospitals specialties and capacities and resources are adjusted. In the second stage a master surgery schedule (MSS) is developed, that is a timetable for D days that specifies the amount of OR time assigned to the specialties on every individual day. After D days this schedule will be repeated without any changes. Hence, developing an MSS is a long-term problem. Finally, specialties will schedule specific surgeries within their assigned OR time. In this work we will focus on the development of the MSS that maximizes the revenue of the hospital. Our main focus will be to ensure that the capacities of the downstream resources, i.e. the bed capacities in the ICU and ward, will not be exceeded. Additionally, we hope that our formulation of the problem will lead to a leveled bed demand without significant peaks. We will incorporate the uncertainty of patient demand and case mix in our model. There have been several approaches on this subject, for example in (Fü15) and (BD07) and this work is in part influenced by these advances.

In Chapter 1 we start by establishing a model of the processes in a hospital that are of interest to our problem. This involves defining an objective and several constraints. Then we will find linear approximations for some of the constraints, such that we can find a close to optimal solution to our problem by solving two linear programs.

In Chapter 2 we find an alternative formulation of the problem as second order cone program, that can be solved without the need of any approximations.

In Chapter 3 we will test our model with real world data and describe how we handled issues arising from missing data. Then we present the results and simulate the performance of a hospital using the found MSS.

Finally, in Chapter 4 we evaluate our approach and identify advantages and limitations.

Notation

Index \in Set	Description
$s \in \{1, \dots, S\}$	Surgery specialties
$d \in \{1, \dots, D\}$	Days in the MSS cycle
$t \in \{1, \dots, LOS_{max}^{IC/W}\}$	Days after surgery that a patient is still in the ICU/ward
$l \in \mathcal{L} = \{l_1, \dots, l_L\}$	Possible lengths of operating hours in the ORs

Parameters	Description
$r_{s,l}$	Revenue for assigning OR time of length l to specialty s
c_s	Capacity of OR staff of specialty s
$C^{IC/W}$	Bed capacity of the ICU/ ward
N_{OR}	Number of operating rooms
$LOS_{max}^{IC/W}$	Maximal LOS in the ICU/ward

Random variables	Description
$U_{s,l} \in \mathbb{N}$	Number of patients operated by specialty s during OR time of length l
$X_{d,s} \in \mathbb{N}$	Number of patients operated by specialty s on day d of the MSS cycle
$Y_{d,s,t}^{ICU/W} \in \mathbb{N}$	Number of patients operated by specialty s on day d who are still in the ICU/ward t days later
$Z_d^{ICU/W} \in \mathbb{N}$	Number of patients in ICU/ward on day d of the MSS cycle
$\mathbb{E}[X]$	Expected value of a random variable X
$\mathbb{V}[X]$	Variance of a random variable X

Probabilities	Description
$\mathbb{P}[A]$	Probability of an event A
$p_{s,t}^{IC/W}$	Probability that a patient operated by specialty s is in the ICU/ward t days after surgery
Φ	Cumulative distribution function of the standard normal distribution

Decision variables etc.	Description
$y_{d,s,l} \in \mathbb{N}$	Number of OR blocks of length l assigned to specialty s on day d of the MSS cycle
$F_{(P)}$	Feasible region of the optimization problem (P)
$p_{(P)}^*$	Optimal value of the optimization problem (P)

1. Problem Statement

In this thesis we address the problem of finding a cyclic master surgery schedule (MSS) ensuring that the bed capacities of the hospital are not exceeded. To do so we link the assignment of operating room (OR) time to the individual specialties to the number of patients in the intensive care unit (ICU) and the ward.

To obtain a manageable model of a real world hospital we have to simplify reality as follows. We model a hospital with S specialties that perform surgeries in N_{OR} operating rooms. We assume that all surgeries can be performed in any OR. Moreover, we assume that all specialties share the same ICU and ward. This is reasonable since it is common practice to compensate bed shortage in one ward by using free resources in the remaining wards. After surgery all patients in our model first pass through the ICU and the ward before being discharged. This way we can model patients going directly to the ward after surgery by denoting their length of stay (LOS) in the ICU to 0 days as well as ambulant patients by denoting their LOS in the ICU and the ward to 0. However, we neglect the case where a patient is transferred from the ward to the ICU.

We only incorporate planned surgeries in our model and exclude any emergency patients. On the one hand the hospital cannot influence emergencies making these cases irrelevant for long term planning, apart from determining the amount of OR time and bed capacity reserved for emergencies. On the other hand (ABD⁺08) remarks that uncertainty in patient arrivals leads to less variability in bed demand than the actual scheduling.

In the model we assign each OR to at most one specialty per day and allow for a finite set of different opening hours \mathcal{L} . For this we introduce variables $y_{d,s,l} \in \mathbb{N}$ denoting the number of ORs open for l hours, that are assigned to specialty s on day d of the MSS. Throughout the following sections it will be convenient to consider the $y_{d,s,l}$ as a vector of decision variables and hence we define $y \in \mathbb{R}^N$ with $N = DSL$:

$$y := (y_{1,1,l_1} \quad \dots \quad y_{1,1,l_L} \quad y_{1,2,l_1} \quad \dots \quad y_{1,S,l_L} \quad y_{2,1,l_1} \quad \dots \quad y_{D,S,l_L})^T. \quad (1.1)$$

Since hospitals compete economically, MSSs that lead to large revenue for the hospital are to be preferred. Hence, we will associate a block of theater time assigned to some specialty with the expected revenue generated during that block and finally maximize the overall revenue of the MSS.

Note that the revenue per operating hour in the OR depends on a variety of parameters. On the one hand the revenue is determined by the number of performed surgeries and the amount of payment per surgery. On the other hand there are more complex processes influencing the revenue, e.g. if OR time and staff shifts are not synchronized, this can lead to high personnel costs. Let $r_{s,l}$ be the revenue to be expected when assigning

1.1. Constraints on Opening Hours

a OR for l hours to specialty s . For our model we assume that $r_{s,l}$ can be retrieved from the available data.

Then the objective of our problem is

$$\max_{y_{d,s,l} \in \mathbb{N}} \sum_{d=1}^D \sum_{s=1}^S \sum_{l \in \mathcal{L}} r_{s,l} y_{d,s,l}.$$

With our notation from 1.1 and appropriately chosen $r \in \mathbb{R}^N$ this reformulates to

$$\max_{y \in \mathbb{N}^N} r^T y. \quad (1.2)$$

Of course there are restrictions on the MSS and therefore on the $y_{d,s,l}$. In the following we will formulate constraints that aim to produce a MSS feasible in a real world setting.

1.1. Constraints on Opening Hours

Without any rules on the assignment of time in the operating rooms to the individual specialties, the optimal solution with respect to maximizing revenue of the hospital would be to open all operating rooms as long as possible and only assigning OR time to the specialty with the highest expected revenue. Of course this solution is not feasible in reality. On the one hand a hospital cannot select its patients without restriction, on the other hand OP staff is limited and can only handle a certain amount of OR time. Therefore, we need to establish some basic rules on the assignment of OR time to the specialties to ensure that any specialty can satisfy the demand of patients needing treatment while keeping overall OR time within bounds that are manageable for the OP staff.

1.1.1. Lower Bounds

To ensure that any specialty is assigned a minimum amount of time in the operating rooms, we model the demand of patients needing surgery by specialty s as a queuing system where patients arrive with some rate λ_s and are treated with mean service time τ_s . To avoid that the number of patients waiting for treatment by any specialty s grows to infinity (GSTH11, p. 9), we require that

$$\lambda_s \tau_s < 1 \quad \text{for all } s \in \{1, \dots, S\}. \quad (1.3)$$

We assume that λ_s can be obtained from the given data. The mean service time τ_s however depends on the number of surgeries performed by specialty s and therefore on the amount of OR time assigned to this specialty.

Let $X_{d,s}$ be the number of patients operated by specialty s on day d of the MSS. To link $X_{d,s}$ and $y_{d,s,l}$ we let $U_{s,l}$ denote the number of surgeries performed by specialty s during OR time of length l and obtain

$$X_{d,s} = \sum_{l \in \mathcal{L}} U_{s,l} y_{d,s,l}. \quad (1.4)$$

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The value of $U_{s,l}$ depends on the lengths of the particular surgeries scheduled during the corresponding OR block. The scheduling of specific surgeries is a short-term problem depending on the current demand and not part of the MSS. Therefore, we model $U_{s,l}$ and $X_{d,s}$ as random variables and assume that the probability distribution of $U_{s,l}$ is given. With this we obtain:

$$\begin{aligned}\frac{1}{\tau_s} &= \frac{1}{D} \sum_{d=1}^D \mathbb{E}[X_{d,s}] \\ &= \frac{1}{D} \sum_{d=1}^D \sum_{l \in \mathcal{L}} \mathbb{E}[U_{s,l}] y_{d,s,l}\end{aligned}$$

and we can reformulate 1.3 as follows:

$$\begin{aligned}\lambda_s \tau_s &< 1 && \text{for all } s \in \{1, \dots, S\} \\ \Leftrightarrow \lambda_s &< \frac{1}{D} \sum_{d=1}^D \sum_{l \in \mathcal{L}} \mathbb{E}[U_{s,l}] y_{d,s,l} && \text{for all } s \in \{1, \dots, S\} \\ \Leftrightarrow A_{LB} y &\leq b_{LB} && \text{(LB)}\end{aligned}$$

for appropriately chosen $A_{LB} \in \mathbb{R}^{S \times N}$ and $b_{LB} \in \mathbb{R}^S$. Hence, we obtain one linear constraint per specialty for the decision variable y .

1.1.2. Upper Bounds

Another naturally occurring constraint concerns the number of available operating rooms. Of course any operating room cannot be assigned to more than one specialty simultaneously. Since we only allow the assignment of ORs for whole days this constraint can be formulated as

$$\sum_{s,l} y_{d,s,l} \leq N_{OR} \quad \text{for all } d \in \{1, \dots, D\}.$$

Again we can simplify this constraint to

$$A_{UB} y \leq b_{UB} \tag{UB}$$

by choosing appropriate $A_{UB} \in \mathbb{R}^{D \times N}$ and $b_{UB} \in \mathbb{R}^D$.

For now we will not consider any constraints regarding the OR staff of the individual specialty and hence assume that every specialty has enough staff to satisfy the demand induced by the final MSS or that if necessary, additional staff can be hired in advance. However, one could alter the model by additionally requiring that

$$\sum_{d,l} l y_{d,s,l} \leq c_s \quad \text{for all } s \in \{1, \dots, S\}. \tag{1.5}$$

to ensure that the weekly work load of the OP staff of specialty s does not exceed its capacity $c_s \in \mathbb{R}_{>0}$.

1.1.3. Weekends

Usually hospitals will avoid to schedule surgeries on weekends and only perform urgent procedures to keep the work load small on weekends.

Therefore, we consider $\mathcal{D} \subset \{1, \dots, D\}$ to be days of the weekends and require

$$y_{d,s,l} = 0 \quad \text{for all } d \in \mathcal{D}, s \in \{1, \dots, S\}, l \in \mathcal{L}. \quad (\text{WE})$$

This constraint leads to irregularities in the amount of patients arriving in ICU and ward. Since we hope to find a MSS that avoids large peaks in bed demand in the ICU and the ward, it will be interesting to investigate the impact of this constraint on the number of patients.

1.2. Constraint for Bed Capacity

Apart from maximizing the hospital's revenue, another main focus of this work is to ensure that the MSS we find will not lead to bed shortage in the ICU or the ward. For this we will now formulate a constraint that only allows for surgery schedules with a small probability of patient overflow. We will assume that the number of patients in the ICU is independent of the number of patients in the ward which simplifies our considerations and allows for independent constraints for ICU and ward. From now on all considerations concentrate on the ICU and work analogously for the ward.

Let Z_d^{IC} be the demand of beds in the ICU on day d of the surgery schedule. Ultimately, we want to obtain a constraint that gives an upper bound for the probability of Z_d^{IC} exceeding the number of available beds in the ICU. For this we will link the value of Z_d^{IC} to the $y_{d,s,l}$.

Recall from Section 1.1.1 the random variable $U_{s,l}$ for the number of surgeries performed by specialty s during OR time of length l and the random variable $X_{d,s}$ for the number of patients operated by specialty s on day d of the MSS. By 1.4 and since the number of surgeries during different OR sessions are independent, the expected value and variance of $X_{d,s}$ can be calculated as follows:

$$\mathbb{E}[X_{d,s}] = \sum_{l \in \mathcal{L}} \mathbb{E}[U_{s,l}] y_{d,s,l}, \quad (1.6)$$

$$\mathbb{V}[X_{d,s}] = \sum_{l \in \mathcal{L}} \mathbb{V}[U_{s,l}] y_{d,s,l}. \quad (1.7)$$

Let $Y_{d,s,t}^{IC}$ be the number of patients operated on day d by specialty s , who are still in the ICU t days later and let $p_{s,t}^{IC}$ be the probability that a patient operated by specialty s is still in the ICU t days after surgery. More precisely, $t = 0$ denotes the day of surgery and $p_{s,0}$ is the probability that a patient has to go to the ICU after surgery. We assume that $p_{s,t}^{IC}$ can be obtained from given data. Then the random variable $Y_{d,s,t}^{IC}$ follows the binomial distribution $\text{Bin}(X_{d,s}, p_{s,t}^{IC})$ or expressed differently:

$$Y_{d,s,t}^{IC} | (X_{d,s} = n) \sim \text{Bin}(n, p_{s,t}^{IC}). \quad (1.8)$$

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By the law of total expectation we obtain

$$\begin{aligned}
\mathbb{E}[Y_{d,s,t}^{IC}] &= \mathbb{E}[\mathbb{E}[Y_{d,s,t}^{IC} | X_{d,s}]] \\
&= \mathbb{E}[X_{d,s} p_{s,t}^{IC}] \\
&= p_{s,t}^{IC} \mathbb{E}[X_{d,s}] \\
&= \sum_{l \in \mathcal{L}} p_{s,t}^{IC} \mathbb{E}[U_{s,l}] y_{d,s,l}
\end{aligned}$$

and the law of total variance implies

$$\begin{aligned}
\mathbb{V}[Y_{d,s,t}^{IC}] &= \mathbb{E}[\mathbb{V}[Y_{d,s,t}^{IC} | X_{d,s}]] + \mathbb{V}[\mathbb{E}[Y_{d,s,t}^{IC} | X_{d,s}]] \\
&= \mathbb{E}[X_{d,s} p_{s,t}^{IC} (1 - p_{s,t}^{IC})] + \mathbb{V}[X_{d,s} p_{s,t}^{IC}] \\
&= p_{s,t}^{IC} (1 - p_{s,t}^{IC}) \mathbb{E}[X_{d,s}] + p_{s,t}^{IC} \mathbb{V}[X_{d,s}] \\
&= \sum_{l \in \mathcal{L}} (p_{s,t}^{IC} (1 - p_{s,t}^{IC}) \mathbb{E}[U_{s,l}] + p_{s,t}^{IC} \mathbb{V}[U_{s,l}]) y_{d,s,l}.
\end{aligned}$$

Now we can calculate the number of patients who are in the ICU on day d of the MSS. Let LOS_{max}^{IC} denote the maximal length of stay in the ICU. We add up all patients who have had surgery at most LOS_{max}^{IC} days prior to day d and are still in the ICU, starting with patients that have had surgery on day d and stay at least one day in the ICU:

$$Z_d^{IC} = \sum_{t=0}^{LOS_{max}^{IC}} \sum_{s=1}^S Y_{((d-t) \bmod D),s,t}^{IC} \quad (1.9)$$

Again the numbers of patients from different blocks left on day d are independent, hence

$$\begin{aligned}
\mathbb{E}[Z_d^{IC}] &= \sum_{t=0}^{LOS_{max}^{IC}} \sum_{s=1}^S \mathbb{E}[Y_{((d-t) \bmod D),s,t}^{IC}] \\
&= \sum_{t=0}^{LOS_{max}^{IC}} \sum_{s=1}^S \sum_{l \in \mathcal{L}} p_{s,t}^{IC} \mathbb{E}[U_{s,l}] y_{((d-t) \bmod D),s,l}.
\end{aligned} \quad (1.10)$$

With

$$\begin{aligned}
T_{d,k}^{IC} &:= \{t \in \{0, \dots, LOS_{max}^{IC}\} | (d-t) \bmod D = k\}, \\
\tilde{m}_{d,s,k,l}^{IC} &:= \sum_{t \in T_{d,k}^{IC}} p_{s,t}^{IC} \mathbb{E}[U_{s,l}]
\end{aligned}$$

we can see that the expected value of Z_d^{IC} is a linear combination of the $y_{d,s,l}$:

$$\begin{aligned}
\mathbb{E}[Z_d^{IC}] &= \sum_{k=0}^D \sum_{s=1}^S \sum_{l \in \mathcal{L}} \tilde{m}_{d,s,k,l}^{IC} y_{d,s,l} \\
&= y^T m_d^{IC}
\end{aligned} \quad (1.11)$$

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for appropriately chosen $m_d^{IC} \in \mathbb{R}^N$.

Additionally, we define

$$\tilde{v}_{d,s,k,l}^{IC} := \sum_{t \in T_{d,k}^{IC}} p_{s,t}^{IC} (1 - p_{s,t}^{IC}) \mathbb{E}[U_{s,l}] + p_{s,t}^{IC} \mathbb{V}[U_{s,l}]$$

and obtain the variance of Z_d^{IC} :

$$\begin{aligned} \mathbb{V}[Z_d^{IC}] &= \sum_{t=0}^{LOS_{max}^{IC}} \sum_{s=1}^S \mathbb{V}[Y_{((d-t) \pmod{D}),st}^{IC}] \\ &= \sum_{t=0}^{LOS_{max}^{IC}} \sum_{s=1}^S \sum_{l \in \mathcal{L}} (p_{s,t}^{IC} (1 - p_{s,t}^{IC}) \mathbb{E}[U_{s,l}] + p_{s,t}^{IC} \mathbb{V}[U_{s,l}]) y_{((d-t) \pmod{D}),s,l} \\ &= \sum_{k=0}^D \sum_{s=1}^S \sum_{l \in \mathcal{L}} \tilde{v}_{d,s,k,l}^{IC} y_{d,s,l} \\ &= y^T v_d^{IC}, \end{aligned}$$

again for appropriately chosen $v_d^{IC} \in \mathbb{R}^N$. Note that it is reasonable to assume $m_d^{IC}, v_d^{IC} > 0$, because $\mathbb{E}[U_{s,l}] \geq 1$ and usually $p_{s,t}^{IC} > 0$ for all $t \leq D$.

The Berry–Esseen theorem states that we can approximate the distribution of Z_d^{IC} by a normal distribution with the same mean and variance (Fel71, p.544):

$$Z_d^{IC} \sim \mathcal{N}(y^T m_d^{IC}, y^T v_d^{IC}). \quad (1.12)$$

To ensure that the number of patients in the ICU on day d will not exceed the number of available beds C^{IC} we formulate the following chance constraint:

$$\mathbb{P}[Z_d^{IC} \leq C^{IC}] \geq 1 - \alpha^{IC} \quad (1.13)$$

for some $0 < \alpha^{IC} \ll 1$. Let Φ denote the cumulative distribution function of the standard normal distribution. Note that Φ is monotone, hence 1.13 is equivalent to

$$\begin{aligned} &\Phi\left(\frac{C^{IC} - y^T m_d^{IC}}{\sqrt{y^T v_d^{IC}}}\right) \geq 1 - \alpha^{IC} \\ \Leftrightarrow &\frac{C^{IC} - y^T m_d^{IC}}{\sqrt{y^T v_d^{IC}}} \geq \underbrace{\Phi^{-1}(1 - \alpha^{IC})}_{=: \phi_{\alpha^{IC}}} \\ \Leftrightarrow &C^{IC} - y^T m_d^{IC} \geq \phi_{\alpha^{IC}} \sqrt{y^T v_d^{IC}}. \end{aligned} \quad (1.14)$$

Since $\phi_{\alpha^{IC}} \sqrt{y^T v_d^{IC}} \geq 0$, it follows that $C^{IC} - y^T m_d^{IC} \geq 0$ and therefore 1.14 is equivalent to

$$(C^{IC} - y^T m_d^{IC})^2 \geq \phi_{\alpha^{IC}}^2 y^T v_d^{IC} \quad \text{and} \quad C^{IC} - y^T m_d^{IC} \geq 0. \quad (1.15)$$

1. Problem Statement

Analogously, we obtain constraints for the ward and finally have the following optimization problem:

$$\begin{aligned}
(P) \quad & \max_{y \in \mathbb{N}^N} \quad r^T y \\
& \text{subject to} \quad A_{LB} y \leq b_{LB} \\
& \quad \quad \quad A_{UB} y \leq b_{UB} \\
& \quad \quad \quad (C^i - y^T m_d^j)^2 \geq \phi_{\alpha^j}^2 y^T v_d^j \quad \text{for } d \in \{1, \dots, D\}, j \in \{IC, W\} \\
& \quad \quad \quad C^j - y^T m_d^j \geq 0 \quad \text{for } d \in \{1, \dots, D\}, j \in \{IC, W\} \\
& \quad \quad \quad y_{d,s,l} = 0 \quad \text{for } d \in \mathcal{D}, s \in \{1, \dots, S\}, l \in \mathcal{L}
\end{aligned}$$

The quadratic constraints are not convex but a simulation suggests that they are close to linear on the positive quadrant. Thus, our next step is the linearization of these constraints to obtain a linear approximation of the problem (P).

1.2.1. Linearization of the Chance Constraint

In Section 1.2 we found a quadratic formulation for the bed capacity constraints. Now we want to approximate each of the quadratic constraints by two linear constraints, one that extends the feasible region of (P) and hence possibly allows for solutions that are not feasible for (P) and one that truncates the feasible region, hence possibly missing the optimal solution of (P). In this way we obtain a lower and upper bound on the optimal value of (P).

Since all considerations apply equally to all days and to the ICU as well as the ward, the superscripts “IC” and “W” and all subscripts will be dropped in this section. This simplifies the notation of the chance constraint 1.15 to

$$(C - y^T m)^2 \geq \phi^2 y^T v \quad \text{and} \quad C - y^T m \geq 0. \quad (1.16)$$

and we consider the following relaxation of (P):

$$\begin{aligned}
(Q) \quad & \max_{y \in \mathbb{R}_{\geq 0}^N} \quad r^T y \\
& \text{subject to} \quad (C - y^T m)^2 \geq \phi^2 y^T v \\
& \quad \quad \quad C - y^T m \geq 0.
\end{aligned}$$

To learn more about the feasible region of (Q) we start by reformulating the quadratic constraint of 1.16:

$$\begin{aligned}
& (C - y^T m)^2 \geq \phi^2 y^T v \\
\Leftrightarrow & C^2 - 2Cm^T y + (m^T y)^2 \geq \phi^2 y^T v \\
\Leftrightarrow & y^T m m^T y - (2Cm^T + \phi^2 v^T) y \geq -C^2.
\end{aligned} \quad (1.17)$$

1.2. Constraint for Bed Capacity

The matrix mm^T is symmetric positive semidefinite implying that

$$K := \{y \mid (C - y^T m)^2 \leq \phi^2 y^T v\}$$

is the sublevel set of a convex function and therefore convex (BV04, p.71, p.75). Now we check whether ∂K has any intersections with the coordinate axes. To find intersections with the i -th axis we set $y_j = 0$ for all $j \neq i$. As mentioned before it is reasonable to assume $m > 0$, thus for $y \in \partial K$ the following must hold:

$$\begin{aligned} (C - y_i m_i)^2 &= \phi^2 y_i v_i \\ \Leftrightarrow y_i^2 + (-2Cm_i - \phi^2 v_i) \frac{1}{m_i^2} y_i + \frac{C^2}{m_i^2} &= 0 \\ \Leftrightarrow y_i &= \frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2} \pm \sqrt{\left(\frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right)^2 - \frac{C^2}{m_i^2}} \end{aligned} \quad (1.18)$$

$$\Leftrightarrow y_i = \underbrace{\frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}}_{>0} \pm \sqrt{\underbrace{\left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2}}_{>0}}. \quad (1.19)$$

Since the radicand of the root is strictly positive, ∂K has two intersections with any axis and we define

$$\begin{aligned} y_i^- &\stackrel{1.19}{:=} \frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2} - \sqrt{\left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2}} \\ y_i^+ &\stackrel{1.18}{:=} \frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2} + \sqrt{\left(\frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right)^2 - \frac{C^2}{m_i^2}}. \end{aligned}$$

It is straightforward that the intersection of the linear constraint from 1.16 with the i -th coordinate axis $\frac{C}{m_i}$ lies in between y_i^- and y_i^+ :

$$y_i^+ > \frac{C}{m_i} = \frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2} - \sqrt{\left(\frac{\phi^2 v_i}{2m_i^2}\right)^2} > y_i^- \quad \text{for all } i. \quad (1.20)$$

Furthermore, we note the following:

$$\begin{aligned} y_i^- > 0 &\Leftrightarrow \frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2} > \sqrt{\left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2}} \\ &\Leftrightarrow \left(\frac{C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right)^2 > \left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2} \\ &\Leftrightarrow \left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2} + \frac{C^2}{m_i^2} > \left(\frac{2C}{m_i} + \frac{\phi^2 v_i}{2m_i^2}\right) \frac{\phi^2 v_i}{2m_i^2} \\ &\Leftrightarrow \frac{C^2}{m_i^2} > 0. \end{aligned}$$

1. Problem Statement

Since we assume $C, m_i > 0$, we have $y_i^- > 0$ for all i , therefore the chance constraint will look somewhat like Figure 1.1 where the feasible region is outside of K and below the linear constraint.

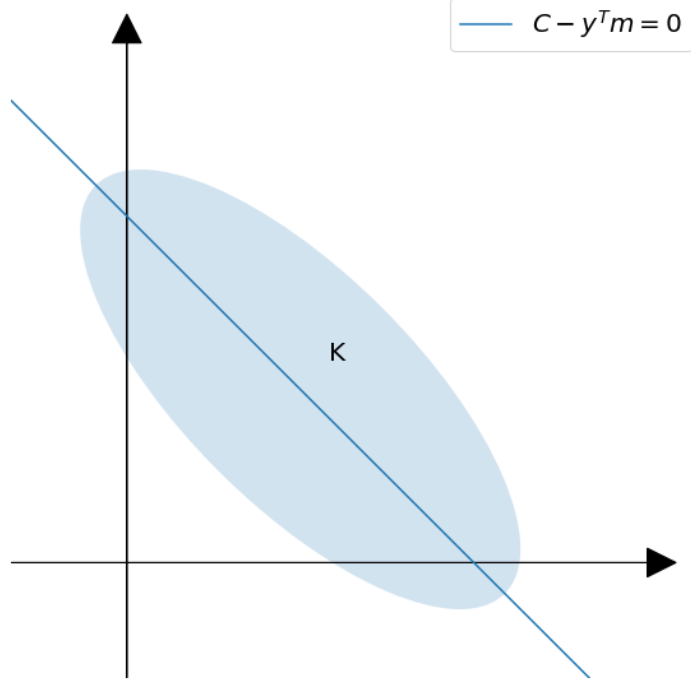


Figure 1.1.: Sketch of the chance constraint

Note that Figure 1.1 is merely a sketch and can be misleading since the matrix mm^T in 1.17 has rank 1 implying that K is unbounded. However, unboundedness of K is of no concern for the following considerations.

Optimistic Linearization

First we aim to replace the quadratic constraints in (Q) by a linear one such that the feasible region is extended. More precisely, we consider

$$\begin{aligned} (Q_+) \quad & \max_{y \in \mathbb{R}_{\geq 0}^N} \quad r^T y \\ & \text{subject to} \quad a_+^T y \leq 1 \\ & \quad \quad \quad C - y^T m \geq 0 \quad (LC) \end{aligned}$$

and search for $a_+ \in \mathbb{R}^N$ such that $F_{(Q)} \subseteq F_{(Q_+)}$, where $F_{(Q)}$ and $F_{(Q_+)}$ denote the feasible region of optimization problems (Q) and (Q_+) respectively.

The idea is to use the smaller intersections y_i^- of ∂K with the axes to span a hyperplane. Since K is convex, this hyperplane will truncate K , while ensuring all points feasible for (Q) will still be feasible for (Q_+) (see figure 1.2).

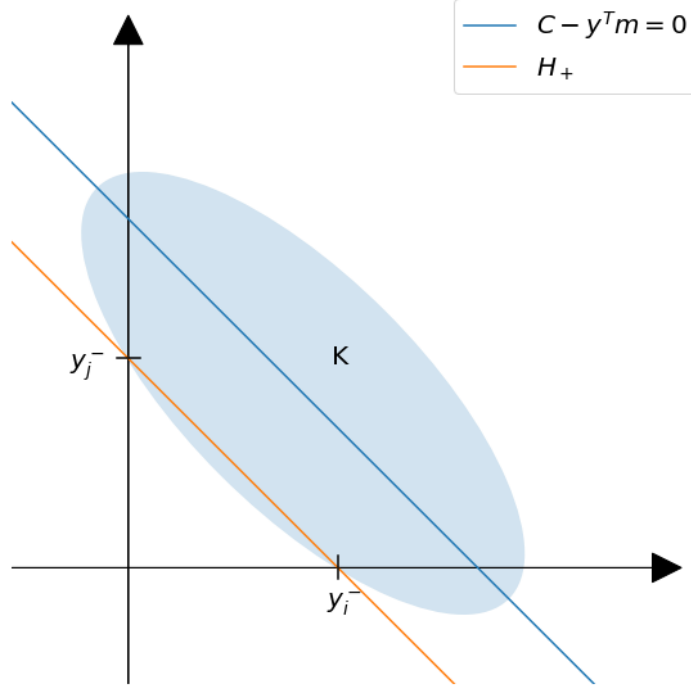


Figure 1.2.: Sketch of the optimistic approximation of the chance constraint

Now we define the hyperplane that contains all y_i^- as:

$$H_+ := \{y \mid a_+^T y = 1\},$$

where $(a_+)_i = \frac{1}{y_i^-} > 0$. Then by convexity of K we have $H_+ \cap \mathbb{R}_{\geq 0} = \text{conv}\{y_i^-\} \subseteq K$ and $F_{(Q)} \subseteq F_{(Q_+)}$. Hence, if $p_{(Q)}^*$ is the optimal value of the optimization problem (Q) , it holds that $p_{(Q)}^* \leq p_{(Q_+)}^*$. Also note that by 1.20 y_i^- is smaller than the intersection of the linear constraint (LC) of (Q) with the i -th axis implying that (LC) is redundant. Therefore, we define a new linear integer program by omitting the linear parts of the chance constraints and replacing each of the quadratic constraints in (P) by a linear approximation as described above:

$$\begin{aligned} (P_+) \quad & \max_{y \in \mathbb{N}^N} \quad r^T y \\ & \text{subject to} \quad A_{LB} y \leq b_{LB} \\ & \quad \quad \quad A_{UB} y \leq b_{UB} \\ & \quad \quad \quad (a_{+,d}^j)^T y \leq 1 \quad \text{for } d \in \{1, \dots, D\}, j \in \{IC, W\} \\ & \quad \quad \quad y_{d,s,l} = 0 \quad \quad \text{for } d \in \mathcal{D}, s \in \{1, \dots, S\}, l \in \mathcal{L}. \end{aligned}$$

Conservative Linearization

Now we want to find another linear approximation of the quadratic constraint such that the feasible region of (Q) is truncated.

1. Problem Statement

For this we can use any supporting hyperplane for K in any point in $\partial K \cap F_{(Q)}$. We choose the supporting hyperplane in a point of the form

$$(x, \dots, x)^T \in \partial K. \quad (1.21)$$

To find the corresponding $x \in \mathbb{R}_{>0}$ we calculate the following:

$$\begin{aligned} & (x, \dots, x)^T \in \partial K \\ \Leftrightarrow & \left(C - x \underbrace{\sum_i m_i}_{=:M} \right)^2 = \phi^2 x \underbrace{\sum_i v_i}_{=:V} \\ \Leftrightarrow & x^2 - x(2CM + \phi^2 V) \frac{1}{M^2} + \frac{C^2}{M^2} = 0 \\ \Leftrightarrow & x = \frac{2CM + \phi^2 V}{2M^2} \pm \sqrt{\frac{(2CM + \phi^2 V)^2}{4M^4} - \frac{C^2}{M^2}} \end{aligned} \quad (1.22)$$

$$\begin{aligned} \Leftrightarrow & x = \frac{2CM + \phi^2 V}{2M^2} \pm \frac{\phi}{2M^2} \sqrt{4CMV + \phi^2 V^2} \\ \Leftrightarrow & x = \frac{C}{M} + \frac{\phi^2 V}{2M^2} \pm \frac{\phi}{2M^2} \underbrace{\sqrt{4CMV + \phi^2 V^2}}_{\geq 0}. \end{aligned} \quad (1.23)$$

We are interested in the solution that is closest to the origin since we want to exclude all of K from the feasible region, hence we define

$$\hat{y} := (x_-, \dots, x_-)^T \quad \text{with} \quad x_- \stackrel{1.23}{:=} \frac{2CM + \phi^2 V}{2M^2} - \frac{\phi}{2M^2} \sqrt{4CMV + \phi^2 V^2}$$

to be the point for which we seek a supporting hyperplane H_- (see Figure 1.3). Note that by 1.22 we have

$$\begin{aligned} x_- &= \frac{2CM + \phi^2 V}{2M^2} - \sqrt{\frac{(2CM + \phi^2 V)^2}{4M^4} - \frac{C^2}{M^2}} \\ &> \frac{2CM + \phi^2 V}{2M^2} - \sqrt{\frac{(2CM + \phi^2 V)^2}{4M^4}} \\ &= 0 \end{aligned}$$

If $\partial K \cap \mathbb{R}_{\geq 0}^{DSL}$ is the graph of a differentiable convex function, the tangent space of ∂K in \hat{y} is a supporting hyperplane for K . We will now look for a function

$$f: \mathbb{R}_{\geq 0}^{DSL-1} \rightarrow \mathbb{R} \quad (1.24)$$

$$\bar{y} \mapsto f(\bar{y}) \quad \text{s.t.} \quad \begin{pmatrix} f(\bar{y}) \\ \bar{y} \end{pmatrix} \in \partial K. \quad (1.25)$$

Let m_i and v_i be the i -th entry of m and v respectively. To simplify the notation for the following calculations we define $\bar{m} := (m_2, \dots, m_N)^T$ and $\bar{v} := (v_2, \dots, v_N)^T$.

1.2. Constraint for Bed Capacity

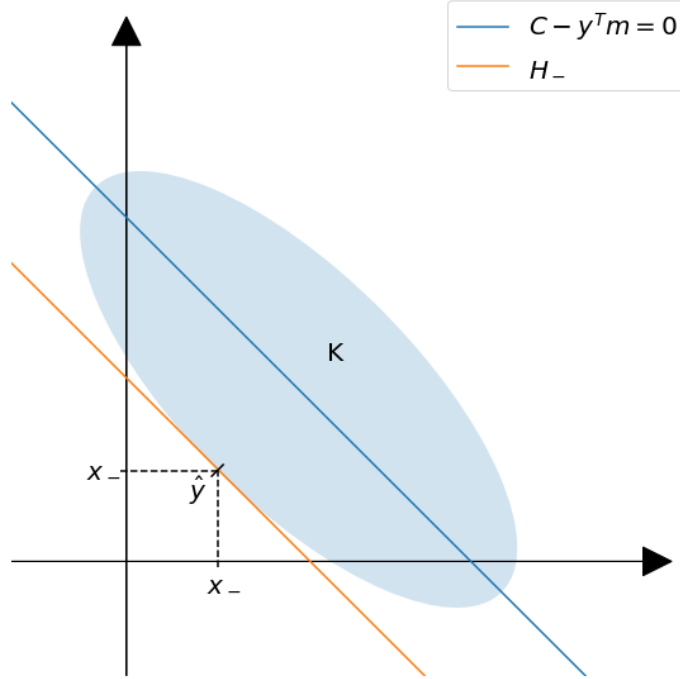


Figure 1.3.: Sketch of the conservative approximation of the chance constraint

$$\begin{aligned}
 & \begin{pmatrix} y_1 \\ \bar{y} \end{pmatrix} \in \partial K \\
 \Leftrightarrow & (C - (m_1 y_1 + \bar{m}^T \bar{y}))^2 = \phi^2 (v_1 y_1 + \bar{v}^T \bar{y}) \\
 \Leftrightarrow & m_1^2 y_1^2 - (2m_1 (C - \bar{m}^T \bar{y}) + \phi^2 v_1) y_1 + (C - \bar{m}^T \bar{y})^2 - \phi^2 \bar{v}^T \bar{y} = 0 \\
 \Leftrightarrow & y_1 = \frac{2m_1 (C - \bar{m}^T \bar{y}) + \phi^2 v_1}{2m_1^2} \\
 & \quad \pm \sqrt{\left(\frac{2m_1 (C - \bar{m}^T \bar{y}) + \phi^2 v_1}{2m_1^2} \right)^2 - \frac{(C - \bar{m}^T \bar{y})^2 - \phi^2 \bar{v}^T \bar{y}}{m_1^2}} \\
 \Leftrightarrow & y_1 = \frac{2m_1 (C - \bar{m}^T \bar{y}) + \phi^2 v_1}{2m_1^2} \pm \frac{\phi}{m_1} \underbrace{\sqrt{\frac{(C - \bar{m}^T \bar{y}) v_1}{m_1} + \frac{\phi^2 v_1^2}{4m_1^2} + \bar{v}^T \bar{y}}}_{=:g(\bar{y})} \quad (1.26)
 \end{aligned}$$

We are only interested in the behavior of f on the set

$$\text{dom}(f) := \{\bar{y} \in \mathbb{R}_{>0}^{N-1} | (f(\bar{y})^T, \bar{y}) \text{ is feasible for } (Q)\}.$$

To ensure that the expression in 1.26 is defined it is sufficient to verify that $C - \bar{m}^T \bar{y} \geq 0$ for $\bar{y} \in \text{dom}(f)$. Remember that $m^T y$ is the expected bed demand (see 1.11) and note

1. Problem Statement

that $\bar{m}^T \bar{y} < m^T y$ for $y \in \mathbb{R}_{>0}^N$. As long as we choose $\alpha < 0.5$, any y with $C - m^T y < 0$ must be infeasible for our problem (Q) (see 1.13). Hence, 1.26 is defined and since we aim to find a linear constraint that excludes all of K from the feasible region, we choose

$$f: \text{dom}(f) \rightarrow \mathbb{R} \quad \text{with} \quad f(\bar{y}) := \frac{2m_1 (C - \bar{m}^T \bar{y}) + \phi^2 v_1}{2m_1^2} - \frac{\phi}{m_1} \sqrt{\frac{(C - \bar{m}^T \bar{y}) v_1}{m_1} + \frac{\phi^2 v_1^2}{4m_1^2} + \bar{v}^T \bar{y}}. \quad (1.27)$$

to parametrize ∂K . Then f is a convex function and differentiable in the interior of $\text{dom}(f)$. Note that by definition $\hat{y} \in \partial K$ and with $\hat{y} > 0$ we have that $(f(\hat{y}), \hat{y})^T$ is in the interior of $\text{dom}(f)$. To determine the tangent space of f at \hat{y} we consider

$$\begin{aligned} \gamma_i: (-\epsilon, \epsilon) &\rightarrow \partial K \\ t &\mapsto \begin{pmatrix} f(z_i(t)) \\ z_i(t) \end{pmatrix}, \end{aligned}$$

with $z_i(t) = (x_-, \dots, x_-)^T + t e_i \in \mathbb{R}_{\geq 0}^{N-1}$, $i = 1, \dots, N-1$ where e_i denotes the i -th unit vector and $\epsilon > 0$ is small enough such that $z_i(\pm\epsilon)$ is in the domain of f . Then $\gamma_i(0) = \hat{y}$ for all i and therefore

$$w_i = \frac{d\gamma_i}{dt}(0) \quad (1.28)$$

$$= \begin{pmatrix} \frac{d}{dt}(f \circ z_i)(0) \\ z_i'(0) \end{pmatrix} \quad (1.29)$$

$$= \begin{pmatrix} \nabla f(z_i(0))^T z_i'(0) \\ z_i'(0) \end{pmatrix} \quad (1.30)$$

$$= \begin{pmatrix} \frac{\partial f}{\partial y_i}((x_-, \dots, x_-)^T) \\ e_i \end{pmatrix} \quad (1.31)$$

is a tangent vector at \hat{y} . The normal vector a_- of the desired hyperplane needs to be perpendicular to all tangent vectors:

$$\begin{aligned} (a_-)^T w_i &= 0 \quad \text{for all } i = 1, \dots, N-1 \\ \Leftrightarrow (a_-)_i \frac{\partial f}{\partial y_i}((x_-, \dots, x_-)^T) + (a_-)_i &= 0 \quad \text{for all } i = 1, \dots, N-1. \end{aligned} \quad (1.32)$$

If we choose $(a_-)_i = 1$, then from 1.32 it follows that

$$(a_-)_i = -\frac{\partial f}{\partial y_i}((x_-, \dots, x_-)^T)$$

and hence

$$a_- = \begin{pmatrix} 1 \\ -\nabla f((x_-, \dots, x_-)^T) \end{pmatrix}.$$

1.2. Constraint for Bed Capacity

Finally, we calculate the gradient of f from 1.27:

$$\nabla f(\bar{y}) = -\frac{1}{m_1}\bar{m} - \left(\bar{v} - \frac{v_1}{m_1}\bar{m}\right) \frac{\phi}{2m_1\sqrt{\frac{(C-\bar{m}^T\bar{y})v_1}{m_1} + \frac{\phi^2 v_1^2}{4m_1^2} + \bar{v}^T\bar{y}}}. \quad (1.33)$$

The hyperplane $H_- := \{y \mid a_-^T y = b_-\}$ is supposed to be supporting for K , therefore we choose b_- such that

$$b_- := (a_-)^T \hat{y}. \quad (1.34)$$

By (BV04, p.77) a_- is an inward pointing normal on the epigraph of f . By definition of f it holds that $K \cap \mathbb{R}_{>0}^N$ is a subset of the epigraph of f . Since no point of K is allowed to be feasible for our conservative linearization of (Q) and a_- is inward pointing for K , the linear approximation of the quadratic constraint must be

$$a_-^T y \leq b_-. \quad (1.35)$$

Hence, we replace every quadratic constraint of (P) by one constraint of the form 1.35. Again, this causes the linear part of the chance constraint to be redundant since the intersections of H_- with the coordinate axes are even smaller than the ones of H_+ and we obtain the conservative linear problem

$$\begin{aligned} (P_-) \quad & \max_{y \in \mathbb{N}^N} && r^T y \\ & \text{subject to} && A_{LB} y \leq b_{LB} \\ & && A_{UB} y \leq b_{UB} \\ & && (a_{-,d}^j)^T y \leq b_{-,d}^j \quad \text{for } d \in \{1, \dots, D\}, j \in \{IC, W\} \\ & && y_{d,s,l} = 0 \quad \text{for } d \in \mathcal{D}, s \in \{1, \dots, S\}, l \in \mathcal{L}. \end{aligned}$$

and we have $p_{(P)}^* \geq p_{(P_-)}^*$.

To find a close to optimal solution for (P) only using linear programs, we can now solve the LPs (P_+) and (P_-) . The solution y^* of (P_-) will also be feasible for (P) and with the optimal value of (P_+) we can evaluate the quality of y^* as solution for (P) .

2. Formulation as an Second-Order Cone Program

Another possibility to model our problem is to use binary decision variables. As we will see, this makes it possible to formulate the chance constraints as second-order cone (SOC) constraints without any need to linearize the chance constraints.

Definition 2.1 (Second-order cone constraint). (BV04, p. 156)

Let $y \in \mathbb{R}^n$ be an optimization variable, $A \in \mathbb{R}^{k \times n}$, $b, c \in \mathbb{R}^n$ and $d \in \mathbb{R}$. A constraint of the form

$$\|Ay + b\|_2 \leq c^T y + d$$

is called *second-order cone constraint*.

To formulate the chance constraint 1.14 in this way we use the same notation as before and introduce new variables $u_{d,s,l,i} \in \{0, 1\}$ such that:

$$y_{d,s,l} = \sum_{i=1}^{\infty} 2^{i-1} u_{d,s,l,i}. \quad (2.1)$$

Recall that N_{OR} is the number of available operating rooms and hence for every feasible solution y we have $y_{d,s,l} \leq N_{OR} = 2^{\log_2(N_{OR})} < 2^{\lfloor \log_2(N_{OR}) \rfloor + 1}$. With $k := \lfloor \log_2(N_{OR}) \rfloor$ we can thus simplify 2.1 to

$$y_{d,s,l} = \sum_{i=1}^{k+1} 2^{i-1} u_{d,s,l,i}$$

or equivalently

$$y = Bu,$$

where

$$B = \begin{pmatrix} 2^0 & \dots & 2^k & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & 2^0 & \dots & 2^k & 0 & \dots & \dots & 0 \\ \vdots & & & & & & \ddots & & & \vdots \\ 0 & & \dots & \dots & & & & 2^0 & \dots & 2^k \end{pmatrix} \in \mathbb{R}^{N \times N(k+1)}$$

and

$$u = \left(u_{1,1,l_1,1} \quad \dots \quad u_{1,1,l_1,k+1} \quad u_{1,1,l_2,1} \quad \dots \quad u_{1,1,l_L,k+1} \quad u_{1,2,l_1,1} \quad \dots \quad u_{D,S,l_L,k+1} \right)^T.$$

Then 1.14 with the simplified notation as before reformulates to

$$\begin{aligned} C - (Bu)^T m &\geq \phi \sqrt{(Bu)^T v} \\ \Leftrightarrow C - u^T \underbrace{(B^T m)}_{=: \hat{m}} &\geq \phi \sqrt{u^T \underbrace{(B^T v)}_{=: \hat{v}}}. \end{aligned}$$

Now note that with $\sigma_i := \sqrt{\hat{v}_i}$ and since u is binary the following holds:

$$u^T \hat{v} = \sum_{i=1}^{k+1} u_i \hat{v}_i = \sum_{i=1}^{k+1} u_i^2 \sigma_i^2 = \left\| \begin{pmatrix} u_1 \sigma_1 \\ \vdots \\ u_{N(k+1)} \sigma_{N(k+1)} \end{pmatrix} \right\|_2^2$$

and we obtain the constraint

$$\left\| \begin{pmatrix} u_1 \sigma_1 \\ \vdots \\ u_{N(k+1)} \sigma_{N(k+1)} \end{pmatrix} \right\|_2 \leq \frac{C - u^T \hat{m}}{\phi}. \quad (2.2)$$

If we define

$$A_{SOC} := \begin{pmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_{N(k+1)} \end{pmatrix}$$

we can reformulate 2.2 to the SOC constraint

$$\|A_{SOC} u\|_2 \leq \frac{C - u^T \hat{m}}{\phi}. \quad (2.3)$$

Finally, we formulate our original problem by replacing every chance constraint of (P) by a constraint of the form 2.3:

$$\begin{aligned} (SOCP) \quad & \max_{u \in \{0,1\}^{N(k+1)}} (B^T r)^T u \\ & \text{subject to} \quad A_{LB} B u \leq b_{LB} \\ & \quad \quad \quad A_{UB} B u \leq b_{UB} \\ & \quad \quad \quad \|A_{SOC,d}^j u\|_2 \leq \frac{C^j - u^T \hat{m}_d^j}{\phi_{\alpha^j}} \quad \text{for } d \in \{1, \dots, D\}, j \in \{IC, W\} \\ & \quad \quad \quad u_{d,s,l,i} = 0 \quad \text{for } d \in \mathcal{D}, s \in \{1, \dots, S\}, l \in \mathcal{L}, \\ & \quad \quad \quad \quad \quad \quad i \in \{1, \dots, k+1\}. \end{aligned}$$

3. Case Study

In this chapter we will apply the model described in Chapter 1 to a real world setting. More precisely, we will use data collected in a real hospital to determine all parameters needed to formulate our model and then use the linear programs (P_+) and (P_-) as well the exact formulation ($SOCP$) from Chapter 2 to find the optimal MSS for this hospital.

We hope to find an MSS with a cycle length of seven days that ensures a certain amount of surgeries can be performed, the specified bed capacities are not exceeded and at the same time leads to a more leveled utilization in the ICU and the ward than the MSS used by the hospital.

3.1. Data

The data used originates from a data collection over 2192 days at Charité Campus Virchow-Klinikum in Berlin. At this hospital there are eight specialties performing surgeries.

For each patient receiving surgery during the period of data collection the dates of admission and release as well as the responsible specialty are known. From this we calculated distributions for the total length of stay of the patients of each specialty (see Figure 3.1).

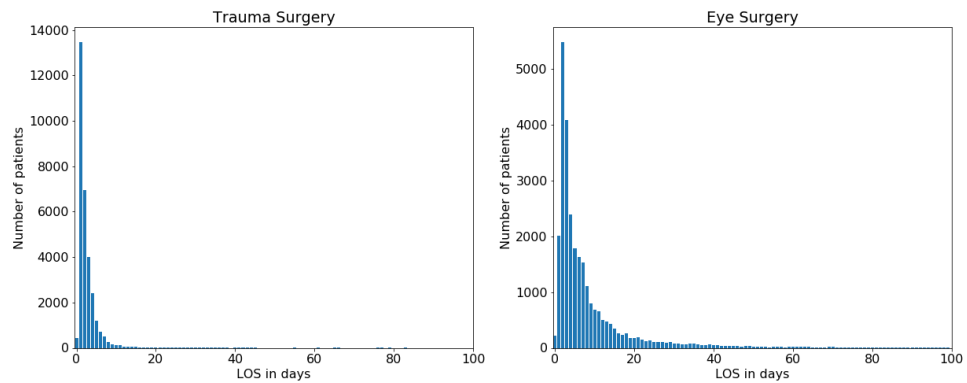


Figure 3.1.: Distribution of the total LOS for two specialties

We did not have any information about whether the patients had to go to the ICU and how long they stayed there. For our model we distinguished between the ICU and the ward, hence it is essential to have separate distributions for the two locations. In

Appendix A we describe our approach to generating separate distributions for the ICU and the ward from the distribution of the total LOS.

At the Virchow hospital the surgeons predict the duration of every surgery they plan to perform. The data included one distribution per specialty showing how often surgeries of a specific length were planned .

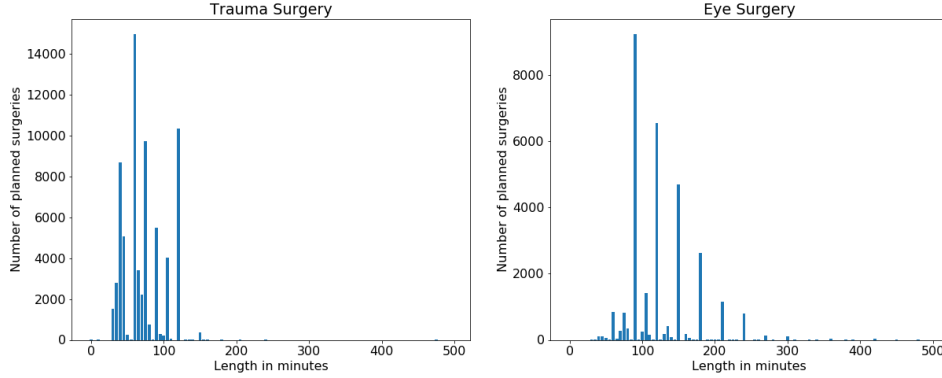


Figure 3.2.: Distribution of the planned surgery durations for two specialties

Despite the data lacking information about the number of surgeries performed during OR time of specific lengths we are able to estimate the needed data based on the planned durations (see Appendix B). This gives us reasonable values for $\mathbb{E}[U_{s,l}]$ required for the constraint (LB) for the lower bound on the OP hours. For the same constraint we need a value for λ_s , the mean arrival rate of the patients of each specialty s . This can be calculated by

$$\lambda_s = \frac{\text{Number of patients of specialty } s}{\text{Length of period of data collection}}.$$

The number of patients who received treatment by specialty s is easily recovered from the given data.

After gathering all parameters for (LB) we realized that for the problem (P) to be feasible the hospital would need an unrealistically large number of operating rooms N_{OR} . Therefore, we decided to set

$$\lambda_s = \frac{3}{4} \cdot \frac{\text{Number of patients of specialty } s}{\text{Length of period of data collection}}.$$

As described in the beginning of Chapter 1 we will exclude emergency surgeries from our model, hence setting λ_s like this might even be more accurate and together with $N_{OR} = 20$ the constraints (LB) and (UB) allow for a non-empty feasible region.

To obtain the vectors $m_d^{IC/W}$ and $v_d^{IC/W}$ for the chance constraint 1.14 we use the probabilities $p_{s,t}^{IC/W}$ as generated in Appendix A and $\mathbb{E}[U_{s,l}]$ and $\mathbb{V}[U_{s,l}]$ estimated as in Appendix B. Additionally, we have to choose $\alpha^{IC/W}$, the probability of exceeding the bed capacity in the ICU/ward. We assume that bed shortage in the ICU is worse than

3. Case Study

in the ward and set $\alpha^{IC} = 1\%$ and $\alpha^W = 2\%$. We have no information about the bed capacity of Virchow hospital, hence our approach on finding reasonable $C^{IC/W}$ was to choose values such that neither the chance constraints nor (UB) are redundant. We found that $C^{IC} = 420$ and $C^W = 250$ do as desired. In Figure 3.3 we see the first two dimensions of the constraints resulting from the chosen parameters.

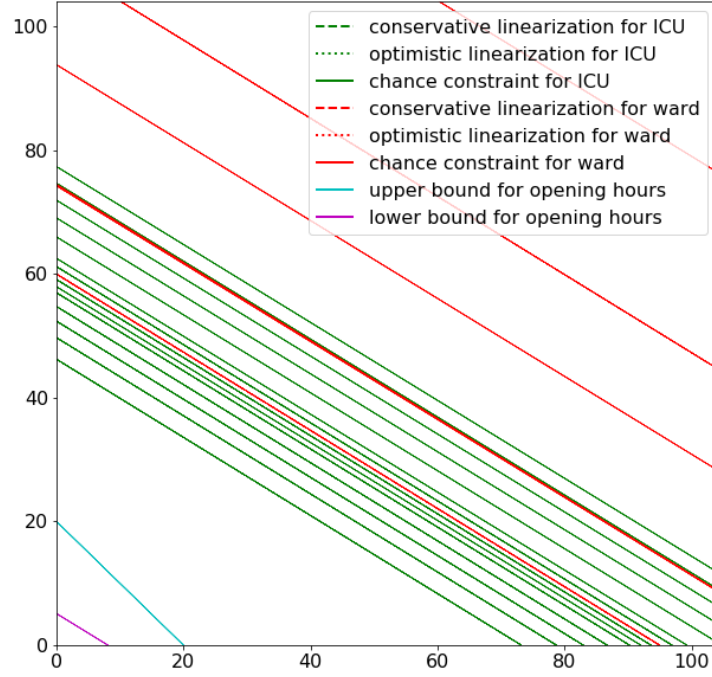


Figure 3.3.: Plot of dimension 1 and 2 of the constraints

As one can see, the chance constraints cannot be distinguished from the corresponding linearizations, suggesting errors induced by the linearization will be minor. If we zoom in on a small section of Figure 3.3 we can at least differentiate between the chance constraint and its conservative linearization (see Figure 3.4).

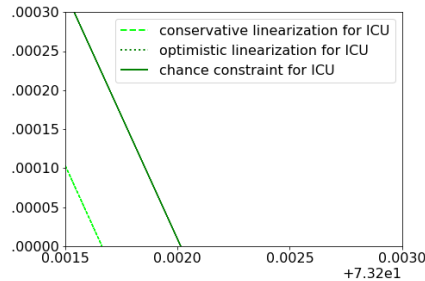


Figure 3.4.: Close up of one chance constraint and its linearizations

The feasible region in Figure 3.3 is between the lower and the upper bound for the

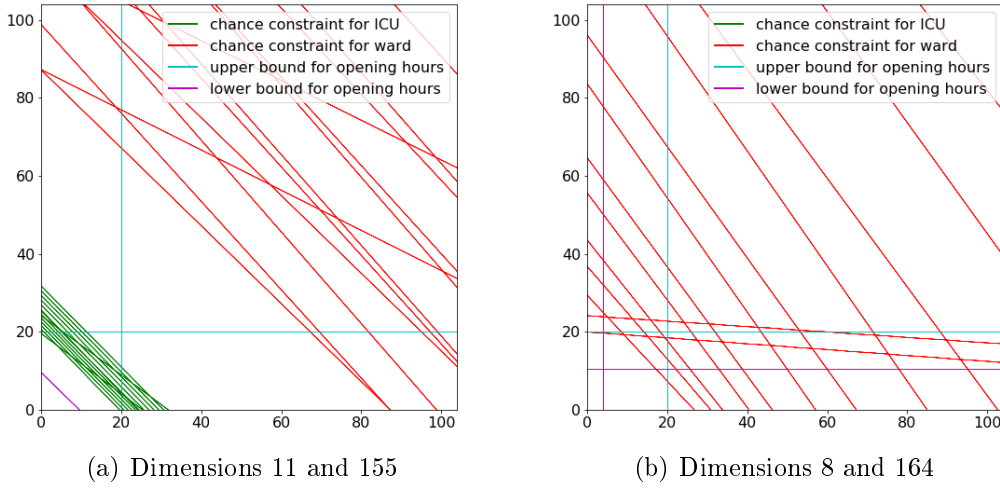


Figure 3.5.: Different dimensions of the constraints

opening hours, while the chance constraints seem to be redundant. Figure 3.5 however suggests that the chance constraints are indeed not redundant.

Finally, we have to decide on the allowed opening hours \mathcal{L} and the revenues $r_{s,l}$. We choose $\mathcal{L} = \{8, 12, 20\}$ and assume that the specialties generate the same revenue during OR time of the same length since there is no corresponding data available. Then we set $r_{s,8} = 0.75$, $r_{s,12} = 1$ and $r_{s,20} = 1.4$. Hence the decision variable y of the problems (P_+) and (P_-) has $7 \cdot 8 \cdot 3 = 168$ dimensions and both problems have a total number of 29 linear inequality constraints for the bound on the OP hours and the linearized chance constraints and 48 linear equality constraints for the weekends. Table 3.1 gives an overview of our chosen parameters.

Parameter	Description	Value
S	Number of specialties	8
D	Number of days in the cycle	7
N_{OR}	Number of operating rooms	20
$\mathcal{L} = \{l_1, l_2, l_3\}$	OR block lengths in hours	$\{8, 12, 20\}$
$C^{IC/W}$	Bed capacity in ICU/ward	420/250
$r_{s,l}$	Revenue per OR block of length 8 hours	0.75
	12 hours	1.00
	20 hours	1.40
$\alpha^{IC/W}$	Allowed probability of overshooting the bed demand in the ICU/ward	1%/2%

Table 3.1.: Overview of chosen parameters

3. Case Study

3.2. Results

We solved the linear programs (P_+) and (P_-) with Gurobi 7.5.2 and CVXPY 0.4.11 on Python 2.7.14 using the data and parameters described above and obtained two MSSs visualized in table 3.6. The entries of the table represent the number of operating rooms assigned to the specialties for the specified length. Note that the MSSs are relatively sparse, giving a large amount of operating rooms to one specialty on the same day. Moreover, the two MSSs differ only marginally and with respect to the objective value they also perform similarly. The optimistic solution has objective value $p_{(P_+)}^* = 98.45$ while the conservative solution has objective value $p_{(P_-)}^* = 98.35$.

Optimal MSS for (P_+)

	Monday			Tuesday			Wednesday			Thursday			Friday		
	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3
s_1			3												
s_2								2		1	9		10		
s_3							2			5		6	1		
s_4					6			10					1		
s_5		3								3	2		5	4	
s_6		2	4										2		
s_7		6	1		4			6							
s_8		1			10										

Optimal MSS for (P_-)

	Monday			Tuesday			Wednesday			Thursday			Friday		
	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3
s_1			3												
s_2										1	12				
s_3							3		1	4		7	1		
s_4		1			6			9		1					
s_5		3					2			3	1		6	5	
s_6			5										1	1	
s_7		6	1		4			6							
s_8		1			10										

Figure 3.6.: Solutions of the linearized problems

Due to the long running time of Gurobi's SOCP solver we were not able to solve the exact problem (*SOCP*). But note that the gap between the objective values of the two linearizations is small:

$$\frac{p_{(P_+)}^* - p_{(P_-)}^*}{p_{(P_-)}^*} \approx 0.1017\%$$

3.2. Results

and hence the optimal solution of (P_+) gives a satisfying solution for (P) as well and it is not absolutely necessary to solve $(SOCP)$.

To evaluate the performance of the found solutions we simulate the processes in a hospital using the corresponding MSSs to assign OR time to their specialties. For this we use the distributions obtained in appendix B to determine the number of surgeries any specialty will perform on a given day. Then for every patient undergoing surgery the LOS in the ICU and the ward are randomly drawn from the distributions generated in appendix A. We monitor the number of surgeries performed and the number of patients in the ICU and the ward over a time span of 2192 days (corresponding to the length of data collection at Virchow hospital) and obtain the following parameters:

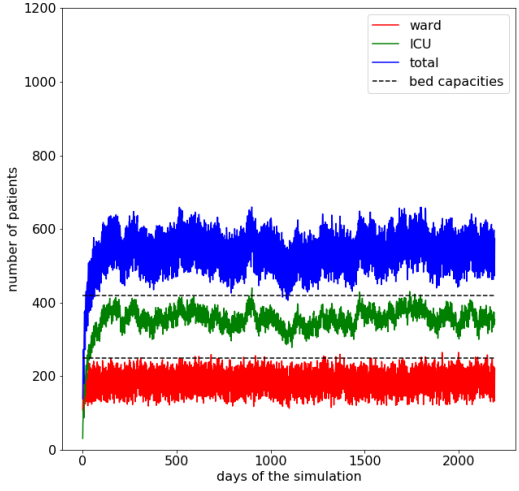
	optimistic	conservative
revenue	98.45	98.35
mean bed demand ICU	363.30	358.42
variance bed demand ICU	662.74	632.60
mean bed demand ward	203.21	203.88
variance bed demand ward	1090.48	1054.45
mean bed demand total	565.44	562.15
variance bed demand total	2568.15	2429.85
number of patients treated	178127	178301
probability of overshooting capacity ICU	0.5018%	0.0456%
probability of overshooting capacity ward	1.0949%	0.8211%

Surprisingly the number of treated patients for the optimistic MSS is smaller than for the conservative one. We believe this is caused by our choice of the objective vector r . Since r just consists of three different values the probability of having multiple optimal solutions is high. The number of treated patients is not part of the objective and hence there might be optimal solutions for (P_+) that would lead to a larger amount of performed surgeries. This particular solution for (P_+) seems to favor a specialty that performs surgeries with long durations and long length of stay in the ICU resulting in fewer procedures overall but a higher mean bed demand in the ICU. One also notes that for the optimistic solution the variances of the bed demand are higher than for the conservative solution. This could again result from the fact, that there are multiple optimal solutions and the variance in bed demand are not considered in the objective function. Another possible explanation is, that the optimistic linearization of the chance constraint allows for more freedom in the induced bed demand. Therefore it is not necessary to use the full capacities at all times to meet the goal of performing sufficiently many surgeries leading to more variance.

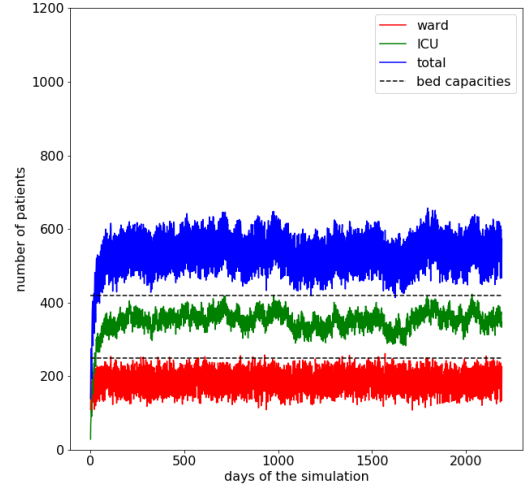
Most importantly, for both MSSs the probabilities of overshooting the bed capacities in the ICU and the ward are even smaller than anticipated and therefore both give satisfying solutions to our problem. Then again, this leaves room for improvement.

In figure 3.7 one can see the bed demand induced by the found MSS. We observe that the bed demand in the ICU is larger than in the ward. This does not seem to be realistic and is induced by our approach on finding separate distributions for the LOS in the ICU and the ward in appendix A that yields large mean LOSs in the ICU for most

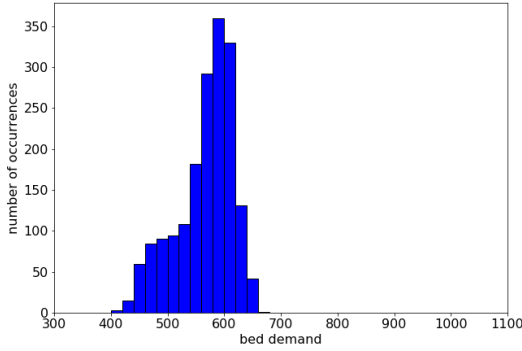
3. Case Study



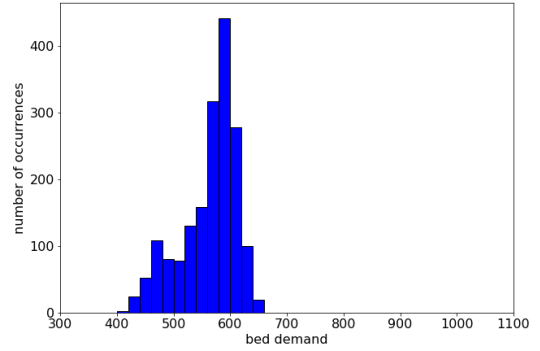
(a) Bed demand for the optimistic solution



(b) Bed demand for the conservative solution



(c) Total bed demand for the optimistic solution



(d) Total bed demand for the conservative solution

Figure 3.7.: Simulation of the bed demand induced by our solutions

of the specialties. Note that the simulation starts with empty ICU and ward and builds up bed demand as more and more patients receive treatment. Hence the first days of the simulated bed demand are biased and were therefore ignored when calculating the histograms and the moments of the corresponding distributions. Figure 3.8 shows the bed demand in Virchow hospital maintained from the data. The only available data with respect to bed demand consisted of the dates of admission and discharge for patients that arrived and left during the period of data collection but information about patients who already were in the hospital at the beginning or who had not been discharged before the end of data collection was missing. This explains the bias in bed demand in the beginning and end of the plot in figure 3.8. Again these days were ignored when calculating histogram and the moments of the distribution.

Comparing figure 3.7 to the bed demand maintained from the data (figure 3.8) we see that overall the bed demand in the simulations is smaller than in reality. This is due to our choice of the constraint (LB) where we only require our MSS to perform $\frac{3}{4}$ of

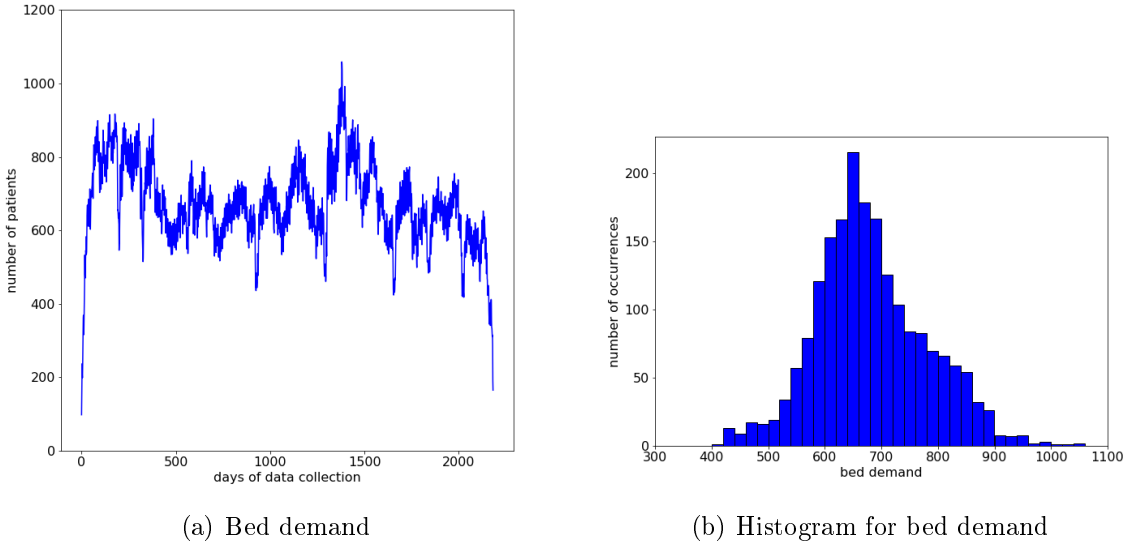


Figure 3.8.: Bed demand in Virchow hospital during the period of data collection

the surgeries that were performed in Virchow hospital. Furthermore the bed demands induced by our MSSs are very leveled compared to figure 3.8. Hence our solutions meet the goal of avoiding peaks in bed demand. To deterministically evaluate the leveling property of our solutions we consider:

	optimistic	conservative	data
mean bed demand total	565.44	562.15	681.40
variance bed demand total	2568.15	2429.85	9753.62
number of treated patients	178127	178301	220540

Both the optimistic as well as the conservative solution perform more than 80% of the number of surgeries in the data but the variance in the bed demand in the data is 3.7 times larger than the variance for the optimistic solution and even 4 times larger than the variance for the conservative solution. Hence compared to the data, our solutions perform very well in terms of leveling the bed demand.

In subsection 1.1.3 we formulated a constraint prohibiting the assignment of OR time on the weekends. This way we induced irregularities in the processes of the operating department. We are interested in the impacts of this constraints on the bed demand. Figure 3.9 shows the bed demand induced by the conservative solution for a period of five weeks. The minimal bed demands occur on Sundays. After that the bed demand increases rapidly in the beginning of the new week and slowly during the last weekdays until it starts decreasing during the weekend. This decrease is of course due to constraint (WE) and inevitable if no scheduled surgeries are performed during weekends. To explain the variation in increase during the week we consider the visualization of our solutions in figure 3.6 again. The longest OR blocks are only assigned on Mondays and throughout

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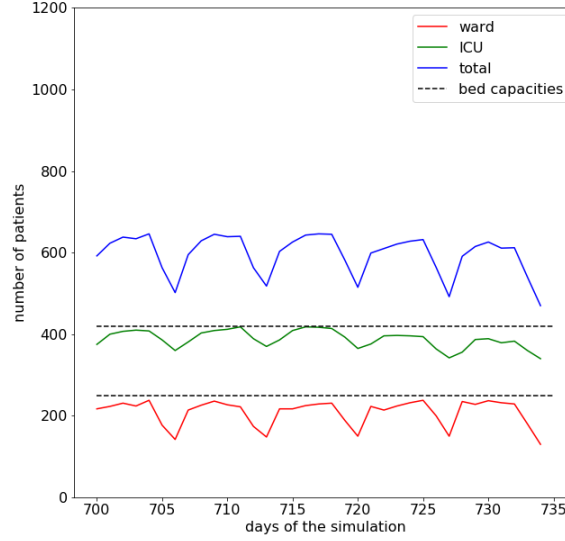


Figure 3.9.: Short period of bed demand for the conservative solution

the week the portion of short OR blocks becomes larger. This way the bed demand is quite leveled during most of the week despite the irregularities in patient arrivals.

3.2.1. Solving the SOCP in Lower Dimensions

We were not able to solve the SOCP formulation of the full problem due to running time issues. However, we reduced the dimension of the problem by only considering two of the eight specialties in this case paediatric surgery and neurosurgery. We chose the same parameters as for the complete problem but reduced the number of operating rooms to 8 and the number of beds in the ICU and the ward to 90 and 70. Moreover, we introduced some normally distributed noise to the revenue to face the problem of having multiple solutions. This approach might also be more realistic, since the generated revenue probably differs amongst the specialties. We were not able to find values for the revenue such that optimistic and conservative solution had different objective values but by adding noise we found the exact same solutions for (P_+) , (P_-) and $(SOCP)$. Again this proves that the linearizations give very good approximations of the chance constraints for this particular problem.

3.2.2. Leveling OR Staff Work Load

As we noted before the MSS visualized in figure 3.6 would not be desirable for a real world application since it assigns many OR blocks to single specialties on one day. This would lead to unlevelled work load for the OR staff and therefore increase costs. To tackle this problem we introduce a simplified version of constraint 1.5:

$$\sum_{l \in \mathcal{L}} y_{d,s,l} \leq c_s \quad \text{for all } d \in \{1, \dots, D\}, s \in \{1, \dots, S\}. \quad (3.1)$$

3.2. Results

This way we give an upper bound for the number of ORs one specialty can occupy on a single day. Reasonable values for c_s are obtained from the MSSs in 3.6:

s_i	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
c_{s_i}	1	3	4	4	4	2	4	3

With the additional constraints we solve the problems (P_+) and (P_-) with the parameters given in table 3.1 again. As in subsection 3.2.1 we add noise to the objective to decrease the probability of multiple optimal solutions and obtain the MSSs visualized in figure 3.10. As desired these MSSs are not as sparse and distribute the work load of the

Optimal MSS for (P_+)

	Monday			Tuesday			Wednesday			Thursday			Friday		
	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3
s_1			1		1			1		1				1	
s_2		1			3			3			3		1	2	
s_3		2	1	1			1			2			4		
s_4			3		3			3			3		1	2	
s_5		4		4			4			4			4		
s_6			2		2		2			1				1	1
s_7			4		4		3	1		2	2			1	
s_8			2		2			2		1	1		1	1	

Optimal MSS for (P_-)

	Monday			Tuesday			Wednesday			Thursday			Friday		
	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3	l_1	l_2	l_3
s_1			1	1				1			1			1	
s_2	1	2			3		3			2	1			3	
s_3	3			2	1									4	
s_4			4		2		4			3	1		4		
s_5			1		4		3	1		3	1		4		
s_6			2	1	1			2		1			1		
s_7			4		4			3			4				
s_8		2				1	1	2		3					

Figure 3.10.: Solutions of the linearized problems with additional constraints on OR staff capacities

OR staff more evenly throughout the week. Again we simulate the bed demand induced by the found MSSs producing the following:

3. Case Study

	optimistic	conservative
revenue	103.07	102.96
mean bed demand ICU	356.07	357.05
variance bed demand ICU	629.22	453.43
mean bed demand ward	198.74	197.84
variance bed demand ward	1176.42	1188.19
mean bed demand total	556.44	555.52
variance bed demand total	2617.45	2438.42
number of patients treated	174018	172914
probability of overshooting capacity ICU	0.1369%	0.0456%
probability of overshooting capacity ward	0.8669%	1.0036%

The revenue cannot be compared to the solutions before since the objective has been altered. But we can see that with the additional constraints considerably less patients were treated while the variance in bed demand increased except for the variance of the bed demand in the ICU induced by the conservative solution.

4. Conclusion

The main goal of this work is to find a cyclic master surgery schedule that optimizes a hospital's revenue while ensuring the bed demand in the ICU and the ward will not exceed the available capacities. In chapter 1 we established a model of the dependencies between the processes in the OR department and the bed demand in the ICU and the ward resulting in a non-convex optimization problem. By approximating the non-convex constraints with linear ones we hoped to obtain tight bounds on the objective value of the optimal solution to the original problem. We were also able to find an SOCP formulation that is equivalent to the original problem. We analyzed the performance of our model with real world data in section 3.2. Unfortunately, we were not able to solve the SOCP formulation due to the long running time of the used solver. However, the gap between the objective values of the linearized problems is remarkably small and hence the solution of the conservative linearization gives a satisfying solution for our problem. Compared to data collected in Virchow hospital the MSS found with our model performed better with respect to leveling the bed demand and the work load of nursing staff in the ICU and the ward. But one needs to consider that our MSSs only performed under simplified conditions in a simulation and hence we cannot draw reliable conclusions about the performance of our solution compared the MSS used by Virchow hospital. By introducing additional constraints on the daily OR time for the individual specialties we were able to achieve improvements with respect to leveling the work load of the OR staff at the price of reducing the total number of surgeries that can be performed and increasing the variance in bed demand. Overall, we presume that the MSS found in subsection 3.2.2 by adding said constraints is feasible in a real world setting and can improve the utilization of the downstream units by leveling the bed demand in the ICU and the ward without decreasing the revenue generated by the OR department. However, one should review the performance more detailed by building a more complex simulation of the processes in a real world hospital that includes for example the dependencies of surgery duration, LOS in the ICU and LOS in the ward. Additionally, we faced the obstacle of missing data that had to be generated. Before implementing our approach in a real world setting one should collect all needed data such as distributions for the LOS in the ICU and the ward, expected revenue and number of performed surgeries per OR block, capacities of staff and other resources, etc.

Moreover, the MSSs found with our approach do not fully utilize the bed capacities, more precisely the probability of exceeding the capacities in the simulation is smaller than demanded via the chance constraints. This could on the one hand be due to the fact that for the formulation of the chance constraint the real distribution of the bed demand has been approximated by a normal distribution. On the other hand our model presumes the bed demand in the ICU and the ward to be independent when in reality

4. *Conclusion*

these random variables are highly correlated. Hence by considering this correlation it might be possible to fully utilize the bed capacities, enabling the OR department to perform more surgeries and increasing the revenue of the hospital.

In section 3.2 we concluded that there probably are multiple solutions that have similar objective values. If leveling the bed demand is of interest for the hospital, one should consider including the variance in bed demand in the objective favoring solutions inducing small variance over slightly greater revenue.

Appendices

A. Seperate Distributions for the LOS in the ICU and the Ward

The available data contained distributions for the LOS after a surgery of any specialty. However it is lacking information about how many days of his stay a patient spend in intensive care. For our model it is necessary to have separate distributions for the LOS in the ICU and the ward.

Let LOS_s^{total} , LOS_s^{IC} and LOS_s^W be random variables denoting the number of days a patient of specialty s has to stay in the hospital overall, in the ICU and in the ward respectively. We want to retrieve reasonable approximations of LOS_s^{IC} and LOS_s^W from the known distribution of LOS_s^{total} . We assume LOS_s^{IC} and LOS_s^W to be negative binomially distributed as proposed in (CP14).

Definition A.1. (DeG75)[p.213ff.] Let $q \in (0, 1)$ and $r \in \mathbb{R}_{>0}$. A random variable X with the probability mass function

$$f(k, r, q) = \mathbb{P}[X = k] = \begin{cases} \binom{k+r-1}{k} q^k (1-q)^r, & \text{if } k \in \mathbb{N}_0 \\ 0, & \text{otherwise} \end{cases}$$

is called *negative binomially distributed* and we write $X \sim \text{NB}(r, q)$. The expected value and variance of X are given by

$$\mathbb{E}[X] = \frac{rq}{1-q} \quad \text{and} \quad \mathbb{V}[X] = \frac{rq}{(1-q)^2}. \quad (\text{A.1})$$

Thus in our case we assume:

$$\begin{aligned} LOS_s^{IC} &\sim \text{NB}(r_s^{IC}, q_s^{IC}) \\ LOS_s^W &\sim \text{NB}(r_s^W, q_s^W) \end{aligned}$$

with probability mass functions $f_s^{IC} = f(\cdot, r_s^{IC}, q_s^{IC})$ and $f_s^W = f(\cdot, r_s^W, q_s^W)$. Of course it must hold, that $LOS_s^{total} = LOS_s^{IC} + LOS_s^W$. If we assume that LOS_s^{IC} and LOS_s^W are independent, this implies that the probability mass function of the total length of stay is the discrete convolution of f_s^{IC} and f_s^W :

$$\mathbb{P}[LOS_s^{total} = k] = (f_s^{IC} * f_s^W)(k). \quad (\text{A.2})$$

From our data we know the value of $\mathbb{P}[LOS_s^{total} = k]$ for any s and k . Hence we search for $r_s^{IC}, q_s^{IC}, r_s^W, q_s^W$ such that the least squares error of A.2 is as small as possible.

The optimization of four parameters using the `curve_fit` function of SciPy version 0.19.0 did not yield acceptable results, therefore we reduced the number of parameters to be optimized by requiring the distributions of LOS_s^{total} and $LOS_s^{IC} + LOS_s^W$ to have the same mean and variance. Since we already assumed LOS_s^{IC} and LOS_s^W to be independent, we know that

$$\underbrace{\mathbb{E}[LOS_s^{total}]}_{:=\mu_s^{LOS}} = \mathbb{E}[LOS_s^{IC}] + \mathbb{E}[LOS_s^W] \quad (\text{A.3})$$

$$\underbrace{\mathbb{V}[LOS_s^{total}]}_{:= (\sigma_s^{LOS})^2} = \mathbb{V}[LOS_s^{IC}] + \mathbb{V}[LOS_s^W] \quad (\text{A.4})$$

must hold as well. With A.1 we can reformulate A.3 to

$$\begin{aligned} \mu_s^{LOS} &= \frac{q_s^{IC} r_s^{IC}}{1 - q_s^{IC}} + \frac{q_s^W r_s^W}{1 - q_s^W} \\ \Leftrightarrow r_s^{IC} &= \left(\mu_s^{LOS} - \frac{q_s^W r_s^W}{1 - q_s^W} \right) (1 - q_s^{IC}) \frac{1}{q_s^{IC}}. \end{aligned} \quad (\text{A.5})$$

Analogously we obtain a representation for r_s^W . We could proceed by reformulating such that we find representations for r_s^{IC} and r_s^W that only depend on q_s^{IC} and q_s^W and then using `curve_fit` to find optimal q_s^{IC} and q_s^W . But again this did not yield satisfying results. Instead we reformulate A.4 to

$$\begin{aligned} (\sigma_s^{LOS})^2 &= \frac{q_s^{IC} r_s^{IC}}{(1 - q_s^{IC})^2} + \frac{q_s^W r_s^W}{(1 - q_s^W)^2} \\ \Leftrightarrow r_s^{IC} &= \left((\sigma_s^{LOS})^2 - \frac{q_s^W r_s^W}{(1 - q_s^W)^2} \right) (1 - q_s^{IC})^2 \frac{1}{q_s^{IC}} \end{aligned} \quad (\text{A.6})$$

and merge A.5 and A.6 to obtain

$$\begin{aligned} \left(\mu_s^{LOS} - \frac{q_s^W r_s^W}{1 - q_s^W} \right) &= \left((\sigma_s^{LOS})^2 - \frac{q_s^W r_s^W}{(1 - q_s^W)^2} \right) (1 - q_s^{IC}) \\ \Leftrightarrow q_s^{IC} &= 1 - \frac{\left(\mu_s^{LOS} - \frac{q_s^W r_s^W}{1 - q_s^W} \right)}{\left((\sigma_s^{LOS})^2 - \frac{q_s^W r_s^W}{(1 - q_s^W)^2} \right)}. \end{aligned} \quad (\text{A.7})$$

A. Seperate Distributions for the LOS in the ICU and the Ward

Finally we replace q_s^{IC} in A.5 by its representation in A.7:

$$\begin{aligned}
 r_s^{IC} &= \frac{\left(\mu_s^{LOS} - \frac{q_s^W r_s^W}{1-q_s^W}\right)^2}{(\sigma_s^{LOS})^2 - \frac{q_s^W r_s^W}{(1-q_s^W)^2}} \cdot \frac{1}{1 - \frac{\mu_s^{LOS} - \frac{q_s^W r_s^W}{1-q_s^W}}{(\sigma_s^{LOS})^2 - \frac{q_s^W r_s^W}{(1-q_s^W)^2}}} \\
 &= \frac{\left(\mu_s^{LOS} - \frac{q_s^W r_s^W}{1-q_s^W}\right)^2}{(\sigma_s^{LOS})^2 - \mu_s^{LOS} + \frac{q_s^W r_s^W}{1-q_s^W} - \frac{q_s^W r_s^W}{(1-q_s^W)^2}} \\
 &= \frac{(\mu_s^{LOS} (1 - q_s^W) - q_s^W r_s^W)^2}{((\sigma_s^{LOS})^2 - \mu_s^{LOS}) (1 - q_s^W)^2 - (q_s^W)^2 r_s^W}. \tag{A.8}
 \end{aligned}$$

Now we use `curve_fit` to find the optimal values for r_s^W and q_s^W . The missing parameters r_s^{IC} and q_s^{IC} can then be calculated by A.7 and A.8 and we obtain distributions for LOS_s^{IC} and LOS_s^W as shown in figure A.1.

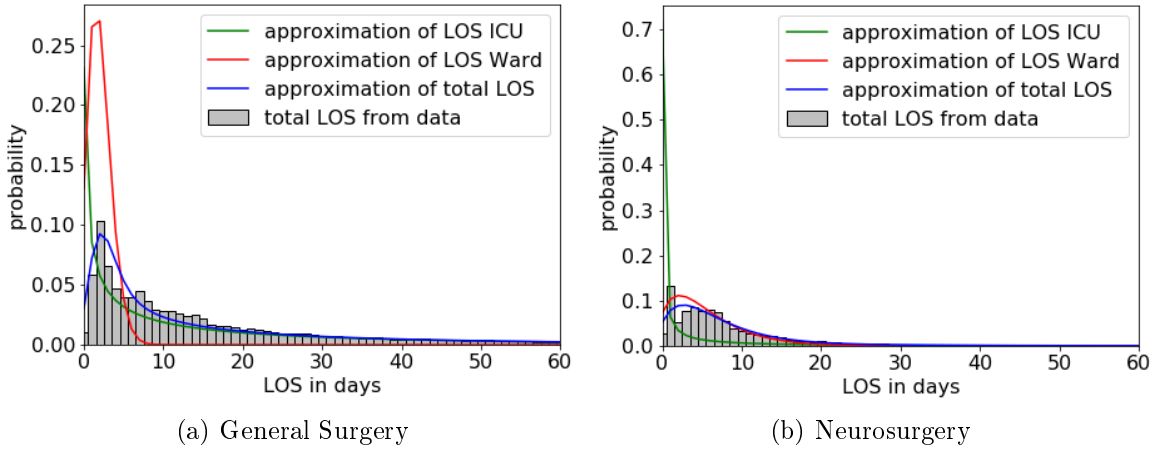


Figure A.1.: Approximation of the distributions for LOS_s^{IC} and LOS_s^W

The success of this approach naturally depends on the compatibility of the negative binomial distribution and the real data. As one can see in figure A.1 we can approximate the total LOS for general surgery, while the approximation for the LOS for neurosurgery does not fit the real data very well. Also note that exchanging the distributions of LOS_s^{IC} and LOS_s^W does not change the distribution of their sum and is therefore consistent with our approach. We decided that the probability of a patient having to go the ICU after a scheduled surgery should be relatively small. Based on this we chose the distributions such that $\mathbb{P}[LOS_s^{IC} = 0] > \mathbb{P}[LOS_s^W = 0]$.

With our previous considerations we are now able to calculate $p_{s,t}^{IC/W}$, the probability that a patient of specialty s is in the ICU or the ward t days after surgery. For the ICU, this is the probability, that a patient has to stay at least t days in the ICU after surgery:

$$p_{s,t}^{IC} = \mathbb{P}[LOS_s^{IC} \geq t]. \tag{A.9}$$

To be in the ward t days after surgery, a patient must have been $\tilde{t} < t$ days in the ICU and at least $t - \tilde{t}$ days in the ward:

$$p_{s,t}^W = \sum_{\tilde{t}=0}^{t-1} f_s^{IC}(\tilde{t}) \mathbb{P}[LOS_s^W \geq t - \tilde{t}]. \quad (\text{A.10})$$

The distributions obtained by A.9 and A.10 is shown in figure A.2. Furthermore, $p_{s,t}^{IC} + p_{s,t}^W$ is our approximation of $\mathbb{P}[LOS_s^{total} \geq t]$, the probability of a patient having to stay at least t days in the hospital given by the data. As one can see in figure A.2 our approach provides adequate results. In the case of general surgery however, our approach yields a

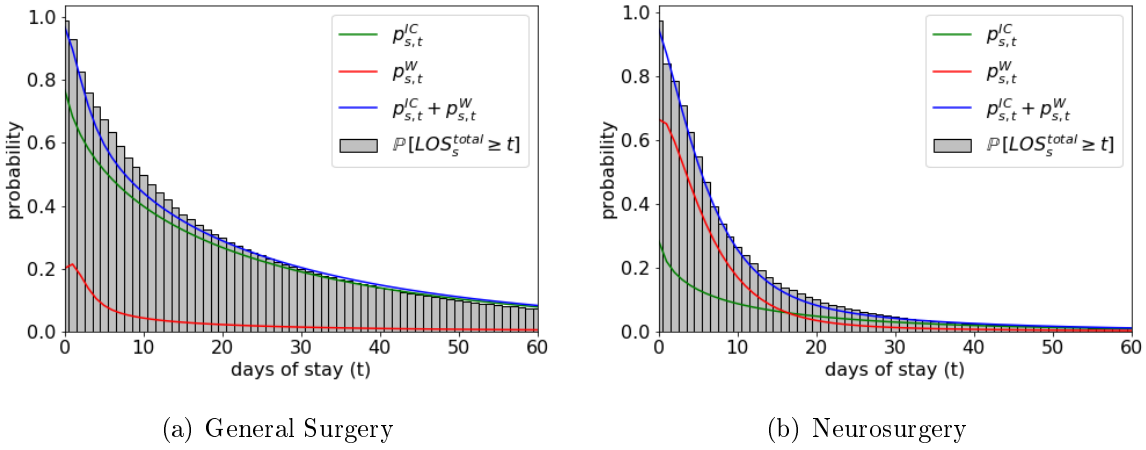


Figure A.2.: Approximation of the probability of a patient being in the ICU/ward t days after surgery

very high probability for a patient to be in the ICU after surgery. Additionally, for some surgery specialties we observe that the mean of the generated LOS in the ICU is larger than the mean LOS in the ward. In our model this will ultimately lead to a higher bed demand in the ICU than in the ward because patients tend to occupy beds in the ICU for long periods of time. We have no data about how close to reality this behavior is.

B. Generating Distributions for the Number of Performed Surgeries

For the formulation of some constraints of our model we need mean and variance of the random variable $U_{s,l}$ that represents the number of surgeries specialty s can perform during an OR block of length l . The data however only provided a distribution for the duration of single surgeries for the individual specialties (see figure 3.2). To generate reasonable distributions for $U_{s,l}$ we randomly draw surgery durations from the given data for specialty s until the combined lengths of the drawn surgeries exceeds the block length l . This process was repeated 1000 times. This way we obtained distributions for $U_{s,l}$ as shown in figure B.1.

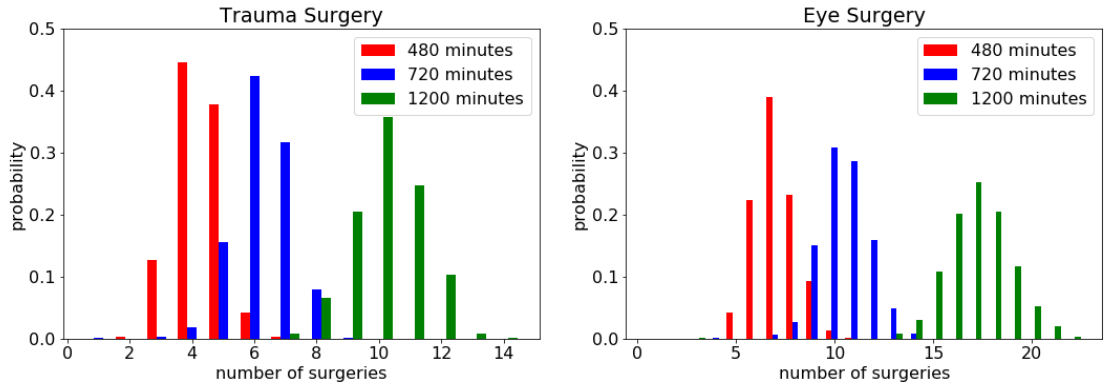


Figure B.1.: Distributions for $U_{s,l}$

This approach is quite greedy and there surely are more intelligent techniques but we assume that for real world applications distributions for $U_{s,l}$ will be available and hence did not focus on this problem in the scope of this work.

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