Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

WERNER BENGER, HANS-CHRISTIAN HEGE,
STEFAN HEUSLER

Visions of Numerical Relativity 1999

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Visions of Numerical Relativity 1999

Werner Benger, Hans-Christian Hege, Stefan Heusler*

Text accompanying the Video

Visual Impressions of Numerical Relativity 1999

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Abstract: We present visualizations of recent supercomputer simulations from numerical relativity, exploiting the progress in visualization techniques and numerical methods also from an artistic point of view. The sequences have been compiled into a video tape, showing colliding black holes, orbiting and merging neutron stars as well as collapsing gravitational waves. In this paper we give some background information and provide a glance at the presented sequences.

Keywords: General Relativity, Data Visualization, Numerical Simulations

1 Introduction

More than 80 years ago Einstein's theory of general relativity revolutionized physics and replaced Newton's theory of space, time and gravitation. Although Newtonian theory provided a very successful framework for centuries, astrophysical observations revealed small deviations, which could not be explained before general relativity was born. (Even an additional planet, called "Vulcan", had been predicted in order to correct for the unexplainable.) General relativity solved these mysterious phenomena and up to now all experimental evidence is in outstanding numerical coincidence with the theory.

On the other side, general relativity is much more complex than Newtonian theory, and suffers from the fact that no general way is known to solve Einstein's field equations analytically. While a couple of explicit solutions have been found, even relatively simple cases like the two-body problem still remain unsolved. For a long time analytic solutions were sought, without much success. Alternatives are:

- find approximate solutions with exact mathematical methods
- find exact solutions with approximate mathematical methods.

The first approach is realized in perturbation theories, which cover linear and post-Newtonian field theories. However, these theories are applicable only for weak gravitational fields and break down in the strong field regime, to which e.g.

*Arts of Science, www.artsofscience.de, Cologne, stefan@khm.de

collisions of black holes belong. The second approach employs numerical methods like finite differences for solving systems of partial differential equations. It is concentrated in the efforts of *Numerical Relativity*, a subfield that emerged during the 1960's, as computers became powerful enough to treat Einstein's field equations numerically [Hahn, Smarr, Seidel&Suen, Seidel].

However, due to limited computational power and not yet matured numerical methods the first attempts were restricted to simple cases, like axisymmetric collisions of black holes. In the 1990's big efforts were undertaken in the US, in particular at the National Center for Supercomputer Applications (NCSA) in Champaign, IL, among other sites, to achieve large steps forward. Some of these initiatives, like the Neutron Star Grand Challenge project, are still in progress [NASA]. The multi-institutional cooperation is now being continued [NumRel] with the recently founded Max-Planck-Institut für Gravitationsphysik Potsdam, also called the Albert-Einstein-Institute (AEI). One contribution of AEI to numerical relativity is the development of the Cactus simulation code [Cactus].

The efforts in numerical relativity culminated in extensive simulation runs performed in June 1999, when a new supercomputer at NCSA prior to official release was dedicated exclusively to numerical relativity simulations. The researchers could utilize this 256 processor SGI Origin 2000 for two weeks [NCSA], to perform previously undone computations of the first large-scale full 3D collision of black holes, of binary systems of neutron stars (i.e. doing general relativistic hydrodynamics) and collapsing gravitational waves [Allen, Butler]. The simulations consumed 140.000 CPU hours and output-ed about 1 terabyte of data.

In contrast to analytical solutions, whose content may be analyzed on a sheet of paper, numerical solutions need to be visualized to convey the results to scientists or to provide a notion of what happens to non-scientists. At this point, advanced visualization techniques, like those developed and implemented at the Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB) come in [Amira]. By employing these methods it was possible to create stunning visualizations of the huge set of simulation data [MPG].

Nowadays, the gap between science and art is very large. However, this has not always been the case. In the old Greek culture, "astronomy" has been regarded as one of the basic human "arts", beside geometry, architecture, theater, sculpture, painting, music and mathematics. With the steady development of scientific methods, the gap between "fine arts" and the "arts" nowadays called "science" grew larger and larger. However, this contrast is only partially caused by a principal gap: When the results of mathematical calculations can be visualized, rational and visual understanding come again very close to each other.

2 Colliding Black Holes

Visualization of data from numerical relativity simulations is not a straightforward task. The fundamental objects describing the gravitational field are four-dimensional tensor fields of high rank. For doing numerical computations, we need to work with the numerous (up to 256!) components of a tensor field. However, these components depend on the coordinates in which they are written

(i.e. which have been chosen for numerical purposes) and hence have no direct physical meaning.

Furthermore, the interpretation of these tensor fields is not straightforward: pure gravity is nothing more than curved "vacuum" and a black hole is "empty space" everywhere, so there is "nothing" to see – as compared to fluid dynamics – no particles, no gas, nothing. But this empty space is curved, according to Einstein's theory, and techniques to understand this curved space must be developed.

2.1 Horizon Information

Some way to look at curved space is to consider the "event horizon". It can be understood as the "surface of no return". Once inside a black hole horizon, nothing – not even light – can ever escape. This is the origin of the name "black hole".

For "usual" static black holes, the horizon is a sphere. But in general, e.g. during mergers of black holes, the horizon will acquire a much more complicated shape. Initially, the black holes carry their own horizons which then merge to form a larger one. This newly formed horizon will become spherical somewhat later, but at first it might be highly irregular due to the extreme gravitational forces acting between the black holes.

In order to locate the event horizon, the entire evolution of the space-time must be known – in numerical simulations this involves a huge amount of data. A concept alternative to the event horizon is the "apparent horizon". It is located always inside or at the event horizon. The apparent horizon can be located in individual time steps, which is much easier to do numerically.

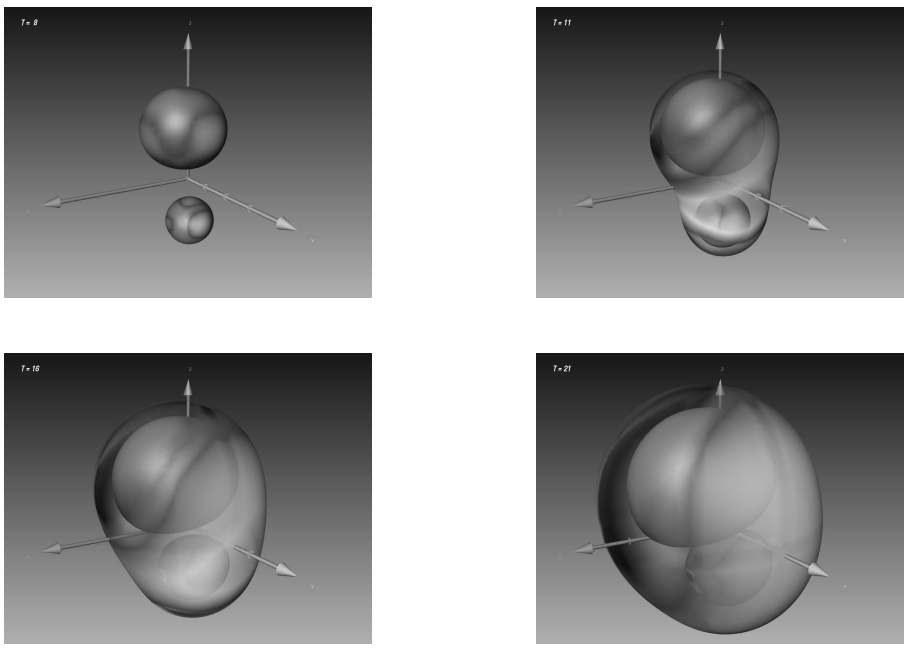

Abbildung 1: *Evolution sequence showing the apparent horizons of two merging black holes. An outer apparent horizon pops up at a certain time.*

2.2 The True Shape of the Apparent Horizon

To get an intuition what is really happening at the apparent horizon, it is useful to look at the Gaussian curvature of spacetime on this surface. The coordinate space, where the computed data reside, usually does not reflect the physical properties in space. Even if the computation points are distributed uniformly in coordinate space, their true physical distance may be completely different, ranging from zero to infinity (both extreme cases leading to numerical troubles for obvious reasons). This is similar to a uniform distribution of points in spherical coordinates – such a distribution is not uniform in Cartesian 3D coordinates. Using general relativistic coordinates, things might be even more complicated, and it is hardly possible to imagine the true shape of physical space. It might be (as an extreme example) that something looking like a sphere in coordinate space in reality is a rectangular box, and vice versa. By "reality" we mean what a physicist would measure locally, whereas the "coordinate space" are numbers which have been chosen for convenient computation. However, during the rendering process these numbers are treated as geometrical information, because depicting the true physical space is highly nontrivial and possibly even impossible within the "just" three dimensions that are available for rendering.

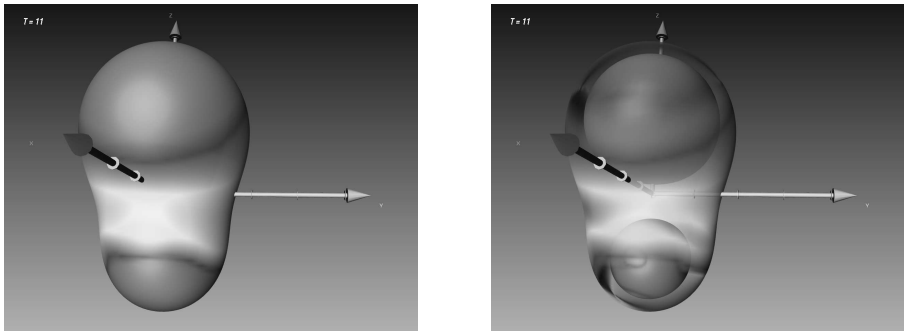

Abbildung 2: *Apparent horizon of two merged black holes, with color-encoded Gaussian curvatures. By assigning transparency to large homogeneous regions (right image), the interior is made visible.*

Nevertheless, by mapping the physical Gaussian curvature on some surface, e.g. the apparent horizon, we get an impression of the physical nature – for example although we might still see a box as a sphere, we might spot it as a box due to its colors. We chose the colors such that slight deviations from flat space or spherical shape jump into the viewer's eye. What we expect from the apparent horizon is to approach a spherical shape during the evolution. So all non-spherical perturbations shall attract the viewers attention, such that oscillations of the curvature stand out prominently. For this purpose, we normalize the curvature by the area of the surface, hence any sphere gets the same curvature value. We map this value to red. Somewhat larger values are mapped to the complementary color green to reveal strong contrast. Similarly, we map flat space (zero curvature) to yellow, the brightest color in the visible spectrum, and negative curvature values to the complementary color blue, see Fig. 2, left. With this color coding, regions of flat space and regions with spherical curvature are easy noticeable.

Additionally, by adjusting transparency appropriately, we may render large, homogeneous subvolumes transparent, thereby enabling a view to their inside, while still preserving significant features (Fig. 2, right).

2.3 The Gravitational Field

We also want to get a visual impression of the gravitational field. Fortunately, most of the 256 tensor field components carry redundant information. The so-called Riemann tensor may be split up in a trace-free part, the Weyl tensor, and the Ricci tensor, which is then further used in the Einstein field equations. In analogy to the 3+1 split of electrodynamics into the well-known Maxwell equations, we may write the Weyl tensor as an "electric" and "magnetic" part. An appropriate linear combination of these quantities reveals five complex quantities, the so-called "Weyl curvature scalars" [Newman&Penrose].

A technique used commonly for visualizing scalar fields is volume rendering. Each value within the three-dimensional data volume is assigned some optical properties like local emission of light and transparency. According to the selected color mappings, the results are "glowing clouds", i.e. bright subvolumes indicating specifically interesting regions of the scalar field, and more transparent areas depicting large homogeneous regions.

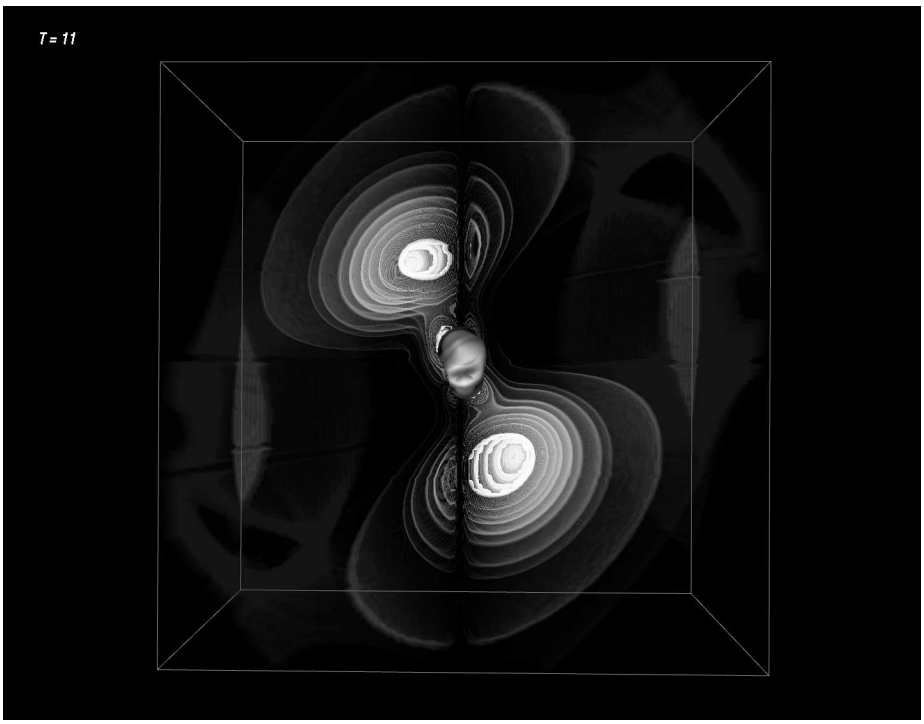

Abbildung 3: *A burst of gravitational energy emerging from two colliding black holes. The onion-like filaments indicate different levels of intensity of the gravitational field.*

However, while standard color maps result in compact clouds that hide inner structures, a special colormap may be chosen which assigns the "glow" property

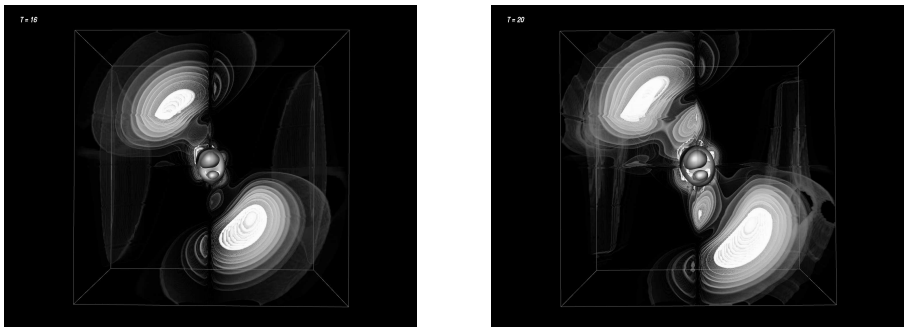

Abbildung 4: *The burst of gravitational energy flowing away...*

only to visually interesting, possibly equidistant levels of the scalar field. This results in tender, self-glowing skins of varying color and brightness, that enclose relevant regions like an onion (Fig. 3, 4 and 8). We call this visualization technique "volumetric isolevel rendering".

3 Orbiting Neutron Stars

3.1 Matter Density and Atmosphere

Visualizing neutron stars is a much easier task than picturing black holes. Neutron stars possess characteristics like matter density, energy per volume per density (internal energy), etc., as known from fluid dynamics. These quantities can be understood quite easily. The density of a neutron star is extremely high and falls off very rapidly at the surface. The "atmosphere" of a neutron star, i.e. the region where the matter density is comparable to the density of air and water, as we are used to from our home planet, can be measured only in millimeters or fractions of a millimeter.

A realistic mapping of a neutron star's matter density to optical properties would reveal just a sphere. However, using computer graphics we have the ability to render physically opaque regions transparent, so we perceive the otherwise invisible high-density regions around the neutron star's core.

By employing the technique of volumetric isolevels it is possible to bring out quite clearly the details of energy fluctuations. Choosing a "glowing" color map is appropriate here, since higher values of the scalar field indicate more energy in nature, so this gives a more physically-correct impression.

Generally speaking, neutron stars are less "abstract" than black holes and can therefore be pictured more realistically. To support this impression, a photo displaying real stars was chosen as background image. So these images touch the border between scientific visualizations and realistic computer graphic renderings, although they are still quite far from photorealism.

If we look at the sequence of the neutron star merger, Fig. 5,9 computed in Newtonian theory, the fluctuations of the atmosphere that come out provide so many details that the eye nearly gets lost. But, since all moving shock fronts fit into a consistent visual context of a global flow, it does not look chaotic. As the rational brain is overflooded by information, rationality is given up and emotionality wins. Indeed a lot of people had the impression that his

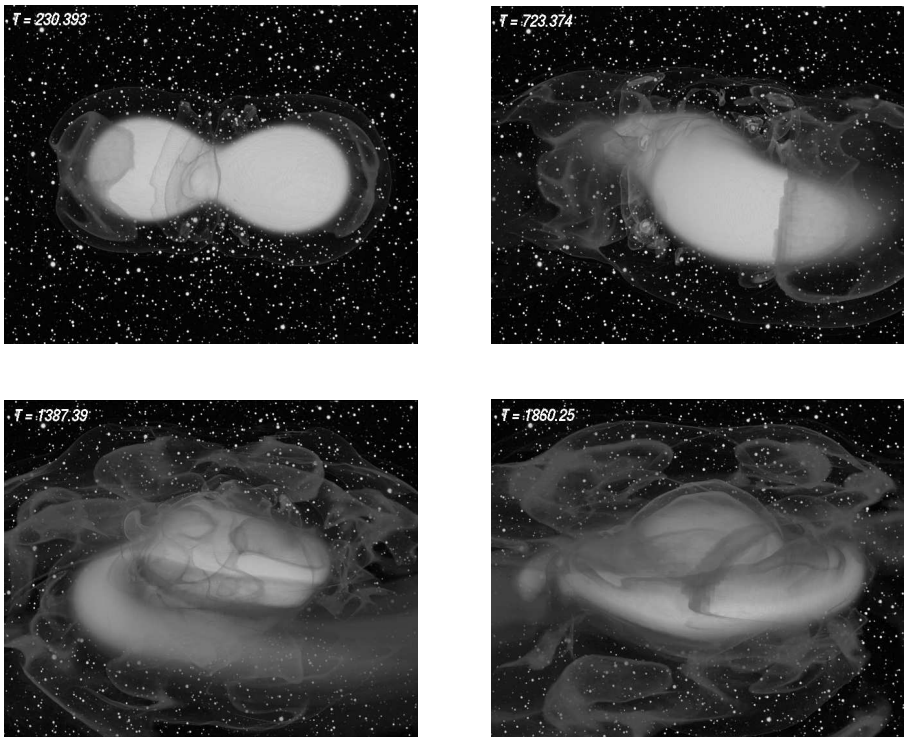

Abbildung 5: *Merging of two neutron stars computed using Newtonian theory. Each star has about the mass of the sun. Depicted are matter density (green) and internal energy (red).*

sequence "cries" for some acoustic accompaniment. Basically, this was one of our motivations to create a music-attributed video.

3.2 Additional Physical Quantities

In addition to matter properties, it is of interest to also visualize the geometrical shape of space around massive neutron stars, i.e. their gravitational field, especially the gravitational waves being emitted during collisions. The gravitational waves created by neutron star mergers are expected to be weaker than those from black hole collisions. Though the computational work to retrieve wave forms with clean physical foundation still needs further development, and the current data are of preliminary character, the first results look already promising, and give first impressions on how gravitational field quantities, often used in black hole spacetimes, might look like around neutron stars (s. Fig. 6).

4 Collapsing Gravitational Waves

In nature, black holes are formed when extreme matter concentrations occur, e.g. during supernova collapses of super-massive stars, or in collisions of neutron stars. Due to $E = mc^2$, extreme concentrations of pure energy may also lead to formations of black holes. This energy can also be gravitational energy ("empty

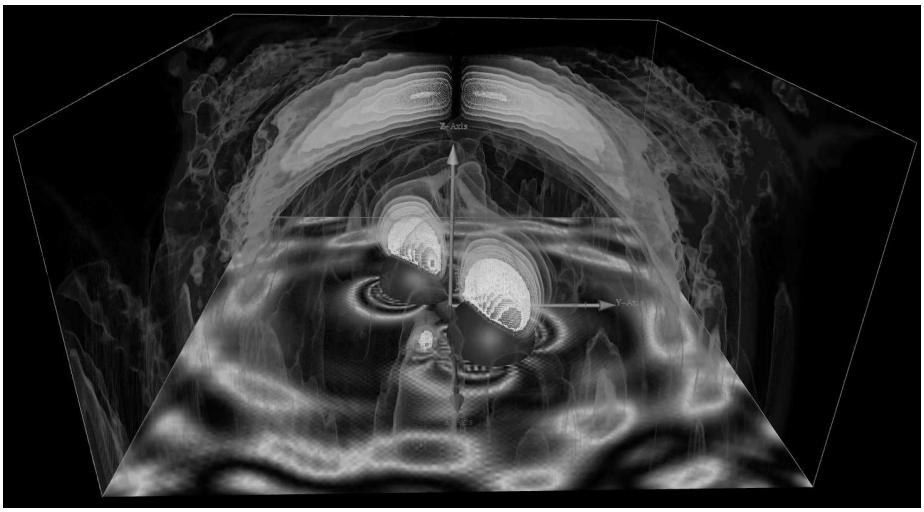

Abbildung 6: *Gravitational waves emitted in the spacetime of orbiting neutron stars. These results from a general relativistic hydrodynamic simulations are still preliminary.*

space"). At the time when it was proven first that such configurations of spacetime can exist [Brill], only initial data could be investigated. Now, by means of numerical relativity, we are able to dynamically evolve these spacetimes, the so-called "Brill Waves", and to study critical phenomena, like the creation of a black hole during the collapse of a gravitational wave.

The big challenge for investigating such spacetimes is that there are lots of physical properties to be visualized at once. We have in-going and outgoing components of the gravitational field, and various polarizations of the gravitational radiations. These are only four quantities of the five complex Weyl Scalar fields, $\Psi_0 \dots \Psi_4$, each of them having a reasonable physical interpretation. In practice it is impossible to display all these properties of spacetime in one image and still get a reasonable impression of what is going on.

Therefore the main task is to bring out – expected or unexpected – correlations between the various physical quantities. In particular, for "supercritical" Brill waves, which form a black hole, we want to see the correlation between the horizon curvature and the gravitational field. For the black hole formation we use the same method as for colliding black holes, i.e. we display the apparent horizon with colormapped Gaussian curvature.

The coincidence between curvature of the horizon and gravitational radiation is of special interest, since some theorem in general relativity states that all non-rotating, uncharged black holes of the same mass are identical, and hence all perturbations are radiated away. Therefore, we expect the largest intensity of gravitational radiations at areas where the curvature of the horizon is vastly varying, as can be seen in Fig. 7 (10).

A special colormap has been chosen for this image, such that the volumetric isolevel rendering reveals symmetry properties of the scalar field Ψ_4 around the value zero. We can therefore detect the wave-like behavior of Ψ_4, as positive regions are displayed in red and negative regions in blue colors. As the scalar

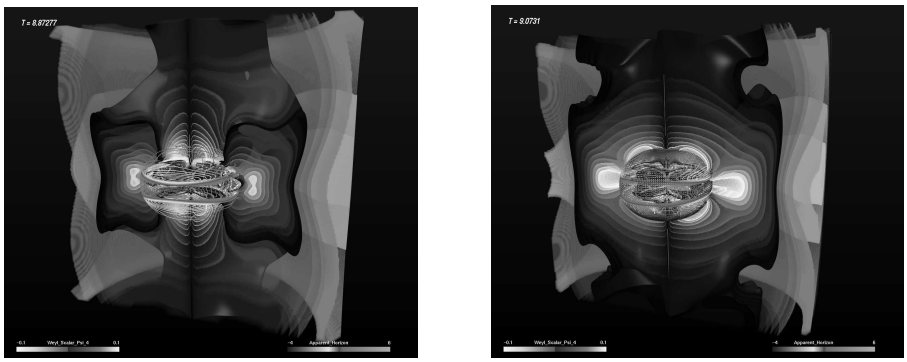

Abbildung 7: *A collapsing gravitational wave with the apparent horizon of the new black hole, formed during the collapse of a gravitational wave; seen as cuts along the xz- and yz-planes.*

field values approach the most frequent value zero, they are rendered more transparent. By this technique, high-intensity regions stand out clearly and the volumetric isolevels give a good impression of the relationship with the horizon's curvature.

5 Video Production Issues

When the visualizations of the numerical simulations became available, it was evident that these sequences are interesting not only due to their scientific content, but also from an artistic point of view. Therefore, the idea was born to produce a film based on these simulations which takes into account both scientific an artistic aspects. Before every sequence starts, the scientific content and the motivations for the numerical simulation are explained.The elucidations for the sequences are produced with very simple techniques, just using a blackboard, a piece of chalk, and classical animation tricks. The sharp contrast between high-tech and simple, but charming low-tech methods is intended to give back a human face to the pictures that have been produced by a record-breaking use of a hyper-modern massively parallel supercomputer.

The aim of this film therefore is to give an intuitive impression and at the same time scientific overview about the current state of the research in the field of numerical relativity. It was a challenge to create a film sequence, which is understandable and attractive for people who have hardly any knowledge about general relativity, and which provides new and interesting information for insiders, too.

6 Acknowledgment

This work was done within the framework of a cooperative research project aiming at new techniques for visualization of general relativistic phenomena performed at ZIB and AEI.

Ed Seidel, head of the numerical relativity group at AEI, contributed lots of ideas and suggestions concerning the physical simulations and the visualization

process. Paul Walker, Joan Masso, Gabrielle Allen, Tom Goodale, Gerd Lanfermann are the primary developers and maintainers of Cactus. Bernd Brügmann and Steve Brandt developed methods for constructing initial data for black hole collisions. Lars Nerger and Takahashi Ryoji performed the large-scale supercomputer simulations in June 1999. Miguel Alcubierre has implemented the numerical treatment of Brill Wave data. Wai-Mo Suen, head of the Neutron Star Grand Challenge project at the Washington University, St.Louis, and Mark Miller did the numerics and performed the simulations of neutron stars and their collapse to a black hole. Phillip Gressman provided the Newtonian Neutron star data.

The visualization system Amira [Amira] which was used to create all images presented here, has been developed by the visualization group at ZIB. We thank especially Detlev Stalling and Malte Zöckler for their significant contributions. Olaf Paetsch (ZIB) and Jeff Carpenter (NCSA) were doing hard work to provide appropriate film postprocessing including film titles. Last but not least, the music was composed by Frank Stanzl from the Music Academy Cologne.

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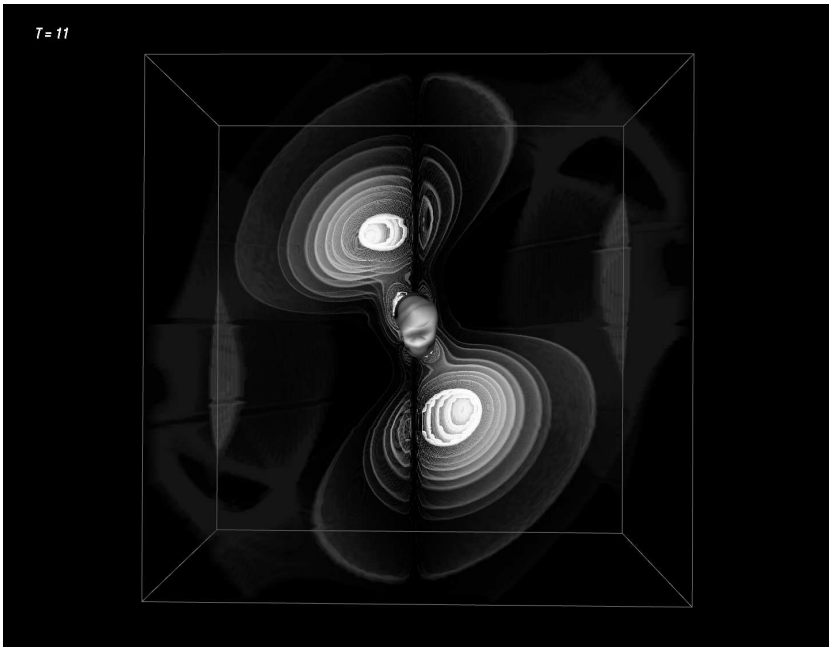

Abbildung 8: *A burst of gravitational energy emerging from colliding black holes.*

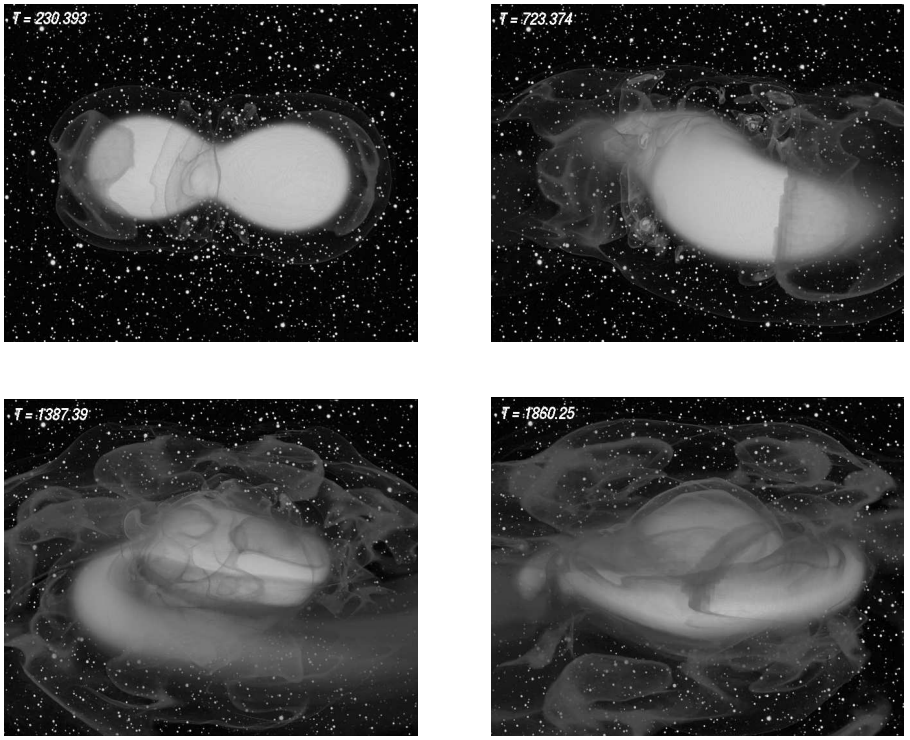

Abbildung 9: *Merging of two neutron stars computed using Newtonian theory.*

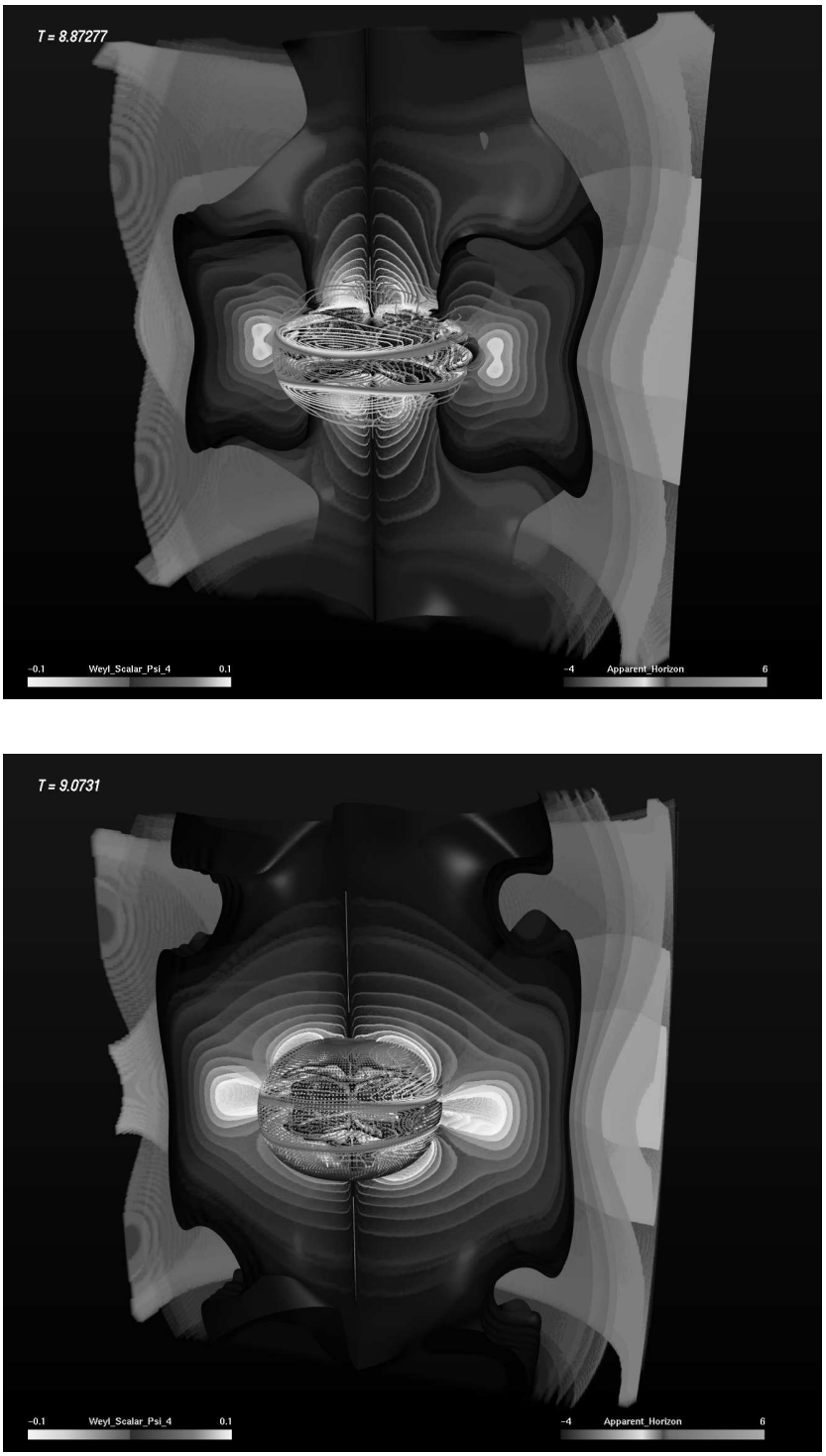

Abbildung 10: *A collapsing gravitational wave with the apparent horizon of the new black hole, formed during the collapse of a gravitational wave; seen as cuts along the xz- and yz-planes.*