

# Application of Multistage Stochastic Programming in Strategic Telecommunication Network Planning

Diplomarbeit  
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Hiermit versichere ich die selbstständige und eigenhändige Anfertigung dieser Arbeit an Eides statt.

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# Contents

<b>Zusammenfassung (German Summary)</b>	<b>iii</b>
<b>Acknowledgments</b>	<b>v</b>
<b>1. Introduction</b>	<b>1</b>
<b>2. Traditional Valuation of Investment Opportunities</b>	<b>5</b>
2.1. Net Present Value . . . . .	6
2.2. Decision Tree Analysis . . . . .	8
<b>3. Real Options</b>	<b>11</b>
3.1. Types of Real Options . . . . .	11
3.2. Real Options in the Literature . . . . .	13
3.3. Types of Financial Options . . . . .	14
3.4. Introduction to Mathematical Finance . . . . .	16
3.4.1. Discrete Time Models . . . . .	17
3.4.2. The Cox-Ross-Rubinstein-Model . . . . .	20
3.4.3. The Black-Scholes Model . . . . .	22
3.5. Revisiting the Assumptions from Finance . . . . .	24
<b>4. Multistage Stochastic Optimization</b>	<b>27</b>
4.1. Introduction . . . . .	27
4.2. The Scenario Formulation . . . . .	30
4.3. The Node Formulation . . . . .	31
4.4. Risk Measures . . . . .	33
4.4.1. Value-at-Risk . . . . .	35
4.4.2. Average Value-at-Risk . . . . .	36
4.4.3. Multi-Period Risk Measures . . . . .	38
<b>5. Modeling UMTS Radio Networks and Demand Evolution Over Time</b>	<b>41</b>
5.1. A Mathematical Model for UMTS Network Coverage and Capacity .	41
5.1.1. Signal Propagation . . . . .	42
5.1.2. Coverage . . . . .	44
5.1.3. Cell Power . . . . .	44
5.1.4. Load Control and Cell Areas . . . . .	49
5.1.5. Adding Capacity Through Carrier and Technology Upgrades	51
5.1.6. Transforming Traffic to Load . . . . .	54

## Contents

5.1.7. Technological and Regulatory Requirements . . . . .	55
5.2. Stochastic Model of Demand Evolution . . . . .	56
5.2.1. Modeling Uncertainty by a Stochastic Process . . . . .	59
5.2.2. Sampling Paths . . . . .	59
5.2.3. Scenario Tree Construction . . . . .	61
<b>6. Optimizing Network Evolution</b>	<b>65</b>
6.1. The Modeling Idea . . . . .	66
6.2. Notation . . . . .	67
6.3. Multistage Stochastic Program for the Expected Profit . . . . .	69
6.3.1. Decision Variables . . . . .	69
6.3.2. The Model . . . . .	71
6.3.3. Assignment of Service- and Coverage Variables . . . . .	72
6.4. Multistage Stochastic Program for Average Value-at-Risk . . . . .	74
6.5. Problem Specific Presolving . . . . .	75
6.5.1. Fixing Service and Coverage Variables Without Impact . . . . .	75
6.5.2. Implicit Integer Variables . . . . .	75
6.5.3. Pixel Aggregation . . . . .	76
6.5.4. Reusing Service Variables as Coverage Variables . . . . .	78
6.5.5. Presolving the Coverage Requirement . . . . .	79
6.5.6. Mandatory Coverage Requirement . . . . .	80
<b>7. Computational Experiments</b>	<b>83</b>
7.1. Planning Scenarios . . . . .	84
7.2. Scenario Trees . . . . .	85
7.3. Parameters . . . . .	86
7.4. The Effect of Presolving . . . . .	90
7.5. Computational Results . . . . .	92
<b>8. Conclusions</b>	<b>101</b>
<b>A. Tables with Detailed Results</b>	<b>105</b>
<b>List of Tables</b>	<b>139</b>
<b>Bibliography</b>	<b>143</b>

# Zusammenfassung

Telekommunikation ist eine wesentliche Grundlage für die Informationsgesellschaft in der wir leben. Sowohl in der Geschäftswelt als auch im Privaten wird die ständige Verfügbarkeit von mobiler Telekommunikation heutzutage vorausgesetzt. Nachdem am Anfang die Sprachnutzung überwog, erzeugen nun Datendienste und mobiles Internet die Hauptlast in den Netzen und bestimmen den Ausbau der Netzinfrastruktur. Im Jahr 2009 erzeugten 19 Millionen Nutzer über 33 Millionen Gigabyte Verkehrsvolumen mit mobilen Datendiensten. UMTS-Netze in Deutschland enthalten über 120 000 Mobilfunkzellen an über 39 000 Standorten. Die vier deutschen Netzbetreiber investierten zwischen 1998 und 2008 über 28 Milliarden Euro in ihre Netzinfrastruktur (alle Zahlen stammen aus [14]).

Diese Zahlen verdeutlichen, dass eine sorgfältige Verwendung der verfügbaren Ressourcen sich wesentlich auf die Profitabilität eines Netzbetreibers auswirken kann. Das Netz sollte möglichst gut an die bestehende Nachfrage angepasst sein. Da sich die Nachfrage mit der Zeit verändert, ergibt sich Bedarf die Infrastruktur entsprechend mitentwickeln. Die Veränderung der Nachfrage ist schwer vorherzusagen und stellt eine starke Unsicherheitsquelle dar. Die strategische Netzplanung hat daher die Unsicherheit zu berücksichtigen, und die geplante Netzevolution sollte sich an veränderte Marktbedingungen anpassen. Die Verwendung von Planungsmethoden unter Berücksichtigung von Unsicherheit kann daher die Profitabilität erhöhen und einen Wettbewerbsvorteil darstellen.

Mathematische Modellierungs- und Optimierungsmethoden sind leistungsfähige Werkzeuge für die Planung von Fest- und Funknetzen gleichermaßen, die in den vergangenen 20 Jahren in diesen Bereich vielfach erfolgreich eingesetzt wurden [36]. Stochastische Optimierung ist ein Ansatz um Planungsprobleme mit ganz oder teilweise unsicheren Planungsdaten anzugehen. Der Ausbau eines Funknetzes wird üblicherweise in Jahres- oder Quartalsabschnitten geplant, so dass wir von mehrstufiger stochastischer Optimierung sprechen.

In dieser Arbeit werden mathematische Modelle und effiziente Optimierungsmethoden für die strategische Planung von zellulären Funknetzen vorgestellt. Wir modellieren die Nachfrageentwicklung als stetigen, stochastischen Prozess und approximieren ihn durch diskrete Szenariobäume. Ein dreistufiger Ansatz wird für die Erzeugung von unregelmäßigen Szenariobäumen verwendet, die die Grundlage der stochastischen Programme sind.

Wir beschreiben ein realistisches Systemmodell für ein UMTS-Funknetz. Unter Berücksichtigung von realistischen Signalausbreitungseigenschaften des Umfeldes und der Interferenz innerhalb des Netzes erfasst das Modell mit Netzabdeckung und Netzkapazität dessen wesentliche Eigenschaften. Passend zu unserem Op-

timierungsansatz leiten wir Zellflächen von Abdeckungseigenschaften und der kapazitiven Auslastung der Funkzellen ab.

Wir formulieren das Netzplanungsproblem als mehrstufiges stochastisches Programm mit Ganzzahligkeitsbedingungen. Die zugehörigen deterministisch-äquivalenten Probleme sind gewöhnliche gemischt-ganzzahlige Programme (MIPs), die wir mit modernen, kommerziellen MIP-Lösern lösen. Dabei verwenden wir speziell entwickelte Presolving-Methoden, um die Problemgröße zu reduzieren, und die Instanzen einer Lösung zugänglich zu machen.

Mit Hilfe von Untersuchungen für realistische Planungsszenarien überprüfen wir unseren Planungsansatz. Wir nutzen sowohl Daten aus öffentlich zugänglichen Quellen als auch ein Planungsszenario eines deutschen Netzbetreibers, um Ausbaupläne und finanzielle Bewertungen für UMTS-Funknetze zu berechnen. Dabei betrachten wir neben dem erwarteten Gewinn auch ein Risikomaß als Zielfunktionen.

Der Planungsansatz soll anhand von Szenarien studiert werden, in denen neben der unsicheren Verkehrsentwicklung möglichst alle weiteren Parameter bekannt sind. Deshalb untersuchen wir die Entwicklung eines UMTS-Netzes zwischen den Jahren 2004 und 2010. Für diesen Zeitraum sind die meisten Parameter öffentlich zugänglich und unterliegen nicht länger der Geheimhaltung durch die Netzbereiber.

Die meisten der formulierten Programme werden innerhalb von 10 Stunden optimal bis auf 0.5 % Optimalitätslücke gelöst. Im Vergleich zur traditionellen Planung auf der erwarteten Bedarfsentwicklung liefert unser Ansatz signifikant bessere Ergebnisse. Die Ergebnisse verdeutlichen den Bedarf nach stochastischen Planungsmethoden, da die berechneten Netzentwicklungen in den einzelnen Szenarien stark von einander abweichen. Die Optimierung mit dem Ziel der Minimierung des Risikomaßes Average Value-at-Risk bringt dagegen nur noch geringe Verbesserungen. Auch die genau Kenntnis über die zukünftige Entwicklung, die einer Relaxierung der Nicht-Antizipativitäts-Bedingungen entsprechen, liefert nur noch wenig Mehrwert.

Der hier untersuchte Planungsansatz lässt sich auf andere Funktechnologien übertragen. Mit der Nachfolgetechnologie von UMTS namens LTE steht bereits ein Kandidat in den Startlöchern. Das Systemmodell ist hierfür geringfügig anzupassen. Auch kann es sinnvoll sein, ein erweitertes Nachfragemodell zu verwenden, das beispielsweise auf anderen stochastischen Prozessen basiert. Weiterhin scheint eine Übertragung für die Planung von leitungsgebundenen Zugangsnetzen wie dem deutschlandweit geplanten Glasfaserausbau möglich. Eine detaillierte Analyse hierzu wird im Rahmen der Arbeit jedoch nicht durchgeführt.



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## *Contents*

# 1. Introduction

Telecommunication is fundamental to the information society we live in. In both, the private and the professional sector, mobile communication is nowadays taken for granted. Having started primarily as a service for speech communication, data service and mobile Internet access are now driving the evolution of network infrastructure. In the year 2009, 19 million users generated over 33 million gigabyte of traffic using mobile data services. The 3rd generation networks (3G or UMTS) in Germany comprises over 39 000 sites with about 120 000 cells. From 1998 to 2008, the four network operators in Germany invested over 28 billion euros in their infrastructure (all figures taken from [14]).

A careful allocation of the resources is thus crucial for the profitability of a network operator: a network should be dimensioned to match customers demand. As this demand evolves over time, the infrastructure has to evolve accordingly. The demand evolution is hard to predict and thus constitutes a strong source of uncertainty. Strategic network planning has to take this uncertainty into account, and the planned network evolution should adapt to changing market conditions. The application of superior planning methods under the consideration of uncertainty can improve the profitability of the network and creates a competitive advantage.

Mathematical modeling and optimization is a powerful tool for network planning [36]. Stochastic programming provides modeling approaches for planning problems where some or all the data is unknown. The evolution of the network can be divided into several stages such that we face a multistage problem. Multistage stochastic programming is, thus, a suitable framework to model strategic telecommunication network planning.

We start by investigating several methods for the monetary valuation of investment opportunities. The most traditional ones, net present value and decision tree analysis, are not satisfying. Net present value is a static framework which leads to poor results in the presence of uncertainty and many possibilities to react unforeseen events. Decision tree analysis can capture flexibility but is limited to discrete time horizons and probability distributions. Moreover, the tree has to be formulated explicitly which is hardly possible in our case. In both framework, future cash-flows are discounted with a risk-adjusted discount rate, which can hardly be determined.

The real options approach proposes to use the market for the monetary valuation of investment opportunities. The results are expected to be more reliable since no risk-adjusted discount rate has to be estimated. The project, however, has to be embedded in an appropriate market to fulfil the conditions. This is hardly possible in our application.

## 1. Introduction

In the literature, stochastic programming is suggested to value real options in the absence of a market embedding. The results is an implicit evaluation of the scenario tree and hence a risk-adjusted discount has to be used. We therefore formulate our network planning problem as a multistage stochastic program.

In statergic telecommunication network design, we identified the future demand as the main source of uncertainty. The future traffic volume is modelled in a stochastic manner. Using a three-stage approach, comprising the modelling as an abstract stochastic process, sampling of paths, and the construction of scenario trees based on the sample path, we obtain a stochastic demand model.

We set up a system model for UMTS radio networks that captures the coverage and capacity behavior of the network. Service by the network is assume to be provided only to mobile devices with cell the cells service area. Both, coverage and capacity, are considered to compute service areas of the cells. By this means, cell capacity is also transferred to a spacial measure which is appropriate for our purpose.

Based on the stochastic demand model and the cell areas for the respective traffic volumes, we formulate deterministic equivalent problems for the network design problem. These deterministic equivalents are ordinay mixed-integer programs which can be solved with state-of-the-art branch-and-cut based solvers. Several presolving routines are applied to reduce the problem size and make the instances computationally tractable.

To demonstrate the practical applicability of the approach, we conduct computational experiments on two realistic planning scenarios. We choose a setting that allows to perform experiments on realistic data without guessing to many parameters. We thus focus on a setting in which most parameters have become publicly accessible and are no longer a trade secret of the industry. This is nowadays the case for the introduction of UMTS, starting around the year 2003. Therefore, the evolution of a UMTS network from the year 2004 to the year 2010 is studied. Historical observation of the traffic volumes are used to calibrate the model.

Strategic planning the next technology generation of cellular networks, called LTE, is eventually ongoing at all German network providers. The technologies developed in this thesis are applicable to this setting as well.

**Outline.** This thesis is structured as follows. In the remainder of this section, we define the notation used in this thesis. We then discuss different frameworks to determine the value of an investment opportunity: Two traditional frameworks in Chapter 2 and the real options framework in Chapter 3. We study the applicability of the powerful tools provided by financial mathematics to value real options, and we explain which nesessary assumptions are not fulfilled for the application of this tool for the problem under consideration.

Therefore, stochastic optimization is the framework of choice to optimize the network evolution such that the expected value of the network is maximized. We introduce the stochastic programming framework in Chapter 4.

The system model of a UMTS network and a stochastic demand model is given in Chapter 5. Based on this, we present optimization models for the network evolution in Chapter 6. Presolving routine to make the problems computationally tractable are also provided.

Computational experiments on realistic datasets and their results are described and analyzed in Chapter 7. Conclusions are presented in Chapter 8.

## Mathematical Programming

This section provides definitions of the types of mathematical programs used in this thesis. A more detailed introduction to linear and mixed-integer programming can be found for example in [65].

**Definition 1:** Let  $m, n \in \mathbb{N}$ ,  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ ,  $c \in \mathbb{R}^n$ , and  $I \subseteq N = \{1, \dots, n\}$ . The optimization problem

$$\min c^T x \quad (1.1)$$

$$\text{s.t. } Ax \leq b \quad (1.2)$$

$$x_i \in \mathbb{Z} \quad \forall i \in I \quad (1.3)$$

$$x \in \mathbb{R}^n \quad (1.4)$$

is called *mixed-integer (linear) program* (MIP). A MIP is called

*an integer program* (IP), if  $I = N$ ,

*a linear program* (LP), if  $I = \emptyset$ .

A minimization problem can be easily transformed into a maximization problem and vice versa. Therefore, a MIP can be equivalently be defined as a maximization problem. Furthermore, the inequality system  $Ax \leq b$  can be transformed into a system of equalities by adding auxiliary *slack variables*.

A vector  $x \in \mathbb{R}^n$  is called *feasible solution* of the MIP if it fulfills the conditions (1.2) and (1.3). We call a feasible solution  $x$  *optimal* if the objective function can not be improved by another feasible solution, i. e., if no feasible  $y \in \mathbb{R}^n$  exists such that  $c^T y > c^T x$ .

An LP can be associated to each MIP by omitting the integrality constraints (1.3). We say that the integrality constraints are *relaxed*. The resulting LP is called *LP-relaxation*. The optimal solution value of the LP-relaxation is a lower bound for the optimal solution value of the MIP and is called *LP-bound*. The relative gap between a lower bound and the objective function value of a feasible solution is a measure for the solution quality.

**Definition 2:** Let  $\bar{c}$  be the objective function value of a feasible solution and  $\underline{c} \leq \bar{c}$

## 1. Introduction

SI prefixes			IEC prefixes		
Value	Abbr.	Name	Value	Abbr.	Name
$1\,000^1$	k	kilo	$1\,012^1$	Ki	kibi
$1\,000^2$	M	mega	$1\,012^2$	Mi	mebi
$1\,000^3$	G	giga	$1\,012^3$	Gi	gibi
$1\,000^4$	T	tera	$1\,012^4$	Ti	tebi
$1\,000^5$	P	peta	$1\,012^5$	Pi	pebi

**Table 1.1.:** Common unit prefixes

the a lower bound on the objective function value. The relative gap is defined by

$$\text{relative gap} = \begin{cases} \infty & \text{if } \bar{c} = 0 \text{ and } \underline{c} \neq \bar{c} \\ 0 & \text{if } \bar{c} = \underline{c} = 0 \\ \left| \frac{\underline{c} - \bar{c}}{\bar{c}} \right| & \text{else} \end{cases} \quad (1.5)$$

Even though general MIP solving is an  $\mathcal{NP}$ -hard optimization problem [5], instances with a large number of variables and constraints can often be solved by state-of-the-art MIP solvers, such as CPLEX [46], GUROBI [37], or SCIP [1, 80].

### Unit Prefixes

Large quantities are commonly scaled by powers of thousands or powers of 1024. Prefixes, like kilo, mega, giga, are used to indicate the scaling. We distinguish the SI decimal prefixes and the IEC binary prefixes. The *International System of Units* (SI from the French *Système international d'unités*) specifies a set of unit prefixes that reflect powers of 1 000. An examples is kilo with is abbreviated by the letter k and means  $1\,000^1$  as the respective unit. The *International Electrotechnical Commission* (IEC) specifies unit prefixes that reflect powers of 1 012 which are primarily used in the electronic and computer related field. Their names are similar to the SI prefixes but still allow a clear distinction. Table 1.1 gives an overview of the most common prefixes in both systems.

## 2. Traditional Valuation of Investment Opportunities

The evaluation of investment opportunities is challenging task that arises in many business situations. An investment opportunity is interpreted as a project that incurs a series of investment outlays and in return generates a series of revenues. Selecting only profitable projects and among several possible projects the one with maximal profit is a common task for management and decision makers. One metric of profitability is the expected value of the investment opportunity at the time the decision has to be taken. Several approaches have been studied and became standard in the management literature.

All approaches rely on the ability to *discount* future cash-flows. Discounting is used to scale cash-flows occurring at different times. Typically, investors are risk-averse; they prefer less risk to more. For taking a risk, investors therefore ask for a risk-premium. This also effects the time preferences for monetary values. Having one unit today is preferred to having one unit in one year. On the other hand, one money unit that an investor gets next year is only worth the fraction  $\frac{1}{r}$  today. The appropriate discount factor  $r$ , also called discount rate, depends on the riskiness of the investment. The riskier the investment is supposed to be, the higher is the discount factor for future cash-flows.

The most prominent approach of the evaluation of investment opportunities is *Net Present Value (NPV) analysis*. The NPV of a project is the sum of all discounted expected cash-flows that occur during the planned lifetime of the project. While this is a feasible approach when no uncertainty is present, difficulties arise when future cash-flows are uncertain and management has the flexibility to alter the operational strategy.

*Decision Tree Analysis (DTA)* is one attempt to capture flexibility. The idea is to construct a tree with event nodes and decision nodes, where discounted cash-flows are associated to the edges. By backwards solving the tree, the expected present value and optimal decision strategies can be computed. In highly uncertain environments with many decisions and alternatives, however, the decision tree suffers from a combinatorial explosion. Finding the appropriate discount is also a problem.

With the advances in mathematical finance another approach emerged: *Real Option Analysis*. The idea is to interpret the options that form flexibility as financial options and value them using the methods from mathematical finance. This gives a powerful tool to model flexibility and gives methods to derive market driven values for investment projects. To capture the abilities and limitations of this ap-

## 2. Traditional Valuation of Investment Opportunities

proach, some background from finance is necessary. We therefore dedicate the real options approach a separate chapter, even though by now it can be called “traditional” as well.

In the remainder of this chapter we present in Section 2.1 the net present value approach and in Section 2.2 the decision tree approach in more detail. The real options approach will be studied in Section 3.

### 2.1. Net Present Value

The most common tool for the evaluation and comparison of investment opportunities or projects is the *Net Present Value* (NPV) *Analysis*. The net present value of a project is the sum of all discounted cash-flows that incur during its lifetime. NPV is a metric for the current value of a project, but is inappropriate in uncertain environments.

In the following, we assume a discrete time model where cash-flows may occur only at a certain points in time. The set of time points is called *time horizon* and is denoted by  $\mathcal{T} = \{0, \dots, T\}$ . To facilitate the notation we assume that the periods between two points are of equal duration.

First, we introduce the NPV if all cash-flows are certain and argue that it is a correct method for valuation. Then, we define the NPV under uncertainty and show the limitations of the approach.

#### Net Present Value Under Certainty

In an environment without uncertainty all cash-flows during the lifetime are known in advance. Let  $C_t$  cash-flow in period  $t$ .

Since the cash-flows are certain, an investor is indifferent of investing in the project or putting the money on a bank account, but he will only invest in the project if the return is at least as high as the return of the bank account.

The value of funds on a bank account increases in every period by a factor of the interest rate  $r$ . A value of  $x$  in period 0, will be worth  $x(1+r)^t$  in period  $t$ . In return, to have  $x$  at  $t$ , only  $\frac{x}{(1+r)^t}$  are needed at time 0. Under the assumption that unlimited amounts can be deposited or borrowed from a bank account at the same interest rate  $r$ , all future cash-flows can be discounted by the rate  $r$ . If the cash-flow  $C_t$  is negative, an investment is required and the amount  $\frac{C_t}{(1+r)^t}$  would have to be deposited at time 0 to cover this expense. If  $C_t$  is positive, the project generates revenues, which are worth  $\frac{C_t}{(1+r)^t}$  at time 0. This amount could be extracted from the bank account at time 0 and the revenue would exactly balance the account.

The NPV of a project is then defined as the sum of all discounted future cash-flows:

$$\text{NPV} := \sum_{t=0}^T \frac{C_t}{(1+r)^t} \quad (2.1)$$



**Interpretation.** Being the sum of all discounted cash-flows, the NPV is the current value of the investment opportunity. Thus, a positive value is the amount we could extract at time 0 from the bank account and the project's revenues would still cover all its expenses. A negative value means that this amount needs to be deposited to cover all future expenses. Consequently, the NPV rule says that projects with positive NPV are favorable and should be executed while projects with negative NPV are unprofitable and should be rejected.

NPV is a metric for project merit and can therefore be used to compare alternative projects.

Trigeorgis [84] puts NPV in a bigger picture and states that in the absence of uncertainty "NPV is the only currently available valuation measure consistent with a firm's objective of maximizing its shareholders' wealth".

### Net Present Value Under Uncertainty

In reality, future market prices for raw materials and final products are often not known with certainty. Many other sources of uncertainty might be present and have an effect on future cash-flows. Cash-flows are no longer deterministic, but stochastic with some underlying probability distribution. Summing up discounted cash-flows is not longer appropriate. Therefore, expectations regarding the future cash-flows are used.

**The Risk-Adjusted Discount Rate.** The uncertainty about the cash-flows generated by a project introduce risk for the investor. Investors, however, tend to be risk-averse and expect a risk-premium in form of a higher expected return of the investing in risky projects. Therefore, discounting has to be done using a *risk-adjusted* discount rate, which is higher than the risk-free interest rate. The risk-adjusted interest rate is also called *opportunity cost of capital*, since it represents the rate of return an investor expects from projects with a similar riskiness. Determining or even estimating riskiness of projects is very difficult. Finding appropriate risk-adjusted discount rates is not always possible and often subject to the risk preference of the decision maker. On the other side, the outcome of a NPV analysis strongly depends on the discount rate and using an inappropriate rate can lead to poor and unprofitable investment decisions. This is one of the mayor back-draws of the NPV approach when future cash-flows are uncertainty.

**Definition.** For a given risk-adjusted discount rate  $\hat{r}$  and a series of (uncertain) cash-flows the NPV is defined by

$$\text{NPV} := \sum_{t=0}^T \frac{\mathbb{E}[C_t]}{(1 + \hat{r})^t} \quad (2.2)$$

By  $\mathbb{E}[\cdot]$  we denote the expectation operator and assume that all expectations exist.

## 2. Traditional Valuation of Investment Opportunities

**Flexibility.** Flexibility is the option to react to observations of unforeseen events by revising and correcting decisions. By using expectations, NPV handles uncertainty in the forecasted cash-flows, but the ability to react to changing market conditions is neglected by NPV analysis. This is a source of valuation errors and might result in poor investment decisions.

### 2.2. Decision Tree Analysis

Another commonly used tool for project valuation and decision support is *decision tree analysis*. The idea is to represent decisions and stochastic events as nodes in a tree. The tree structure is suitable to visualize the different alternatives in the lifetime of a project and to determine optimal decision strategies, depending on the outcome of the stochastic events.

The path from the root to a leaf contains all the information on the events occurred and the decisions taken.

The evaluation of the tree is done from the leaves to the root. At each node, the expected remaining NPV is calculated. At event nodes, the expectation over all events and at decision node, the most profitable decision is taken.

**Example.** Suppose a company has the option to build a production plant. It comes in two sizes; a big and a small size. If the small size is built, an upgrade can be built after one period. The sales price of the products is uncertain. Starting at 100 at time 0, the price increases with probability 0.4 by 40 percent and decreases with probability 0.6 by 30 percent in each of the two periods. The big production size can produce three units of the product and the small only one. No capacity downgrade or shutdown is possible. Building the big plant involves cost of 450, while building the small plant only costs 150, both occurring at time 0. The upgrade costs another 150 which has to be paid in period one. Products are sold in periods one and two. All cash-flows have to be discounted with a risk-adjusted rate of 0.15. The decision tree for this project is depicted in Figure 2.1.

Typically decision nodes are drawn rectangular and event (or chance) nodes round. Terminal nodes are drawn as triangles. Edges from event nodes are labeled with the probability  $p$  of the event. Edges from decision nodes are labeled with the decision. The number in brackets is the discounted cash-flow generated by the edge; either revenues from product sales or expenses for construction works. Numbers above nodes show the sum of the NPV of the subtree. The highlighted edges show which decisions are taken.

Let us first look at the subtree corresponding to building the small plant. If the price goes up, upgrading the facilities is optimal since the value of the “yes” branch is  $311.2 - 130.4 = 180.8 > 103.7$ . The sum if the expected discounted cash-flows at the decision to upgrade is thus 180.8. If the price decreases, upgrading is no longer optimal since the value of the “yes” branch in this case is  $155.6 - 130.4 = 25.2$ , which is smaller than 51.9. At the time of the decision, the sum of the

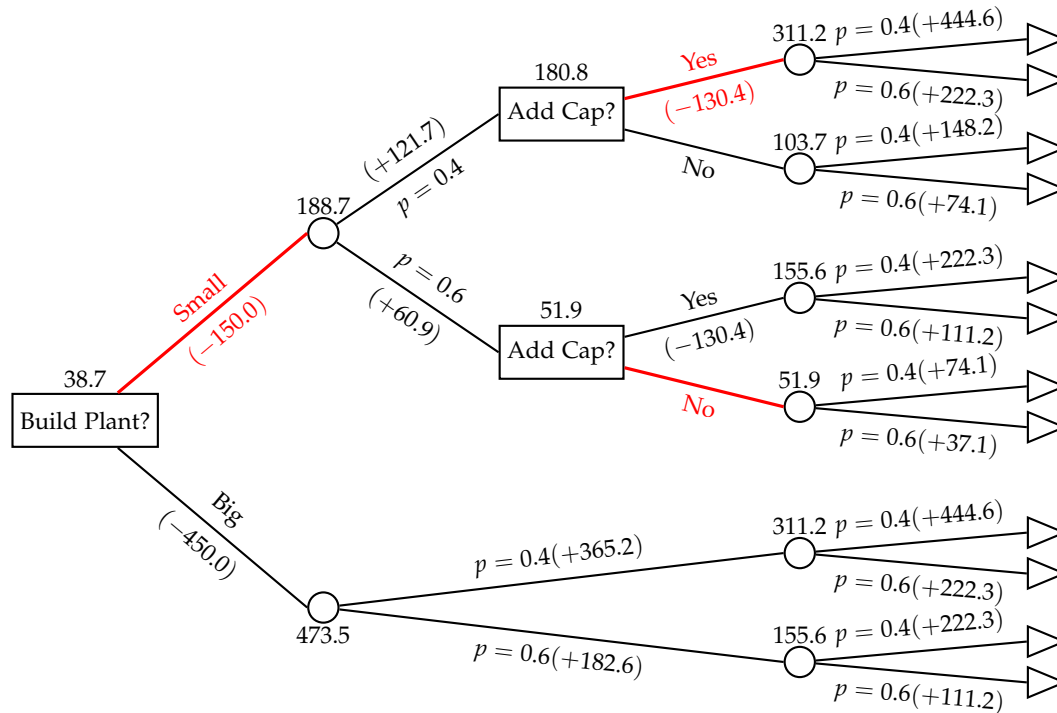


Figure 2.1.: Example of a decision tree, decisions taken indicated by red arcs

expected discounted cash-flows is 51.9. Adding the discounted sales incomes and weighting with the probability of the price going up or down in the first period, the value of building the small plant is 188.7 minus the construction cost, thus 38.7.

If the big plant is build, no more flexibility exists. The sum of the expected, discounted cash-flows is 473.5. Subtracting the investment cost of 450 yield a value of building the big plant of 23.5.

The maximize the value, the small plant should be build and upgraded, if the price develops favorable. The value of the project is then 38.7.

**Disadvantages.** Even though decision tree analysis is superior to NPV analysis, since it can capture flexibility to a certain degree, it still has the same major back-draw: Choosing the appropriate discount rate. What is the appropriate discount rate in this case anyway? In the example above we compare two projects; Building the big or the small plant. In the worst case, the big plant generates a discounted loss of 156.2 if the price decreases twice. In the best case a discounted win of 359.8 can be realized. Building the small plant and eventually the upgrade generates a win of 285.9 in the best and a loss of 52.0 in the worst case. Clearly, building the big plant is the riskier project and should have been discounted with a higher rate.

## *2. Traditional Valuation of Investment Opportunities*

Using the same discount rate over time is also critical, since it implies that uncertainty is revealed continuously over time. In the worst case, at every node in the tree a different discount rate has to be chosen.

Another problem is the combinatorial explosion of decision trees with many decisions and many different outcomes of the random variables. Constructing the tree can be non trivial as well if more complicated constraints on the possible decisions are present.

The last back-draw we want to mention is the discrete time setting. Not only does the tree get bigger and bigger if the discretization gets finer, but some processes can be monitored continuously and decision can be taken at any point in time. Infinite time horizons, are also not possible in this setting.

### 3. Real Options

With the success of mathematical finance with its abilities to compute prices for financial derivatives, another approach to value projects arose: *Real Options*. An investment opportunity is viewed as a collection of options on future cash flows and methods from finance are used to compute prices for these options on “real” assets, thus the term “real option”. The Flexibility to modify the operational strategy is considered by adding appropriate options to the collection of options that form the project. These options are valued in a suitable market model. The resulting values are driven by the market and thus risk-neutral. In particular, they do not rely on risk-adjusted discount rates, which was one of the main drawbacks of net present value and decision tree analysis.

Even though the approach is appealing, its correct application is elaborate. One sticking point is that typically neither the real option itself nor the underlying assets are tradable. In the standard mathematical finance models a perfect and complete market is assumed, which can be used to hedge against all risks. This allows to calculate risk-neutral prices with the property of being arbitrage-free in the market model. To make the valuation approach work, the real options have to be embedded in such a market. Several ways to do this have been proposed, but apparently there are situation where no suitable embedding can be found.

In the remainder of this chapter we present the most common types of real options and give a literature review about application areas of the approach. Then, standard terminology and notation from mathematical finance is introduced together with common market models. A critical summary of the assumptions from finance, together with a discussion about usability of the real options approach will end this chapter.

#### 3.1. Types of Real Options

The collection of real options that form an investment opportunity typically contains several types of options. The most common types will be presented in the following.

**The Option to Wait.** In many situations investments can be undertaken not only at some unique point in time, but during a time window. Thus, management has the option to wait until conditions are favorable to decide on the execution. Delaying has the advantage that more information regarding market developments might be revealed.

### 3. Real Options

In terms of the real options view on investment opportunities, the option to wait is the option to purchase the remaining options in the basket by paying the initial investment needed to start the project. The option is only exercised if the market conditions are favorable and the initial investment will pay off by the remaining options and their corresponding cash-flows. Otherwise, the project is not started and all the remaining options turn worthless.

The option to wait and defer an investment is especially valuable in the absence of competition. Generally, competition is important to take into account when modeling real options.

**The Option to Default During Construction.** Often projects are run in several stages and an abandonment of the project is possible before starting the next stage. The option to default during construction has a value, when the investment does not require one outlay, but a series of expenses. The option is exercised if the remaining expenses are expected to be higher than the expected revenues. In staged projects, the outlay in each stage is the price for acquiring the option to continue in the next stage.

**The Option to Expand or Contract.** This option reflects the scalability of many projects. Production plants are often scalable to some degree. This might concern buying additional machinery to extend production or to shut down parts of the plant.

Capacity expansion typically involves costs and thus the option to expand is only exercised if the conditions are favorable. The option to contract enables the management to reduce the scale of the project by, for example, reducing the production in a factory. This causes the abandonment of future revenues to save operational expenses.

**The Option to Abandon.** Many projects can be abandoned before the planned lifetime ends. Facilities and technologies are often still of value and can be sold on the market.

**Interdependent and Compound Options.** Typical investment projects comprise several options effecting each other. The option to default is an example of a compound option, since it involves several options: one for each period when defaulting is possible, but they are highly interdependent. Each of them has to be exercised to acquire the next. The value of the option to default in some period, hence, depends on the exercise strategy in the preceeding periods.

When options interact with each other they cannot be valued independently anymore. They have to be seen as one compound option. The exercise decision of an option in the compound depends also on the exercise strategy of the other options; in particular, on the exercise history of the other options. When the

decision to exercise does not only depend on the current state of the system but on the history, we call an option *path-dependent*. Interactions among options and path-dependence add significant complexity to the valuation. The determination of optimal exercise strategies is also very important.

## 3.2. Real Options in the Literature

The term “real option” was coined by Myers [62] in 1977. Since then, the study and application of real option has generated innumerable publications. Anyhow, we try to give an overview without claiming completeness.

In the mid 1980s, people started to investigate on real options by considering one option at a time.

Several authors contributed in the area of investment timing and stages investments. McDonald and Siegel [57] focus on the option to wait. Staged investment and the option to default during construction is discussed by Majd and Pindyck [55]. They assume that an investment has to be made with a limited investment level per time period. Therefore, investment has to be done continuously (but with possible breaks) over a time period and investment levels have to be determined. Bar-Ilan and Strange [6] model investments with several stages that take time to complete and where the staged investments have to be done at once, in contrast to the continuous model from Majd and Pindyck. Paddock, Siegel and Smith [68] investigated on staged investment in the context of offshore petroleum leases, where the three stages are: Exploration, development and extraction.

Options concerning the scalability of projects are another big branch. Pindyck [71] studies capacity choice where the demand is stochastic; an idea which is extended by Dixit and Pindyck [24]. The option to shut down and restart can also be seen in the light of scalability of projects. It has been studied by McDonald and Siegel [56]. Project abandonment is investigated by Myers and Majd [63].

Several publications target to present the real options to a broader public, especially corporate managers. Examples are publication by Luehrman [53,54], where the first runs under heading “Manager’s Tool Kit”, and Copeland and Tufano [19].

Various books about real options have been published. The prominent ones are by Trigeorgis [84] and Dixit and Pindyck [23], but there are several more, i.e., [17,18].

The real options approach has been used in several applications, such as natural resources [9, 68, 82], telecommunication [38, 42], technology adoption [3, 47, 58], electricity [21, 48, 76], research and development (R&D) [44, 89].

Several publications, i.e., [2, 8, 52], deal with the problems in the practical implementation of real option analysis and show the limitations.

The idea to model and value real options by means of multistage stochastic programming can be found in [85, 87] and in the PhD-thesis of Wang [86], which is our main reference in this context.

### 3. Real Options

#### 3.3. Types of Financial Options

Financial instruments that are derived from traded assets, such as stocks or bond, are called *derivatives*. *Options* are a special class of derivatives, where the buyer has the right, but not the obligation, to take some specified action.

Let us first review the definitions of the most common options in practice: *call* and *put* options. These products are often called *plain vanilla options*. In contrast to these standard options a variety of *exotic options* are traded, among them options depending on the average, minimum or maximum price of the underlying and barrier products.

Two types of options are distinguished: *European* and *American*. European options can only be exercised at a specified point in time, called expiry. American options, in contrast, can be exercised anytime within a time window, normally from the emission until the expiry. American options are more sophisticated since different exercise strategies have to be considered. Options are assumed to be of European type if not specified otherwise.

##### Plain Vanilla Options

Let us examine the most common types of options.

**The European Call Option.** We will first give a formal definition of the call option.

**Definition 3 ( Call Option):** The holder of a call option with underlying  $S$ , strike  $K$ , and expiry  $T$  has the right but not the obligation to buy one unit of  $S$  for the price  $K$ . European call options can be exercised only at time  $T$ , American call options anytime before  $T$ .

Any rationally acting investor exercises the option if the price  $S_t$  is higher than then the strike, otherwise he gets the asset at the same or a smaller price at the market. In the case of exercise, the asset can be sold directly and the holder of the option has an instant win of

$$S_t - K. \quad (3.1)$$

In practice options are mostly not settled by exchanging the asset, but by paying the difference (3.1). The resulting payoff profile for the call option is

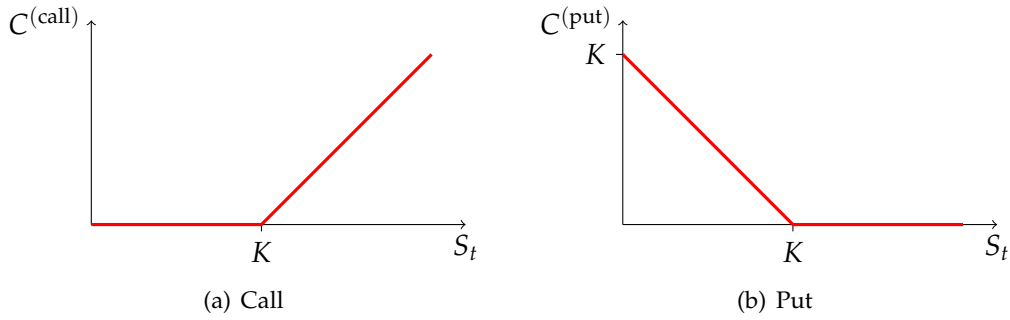
$$C^{(\text{call})} := (S_t - K)^+ \quad (3.2)$$

where  $(.)^+$  is the positive part defined by

$$(x)^+ := \max(0, x).$$

Figure 3.1(a) shows a plot of the payoff profile.





**Figure 3.1:** Payoff profiles of plain vanilla options with strike  $K$

**The Put Option.** The counterpart to call options are put options. They allow participation if the value of the asset decreases.

**Definition 4 (Put Option):** The holder of a put option with underlying  $S$ , strike  $K$ , and expiry  $T$  has the right but not the obligation to sell one unit of  $S$  for the price  $K$ . European put options can be exercised only at time  $T$ , American put options anytime before  $T$ .

The payoff profile for the put is

$$C^{(\text{put})} := (K - S_t)^+ \quad (3.3)$$

The plot in Figure 3.1(b) shows the payoff profile. We see that the holder profits from decreases in the value of the underlying asset.

### Exotic Options

Innumerable types of non-standard options exist. To get an impression, we present some of them. Let  $S$  be an asset. We present only the European version options.

Options whose payoff is based on the average price of an asset are called *Asian option*. Often discrete monitoring dates are used of average, such that the average price is defined by

$$S_t^{\text{av}} = \frac{1}{|\mathbb{T}(t)|} \sum_{k \in \mathbb{T}(t)} S_k,$$

where  $\mathbb{T}(t)$  is the set of monitoring dates that is smaller than  $t$ . The average price can now be used to define payoff profiles for call and put option

$$\begin{aligned} C_{\text{avP}}^{\text{call}} &:= (S_t^{\text{av}} - K)^+ \\ C_{\text{avP}}^{\text{put}} &:= (K - S_t^{\text{av}})^+ \end{aligned}$$

To make the confusion complete, Asian options can be of European or American type, referring to the possible exercise only at expiry or anytime before that.

### 3. Real Options

The payoff of *Lookback options* is based on the minimum or maximum price of an asset. Examples are the lookback put and lookback call with the respective payoff profiles

$$C_{\text{look}}^{\text{call}} := S_T - \min_{t \in [0, T]} S_t$$
$$C_{\text{look}}^{\text{put}} := \max_{t \in [0, T]} S_t - S_T.$$

Another mechanism to create exotic options are *barriers*. If the value of the underlying hits or passes a thresholds, the payoff profile of the option is changed. Classical barrier mechanisms are *up-and-in*, which only activates a payoff if the value of the underlying was at some point on or higher than the barrier, *up-and-out*, which only pays if the value of the underlying was never superior of the barrier, *down-and-in* and *down-and-out* which work similarly.

#### American options

In contrast to European options that can be exercised only at the expiry, American options can be exercised any time before at at the expiry. Pricing is therefore much more complicated and will not be discussed in detail. We refer to the textbooks [29, 45, 81].

Anyhow, we want to point out the key issues. The value of an option can be seen from two perspectives: from the investors and the sellers point of view. An investor is not willing to pay more for an American option than the expected payoff if the option is exercise optimally. In probability theory this is formulated using optimal *stopping times*. Since exercise can be anytime between the emission and the expiry, the subjective preference is also involved. Fortunately, in complete markets it suffices to maximize the expected payoff under the equivalent martingale measure. The seller of the option, on the other side, has to hedge the option. The hedging strategy must be chosen such that it covers the claims for *all* optimal stopping strategies.

#### Asymmetry of Risk

The most important point in the definitions of options is that the holder has the right but not the obligation to perform some action. This results in an asymmetry of risk. The holder profits from movements in one direction—up for calls, down for puts—, but has limited downside risk if the value of the underlying goes in the opposite direction.

### 3.4. Introduction to Mathematical Finance

The idea in valuing real options is to interpret them as options on traded assets and apply methods from mathematical finance to calculate values and prices.

Therefore, we want to give a brief introduction to pricing methods in mathematical finance. An understanding of the ideas and methods and especially of the assumptions in this field is useful in the application of real options.

First, we want to make some assumptions that are common to all the market models described here.

- No transaction costs
- No taxes
- No limitations in short-selling
- Money can be borrowed and deposited in unlimited amounts at the same interest rate
- Arbitrarily sharable assets

Short-selling means selling an asset that the trader does not own. The trader borrows it and sells it on the market. When he buys it back, the price difference is his profit. This way, it is possible to participate if the value of the asset goes down. The last point says that an investor can hold any fractional quantity of an asset.

Throughout this presentation, we assume that the assets do not pay dividends and that the interest rate is constant over time. All options are of European type.

### 3.4.1. Discrete Time Models

In this section, we present the results from the chapters 1 and 5 from Föllmer and Schied [29]. We also use the notation from this book. We do not aim for a complete review of the topic, but for an intuitive introduction and overview on the most interesting phenomena and principles in stochastic finance in discrete time. Of particular importance is the “fundamental theorem of asset pricing” which ensures the existence of probability measures under which expectations result in fair prices. We start from a description of the market model.

#### The market model.

A finite time horizon  $\mathcal{T} = \{0, \dots, T\}$  is assumed in this discrete setting. The market consists of  $d + 1$  assets. Assets can be bonds, stocks, currencies, commodities, and others. We assume that the initial prices at time 0 are deterministic.

**Prices.** The prices of the assets are described by non-negative random variables on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . The random variable  $S_t^k(\omega)$  denotes the value of 1 unit of asset  $k$  at time  $t$  in scenario  $\omega \in \Omega$ . If no confusion can occur, we omit the specification of the scenario and identify  $S_t^k = S_t^k(\omega)$ .

The prices at time  $t$  are combined to a random vector  $S_t = (S_t^0, S_t^1, \dots, S_t^d)$ . The stochastic process  $(S_t)_{t \in \mathcal{T}}$  describes the price system.

The 0th asset  $S_t^0$  is the risk-free bond. It yields the risk-free interest rate  $r$ . Let  $S_0^0 = 1$  be the initial value of the bond at time  $t = 0$ . Then the value in the later

### 3. Real Options

periods is

$$S_t^0 = (1 + r)^t \quad (3.4)$$

**Trading Strategies.** A *trading strategy* has to specify the quantities of the assets in the portfolio. In every period the portfolio can be restructured depending on the development of the prices. The trading strategy, thus, also depends on the realization of the price process.

To understand the mechanics, we start with an example. At time 0 the prices are known and an initial portfolio  $\xi_1 = (\xi_1^0, \dots, \xi_1^d)$  has to be chosen. The value  $\xi_1^k$  specifies the quantity of asset  $k$  in the portfolio at this point. The initial value of the portfolio is given by the scalar product  $\xi_1 \cdot S_0 = \sum_{i=0}^d \xi_1^i S_0^i$ . In period 1, after the prices have changed, the value of the portfolio is  $\xi_1 \cdot S_1 = \sum_{i=0}^d \xi_1^i S_1^i$ . At this point the portfolio can be restructured. The new quantities are given by the random vector  $\xi_2$ . Following this pattern in each period  $t \in \{0, \dots, T-1\}$  quantities for the portfolio  $\xi_{t+1} = (\xi_{t+1}^0, \dots, \xi_{t+1}^d)$  are chosen. The trading strategy  $(\xi_t)_{t \in \{1, \dots, T\}}$  is also a stochastic process since it depends on the development of the prices.

The selection of the portfolio has to fulfill two important properties. First, the selection is allowed to depend only on the observation of the prices up to time  $t$ . No at time  $t$  unknown events can influence the selection. Second, the selection has to be *self-financing*. This means that the restructuring has to preserve the portfolios value. This is expressed by the condition

$$\xi_t \cdot S_t = \xi_{t+1} \cdot S_t \quad \text{for all } t = 1, \dots, T-1 \quad (3.5)$$

The left-hand side is the value before and the right-hand side the value after restructuring.

**Discounting.** The risk-free bond  $S_t^0$  is used for discounting. The discounted prices process  $X$  is defined by the component-wise division

$$X_t := \frac{S_t}{(1+r)^t} \quad (3.6)$$

We define the discounted value process  $V$  for a trading strategy  $\xi$  by

$$V_0 := \xi_1 \cdot X_0 \quad (3.7)$$

$$V_t := \xi_t \cdot X_t \quad \text{for all } t = 1, \dots, T \quad (3.8)$$

**Arbitrage.** Let us now formally define what arbitrage is.

**Definition 5** (Arbitrage opportunity): A self-financing trading strategy is called an *arbitrage opportunity* if its discounted value process  $V$  satisfies

$$V_0 \leq 0, \quad V_T \geq 0 \text{ almost surely,} \quad \text{and} \quad \mathbb{P}(V_T > 0) > 0. \quad (3.9)$$

A market model is said to be *arbitrage-free* if it permits no arbitrage opportunities.

According to definition 5 an arbitrage opportunity is a trading strategy, that has zero or even negative value in the beginning, but non-negative value in the end and with positive probability the strategy yields a strictly positive profit. In the following we assume the market model to be arbitrage-free.

**European Contingent Claims.** With this description of the market, we can define financial derivatives. We consider *European contingent claims* which are non-negative random variables on  $(\Omega, \mathcal{F}_T, \mathbb{P})$ . At expiry  $T$  the claim is settled by paying the payoff according to the payoff profile. Contingent claims are general options whose payoff profile can depend on the development of the market but also of other factors, such as the weather and outcomes of sports events. We are interested in claims whose payoff is a function underlying asset. A European contingent claim  $C$  is called a *derivative* of the underlying assets  $S^0, S^1, \dots, S^d$ , if its payoff profile is a function of the assets prices, i.e., there exists a function  $f$  such that

$$C = f((S_t^0)_{t \in \mathcal{T}}, \dots, (S_t^d)_{t \in \mathcal{T}}) \quad (3.10)$$

If there is a trading strategy  $\xi$  such that the final value of the portfolio equals the payoff of the claim  $C$

$$C = \xi_T \cdot S_T \quad \text{almost surely.} \quad (3.11)$$

the contingent claim is replicable. A trading strategy fulfilling (3.11) is called *replicating portfolio*. The arbitrage-free price of a replicable claim is the initial value of the replicating portfolio. For non-replicable claims more than one fair price can exist without generating arbitrage.

Market models in which all contingent claims can be replicated are called *complete*.

From the sellers point of view, replicating portfolios is of great importance. They allow *hedging*, which means that the seller of a claim can source the risk arising from the potential future payoff out to the market. In this context the replicating portfolio is called *hedge*.

### Arbitrage-Free Pricing

In the following, we present the principles of risk-neutral pricing. Since a full presentation of all the details goes beyond the scope of this work, we present the key ideas and refer to Föllmer and Schied [29] for the details.

The absence of arbitrage in a market model is strongly related to the existence of special measures: *martingale measures*. They have the nice property that the discounted price processes of all assets are martingales under this measure. Martingales are the mathematical representation of fair games.

The following theorem states the relationship between martingale measures and arbitrage.

### 3. Real Options

**Theorem 3.1** (Fundamental Theorem of Asset Pricing): *A market model is arbitrage-free if and only if the set  $\mathcal{P}$  of equivalent martingale measures is non-empty.*

The impact of martingale measure lies in their pricing capability. The set  $\Pi(C)$  of arbitrage-free prices of a claim  $C$  is described by expectations under martingale measures.

**Theorem 3.2:** *The set of arbitrage-free prices of a claim  $C$  is non-empty and given by*

$$\Pi(C) = \left\{ \frac{1}{(1+r)^t} \mathbb{E}^*[C] \mid \mathbb{P}^* \in \mathcal{P} \text{ and } \mathbb{E}^*[C] < \infty \right\} \quad (3.12)$$

where  $\mathbb{E}^*[\cdot]$  denotes the expectation with respect to  $\mathbb{P}^*$ .

Theorem 3.2 is particularly useful in complete markets. In these markets, the martingale measure is unique which by Eq. (3.12) implies that the price is also unique and can be calculated as the discounted expected payoff of the claim.

#### 3.4.2. The Cox-Ross-Rubinstein-Model

In the time discrete setting, complete markets have a simple structure. Not only can all claims can be replicated, there is exactly one martingale measure and one unique arbitrage-free price. The probabilistic structure is also simple. A set  $A \in \mathcal{F}_T$  is an *atom*, if for every  $B \in \mathcal{F}_T$  with  $B \subseteq A$ , we have either  $\mathbb{P}(B) = 0$  or  $\mathbb{P}(B) = \mathbb{P}(A)$ . It can thus be interpreted as an scenario that cannot be split. The following theorem states that a complete market only distinguishes finitely many scenarios.

**Theorem 3.3:** *If a market model is complete, then the number of atoms in  $(\Omega, \mathcal{F}_T, \mathbb{P})$  is bounded by  $(d+1)^T$ .*

On the basis of this theorem, John Cox, Stephen Ross and Mark Rubinstein [20] described a market model, which consists of only two assets: the risk-free bond and one risky asset. The bond has an interest rate  $r$  and its value is given by

$$S_t^0 = (1+r)^t$$

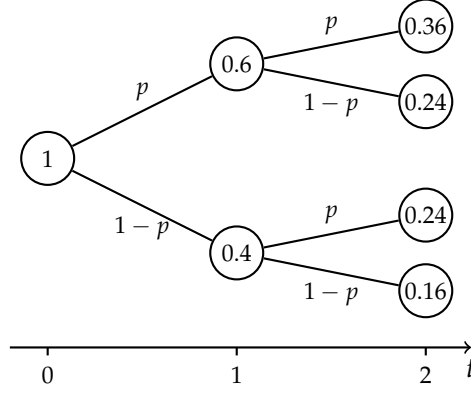
The asset behaves in a tree-like fashion. From  $S_t^1$  it moves with probability  $p$  to the high value  $S_t^1(1+a)$  and with probability  $1-p$  to the low value  $S_t^1(1+b)$ . Like this, it spans a binary tree with probabilities on the arcs as depicted in Figure 3.2. The model is often called *CRR-model* or *binomial model*.

Let us describe the model by first defining the sample space

$$\Omega = \{a, b\}^T. \quad (3.13)$$

The elementary events  $\omega \in \Omega$  are called *scenarios*. Let

$$Y_t(\omega) := y_t \quad \text{for } \omega = (y_1, y_2, \dots, y_T) \quad (3.14)$$



**Figure 3.2.:** Representation of a binary scenario tree with time horizon  $\mathcal{T} = \{0, 1, 2\}$ , nodes are labeled with their probabilities for  $p = 0.6$

be the projection on the  $t$ th coordinate. The price of the risky asset at time  $t$  is given by

$$S_t^1(\omega) = S_{t-1}^1(\omega)(1 + Y_t(\omega)) = S_0^1 \prod_{k=1}^t (1 + Y_k(\omega)) \quad (3.15)$$

Like that the  $t$ th coordinate of  $\omega$  specifies whether the price goes from  $S_{t-1}^1$  to the high value or to the low value.

**Arbitrage-free Pricing.** The following theorem specifies under which condition the CRR-model is arbitrage-free and also give a martingale measure.

**Theorem 3.4:** *The CRR-model is arbitrage-free if and only if  $a < r < b$ . In this case it is complete, and there is a unique martingale measure  $\mathbb{P}^*$ . The martingale measure is characterized by the fact that the random variables  $Y_1, \dots, Y_T$  are independent under  $\mathbb{P}^*$  with common distribution*

$$\mathbb{P}^*(Y_t = a) = p^* = \frac{r - b}{a - b} \quad \text{for } t = 1, \dots, T \quad (3.16)$$

A formal proof can be found in [29]. The risk-free probability of a scenario  $\omega \in \Omega$  is now easily calculated by

$$\mathbb{P}^*(\omega) = p^{*|\{t \mid Y_t(\omega)=a\}|} (1 - p^*)^{|\{t \mid Y_t(\omega)=b\}|} \quad (3.17)$$

Pricing in the CRR-model is now an easy task. The price is just the expectation of the discounted payoff with respect to the martingale measure. If the payoff is  $C(\omega)$  then is price  $\pi(C)$  is:

$$\pi(C) = \sum_{\omega \in \Omega} \mathbb{P}^*(\omega) \frac{C(\omega)}{(1 + r)^T} \quad (3.18)$$

### 3. Real Options

The CRR-model is of great importance not only because it is an easy model which allows to calculate arbitrage-free prices even for path dependent options, but it also has the nice property that the prices converge against the prices given by the continuous Black-Scholes Model if some regularity assumption are fulfilled.

#### 3.4.3. The Black-Scholes Model

The Black-Scholes model is the most famous pricing model in continuous time. It was published by Fischer Black and Myron Scholes [7] in 1973. Robert Merton was also working with them on the topic but published a separate paper [59]. Merton and Scholes were awarded with the Nobel Prize in Economics in 1995 for their work. Black could not be awarded, since he died in 1993 and the Nobel Prize can only be awarded to living persons.

**The Market Model.** The Black-Scholes model assumes a continuous time horizon  $[0, T]$ , where  $T$  normally is the maturity of the option which is to be priced. In its easiest form, the market contains only a riskless bond and one risky asset. The dynamics of the bond are characterized by continuous compounding with rate  $r$  such that the value at time  $t$  is given by

$$S_t^0 = e^{rt} \quad (3.19)$$

where  $S_0^0 = 1$  is the initial value of the bond.

The dynamic of the risky asset is given by a stochastic differential equation (SDE):

$$dS_t^1 = \mu S_t^1 dt + \sigma S_t^1 dW_t \quad (3.20)$$

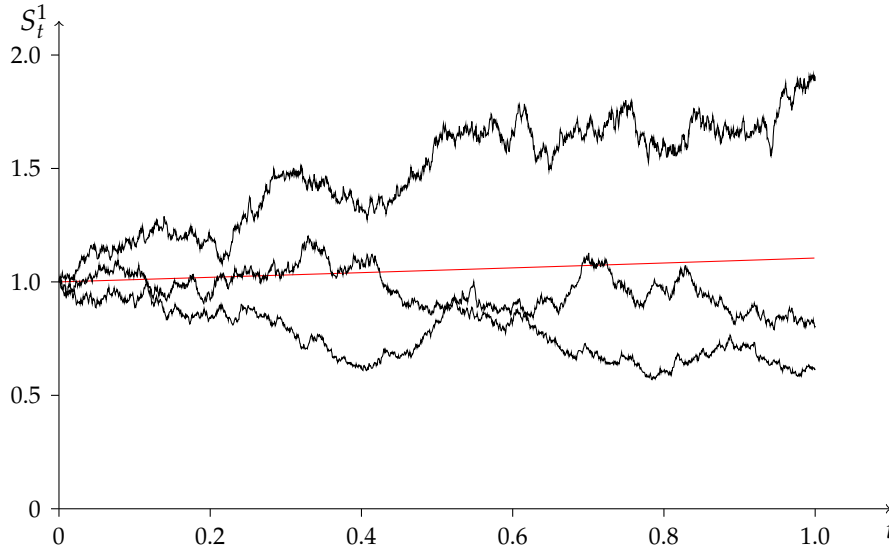
The left side of equation (3.20) is the infinitesimal change in the stock price. The change consists of a deterministic drift  $\mu S_t^1 dt$  with drift coefficient  $\mu$  and a stochastic perturbation  $\sigma S_t^1 dW_t$ . The parameter  $\sigma$  is called *volatility* and describes the degree of fluctuation in the price. By  $W_t$  we denote a standard Brownian motion in the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . For theory of stochastic processes, we refer to text books from probability theory [49] or from finance [81]. Theory and solution methods for stochastic differential equation can be found in [66].

One solution of the SDE (3.20) with start value  $S_0^1$  is

$$S_t^1 = S_0^1 \exp \left( \sigma W_t + \left( \mu - \frac{1}{2} \sigma^2 \right) t \right). \quad (3.21)$$

This process is called *geometric Brownian motion*. Figure 3.3 shows three realizations of the geometric Brownian motion on the interval  $[0, 1]$ . The smooth line shows the drift component. Deviations from the drift are caused by the perturbing effect of the Brownian motion.





**Figure 3.3.:** Three realizations of geom. Brownian motion with  $\sigma = 0.4$  and  $\mu = 0.1$ , the red line indicates the drift  $e^{\mu t}$

**The Black-Scholes Formula.** In this model several options can be priced explicitly and it is very useful to approximate the value of exotic options numerically. In particular Black and Scholes derived explicit formulas to price the European call with maturity  $T$  and strike  $K$ . They showed that the price process  $c(t, x)$  of the call, which takes the time  $t$  as first and value of the underlying asset as second argument, is a continuous function that satisfies the Black-Scholes partial differential equation

$$c_t(t, x) + rxc_x(t, x) + \frac{1}{2}\sigma^2 x^2 c_{xx}(t, x) = rc(t, x) \quad \text{for all } t \in [0, T), x \geq 0. \quad (3.22)$$

The final value of the price process has to be the payoff profile of the call, so that the terminal condition is

$$c(T, x) = (x - K)^+ \quad (3.23)$$

This partial differential equation can be solved and  $c(t, x)$  is given by

$$c(t, x) = xN(d_+(T - t, x)) - Ke^{-r(T-t)}N(d_-(T - t, x)), \quad \text{for all } t \in [0, T), x \geq 0 \quad (3.24)$$

where

$$d_{\pm}(T - t, x) = \frac{1}{\sqrt{T-t}} \left[ \log \frac{x}{K} + \left( r \pm \frac{\sigma^2}{2} \right) (T - t) \right]$$

### 3. Real Options

and  $N$  is the cumulative standard normal distribution

$$N(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{z^2}{2}} dz$$

This shows various things. The price of the option does not depend on the drift parameter  $\mu$  if the geometric Brownian motion, which in practice is very hard to determine or even estimate. The volatility is much easier to observe. Furthermore, a hedging rule can be found in the derivation of the PDE (3.22). The seller of the option has to hold  $c_x(t, S_t^1)$  units of the asset. The first partial derivative of the option price in the direction of the underlying price is called *delta*, thus the name *Delta-Hedge*. In theory this trading strategy allows a perfect replication of the European call, but adjusting of the positions have to be done continuously, which is obviously not applicable in practice.

**Risk-Neutral Pricing.** The Black-Scholes Model is complete, which implies the existence of a unique martingale measure. We do not go into the details here, but we want to state how this can be used to approximate the value of options. Details can be found in [81].

The theorem of Girsanov is used to derive a description of the behavior of the risky asset under the martingale measure  $\mathbb{P}^*$ . The dynamic is given by

$$dS_t^1 = rS_t^1 dt + \sigma S_t^1 d\tilde{W}_t \quad (3.25)$$

where  $\tilde{W}_t$  is a standard Brownian motion under the martingale measure  $\mathbb{P}^*$ . Note that the drift coefficient from (3.20) has been replaced by the risk-free interest rate and has no longer influence on the dynamics. This reflects that the drift does not have influence on the price of the call option.

Prices can be computed by a similar pricing formula to the discrete time model

$$\pi(C) = e^{-rt} \mathbb{E}^*[C] \quad (3.26)$$

is still valid. The paths of the assets can be approximated using (3.25), such that the expected value can be approximated by Monte Carlo simulations or other simulation techniques.

## 3.5. Revisiting the Assumptions from Finance

In section 3.1 we have seen that options arising from flexibility in project management have a similar structure than financial option, but can they be priced with the same models?

Borison [8] gives a summary over the different approaches to justify the applicability of the results from mathematical finance. We adopt his criticism and

present multistage stochastic programming as a framework to maximize the expected profit in a net present value sense in incomplete markets. By that execution strategies for compound real options are computed.

In the previous section, we have seen that risk-neutral prices can be computed in markets where all risks can be hedges by trading the markets' assets. The CRR-model and the Black-Scholes Model are complete, meaning that every derivative can be replicated by a trading strategy in the market. In particular, the underlying of the derivative is traded in a hedging strategy. In most real options, the project value is the underlying. To make it tradable, a *twin security* as to be identified. It is a traded asset whose value is perfectly or at least highly correlated to the value of the project. Portfolios of traded assets are possible to be used as twin securities as well. Having a twin security, a lot a real options can be priced by financial methods, since their payoffs can be replicated with trading the twin security. Finding reasonable twin securities is however difficult, if not impossible, in most cases.

This shows two problems in the application of real option analysis: Finding twin securities is difficult and sometimes not possible. Even if a twin security can be found, its stochastic behavior has to be modeled. In the Black-Scholes Model assumed that the underlying behaves like geometric Brownian motion. Hence, to apply the standard Black-Scholes Model, the twin security has to follow a geometric Brownian motion.

In the absence of a suitable twin security, the parameter needed to apply the Black-Scholes Model can be estimated. This however is contradictory to the assumptions of the model and the prices will no longer have an interpretation. Some authors (i.e. [53]) argue that even though the prices are not correct, qualitative insights are still possible.

Another approach to justify the use of simple financial models to price real options is the *Marketed Asset Disclaimer* (MAD). In this concept, which is described for example in [18], the NPV of the project without flexibility is used as twin security. The parameter required of the financial pricing model are estimated by Monte-Carlo Simulations or other techniques and then prices are calculated on this basis. Again, the condition that risks can be hedged in the market is violated. Additionally, the pricing model makes assumptions about the assets probability distribution. If the Black-Scholes Model is used, it is assumed that the value of the project is a geometric Brownian motion. This assumption is not fulfilled in most applications.

If the real options refers to applying standard methods from finance, in particular the Black-Scholes Model, to the problem of valuing investment opportunities in uncertain environments, the method is of little practical use or at least has to be applied very carefully. Anyhow, we think it is more than that. The view on flexibility, namely as option to perform some action in the future, is very natural and appealing. This explains the great success of methods carrying the label "real options".

### 3. *Real Options*

Big efforts have been made to use the real options idea in incomplete environments, which means in environments where risks from the project cannot be hedged with traded assets.

Various authors distinguish two types of risks. Market risks can be hedged within the market, while idiosyncratic or private risks are company specific, such that no hedging is possible. The idiosyncratic risks make the market incomplete. Typically the uncertainty in future cash-flows is decomposed by risk type. The market part is priced using no-arbitrage pricing and the private part is valued by subjective estimates on probabilities and risk-preferences. This results in an integration of financial pricing theory and decision analysis. Examples of this approach are [83] and [60]

The market in the telecommunication sector usually only consists of a few competitors; many of them international companies present in different markets and regions. The competitors typically follow different business strategies that are often expressed by different network evolution strategies. The technology leader typically adopts the latest technologies early and builds a network with large coverage. The market follower typically starts the construction at a later point and concentrates on the locations where a high usage is expected. Finding a twin asset for the value of a telecommunication network of a specific technology and market seems difficult under these circumstances. A market driven valuation of the investment opportunity can thus not be expected. The real options framework, however, is still a powerful tool to model and conceptualize options in investment opportunities.

Richard de Neufville and Tao Wang [86–88] distinguish two types of real options: real options “in” and “on” projects. Their definition of real options “on” projects concerns investment opportunities which are present for various participants in the market and can thus be replicated. The project itself with all the technological aspects is seen as a black box. This corresponds to the traditional approach to real options in a complete market. Real options “in” project, on the other side, describe the flexibility in the project including technological details. The types of risk and flexibility cannot be valued by the market. This is the classical situation where no suitable twin security can be found. In his PhD-thesis [86] Wang states that these options are often highly interdependent and compound options, which also makes the classical pricing approach difficult to apply.

He proposes to use stochastic mixed-integer programming to evaluate real options “in” projects. The objective is to compute execution strategies for real options that maximize the discounted profit of the investment. In the absence of a complete market, the value is not market driven, but in a net present value sense.

## 4. Multistage Stochastic Optimization

The evolution of telecommunication networks takes place in a volatile, rapidly changing environment. The strategic planning of the network evolution should consider the uncertainty in the planning parameters. In the previous section, the real options framework is presented as a framework to model investment opportunities in uncertain environments and evaluate its profitability. Unfortunately, the assumption of tradability of the underlying or the existence of a twin asset is not fulfilled in our application. Since the construction of a telecommunication network is a stagewise process that involved large time horizons, we use multistage stochastic programming to compute network evolution strategies that are adapted to the evolution of the uncertain parameters and that maximize the profit in a net present value sense.

In the chapter, we introduce some concepts of multistage stochastic programming and common deterministic equivalent formulations. More details on (multistage) stochastic programming can be found for example in [22] and [77]. In the remainder of this chapter we first present the general concept of multistage stochastic programming and introduce some notation. Next, we present two formulations; the scenario and the node formulation. Last, we introduce the concept of risk measures and present two representatives in more detail.

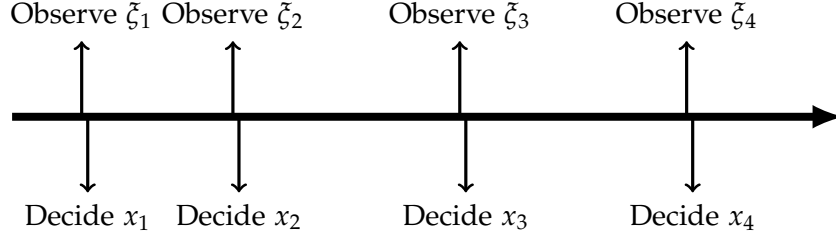
### 4.1. Introduction

Traditionally, optimization was performed in a deterministic setting. All parameters are assumed to be known in advance, otherwise, some expectation is used. In many areas, however, uncertainty is an important part of the problem and neglecting it leads to poor planning decisions. *Stochastic programming* is a framework that allows optimization under uncertainty. In *multistage stochastic programming* uncertainty is resolved gradually. The random parameters are described by a stochastic process over time, whose distribution is known. The realization of the process can be observed at discrete points in time and recourse actions are possible. This results in an iterated observation-decision scheme, where a decision on recourse actions follows each observation of the stochastic process. Figure 4.1 gives a graphical representation of the observation-decision scheme.

We consider a discrete time horizon  $\mathcal{T}$  and assume that the set is either finite or countable infinite. Later, we will restrict ourselves to finite time horizons. We identify the time points by their index and write  $\mathcal{T} = \{1, 2, \dots, T\}$  or  $\mathcal{T} = \{1, 2, \dots\}$ .

The stochastic parameters are described by a stochastic process on a probability

#### 4. Multistage Stochastic Optimization



**Figure 4.1.:** Graphical representation of the iterated decision-observation scheme

space  $(\Omega, \mathcal{F}, \mathbb{P})$ . We denote the random data process by  $(\mathbb{X}_t)_{t \in \mathcal{T}}$ . The realization in scenario  $\omega \in \Omega$  is thus  $(\mathbb{X}_t(\omega))_{t \in \mathcal{T}}$ . With  $\mathbb{X}_{[t]} = (\mathbb{X}_1, \dots, \mathbb{X}_t)$ , we denote the history of the data process up to time  $t$ . The data for the first period is assumed to be deterministic. The information incorporated in the stochastic process is described by a series of  $\sigma$ -algebras

$$\mathcal{F}_t := \sigma(\mathbb{X}_{[t]}) = \sigma(\mathbb{X}_1, \dots, \mathbb{X}_t), \quad t \in \mathcal{T} \quad (4.1)$$

Since  $\mathbb{X}_1$  is deterministic, corresponding  $\sigma$ -algebra  $\mathcal{F}_1$  is trivial, i. e.,

$$\mathcal{F}_1 = \{\emptyset, \Omega\}.$$

The information that stem from observing the process is increasing. This is expressed by successive inclusions of the  $\sigma$ -algebras:

$$\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \dots \subseteq \mathcal{F}_t \subseteq \dots \quad (4.2)$$

A series of  $\sigma$ -algebras satisfying (4.2) is called filtration.

At every point  $t \in \mathcal{T}$  in time, recourse actions are possible. Recourse actions are represented by vector valued random variables, i. e., different recourse actions are possible for distinct realizations. We denote the random variables corresponding to period  $t$  with  $x_t$ . The decision strategy up to time  $t$  is denoted by  $x_{[t]} = (x_1, \dots, x_t)$ . We assume without loss of generality, that the number  $d_t$  of decisions in stage  $t$  is independent of the realization of  $\mathbb{X}_t$ .

The decisions have to be non-anticipative, meaning that  $x_t$  only depends on the realization of the data process up to period  $t$  and not on future events. In terms of  $\sigma$ -algebras, we demand that the random variable  $x_t$  is measurable with respect to  $\mathcal{F}_t$ .

Typically, not all decisions are feasible. The set of feasible decisions in period  $t$  may depend on the decisions taken in the previous periods and on the evolution of the random data process. We, thus, denote the set of feasible solutions in stage  $t$  with  $\mathcal{X}_t(x_{[t-1]}, \mathbb{X}_t)$ . Since first stage decisions are deterministic, we denote the set of feasible first stage decisions by  $\mathcal{X}_1$ .

Costs are associated with the decisions. The objective in stochastic programming is often a function of the costs occurring in all stages and scenarios. Costs can be

random, too. For example, prices for raw material might not be known in advance. The cost occurring in period  $t$  depends on the decision  $x_t$  and on the stochastic data process  $\mathbb{X}_t$  and is denoted by  $f_t(x_t, \mathbb{X}_t)$ . Since  $\mathbb{X}_1$  is deterministic, the cost for the first stage decision are not random. The cost function is denoted by  $f_1(x_1)$ .

Several ways for aggregating the cost into one objective function are known. Traditionally, the expected value of the cost is optimized. Other functionals that are more adverse to risk are also widely employed, see Section 4.4 for an introduction to risk measures. For now, we assume that the objective is to minimize the expected costs. Revenues are interpreted as cost with a negative sign, thus, expected profit can be maximized equivalently.

A generic *multistage stochastic optimization problem* can be written as

$$\text{Min}_{x_1 \in \mathcal{X}_1} f_1(x_1) + \mathbb{E} \left[ \inf_{x_2 \in \mathcal{X}_2(x_{[1]}, \mathbb{X}_2)} \left\{ f_2(x_2, \mathbb{X}_2) + \mathbb{E} \left[ \inf_{x_3 \in \mathcal{X}_3(x_{[2]}, \mathbb{X}_3)} \left\{ f_3(x_3, \mathbb{X}_3) + \mathbb{E}[\dots] \right\} \right] \right\} \right] \quad (4.3)$$

By recursively defining the *cost-to-go* function

$$\mathcal{Q}_t(x_{[t-1]}, \mathbb{X}_t) := \inf_{x_t \in \mathcal{X}_t(x_{[t-1]}, \mathbb{X}_t)} f_t(x_t, \mathbb{X}_t) + \mathbb{E} \left[ \mathcal{Q}_{t+1}(x_{[t]}, \mathbb{X}_{t+1}) \right] \quad (4.4)$$

we can reformulate (4.3) to

$$\text{Min}_{x_1 \in \mathcal{X}_1} f_1(x_1) + \mathbb{E} \left[ \mathcal{Q}_2(x_{[1]}, \mathbb{X}_2) \right] \quad (4.5)$$

Equations (4.4) and (4.5) hide the complexity of the multistage problem, but the interpretation fits nicely in the decision-observation scheme depicted in Figure 4.1; in every stage the decision is taken such that the sum of current and expected future cost is minimized.

The problem is said to be *linear* if the objective function is linear and the sets of feasible solutions are described by linear constraints. The random data process can thus be interpreted as a series of random vectors and matrices of suitable dimensions

$$\mathbb{X}_t = (c_t, (B_\tau^{(t)})_{\tau=1, \dots, t-1}, b_t),$$

such that cost and feasible decision sets have the form

$$f_t(x_t, \mathbb{X}_t) = c_t x_t \quad (4.6)$$

$$\mathcal{X}_t(x_{[t-1]}, \mathbb{X}_t) = \left\{ x_t \in \mathbb{R}^{d_t} \mid \sum_{\tau=1}^t B_\tau^{(t)} x_\tau \leq b_t \right\}. \quad (4.7)$$

A problem of this form is called *(Multistage) Stochastic Linear Program* (SLP). Models containing equality constraints can be easily transformed to a system of inequalities using standard transformations.

#### 4. Multistage Stochastic Optimization

Integer requirements on some of the variables yield a *(Multistage) Stochastic Mixed-Integer (Linear) Program* (SMIP). Let  $\mathcal{J}_t$  denote the index set of the variables at time  $t$  that have to meet integer requirements and  $x_{jt}$  the  $j$ th component of the vector  $x_t$ . Then, the feasible sets in SMIPs have the form

$$\mathcal{X}_t(x_{[t-1]}, \mathbb{X}_t) = \left\{ x_t \in \mathbb{R}^{d_t} \mid \sum_{\tau=1}^t B_\tau^{(t)} x_\tau \leq b_t, x_{jt} \in \mathbb{Z}, j \in \mathcal{J}_t \right\}. \quad (4.8)$$

Of course, stochastic linear programs are the special form of stochastic mixed-integer programs, where the sets  $\mathcal{J}_t$  are empty for all  $t$ .

In the following two sections, we present two common formulations of deterministic equivalents of (4.3), as they will be necessary to approach the solution of (4.3) numerically.

### 4.2. The Scenario Formulation

For each scenario  $\omega \in \Omega$ , the random data process is realized and a decision strategy has to be found. To emphasize the dependence on the scenario  $\omega \in \Omega$ , we denote the random data by  $(\mathbb{X}_t(\omega))_{t \in \mathcal{T}}$  and the corresponding decision strategy by  $(x_{t\omega})_{t \in \mathcal{T}}$ .

This notation obfuscates that the decision at stage  $t$  depends only on the observation and decision up to this time and not on the entire scenario. The variable  $x_{t\omega}$  as mapping from  $\Omega$  to  $\mathbb{R}^{d_t}$  has to be measurable with respect  $\mathcal{F}_t = \sigma(\mathbb{X}_{[t]})$ . This implies that decisions which base on the same observation of the stochastic process have to coincide. This is ensured by the *non-anticipativity constraints*:

$$x_{t\omega} = x_{t\tilde{\omega}} \quad \text{for all } \omega \neq \tilde{\omega} \text{ with } \mathbb{X}_{[t]}(\omega) = \mathbb{X}_{[t]}(\tilde{\omega}), t \in \mathcal{T} \quad (4.9)$$

This allows to state the scenario formulation of a stochastic program:

$$\text{Min}_{x_1, x_2, \dots, x_n} \mathbb{E}[f_1(x_{1\omega}) + f_2(x_{1\omega}, \mathbb{X}_2(\omega)) + f_3(x_{2\omega}, \mathbb{X}_3(\omega)) + \dots] \quad (4.10a)$$

$$\text{s. t. } x_{1\omega} \in \mathcal{X}_1 \quad \text{for all } \omega \in \Omega \quad (4.10b)$$

$$x_{t\omega} \in \mathcal{X}_t(x_{[t-1]\omega}, \mathbb{X}_t(\omega)) \quad \text{for all } t \in \mathcal{T}, t > 1, \omega \in \Omega \quad (4.10c)$$

$$x_{t\omega} = x_{t\tilde{\omega}} \quad \text{for all } \omega \neq \tilde{\omega} \text{ with } \mathbb{X}_{[t]}(\omega) = \mathbb{X}_{[t]}(\tilde{\omega}), t \in \mathcal{T} \quad (4.10d)$$

Minimizing the expected cost is the objective (4.10a). Constraints (4.10b) and (4.10c) ensure feasibility and (4.10d) ensures non-anticipativity. The non-anticipativity constraints couple different scenarios. In the absence of these constraints, the problem decomposes to  $|\Omega|$  independent subproblems. This makes this formulation very amenable for decomposition approaches.



Let us in the following assume a finite time horizon ( $|\mathcal{T}| < \infty$ ) and a finite probability space ( $|\Omega| < \infty$ ). The expectation can then be easily computed as

$$\sum_{\omega \in \Omega} \mathbb{P}(\omega) \left( f_1(x_{1\omega}) + f_2(x_{1\omega}, \mathbb{X}_2(\omega)) + \dots + f_T(x_{T-1\omega}, \mathbb{X}_T(\omega)) \right)$$

In the linear case the stochastic program can then be formulated as a MIP. The stochastic process consists of matrices and vectors of appropriate dimension

$$\mathbb{X}_t(\omega) = (c_{t\omega}, (B_{\tau\omega}^{(t)})_{\tau=1,\dots,t}, b_{t\omega})$$

The SMIP can now be stated as a regular MIP, which is called *Deterministic Equivalent Model* (DEM):

$$\text{Min} \quad \sum_{\omega \in \Omega} \mathbb{P}(\omega) \sum_{t=1}^T c_{t\omega} x_{t\omega} \quad (4.11a)$$

$$\text{s. t.} \quad \sum_{\tau=1}^t B_{\tau\omega}^{(t)} x_{\tau\omega} \leq b_{t\omega} \quad \text{for all } \omega \in \Omega \quad (4.11b)$$

$$x_{t\omega} = x_{t\tilde{\omega}} \quad \text{for all } \omega \neq \tilde{\omega} \text{ with } \mathbb{X}_{[t]}(\omega) = \mathbb{X}_{[t]}(\tilde{\omega}), t = 1, \dots, T \quad (4.11c)$$

$$x_{jt\omega} \in \mathbb{Z}, \quad \text{for all } \omega \in \Omega, j \in \mathcal{J}_t, t \in \mathcal{T} \quad (4.11d)$$

$$x_{jt\omega} \in \mathbb{R} \quad \text{for all } \omega \in \Omega, j \notin \mathcal{J}_t, t \in \mathcal{T} \quad (4.11e)$$

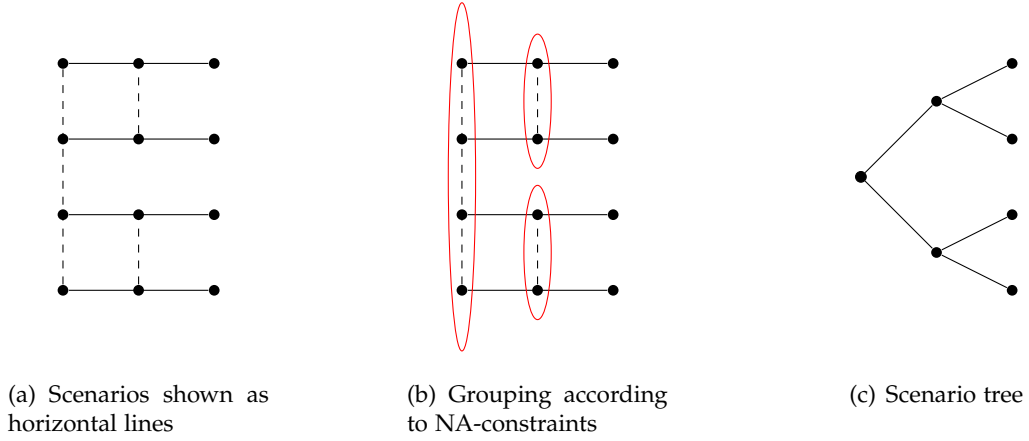
### 4.3. The Node Formulation

The scenario formulation with explicit non-anticipativity constraints is intuitive, but treating the non-anticipativity constraints implicitly allows for a much more compact formulation. The idea is to group variables that have to be equal due to the non-anticipativity constraints and add just one representative to the model.

Nodes represent the different information states, i.e., the realizations of  $\mathbb{X}_{[t]}$ . The node belonging to the deterministic initial information is the root. Each node, except the root, is connected with the node corresponding to its information history up to the previous period. The information history is unique such that each node, except the root, has exactly one direct predecessor and the result is a tree. The tree completely describes the probability distribution and is called *scenario tree*. Each scenario is represented by a path from the root to a leaf. The transformation from scenarios with non-anticipativity constraints to a scenario tree is depicted in Figure 4.2.

This procedure is also feasible if the distribution of  $\mathbb{X}_t$  is continuous or infinitely many scenarios exist. Then the tree has infinitely many nodes and edges. In the following, we restrict ourselves to probability spaces with finitely many scenarios and finite time horizons.

#### 4. Multistage Stochastic Optimization



**Figure 4.2.:** Transformation from scenario with non-anticipativity (NA) constraints as dashed lines to scenario tree.

Let us introduce some notation. Let  $(\mathcal{N}, \mathcal{E})$  be a scenario tree with node set  $\mathcal{N}$  and edge set  $\mathcal{E}$ . For a node  $n \in \mathcal{N}$  we denote with  $t(n)$  the corresponding stage, with  $p(n)$  its father, and with  $p^k(n)$  the  $k$ th predecessor of  $n$ . The history of a node  $n$  is represented by the path from  $n$  to the root. The path is denoted by  $\mathcal{P}(n)$ . The set of nodes belonging to stage  $t$  is denoted by  $\mathcal{N}(t)$ . With  $\mathbb{P}(n)$ , we denote the probability that node  $n$  is realized. Table 4.1 gives an overview on the notation used.

A realization of the stochastic data process corresponds to each node of the tree. The data process and the variables are, thus, not given per scenario  $\omega$ , but per node  $n$ . The formulation of a multistage stochastic program which is based on a scenario tree is thus called *node formulation*. We omit the general version and present directly the formulation for the linear case. In this case, the random data process again contains vectors and matrices of suitable dimensions:

$$\mathbb{X}(n) = (c_n, (B_{\bar{n}}^{(n)})_{\bar{n} \in \mathcal{P}(n)}, b_n)$$

The decisions in node  $n$  might be constrained by decisions in former periods, i. e., decisions corresponding to the nodes on the path from the root to  $n$ . Therefore, a constraint matrix  $B_{\bar{n}}^{(n)}$  for each node  $\bar{n}$  on the path from  $n$  to the root is given. The expected cost is calculated by weighting the cost in each node according to its probability:

$$\sum_{n \in \mathcal{N}} \mathbb{P}(n) c_n x_n \tag{4.12}$$

Name	Domain	Interpretation
$(\mathcal{N}, \mathcal{E})$		The scenario tree
$\mathcal{N}$		Nodes in scenario tree
$\mathcal{S}$	$\subseteq \mathcal{N}$	Set of scenarios, i.e., the leaves of the scenario tree
$\mathcal{P}(n)$	$\subseteq \mathcal{N}$	The path from node $n$ to the root including the node itself
$t(n)$	$\in \mathcal{T}$	The stage of node $n$
$r$	$\in \mathcal{N}$	The root of the tree
$p(n)$	$\in \mathcal{N}$	The unique father of node $n$ in the scenario tree
$p^i(n)$	$\in \mathcal{N}$	The $i$ th ancestor of node $n$ in the scenario tree
$\mathbb{P}(n)$	$\in [0, 1]$	Probability of node $n$

Table 4.1.: Notation related to scenario trees

The deterministic equivalent program in the node formulation is then:

$$\text{Min} \quad \sum_{n \in \mathcal{N}} \mathbb{P}(n) c_n x_n \quad (4.13a)$$

$$\text{s. t.} \quad \sum_{\bar{n} \in \mathcal{P}(n)} B_{\bar{n}}^{(n)} x_{\bar{n}} \leq b_n \quad \text{for all } n \in \mathcal{N} \quad (4.13b)$$

$$x_{jn} \in \mathbb{Z} \quad \text{for all } j \in \mathcal{J}_{t(n)}, n \in \mathcal{N} \quad (4.13c)$$

$$x_{jn} \in \mathbb{R} \quad \text{for all } j \notin \mathcal{J}_{t(n)}, n \in \mathcal{N} \quad (4.13d)$$

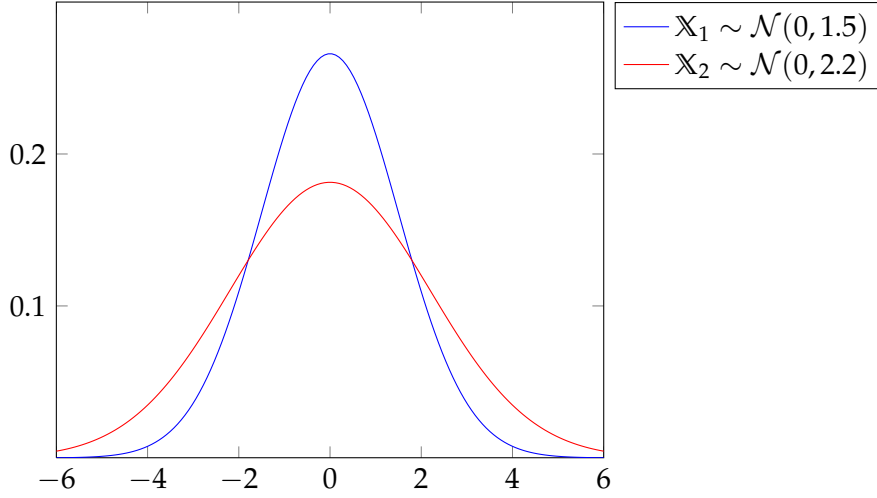
Minimizing the expected cost is the objective (4.13a). The decisions in node  $n$  are possibly constrained by all decisions in the previous stages. Thus, in (4.13b) all decisions corresponding to nodes in the path  $\mathcal{P}(n)$  appear in the constraint. The domains of the variables constitute the last two rows of the program.

We see that the non-anticipativity constraints are not stated explicitly anymore but that decisions are non-anticipative by construction. The compactness of the node formulation makes this formulation favorable for the solution with general MIP solvers.

## 4.4. Risk Measures

The quantification of risk is an important subject in stochastic optimization. The meaning of the word risk is not clearly defined. Generally, decisions that incorporate a higher probability of high losses are considered riskier. Consider, for example, the two normal distributions from Figure 4.3. Both have the same mean, but different variance. We use the notation  $\mathbb{X}_1 \sim \mathcal{N}(0, 1.5)$  to express that  $\mathbb{X}_1$  follows a normal distribution with mean 0 and variance 1.5.  $\mathbb{X}_2$  has variance 2.2, such that  $\mathbb{X}_2 \sim \mathcal{N}(0, 2.2)$ . The density of  $\mathbb{X}_1$  with the low variance is more concentrated around the mean. If the corresponding random variables represent profit, the probability of obtaining a high loss (profit  $< -4$ , say) is much higher for  $\mathbb{X}_2$

#### 4. Multistage Stochastic Optimization



**Figure 4.3.:** Density functions of two Gaussian random variables with mean  $\mu = 0$  and different variance  $\sigma$

than for  $\mathbb{X}_1$ . The random variable with the higher variance is therefore considered more risky. Apparently, the expected value is not suitable to capture the different risk profiles.

*Risk measures* serve to quantify risk. A risk measure  $\rho$  is a mapping from the random variables to the real numbers:

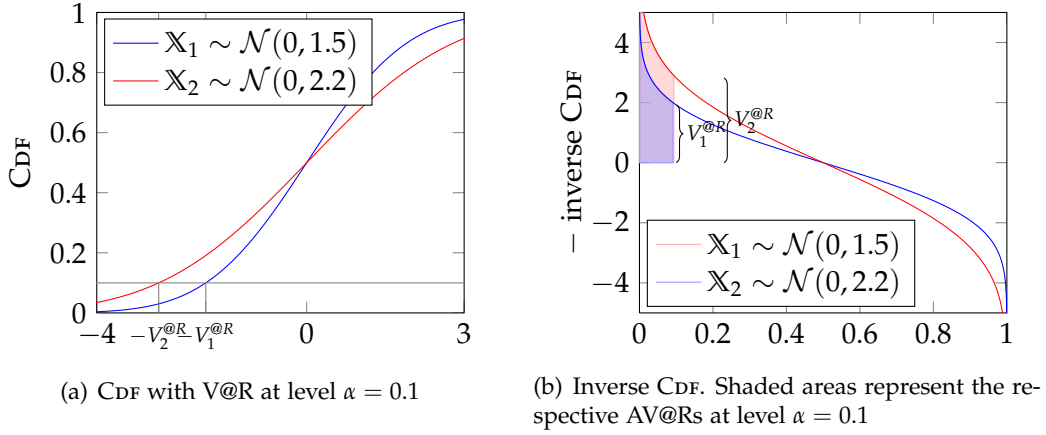
$$\rho : \mathcal{L}^1(\Omega, \mathcal{F}; \mathbb{R}) \rightarrow \mathbb{R} \quad (4.14)$$

The space  $\mathcal{L}^1(\Omega, \mathcal{F}; \mathbb{R})$  consists of all integratable functions from the measurable space  $(\Omega, \mathcal{F})$  to the real numbers. Unless otherwise specified, we assume all random variables to be in  $\mathcal{L}^1(\Omega, \mathcal{F}; \mathbb{R})$ . Furthermore, we assume that the random variables describe profits, i. e., larger values are better. This point is not handled uniformly throughout the literature. Even though both notions are equivalent (costs are just negative profits), the formulas differ if smaller values are considered favorable.

A decent risk measure should fulfill some desirable properties. Artzner, Delbaen, Eber and Heath [4] specify four desirable properties of a risk measure and call such risk measures *coherent*.

**Definition 6:** A mapping  $\rho : \mathcal{L}^1(\Omega, \mathcal{F}; \mathbb{R}) \rightarrow \mathbb{R}$  is called *coherent risk measure* if it fulfills the following four properties:

1. *Monotonicity:* If  $\mathbb{X} \leq \mathbb{Y}$  almost surely, then  $\rho(\mathbb{Y}) \leq \rho(\mathbb{X})$ .
2. *Translation invariance:* If  $m \in \mathbb{R}$ , then  $\rho(\mathbb{X} + m) = \rho(\mathbb{X}) - m$ .
3. *Positive homogeneity:* If  $\lambda \geq 0$ , then  $\rho(\lambda \mathbb{X}) = \lambda \rho(\mathbb{X})$ .
4. *Subadditivity:*  $\rho(\mathbb{X} + \mathbb{Y}) \leq \rho(\mathbb{X}) + \rho(\mathbb{Y})$ .



**Figure 4.4.:** CDFs and inverse CDFs for two Gaussian random variables. Short notation:  $V_i^{\text{@R}} := \text{V@R}_{0.1}(\mathbb{X}_i)$

For a detailed introduction to risk measures, we refer to [29] and [70]. In the following, we present two risk measures; Value-at-Risk and Average Value-at-Risk.

#### 4.4.1. Value-at-Risk

The Value-at-Risk (V@R) is a risk measure that is widely used in economics. It specifies a loss that is not exceeded with a given probability.

Let  $\mathbb{X}$  be a random variable on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  representing profit. We use the usual short notation

$$\mathbb{P}(\mathbb{X} < t) := \mathbb{P}(\{\omega \in \Omega \mid \mathbb{X}(\omega) < t\})$$

to denote the probability that  $\mathbb{X}$  is smaller than  $t$ . Similar notations are used for all other comparison operators.

We adopt the definition for the Value-at-Risk from [29].

**Definition 7:** The *Value-at-Risk* at confidence level  $\alpha \in (0, 1)$  is defined as

$$\text{V@R}_\alpha(\mathbb{X}) = -\sup \{m \mid \mathbb{P}(\mathbb{X} < m) \leq \alpha\} = -\inf \{m \mid \mathbb{P}(\mathbb{X} \leq m) > \alpha\}$$

The meaning of the V@R is that losses larger than  $\text{V@R}_\alpha(\mathbb{X})$  occur only with probability not exceeding  $\alpha$ . The size of the losses in these cases is, however, not quantified.

The V@R is strongly related with the cumulative distribution function (CDF) of  $\mathbb{X}$ . The CDF of  $\mathbb{X}$  is defined by

$$F_{\mathbb{X}}(t) := \mathbb{P}(\mathbb{X} \leq t)$$

The V@R can now be expressed by the CDF:

$$\text{V@R}_\alpha(\mathbb{X}) = -\inf \{t \mid F_{\mathbb{X}}(t) > \alpha\}$$

#### 4. Multistage Stochastic Optimization

If the CDF is continuous and strictly increasing, the V@R is the negative of the unique solution  $t$  of  $F_X(t) = \alpha$ . In this case, the cumulative distribution function is invertible and we have

$$V@R_\alpha(X) = -F_X^{-1}(\alpha)$$

The CDFs for the two Gaussian distributed random variables are depicted in Figure 4.4(a). The V@R with  $\alpha = 0.1$  is significantly higher for  $X_2$ , which has a higher variance. If, however, the confidence level is choose 0.5, both variables have the same V@R of 0 and thus have the same riskiness in this measure. This is because the V@R only captures the probability of a loss, but does not quantify the amount of capital loss in the tail of the distribution. Furthermore, the Value-at-Risk is not subadditive and hence not coherent (cf. [69]). A coherent risk measure, which is closely related to V@R, is the Average Value-at-Risk.

##### 4.4.2. Average Value-at-Risk

The *Average Value-at-Risk* (AV@R) overcomes the shortcomings of V@R. The AV@R is the expectation of the losses that exceed the V@R and therefore accounts for all unfavorable realizations.

**Definition 8:** The *Average Value-at-Risk* at confidence level  $\alpha \in (0, 1)$  is defined as

$$AV@R_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha V@R_\gamma(X) d\gamma$$

The integration ensures that the tail of the distribution enters into the measurement of risk. Other common names for the Average Value-at-Risk are “Conditional Value-at-Risk” or “Expected Shortfall”. In [69] the AV@R is shown to be a coherent risk measure.

If the cumulative density function is invertible, the AV@R can be obtained by integrating the inverse CDF:

$$AV@R_\alpha(X) = -\frac{1}{\alpha} \int_0^\alpha F_X^{-1}(\gamma) d\gamma$$

The negative inverse CDFs for two Gaussian random variables with different variance is plotted in Figure 4.4(b). The area under the plot is larger for the variable with higher variance, indicating a higher AV@R.

##### AV@R as the Solution of a Minimization Problem

Rockafella and Uryasev [73] showed that the Average Value-at-Risk is the solution of a minimization problem. This is particularly interesting since the problem can be formulated as a linear program. Adding a few auxiliary variables allows us to use the non-linear risk functional AV@R as objective function for our stochastic programs.

First, we present the minimization representation of AV@R.

**Theorem 4.1** (Fundamental minimization formula [73]): *Let  $\mathbb{X}$  be a random variable with finite expectation representing profit and  $\alpha \in (0, 1)$  a fixed confidence level. Then the function*

$$\phi_\alpha(\mathbb{X}, y) = y + \frac{1}{\alpha} \mathbb{E}[(y + \mathbb{X})^-] \quad (4.15)$$

*as a function of  $y$  is finite, convex, and hence continuous. The Average Value-at-Risk is given by*

$$\text{AV@R}_\alpha(\mathbb{X}) = \min_y \phi_\alpha(\mathbb{X}, y) \quad (4.16)$$

*The V@R is a minimizer, such that,*

$$\text{AV@R}_\alpha(\mathbb{X}) = \phi_\alpha(\mathbb{X}, \text{V@R}_\alpha(\mathbb{X}))$$

The Average Value-at-Risk can, thus, be calculated by solving the optimization problem (4.16). Even though  $\phi_\alpha(\mathbb{X}, y)$  is non-linear, the optimization problem can be linearized by adding additional variables and constraints. The AV@R is the optimal value of the following stochastic program:

$$\text{AV@R}_\alpha(\mathbb{X}) = \min y^0 + \frac{1}{\alpha} \mathbb{E}[y^-] \quad (4.17a)$$

$$\text{s.t. } y^0 + \mathbb{X}(\omega) = y_\omega^+ - y_\omega^- \quad \text{for all } \omega \in \Omega \quad (4.17b)$$

$$y^0 \in \mathbb{R}, (y^+, y^-) \in \mathbb{R}_{\geq 0}^{\Omega \times \Omega} \quad (4.17c)$$

The program (4.17) is a stochastic two-stage linear program with random right hand side. Risk measures that are expressed as the solution of certain two-stage stochastic programs are called *polyhedral*. For the precise definition and structural properties of polyhedral risk measures, we refer to [25]. Polyhedral risk measures have the nice property that, when used as objective function for stochastic programs, the resulting programs are still stochastic programs with additional variables and constraints. Thus, the standard MIP technology can be applied to their deterministic equivalents.

Consider for example a multi-stage stochastic program in node formulation (cf. program (4.13)). The profit is defined as a random variable  $v$  in the last stage of the problem. The corresponding stochastic program with the Average Value-at-Risk

#### 4. Multistage Stochastic Optimization

of the profit as objective function is:

$$\text{Min } y^0 + \frac{1}{\alpha} \mathbb{E}[y^-] \quad (4.18a)$$

$$\text{s. t. } Ax_r \leq b \quad (4.18b)$$

$$\sum_{\bar{n} \in \mathcal{P}(n)} B_{\bar{n}}^{(n)} x_{\bar{n}} \leq b_n \quad \text{for all } n \in \mathcal{N} \setminus \{r\} \quad (4.18c)$$

$$v_n = v_{p(n)} - c_n x_n \quad \text{for all } n \in \mathcal{S} \quad (4.18d)$$

$$y^0 + v_n = y_n^+ - y_n^- \quad \text{for all } n \in \mathcal{S} \quad (4.18e)$$

$$x_{jn} \in \mathbb{Z} \quad \text{for all } j \in \mathcal{J}_n, n \in \mathcal{N} \quad (4.18f)$$

$$x_{jn} \in \mathbb{R} \quad \text{for all } j \notin \mathcal{J}_n, n \in \mathcal{N} \quad (4.18g)$$

$$v_n \in \mathbb{R} \quad \text{for all } n \in \mathcal{S} \quad (4.18h)$$

$$y^0 \in \mathbb{R} \quad (4.18i)$$

$$(y_n^+, y_n^-) \in \mathbb{R}_{\geq 0}^2 \quad \text{for all } n \in \mathcal{S} \quad (4.18j)$$

Apart from the constraints (4.18b), (4.18c) that ensure feasible decision and the domain constraints (4.18f) and (4.18g), the model has constraints to compute the AV@R. Constraint (4.18d) defines the value at the leaf nodes and constraint (4.18e) corresponds to constraint (4.17b). The value  $v_n$  is calculated for all scenarios  $n \in \mathcal{S}$ , i. e., all leaves of the tree. (4.18h), (4.18i), and (4.18j) specify the domains of the auxiliary variables. According to (4.17a), the objective is to minimize the minimum of  $y^0 + \frac{1}{\alpha} \mathbb{E}[y^-]$  over all admissible decisions. Eichhorn and Römisch [25] have shown that both minima can be computed jointly. They also study the structure of such problems. Studies of structural properties of Average Value-at-Risk based stochastic programs can also be found in [79].

##### 4.4.3. Multi-Period Risk Measures

The Average Value-at-Risk is designed to measure the risk for one random variable. It is therefore a suitable measure for the risk of a value process in a single period, i. e., at the end of the time horizon. In multistage stochastic programs, however, a non-anticipative value process might carry risk in all periods. Several extensions have been proposed to measure multi-period risk. One idea is to combine several single-period risk measures to a multi-period risk measure by scalarization. Consider, for example, a value process  $v_t$ , confidence levels  $\alpha_t$ , and weights  $\gamma_t$ . The weighted sum of the single-period AV@Rs

$$\sum_{t=2}^T \gamma_t \text{AV@R}_{\alpha_t}(v_t) \quad (4.19)$$

constitutes a multi-period risk measure. The first stage is not considered in the sum, since the value is deterministic and, thus, does not carry risk. The weighted



average (4.19) can be used as objective function for a multistage stochastic program. The linearization and the deterministic equivalent program is similar to (4.17) with auxiliary variables and the respective constraints for all periods. The multistage stochastic program with (4.19) as objective function is given by:

$$\text{Min} \quad \sum_{t=2}^T \gamma_t \left( y_t^0 + \frac{1}{\alpha_t} \sum_{n \in \mathcal{N}(t)} \mathbb{P}(n) y_n^- \right) \quad (4.20a)$$

$$\text{s. t.} \quad Ax_r \leq b \quad (4.20b)$$

$$\sum_{\bar{n} \in \mathcal{P}(n)} B_{\bar{n}}^{(n)} x_{\bar{n}} \leq b_n \quad \text{for all } n \in \mathcal{N} \setminus \{r\} \quad (4.20c)$$

$$v_n = v_{p(n)} - c_n x_n \quad \text{for all } n \in \mathcal{N} \quad (4.20d)$$

$$y_{t(n)}^0 + v_n = y_n^+ - y_n^- \quad \text{for all } n \in \mathcal{N} \quad (4.20e)$$

$$x_{jn} \in \mathbb{Z} \quad \text{for all } j \in \mathcal{J}_n, n \in \mathcal{N} \quad (4.20f)$$

$$x_{jn} \in \mathbb{R} \quad \text{for all } j \notin \mathcal{J}_n, n \in \mathcal{N} \quad (4.20g)$$

$$v_n \in \mathbb{R} \quad \text{for all } n \in \mathcal{N} \quad (4.20h)$$

$$y_t^0 \in \mathbb{R} \quad \text{for all } t \in \mathcal{T}, t > 1 \quad (4.20i)$$

$$(y_n^+, y_n^-) \in \mathbb{R}_{\geq 0}^2 \quad \text{for all } n \in \mathcal{N} \quad (4.20j)$$

The inner sum in the objective calculates the expectation of the negative part of the  $y_t^0 + v_t$  as required for the AV@R calculation:

$$\mathbb{E}[(y_t^0 + v_t)^-] = \sum_{n \in \mathcal{N}(t)} \mathbb{P}(n) y_n^-$$

This and several other multi-period risk measures are presented in [25] and [26].

#### *4. Multistage Stochastic Optimization*

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

Mobile telecommunication network are complex systems. In this chapter, we describe a widely used system model for UMTS radio networks and present a stochastic model for the demand evolution over time. The objective is to provide a realistic model to be used for optimization. We therefore do not focus on predictions of the future, but adopt the perspective of a network provider before the commercial introduction of UMTS. This focus allows us to set up a realistic model that allows to demonstrate the applicability of our approach.

In the following section, we present a system model for UMTS networks. Even though with LTE the next generation of mobile telecommunication networks is waiting in the wings, we present a system model for the UMTS technology. We do not aim for the most detailed system model but choose a level of abstraction that is suitable to describe the aspects of coverage and capacity needed in the optimization model presented in Chapter 6.

After the system model is set up in Section 5.1, a stochastic model for the demand evolution is developed in Section 5.2. The focus for the demand model is not to predict the future demand evolution. Instead we present observations of historical traffic volumes and choose a model that adopts to that observations. This allows us to set up a realistic stochastic model without predictions about the future since this would lead us to much in the direction of economics.

### 5.1. A Mathematical Model for UMTS Network Coverage and Capacity

The system model is used to evaluate the optimization of the network evolution over time. Therefore, we choose a high level network description. The focus is on the UMTS technology and its extensions. Coverage and capacity are the main determinantes of 3G network performance. Coverage is primarily treated as a function of the received signal level, while (soft) capacity is a matter of resources and interference. The interdependence between both aspects makes the problem hard. This work is based on Geerdes [33], and we adopt the notation and definitions from there. For detailed descriptions of 3G networks we refer to [43,51,64]

We define a mobile telecommunication network as a collection  $\mathcal{C}$  of *cells*. A particular selection of cells and their parameters is referred to as *network configuration*. Each cell has a *transmission/reception unit* that provides access to the radio

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

network. The unit is part of a *base station* installed at a *site*. Typically a site hosts between 1 and 3 cells. The radio signal emitted by the transmission unit of a cell  $i \in \mathcal{C}$  provides network access to the mobile devices located within the cell area  $\mathcal{A}_i$ . Depending on the occasion the term cell either refers the cell area or the transmission/reception unit.

Two communication directions exist within a cell; the direction from the base station to the mobile devices is called *downlink*, while the reverse direction is called *uplink*. The uplink is considered the limiting direction for coverage, the downlink is considered the limiting direction for capacity (given higher downlink than uplink traffic). Since capacity is the limiting factor in our model, we only consider the downlink.

Signals are transmitted on different *channels*. *Common channels* and *dedicated channels* are distinguished. The common channels are used to spread information to all the mobile devices in the cell. Signals on a dedicated channels are specific to that channel and can only be decoded by the addressed mobile device in the cell. The mayor part of the data is transmitted on dedicated channels.

**Services.** A cell can provide different services, such as voice or video telephony, data streaming or Internet browsing. The set of available services in a planning scenario is denoted by  $\mathcal{R}$ . Depending on the service properties, the supporting dedicated channel needs to fulfill bandwidth and quality requirements.

The *Carrier-to-Interference Ratio* (CIR) is a measure for the quality for the dedicated channel. The CIR is defined as the fraction of the received signal strength on the dedicated channel and the total interference:

$$\text{CIR} := \frac{\text{Received Signal}}{\text{Noise} + \text{Interference}}$$

In this work, we forgo the exact definition, which can be found in [33].

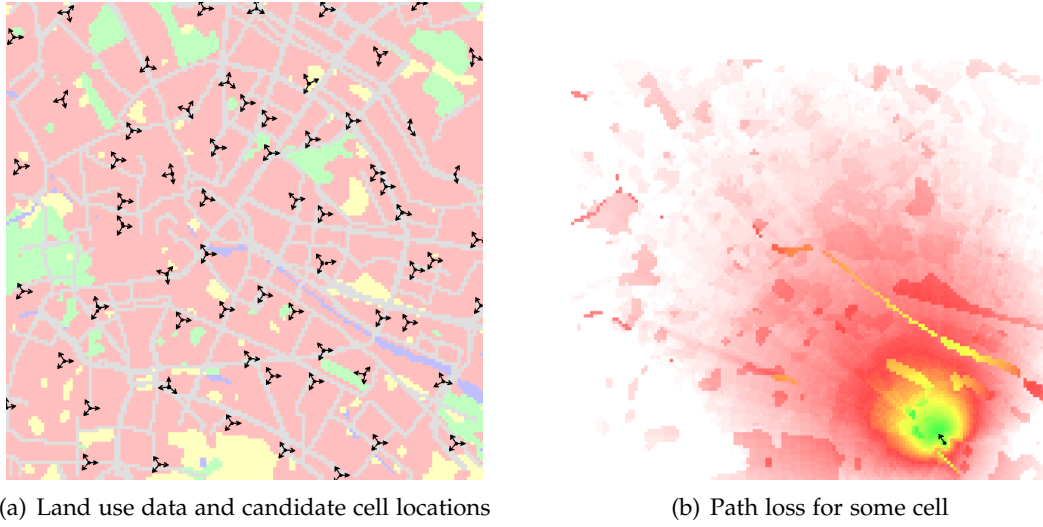
Each service  $s$  requires a minimum channel quality in form of a CIR target  $\mu_s$ . Successful operation of the service requires

$$\text{CIR} \geq \mu_s. \quad (5.1)$$

**Perfect Power Control.** Power control regulates the transmission power on the dedicated channels such that CIR requirements are met (if possible). We assume *perfect power control*, which regulates the transmission power such that Ineq. (5.1) is always met with equality, i. e., the quality requirement of the dedicated channel is exactly met but not oversatisfied. The assumption of perfect power control ensures a proper coupling of the powers of the cells in the network and is a requirement for the formulation of the interference coupling system present in Section 5.1.3.

### 5.1.1. Signal Propagation

When traveling from the transmitter to the receiver, radio signals loose intensity. The amount of attenuation the signal suffers depends on the distance between the



**Figure 5.1.:** Data from the MOMENTUM Berlin scenario

sender and the receive as well as on the propagation properties of the environment. Several components influence the attenuation.

*Path loss* here refers to the median signal attenuation. Usually it is modeled as a deterministic function of the distance and the propagation properties of the environment. The dynamic components of attenuation are called *fading*. Fading components further contribute to attenuation. The effect of obstacles in the transmission path is captured by *shadow fading*. For moving users, the shadow fading varies in the time scale of seconds. Signal variations in the time scale of a few milliseconds are referred to as *fast fading*. Fast fading mainly constitutes of multi-path fading, i.e., signal taking different paths from the transmitter to the receiver [33]. Fading is difficult to capture in detail and thus usually modeled by random variables. For network planning purposes fading effects are typically neglected. We follow this habit in this work.

The inverse of attenuation is called *channel gain*. Quantities, such as attenuation and channel gain, have a high dynamic range and are commonly stated on a logarithmic scale in *decibel* (dB). The decibel value of the unit-less ratio of two positive quantities  $a > 0$  and  $b > 0$  is:

$$10 \log_{10}(a/b) \text{ dB}$$

A path loss of  $10^{10}$  thus corresponds to a channel gain of  $10^{-10}$  or 100 dB. All formulas in this thesis are given in linear scale unless stated otherwise. In the following, the term path loss refers to the path loss component of the channel gain, and is denoted with  $\gamma_i(x) \in [0, 1]$  for a signal from cell  $i$  to some point  $x \in \mathcal{A}$ . The values of the path loss function for some antenna in the Berlin scenario are depicted in Figure 5.1(b).

### 5.1.2. Coverage

Locations that receive the signal from a cell with sufficient strength and quality are *covered* by the cell. While the signal strength only depends on the emitted power and the signal attenuation, the signal quality also depends on the interference from other transmissions in the network and noise at the receiver. The signal quality hence depends on the network configuration.

Mobile devices determine the signal strength and quality of the surrounding cell by sensing the cell's *pilot channels*. The pilot channel is a common channel on which each cell sends essential cell information. If  $p_i^{(P)}$  denotes the power a cell  $i$  emits on the pilot channel, the received signal strength in  $x \in \mathcal{A}$  is  $\gamma_i(x)p_i^{(P)}$ .

The received pilot signal strength, also referred to as  $E_c$ , is a measure for signal strength. An  $E_c$ -level greater or equal to some threshold  $\pi_{E_c}$  is required for successful receptions:

$$\gamma_i(x)p_i^{(P)} \geq \pi_{E_c} \quad (5.2)$$

A point  $x \in \mathcal{A}$  has  $E_c$ -coverage if condition (5.2) is fulfilled for some cell  $i$ . The  $E_c$ -covered area by cell  $i$  is denoted by  $\mathcal{A}_i^{E_c}$ .

Signal quality is measured through the  $E_c/I_0$ -level, which describes the ratio of received  $E_c$ -level and interfering signals. While the  $E_c$ -level and consequently the  $E_c$ -coverage can be calculated for each individual cell, the calculation of the  $E_c/I_0$ -level requires knowledge about the interfering signals. For this reason, the  $E_c/I_0$ -level can only be computed if the cell powers for all cells in the network are known. A point has  $E_c/I_0$ -coverage if the  $E_c/I_0$ -level greater or equal to the threshold  $\pi_{E_c/I_0}$ .

A pixel has coverage if it has  $E_c$ - and  $E_c/I_0$ -coverage. The pixel covered by a cell form the cell area. In the following we assign each cell  $i$  a cell area  $\mathcal{A}_i$  where  $E_c$ - and  $E_c/I_0$ -coverage is given. Due to the complexity of  $E_c/I_0$ -level computations,  $E_c/I_0$ -coverage is not considered explicitly in the site selection planning process. The cell areas used in the planning process will be subsets of the  $E_c$ -covered area, whose shape depends on the traffic intensity. This can be viewed as an implicit assumption of  $E_c/I_0$ -coverage. Geerdes [33] provides procedures that optimize the cell configurations for a given site selection. This should be done in a second step considering  $E_c/I_0$ -coverage as well.

### 5.1.3. Cell Power

Power values are commonly specified in dBm, which is decibel over 1 mW. The power of  $a$  W corresponds to

$$10 \log_{10} a + 30 \text{ dBm}$$

Unless explicitly stated otherwise, all formulas are in linear form throughout this thesis.

### 5.1. A Mathematical Model for UMTS Network Coverage and Capacity

The total output powers of the transmission units are stored in the column vector  $\mathbf{p} = (p_i)_{i \in \mathcal{C}}$ . The total transmission power  $p_i$  of a cell  $i$  is split into the power used for common channels  $p_i^{(C)}$  and the power used for dedicated channels  $p_i^{(D)}$ . The vectors  $\mathbf{p}^{(C)}$  and  $\mathbf{p}^{(D)}$  hold the values for common and dedicated channel powers, respectively. Power control mechanisms regulate the power level of each dedicated channel of a cell. This affects also the total transmission power of the cell. The total powers are given by:

$$\mathbf{p} = \mathbf{p}^{(C)} + \mathbf{p}^{(D)} \quad (5.3)$$

Of course, the total power is limited. The maximum transmit power is denoted with  $p_{\max}$ , and the powers have to fulfill the constraint

$$p_i \leq p_{\max}$$

#### The Interference-Coupling System

In UMTS networks, interference is a key driver for network performance in general and cell power in particular. The assumption of perfect power control is the fundament of interference-coupling. In the following, we briefly introduce a model for the interference-coupling of the cells in a network. The resulting coupling system can be used to approximate the expected powers of the cells in the network. The level of detail in the presentation focuses on the needs of the application in the network planning procedure. A detailed derivation and description can be found in [33].

The effect of the CIR-inequality (5.1) is that for a successful transmission on the dedicated channels, the received signal strength has to achieve some ratio to the interfering signals. We assume perfect power control such that the CIR-inequality is always met with equality. From this equality an interference-coupling system can be derived that models the interdependence of the cell powers of the cells in the network. The fraction of the power of cell  $j$  that is received in cell  $i$  as interference is stated by the *interference-coupling coefficient*  $c_{ij}$ . The interference-coupling system relates the power of cell  $i$  the powers of all other cells:

$$p_i = c_{ii}p_i + \sum_{j \neq i} c_{ij}p_j + p_i^{(\eta)} + p_i^{(C)} \quad (5.4)$$

The first summand on the right side of Eq. (5.4) is the amount of *intra-cell interference*. The second summand specifies the interference from the other cells, called *inter-cell interference*.

Arranging the coupling coefficients in a *interference-coupling matrix*  $\mathbf{C} = (c_{ij})_{i,j \in \mathcal{C}}$  and writing Eq. (5.4) in vector notation gives the *interference-coupling system*:

$$\mathbf{p} = \mathbf{C}\mathbf{p} + \mathbf{p}^{(\eta)} + \mathbf{p}^{(C)} \quad (5.5)$$

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

The coupling matrix  $\mathbf{C}$  and the noise power  $\mathbf{p}^{(\eta)}$  depend on the traffic situation such that the system (5.5) might be infeasible or undefined. In this case, the traffic load cannot be served with the available transmission power and load control kicks in to reduce the traffic intensity. We assume *perfect load control*. Perfect load control scales the traffic, such that the remaining traffic is served with maximum cell power. Therefore, traffic scaling factors  $\lambda_i \in [0, 1]$  are introduced. The traffic scaling reduces the power on the dedicated channel such that the coupling equation (5.4) becomes

$$p_i = \lambda_i \left( c_{ii}p_i + \sum_{j \neq i} c_{ij}p_j + p_i^{(\eta)} \right) + p_i^{(C)} \quad (5.6)$$

In the vector notation this reads

$$\mathbf{p} = \text{diag}(\boldsymbol{\lambda}) (\mathbf{C}\mathbf{p} + \mathbf{p}^{(\eta)}) + \mathbf{p}^{(C)} \quad (5.7)$$

where  $\text{diag}(\boldsymbol{\lambda})$  is the matrix that has the elements of  $\boldsymbol{\lambda}$  as diagonal entries.

Perfect load control says that if the traffic is scaled down, the cell must transmit at its maximum power. This implies complementarity of the two constraints

$$p_i \leq p_{\max} \quad \text{and} \quad \lambda_i \leq 1 \quad (5.8)$$

such that one of the two inequalities has to be met with equality.

For a more detailed presentation, we refer to Geerdes [33], where also results on the uniqueness and existence of solutions for the system comprising of the coupling system (5.7) and the complementarity constraints (5.8) are given.

### Generalized Pole Equations for Cell Power

By introducing the *other-to-own interference ratio* as the quotient of inter- and intra-cell interference, the coupling equations (5.6) can be simplified and the inter-cell interference is treated as an affine function of the cell's own transmission power. The other-to-own interference ratio is often called *little i* or just *i*. To avoid confusion with the index, we use the symbol  $\iota$  instead. The other-to-own interference ratio  $\iota_i$  for cell  $i$  is defined by

$$\iota_i := \frac{\sum_{j \neq i} c_{ij}p_j}{c_{ii}p_i} \quad (5.9)$$

The ratio  $\iota_i$  is well-defined for all cells  $i$  with  $c_{ii} > 0$ . The cell power is positive since  $p_i \geq p_i^{(C)} > 0$ . Substituting the interference terms in Eq. (5.6) gives

$$p_i = (1 + \iota_i)\lambda_i c_{ii}p_i + \lambda_i p_i^{(\eta)} + p_i^{(C)} \quad (5.10)$$



### 5.1. A Mathematical Model for UMTS Network Coverage and Capacity

Isolating  $p_i$  provides a formula for the power value

$$p_i = \frac{\lambda_i p_i^{(\eta)} + p_i^{(C)}}{(1 + \iota_i) \lambda_i c_{ii}} \quad (5.11)$$

Using the complementarity of the two inequalities in (5.8) yield the following lemma:

**Lemma 5.1:** *For any cell  $i \in \mathcal{C}$  with  $c_{ii} > 0$ , the following identities hold:*

$$p_i = \begin{cases} \frac{p_i^{(\eta)} + p_i^{(C)}}{(1 + \iota_i) c_{ii}} & \text{if } (1 + \iota_i) c_{ii} < \frac{p_{\max} - p_i^{(C)} - p_i^{(\eta)}}{p_{\max}}, \\ p_{\max} & \text{otherwise.} \end{cases} \quad (5.12a)$$

$$\lambda_i = \begin{cases} 1 & \text{if } (1 + \iota_i) c_{ii} < \frac{p_{\max} - p_i^{(C)} - p_i^{(\eta)}}{p_{\max}}, \\ \frac{p_{\max} - p_i^{(C)}}{p_{\max}(1 + \iota_i) c_{ii} + p_i^{(\eta)}} & \text{otherwise.} \end{cases} \quad (5.12b)$$

The equations (5.12) are referred to as *generalized pole equations*. Eq. (5.12a) is suitable to approximate the cell power when the rest of the network, in particular the inter-cell interference, is not known. In this case, an estimate for the other-to-own interference ratio  $\iota_i$  is used and just the coupling element  $c_{ii}$  and the noise power  $p_i^{(\eta)}$  have to be calculated.

#### Expected Interference-Coupling and Cell Power

Noise power and the coupling matrix depend on the traffic in the network and can be computed for each traffic snapshot of the network. The coupling system or the pole equation can then be used to compute the cell powers. Sampling traffic snapshots in a Monte-Carlo fashion can be used to approximate the expectation of this values. We avoid the time consuming task by computing the expected coupling matrix and the expected noise power directly. This procedure is also presented in Geerdes [33].

**Orthogonality.** *Code division multiple access* (CDMA) is used to separate the signal from different transmission links. Within the same cell, separation is strengthened via *orthogonal codes* in the downlink. The base station encodes the original signals destined for the different users in the cell using distinct codes (scrambling codes). The encoded signals are added and transmitted. Each receiver can decode the original signal it is supposed to receive from the transmitted data stream. The orthogonality of the codes ensures signals encoded with different codes can (ideally) be perfectly separated in the decoding procedure such that the signals do not interfere.

In practice, however, orthogonality is partly lost during transmission due to disturbances in the transmitted data stream. The amount of orthogonality loss

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

depends on the signal propagation properties of the environment. The loss is greater in areas where the signals are reflected and refracted by obstacles and thus might take different paths from the sender to the receiver, such as densely build areas. The loss is smaller in less cluttered areas. We model the orthogonality loss as a parameter  $\omega(x) \in [0, 1]$  that depends on the location  $x$  of the receiver. A factor of  $\omega(x) = 0$  means that orthogonality is perfectly preserved, while  $\omega(x) = 1$  means that orthogonality is completely lost. Further details can be found in [33].

**User Load Intensity.** For each service  $s \in \mathcal{R}$  a *user intensity function*  $T_s(x)$  specifies the number of users that are expected to use the service in pixel  $x \in \mathcal{A}$ . The location and number of the users are modeled as an inhomogeneous spatial Poisson point process. The number of users in any measurable subset  $\mathcal{S} \subseteq \mathcal{A}$  is Poisson-distributed with parameter  $\int_{\mathcal{S}} T_s(x) dx$ . With this data, traffic snapshots can be sampled. We use the user intensity function to calculate user load intensities.

The load generated by one user of service  $s$  in point  $x$  is specified by the *user loading factor*  $l_s(x)$ . The load of a user depends on the loss of orthogonality, a channel-activity factor  $\alpha_s$  and the CIR-target of the service. The activity factor reflects the fraction of time, the service maintains the channel busy. For speech telephony, control channels suggest values of  $\alpha_s \geq 0.5$  since on average each party is speaking half of the time. For video telephony we have  $\alpha_s = 1$ , since video is transmitted continuously. The downlink user loading factor is defined by

$$l_s(x) := \frac{\mu_s \alpha_s}{1 + \omega(x) \mu_s \alpha_s} \quad (5.13)$$

The user load intensity combines the user intensity function with the user loading factor. The *total normalized user load intensity*  $T_l(x)$  specified the total load intensity in each point  $x \in \mathcal{A}$  and is defined by

$$T_l(x) := \sum_{s \in \mathcal{R}} l_s(x) T_s(x) \quad (5.14)$$

The expectation of the coupling elements  $c_{ij}$  can now be computed as integrals over the cell area  $\mathcal{A}_i$ :

$$\mathbb{E}[c_{ii}] = \int_{\mathcal{A}_i} \omega(x) T_l(x) dx \quad (5.15)$$

$$\mathbb{E}[c_{ij}] = \int_{\mathcal{A}_i} \frac{\gamma_i(x)}{\gamma_j(x)} T_l(x) dx \quad (5.16)$$

Indeed, there would be not intra-cell interference if orthogonality were perfectly preserved ( $\omega(x) = 0$ ).

**Expected Noise Power.** Noise from external sources is aggregated in a noise function  $\eta(x)$ . Noise comprises thermal noise, equipment noise and interference from sources other than the UMTS network. The noise power is the power that would be needed to achieve a proper reception in the case of complete absence of interference. Without intra system interference (intra- and inter cell interference), perfect power control would ensure that the transmission power is just the noise power. The expected noise power is given by

$$\mathbb{E}[p_i^{(\eta)}] = \int_{\mathcal{A}_i} \frac{\eta(x)}{\gamma_i(x)} T_l(x) dx \quad (5.17)$$

**Approximating Expected Cell Powers.** The ability to compute expected coupling coefficients and noise power gives us the main ingredients to approximate the cell powers by applying the pole equation (5.12a). We assume that the cell is not in overload ( $\lambda_i = 1$ ). How the cell areas  $\tilde{\mathcal{A}}_i$  are determined in order to meet this assumption is explained in Section 5.1.4. Moreover, an estimation  $\tilde{t}_i$  of the other-to-own interference ratio  $t_i$  is used.

Given an approximation for the cell area  $\tilde{\mathcal{A}}_i$  and an estimate other-to-own interference ratio  $\tilde{t}_i$ , we can now use the pole equation to approximate the expected cell power. Unfortunately, the right-hand side of Eq. (5.12a) is not linear in  $p_i^{(\eta)}$  and  $c_{ii}$ , so that we cannot calculate the expected power by replacing  $p_i^{(\eta)}$  and  $c_{ii}$  with its expectations. The approximation, anyhow, is reasonable well as Geerdes demonstrates [33]. Therefore, we define the *approximated expected cell power*  $\tilde{p}_i$  of a not overloaded cell by

$$\tilde{p}_i := \frac{p_i^{(P)} + \mathbb{E}[p_i^{(\eta)}]}{1 - (1 + \tilde{t}_i)\mathbb{E}[c_{ii}]} \quad (5.18)$$

**Cell Load.** Finally, we can define the *approximated load*  $\tilde{L}_i$  of cell  $i$  as the fraction of the maximum power that is used

$$\tilde{L}_i := \frac{\tilde{p}_i}{\bar{p}_{\max}}. \quad (5.19)$$

Cell load describes the amount by which the capacity of a cell is used and is an important performance indicator for wireless networks. Cells with a load above some threshold are considered overloaded and do likely not satisfy the traffic demand. The service quality for the users will be reduced, no new users will be accepted and eventually users will be dropped. For planning purposes cell load not higher than 60 percent is aimed at.

#### 5.1.4. Load Control and Cell Areas

A key driver for the development of the network over time is the (anticipated) evolution of the traffic demand. A network design that is fit to service a specific

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

traffic load intensity will eventually become overloaded when the traffic scales up. For the purpose of strategic network planning, the network shall be evolved in such a way that overload is penalized.

We therefore do not use an approach that relies on servicing traffic from a fixed cell area fractionally (as proposed in Section 5.1.3). Instead, we reduce the effective cell area in order to confine traffic and thus to prevent overload. To this end, we define *cell service areas* as the subset of the  $E_c$ -covered area wherein all traffic can be serviced without exceeding a cell load target. When the cell is empty, the entire traffic in the cell can be serviced and the cell service area is equal the area covered by the cell. When the cell gets loaded, first the users from the cell border are excluded from service. The reduction of the service area is achieved by strengthening the requirement on the received pilot power that is needed to access the network. Figure 5.2(a) shows the service areas for different traffic intensities in a sample cell in the MOMENTUM Berlin scenario.

With the introduction of cell service area, we distinguish covered and serviced pixels. Our notion of coverage refers to pure  $E_c$ -coverage, which is not effected by variations of the traffic. In practice, however, coverage consists of  $E_c$ - and  $E_c/I_0$ -coverage. The cell borders suffer most from interference and are the first places to loose  $E_c/I_0$ -coverage when the network gets loaded. The service areas can thus be seen as a way to mimic  $E_c/I_0$ -coverage as well.

Let  $L^{(t)} \in [0, 1]$  be the load target, i. e., a load that is desirable for operation. The service area for cell  $i$  is denoted by  $\mathcal{A}_i^{(s)}$ . The power and load of a cell depends on the traffic but also on the area where service is provided. For a given traffic pattern, we denote by  $L_{\mathcal{A}_i^{(s)}}$  the approximated expected load for the cell area  $\mathcal{A}_i^{(s)}$ . The sets  $\mathcal{A}_i^{(s)}$  are constructed in such a way that

$$\mathcal{A}_i^{(s)} \subseteq \mathcal{A}_i^{E_c} \quad (5.20)$$

$$L_{\mathcal{A}_i^{(s)}} \leq L^{(t)} \quad (5.21)$$

We construct the sets by including pixel with decreasing received power until the load limit is reached or until the pixels are not  $E_c$ -covered anymore. If constructed by this algorithm, the sets have the form

$$\begin{aligned} \mathcal{A}_i^{(s)} &= \left\{ x \in \mathcal{A} \mid \gamma_i(x) p_i^{(P)} \geq \max(\pi_i^{(s)}, \pi_{E_c}) \right\} \\ &= \left\{ x \in \mathcal{A}_i^{E_c} \mid \gamma_i(x) p_i^{(P)} \geq \pi_i^{(s)} \right\} \end{aligned} \quad (5.22)$$

The threshold  $\pi_i^{(s)}$  is the minimal received power needed to satisfy the load target condition (5.21). With increasing traffic, the load generated in the service area increases. As a consequence the minimum feasible received power  $\pi_i^{(s)}$  increases and the service area shrinks. Figure 5.2(a) illustrates the service areas for different load intensities.

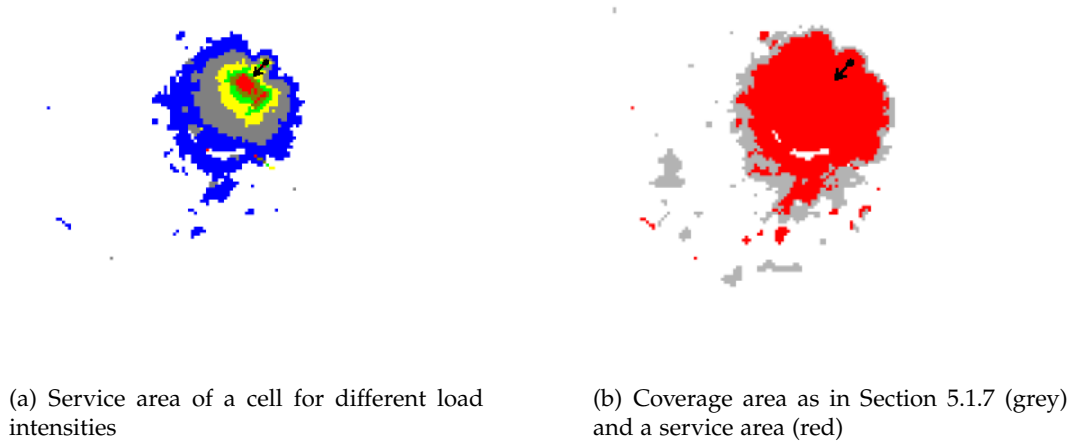


Figure 5.2.: Cell areas for different load intensities

#### 5.1.5. Adding Capacity Through Carrier and Technology Upgrades

Traffic volumes observed over the last years in the mobile networks would be impossible without techniques to improve plain UMTS cell capacity. The use of more radio spectrum and advanced transmission techniques is standard in state-of-the-art 3G networks. In the following, we describe the most important steps for cell capacity improvements and how we mimic these measures in our model.

In Section 5.1, we presented a system model for a UMTS network working on a single carrier. In practice, however, network operator have several methods to enhance the system such that it provides more capacity. One method is to activate additional frequency blocks, so called *carriers*, for transmissions. Since different frequency bands are used, both carriers do not interfere. The formulas from Section 5.1 remain valid for calculating cell powers, just the load that is handled by each carrier is reduced. In a system that is upgraded from one to two carriers, each carrier handles half the load such that the total capacity is doubled.

The UMTS standard itself also evolves. Nowadays, all providers in Germany use *High Speed Downlink Packet Access* (HSDPA, sometimes also called 3.5G or UMTS-broadband) to enhance downlink speed and capacity. HSDPA uses different modulation and coding techniques and different power management mechanisms that allow substantial speedups. At the same time, HSDPA is still an interference limited system. A complementarity system similar to the one for UMTS can be derived [32]. With HSDPA+, the network operator already have the next evolution step at hand. In Germany, one network provider already offers HSDPA+ and two of the other three plan on introducing HSDPA+ by the end of 2010 [67]. With *High Speed Uplink Packet Access* (HSUPA) a technology update for the uplink

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

is also available and widely employed. Details on technical aspects and speed measurements in HSDPA and HSUPA can be found in [43].

We refer to additional carriers and technology updates as *technology upgrades*. Both types of upgrades are applied at cell level. In several cases, upgrading involves only a software update at the base station. We therefore neglect expenses upgrades might cause.

Technology upgrades reduce the impact of a single user to the cell load and, thus, add capacity to the system. We take account for this by scaling the user intensity. Therefore, we introduce a *technology scaling parameter*  $f_i^{(\text{tech})} \in (0, 1]$  for each cell  $i \in \mathcal{C}$ . Each upgrade accounts for a reduction of the traffic scaling parameter. For standard UMTS on one carrier we have  $f_i^{(\text{tech})} = 1$ . If an additional carrier is activated, each carrier takes half the traffic and the scaling parameter is  $f_i^{(\text{tech})} = \frac{1}{2}$ .

We assume a finite, ordered set  $\{u_1, \dots, u_k\}$  of upgrades. The upgrades can only be installed in order, such that  $u_i$  can only be installed if  $u_{i-1}$  is already installed. Each upgrade  $u_i$  constitutes a factor of  $f_{u_i}$  to the technology scaling parameter. If the first  $i$  upgrades are installed, the scaling parameter is given by

$$f_i^{(\text{tech})} = \prod_{l=1}^i f_{u_l}$$

We assume the entire set of upgrades is available from the beginning. This can be extended such that the number of available upgrades increases as time passes.

The technology scaling parameter reduces impact of the traffic by scaling the user intensity. For the power calculation of cell  $i$  the scaled user intensity function

$$\bar{T}_{s^*}(x) := f_i^{(\text{tech})} T_{s^*}(x) \quad (5.23)$$

is used. It is, however, easier to incorporate the scaling factor into Eq. (5.18), i.e., the formula for approximated cell power. Both, the coupling elements and the noise power, are linear in  $T_{s^*}(x)$ , see (5.15) and (5.17). Instead of applying the scaling to the user intensity function, we can equivalently modify the approximation formula (5.18) to

$$\tilde{p}_i := \frac{p_i^{(\text{P})} + f_i^{(\text{tech})} \mathbb{E}[p_i^{(\eta)}]}{1 - (1 + \tilde{t}_i) f_i^{(\text{tech})} \mathbb{E}[c_{ii}]} \quad (5.24)$$

Of course, in this case coupling elements and noise power have to be calculated using the unscaled user intensity function given by Eq. (5.27).

### Upgrade strategy

The technology upgrade strategy is not subject to the optimization procedure, but done by a simple rule in a preprocessing step. In Section 5.1.4, we introduced the

### 5.1. A Mathematical Model for UMTS Network Coverage and Capacity

cell service areas, whose size depends on the cell load. The service area is the area where service can be provided and the cell load stays within limits. A shrinking service area, thus, indicates that the cell is at its capacity limits. The update rule says that a cell is upgraded every time, the service area shrinks by a certain degree. This procedure is repeated until the maximum number of upgrades is reached.

The service area contains all pixel whose received pilot power are sufficiently large:

$$\mathcal{A}_i^{(s)} = \left\{ x \in \mathcal{A} \mid \gamma_i(x) p_i^{(P)} \geq \max(\pi_i^{(s)}, \pi_{E_c}) \right\}. \quad (5.22)$$

In a loaded cell ( $L_{\mathcal{A}_i^{E_c}} > L^{(t)}$ ), the maximum is taken in  $\pi_i^{(s)}$ , i. e., the service area does not have its maximum range anymore. Dividing the inequality in (5.22) by the cell power yields a condition on the pathloss

$$\mathcal{A}_i^{(s)} = \left\{ x \in \mathcal{A} \mid \gamma_i(x) \geq \frac{\max(\pi_i^{(s)}, \pi_{E_c})}{p_i^{(P)}} \right\}$$

When  $\pi_i^{(s)}$  increases, the required  $\gamma_i(x)$  also increases. This quantity is the measure for our updating rule. Whenever the required  $\gamma_i(x)$  increases by a factor  $U$ , an upgrade is installed (as long as the maximum number of upgrades has not been reached).

To put this into formulas, denote with  $\gamma_i^{E_c}$  the pathloss that is needed to meet the  $E_c$ -coverage threshold

$$\gamma_i^{E_c} = \frac{\pi_{E_c}}{p_i^{(P)}}$$

and with  $\gamma_i^s$  the pathloss that is needed to be within the service area at the current upgrade level

$$\gamma_i^s = \frac{\max(\pi_i^{(s)}, \pi_{E_c})}{p_i^{(P)}}$$

An upgrade is installed, if

$$\frac{\gamma_i^s}{\gamma_i^{E_c}} > U$$

and another update is possible. Figure 5.2(a) illustrates the service areas for different numbers of upgrades. The service area is more extended if more updates are applied.

### 5.1.6. Transforming Traffic to Load

The traffic that is expected to be generated by mobile data services describes the demand evolution. In Section 5.2, a stochastic model for the evolution of the monthly traffic per capita is developed. In the following we describe a model to transform a traffic volume generated in a fixed period of time into a load intensity. This transformation allows for an evaluation of the coverage and capacity properties of the cells given a fixed traffic volume.

Increasing traffic should increase the load in the network. The increased load is caused by two effects: The number of users increases and the users use the service more intensively and generate more traffic. In a growing market both effects should be present and cannot be separated by just examining the traffic volumes. The user intensity function  $T_s(x)$  describes the number of users that are expected to use the service  $s$  in pixel  $x$ . Via the total normalized user load intensity  $T_l(x)$ , the user intensity function is one input of the calculation of cell powers (cf. (5.14)). Therefore, the traffic volume per capita is expressed in terms of users of a reference service. In combination with the population density, this allows to calculate a user intensity function for the reference service that reflects the load caused by that traffic volume.

Let  $V$  denote the traffic volume per capita in KiB per month. Assume a fixed reference service  $s^*$ , i. e., speech telephony. With  $\nu_{s^*}$  we denote the average data rate of the reference service specified in bit/s. The traffic is not generated uniformly over the day. We focus on the so called *busy hour*, which is the hour during the day in which on average the most traffic is generated and assume the same busy hour for all cells. Measuring the performance in the busy hour is desired since it represents the average peak load. Let  $\beta \in [0, 1]$  be the fraction of the traffic that occurs in the busy hour. The traffic volume generated in the busy hour of one day is thus given by

$$\frac{\beta V}{30}$$

assuming that a month has 30 days on average. The fraction

$$\frac{1024 \cdot 8 \beta V}{3600 \cdot 30} \quad (5.25)$$

represents the data rate in the busy hour per capita in bit/s. We introduce the notion of *Reference Service Equivalents* (RSE). One RSE is the amount of traffic that is equivalent to one user of the reference service in the busy hour and is thus a measure for traffic volume. The number of reference service users that are equivalent to a traffic volume of  $T$  KiB is denoted with  $\text{RSE}(T)$ . Dividing (5.25) by the average data rate of the reference services gives

$$\text{RSE}(V) = \frac{1024 \cdot 8 \beta V}{3600 \cdot 30 \nu_{s^*}} \quad (5.26)$$



### 5.1. A Mathematical Model for UMTS Network Coverage and Capacity

Symbol	Unit	Meaning
$V$	KiB/Month/Capita	Average traffic volume per month per capita
$\beta$		Fraction of total traffic volume spent in busy hour
$m$		Market share
$s^*$		Reference service
$v_{s^*}$	bit/s	Data rate of reference service
RSE		Reference Service Equivalents
$\text{RSE}(T)$	RSE	Number of reference service users equivalent to $T$ KiB
$\rho(x)$		Population in pixel $x$
$T_{s^*}(x)$	RSE	Reference service users in pixel $x$

**Table 5.1.:** Overview of symbols regarding transformation from traffic volume to load

Since  $V$  specifies the traffic volume per capita,  $\text{RSE}(V)$  specifies the traffic volume in RSE/capita. Multiplying  $\text{RSE}(V)$  with the population  $\rho(x)$  in a pixel and the market share  $m$  of the operator yields the number of reference users that would cause the given traffic in the network of the operator. This forms the desired user intensity function  $T_{s^*}(x)$ :

$$T_{s^*}(x) := m \text{RSE}(V) \rho(x) \quad (5.27)$$

#### 5.1.7. Technological and Regulatory Requirements

##### Limited Construction Activities

The activation of new sites is assumed to involve construction activities; antennas have to be mounted, equipped with transmission units, and connected to the network. The same holds true if a new cell is to be opened on an existing site; technicians have to mount and configure the transmission unit. We assume that the capacity of constructing new sites and activating new cells is limited. This is a reasonable assumption, since the deployment of a network involves the construction of ten-thousands of cells.

##### Coverage Requirements

From participants in the mobile telecommunication mass market a minimum coverage is expected and stipulated by license terms as for example in Germany [12]. In Section 5.1.4 we introduced the cell service area, which is the part of the covered area from which all traffic can be serviced without exceeding some load threshold. The area where the cell provides coverage might be larger. The coverage constraints are thus not formulated on basis of the service area but on the basis of *coverage areas*. The coverage areas is defined as that part of the  $E_c$ -covered area that can be serviced assuming a base-line traffic intensity. We therefore choose the coverage area of a cell as the service area for some base-line traffic intensity. Figure 5.2

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time

shows the coverage area and a service area of a cell. Using this definition, the service area is a superset of the coverage area if the traffic is below the base-line traffic.

We denote the coverage area of cell  $i$  with  $\mathcal{A}_i^{(c)}$ . The total coverage area  $\mathcal{A}^{(c)}$  of a network configuration is the union of all coverage areas of cell installed:

$$\mathcal{A}^{(c)} = \bigcup_{i \text{ is installed}} \mathcal{A}_i^{(c)}$$

Let  $w : \mathcal{A} \rightarrow \mathbb{R}_{\geq 0}$  be a function weighting the planning area, such as a some population density or constant value. The coverage requirement is fulfilled, if the weight of the total coverage area exceeds the required percentage of the weight of the planning area. The required percentage is expressed by the parameter  $\alpha \in [0, 1]$ . The coverage requirement is satisfied if

$$\int_{\mathcal{A}^{(c)}} w(x) \, dx \geq \alpha \int_{\mathcal{A}} w(x) \, dx. \quad (5.28)$$

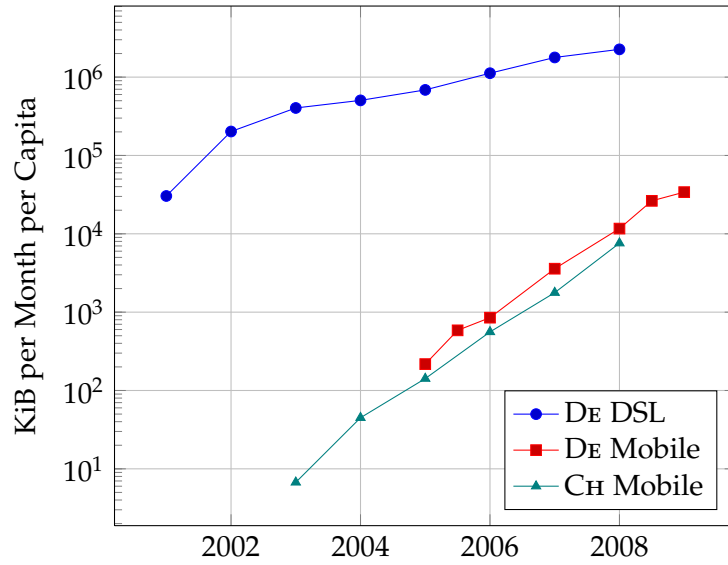
Coverage requirements may also apply only in certain parts of the area, as for example in Germany. There, the LTE technology is suitable to replace wired broadband Internet connections in poorly populated areas. Therefore, the providers are forced to provide access to communities with up to 5 000 habitants, then to the ones with 5 000 to 20 000, then with cities with up to 50 000 habitants, and so on. Each new step can be made if the area of the previous has 90 percent coverage [28]. The coverage requirement holds in a region  $\mathcal{S} \subseteq \mathcal{A}$  if

$$\int_{\mathcal{A}^{(c)} \cap \mathcal{S}} w(x) \, dx \geq \alpha \int_{\mathcal{A} \cap \mathcal{S}} w(x) \, dx. \quad (5.29)$$

$\mathcal{S}$  could be the area of communities with up to 5 000 habitants. In our computational experiments, however, we only include constraints of the form (5.28).

### 5.2. Stochastic Model of Demand Evolution

In many situations decision have to be taken while the future evolution of influencing parameters is uncertain. Traditional optimization models do not account for uncertainty but assume deterministic data, e. g., by optimizing for only one possible realization of the stochastic parameters at a time or for some expectation. The stochastic optimization framework explicitly accounts for uncertainty by including several scenario (weighted by probabilities) into the optimization model. The dynamics in the mobile telecommunication markets are difficult to capture and predict. A variety of factors, such as prizes, attractiveness, market acceptance of the services, and competition, influence the market. Especially when a new mobile telecommunication technology is introduced, it is unclear how the market will accept the new products which then become possible.



**Figure 5.3.:** Monthly traffic volume per capita in Germany (DE) and Switzerland (CH) through fixed DSL and mobile networks

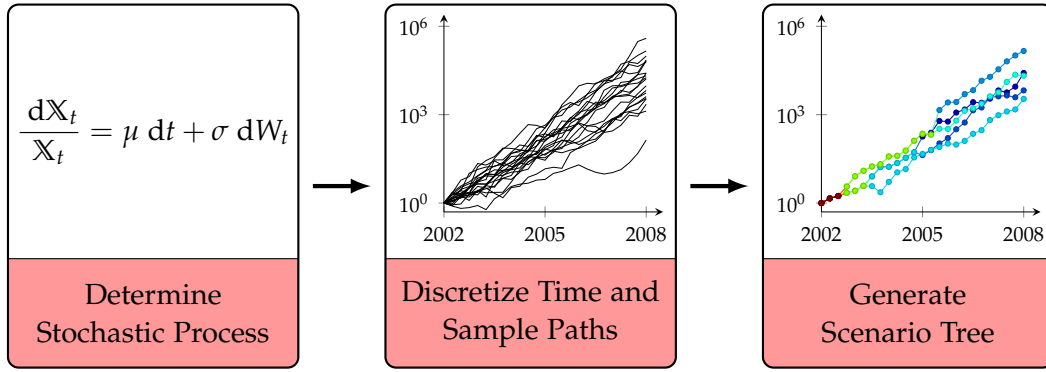
Capturing the stochastic nature of the parameters is a difficult task. Pflug and Römisch [70] determine three sources of information for the modeling of uncertainty:

- historical data
- theoretical considerations
- expert opinion

All three sources have their strengths and weaknesses. The information of historical data is of arguable value for future realizations, especially in changing environments such as developing markets. Theoretical considerations are important, but in complex environments with lots of interrelations hardly suitable as the only source of a model. Expert opinion can anticipate developments that cannot be captured by the other sources. Incorporating the decision makers expectation regarding future developments into the stochastic model also strengthens the acceptance of the solution of the optimization procedure and should, thus, not be disregarded.

The development of proper models (stochastic or deterministic) to predict future market developments is subject to economics and not the scope of this work. We therefore take the perspective of a mobile network operator prior to launching a UMTS network. This allows to demonstrate the methodology in a setting where less parameter need to be guessed since historical data is present. The stochastic demand model is set up such that the observed traffic development could be a sample path of the stochastic process.

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time



**Figure 5.4.:** Three stage approach for scenario tree construction

We assume that the traffic evolution is independent of our actions and, thus, neglect the existence of endogenous uncertainty. The traffic model is thus more suitable for a market follower than for the market leader.

From the network planning point of view, demand is the anticipated traffic from (future) customers. Two key parameters thus influence the demand in a network: The number of customers and the amount of traffic generated each user. The average traffic per capita over a time period is a quantity that aggregates the number of users and the average traffic volume. Figure 5.3 shows the development of the average monthly traffic volume per capita over several years in the DSL network in Germany and the mobile networks in Germany and Switzerland. The traffic data was extracted from official statistics provided by the German *Bundesnetzagentur* [13–16] and the Swiss *Amtliche Fernmeldestatistik* [10, 11]. Statistics of the total population in the respective countries was retrieved from the European statistic service “Eurostat” [27] and the Swiss “Bundesamt für Statistik” [30]. The mobile traffic levels in both countries show a similar behavior. The volume generated through the mobile networks is still two orders of magnitude smaller than for DSL, but still increasing steeply. Similar to the DSL plot a growth slow down can be expected for the mobile services, where the volume was increasing at more moderate rates from 2003 onwards. The prediction of the exact growth curve or even the saturation level is however difficult.

In the following, a three stage approach to compute scenario trees is described. Scenario trees are the basis of stochastic optimization. In the first stage, we model random influences as a stochastic process. Second, the time horizon is discretized and sample paths from the stochastic process are drawn. A large number of samples is usually necessary to capture the probabilistic nature of the process. Third, a scenario tree is constructed. The tree approximates the information structure (i. e., the increase of information over time) inherent in the scenarios (and the probabilities) with as few nodes and scenarios as possible. The scenario tree is the input for a stochastic program that can be solved numerically. Figure 5.4 shows the scheme of the scenario tree construction.

### 5.2.1. Modeling Uncertainty by a Stochastic Process

The first step is to determine the random influences. In practice, almost all quantities are subject to uncertainty. For stochastic optimization, however, it is reasonable to focus on the parameters with the highest impact. In this way, the effect of uncertainty on the optimization problem and, especially, on the solution of the problem can be studied (and hopefully understood). The selection of random influences in the model is a modeling decision and can be discussed like any other modeling decision.

In our application, we assume future demand to be the main source of uncertainty. The large dynamic range in the traffic observations, depicted in Figure 5.3, indicates that prediction errors can lead to poor planning decisions. The demand can be further decomposed into two factors. The intensity is a measure for the acceptance of the technology in the market. The spatial distribution of the demand is the other factor. The spatial distribution might vary over time. In this work, we focus on the demand intensity, i.e., the total demanded traffic volume. The spacial distribution is given by a non-uniform population distribution which is scaled according to the demand intensity.

Next, the uncertain parameters are modeled as a stochastic process. The traffic in the mobile networks has commonly been perceived as exponentially growing in the timehorizon under consideration (cf. Figure 5.3). The geometric Brownian motion is appropriate to model exponential growth with random disturbance. The geometric Brownian motion was introduced in Section 3.4.3 and is a stochastic process  $X_t$ , that solves the stochastic differential equation:

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad (5.30)$$

The change of the value of  $X_t$  consists of a deterministic term  $\mu X_t dt$  and a random term  $\sigma X_t dW_t$  with a standard Brownian motion  $W_t$ .

### 5.2.2. Sampling Paths

The geometric Brownian motion is used to sample paths. A solution of the stochastic differential equation (5.30) with boundary condition  $X_0 = s$  is

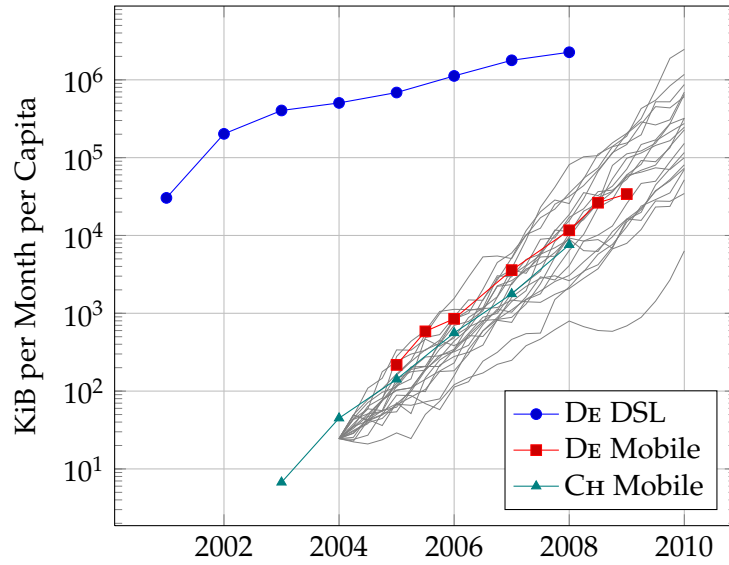
$$X_t = s e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \quad (5.31)$$

A standard Brownian motion can thus be transformed into a geometric Brownian motion using Eq. (5.31).

Several methods for the simulation of Brownian motions exist. Glasserman [35] gives an overview over sampling methods for path of geometric and standard Brownian motions. We use the straight forward approach and sample the random increments. To describe the approach, first we state the definition of the standard Brownian motion:

**Definition 9:** A stochastic process  $(W_t)_{t \in [0, T]}$  satisfying

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time



**Figure 5.5:** Observed traffic and 20 sample paths of geometric Brownian motion (grey)

- i)  $W_0 = 0$
- ii) the mapping  $t \mapsto W_t$  is almost surely continuous on  $[0, T]$
- iii) the increments  $\{W_{t_1} - W_{t_0}, W_{t_2} - W_{t_1}, \dots, W_{t_k} - W_{t_{k-1}}\}$  are independent for any  $k$  and any  $0 \leq t_0 < t_1 < \dots < t_k \leq T$
- iv) the increment  $W_t - W_s$  is normally distributed with mean 0 and variance  $t - s$  for any  $0 \leq s < t \leq T$

is called (*standard*) *Brownian motion* on  $[0, T]$ .

The Brownian motion is a stochastic process on a filtered probability space  $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ . Definition 9 instructs how the paths of the Brownian motion can be approximated: Starting at  $\tilde{W}_0 = 0$  the increments are drawn and the resulting points are connected to a piecewise linear, continuous function. We denote the approximated path with  $\tilde{W}_t$ . For a discretization  $0 = t_0 < t_1 < \dots < t_k = T$  of  $[0, T]$ , the Brownian motion at  $t_i$  for  $1 \leq i \leq k$  is sampled by

$$\tilde{W}_{t_i} = \tilde{W}_{t_{i-1}} + \sqrt{t_i - t_{i-1}} Z_i \quad (5.32)$$

where the  $(Z_i)_{i \in \{1, \dots, k\}}$  is a sequence of independent normally distributed random variables with mean 0 and variance 1. The value at the intermediate points is the linear interpolation between the two neighboring points. At the support points  $t_i, i \in \{0, \dots, k\}$ , the sample is exact in the sense that the joint distribution of the sampled points coincides with the joint distribution of the Brownian motion at this points. Between the support points, the Brownian motion is only approximated.

Each path represents a possible evolution of the demand. The discretization is chosen such that the value of process is sampled at the points in time where decision have to be made. Historical data helps to get a reasonable choice of the parameters of the geometric Brownian motion. Figure 5.5 shows the observed traffic data and 20 sample paths of the geometric Brownian motion. The sampled paths all have the same probability, so a large number of samples is needed to capture the characteristics of the distribution.

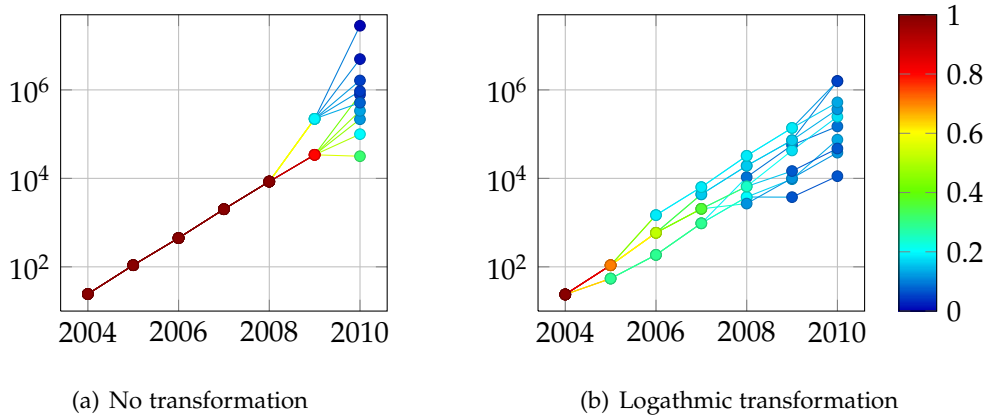
### 5.2.3. Scenario Tree Construction

The power of stochastic optimization lays in the consideration of different future evolutions of key parameters in the optimization model. In multistage stochastic optimization, tree structured processes are most suitable to describe randomness. They have the property that scenarios share the first part of the random path until the path splits. Until the splitting point, these parts are not distinguishable and the future evolution cannot be predicted. Only the conditional distribution of future events is known at that time. Repeatedly splitting of bundles of scenarios makes the problem multistage.

The path of two Geometric Brownian motion coincide almost surely only at time  $t_0 = 0$ . For all times  $t > 0$ , two path take almost surely distinct values. The same hold true for our discrete approximation, since two independent normally distributed random variables almost surely take distinct values. The formulation of multistage stochastic program as described in Section 4 with the sample paths as stochastic process yield a two-stage program. The decisions to be taken at  $t_0 = 0$  are first stage decisions. The decisions at times  $t > 0$  do not interact anymore and are regarded as second stage decisions.

To regain a multistage program with more than two stages, the information structure described by the sample paths is approximated by a tree structured stochastic process, i. e., a scenario tree. The construction of scenario trees on the basis of sample paths is done by SCENRED, a tool for scenario reduction and scenario tree construction. SCENRED constructs a scenario tree from a given set of scenarios by minimizing an appropriate metric on the space of probability distributions. The choice of the metric and the construction method is based on stability results that have been obtained for certain classes of optimization problems. Stability results relate the optimal value of the optimization problems for different input distributions with the distance of the respective distributions. Thereby, the difference in the optimal values and the solution sets can be bounded by a function of the distance of the respective input distributions. This is referred to as *stability*. Stability results are known for some classes of multistage stochastic programs, such as linear programs with random objective function and random right hand side [39–41, 74]. For stability results for two-stage stochastic mixed-integer programmes, see [75, 78]. For multistage mixed-integer stochastic programs with more than two stages, stability results are difficult to obtain and for the optimiza-

## 5. Modeling UMTS Radio Networks and Demand Evolution Over Time



**Figure 5.6.:** Scenario trees constructed with SCENRED on original and logarithmic transformed paths, colors represent the probabilities

tion models that are examined in this work, no suitable stability results are known so far. SCENRED was written by Holger Heitsch [39] and is distributed as part of the GAMS package [31] or can be obtained by the authors on request.

The focus on the choice of the input parameters for the tree construction with SCENRED is on obtaining trees that cover large parts of the value range of the sample paths. At the same time, we seek a moderate number of nodes in the tree, since additional nodes can lead to mathematical programs that are computational intractable. Since the dynamic range of the values of the paths increases, SCENRED judges the last periods much more important than the beginning and generates the mayor part of the path splits and tree nodes in the last periods, see Figure 5.6(a). To achieve a more balanced tree, SCENRED is applied to the logarithm of the sample paths. The inverse transformation is applied to the resulting scenario tree. The result of this procedure is visualized in Figure 5.6(b).

Figure 5.7 shows a graphical representation of a scenario tree which approximates 300 sampled paths. The colors of the nodes indicate their probability.

### The Expected Demand Evolution

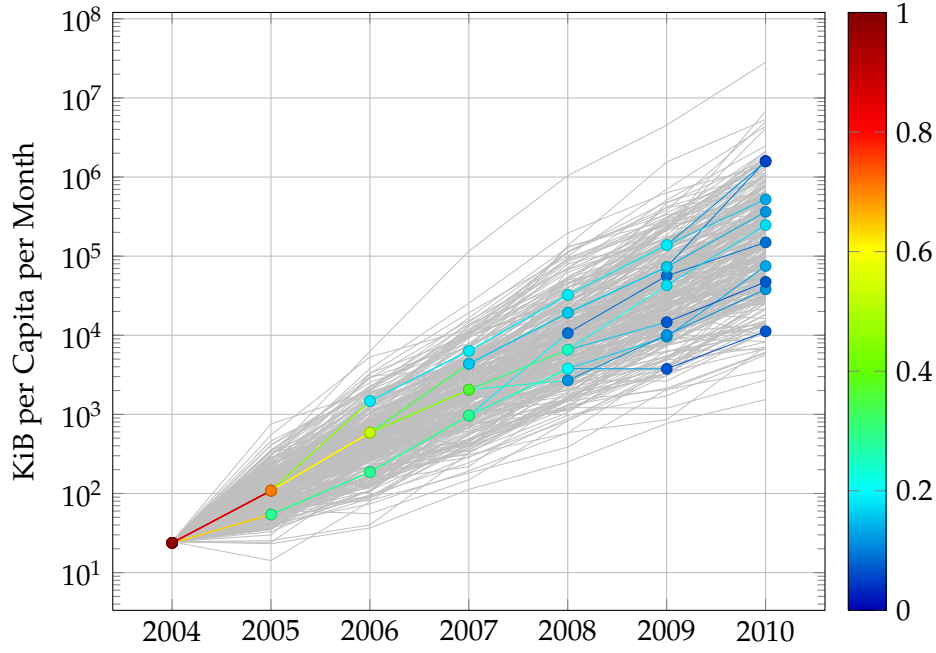
The expected demand evolution is described by the expectation of the geometric Brownian motion. The expected path of the geometric Brownian motion is the solution of Eq. (5.30) with  $\sigma = 0$ :

$$d\mathbb{X}_t = \mu\mathbb{X}_t dt$$

This is an ordinary differential equation which for the start value  $s$  has the solution

$$\mathbb{X}_t = s e^{\mu t}$$





**Figure 5.7.:** 300 sample paths (grey) and the scenario tree constructed by SCENRED. The node colors indicate their probability.

### Notation

Let  $\mathcal{N}$  denote the set of nodes in the scenario tree. For a node  $n$ , let  $p(n)$  denote its father and  $p^k(n)$  the  $k$ th ancestor along the path from  $n$  to the root of the tree such that  $p(n) = p^1(n)$ . Each node in the tree has a probability to be realized, which is denoted by  $\mathbb{P}(n)$ . Each path from the root to a leaf represents a scenario, i. e., a realization of the entire stochastic process. The leaves of the scenario tree thus characterize a scenario and the set of leaves is denoted by  $\mathcal{S}$ . The path from a node  $n$  to the root of the tree is denoted by  $\mathcal{P}(n)$ .

## *5. Modeling UMTS Radio Networks and Demand Evolution Over Time*

## 6. Optimizing Network Evolution

The construction of a mobile telecommunication network is a process that takes years of time and binds a lot of capital. The demand for mobile telecommunication services is uncertain such that careful planning of the evolution of the network under consideration of uncertainty is beneficial. The main decision in the evolution of a network is where and when to build new cells.

We follow two objectives which go hand in hand. First, we seek a monetary valuation for the opportunity to invest into a mobile telecommunication network. Second, we seek optimal strategic decision regarding the evolution of the network. The strategic decisions are the placement of sites and cells. Planning network evolution involves a large planning horizon since the network cannot be build from one day to the other but has to be build stagewise and over years. Therefore planning decision in the later periods should depend on all information available in that moment. The decisions in the beginning should be made to have flexibility to react to foreseen and unforeseen events and developments in the future.

In Section 2 and Section 3 we introduced three valuation methods for investment opportunities.

**Net Present Value Analysis.** We have a highly uncertain environment with lots of flexibility such that net present value analysis on the expected cash flows is not the appropriate valuation method. What are the expected cash flows in this case anyway? They highly depend on the evolution of the network. Net present value analysis has to be complemented by a procedure to compute plans for network evolution.

**Decision Tree Analysis.** Decision tree analysis allows to consider uncertainty and flexibility but suffers from a combinatorial explosion. Each path from the root to a leaf represents a realization of the random parameters and a particular choice for all decisions.

Consider the special case of the problem to select up to  $n$  cells out of  $k$  candidates. This is a deterministic problem but there are  $\binom{k}{n}$  possible selections. For 120 candidate locations and 10 cells to select this gives more than  $10^{14}$  possibilities that would have to be considered. Generating the entire tree involves the enumeration of all possible combinations of decisions. Clearly, this is computationally not feasible for realistic problem sizes.

## 6. *Optimizing Network Evolution*

**Real Options Analysis.** The real options framework, presented in Chapter 3, seems promising. The option to build a telecommunication network is a compound option. The compound option consists of the options to build cells from a candidate set. These options are american options on the revenues generated in the cells with the installation cost as strike. The revenue that is generated in a cell depends on the neighboring cells since a cell is not assumed to generate extra revenue if the cell area is already covered. The option is path dependent and compound with the options to build the other cells in the network. This makes the evaluation very difficult.

Another issue arises if we recall the argumentation of Section 3.5 where the assumptions of the mathematical model for the valuation of real options are discussed. The mathematical model assumes that the underlyings of the options are traded in a complete market and can be used for hedging. For a correct valuation of the option, a tradable twin security has to be identified that replicates the value of the underlying. For the option on the entire investment opportunity, the stock of a competitor could serve as twin asset. This will be the case if the value of the stock is perfectly correlated with value of the telecommunication network. Since competitors follow different business strategies this correlation is difficult to obtain.

For the real options on the construction of an individual cell, the application of the financial toolbox (cf. Section 3.4) is even harder to justify. The underlying is the value of the revenues generated in a cell. This can hardly be replicated by traded assets in the market.

Richard de Neufville and Tao Wang distinguish real options “on” and “in” projects. The option to construct a cell is an option “in” the project of constructing the network. De Neufville and Wang argue that for real options “in” project, market-driven values cannot be expected. They propose the application of stochastic mixed-integer programming to value real options. Risk-adjusted interest rates have to be used to discount the future cash flows. The result is a procedure that maximizes the net present value of the project. This is done by implicitly evaluating the decision tree.

### 6.1. The Modeling Idea

Before we start with the mathematical formulation of the model, we want to present the key ideas. We seek a monetary valuation for the investment opportunity to build and operate a mobile telecommunication network. The aim is therefore to find a construction plan that maximizes the profit. Two objective functions are investigated and compared; the expected discounted profit and the conditional value at risk of discounted value process at the end of the planning horizon. The expected profit is just the sum of the discounted cash-flows. The discounted value is the aggregation of all discounted cash-flows.

We distinguish two types of cash-flows. The outgoing cash-flows are the

costs for the construction and the operation and maintenance of the network installations. The calculation of the cost is straight-forward (see Section 7.3). All other sources of outgoing cash-flows, such as cost for the administration and the backbone network, are neglected.

Modelling the incoming cash-flows is more challenging. We consider charges for telecommunication services as only source of income. The income that a network operator can generate depends on the quality of the network it offers. From a costumers point of view, lacks of network coverage and lacks of capacity are two reasons for dissatisfaction with the network quality. An operator with poor quality in any of the two dimensions will loose clients and income. We therefore introduced the load dependent service areas and assume that service is provided only to costumers within the service areas of activated cells. Clients using volume based contracts will use the service and get charges for the traffic caused. Clients with a flat rate will likely choose an operator that can provide service in the locations the clients use to populate. We assume that income is generated in the service areas. The hight of the incoming cash-flow in a pixel depends on the population in the pixel and the network traffic generated per habitant. We present the exact calculation used for the computational experiments in Section 7.3.

A coverage model is set up. A variable per cell and time point describes if the cell is active at that time. In this case, the pixels within the service area of the cell generate revenues. Pixels that are in the service area of more than one cell generate the revenues only once. Binary variables indicate if a pixel is in the service area of an actived cell.

## 6.2. Notation

Before we can formulate the model, some notation has to be introduced. The notation used in the optimization model is summarized in Table 6.1. In Section 5.2 we introduced the traffic volume per capita as measure for the demand. We consider a scenario tree describing the evolution of the uncertain traffic volume.

**Cells and Cell Areas.** The planning area is discretized into three-dimensional pixel such that  $\mathcal{A}$  in the following represents the set of pixel and  $x \in \mathcal{A}$  refers to a *pixel*. We assume a set  $\mathcal{C}$  of *candidate locations* for cells.

The service area  $\mathcal{A}_{ni}^{(s)}$  in node  $n$  is a subset of the planning area. We assume that a cell  $i \in \mathcal{C}$  can satisfy the demand for telecommunication services only within the service area. The shape of the service area is determined by the load control mechanism introduced in Section 5.1.4 und thus dependent of the uncertain traffic evolution. The set of potentially servicing cells  $\mathcal{C}_{nx}^{(s)}$  of a pixel  $x$  in a node  $n$  is given by

$$\mathcal{C}_{nx}^{(s)} = \left\{ i \in \mathcal{C} \mid x \in \mathcal{A}_{ni}^{(s)} \right\}$$

## 6. Optimizing Network Evolution

The coverage area is the area where the signal has sufficient strength and that permits the service to all pixel under a medium traffic assumption. The coverage area is used to formulate the constraint on the required coverage. Under load control, service can be offered to all users in the service area. The coverage area of cell  $i$  in node  $n$  is denoted by  $\mathcal{A}_{ni}^{(c)}$  and the set of servers that cover a pixel is given by

$$\mathcal{C}_{nx}^{(c)} = \left\{ i \in \mathcal{C} \mid x \in \mathcal{A}_{ni}^{(c)} \right\}$$

**Cost.** The main sources of costs occurs at the cells. Capital expenditures (CAPEX) are spent to install the cell. This includes the cost for buying the land, construction if the mast and other facilities, the acquisition and installation of the transmission equipment and all other expenditures related to the initial setup of the cell. Operational expenditures (OPEX) incur during the lifetime of the cell. OPEX include maintenance, electricity and all other expenditures related to operation of the cell. We denote CAPEX and OPEX for cell  $i$  in node  $n$  by  $c_{ni}^{(cap)}$  and  $c_{ni}^{(op)}$ , respectively. All costs are denoted by non-negative numbers. We assume that cells are installed at the beginning of a period. Construction delay is not considered. This means that the cells can provide service and OPEX have to be spent for the entire period. While CAPEX is assumed to occur at the beginning of a period, OPEX is charged at the end. This is reflected in the discount factors of the discounted costs. Cost have to be discounted by the risk-adjusted interest rate in this model. Throughout the chapter quantities discounted at the risk-adjusted interest rate are denoted by a hat. The discounted cost are defined by

$$\begin{aligned} \hat{c}_{ni}^{(cap)} &= \frac{c_{ni}^{(cap)}}{(1 + \hat{r})^{t(n)-1}} \\ \hat{c}_{ni}^{(op)} &= \frac{c_{ni}^{(op)}}{(1 + \hat{r})^{t(n)}} \end{aligned}$$

In fact, we assume deterministic costs. This means that CAPEX and OPEX do not depend on the node in the tree, but on the stage. For ease of notation we index it by the node as above.

**Revenues.** The revenue  $C_{nx}$  in pixel  $x$  and in node  $n$  is modeled as a function dependent of the uncertain traffic volume  $V_n$ , the population density  $\rho(x)$ , and the time  $t(n)$ :

$$C_{nx} = g(V_n, \rho(x), t(n))$$

We assume that the revenue in each pixel is non-negative, hence  $C_{nx} \geq 0$ .

The revenues have to be discounted with a risk-adjusted interest rate  $\hat{r}$ . The discounted revenue  $\hat{C}_{nx}$  of pixel  $x$  in node  $n$  is denoted by  $\hat{C}_{nx}$ . Costumers are

typically charged after they received the services such that we assume that the revenues are generated at the end of each period and discounted accordingly

$$\hat{C}_{nx} = \frac{C_{nx}}{(1 + \hat{r})^{t(n)}}.$$

### 6.3. Multistage Stochastic Program for the Expected Profit

The aim of the optimization procedure is to determine an optimal network evolution plan, i. e., a plan when to activate which cell. All technical and regulatory requirements described in Section 5.1 have to be fulfilled by a feasible solution and have to be included in the model. We formulate a multistage stochastic integer program that optimizes the expected net present value. The net present value is the sum of all future discounted cash-flows, i. e., the discounted revenues and the discounted expenditures.

#### 6.3.1. Decision Variables

The model includes four types of decision variables.

The decision for the activation of the cells is represented by the binary *activation variable*  $y_{ni}$ . The model sets  $y_{ni} = 1$  if cell  $i$  is already built and active in node  $n$  and  $y_{ni} = 0$  otherwise. We only consider cell construction, removing already constructed cells is not allowed. The difference  $y_{ni} - y_{p(n)i}$  is thus non-negative and is one if and only if the cell is built in node  $n$  and 0 otherwise.

According to the coverage requirement introduced in Section 5.1.7, the provider can start the operation of the network if sufficient coverage is provided. We assume that cells can be built in all periods, but the network can start operation and provide service only if the coverage requirement is fulfilled. Binary variables  $c_n$  indicate whether enough coverage is provided in node  $n$ . The coverage requirement is formulated on the bases of the coverage areas of the cells. Binary *coverage variables*  $x_{nx}^{(c)}$  are set to 1 if and only if the pixel  $x$  is within the coverage area of an active cell in node  $n$ . This definition allows a close form expression for the coverage variable

$$x_{nx}^{(c)} = \min \left( 1, \sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \right) \quad (6.1)$$

Binary *service variables*  $x_{nx}^{(s)}$  indicate if a pixel is serviced by the network. Service is provided in pixels that are within the service area of an active cell and only if sufficient coverage is provided. The value of the service variable is thus given by

$$x_{nx}^{(s)} = \min \left( c_n, \sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \right) \quad (6.2)$$

## 6. Optimizing Network Evolution

Name	Domain	Interpretation
<b>Sets</b>		
$\mathcal{A}$	$\subseteq \mathbb{R}^3$	Planning area, usually divided into three-dimensional pixel
$\mathcal{C}$		Candidate set of cells
$\mathcal{A}_{ni}^{(s)}$	$\subseteq \mathcal{A}$	Service area of cell $i$ in node $n$
$\mathcal{A}_{ni}^{(c)}$	$\subseteq \mathcal{A}$	Coverage area of cell $i$ in node $n$
$\mathcal{C}_{nx}^{(s)}$	$\subseteq \mathcal{C}$	Cells that can provide service to pixel $x$ in node $n$
$\mathcal{C}_{nx}^{(c)}$	$\subseteq \mathcal{C}$	Cells that can cover pixel $x$ in node $n$
<b>Scenario Tree</b>		
$\mathcal{N}$		Nodes in scenario tree
$\mathcal{S}$		Set of scenarios, i. e., the leaves of the scenario tree
$\mathcal{P}(n)$	$\subseteq \mathcal{N}$	The path from node $n$ to the root
$p(n)$	$\in \mathcal{N}$	The unique father of node $n$ in the scenario tree
$p^k(n)$	$\in \mathcal{N}$	The $k$ th ancestor of node $n$ in the scenario tree
$\mathbb{P}(n)$	$\in (0, 1]$	Probability of node $n$
<b>Coefficients and Parameters</b>		
$\hat{r}$	$\in \mathbb{R}_{\geq 0}$	Risk adjusted interest rate
$K_n$	$\in \mathbb{N}$	construction limit per timeperiod
$\alpha_n$	$\in [0, 1]$	Coverage requirement parameter
$w_{nx}$	$\in \mathbb{R}_{\geq 0}$	Weights on pixel
$C_{nx}$	$\in \mathbb{R}_{\geq 0}$	Revenue generated by pixel $x$ at node $n$
$\hat{C}_{nx}$	$\in \mathbb{R}_{\geq 0}$	Discounted revenue generated by pixel $x$ at node $n$
$c_{ni}^{(cap)}$	$\in \mathbb{R}_{\geq 0}$	Capital expenses for cell $i$ at node $n$
$\hat{c}_{ni}^{(cap)}$	$\in \mathbb{R}_{\geq 0}$	Discounted capital expenses for cell $i$ at node $n$
$c_{ni}^{(op)}$	$\in \mathbb{R}_{\geq 0}$	Operational expenses for cell $i$ at node $n$
$\hat{c}_{ni}^{(op)}$	$\in \mathbb{R}_{\geq 0}$	Discounted operational expenses for cell $i$ at node $n$
<b>Decision Variables</b>		
$y_{ni}$	$\in \{0, 1\}$	Cell $i$ is active at node $n$
$x_{nx}^{(s)}$	$\in \{0, 1\}$	Pixel $x$ has service by any active cell at node $n$
$x_{nx}^{(c)}$	$\in \{0, 1\}$	Pixel $x$ is covered by any active cell at node $n$
$c_n$	$\in \{0, 1\}$	Coverage requirement is met at node $n$

**Table 6.1.:** Variables and coefficients in optimization models



### 6.3. Multistage Stochastic Program for the Expected Profit

The coverage requirement is reflected by the complementarity of the following equations:

$$\sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \geq \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \quad (6.3)$$

$$x_{nx}^{(s)} = 0 \quad \text{for all } x \in \mathcal{A} \quad (6.4)$$

Ineq. (6.3) is the discrete version of the coverage condition (5.28). The left side determines the weight of the covered area in node  $n$ . The covered area has to be greater than  $\alpha_n$  times the total weight for the coverage requirement to be fulfilled. Complementarity means that either of the two conditions has to hold true. The complementarity has to be ensured independently in all nodes of the scenario tree.

#### 6.3.2. The Model

We now formulate a deterministic equivalent model for the multistage stochastic optimization problem. A summary of the decision variables and parameters in the model can be found in Table 6.1.

$$\max \sum_{n \in \mathcal{N}} \mathbb{P}(n) \left[ \sum_{x \in \mathcal{A}} \hat{C}_{nx} x_{nx}^{(s)} - \sum_{i \in \mathcal{C}} (\hat{c}_{ni}^{(cap)} (y_{ni} - y_{p(n)i}) + \hat{c}_{ni}^{(op)} y_{ni}) \right] \quad (\text{DEMExp})$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nx}^{(s)} \quad \text{for all } n \in \mathcal{N}, x \in \mathcal{A} \quad (6.5a)$$

$$\sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \geq x_{nx}^{(c)} \quad \text{for all } n \in \mathcal{N}, x \in \mathcal{A} \quad (6.5b)$$

$$y_{ni} \geq y_{p(n)i} \quad \text{for all } n \in \mathcal{N}, i \in \mathcal{C} \quad (6.5c)$$

$$\sum_{i \in \mathcal{C}} (y_{ni} - y_{p(n)i}) \leq K_n \quad \text{for all } n \in \mathcal{N} \quad (6.5d)$$

$$\sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \geq \left( \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \right) c_n \quad \text{for all } n \in \mathcal{N} \quad (6.5e)$$

$$\sum_{x \in \mathcal{A}} x_{nx}^{(s)} \leq |\mathcal{A}| c_n \quad \text{for all } n \in \mathcal{N} \quad (6.5f)$$

$$y \in \{0, 1\}^{\mathcal{N} \times \mathcal{C}}, x^{(s)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}}, x^{(c)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}}, c \in \{0, 1\}^{\mathcal{N}} \quad (6.5g)$$

The objective is to maximize the expected net present value. The expectation operator is easily evaluated for discrete probability distributions. In this case we sum the discounted cash-flows of all nodes and weight them by their probability. Within the brackets the total discounted cash-flow in node  $n$  is computed. The first sum within the brackets computes the revenues while the second computes the CAPEX and the OPEX in the node.

Constraint (6.5a) ensures the correct setting of the service variables. The variables  $x_{nx}^{(s)}$  can only be set to 1, if at least one of the potential services is active,

## 6. Optimizing Network Evolution

i. e.,  $\sum_{i \in C_{nx}^{(s)}} y_{ni} \geq 1$ . The following constraint (6.5b) does the same for the coverage variable  $x_{nx}^{(c)}$ .

Deactivation of cells is not allowed. This is reflected by monotonicity of the activation variables  $y_{ni}$  and is ensured by constraint (6.5c). For the father of the root node  $r$  of the scenario tree, artificial variables  $y_{p(r)i} = 0$  are included. Constraint (6.5d) ensures that the construction limit is not violated. The difference  $y_{ni} - y_{p(n)i}$  is 1 if the cell is build in node  $n$ . The sum, thus, counts the number of cells built in node  $n$  which is limited by the construction limit  $K_n$ .

The constraints (6.5e) and (6.5f) ensure the complementarity of condition (6.3) and (6.4). If enough coverage is provided,  $c_n = 1$  and (6.3) holds true. Constraint (6.5f) is void in this case. If in return  $c_n = 0$ , constraint (6.5e) is always fulfilled and constraint (6.5f) forces all  $x_{nx}^{(s)}$  to 0.

The domains of the decision variables are specified in (6.5g).

### 6.3.3. Assignment of Service- and Coverage Variables

With Eq. (6.1) and Eq. (6.2) we provided explicit formulas for the coverage and service variables. However, the values of the variables set by the model might not fulfill the equations, even in optimal solutions.

The coverage variables are driven to one by the coverage condition (6.5e). The model, however, only ensures that Ineq. (6.5e) is met with equality. Setting the coverage variable like in Eq. (6.1) is always feasible and optimal.

**Lemma 6.1:** *Let  $(x^{(s)}, x^{(c)}, y, c)$  be a feasible (possibly suboptimal) solution of (DEMExp). Let  $\bar{x}^{(c)}$  be defined by Eq. (6.1). Then  $(x^{(s)}, \bar{x}^{(c)}, y, c)$  is feasible with the same objective function value.*

*Proof.* Consider a coverage variable  $x_{nx}^{(c)}$ . The variable appears in exactly two constraints:

$$\sum_{i \in C_{nx}^{(c)}} y_{ni} \geq x_{nx}^{(c)} \quad (6.5b)$$

$$\sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \geq \left( \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \right) c_n \quad (6.5e)$$

If Eq. (6.1) is not fulfilled for the variable, then  $\sum_{i \in C_{nx}^{(c)}} y_{ni} \geq 1$ , but  $x_{nx}^{(c)} = 0$ . Replacing  $x_{nx}^{(c)}$  by  $\bar{x}_{nx}^{(c)}$  is feasible without changing any other variable. Furthermore, the coverage variables do not appear in the objective function which, thus, is not changed by the substitution. Repeating the argument for all coverage variables proofs the lemma.  $\square$

**Corollary 6.2:** *Let  $(x^{(s)*}, x^{(c)*}, y^*, c^*)$  be an optimal solution of (DEMExp). Let  $\bar{x}^{(c)}$  be defined by Eq. (6.1). Then  $(x^{(s)*}, \bar{x}^{(c)}, y^*, c^*)$  is also an optimal solution.*

### 6.3. Multistage Stochastic Program for the Expected Profit

Similar conclusions can be drawn regarding the service variables. The objective function is the driver of the service variables. In a suboptimal solution and for service variables with  $\hat{C}_{nx} = 0$ , Eq. (6.2) might not hold.

**Lemma 6.3:** *Let  $(x^{(s)}, x^{(c)}, y, c)$  be a feasible solution of (DEMExp). Let  $\bar{x}^{(s)}$  be defined by Eq. (6.1). Then  $(\bar{x}^{(s)}, x^{(c)}, y, c)$  is feasible. If  $\bar{x}_{nx}^{(s)} \neq x_{nx}^{(s)}$  for a node  $n$  and a pixel  $x$  with  $\hat{C}_{nx} > 0$ , then the solution is not optimal and  $(\bar{x}^{(s)}, x^{(c)}, y, c)$  has a larger objective function value.*

*Proof.* Consider a service variable  $x_{nx}^{(s)}$ . The variable occurs in exactly two constraints:

$$\sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nx}^{(s)} \quad (6.5a)$$

$$\sum_{x \in \mathcal{A}} x_{nx}^{(s)} \leq |\mathcal{A}| c_n \quad (6.5f)$$

Suppose Eq. (6.2) does not hold for  $x_{nx}^{(s)}$ . Then  $\sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} = c_n = 1$ , but  $x_{nx}^{(s)} = 0$ . This implies that setting  $x_{nx}^{(s)} = 1$  is feasible. If the revenue  $\hat{C}_{nx}$  is positive, this increases the objective function value.  $\square$

**Corollary 6.4:** *Let  $(x^{(s)*}, x^{(c)*}, y^*, c^*)$  be an optimal solution of (DEMExp). Eq. (6.2) holds for all  $x_{nx}^{(s)}$  with  $\hat{C}_{nx} > 0$ . Let  $\bar{x}^{(s)}$  be defined by Eq. (6.2). Then  $(\bar{x}^{(s)}, x^{(c)*}, y^*, c^*)$  is also an optimal solution.*

These results show that given an optimal solution of (DEMExp), a simple post-processing step suffices to obtain an optimal solution where the coverage and service variables have the form (6.1) and (6.2), respectively.

The objective is maximization the expected profit over the lifetime of the network. Setting all variables to 0 is a feasible solution with objective function value 0 which corresponds to the strategy not to build a network. The model would thus never deliver a solution with expected loss. This does not mean the NPV has to be non-negative in all scenarios. As long as the expectation is non-negative, losses can occur in single scenarios.

The expectation for profits is the driver for network evolution. This is also reflected in the model. The service variables  $x_{nx}^{(s)}$  are the only variables with positive objective function value. Setting the service variables to one is thus the only way to generate profit. Constraint (6.5a) says that one server has to be activated for a pixel to be serviced. The constraint, thus, connects the profit with the availability and construction of cells.

### 6.4. Multistage Stochastic Program for Average Value-at-Risk

In Section 6.3, the aim was to obtain a network evolution plan which maximizes the expected profit. In this section, the model is extended to the optimization of a more risk-adverse risk measure. Therefore, a *value process* is defined and the multi-period AV@R presented in Section 4.4.3 is applied.

The *discounted value process* is used to apply the AV@R in each period. The (discounted) value process  $v_n$  for node  $n$  is defined by

$$v_n = v_{p(n)} + \sum_{x \in \mathcal{A}} \hat{C}_{nx} x_{nx}^{(s)} - \sum_{i \in \mathcal{C}} (\hat{c}_{ni}^{(cap)} (y_{ni} - y_{p(n)i}) + \hat{c}_{ni}^{(op)} y_{ni}) \quad (6.6)$$

The value composes of the value at the parent node and the discounted additional value in node  $n$ . Now, we state the deterministic equivalent optimization model with the multi-period Average Value-at-Risk as objective function. All constraints are from the expectation based model (DEMExp) are present.

$$\max \sum_{t=2}^T \gamma_t \left( y_t^0 + \frac{1}{\alpha_t} \sum_{n \in \mathcal{N}(t)} \mathbb{P}(n) y_n^- \right) \quad (\text{DEMAV@R})$$

$$\text{s.t. } v_n = v_{p(n)} + \sum_{x \in \mathcal{A}} \hat{C}_{nx} x_{nx}^{(s)} - \sum_{i \in \mathcal{C}} (\hat{c}_{ni}^{(cap)} (y_{ni} - y_{p(n)i}) + \hat{c}_{ni}^{(op)} y_{ni}) \quad \text{for all } n \in \mathcal{N} \quad (6.7a)$$

$$y_{t(n)}^0 + v_n = y_n^+ + y_n^- \quad \text{for all } n \in \mathcal{N} \quad (6.7b)$$

$$\sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nx}^{(s)} \quad \text{for all } n \in \mathcal{N}, x \in \mathcal{A} \quad (6.5a)$$

$$\sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \geq x_{nx}^{(c)} \quad \text{for all } n \in \mathcal{N}, x \in \mathcal{A} \quad (6.5b)$$

$$y_{ni} \geq y_{p(n)i} \quad \text{for all } n \in \mathcal{N}, i \in \mathcal{C} \quad (6.5c)$$

$$\sum_{i \in \mathcal{C}} (y_{ni} - y_{p(n)i}) \leq K_n \quad \text{for all } n \in \mathcal{N} \quad (6.5d)$$

$$\sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \geq \left( \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \right) c_n \quad \text{for all } n \in \mathcal{N} \quad (6.5e)$$

$$\sum_{x \in \mathcal{A}} x_{nx}^{(s)} \leq |\mathcal{A}| c_n \quad \text{for all } n \in \mathcal{N} \quad (6.5f)$$

$$y \in \{0, 1\}^{\mathcal{N} \times \mathcal{C}}, x^{(s)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}}, x^{(c)} \in \{0, 1\}^{\mathcal{N} \times \mathcal{A}}, c \in \{0, 1\}^{\mathcal{N}} \quad (6.5g)$$

$$v \in \mathbb{R}^{\mathcal{N}}, y^0 \in \mathbb{R}^T, y^+ \in \mathbb{R}_{\geq 0}^{\mathcal{N}}, y^- \in \mathbb{R}_{\geq 0}^{\mathcal{N}} \quad (6.7c)$$

The objective function is the weighted average of the single period AV@Rs. Constraint (6.7a) sets the value variable  $v_n$  and constraint (6.7b) sets the auxiliary vari-

ables for the AV@R calculation. The domains of the auxiliary variables are given in (6.7c).

The results from Section 6.3.3 can be easily adjusted to this model, but not presented here.

## 6.5. Problem Specific Presolving

In the following, we present several presolving routines to reduce the problem size in terms of variables and constraints.

### 6.5.1. Fixing Service and Coverage Variables Without Impact

Variables and constraints that do not have an impact in the model can be eliminated in a presolving step. Service variables for pixel that do not generate cash-flow have no impact on the objective function. Fixing these variables to 0 in an optimal solution is always feasible and the resulting solution is also optimal. This fixing should hence be done in a presolving step and the variables should not be included into the model. In this case, the corresponding service constraints (6.5a) are redundant and also deleted. The same applies to coverage variables with weight 0. In the following, we therefore assume  $\hat{C}_{nx} > 0$  and  $w_{nx} > 0$ .

### 6.5.2. Implicit Integer Variables

In the model (DEMExp), all variables are explicitly declared binary. The integrality condition, however, can be relaxed for many variables since integrality is ensured by the structure of the problem in all optimal solutions. Variables with this property are referred to as *implicit integer variables*. A binary variable  $x \in \{0, 1\}$  can thus be replaced by a continuous variable  $x \in [0, 1]$ . Implicit integer variables do not need to be considered for branching. They will take integral values in the LP-optimum, if all integer variables take integral values.

Branching on these variables can still be beneficial for the solution process. Therefore, most solvers identify implicit integer variables during preprocessing. To some solvers, like SCIP, the information that a variable is implicit integer can be passed directly and possibly speed up the solution process. In the following, we investigate the effect of relaxing the integrality condition of service and coverage variables.

**Lemma 6.5:** *Let  $(x^{(s)*}, x^{(c)*}, y^*, c^*)$  be an optimal solution of program (DEMExp) where the integrality conditions for all service variables and all coverage variables are relaxed. Let  $\pi$  be the objective function value. Then the following holds:*

- i) *All service variables  $x_{nx}^{(s)*}$  have integral values.*

## 6. Optimizing Network Evolution

- ii) Coverage variables can take values between 0 and  $\min \left( 1, \sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni}^* \right)$ . Rounding up or setting according to Eq. (6.1) yields an optimal solution to (DEMExp).

*Proof.* Consider an optimal solution  $(x^{(s)*}, x^{(c)*}, y^*, c^*)$ . A service variable  $x_{nx}^{(s)*}$  occurs in exactly two constraints:

$$\sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nx}^{(s)*} \quad (6.5a)$$

$$\sum_{x \in \mathcal{A}} x_{nx}^{(s)*} \leq |\mathcal{A}| c_n \quad (6.5f)$$

The positive objective function value pushes the variable to the upper bound provided by these constraints:

$$x_{nx}^{(s)*} = \min \left( c_n^*, \sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni}^* \right)$$

Since  $c_n^*$  and  $y_{ni}^*$  are integral,  $x_{nx}^{(s)*}$  is integral.

Consider a node  $n$ . If the coverage indicator variable  $c_n^*$  is 0, all coverage variables are forced to 0 and take the value specified in Eq. (6.1).

If  $c_n^* = 1$ , sufficiently many coverage variables take positive values as to fulfill the coverage requirement Eq. (6.5e). The coverage variables are only restricted by constraint

$$\sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni}^* \geq x_{nx}^{(c)*} \quad (6.5b)$$

Since the right side is integral, rounding up is always feasible as is setting according to Eq. (6.1). This does not change the objective function value and, hence, the resulting solution is optimal.  $\square$

The lemma states that omitting the integrality condition of the service and the coverage variables does not change the structure of the solution. The obtained network evolution plan is still optimal and an integer feasible solution can be computed by rounding. Service variables corresponding to pixel which generate no revenues ( $\hat{C}_{nx} = 0$ ) can be fixed to 0. The variables and the corresponding constraints are, therefore, not included in the model.

### 6.5.3. Pixel Aggregation

The planning area is usually divided into rectangular pixels by a grid. A high resolution in the pixel grid results in a large number of pixels. For every pixel the model needs to determine if it has coverage and service. The model thus comprises one variable and one constraint for each of this decisions. The number of pixels

$|\mathcal{A}|$  clearly dominates the number of candidate cells  $|\mathcal{C}|$  and the number of nodes in the scenario tree  $|\mathcal{N}|$  such that the size of the model is mainly determined by  $|\mathcal{A}|$ . Significant reductions in the problem size are possible if pixels with the same covering or servicing cell are merged. The service variables corresponding to two pixels with the same set of serving cells will have the same value in an optimal solution. The pixel can thus be merged and the corresponding variables can be aggregated in the model. The same holds for the coverage variables. This results in a different discretization of the planning area depending of whether service or coverage is considered. Since the service areas change over time, a different discretization is computed for each node.

To formalize the idea of pixel aggregation, let us first consider coverage. In Section 6.3.3, we show that the values of coverage variables in an optimal solution can be set according to Eq. (6.1):

$$x_{nx}^{(c)} = \min \left( 1, \sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \right).$$

This implies that variables with the same covering cells have the same values:

$$\mathcal{C}_{nx}^{(c)} = \mathcal{C}_{ny}^{(c)} \implies x_{nx}^{(c)} = x_{ny}^{(c)}. \quad (6.8)$$

This allows the aggregation of all pixels with the same set of covering cells. Let  $\mathcal{G}_n^{(c)} \subseteq 2^{\mathcal{C}}$  denote the set consisting of the sets of potential covering cells in node  $n$ :

$$\mathcal{G}_n^{(c)} = \left\{ \mathcal{C}_{nx}^{(c)} \mid x \in \mathcal{A} \right\}.$$

For a set of cells  $G \in \mathcal{G}_n^{(c)}$ , let  $\mathcal{A}_{nG}^{(c)}$  denote the set of pixels, which can be covered by all cells in  $G$  and by not other cell:

$$\mathcal{A}_{nG}^{(c)} = \left\{ x \in \mathcal{A} \mid \mathcal{C}_{nx}^{(c)} = G \right\}$$

According to (6.8), the values of the coverage variables coincide of all pixel in  $\mathcal{A}_{nG}^{(c)}$ :

$$x_{nx}^{(c)} = x_{ny}^{(c)} \quad \text{for all } x, y \in \mathcal{A}_{nG}^{(c)}$$

Therefore, coverage variables can be indexed by the set of potential covering cell rather then by pixel:

$$x_{nx}^{(c)} \longrightarrow x_{nG}^{(c)}$$

This yields the following modifications in the constraints (6.5b) and (6.5e):

$$\sum_{i \in G} y_{ni} \geq x_{nG}^{(c)} \quad \text{for all } n \in \mathcal{N}, G \in \mathcal{G}_n^{(c)}, \quad (6.9)$$

$$\sum_{G \in \mathcal{G}_n^{(c)}} w_{nG} x_{nG}^{(c)} \geq \left( \alpha_n \sum_{x \in \mathcal{A}} w_{nx} \right) c_n \quad \text{for all } n \in \mathcal{N}, \quad (6.10)$$

## 6. Optimizing Network Evolution

where  $w_{nG}$  is defined by

$$w_{nG} = \sum_{x \in \mathcal{A}_{nG}^{(c)}} w_{nx}$$

Typically,  $|\mathcal{G}_n^{(c)}|$  is much smaller than  $|\mathcal{A}|$ . Hence the size of the model is decreased significantly.

The same procedure can be applied to the service variables. Their value in an optimal solution is given by Eq. (6.2):

$$x_{nx}^{(s)} = \min \left( c_n, \sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \right)$$

Corresponding to (6.8), this implies

$$C_{nx}^{(s)} = C_{ny}^{(s)} \implies x_{nx}^{(s)} = x_{ny}^{(s)} \quad (6.11)$$

Constructing  $\mathcal{G}_n^{(s)}$  and  $\mathcal{A}_{nG}^{(s)}$  similar to  $\mathcal{G}_n^{(c)}$  and  $\mathcal{A}_{nG}^{(c)}$ , we can also re-index the service variables:

$$x_{nx}^{(s)} \longrightarrow x_{nG}^{(s)}$$

This has effects on the objective function and the constraints (6.5a) and (6.5f). They get changed as follows:

$$\max \sum_{n \in \mathcal{N}} \mathbb{P}(n) \left[ \sum_{G \in \mathcal{G}_n^{(s)}} \widehat{C}_{nG} x_{nG}^{(s)} - \sum_{i \in \mathcal{C}} (\widehat{c}_{ni}^{(cap)} (y_{ni} - y_{p(n)i}) + \widehat{c}_{ni}^{(op)} y_{ni}) \right] \quad (6.12)$$

$$\sum_{i \in \mathcal{C}_{nx}^{(s)}} y_{ni} \geq x_{nG}^{(s)} \quad \text{for all } n \in \mathcal{N}, G \in \mathcal{G}_n^{(s)} \quad (6.13)$$

$$\sum_{G \in \mathcal{G}_n^{(s)}} x_{nG}^{(s)} \leq |\mathcal{G}_n^{(s)}| c_n \quad \text{for all } n \in \mathcal{N} \quad (6.14)$$

Again, this transformation typically yields a large reduction of the number of variables and constraints.

The aggregated coverage and service variables behave exactly like the disaggregated ones such that aggregated versions of Eq. (6.1) and (6.2) are valid. Therefore, the aggregated service variables are also implicit integer and the results from Section 6.3.3 and Section 6.5.2 are valid for aggregated model.

### 6.5.4. Reusing Service Variables as Coverage Variables

The equations (6.1) and (6.2) reveal that the values of coverage and service variables are mainly determined by the cell activation variables of the cells that can



provide coverage or service, respectively. Suppose a node  $n$  and two pixel  $x$  and  $y$ , where the cell of possible servers of  $x$  is equal to the set of possibly covering cell of  $y$ :

$$\mathcal{C}_{nx}^{(s)} = \mathcal{C}_{ny}^{(c)}$$

Equations (6.1) and (6.2) say that the values of the corresponding variables  $x_{nx}^{(s)}$  and  $x_{ny}^{(c)}$  coincide if the coverage requirement is fulfilled, i. e.,  $c_n = 1$ . If the coverage requirement is not fulfilled, the service variables are forced to 0. In this case, setting the coverage variables to 0 is always feasible. Therefore, there is an optimal solution that features the following implication:

$$\mathcal{C}_{nx}^{(s)} = \mathcal{C}_{ny}^{(c)} \longrightarrow x_{nx}^{(s)} = x_{ny}^{(c)} \quad (6.15)$$

This becomes even clearer if we consider the aggregated form for the service and coverage variables. For each group  $G$  of cells, for an optimal solution we can assume:

$$x_{nG}^{(s)} = x_{nG}^{(c)}$$

The values of coverage variables are less restricted than those of service variables. If the coverage requirement not is fulfilled, coverage variables can take arbitrary values while service variables are forced to 0. An identification of coverage and service variables is thus only beneficial for a set  $G$  of cells for which both, the coverage and the service variable are included in the model. In this case, the coverage variable is substituted by the corresponding service variable in constraint (6.10) and the corresponding constraints (6.9) are eliminated. By this transformation  $\sum_{n \in \mathcal{N}} |\mathcal{G}_n^{(s)} \cap \mathcal{G}_n^{(c)}|$  constraints and variables can be eliminated from the model.

#### 6.5.5. Presolving the Coverage Requirement

Before a network can start operation, an appropriate level of coverage has to be provided. The decision variable  $c_n$  determines if the coverage requirement is fulfilled in scenario node  $n$ . If  $c_n = 0$ , then the service and coverage variables corresponding to that node have to be set to 0 as well.

A large number of cells might be necessary to provide enough coverage. The aim of this presolving procedure is to determine nodes, whose coverage requirement cannot be fulfilled. In this case, we can fix  $c_n = 0$  and consequently the corresponding coverage and service variables. Hence, these variables and the corresponding constraints can be eliminated from the model.

The effect is twofold. First, the problem size is reduced. Second, the LP-relaxation is strengthened. In the LP-relaxation,  $c_n$  may take fractional values and, hence, allows positive values for some service variables. The fixing to 0, therefore, potentially decreases the objective of the LP-relaxation and provides a stronger bound.

## 6. Optimizing Network Evolution

Consider a node  $n \in \mathcal{N}$ . The coverage provided in  $n$  depends on the cells that are active. The decision whether sufficient coverage can be provided in a node only depends on the construction activities on the path from the node to the root and is deterministic. Since the order of the construction of cells is not important, we look for a set of cells that provides maximal coverage. We have to take the total construction limit until node  $n$  into account. Service decisions can be ignored. Coverage variables are just needed for the considered node. The following model, thus, computes the maximal achievable coverage in node  $n \in \mathcal{N}$ :

$$\max \sum_{x \in \mathcal{A}} w_{nx} x_{nx}^{(c)} \quad (\text{MaxCov})$$

$$\text{s.t. } \sum_{i \in \mathcal{C}_{nx}^{(c)}} y_{ni} \geq x_{nx}^{(c)} \quad \text{for all } x \in \mathcal{A} \quad (6.16a)$$

$$\sum_{i \in \mathcal{C}} y_{ni} \leq \sum_{\bar{n} \in \mathcal{P}(n)} K_{\bar{n}} \quad (6.16b)$$

$$y \in \{0, 1\}^{\mathcal{C}}, x^{(c)} \in \{0, 1\}^{\mathcal{A}} \quad (6.16c)$$

The objective function is exactly the right-hand side of the coverage constraint (6.5e). Constraint (6.16a) ensures that coverage variables corresponding uncovered pixels are set to 0. Constraint (6.16b) ensures that the total construction limit is not violated.

If the objective function value of (MaxCov) is smaller than  $\alpha_n \sum_{x \in \mathcal{A}} w_{nx}$ , the coverage requirement cannot be fulfilled in node  $n$ . In a presolving routine, model (MaxCov) should be run for every node in the scenario tree.

In model (MaxCov), pixel aggregation is also possible and further reduces the problem size. In all optimization settings studied in Chapter 7, these models are solved to optimality within a few seconds.

### 6.5.6. Mandatory Coverage Requirement

Sometimes the coverage requirement is mandatory, i. e., forced by the regulation authorities. The choice of not fulfilling the coverage requirement and not operating the network is not given.

The coverage requirement variables in Model (DEMExp) are fixed to 1 in those nodes for which the coverage requirement is demanded. This fixing can make the problem infeasible if the required coverage can not be provided. In this case model (MaxCov) can be used to determine the nodes where the coverage requirement can be fulfilled.

The coverage requirement is monotone in the sense that if it is fulfilled in a node, it will also be fulfilled in all nodes in the underlying subtree. The constraints (6.5e) and (6.5f) are thus only needed for the first node on each path from the root to a leaf where the requirement has to be fulfilled. The respective constraints

### 6.5. *Problem Specific Presolving*

corresponding to nodes in the later stages are redundant and can thus be deleted from the model. The corresponding coverage variables do not appear anymore and can also be deleted from the model.

## 6. *Optimizing Network Evolution*

## 7. Computational Experiments

In this chapter, we report about computational experiments. Two realistic planning scenarios, presented in Section 7.1, are used to study our approach. The planning procedure is conducted for two time horizons with period lengths of 3 and 12 months, respectively. Two different scenario trees for the annual planning are compared. The scenario trees are presented in Section 7.2. Before coming to the results, the remaining parameters are specified in Section 7.3. Reducing the problem size of the deterministic equivalent problems by presolving routines is essential to make the problem computationally tractable. The problem size in terms of variables and constraints can be reduced by a factor 200, yielding mixed-integer programs with about 620 000 variables and constraints (from 127 mill. variables and constraints without presolving). The results are presented in detail in Section 7.4.

Most instances are solved within 10 hours up to a 0.5 % optimality gap by the CPLEX MIP solver. Four variations of the models presented in Chapter 6 with different optimization goals are used :

- Maximization of the expected profit
- Minimization of the Average Value-at-Risk
- Relaxation of the non-anticipativity constraints and maximization of the profit for each scenario
- Maximization of the profit of the expected demand evolution and evaluation of the resulting network evolution plan in the different scenarios

Since we use a node formulation (cf. Section 4.3), the non-anticipativity constraints are not stated explicitly. A relaxation of the non-anticipativity constraints in the scenario formulation causes a decompositions into independent subproblems for each scenario. This corresponds to solving the deterministic problem corresponding to each root-leaf path individually.

The results from all four approaches are compared and presented in Section 7.5. The construction of a UMTS network is profitable in all planning scenarios and parameter settings. The positive 40 %-AV@R in two parameter setting, however, indicates that the risk of losses is present.

Comparing the different optimization goals, we observe that the difference in the objectives of the first three is rather small. The expected profit increases only by at most 2.2 % if the non-anticipativity constraints are relaxed. In the Berlin

## 7. Computational Experiments

	Area	Pixel	Number	Pixel with	Population	Candidates	
	[km <sup>2</sup> ]	Size [m]	of Pixel	$\rho(x) > 0$	Estimate	Sites	Cells
Berlin	56.25	50	22 500	22 211	750 000	65	193
Hamburg	4 682.34	50	1 872 936	296 538	1 800 000	477	1 476

**Table 7.1.:** Key properties of planning scenarios

scenario, the AV@R can be decreased by 4.4% compared to the solution of the maximization of the expected profit by relaxing the non-anticipativity constraints. Optimizing for AV@R almost reaches this bound. In the Hamburg scenario, the AV@R of the expected profit maximization solution is improved by at most 2.5% by maximizing for AV@R and by at most 2.5% by relaxing the non-anticipativity constraints. In Hamburg the absolute AV@R difference between the different approaches is, however, small compared to the expected profit.

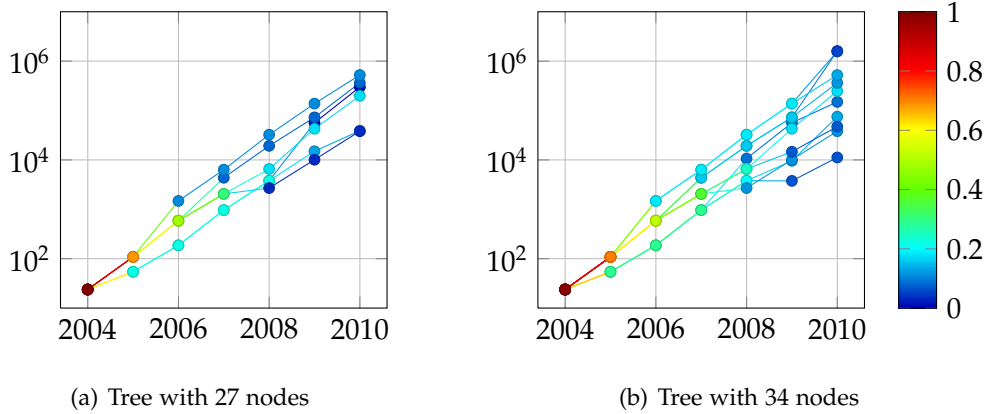
In traditional planning, the expectation of uncertain parameters is often used to perform deterministic optimization. The resulting configuration evaluated in the different scenarios delivers an (often significantly) poorer expected profit and AV@R than our stochastic programming approach. In practice, however, the network evolution plan should be recalculated regularly and this approach should yield better results.

### 7.1. Planning Scenarios

We study our methods in two realistic planning scenarios. The first is a publicly available dataset of the inner city area of Berlin, which was developed in the MOMENTUM project. The second is a realistic dataset of Hamburg, which is provided by a German network operator. Table 7.1 gives an overview over the scenario data.

**Berlin.** The Berlin scenario is published as part of the MOMENTUM project [61] and can be downloaded from <http://momentum.zib.de>. The MOMENTUM project provides three publicly available UMTS planning scenarios from which we use the Berlin scenario. The scenario contains realistic data, such as equipment data, average load grids, land use as well as land cover information, and a reference network design.

The planning area of this scenario covers the inner city of Berlin. The area comprises 56.25 km<sup>2</sup> and is divided into 22 500 square pixel of 50 m width and height. It reaches from Tiergarten in the West, the former airport Tempelhof in the South, Lichtenberg in the East to Prenzlauer Berg in the North. In total about 750 000 persons live in this area. The land use in the planning area is visualized in Figure 5.1(a).



**Figure 7.1.:** The scenario trees for annual planning, colors visualize probabilities

As candidate sites we use the sites in the reference network. The antennas are mounted without mechanical but with a 6 degree electrical tilt. Since the numbers of sites and cells are relatively small, we plan on cell basis, i.e., we face a cell selection problem rather than a site selection problem.

**Hamburg.** The second scenario describes the city of Hamburg and the surrounding rural area with some smaller cities. The population distribution and the candidate site locations are provided by a German network provider. The remaining scenario parameters, such as noise ratios, information on the services and equipment are taken from the MOMENTUM project. For reasons of confidentiality, this thesis does not contain visualizations of the scenario data.

The planning area is considerably larger than in the Berlin scenario; it comprises  $4\,682.34\text{ km}^2$  divided into  $1\,872\,936$  pixel. The pixel also have a resolution of  $50\text{ m}$  in both directions. Since the planning area contains large rural areas, the population is concentrated in the city of Hamburg and some smaller cities and villages. The total population of 1.8 million habitants is distributed among only 16 percent of the pixels. The number of pixel containing population is displayed in the fifth column of Table 7.1.

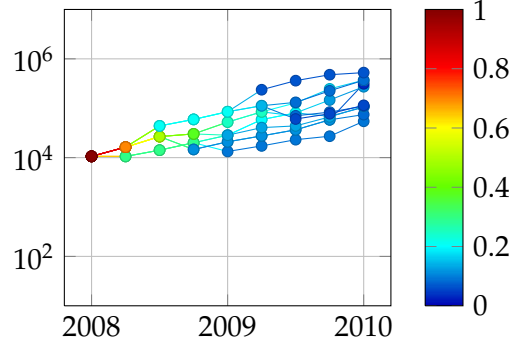
The scenario contains 1 476 cells at 477 sites in a reference network. Due to the large number of cells, we plan on site basis, i.e., we decide which sites to build.

## 7.2. Scenario Trees

All scenario trees are generated with the methods described in Section 5.2.3. The logarithmic transformation proposed there is applied.

We conduct experiments for two time horizons. First, we perform an annual planning from 2004 to 2010. There, we compare two different scenario trees, which

## 7. Computational Experiments



**Figure 7.2.:** The scenario tree for quarterly planning, colors visualize probabilities

are illustrated in Figure 7.1. The trees have a similar structure until the year 2007 and then the tree with 34 nodes spreads more and covers a larger value range. The second time horizon ranges from 2008 to 2010 and we plan on a quarterly basis. Experiments are only conducted on one scenario tree with 53 nodes, which is depicted in Figure 7.2.

All trees result from a tree construction on the basis of 300 sampled paths. A drift parameter of  $\mu = 1.6$  and different volatilities  $\sigma$  are used;  $\sigma = 0.6$  for the annual planning and  $\sigma = 0.8$  for the quarterly planning.

### 7.3. Parameters

Several parameters need specification. Let  $\mathcal{N}$  be the set of nodes in the scenario tree and  $V_n$  the monthly traffic per capita in KiB in node  $n \in \mathcal{N}$ . With  $T_n^{s*}(x)$  we denote the user intensity function of the reference service that corresponds to the traffic in node  $n$ . With  $t(n)$  we denote the year that node  $n$  represents. We start with the specification of costs and revenues. Table 7.2 summarizes the remaining parameters.

#### Costs

For the monetary valuation, knowledge of the costs for the construction and the maintenance of network components, such as base stations and cells, is needed. In a case-study supported by E-Plus Gerpott [34] estimated capital and operational expenses for UMTS sites. He distinguished whether the UMTS equipment is mounted on top of an existing GSM site or built from scratch. Costs are, of course, considerably higher in the latter case. In both scenarios, the candidates are taken from the reference network, hence we assume that GSM sites are installed in all candidate locations. Gerpott gives a total investment volume of 60 900 € in this case. We assume this is the investment volume in the year 2008. Of course,



prices for equipment decrease over the years. We assume that in the year 2004 a CAPEX investment of 100 000 € was necessary and inter- and extrapolate between these values linearly. The CAPEX  $c_{ni}^{(cap)}$  for site  $i$  in year  $t$  is thus given by

$$c_{ni}^{(cap)} = 60\,900 + \frac{t(n) - 2008}{2004 - 2008} (100\,000 - 60\,900). \quad (7.1)$$

This numbers, however, refer to the construction of a site. Since we plan on cell basis in the Berlin scenario, the costs for a cell is the costs for a site divided by the number of candidate cells at the respective site. We assume that the operational expenses are constant over time. Gerpott gives annual OpEx for a site of 8 700 € such that

$$c_{ni}^{(op)} = 8\,700. \quad (7.2)$$

Again this amount is divide among the cells for cell based planning. Note, that all expenses do not depend on the node. Operational expenses remain constant over time and capital expenses depend only on the year in which the expenses occur. The expenses are therefore deterministic in our model. However, the notation indicates that an extension of the model to stochastic expenses is possible.

## Revenues

After costs, incoming cash-flow is the other component in monetary valuation. Revenues are only generated in the service areas of active cells (cf. Section 6.1) and depend on the traffic per capita as measure for the market development and on the population in the covered area.

Similar to the argument given in Section 5.1.6, we relate revenue and traffic volume by means of reference service equivalents (RSE), i. e., to the number of reference service users in the busy hour that would cause this traffic volume. The number of RSE per pixel is given by the user intensity function  $T_n^{s^*}(x)$  of the reference service  $s^*$  in node  $n$ . The missing part is to determine the revenue per RSE. This is calculated from data on the overall traffic generation in Germany [16] and public data from the network operator Vodafone D2. The Vodafone Group, whose affiliated company Vodafone D2 is, publishes revenues for data services in Germany its annual reports [72]. We refer to 2008 as reference year  $t^*$ .

The first step is to determine the total number of RSE. We therefore recall the function  $\text{RSE}(\cdot)$ , which transforms a monthly traffic volume (in KiB) into reference service users equivalents. Let  $V^*$  denote the total traffic volume in KiB in the reference year. The number of reference users that corresponds to that traffic volume is  $\text{RSE}(\frac{V^*}{12})$ , where the division by 12 converts annual to monthly traffic.

Next, we determine the revenue per RSE. We derive the corresponding formula and substitute for the official Vodafone numbers. Let  $C^{(\text{tot})}$  denote the total data revenue in the reference year of the reference operator (Vodafone). Assuming that all operators in the market have the same average revenue per user (ARPU), the

## 7. Computational Experiments

Parameter	Symbol	Value	
Busy hour fraction	$\beta$	25 %	
Market share	$m$	20 %	
Reference year	$t^*$	2008	
Reference operator		Vodafone D2	
Market share of reference operator	$m^*$	35 %	
Data revenue of reference operator	$C^{(\text{tot})}$	736.33 mill. €	
Total traffic in Germany 2008	$V^*$	11.5 mill. GiB	
Reference service	$s^*$	Speech telephony	
Base-line traffic for coverage		50 MiB per Cap. per Month	
Risk-adjusted interest rate (annual)	$\hat{r}$	20 %	
Maximum transmission power	$p_{\max}$	20 W	
Other-to-own interference ratio	$\iota$	60 %	
Cell load limit	$L^{(t)}$	60 %	
		Hamburg	Berlin
Candidates		Sites	Cells
Average data rate of reference service	$v_{s^*}$	6 000 bit/s	3 000 bit/s
Construction limit		60 and 100	10
Maximum number of technology upgrades		5	0
Technology upgrade scaling factor	$f_{u_i}$	0.5	–
Pathloss difference for technology upgrade	$U$	6 dB	–

**Table 7.2.:** Parameter values used for optimization

total data revenue in Germany is obtained by dividing  $C^{(\text{tot})}$  by the market share  $m^*$  of the reference operator. The annual revenue  $C^{(\text{RSE})}$  of one RSE is thus given by

$$C^{(\text{RSE})} = \frac{C^{(\text{tot})}}{m^* \text{RSE}(\frac{V^*}{12})}. \quad (7.3)$$

The prices for mobile traffic have decreased significantly since the service started. Clients on flat rate contracts, for example, get used to more bandwidth and use the service more intensively for the same or even a decreasing fee. This causes a decrease in the revenue per traffic volume unit. Reference service equivalent as unit for traffic volume is constant over time. The revenue per RSE, however, decreases significantly. No public data about the development of the revenue per traffic unit (KiB or RSE) is available. We therefore assume the revenue per RSE decreases each year by a multiplicative factor  $\lambda$  during the time horizon under consideration. The scaled revenue  $C^{(\text{RSE})}(t)$  in year  $t$  per RSE is thus given by

$$C^{(\text{RSE})}(t) = \lambda^{t^* - t} C^{(\text{RSE})}. \quad (7.4)$$

Note the consistent valuation for  $t = t^*$ . We can now specify the traffic dependent revenue per pixel as

$$C_{nx} = C^{(\text{RSE})}(t(n)) T_n^{s^*}(x). \quad (7.5)$$

The model can be easily be refined if appropriate data is at hand.

#### Comments on parameters

Table 7.2 summarizes the parameters used for the optimization procedure. A few comments are in place.

**Currency.** The currency for our valuation is Euro. Being a British company, the Vodafone Group reports in British Pound. The data revenue of 583 mill.£ is converted to Euros using the exchange rate 1.26 of the 1st of July 2008.

**Unit Prefixes.** The German Bundesnetzagentur publishes their numbers in GB, but since a clear distinction between the SI and IEC prefixes (cf., Section 1) is not yet established, we assume the figure to be in GiB.

**Reference Service.** We choose speech telephony as reference service. Typically, a data rate of 12 000 bit/s is assumed for voice transmissions. The typical speech telephony usage is that each participant speaks half of the time on average. This is reflected by an activity factor of  $\alpha_s = 0.5$ . The average data rate in one direction is thus 6 000 bit/s.

The traffic in the Berlin scenario is rather homogeneously distributed across the planning area. Since there are no peak areas, all cells have the same (low) traffic intensity such that the service areas do not shrink with the traffic present in the scenario trees considered. We therefore decided not to consider technology updates in the Berlin scenario and increase the traffic level by decreasing the data rate for the average service to 3 000 bit/s.

**Cell Load Limit.** We assume a cell load limit of 60 %. This choice ensures a stable operation of the network and is common in the literature [43, 51].

**Average Value-at-Risk.** Even though a multi-stage problem is given, we optimize for the AV@R of the final discounted value, i. e., the sum of all discounted cash-flows in a scenario. A confidence level of  $\alpha = 0.4$  is used.

## 7. Computational Experiments

	34 Nodes		27 Nodes	
	Cons	Vars	Cons	Vars
Without presolving	127 375 491	127 375 900	101 151 027	101 151 450
Only $\rho(x) > 0$	20 180 427	20 180 836	16 025 535	16 025 958
Pixel aggregation	834 905	835 314	692 543	692 966
Variable reuse	619 944	620 353	501 426	501 849
Fix coverage requirement	430 737	430 295	357 897	357 448

**Table 7.3.:** Effect of presolving in the Hamburg scenario for two scenario trees

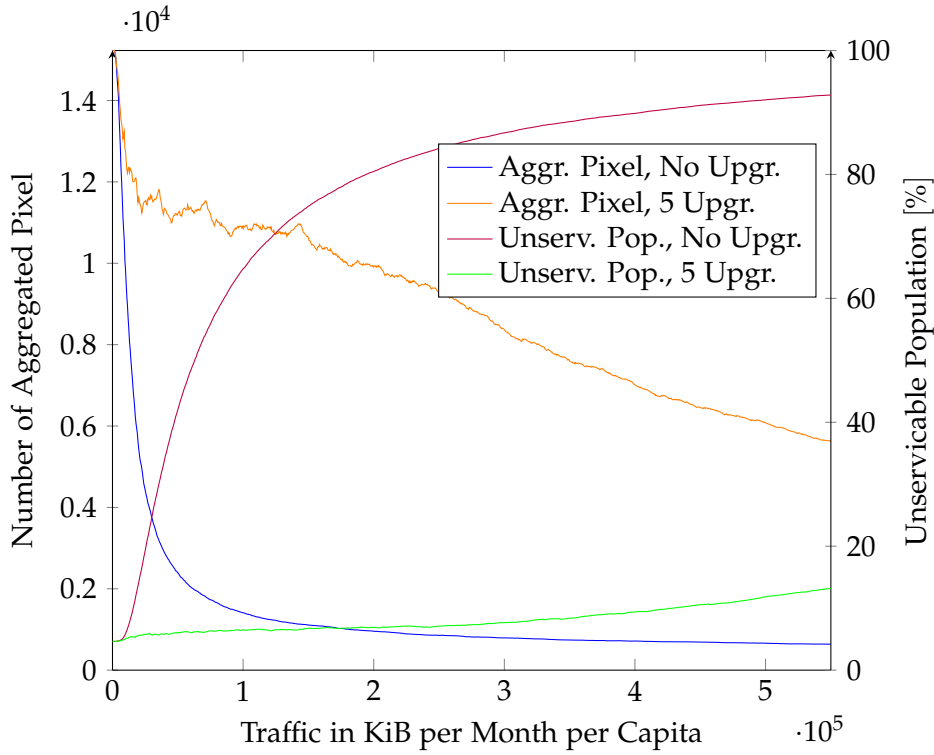
### 7.4. The Effect of Presolving

A reduction of the problem size in terms of variables and constraints does not always result in a reduction of the solution time of mixed-integer program. Removing redundant variables and constraints, however, usually leads to speed-ups of the solution process. In Section 6.5, several presolving ideas are presented. In this section, the effect on the problem size is studied.

The number of nodes  $|\mathcal{N}|$  and the number of candidate cells  $|\mathcal{C}|$  is small in relation to the number of pixels  $|\mathcal{A}|$ . The model contains  $2|\mathcal{N}||\mathcal{A}| + (|\mathcal{N}| - 1)|\mathcal{C}| + 3|\mathcal{N}|$  constraints and  $2|\mathcal{A}||\mathcal{N}| + |\mathcal{C}||\mathcal{N}| + |\mathcal{N}|$  variables. The term  $(|\mathcal{N}| - 1)|\mathcal{C}|$  stems from the monotonicity constraints (6.5c) that ensure the sites cannot be deactivated. They are not needed for the root node. All constraints are inequalities. Several presolve routines to reduce the number of pixels and corresponding variables are proposed in Section 6.5. The resulting numbers of variables and constraints after the different presolve steps are displayed in Table 7.3. The first rows gives the problem size for the problem without any presolving.

**Fixing Service and Coverage Variables Without Impact.** Pixel with a population density of 0 do neither increase the covered population or do they generate profit, if serviced. The respective coverage and service variables can therefore be fixed to 0 and not included in the model. In the original model without pixel aggregation, the effect can be substantial. In the Berlin scenario, 289 out of 22 500 pixel have a population density of 0. In the Hamburg scenario, the effect is much stronger; 1 576 398 out of 1 872 936 pixel carry no population (cf. Table 7.1). The resulting problem size is given in the second row of Table 7.3. The reductions are significant.

**Pixel Aggregation.** Two pixel that can be serviced by the same set of servers are either both covered or uncovered. The pixel are not distinguishable by the model and can be aggregated. Since the shapes of the service areas depend on the traffic, the discretization of the planning area might be different in each node. The sizes of the models with aggregated pixels are given in the third row of Table 7.3.



**Figure 7.3.:** Number of aggregated pixels and percentage of unserviceable pixels with increasing traffic intensity in the Hamburg scenario

**Technology Upgrades.** The traffic increases with the time and causes a reduction of the service areas. As a consequence, fewer service areas overlap. The blue and the green line in Figure 7.3 represent the number of aggregated pixel against the traffic intensity in the Hamburg scenario with and without 5 technology upgrades, respectively. The number of aggregated pixel decreases much faster without upgrades. If upgrades are present, the number of aggregated pixel decreases steeply in the beginning and from about 25 000 Kib per month and capita onwards, cells obtain technology upgrades and the reduction in aggregated pixels slows down considerably. At the same time, the part of the population that can not be serviced increases, as in Figure 7.3. At peak traffic over 90 % of the population can not be serviced if no upgrades are allowed. Since upgrades counteract a reduction of the service areas, at most 13 % of the population is unserviceable if 5 upgrades are allowed.

**Reusing Service Variables For Coverage Observation.** Coverage and service variables constitute the major part of the variables in the model. Similar to the aggregation, coverage pixel with the same servers as a service pixel can be replaced by the service pixel and deleted from the model. In our experiments, about half of

## 7. Computational Experiments

the coverage pixel can be replaced. This allows a reduction of the model of about 25 %. The figures are displayed in the fourth row of Table 7.3.

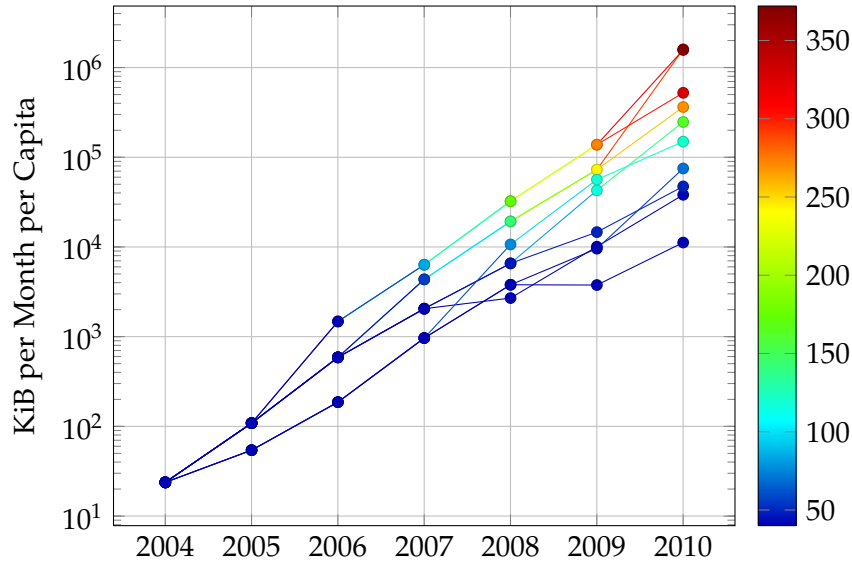
**Presolving the Coverage Requirement.** By means of model (MaxCov), the maximum achievable coverage in a node can be determined. If the coverage requirement cannot be fulfilled, the service and coverage variables for the respective node can be fixed to 0 and eliminated from the model. In the planning scenario with annual time discretization, the coverage requirement can be fulfilled already in the root node and hence no savings are possible. In contrast, only 38 % of the population can be covered in the root node of the quarterly discretization in the Hamburg scenario with a construction limit of 100. The coverage requirement cannot be fulfilled and the problem size can be reduced from 885 457 variables and 885 086 constraints to 867 691 variables and 867 319 constraints. This presolving step also strengthens the LP-relaxation. A decrease of 3.9 % is observed in quarterly Hamburg scenario with a construction limit of 100 and optimization of expected profit.

**Mandatory Coverage Requirement.** If the coverage requirement is obligatory in some stage, the coverage requirement variables for the respective stages are fixed to one. If the coverage requirement is fulfilled once, it is fulfilled in all following stages. A monitoring is thus only needed in the first stage the coverage requirement is demanded. The respective constraints and coverage variables can be deleted from the model. The result is a further reduction of the number of variables and constraints as displayed in the fifth row of Table 7.3.

### 7.5. Computational Results

We conduct our computational experiments on quad-core PCs with 16 GiB main memory. The implementation is based on an existing JAVA environment at ZIB [33] for UMTS network visualization, analysis, and optimization from which primarily the data handling routines are used. A part of this thesis, the author implemented all presolving routines, data exports as well as analysis methods using JAVA, JYTHON, and PYTHON. All MIPs are generated with ZIMPL [50] and solved using CPLEX Version 12.2 [46] with a time limit of 10 hours and a relative gap limit of 0.5 %.

In the Berlin scenario, all deterministic equivalent models are solved to a 0.5 % gap within 4.5 hours. Most models can be solved to optimality within 10 hours running time. In the Hamburg scenario, all deterministic equivalent problems corresponding to the maximization of the expected profit are solved to a 0.5 % gap within 10 hours except for one with a remaining gap of 0.58 %. Surprisingly, the deterministic optimization problems of the individual scenarios do not solve equally well. Even though the solution of the entire tree was used as MIP start,



**Figure 7.4.:** Evolution of the number of installed sites (represented by the color of the nodes) in Hamburg in the scenario tree with 34 nodes and a construction limit of 100

CPLEX hit the time limit or ran out of memory. In these cases, the gap ranged from 2-5 %; 34 % in one scenario. The absolute differences, however, are all in the same order of magnitude, such that the big gaps are mainly due to scaling effects.

Generally, the deterministic equivalents corresponding to the model where the AV@R is to be minimized are more difficult to solve to the required gap. In the Hamburg scenario two instances ended with a 13 % the gap after 10 hours. However, the absolute gaps in the AV@R are rather small compared to the expected profit (15 000 euros absolute compared to an expectation of almost 13 mill. euros). The relaxation of the non-anticipativity constraints provides a lower bound on the AV@R. This bound gives a gap of less than 4 %. Due to the small absolute value and the moderate relative gap, no additional effort is put into closing the gap.

Generally, large differences in the network evolution across the different scenarios are observed. Consider, for example, the Hamburg annually planning scenario optimized on the scenario tree with 34 nodes and a maximum construction limit of 100. The evolution of the number of installed sites is visualized in Figure 7.5(a). At the root node, 40 sites are built to ensure the coverage requirement (57 % of the population are covered in the root). In the scenarios with poor traffic evolution, very few further sites are activated after that; less than 10 with a probability of 26 %. In the scenarios with a high traffic increase, considerably more sites are activated; 327 or more are activated in 2010 with a probability of 22 %. The connection of the monthly traffic volume and the number of constructed sites is also revealed in Figure 7.4, where the number of installed sites is visualized by the color of

## 7. Computational Experiments

	Opt. over scenario tree		Relaxation of non-anticipativity		EVPI
	Expectation	AV@R	Expectation	AV@R	
<i>Berlin annually</i>					
27 Nodes, 10 Cap	9 870 175 €	-3 205 609 €	9 944 123 € ( +0.7 %)	-3 254 751 € ( -1.5 %)	73 949 €
34 Nodes, 10 Cap	9 868 577 €	-2 356 112 €	10 036 282 € ( +1.7 %)	-2 460 744 € ( -4.4 %)	167 705 €
<i>Berlin annually with mandatory coverage in root</i>					
27 Nodes, 10 Cap	9 871 149 €	-3 204 851 €	9 957 594 € ( +0.9 %)	-3 257 863 € ( -1.7 %)	86 445 €
34 Nodes, 10 Cap	9 873 531 €	-2 359 760 €	10 040 086 € ( +1.7 %)	-2 460 765 € ( -4.3 %)	166 555 €
<i>Hamburg annually</i>					
27 Nodes, 100 Cap	15 286 450 €	-2 917 462 €	15 291 819 € ( +0.0 %)	-2 930 883 € ( -0.5 %)	5 368 €
27 Nodes, 60 Cap	15 233 750 €	-2 921 662 €	15 245 549 € ( +0.1 %)	-2 931 112 € ( -0.3 %)	11 798 €
34 Nodes, 100 Cap	16 659 934 €	-1 601 728 €	16 895 924 € ( +1.4 %)	-1 658 141 € ( -3.5 %)	235 991 €
34 Nodes, 60 Cap	16 198 435 €	-1 607 326 €	16 510 183 € ( +1.9 %)	-1 657 085 € ( -3.1 %)	311 749 €
<i>Hamburg annually with mandatory coverage in root</i>					
27 Nodes, 100 Cap	12 972 933 €	-431 222 €	13 024 427 € ( +0.4 %)	-446 821 € ( -3.6 %)	51 494 €
27 Nodes, 60 Cap	12 906 688 €	-428 591 €	12 969 340 € ( +0.5 %)	-445 501 € ( -3.9 %)	62 652 €
34 Nodes, 100 Cap	14 410 188 €	936 882 €	14 595 176 € ( +1.3 %)	910 019 € ( -2.9 %)	184 988 €
34 Nodes, 60 Cap	13 969 286 €	938 999 €	14 279 881 € ( +2.2 %)	909 088 € ( -3.2 %)	310 595 €
<i>Hamburg quarterly with mandatory coverage in the second stage</i>					
53 Nodes, 100 Cap	18 805 586 €	-7 742 900 €	18 955 382 € ( +0.8 %)	-7 883 913 € ( -1.8 %)	149 796 €
53 Nodes, 60 Cap	16 416 908 €	-6 641 045 €	16 468 341 € ( +0.3 %)	-6 701 818 € ( -0.9 %)	51 433 €

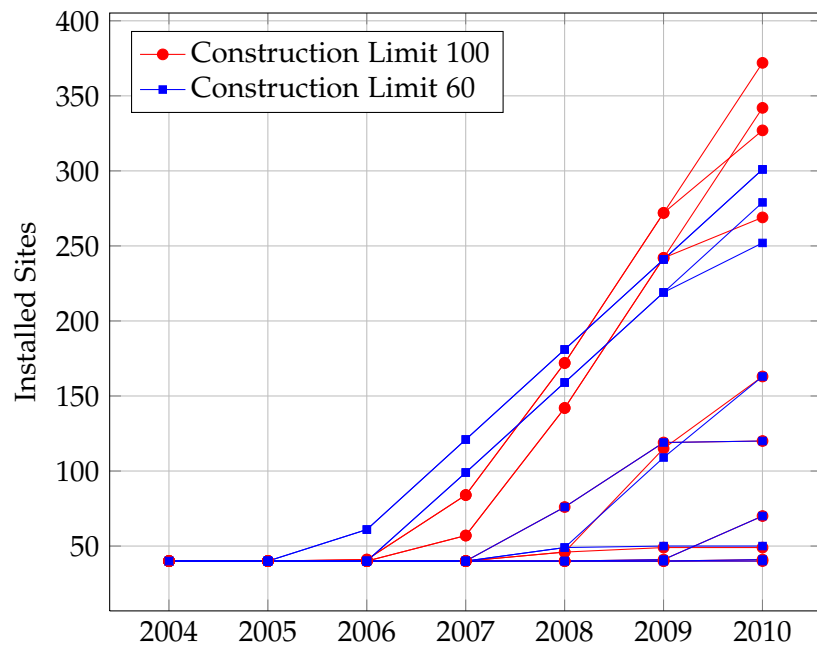
**Table 7.4.:** Comparison of optimization over the scenario tree and and over the individual scenarios, i. e., the relaxation of the non-anticipativity constraints

the nodes in the scenario tree. The complete data for this setting is displayed in Table A.12. There, we also see that negative cash-flows occur only in 3 nodes. The aggregated cash-flow is only negative in the scenario with the poorest traffic evolution (probability 7 %). In this case, the initial investment of 4 mill. is never recovered. Discounting changes this situation noticeably. Four scenarios with a total probability of 39 % have a negative final aggregated discounted cash-flow. In these scenarios, the investment in an alternative project with the same return is favorable. This is also reflected by a 40 %-AV@R on the final aggregated discounted cash-flow of 936 882 €. Taking all scenarios into account, however, the project is profitable with an expected discounted profit of 14 410 188 €.

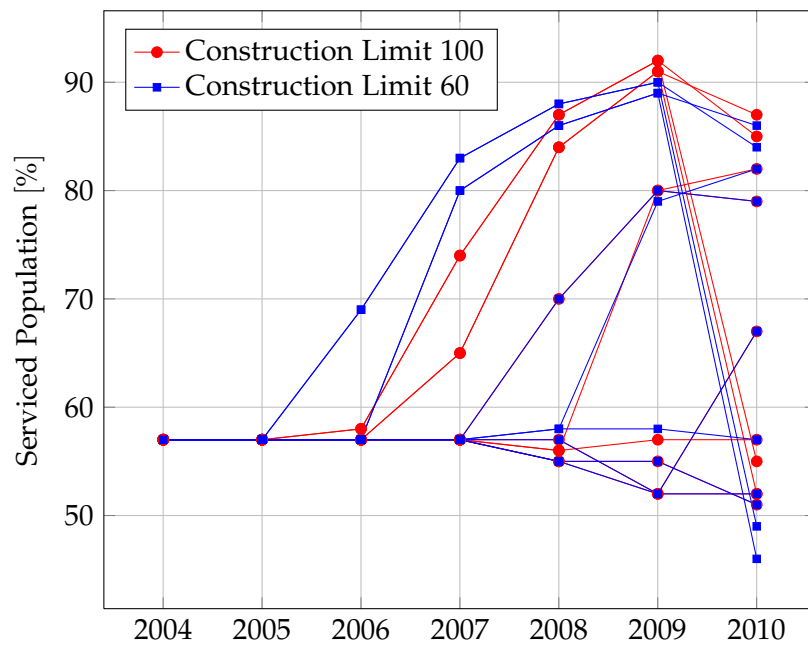
**Optimizing for Expected Profit.** The optimization results for the maximization of the expected profit are displayed in the second and the third column of Table 7.4. In all settings, we notice a considerable difference between expectation and the AV@R. High profits are thus expected in the best 60 % of the scenarios.

In the Berlin scenario, a mandatory coverage of 50 % of the population in the root node almost does not change the resulting figures. The expected profit, furthermore, is almost constant for the two different trees. The AV@R, however, is significantly higher in the tree with 34 nodes. In any case the AV@R is below -2.3 mill. euros. The project seems thus not very risky, since even in the worst 40 percent a profit of 2.3 mill. euros is expected.





(a) Installed Sites



(b) Covered Population

**Figure 7.5.:** Visualization of optimization results in Hamburg using the scenario tree with 34 nodes and different construction limits

## 7. Computational Experiments

In the Hamburg scenario demanding for coverage in the root node considerably decreases the performance in terms of expected profit and AV@R. Here, the figures for the different trees differ. The Expectation is about 10% higher for the tree with 34 nodes. The higher expectation is traded for a higher risk. The AV@R is negative for the tree with 27 nodes and positive for the tree with 34 nodes. In the latter case, a loss of almost 1 mill. euros is expected in the worst 40 percent of the scenarios. The reduction of the construction limit from 100 to 60 has only a minor effect in the scenario tree with 27 nodes. In the setting with the tree with 34 nodes, the expected profit suffers from the reduced construction limit, while the AV@R remains constant. The limit is exhausted only in the scenarios with high increase in the traffic, which are the most profitable. In the scenarios with poor profitability, the construction limit is not exhausted. In the quarterly planning the reduction has a significant impact on the expectation and AV@R. Furthermore, the AV@R is significantly lower in the quarterly planning than in the annual. Even with a construction limit of 60, more than 6.6 mill. euros profit are expected in the worst 40 % of the scenarios. Figure 7.5(a) shows the number of sites activated in the Hamburg scenario on the different scenarios in the tree with 34 nodes. Different construction limits are used. We observe that the numbers of sites in the scenarios with poor traffic evolution hardly differ. In the scenarios with more construction activities, the construction begins in earlier periods in the case of lower construction limits.

The serviced population in the same settings is shown in Figure 7.5(b). Focusing on the scenarios with big differences in the percentage of serviced population, we observe that much more of the population is in the service area of constructed sites in the years 2006 and 2007 if the construction limit is 60. This is because we started construction of additional sites in these years. In the years 2009 and 2010, the service level is higher if 100 sites can be built per year. In 2010 the services population decreases in some scenarios due to shrinking service areas. Since the expected profit is about half a million higher if 100 sites can be built, the superior service levels in the early years do not have such a strong impact on the profit as service in the later years. Indeed, in the best 9 % of the scenarios the discounted cash-flow in the last period is above 41 mill. euros if the construction limit is 100 and only between 37 and 39 mill. euros if 60 sites can be built per year.

**Comparison to a Clairvoyant.** The solution of our stochastic program is a non-anticipative network evolution plan for each root-leaf path in the scenario tree. If the non-anticipativity constraints are relaxed, the model decomposes into independent subproblems for the leaves which can be optimized individually. This corresponds to the perspective of a clairvoyant. The clairvoyant is expected to achieve better results in terms of expected profit and AV@R. The expected profit of the clairvoyant's strategy provides an upper bound for the expected profit of the stochastic program. For the AV@R, the clairvoyant provides a lower bound. The expected profit and AV@R of the clairvoyant in the different settings are displayed

## 7.5. Computational Results

	Opt. for expected profit	Opt. over AV@R	Clairvoyant
<i>Berlin annually with mandatory coverage in root</i>			
27 Nodes, 10 Cap	-3 204 851 €	-3 255 613 € ( -1.6 %)	-3 257 863 € ( -1.7 %)
34 Nodes, 10 Cap	-2 359 760 €	-2 459 457 € ( -4.2 %)	-2 460 765 € ( -4.3 %)
<i>Berlin annually</i>			
27 Nodes, 10 Cap	-3 205 609 €	-3 255 613 € ( -1.6 %)	-3 254 751 € ( -1.5 %)
34 Nodes, 10 Cap	-2 356 112 €	-2 458 983 € ( -4.4 %)	-2 460 744 € ( -4.4 %)
<i>Hamburg annually with mandatory coverage in root</i>			
27 Nodes, 100 Cap	-431 222 €	-438 100 € ( -1.6 %)	-446 821 € ( -3.6 %)
27 Nodes, 60 Cap	-428 591 €	-436 112 € ( -1.8 %)	-445 501 € ( -3.9 %)
34 Nodes, 100 Cap	936 882 €	915 089 € ( -2.3 %)	910 019 € ( -2.9 %)
34 Nodes, 60 Cap	938 999 €	915 653 € ( -2.5 %)	909 088 € ( -3.2 %)
<i>Hamburg annually</i>			
27 Nodes, 100 Cap	-2 917 462 €	-2 920 267 € ( -0.1 %)	-2 930 883 € ( -0.5 %)
27 Nodes, 60 Cap	-2 921 662 €	-2 924 311 € ( -0.1 %)	-2 931 112 € ( -0.3 %)
34 Nodes, 100 Cap	-1 601 728 €	-1 611 618 € ( -0.6 %)	-1 658 141 € ( -3.5 %)
34 Nodes, 60 Cap	-1 607 326 €	-1 607 326 € ( +0.0 %)	-1 657 085 € ( -3.1 %)
<i>Hamburg quarterly with mandatory coverage in the second stage</i>			
53 Nodes, 100 Cap	-7 742 900 €	-7 836 117 € ( -1.2 %)	-7 883 913 € ( -1.8 %)
53 Nodes, 60 Cap	-6 641 045 €	-6 653 595 € ( -0.2 %)	-6 701 818 € ( -0.9 %)

**Table 7.5.:** Comparison of the AV@Rs for the different objectives

in the third and fourth column of Table 7.4.

The difference between the solution of the stochastic program and the clairvoyant's solutions is surprisingly small. In all settings, the clairvoyant expects at most 2.2 % more than we do. The relative reduction of the AV@R is larger; at most 4.4 % in Berlin and at most 3.9 % in the Hamburg scenario. In the Berlin scenario this means a reduction by 100 000 €, which is about 1 % of the expected profit. In the Hamburg scenario the impact of perfect information is even smaller.

The absolute difference between the expectations of the clairvoyant's strategy and the non-anticipative strategy is called *expected value of perfect information* (EVPI). Since the clairvoyant's problem is a relaxation of the stochastic program, the EVPI is always non-negative. The EVPI in the different settings is stated in the last column of Table 7.4. We observe that the EVPI is considerable higher in the tree with 34 nodes than in the tree with 27 nodes. However, the EVPI always represents less than 2.2 % of the expected profit.

**Optimizing the Average Value-at-Risk.** Table 7.5 compares the AV@R for different optimization settings: Optimizing for the expected profit, optimizing for the AV@R, and relaxing the non-anticipativity constraints and optimize the individual scenarios. The differences between the three are small. This does not surprise anymore. The clairvoyant's strategy provides a lower bound for the AV@R. In the

## 7. Computational Experiments

	Opt. over scenario tree		Opt. over exp. traffic evol.	
	Expectation	AV@R	Expectation	AV@R
<i>Berlin annually with mandatory coverage in root</i>				
27 Nodes, 10 Cap	9 871 149 €	-3 204 851 €	9 622 943 € ( -2.5 %)	-2 765 367 € ( +13.7 %)
34 Nodes, 10 Cap	9 873 531 €	-2 359 760 €	9 505 135 € ( -3.7 %)	-1 934 687 € ( +18.0 %)
<i>Berlin annually</i>				
27 Nodes, 10 Cap	9 870 175 €	-3 205 609 €	9 617 548 € ( -2.6 %)	-2 765 367 € ( +13.7 %)
34 Nodes, 10 Cap	9 868 577 €	-2 356 112 €	9 499 740 € ( -3.7 %)	-1 934 687 € ( +17.9 %)
<i>Hamburg annually with mandatory coverage in root</i>				
27 Nodes, 100 Cap	12 972 933 €	-431 222 €	11 130 202 € ( -14.2 %)	2 575 456 € ( +697.2 %)
27 Nodes, 60 Cap	12 906 688 €	-428 591 €	11 147 457 € ( -13.6 %)	2 484 495 € ( +679.7 %)
34 Nodes, 100 Cap	14 410 188 €	936 882 €	11 681 093 € ( -18.9 %)	4 298 859 € ( +358.8 %)
34 Nodes, 60 Cap	13 969 286 €	938 999 €	11 592 296 € ( -17.0 %)	4 195 610 € ( +346.8 %)
<i>Hamburg annually</i>				
27 Nodes, 100 Cap	15 286 450 €	-2 917 462 €	13 295 759 € ( -13.0 %)	131 243 € ( +104.5 %)
27 Nodes, 60 Cap	15 233 750 €	-2 921 662 €	13 286 692 € ( -12.8 %)	64 594 € ( +102.2 %)
34 Nodes, 100 Cap	16 659 934 €	-1 601 728 €	13 843 274 € ( -16.9 %)	1 834 586 € ( +214.5 %)
34 Nodes, 60 Cap	16 198 435 €	-1 607 326 €	13 721 703 € ( -15.3 %)	1 756 479 € ( +209.3 %)
<i>Hamburg quarterly with mandatory coverage in the second stage</i>				
53 Nodes, 100 Cap	18 805 586 €	-7 742 900 €	17 889 615 € ( -4.9 %)	-6 340 598 € ( +18.1 %)
53 Nodes, 60 Cap	16 416 908 €	-6 641 045 €	15 904 370 € ( -3.1 %)	-5 980 380 € ( +9.9 %)

**Table 7.6.:** Comparison of stochastic optimization over scenario tree and optimization with the expected traffic evolution and evaluated on scenario tree.

discussion of Table 7.4 already stated that the gap between the clairvoyant's AV@R and the AV@R obtained by the solution of maximizing the profit is at most 4.4%. By minimization of the AV@R with non-anticipativity only reductions of at most 2.5% are achieved.

**Deterministic Optimization for the Expected Traffic Evolution.** Without stochastic programming, uncertain data is often treated by optimizing for the expected data evolution. The resulting network evolution can then be evaluated in the different scenarios of a scenario tree, and the expected profit and AV@R of that evolution strategy can be calculated. The results of this experiment are displayed in the last two columns of Table 7.6 and compared to the solution of the stochastic optimization in the second and third column.

A poorer performance is observed. For the annual planning in the Hamburg scenario the difference is significant. The expected profit decreases between 13% and 19%. This strategy is also much riskier, which is reflected by a much higher AV@R. The AV@R increases by about 3 mill. euros in these cases. In the Berlin scenario and in the quarterly planned Hamburg scenario the expected profit decreases by at most 5% and in the AV@R by less than 20%.

This comparison shows that the stochastic programming approach yields signif-

### 7.5. Computational Results

icantly better results than planning on the expected demand evolution. In practice, however, the network operator would probably not stick to the planning on the basis of the expected traffic evolution, but would re-optimize regularly to adjust the planning to the shifted expectation of the future demand evolution. The procedure in practice would be a rolling horizon approach and would probably yield better solutions.

## 7. *Computational Experiments*

## 8. Conclusions

In this thesis, we propose, implement, and analyze the application of multistage stochastic programming in strategic cellular network planning. Network operators in the telecommunication industry face strong, hardly predictable demand evolution. This applies to the types of the services requested as well as to the demand intensity. Strategic planning is substantial in this highly uncertain market environment, since huge financial investments have to be mastered.

The classical approaches for strategic decision taking, such as net present value and decision tree analysis, have considerable shortcomings. The popular real options approach addresses most of them, but the theoretical barrier for the correct application is high and the conditions often not fulfilled.

Recently, stochastic programming has been proposed as an alternative to real options, which avoids the assumptions from financial mathematics [86]. The steady advances in MIP technology as well as computer technology render the routine solution of large-scale mixed-integer programs possible. These two observations are at the starting point of this thesis. The goal is to analyze how stochastic programming can be used to tackle realistic strategic network planning problems.

We use a realistic system model of UMTS radio cells, which takes signal propagation and interferences into account. We devise an approach for strategic UMTS radio network planning that maps both, cell coverage as well as a cell's capability to serve demand, into the notion of a cell's service area. This also allows to model the increase of cell capacity through technology upgrades.

The demand evolution is modeled as a continuous stochastic process, which is approximated by a discrete scenario tree. We use a three-stage approach for the construction of non-uniform scenario trees that serve as input of the stochastic program. Our aggregation technique allows to substantially reduce the problem sizes of the corresponding deterministic equivalent programs; on realistic input data a reduction from about 120 000 000 to 600 000 variables and constraints is achieved in some instances (see Table 7.3).

We conduct computational experiments on two realistic planning scenarios: a medium-scale scenario based in the city of Berlin (based on publicly available data) and a large-scale scenario covering the city of Hamburg (based on the network of a German network provider). We study 14 different planning settings in total. The corresponding deterministic equivalent problems are solved using a commercial MIP solver to small optimality gaps within 10 hours running time. The optimization result provides a tree-like network deployment plan, where construction over time is tuned to the demand evolution. The evolution paths differ considerable across the tree. In the Hamburg scenario, for example, the number of installed sites

## 8. Conclusions

ranges from 40 to 372 and the net present value for the paths from  $-3\,208\,595\text{€}$  to  $59\,930\,638\text{€}$ . Compared to deterministic optimization for the expected demand evolution, the expected profit increases from  $11\,681\,093\text{€}$  by 18.9 % to  $14\,410\,188\text{€}$  if our approach is applied. In conclusion, the acceptable solution times as well as the obtained results encourage the application of stochastic methods in this setting.

Furthermore, we compare the results of the stochastic programs for different objective functions, namely, the expected profit and the risk measure Average Value-at-Risk. The risk measure focuses on the scenarios with poor performance. This is a typical approach to trade in a decrease of the expected profit for a decrease in risk. In our application, however, no considerable effect is observed.

Relaxing the non-anticipativity of the network evolution yield results from a clairvoyant's perspective. We observe improvements of at most 2 % in the expected profit and at most 4.4 % in the AV@R. The value of perfect information is hence rather small in the planning scenarios under consideration.

Our approach can be applied to plan the evolution of other cellular radio network technologies as well. Planning for LTE requires merely small changes in the system model and an update of model parameters. The market model should be updated to match the present market conditions with a high degree of penetration (approaching saturation) and flat rate contracts. An application to planning optical fiber access networks (FTTx) may also be feasible, but would require an in-depth analysis not conducted here.



# Appendix




## A. Tables with Detailed Results

In the following, we display detailed results of the all optimization procedures. Table A.1 provides a description of the columns. Arrows assist with the orientation in the trees.

Column	Description
node	Name of the node
father	Name of the father (The root node is its own father.)
stage	Stage of corresponding to the node
year	Year of corresponding to the node
KiB/cap	Traffic volume in monthly KiB per capita
prop	Probability of the node
totSites	Total number of installed sites (cells in the Berlin scenario)
coverPop	Percentage of the population that are serviced
inco	Undiscounted revenues in euros
capE	Undiscounted CAPEX in euros
opE	Undiscounted OPEX in euros
CFDis	Total discounted cash-flow in euros in the node
aggCF	Aggregated undiscounted cash-flow in euros up to the node
aggCFDis	Aggregated discounted cash-flow in euros up to the node

**Table A.1.:** Description of the columns



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.53	62 962	166 667	14 500	-126 281	-118 204	-126 281
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.53	71 920	0	14 500	39 875	-60 784	-86 406
4	2	3	2006	$1.86 \cdot 10^2$	0.29	8	0.67	156 466	80 450	23 200	21 253	-7 968	-65 153
7	4	4	2007	$9.64 \cdot 10^2$	0.29	18	0.93	563 898	235 583	52 200	110 435	268 146	45 282
11	7	5	2008	$3.79 \cdot 10^3$	0.29	28	0.99	1 178 660	203 000	81 200	343 147	1 162 607	388 429
16	11	6	2009	$5.64 \cdot 10^4$	0.09	38	1.00	8 809 835	170 417	110 200	2 845 004	9 691 825	3 233 433
22	16	7	2010	$3.00 \cdot 10^5$	0.09	48	0.94	22 170 352	137 833	139 200	6 102 330	31 585 144	9 335 763
17	11	6	2009	$1.50 \cdot 10^4$	0.20	28	0.99	2 330 015	0	81 200	753 123	3 411 421	1 141 553
23	17	7	2010	$3.89 \cdot 10^4$	0.20	28	0.99	3 024 326	0	81 200	821 373	6 354 547	1 962 925
3	1	2	2005	$1.09 \cdot 10^2$	0.70	15	0.86	234 971	300 750	43 500	-117 659	-227 484	-243 941
5	3	3	2006	$5.88 \cdot 10^2$	0.52	21	0.96	705 521	160 900	60 900	261 308	256 237	17 368
8	5	4	2007	$2.05 \cdot 10^3$	0.36	21	0.96	1 229 645	0	60 900	563 631	1 424 982	580 998
13	8	5	2008	$6.56 \cdot 10^3$	0.24	26	0.98	2 020 706	101 500	75 400	732 826	3 268 788	1 313 824
19	13	6	2009	$4.28 \cdot 10^4$	0.24	36	1.00	6 702 596	170 417	104 400	2 141 236	9 696 567	3 455 060
25	19	7	2010	$1.99 \cdot 10^5$	0.24	46	0.99	15 428 921	137 833	133 400	4 222 539	24 854 255	7 677 600
12	8	5	2008	$2.69 \cdot 10^3$	0.12	21	0.96	807 885	0	60 900	300 197	2 171 967	881 195
18	12	6	2009	$1.01 \cdot 10^4$	0.12	22	0.96	1 523 428	17 042	63 800	481 978	3 614 553	1 363 173
24	18	7	2010	$3.82 \cdot 10^4$	0.12	22	0.96	2 883 314	0	63 800	786 875	6 434 068	2 150 047
9	5	4	2007	$4.35 \cdot 10^3$	0.16	31	1.00	2 720 106	235 583	89 900	1 132 092	2 650 859	1 149 459
14	9	5	2008	$1.93 \cdot 10^4$	0.16	41	1.00	6 040 514	203 000	118 900	2 281 867	8 369 473	3 431 326
20	14	6	2009	$7.28 \cdot 10^4$	0.16	51	1.00	11 411 531	170 417	147 900	3 703 681	19 462 687	7 135 006
26	20	7	2010	$3.63 \cdot 10^5$	0.16	61	0.95	27 141 366	137 833	176 900	7 479 128	46 289 320	14 614 134
6	3	3	2006	$1.48 \cdot 10^3$	0.18	25	0.97	1 805 150	268 167	72 500	816 464	1 236 999	572 523
10	6	4	2007	$6.34 \cdot 10^3$	0.18	35	1.00	3 970 292	247 363	102 950	1 721 888	4 856 979	2 294 411
15	10	5	2008	$3.24 \cdot 10^4$	0.18	45	1.00	10 141 680	203 000	131 950	3 924 789	14 663 709	6 219 200
21	15	6	2009	$1.38 \cdot 10^5$	0.18	55	1.00	21 612 480	170 417	160 950	7 115 587	35 944 822	13 334 787
27	21	7	2010	$5.23 \cdot 10^5$	0.18	65	0.85	34 731 866	137 833	189 950	9 593 855	70 348 905	22 928 642

**Table A.2.:** Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, and optimization for expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.51	61 446	166 667	14 500	-127 545	-119 720	-127 545
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.51	70 188	0	14 500	38 673	-64 032	-88 872
4	2	3	2006	$1.86 \cdot 10^2$	0.29	8	0.67	156 466	80 450	23 200	21 253	-11 216	-67 619
7	4	4	2007	$9.64 \cdot 10^2$	0.29	18	0.93	563 898	235 583	52 200	110 435	264 898	42 816
11	7	5	2008	$3.79 \cdot 10^3$	0.29	28	0.99	1 178 660	203 000	81 200	343 147	1 159 359	385 963
16	11	6	2009	$5.64 \cdot 10^4$	0.09	38	1.00	8 809 835	170 417	110 200	2 845 004	9 688 577	3 230 967
22	16	7	2010	$3.00 \cdot 10^5$	0.09	48	0.94	22 170 352	137 833	139 200	6 102 330	31 581 896	9 333 297
17	11	6	2009	$1.50 \cdot 10^4$	0.20	28	0.99	2 330 015	0	81 200	753 123	3 408 173	1 139 087
23	17	7	2010	$3.89 \cdot 10^4$	0.20	28	0.99	3 024 326	0	81 200	821 373	6 351 300	1 960 459
3	1	2	2005	$1.09 \cdot 10^2$	0.70	15	0.85	232 089	300 750	43 500	-119 660	-231 881	-247 205
5	3	3	2006	$5.88 \cdot 10^2$	0.52	20	0.95	698 588	134 083	58 000	277 597	274 623	30 392
8	5	4	2007	$2.05 \cdot 10^3$	0.36	21	0.96	1 229 645	23 558	60 900	549 997	1 419 809	580 389
13	8	5	2008	$6.56 \cdot 10^3$	0.24	26	0.98	2 020 706	101 500	75 400	732 826	3 263 615	1 313 215
19	13	6	2009	$4.28 \cdot 10^4$	0.24	36	1.00	6 704 218	170 417	104 400	2 141 779	9 693 016	3 454 994
25	19	7	2010	$1.99 \cdot 10^5$	0.24	46	0.99	15 440 513	137 833	133 400	4 225 774	24 862 296	7 680 769
12	8	5	2008	$2.69 \cdot 10^3$	0.12	21	0.96	807 885	0	60 900	300 197	2 166 795	880 586
18	12	6	2009	$1.01 \cdot 10^4$	0.12	22	0.96	1 523 428	17 042	63 800	481 978	3 609 381	1 362 564
24	18	7	2010	$3.82 \cdot 10^4$	0.12	22	0.96	2 883 314	0	63 800	786 875	6 428 895	2 149 438
9	5	4	2007	$4.35 \cdot 10^3$	0.16	30	0.99	2 716 078	235 583	87 000	1 131 548	2 668 117	1 161 940
14	9	5	2008	$1.93 \cdot 10^4$	0.16	40	1.00	6 031 248	203 000	116 000	2 279 308	8 380 365	3 441 248
20	14	6	2009	$7.28 \cdot 10^4$	0.16	50	1.00	11 394 539	170 417	145 000	3 698 961	19 459 487	7 140 209
26	20	7	2010	$3.63 \cdot 10^5$	0.16	60	0.95	27 067 171	137 833	174 000	7 459 230	46 214 824	14 599 439
6	3	3	2006	$1.48 \cdot 10^3$	0.18	25	0.97	1 802 393	268 167	72 500	814 869	1 229 845	567 664
10	6	4	2007	$6.34 \cdot 10^3$	0.18	35	1.00	3 969 450	235 583	101 500	1 728 998	4 862 211	2 296 661
15	10	5	2008	$3.24 \cdot 10^4$	0.18	45	1.00	10 137 202	203 000	130 500	3 923 572	14 665 914	6 220 233
21	15	6	2009	$1.38 \cdot 10^5$	0.18	55	1.00	21 602 963	170 417	159 500	7 112 886	35 938 960	13 333 119
27	21	7	2010	$5.23 \cdot 10^5$	0.18	65	0.85	34 800 750	137 833	188 500	9 613 484	70 413 376	22 946 602

**Table A.3.:** Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, mandatory coverage, and optimization for expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	8	0.66	78 468	266 667	23 200	-220 610	-211 398	-220 610
2	1	2	2005	$5.43 \cdot 10^1$	0.29	8	0.66	89 632	0	23 200	46 133	-144 966	-174 476
4	2	3	2006	$1.86 \cdot 10^2$	0.29	9	0.72	168 195	26 817	26 100	63 608	-29 688	-110 868
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.90	548 035	164 908	46 400	146 482	307 038	35 614
11	7	5	2008	$1.07 \cdot 10^4$	0.09	26	0.99	3 309 230	203 000	75 400	1 201 706	3 337 869	1 237 320
17	11	6	2009	$5.64 \cdot 10^4$	0.09	30	1.00	8 818 907	68 167	87 000	2 896 903	12 001 609	4 134 223
25	17	7	2010	$1.50 \cdot 10^5$	0.09	40	0.99	11 664 622	137 833	116 000	3 176 848	23 412 397	7 311 072
12	7	5	2008	$3.79 \cdot 10^3$	0.20	20	0.96	1 138 735	81 200	58 000	395 164	1 306 574	430 778
19	12	6	2009	$9.58 \cdot 10^3$	0.13	21	0.96	1 449 737	17 042	60 900	458 270	2 678 369	889 048
27	19	7	2010	$7.50 \cdot 10^4$	0.13	27	0.98	5 793 888	82 700	78 300	1 567 420	8 311 257	2 456 468
18	12	6	2009	$3.76 \cdot 10^3$	0.07	20	0.96	565 318	0	58 000	169 900	1 813 892	600 678
26	18	7	2010	$1.12 \cdot 10^4$	0.07	20	0.96	841 904	0	58 000	218 773	2 597 796	819 451
3	1	2	2005	$1.09 \cdot 10^2$	0.70	18	0.92	251 841	300 750	52 200	-111 986	-312 508	-332 595
5	3	3	2006	$5.88 \cdot 10^2$	0.52	28	0.99	729 824	268 167	81 200	189 134	67 949	-143 461
8	5	4	2007	$2.05 \cdot 10^3$	0.36	28	0.99	1 272 003	0	81 200	574 268	1 258 752	430 807
14	8	5	2008	$6.56 \cdot 10^3$	0.24	31	1.00	2 053 214	60 900	89 900	759 643	3 161 166	1 190 450
21	14	6	2009	$1.46 \cdot 10^4$	0.07	31	1.00	2 289 835	0	89 900	736 754	5 361 102	1 927 204
29	21	7	2010	$4.73 \cdot 10^4$	0.07	31	1.00	3 702 572	0	89 900	1 008 231	8 973 774	2 935 434
22	14	6	2009	$4.28 \cdot 10^4$	0.17	41	1.00	6 705 616	170 417	118 900	2 137 391	9 577 465	3 327 841
30	22	7	2010	$2.47 \cdot 10^5$	0.17	51	0.99	19 147 990	137 833	147 900	5 256 416	28 439 722	8 584 257
13	8	5	2008	$2.69 \cdot 10^3$	0.12	28	0.99	835 715	0	81 200	303 222	2 013 267	734 029
20	13	6	2009	$1.01 \cdot 10^4$	0.12	28	0.99	1 563 968	0	81 200	496 576	3 496 035	1 230 606
28	20	7	2010	$3.82 \cdot 10^4$	0.12	28	0.99	2 960 043	0	81 200	803 432	6 374 878	2 034 038
9	5	4	2007	$4.35 \cdot 10^3$	0.16	38	1.00	2 730 637	235 583	110 200	1 127 381	2 452 803	983 920
15	9	5	2008	$1.93 \cdot 10^4$	0.16	48	1.00	6 047 525	203 000	139 200	2 276 526	8 158 129	3 260 446
23	15	6	2009	$7.28 \cdot 10^4$	0.16	58	1.00	11 424 776	170 417	168 200	3 701 318	19 244 288	6 961 764
31	23	7	2010	$3.63 \cdot 10^5$	0.12	68	0.97	27 605 494	137 833	197 200	7 602 992	46 514 748	14 564 755
32	23	7	2010	$1.60 \cdot 10^6$	0.04	68	0.37	45 759 879	137 833	197 200	12 669 547	64 669 134	19 631 311
6	3	3	2006	$1.48 \cdot 10^3$	0.18	28	0.99	1 832 243	268 167	81 200	827 108	1 170 369	494 513
10	6	4	2007	$6.34 \cdot 10^3$	0.18	38	1.00	3 973 654	235 583	110 200	1 726 829	4 798 239	2 221 342
16	10	5	2008	$3.24 \cdot 10^4$	0.18	48	1.00	10 145 642	203 000	139 200	3 923 467	14 601 681	6 144 810
24	16	6	2009	$1.38 \cdot 10^5$	0.18	58	1.00	21 620 070	170 417	168 200	7 115 701	35 883 135	13 260 511
34	24	7	2010	$1.57 \cdot 10^6$	0.05	68	0.37	45 032 884	137 833	197 200	12 466 657	80 580 986	25 727 168
33	24	7	2010	$5.23 \cdot 10^5$	0.13	68	0.86	35 239 181	137 833	197 200	9 733 414	70 787 283	22 993 925

**Table A.4.:** Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, and optimization for expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	8	0.66	78 468	266 667	23 200	-220 610	-211 398	-220 610
2	1	2	2005	$5.43 \cdot 10^1$	0.29	8	0.66	89 632	0	23 200	46 133	-144 966	-174 476
4	2	3	2006	$1.86 \cdot 10^2$	0.29	9	0.72	168 195	26 817	26 100	63 608	-29 688	-110 868
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.91	549 010	164 908	46 400	146 952	308 013	36 084
11	7	5	2008	$1.07 \cdot 10^4$	0.09	26	0.99	3 299 483	203 000	75 400	1 197 789	3 329 096	1 233 873
17	11	6	2009	$5.64 \cdot 10^4$	0.09	30	1.00	8 810 121	68 167	87 000	2 893 961	11 984 051	4 127 834
25	17	7	2010	$1.50 \cdot 10^5$	0.09	40	0.99	11 676 825	137 833	116 000	3 180 254	23 407 042	7 308 088
12	7	5	2008	$3.79 \cdot 10^3$	0.20	19	0.95	1 127 515	60 900	55 100	401 610	1 319 528	437 694
19	12	6	2009	$9.58 \cdot 10^3$	0.13	21	0.96	1 449 632	34 083	60 900	451 386	2 674 176	889 080
27	19	7	2010	$7.50 \cdot 10^4$	0.13	27	0.98	5 795 836	82 700	78 300	1 567 963	8 309 012	2 457 043
18	12	6	2009	$3.76 \cdot 10^3$	0.07	19	0.95	559 748	0	55 100	169 005	1 824 176	606 699
26	18	7	2010	$1.12 \cdot 10^4$	0.07	19	0.95	833 609	0	55 100	217 267	2 602 684	823 967
3	1	2	2005	$1.09 \cdot 10^2$	0.70	18	0.92	251 122	300 750	52 200	-112 485	-313 227	-333 095
5	3	3	2006	$5.88 \cdot 10^2$	0.52	28	0.99	730 917	268 167	81 200	189 767	68 323	-143 328
8	5	4	2007	$2.05 \cdot 10^3$	0.36	28	0.99	1 273 908	0	81 200	575 187	1 261 031	431 859
14	8	5	2008	$6.56 \cdot 10^3$	0.24	30	1.00	2 050 595	40 600	87 000	769 545	3 184 026	1 201 404
21	14	6	2009	$1.46 \cdot 10^4$	0.07	30	1.00	2 286 914	0	87 000	736 747	5 383 941	1 938 151
29	21	7	2010	$4.73 \cdot 10^4$	0.07	30	1.00	3 697 848	0	87 000	1 007 722	8 994 789	2 945 873
22	14	6	2009	$4.28 \cdot 10^4$	0.17	40	1.00	6 705 616	170 417	116 000	2 138 362	9 603 225	3 339 767
30	22	7	2010	$2.47 \cdot 10^5$	0.17	50	0.98	19 109 307	137 833	145 000	5 246 430	28 429 698	8 586 196
13	8	5	2008	$2.69 \cdot 10^3$	0.12	28	0.99	836 966	0	81 200	303 725	2 016 797	735 584
20	13	6	2009	$1.01 \cdot 10^4$	0.12	28	0.99	1 566 311	0	81 200	497 361	3 501 908	1 232 945
28	20	7	2010	$3.82 \cdot 10^4$	0.12	28	0.99	2 964 476	0	81 200	804 669	6 385 184	2 037 614
9	5	4	2007	$4.35 \cdot 10^3$	0.16	38	1.00	2 731 914	235 583	110 200	1 127 997	2 454 454	984 669
15	9	5	2008	$1.93 \cdot 10^4$	0.16	48	1.00	6 047 578	203 000	139 200	2 276 547	8 159 832	3 261 216
23	15	6	2009	$7.28 \cdot 10^4$	0.16	58	1.00	11 424 875	170 417	168 200	3 701 351	19 246 090	6 962 567
31	23	7	2010	$3.63 \cdot 10^5$	0.12	68	0.97	27 583 398	137 833	197 200	7 596 825	46 494 455	14 559 392
32	23	7	2010	$1.60 \cdot 10^6$	0.04	68	0.36	45 696 028	137 833	197 200	12 651 728	64 607 085	19 614 294
6	3	3	2006	$1.48 \cdot 10^3$	0.18	28	0.99	1 829 998	268 167	81 200	825 809	1 167 405	492 714
10	6	4	2007	$6.34 \cdot 10^3$	0.18	38	1.00	3 976 467	247 363	111 650	1 720 670	4 784 859	2 213 385
16	10	5	2008	$3.24 \cdot 10^4$	0.18	48	1.00	10 152 413	203 000	140 650	3 925 606	14 593 622	6 138 990
24	16	6	2009	$1.38 \cdot 10^5$	0.18	58	1.00	21 644 675	170 417	169 650	7 123 456	35 898 230	13 262 446
34	24	7	2010	$1.57 \cdot 10^6$	0.05	68	0.37	45 177 313	137 833	198 650	12 506 559	80 739 059	25 769 005
33	24	7	2010	$5.23 \cdot 10^5$	0.13	68	0.86	35 300 806	137 833	198 650	9 750 207	70 862 553	23 012 653

**Table A.5.:** Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, mandatory coverage, and optimization for expected profit

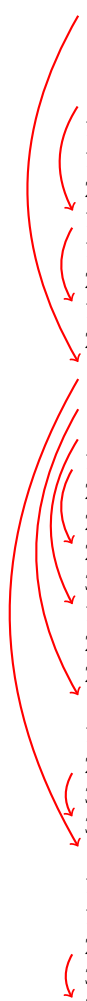
node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.53	63 136	166 667	14 500	-126 137	-118 031	-126 137
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.53	72 119	0	14 500	40 013	-60 412	-86 124
4	2	3	2006	$1.86 \cdot 10^2$	0.29	6	0.59	137 797	26 817	17 400	51 052	33 169	-35 072
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.91	548 766	235 583	46 400	105 935	299 952	70 863
11	7	5	2008	$3.79 \cdot 10^3$	0.29	20	0.96	1 140 172	81 200	58 000	395 742	1 300 923	466 604
16	11	6	2009	$5.64 \cdot 10^4$	0.09	30	0.98	8 666 183	170 417	87 000	2 804 664	9 709 689	3 271 269
22	16	7	2010	$3.00 \cdot 10^5$	0.09	38	0.80	18 806 756	110 267	110 200	5 180 938	28 295 979	8 452 206
17	11	6	2009	$1.50 \cdot 10^4$	0.20	24	0.98	2 314 042	68 167	69 600	724 265	3 477 199	1 190 869
23	17	7	2010	$3.89 \cdot 10^4$	0.20	24	0.98	3 003 595	0	69 600	818 824	6 411 193	2 009 693
3	1	2	2005	$1.09 \cdot 10^2$	0.70	8	0.69	188 270	90 225	23 200	39 445	-43 185	-86 692
5	3	3	2006	$5.88 \cdot 10^2$	0.52	16	0.90	667 695	214 533	46 400	210 564	363 576	123 872
8	5	4	2007	$2.05 \cdot 10^3$	0.36	20	0.96	1 232 244	94 233	58 000	511 750	1 443 587	635 622
13	8	5	2008	$6.56 \cdot 10^3$	0.24	26	0.99	2 037 396	121 800	75 400	729 744	3 283 783	1 365 366
19	13	6	2009	$4.28 \cdot 10^4$	0.24	36	1.00	6 697 044	170 417	104 400	2 139 377	9 706 011	3 504 742
25	19	7	2010	$1.99 \cdot 10^5$	0.24	46	0.99	15 405 519	137 833	133 400	4 216 008	24 840 297	7 720 750
12	8	5	2008	$2.69 \cdot 10^3$	0.12	20	0.96	809 593	0	58 000	302 048	2 195 180	937 670
18	12	6	2009	$1.01 \cdot 10^4$	0.12	21	0.96	1 526 122	17 042	60 900	483 851	3 643 360	1 421 521
24	18	7	2010	$3.82 \cdot 10^4$	0.12	21	0.96	2 888 413	0	60 900	789 107	6 470 873	2 210 628
9	5	4	2007	$4.35 \cdot 10^3$	0.16	16	0.90	2 470 290	0	46 400	1 168 928	2 787 466	1 292 800
14	9	5	2008	$1.93 \cdot 10^4$	0.16	16	0.90	5 467 867	0	46 400	2 178 766	8 208 933	3 471 566
20	14	6	2009	$7.28 \cdot 10^4$	0.16	19	0.87	9 933 088	51 125	55 100	3 287 572	18 035 796	6 759 139
26	20	7	2010	$3.63 \cdot 10^5$	0.16	25	0.45	12 858 927	89 592	73 950	3 538 048	30 731 182	10 297 187
6	3	3	2006	$1.48 \cdot 10^3$	0.18	11	0.73	1 350 306	80 450	31 900	707 099	1 194 771	620 407
10	6	4	2007	$6.34 \cdot 10^3$	0.18	11	0.73	2 898 027	0	31 900	1 382 198	4 060 898	2 002 605
15	10	5	2008	$3.24 \cdot 10^4$	0.18	12	0.74	7 483 836	20 300	34 800	2 983 811	11 489 634	4 986 416
21	15	6	2009	$1.38 \cdot 10^5$	0.18	13	0.48	10 321 821	17 042	37 700	3 437 283	21 756 713	8 423 699
27	21	7	2010	$5.23 \cdot 10^5$	0.18	16	0.20	8 042 350	48 242	47 850	2 214 962	29 702 971	10 638 661

**Table A.6.:** Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, and optimization for AV@R



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.53	63 136	166 667	14 500	-126 137	-118 031	-126 137
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.53	72 119	0	14 500	40 013	-60 412	-86 124
4	2	3	2006	$1.86 \cdot 10^2$	0.29	6	0.59	137 797	26 817	17 400	51 052	33 169	-35 072
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.91	548 766	235 583	46 400	105 935	299 952	70 863
11	7	5	2008	$3.79 \cdot 10^3$	0.29	20	0.96	1 140 172	81 200	58 000	395 742	1 300 923	466 604
16	11	6	2009	$5.64 \cdot 10^4$	0.09	21	0.96	8 452 358	17 042	60 900	2 803 434	9 675 339	3 270 038
22	16	7	2010	$3.00 \cdot 10^5$	0.09	31	0.71	16 632 604	137 833	89 900	4 570 605	26 080 210	7 840 643
17	11	6	2009	$1.50 \cdot 10^4$	0.20	24	0.98	2 314 042	68 167	69 600	724 265	3 477 199	1 190 869
23	17	7	2010	$3.89 \cdot 10^4$	0.20	24	0.98	3 003 595	0	69 600	818 824	6 411 193	2 009 693
3	1	2	2005	$1.09 \cdot 10^2$	0.70	8	0.69	188 270	90 225	23 200	39 445	-43 185	-86 692
5	3	3	2006	$5.88 \cdot 10^2$	0.52	16	0.90	667 695	214 533	46 400	210 564	363 576	123 872
8	5	4	2007	$2.05 \cdot 10^3$	0.36	20	0.96	1 232 244	94 233	58 000	511 750	1 443 587	635 622
13	8	5	2008	$6.56 \cdot 10^3$	0.24	26	0.99	2 037 396	121 800	75 400	729 744	3 283 783	1 365 366
19	13	6	2009	$4.28 \cdot 10^4$	0.24	36	1.00	6 697 044	170 417	104 400	2 139 377	9 706 011	3 504 742
25	19	7	2010	$1.99 \cdot 10^5$	0.24	46	0.99	15 405 519	137 833	133 400	4 216 008	24 840 297	7 720 750
12	8	5	2008	$2.69 \cdot 10^3$	0.12	20	0.96	809 593	0	58 000	302 048	2 195 180	937 670
18	12	6	2009	$1.01 \cdot 10^4$	0.12	21	0.96	1 526 122	17 042	60 900	483 851	3 643 360	1 421 521
24	18	7	2010	$3.82 \cdot 10^4$	0.12	21	0.96	2 888 413	0	60 900	789 107	6 470 873	2 210 628
9	5	4	2007	$4.35 \cdot 10^3$	0.16	16	0.90	2 470 290	0	46 400	1 168 928	2 787 466	1 292 800
14	9	5	2008	$1.93 \cdot 10^4$	0.16	18	0.91	5 496 586	40 600	52 200	2 168 397	8 191 252	3 461 197
20	14	6	2009	$7.28 \cdot 10^4$	0.16	23	0.93	10 638 016	85 208	66 700	3 506 069	18 677 359	6 967 266
26	20	7	2010	$3.63 \cdot 10^5$	0.16	33	0.63	17 861 769	137 833	95 700	4 912 024	36 305 595	11 879 290
6	3	3	2006	$1.48 \cdot 10^3$	0.18	8	0.69	1 279 239	0	23 200	726 874	1 212 854	640 183
10	6	4	2007	$6.34 \cdot 10^3$	0.18	8	0.69	2 745 502	0	23 200	1 312 838	3 935 156	1 953 021
15	10	5	2008	$3.24 \cdot 10^4$	0.18	9	0.69	7 050 254	20 300	26 100	2 813 060	10 939 010	4 766 081
21	15	6	2009	$1.38 \cdot 10^5$	0.18	13	0.55	11 935 689	68 167	37 700	3 957 218	22 768 833	8 723 299
27	21	7	2010	$5.23 \cdot 10^5$	0.18	23	0.34	13 987 013	137 833	66 700	3 838 744	36 551 313	12 562 043

**Table A.7.:** Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, mandatory coverage, and optimization for AV@R



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.53	63 136	166 667	14 500	-126 137	-118 031	-126 137
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.53	72 119	0	14 500	40 013	-60 412	-86 124
4	2	3	2006	$1.86 \cdot 10^2$	0.29	6	0.59	137 797	26 817	17 400	51 052	33 169	-35 072
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.91	549 741	235 583	46 400	106 405	300 926	71 333
11	7	5	2008	$1.07 \cdot 10^4$	0.09	26	0.99	3 302 493	203 000	75 400	1 198 999	3 325 019	1 270 332
17	11	6	2009	$5.64 \cdot 10^4$	0.09	30	0.99	8 802 823	68 167	87 000	2 891 517	11 972 675	4 161 848
25	17	7	2010	$1.50 \cdot 10^5$	0.09	40	0.99	11 663 788	137 833	116 000	3 176 616	23 382 631	7 338 464
12	7	5	2008	$3.79 \cdot 10^3$	0.20	19	0.95	1 128 951	60 900	55 100	402 187	1 313 877	473 520
19	12	6	2009	$9.58 \cdot 10^3$	0.13	21	0.97	1 451 425	34 083	60 900	451 987	2 670 319	925 507
27	19	7	2010	$7.50 \cdot 10^4$	0.13	26	0.98	5 777 811	68 917	75 400	1 568 358	8 303 813	2 493 865
18	12	6	2009	$3.76 \cdot 10^3$	0.07	19	0.95	560 461	0	55 100	169 244	1 819 238	642 765
26	18	7	2010	$1.12 \cdot 10^4$	0.07	19	0.95	834 671	0	55 100	217 564	2 598 809	860 328
3	1	2	2005	$1.09 \cdot 10^2$	0.70	8	0.69	188 270	90 225	23 200	39 445	-43 185	-86 692
5	3	3	2006	$5.88 \cdot 10^2$	0.52	16	0.90	667 695	214 533	46 400	210 564	363 576	123 872
8	5	4	2007	$2.05 \cdot 10^3$	0.36	20	0.96	1 232 403	94 233	58 000	511 826	1 443 746	635 699
14	8	5	2008	$6.56 \cdot 10^3$	0.24	24	0.98	2 026 755	81 200	69 600	747 378	3 319 701	1 383 076
21	14	6	2009	$1.46 \cdot 10^4$	0.07	24	0.98	2 260 327	0	69 600	733 670	5 510 428	2 116 746
29	21	7	2010	$4.73 \cdot 10^4$	0.07	24	0.98	3 654 851	0	69 600	1 000 578	9 095 679	3 117 324
22	14	6	2009	$4.28 \cdot 10^4$	0.17	34	0.99	6 643 721	178 938	100 050	2 119 551	9 684 434	3 502 627
30	22	7	2010	$2.47 \cdot 10^5$	0.17	44	0.92	17 872 432	137 833	129 050	4 905 692	27 289 983	8 408 320
13	8	5	2008	$2.69 \cdot 10^3$	0.12	20	0.96	809 698	0	58 000	302 090	2 195 444	937 789
20	13	6	2009	$1.01 \cdot 10^4$	0.12	22	0.97	1 538 034	34 083	63 800	480 021	3 635 595	1 417 810
28	20	7	2010	$3.82 \cdot 10^4$	0.12	22	0.97	2 910 958	0	63 800	794 590	6 482 753	2 212 399
9	5	4	2007	$4.35 \cdot 10^3$	0.16	25	0.99	2 692 801	212 025	72 500	1 140 948	2 771 852	1 264 821
15	9	5	2008	$1.93 \cdot 10^4$	0.16	29	1.00	6 024 850	81 200	84 100	2 348 295	8 631 402	3 613 116
23	15	6	2009	$7.28 \cdot 10^4$	0.16	33	0.99	11 365 642	76 688	97 150	3 742 976	19 823 207	7 356 092
31	23	7	2010	$3.63 \cdot 10^5$	0.12	41	0.62	17 715 333	110 267	120 350	4 873 509	37 307 923	12 229 601
32	23	7	2010	$1.60 \cdot 10^6$	0.04	33	0.17	21 207 294	0	97 150	5 891 454	40 933 351	13 247 546
6	3	3	2006	$1.48 \cdot 10^3$	0.18	14	0.82	1 521 830	160 900	40 600	745 457	1 277 145	658 765
10	6	4	2007	$6.34 \cdot 10^3$	0.18	14	0.82	3 266 150	0	40 600	1 555 531	4 502 694	2 214 296
16	10	5	2008	$3.24 \cdot 10^4$	0.18	14	0.82	8 338 849	0	40 600	3 334 880	12 800 943	5 549 177
24	16	6	2009	$1.38 \cdot 10^5$	0.18	16	0.63	13 629 181	34 083	46 400	4 535 148	26 349 641	10 084 325
34	24	7	2010	$1.57 \cdot 10^6$	0.05	17	0.09	10 794 381	13 783	49 300	2 994 139	37 080 938	13 078 464
33	24	7	2010	$5.23 \cdot 10^5$	0.13	23	0.30	12 421 590	96 483	66 700	3 415 711	38 608 047	13 500 036

**Table A.8.:** Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	5	0.53	63 136	166 667	14 500	<b>-126 137</b>	<b>-118 031</b>	<b>-126 137</b>
2	1	2	2005	$5.43 \cdot 10^1$	0.29	5	0.53	72 119	0	14 500	40 013	<b>-60 412</b>	<b>-86 124</b>
4	2	3	2006	$1.86 \cdot 10^2$	0.29	6	0.59	137 797	26 817	17 400	51 052	33 169	<b>-35 072</b>
7	4	4	2007	$9.64 \cdot 10^2$	0.29	16	0.91	549 741	235 583	46 400	106 405	300 926	71 333
11	7	5	2008	$1.07 \cdot 10^4$	0.09	26	0.99	3 302 493	203 000	75 400	1 198 999	3 325 019	1 270 332
17	11	6	2009	$5.64 \cdot 10^4$	0.09	30	0.99	8 802 823	68 167	87 000	2 891 517	11 972 675	4 161 848
25	17	7	2010	$1.50 \cdot 10^5$	0.09	40	0.99	11 669 351	137 833	116 000	3 178 168	23 388 193	7 340 017
12	7	5	2008	$3.79 \cdot 10^3$	0.20	19	0.95	1 128 951	60 900	55 100	402 187	1 313 877	473 520
19	12	6	2009	$9.58 \cdot 10^3$	0.13	21	0.97	1 451 425	34 083	60 900	451 987	2 670 319	925 507
27	19	7	2010	$7.50 \cdot 10^4$	0.13	26	0.98	5 777 811	68 917	75 400	1 568 358	8 303 813	2 493 865
18	12	6	2009	$3.76 \cdot 10^3$	0.07	19	0.95	560 461	0	55 100	169 244	1 819 238	642 765
26	18	7	2010	$1.12 \cdot 10^4$	0.07	19	0.95	834 671	0	55 100	217 564	2 598 809	860 328
3	1	2	2005	$1.09 \cdot 10^2$	0.70	8	0.69	188 270	90 225	23 200	39 445	<b>-43 185</b>	<b>-86 692</b>
5	3	3	2006	$5.88 \cdot 10^2$	0.52	16	0.90	667 695	214 533	46 400	210 564	363 576	123 872
8	5	4	2007	$2.05 \cdot 10^3$	0.36	20	0.96	1 232 403	94 233	58 000	511 826	1 443 746	635 699
14	8	5	2008	$6.56 \cdot 10^3$	0.24	24	0.98	2 026 755	81 200	69 600	747 378	3 319 701	1 383 076
21	14	6	2009	$1.46 \cdot 10^4$	0.07	24	0.98	2 260 327	0	69 600	733 670	5 510 428	2 116 746
29	21	7	2010	$4.73 \cdot 10^4$	0.07	24	0.98	3 654 851	0	69 600	1 000 578	9 095 679	3 117 324
22	14	6	2009	$4.28 \cdot 10^4$	0.17	34	0.99	6 634 822	170 417	98 600	2 120 481	9 685 506	3 503 557
30	22	7	2010	$2.47 \cdot 10^5$	0.17	44	0.89	17 385 994	137 833	127 600	4 770 341	26 806 067	8 273 898
13	8	5	2008	$2.69 \cdot 10^3$	0.12	20	0.96	809 698	0	58 000	302 090	2 195 444	937 789
20	13	6	2009	$1.01 \cdot 10^4$	0.12	22	0.97	1 538 034	34 083	63 800	480 021	3 635 595	1 417 810
28	20	7	2010	$3.82 \cdot 10^4$	0.12	22	0.97	2 910 958	0	63 800	794 590	6 482 753	2 212 399
9	5	4	2007	$4.35 \cdot 10^3$	0.16	19	0.91	2 481 709	82 454	56 550	1 121 824	2 706 280	1 245 696
15	9	5	2008	$1.93 \cdot 10^4$	0.16	23	0.93	5 648 678	91 350	69 600	2 198 053	8 194 009	3 443 748
23	15	6	2009	$7.28 \cdot 10^4$	0.16	29	0.91	10 445 753	102 250	87 000	3 428 033	18 450 512	6 871 782
31	23	7	2010	$3.63 \cdot 10^5$	0.12	39	0.67	19 246 770	137 833	116 000	5 292 887	37 443 449	12 164 669
32	23	7	2010	$1.60 \cdot 10^6$	0.04	39	0.20	24 919 639	137 833	116 000	6 876 080	43 116 318	13 747 862
6	3	3	2006	$1.48 \cdot 10^3$	0.18	18	0.82	1 516 976	268 167	52 200	661 444	1 153 424	574 752
10	6	4	2007	$6.34 \cdot 10^3$	0.18	28	0.90	3 574 115	235 583	81 200	1 548 136	4 410 755	2 122 888
16	10	5	2008	$3.24 \cdot 10^4$	0.18	38	0.95	9 623 932	213 150	111 650	3 719 981	13 709 888	5 842 869
24	16	6	2009	$1.38 \cdot 10^5$	0.18	48	0.97	21 075 040	178 938	142 100	6 938 488	34 463 890	12 781 357
34	24	7	2010	$1.57 \cdot 10^6$	0.05	58	0.28	34 243 390	137 833	171 100	9 462 791	68 398 347	22 244 148
33	24	7	2010	$5.23 \cdot 10^5$	0.13	58	0.65	26 837 217	137 833	171 100	7 395 864	60 992 174	20 177 221

**Table A.9.:** Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, mandatory coverage, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	6	0.53	65 657	200 000	17 400	−159 786	−151 743	−159 786
2	1	2	2005	$1.22 \cdot 10^2$	1.00	14	0.82	249 981	240 600	40 600	−55 096	−182 961	−214 882
3	2	3	2006	$6.02 \cdot 10^2$	1.00	24	0.96	726 014	268 167	69 600	193 643	205 286	−21 239
4	3	4	2007	$2.98 \cdot 10^3$	1.00	34	0.99	1 862 121	235 583	98 600	714 130	1 733 224	692 891
5	4	5	2008	$1.48 \cdot 10^4$	1.00	44	1.00	4 626 532	203 000	127 600	1 710 123	6 029 156	2 403 014
6	5	6	2009	$7.31 \cdot 10^4$	1.00	54	1.00	11 458 443	170 417	156 600	3 716 478	17 160 582	6 119 491
7	6	7	2010	$3.62 \cdot 10^5$	1.00	64	0.97	27 586 205	137 833	185 600	7 600 846	44 423 353	13 720 337

**Table A.10.:** Results of Berlin planning scenario using the expected traffic evolution, a construction limit of 10 and mandatory coverage

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	6	0.53	65 657	200 000	17 400	−159 786	−151 743	−159 786
2	1	2	2005	$1.22 \cdot 10^2$	1.00	14	0.82	249 981	240 600	40 600	−55 096	−182 961	−214 882
3	2	3	2006	$6.02 \cdot 10^2$	1.00	24	0.96	726 014	268 167	69 600	193 643	205 286	−21 239
4	3	4	2007	$2.98 \cdot 10^3$	1.00	34	0.99	1 862 121	235 583	98 600	714 130	1 733 224	692 891
5	4	5	2008	$1.48 \cdot 10^4$	1.00	44	1.00	4 626 532	203 000	127 600	1 710 123	6 029 156	2 403 014
6	5	6	2009	$7.31 \cdot 10^4$	1.00	54	1.00	11 458 443	170 417	156 600	3 716 478	17 160 582	6 119 491
7	6	7	2010	$3.62 \cdot 10^5$	1.00	64	0.97	27 586 205	137 833	185 600	7 600 846	44 423 353	13 720 337

**Table A.11.:** Results of Berlin planning scenario using the expected traffic evolution, a construction limit of 10

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.57	164 314	4 000 000	348 000	-4 153 071	-4 183 686	-4 153 071
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.57	187 692	0	348 000	-111 325	-4 343 994	-4 264 397
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.57	322 118	0	348 000	-14 978	-4 369 876	-4 279 374
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.57	833 845	0	348 000	234 300	-3 884 031	-4 045 074
11	7	5	2008	$1.07 \cdot 10^4$	0.09	76	0.70	5 665 424	2 192 400	661 200	953 794	-1 072 207	-3 091 280
17	11	6	2009	$5.64 \cdot 10^4$	0.09	119	0.80	16 975 733	2 198 375	1 035 300	4 454 941	12 669 851	1 363 661
25	17	7	2010	$1.50 \cdot 10^5$	0.09	120	0.79	22 198 505	41 350	1 044 000	5 889 986	33 783 006	7 253 647
12	7	5	2008	$3.79 \cdot 10^3$	0.20	40	0.55	1 564 487	0	348 000	488 879	-2 667 543	-3 556 195
19	12	6	2009	$9.58 \cdot 10^3$	0.13	41	0.52	1 884 431	51 125	356 700	491 088	-1 190 938	-3 065 107
27	19	7	2010	$7.50 \cdot 10^4$	0.13	70	0.67	9 479 783	1 199 150	609 000	2 074 080	6 480 695	-991 028
18	12	6	2009	$3.76 \cdot 10^3$	0.07	40	0.55	777 265	0	348 000	143 760	-2 238 279	-3 412 435
26	18	7	2010	$1.12 \cdot 10^4$	0.07	40	0.51	1 078 398	0	348 000	203 841	-1 507 880	-3 208 595
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.57	375 877	0	348 000	19 359	-4 155 809	-4 133 713
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.57	1 016 753	0	348 000	387 010	-3 487 057	-3 746 703
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 765 495	0	348 000	683 592	-2 069 561	-3 063 112
14	8	5	2008	$6.56 \cdot 10^3$	0.24	46	0.56	2 781 919	365 400	400 200	780 944	-53 242	-2 282 168
21	14	6	2009	$1.46 \cdot 10^4$	0.07	49	0.57	3 140 631	153 375	426 300	847 386	2 507 714	-1 434 781
29	21	7	2010	$4.73 \cdot 10^4$	0.07	49	0.57	5 044 430	0	426 300	1 288 835	7 125 844	-145 946
22	14	6	2009	$4.28 \cdot 10^4$	0.17	115	0.80	12 867 855	3 527 625	1 000 500	2 556 680	8 286 488	274 512
30	22	7	2010	$2.47 \cdot 10^5$	0.17	163	0.82	38 455 049	1 984 800	1 418 100	9 671 627	43 338 637	9 946 140
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 148 836	0	348 000	321 838	-1 268 725	-2 741 273
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 970 733	0	348 000	543 450	354 008	-2 197 823
28	20	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 726 738	41 350	356 700	926 668	3 682 696	-1 271 156
9	5	4	2007	$4.35 \cdot 10^3$	0.16	57	0.65	4 261 512	1 201 475	495 900	1 120 680	-922 919	-2 626 023
15	9	5	2008	$1.93 \cdot 10^4$	0.16	142	0.84	12 197 271	5 176 500	1 235 400	1 908 947	4 862 452	-717 076
23	15	6	2009	$7.28 \cdot 10^4$	0.16	242	0.91	24 840 456	5 112 500	2 105 400	5 559 325	22 485 007	4 842 249
31	23	7	2010	$3.63 \cdot 10^5$	0.12	269	0.87	59 650 728	1 116 450	2 340 300	15 620 392	78 678 985	20 462 641
32	23	7	2010	$1.60 \cdot 10^6$	0.04	342	0.52	157 597 838	4 135 000	2 975 400	41 767 482	172 972 446	46 609 731
6	3	3	2006	$1.48 \cdot 10^3$	0.18	41	0.58	2 591 040	80 450	356 700	1 237 153	-2 001 920	-2 896 560
10	6	4	2007	$6.34 \cdot 10^3$	0.18	84	0.74	7 093 357	3 039 025	730 800	1 309 668	1 321 612	-1 586 893
16	10	5	2008	$3.24 \cdot 10^4$	0.18	172	0.87	21 282 161	5 359 200	1 496 400	5 366 963	15 748 173	3 780 070
24	16	6	2009	$1.38 \cdot 10^5$	0.18	272	0.92	47 734 497	5 112 500	2 366 400	13 139 085	56 003 770	16 919 155
34	24	7	2010	$1.57 \cdot 10^6$	0.05	372	0.55	162 316 321	4 135 000	3 236 400	43 011 483	210 948 691	59 930 638
33	24	7	2010	$5.23 \cdot 10^5$	0.13	327	0.85	84 050 574	2 274 250	2 844 900	21 901 371	134 935 194	38 820 527

**Table A.12.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, mandatory coverage, and optimization for expected profit

A. Tables with Detailed Results

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.57	164 129	4 000 000	348 000	-4 153 225	-4 183 871	-4 153 225
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.57	187 481	0	348 000	-111 472	-4 344 390	-4 264 697
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.57	321 756	0	348 000	-15 188	-4 370 634	-4 279 885
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.57	832 907	0	348 000	233 848	-3 885 727	-4 046 037
11	7	5	2008	$1.07 \cdot 10^4$	0.09	76	0.70	5 665 424	2 192 400	661 200	953 794	-1 073 902	-3 092 243
17	11	6	2009	$5.64 \cdot 10^4$	0.09	119	0.80	16 975 733	2 198 375	1 035 300	4 454 941	12 668 155	1 362 698
25	17	7	2010	$1.50 \cdot 10^5$	0.09	120	0.79	22 198 505	41 350	1 044 000	5 889 986	33 781 310	7 252 684
12	7	5	2008	$3.79 \cdot 10^3$	0.20	40	0.55	1 560 771	0	348 000	487 385	-2 672 956	-3 558 651
19	12	6	2009	$9.58 \cdot 10^3$	0.13	41	0.52	1 880 992	51 125	356 700	489 936	-1 199 789	-3 068 715
27	19	7	2010	$7.50 \cdot 10^4$	0.13	70	0.67	9 489 175	1 199 150	609 000	2 076 701	6 481 236	-992 014
18	12	6	2009	$3.76 \cdot 10^3$	0.07	40	0.55	775 420	0	348 000	143 142	-2 245 536	-3 415 509
26	18	7	2010	$1.12 \cdot 10^4$	0.07	40	0.51	1 072 733	0	348 000	202 260	-1 520 803	-3 213 250
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.57	375 454	0	348 000	19 065	-4 156 417	-4 134 160
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.57	1 015 609	0	348 000	386 348	-3 488 807	-3 747 812
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 763 503	0	348 000	682 631	-2 073 305	-3 065 182
14	8	5	2008	$6.56 \cdot 10^3$	0.24	49	0.58	2 867 308	548 100	426 300	716 663	-180 397	-2 348 518
21	14	6	2009	$1.46 \cdot 10^4$	0.07	50	0.58	3 174 385	51 125	435 000	896 869	2 507 863	-1 451 650
29	21	7	2010	$4.73 \cdot 10^4$	0.07	50	0.57	5 106 051	0	435 000	1 303 605	7 178 914	-148 045
22	14	6	2009	$4.28 \cdot 10^4$	0.17	109	0.79	12 683 298	3 067 500	948 300	2 697 268	8 487 101	348 749
30	22	7	2010	$2.47 \cdot 10^5$	0.17	163	0.82	38 450 249	2 232 900	1 418 100	9 587 199	43 286 350	9 935 949
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 147 527	0	348 000	321 312	-1 273 778	-2 743 870
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 969 464	0	348 000	543 025	347 686	-2 200 845
28	20	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 729 886	41 350	356 700	927 546	3 679 523	-1 273 298
9	5	4	2007	$4.35 \cdot 10^3$	0.16	99	0.80	5 244 309	4 169 825	861 300	-299 374	-3 275 623	-4 047 186
15	9	5	2008	$1.93 \cdot 10^4$	0.16	159	0.86	12 477 776	3 654 000	1 383 300	2 696 468	4 164 853	-1 350 717
23	15	6	2009	$7.28 \cdot 10^4$	0.16	219	0.89	24 380 705	3 067 500	1 905 300	6 294 208	23 572 758	4 943 491
31	23	7	2010	$3.63 \cdot 10^5$	0.12	252	0.86	58 769 916	1 364 550	2 192 400	15 332 761	78 785 724	20 276 252
32	23	7	2010	$1.60 \cdot 10^6$	0.04	279	0.46	139 270 151	2 481 000	2 427 300	37 359 446	157 934 609	42 302 937
6	3	3	2006	$1.48 \cdot 10^3$	0.18	61	0.69	3 094 281	1 689 450	530 700	310 325	-3 282 286	-3 823 836
10	6	4	2007	$6.34 \cdot 10^3$	0.18	121	0.83	7 918 072	4 240 500	1 052 700	856 854	-657 414	-2 966 982
16	10	5	2008	$3.24 \cdot 10^4$	0.18	181	0.88	21 468 085	3 654 000	1 574 700	6 232 553	15 581 972	3 265 571
24	16	6	2009	$1.38 \cdot 10^5$	0.18	241	0.90	46 933 306	3 067 500	2 096 700	13 782 929	57 351 078	17 048 500
34	24	7	2010	$1.57 \cdot 10^6$	0.05	301	0.49	145 029 090	2 481 000	2 618 700	38 913 244	197 280 468	55 961 744
33	24	7	2010	$5.23 \cdot 10^5$	0.13	301	0.84	82 633 076	2 481 000	2 618 700	21 499 662	134 884 454	38 548 162

**Table A.13.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, mandatory coverage, and optimization for expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	165 478	4 000 000	348 000	-4 152 101	-4 182 522	-4 152 101
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	189 022	0	348 000	-110 402	-4 341 500	-4 262 503
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	324 400	0	348 000	-13 657	-4 365 100	-4 276 160
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	839 752	0	348 000	237 149	-3 873 348	-4 039 011
11	7	5	2008	$1.07 \cdot 10^4$	0.09	76	0.71	5 667 060	2 192 400	661 200	954 451	-1 059 888	-3 084 560
17	11	6	2009	$5.64 \cdot 10^4$	0.09	120	0.80	16 980 117	2 249 500	1 044 000	4 432 950	12 626 730	1 348 390
25	17	7	2010	$1.50 \cdot 10^5$	0.09	121	0.79	22 258 213	41 350	1 052 700	5 904 222	33 790 893	7 252 611
12	7	5	2008	$3.79 \cdot 10^3$	0.20	40	0.55	1 578 496	0	348 000	494 509	-2 642 851	-3 544 502
19	12	6	2009	$9.58 \cdot 10^3$	0.13	40	0.52	1 873 628	0	348 000	510 930	-1 117 223	-3 033 573
27	19	7	2010	$7.50 \cdot 10^4$	0.13	70	0.67	9 499 766	1 240 500	609 000	2 065 809	6 533 043	-967 764
18	12	6	2009	$3.76 \cdot 10^3$	0.07	40	0.55	784 219	0	348 000	146 089	-2 206 632	-3 398 414
26	18	7	2010	$1.12 \cdot 10^4$	0.07	40	0.51	1 079 861	0	348 000	204 249	-1 474 771	-3 194 165
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	378 540	0	348 000	21 208	-4 151 982	-4 130 893
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 023 956	0	348 000	391 178	-3 476 026	-3 739 715
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 778 050	0	348 000	689 646	-2 045 976	-3 050 069
14	8	5	2008	$6.56 \cdot 10^3$	0.24	47	0.57	2 804 018	426 300	408 900	756 960	-77 158	-2 293 109
21	14	6	2009	$1.46 \cdot 10^4$	0.07	49	0.57	3 142 341	102 250	426 300	868 505	2 536 633	-1 424 605
29	21	7	2010	$4.73 \cdot 10^4$	0.07	49	0.57	5 052 498	0	426 300	1 291 087	7 162 831	-133 518
22	14	6	2009	$4.28 \cdot 10^4$	0.17	115	0.75	12 110 908	3 476 500	1 000 500	2 323 726	7 556 750	30 616
30	22	7	2010	$2.47 \cdot 10^5$	0.17	215	0.85	39 580 619	4 135 000	1 870 500	9 139 399	41 131 869	9 170 015
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 153 071	0	348 000	323 540	-1 240 905	-2 726 529
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 986 714	0	348 000	548 802	397 809	-2 177 727
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 715 859	0	348 000	939 908	3 765 668	-1 237 820
9	5	4	2007	$4.35 \cdot 10^3$	0.16	135	0.74	4 828 796	6 714 125	1 174 500	-2 123 194	-6 535 856	-5 862 909
15	9	5	2008	$1.93 \cdot 10^4$	0.16	235	0.81	11 696 989	6 090 000	2 044 500	942 197	-2 973 367	-4 920 711
23	15	6	2009	$7.28 \cdot 10^4$	0.16	335	0.90	24 798 902	5 112 500	2 914 500	5 274 443	13 798 535	353 731
31	23	7	2010	$3.63 \cdot 10^5$	0.12	348	0.89	60 930 045	537 550	3 027 600	15 979 485	71 163 430	16 333 217
32	23	7	2010	$1.60 \cdot 10^6$	0.04	435	0.54	161 189 457	4 135 000	3 784 500	42 544 032	167 068 493	42 897 763
6	3	3	2006	$1.48 \cdot 10^3$	0.18	138	0.73	3 244 694	7 884 100	1 200 600	-4 292 145	-9 991 988	-8 423 038
10	6	4	2007	$6.34 \cdot 10^3$	0.18	238	0.80	7 613 762	7 067 500	2 070 600	-1 416 782	-11 516 326	-9 839 820
16	10	5	2008	$3.24 \cdot 10^4$	0.18	337	0.85	20 697 054	6 029 100	2 931 900	4 231 865	219 728	-5 607 955
24	16	6	2009	$1.38 \cdot 10^5$	0.18	437	0.92	47 884 445	5 112 500	3 801 900	12 708 556	39 189 773	7 100 601
34	24	7	2010	$1.57 \cdot 10^6$	0.05	468	0.57	169 483 185	1 281 850	4 071 600	45 734 049	203 319 508	52 834 650
33	24	7	2010	$5.23 \cdot 10^5$	0.13	446	0.87	85 920 335	372 150	3 880 200	22 771 264	120 857 758	29 871 865

**Table A.14.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R

A. Tables with Detailed Results

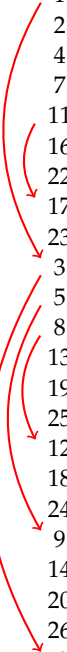
node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	165 214	4 000 000	348 000	-4 152 322	-4 182 786	-4 152 322
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	188 720	0	348 000	-110 611	-4 342 066	-4 262 933
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	323 882	0	348 000	-13 957	-4 366 184	-4 276 890
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	838 411	0	348 000	236 502	-3 875 773	-4 040 388
11	7	5	2008	$1.07 \cdot 10^4$	0.09	75	0.70	5 636 540	2 131 500	652 500	975 051	-1 023 234	-3 065 337
17	11	6	2009	$5.64 \cdot 10^4$	0.09	121	0.80	17 006 716	2 351 750	1 052 700	4 397 852	12 579 033	1 332 516
25	17	7	2010	$1.50 \cdot 10^5$	0.09	122	0.79	22 275 377	41 350	1 061 400	5 906 584	33 751 660	7 239 099
12	7	5	2008	$3.79 \cdot 10^3$	0.20	40	0.55	1 573 603	0	348 000	492 542	-2 650 171	-3 547 846
19	12	6	2009	$9.58 \cdot 10^3$	0.13	40	0.52	1 876 461	0	348 000	511 878	-1 121 710	-3 035 967
27	19	7	2010	$7.50 \cdot 10^4$	0.13	69	0.67	9 447 841	1 199 150	600 300	2 067 593	6 526 681	-968 374
18	12	6	2009	$3.76 \cdot 10^3$	0.07	40	0.55	781 790	0	348 000	145 275	-2 216 381	-3 402 570
26	18	7	2010	$1.12 \cdot 10^4$	0.07	40	0.51	1 086 659	0	348 000	206 146	-1 477 722	-3 196 424
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	377 935	0	348 000	20 788	-4 152 851	-4 131 534
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 022 320	0	348 000	390 232	-3 478 531	-3 741 302
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 775 971	0	348 000	688 643	-2 050 560	-3 052 659
14	8	5	2008	$6.56 \cdot 10^3$	0.24	47	0.57	2 801 396	426 300	408 900	755 906	-84 364	-2 296 752
21	14	6	2009	$1.46 \cdot 10^4$	0.07	49	0.57	3 130 857	102 250	426 300	864 659	2 517 943	-1 432 094
29	21	7	2010	$4.73 \cdot 10^4$	0.07	49	0.57	5 079 089	0	426 300	1 298 508	7 170 732	-133 586
22	14	6	2009	$4.28 \cdot 10^4$	0.17	107	0.71	11 385 694	3 067 500	930 900	2 268 530	7 302 930	-28 223
30	22	7	2010	$2.47 \cdot 10^5$	0.17	167	0.78	36 200 409	2 481 000	1 452 900	8 866 510	39 569 439	8 838 288
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 152 476	0	348 000	323 301	-1 246 084	-2 729 358
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.53	1 995 144	0	348 000	551 625	401 060	-2 177 732
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 723 359	0	348 000	942 001	3 776 419	-1 235 732
9	5	4	2007	$4.35 \cdot 10^3$	0.16	100	0.68	4 448 627	4 240 500	870 000	-728 189	-4 140 404	-4 469 491
15	9	5	2008	$1.93 \cdot 10^4$	0.16	159	0.72	10 447 622	3 593 100	1 383 300	1 909 964	1 330 818	-2 559 527
23	15	6	2009	$7.28 \cdot 10^4$	0.16	218	0.79	21 562 370	3 016 375	1 896 600	5 373 813	17 980 212	2 814 286
31	23	7	2010	$3.63 \cdot 10^5$	0.12	278	0.84	57 247 222	2 481 000	2 418 600	14 470 780	70 327 834	17 285 066
32	23	7	2010	$1.60 \cdot 10^6$	0.04	278	0.38	114 859 082	2 481 000	2 418 600	30 549 193	127 939 694	33 363 479
6	3	3	2006	$1.48 \cdot 10^3$	0.18	100	0.69	3 092 113	4 827 000	870 000	-2 066 138	-6 757 738	-6 197 672
10	6	4	2007	$6.34 \cdot 10^3$	0.18	160	0.75	7 184 966	4 240 500	1 392 000	339 683	-5 205 272	-5 857 989
16	10	5	2008	$3.24 \cdot 10^4$	0.18	220	0.78	19 110 479	3 654 000	1 914 000	5 148 726	8 337 207	-709 263
24	16	6	2009	$1.38 \cdot 10^5$	0.18	280	0.83	42 984 913	3 067 500	2 436 000	12 346 990	45 818 621	11 637 727
34	24	7	2010	$1.57 \cdot 10^6$	0.05	340	0.46	135 714 871	2 481 000	2 958 000	36 219 124	176 094 492	47 856 851
33	24	7	2010	$5.23 \cdot 10^5$	0.13	340	0.79	77 419 500	2 481 000	2 958 000	19 949 956	117 799 121	31 587 683

**Table A.15.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	164 940	4 000 000	348 000	-4 152 550	-4 183 060	-4 152 550
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	188 406	0	348 000	-110 829	-4 342 654	-4 263 379
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	323 345	0	348 000	-14 268	-4 367 309	-4 277 647
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	837 020	0	348 000	235 831	-3 878 289	-4 041 816
11	7	5	2008	$3.79 \cdot 10^3$	0.29	40	0.55	1 570 684	0	348 000	491 369	-2 655 605	-3 550 446
16	11	6	2009	$5.64 \cdot 10^4$	0.09	140	0.83	17 628 064	5 112 500	1 218 000	3 441 098	8 641 960	-109 348
22	16	7	2010	$3.00 \cdot 10^5$	0.09	200	0.84	47 358 034	2 481 000	1 740 000	11 900 274	51 778 994	11 790 926
17	11	6	2009	$1.50 \cdot 10^4$	0.20	44	0.54	3 052 347	204 500	382 800	811 842	-190 558	-2 738 604
23	17	7	2010	$3.89 \cdot 10^4$	0.20	44	0.54	3 976 106	0	382 800	1 002 826	3 402 748	-1 735 779
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	377 308	0	348 000	20 353	-4 153 752	-4 132 197
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 020 624	0	348 000	389 250	-3 481 128	-3 742 947
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 772 243	0	348 000	686 846	-2 056 885	-3 056 102
13	8	5	2008	$6.56 \cdot 10^3$	0.24	48	0.57	2 838 834	487 200	417 600	738 086	-122 852	-2 318 016
19	13	6	2009	$4.28 \cdot 10^4$	0.24	116	0.80	12 897 164	3 476 500	1 009 200	2 584 128	8 288 613	266 112
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 361 032	1 116 450	1 244 100	7 752 105	36 289 095	8 018 216
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 149 278	0	348 000	322 016	-1 255 608	-2 734 086
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 973 709	0	348 000	544 447	370 101	-2 189 640
24	18	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 747 180	41 350	356 700	932 373	3 719 231	-1 257 267
9	5	4	2007	$4.35 \cdot 10^3$	0.16	56	0.64	4 227 466	1 130 800	487 200	1 149 357	-871 662	-2 593 591
14	9	5	2008	$1.93 \cdot 10^4$	0.16	109	0.79	11 441 003	3 227 700	948 300	2 660 214	6 393 341	66 623
20	14	6	2009	$7.28 \cdot 10^4$	0.16	160	0.86	23 534 570	2 607 375	1 392 000	6 367 656	25 928 536	6 434 279
26	20	7	2010	$3.63 \cdot 10^5$	0.16	241	0.85	58 231 412	3 349 350	2 096 700	14 544 477	78 713 898	20 978 757
6	3	3	2006	$1.48 \cdot 10^3$	0.18	40	0.58	2 563 692	0	348 000	1 282 229	-1 938 061	-2 849 969
10	6	4	2007	$6.34 \cdot 10^3$	0.18	84	0.74	7 095 401	3 109 700	730 800	1 269 754	1 316 840	-1 580 215
15	10	5	2008	$3.24 \cdot 10^4$	0.18	149	0.85	20 677 654	3 958 500	1 296 300	5 879 932	16 739 694	4 299 718
21	15	6	2009	$1.38 \cdot 10^5$	0.18	225	0.90	46 835 239	3 885 500	1 957 500	13 467 969	57 731 933	17 767 686
27	21	7	2010	$5.23 \cdot 10^5$	0.18	312	0.84	83 287 785	3 597 450	2 714 400	21 281 774	134 707 868	39 049 461

**Table A.16.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, mandatory coverage, and optimization for expected profit

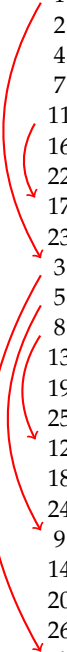


node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	165 346	4 000 000	348 000	-4 152 212	-4 182 654	-4 152 212
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	188 870	0	348 000	-110 507	-4 341 784	-4 262 718
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	324 141	0	348 000	-13 807	-4 365 643	-4 276 526
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	839 080	0	348 000	236 825	-3 874 563	-4 039 701
11	7	5	2008	$3.79 \cdot 10^3$	0.29	42	0.57	1 617 980	121 800	365 400	444 645	-2 743 783	-3 595 055
16	11	6	2009	$5.64 \cdot 10^4$	0.09	102	0.76	16 165 950	3 067 500	887 400	3 883 996	9 467 268	288 941
22	16	7	2010	$3.00 \cdot 10^5$	0.09	162	0.79	44 747 393	2 481 000	1 409 400	11 263 957	50 324 261	11 552 897
17	11	6	2009	$1.50 \cdot 10^4$	0.20	44	0.54	3 047 789	102 250	382 800	851 407	-181 044	-2 743 648
23	17	7	2010	$3.89 \cdot 10^4$	0.20	44	0.54	3 973 997	0	382 800	1 002 237	3 410 153	-1 741 411
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	378 237	0	348 000	20 998	-4 152 417	-4 131 214
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 023 136	0	348 000	390 704	-3 477 281	-3 740 510
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 776 622	0	348 000	688 957	-2 048 659	-3 051 553
13	8	5	2008	$6.56 \cdot 10^3$	0.24	53	0.60	2 990 898	791 700	461 100	634 869	-310 562	-2 416 684
19	13	6	2009	$4.28 \cdot 10^4$	0.24	113	0.79	12 799 660	3 067 500	983 100	2 724 583	8 438 498	307 899
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 334 125	1 240 500	1 244 100	7 703 051	36 288 023	8 010 950
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 152 155	0	348 000	323 172	-1 244 505	-2 728 381
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 976 926	0	348 000	545 524	384 422	-2 182 857
24	18	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 744 146	41 350	356 700	931 526	3 730 518	-1 251 331
9	5	4	2007	$4.35 \cdot 10^3$	0.16	57	0.65	4 263 439	1 201 475	495 900	1 121 609	-911 217	-2 618 901
14	9	5	2008	$1.93 \cdot 10^4$	0.16	112	0.79	11 526 897	3 349 500	974 400	2 625 505	6 291 780	6 604
20	14	6	2009	$7.28 \cdot 10^4$	0.16	172	0.87	23 843 697	3 067 500	1 496 400	6 251 305	25 571 576	6 257 909
26	20	7	2010	$3.63 \cdot 10^5$	0.16	232	0.84	57 678 224	2 481 000	2 018 400	14 702 753	78 750 400	20 960 663
6	3	3	2006	$1.48 \cdot 10^3$	0.18	41	0.58	2 603 524	80 450	356 700	1 244 377	-1 986 044	-2 886 837
10	6	4	2007	$6.34 \cdot 10^3$	0.18	101	0.79	7 518 191	4 240 500	878 700	747 922	412 948	-2 138 915
15	10	5	2008	$3.24 \cdot 10^4$	0.18	161	0.86	20 997 853	3 654 000	1 400 700	6 113 503	16 356 100	3 974 589
21	15	6	2009	$1.38 \cdot 10^5$	0.18	221	0.90	46 578 055	3 067 500	1 922 700	13 722 229	57 943 955	17 696 817
27	21	7	2010	$5.23 \cdot 10^5$	0.18	281	0.82	81 142 413	2 481 000	2 444 700	21 132 206	134 160 668	38 829 023

**Table A.17.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, mandatory coverage, and optimization for expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	165 850	4 000 000	348 000	-4 151 792	-4 182 150	-4 151 792
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	189 446	0	348 000	-110 107	-4 340 705	-4 261 899
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	325 128	0	348 000	-13 236	-4 363 577	-4 275 135
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	841 636	0	348 000	238 058	-3 869 940	-4 037 078
11	7	5	2008	$3.79 \cdot 10^3$	0.29	40	0.55	1 581 570	0	348 000	495 744	-2 636 370	-3 541 333
16	11	6	2009	$5.64 \cdot 10^4$	0.09	139	0.74	15 791 381	5 061 375	1 209 300	2 849 456	6 884 336	-691 877
22	16	7	2010	$3.00 \cdot 10^5$	0.09	239	0.83	46 804 200	4 135 000	2 079 300	11 097 096	47 474 236	10 405 219
17	11	6	2009	$1.50 \cdot 10^4$	0.20	43	0.53	3 018 138	153 375	374 100	823 845	-145 707	-2 717 488
23	17	7	2010	$3.89 \cdot 10^4$	0.20	43	0.54	3 923 432	0	374 100	990 553	3 403 625	-1 726 935
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	379 389	0	348 000	21 798	-4 150 762	-4 129 994
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 026 253	0	348 000	392 508	-3 472 509	-3 737 487
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 782 054	0	348 000	691 577	-2 038 455	-3 045 910
13	8	5	2008	$6.56 \cdot 10^3$	0.24	48	0.57	2 830 610	487 200	417 600	734 781	-112 644	-2 311 129
19	13	6	2009	$4.28 \cdot 10^4$	0.24	117	0.80	12 903 413	3 527 625	1 017 900	2 562 761	8 245 244	251 632
25	19	7	2010	$1.99 \cdot 10^5$	0.24	144	0.81	30 380 710	1 116 450	1 252 800	7 755 168	36 256 704	8 006 801
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 155 724	0	348 000	324 606	-1 230 731	-2 721 304
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 978 502	0	348 000	546 052	399 771	-2 175 252
24	18	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 755 211	41 350	356 700	934 614	3 756 932	-1 240 638
9	5	4	2007	$4.35 \cdot 10^3$	0.16	140	0.74	4 840 860	7 067 500	1 218 000	-2 342 853	-6 917 149	-6 080 340
14	9	5	2008	$1.93 \cdot 10^4$	0.16	240	0.81	11 825 380	6 090 000	2 088 000	976 313	-3 269 769	-5 104 026
20	14	6	2009	$7.28 \cdot 10^4$	0.16	340	0.88	24 127 762	5 112 500	2 958 000	5 035 111	12 787 493	-68 915
26	20	7	2010	$3.63 \cdot 10^5$	0.16	439	0.90	61 410 543	4 093 650	3 819 300	14 701 704	66 285 086	14 632 789
6	3	3	2006	$1.48 \cdot 10^3$	0.18	139	0.73	3 250 096	7 964 550	1 209 300	-4 349 921	-10 074 516	-8 479 915
10	6	4	2007	$6.34 \cdot 10^3$	0.18	239	0.81	7 695 825	7 067 500	2 079 300	-1 381 402	-11 525 491	-9 861 318
15	10	5	2008	$3.24 \cdot 10^4$	0.18	339	0.86	20 953 232	6 090 000	2 949 300	4 298 455	388 441	-5 562 862
21	15	6	2009	$1.38 \cdot 10^5$	0.18	439	0.92	47 721 129	5 112 500	3 819 300	12 648 035	39 177 770	7 085 172
27	21	7	2010	$5.23 \cdot 10^5$	0.18	461	0.87	85 335 545	909 700	4 010 700	22 391 615	119 592 915	29 476 787

**Table A.18.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	40	0.58	165 781	4 000 000	348 000	-4 151 849	-4 182 219	-4 151 849
2	1	2	2005	$5.43 \cdot 10^1$	0.29	40	0.58	189 368	0	348 000	-110 161	-4 340 851	-4 262 010
4	2	3	2006	$1.86 \cdot 10^2$	0.29	40	0.58	324 994	0	348 000	-13 313	-4 363 857	-4 275 324
7	4	4	2007	$9.64 \cdot 10^2$	0.29	40	0.58	841 290	0	348 000	237 891	-3 870 567	-4 037 433
11	7	5	2008	$3.79 \cdot 10^3$	0.29	40	0.55	1 579 080	0	348 000	494 743	-2 639 487	-3 542 690
16	11	6	2009	$5.64 \cdot 10^4$	0.09	99	0.68	14 390 151	3 016 375	861 300	3 318 571	7 872 989	-224 118
22	16	7	2010	$3.00 \cdot 10^5$	0.09	159	0.74	41 685 867	2 481 000	1 383 300	10 416 825	45 694 555	10 192 706
17	11	6	2009	$1.50 \cdot 10^4$	0.20	43	0.53	3 016 076	153 375	374 100	823 154	-150 886	-2 719 535
23	17	7	2010	$3.89 \cdot 10^4$	0.20	43	0.53	3 918 962	0	374 100	989 306	3 393 976	-1 730 229
3	1	2	2005	$1.09 \cdot 10^2$	0.70	40	0.58	379 233	0	348 000	21 689	-4 150 986	-4 130 160
5	3	3	2006	$5.88 \cdot 10^2$	0.52	40	0.58	1 025 831	0	348 000	392 263	-3 473 155	-3 737 896
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 781 318	0	348 000	691 222	-2 039 837	-3 046 674
13	8	5	2008	$6.56 \cdot 10^3$	0.24	52	0.60	2 962 357	730 800	452 400	656 265	-260 680	-2 390 409
19	13	6	2009	$4.28 \cdot 10^4$	0.24	112	0.79	12 769 146	3 067 500	974 400	2 717 277	8 466 565	326 868
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 333 037	1 281 850	1 244 100	7 688 899	36 273 652	8 015 767
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 155 240	0	348 000	324 412	-1 232 597	-2 722 263
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 972 342	0	348 000	543 989	391 744	-2 178 274
24	18	7	2010	$3.82 \cdot 10^4$	0.12	41	0.52	3 738 799	41 350	356 700	930 034	3 732 493	-1 248 241
9	5	4	2007	$4.35 \cdot 10^3$	0.16	99	0.68	4 427 575	4 169 825	861 300	-693 246	-4 076 706	-4 431 143
14	9	5	2008	$1.93 \cdot 10^4$	0.16	158	0.73	10 525 161	3 593 100	1 374 600	1 944 622	1 480 755	-2 486 521
20	14	6	2009	$7.28 \cdot 10^4$	0.16	218	0.79	21 571 271	3 067 500	1 896 600	5 356 248	18 087 926	2 869 727
26	20	7	2010	$3.63 \cdot 10^5$	0.16	278	0.79	53 983 256	2 481 000	2 418 600	13 559 867	67 171 582	16 429 594
6	3	3	2006	$1.48 \cdot 10^3$	0.18	100	0.70	3 104 299	4 827 000	870 000	-2 059 086	-6 743 687	-6 189 246
10	6	4	2007	$6.34 \cdot 10^3$	0.18	160	0.75	7 158 496	4 240 500	1 392 000	326 917	-5 217 691	-5 862 329
15	10	5	2008	$3.24 \cdot 10^4$	0.18	220	0.78	19 003 864	3 654 000	1 914 000	5 105 880	8 218 173	-756 448
21	15	6	2009	$1.38 \cdot 10^5$	0.18	280	0.81	42 354 680	3 067 500	2 436 000	12 135 926	45 069 352	11 379 477
27	21	7	2010	$5.23 \cdot 10^5$	0.18	340	0.76	75 234 213	2 481 000	2 958 000	19 340 083	114 864 565	30 719 560

**Table A.19.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	40	0.58	170 146	4 000 000	348 000	-4 148 212	-4 177 854	-4 148 212
2	1	2	2005	$1.22 \cdot 10^2$	1.00	40	0.58	421 397	0	348 000	50 970	-4 104 457	-4 097 242
3	2	3	2006	$6.02 \cdot 10^2$	1.00	40	0.58	1 043 665	0	348 000	402 584	-3 408 792	-3 694 658
4	3	4	2007	$2.98 \cdot 10^3$	1.00	41	0.57	2 550 341	70 675	356 700	1 016 990	-1 285 826	-2 677 667
5	4	5	2008	$1.48 \cdot 10^4$	1.00	91	0.74	8 282 645	3 045 000	791 700	1 541 982	3 160 119	-1 135 685
6	5	6	2009	$7.31 \cdot 10^4$	1.00	159	0.86	23 581 856	3 476 500	1 383 300	6 037 124	21 882 174	4 901 439
7	6	7	2010	$3.62 \cdot 10^5$	1.00	241	0.85	58 067 368	3 390 700	2 096 700	14 484 848	74 462 143	19 386 286

**Table A.20.:** Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 100, mandatory coverage

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$1.22 \cdot 10^2$	1.00	0	0.00	0	0	0	0	0	0
3	2	3	2006	$6.02 \cdot 10^2$	1.00	0	0.00	0	0	0	0	0	0
4	3	4	2007	$2.98 \cdot 10^3$	1.00	41	0.57	2 550 356	2 897 675	356 700	-618 998	-704 019	-618 998
5	4	5	2008	$1.48 \cdot 10^4$	1.00	89	0.74	8 274 983	2 923 200	774 300	1 604 634	3 873 464	985 636
6	5	6	2009	$7.31 \cdot 10^4$	1.00	156	0.85	23 504 036	3 425 375	1 357 200	6 040 349	22 594 925	7 025 985
7	6	7	2010	$3.62 \cdot 10^5$	1.00	240	0.85	58 028 753	3 473 400	2 088 000	14 448 803	75 062 277	21 474 788

**Table A.21.:** Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 100

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	40	0.57	169 574	4 000 000	348 000	−4 148 688	−4 178 426	−4 148 688
2	1	2	2005	$1.22 \cdot 10^2$	1.00	40	0.57	419 981	0	348 000	49 987	−4 106 445	−4 098 701
3	2	3	2006	$6.02 \cdot 10^2$	1.00	40	0.57	1 040 159	0	348 000	400 555	−3 414 286	−3 698 147
4	3	4	2007	$2.98 \cdot 10^3$	1.00	43	0.58	2 604 880	212 025	374 100	953 101	−1 395 532	−2 745 046
5	4	5	2008	$1.48 \cdot 10^4$	1.00	103	0.78	8 628 552	3 654 000	896 100	1 345 346	2 682 921	−1 399 700
6	5	6	2009	$7.31 \cdot 10^4$	1.00	163	0.86	23 684 163	3 067 500	1 418 100	6 224 100	21 881 483	4 824 400
7	6	7	2010	$3.62 \cdot 10^5$	1.00	223	0.83	56 937 367	2 481 000	1 940 100	14 517 846	74 397 750	19 342 246

**Table A.22.:** Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 60, mandatory coverage

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.45 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$1.22 \cdot 10^2$	1.00	0	0.00	0	0	0	0	0	0
3	2	3	2006	$6.02 \cdot 10^2$	1.00	0	0.00	0	0	0	0	0	0
4	3	4	2007	$2.98 \cdot 10^3$	1.00	42	0.57	2 581 719	2 968 350	365 400	−648 969	−752 031	−648 969
5	4	5	2008	$1.48 \cdot 10^4$	1.00	102	0.78	8 645 923	3 654 000	887 400	1 355 824	3 352 492	706 855
6	5	6	2009	$7.31 \cdot 10^4$	1.00	162	0.86	23 644 214	3 067 500	1 409 400	6 213 635	22 519 807	6 920 490
7	6	7	2010	$3.62 \cdot 10^5$	1.00	222	0.83	56 914 708	2 481 000	1 931 400	14 513 950	75 022 115	21 434 440

**Table A.23.:** Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 60

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.06 \cdot 10^4$	1.00	25	0.33	0	1 196 667	54 375	-1 248 619	-1 251 042	-1 248 619
2	1	2	2005	$1.06 \cdot 10^4$	0.29	50	0.58	966 580	1 115 208	108 750	-282 430	-1 508 420	-1 531 048
4	2	3	2006	$1.42 \cdot 10^4$	0.29	65	0.65	1 223 414	620 250	141 375	377 541	-1 046 631	-1 153 507
7	4	4	2007	$2.00 \cdot 10^4$	0.29	66	0.66	1 473 310	38 092	143 550	1 074 910	245 037	-78 597
11	7	5	2008	$1.33 \cdot 10^4$	0.09	66	0.65	810 236	0	143 550	530 817	911 724	452 220
17	11	6	2009	$1.73 \cdot 10^4$	0.09	66	0.65	893 284	0	143 550	570 342	1 661 458	1 022 563
24	17	7	2010	$2.31 \cdot 10^4$	0.09	66	0.64	990 472	0	143 550	615 569	2 508 380	1 638 131
33	24	8	2011	$2.71 \cdot 10^4$	0.09	66	0.65	985 265	0	143 550	584 525	3 350 096	2 222 656
43	33	9	2012	$5.44 \cdot 10^4$	0.09	66	0.65	1 658 910	0	143 550	1 005 444	4 865 456	3 228 100
12	7	5	2008	$2.85 \cdot 10^4$	0.20	91	0.74	1 984 335	870 833	197 925	696 649	1 160 614	618 052
18	12	6	2009	$5.89 \cdot 10^4$	0.20	113	0.79	3 685 985	694 650	245 775	2 063 974	3 906 173	2 682 027
25	18	7	2010	$8.04 \cdot 10^4$	0.20	116	0.79	4 250 604	84 950	252 300	2 841 463	7 819 527	5 523 490
35	25	8	2011	$6.85 \cdot 10^4$	0.08	116	0.80	3 052 691	0	252 300	1 944 716	10 619 918	7 468 206
45	35	9	2012	$1.05 \cdot 10^5$	0.08	116	0.79	3 897 213	0	252 300	2 418 407	14 264 831	9 886 613
34	25	8	2011	$1.47 \cdot 10^5$	0.12	141	0.82	6 746 306	626 458	306 675	4 016 637	13 632 700	9 540 127
44	34	9	2012	$2.72 \cdot 10^5$	0.12	148	0.78	10 054 465	152 600	321 900	6 351 604	23 212 665	15 891 731
3	1	2	2005	$1.63 \cdot 10^4$	0.70	50	0.57	1 471 489	1 115 208	108 750	178 487	-1 003 511	-1 070 132
5	3	3	2006	$2.67 \cdot 10^4$	0.49	75	0.68	2 410 362	1 033 750	163 125	1 016 351	209 976	-53 781
8	5	4	2007	$1.46 \cdot 10^4$	0.11	76	0.69	1 131 189	38 092	165 300	771 684	1 137 774	717 903
13	8	5	2008	$2.09 \cdot 10^4$	0.11	82	0.72	1 409 971	209 000	178 350	806 453	2 160 395	1 524 356
19	13	6	2009	$2.75 \cdot 10^4$	0.11	88	0.73	1 585 893	189 450	191 400	909 987	3 365 438	2 434 343
26	19	7	2010	$3.66 \cdot 10^4$	0.11	90	0.73	1 786 982	56 633	195 750	1 113 473	4 900 037	3 547 815
36	26	8	2011	$5.83 \cdot 10^4$	0.11	90	0.73	2 395 103	0	195 750	1 527 328	7 099 389	5 075 143
47	36	9	2012	$3.13 \cdot 10^5$	0.03	115	0.68	9 989 663	545 000	250 125	6 083 730	16 293 927	11 158 873
46	36	9	2012	$7.44 \cdot 10^4$	0.08	90	0.73	2 564 841	0	195 750	1 571 896	9 468 480	6 647 040
9	5	4	2007	$3.02 \cdot 10^4$	0.38	96	0.76	2 555 198	799 925	208 800	1 257 640	1 756 449	1 203 859
14	9	5	2008	$2.85 \cdot 10^4$	0.13	97	0.75	2 018 449	34 833	210 975	1 410 087	3 529 090	2 613 947
20	14	6	2009	$4.05 \cdot 10^4$	0.13	98	0.75	2 421 243	31 575	213 150	1 654 613	5 705 608	4 268 560
27	20	7	2010	$4.39 \cdot 10^4$	0.13	98	0.75	2 204 496	0	213 150	1 447 370	7 696 954	5 715 930
37	27	8	2011	$6.67 \cdot 10^4$	0.13	98	0.75	2 817 632	0	213 150	1 808 668	10 301 436	7 524 598
48	37	9	2012	$1.08 \cdot 10^5$	0.13	99	0.75	3 834 872	21 800	215 325	2 386 438	13 899 183	9 911 036
15	9	5	2008	$5.25 \cdot 10^4$	0.25	121	0.80	3 951 725	870 833	263 175	2 211 138	4 574 166	3 414 997
21	15	6	2009	$8.44 \cdot 10^4$	0.25	136	0.83	5 523 960	473 625	295 800	3 600 095	9 328 701	7 015 092
29	21	7	2010	$1.28 \cdot 10^5$	0.17	161	0.85	7 243 621	707 917	350 175	4 471 832	15 514 230	11 486 924
39	29	8	2011	$2.47 \cdot 10^5$	0.17	186	0.85	11 726 726	626 458	404 550	7 407 293	26 209 948	18 894 218
50	39	9	2012	$3.69 \cdot 10^5$	0.17	197	0.80	13 883 376	239 800	428 475	8 760 825	39 425 049	27 655 043
28	21	7	2010	$7.03 \cdot 10^4$	0.08	136	0.82	3 860 001	0	295 800	2 590 568	12 892 903	9 605 660
38	28	8	2011	$8.34 \cdot 10^4$	0.08	136	0.83	3 856 497	0	295 800	2 472 706	16 453 599	12 078 366
49	38	9	2012	$1.10 \cdot 10^5$	0.08	136	0.82	4 267 443	0	295 800	2 635 193	20 425 243	14 713 559
6	3	3	2006	$4.39 \cdot 10^4$	0.21	75	0.68	3 999 155	1 033 750	163 125	2 402 089	1 798 769	1 331 958
10	6	4	2007	$5.94 \cdot 10^4$	0.21	100	0.76	5 053 960	952 292	217 500	3 199 798	5 682 937	4 531 756
16	10	5	2008	$8.50 \cdot 10^4$	0.21	125	0.81	6 466 014	870 833	271 875	4 206 093	11 006 242	8 737 849
22	16	6	2009	$1.12 \cdot 10^5$	0.14	139	0.82	7 338 254	442 050	302 325	5 000 451	17 600 121	13 738 300
30	22	7	2010	$1.32 \cdot 10^5$	0.09	164	0.85	7 446 056	707 917	356 700	4 614 226	23 981 560	18 352 526
40	30	8	2011	$2.25 \cdot 10^5$	0.09	189	0.86	10 805 214	626 458	411 075	6 762 823	33 749 241	25 115 349
51	40	9	2012	$3.69 \cdot 10^5$	0.09	201	0.80	13 944 570	261 600	437 175	8 780 516	46 995 035	33 895 865
31	22	7	2010	$6.05 \cdot 10^4$	0.06	139	0.83	3 355 803	0	302 325	2 219 359	20 653 599	15 957 659
41	31	8	2011	$7.74 \cdot 10^4$	0.06	139	0.82	3 572 198	0	302 325	2 270 745	23 923 472	18 228 405
52	41	9	2012	$1.14 \cdot 10^5$	0.06	139	0.82	4 441 586	0	302 325	2 746 408	28 062 734	20 974 813
23	16	6	2009	$2.37 \cdot 10^5$	0.07	150	0.81	15 152 503	789 375	326 250	10 650 211	25 043 120	19 388 060
32	23	7	2010	$3.62 \cdot 10^5$	0.07	175	0.77	18 655 429	707 917	380 625	12 744 145	42 610 008	32 132 204
42	32	8	2011	$4.77 \cdot 10^5$	0.07	200	0.76	20 181 034	626 458	435 000	13 257 195	61 729 583	45 389 399
53	42	9	2012	$5.26 \cdot 10^5$	0.07	225	0.76	18 950 228	545 000	489 375	11 870 340	79 645 436	57 259 739

**Table A.24.:** Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 100, mandatory coverage, and optimization for the expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.06 \cdot 10^4$	1.00	15	0.19	0	718 000	32 625	-749 171	-750 625	-749 171
2	1	2	2005	$1.06 \cdot 10^4$	0.29	30	0.37	0	669 125	65 250	-698 875	-1 485 000	-1 448 047
4	2	3	2006	$1.42 \cdot 10^4$	0.29	45	0.54	1 025 734	620 250	97 875	243 067	-1 177 391	-1 204 980
7	4	4	2007	$2.00 \cdot 10^4$	0.29	60	0.63	1 417 097	571 375	130 500	573 813	-462 169	-631 167
11	7	5	2008	$1.33 \cdot 10^4$	0.09	60	0.62	774 472	0	130 500	512 732	181 803	-118 435
17	11	6	2009	$1.73 \cdot 10^4$	0.09	60	0.63	857 682	0	130 500	553 186	908 986	434 752
24	17	7	2010	$2.31 \cdot 10^4$	0.09	60	0.62	946 663	0	130 500	593 212	1 725 149	1 027 964
33	24	8	2011	$2.71 \cdot 10^4$	0.09	60	0.62	945 383	0	130 500	565 891	2 540 032	1 593 854
43	33	9	2012	$5.44 \cdot 10^4$	0.09	60	0.62	1 591 673	0	130 500	969 491	4 001 205	2 563 346
12	7	5	2008	$2.85 \cdot 10^4$	0.20	75	0.69	1 853 031	522 500	163 125	910 091	705 237	278 924
18	12	6	2009	$5.89 \cdot 10^4$	0.20	90	0.74	3 457 049	473 625	195 750	2 103 853	3 492 911	2 382 776
25	18	7	2010	$8.04 \cdot 10^4$	0.20	105	0.78	4 150 608	424 750	228 375	2 527 678	6 990 394	4 910 455
35	25	8	2011	$6.85 \cdot 10^4$	0.08	105	0.77	2 967 496	0	228 375	1 902 167	9 729 515	6 812 622
45	35	9	2012	$1.05 \cdot 10^5$	0.08	105	0.77	3 788 472	0	228 375	2 362 131	13 289 612	9 174 753
34	25	8	2011	$1.47 \cdot 10^5$	0.12	120	0.79	6 509 375	375 875	261 000	4 065 952	12 862 894	8 976 407
44	34	9	2012	$2.72 \cdot 10^5$	0.12	135	0.76	9 736 709	327 000	293 625	6 038 421	21 978 978	15 014 828
3	1	2	2005	$1.63 \cdot 10^4$	0.70	30	0.40	0	669 125	65 250	-698 875	-1 485 000	-1 448 047
5	3	3	2006	$2.67 \cdot 10^4$	0.49	45	0.54	1 906 709	620 250	97 875	1 011 449	-296 416	-436 598
8	5	4	2007	$1.46 \cdot 10^4$	0.11	60	0.63	1 027 800	571 375	130 500	249 399	29 509	-187 199
13	8	5	2008	$2.09 \cdot 10^4$	0.11	75	0.70	1 374 954	522 500	163 125	529 445	718 838	342 246
19	13	6	2009	$2.75 \cdot 10^4$	0.11	84	0.72	1 562 812	284 175	182 700	823 626	1 814 775	1 165 872
26	19	7	2010	$3.66 \cdot 10^4$	0.11	87	0.73	1 777 703	84 950	189 225	1 089 930	3 318 303	2 255 801
36	26	8	2011	$5.83 \cdot 10^4$	0.11	88	0.73	2 376 442	25 058	191 400	1 499 177	5 478 286	3 754 979
47	36	9	2012	$3.13 \cdot 10^5$	0.03	103	0.64	9 499 409	327 000	224 025	5 927 152	14 426 671	9 682 131
46	36	9	2012	$7.44 \cdot 10^4$	0.08	88	0.73	2 551 708	0	191 400	1 566 069	7 838 594	5 321 047
9	5	4	2007	$3.02 \cdot 10^4$	0.38	60	0.63	2 119 139	571 375	130 500	1 158 848	1 120 848	722 251
14	9	5	2008	$2.85 \cdot 10^4$	0.13	75	0.69	1 843 285	522 500	163 125	902 331	2 278 508	1 624 582
20	14	6	2009	$4.05 \cdot 10^4$	0.13	85	0.73	2 333 312	315 750	184 875	1 382 970	4 111 195	3 007 552
27	20	7	2010	$4.39 \cdot 10^4$	0.13	85	0.73	2 121 735	0	184 875	1 407 768	6 048 055	4 415 320
37	27	8	2011	$6.67 \cdot 10^4$	0.13	86	0.72	2 694 251	25 058	187 050	1 722 899	8 530 198	6 138 219
48	37	9	2012	$1.08 \cdot 10^5$	0.13	86	0.72	3 669 977	0	187 050	2 310 929	12 013 125	8 449 148
15	9	5	2008	$5.25 \cdot 10^4$	0.25	75	0.69	3 388 364	522 500	163 125	2 132 526	3 823 587	2 854 777
21	15	6	2009	$8.44 \cdot 10^4$	0.25	90	0.74	4 961 671	473 625	195 750	3 248 458	8 115 883	6 103 235
29	21	7	2010	$1.28 \cdot 10^5$	0.17	105	0.76	6 506 210	424 750	228 375	4 239 800	13 968 968	10 343 035
39	29	8	2011	$2.47 \cdot 10^5$	0.17	120	0.75	10 350 924	375 875	261 000	6 733 694	23 683 017	17 076 730
50	39	9	2012	$3.69 \cdot 10^5$	0.17	135	0.69	12 069 229	327 000	293 625	7 586 052	35 131 621	24 662 782
28	21	7	2010	$7.03 \cdot 10^4$	0.08	96	0.75	3 522 704	169 900	208 800	2 279 397	11 259 887	8 382 632
38	28	8	2011	$8.34 \cdot 10^4$	0.08	96	0.76	3 530 784	0	208 800	2 306 933	14 581 871	10 689 565
49	38	9	2012	$1.10 \cdot 10^5$	0.08	96	0.75	3 879 455	0	208 800	2 435 486	18 252 526	13 125 052
6	3	3	2006	$4.39 \cdot 10^4$	0.21	45	0.54	3 181 386	620 250	97 875	2 123 218	978 261	675 171
10	6	4	2007	$5.94 \cdot 10^4$	0.21	60	0.62	4 155 355	571 375	130 500	2 855 695	4 431 742	3 530 866
16	10	5	2008	$8.50 \cdot 10^4$	0.21	75	0.69	5 521 094	522 500	163 125	3 830 611	9 267 210	7 361 477
22	16	6	2009	$1.12 \cdot 10^5$	0.14	90	0.73	6 500 059	473 625	195 750	4 418 749	15 097 894	11 780 225
30	22	7	2010	$1.32 \cdot 10^5$	0.09	105	0.76	6 662 164	424 750	228 375	4 353 152	21 106 933	16 133 378
40	30	8	2011	$2.25 \cdot 10^5$	0.09	120	0.76	9 582 140	375 875	261 000	6 199 817	30 052 199	22 333 195
51	40	9	2012	$3.69 \cdot 10^5$	0.09	135	0.69	12 075 424	327 000	293 625	7 590 163	41 506 998	29 923 358
31	22	7	2010	$6.05 \cdot 10^4$	0.06	91	0.74	2 986 616	28 317	197 925	2 005 363	17 858 268	13 785 589
41	31	8	2011	$7.74 \cdot 10^4$	0.06	91	0.74	3 191 063	0	197 925	2 078 568	20 851 407	15 864 157
52	41	9	2012	$1.14 \cdot 10^5$	0.06	91	0.73	3 952 751	0	197 925	2 491 334	24 606 233	18 355 491
23	16	6	2009	$2.37 \cdot 10^5$	0.07	90	0.67	12 513 748	473 625	195 750	8 993 517	21 111 583	16 354 994
32	23	7	2010	$3.62 \cdot 10^5$	0.07	105	0.62	14 897 170	424 750	228 375	10 338 601	35 355 628	26 693 595
42	32	8	2011	$4.77 \cdot 10^5$	0.07	120	0.58	15 590 458	375 875	261 000	10 372 260	50 309 211	37 065 855
53	42	9	2012	$5.26 \cdot 10^5$	0.07	135	0.59	14 747 300	327 000	293 625	9 362 958	64 435 886	46 428 813

**Table A.25.:** Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 60, mandatory coverage, and optimization for the expected profit



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.06 \cdot 10^4$	1.00	25	0.34	0	1 196 667	54 375	-1 248 619	-1 251 042	-1 248 619
2	1	2	2005	$1.06 \cdot 10^4$	0.29	50	0.58	974 957	1 115 208	108 750	-274 783	-1 500 043	-1 523 401
4	2	3	2006	$1.42 \cdot 10^4$	0.29	61	0.63	1 192 301	454 850	132 675	508 982	-895 267	-1 014 419
7	4	4	2007	$2.00 \cdot 10^4$	0.29	61	0.64	1 431 055	0	132 675	1 081 983	403 113	67 564
11	7	5	2008	$1.33 \cdot 10^4$	0.09	61	0.63	786 382	0	132 675	520 483	1 056 820	588 047
17	11	6	2009	$1.73 \cdot 10^4$	0.09	61	0.63	864 117	0	132 675	556 426	1 788 262	1 144 474
24	17	7	2010	$2.31 \cdot 10^4$	0.09	61	0.62	956 900	0	132 675	599 071	2 612 486	1 743 545
33	24	8	2011	$2.71 \cdot 10^4$	0.09	61	0.63	954 157	0	132 675	570 474	3 433 969	2 314 019
43	33	9	2012	$5.44 \cdot 10^4$	0.09	61	0.63	1 606 476	0	132 675	977 870	4 907 770	3 291 888
12	7	5	2008	$2.85 \cdot 10^4$	0.20	86	0.72	1 944 761	870 833	187 050	673 799	1 289 991	741 363
18	12	6	2009	$5.89 \cdot 10^4$	0.20	101	0.76	3 561 507	473 625	219 675	2 165 116	4 158 198	2 906 480
25	18	7	2010	$8.04 \cdot 10^4$	0.20	101	0.77	4 099 382	0	219 675	2 819 887	8 037 905	5 726 367
35	25	8	2011	$6.85 \cdot 10^4$	0.08	101	0.76	2 930 002	0	219 675	1 882 172	10 748 232	7 608 538
45	35	9	2012	$1.05 \cdot 10^5$	0.08	101	0.76	3 748 950	0	219 675	2 341 681	14 277 507	9 950 219
34	25	8	2011	$1.47 \cdot 10^5$	0.12	126	0.77	6 370 939	626 458	274 050	3 778 622	13 508 336	9 504 989
44	34	9	2012	$2.72 \cdot 10^5$	0.12	151	0.73	9 371 981	545 000	328 425	5 621 945	22 006 892	15 126 934
3	1	2	2005	$1.63 \cdot 10^4$	0.70	50	0.58	1 491 869	1 115 208	108 750	197 092	-983 131	-1 051 527
5	3	3	2006	$2.67 \cdot 10^4$	0.49	75	0.68	2 428 138	1 033 750	163 125	1 031 855	248 132	-19 672
8	5	4	2007	$1.46 \cdot 10^4$	0.11	75	0.70	1 140 498	0	163 125	814 478	1 225 506	794 805
13	8	5	2008	$2.09 \cdot 10^4$	0.11	78	0.71	1 390 287	104 500	169 650	884 790	2 341 642	1 679 596
19	13	6	2009	$2.75 \cdot 10^4$	0.11	80	0.71	1 539 278	63 150	174 000	988 322	3 643 770	2 667 918
26	19	7	2010	$3.66 \cdot 10^4$	0.11	81	0.71	1 727 438	28 317	176 175	1 105 963	5 166 717	3 773 881
36	26	8	2011	$5.83 \cdot 10^4$	0.11	81	0.71	2 324 763	0	176 175	1 492 075	7 315 304	5 265 956
47	36	9	2012	$3.13 \cdot 10^5$	0.03	106	0.65	9 610 887	545 000	230 550	5 845 400	16 150 642	11 111 356
46	36	9	2012	$7.44 \cdot 10^4$	0.08	81	0.71	2 481 150	0	176 175	1 529 355	9 620 279	6 795 311
9	5	4	2007	$3.02 \cdot 10^4$	0.38	89	0.74	2 503 121	533 283	193 575	1 459 495	2 024 395	1 439 822
14	9	5	2008	$2.85 \cdot 10^4$	0.13	89	0.74	1 976 296	0	193 575	1 419 406	3 807 116	2 859 228
20	14	6	2009	$4.05 \cdot 10^4$	0.13	90	0.74	2 365 463	31 575	195 750	1 625 416	5 945 254	4 484 645
27	20	7	2010	$4.39 \cdot 10^4$	0.13	90	0.74	2 154 003	0	195 750	1 423 317	7 903 507	5 907 962
37	27	8	2011	$6.67 \cdot 10^4$	0.13	90	0.74	2 753 510	0	195 750	1 776 222	10 461 267	7 684 184
48	37	9	2012	$1.08 \cdot 10^5$	0.13	91	0.74	3 751 909	21 800	197 925	2 342 937	13 993 452	10 027 121
15	9	5	2008	$5.25 \cdot 10^4$	0.25	111	0.75	3 724 744	766 333	241 425	2 134 816	4 741 381	3 574 638
21	15	6	2009	$8.44 \cdot 10^4$	0.25	136	0.77	5 182 117	789 375	295 800	3 088 645	8 838 323	6 663 283
29	21	7	2010	$1.28 \cdot 10^5$	0.17	161	0.78	6 667 016	707 917	350 175	4 052 739	14 447 248	10 716 022
39	29	8	2011	$2.47 \cdot 10^5$	0.17	186	0.76	10 558 135	626 458	404 550	6 595 772	23 974 375	17 311 794
50	39	9	2012	$3.69 \cdot 10^5$	0.17	211	0.73	12 701 393	545 000	458 925	7 744 429	35 671 843	25 056 223
28	21	7	2010	$7.03 \cdot 10^4$	0.08	160	0.81	3 778 788	679 600	348 000	1 976 610	11 589 512	8 639 893
38	28	8	2011	$8.34 \cdot 10^4$	0.08	184	0.83	3 856 566	601 400	400 200	1 963 139	14 444 477	10 603 032
49	38	9	2012	$1.10 \cdot 10^5$	0.08	193	0.83	4 324 149	196 200	419 775	2 454 310	18 152 652	13 057 342
6	3	3	2006	$4.39 \cdot 10^4$	0.21	75	0.63	3 666 527	1 033 750	163 125	2 111 973	1 486 522	1 060 446
10	6	4	2007	$5.94 \cdot 10^4$	0.21	100	0.67	4 451 717	952 292	217 500	2 697 929	4 768 447	3 758 375
16	10	5	2008	$8.50 \cdot 10^4$	0.21	125	0.71	5 663 742	870 833	271 875	3 567 372	9 289 480	7 325 697
22	16	6	2009	$1.12 \cdot 10^5$	0.14	150	0.74	6 599 699	789 375	326 250	4 143 822	14 773 555	11 469 570
30	22	7	2010	$1.32 \cdot 10^5$	0.09	175	0.76	6 676 729	707 917	380 625	4 037 667	20 361 742	15 507 236
40	30	8	2011	$2.25 \cdot 10^5$	0.09	200	0.76	9 533 437	626 458	435 000	5 863 031	28 833 721	21 370 267
51	40	9	2012	$3.69 \cdot 10^5$	0.09	225	0.72	12 487 348	545 000	489 375	7 582 206	40 286 694	28 952 473
31	22	7	2010	$6.05 \cdot 10^4$	0.06	174	0.79	3 196 494	679 600	378 450	1 531 249	16 911 998	13 000 819
41	31	8	2011	$7.74 \cdot 10^4$	0.06	198	0.81	3 508 246	601 400	430 650	1 700 104	19 388 194	14 700 923
52	41	9	2012	$1.14 \cdot 10^5$	0.06	223	0.83	4 451 902	545 000	485 025	2 253 558	22 810 071	16 954 481
23	16	6	2009	$2.37 \cdot 10^5$	0.07	150	0.68	12 780 889	789 375	326 250	8 846 063	20 954 745	16 171 761
32	23	7	2010	$3.62 \cdot 10^5$	0.07	175	0.64	15 495 123	707 917	380 625	10 447 139	35 361 326	26 618 900
42	32	8	2011	$4.77 \cdot 10^5$	0.07	200	0.61	16 373 838	626 458	435 000	10 613 309	50 673 706	37 232 209
53	42	9	2012	$5.26 \cdot 10^5$	0.07	225	0.63	15 726 152	545 000	489 375	9 731 159	65 365 483	46 963 368

**Table A.26.:** Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.06 \cdot 10^4$	1.00	12	0.15	0	574 400	26 100	−599 337	−600 500	−599 337
2	1	2	2005	$1.06 \cdot 10^4$	0.29	26	0.34	0	624 517	56 550	−648 313	−1 281 567	−1 247 650
4	2	3	2006	$1.42 \cdot 10^4$	0.29	41	0.52	978 337	620 250	89 175	209 316	−1 012 654	−1 038 334
7	4	4	2007	$2.00 \cdot 10^4$	0.29	56	0.61	1 376 481	571 375	121 800	547 216	−329 348	−491 118
11	7	5	2008	$1.33 \cdot 10^4$	0.09	56	0.60	747 321	0	121 800	498 041	296 173	6 924
17	11	6	2009	$1.73 \cdot 10^4$	0.09	56	0.61	830 772	0	121 800	539 334	1 005 145	546 257
24	17	7	2010	$2.31 \cdot 10^4$	0.09	56	0.60	917 069	0	121 800	578 025	1 800 414	1 124 283
33	24	8	2011	$2.71 \cdot 10^4$	0.09	56	0.60	911 630	0	121 800	548 493	2 590 245	1 672 776
43	33	9	2012	$5.44 \cdot 10^4$	0.09	56	0.60	1 534 769	0	121 800	937 508	4 003 213	2 610 283
12	7	5	2008	$2.85 \cdot 10^4$	0.20	71	0.67	1 807 228	522 500	154 425	880 549	800 955	389 431
18	12	6	2009	$5.89 \cdot 10^4$	0.20	86	0.73	3 387 989	473 625	187 050	2 057 936	3 528 269	2 447 367
25	18	7	2010	$8.04 \cdot 10^4$	0.20	94	0.75	4 019 361	226 533	204 450	2 600 462	7 116 646	5 047 829
35	25	8	2011	$6.85 \cdot 10^4$	0.08	94	0.75	2 859 647	0	204 450	1 843 887	9 771 844	6 891 716
45	35	9	2012	$1.05 \cdot 10^5$	0.08	94	0.74	3 667 068	0	204 450	2 297 454	13 234 462	9 189 169
34	25	8	2011	$1.47 \cdot 10^5$	0.12	109	0.75	6 198 181	375 875	237 075	3 866 460	12 701 878	8 914 289
44	34	9	2012	$2.72 \cdot 10^5$	0.12	124	0.70	8 923 707	327 000	269 700	5 514 867	21 028 884	14 429 155
3	1	2	2005	$1.63 \cdot 10^4$	0.70	27	0.37	0	669 125	58 725	−692 919	−1 328 350	−1 292 256
5	3	3	2006	$2.67 \cdot 10^4$	0.49	42	0.52	1 831 172	620 250	91 350	951 257	−208 778	−340 999
8	5	4	2007	$1.46 \cdot 10^4$	0.11	57	0.61	1 007 838	571 375	123 975	238 202	103 710	−102 797
13	8	5	2008	$2.09 \cdot 10^4$	0.11	72	0.68	1 347 574	522 500	156 600	512 840	772 184	410 042
19	13	6	2009	$2.75 \cdot 10^4$	0.11	79	0.70	1 529 376	221 025	171 825	856 743	1 908 709	1 266 786
26	19	7	2010	$3.66 \cdot 10^4$	0.11	81	0.71	1 736 630	56 633	176 175	1 091 103	3 412 531	2 357 889
36	26	8	2011	$5.83 \cdot 10^4$	0.11	82	0.71	2 328 383	25 058	178 350	1 474 865	5 537 506	3 832 754
47	36	9	2012	$3.13 \cdot 10^5$	0.03	97	0.62	9 177 231	327 000	210 975	5 722 045	14 176 761	9 554 798
46	36	9	2012	$7.44 \cdot 10^4$	0.08	82	0.71	2 492 025	0	178 350	1 535 128	7 851 181	5 367 882
9	5	4	2007	$3.02 \cdot 10^4$	0.38	57	0.61	2 078 869	571 375	123 975	1 130 727	1 174 740	789 728
14	9	5	2008	$2.85 \cdot 10^4$	0.13	72	0.68	1 822 829	522 500	156 600	891 239	2 318 469	1 680 967
20	14	6	2009	$4.05 \cdot 10^4$	0.13	85	0.72	2 323 956	410 475	184 875	1 300 433	4 047 075	2 981 400
27	20	7	2010	$4.39 \cdot 10^4$	0.13	85	0.72	2 118 990	0	184 875	1 405 773	5 981 190	4 387 172
37	27	8	2011	$6.67 \cdot 10^4$	0.13	86	0.72	2 699 910	25 058	187 050	1 726 829	8 468 992	6 114 001
48	37	9	2012	$1.08 \cdot 10^5$	0.13	86	0.72	3 681 504	0	187 050	2 318 577	11 963 446	8 432 578
15	9	5	2008	$5.25 \cdot 10^4$	0.25	72	0.63	3 110 682	522 500	156 600	1 916 630	3 606 322	2 706 358
21	15	6	2009	$8.44 \cdot 10^4$	0.25	87	0.67	4 454 787	473 625	189 225	2 867 822	7 398 259	5 574 180
29	21	7	2010	$1.28 \cdot 10^5$	0.17	102	0.68	5 827 923	424 750	221 850	3 751 543	12 579 582	9 325 723
39	29	8	2011	$2.47 \cdot 10^5$	0.17	117	0.64	8 903 900	375 875	254 475	5 733 348	20 853 132	15 059 071
50	39	9	2012	$3.69 \cdot 10^5$	0.17	132	0.57	9 926 379	327 000	287 100	6 168 597	30 165 411	21 227 668
28	21	7	2010	$7.03 \cdot 10^4$	0.08	97	0.70	3 256 079	283 167	210 975	1 997 860	10 160 196	7 572 040
38	28	8	2011	$8.34 \cdot 10^4$	0.08	105	0.71	3 339 710	200 467	228 375	2 014 944	13 071 064	9 586 984
49	38	9	2012	$1.10 \cdot 10^5$	0.08	119	0.72	3 755 300	305 200	258 825	2 107 973	16 262 339	11 694 958
6	3	3	2006	$4.39 \cdot 10^4$	0.21	42	0.51	2 973 084	620 250	91 350	1 947 229	933 134	654 973
10	6	4	2007	$5.94 \cdot 10^4$	0.21	57	0.55	3 629 995	571 375	123 975	2 423 332	3 867 779	3 078 305
16	10	5	2008	$8.50 \cdot 10^4$	0.21	72	0.57	4 598 511	522 500	156 600	3 101 243	7 787 190	6 179 548
22	16	6	2009	$1.12 \cdot 10^5$	0.14	87	0.60	5 348 196	473 625	189 225	3 547 461	12 472 535	9 727 009
30	22	7	2010	$1.32 \cdot 10^5$	0.09	102	0.63	5 551 789	424 750	221 850	3 550 841	17 377 725	13 277 850
40	30	8	2011	$2.25 \cdot 10^5$	0.09	117	0.61	7 761 485	375 875	254 475	4 940 004	24 508 859	18 217 854
51	40	9	2012	$3.69 \cdot 10^5$	0.09	132	0.53	9 208 662	327 000	287 100	5 692 391	33 103 422	23 910 244
31	22	7	2010	$6.05 \cdot 10^4$	0.06	102	0.64	2 572 410	424 750	221 850	1 385 339	14 398 345	11 112 348
41	31	8	2011	$7.74 \cdot 10^4$	0.06	117	0.67	2 925 210	375 875	254 475	1 581 480	16 693 205	12 693 827
52	41	9	2012	$1.14 \cdot 10^5$	0.06	131	0.69	3 726 997	305 200	284 925	2 071 877	19 830 077	14 765 704
23	16	6	2009	$2.37 \cdot 10^5$	0.07	87	0.53	9 962 993	473 625	189 225	7 058 056	17 087 332	13 237 603
32	23	7	2010	$3.62 \cdot 10^5$	0.07	102	0.46	11 057 250	424 750	221 850	7 552 375	27 497 983	20 789 979
42	32	8	2011	$4.77 \cdot 10^5$	0.07	117	0.43	11 394 589	375 875	254 475	7 462 993	38 262 222	28 252 972
53	42	9	2012	$5.26 \cdot 10^5$	0.07	132	0.44	10 828 247	327 000	287 100	6 766 988	48 476 369	35 019 960

**Table A.27.:** Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$1.07 \cdot 10^4$	0.09	87	0.73	5 841 775	5 298 300	756 900	-511 625	-213 425	-511 625
17	11	6	2009	$5.64 \cdot 10^4$	0.09	130	0.81	17 107 208	2 198 375	1 131 000	4 466 922	13 564 408	3 955 298
25	17	7	2010	$1.50 \cdot 10^5$	0.09	132	0.80	22 482 131	82 700	1 148 400	5 926 157	34 815 439	9 881 455
12	7	5	2008	$3.79 \cdot 10^3$	0.20	0	0.00	0	0	0	0	0	0
19	12	6	2009	$9.58 \cdot 10^3$	0.13	42	0.53	1 923 920	2 147 250	365 400	-340 986	-588 730	-340 986
27	19	7	2010	$7.50 \cdot 10^4$	0.13	65	0.66	9 283 275	951 050	565 500	2 114 466	7 177 995	1 773 480
18	12	6	2009	$3.76 \cdot 10^3$	0.07	0	0.00	0	0	0	0	0	0
26	18	7	2010	$1.12 \cdot 10^4$	0.07	0	0.00	0	0	0	0	0	0
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 778 326	2 827 000	348 000	-946 216	-1 396 674	-946 216
14	8	5	2008	$6.56 \cdot 10^3$	0.24	47	0.57	2 806 301	426 300	408 900	757 877	574 427	-188 339
21	14	6	2009	$1.46 \cdot 10^4$	0.07	49	0.57	3 141 180	102 250	426 300	868 116	3 187 057	679 777
29	21	7	2010	$4.73 \cdot 10^4$	0.07	49	0.57	5 056 232	0	426 300	1 292 129	7 816 988	1 971 906
22	14	6	2009	$4.28 \cdot 10^4$	0.17	115	0.80	12 867 855	3 476 500	1 000 500	2 577 226	8 965 282	2 388 887
30	22	7	2010	$2.47 \cdot 10^5$	0.17	163	0.82	38 455 049	1 984 800	1 418 100	9 671 627	44 017 431	12 060 514
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 153 253	0	348 000	323 613	-591 422	-622 603
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 987 332	0	348 000	549 009	1 047 911	-73 594
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 713 333	0	348 000	939 203	4 413 244	865 609
9	5	4	2007	$4.35 \cdot 10^3$	0.16	64	0.68	4 451 664	4 523 200	556 800	-739 282	-628 336	-739 282
15	9	5	2008	$1.93 \cdot 10^4$	0.16	152	0.84	12 262 746	5 359 200	1 322 400	1 812 189	4 952 810	1 072 906
23	15	6	2009	$7.28 \cdot 10^4$	0.16	252	0.91	24 885 476	5 112 500	2 192 400	5 545 266	22 533 385	6 618 173
31	23	7	2010	$3.63 \cdot 10^5$	0.12	278	0.88	59 951 257	1 075 100	2 418 600	15 696 260	78 990 943	22 314 432
32	23	7	2010	$1.60 \cdot 10^6$	0.04	352	0.53	160 187 041	4 135 000	3 062 400	42 465 801	175 523 027	49 083 973
6	3	3	2006	$1.48 \cdot 10^3$	0.18	44	0.59	2 644 006	3 539 800	382 800	-1 149 626	-1 278 594	-1 149 626
10	6	4	2007	$6.34 \cdot 10^3$	0.18	89	0.74	7 111 384	3 180 375	774 300	1 215 583	1 878 115	65 957
16	10	5	2008	$3.24 \cdot 10^4$	0.18	183	0.88	21 363 070	5 724 600	1 592 100	5 184 804	15 924 485	5 250 761
24	16	6	2009	$1.38 \cdot 10^5$	0.18	283	0.92	47 791 766	5 112 500	2 462 100	13 126 214	56 141 651	18 376 975
34	24	7	2010	$1.57 \cdot 10^6$	0.05	383	0.56	164 366 343	4 135 000	3 332 100	43 556 899	213 040 894	61 933 874
33	24	7	2010	$5.23 \cdot 10^5$	0.13	332	0.86	84 280 945	2 026 150	2 888 400	22 036 612	135 508 047	40 413 587

**Table A.28.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, and optimization for the expected profit

A. Tables with Detailed Results

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$1.07 \cdot 10^4$	0.09	60	0.63	5 093 426	3 654 000	522 000	75 001	917 426	75 001
17	11	6	2009	$5.64 \cdot 10^4$	0.09	117	0.80	16 896 636	2 914 125	1 017 900	4 146 635	13 882 036	4 221 636
25	17	7	2010	$1.50 \cdot 10^5$	0.09	120	0.79	22 185 723	124 050	1 044 000	5 858 723	34 899 709	10 080 358
12	7	5	2008	$3.79 \cdot 10^3$	0.20	0	0.00	0	0	0	0	0	0
19	12	6	2009	$9.58 \cdot 10^3$	0.13	42	0.53	1 928 738	2 147 250	365 400	−339 373	−583 912	−339 373
27	19	7	2010	$7.50 \cdot 10^4$	0.13	65	0.66	9 283 275	951 050	565 500	2 114 466	7 182 813	1 775 093
18	12	6	2009	$3.76 \cdot 10^3$	0.07	0	0.00	0	0	0	0	0	0
26	18	7	2010	$1.12 \cdot 10^4$	0.07	0	0.00	0	0	0	0	0	0
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 770 580	2 827 000	348 000	−949 952	−1 404 420	−949 952
14	8	5	2008	$6.56 \cdot 10^3$	0.24	50	0.59	2 895 769	609 000	435 000	695 236	447 349	−254 716
21	14	6	2009	$1.46 \cdot 10^4$	0.07	51	0.58	3 201 969	51 125	443 700	903 193	3 154 493	648 477
29	21	7	2010	$4.73 \cdot 10^4$	0.07	51	0.58	5 155 169	0	443 700	1 314 884	7 865 962	1 963 361
22	14	6	2009	$4.28 \cdot 10^4$	0.17	109	0.79	12 675 242	3 016 375	948 300	2 715 116	9 157 916	2 460 400
30	22	7	2010	$2.47 \cdot 10^5$	0.17	163	0.82	38 454 800	2 232 900	1 418 100	9 588 470	43 961 716	12 048 869
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 148 164	0	348 000	321 568	−604 256	−628 384
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 986 081	0	348 000	548 590	1 033 825	−79 794
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 710 648	0	348 000	938 453	4 396 473	858 660
9	5	4	2007	$4.35 \cdot 10^3$	0.16	60	0.66	4 352 429	4 240 500	522 000	−606 757	−410 071	−606 757
15	9	5	2008	$1.93 \cdot 10^4$	0.16	120	0.80	11 561 629	3 654 000	1 044 000	2 464 646	6 453 558	1 857 889
23	15	6	2009	$7.28 \cdot 10^4$	0.16	179	0.85	23 394 751	3 016 375	1 557 300	6 101 105	25 274 633	7 958 994
31	23	7	2010	$3.63 \cdot 10^5$	0.12	238	0.84	57 379 502	2 439 650	2 070 600	14 618 666	78 143 885	22 577 660
32	23	7	2010	$1.60 \cdot 10^6$	0.04	239	0.42	125 396 496	2 481 000	2 079 300	33 584 684	146 110 829	41 543 678
6	3	3	2006	$1.48 \cdot 10^3$	0.18	60	0.69	3 061 925	4 827 000	522 000	−1 882 219	−2 287 075	−1 882 219
10	6	4	2007	$6.34 \cdot 10^3$	0.18	120	0.82	7 853 551	4 240 500	1 044 000	829 934	281 976	−1 052 285
16	10	5	2008	$3.24 \cdot 10^4$	0.18	180	0.88	21 402 457	3 654 000	1 566 000	6 209 675	16 464 433	5 157 389
24	16	6	2009	$1.38 \cdot 10^5$	0.18	240	0.90	46 890 223	3 067 500	2 088 000	13 771 415	58 199 157	18 928 804
34	24	7	2010	$1.57 \cdot 10^6$	0.05	300	0.49	145 225 071	2 481 000	2 610 000	38 970 367	198 333 227	57 899 171
33	24	7	2010	$5.23 \cdot 10^5$	0.13	300	0.84	82 689 250	2 481 000	2 610 000	21 517 767	135 797 407	40 446 571

**Table A.29.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, and optimization for the expected profit

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$1.07 \cdot 10^4$	0.09	77	0.71	5 695 369	4 689 300	669 900	-241 806	336 169	-241 806
17	11	6	2009	$5.64 \cdot 10^4$	0.09	118	0.80	16 919 429	2 096 125	1 026 600	4 480 091	14 132 874	4 238 285
25	17	7	2010	$1.50 \cdot 10^5$	0.09	119	0.79	22 152 349	41 350	1 035 300	5 879 533	35 208 573	10 117 818
12	7	5	2008	$3.79 \cdot 10^3$	0.20	0	0.00	0	0	0	0	0	0
19	12	6	2009	$9.58 \cdot 10^3$	0.13	42	0.53	1 915 144	2 147 250	365 400	-343 925	-597 506	-343 925
27	19	7	2010	$7.50 \cdot 10^4$	0.13	66	0.66	9 328 289	992 400	574 200	2 110 753	7 164 184	1 766 828
18	12	6	2009	$3.76 \cdot 10^3$	0.07	0	0.00	0	0	0	0	0	0
26	18	7	2010	$1.12 \cdot 10^4$	0.07	0	0.00	0	0	0	0	0	0
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.58	1 778 421	2 827 000	348 000	-946 170	-1 396 579	-946 170
14	8	5	2008	$6.56 \cdot 10^3$	0.24	47	0.57	2 801 396	426 300	408 900	755 906	569 618	-190 264
21	14	6	2009	$1.46 \cdot 10^4$	0.07	49	0.57	3 130 857	102 250	426 300	864 659	3 171 924	674 394
29	21	7	2010	$4.73 \cdot 10^4$	0.07	49	0.57	5 079 089	0	426 300	1 298 508	7 824 714	1 972 903
22	14	6	2009	$4.28 \cdot 10^4$	0.17	146	0.80	12 848 371	5 061 375	1 270 200	1 843 453	7 086 414	1 653 189
30	22	7	2010	$2.47 \cdot 10^5$	0.17	246	0.87	40 600 769	4 135 000	2 140 200	9 348 836	41 411 983	11 002 025
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 154 064	0	348 000	323 939	-590 514	-622 231
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.53	1 995 286	0	348 000	551 673	1 056 772	-70 558
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 724 157	0	348 000	942 223	4 432 928	871 665
9	5	4	2007	$4.35 \cdot 10^3$	0.16	100	0.59	3 852 625	7 067 500	870 000	-2 651 609	-4 084 875	-2 651 609
15	9	5	2008	$1.93 \cdot 10^4$	0.16	200	0.83	12 044 179	6 090 000	1 740 000	1 204 097	129 304	-1 447 511
23	15	6	2009	$7.28 \cdot 10^4$	0.16	300	0.88	24 257 183	5 112 500	2 610 000	5 194 999	16 663 987	3 747 487
31	23	7	2010	$3.63 \cdot 10^5$	0.12	381	0.90	61 404 267	3 349 350	3 314 700	15 090 042	71 404 204	18 837 529
32	23	7	2010	$1.60 \cdot 10^6$	0.04	400	0.52	155 155 509	4 135 000	3 480 000	40 945 048	164 204 496	44 692 535
6	3	3	2006	$1.48 \cdot 10^3$	0.18	93	0.75	3 340 352	7 481 850	809 100	-3 730 884	-4 950 598	-3 730 884
10	6	4	2007	$6.34 \cdot 10^3$	0.18	165	0.83	7 963 063	5 088 600	1 435 500	203 146	-3 511 635	-3 527 739
16	10	5	2008	$3.24 \cdot 10^4$	0.18	252	0.87	21 290 196	5 298 300	2 192 400	5 119 854	10 287 861	1 592 116
24	16	6	2009	$1.38 \cdot 10^5$	0.18	305	0.89	46 174 087	2 709 625	2 653 500	13 486 019	51 098 823	15 078 135
34	24	7	2010	$1.57 \cdot 10^6$	0.05	307	0.43	127 567 752	82 700	2 670 900	34 828 723	175 912 975	49 906 858
33	24	7	2010	$5.23 \cdot 10^5$	0.13	405	0.85	83 797 568	4 135 000	3 523 500	21 018 216	127 237 891	36 096 351

**Table A.30.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, and optimization for AV@R

A. Tables with Detailed Results

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$1.07 \cdot 10^4$	0.09	60	0.63	5 093 426	3 654 000	522 000	75 001	917 426	75 001
17	11	6	2009	$5.64 \cdot 10^4$	0.09	117	0.80	16 896 636	2 914 125	1 017 900	4 146 635	13 882 036	4 221 636
25	17	7	2010	$1.50 \cdot 10^5$	0.09	120	0.79	22 185 723	124 050	1 044 000	5 858 723	34 899 709	10 080 358
12	7	5	2008	$3.79 \cdot 10^3$	0.20	0	0.00	0	0	0	0	0	0
19	12	6	2009	$9.58 \cdot 10^3$	0.13	42	0.53	1 928 738	2 147 250	365 400	−339 373	−583 912	−339 373
27	19	7	2010	$7.50 \cdot 10^4$	0.13	65	0.66	9 283 275	951 050	565 500	2 114 466	7 182 813	1 775 093
18	12	6	2009	$3.76 \cdot 10^3$	0.07	0	0.00	0	0	0	0	0	0
26	18	7	2010	$1.12 \cdot 10^4$	0.07	0	0.00	0	0	0	0	0	0
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 770 580	2 827 000	348 000	−949 952	−1 404 420	−949 952
14	8	5	2008	$6.56 \cdot 10^3$	0.24	50	0.59	2 895 769	609 000	435 000	695 236	447 349	−254 716
21	14	6	2009	$1.46 \cdot 10^4$	0.07	51	0.58	3 201 969	51 125	443 700	903 193	3 154 493	648 477
29	21	7	2010	$4.73 \cdot 10^4$	0.07	51	0.58	5 155 169	0	443 700	1 314 884	7 865 962	1 963 361
22	14	6	2009	$4.28 \cdot 10^4$	0.17	109	0.79	12 675 242	3 016 375	948 300	2 715 116	9 157 916	2 460 400
30	22	7	2010	$2.47 \cdot 10^5$	0.17	163	0.82	38 454 800	2 232 900	1 418 100	9 588 470	43 961 716	12 048 869
13	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 148 164	0	348 000	321 568	−604 256	−628 384
20	13	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 986 081	0	348 000	548 590	1 033 825	−79 794
28	20	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 710 648	0	348 000	938 453	4 396 473	858 660
9	5	4	2007	$4.35 \cdot 10^3$	0.16	60	0.66	4 352 429	4 240 500	522 000	−606 757	−410 071	−606 757
15	9	5	2008	$1.93 \cdot 10^4$	0.16	120	0.80	11 561 629	3 654 000	1 044 000	2 464 646	6 453 558	1 857 889
23	15	6	2009	$7.28 \cdot 10^4$	0.16	179	0.85	23 394 751	3 016 375	1 557 300	6 101 105	25 274 633	7 958 994
31	23	7	2010	$3.63 \cdot 10^5$	0.12	238	0.84	57 379 502	2 439 650	2 070 600	14 618 666	78 143 885	22 577 660
32	23	7	2010	$1.60 \cdot 10^6$	0.04	239	0.42	125 396 496	2 481 000	2 079 300	33 584 684	146 110 829	41 543 678
6	3	3	2006	$1.48 \cdot 10^3$	0.18	60	0.69	3 061 925	4 827 000	522 000	−1 882 219	−2 287 075	−1 882 219
10	6	4	2007	$6.34 \cdot 10^3$	0.18	120	0.82	7 853 551	4 240 500	1 044 000	829 934	281 976	−1 052 285
16	10	5	2008	$3.24 \cdot 10^4$	0.18	180	0.88	21 402 457	3 654 000	1 566 000	6 209 675	16 464 433	5 157 389
24	16	6	2009	$1.38 \cdot 10^5$	0.18	240	0.90	46 890 223	3 067 500	2 088 000	13 771 415	58 199 157	18 928 804
34	24	7	2010	$1.57 \cdot 10^6$	0.05	300	0.49	145 225 071	2 481 000	2 610 000	38 970 367	198 333 227	57 899 171
33	24	7	2010	$5.23 \cdot 10^5$	0.13	300	0.84	82 689 250	2 481 000	2 610 000	21 517 767	135 797 407	40 446 571

**Table A.31.:** Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$3.79 \cdot 10^3$	0.29	40	0.55	1 569 341	2 436 000	348 000	−683 939	−1 214 659	−683 939
16	11	6	2009	$5.64 \cdot 10^4$	0.09	138	0.83	17 615 464	5 010 250	1 200 600	3 483 798	10 189 955	2 799 859
22	16	7	2010	$3.00 \cdot 10^5$	0.09	198	0.83	47 152 773	2 481 000	1 722 600	11 847 846	53 139 128	14 647 704
17	11	6	2009	$1.50 \cdot 10^4$	0.20	44	0.54	3 044 445	204 500	382 800	809 196	1 242 487	125 257
23	17	7	2010	$3.89 \cdot 10^4$	0.20	44	0.54	3 965 198	0	382 800	999 781	4 824 884	1 125 038
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 770 387	2 827 000	348 000	−950 045	−1 404 613	−950 045
13	8	5	2008	$6.56 \cdot 10^3$	0.24	50	0.59	2 893 191	609 000	435 000	694 200	444 578	−255 845
19	13	6	2009	$4.28 \cdot 10^4$	0.24	116	0.80	12 897 164	3 374 250	1 009 200	2 625 220	8 958 293	2 369 375
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 361 032	1 116 450	1 244 100	7 752 105	36 958 775	10 121 479
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 148 037	0	348 000	321 517	−604 577	−628 528
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 978 585	0	348 000	546 080	1 026 009	−82 449
24	18	7	2010	$3.82 \cdot 10^4$	0.12	40	0.51	3 696 347	0	348 000	934 462	4 374 356	852 014
9	5	4	2007	$4.35 \cdot 10^3$	0.16	57	0.65	4 288 660	4 028 475	495 900	−502 223	−235 715	−502 223
14	9	5	2008	$1.93 \cdot 10^4$	0.16	107	0.79	11 426 226	3 045 000	930 900	2 749 375	7 214 611	2 247 152
20	14	6	2009	$7.28 \cdot 10^4$	0.16	158	0.86	23 495 547	2 607 375	1 374 600	6 360 415	26 728 183	8 607 567
26	20	7	2010	$3.63 \cdot 10^5$	0.16	240	0.85	58 194 590	3 390 700	2 088 000	14 522 781	79 444 073	23 130 348
6	3	3	2006	$1.48 \cdot 10^3$	0.18	42	0.59	2 616 332	3 378 900	365 400	−1 043 836	−1 127 968	−1 043 836
10	6	4	2007	$6.34 \cdot 10^3$	0.18	84	0.74	7 103 119	2 968 350	730 800	1 355 275	2 276 001	311 440
15	10	5	2008	$3.24 \cdot 10^4$	0.18	148	0.85	20 645 839	3 897 600	1 287 600	5 900 012	17 736 639	6 211 452
21	15	6	2009	$1.38 \cdot 10^5$	0.18	225	0.90	46 826 410	3 936 625	1 957 500	13 444 466	58 668 924	19 655 918
27	21	7	2010	$5.23 \cdot 10^5$	0.18	311	0.85	83 307 055	3 556 100	2 705 700	21 303 428	135 714 179	40 959 346

**Table A.32.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, and optimization for the expected profit


node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$3.79 \cdot 10^3$	0.29	41	0.56	1 598 164	2 496 900	356 700	-705 221	-1 255 436	-705 221
16	11	6	2009	$5.64 \cdot 10^4$	0.09	101	0.76	16 206 075	3 067 500	878 700	3 900 347	11 004 439	3 195 126
22	16	7	2010	$3.00 \cdot 10^5$	0.09	161	0.79	44 698 885	2 481 000	1 400 700	11 252 847	51 821 624	14 447 973
17	11	6	2009	$1.50 \cdot 10^4$	0.20	45	0.55	3 100 232	204 500	391 500	824 965	1 248 795	119 743
23	17	7	2010	$3.89 \cdot 10^4$	0.20	45	0.55	4 010 562	0	391 500	1 010 014	4 867 857	1 129 757
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 776 491	2 827 000	348 000	-947 101	-1 398 509	-947 101
13	8	5	2008	$6.56 \cdot 10^3$	0.24	53	0.61	2 997 109	791 700	461 100	637 366	345 800	-309 736
19	13	6	2009	$4.28 \cdot 10^4$	0.24	113	0.79	12 800 281	3 067 500	983 100	2 724 791	9 095 482	2 415 055
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 357 623	1 240 500	1 244 100	7 709 609	36 968 504	10 124 664
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 151 822	0	348 000	323 038	-594 687	-624 063
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 975 108	0	348 000	544 915	1 032 421	-79 148
24	18	7	2010	$3.82 \cdot 10^4$	0.12	40	0.51	3 698 950	0	348 000	935 189	4 383 371	856 041
9	5	4	2007	$4.35 \cdot 10^3$	0.16	57	0.65	4 288 660	4 028 475	495 900	-502 223	-235 715	-502 223
14	9	5	2008	$1.93 \cdot 10^4$	0.16	109	0.79	11 487 273	3 166 800	948 300	2 708 178	7 136 458	2 205 955
20	14	6	2009	$7.28 \cdot 10^4$	0.16	169	0.87	23 801 777	3 067 500	1 470 300	6 246 007	26 400 435	8 451 962
26	20	7	2010	$3.63 \cdot 10^5$	0.16	229	0.84	57 494 437	2 481 000	1 992 300	14 658 746	79 421 572	23 110 708
6	3	3	2006	$1.48 \cdot 10^3$	0.18	41	0.58	2 603 928	3 298 450	356 700	-990 111	-1 051 222	-990 111
10	6	4	2007	$6.34 \cdot 10^3$	0.18	101	0.79	7 549 354	4 240 500	878 700	762 951	1 378 933	-227 160
15	10	5	2008	$3.24 \cdot 10^4$	0.18	161	0.86	21 009 493	3 654 000	1 400 700	6 118 181	17 333 725	5 891 021
21	15	6	2009	$1.38 \cdot 10^5$	0.18	221	0.89	46 526 621	3 067 500	1 922 700	13 705 003	58 870 146	19 596 024
27	21	7	2010	$5.23 \cdot 10^5$	0.18	281	0.82	81 298 802	2 481 000	2 444 700	21 175 851	135 243 248	40 771 875

**Table A.33.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, and optimization for the expected profit



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$3.79 \cdot 10^3$	0.29	41	0.56	1 596 390	2 496 900	356 700	−705 934	−1 257 210	−705 934
16	11	6	2009	$5.64 \cdot 10^4$	0.09	131	0.71	15 170 707	4 601 250	1 139 700	2 849 817	8 172 547	2 143 883
22	16	7	2010	$3.00 \cdot 10^5$	0.09	229	0.79	44 818 797	4 052 300	1 992 300	10 594 982	46 946 744	12 738 865
17	11	6	2009	$1.50 \cdot 10^4$	0.20	44	0.54	3 064 569	153 375	382 800	836 481	1 271 184	130 547
23	17	7	2010	$3.89 \cdot 10^4$	0.20	44	0.55	3 994 800	0	382 800	1 008 043	4 883 184	1 138 590
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 762 986	2 827 000	348 000	−953 614	−1 412 014	−953 614
13	8	5	2008	$6.56 \cdot 10^3$	0.24	50	0.58	2 891 039	609 000	435 000	693 335	435 025	−260 279
19	13	6	2009	$4.28 \cdot 10^4$	0.24	117	0.80	12 915 567	3 425 375	1 017 900	2 607 923	8 907 317	2 347 644
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 341 277	1 075 100	1 244 100	7 760 439	36 929 394	10 108 083
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.56	1 146 904	0	348 000	321 061	−613 111	−632 553
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 970 924	0	348 000	543 514	1 009 813	−89 039
24	18	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 704 714	0	348 000	936 797	4 366 527	847 758
9	5	4	2007	$4.35 \cdot 10^3$	0.16	100	0.77	5 036 283	7 067 500	870 000	−2 080 786	−2 901 217	−2 080 786
14	9	5	2008	$1.93 \cdot 10^4$	0.16	200	0.82	11 940 484	6 090 000	1 740 000	1 162 424	1 209 267	−918 361
20	14	6	2009	$7.28 \cdot 10^4$	0.16	300	0.86	23 703 511	5 112 500	2 610 000	5 009 575	17 190 278	4 091 214
26	20	7	2010	$3.63 \cdot 10^5$	0.16	400	0.88	60 221 992	4 135 000	3 480 000	14 450 845	69 797 270	18 542 059
6	3	3	2006	$1.48 \cdot 10^3$	0.18	100	0.60	2 677 456	8 045 000	870 000	−4 540 824	−6 237 544	−4 540 824
10	6	4	2007	$6.34 \cdot 10^3$	0.18	200	0.84	8 057 638	7 067 500	1 740 000	−1 043 288	−6 987 405	−5 584 112
15	10	5	2008	$3.24 \cdot 10^4$	0.18	300	0.89	21 594 745	6 090 000	2 610 000	4 692 622	5 907 339	−891 490
21	15	6	2009	$1.38 \cdot 10^5$	0.18	400	0.91	47 557 597	5 112 500	3 480 000	12 706 899	44 872 436	11 815 409
27	21	7	2010	$5.23 \cdot 10^5$	0.18	459	0.86	85 084 060	2 439 650	3 993 300	21 813 909	123 523 546	33 629 318

**Table A.34.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, and optimization for AV@R



node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$2.38 \cdot 10^1$	1.00	0	0.00	0	0	0	0	0	0
2	1	2	2005	$5.43 \cdot 10^1$	0.29	0	0.00	0	0	0	0	0	0
4	2	3	2006	$1.86 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
7	4	4	2007	$9.64 \cdot 10^2$	0.29	0	0.00	0	0	0	0	0	0
11	7	5	2008	$3.79 \cdot 10^3$	0.29	40	0.55	1 580 492	2 436 000	348 000	−679 458	−1 203 508	−679 458
16	11	6	2009	$5.64 \cdot 10^4$	0.09	100	0.68	14 510 318	3 067 500	870 000	3 335 355	9 369 309	2 655 898
22	16	7	2010	$3.00 \cdot 10^5$	0.09	160	0.71	40 212 394	2 481 000	1 392 000	10 003 178	45 708 704	12 659 075
17	11	6	2009	$1.50 \cdot 10^4$	0.20	42	0.53	2 973 667	102 250	365 400	832 411	1 302 509	152 954
23	17	7	2010	$3.89 \cdot 10^4$	0.20	42	0.53	3 876 544	0	365 400	979 896	4 813 652	1 132 849
3	1	2	2005	$1.09 \cdot 10^2$	0.70	0	0.00	0	0	0	0	0	0
5	3	3	2006	$5.88 \cdot 10^2$	0.52	0	0.00	0	0	0	0	0	0
8	5	4	2007	$2.05 \cdot 10^3$	0.36	40	0.57	1 773 823	2 827 000	348 000	−948 388	−1 401 177	−948 388
13	8	5	2008	$6.56 \cdot 10^3$	0.24	53	0.60	2 989 272	791 700	461 100	634 216	335 295	−314 172
19	13	6	2009	$4.28 \cdot 10^4$	0.24	113	0.79	12 806 134	3 067 500	983 100	2 726 751	9 090 829	2 412 579
25	19	7	2010	$1.99 \cdot 10^5$	0.24	143	0.81	30 356 801	1 240 500	1 244 100	7 709 380	36 963 030	10 121 958
12	8	5	2008	$2.69 \cdot 10^3$	0.12	40	0.57	1 150 374	0	348 000	322 456	−598 803	−625 932
18	12	6	2009	$1.01 \cdot 10^4$	0.12	40	0.52	1 986 510	0	348 000	548 734	1 039 707	−77 198
24	18	7	2010	$3.82 \cdot 10^4$	0.12	40	0.52	3 712 506	0	348 000	938 972	4 404 213	861 774
9	5	4	2007	$4.35 \cdot 10^3$	0.16	60	0.49	0	4 240 500	522 000	−2 705 729	−4 762 500	−2 705 729
14	9	5	2008	$1.93 \cdot 10^4$	0.16	120	0.75	10 924 044	3 654 000	1 044 000	2 208 415	1 463 544	−497 314
20	14	6	2009	$7.28 \cdot 10^4$	0.16	180	0.79	21 767 331	3 067 500	1 566 000	5 532 625	18 597 375	5 035 311
26	20	7	2010	$3.63 \cdot 10^5$	0.16	240	0.77	52 737 145	2 481 000	2 088 000	13 304 365	66 765 520	18 339 677
6	3	3	2006	$1.48 \cdot 10^3$	0.18	60	0.47	0	4 827 000	522 000	−3 654 167	−5 349 000	−3 654 167
10	6	4	2007	$6.34 \cdot 10^3$	0.18	120	0.65	6 215 210	4 240 500	1 044 000	39 839	−4 418 290	−3 614 328
15	10	5	2008	$3.24 \cdot 10^4$	0.18	180	0.82	19 925 494	3 654 000	1 566 000	5 616 116	10 287 204	2 001 788
21	15	6	2009	$1.38 \cdot 10^5$	0.18	240	0.84	43 718 663	3 067 500	2 088 000	12 709 265	48 850 367	14 711 054
27	21	7	2010	$5.23 \cdot 10^5$	0.18	300	0.75	73 704 775	2 481 000	2 610 000	19 010 365	117 464 142	33 721 419

A. Tables with Detailed Results

**Table A.35.:** Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, and optimization for AV@R

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.17 \cdot 10^4$	1.00	25	0.31	0	1 196 667	54 375	−1 248 619	−1 251 042	−1 248 619
2	1	2	2005	$1.74 \cdot 10^4$	1.00	50	0.58	1 606 814	1 115 208	108 750	302 021	−868 186	−946 598
3	2	3	2006	$2.59 \cdot 10^4$	1.00	75	0.68	2 358 726	1 033 750	163 125	971 314	293 665	24 717
4	3	4	2007	$3.87 \cdot 10^4$	1.00	100	0.77	3 320 504	952 292	217 500	1 755 251	2 444 377	1 779 968
5	4	5	2008	$5.77 \cdot 10^4$	1.00	125	0.81	4 404 768	870 833	271 875	2 564 924	5 706 437	4 344 892
6	5	6	2009	$8.61 \cdot 10^4$	1.00	150	0.84	5 762 124	789 375	326 250	3 506 707	10 352 935	7 851 599
7	6	7	2010	$1.28 \cdot 10^5$	1.00	163	0.85	7 271 498	368 117	354 525	4 747 428	16 901 792	12 599 027
8	7	8	2011	$1.92 \cdot 10^5$	1.00	169	0.84	9 033 675	150 350	367 575	5 908 846	25 417 542	18 507 873
9	8	9	2012	$2.86 \cdot 10^5$	1.00	172	0.81	10 953 945	65 400	374 100	6 974 331	35 931 987	25 482 204

**Table A.36.:** Results of quarterly Hamburg planning scenario using the expected traffic evolution, a construction limit of 100, mandatory coverage

node	father	stage	year	KiB/cap	prob	totSites	coverPop	inco	capE	opE	CFDis	aggCF	aggCFDis
1	1	1	2004	$1.17 \cdot 10^4$	1.00	15	0.21	0	718 000	32 625	−749 171	−750 625	−749 171
2	1	2	2005	$1.74 \cdot 10^4$	1.00	30	0.37	0	669 125	65 250	−698 875	−1 485 000	−1 448 047
3	2	3	2006	$2.59 \cdot 10^4$	1.00	45	0.53	1 847 478	620 250	97 875	959 789	−355 647	−488 258
4	3	4	2007	$3.87 \cdot 10^4$	1.00	60	0.63	2 718 168	571 375	130 500	1 658 039	1 660 647	1 169 781
5	4	5	2008	$5.77 \cdot 10^4$	1.00	75	0.69	3 759 811	522 500	163 125	2 428 273	4 734 832	3 598 054
6	5	6	2009	$8.61 \cdot 10^4$	1.00	90	0.74	5 042 584	473 625	195 750	3 310 010	9 108 041	6 908 064
7	6	7	2010	$1.28 \cdot 10^5$	1.00	105	0.76	6 534 816	424 750	228 375	4 260 592	14 989 732	11 168 656
8	7	8	2011	$1.92 \cdot 10^5$	1.00	120	0.77	8 261 167	375 875	261 000	5 282 474	22 614 024	16 451 130
9	8	9	2012	$2.86 \cdot 10^5$	1.00	135	0.75	10 157 059	327 000	293 625	6 317 324	32 150 458	22 768 454

**Table A.37.:** Results of quarterly Hamburg planning scenario using the expected traffic evolution, a construction limit of 60, mandatory coverage

*A. Tables with Detailed Results*

# List of Tables

1.1. Common unit prefixes . . . . .	4
4.1. Notation related to scenario trees . . . . .	33
5.1. Overview of symbols regarding transformation from traffic volume to load . . . . .	55
6.1. Variables and coefficients in optimization models . . . . .	70
7.1. Key properties of planning scenarios . . . . .	84
7.2. Parameter values used for optimization . . . . .	88
7.3. Effect of presolving in the Hamburg scenario for two scenario trees . . . . .	90
7.4. Comparison of optimization over the scenario tree and over the individual scenarios, i. e., the relaxation of the non-anticipativity constraints . . . . .	94
7.5. Comparison of the AV@Rs for the different objectives . . . . .	97
7.6. Comparison of stochastic optimization over scenario tree and optimization with the expected traffic evolution and evaluated on scenario tree. . . . .	98
A.1. Description of the columns . . . . .	105
A.2. Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, and optimization for expected profit . . . . .	106
A.3. Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, mandatory coverage, and optimization for expected profit . . . . .	107
A.4. Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, and optimization for expected profit . . . . .	108
A.5. Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, mandatory coverage, and optimization for expected profit . . . . .	109
A.6. Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, and optimization for AV@R . . . . .	110

## List of Tables

A.7. Results of Berlin planning scenario using the scenario tree with 27 nodes, a construction limit of 10, mandatory coverage, and optimization for AV@R . . . . .	111
A.8. Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, and optimization for AV@R . . . .	112
A.9. Results of Berlin planning scenario using the scenario tree with 32 nodes, a construction limit of 10, mandatory coverage, and optimization for AV@R . . . . .	113
A.10. Results of Berlin planning scenario using the expected traffic evolution, a construction limit of 10 and mandatory coverage . . . . .	114
A.11. Results of Berlin planning scenario using the expected traffic evolution, a construction limit of 10 . . . . .	114
A.12. Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, mandatory coverage, and optimization for expected profit . . . . .	115
A.13. Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, mandatory coverage, and optimization for expected profit . . . . .	116
A.14. Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R . . . . .	117
A.15. Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R . . . . .	118
A.16. Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, mandatory coverage, and optimization for expected profit . . . . .	119
A.17. Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, mandatory coverage, and optimization for expected profit . . . . .	120
A.18. Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R . . . . .	121
A.19. Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R . . . . .	122
A.20. Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 100, mandatory coverage . .	123
A.21. Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 100 . . . . .	123
A.22. Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 60, mandatory coverage . .	124

A.23.Results of annual Hamburg planning scenario using the expected traffic evolution, a construction limit of 60 . . . . .	124
A.24.Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 100, mandatory coverage, and optimization for the expected profit . . . . .	125
A.25.Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 60, mandatory coverage, and optimization for the expected profit . . . . .	126
A.26.Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 100, mandatory coverage, and optimization for AV@R . . . . .	127
A.27.Results of quarterly Hamburg planning scenario using the scenario tree with 53 nodes, a construction limit of 60, mandatory coverage, and optimization for AV@R . . . . .	128
A.28.Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, and optimization for the expected profit . . . . .	129
A.29.Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, and optimization for the expected profit . . . . .	130
A.30.Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 100, and optimization for AV@R . . . . .	131
A.31.Results of Hamburg planning scenario using the scenario tree with 34 nodes, a construction limit of 60, and optimization for AV@R . . . . .	132
A.32.Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, and optimization for the expected profit . . . . .	133
A.33.Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, and optimization for the expected profit . . . . .	134
A.34.Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 100, and optimization for AV@R . . . . .	135
A.35.Results of Hamburg planning scenario using the scenario tree with 27 nodes, a construction limit of 60, and optimization for AV@R . . . . .	136
A.36.Results of quarterly Hamburg planning scenario using the expected traffic evolution, a construction limit of 100, mandatory coverage . . . . .	137
A.37.Results of quarterly Hamburg planning scenario using the expected traffic evolution, a construction limit of 60, mandatory coverage . . . . .	137

*List of Tables*



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