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Balancing Efficiency and Robustness – A Bi-criteria Optimization Approach to Railway Track Allocation

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Balancing Efficiency and Robustness – A Bi-criteria Optimization Approach to Railway Track Allocation*

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Abstract

Technical restrictions and challenging details let railway traffic become one of the most complex transportation systems. Routing trains in a conflict-free way through a track network is one of the basic scheduling problems for any railway company, also known as the train timetabling problem (TTP). This article focuses on a robust extension of the TTP, which typically consists in finding a conflict free set of train routes of maximum value for a given railway network.

Timetables are, however, not only required to be profitable. Railway companies are also interested in reliable and robust solutions. Intuitively, we expect a more robust track allocation to be one where disruptions arising from delays are less likely to propagate and cause delays to subsequent trains. This trade-off between an efficient use of railway infrastructure and the prospects of recovery leads us to a bi-criteria optimization approach. On the one hand, we want to maximize the profit of a schedule, that is the number of routed trains. On the other hand, if two trains are scheduled with a minimum gap the delay of the first one will affect the subsequent train.

We present extensions of the standard integer programming formulation for solving the TTP. These models incorporate both aspects with additional track configuration variables. We discuss how these variables reflect a certain robustness measure. These models can be solved by column generation techniques. We propose scalarization techniques to determine efficient, i.e. the decisions Pareto optimal, solutions. We prove that the LP-relaxation of the TTP including an additional ε-constraint remains solvable in polynomial time. Finally, we present some preliminary computational results on macroscopic real-world data of a part of the German long distance railway network.

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1 Introduction

Constructing a maximum number of train routes in a conflict-free way through a track network is one of the major scheduling problems a railway company faces. From a complexity point of view, this problem turns out to be $\mathcal{NP}$-hard, see Caprara *et al.* (2002), but nevertheless in the literature several optimization models were discussed which are able to solve real-life instances to near-optimality, see Brannlund *et al.* (1998), Caprara *et al.* (2002), Borndörfer *et al.* (2006), Cacchiani *et al.* (2008), Cacchiani (2007) and Fischer *et al.* (2008) for more details. All of these articles model the track allocation problem in terms of a multi-commodity flow of trains in an appropriate time expanded digraph. Feasibility is ensured by additional packing constraints, which rule out conflicts between the routes.

In Borndörfer and Schlechte (2007), a different approach was introduced, which handles conflicts not in terms of constraints, but in terms of additional variables. Its path formulation is amenable to standard column generation techniques and therefore suited for large-scale computation. The major contribution of this paper is that these additional variables can be used to measure robustness in terms of implicit available buffer times of a timetable. We refrain from supporting this by recent statistics to punctuality and reliability of any railway company. But obviously, decision makers are more and more sensitive to the importance of finding a good compromise between profitable and reliable timetables.

Robust optimization, that means the incorporation of data uncertainties through mathematical models in its original definition as proposed by Soyster (1973), is not applicable to large scale optimization problems. Moreover these models produce too conservative solutions, which are resistant against all considered eventualities, but far away from implementable in real world. Robust optimization, however, has become a fruitful field recently because more and more optimization problems can be solved in adequate time. This opens the door to additionally deal with stochastic assumptions instead of only nominal given data. In Ben-Tal and Nemirovski (1998) and El-Ghaoui *et al.* (1998), less conservative models were introduced, which adjust the robustness of the solution by some protection level parameters. Bertsimas and Sim (2003) survey robust optimization theory and its network flow applications. Kroon *et al.* (2006), Liebchen *et al.* (2007) and Fischetti *et al.* (2007) apply these robust considerations to the world of railways. They investigate a cyclic version of the timetabling problem, modelled as Periodic Event Scheduling Problem and introduce a stochastic methodology of Light Robustness. For the detailed routing through stations or junctions, Caimi *et al.* (2004) and Delorme *et al.* (2009) proposed approaches to find delay resistant and stable routings. The aim of these considerations is to gain more insights into the trade-off between efficiency and robustness of solutions and find the 'price of robustness'.

We focus on a pure combinatorial optimization approach, which is somehow related to Ehrgott and Ryan (2002) and Ehrgott *et al.* (2007), broaching the issue of robustness in airline crew scheduling. We consider robustness (available buffer times, quality of day-to-day operations) and efficiency (used track kilometers, planned capacity utilization) to be incomparable entities and consequently favor a bi-criteria optimization approach.

The organization of this article is as follows: In Section 2, we briefly recall a version of the train timetabling or track allocation problem. In Section 3, we introduce the corresponding linear programming formulation ACP using arc variables and PCP using path variables, respectively. Then, we extend these models to measure robustness, which leads directly to an bi-criteria optimization approach of the problem. To determine efficient solutions of the bi-criteria models we propose so-called scalarization techniques, see Ehrgott (2005). Section 4 discusses details on a straight-forward column generation approach to solve the scalarized optimization problem. We prove that the LP-relaxation of the PCP including an additional ε-constraint remains solvable in polynomial time. Finally, we present some preliminary computational results for the model ACP on a part of the German Railway Network in Section 5. Let us point out explicitly that we do not claim these results are already practically significant; we only want to show the potential of our approach on real-world-data as a proof of concept.

2 The Track Allocation Problem

The track allocation problem in its single objective version can be formally defined in terms of several digraphs $D = (V,A)$. By $\delta_{\text{in}}(v)$ we denote the set of incoming arcs $a \in A$ for $v \in V$, by $\delta_{\text{out}}(v)$ the set of outgoing arcs, respectively. Arrivals and departures of trains at a set S of stations at discrete times $T \subseteq \mathbf{N}$ are represented by the nodes $v \in V$, arcs model activities like runs of trains between stations or turnovers and dwelling inside a station. An explicit running time supplement is usually included to assure operational feasibility. For each $v \in V$, we denote by $\sigma(v) \in S$ the station associated with departure or arrival and by $\tau(v) \in T$ the time of this event; we assume $\tau(u) < \tau(v)$ for each arc $uv \in A$ such that D is acyclic; denote by $J = \{\sigma(u)\sigma(v) : uv \in A\}$ the set of all railway tracks.

We are further given a set I of requests to route trains through D. More precisely, train $i \in I$ can be routed on a path through some suitably defined digraph $D_i = (V_i,A_i)$ from a source node $s_i \in V_i$ to a sink node $t_i \in V_i$; let P_i be the set of all routes for train $i \in I$, and $P = \bigcup_{i \in I} P_i$ the set of all train routes (taking the disjoint union). An arc $uv \in A$ *blocks* the underlying track $\sigma(u)\sigma(v)$ during the time interval $[\tau(u), \tau(v) - 1]$. Two arcs $a,b \in A$ are *in conflict* if their respective blocking intervals overlap, and two routes $p,q \in P$ are in conflict if any of their arcs are in conflict.*

A *track allocation* or *timetable* is a set of conflict-free routes, at most one for each train. Given arc weights w_a, $a \in A$, the weight of route $p \in P$ is $w_p = \sum_{a \in p} w_a$, and the weight of a track allocation $X \subseteq P$ is $w(X) = \sum_{p \in X} w_p$. The *train*

*In reality, train conflicts are more complex. For simpler notation, though, we avoid the introduction of headway matrices and train types.

timetabling problem (TTP) in its single objective version is to find a conflict-free track allocation of maximum weight.

TTP can be seen as a multi-commodity flow problem with additional packing constraints usually ensured by inequalities in the models. In Borndörfer and Schlechte (2007), we have proposed an alternative formulation that is based on valid 'configurations'. These are sets of arcs on the same underlying track that are mutually conflict-free. Formally, let $A_{st} = \{uv \in A : \sigma(u)\sigma(v) = st\}$ be the set of all arcs associated with some track $st \in J$; a *configuration* for this track st is a set of arcs $q \subseteq A_{st}$ that are mutually not in conflict, i.e. in our simplified case not overlapping. Let Q_j denote the set of all configuration associated with track $j \in J$, and $Q = \bigcup_{j \in J} Q_j$ the set of all configurations. By $A_{LR} = \bigcup_{st \in J} A_{st}$, we denote the set of all 'forward' arcs.

For the construction of configurations, we introduce *track digraphs* $D_j = (V_j, A_j)$ on each track $j \in J$. Consider the forward arcs $A_{xy} = \{uv \in A : \sigma(u)\sigma(v) = xy\}$ on a track $j = xy \in J$. Denote by $L_{xy} := \{u : uv \in A_{xy}\}$ and $R_{xy} := \{v : uv \in A_{xy}\}$ the associated set of departure and arrival nodes. Construct two new, additional nodes s_{xy} and t_{xy} by setting $\sigma(s_{xy}) = y$, $\tau(s_{xy}) := \min \tau(R_{xy}) - 1$, and $\sigma(t_{xy}) = x$, $\tau(t_{xy}) := \max \tau(R_{xy}) + 1$, i.e., s_{xy} marks an artificial source node at station y before the departure of the earliest trip on xy, and t_{xy} marks an artificial sink node at station x after the arrival of the latest trip on xy. Let $\overline{L}_{xy} := L_{xy} \cup \{t_{xy}\}$ and $\overline{R}_{xy} := R_{xy} \cup \{s_{xy}\}$; note that all arcs in A_{xy} go from L_{xy} to R_{xy}. Now let $A_{RL} := \{vu : \tau(v) \leq \tau(u), v \in \overline{R}_{st}, u \in \overline{L}_{st}\}$ be a set of 'backward' arcs that go in the opposite direction; they connect the arrival of a trip on xy (or node s_{xy}) with all possible follow-on trips (or node t_{xy}) on that track. Table 1 summarizes the notation. Figure 1 illustrates the construction. On the left, a complete track digraph without s_{xy} and t_{xy} is shown. The three subgraphs on the right correspond to feasible configurations on that track digraph. They are, from left to right gradually more robust.

Finally, it is easy to observe, that per definition:

- each train digraph D_i is acyclic
- each track digraph D_j is acyclic and bipartite
- each arc $a \in A_{LR}$ is part of exactly one train digraph D_i and one track digraph D_j
- there is an isomorphism between Q_j and the set of all $s_{xy}t_{xy}$-paths in D_j.

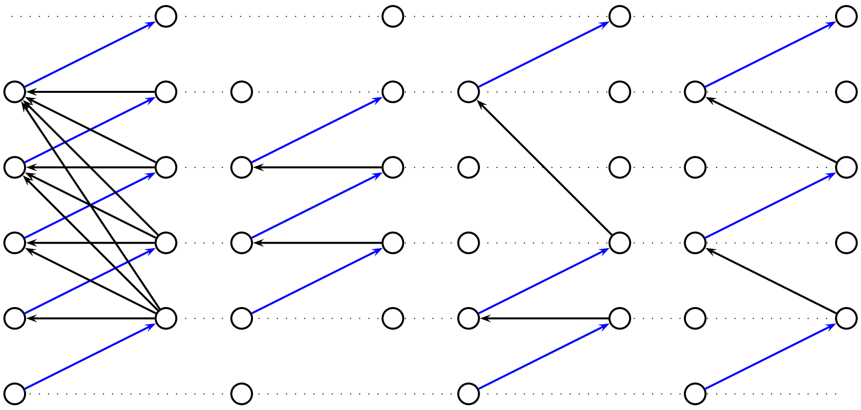

Figure 1: Routing digraph D_j and from fragile q_1 and q_2 to robust configuration q_3.

3 Towards a Bi-criteria Optimization Approach

Introducing 0/1-variables x_p, $p \in P$, and y_q, $q \in Q$, the single objective TTP can be stated as the following integer program:

Table 1: Notation for the Train Timetabling Problem.

symbol	meaning
S	stations
J	tracks
$G=(S,J)$	infrastructure digraph
I	train requests
w	arc weights (profit)
r	arc weights (robustness)
$\sigma : V \mapsto S$	mapping of nodes to stations
$\tau : V \mapsto \mathbf{N}$	mapping of nodes to time
s_i, t_i	source, sink of train $i \in I$
$D_i = (V_i, A_i)$	train digraph of $i \in I$
s_{xy}, t_{xy}	source, sink of track $j = xy \in J$
$D_j = (V_j, A_j)$	track digraph of $j \in J$
P_i	set of s_i, t_i -paths in D_i
Q_j	set of s_{xy}, t_{xy} -paths in D_j
A_{LR}	coupling arcs
A_{RL}	backward arcs
$A := A_I \cup A_J$	all arcs

$$
\begin{array}{llllr}
\text{(PCP)} & & & & \\
\max & \sum\limits_{p \in P} w_p x_p & & & \text{(i)} \\
\text{s.t.} & \sum\limits_{p \in P_i} x_p & \leq 1, & \forall\, i \in I & \text{(ii)} \\
& \sum\limits_{q \in Q_j} y_q & \leq 1, & \forall\, j \in J & \text{(iii)} \\
& \sum\limits_{p \in P, a \in p} x_p - \sum\limits_{q \in Q, a \in q} y_q & \leq 0, & \forall\, a \in A_{LR} & \text{(iv)} \\
& x_p, y_q & \geq 0, & \forall\, p \in P,\ q \in Q & \text{(v)} \\
& x_p, y_q & \in \{0,1\}, & \forall\, p \in P,\ q \in Q. & \text{(vi)}
\end{array}
$$

The objective PCP (i) maximizes the weight of the track allocation. Constraints (ii) state that a train can run on one route only, Constraints (iii) allow at most one configuration for each track. Inequalities (iv) couple train routes and track configurations to guarantee a conflict-free allocation and (v) and (vi) are the non-negativity and integrality constraints. Note that the upper bounds $x_p \leq 1$, $p \in P$, and $y_q \leq 1$, $q \in Q$, hold implicitly.

An arc based version can be formulated as well. Variables x_a, $a \in A_i$, $i \in I$

control the use of trip a in D_i and y_a, $a \in A_j$, $j \in J$ in D_j, respectively;

$$
\begin{array}{llll}
\text{(ACP)} & & & \\
\max & \sum\limits_{a \in A} w_a x_a & & \text{(i)} \\
\text{s.t.} & \sum\limits_{a \in \delta^i_{\text{out}}(v)} x_a - \sum\limits_{a \in \delta^i_{\text{in}}(v)} x_a = 0, & \forall\, i \in I, v \in V_i \backslash \{s_i, t_i\} & \text{(ii)} \\
& \sum\limits_{a \in \delta^i_{\text{out}}(s_i)} x_a \leq 1, & \forall\, i \in I & \text{(iii)} \\
& \sum\limits_{a \in \delta^i_{\text{out}}(v)} y_a - \sum\limits_{a \in \delta^i_{\text{in}}(v)} y_a = 0, & \forall\, j \in J, v \in V_j \backslash \{s_j, t_j\} & \text{(iv)} \\
& \sum\limits_{a \in \delta^i_{\text{out}}(s_j)} x_a \leq 1, & \forall\, j \in J & \text{(v)} \\
& x_a - y_a \leq 0, & \forall\, a \in A_{LR} & \text{(vi)} \\
& x_a, y_a \geq 0, & \forall\, a \in A & \text{(vii)} \\
& x_a, y_a \in \{0,1\}, & \forall\, a \in A. & \text{(viii)}
\end{array}
$$

As before, the objective, denoted in ACP (i), is to maximize the weight of the track allocation. Equalities (ii) and (iii) are well-known *flow conservation constraints* for all trains $i \in I$, (iv) and (v) for all tracks $j \in J$, respectively. Inequalities (vi) link arcs used by train routes and track configurations to ensure a conflict-free allocation, (vii) and (viii) are the non-negativity and the integrality constraints.

Let us explain the incorporation of robustness on a simple example. By r_q we denote a robustness value for each configuration $q \in Q$. We assume that a high robustness value r_q means configuration q is robust and a smaller the contrary. As a simplificiation, we expect $r_q = \sum_{a \in q} r_a$, i.e. the robustness of a track can be expressed as the sum of the robustness.

Figure 1 should illustrate the idea on an a single track. Considering the track digraph D_j on the left induced by three train requests. Straight forwardly maximizing the number of scheduled trains in our setting will always lead to a schedule with profit value 3, but as you can see this can result in a lot of varying schedules, in fact all *LR*-paths of length 5, i.e. the three shown in Figure 1. We are given a desired implicit buffer $b \in \mathbf{N}$, i.e. 5 minutes, which we maximally want to hedge against. Then the following robustness function $r : \mathbf{R}^{|A|} \to \mathbf{R}$ with

$$
r((u,v)) := \begin{cases} \sqrt{b} & (u,v) \in A_{RL} \text{ and } t(v) - t(u) > b, \\ \sqrt{t(v) - t(u)} & (u,v) \in A_{RL} \text{ and } t(v) - t(u) \leq b, \\ 0 & \text{otherwise} \end{cases}
$$

will measure the available buffers appropriately. The robustness function r benefits values near to b and balances the partition of the available implicit buffer times by its concaveness. Assume $b = 2$ in our example in Figure 1. Then the first configuration q_1 has value $r_{q_1} = 0$, for the second configuration r_{q_2} is $\sqrt{2}$ and the third one has $r_{q_3} = 2$. (For the sake of completeness we set r_q to a sufficiently big M for an empty configuration q.)

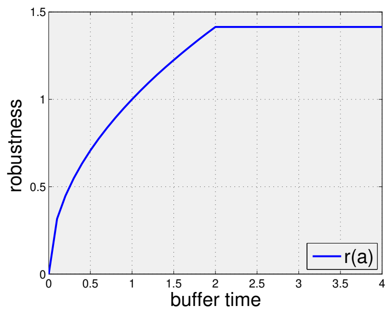

Figure 2: Function $r(a)$ of a single buffer arc $a \in A_{RL}$.

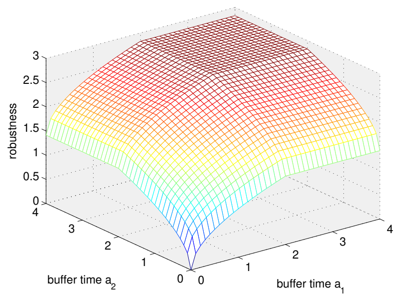

Figure 3: Robustness function of two buffer arcs.

We can easily extend ACP and PCP to bi-criteria models by taking this second, obviously contradictory, objective into account:

$$\text{PCP (i')} \quad \max \sum_{q \in Q} r_q y_q \quad \text{or analogously} \quad \text{ACP (i')} \quad \max \sum_{a \in A} r_a y_a.$$

To find all efficient solutions, we propose a straight-forward combined weighted sum and ε-constraint hybrid method, see Ehrgott (2005). Considering model PCP, this leads to the following objective function with a scalar $\alpha \in [0,1]$:

$$\max \ \alpha(\sum_{p \in P} w_p x_p) + (1-\alpha)(\sum_{q \in Q} r_q y_q)$$

and to an additional constraint on one of the objectives, i.e.

$$\sum_{p \in P} w_p x_p \geq \varepsilon \quad \text{or} \quad \sum_{q \in Q} r_q y_q \geq \varepsilon.$$

In practice, we could choose an allowed deviation from a profit maximizing schedule. Let v_{opt} be the optimal value derived by solving ACP or PCP. By ν we denote a given percentage of decrease in the profit function. Considering $\varepsilon = (1-\nu)v_{opt}$ and adding an ε-constraint for the first objective, we receive:

$$
\begin{array}{llllr}
((\alpha,\varepsilon)-\text{PCP}) & & & & \\
\max & \alpha(\sum_{p\in P} w_p x_p) + (1-\alpha)(\sum_{q\in Q} r_q y_q) & & & \text{(i)} \\
\text{s.t.} & \sum_{p\in P_i} x_p & \leq 1, & \forall i\in I & \text{(ii)} \\
& \sum_{q\in Q_j} y_q & \leq 1, & \forall j\in J & \text{(iii)} \\
& \sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q & \leq 0, & \forall a\in A_{LR} & \text{(iv)} \\
& \sum_{p\in P} w_p x_p & \geq \varepsilon, & & \text{(v)} \\
& x_p, y_q & \geq 0, & \forall p\in P,\ q\in Q & \text{(vi)} \\
& x_p, y_q & \in \{0,1\}, & \forall p\in P,\ q\in Q. & \text{(vii)}
\end{array}
$$

4 Details on Column Generation

The LP-relaxation PLP of PCP, i.e., PLP = PCP (i)–(v) can be solved efficiently by column generation. The added ε-constraint changes the structure, only slightly, as we will show. Fortunately, it will turn out that the pricing problems remain solvable in polynomial time (by computing longest paths in acyclic digraphs D_i and D_j). To see this, consider the dual DLP of the LP-relaxation of $((\alpha,\varepsilon)-\text{PCP})$, i.e. neglecting constraints $((\alpha,\varepsilon)-\text{PCP})$ (vii):

$$
\begin{array}{llllr}
\text{(DLP)} & & & & \\
\min & \sum_{j\in J}\pi_j + \sum_{i\in I}\gamma_i - \varepsilon\rho & & & \text{(i)} \\
\text{s.t.} & \gamma_i + \sum_{a\in p}\lambda_a - w_p\rho & \geq \alpha w_p & \forall p\in P_i,\ i\in I & \text{(ii)} \\
& \pi_j - \sum_{a\in q}\lambda_a & \geq (1-\alpha) r_q & \forall q\in Q_j,\ j\in J & \text{(iii)} \\
& \gamma_i, \pi_j, \lambda_a, \rho & \geq 0 & \forall i\in I,\ j\in J,\ a\in A_{LR}. & \text{(iv)}
\end{array}
$$

Here, γ_i, $i\in I$, π_j, $j\in J$, λ_a, $a\in A_{LR}$ and ρ, are the dual variables associated with constraints $((\alpha,\varepsilon)-\text{PCP})$ (ii), (iii),(iv) and (v), respectively. The pricing problem for a route $p\in P_i$ for train $i\in I$ is then:

$$
\exists\, p\in P_i : \gamma_i + \sum_{a\in p}\lambda_a - w_p\rho < \alpha w_p \iff \sum_{a\in p}((\alpha+\rho)w_a - \lambda_a) > \gamma_i.
$$

This is the same as finding a longest s_it_i-path in D_i w.r.t. arc weights $(\alpha+\rho)w_a - \lambda_a$; as D_i is acyclic, this problem can be solved in polynomial time.

The pricing problem for a configuration $q\in Q_j$ for track $j\in J$ (w.r.t. the additional ε-constraint (v)) is:

$$
\exists\, q\in Q_j : \pi_j - \sum_{a\in q}\lambda_a < (1-\alpha) r_q \iff \sum_{a\in q}(\lambda_a + (1-\alpha) r_a) > \pi_j.
$$

Using arc weights $(\lambda_a + (1-\alpha)r_a)$, $a \in A_{LR}$, and 0 otherwise, pricing configurations Q_j is equivalent to finding a longest $s_j t_j$-path in D_j. As D_j is acyclic, this is polynomial. By the polynomial equivalence of separation and optimization, see Grötschel *et al.* (1988), here applied to the DLP, we obtain:

Theorem 1 *The LP-relaxation of* $((\alpha,\varepsilon)-\mathrm{PCP})$ *is solvable in polynomial time.*

5 Preliminary Computational Results

We consider the Hanover-Kassel-Fulda area of the German long-distance railway network. All instances are based on the macroscopic infrastructure network provided by our project partners from departments for railway track and operations. The data was produced by suited aggregation to minutes based on detailed microscopic simulation results (with a precision of seconds). The network consists of 37 stations, 120 tracks and 6 different train types (ICE, IC, RE, RB, S, ICG). Because of various possible turnover and driving times for each train type, this produces an infrastructure digraph with 198 nodes and 1140 arcs. For the construction of correct track digraphs, we stick to 4320 realistic headway times. We present here the results for one scenario with 285 requested trains.[†] Based on the 2002 timetable of Deutsche Bahn AG, we considered all trains in a time interval of about 6 hours at a regular weekday from 9:00 to 15:00. This leads to a representable mix of long distance trains (IC, ICE), synchronized regional and suburban passenger trains (S, RE, RB), and freight trains (ICG). The flexibility to reroute trains is set to departure and arrival time windows of 6 minutes length. Maximizing the total number of trains in the schedule is our first objective[‡]; the second goal is to maximize our defined robustness measure, choosing $b = 6$ minutes.

Table 2: Sizes of the digraph for the ACP instance.

$\|I\|$	$\|J\|$	$\|V_I\|$	$\|A_I\|$	$\|V_J\|$	$\|A_J\|$
285	111	14147	20125	16014	59467

Table 2 lists the key numbers for the instance after some trivial graph preprocessing. We only present results for the linear relaxation of model ACP, i.e. an LP with 79592 columns and 38057 rows. We computed the solutions of the LP relaxation with the barrier method of CPLEX 11.0, see CPLEX (2007), for 21 different values of α, taking $\alpha = \frac{i}{20}$ for $i = 0 \ldots 20$.; each run takes about 30 seconds. [§] All

[†]This scenario can be downloaded as part of the TTPlib 2008, see Erol *et al.* (2008), at `ttplib.zib.de`, i.e HAKAFU_SIMPLE_37_120_6_REQ02_0285_0331_6.XML.

[‡]Furthermore, we slightly penalize deviations from certain desired departure and arrival times at visiting stations.

[§]In addition CPLEX MIPSolve needs only some minutes and a few hundred branch and bound nodes to find an IP solution with an optimality gap of at most 2 %.

computations were made single threaded on a Dell Precision 650 PC with 2GB of main memory and a dual Intel Xeon 3.8 GHz CPU running SUSE Linux.

The right part of Figure 4 shows both objectives depending on α. The extreme cases are as expected: For $\alpha = 0$, only the robustness measure contributes to the objective and is therefore maximized as much as possible at the expense of scheduling only some trains. For $\alpha = 1$, the robustness measure does not contribute to the objective and is therefore low, while the total profit is maximal. With increasing α, the total robustness monotonically decreases, while the total profit increases. On the left part of Figure 4 the Pareto frontier can be seen. Note that each computed pair of total robustness and profit constitutes a Pareto optimal point, i.e., is not dominated by any other attainable combination. Conversely, any Pareto optimal solution of the LP relaxation can be obtained as the solution for some $\alpha \in [0, 1]$, see, e.g., Ehrgott (2005).

Some extensive computational studies, especially for the IP case, are needed to decide whether the results presented in this paper are useful in practice. Nevertheless, we are confident that our contribution helps to balance efficiency and robustness in the near railway future.

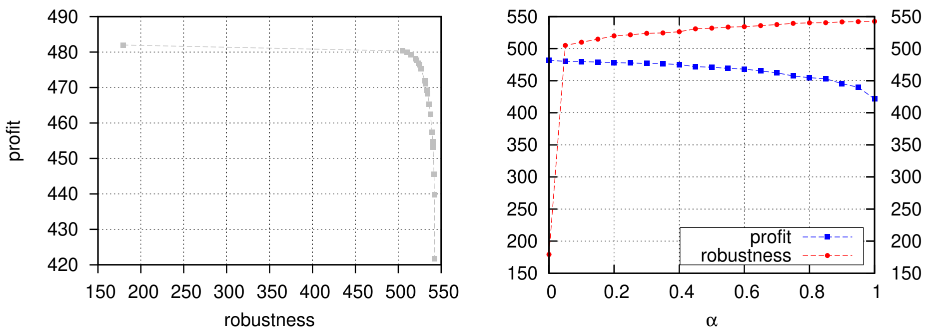

Figure 4: Total profit objective (blue, left axis) and total robustness objective (green, right axis) in dependence on α.

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