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Estimating trenching costs in FTTx network planning

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Abstract In this paper we assess to which extent trenching costs of an FTTx network are unavoidable, even if technical side constraints are neglected. For that purpose we present an extended Steiner tree model. Using a variety of realistic problem instances we demonstrate that the total trenching cost can only be reduced by about 5 percent in realistic scenarios. This work has been funded by BMBF (German Federal Ministry of Education and Research) within the program “KMU-innovativ”.

1 Introduction

Broadband internet access is a key infrastructure since both the demand for high-data services rates and the private and commercial dependency on all-time broadband connectivity are continuously rising. In this light, several telecommunication carriers are already realizing *fiber-to-the-home* (FTTH) or *fiber-to-the-building* (FTTB) projects, and in addition, many companies are seriously investigating how such a network could be deployed. Given the vast investments that have to be made – an estimated amount of 40 to 60 billion Euro in total for Germany alone – the careful preparation of such a deployment project is indispensable.

The planning of an *FTTx network* is a highly complex task comprising numerous interesting optimization problems. In the German BMBF-project FTTX-PLAN (see [2], cf. [8], [9]), we developed methods to compute technologically feasible and cost-optimized FTTx networks including hardware, installation and trenching cost. Since the latter cost are considered to be the lion’s share of the expenses involved in the roll-out of an FTTx network (cf. [11, Section 6.1]), we focus in this paper on

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an evaluation of the incurred trenching cost in overall cost-optimized networks as computed by the FTTX-PLAN software.

To this end, we use an extended Steiner tree model to determine lower bounds on the trenching costs for a given FTTx instance. This model is presented in Section 3, after a high-level description of the FTTx network planning problem is given in Section 2. Finally, Section 4 compares trenching costs in solutions obtained from FTTX-PLAN with the lower bounds obtained by the Steiner tree model on various realistic test instances.

2 Problem formulation

The FTTx network planning problem can be described as the task to connect a number of given customers to central offices of the telecommunication carrier, using optical fibers and various active and passive components. Fibers can be laid out in different types of cables which themselves are direct-buried or embedded into different types of ducts, which are eventually buried into the ground along specified trails of the deployment area.

The topological structure of the input is represented by the *trail network*, an undirected graph whose edges designate the trails along which connections (fibers, cables, ducts) can be laid. To each edge is assigned a cost value that determines the trenching costs for the trail in question. The trails can also have existing infrastructure, such as dark fibers or ducts, which can be used for planning.

Some nodes in the trail network can be of a special type – each such node is associated with a setup cost which is incurred if it is included in the final network. Figure 1 shows an example of a trail network with special nodes BTPs and COs.

Fig. 1 Trail network of an instance, projected onto a satellite image of the deployment area. Yellow dots indicate BTPs, red diamonds possible CO locations; trails are colored according to their trenching costs – green for low, red for high costs.



- BTPs (*Building Termination Points*) represent potential customer locations. Each such location contains a number of residents of (possibly) different types, with certain demand values. These comprise several parameters, such as the number of required fibers.
- DPs (*Distribution Points*) are intermediate locations to (logically) connect customer locations to central offices. At DPs joint closures and splitters can be installed in order to concentrate fibers from individual customers into cables with a higher number of fibers.
- COs (*Central Offices*) are locations where customer connections are terminated using active equipment.

Global parameters formulate requirements to be fulfilled by any feasible network. These include length restrictions for optical connections, bounds on the number of customers connected to a CO or another concentrator location, or targeted percentage rates of connected customers.

Eventually, a hardware catalogue describes the usable active and passive hardware components – with investment and installation costs. These components impose capacity restrictions, for instance, on the number of cables or fibers that can be attached to a joint closure or the number of downstream ports that are available at a splitter. This further restricts the feasibility of a technical valid solution network.

The software developed within FTTX-PLAN computes cost-optimized solutions to the described planning problem using a variety of methods in different stages, including IP models and heuristics.

3 Estimating trenching costs

Costs for trenching are the most substantial budget item in the deployment of an FTTx network. Given such a network it can naturally be asked to which extent the incurred trenching costs are unavoidable since all BTP locations must be topologically connected to a CO. To answer this question, we compute lower bounds on the trenching costs by solving an extended Steiner tree problem that connects all BTPs to COs, where the CO locations must obey additional capacity constraints and incur setup cost.

Let (V, A) be the directed graph that is constructed from the trail network by replacing each edge $e \in E$ with two arcs (with the end-nodes of e) in opposite directions and augmenting this graph with an artificial root node (representing a virtual CO) connected by artificial arcs to every potential CO location. Each artificial arc a has a capacity k_a (the maximal number of BTPs to be connected to the CO in question) and a cost c_a (the setup cost of the CO). Let $A_0 \subset A$ denote the set of artificial arcs. The problem to solve is to find a cost-minimal tree in (V, A) connecting all BTPs to the artificial root node.

The following mixed-integer-programming flow formulation for this problem can be given, where V_B denotes the set of BTP locations, all of which have to be connected, N_C the maximal number of COs to open, $\delta^+(v)$ and $\delta^-(v)$ the set of outgo-

ing and incoming arcs of node $v \in V$, respectively, and e^+ and e^- the two opposite arcs that originated from an edge $e \in E$.

$$\begin{aligned}
& \min \quad \sum_{e \in E} c_e w_e + \sum_{a \in A_0} c_a x_a \\
& \text{s.t.} \quad \sum_{a \in \delta^-(v)} f_a - \sum_{a \in \delta^+(v)} f_a = \begin{cases} 1 & \text{if } v \in V_B \\ 0 & \text{otherwise} \end{cases} \quad \forall v \in V \quad (1) \\
& \quad \quad \quad f_a \leq |V_B| x_a \quad \forall a \in A \quad (2) \\
& \quad \quad \quad x_{e^+} + x_{e^-} = w_e \quad \forall e \in E \quad (3) \\
& \quad \quad \quad \sum_{a \in \delta^-(v)} x_a = 1 \quad \forall v \in V_B \quad (4) \\
& \quad \quad \quad \sum_{a \in \delta^-(v)} x_a \leq 1 \quad \forall v \in V \setminus V_B \quad (5) \\
& \quad \quad \quad \sum_{a \in \delta^-(v)} x_a \leq \sum_{a \in \delta^+(v)} x_a \quad \forall v \in V \setminus V_B \quad (6) \\
& \quad \quad \quad \sum_{a \in \delta^-(v)} x_a \geq x_{a'} \quad \forall v \in V \setminus V_B, a' \in \delta^+(v) \quad (7) \\
& \quad \quad \quad f_a \leq k_a \quad \forall a \in A_0 \quad (8) \\
& \quad \quad \quad \sum_{a \in A_0} x_a \leq N_C \quad (9) \\
& \quad \quad \quad f_a \geq 0 \quad \forall a \in A \quad (10) \\
& \quad \quad \quad x_a \in \{0, 1\} \quad \forall a \in A \quad (11) \\
& \quad \quad \quad w_e \in \{0, 1\} \quad \forall e \in E \quad (12)
\end{aligned}$$

Constraints (4) – (7) are the so-called *flow-balance inequalities* from [5]. Note that most planning and capacity restrictions are relaxed in this model, with the sole exception of the number of BTPs connected to a CO location and the total number of COs to open (constraints (8) and (9)). Furthermore, no kind of length restrictions are respected; since connection lengths have to have a strict upper bound in an optical fiber network, this is quite a severe relaxation from a practical point of view. Therefore we cannot expect that lower bounds on trenching costs obtained with the extended Steiner tree model can be realized in practically feasible FTTx networks.

Finally, we note that similar models have been presented for a slightly different problem involving existing copper infrastructure, the (Capacitated) Connected Facility Location Problem; see [6] for a recent contribution and [4] for an extensive treatment of this subject. Steiner tree problems themselves have been studied for quite some time and models similar to the one used here have been given in the literature; see [3, Section 9.6] or [10]. In [7] a multi-commodity flow formulation is mentioned, which can be seen as the basis of the model presented above.

4 Results and conclusion

We assessed trenching costs in 7 instances, given in Table 1. The first three trail networks were artificially generated using GIS information from OpenStreetMap [1]. The last four are modified networks provided by different German city carriers. Figure 2 shows an example of an FTTX-PLAN solution and the trenched trails attaining the lower bound for the instance a2.

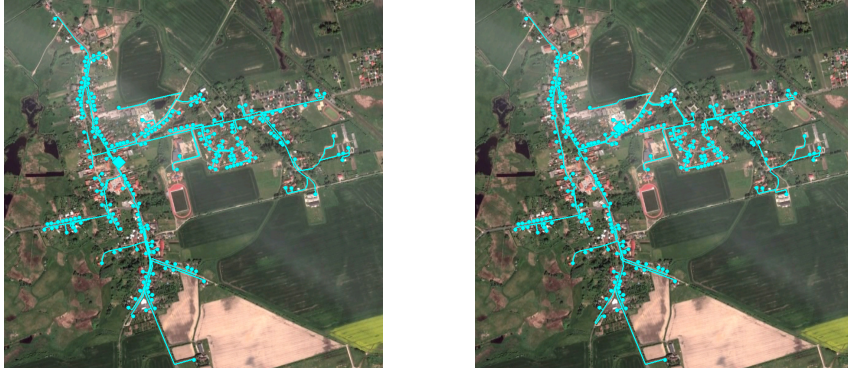


Fig. 2 Solution network (left) to instance a2 and trenched trails attaining the lower bound (right).

Table 1 shows the results from the computations. Besides the size of the instances, we have given the trenching costs incurred by the solution networks, the lower bound obtained with the model in Section 3, and the relative gap between these two values. Additionally, we list the average and maximal connection lengths for the networks and the lower bound.

Instance:	a1	a2	a3	c1	c2	c3	c4
# nodes	637	1229	4110	1051	1151	2264	6532
# edges	826	1356	4350	1079	1199	2380	7350
# customers	39	238	1670	345	315	475	1947
# potential COs	4	5	6	4	5	1	1
network trenching cost	235640	598750	2114690	322252	1073784	2788439	4408460
average connection length	717.1	1186.6	733.4	589.6	1969.2	995.3	2094.1
maximal connection length	1320	2257	2334	1180	4369	2452	4809
lower bound	224750	575110	2066190	312399	1063896	2743952	4323196
relative gap	4.8%	4.1%	2.3%	3.2%	0.9%	1.6%	2.0%
average connection length	299.2	887.5	504.6	363.7	1531.0	590.2	1092.9
maximal connection length	1049	3142	3215	1394	7315	2586	4786

Table 1 Comparison of trenching costs for different instances.

It can be observed that trenching costs in the solution networks are in an acceptable range with respect to the obtained lower bounds (up to 5 percent). Furthermore,

length restrictions can be the major problem in the extended Steiner tree model. The maximal length from a BTP to its CO is in extreme cases almost doubled with respect to the solution networks.

Additionally, we compared trenching costs for an instance where existing infrastructure is provided. As can be seen from Table 2, both network trenching costs as well as the lower bound give smaller values, and the solution network uses a fair amount of the provided empty ducts.

	trenching costs	used existing infrastructure		
		total length	# trails used	% trails used
solution network	4063839	7131	352	95%
lower bound	3990065	7531	360	97%
relative gap	1.8%			

Table 2 Trenching costs with existing infrastructure (instance c_4 with empty ducts on 371 trails).

Further research. Next we will investigate which and how many practical restrictions are violated by the solution computed with the extended Steiner tree model. Based on this analysis, further extensions of the model will be formulated in order to tighten the lower bound beyond the bounds presented in this paper.

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