

Essays on the Valuation of Commodity Derivatives

Inaugural Dissertation

in Partial Fulfillment of the Requirements for the Degree of
Doctor Rerum Politicarum (Dr. rer. pol.)

WHU – Otto Beisheim School of Management

Submitted by
Janis Back

July 4, 2011

Referee: Prof. Dr. Markus Rudolf
Co-Referee: Prof. Dr. Christian Koziol

Acknowledgements

I am deeply indebted to my advisor Professor Markus Rudolf for helpful guidance, important advice and constant encouragement and support during this work. I also wish to thank Professor Christian Koziol for valuable comments and for co-refereeing this dissertation. Moreover, I want to express my profound appreciation to Professor Stephen Figlewski and Professor John Mulvey for giving me the possibility to work under their guidance and hosting me as visiting scholar at the Stern School of Business (NYU) and Princeton University, respectively. These visits helped to shape my research project and have been valuable experiences. I also want to thank Dr. Marcel Prokopczuk and my colleagues from the Chair of Finance for fruitful discussions and constructive comments.

Contents

Contents	ii
List of Tables	v
List of Figures	vi
List of Abbreviations	vii
List of Essential Symbols	ix
1 Introduction	1
2 Commodity Price Dynamics and Derivatives Valuation:	
A Review	5
2.1 Introduction	5
2.2 Theoretical Background and Empirical Price Behavior	6
2.2.1 Backwardation and Contango	6
2.2.2 Mean-Reversion	11
2.2.3 Samuelson Effect	13
2.2.4 Seasonality	15
2.3 Valuation Models for Commodity Derivatives	17
2.3.1 Spot Price Models	18
2.3.2 No-Arbitrage Models of the Futures Curve	22
2.3.3 Jumps	22
2.3.4 Constant, Deterministic, and Stochastic Volatility	24
2.3.5 Seasonality	26

2.3.6	Further Extensions	27
2.4	Conclusion	28
3	Seasonality and the Valuation of Commodity Options	30
3.1	Introduction	30
3.2	Empirical Evidence on Seasonality in Commodity Markets	33
3.3	Valuation Models	39
3.3.1	Price Dynamics	39
3.3.2	Valuation of Futures and Options	42
3.4	Data Description and Estimation Procedure	44
3.4.1	Data	44
3.4.2	Model Estimation	47
3.5	Empirical Model Comparison	49
3.5.1	Estimated Parameters	49
3.5.2	In-Sample Model Comparison	49
3.5.3	Out-of-Sample Model Comparison	54
3.5.4	Analysis of Pricing Errors	60
3.5.5	Robustness Check	64
3.6	Conclusion	65
3.7	Appendix	66
4	Seasonal Stochastic Volatility: Implications for the Pricing of Commodity Options	68
4.1	Introduction	68
4.2	Model Description	71
4.2.1	Commodity Futures Price Dynamics	72
4.2.2	Valuation of Options	73
4.3	Data Description and Estimation Procedure	75
4.3.1	Data	75
4.3.2	Estimation Approach	78
4.4	Results	81
4.4.1	Estimated Parameters	81
4.4.2	Pricing Performance	86
4.4.3	Robustness Checks	93

4.5	Conclusion	97
4.6	Appendix	99
5	Conclusion	101
	Bibliography	xii

List of Tables

3.1	Sample Description of Soybean Futures Options	45
3.2	Sample Description of Heating Oil Futures Options	46
3.3	Implied Parameter Estimates: Soybean Options	50
3.4	Implied Parameter Estimates: Heating Oil Options	51
3.5	In-Sample Pricing Errors: Soybean Options	52
3.6	In-Sample Pricing Errors: Heating Oil Options	53
3.7	Out-of-Sample Pricing Errors: Soybean Options	55
3.8	Out-of-Sample Pricing Errors: Heating Oil Options	56
3.9	Reduction of Pricing Errors: Soybean Options	58
3.10	Reduction of Pricing Errors: Heating Oil Options	59
3.11	Regression Analysis of Out-of-Sample Pricing Errors	63
4.1	Sample Description of Natural Gas Futures Options	77
4.2	Physical Measure Parameter Estimates	82
4.3	Risk-Neutral Measure Parameter Estimates	83
4.4	In-Sample Pricing Errors: Fitted to Implied Volatilities	88
4.5	In-Sample Pricing Errors: Fitted to Prices	89
4.6	Out-of-Sample Pricing Errors: Fitted to Implied Volatilities	91
4.7	Out-of-Sample Pricing Errors: Fitted to Prices	92
4.8	Reduction of Pricing Errors: Fitted to Implied Volatilities	94
4.9	Reduction of Pricing Errors: Fitted to Prices	95

List of Figures

2.1	Different Shapes of Futures Curves for Crude Oil	7
2.2	Inventory Levels and Basis for Copper Futures	11
2.3	Term Structure of Volatility in Crude Oil Futures	14
3.1	Seasonal Pattern of Futures Prices	35
3.2	Seasonal Pattern of Historical Volatility	37
3.3	Seasonal Pattern of Implied Volatility	38
3.4	Out-of-Sample Mean Pricing Errors by Maturity Months	61
4.1	Historical Volatility of Natural Gas Futures	70
4.2	Estimated Current Volatility Level	84
4.3	Estimated Market Price of Variance Risk	85

List of Abbreviations

ATM	At-the-money
B.C.	Before Christ
CARMA	Continuous autoregressive moving average
CBOT	Chicago Board of Trade
CME	Chicago Mercantile Exchange
e.g.	exempli gratia; for example
i.e.	id est; that is
ITM	In-the-money
IV-RMSE	Root mean squared error of implied volatilities
kt	Kilotonnes
Libor	London interbank offered rate
log	Logarithm
LME	London Metal Exchange
MCMC	Markov chain Monte Carlo
mmBtu	Million British thermal units
MPE	Mean percentage error
NYMEX	New York Mercantile Exchange
ODE	Ordinary differential equation
OTM	Out-of-the-money
p.	Page
RMSE	Root mean squared error
RRMSE	Relative root mean squared error
\$_RMSE	Root mean squared error of prices
SSV	Seasonal stochastic volatility
SV	Stochastic volatility

UK	United Kingdom of Great Britain and Northern Ireland
US	United States
USD	United States Dollar
vol	Volatility
WTI	West Texas Intermediate

List of Essential Symbols

$\arg \min(\cdot)$	Argument of the minimum
β_0	Regression constant
β_i	Regression coefficient for variable i
c_t	Price of a call option at time t
$CALL$	Dummy variable: Call option
dt	Instantaneous step in time
Δt	Unit of time
$E(\cdot)$	Expected value
e	Euler's number
ϵ	Error term
$\epsilon_t^V, \epsilon_t^Y$	Normal random variables
η	Amplitude of the seasonality function (Chapter 4)
f	Characteristic function
$F_t(T)$	Price of futures contract with maturity T at time t
$Feb \dots Dec$	Monthly dummy variables: February ... December
G	Number of draws in MCMC estimation procedure
i	Imaginary unit
$IV_{t,i}$	Implied volatility for market price of option i at time t
$\hat{IV}_{t,i}$	Implied volatility for model price of option i at time t
K	Strike price
κ	Mean-reversion speed
\ln	Natural logarithm
λ	Market price of variance risk
MAT	Independent variable: Time to maturity
$\max(\cdot)$	Maximum

$\min(\cdot)$	Minimum
MON	Independent variable: Moneyness
μ	Drift parameter
N_t	Number of options at time t
$N(\cdot)$	Standard normal cumulative distribution function
p_t	Price of a put option at time t
P_i	Weighting function in Heston formula
$P_{t,i}$	Market price of option i at time t
$\hat{P}_{t,i}$	Model price of option i at time t
PE	Absolute pricing error
$\varphi(t)$	Deterministic function for seasonality in volatility
Φ	Vector with parameters
Φ^*	Vector with optimal parameters
Q	Risk-neutral measure
r	Risk-free rate
R^2	Coefficient of determination
$\text{Re} [\cdot]$	Returns the real part of a complex expression
ρ	Correlation
$s(t)$	Deterministic function for seasonality at the price level
S_t	Spot price at time t
\sin	Sinus
σ	Volatility
Σ	Sum
t	Time index
T	Maturity
T_{max}	Maximum time to maturity
τ	Time index
θ	Amplitude of the seasonality function (Chapter 3)
$\theta(t)$	Time dependent long-term variance level (Chapter 4)
$\bar{\theta}$	Long-term mean variance level
Θ	Set of model parameters
V_t	Variance at time t
$\text{Var}(\cdot)$	Variance

W_t	Standard Brownian motion
X_t	State variable at time t
y	Convenience yield
Y	Vector of observed log futures prices
Y_t	State Variable at time t
Z_t	Standard Brownian motion
ζ	Parameter describing the shift of the seasonality function along the time axis
¢	US cent
$\text{\$}$	US dollar
\in	Element of
∞	Infinity
\propto	Proportional to

Chapter 1

Introduction

Commodities are consumable physical assets that play a central role in economic growth and welfare. Trading in physical assets often involves high transaction costs. For this reason, trading in commodities takes place primarily in futures markets. Futures contracts reflect expectations of market participants regarding future spot prices and are subject to the interaction of demand and supply for different delivery dates. Intertemporal consumption smoothing is facilitated by inventories, which can absorb disruptions in demand or supply. Accordingly, there exists a close relationship between spot and futures markets and inventory levels. Commodity derivatives, like futures contracts, have the function of making commodity risks tradable and allowing for a more efficient risk allocation.

From a portfolio management perspective, commodities are frequently considered to have an attractive risk-return profile. Analyzing the properties of commodities as an asset class, it is usually found that adding commodity investments to a portfolio leads to a more efficient asset allocation. Often, a significant diversification potential of commodities is noted due to a low or negative correlation with other traditional asset classes. Furthermore, commodities are found to have the property of being a good inflation hedge. Studies regarding the benefits from commodity investments comprise the work of, e.g., Bodie and Rosansky (1980), Rudolf et al. (1993), Jensen et al. (2000, 2002), Mulvey et al. (2004), Gorton and Rouwenhorst (2006), Erb and Harvey (2006), Miffre and Rallis (2007), and Geman and Kharoubi (2008).

In comparison to other financial markets, commodity markets exhibit several peculiarities. From a financial perspective, e.g., storage and transportation costs and the perishability of goods hinder the application of standard arbitrage relationships for the valuation of commodity contingent claims. Furthermore, commodity price dynamics are regularly characterized by mean-reversion and can display jumps and high levels of volatility – due to scarcity for a commodity while demand is relatively inelastic, for example. The special nature of commodities implies that their price dynamics exhibit some unique characteristics which need to be considered when deciding about hedging strategies or when valuing commodity contingent claims. Thereby, a thorough understanding of commodity price dynamics is important for hedgers and investors alike. Chapter 2 of this thesis is devoted to highlighting and discussing these aspects.

A further stylized fact of commodities is that price movements in many commodity markets show significant seasonal patterns. The seasonal behavior of commodity price dynamics can be induced either by the demand or by the supply side. Prominent examples are weather-related demand patterns for energy commodities and the varying supply of agricultural commodities according to harvesting cycles. As such, seasonality is present at the price level and, furthermore, many commodity markets contain a strong seasonal component in volatility. While seasonality at the price level and its implications for the valuation of commodity futures is widely recognized and relatively well understood, literature considering seasonality in volatility is very limited. This doctoral thesis aims to shed light on the seasonal variations in volatility and their implications for commodity derivatives prices.

It is of vital importance that the pronounced seasonal variations in volatility be taken into account, for both risk management and the pricing of commodity derivatives. This thesis concentrates on the latter aspect and contributes to the literature concerning the valuation of commodity contingent claims. Since assumptions regarding volatility are especially important for options prices, this thesis focuses on the role of seasonal volatility for the pricing of commodity options.

Motivated by the observation of these pronounced seasonal patterns in volatility, we propose different commodity derivatives valuation models, which capture

this seasonal behavior. Using extensive data samples of commodity options, we empirically study the pricing accuracy of the presented models. In a first study, we generalize one- and two-factor models by allowing for time-varying, seasonal volatility. In a second study, we propose a stochastic volatility model where the variance process contains a seasonal component. We contribute to the literature by documenting the importance of seasonal volatility and, most importantly, by presenting how this can be considered in commodity options pricing models.

Organization of the Thesis

Chapter 2 reviews the literature on commodity price dynamics and the valuation of commodity derivatives. It comprises two main parts: First, we discuss the theoretical background and empirical observations with regard to the relationship between commodity spot and futures contracts and their price dynamics. Second, we give a comprehensive overview of valuation models for commodity derivatives. In this manner, Chapter 2 identifies the research gap in the literature which will be addressed in the empirical studies presented in Chapters 3 and 4. In these studies, the importance of the seasonal behavior of commodities for the pricing of commodity futures options is analyzed.

Specifically, Chapter 3 considers the extension of one- and two-factor spot price models by including deterministic time-dependent components in order to take the seasonal behavior of commodities into account. While a deterministic seasonal component at the price level can be neglected in terms of options pricing, this is not true for the seasonal pattern observed in the volatility of commodity prices. Analyzing an extensive sample of soybean and heating oil options, we find that the inclusion of an appropriate seasonality adjustment significantly reduces pricing errors and yields more improvement in valuation accuracy than increasing the number of stochastic factors.

In Chapter 4, we extend this analysis and propose a stochastic volatility model where the drift term of the variance process captures the observed seasonal pattern in volatility. This framework allows us to derive semi closed-form pricing formulas for the valuation of options on commodity futures in the spirit of Heston (1993). We empirically study the impact of the proposed seasonal stochastic volatility model on the pricing accuracy of natural gas futures options. Our

results demonstrate that allowing stochastic volatility to fluctuate seasonally significantly reduces pricing errors for these contracts. We conclude that seasonality in volatility is an important aspect to consider when valuing these contracts.

Chapter 5 contains concluding remarks and presents potential directions for further research.

Chapter 2

Commodity Price Dynamics and Derivatives Valuation: A Review

2.1 Introduction

Commodity markets and commodity price dynamics are of fundamental importance for many industries. A profound understanding of these markets is important for production companies seeking to hedge unwanted commodity exposures and for investors considering commodities as investments. Thereby, financial markets, e.g. for commodity futures and options, serve as a means to make risks tradable and to allow for an efficient allocation of commodity price risks among market participants.

Hedging commodity exposures as well as valuing commodity contingent claims requires a thorough knowledge of the commodities' price behavior. Since commodity contingent claims are based on consumable physical assets, commodity markets differ from other financial markets, as we explain in this chapter. This implies that it is generally not possible to apply standard equity models to commodity markets. Hence, an individual literature stream, both theoretical and empirical in nature, evolved to foster the understanding of commodity markets and the valuation of their instruments. This chapter gives an overview of the literature on commodities and in particular on the valuation of commodity derivatives. Thereby, we proceed as follows. In a first step, the empirically observed price behavior of commodities is presented and corresponding economic rationals

are discussed in Section 2.2. Building on these ideas, models for the valuation of commodity derivatives are discussed thereafter in Section 2.3. Section 2.4 concludes and contains directions for further research.

2.2 Theoretical Background and Empirical Price Behavior

Distinct from most other asset classes, commodities exhibit certain peculiarities which are in the center of interest in this section. Trading in commodity spot markets is limited due to extremely high transaction costs. Therefore, trading and price discovery take place primarily in the futures¹ markets. In the following, the discussion will focus on the shape of the futures curves, the mean-reverting behavior of commodity prices, the relationship between time to maturity and the volatility of futures contracts, and the role of seasonality.

2.2.1 Backwardation and Contango

The most well-known peculiarity of commodity markets is the fact that futures curves, which display prices of futures contracts for different maturities, can either be upward or downward sloping. Thereby, the situation when futures prices are below the current spot price is called *backwardation*, while a futures curve which is upward sloping for increasing times to maturity is referred to as being in *contango*. Figure 2.1 illustrates different shapes of futures curves observed over the course of the last years for crude oil futures traded at the New York Mercantile Exchange (NYMEX). Besides the situation of backwardation and contango, futures curves can also exhibit a *humped* shape as it becomes apparent in Figure 2.1. Moreover, the shape of the term structure of futures prices is changing over time.

¹Since forward and futures prices are the same for non-stochastic interest rates and in the absence of credit risk, the terms can be used interchangeably for our discussion focusing on the market risk of commodities. Since our data employed for empirical considerations consists of futures prices, we will generally speak of futures contracts. See Cox et al. (1981) for a discussion regarding the relationship between forward and futures prices.

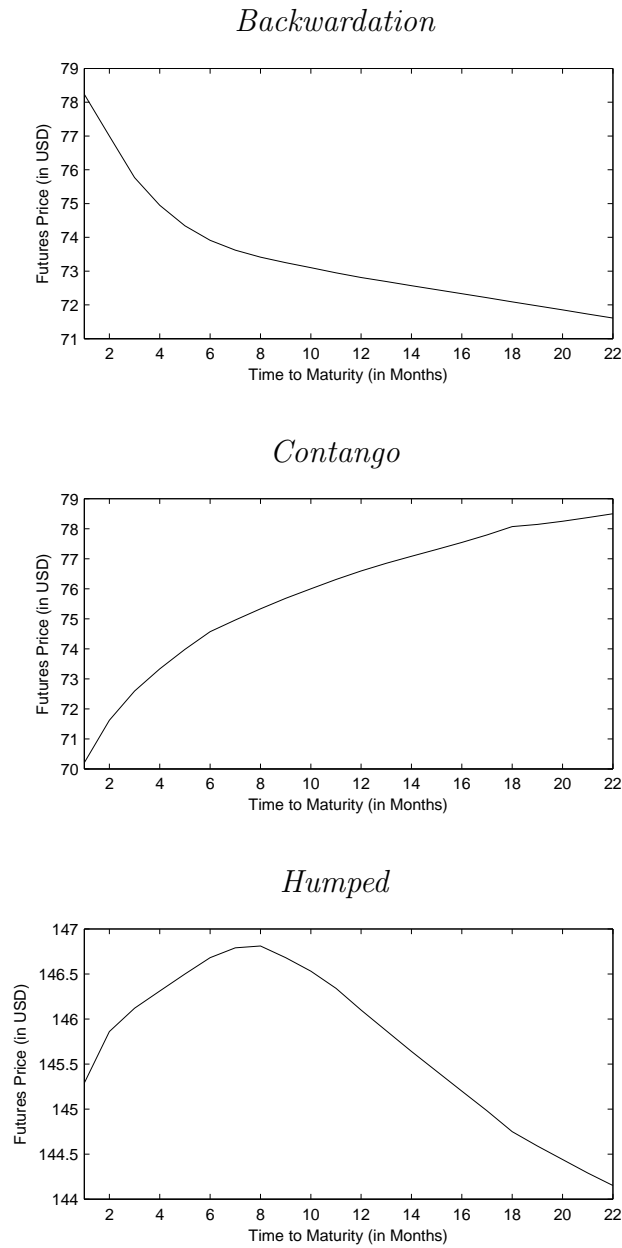


Figure 2.1: Different Shapes of Futures Curves for Crude Oil

This figure illustrates different shapes of futures curves for light sweet crude oil (WTI) futures observed at the following dates: September 11, 2007 (backwardation); May 24, 2010 (contango); July 3, 2008 (humped). The crude oil futures at the NYMEX are traded in USD per barrel. Prices are from Bloomberg.

A very popular metric to describe the shape of the futures curve is the *basis*, which is defined as²

$$Basis = Spot Price - Futures Price. \quad (2.1)$$

The basis is positive when the market is in backwardation and negative when the market is in contango. Naturally, changes in the shape of the futures curve have important consequences for risk management and investment decisions and, therefore, have been in the center of interest in numerous studies.

The literature trying to explain the shapes of commodity futures curves is traditionally divided into two broad strands. The *theory of storage*, originally proposed by Kaldor (1939), Working (1949), Brennan (1958), and Telser (1958), focuses on the aspects of inventories and the benefits of holding the physical commodity. In contrast, the *hedging pressure* literature, going back to Keynes (1930) and his rationales for *normal backwardation*, concentrates on the role of risk premia.

Keynes' original *theory of normal backwardation* is based on the assumption that hedgers hold on average a short position in the futures market, e.g. a commodity producer who wants to secure a certain price level for future deliveries. Since these market participants are willing to pay a risk premium in order to hedge their positions, the price of a futures contract will be a downward biased estimator of future spot prices in the normal backwardation framework. Since the assumption of hedgers being on average net short might not be universally appropriate, the theory is generalized in the sense that futures prices may carry either a positive or a negative risk premium depending on the net position of hedgers as proposed by Cootner (1960).

Dusak (1973), Breeden (1980), and Hazuka (1984) examine the role and existence of risk premia in commodity markets in capital asset pricing model frameworks and obtain mixed results. In addition to these studies of systematic risk as determinant of futures risk premia, there exists a vast amount of literature on the role of hedging pressure as determinant of risk premia or a combination of both systematic risk and hedging pressure. Important studies in these direc-

²See Geman (2005), p. 14.

tions comprise the work of Stoll (1979), Chang (1985), Hirshleifer (1989, 1990), Bessembinder (1992), and de Roon et al. (2000). Recently, risk premia on electricity forward markets have been analyzed by Bessembinder and Lemmon (2002), Longstaff and Wang (2004), and Ronn and Wimschulte (2008). Considering the heterogeneity of the obtained results, it can be summarized that the role and the determinants of risk premia in commodity futures markets yet need to be explored in greater detail and are subject to an ongoing debate.

Contrary to the risk premium literature, there is general agreement on the ideas of the theory of storage. The theory of storage relates spot and futures contract prices to inventories while emphasizing that holding the physical asset implies costs but also certain benefits. Accordingly, the so called *convenience yield* is in the center of interest, which is defined by Brennan and Schwartz (1985) as follows:

“The convenience yield is the flow of services that accrues to an owner of the physical commodity but not to the owner of a contract for future delivery of the commodity.”

Usually, the notion of convenience yield refers to the net convenience yield, which equals the gross convenience yield less the costs of carriage for holding the physical asset, comprising, e.g., storage and transportation costs. The convenience yield can be interpreted to be similar to a dividend yield on a stock, but in a more comprehensive sense, where the value is not of direct monetary nature but arises, e.g., from the flexibility in the production process. Owning the physical commodity has a value to meet unexpected demand and to avoid production disruptions. Telser (1958) describes that there is an implied timing option of holding commodities in inventory rather than being dependent on the market and potentially being confronted with unfavorable price developments. Articles building on rational intertemporal consumption decisions in order to explain the relation between inventories and the shape of the futures curve include the work of Deaton and Laroque (1992, 1996), Chambers and Bailey (1996), Chavas et al. (2000), and Routledge et al. (2000).

According to the theory of storage, the relationship between spot and futures prices can be described by

$$F(T) = S e^{(r-y)T} \tag{2.2}$$

when assuming a constant net convenience yield y and risk-free rate r . $F(T)$ is the price of a futures contract with maturity T and S denotes the current spot price. Consequently, when the benefits of holding the physical asset (convenience yields) are higher than the financing costs (interest rates), the futures curve will be in backwardation, while the futures curve will be in contango when interest rates exceed the net convenience yield.

Naturally, following supply and demand arguments, a negative relationship between inventory levels and prices exists. Furthermore, benefiting from commodities held in stock, i.e. exercising the implied real option and earning a convenience yield, is more likely when inventories are low. Hence, the theory of storage predicts a negative relationship between inventory levels and convenience yields. This is confirmed, e.g., by the empirical study of Fama and French (1987). The relationship between inventory levels and the shape of the futures curve is shown in Figure 2.2 for copper futures traded at the London Metal Exchange (LME). As implied by the theory of storage, the futures curve exhibits a negative basis (contango) when inventory levels are high and a positive basis (backwardation) when inventory levels are low. Thereby, the substantial economic importance of the convenience yield in periods of scarcity is reflected in the futures curves showing a high positive basis in dollar terms. Combining, on the one hand, the negative relationship between inventory levels and prices and, on the other hand, the negative relationship between inventory levels and convenience yields, it becomes clear that price level and convenience yields are positively correlated.

Inventory levels play an important role not only with regard to the shape of the futures curve but also with regard to the volatility of commodity prices. Many commodity markets are characterized by occasional sharp price spikes due to scarcity as discussed by Deaton and Laroque (1992). When inventory levels are low and there exists scarcity for a commodity, supply disruptions or unexpected changes in demand cannot be absorbed and lead to higher price fluctuations (Fama and French, 1988). Hence, inventory levels and price volatility tend to be negatively correlated. This relationship has been subject to extensive analyses by Fama and French (1987), Ng and Pirrong (1994), Litzemberger and Rabinowitz (1995), Geman and Nguyen (2005), and Geman and Ohana (2009). Since at

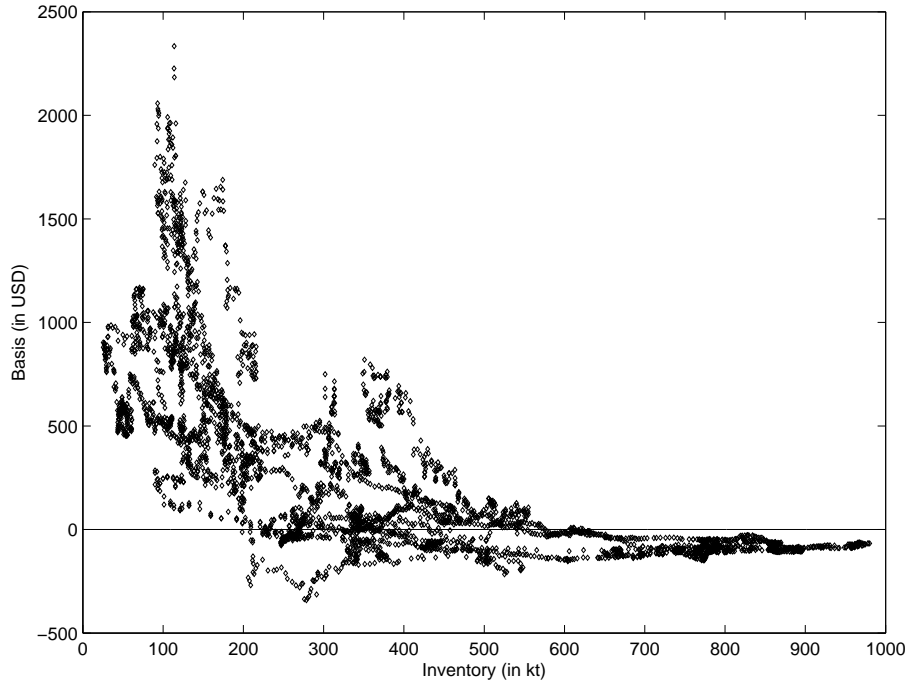


Figure 2.2: Inventory Levels and Basis for Copper Futures

This figure shows the relationship between inventory levels and the basis for copper futures traded at the LME from January 1995 to December 2010. Here, the basis is defined as the price of the 3-months futures (as a proxy for the spot price) less the price of the 27-months futures contract. Copper futures traded at the LME are traded in USD per tonne; the inventory is specified in kilotonnes (kt). All data are from Thomson Reuters Datastream.

the same time lower inventory levels usually imply a higher price, the observed correlation between commodity prices and volatility is often positive.³ This contrasts the findings for equity markets which usually exhibit a negative correlation, traditionally explained by the leverage effect.⁴

2.2.2 Mean-Reversion

Due to the interaction of demand and supply, commodity markets typically exhibit a mean-reverting price behavior. Prices will spike up when shortages occur,

³See, e.g., the results of the empirical study for the natural gas market in Section 4.4.1.

⁴Empirical studies on the leverage effect include the work of Christie (1982) and Cheung (1992). For more recent discussions challenging the traditional leverage explanation, refer to Figlewski and Wang (2000), Bollerslev et al. (2006), and Hasanhodzic and Lo (2010).

leading to higher investments in production facilities or causing more producers to enter the market, which will then lead to a higher supply, even though with a certain lag in time. The higher supply will then bring prices down again and vice versa.

In their seminal work, Bessembinder et al. (1995) provide strong evidence for the presence of mean-reversion in commodity markets. Based on the idea that mean-reversion can be explained by the positive correlation between price level and convenience yield, they essentially test if investors expect spot prices to be mean-reverting under the risk-neutral measure. Following the arguments of the theory of storage, decreases in inventory will lead to higher spot prices while the futures price does not change as much since the resulting higher convenience yield has an offsetting effect. Since futures prices reflect the market participants' expectations regarding future spot prices, this implies the anticipation of a mean-reverting price behavior.

Subsequently, several studies elaborated on the question if commodity prices follow a random walk or if they are indeed mean-reverting. For a broad sample of different commodity markets, Barkoulas et al. (1997) employ unit root tests in their analysis and obtain results which favor the random walk model over the mean-reversion model for most commodities. In contrast, Schwartz (1997) finds strong mean-reversion for the crude oil and copper markets. For the same markets, Casassus and Collin-Dufresne (2005) provide strong evidence for convenience yields being a function of spot prices which explains mean-reversion under the risk-neutral measure. Outlining that traditional unit root tests have only very low power for these applications, Andersson (2007) proposes a test on the basis of hedging errors and concludes that commodity prices are better described by a mean-reversion model than by a random walk. Similarly, Bernard et al. (2008) find that mean-reversion models better capture the dynamics of commodity spot and futures prices. In a recent study, Tang (2010) finds that commodity prices are mean-reverting but prices are reverting to a time-varying long-run mean and not to a constant mean.

While the economic rationales and the obtained empirical evidence provide strong support for the presence of mean-reversion in commodity prices, it is unclear if prices revert to some constant or to some stochastic equilibrium level.

Accordingly, different approaches can be found in the literature concerning the modeling of commodity price dynamics. For example, the one-factor model of Schwartz (1997) is based on the constant mean assumption while, e.g., the model of Schwartz and Smith (2000) explicitly assumes the long-term mean to be stochastic.

Overall, it can be concluded that mean-reversion is an important property which needs to be considered when analyzing the behavior of commodity prices.

2.2.3 Samuelson Effect

Another stylized fact of commodity markets which received considerable attention in the academic literature is the observation that volatility of futures prices tends to increase when the contracts come closer to expiration. Since Samuelson (1965) was the first to give a theoretical explanation for this observed price behavior, this phenomenon is commonly referred to as the *Samuelson effect* or the *Samuelson hypothesis*.⁵

The theoretical argument of Samuelson (1965) is that futures prices react more quickly to new information when they are close to expiration since spot and futures prices have to converge when the futures contract matures. Hence, new information will primarily impact the short-end of the futures curve and will influence long-term contracts only to a lower extent. This property of commodity futures has important implications for hedging strategies and options pricing.

Early studies which find evidence for the Samuelson effect include the work of Castelino and Francis (1982), Anderson (1985), Milonas (1986), Khoury and Yourougou (1993), Ng and Pirrong (1994), and Galloway and Kolb (1996). Especially for agricultural commodity markets, empirical evidence for the Samuelson effect is strong while there is mixed evidence for other commodities. In recent studies, Movassagh and Modjtahedi (2005), Mu (2007) and Suenaga et al. (2008) document the Samuelson effect in the natural gas market. Figure 2.3 shows the historical term structure of volatility for crude oil futures calculated over the time period January 2000 to November 2010. It becomes clear that futures contracts

⁵Alternatively, the notion of the *maturity effect* is used by some authors. Even though the maturity effect in the volatility of futures prices is named after Samuelson (1965), this was not the focus of Samuelson's article. For a discussion, refer to Bessembinder et al. (1996).

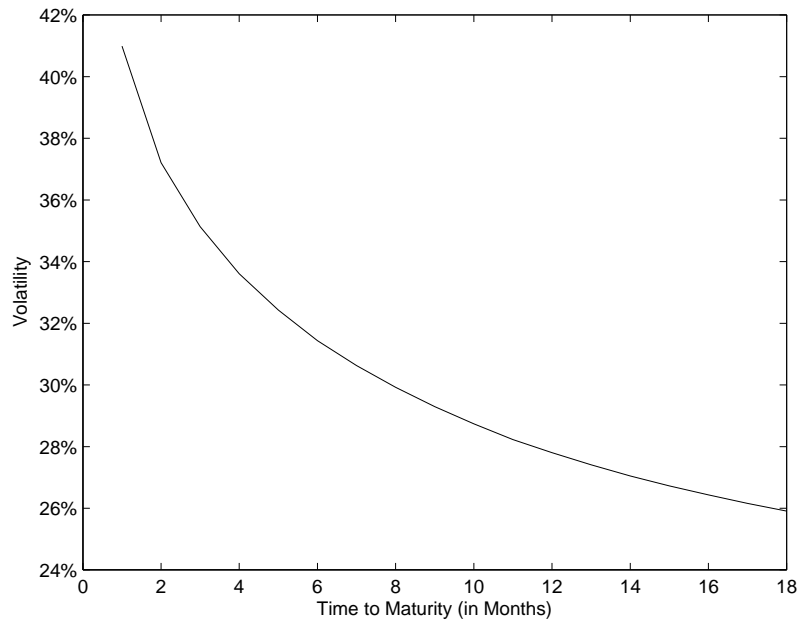


Figure 2.3: Term Structure of Volatility in Crude Oil Futures

This figure shows the volatility of light sweet crude oil (WTI) futures for different maturities traded on the NYMEX during the time period from January 2000 to November 2010. Volatilities are calculated as annualized standard deviations of returns in the individual maturity categories. Futures prices used for the calculations are from Bloomberg.

close to expiration are far more volatile than contracts with a long time to maturity. For stock markets and other financial markets, the Samuelson hypothesis is usually rejected.⁶

Fama and French (1988) confirm that for the markets of industrial metals spot prices are more volatile than prices of futures contracts. However, when inventory levels are high, they find that spot price shocks are reflected one for one in futures prices. This is also consistent with the equilibrium model of Routledge et al. (2000). This model shows that the conditional volatility for short-term contracts can be lower than for longer maturity contracts when inventory levels

⁶For example, Grammatikos and Saunders (1986) and Chen et al. (1999) find no support for the Samuelson effect for currency futures and futures on the Nikkei index, respectively.

are sufficiently high and, hence, the probability of a stock-out in the short-run is low.

Bessembinder et al. (1996) argue that the Samuelson effect will only be present in markets with a negative co-variation between changes in the spot price and slope changes of the futures curve (changes in the convenience yield). Hence, this is often referred to as the *negative variance* hypothesis. This hypothesis is consistent with price mean-reversion due to the positive correlation between commodity prices and convenience yields as predicted by the theory of storage and outlined in Section 2.2.2. While this is applicable for commodities where the owner of the physical asset earns a convenience yield, it does generally not hold true for financial assets. Accordingly, Bessembinder et al. (1996) find support for the Samuelson effect for agricultural commodities and crude oil and to a lower extent for metals. In contrast, they find no support for the Samuelson effect on financial markets.⁷

2.2.4 Seasonality

Supply and demand for commodities are driven by fundamental factors, which often follow seasonal cycles. For many agricultural commodities, supply obeys specific harvesting cycles and is especially dependent on the weather. After the harvest, inventory levels and thus supply are high while prices tend to be relatively low. In contrast, preceding the harvest, inventory levels are low and commodities will exhibit higher prices due to their scarcity. Whereas for agricultural commodities a seasonal price pattern is induced primarily by the supply side, seasonality for some energy markets, in particular natural gas and heating oil, originates mainly from the demand side. For heating oil and natural gas, demand is higher in the cold season and, therefore, prices tend to be higher during the heating period.⁸

⁷Contrary to Daal et al. (2006), who find only very weak evidence in favor of the negative variance hypothesis of Bessembinder et al. (1996), Duong and Kalev (2008) find strong support for the hypothesis in their study.

⁸The discussion here focuses on seasonality induced by persistent economic factors while there might additionally be seasonality in the sense of a day-of-the-week or a turn-of-the-year effect as discussed by, e.g., Gay and Kim (1987) and Lucey and Tully (2006). Also, there is evidence for increased volatility due to the periodic release of market reports as documented for the natural gas markets by Linn and Zhu (2004).

Whereas seasonality is prevalent on many agricultural, animal product, and energy commodity markets, evidence for the markets of metals or crude oil is weak. In particular for the agricultural commodities, inventory levels vary over the course of the calendar year and, thus, the observed seasonality in prices is also consistent with the theory of storage. For example, Fama and French (1987) find a strong seasonal variation in convenience yields for most agricultural and animal products but not for metals. Sørensen (2002) documents pronounced seasonal patterns in prices for the soybean, corn, and wheat markets, while Manoliu and Tompaidis (2002) find strong seasonal effects in natural gas prices.

Even though the presence of known seasonal price patterns would imply arbitrage opportunities on other financial markets, this is not necessarily the case for commodities. While market participants anticipate that, e.g., grain prices will subsequently increase after one year's harvest until prior to next year's harvest, storage costs and the perishability of goods preclude exploitation of this price pattern by the means of cash-and-carry arbitrage.

In addition to a seasonal variation in the price level, many commodity markets exhibit a seasonal pattern in volatility. Price uncertainty and, thus, volatility is the highest shortly before the harvest for agricultural commodities. In turn, during the winter, usually little new information enter the market and prices tend to be less volatile. In contrast, high demand fluctuations for heating oil and natural gas are more often observed during winter when weather shocks occur. In combination with a relatively inelastic supply, this is the economic rationale for the observed seasonal pattern in volatility of natural gas and heating oil prices showing the highest variations during the winter months.

In his study, which comprises primarily agricultural commodities, Anderson (1985) finds seasonality to be the primary factor leading to variations in volatility and to be more important than the Samuelson effect. Choi and Longstaff (1985) document a seasonal variation in volatility for soybean futures. Kenyon et al. (1987), Milonas (1991), and Karali and Thurman (2010) find a statistically significant seasonal pattern in volatility for the soybean, corn and wheat markets. Also, Streeter and Tomek (1992), Richter and Sørensen (2002), and Geman and Nguyen (2005) analyze seasonal volatility for the soybean market. In an empir-

ical study based on inventory data, Geman and Nguyen (2005) document that volatility is an inverse function of inventory levels.

Another market which received considerable attention in the literature is the market for natural gas. Mu (2007) discusses how weather affects natural gas price volatility and documents a strong seasonal pattern. Similarly, Suenaga et al. (2008) and Geman and Ohana (2009) find that price volatility on the natural gas market is significantly higher during winter than during summer and explain this by the higher demand variability during the cold season and declining inventory levels, which is in agreement with the theory of storage. Besides heating oil and natural gas, electricity is another energy commodity exhibiting a strong seasonal behavior not only with regard to an annual price pattern but also with regard to intra-week and intra-day fluctuations.⁹

Additionally, Doran and Ronn (2005) observe a seasonal pattern in market prices of volatility risk for natural gas. Extending this analysis, Doran and Ronn (2008) show that natural gas and heating oil markets exhibit a strong seasonal component in volatility and also in volatility risk premia. In a related study, Trolle and Schwartz (2010) find evidence, even though statistically not significant, for a seasonal variation in risk premia in the case of natural gas.

2.3 Valuation Models for Commodity Derivatives

Over the last years, an extensive literature on the modeling of commodity price dynamics has emerged. In contrast to the previous section, the focus of this section rests on the valuation of financial contracts rather than on the analysis of commodity markets' fundamentals. Most of the considered commodity derivatives pricing models follow the same principles: Relevant risk factors are identified and, next, the dynamics of these factors, e.g. prices, convenience yields or interest rates, are *exogenously* specified.¹⁰ This then allows to derive prices for contin-

⁹Refer to Lucia and Schwartz (2002) and Cartea and Villaplana (2008) for studies on the seasonal behavior of electricity prices.

¹⁰For latent factor models, often no interpretation of the factors is provided or an interpretation is attempted afterwards based on the obtained empirical results.

gent claims on the modeled underlying. This contrasts the idea of equilibrium models like, e.g., the model of Bessembinder and Lemmon (2002) or the model of Routledge et al. (2000) where convenience yield is derived *endogenously*. Instead, reduced-form Gaussian models are widely used for valuation purposes since they often allow for closed-form pricing formulas for futures and European options.

A very popular model for options pricing, especially among practitioners, is the model of Black (1976). When relying on the Black (1976) framework for the valuation of commodity futures, it is assumed that the cost of carry formula holds and that net convenience yields are constant. While this model is very tractable and easy to implement, it is unable to capture the observed price dynamics of commodity futures due to its simplifying assumptions. Describing the futures price dynamics only by a geometric Brownian motion does not capture commodity price properties like changes in the futures curves' shape, mean-reversion or the Samuelson effect. Subsequently, more sophisticated models were developed in the literature. An overview of different term structure models for commodities can also be found in Lautier (2005).

2.3.1 Spot Price Models

Naturally, the current spot price is a major determinant of the price of a futures contract. Accordingly, many models follow the approach to exogenously specify a stochastic process for the spot price dynamics and derive prices of futures contracts with respect to this spot price dynamics. Thereby, the price of a futures contract equals the expected future spot price under the risk-neutral measure. Models following the approach of modeling the stochastic dynamics of the spot price are therefore referred to as *spot price models*. Since spot prices have a high explanatory power for futures prices, the most common models to describe the commodity price dynamics are spot price models.

Brennan and Schwartz (1985) are the first to present a valuation model which is based on the spot price dynamics and considers convenience yields. In their model, convenience yield is simply modeled as a deterministic function of the spot price. The one-factor model of Brennan and Schwartz (1985) can be seen as the starting point for most of the valuation models which were developed since then.

In their model, the spot price is assumed to follow a geometric Brownian motion. Accordingly, their model is neither able to capture the empirically observed mean-reversion behavior of commodity prices, nor does it consider the Samuelson effect.

To overcome these shortcomings, a mean-reverting Ornstein-Uhlenbeck process instead of a geometric Brownian motion is a prevalent choice to describe the spot price dynamics. A popular model following this approach is the one-factor model of Schwartz (1997), where the log spot price follows an Ornstein-Uhlenbeck process. However, a problematic property of one-factor models, which is unwarranted by empirical observations, is that all futures prices are perfectly correlated since there is only one source of uncertainty. In general, one-factor models seem not to be able to capture the empirical properties of commodity futures price dynamics as pointed out by, e.g., Schwartz (1997). For this reason, several extensions have been presented in the literature introducing one or more additional stochastic factors.

A very popular class of models are *convenience yield models*, which assume that the convenience yield follows a stochastic process instead of being constant or deterministic. The inclusion of convenience yield as an additional stochastic factor to explain commodity price dynamics is in line with the theory of storage. The other important class of models describes the spot price or its logarithm as the sum of stochastic factors, hence *latent factor models*. In general, introducing additional factors allows for a greater flexibility with regard to the dynamics of the term structure of futures prices and the term structure of volatilities. However, it comes at the cost of additional complexity.

Convenience Yield Models

The Gibson and Schwartz (1990) model is the first two-factor model for the pricing of commodity derivatives. The model dynamics rely on a joint diffusion process of the spot price and the net convenience yield. It is based on the idea that the spot price and the convenience yield together can explain the futures price dynamics. While a geometric Brownian motion is assumed for the spot price dynamics, the convenience yield is described through an Ornstein-Uhlenbeck process. Thereby, the mean-reversion property of commodity prices is induced by the convenience yield process. In this model framework, the market price of convenience yield risk

needs to be estimated. Utilizing a two-factor model with stochastic convenience yield as proposed by Gibson and Schwartz (1990), Schwartz (1997) finds that the inclusion of the second stochastic factor greatly improves the ability to describe the empirically observed price behavior of copper, crude oil, and gold.

Furthermore, Schwartz (1997) presents a three-factor model where additionally interest rates are modeled as a stochastic factor. However, stochastic interest rates seem not to enhance futures pricing accuracy significantly. In particular, for commodity derivatives with a short time to maturity, interest rates are only of minor importance.

All the presented mean-reversion models endogenously produce a term structure of volatilities according to the Samuelson hypothesis. However, they differ in important ways. In the two- or three-factor models, volatility converges to some fixed level when the maturity approaches infinity. In contrast, futures price volatility converges to zero in the one-factor model of Schwartz (1997).

Convenience yield models have been extended in several ways, e.g. by considering jumps or stochastic volatility. Thereby, the Gibson and Schwartz (1990) and Schwartz (1997) models serve as the starting point and are reference models for, e.g., Hilliard and Reis (1998) and Yan (2002). Furthermore, Cortazar and Schwartz (2003) extend the work of Schwartz (1997) by introducing a third stochastic factor, the long-term return of the spot price. The presented models can be extended and generalized not only in terms of the spot price dynamics but also with regard to the assumptions concerning convenience yields. For example, Casassus and Collin-Dufresne (2005) present a three-factor model where convenience yield can be dependent on the other two factors, namely spot prices and interest rates. In a related study, Liu and Tang (2011) present a three-factor model which captures an observed heteroskedasticity in the convenience yield.

Latent Factor Models

A different approach to modeling the convenience yield is taken by Schwartz and Smith (2000) who describe the log spot price as the sum of two latent factors. The first factor is termed the long-term equilibrium price which follows a geometric Brownian motion while the second factor captures short-term deviations from the long-term equilibrium trend and is modeled through an Ornstein-Uhlenbeck

process reverting to zero. Schwartz and Smith (2000) show that this model is equivalent to the stochastic convenience yield model of Gibson and Schwartz (1990) in the sense that the state variables of one model can be represented by a linear combination of the state variables of the other model.

The so called *short-term/long-term model* of Schwartz and Smith (2000) is often preferred for empirical studies since it is econometrically advantageous. In particular, it eases an estimation of the model parameters based on Kalman filter techniques, which became the standard approach in the literature.¹¹ Specifically, the two factors are more orthogonal than in a stochastic convenience yield model expressed by a much lower correlation.¹² Furthermore, the two factors are only related through their correlation. This contrasts the convenience yield models where the convenience yield directly enters the spot price process, which complicates the analysis of each factor's influence. As the convenience yield models, the latent factor models can capture the Samuelson effect and the mean-reversion property of commodity prices.

The framework of the short-term/long-term model can easily be extended to incorporate additional factors. Several models subsequently presented in the literature build on the influential work of Schwartz and Smith (2000) and utilize a latent factor framework. For example, Sørensen (2002) and Lucia and Schwartz (2002) incorporate seasonal components. A generalized N-factor model for commodity prices is presented by Cortazar and Naranjo (2006). Paschke and Prokopczuk (2010) generalize the model of Schwartz and Smith (2000) by developing a model where the short-term deviations from the long-term equilibrium are described by the means of a continuous autoregressive moving average (CARMA) process instead of utilizing an Ornstein-Uhlenbeck process, which is a special case of the CARMA process.

Similar to the Schwartz and Smith (2000) model, Korn (2005) presents a model where both stochastic factors follow mean-reverting processes. Hence, commodity futures prices are stationary in contrast to the non-stationarity in the short-term/long-term model of Schwartz and Smith (2000). He claims that the

¹¹For example, Schwartz (1997), Schwartz and Smith (2000), Sørensen (2002), Manoliu and Tompaidis (2002), Geman and Nguyen (2005), and Liu and Tang (2011) use Kalman filter techniques in their estimations.

¹²Refer to Schwartz and Smith (2000) for a discussion on this issue.

proposed specification facilitates an improved pricing performance for long-term crude oil futures contracts.

In general, the assumptions regarding the long-term futures price level are different in the presented models. In contrast to the one-factor model of Schwartz (1997), the long-term mean is stochastic in the two-factor model of Schwartz and Smith (2000) and follows a geometric Brownian motion.

2.3.2 No-Arbitrage Models of the Futures Curve

Since in spot price models futures prices are derived endogenously according to the spot price dynamics, these models do not necessarily fit the observed term structure of futures prices. An alternative approach avoiding this problem is found in *no-arbitrage models of the futures curve*. Here, the term structure of commodity futures serves as an input and the stochastic movement of the term structure is described according to no-arbitrage ideas. Hence, futures price models are in similar spirit as the no-arbitrage term structure models for interest rates as proposed by Ho and Lee (1986) and Heath et al. (1992). Early models for commodities in this framework were proposed by Reisman (1992) and Cortazar and Schwartz (1994).

Miltersen and Schwartz (1998) develop a general model for options pricing when interest rates and convenience yields are stochastic. Similar to the concept of forward rates in interest rate models, they propose the concept of forward and future convenience yields and derive a model along the lines of Heath et al. (1992). Similarly, Miltersen (2003) outlines a model which matches the current term structure of futures prices and of futures volatilities. He implements his model according to the ideas of Hull and White (1993). Crosby (2008) presents an arbitrage-free model which is consistent with the initial term structure of futures prices and includes jumps. However, the proposed model requires Monte Carlo simulation techniques for valuation purposes.

2.3.3 Jumps

In order to account for sudden price changes, jumps can be considered in the stochastic process of the commodity price. Especially for financial contracts with

an asymmetric payoff profile like options, jumps can have an important price effect. For example, Hilliard and Reis (1999) compare the pricing performance of the standard Black (1976) model and the Bates (1991) jump-diffusion model for soybean futures options and find that the jump-diffusion model yields a significantly better pricing performance. Similarly, Koekebakker and Lien (2004) propose a modified version of the Bates (1991) model and find for the wheat market that their model yields a better pricing performance than other models neglecting a jump component.

With regard to valuation models particularly for commodity markets, Hilliard and Reis (1998) propose a model which allows for jumps in the spot price process in addition to spot price, convenience yield and interest rates being stochastic. While the incorporated jump component in their model is very important for the valuation of commodity futures options, it has no influence on futures and forward prices. Yan (2002) proposes a model for the valuation of commodity derivatives which considers simultaneous jumps in spot price and volatility while also allowing volatility to be stochastic. Again, these extensions are found to be of high importance for options pricing but are irrelevant for the valuation of commodity futures.

The incorporation of jumps is especially important for financial contracts written on electricity as underlying. Due to the non-storability of electricity, cash-and-carry is usually not possible, which can lead to extreme price spikes when demand is unexpectedly high. Accordingly, jumps have been considered for electricity markets in several studies, e.g. by Deng (2000), Benth et al. (2003), Cartea and Figueroa (2005), Geman and Roncoroni (2006), Benth et al. (2007), Seifert and Uhrig-Homburg (2007), Nomikos and Soldatos (2008), and Cartea et al. (2009).¹³ Thereby, different aspects of the market characteristics can be considered. For example, Nomikos and Soldatos (2008) present a model in which jumps are driven by seasonality, whereby jump intensity and jump size are dependent on the time of the year.

¹³Since classical arbitrage arguments are hindered due to the non-storability of electricity, a no-arbitrage model framework might be unfavorable for the valuation of electricity futures contracts as argued by, e.g., Muck and Rudolf (2008). An alternative is found in equilibrium models, which do not model the price dynamics directly but rather rely on fundamental factors as presented by Bessembinder and Lemmon (2002), Barlow (2002), Cartea and Villaplana (2008), and Pirrong and Jermakyan (2008).

2.3.4 Constant, Deterministic, and Stochastic Volatility

In the popular spot price models of Schwartz (1997) and Schwartz and Smith (2000), the state variables follow stochastic processes with a constant volatility. Due to their mean-reversion properties, these models nevertheless imply that volatility varies with the time to maturity of futures contracts, i.e. they endogenously capture the Samuelson effect. However, several authors questioned the constant volatility assumption for the state variables proposing either deterministic variations in volatility or describing volatility through an additional stochastic process.

Deterministic variations in volatility comprise in particular seasonal patterns which can be expected to repeat on a regular basis due to fundamental factors like harvesting cycles or demand patterns. Furthermore, for a model where the Samuelson effect is not taken into account implicitly, allowing volatility to be a function of the futures' time to maturity is a way to adjust for this. For example, Doran and Ronn (2005) follow this approach since they directly model the dynamics of the futures price instead of the spot price and, hence, the Samuelson effect does not arise implicitly from the spot price dynamics. Similarly, Koekebakker and Lien (2004) propose a model of the futures price dynamics in which they consider time-dependent volatility to capture the Samuelson effect and seasonal variations. They document the high importance of such time-varying volatility in a numerical example for wheat options.

Following the rationales of the theory of storage, a positive relationship between volatility and convenience yield exists. This is true since commodity prices and convenience yields are high when inventory levels are low, while at the same time volatility tends to be higher and vice versa. According to this idea, Nielsen and Schwartz (2004) present a model in which the spot volatility is a function of the convenience yield level. The proposed model is a generalization of the Gibson and Schwartz (1990) model. Nielsen and Schwartz (2004) argue that reliable and accurate information on inventory levels is not readily available and, therefore, decided not to model inventory levels explicitly but rather describe the volatility of the spot price process through its relationship to the convenience yield. In contrast, Geman and Nguyen (2005) present a model where they include scarcity as

a separate state variable and the volatility of the spot price is a function thereof and is, thus, stochastic.

Stochastic volatility models for commodities are also presented in Deng (2000) and Yan (2002). Both describe the volatility dynamics through a square root process as in Heston (1993). Additionally, they consider jumps in both spot price and spot volatility. As with jumps, the assumptions regarding volatility are especially important for options pricing and often irrelevant for the valuation of futures as in the model of Yan (2002). Stochastic volatility models which additionally acknowledge a seasonal variation are proposed by Richter and Sørensen (2002) and Geman and Nguyen (2005).

While introducing a stochastic process for the volatility of the spot price in the popular setting of spot price models is a natural step, it significantly complicates the valuation of commodity derivatives. No closed-form solutions for futures and options on futures are available in, e.g., the models of Richter and Sørensen (2002) and Geman and Nguyen (2005). Accordingly, this numerical burden hampers real-world applications. To approach this difficulty, Hikspoors and Jaimungal (2008) present spot price models where the volatility of the spot price follows a fast mean-reverting Ornstein Uhlenbeck process and for which they derive asymptotic prices for commodity futures and options on futures.

Recently, more general stochastic volatility models were presented by, e.g., Huguen (2010) and Trolle and Schwartz (2009). In particular, Trolle and Schwartz (2009) present a Heath, Jarrow, and Morton (1992)-style model which takes unspanned volatility into account.¹⁴ In an empirical study of the crude oil market, they find that two volatility factors are needed to describe options prices and that these factors are largely unspanned by the futures contracts.

Even though the assumptions regarding volatility are especially important for option prices, empirical studies based on commodity futures options prices are rare. Particularly, the literature lacks analyses regarding the options pricing accuracy of proposed valuation models.

¹⁴Unspanned volatility implies that volatility risk cannot be completely hedged by a position in the underlying security.

2.3.5 Seasonality

Considering the strong seasonal patterns on many commodity markets, Sørensen (2002) extends the model of Schwartz and Smith (2000) and includes a deterministic seasonal component to describe the seasonal variations in price levels. Thereby, the log spot price is the sum of a deterministic calendar time dependent trigonometric function and two latent factors as in Schwartz and Smith (2000). He considers the markets for soybeans, corn, and wheat and finds strong support for the inclusion of the proposed seasonality adjustment. Similar approaches can also be found in, e.g., Richter and Sørensen (2002) and Geman and Nguyen (2005).

In general, describing a seasonal pattern by trigonometric functions has the advantage that only few additional parameters are needed and, additionally, trigonometric functions are continuous in time. Seasonal dummy variables represent a different modeling approach as used, e.g., by Manoliu and Tompaidis (2002) and Todorova (2004). While this approach offers the advantage of being very flexible, a higher number of parameters is usually needed and the approach is more sensitive to outliers potentially distorting the obtained results as pointed out by Lucia and Schwartz (2002). Lucia and Schwartz (2002) present one- and two-factor models which are extended by a deterministic component to capture market regularities like working and non-working day effects as well as the seasonal behavior of electricity prices during the calendar year. In their study, they utilize both dummy variables as well as a trigonometric function approach. In her paper, Todorova (2004) proposes a model where seasonality is not modeled deterministically but as a distinct third stochastic factor in addition to the latent two-factor short-term/long-term model.

All these studies rely on a spot price model framework where seasonality enters the dynamics of the spot price. Based on the seasonal spot price dynamics, futures prices which are consistent with these assumptions and reflect the seasonal behavior can be derived. A different approach is presented by Borovkova and Geman (2006), who propose a model where the first state-variable represents the average forward price instead of the spot price. The second factor should capture changes in the shape of the forward curve and, thus, changes in the convenience yield. Additionally, a deterministic seasonal premium is considered. Hence, in

contrast to other models, they model seasonality as a function of the future's maturity date and not with respect to seasonal changes in the spot price over time until the future's expiration.

As discussed in Section 2.2.4, many commodity markets exhibit seasonal variations not only in the price level but also in volatility. To account for this, Richter and Sørensen (2002) and Geman and Nguyen (2005) include a seasonal component to govern volatility in their stochastic volatility models. Analogous to modeling seasonality in the price level, the seasonal pattern in volatility is described through trigonometric functions. This approach has also been adapted in subsequent studies, e.g. in Koekebakker and Lien (2004) and Geman and Ohana (2009). The market for electricity is also characterized by strong seasonal variations in volatility as acknowledged for example in the model of Cartea and Villaplana (2008) where the seasonality is captured by a dummy variable specification.

2.3.6 Further Extensions

Regime-switching models for commodity prices are based on the idea that there exist distinct regimes which are characterized through different price dynamics. For example, Fong and See (2002) and Vo (2009) analyze the price behavior on the crude oil market by the means of regime-switching models for volatility and find that price dynamics switch between high and low volatility regimes. When valuing commodity derivatives, regime-switching can be used not only to characterize regimes with regard to volatility; different regimes can be characterized, e.g., by different jump intensities as proposed by Cartea et al. (2009) or by completely distinct processes as in Chen and Forsyth (2010). Furthermore, Nomikos and Soldatos (2008) propose a jump-diffusion model which considers regime-switching in the long-run equilibrium level according to a seasonal variable. In their study, the regimes are a high and a low water level regime since the level in water reservoirs is an important determinant for the analyzed hydropower-dominated electricity market of Scandinavia. While in these regime-switching models a change to a different regime is an exogenous event occurring with a certain probability, Ribeiro and Hodges (2005) present a model where the spot price dynamics switch

between two distinct processes depending on the current spot price level being above or below a critical threshold.

Another fact which has important implications for risk management and especially for the valuation of commodity spread options is that many commodity markets are closely related and that there are dependencies between these markets. This is especially true for agricultural and energy markets where e.g. heating oil, gasoline and other products are produced from crude oil. In fact, energy markets are often not only related, they are co-integrated as pointed out by Asche et al. (2003) and Paschke and Prokopczuk (2009). Accordingly, Paschke and Prokopczuk (2009) propose a continuous time multi-factor model which integrates price dynamics of related commodities in a single model. Also, due to the co-integration between crude oil and heating oil prices, Dempster et al. (2008) propose to model the spread between the two commodities directly instead of modeling the price dynamics of the individual commodities when valuing, e.g., crack spread options.¹⁵

2.4 Conclusion

Commodity markets are characterized by some unique stylized facts which received a significant amount of attention in the academic literature. Understanding and taking into account these peculiarities like mean-reversion, the Samuelson effect, convenience yields or seasonality is crucial for the analysis of commodity price dynamics. Naturally, these unique characteristics are of utmost importance for risk management decisions and for the valuation of commodity derivatives. With regard to valuation models for commodity derivatives, we discussed different approaches and developments in the literature with a particular emphasis on questioning if and how the peculiarities of commodity markets are taken into account. Interestingly, most of the literature considers only the markets for commodity futures while options on these contracts received relatively little attention. Accordingly, when assessing the properties and the pricing performance of

¹⁵The crack spread is the price spread between a commodity and a derivative of this commodity, for example the price spread between crude oil and heating oil.

commodity valuation models, the analysis often concentrates only on commodity futures markets.

Naturally, the assumptions regarding volatility are of much higher importance for the valuation of options than for the valuation of futures contracts. Hence, correctly modeling the volatility dynamics will play a significant role in any option pricing application. Despite the fact that seasonal variation in volatility is a well-known property of many commodity markets, there exists only very limited literature on how to account for this in a valuation model.

The following two essays contribute to the literature by being the first studies on the role of seasonal volatility concerning the pricing accuracy of commodity option pricing models. In particular, in Chapter 3, we incorporate seasonality in a deterministic volatility setting of spot price models. In Chapter 4, a stochastic volatility model which takes seasonal volatility into account is presented. The role of seasonality in volatility and its influence on option pricing accuracy of the proposed valuation models will be analyzed in extensive empirical studies for different commodity markets.

Chapter 3

Seasonality and the Valuation of Commodity Options

3.1 Introduction¹

Commodity options have a long history. One of the first usages was documented by Aristotle, who reported in his book *Politics* (published 332 B.C.) a story about the philosopher Thales, who was able to make good predictions on the next year's olive harvest, but did not have sufficient money to make direct use of his forecasts. Therefore, Thales bought options on the usage of olive presses, which were available for small premiums early in the year. When the harvest season arrived, and the crop yield was, as expected by Thales, high, olive presses were in huge demand, and he was able to sell his usage options for a small fortune.² In contrast, modern commodity options, as we know them today, are quite recent innovations. The first commodity options traded at the Chicago Board of Trade (CBOT) were live cattle and soybean contracts, both introduced in October 1984.³ As distinguished from the ancient contracts, modern commodity options are generally not

¹This chapter is based on a homonymous paper co-authored by Marcel Prokopczuk and Markus Rudolf.

²See Williams and Hoffman (2001), Chapter 1. The interested reader might also refer to the translated original text, e.g. Aristotle (1981), p. 88–90, where Aristotle refers to the strategy of Thales as a '*money-spinning device*'.

³See the CME Group website: www.cmegroup.com.

written on the commodity itself, but on a futures contract. This ensures liquidity of the underlying, as most of the trading takes place in the futures market.

When considering the pricing of commodity options contracts, the special features of these markets should be taken into account. One of the earliest, and perhaps today's most popular commodity options pricing formula among practitioners, was derived by Black (1976). Black's formula can basically be regarded as a straightforward advancement of the well-known Black and Scholes (1973) stock options pricing formula, taking into account the fact that no initial outlay is needed when entering a futures position. However, other stylized facts present in commodity markets are not considered in Black's approach. These issues have been addressed in more recent research. Gibson and Schwartz (1990), Brennan (1991), Ross (1997), and Schwartz (1997) point out that the dynamics of supply and demand result in a mean-reverting behavior of commodity prices. Schwartz (1997) tests three different model variants, incorporating mean-reversion (a one-, two-, and three-factor model), in terms of their ability to price futures contracts on crude oil, copper, and gold. All of these commodities belong to the part of the commodity universe not showing seasonality in their price dynamics.⁴

Seasonality can be considered as another stylized fact of many commodity markets, distinguishing them from traditional financial assets. The seasonal behavior of many commodity prices has been documented in numerous studies, e.g. Fama and French (1987), and, thus, should be considered in a valuation model. Sørensen (2002) considers the pricing of agricultural commodity futures (corn, soybeans, and wheat) by adding a deterministic seasonal price component to the two-factor model of Schwartz and Smith (2000). Similarly, Lucia and Schwartz (2002) and Manoliu and Tompaidis (2002) consider the electricity and natural gas futures markets, respectively. Furthermore, Dempster et al. (2008) analyze the crack spread between heating oil and crude oil. Thus, the modeling of seasonality at the price level is relatively well understood.

When it comes to options pricing, price level seasonality is, however, of no importance. In a standard setting, the deterministic component of the price pro-

⁴More recent advancements in pricing commodity futures contracts include, e.g., Casassus and Collin-Dufresne (2005) and Paschke and Prokopcuk (2010).

cess does not enter the options valuation formula.⁵ However, as noted by Choi and Longstaff (1985), there exists a second type of seasonality which can have a great influence on the value of a commodity option. As the degree of price uncertainty changes through the year, the standard deviation – i.e. the volatility of a commodity futures’ return – shows strong seasonal patterns. A good example is provided by most agricultural markets, where the harvesting cycles determine the supply of goods. Shortly before the harvest, the price uncertainty is higher than after the harvest when crop yields are known to the market participants resulting in a seasonal pattern in volatility in addition to the price level seasonality.

Surprisingly, the impact of seasonal volatility on commodity options valuation has attracted very little academic attention. Due to the lack of available options data, Choi and Longstaff (1985) do not conduct any empirical study. Richter and Sørensen (2002) and Geman and Nguyen (2005) consider the soybean market and acknowledge the time-varying volatility by including a deterministic component in their model, but do not study the impact on the models’ options pricing performance.

We contribute to the literature by filling this gap. Two commodity pricing models, a one-factor and a two-factor model, are extended by allowing for seasonal changes of volatility throughout the calendar year. These models are estimated using an extensive sample of options prices for two different commodity markets. First, we consider soybean options traded at the CBOT. Being the biggest agricultural derivatives market, soybean contracts provide a prominent example of a commodity with seasonality effects mainly induced from the supply side of the market. Second, we study the impact of seasonalities on heating oil options traded at the New York Mercantile Exchange (NYMEX). In contrast to the soybean market, the seasonality in this market is mainly driven by the demand side. The considered options pricing models are calibrated on a daily basis

⁵Intuitively, this can be seen by the fact that the deterministic price seasonality only affects the drift of the underlying. As the risk-free hedge portfolio must earn the risk-free rate, the price seasonality cannot have any influence on the option price. More formally, this argument can be seen in the model description in Section 3.3. One should note, however, that a predictable component in the price process might have an influence on the estimation of the model parameters. If one estimates the volatility using a historical time series of asset prices, one must clearly account for seasonal price variations as changes in the mean return affect the variance. As we estimate our model implicitly using option prices, this problem does not arise. See also Lo and Wang (1995) on this issue.

and then tested with respect to their in- and out-of-sample pricing performance. Our results show that the pricing performance can be greatly improved by including seasonality components in the volatility. This demonstrates that considering the seasonality of volatility is of great importance when dealing with options or option-like products in seasonally behaving commodity markets.

The remainder of this chapter is organized as follows. Section 3.2 provides an overview of seasonality in commodity markets in general and the two considered markets in particular. In Section 3.3, we describe the considered model dynamics and provide futures and options valuation formulas. Section 3.4 describes the sample of options data and the estimation procedure employed, while the empirical results of our study are presented in Section 3.5. Section 3.6 contains concluding remarks.

3.2 Empirical Evidence on Seasonality in Commodity Markets

Hylleberg (1992) defines seasonality as “... *the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy.*”

Following this definition, agricultural commodity markets clearly show seasonal patterns induced by the supply side mainly due to harvesting cycles, the perishability of agricultural goods, and the effects of weather. In contrast, many energy commodity markets show seasonal patterns induced from the demand side, which are due to regular climatic changes as well as regular calendar patterns, such as holidays.⁶ Furthermore, inventories of these commodity markets undergo a seasonal pattern. Thus, the presence of seasonality in commodity markets is also predicted by the theory of storage (Kaldor (1939), Working (1949), Brennan

⁶In the case of electricity markets, varying demand levels induce regular intra-day and intra-week price patterns in addition to a calendar year effect as shown by Longstaff and Wang (2004) and Lucia and Schwartz (2002), respectively.

(1958), and Telser (1958)), which states that the convenience yield and, thus, the commodity price are negatively related to the level of inventory.

In this chapter, we consider two commodity markets: soybeans and heating oil. The soybean market is the largest agricultural commodity market in the world, whereas heating oil is, together with gasoline, the most important refined oil product market.⁷

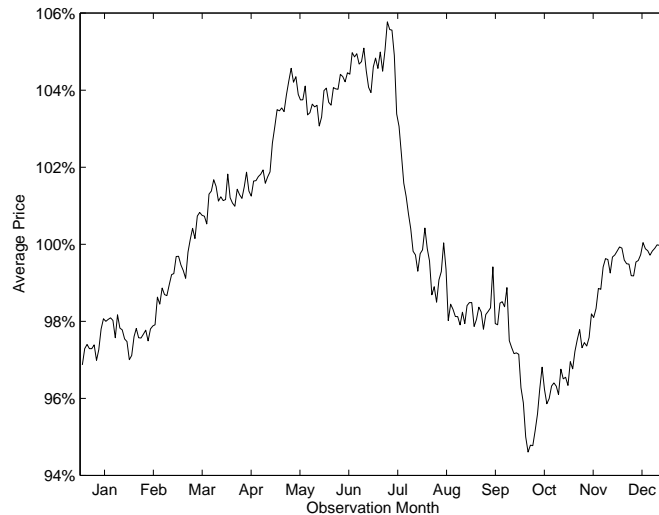
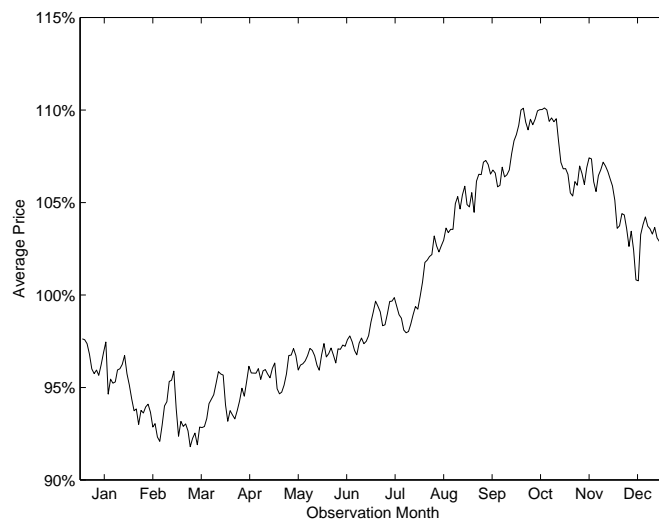
Although not the main focus of this essay, we first provide empirical evidence on seasonal patterns at the price *level* to draw a complete picture with respect to seasonality in the two considered markets. In order to illustrate the seasonal pattern at the price level, we consider front-month futures prices as an approximation of spot prices. We standardize each daily price observation relative to the annual average. Thereby, we obtain a price series describing the price pattern for each year considered in our sample which spans January 1990 to December 2009. In the next step, we calculate average values of the annual patterns to derive the historical seasonal pattern of the two considered commodities. Following the economic rationales outlined above, we expect soybean prices to increase before the harvests in South America and the United States, which take place during spring and summer.⁸ In the case of heating oil, we expect the price to increase during the fall when demand is higher relative to the spring and summer months. These expected price patterns can be observed in Figure 3.1, which displays the estimated seasonal price paths.

As discussed in the introduction, this essay focuses on a second type of seasonality present in the price dynamics of commodity markets. According to Anderson (1985), the *volatility* of commodity futures prices will be high during periods when new information enters the market and significant amounts of supply or demand uncertainty are resolved.⁹ For heating oil, this is the case during the winter when heating is needed. In agricultural markets, this is true shortly before and during

⁷Details on these markets can be found in Geman (2005). The seasonal behavior of prices is documented by Milonas (1991), Frechette (1997), and Geman and Nguyen (2005) for the soybean market, and Girma and Paulson (1998) and Borovkova and Geman (2006) for the heating oil market.

⁸South America, in particular Argentina and Brazil, and the United States are the world's biggest producers of soybeans.

⁹Note that there exists a second effect on volatility which is usually referred to as the "Samuelson effect" because it was first introduced by Samuelson (1965). This effect describes the empirical fact that the volatility of futures increases as maturity approaches, which can be explained

Panel A: Soybeans*Panel B: Heating Oil***Figure 3.1: Seasonal Pattern of Futures Prices**

This figure shows the seasonal pattern of front-month futures prices from January 1990 to December 2009 for soybeans and heating oil. The empirical figure was derived by first standardizing every observed price relative to the annual average and second taking the average of the price patterns of the considered years. Prices are from Bloomberg.

the harvesting period. Information regarding the subsequent harvest becomes available during this time, causing a higher fluctuation in prices, while a minimum is typically reached during the winter months. These effects have been documented empirically for various commodity markets.¹⁰

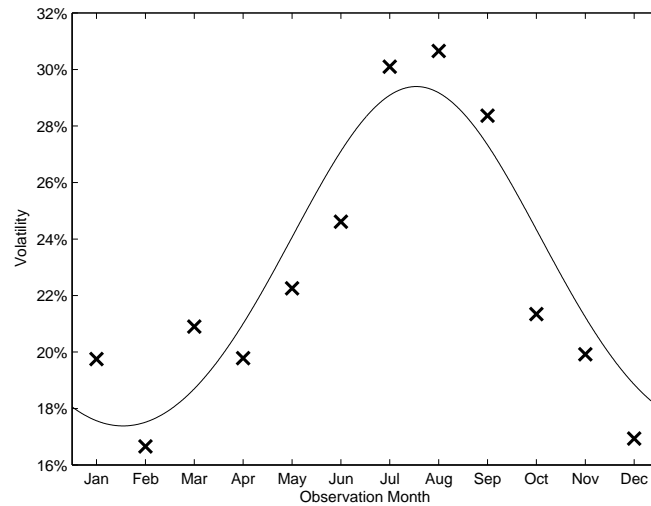
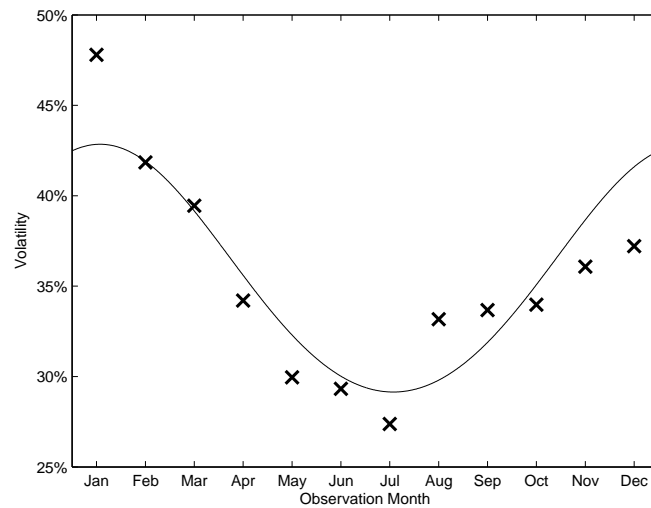
To analyze seasonality in volatility in the soybean and heating oil markets, we calculate two different types of volatility: historical and option implied volatility. To obtain historical (realized) volatilities for the two considered commodities, we first calculate daily returns for the front-month futures prices during the same sample periods. In the next step, the daily returns are grouped by their observation months separately for each year. The standard deviation of the daily returns is then calculated for each observation month and annualized to make the results easier to interpret. We then take the average values of volatilities in the different calendar months of the calculated time series to obtain an estimation of the average volatility with regard to the time of the year. These historical volatility estimates are displayed in Figure 3.2. One can clearly observe that the realized volatility varies considerably throughout the year, ranging from 17% to 30% in the case of soybeans, and 27% to 48% in the case of heating oil. The shapes of the patterns are very similar to the ones observed for the price level. Furthermore, Figure 3.2 shows that the seasonal volatility pattern can be reasonably approximated by a trigonometric function which serves as motivation for the valuation models presented in the next section.

Besides historical volatilities, the volatilities implied in options prices are of interest since they reflect how market participants assess the future volatility pattern. For that reason, we compute implied volatilities employing the standard model of Black (1976).¹¹ The obtained implied volatility estimates are then grouped by the options' months of maturity, and average values are calculated as shown in Figure 3.3. Again, it can be observed that a trigonometric function works reasonably well to describe the seasonal volatility pattern. Furthermore, the seasonal pattern of implied volatilities is very similar to the pattern of his-

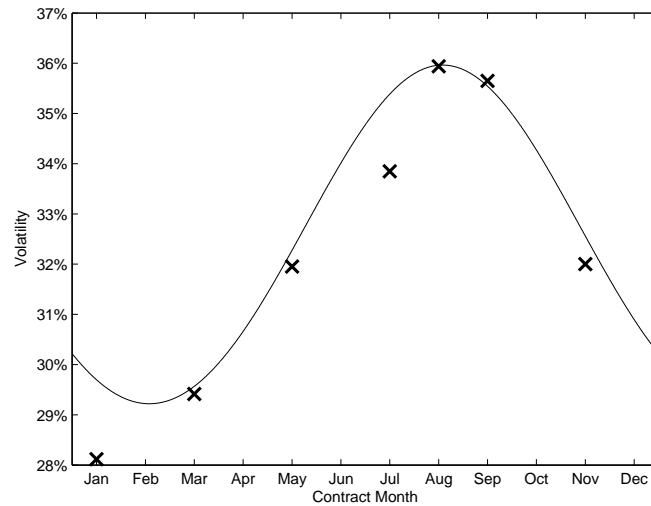
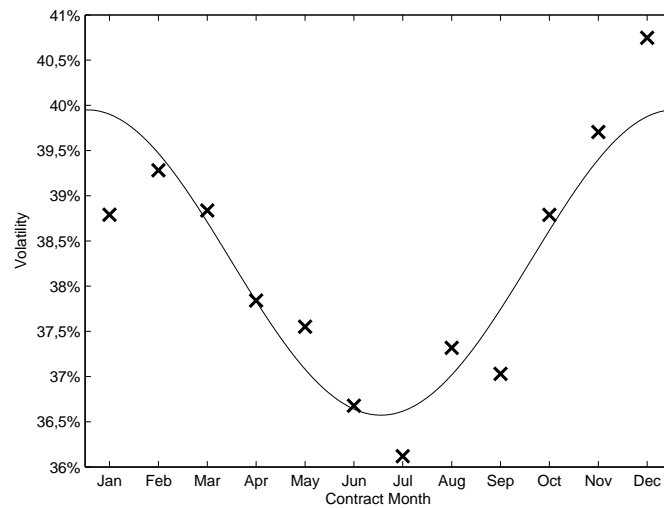
by decreasing supplier flexibility. The Samuelson effect is implicitly accounted for by the commodity pricing models considered in this study.

¹⁰See Anderson (1985), Choi and Longstaff (1985), Khoury and Yourougou (1993), Suenaga et al. (2008), and Karali and Thurman (2010) on seasonality in the volatility of commodity prices.

¹¹The options data set used is described in Section 3.4.

Panel A: Soybeans*Panel B: Heating Oil***Figure 3.2: Seasonal Pattern of Historical Volatility**

This figure shows the seasonal pattern of front-month futures volatilities from January 1990 to December 2009 for soybeans and heating oil. Furthermore, the seasonal volatility pattern was approximated by a trigonometric function as proposed for the price dynamics of the considered models (see Section 3.3). The historical volatilities were derived by first grouping the daily returns by observation month and calculating their standard deviation for each year separately and second taking the average of the annualized standard deviations of the considered years. Prices are from Bloomberg.

Panel A: Soybeans*Panel B: Heating Oil***Figure 3.3: Seasonal Pattern of Implied Volatility**

This figure shows the seasonal pattern of the implied volatilities of futures options grouped by maturity month. The sample spans July 29, 2004 to June 22, 2009 for soybeans and from October 21, 2004 to October 2, 2009 for heating oil. Furthermore, the seasonal volatility pattern was approximated by a trigonometric function as proposed for the price dynamics of the considered models (see Section 3.3). The implied volatilities were derived by first transforming the American style options into European style options according to the approximation suggested by Barone-Adesi and Whaley (1987), and second calculating the implied volatilities by the Black (1976) formula. Only ATM options ($90\% \leq S/K \leq 110\%$) with 6 to 120 days to maturity were considered. Prices are from Bloomberg.

torical volatilities.¹² Please note that the considered time period is different to the analysis of historical volatilities, as the available options data cover a shorter period of time compared to the futures data set available, and, therefore, the level of the volatilities is somewhat different. Still, the overall picture of a strong seasonal pattern, with volatility reaching a maximum in late summer and early fall and a minimum in winter for soybeans and vice versa for heating oil, remains the same.

3.3 Valuation Models

In this section, we describe the dynamics of the pricing models used in the empirical study. We then provide the futures and European options pricing formulas.

3.3.1 Price Dynamics

Due to the interaction of supply and demand, commodity prices are usually considered to exhibit mean-reversion characteristics.¹³ We consider two main model specifications, both of which include mean-reverting behavior; however, the two differ with respect to the assumptions regarding the long-term equilibrium price of the considered commodity. All models are specified directly under the risk-neutral measure. This approach is motivated by the observation of Schwartz and Smith (2000) and Geman and Nguyen (2005) that the market price of risk can only be estimated with very low precision from derivatives data. As the focus of our study is options pricing, we prefer to work directly under the risk-neutral measure making a change of measure dispensable.

The first model we consider is a one-factor model in which the logarithm of the spot price, $\ln S_t$, of a commodity is assumed to follow an Ornstein–Uhlenbeck process with seasonality in level and volatility. Thus, the long-term equilibrium

¹²Doran and Ronn (2008) also find a seasonal variation in implied and historical volatility levels for natural gas and heating oil.

¹³See, e.g., Brennan (1991), Gibson and Schwartz (1990), and Schwartz (1997) on the mean-reverting behavior of commodity prices.

price is assumed to be deterministic. Let

$$\ln S_t = X_t + s(t), \quad (3.1)$$

where $s(t)$ is a deterministic function of time capturing the seasonality of a commodity's price level. Note that this component is added for the sake of completeness only. As it has no impact on option prices, which are the subject of the study in this essay, we refrain from specifying the function $s(t)$ explicitly.

Let Z_t^X be a standard Brownian motion. The stochastic component X_t is assumed to follow the dynamics

$$dX_t = \kappa(\mu - X_t)dt + \sigma_X e^{\varphi(t)} dZ_t^X, \quad (3.2)$$

with $\kappa > 0$ denoting the degree of mean-reversion towards the long-run mean μ of the process. The volatility of the process is characterized by σ_X and the function $\varphi(t)$, which describes the seasonal behavior of the asset's volatility. In contrast to $s(t)$, $\varphi(t)$ impacts the price of an option by directly affecting the underlying asset's volatility. Considering the empirical volatility patterns in Figures 3.2 and 3.3, we follow Geman and Nguyen (2005) and specify the function $\varphi(t)$ as

$$\varphi(t) = \theta \sin(2\pi(t + \zeta)) \quad (3.3)$$

with $\theta \geq 0$ and $\zeta \in [-0.5, 0.5]$ in order to ensure the parameters' uniqueness. Thereby, January 1 serves as the time origin. We refer to this model as *Model 1-S* throughout the rest of the chapter, indicating it as a one-factor model with seasonal volatility.

The proposed one-factor model is closely linked to existing commodity pricing models. By setting $\varphi(t) = 0$ and $s(t) = 0$, the model nests the one-factor model proposed by Schwartz (1997).¹⁴ Thus, the model of Schwartz (1997) serves as a natural benchmark and will be referred to as *Model 1*.¹⁵

¹⁴Note that Schwartz (1997) also considers two- and three-factor models in his study.

¹⁵The proposed model can also be considered as a simpler version of the model considered by Geman and Nguyen (2005), who studied the influence of inventory levels on the pricing of futures contracts. As our main purpose is to investigate the benefits of modeling seasonality of volatility in the context of empirical option pricing, we keep the model parsimonious to enhance implementation and interpretation.

In the second model considered, the assumption regarding the long-term equilibrium price level is changed. Following the ideas presented by Schwartz and Smith (2000), a second latent risk factor is added, representing the fact that uncertainty about the long-term equilibrium price exists in the economy. The following model will be referred to as *Model 2-S*, i.e. a two-factor model with seasonal volatility. Let

$$\ln S_t = X_t + Y_t + s(t), \quad (3.4)$$

with

$$dX_t = \mu dt + \sigma_X e^{\varphi(t)} dZ_t^X, \quad (3.5)$$

$$dY_t = -\kappa Y_t dt + \sigma_Y dZ_t^Y, \quad (3.6)$$

where $s(t)$ is again a deterministic function of time capturing seasonality effects at the price level. The first stochastic component X_t describes the non-stationary long-term equilibrium price process. The parameter μ captures the drift and σ_X together with the deterministic function $\varphi(t)$ capture the volatility of the process. As for the one-factor model, $\varphi(t)$ governs the seasonality of volatility and is again assumed to be described by Equation (3.3). The zero mean Ornstein–Uhlenbeck process Y_t captures short-term deviations from the long-term equilibrium. The parameter $\kappa > 0$ governs the speed of mean-reversion, while σ_Y governs the volatility of the process. Z_t^X and Z_t^Y are standard Brownian motions with instantaneous correlation ρ .

Note that for $\varphi(t) = 0$ and specifying $s(t)$ accordingly, the model is identical to the model proposed by Sørensen (2002). When also imposing $s(t) = 0$, one obtains the well-known two-factor model of Schwartz and Smith (2000), which we call *Model 2* throughout the chapter. This model has been studied extensively and, thus, provides an ideal basis to build on our empirical analysis.¹⁶

One might argue that more complex pricing models exist compared to the ones we use in this study – for example, these models might include jumps or

¹⁶Note that, although not labeling one of the factors as convenience yield, Schwartz and Smith (2000) showed that their latent factor approach is equivalent to the two-factor model of Gibson and Schwartz (1990), which explicitly models the convenience yield. As the latent factor model of Schwartz and Smith (2000) is more convenient for estimation, it is usually preferred in empirical studies.

stochastic volatility.¹⁷ However, as our main focus is on the influence of the impact of deterministic changes of volatility on the pricing of options, we decided to employ well-established and understood models as benchmarks for our empirical study.

3.3.2 Valuation of Futures and Options

As the price dynamics are directly specified under the risk-neutral measure, the futures price is equal to the expected spot price at the contract's maturity.¹⁸ Since all state variables are normally distributed, the spot price follows a log-normal distribution. Thus, conditional on information available at time zero, the futures price with maturity T at time zero, denoted by $F_0(T)$, is given by

$$\begin{aligned}\ln F_0(T) &= \ln \mathbb{E}[S_T] \\ &= \mathbb{E}[\ln(S_T)] + \frac{1}{2}\text{Var}[\ln(S_T)].\end{aligned}\quad (3.7)$$

For the one-factor model, *Model 1-S*, the futures price is therefore given by¹⁹

$$\begin{aligned}\ln F_0(T) &= e^{-\kappa T} X_0 + \mu(1 - e^{-\kappa T}) + s(T) \\ &\quad + \frac{1}{2}\sigma_X^2 \int_0^T e^{2\theta \sin(2\pi(u+\zeta))} e^{-2\kappa(T-u)} du.\end{aligned}\quad (3.8)$$

Analogously, the futures price in the two-factor model, *Model 2-S*, can be obtained as

$$\begin{aligned}\ln F_0(T) &= X_0 + \mu T + Y_0 e^{-\kappa T} + s(T) + \frac{1}{2}\sigma_X^2 \int_0^T e^{2\theta \sin(2\pi(u+\zeta))} du \\ &\quad + (1 - e^{-2\kappa T}) \frac{\sigma_Y^2}{4\kappa} + \sigma_X \sigma_Y \rho \int_0^T e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(T-u)} du.\end{aligned}\quad (3.9)$$

Similarly, the value of a European option on a futures contract can be immediately calculated as the expected pay-off discounted at the risk-free rate r . There-

¹⁷Refer to, e.g., Geman and Nguyen (2005) for a model with stochastic volatility and Hilliard and Reis (1998) for a jump-diffusion model.

¹⁸See, e.g., Cox et al. (1981).

¹⁹For more detailed information on the pricing formulas for futures and options presented in this section, please refer to the Appendix in Section 3.7.

fore, the price of a call option, with exercise price K and maturity t written on a future with maturity T at time zero, is given by $c_0 = e^{-rt} E[\max(F_t(T) - K, 0)]$.

As all state variables are normally distributed, the log futures price $\ln F_t(T)$ is also normally distributed. The variance $\sigma_F^2(t, T)$ of $\ln F_t(T)$ for *Model 1-S* is given by

$$\sigma_F^2(t, T) = \sigma_X^2 e^{-2\kappa(T-t)} \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} e^{-2\kappa(t-u)} du, \quad (3.10)$$

and for *Model 2-S* by

$$\begin{aligned} \sigma_F^2(t, T) &= \sigma_X^2 \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} du + \frac{\sigma_Y^2}{2\kappa} e^{-2\kappa(T-t)} (1 - e^{-2\kappa t}) \\ &\quad + 2\sigma_X \sigma_Y \rho e^{-\kappa(T-t)} \int_0^t e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(t-u)} du. \end{aligned} \quad (3.11)$$

Thus, $F_t(T)$ follows a log-normal distribution and European option pricing formulas can be obtained by following the arguments provided in Black (1976). Therefore, the price of a European call option is given by

$$c_0 = e^{-rt} \left(F_0(T) N(d_1) - K N(d_1 - \sigma_F(t, T)) \right), \quad (3.12)$$

where r denotes the risk-free rate which is assumed constant, N denotes the cumulative distribution function of the standard normal distribution, and d_1 is defined as

$$d_1 = \frac{\ln(F_0(T)/K) + \frac{1}{2}\sigma_F^2(t, T)}{\sigma_F(t, T)}. \quad (3.13)$$

The formula of a European put can be derived accordingly and is given by

$$p_0 = e^{-rt} \left(K N(-d_1 + \sigma_F(t, T)) - F_0(T) N(-d_1) \right). \quad (3.14)$$

Note that, by the inclusion of seasonal volatility, the resulting pricing formulas are only semi-analytical, i.e. the remaining integral has to be computed numerically; this can be done with high speed and precision.

3.4 Data Description and Estimation Procedure

3.4.1 Data

The data set used for our empirical study consists of daily prices of American style options and corresponding futures contracts written on soybeans and heating oil. All data are obtained from Bloomberg. In the case of soybeans, the data set includes prices for call and put options on futures traded at the CBOT maturing between January 2005 and November 2010. CBOT soybean futures and options are available for seven different maturity months: January, March, May, July, August, September, and November. For the heating oil options, the data set includes prices for call and put options traded at the NYMEX with maturity months between January 2005 and December 2010. Heating oil futures and options are available with maturities in all twelve calendar months.

Several exclusion criteria were applied when constructing our data set. In order to avoid liquidity related biases, we only consider options with strike prices between 90 % and 110 % of the underlying futures prices. Following Bakshi et al. (1997), we furthermore only consider options with at least six days to maturity for the same reason. Due to discreteness in the reported prices, we excluded options with values of less than \$0.50. Additionally, price observations allowing for immediate arbitrage profits by exercising the American option are excluded from our sample.

Since we want to assess the effects of seasonal volatility, it is necessary to ensure that the seasonal pattern over the course of the calendar year is reflected in our data. Hence, prices for options maturing in the various contract months need to be available. Taking this into account, the time periods for our empirical study extend from July 29, 2004 through June 22, 2009 for the soybean options, and October 21, 2004 to October 2, 2009 for the heating oil options.

Tables 3.1 and 3.2 summarize the properties of our data set consisting of daily put and call options prices. The data set covers a total of 156,129 observations for the soybean options, and 475,472 observations for the heating oil options. The considered observations consist of options within different moneyness and maturity categories. When the price of the futures contract is between 90% and 95% of the option's strike price, call (put) options are considered as out-

Table 3.1: Sample Description of Soybean Futures Options

This table shows average prices for the soybean options data set grouped according to moneyness and time to maturity for both call and put options. The numbers of observations are reported in parentheses. Prices are obtained from Bloomberg for the period from July 29, 2004 to June 22, 2009.

Call Options		Days-to-Expiration			Subtotal
	S/K	<60	60–180	>180	
ITM	1.05–1.1	\$ 76.45 (3,753)	\$ 96.74 (4,312)	\$ 108.74 (7,728)	(15,793)
ATM	0.95–1.05	\$ 36.94 (9,740)	\$ 66.77 (11,069)	\$ 83.11 (19,967)	(40,776)
OTM	0.9–0.95	\$ 15.29 (4,734)	\$ 42.26 (6,401)	\$ 58.28 (11,525)	(22,660)
Subtotal		(18,227)	(21,782)	(39,220)	(79,229)
Put Options					
ITM	0.9–0.95	\$ 85.86 (4,440)	\$ 110.47 (5,442)	\$ 117.87 (10,476)	(20,358)
ATM	0.95–1.05	\$ 39.13 (9,660)	\$ 67.08 (10,636)	\$ 84.23 (19,459)	(39,755)
OTM	1.05–1.1	\$ 13.44 (3,861)	\$ 36.69 (4,678)	\$ 56.14 (8,248)	(16,787)
Subtotal		(17,961)	(20,756)	(38,183)	(76,900)
Total		(36,188)	(42,538)	(77,403)	(156,129)

of-the-money (in-the-money). Both call and put options are considered to be at-the-money when the price of the futures contract is between 95% and 105% of the option's strike price. When the price of the futures contract is between 105% and 110% of the option's strike price, call (put) options are considered as in-the-money (out-of-the-money). Options with less than 60 days to expiration are considered to be short-term, while those with 60 to 180 days to expiration are medium-term and options with more than 180 days to expiration are long-term

Table 3.2: Sample Description of Heating Oil Futures Options

This table shows average prices for the heating oil options data set grouped according to moneyness and time to maturity for both call and put options. The numbers of observations are reported in parentheses. Prices are obtained from Bloomberg for the period from October 21, 2004 to October 2, 2009.

Call Options		Days-to-Expiration			Subtotal
	S/K	<60	60–180	>180	
ITM	1.05–1.1	\$ 18.76 (13,597)	\$ 24.92 (23,244)	\$ 32.04 (15,549)	(52,390)
ATM	0.95–1.05	\$ 9.52 (35,417)	\$ 16.67 (65,530)	\$ 23.73 (52,460)	(153,407)
OTM	0.9–0.95	\$ 4.29 (17,600)	\$ 10.89 (35,863)	\$ 18.33 (25,873)	(79,336)
Subtotal		(66,614)	(124,637)	(93,882)	(285,133)
Put Options					
ITM	0.9–0.95	\$ 19.57 (10,421)	\$ 28.08 (13,852)	\$ 38.25 (6,174)	(30,447)
ATM	0.95–1.05	\$ 9.40 (29,785)	\$ 16.50 (44,513)	\$ 23.51 (28,530)	(102,828)
OTM	1.05–1.1	\$ 3.95 (13,430)	\$ 9.78 (25,282)	\$ 16.16 (18,352)	(57,064)
Subtotal		(53,636)	(83,647)	(53,056)	(190,339)
Total		(120,250)	(208,284)	(146,938)	(475,472)

contracts. Interest rates used in our empirical study are the 3-month USD Libor rates published by the British Bankers' Association.

The closed- or semi closed-form solutions presented for the different valuation models in Section 3.3 are only available for European style options. However, all options in our data set are American style contracts. To deal with this issue, we follow the approach taken by Trolle and Schwartz (2009). Using the analytical approximation of the early exercise premium developed by Barone-Adesi

and Whaley (1987), we transform each American option price into its European counterpart.

The approach of Barone-Adesi and Whaley (1987) relies on the constant volatility Black (1976) framework. The seeming inconsistency of this approach with the valuation models described above is remedied by the fact that each option is transformed separately. Therefore, the price characteristics regarding the influence of maturity, moneyness, volatility, and so on should be reflected in the transformed prices as well.²⁰

Furthermore, it should be noted that the early exercise feature is of minor importance in our study. For the soybean futures options, the average correction for the early exercise feature is only 0.68 % for both call and put options. In the case of the options on heating oil futures, the average premium for early exercise is estimated to be 0.32 % for the call options and 0.30 % for the put options.

3.4.2 Model Estimation

The four different valuation models presented in Section 3.3 are the subject of our empirical analysis. In order to compare these model specifications with regard to their ability to price commodity options, we need to specify the models' parameters. While contract characteristics like the maturity and strike price of the options to be priced are given and the price of the underlying asset and the risk-free rate are observable, the model parameters are not. They need to be estimated from market data.

To do this, we employ an option-implied parameter estimation approach rather than relying on historical estimation, as the forward-looking implied estimation of option valuation model parameters can be regarded as standard in both the academic literature and in practice. The four models are re-estimated on a daily basis and only the most liquid at-the-money (ATM) options with strike prices between 95% and 105% of the corresponding futures price are used for the estimation, which make up slightly more than half of our overall sample. Hence, we are able to assess the models' in-sample fit as well as the models' ability to consistently price the cross-section of options out-of-sample.

²⁰See also Trolle and Schwartz (2009) for a discussion regarding the justification of this approach.

In the case of the soybean options, the minimum number of contracts used for the estimation is 27, while the average number is 65. For the heating oil options, the minimum number of observable ATM prices is 15,²¹ while on average 206 observations are used for the estimation. The maximum number of parameters to be estimated is six parameters for the two-factor seasonal volatility model, *Model 2-S*. Thus, it is ensured that for each observation day the number of observable prices is not less than the model parameters to be estimated.

Parameters are estimated by numerically minimizing a loss function describing the pricing errors between theoretical model prices and observed market prices. As common in the literature, we use the root mean squared error (RMSE) as the objective function.²²

Theoretical model prices are obtained by using the formulas for pricing call and put options on futures contracts presented in Equations (3.12) and (3.14). Accordingly, the parameters to be estimated for *Model 1* are $\Phi \equiv \{\kappa, \sigma_X\}$, and for the extended one-factor model, *Model 1-S*, $\Phi \equiv \{\kappa, \sigma_X, \theta, \zeta\}$ must be estimated. The standard two-factor model, *Model 2*, requires the estimation of $\Phi \equiv \{\kappa, \sigma_X, \sigma_Y, \rho\}$, while the parameters $\Phi \equiv \{\kappa, \sigma_X, \sigma_Y, \rho, \theta, \zeta\}$ must be estimated for *Model 2-S* in order to additionally take the seasonal pattern of the volatility into account. The procedure to obtain the parameter estimates Φ_t^* for every observation date t can be summarized as follows:

$$\Phi_t^* = \arg \min_{\Phi_t} RMSE_t(\Phi_t) = \arg \min_{\Phi_t} \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{P}_{t,i}(\Phi_t) - P_{t,i})^2}. \quad (3.15)$$

Thereby, $P_{t,i}$ is the observed market price of option i out of N_t option prices used for the estimation at time t and $\hat{P}_{t,i}(\Phi_t)$ is the theoretical model price based on a set of parameters Φ_t . Parameters are not allowed to take values inconsistent with the model frameworks. In detail, the following restrictions were applied:

²¹One should note, however, that there are only 15 out of the 1244 days with less than 50 observations.

²²See, e.g., Bakshi et al. (1997). A different approach would have been to minimize the relative root mean squared errors (RRMSE). Since the RMSE minimizes dollar pricing errors rather than percentage pricing errors, it gives in-the-money options relatively more weight compared to the RRMSE.

$\kappa, \sigma_X, \sigma_Y > 0$ and $\rho \in (-1, 1)$. Furthermore, the parameters governing the seasonal pattern of volatility were restricted to ensure their uniqueness: $\theta \geq 0$ and $\zeta \in [-0.5, 0.5]$.²³

3.5 Empirical Model Comparison

In this section, we report the results of our empirical study. First, we briefly discuss the implied parameter estimates, and then present the in-sample and out-of-sample pricing results of the valuation models when both including seasonal volatility and excluding it. We end the section with an analysis of the pricing errors and a robustness check of the obtained results.

3.5.1 Estimated Parameters

The mean values and standard errors of the daily re-estimated implied parameters for the soybean and heating oil options are reported in Tables 3.3 and 3.4, respectively. Although the standard errors are reasonably small, it should be noted that the obtained parameter estimates are far from being constant over time. This observation, however, is not peculiar to our study. For example, time-varying parameter estimates when estimating models implicitly have been observed by de Munnik and Schotman (1994) in the case of term structure models and Bakshi et al. (1997) in the case of equity index options. As a robustness check we repeat the study by estimating parameters using one week of options data, see Section 3.5.5.

3.5.2 In-Sample Model Comparison

For each day, we calculate the model prices of each option given by the respective valuation model and the implied parameter values and compare them with their observed counterparts. It is worth noting that this is only a true in-sample test for the ATM options, as only these contracts have been used for the parameter

²³For the numerical estimation of the parameters, ρ was limited to -0.999 and 0.999 , and for κ, σ_X , and σ_Y the lower boundaries of 0.001 were assumed. Furthermore, 10 was used as an artificial upper boundary for the parameters in the numerical estimation procedure.

Table 3.3: Implied Parameter Estimates: Soybean Options

This table reports the mean values and standard errors [in brackets] of the implied parameter estimates for the soybean options data. For the estimation, the cross-section of observed ATM options with $0.95 \leq S/K \leq 1.05$ is used. Implied parameter estimates are obtained by minimizing the root mean squared errors (RMSE) between observed market prices and theoretical model prices. The models are re-estimated daily.

	<i>Model 1</i>	<i>Model 1-S</i>	<i>Model 2</i>	<i>Model 2-S</i>
κ	0.3331 [0.0127]	0.4782 [0.0200]	1.1743 [0.0696]	2.3184 [0.0817]
σ_X	0.3231 [0.0027]	0.3321 [0.0030]	1.4080 [0.0612]	0.2713 [0.0168]
σ_Y	- -	- -	1.6203 [0.0619]	0.5375 [0.0178]
ρ	- -	- -	-0.5487 [0.0182]	-0.1580 [0.0230]
θ	- -	0.1488 [0.0031]	- -	1.3985 [0.0500]
ζ	- -	-0.2073 [0.0065]	- -	-0.1448 [0.0070]

estimation. For the in-the-money (ITM) and out-of-the-money (OTM) contracts one might speak of an out-of-sample test, although not with respect to time, but with respect to moneyness.

As the models without seasonal volatility are nested in their counterparts with seasonal volatility, a higher number of parameters must result in a better in-sample model fit for the latter ones. In contrast, *Model 1-S* and *Model 2* are not nested in each other, and it will be interesting to see what in-sample gain in valuation precision can be achieved by incorporating seasonal volatility versus a second stochastic factor.

Tables 3.5 and 3.6 display the in-sample results. We report average pricing errors according to two different error metrics: the root mean squared error, $RMSE_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{P}_{t,i} - P_{t,i})^2}$, and the relative root mean squared error, $RRMSE_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left(\frac{\hat{P}_{t,i} - P_{t,i}}{P_{t,i}}\right)^2}$. Thereby, $P_{t,i}$ is the observed market price of

Table 3.4: Implied Parameter Estimates: Heating Oil Options

This table reports the mean values and standard errors [in brackets] of the implied parameter estimates for the heating oil options data. For the estimation, the cross-section of observed ATM options with $0.95 \leq S/K \leq 1.05$ is used. Implied parameter estimates are obtained by minimizing the root mean squared errors (RMSE) between observed market prices and theoretical model prices. The models are re-estimated daily.

	<i>Model 1</i>	<i>Model 1-S</i>	<i>Model 2</i>	<i>Model 2-S</i>
κ	0.5836 [0.0161]	0.6201 [0.0131]	2.2694 [0.0858]	2.2756 [0.0779]
σ_X	0.4122 [0.0037]	0.4125 [0.0036]	0.9187 [0.0344]	0.2940 [0.0164]
σ_Y	- -	- -	1.2090 [0.0376]	0.5261 [0.0165]
ρ	- -	- -	-0.3369 [0.0203]	-0.0079 [0.0216]
θ	- -	0.1137 [0.0037]	- -	1.0694 [0.0430]
ζ	- -	0.1755 [0.0074]	- -	0.1946 [0.0071]

option i , $\hat{P}_{t,i}$ is the theoretical model price, and N_t is the number of observations at date t . As RMSE was employed as the objective function in the estimation, it is most appropriate to compare the in-sample fit with respect to this error metric.²⁴ However, due to the non-linear pay-off profile of options contracts, it is also interesting to see how this relates to relative pricing errors. Furthermore, we present the results for the nine different maturity and moneyness brackets.

Comparing the models' pricing fit between the soybean and heating oil options, one can observe that the RMSE for the former is substantially higher in every instance. However, this is a direct consequence of different trading units and price levels of the underlying assets.²⁵ When considering RRMSE, the errors for

²⁴See Christoffersen and Jacobs (2004) on the selection of appropriate error metrics.

²⁵The average price of the front-month futures during the considered time periods is \$ 820.08 for soybeans and \$ 198.79 for heating oil.

Table 3.5: In-Sample Pricing Errors: Soybean Options

This table displays in-sample pricing errors of options on soybean futures. Pricing errors are reported as root mean squared errors (RMSE) and relative root mean squared errors (RRMSE) of option prices for the four valuation models. The reported pricing errors are calculated as average values over the period from July 29, 2004 to June 22, 2009. Pricing errors are grouped by maturity and moneyness of the options. Only ATM options were used for the parameter estimation.

	Days-to-Expiration						Subtotal	
	<60		60-180		>180		RMSE	RRMSE
	RMSE	RRMSE	RMSE	RRMSE	RMSE	RRMSE	RMSE	RRMSE
	[\$]	[%]	[\$]	[%]	[\$]	[%]	[\$]	[%]
<i>Model 1</i>	2.15	36.97	3.32	14.29	5.47	14.65	4.49	22.58
<i>Model 1-S</i>	1.66	27.78	2.98	12.64	4.80	12.33	3.91	17.86
<i>Model 2</i>	1.95	36.17	3.15	13.61	5.07	14.20	4.22	21.93
<i>Model 2-S</i>	1.52	26.89	2.87	12.34	4.27	11.77	3.56	16.87
<i>Model 1</i>	2.62	16.15	2.72	6.12	4.67	8.25	4.05	11.59
<i>Model 1-S</i>	1.80	11.09	2.10	4.46	3.76	6.72	3.23	8.59
<i>Model 2</i>	2.44	15.33	2.48	5.46	4.11	7.74	3.68	10.87
<i>Model 2-S</i>	1.57	10.15	1.87	4.00	3.18	6.14	2.78	7.69
<i>Model 1</i>	2.46	3.20	3.48	3.80	6.27	5.95	5.10	5.36
<i>Model 1-S</i>	2.04	2.58	3.23	3.45	5.54	5.20	4.53	4.67
<i>Model 2</i>	2.34	3.08	3.34	3.65	5.74	5.65	4.79	5.14
<i>Model 2-S</i>	1.94	2.49	3.16	3.38	4.93	4.83	4.18	4.42
<i>Model 1</i>	2.55	22.09	3.13	8.77	5.38	10.25	4.51	14.84
<i>Model 1-S</i>	1.90	16.34	2.69	7.46	4.61	8.62	3.83	11.56
<i>Model 2</i>	2.38	21.47	2.94	8.25	4.89	9.84	4.19	14.27
<i>Model 2-S</i>	1.74	15.52	2.55	7.18	4.04	8.12	3.44	10.81

Table 3.6: In-Sample Pricing Errors: Heating Oil Options

This table displays in-sample pricing errors of options on heating oil futures. Pricing errors are reported as root mean squared errors (RMSE) and relative root mean squared errors (RRMSE) of option prices for the four valuation models. The reported pricing errors are calculated as average values over the period from October 21, 2004 to October 2, 2009. Pricing errors are grouped by maturity and moneyness of the options. Only ATM options were used for the parameter estimation.

	Days-to-Expiration						Subtotal	
	<60		60-180		>180		RMSE	RRMSE
	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]
<i>Model 1</i>	0.53	15.13	0.58	13.25	0.77	5.22	0.72	14.99
<i>Model 1-S</i>	0.40	10.39	0.51	12.46	0.63	4.32	0.61	12.52
<i>Model 2</i>	0.38	10.16	0.55	12.67	0.65	4.51	0.63	12.60
<i>Model 2-S</i>	0.33	8.32	0.50	12.35	0.58	4.02	0.57	11.71
<i>Model 1</i>	0.55	8.78	0.49	6.74	0.68	3.89	0.64	8.71
<i>Model 1-S</i>	0.35	6.07	0.36	5.75	0.48	2.93	0.47	7.06
<i>Model 2</i>	0.38	6.51	0.40	5.95	0.53	3.19	0.51	7.36
<i>Model 2-S</i>	0.26	4.99	0.31	5.25	0.41	2.65	0.40	6.37
<i>Model 1</i>	0.42	2.23	0.46	1.92	0.65	2.24	0.54	2.24
<i>Model 1-S</i>	0.28	1.42	0.37	1.51	0.47	1.59	0.40	1.62
<i>Model 2</i>	0.29	1.51	0.40	1.62	0.53	1.79	0.44	1.75
<i>Model 2-S</i>	0.23	1.12	0.34	1.40	0.43	1.47	0.37	1.45
<i>Model 1</i>	0.53	10.65	0.54	10.30	0.73	4.44	0.67	11.65
<i>Model 1-S</i>	0.36	7.55	0.44	9.53	0.56	3.61	0.53	9.91
<i>Model 2</i>	0.38	7.67	0.47	9.68	0.60	3.79	0.56	10.06
<i>Model 2-S</i>	0.29	6.28	0.41	9.23	0.51	3.34	0.48	9.28

heating oil are still smaller than for the soybean options, but the difference is smaller than for the RMSE.

The overall RMSE yields \$4.51 for *Model 1*, \$3.83 for *Model 1-S*, \$4.19 for *Model 2*, and \$3.44 for *Model 2-S* for the soybean options. The corresponding values for the heating oil options are \$0.67, \$0.53, \$0.56, and \$0.48. In both cases, the models incorporating seasonal volatility outperform their counterparts which do not include this adjustment. Interestingly, *Model 1-S*, the one-factor model with seasonality adjustment, yields lower errors than *Model 2*, the standard two-factor model. Thus, allowing for seasonally varying volatility seems to be more important than adding additional stochastic factors.

Considering the different moneyness categories, the ranking of the models is retained. The OTM and ITM RMSE are slightly higher than their ATM counterparts for soybeans, while for heating oil the ITM RMSE is slightly lower. Naturally, the RRMSE increases for OTM and decreases for ITM options. In both markets, the RMSE is increasing along the maturity brackets. This can be regarded as a direct consequence of the higher average prices of longer maturity options in our sample, as can be seen in Tables 3.1 and 3.2. The RRMSE does not show any clear pattern with respect to maturity.

3.5.3 Out-of-Sample Model Comparison

The most conclusive way to compare different valuation models with respect to their pricing accuracy is their out-of-sample performance. We thus proceed as follows: on each day, we compare the observed market prices with the respective model prices using the parameters estimated on the previous day. Using this approach, only information from the previous day enters the model evaluation. As for the in-sample comparison, we report the results for RMSE and RRMSE, for OTM, ATM, and ITM options, and for the three considered maturity brackets. The results are provided in Table 3.7 for the soybean contracts, and in Table 3.8 for the heating oil contracts.

The overall RMSE for the four models are \$4.86, \$4.33, \$4.63, and \$4.05 for the soybean sample, and \$0.75, \$0.63, \$0.66, and \$0.60 for the heating oil sample. Compared to their in-sample counterparts, one can observe that these errors are

Table 3.7: Out-of-Sample Pricing Errors: Soybean Options

This table reports out-of-sample pricing errors of options on soybean futures. Pricing errors are reported as root mean squared errors (RMSE) and relative root mean squared errors (RRMSE) of option prices for the four valuation models. The reported pricing errors are calculated as average values during the period from July 30, 2004 to June 22, 2009. Pricing errors are grouped by maturity and moneyness of the options. Theoretical model-based prices are derived by using parameters estimated the day before.

	Days-to-Expiration								Subtotal	
	<60		60-180				>180		RMSE	RRMSE
	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]	RMSE [\$]	RRMSE [%]
<i>Model 1</i>	2.31	38.15	3.58	15.07	5.80	15.24	4.79	23.50		
<i>Model 1-S</i>	1.93	29.77	3.28	13.54	5.26	13.11	4.30	19.24		
<i>Model 2</i>	2.18	38.22	3.44	14.52	5.51	14.97	4.58	23.34		
<i>Model 2-S</i>	1.85	29.29	3.20	13.30	4.84	12.75	4.03	18.63		
<i>Model 1</i>	2.83	16.82	3.08	6.71	5.22	8.81	4.50	12.25		
<i>Model 1-S</i>	2.19	12.27	2.62	5.35	4.61	7.59	3.90	9.68		
<i>Model 2</i>	2.74	16.48	2.90	6.19	4.83	8.47	4.25	11.86		
<i>Model 2-S</i>	2.10	12.03	2.50	5.06	4.21	7.20	3.60	9.18		
<i>Model 1</i>	2.60	3.35	3.74	4.04	6.59	6.19	5.38	5.61		
<i>Model 1-S</i>	2.28	2.84	3.53	3.74	6.00	5.57	4.91	5.03		
<i>Model 2</i>	2.54	3.29	3.64	3.94	6.15	5.96	5.13	5.46		
<i>Model 2-S</i>	2.24	2.82	3.49	3.69	5.50	5.28	4.63	4.84		
<i>Model 1</i>	2.72	22.84	3.44	9.36	5.80	10.75	4.86	15.51		
<i>Model 1-S</i>	2.22	17.65	3.09	8.19	5.22	9.32	4.33	12.61		
<i>Model 2</i>	2.64	22.78	3.29	8.93	5.43	10.48	4.63	15.29		
<i>Model 2-S</i>	2.15	17.41	3.00	7.99	4.79	8.99	4.05	12.17		

higher which is, of course, not surprising. More importantly, the ranking of the four models remains identical to the in-sample case: the models including seasonal volatility outperform their counterparts with constant volatility. Again, the one-factor model including seasonality, *Model 1-S*, even beats *Model 2*, the two-factor model without seasonality, in terms of RMSE and RRMSE.

Inspecting the results with respect to moneyness and maturity, it becomes evident that the ranking of the models remains the same in almost all cases and for both the RMSE and the RRMSE criterion. Only in the case of short-term OTM options can we observe one exception for soybeans, where the relative error for *Model 2* is marginally lower than for *Model 1*, and one exception for heating oil where *Model 2* performs better than *Model 1-S*. Notably, the seasonal volatility model variants always beat their constant volatility counterparts. Overall, the obtained results indicate that both a second stochastic factor and seasonal volatility are important for increasing the pricing accuracy of the models. Combining these components in *Model 2-S* yields the best out-of-sample performance in every case.

Lastly, to see whether the observed differences are also statistically significant, we perform Wilcoxon signed-rank tests to compare several model variants. Tables 3.9 (Soybeans) and 3.10 (Heating Oil) present the percentage reductions of the RMSE when introducing a seasonal volatility component (upper part) and when adding a second stochastic factor (lower part). The non-parametric Wilcoxon signed-rank statistic tests whether the median of the differences is significantly different from zero.

One can observe that incorporating seasonal volatility reduces the RMSE in every instance, i.e. for both markets, both models, in-sample and out-of-sample, for every maturity bracket, and for every moneyness category at a 1% significance level. To keep the presentation manageable, we do not report the ITM and OTM results separately; however, they do not deviate qualitatively from the results presented. The out-of-sample pricing errors of all options for the one- and two-factor models are reduced by 11.04% and 12.61% for the soybean options, and by 15.33% and 9.42% for the heating oil options, respectively. The greatest improvements are observed for short-term heating oil contracts, with a maximal improvement of 28.17% for the ATM options and the one-factor model.

Table 3.9: Reduction of Pricing Errors: Soybean Options

This table shows the percentage reduction in root mean squared errors (RMSE) for options on soybean futures due to model extensions. The respective RMSE are reported in Tables 3.5 and 3.7. The figures are grouped by maturity and moneyness of the options and are reported for the in-sample and out-of-sample pricing errors. * indicates significance at the 1 % level according to the Wilcoxon signed-rank test.

		In-Sample				Out-of-Sample				
		Days-to-Expiration				Days-to-Expiration				
		<60	60-180	>180	All	<60	60-180	>180	All	
Introduction of Seasonal Volatility	One- Factor	ATM	31.24%*	22.76%*	19.53%*	20.36%*	22.54%*	14.79%*	11.62%*	13.31%*
		All	25.41%*	14.09%*	14.41%*	15.10%*	18.51%*	10.18%*	10.07%*	11.04%*
	Two- Factor	ATM	35.33%*	24.58%*	22.48%*	24.58%*	23.14%*	14.06%*	12.77%*	15.22%*
		All	27.00%*	13.22%*	17.34%*	17.85%*	18.51%*	8.82%*	11.93%*	12.61%*
Introduction of second Factor	w/o Seasonal Volatility	ATM	7.05%*	8.78%*	12.05%*	9.12%*	3.16%*	5.60%*	7.50%*	5.53%*
		All	6.56%*	6.19%*	9.13%*	7.10%*	3.24%*	4.29%*	6.33%*	4.82%*
	with Seasonal Volatility	ATM	12.57%*	10.93%*	15.27%*	13.94%*	3.90%*	4.79%*	8.71%*	7.62%*
		All	8.56%*	5.24%*	12.25%*	10.11%*	3.24%*	2.84%*	8.28%*	6.50%*

Table 3.10: Reduction of Pricing Errors: Heating Oil Options

This table shows the percentage reduction in root mean squared errors (RMSE) for options on heating oil futures due to model extensions. The respective RMSE are reported in Tables 3.6 and 3.8. The figures are grouped by maturity and moneyness of the options and are reported for the in-sample and out-of-sample pricing errors. * indicates significance at the 1 % level according to the Wilcoxon signed-rank test.

		In-Sample				Out-of-Sample				
		Days-to-Expiration				Days-to-Expiration				
		<60	60-180	>180	All	<60	60-180	>180	All	
Introduction of Seasonal Volatility	One-Factor	ATM	36.62%*	26.12%*	29.43%*	26.45%*	28.17%*	16.87%*	20.12%*	18.53%*
		All	31.75%*	18.57%*	22.74%*	20.49%*	24.84%*	12.49%*	17.13%*	15.33%*
	Two-Factor	ATM	30.62%*	21.47%*	22.51%*	21.22%*	18.47%*	10.48%*	14.58%*	12.57%*
		All	22.65%*	13.68%*	14.97%*	14.18%*	14.23%*	7.80%*	10.97%*	9.42%*
Introduction of second Factor	w/o Seasonal Volatility	ATM	30.81%*	18.95%*	21.92%*	20.25%*	23.81%*	12.83%*	15.19%*	14.35%*
		All	28.94%*	12.65%*	18.17%*	16.13%*	22.96%*	8.82%*	13.34%*	12.05%*
	with Seasonal Volatility	ATM	24.26%*	13.85%*	14.26%*	14.58%*	13.53%*	6.13%*	9.30%*	8.09%*
		All	19.47%*	7.40%*	9.95%*	9.47%*	12.08%*	3.94%*	6.90%*	5.91%*

The introduction of the second stochastic factor also significantly improves the pricing accuracy. Only the out-of-sample results in the soybean case for options with a short time to maturity are less substantial: the observed improvements are small and, although statistically significant, economically less important.

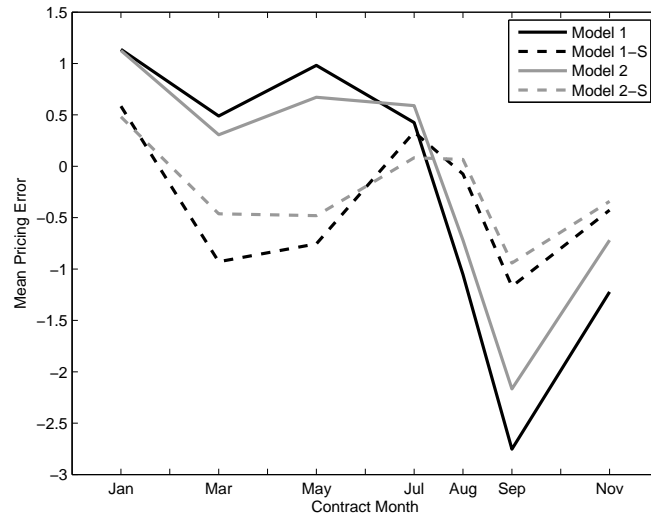
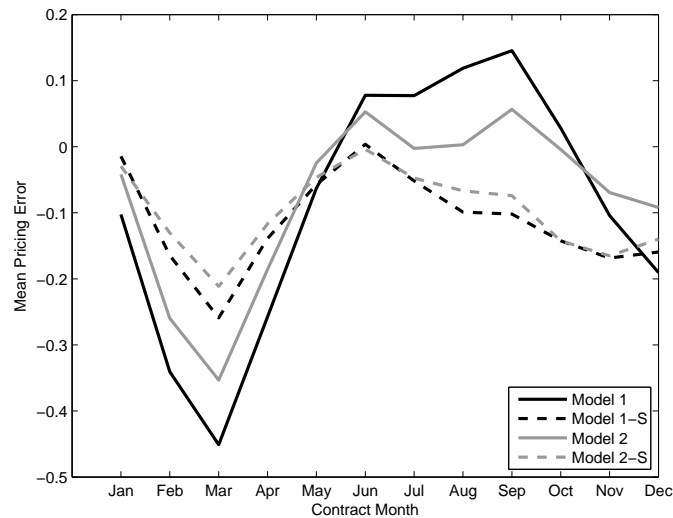
Overall, our empirical findings provide clear evidence for the benefits of valuation models including a seasonal adjustment to the volatility specification when considering the pricing of soybean and heating oil futures options. The inclusion of such a component, which is very simple from the modeling point of view, greatly improves the in-sample and, most importantly, the out-of-sample pricing accuracy.

3.5.4 Analysis of Pricing Errors

We want to further understand the structure of remaining pricing errors and therefore analyze these in more detail. Due to the seasonal pattern of volatility which is documented in Section 3.2, we expect the models without seasonality adjustment (*Model 1* and *Model 2*) to underestimate the prices of options contracts maturing in high-volatility months and overestimate the prices in low-volatility months. For the extended seasonal volatility models (*Model 1-S* and *Model 2-S*) we expect that this systematic mispricing with respect to maturity months is substantially reduced, although most likely not perfectly, as the seasonal structure is modeled in a rather restrictive parametric form.

Let $PE_{t,i}$ denote the absolute out-of-sample pricing errors, i.e. $PE_{t,i} = \hat{P}_{t,i} - P_{t,i}$. Again, $P_{t,i}$ denotes the observed market price of option i at time t and $\hat{P}_{t,i}$ is the corresponding model price. Figure 3.4 displays the mean absolute pricing errors depending on the options' maturity months for soybean and heating oil options and for the four valuation models. As expected, we can observe a strong seasonal pattern in the pricing errors with respect to the maturity months for the case of constant volatility, i.e. *Model 1* and *Model 2*. Allowing volatility to be seasonal substantially reduces this systematic mispricing.

In particular, for soybeans, it can be observed that until July the mean pricing errors are positive in the case of the models without seasonal volatility. This means that these models overestimate the price as the model volatility is higher

Panel A: Soybeans*Panel B: Heating Oil***Figure 3.4: Out-of-Sample Mean Pricing Errors by Maturity Months**

This figure shows the mean absolute pricing errors for each contract month of the soybean and heating oil futures options in our data set. The pricing errors are defined as the difference between theoretical model and observed market prices, where model prices are calculated out-of-sample based on parameters estimated on the previous day for the four different valuation models.

than the actual one for these maturity months. In contrast, pricing errors from August on, in particular for *Model 1* and *Model 2*, are strongly negative since actual volatility spikes up and exceeds the annual average once the harvest in the US commences. In the case of heating oil, one can observe the same inverse relationship between actual implied volatility (see Figure 3.3) and changes in pricing errors over the different maturity months.

To study the relationship of pricing errors and other contract specific factors, we run the following regression:

$$\begin{aligned}
 PE = & \beta_0 + \beta_1 CALL + \beta_2 MAT + \beta_3 MON + \beta_4 MON^2 + \beta_5 Feb \\
 & + \beta_6 Mar + \beta_7 Apr + \beta_8 May + \beta_9 Jun + \beta_{10} Jul + \beta_{11} Aug \\
 & + \beta_{12} Sep + \beta_{13} Oct + \beta_{14} Nov + \beta_{15} Dec + \epsilon.
 \end{aligned} \tag{3.16}$$

The considered independent variables include a constant, a dummy variable for call options (*CALL*) which takes the value 1 for calls and 0 for puts, the time to maturity in years (*MAT*), the moneyness of the option (*MON*), and the moneyness squared (*MON*²). Furthermore, to capture the maturity month effect documented above, dummy variables for all contract months but January are included.²⁶

The results of the regression analysis are summarized in Table 3.11. Significance of the coefficients is evaluated based on heteroscedasticity-robust t-statistics computed by the method of Newey–West.

The coefficients of the *CALL* variable are negative and significant at the 1% level for heating oil while not being significant for soybeans. The time to maturity, *MAT*, seems to be negatively related with the pricing errors for the majority of specifications. The coefficients for moneyness and moneyness squared are all highly significantly positive (*MON*) and negative (*MON*²), respectively. This indicates a “smile effect” across strikes which is not captured by the models.

Most of the maturity month dummies are significant for all model specifications and for both soybeans and heating oil options.²⁷ As noted above, the trigonometric seasonality function is not able to remove the seasonality effect en-

²⁶Since for soybeans there are no contracts with maturity in February, April, June, October, or December, these terms are dropped in the soybean case.

²⁷Please note that the month dummies capture the pricing errors relative to the January contract since there is no dummy for January.

Table 3.11: Regression Analysis of Out-of-Sample Pricing Errors

This table reports the results of the regression analysis for the out-of-sample absolute pricing errors for options on soybean and heating oil futures. The pricing errors are defined as the difference between theoretical model and observed market prices. Independent variables include a constant, a dummy for call options (CALL), the time to maturity in years (MAT), the moneyness of the option (MON) and moneyness squared (MON²). Thereby, moneyness is defined as futures price/strike price for call options and as strike price/futures price for put options. Furthermore, dummy variables for the contract months are considered. *, **, and *** indicate significance at the 1%, 5%, and 10% level, respectively, according to heteroscedasticity-robust *t*-statistics.

	Soybeans					Heating Oil				
	Model 1	Model 1-S	Model 2	Model 2-S		Model 1	Model 1-S	Model 2	Model 2-S	
Constant	-199.323***	-184.100***	-191.732***	-173.027***		-40.302***	-42.619***	-44.284***	-43.421***	
CALL	0.005	0.041	0.023	0.051		-0.033***	-0.036***	-0.047***	-0.039***	
MAT	-0.199*	-0.598***	-0.174*	-0.081		0.065***	-0.012	-0.060***	-0.041**	
MON	401.722***	370.451***	386.351***	347.731***		79.589***	84.500***	87.760***	86.113***	
MON ²	-200.518***	-184.901***	-192.795***	-173.645***		-39.259***	-41.731***	-43.323***	-42.542***	
Feb	-	-	-	-		-0.243***	-0.153***	-0.220***	-0.103***	
Mar	-0.643***	-1.508***	-0.814***	-0.940***		-0.352***	-0.245***	-0.310***	-0.181***	
Apr	-	-	-	-		-0.159***	-0.124***	-0.140***	-0.085***	
May	-0.136	-1.309***	-0.436***	-0.949***		0.034*	-0.045***	0.016	-0.019	
Jun	-	-	-	-		0.174***	0.012	0.089***	0.020	
Jul	-0.681***	-0.191*	-0.507***	-0.378***		0.177***	-0.045***	0.029*	-0.028**	
Aug	-2.187***	-0.663***	-1.843***	-0.412***		0.219***	-0.091***	0.035**	-0.046***	
Sep	-3.883***	-1.750***	-3.286***	-1.417***		0.246***	-0.093***	0.090***	-0.052***	
Oct	-	-	-	-		0.136***	-0.126***	0.038**	-0.112***	
Nov	-2.260***	-0.753***	-1.755***	-0.772***		0.004	-0.150***	-0.023	-0.132***	
Dec	-	-	-	-		-0.095***	-0.145***	-0.047**	-0.108***	
Adj. R ²	0.0519	0.0237	0.0387	0.0163		0.0546	0.0312	0.0458	0.0322	

tirely. However, the size of the coefficients for the different maturity months is substantially reduced.

Finally, the reported values of the adjusted R^2 show how much of the variation in pricing errors can be explained by the applied regression model. In this context, a lower R^2 implies a lower systematic pricing error. In the case of soybeans, when introducing seasonal volatility, the adjusted R^2 decreases from 0.0519 to 0.0237 and from 0.0387 to 0.0163 for the one- and two-factor models, respectively. For heating oil, the adjusted R^2 is reduced from 0.0546 to 0.0312 for the one-factor models and from 0.0458 to 0.0322 for the two-factor models. Hence, the model extensions, accounting for seasonal volatility patterns, reduce systematic mispricing.

3.5.5 Robustness Check

As a robustness check with respect to the daily estimation approach, we repeat the entire study on a weekly basis, i.e. we estimate model parameters using a week of option prices and consider the pricing of the next week's options in the out-of-sample analysis. Naturally, pricing errors are higher, but the overall result of the ranking of the different models remains unchanged.

Due to space constraints, we do not report detailed results, but summarize the main findings. For the complete data set, the in-sample (out-of-sample) reduction of RMSE due to the introduction of seasonal volatility is 12.21 % (7.00 %) for the one-factor and 11.74 % (5.06 %) for the two-factor model. In the case of heating oil, the reductions are 15.40 % (7.13 %) and 9.30 % (3.91 %) for the one- and two-factor model, respectively. All these reductions are significant at the 1 % level. The introduction of a second stochastic factor reduces the overall RMSE in the soybean sample by 6.77 % (3.79 %) and by 6.27 % (1.78 %) for the model without and with seasonal volatility, respectively. Thereby, it is worth noting that the error reductions for the short-term contracts are not significant, fostering the impression that the second stochastic factor is more important in explaining the dynamics at the long end of the futures curve. For heating oil, the two-factor models show a RMSE which, in comparison to the one-factor models, is reduced by 12.36 % (5.77 %) for the case without seasonal volatility and by 6.04 % (2.50 %)

for the case with seasonal volatility. Again, these results are significant at the 1% level. The reductions in pricing errors are, with few exceptions, significant for all time to maturity and moneyness brackets.

3.6 Conclusion

In this chapter, we study the impact of seasonally fluctuating volatility in commodity markets on the pricing of options. These seasonal effects are well-known in the literature, but their impact on commodity options pricing has never been investigated. We extend two standard continuous time commodity derivatives valuation models to incorporate seasonality in volatility. Using an extensive data set of soybean and heating oil options, we compare the empirical options pricing accuracy of these models with their constant volatility counterparts. The results show that incorporating the stylized fact of seasonally fluctuating volatility significantly improves options valuation performance and is more important than introducing a second stochastic factor. This leads to the conclusion that seasonality in volatility should be accounted for when dealing with options on seasonal commodities.

Future research could extend our results in various ways. With respect to the modeling of seasonality, it might be worth investigating which parametric assumption performs best for different markets. Compared to the trigonometric approach taken in this chapter, one might model this component in other ways, e.g. by using simple step functions, allowing for more complex seasonality patterns while relying on a higher number of parameters. As a next step, the importance of seasonality in a stochastic volatility setting can be analyzed. It is not clear what fraction of the fluctuation in volatility can be captured by seasonality and what fraction remains stochastic. The next chapter, emphasizes this and presents a stochastic volatility model which takes into account seasonality in the variance process.

3.7 Appendix

As outlined in Section 3.3, the logarithm of the spot price is defined as $\ln S_t = X_t + s(t)$ and $\ln S_t = X_t + Y_t + s(t)$ for the one- and two-factor models, respectively. Applying Ito's Lemma to Equation (3.2) and to Equations (3.5) and (3.6), respectively, yields for the one-factor model, *Model 1-S*,

$$\ln S_t = X_0 e^{-\kappa t} + \mu(1 - e^{-\kappa t}) + \sigma_X \int_0^t e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(t-u)} dZ_u^X + s(t) \quad (3.17)$$

and for the two-factor model, *Model 2-S*,

$$\begin{aligned} \ln S_t = & X_0 + \mu t + \sigma_X \int_0^t e^{\theta \sin(2\pi(u+\zeta))} dZ_u^X \\ & + Y_0 e^{-\kappa t} + \sigma_Y \int_0^t e^{-\kappa(t-u)} dZ_u^Y + s(t). \end{aligned} \quad (3.18)$$

The mean and variance of $\ln S_t$ can be obtained for the one-factor model as

$$E[\ln(S_t)] = X_0 e^{-\kappa t} + \mu(1 - e^{-\kappa t}) + s(t) \quad (3.19)$$

and

$$\text{Var}[\ln(S_t)] = \sigma_X^2 \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} e^{-2\kappa(t-u)} du \quad (3.20)$$

and for the two-factor model as

$$E[\ln(S_t)] = X_0 + \mu t + Y_0 e^{-\kappa t} + s(t) \quad (3.21)$$

and

$$\begin{aligned} \text{Var}[\ln(S_t)] = & \sigma_X^2 \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} du + (1 - e^{-2\kappa t}) \frac{\sigma_Y^2}{2\kappa} \\ & + 2\sigma_X \sigma_Y \rho \int_0^t e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(t-u)} du. \end{aligned} \quad (3.22)$$

Since all state variables are normally distributed, the logarithm of the spot price is also normally distributed. Our model is formulated directly under the risk-neutral measure, so the price of the futures contract equals the expected spot

price. For the one-factor model, the futures price is therefore given by

$$\begin{aligned}
\ln F_0(T) &= \ln E[S_T] \\
&= E[\ln(S_T)] + \frac{1}{2} \text{Var}[\ln(S_T)] \\
&= X_0 e^{-\kappa T} + \mu(1 - e^{-\kappa T}) + s(T) \\
&\quad + \frac{1}{2} \sigma_X^2 \int_0^T e^{2\theta \sin(2\pi(u+\zeta))} e^{-2\kappa(T-u)} du
\end{aligned} \tag{3.23}$$

and for the two-factor model by

$$\begin{aligned}
\ln F_0(T) &= X_0 + \mu T + Y_0 e^{-\kappa T} + s(T) + \frac{1}{2} \sigma_X^2 \int_0^T e^{2\theta \sin(2\pi(u+\zeta))} du \\
&\quad + (1 - e^{-2\kappa T}) \frac{\sigma_Y^2}{4\kappa} + \sigma_X \sigma_Y \rho \int_0^T e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(T-u)} du.
\end{aligned} \tag{3.24}$$

Analogous to Schwartz and Smith (2000), we refer to $\ln F_t(T)$ in terms of the time t state variables. Applying this to (3.23) and (3.24), all terms except the state variables are deterministic and defined as constant c . Hence, the variance $\sigma_F^2(t, T)$ of $\ln F_t(T)$ for the one-factor model can be derived as

$$\begin{aligned}
\sigma_F^2(t, T) &= \text{Var}[\ln F_t(T)] \\
&= \text{Var}[X_t e^{-\kappa(T-t)} + c] \\
&= \sigma_X^2 e^{-2\kappa(T-t)} \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} e^{-2\kappa(t-u)} du,
\end{aligned} \tag{3.25}$$

and for the two-factor model as

$$\begin{aligned}
\sigma_F^2(t, T) &= \text{Var}[X_t + Y_t e^{-\kappa(T-t)} + c] \\
&= \sigma_X^2 \int_0^t e^{2\theta \sin(2\pi(u+\zeta))} du + \frac{\sigma_Y^2}{2\kappa} e^{-2\kappa(T-t)} (1 - e^{-2\kappa t}) \\
&\quad + 2\sigma_X \sigma_Y \rho e^{-\kappa(T-t)} \int_0^t e^{\theta \sin(2\pi(u+\zeta))} e^{-\kappa(t-u)} du.
\end{aligned} \tag{3.26}$$

Chapter 4

Seasonal Stochastic Volatility: Implications for the Pricing of Commodity Options

4.1 Introduction¹

Trading in commodity derivatives markets has experienced a tremendous growth over the last decade. Increased volatility of commodity prices created the need for efficient risk management strategies. This is especially true for energy markets, as energy is a critical input factor for many industrial firms. The ability to efficiently manage these price risks has direct consequences for the profitability of firms and economic growth. Commodity options provide a powerful risk management tool, but accurately pricing these contracts is not a trivial task as the main input factor, the volatility, is not observable. Therefore, the accurate modeling of volatility in these markets is of critical importance.

Although many commodities exhibit significant seasonal variations, most of the existing literature concerning the pricing of commodity contingent claims solely considers the crude oil market or other markets without seasonality, such as copper or gold. Brennan and Schwartz (1985), Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000), and Casassus and Collin-Dufresne

¹This chapter is based on a homonymous paper co-authored by Marcel Prokopczuk and Markus Rudolf.

(2005) develop one-, two-, and three-factor models in a constant volatility framework and study the empirical performance for pricing crude oil, copper, gold, and silver futures.

Sørensen (2002) adds a seasonal component at the price level to the constant volatility two-factor model of Schwartz and Smith (2000) and applies it to the wheat, corn, and soybean markets. Similarly, Manoliu and Tompaidis (2002) and Cartea and Williams (2008) apply this model to the US and UK natural gas futures market, respectively.

Back et al. (2010) extend this work by allowing the volatility to vary seasonally.² They show that considering seasonality in the volatility greatly improves the pricing performance for heating oil and soybean options. However, all the previously outlined work assumes that volatility is deterministic which is clearly a very strong assumption as it cannot generate the volatility smile, which is also observed in commodity markets.³

Trolle and Schwartz (2009) develop a Heath, Jarrow, and Morton (1992)-type stochastic volatility model for the pricing of commodity futures and options but do not consider seasonality as they apply their model only to the crude oil market. The only articles allowing for seasonal and stochastic volatility we are aware of are Geman and Nguyen (2005) and Richter and Sørensen (2002), who consider the pricing of soybean futures and options. However, their models only facilitate pricing formulas for futures and options that are computationally very burdensome. Accordingly, they do not study the options pricing ability of their models at all.

In this essay, we suggest an extension of the Heston (1993) stochastic volatility model that reflects the seasonal nature of volatility. In contrast to the models proposed by Geman and Nguyen (2005) and Richter and Sørensen (2002), our model has the crucial advantage of enabling us to compute option values in an efficient way which is of significant importance if one wants to apply the model in practice. This fact allows us to empirically study the pricing performance of our model using an extensive data set of options prices.

²See also Chapter 3.

³See, e.g., Trolle and Schwartz (2009) and Liu and Tang (2011).

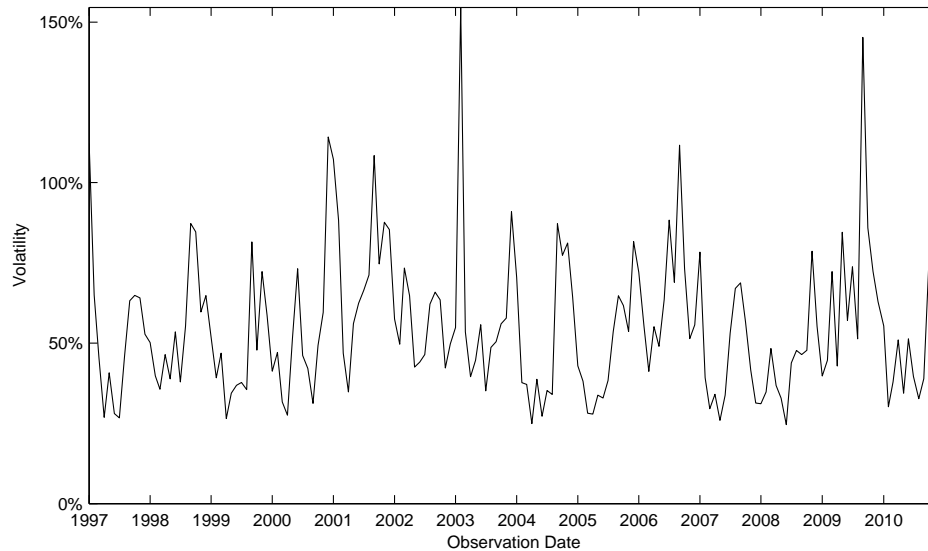


Figure 4.1: Historical Volatility of Natural Gas Futures

This figure shows historical volatilities of natural gas front-month futures from January 1997 to December 2010. The historical volatilities were derived by calculating the annualized standard deviation of the daily returns for each observation month. Prices are from Bloomberg.

Our model is applicable to every commodity market exhibiting seasonality in volatility. In our empirical analysis, we focus on the natural gas market, which is a prominent example of a market with stochastic and seasonal volatility. Historical volatilities of natural gas front-month futures are shown in Figure 4.1. It can be seen that volatility is far from being constant over time. In fact, volatility seems to fluctuate stochastically while following a very pronounced seasonal pattern.⁴ For energy markets like the natural gas market, weather-induced demand shocks lead to a higher volatility of futures prices during the winter, whereas for agricultural commodities, volatility is usually highest during the summer prior and throughout the harvesting period when inventory levels are low and significant uncertainty regarding the new harvest is resolved.⁵

⁴See also Suenaga et al. (2008) and Doran and Ronn (2008) for empirical studies of the natural gas market. Both find that volatility is highly seasonal and varies over time.

⁵Refer to Anderson (1985), Khoury and Yourougou (1993), and Karali and Thurman (2010) for empirical studies on the seasonal behavior of commodity price volatility.

We use a large data set of New York Mercantile Exchange (NYMEX) natural gas options and corresponding futures contracts spanning the time period from January 2007 to December 2010 which consists of 367,469 option price observations. Additionally, we employ ten years of futures data spanning the period from January 1997 to December 2006 to estimate our model under the physical measure using a Bayesian Markov Chain Monte Carlo (MCMC) approach. In doing so, we follow Bates (2000) and Broadie et al. (2007) and abstain from a pure cross-sectional (re)-calibration exercise as in Bakshi et al. (1997) but estimate all parameters that should be equal under the physical and the risk-neutral measure from historical data.

The results of our empirical study show that our model is superior for the pricing of commodity options with seasonalities. Compared to the standard stochastic volatility model of Heston (1993), our model yields substantial improvements in pricing accuracy. The obtained results are both statistically and economically significant and consistent for different robustness checks, implying that the proposed seasonal model should be considered when valuing options on commodities that undergo a seasonal cycle.

The remainder of this chapter is organized as follows. Section 4.2 lays out the model for pricing options under seasonal volatility. Section 4.3 describes the data set and the estimation approach. Section 4.4 presents and discusses the empirical results. Section 4.5 concludes. The Appendix in Section 4.6 contains additional details on the numerical implementation of the model.

4.2 Model Description

In this section, we present a stochastic volatility model that incorporates a seasonal adjustment of the variance process to capture the empirically observed seasonal behavior of many commodities. After introducing the price and variance dynamics, we derive the valuation formula for European call options.

4.2.1 Commodity Futures Price Dynamics

The underlying of almost any exchange traded commodity option is not the commodity's spot price but the price of a corresponding futures contract. We therefore start by specifying the dynamics of the futures price. The alternative approach would be to make an assumption on the dynamics of the spot price and derive the futures price dynamics within this model. However, this approach has the severe disadvantage that it is generally not possible to derive a closed-form solution for the commodity futures price in a stochastic volatility framework, which hinders the derivation of a computationally efficient options pricing formula.⁶

The commodity futures price dynamics under the physical measure are assumed to follow

$$dF_t(T) = \mu F_t(T)dt + F_t(T)\sqrt{V_t}dW_{F,t} \quad (4.1)$$

$$dV_t = \kappa(\theta(t) - V_t)dt + \sigma\sqrt{V_t}dW_{V,t} \quad (4.2)$$

$$\theta(t) = \bar{\theta} e^{\eta \sin(2\pi(t+\zeta))} \quad (4.3)$$

where $F_t(T)$ is the futures price at t with maturity T and μ is the drift of the futures price process under the physical measure. V_t is the instantaneous variance of the futures returns described through a square-root process as used by Cox et al. (1985), κ is the mean-reversion speed of the variance process, $\theta(t)$ is the long-term variance level to which the process reverts, and σ is the volatility-of-volatility parameter. $W_{F,t}$ and $W_{V,t}$ are two standard Brownian motions with instantaneous correlation ρ .

If we set $\theta(t)$ to be constant, the model is identical to the stochastic volatility model of Heston (1993). However, in contrast to Heston's model, the long-term variance parameter $\theta(t)$ is generalized to be a deterministic function of time. The long-term mean variance level is assumed to be $\bar{\theta}$, which is superimposed by a

⁶The reason that it is not possible to derive a closed-form futures pricing formula is that the spot commodity is usually assumed to be non-tradable and therefore the market is incomplete (see, e.g., Schwartz (1997)). Furthermore, empirical studies have demonstrated that a second, mean-reverting factor is needed to properly price futures contracts. The mean-reversion property together with volatility being stochastic prohibits the derivation of a closed-form futures pricing formula (see Richter and Sørensen (2002) and Geman and Nguyen (2005)). In contrast, as the futures contract is clearly tradable, no mean-reversion can prevail under the risk-neutral measure as otherwise arbitrage opportunities would exist.

seasonal component as defined in Equation (4.3). The shape of the seasonal adjustment is specified by two parameters: the size of the seasonal effect is governed by η (amplitude of the sine-function) and ζ (shift of the sine-function along the time-dimension). To ensure the parameters' uniqueness, we impose $\eta \geq 0$ and $\zeta \in [0, 1]$, while January 1 represents the time origin. In general, the model setup allows $\theta(t)$ to be of any functional form. We use the simple trigonometric function as it provides a reasonable compromise of good fit to the observed volatility pattern for many seasonal commodity markets while introducing only two additional parameters, facilitating model estimation in empirical applications.⁷ In the following, we will refer to this Seasonal Stochastic Volatility Model as SSV MODEL. For $\eta = 0$, the SSV MODEL nests a non-seasonal specification of this Stochastic Volatility Model, labeled as SV MODEL.

4.2.2 Valuation of Options

To derive the pricing formula for European call options, we change to the risk-neutral measure. Assuming constant market prices of risk, we obtain

$$dF_t(T) = F_t(T)\sqrt{V_t}dW_{F,t}^Q \quad (4.4)$$

$$dV_t = [\kappa(\theta(t) - V_t) - \lambda V_t] dt + \sigma\sqrt{V_t}dW_{V,t}^Q \quad (4.5)$$

$$\theta(t) = \bar{\theta} e^{\eta \sin(2\pi(t+\zeta))}. \quad (4.6)$$

Thereby, λ denotes the market price of risk for the variance process and $W_{F,t}^Q$ and $W_{V,t}^Q$ are standard Brownian motions under the risk-neutral measure with instantaneous correlation ρ . Under the risk-neutral measure, the futures price has to be a martingale and hence, the price process exhibits a drift of zero.

We have extended Heston's model by allowing the long-term variance level to vary over the calendar year in a deterministic fashion. Therefore, the fundamental partial differential equation is, except for the time dependence, identical to Heston's solution. Any claim U on F must satisfy

$$\frac{\partial U}{\partial t} + \frac{1}{2}F^2V\frac{\partial^2 U}{\partial F^2} + [\kappa(\theta(t) - V) - \lambda V]\frac{\partial U}{\partial V} + \frac{1}{2}V\sigma^2\frac{\partial^2 U}{\partial V^2} + \sigma\rho FV\frac{\partial^2 U}{\partial V\partial F} = 0. \quad (4.7)$$

⁷We have also tried to use a more complex specification introducing four additional parameters; however, our empirical results show very little or no benefit from doing so.

Heston derives a quasi closed-form solution for European call options in terms of characteristic functions, which for futures contracts is given as

$$c(F, K, V, T) = e^{-r(T-t)}[FP_1 - KP_2] \quad (4.8)$$

with

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left[\frac{e^{i\phi \ln K} f_j(F, V, t, T, \phi)}{i\phi} \right] d\phi, \quad j = 1, 2 \quad (4.9)$$

where c is the price of a European call option on a futures contract F at time t with strike price K and maturity T ; i denotes the imaginary unit, $\operatorname{Re}[\cdot]$ returns the real part of a complex expression, and f_j is a characteristic function.

As shown by Heston (1993) and more generally by Duffie et al. (2000), the characteristic function solution is of the form

$$f_j = e^{C_j(T-t, \phi) + D_j(T-t, \phi)V + i\phi \ln F}. \quad (4.10)$$

With $\tau = T - t$, the resulting system of ordinary differential equations (ODE) for $C_j(\tau, \phi)$ and $D_j(\tau, \phi)$ to be solved reads

$$\frac{\partial D_j}{\partial \tau} = \frac{1}{2}\sigma^2 D_j^2 - (b_j - \rho\sigma\phi i)D_j + u_j\phi i - \frac{1}{2}\phi^2 \quad (4.11)$$

$$\frac{\partial C_j}{\partial \tau} = \kappa\theta(\tau)D_j \quad (4.12)$$

where $u_1 = \frac{1}{2}$, $u_2 = -\frac{1}{2}$, $b_1 = \kappa + \lambda - \rho\sigma$, and $b_2 = \kappa + \lambda$.

The important aspect to note is that only the second, simple ODE is affected by our model extension as the long-term variance level does not appear in the first ODE. Consequently, the solution of Equation (4.11) remains unchanged from Heston's solution and is given by

$$D_j(\tau, \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2} \left[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}} \right] \quad (4.13)$$

with

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d} \quad (4.14)$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}. \quad (4.15)$$

The solution of Equation (4.12) can be expressed by means of the hypergeometric function. However, we found that a direct numerical integration is the fastest way to solve this ODE while maintaining high precision.⁸ In general, it should be noted that the proposed model extension is well tractable with regard to its computational demand, rendering real-world applications feasible. Details on the implementation are given in the Appendix. Prices for European put options can easily be obtained through the put-call-parity.

4.3 Data Description and Estimation Procedure

4.3.1 Data

For our empirical study, we use a data set consisting of daily prices of physically settled natural gas futures and American style options written on these futures contracts traded at the NYMEX. A short position in the futures contract commits the holder to deliver 10,000 million British thermal units (mmBtu) of natural gas at Sabine Pipe Line Co.'s Henry Hub in Louisiana. Prices are quoted as US dollars and cents per mmBtu. Delivery has to take place between the first and the last calendar day of the delivery month and should be made at an uniform daily and hourly rate.⁹ As interest rates, we use the 3-month USD Libor rates published by the British Bankers' Association. All data are obtained from Bloomberg.

The futures data set spans the time period from January 2, 1997 to December 31, 2010, whereas the available options data set spans the period January 3, 2007 to December 31, 2010 and comprises 1,008 trading days. Call and put options and

⁸In the case of the SV MODEL, the closed-form solution for this ODE is of the form $C_j(\tau, \phi) = \frac{\kappa\theta}{\sigma^2} \left[(b_j - \rho\sigma\phi i + d)\tau - 2 \ln \left(\frac{1 - ge^{d\tau}}{1 - g} \right) \right]$.

⁹See the webpage of the CME group, www.cmegroup.com, for details on the contract specifications.

the corresponding futures contracts are available with maturities in each calendar month. Therefore, we use options with delivery months from February 2007 to December 2011. While trading in the futures contract ceases three business days prior to the first day of the delivery month, trading in the options written on this futures contracts ends on the business day before the last trading day of the futures.

The minimum price fluctuation for the natural gas options is \$0.001. Due to this discreteness in reported prices, we exclude options with a price of less than \$0.01. Furthermore, following Doran and Ronn (2008) and Trolle and Schwartz (2009), we exclude options being very close to expiration and long-term contracts since for these open interest is usually lower and liquidity tends to be low as well, i.e. we consider options with a maturity of at least 15 and not more than 365 days. For the same reasons, we only consider options with a moneyness between 90 % and 110 %.

Table 4.1 summarizes the properties of the call and put options comprising our data set. The total number of observations is 367,469 which we divide in different moneyness and maturity brackets for the subsequent analysis. We refer to a call (put) option as out-of-the-money, OTM, (in-the-money, ITM) when the price of the futures contract is between 90 % and 95 % of the option's strike price. When the price of the futures contract is between 95 % and 105 % of the option's strike price, options are considered to be at-the-money, ATM. Finally, for futures prices between 105 % and 110 % of the option's strike price, call (put) options are referred to as ITM (OTM). We consider options with less than 60 days to expiration as short-term, options with 60 to 180 days to expiration as medium-term, and options with 180 to 365 days to expiration as long-term.

The pricing formulas obtained in Section 4.2 are for European options while the options in our data set are of the American style. To take this aspect into account, we follow Trolle and Schwartz (2009) and transform each American option price into its European counterpart by approximating the early exercise premium using the procedure developed by Barone-Adesi and Whaley (1987). Since the adjustment is carried out for each option separately, the options' price characteristics should not be altered and our analysis should not be affected, even though the analytical approximation approach of Barone-Adesi and Whaley

Table 4.1: Sample Description of Natural Gas Futures Options

This table shows average prices for the natural gas futures options grouped according to moneyness and time to maturity for both call and put options. The numbers of observations are reported in parentheses. Prices are obtained from Bloomberg for the period from January 3, 2007 to December 31, 2010.

Call Options	S/K	Days-to-Expiration			Subtotal
		<60	60–180	180–365	
ITM	1.05–1.1	\$ 0.7191 (6,105)	\$ 1.0140 (13,082)	\$ 1.2375 (15,922)	(35,109)
ATM	0.95–1.05	\$ 0.4309 (16,688)	\$ 0.7490 (36,902)	\$ 0.9792 (41,978)	(95,568)
OTM	0.9–0.95	\$ 0.2354 (10,039)	\$ 0.5318 (23,028)	\$ 0.7225 (23,257)	(56,324)
Subtotal		(32,832)	(73,012)	(81,157)	(187,001)
Put Options					
ITM	0.9–0.95	\$ 0.7452 (7,947)	\$ 1.0263 (17,562)	\$ 1.2171 (19,728)	(45,237)
ATM	0.95–1.05	\$ 0.4349 (16,470)	\$ 0.7163 (35,320)	\$ 0.9423 (41,109)	(92,899)
OTM	1.05–1.1	\$ 0.2174 (7,410)	\$ 0.4784 (16,185)	\$ 0.6989 (18,737)	(42,332)
Subtotal		(31,827)	(69,067)	(79,574)	(180,468)
Total		(64,659)	(142,079)	(160,731)	(367,469)

(1987) is based on a constant volatility framework in contrast to the present stochastic volatility setting.¹⁰ As we only consider options with a time to maturity of not more than one year and the considered strike range excludes options which are deep ITM, the American style feature is of limited importance. Based on the approximation of Barone-Adesi and Whaley (1987), the average premium for the

¹⁰Refer to Trolle and Schwartz (2009) for a more detailed discussion and justification of this approach.

right of early exercise amounts to only 0.29 % of the options value for calls and 0.28 % for puts.

4.3.2 Estimation Approach

Every stochastic volatility model poses a substantial estimation problem as the volatility path is not observable. Therefore, one needs to estimate not only the model parameters but also the latent volatility. A standard approach found in numerous articles is based on a pure cross-sectional calibration, such as in Bakshi et al. (1997). For each observation date, one minimizes an objective function to fit the observed option prices on that particular date. This procedure is repeated for every observation date and, thus, allows the parameters to fluctuate freely through time, which is, of course, inconsistent with the assumed model dynamics in which the parameters are assumed to be constant.

To reduce this inconsistency and to make better use of available information, we follow a different approach which has been suggested by Bates (2000) and Broadie et al. (2007) and comprises a two-step procedure. The first step consists of estimating all parameters that should be equal under the physical and the risk-neutral probability measure using return observations. We therefore make use of a long time series of data to infer most of the model parameters. Given these parameters, we use in a second step the cross-section of options data to estimate the risk premium λ and the current variance level V_t .¹¹

Since the volatility process is not observable, simple estimation methods such as maximum likelihood methods cannot be applied for the first step. Therefore, we follow Jacquier et al. (1994) and Eraker et al. (2003) and apply a Markov Chain Monte Carlo (MCMC) estimation approach, which is a Bayesian simulation-based technique. This approach allows us to estimate the unknown model parameters and the unobservable state variables, i.e. the volatility path, simultaneously.¹²

¹¹A third possibility is to estimate all parameters jointly from a time series of returns and options prices, as in Eraker (2004). However, as Broadie et al. (2007) point out, this approach is hindered by the computational burden and substantially constraints the amount of data that can be used. For example, Eraker (2004) restricts his analysis to an average of three options per day.

¹²For an excellent overview of MCMC estimation techniques with financial applications, see Johannes and Polson (2006).

In order to be able to estimate the models, it is necessary to express them in discretized form. Defining $Y_t = \ln F_t$ and using a simple Euler discretization, we get¹³

$$Y_t = Y_{t-\Delta t} + \mu(t)\Delta t + \sqrt{V_{t-\Delta t}} \varepsilon_t^Y \quad (4.16)$$

and for the variance process

$$V_t = V_{t-\Delta t} + \kappa(\theta(t) - V_{t-\Delta t})\Delta t + \sigma\sqrt{V_{t-\Delta t}} \varepsilon_t^V. \quad (4.17)$$

The innovations ε_t^Y and ε_t^V are normal random variables, i.e. $\varepsilon_t^Y \sim N(0, \Delta t)$ and $\varepsilon_t^V \sim N(0, \Delta t)$ with correlation ρ . The series Y_t is constructed by concatenating futures prices with different maturity months yielding a series of futures prices with almost constant maturity. As this price series also contains a seasonal component, we allow the mean drift to fluctuate seasonally by setting $\mu(t) = \bar{\mu} + \phi \sin(2\pi(t + \xi))$. For the SV MODEL, we set $\theta(t) = \bar{\theta}$ and for the SSV MODEL we set $\theta(t) = \bar{\theta} e^{\eta \sin(2\pi(t+\zeta))}$. In the following implementation, we estimate both models using daily data.

The main piece of interest in Bayesian inference is the posterior distribution $p(\Theta, V|Y)$ which can be factorized as

$$p(\Theta, V|Y) \propto p(Y|V, \Theta)p(V|\Theta)p(\Theta) \quad (4.18)$$

where Y is the vector of observed log prices, V contains the time series of volatility, Θ is the set of model parameters, $p(Y|V, \Theta)$ is called the likelihood, $p(V|\Theta)$ provides the distribution of the latent volatility, and $p(\Theta)$ is the prior, reflecting the researcher's beliefs regarding the unknown parameters. The MCMC method provides a way to sample from this high-dimensional complex distribution. The main idea is to break down the high-dimensional posterior distribution into its low-dimensional complete conditionals of parameters and latent factors which can be efficiently sampled from. The output of the simulation procedure is a set of G draws, $\{\Theta^{(g)}, V^{(g)}\}_{g=1:G}$, that forms a Markov chain and converges to $p(\Theta, V|Y)$. Given the sample from $p(\Theta, V|Y)$, information about individual parameters can then be obtained from the respective marginals of the posterior distribution.

¹³As we work with daily data, the discretization bias is negligible.

Whenever possible, we use conjugate priors and apply a Gibbs sampler.¹⁴ The basic SV MODEL is identical to the model analyzed in Eraker et al. (2003); we therefore follow their prior specifications. The distribution of V_t is non-standard but can be sampled using a random walk Metropolis algorithm which is calibrated to yield an acceptance probability between 30 % and 50 %.¹⁵ For the seasonal parameters, we use a Gibbs sampler with an exponential prior for η , and an independence Metropolis algorithm with a uniform density over the unit interval as the proposal density for ζ . We use 100,000 simulations, i.e. $G = 100,000$, and discard the first 30,000 as ‘burn-in’ period of the algorithm.

Given the structural model parameters estimated under the physical measure according to the MCMC approach, the market price of risk λ and the current variance level V_t can be inferred from options data in the second step of our estimation procedure. Thereby, theoretical option prices can be obtained using the pricing formulas presented in Section 4.2.

The two quantities λ and V_t are estimated by minimizing a loss function capturing the fit between the theoretical model prices and the prices observed at the market. For robustness reasons, we employ two different objective functions, both of them popular in the literature: the first metric, e.g. used by Bakshi et al. (1997), is the root mean squared error of prices ($\$$ -RMSE), i.e.

$$\Phi_t^* = \arg \min_{\Phi_t} \$\text{-RMSE}(\Phi_t) = \arg \min_{\Phi_t} \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{P}_{t,i}(\Phi_t) - P_{t,i})^2}. \quad (4.19)$$

Hence, squared differences between observed market prices, denoted by $P_{t,i}$, and obtained model prices, $\hat{P}_{t,i}(\Phi_t)$, are minimized. N_t denotes the number of contracts available at date t and $\Phi_t = \{\lambda, V_t\}$ the unknown quantities to be estimated.

The second metric, e.g. used by Broadie et al. (2007), is the RMSE of implied volatilities (IV-RMSE), i.e.

$$\Phi_t^* = \arg \min_{\Phi_t} \text{IV-RMSE}(\Phi_t) = \arg \min_{\Phi_t} \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{IV}_{t,i}(\Phi_t) - IV_{t,i})^2}. \quad (4.20)$$

¹⁴See Geman and Geman (1984).

¹⁵See Johannes and Polson (2006).

Here, $IV_{t,i}$ is the implied Black (1976) volatility of the observed market price and $\hat{IV}_{t,i}(\Phi_t)$ is the implied volatility of the theoretical model price.

The first approach is more natural as one uses the observed quantities directly and is therefore ‘model-free’. However, it puts more weight on the more expensive ITM options and on options having a longer time to maturity. Minimizing the implied volatility metric provides an intuitive way of weighting all observations more or less equally.¹⁶ In Section 4.4, we present the results for both objective functions, but base the discussion mainly on the IV-RMSE.

4.4 Results

In this section, we report the results of our empirical study. After discussing the obtained parameter estimates for the two models, we present in-sample and out-of-sample results regarding the models’ options pricing performance. At the end of this section, we provide information on several robustness checks conducted.

4.4.1 Estimated Parameters

In the first step of the estimation procedure, the structural model parameters are estimated under the physical measure using the time series of futures prices with the presented MCMC approach. To do this, we have to select a futures time series. The average time to maturity of our options data set is 170 days, which is approximately 6 months. Therefore, we use the time series of the futures contract with 6 months to maturity to estimate our model.

The obtained parameter estimates and the corresponding standard errors are reported in Table 4.2. Overall, the parameter estimates are of reasonable magnitude. We find a positive correlation ρ of 0.29 and 0.40 between the natural gas futures price and the variance processes for the SV and the SSV MODEL,

¹⁶For the numerical estimation of the two parameters, V_t was limited to the interval 0 to 10 and λ was restricted not to exceed an upper boundary of 100 while the lower boundary is given by $-\kappa$ to ensure the mean-reversion property of the variance process. One should note, however, that these artificial boundaries were non-binding in almost all cases. Only the artificial upper boundary for λ was binding once for the SV MODEL and never for the SSV MODEL in the case of the estimation minimizing implied volatility errors, and for the estimation minimizing price errors this boundary was binding three times for the SV MODEL and once for the SSV MODEL during the 1,008 trading days in our sample.

Table 4.2: Physical Measure Parameter Estimates

This table provides MCMC estimates of model parameters using a continuous series of six-months futures prices from 1997 to 2006. Parameter estimates and standard errors [in brackets] are the mean and standard deviation of the posterior distributions. The estimation is based on 100,000 replications, the first 30,000 are discarded as burn-in.

	SV Model	SSV Model
κ	7.7364 [2.7110]	2.1748 [1.0762]
$\bar{\theta}$	0.1037 [0.0313]	0.1604 [0.0325]
σ	0.7717 [0.1353]	0.5584 [0.0871]
ρ	0.2916 [0.0887]	0.3981 [0.0837]
η	- -	0.3147 [0.1278]
ζ	- -	0.4984 [0.0823]

respectively. This result is in line with Trolle and Schwartz (2010), who also observe a moderately positive correlation in the case of natural gas, although for a different time period.

The long-run mean of the variance process, $\bar{\theta}$, is lower for the SV MODEL than for the SSV MODEL. Specifically, the estimated $\bar{\theta}$ value for the SV MODEL corresponds to a long-run average volatility of 32.2%. In the case of the SSV MODEL, the obtained parameter estimates translate into a minimum of 34.2% and a maximum of 46.9% for the time-varying seasonal long-run mean volatility. On the other hand, the vol-of-vol parameter, σ , is estimated higher for the SV MODEL increasing the volatility of the variance process. Therefore, it seems that the error induced by ignoring the seasonal fluctuations of the variance levels is captured by a higher variability while inducing a downward bias in the long-term level estimate.

Thereby, the estimation result of ζ , the parameter describing the shift of the seasonality function along the time-axis, implies $\theta(t)$ to be the highest in late September and early October, while reaching a minimum in late March and early April. This result fits the empirical observations regarding a higher volatility during the winter than during the summer months by, e.g., Suenaga et al. (2008) and Geman and Ohana (2009). The economic rationale for this pattern is the high sensitivity of natural gas prices to weather-related demand shocks during the winter since supply and demand are relatively price inelastic. The high values of $\theta(t)$ during the fall pull up the volatility, while in early spring, by the end of the cold season, the drift component brings the volatility down again.

The values of the current volatility \sqrt{V}_t and the variance risk premium λ , which are re-estimated daily from the cross-section of observed option prices in the second step of the employed estimation procedure, are summarized in Table 4.3. The minimum number of option prices employed for the estimation procedure on an individual day is 106 while the average number is 365. The current volatility level for both models, with and without the seasonal component, is on average approximately around 60%; only when using the \$-RMSE as the objective function does the SV MODEL yield a slightly higher estimate of 68%.

Table 4.3: Risk-Neutral Measure Parameter Estimates

This table reports mean values and standard errors [in brackets] obtained in the estimation procedure according to both the IV-RMSE and the \$-RMSE criterion for the SV and the SSV MODEL. For the estimation, the cross-section of observed natural gas futures options is used as outlined in Section 4.3. Given the physical measure parameter estimates as reported in Table 4.2, the current volatility level \sqrt{V}_t and the risk premium λ are estimated for each observation day during the period January 3, 2007 to December 31, 2010.

	IV-RMSE		\$-RMSE	
	SV Model	SSV Model	SV Model	SSV Model
\sqrt{V}_t	0.6256 [0.0072]	0.5989 [0.0061]	0.6795 [0.0088]	0.6197 [0.0065]
λ	0.4542 [0.1638]	2.9424 [0.1066]	1.7451 [0.2479]	3.3420 [0.1400]

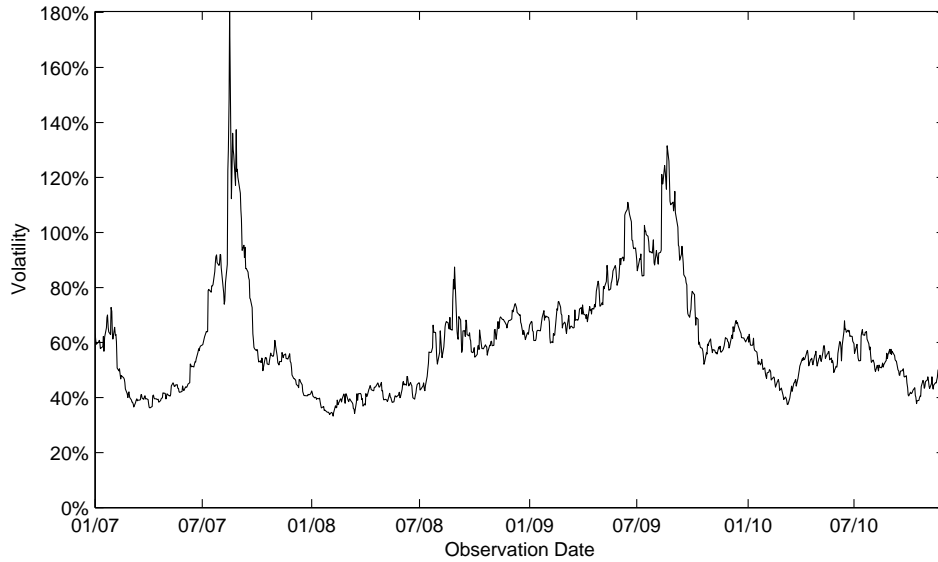


Figure 4.2: Estimated Current Volatility Level

This figure shows the obtained current volatility level \sqrt{V}_t during the considered time period January 3, 2007 to December 31, 2010 for the SSV MODEL using the IV-RMSE criterion for the estimation from the cross-section of observed natural gas futures option prices.

Figure 4.2 shows the obtained path of the current volatility level \sqrt{V}_t during the considered time period for the SSV MODEL when parameters are estimated according to the IV-RMSE criterion. Since option prices are very sensitive to the current volatility level, estimated values of \sqrt{V}_t are very similar for the SV and SSV MODELS and follow the same pattern over time for both loss function specifications. It becomes obvious that volatility of natural gas varies significantly over time. Additionally, it can be seen that during the considered time period, 2007 to 2010, *realized* instantaneous volatility seems to be primarily driven by other factors like, e.g., the economic downturn and turbulences on the financial markets rather than by the normal seasonal demand cycle. However, for options pricing purposes, the market anticipated *implied* volatility is of relevance, not realized volatility. Yet, compared to other time periods with a more pronounced seasonal pattern, the relative performance of the SSV MODEL could potentially be downward biased and it will be interesting to see how the SSV MODEL performs in comparison to the SV MODEL in our study.

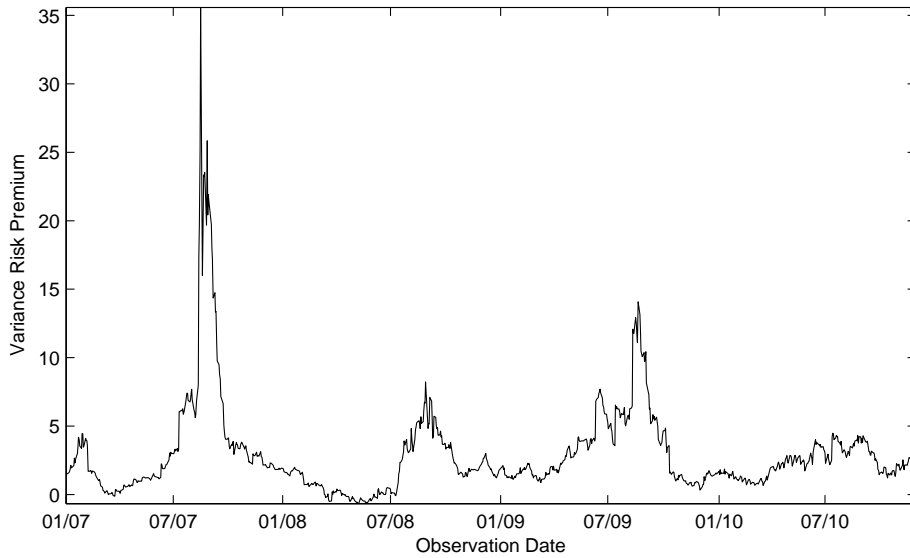


Figure 4.3: Estimated Market Price of Variance Risk

This figure shows the obtained market price of variance risk λ during the considered time period January 3, 2007 to December 31, 2010 for the SSV MODEL using the IV-RMSE criterion for the estimation from the cross-section of observed natural gas futures option prices.

The average market prices of variance risk, λ , are positive. This is somewhat surprising since Doran and Ronn (2008) and Trolle and Schwartz (2010) find a negative market price of variance risk for natural gas. However, one should recall that the structural model parameters were estimated out-of-sample under the physical measure from the time series of futures prices. Therefore, potential changes in the market environment, and hence model parameters, will be reflected in the values obtained from the cross-section of options and hinder interpretation of the absolute values of the obtained risk premium estimates.¹⁷ While the interpretation of the average absolute magnitude of the risk premia is therefore

¹⁷In particular, changes in the mean-reversion speed, κ , or in the long-run mean, $\bar{\theta}$, might be absorbed by a then biased estimate of the risk premium since the risk-neutral version of the variance process has a mean-reversion speed of $\kappa^Q = \kappa + \lambda$ and a long-run mean of $\bar{\theta}^Q = \frac{\kappa \bar{\theta}}{\kappa + \lambda}$. When estimating the model under the physical measure for different time periods, one can indeed observe that the obtained estimates for κ and $\bar{\theta}$ vary somewhat over time, which can also be seen, e.g., in the study of Trolle and Schwartz (2009) for crude oil.

hampered due to our out-of-sample setup, one can assess the risk premium dynamics over time during the period of our study.

The estimated pattern for λ according to the IV-RMSE criterion is shown in Figure 4.3 for the SSV MODEL. Even though this is not the focus of this essay, it is interesting to note that risk premia are on average far lower during the winter months than during the summer. For the SV MODEL, the average risk premium is -0.75 (-0.57) for the period October to March and 1.64 (4.02) for the period April to September for the estimation carried out with the IV-RMSE (\$-RMSE) criterion. For the SSV MODEL, the corresponding values are 1.77 (1.82) for the winter and 4.10 (4.84) for the summer period. This supports the results of Doran and Ronn (2005), Doran and Ronn (2008) and Trolle and Schwartz (2010) who find evidence of a more negative risk premium during the more volatile winter than during the summer for the natural gas market.

4.4.2 Pricing Performance

Ultimately, we are interested in the pricing accuracy of an options valuation model. In particular, we want to see how the pricing ability of the SSV MODEL incorporating a seasonal drift as the proposed model extension compares to the nested benchmark stochastic volatility model, the SV MODEL.

As outlined before, the structural model parameters are estimated from the time series of futures prices from 1997 to 2006. Since this time period is chosen not to overlap with the 2007 to 2010 time period for our options pricing application, no in-sample information is reflected in the obtained structural parameters which are estimated in the first step of our estimation approach. In the second step, the current variance level, V_t , and the variance risk premium, λ , are estimated for each observation day t from the cross-section of observed option prices. Even though the SSV MODEL nests its non-seasonal counterpart and is therefore more flexible, the structural parameters are already determined at this point and only these two values, V_t and λ , are estimated from the options data – for both models. In this sense, the models have the same degrees of freedom to fit observed option prices and the SSV MODEL will only yield a superior performance if the model extension picks up valuable information regarding the price dynamics in the first

step of the estimation. Additionally, these price dynamics need to be persistent over time. Given the estimated parameters, we construct a time series of pricing errors according to different error metrics for the two models. Since V_t and λ are estimated from the option contracts that are used to assess the pricing accuracy, we will refer to the obtained pricing errors as in-sample pricing errors.

To analyze the pricing accuracy of the two models, we report four different error metrics: The Root Mean Squared Error of Black (1976) implied volatilities, $IV\text{-}RMSE = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{IV}_{t,i} - IV_{t,i})^2}$, the Root Mean Squared Error of option prices, $\$\text{-}RMSE = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{P}_{t,i} - P_{t,i})^2}$, the Relative Root Mean Squared Error, $RRMSE = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left(\frac{\hat{P}_{t,i} - P_{t,i}}{P_{t,i}}\right)^2}$, and the Mean Percentage Error, $MPE = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\hat{P}_{t,i} - P_{t,i}}{P_{t,i}}$. Thereby, $P_{t,i}$ denotes the observed market price and $IV_{t,i}$ the implied volatility of option i , $\hat{P}_{t,i}$ is the theoretical model price with implied volatility $\hat{IV}_{t,i}$, and N_t is the number of observations at date t .

As Christoffersen and Jacobs (2004) point out, the most appropriate error metric to assess the performance of an options pricing model is the one employed as the loss function during the estimation. Hence, in this study, the IV-RMSE and the $\$\text{-}RMSE$ are the error metrics which are at the center of interest. Additionally, we report the RRMSE to assess the relative pricing errors and the MPE to look for systematic biases in obtained model prices. All results are reported for the nine different maturity and moneyness brackets as defined in Section 4.3.

The in-sample results are provided in Tables 4.4 and 4.5 as average pricing errors according to the different metrics over the considered time period from January 3, 2007 to December 31, 2010 for the estimation with the IV-RMSE and the $\$\text{-}RMSE$ criterion, respectively. When estimated according to the IV-RMSE, the resulting overall IV-RMSE is 3.16% for the SV MODEL and 2.98% for the SSV MODEL, while the overall $\$\text{-}RMSE$ amounts to 6.01¢ and 5.59¢, respectively. For the $\$\text{-}RMSE$ -based estimation, the obtained overall IV-RMSE is 3.73% for the SV MODEL and 3.45% for the SSV MODEL and the $\$\text{-}RMSE$ is 5.52¢ and 5.17¢, respectively. As expected, the $\$\text{-}RMSE$ criterion puts more weight on the more expensive long-term options in the estimation and, hence, yields lower pricing errors for these contracts. This holds true for all moneyness categories and

Table 4.4: In-Sample Pricing Errors: Fitted to Implied Volatilities

This table displays in-sample pricing errors of options on natural gas futures. Pricing errors are reported as root mean squared errors of the implied volatilities (IV-RMSE), root mean squared errors of option prices (\$-RMSE), relative root mean squared errors (RRMSE), and mean percentage errors (MPE) of option prices for the SV and the SSV MODEL. The reported pricing errors are calculated as average values over the period from January 3, 2007 to December 31, 2010. Pricing errors are grouped by maturity and moneyness of the options. The risk premium and the current volatility level are estimated with regard to the IV-RMSE criterion.

		Days-to-Expiration						Subtotal		
		<60		60-180		180-365		SV	SSV	
		SV	SSV	SV	SSV	SV	SSV	SV	SSV	
IV-RMSE		OTM	2.78	2.70	3.12	2.95	3.26	3.06	3.17	3.00
	[%]	ATM	2.69	2.57	3.00	2.81	3.22	2.99	3.12	2.93
		ITM	2.77	2.69	3.13	2.97	3.24	3.05	3.22	3.05
		Subtotal	2.75	2.65	3.06	2.89	3.24	3.03	3.16	2.98
\$-RMSE		OTM	1.88	1.81	4.43	4.13	8.22	7.65	5.90	5.50
	[¢]	ATM	1.99	1.88	4.37	4.04	8.28	7.65	6.01	5.57
		ITM	1.86	1.79	4.46	4.19	8.19	7.66	6.08	5.68
		Subtotal	1.94	1.86	4.41	4.10	8.26	7.66	6.01	5.59
RRMSE		OTM	9.80	9.57	8.29	7.92	10.91	10.04	9.85	9.29
	[%]	ATM	5.39	5.18	5.87	5.51	8.35	7.62	7.06	6.54
		ITM	2.71	2.63	4.23	4.02	6.20	5.75	5.07	4.74
		Subtotal	6.57	6.38	6.37	6.04	8.72	8.00	7.60	7.11
MPE		OTM	5.48	4.51	-2.90	-1.71	3.33	2.00	1.33	1.11
	[%]	ATM	3.09	2.56	-2.26	-1.41	2.60	1.55	0.81	0.63
		ITM	1.36	1.05	-1.68	-1.09	1.97	1.23	0.42	0.31
		Subtotal	3.38	2.78	-2.31	-1.42	2.66	1.60	0.87	0.69

Table 4.5: In-Sample Pricing Errors: Fitted to Prices

This table displays in-sample pricing errors of options on natural gas futures. Pricing errors are reported as root mean squared errors of the implied volatilities (IV-RMSE), root mean squared errors of option prices (\$-RMSE), relative root mean squared errors (RRMSE), and mean percentage errors (MPE) of option prices for the SV and the SSV MODEL. The reported pricing errors are calculated as average values over the period from January 3, 2007 to December 31, 2010. Pricing errors are grouped by maturity and moneyness of the options. The risk premium and the current volatility level are estimated with regard to the \$-RMSE criterion.

		Days-to-Expiration						Subtotal		
		<60		60-180		180-365		SV	SSV	
		SV	SSV	SV	SSV	SV	SSV	SV	SSV	
IV-RMSE	[%]	OTM	5.96	5.31	2.92	2.75	2.78	2.68	3.71	3.45
		ATM	6.02	5.29	2.79	2.60	2.72	2.58	3.72	3.42
		ITM	5.99	5.34	2.93	2.77	2.73	2.64	3.78	3.52
		Subtotal	6.01	5.32	2.86	2.69	2.74	2.62	3.73	3.45
\$-RMSE	[¢]	OTM	3.81	3.34	4.10	3.77	7.01	6.70	5.41	5.10
		ATM	4.19	3.61	4.00	3.65	7.01	6.62	5.54	5.16
		ITM	3.80	3.33	4.12	3.81	6.92	6.62	5.56	5.24
		Subtotal	4.02	3.49	4.06	3.73	7.00	6.65	5.52	5.17
RRMSE	[%]	OTM	21.74	18.94	8.01	7.64	9.01	8.60	12.54	11.42
		ATM	12.20	10.58	5.56	5.19	6.86	6.44	8.00	7.29
		ITM	5.83	5.14	3.97	3.75	5.11	4.88	5.06	4.74
		Subtotal	14.63	12.74	6.09	5.76	7.18	6.80	9.03	8.25
MPE	[%]	OTM	15.33	10.69	-1.41	-0.47	1.33	0.92	2.75	2.26
		ATM	8.42	5.74	-1.22	-0.55	1.06	0.72	1.44	1.14
		ITM	3.93	2.49	-1.04	-0.56	0.82	0.60	0.57	0.44
		Subtotal	9.35	6.44	-1.23	-0.53	1.07	0.74	1.60	1.29

for all error metrics. Conversely, the IV-RMSE criterion leads to lower pricing errors for the short-term options than the \$-RMSE criterion does.

With 7.60% for the SV and 7.11% for the SSV MODEL, the observed overall RRMSE is lower when the parameters are estimated according to the IV-RMSE criterion since the IV-RMSE metric provides a more equal weighting of the options during the estimation than the \$-RMSE and, hence, is more similar to the RRMSE than the \$-RMSE is. Not surprisingly, the RRMSE is higher for OTM and lower for ITM options.

The MPE results reveal that on average both models tend to slightly overprice the options in our data set. Particularly, for the estimation with the IV-RMSE (\$-RMSE) criterion, the MPE is 0.87% (1.60%) and 0.69% (1.29%) for the SV and the SSV MODEL, respectively. Thereby, it is noteworthy that short-term options are on average overpriced, especially when employing the \$-RMSE criterion for the estimation. In contrast, medium-term options are underpriced, while long-term options are moderately overpriced. Most importantly, it can be observed that mispricing is in every instance lower for the model including the seasonality component. This holds true for both loss functions and for every maturity and moneyness bracket.

To this point, it can be summarized that the SSV MODEL outperforms the SV MODEL with respect to all four error metrics, for all moneyness and maturity categories, and for the two different loss functions employed in the estimation.

To see whether these results hold in a true out-of-sample case, we conduct the following analysis: For each day t , the current variance level V_t and the risk premium λ are estimated with option price observations as in the previous case. These estimates are now used to price all options of the subsequent day, $t + 1$. Hence, no information from the day of the actual pricing comparison is utilized when calculating the theoretical option prices.

The out-of-sample results are summarized in Tables 4.6 and 4.7. Naturally, the obtained average pricing errors are somewhat higher than in the in-sample case. However, as in the in-sample study, it can be observed that the SSV MODEL outperforms the SV MODEL in every instance. In particular, the overall IV-RMSE is 3.34% for the SV MODEL and 3.16% for the SSV MODEL with the IV-RMSE-based estimation, while the overall \$-RMSE amount to 6.19¢ and

Table 4.6: Out-of-Sample Pricing Errors: Fitted to Implied Volatilities

This table displays out-of-sample pricing errors of options on natural gas futures. Pricing errors are reported as root mean squared errors of the implied volatilities (IV-RMSE), root mean squared errors of option prices (\$-RMSE), relative root mean squared errors (RRMSE), and mean percentage errors (MPE) of option prices for the SV and the SSV MODEL. The reported pricing errors are calculated as average values over the period from January 4, 2007 to December 31, 2010. Pricing errors are grouped by maturity and moneyness of the options. The risk premium and the current volatility level are estimated with regard to the IV-RMSE criterion. Theoretical model-based prices are derived by using parameters estimated the day before.

		Days-to-Expiration											
		<60			60-180			180-365			Subtotal		
		SV	SSV	SV	SSV	SV	SSV	SV	SSV	SV	SSV	SV	SSV
IV-RMSE		OTM	3.14	3.02	3.28	3.13	3.31	3.11	3.35	3.17			
	[%]	ATM	3.04	2.90	3.16	2.99	3.27	3.04	3.31	3.11			
		ITM	3.12	3.00	3.28	3.14	3.28	3.09	3.40	3.23			
		Subtotal	3.10	2.97	3.22	3.07	3.29	3.07	3.34	3.16			
\$-RMSE		OTM	2.11	2.03	4.62	4.35	8.34	7.77	6.07	5.67			
	[¢]	ATM	2.23	2.13	4.58	4.29	8.40	7.77	6.20	5.75			
		ITM	2.08	2.00	4.65	4.41	8.31	7.77	6.25	5.86			
		Subtotal	2.18	2.09	4.61	4.34	8.38	7.78	6.19	5.77			
RRMSE		OTM	10.91	10.56	8.74	8.44	11.08	10.21	10.45	9.88			
	[%]	ATM	6.04	5.79	6.18	5.88	8.48	7.74	7.42	6.90			
		ITM	3.03	2.92	4.43	4.25	6.29	5.85	5.27	4.94			
		Subtotal	7.32	7.07	6.70	6.43	8.86	8.14	8.01	7.51			
MPE		OTM	5.72	4.71	-2.82	-1.62	3.38	2.06	1.45	1.22			
	[%]	ATM	3.19	2.64	-2.21	-1.35	2.64	1.59	0.88	0.69			
		ITM	1.40	1.08	-1.65	-1.06	2.00	1.26	0.47	0.34			
		Subtotal	3.50	2.89	-2.25	-1.36	2.70	1.64	0.95	0.76			

Table 4.7: Out-of-Sample Pricing Errors: Fitted to Prices

This table displays out-of-sample pricing errors of options on natural gas futures. Pricing errors are reported as root mean squared errors of the implied volatilities (IV-RMSE), root mean squared errors of option prices (\$-RMSE), relative root mean squared errors (RRMSE), and mean percentage errors (MPE) of option prices for the SV and the SSV MODEL. The reported pricing errors are calculated as average values over the period from January 4, 2007 to December 31, 2010. Pricing errors are grouped by maturity and moneyness of the options. The risk premium and the current volatility level are estimated with regard to the \$-RMSE criterion. Theoretical model-based prices are derived by using parameters estimated the day before.

		Days-to-Expiration						Subtotal		
		<60		60-180		180-365		SV	SSV	
		SV	SSV	SV	SSV	SV	SSV	SV	SSV	
IV-RMSE	[%]	OTM	6.06	5.39	3.06	2.91	2.84	2.74	3.83	3.56
		ATM	6.10	5.37	2.94	2.77	2.78	2.64	3.84	3.53
		ITM	6.08	5.41	3.06	2.92	2.79	2.69	3.91	3.63
		Subtotal	6.09	5.40	3.00	2.85	2.80	2.68	3.85	3.56
\$-RMSE	[¢]	OTM	3.87	3.41	4.28	3.99	7.17	6.85	5.57	5.26
		ATM	4.24	3.69	4.21	3.88	7.17	6.77	5.71	5.33
		ITM	3.86	3.39	4.29	4.02	7.07	6.77	5.72	5.41
		Subtotal	4.07	3.57	4.25	3.95	7.15	6.80	5.69	5.34
RRMSE	[%]	OTM	22.02	19.18	8.42	8.11	9.23	8.80	12.90	11.77
		ATM	12.34	10.74	5.85	5.52	7.03	6.60	8.24	7.53
		ITM	5.91	5.21	4.16	3.96	5.22	5.00	5.23	4.89
		Subtotal	14.81	12.90	6.40	6.11	7.35	6.96	9.30	8.51
MPE	[%]	OTM	15.45	10.79	-1.34	-0.39	1.38	0.98	2.85	2.34
		ATM	8.48	5.79	-1.17	-0.50	1.10	0.76	1.50	1.20
		ITM	3.95	2.51	-1.02	-0.53	0.85	0.63	0.61	0.47
		Subtotal	9.41	6.50	-1.18	-0.47	1.11	0.79	1.67	1.35

5.77 ¢, respectively. In the case of the \$-RMSE-based estimation, the overall IV-RMSE amount to 3.85 % and 3.56 % and the \$-RMSE to 5.69 ¢ and 5.34 ¢ for the SV and SSV MODEL, respectively.

In a last step, we perform Wilcoxon signed-rank tests to inspect whether the observed differences in the pricing errors are also statistically significant. Specifically, the non-parametric Wilcoxon signed-rank test statistic tests whether the median of the differences is significantly different from zero. The percentage reductions in pricing errors in terms of IV-RMSE and \$-RMSE are provided in Tables 4.8 and 4.9. It can be observed that the pricing error reductions due to the proposed model extension are always significant at the 1 % level – for both loss functions, for every moneyness and maturity bracket, and for the in-sample as well as the out-of-sample study.

When the estimation is based on the IV-RMSE criterion, the inclusion of seasonality in the variance process reduces the in-sample (out-of-sample) IV-RMSE by 5.75 % (5.48 %) and the \$-RMSE by 7.07 % (6.83 %). For the \$-RMSE-based estimation, the reduction in terms of IV-RMSE yields 7.49 % (7.44 %) and in terms of \$-RMSE 6.27 % (6.09 %). The greatest improvements can be observed for short-term options when the estimation is carried out with regard to the \$-RMSE criterion: \$-RMSE reductions for ATM options amount to 13.82 % and 12.92 % in the in- and out-of-sample case, respectively.

Overall, we find clear empirical evidence that the proposed model extension of incorporating a seasonal component in the drift term of the variance process significantly improves the pricing accuracy for natural gas options.

4.4.3 Robustness Checks

We conducted a number of robustness checks. Due to space constraints, we refrain from presenting detailed results of these analyses, but summarize them below.

(i) In order to assess the influence of volatility being stochastic on the pricing performance, we compared the stochastic volatility models to the constant volatility model of Black (1976). We found that the pricing accuracy of the Black (1976) model is significantly lower compared to the stochastic volatility models. For example, when estimating the models according to the \$-RMSE criterion,

Table 4.8: Reduction of Pricing Errors: Fitted to Implied Volatilities

*This table shows the percentage reduction in root mean squared errors of implied volatilities (IV-RMSE) and of prices (\$-RMSE) for options on natural gas futures due to the seasonal volatility extension. The corresponding pricing errors are reported in Tables 4.4 and 4.6. The figures are grouped by maturity and moneyness of the options and are reported for the in-sample and out-of-sample pricing errors. Estimation was carried out with regard to the IV-RMSE criterion. * indicates significance at the 1% level according to the Wilcoxon signed-rank test.*

	In-Sample				Out-of-Sample				
	Days-to-Expiration				Days-to-Expiration				
	<60	60-180	>180	Subtotal	<60	60-180	>180	Subtotal	
IV-RMSE	OTM	3.03%*	5.56%*	6.11%*	5.35%*	3.82%*	4.62%*	6.05%*	5.10%*
	ATM	4.47%*	6.33%*	7.13%*	6.25%*	4.74%*	5.30%*	7.03%*	5.92%*
	ITM	2.91%*	5.03%*	5.79%*	5.21%*	3.66%*	4.17%*	5.72%*	5.03%*
	Subtotal	3.60%*	5.80%*	6.57%*	5.75%*	4.17%*	4.84%*	6.49%*	5.48%*
\$-RMSE	OTM	3.78%*	6.82%*	6.88%*	6.80%*	3.76%*	5.76%*	6.84%*	6.56%*
	ATM	5.24%*	7.57%*	7.55%*	7.44%*	4.51%*	6.38%*	7.48%*	7.17%*
	ITM	3.74%*	6.04%*	6.48%*	6.48%*	3.68%*	5.03%*	6.42%*	6.30%*
	Subtotal	4.38%*	7.01%*	7.18%*	7.07%*	4.06%*	5.91%*	7.11%*	6.83%*

Table 4.9: Reduction of Pricing Errors: Fitted to Prices

*This table shows the percentage reduction in root mean squared errors of implied volatilities (IV-RMSE) and of prices (\$-RMSE) for options on natural gas futures due to the seasonal volatility extension. The corresponding pricing errors are reported in Tables 4.5 and 4.7. The figures are grouped by maturity and moneyness of the options and are reported for the in-sample and out-of-sample pricing errors. Estimation was carried out with regard to the \$-RMSE criterion. * indicates significance at the 1 % level according to the Wilcoxon signed-rank test.*

	In-Sample				Out-of-Sample				
	Days-to-Expiration				Days-to-Expiration				
	<60	60-180	>180	Subtotal	<60	60-180	>180	Subtotal	
IV-RMSE	OTM	10.97%*	5.60%*	3.63%*	7.00%*	11.09%*	4.81%*	3.66%*	6.96%*
	ATM	12.19%*	6.70%*	5.09%*	8.07%*	11.96%*	5.83%*	5.03%*	7.97%*
	ITM	10.93%*	5.35%*	3.51%*	6.94%*	11.04%*	4.65%*	3.52%*	6.97%*
	Subtotal	11.48%*	6.04%*	4.32%*	7.49%*	11.44%*	5.24%*	4.31%*	7.44%*
\$-RMSE	OTM	12.40%*	7.91%*	4.46%*	5.77%*	11.99%*	6.89%*	4.46%*	5.60%*
	ATM	13.82%*	8.97%*	5.55%*	6.80%*	12.92%*	7.81%*	5.47%*	6.59%*
	ITM	12.45%*	7.32%*	4.26%*	5.67%*	12.05%*	6.36%*	4.24%*	5.55%*
	Subtotal	13.06%*	8.29%*	4.98%*	6.27%*	12.41%*	7.22%*	4.94%*	6.09%*

the overall in-sample IV-RMSE for the Black (1976) model yields 7.51 % and the \$-RMSE amounts to 9.16¢ in comparison to 3.73 % and 5.52¢ for the standard stochastic volatility model, the SV MODEL. These results confirm that a stochastic volatility setting is necessary to capture the dynamics of natural gas futures prices.

(ii) Since the structural model parameters, which are obtained out-of-sample under the physical measure from the historical futures prices, are different for the SV and SSV MODEL, the higher pricing accuracy of the seasonal volatility model might stem from the different parameter set and not from the seasonality extension. In order to control for this, we repeated the second step of the estimation procedure and obtained optimal V_t and λ values given the structural parameter values from the SSV MODEL while restricting η , the amplitude of the seasonality function, to be zero. We then compared the pricing accuracy of the SSV and SV MODELS when having an identical set of structural parameters with the only difference being that η is equal to zero for the SV MODEL. The results show that the pricing accuracy of the SV MODEL with these parameters is somewhat improved. Yet, the SSV MODEL consistently outperforms its non-seasonal counterpart in terms of both IV-RMSE and \$-RMSE for every moneyness and maturity category, for both the in- and the out-of-sample study. As before, all results are significant at the 1 % level.

(iii) We estimated the current variance level V_t and the variance risk premium λ according to the RRMSE as loss function as an alternative to the IV-RMSE and \$-RMSE criteria. We found that the obtained results are robust with respect to this alternative loss function. Furthermore, the observed pricing error reductions due to the model extension are all significant at the 1 % level and are of similar magnitude as for the other two loss functions.

(iv) As a final robustness check, we divided the data set in four sub-samples and considered each of the four years covered in our study separately. For the years 2007 and 2008, the SSV MODEL consistently outperforms the SV MODEL for every error metric, for every moneyness and maturity bracket, and for the in- and out-of-sample study at the 1 % significance level. For the year 2009, we obtained mixed results and observed that in large parts differences in pricing errors are economically negligible with the average overall IV-RMSE and \$-RMSE

differences between the two models being less than 1%. A possible explanation for this could be the extremely high volatility level in 2009 leading to an average estimate for \sqrt{V}_t of 82.2% and 79.5% for the SV and SSV MODEL, respectively (according to the IV-RMSE loss function).¹⁸ Hence, both models imply in the short-run a declining volatility and yield a negative drift term – regardless of the seasonality extension. For the year 2010, we found again that the SSV MODEL performs significantly better than the SV MODEL. With very few exceptions, the seasonality extension leads to lower pricing errors which are economically and statistically significant. Therefore, we can confirm that the significantly improved pricing accuracy due to the proposed seasonality extension is also robust in the sense that the results are not driven by a particular sub-sample.

4.5 Conclusion

Volatility in many commodity markets follows a pronounced seasonal pattern while also fluctuating stochastically. In this chapter, we extend the stochastic volatility model of Heston (1993) to allow volatility to vary with the seasonal cycle. The proposed model framework enables us to derive semi closed-form solutions for pricing futures options. We then study the empirical performance in pricing natural gas options. In contrast to other studies, we estimate our model using not only the cross-section of options prices but also considering the time series of futures contracts. The empirical results show that the suggested model indeed increases the accuracy of pricing natural gas contracts, in terms of both statistical and economic significance.

Finally, we conclude this chapter by outlining areas for further research. Many financial data exhibit jumps in prices and volatilities. This is also true for many commodity markets, and especially true for the natural gas market considered in this study. Extending our model by including jump components is therefore a natural next step. Compared to equity markets in which the jump frequency

¹⁸Indeed, in 2009, we observe the highest historical volatility of front-month futures returns (75.4%) during the time period 1997 to 2010 which is covered by our data set. Only the year 2001 shows a similarly high volatility level, being well above the average of 57.7%.

is usually assumed to be constant, one might also consider modeling the jump intensity according to a seasonal function.

4.6 Appendix

For the practical application of any options pricing model, computational efficiency and robustness are of high importance. In order to facilitate the implementation, one can reformulate the valuation formula which was presented according to the standard terminology in Section 4.2. For our empirical study, we employed the characteristic function formulation as proposed, e.g., by Albrecher et al. (2007) to overcome the branch cut problem with the original solution of Heston (1993).¹⁹ Furthermore, following the idea of Attari (2004), we rewrite the pricing formula in a way that the ‘Heston-Integral’ in Equation (4.9) has to be evaluated only once instead of twice and that the integrand contains a square term in the denominator, causing the integral to converge faster. The obtained numerically more efficient formula for the price of a European call option on a futures contract is given by

$$c(F, K, V, T) = F e^{-r(T-t)} - \frac{K}{2} e^{-r(T-t)} + \frac{K}{\pi} e^{-r(T-t)} \int_0^\infty \operatorname{Re} \left[\frac{f(\phi)}{i} [\cos(\phi \ln K) - i \sin(\phi \ln K)] \frac{(\phi - i) e^{-r(T-t) - \phi - \frac{1}{\phi}}}{\phi^2 + 1} \right] d\phi. \quad (4.21)$$

The characteristic function has the same form as before and the corresponding system of ODEs is given by

$$\frac{\partial D}{\partial \tau} = \frac{1}{2} \sigma^2 D^2 - (\kappa + \lambda - \rho \sigma \phi i) D - \frac{\phi i + \phi^2}{2} \quad (4.22)$$

$$\frac{\partial C}{\partial \tau} = \kappa \theta(\tau) D \quad (4.23)$$

with $\tau = T - t$. While Equation (4.23) has to be solved numerically,²⁰ the solution of Equation (4.22) reads

$$D(\tau, \phi) = \frac{\kappa + \lambda - \rho \sigma \phi i - d}{\sigma^2} \left[\frac{1 - e^{-d\tau}}{1 - g e^{-d\tau}} \right] \quad (4.24)$$

¹⁹See also Lord and Kahl (2010) on this issue.

²⁰For the SV MODEL, the solution for Equation (4.23) is given by $C(\tau, \phi) = \frac{\kappa \theta}{\sigma^2} \left[(\kappa + \lambda - \rho \sigma \phi i - d) \tau - 2 \ln \left(\frac{1 - g e^{-d\tau}}{1 - g} \right) \right]$.

with

$$g = \frac{\kappa + \lambda - \rho\sigma\phi i - d}{\kappa + \lambda - \rho\sigma\phi i + d} \quad (4.25)$$

$$d = \sqrt{(\rho\sigma\phi i - \kappa - \lambda)^2 + \sigma^2(\phi i + \phi^2)}. \quad (4.26)$$

Furthermore, the choice of the numerical integration procedure is of high importance for the implementation of any stochastic volatility model. Since the ODE in Equation (4.23) has to be solved for each evaluation within the numerical integration scheme of the ‘Heston-Integral’, this double integral is potentially computationally very costly. In contrast to adaptive methods like Gauss-Lobatto or the Simpson-Quadrature, a simple trapezoidal integration scheme brings the advantage that we can span a matrix with integral evaluations which can then be kept in memory and be called when needed for the next evaluation. Similar to the caching technique of Kilin (2011), this approach dramatically reduces computing time for the option valuation in the proposed SSV MODEL.

In particular, Kilin (2011) notes that the characteristic function is independent of the strike price and hence should be evaluated only once for each sub-sample of options having an equal time to maturity. Similarly, only the upper integration limit τ is different for each maturity sub-sample when solving the ODE in Equation (4.23). For a given grid of the ‘Heston-Integral’, all evaluations of this ODE up to the integration limit yield the same values and, hence, it is possible to evaluate this integral only once for the longest maturity T_{max} and store the obtained values in the computer’s memory. When evaluating the characteristic function for options with shorter maturities T , where $T < T_{max}$, the needed function evaluations can be called from the stored values. Interpolation methods can be used if the matrix of stored values does not contain an evaluation corresponding exactly to the shorter maturity T . Hence, in our empirical study with an average number of 365 options with 12 different maturity months for a given observation day, this yields 12 characteristic function evaluations for the SV MODEL and one additional numerical evaluation of the ODE in Equation (4.23) for the SSV MODEL. In this fashion, the proposed seasonal model extension can be implemented in a computationally efficient way similar to the Heston model.

Chapter 5

Conclusion

This doctoral thesis focuses on the valuation of commodity derivatives. In Chapter 2, we lay the foundations of the subsequent analyses by providing a broad overview of the literature on commodity price dynamics and valuation models. The unique characteristics of commodities and their implications for the valuation of commodity contingent claims are described. In particular, a seasonal price behavior can be observed in many commodity markets that is in the center of interest in this thesis. We present commodity valuation models, which take this seasonal behavior into account and study the pricing accuracy of these models. The importance of seasonal variations in volatility for the valuation of commodity options is analyzed in two empirical studies.

In Chapter 3, we present our first empirical study and investigate the role of seasonal volatility in the context of one- and two-factor spot price models. The study considers the soybean and heating oil markets where a seasonal volatility pattern is induced from harvesting cycles and temperature-related demand variations, respectively. It is found that the inclusion of a deterministic seasonal function in volatility significantly reduces options pricing errors. Thereby, the seasonality adjustment is more important in terms of options pricing accuracy than adding a second stochastic factor. Furthermore, in a regression analysis of pricing errors, we document that systematic mispricing of soybean and heating oil futures options is reduced due to the seasonality adjustment.

Chapter 4 contains our second empirical study on seasonal volatility. In contrast to the study in Chapter 3, we follow a different modeling approach and

specify the price dynamics of the futures contract directly instead of deriving futures prices from the spot price dynamics. Particularly, we propose a stochastic volatility model where the long-term mean of the variance process is described through a seasonal function. The model framework allows for the derivation of semi closed-form options pricing formulas in the spirit of Heston (1993). In an empirical study, we apply our model to the natural gas market. The natural gas market is characterized by high volatility, which evolves stochastically while following a pronounced seasonal pattern. Due to inelastic demand and supply, demand variations during the heating period can cause large price changes and lead to a higher volatility during the cold season. Analyzing an extensive data set of natural gas futures options, we find that our seasonal stochastic volatility model yields significantly lower pricing errors than its nonseasonal counterpart.

Overall, we conclude that seasonal variations in volatility need to be considered when valuing options written on futures for commodities exhibiting seasonality. This is true for both a spot price model setting with deterministic volatility and the presented stochastic volatility framework. Naturally, considering seasonality in volatility is of crucial importance not only for the valuation of commodity contingent claims but also for hedging strategies and risk management in general.

Future research can extend our analyses in several directions. First, further commodity markets can be considered in order to document which markets display a seasonal behavior and to assess the implications for the valuation of derivatives traded in these markets. Second, different parametric assumptions regarding the seasonality function can be tested. Our proposed models are flexible with regard to the specifications of the deterministic function describing the seasonal figure. Third, from a modeling perspective, price dynamics can be generalized by, e.g., the inclusion of jumps, both in price level and volatility. In this context, the jump intensity can be described according to a seasonal function. However, balancing flexibility in price dynamics against a model's complexity and, hence, risk of misspecification will always be a challenge for empirical applications.

Bibliography

- H. Albrecher, P. Mayer, W. Schoutens, and J. Tistaert. The little Heston trap. *Wilmott Magazine*, (1):83–92, 2007.
- R. W. Anderson. Some determinants of the volatility of futures prices. *Journal of Futures Markets*, 5:331–348, 1985.
- H. Andersson. Are commodity prices mean reverting? *Applied Financial Economics*, 17:769–783, 2007.
- Aristotle. *The Politics - Translated by T.A. Sinclair*. Prentice Hall, 1981.
- F. Asche, O. Gjølborg, and T. Völker. Price relationships in the petroleum market: An analysis of crude oil and refined product prices. *Energy Economics*, 25:289–301, 2003.
- M. Attari. Option pricing using Fourier transforms: A numerically efficient simplification. Working Paper, March 2004.
- J. Back, M. Prokopczuk, and M. Rudolf. Seasonality and the valuation of commodity options. Working Paper, March 2010.
- G. Bakshi, C. Cao, and Z. Chen. Empirical performance of alternative option pricing models. *Journal of Finance*, 52:2003–2049, 1997.
- J. Barkoulas, W. C. Labys, and J. Onochie. Fractional dynamics in international commodity prices. *Journal of Futures Markets*, 17:161–189, 1997.
- M. T. Barlow. A diffusion model for electricity prices. *Mathematical Finance*, 12:287–298, 2002.

- G. Barone-Adesi and R. E. Whaley. Efficient analytic approximation of American option values. *Journal of Finance*, 42:301–320, 1987.
- D. Bates. The crash of '87: Was it expected? The evidence from options markets. *Journal of Finance*, 46:1009–1044, 1991.
- D. Bates. Post-'87 crash fears in S&P 500 futures options. *Journal of Econometrics*, 94:181–238, 2000.
- F. E. Benth, L. Ekeland, R. Hauge, and B. F. Nielsen. A note on arbitrage-free pricing of forward contracts in energy markets. *Applied Mathematical Finance*, 10:325–336, 2003.
- F. E. Benth, J. Kallsen, and T. Meyer-Brandis. A Non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing. *Applied Mathematical Finance*, 14:153–169, 2007.
- J.-T. Bernard, L. Khalaf, M. Kichian, and S. McMahon. Forecasting commodity prices: GARCH, jumps, and mean reversion. *Journal of Forecasting*, 27:279–291, 2008.
- H. Bessembinder. Systematic risk, hedging pressure, and risk premiums in futures markets. *Review of Financial Studies*, 5:637–667, 1992.
- H. Bessembinder and M. L. Lemmon. Equilibrium pricing and optimal hedging in electricity forward markets. *Journal of Finance*, 57:1347–1382, 2002.
- H. Bessembinder, J. F. Coughenour, P. J. Seguin, and M. M. Smoller. Mean reversion in equilibrium asset prices: Evidence from the futures term structure. *Journal of Finance*, 50:361–375, 1995.
- H. Bessembinder, J. F. Coughenour, P. J. Seguin, and M. M. Smoller. Is there a term structure of futures volatilities? Reevaluating the Samuelson hypothesis. *Journal of Derivatives*, 4:45–58, 1996.
- F. Black. The pricing of commodity contracts. *Journal of Financial Economics*, 3:167–179, 1976.

- F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–654, 1973.
- Z. Bodie and V. I. Rosansky. Risk and return in commodity futures. *Financial Analysts Journal*, 36:27–39, 1980.
- T. Bollerslev, J. Litvinova, and G. Tauchen. Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics*, 4:353–384, 2006.
- S. Borovkova and H. Geman. Seasonal and stochastic effects in commodity forward curves. *Review of Derivatives Research*, 9:167–186, 2006.
- D. T. Breeden. Consumption risk in futures markets. *Journal of Finance*, 35:503–520, 1980.
- M. J. Brennan. The supply of storage. *American Economic Review*, 47:50–72, 1958.
- M. J. Brennan. The price of convenience and the valuation of commodity contingent claims. In D. Lund and B. Oksendal, editors, *Stochastic Models and Option Values*. Elsevier Science, 1991.
- M. J. Brennan and E. S. Schwartz. Evaluating natural resource investments. *Journal of Business*, 58:135–157, 1985.
- M. Broadie, M. Chernov, and M. Johannes. Model specification and risk premia: Evidence from futures options. *Journal of Finance*, 62:1453–1490, 2007.
- Á. Cartea and M. G. Figueroa. Pricing in electricity markets: A mean reverting jump diffusion model with seasonality. *Applied Mathematical Finance*, 12:313–335, 2005.
- Á. Cartea and P. Villaplana. Spot price modeling and the valuation of electricity forward contracts: The role of demand and capacity. *Journal of Banking & Finance*, 32:2502–2519, 2008.

- Á. Cartea and T. Williams. UK gas market: The market price of risk and applications to multiple interruptible supply contracts. *Energy Economics*, 30: 829–846, 2008.
- Á. Cartea, M. G. Figueroa, and H. Geman. Modelling electricity prices with forward looking capacity constraints. *Applied Mathematical Finance*, 16:103–122, 2009.
- J. Casassus and P. Collin-Dufresne. Stochastic convenience yield implied from commodity futures and interest rates. *Journal of Finance*, 60:2283–2331, 2005.
- M. G. Castelino and J. C. Francis. Basis speculation in commodity futures: The maturity effect. *Journal of Futures Markets*, 2:195–206, 1982.
- M. J. Chambers and R. E. Bailey. A theory of commodity price fluctuations. *Journal of Political Economy*, 104:924–957, 1996.
- E. C. Chang. Returns to speculators and the theory of normal backwardation. *Journal of Finance*, 40:193–208, 1985.
- J.-P. Chavas, P. M. Despins, and T. R. Fortenbery. Inventory dynamics under transaction costs. *American Journal of Agricultural Economics*, 82:260–273, 2000.
- Y.-J. Chen, J.-C. Duan, and M.-W. Hung. Volatility and maturity effects in the Nikkei index futures. *Journal of Futures Markets*, 19:895–909, 1999.
- Z. Chen and P. A. Forsyth. Implications of a regime-switching model on natural gas storage valuation and optimal operation. *Quantitative Finance*, 10:159–176, 2010.
- Y.-W. Cheung. Stock price dynamics and firm size: An empirical investigation. *Journal of Finance*, 47:1985–1997, 1992.
- J. W. Choi and F. A. Longstaff. Pricing options on agricultural futures: An application of the constant elasticity of variance option pricing model. *Journal of Futures Markets*, 5:247–258, 1985.

- A. A. Christie. The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics*, 10:407–432, 1982.
- P. Christoffersen and K. Jacobs. The importance of the loss function in option valuation. *Journal of Financial Economics*, 72:291–318, 2004.
- P. H. Cootner. Returns to speculators: Telser versus Keynes. *Journal of Political Economy*, 68:396–404, 1960.
- G. Cortazar and L. Naranjo. An N-factor Gaussian model of oil futures prices. *Journal of Futures Markets*, 26:243–268, 2006.
- G. Cortazar and E. S. Schwartz. The valuation of commodity-contingent claims. *Journal of Derivatives*, 1:27–39, 1994.
- G. Cortazar and E. S. Schwartz. Implementing a stochastic model for oil futures prices. *Energy Economics*, 25:215–238, 2003.
- J. C. Cox, J. E. Ingersoll Jr., and S. A. Ross. The relation between forward prices and futures prices. *Journal of Financial Economics*, 9:321–346, 1981.
- J. C. Cox, J. E. Ingersoll Jr., and S. A. Ross. A theory of the term structure of interest rates. *Econometrica*, 53:385–407, 1985.
- J. Crosby. A multi-factor jump-diffusion model for commodities. *Quantitative Finance*, 8:181–200, 2008.
- E. Daal, J. Farhat, and P. P. Wei. Does futures exhibit maturity effect? New evidence from an extensive set of US and foreign futures contracts. *Review of Financial Economics*, 15:113–128, 2006.
- J. F. J. de Munnik and P. C. Schotman. Cross-sectional versus time series estimation of term structure models: Empirical results for the Dutch bond market. *Journal of Banking & Finance*, 18:997–1025, 1994.
- F. de Roon, T. Nijman, and C. Veld. Hedging pressure effects in futures markets. *Journal of Finance*, 55:1438–1456, 2000.

- A. Deaton and G. Laroque. On the behaviour of commodity prices. *Review of Economic Studies*, 59:1–23, 1992.
- A. Deaton and G. Laroque. Competitive storage and commodity price dynamics. *Journal of Political Economy*, 104:896–923, 1996.
- M.A.H. Dempster, E. Medova, and K. Tang. Long term spread option valuation and hedging. *Journal of Banking & Finance*, 32:2530–2540, 2008.
- S. Deng. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. Working Paper, 2000.
- J. S. Doran and E. I. Ronn. The bias in Black-Scholes/Black implied volatility: An analysis of equity and energy markets. *Review of Derivatives Research*, 8: 177–198, 2005.
- J. S. Doran and E. I. Ronn. Computing the market price of volatility risk in the energy commodity markets. *Journal of Banking & Finance*, 32:2541–2552, 2008.
- D. Duffie, J. Pan, and K. Singleton. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68:1343–1376, 2000.
- H. N. Duong and P. S. Kalev. The Samuelson hypothesis in futures markets: An analysis using intraday data. *Journal of Banking & Finance*, 32:489–500, 2008.
- K. Dusak. Futures trading and investor returns: An investigation of commodity market risk premiums. *Journal of Political Economy*, 81:1387–1406, 1973.
- B. Eraker. Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *Journal of Finance*, 59:1367–1403, 2004.
- B. Eraker, M. Johannes, and N. G. Polson. The impact of jumps in volatility and returns. *Journal of Finance*, 58:1269–1300, 2003.
- C. B. Erb and C. R. Harvey. The strategic and tactical value of commodity futures. *Financial Analysts Journal*, 62:69–97, 2006.

- E. F. Fama and K. R. French. Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. *Journal of Business*, 60: 55–74, 1987.
- E. F. Fama and K. R. French. Business cycles and the behavior of metals prices. *Journal of Finance*, 43:1075–1094, 1988.
- S. Figlewski and X. Wang. Is the 'leverage effect' a leverage effect? Working Paper, 2000.
- W. M. Fong and K. H. See. A Markov switching model of the conditional volatility of crude oil futures prices. *Energy Economics*, 24:71–95, 2002.
- D. L. Frechette. The dynamics of convenience and the Brazilian soybean boom. *American Journal of Agricultural Economics*, 79:1108–1118, 1997.
- T. M. Galloway and R. W. Kolb. Futures prices and the maturity effect. *Journal of Futures Markets*, 16:809–828, 1996.
- G. D. Gay and T.-H. Kim. An investigation into seasonality in the futures market. *Journal of Futures Markets*, 7:169–181, 1987.
- H. Geman. *Commodities and Commodity Derivatives*. John Wiley, Chichester, 2005.
- H. Geman and C. Kharoubi. WTI crude oil futures in portfolio diversification: The time-to-maturity effect. *Journal of Banking & Finance*, 32:2553–2559, 2008.
- H. Geman and V.-N. Nguyen. Soybean inventory and forward curve dynamics. *Management Science*, 51:1076–1091, 2005.
- H. Geman and S. Ohana. Forward curves, scarcity and price volatility in oil and natural gas markets. *Energy Economics*, 31:576–585, 2009.
- H. Geman and A. Roncoroni. Understanding the fine structure of electricity prices. *Journal of Business*, 79:1225–1261, 2006.

- S. Geman and D. Geman. Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6:721–741, 1984.
- R. Gibson and E. S. Schwartz. Stochastic convenience yield and the pricing of oil contingent claims. *Journal of Finance*, 45:959–976, 1990.
- P. B. Girma and A. S. Paulson. Seasonality in petroleum futures spreads. *Journal of Futures Markets*, 18:581–598, 1998.
- G. Gorton and K. G. Rouwenhorst. Facts and fantasies about commodity futures. *Financial Analysts Journal*, 62:47–68, 2006.
- T. Grammatikos and A. Saunders. Futures price variability: A test of maturity and volume effects. *Journal of Business*, 59:319–330, 1986.
- J. Hasanhodzic and A. W. Lo. Black’s leverage effect is not due to leverage. Working Paper, 2010.
- T. B. Hazuka. Consumption betas and backwardation in commodity markets. *Journal of Finance*, 39:647–655, 1984.
- D. Heath, R. A. Jarrow, and A. Morton. Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica*, 60:77–105, 1992.
- S. L. Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6: 327–343, 1993.
- S. Hiksloops and S. Jaimungal. Asymptotic pricing of commodity derivatives using stochastic volatility spot models. *Applied Mathematical Finance*, 15: 449–477, 2008.
- J. E. Hilliard and J. Reis. Valuation of commodity futures and options under stochastic convenience yields, interest rates, and jump diffusions in the spot. *Journal of Financial and Quantitative Analysis*, 33:61–86, 1998.

- J. E. Hilliard and J. Reis. Jump processes in commodity futures prices and options pricing. *American Journal of Agricultural Economics*, 81:273–286, 1999.
- D. Hirshleifer. Determinants of hedging and risk premia in commodity futures markets. *Journal of Financial and Quantitative Analysis*, 24:313–331, 1989.
- D. Hirshleifer. Hedging pressure and future price movements in a general equilibrium model. *Econometrica*, 58:411–428, 1990.
- T. S. Y. Ho and S.-B. Lee. Term structure movements and pricing interest rate contingent claims. *Journal of Finance*, 41:1011–1029, 1986.
- W. K. Huguen. A maximal affine stochastic volatility model of oil prices. *Journal of Futures Markets*, 30:101–133, 2010.
- J. Hull and A. White. One-factor interest-rate models and the valuation of interest-rate derivative securities. *Journal of Financial and Quantitative Analysis*, 28:235–2354, 1993.
- S. Hylleberg. *Modelling Seasonality*. Oxford University Press, 1. edition, 1992.
- E. Jacquier, N. G. Polson, and P. E. Rossi. Bayesian analysis of stochastic volatility models. *Journal of Business and Economic Statistics*, 12:371–389, 1994.
- G. R. Jensen, R. R. Johnson, and J. M. Mercer. Efficient use of commodity futures in diversified portfolios. *Journal of Futures Markets*, 20:489–506, 2000.
- G. R. Jensen, R. R. Johnson, and J. M. Mercer. Tactical asset allocation and commodity futures. *Journal of Portfolio Management*, 28:100–111, 2002.
- M. Johannes and N. G. Polson. MCMC methods for financial econometrics. In Y. Aït-Sahalia and L. Hansen, editors, *Handbook of Financial Econometrics*. Elsevier, 2006.
- N. Kaldor. Speculation and economic stability. *Review of Economic Studies*, 7: 1–27, 1939.
- B. Karali and W. N. Thurman. Components of grain futures price volatility. *Journal of Agricultural and Resource Economics*, 35:167–182, 2010.

- D. Kenyon, K. Kling, J. Jordan, W. Seale, and N. McCabe. Factors affecting agricultural futures price variance. *Journal of Futures Markets*, 7:73–91, 1987.
- J. M. Keynes. *A Treatise on Money Vol. II: The Applied Theory of Money*. Macmillan & Co., 1930.
- N. Khoury and P. Yourougou. Determinants of agricultural futures price volatilities: Evidence from Winnipeg Commodity Exchange. *Journal of Futures Markets*, 13:345–356, 1993.
- F. Kilin. Accelerating the calibration of stochastic volatility models. *Journal of Derivatives*, 18:7–16, 2011.
- S. Koekebakker and G. Lien. Volatility and price jumps in agricultural futures prices – evidence from wheat options. *American Journal of Agricultural Economics*, 86:1018–1031, 2004.
- O. Korn. Drift matters: An analysis of commodity derivatives. *Journal of Futures Markets*, 25:211–241, 2005.
- D. Lautier. Term structure models of commodity prices: A review. *Journal of Alternative Investments*, 8:42–64, 2005.
- S. C. Linn and Z. Zhu. Natural gas prices and the gas storage report: Public news and volatility in energy futures markets. *Journal of Futures Markets*, 24: 283–313, 2004.
- R. H. Litzenberger and N. Rabinowitz. Backwardation in oil futures markets: Theory and empirical evidence. *Journal of Finance*, 50:1517–1545, 1995.
- P. Liu and K. Tang. The stochastic behavior of commodity prices with heteroskedasticity in the convenience yield. *Journal of Empirical Finance*, 18: 211–224, 2011.
- A. W. Lo and J. Wang. Implementing option pricing models when asset returns are predictable. *Journal of Finance*, 50:87–129, 1995.
- F. Longstaff and A. Wang. Electricity forward prices: A high-frequency empirical analysis. *Journal of Finance*, 59:1877–1900, 2004.

- R. Lord and C. Kahl. Complex logarithms in Heston-like models. *Mathematical Finance*, 20:671–694, 2010.
- B. M. Lucey and E. Tully. Seasonality, risk and return in daily COMEX gold and silver data 1982-2002. *Applied Financial Economics*, 16:319–333, 2006.
- J. J. Lucia and E. S. Schwartz. Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research*, 5:5–50, 2002.
- M. Manoliu and S. Tompaidis. Energy futures prices: Term structure models with Kalman filter estimation. *Applied Mathematical Finance*, 9:21–43, 2002.
- J. Miffre and G. Rallis. Momentum strategies in commodity futures markets. *Journal of Banking & Finance*, 31:1863–1886, 2007.
- N. T. Milonas. Price variability and the maturity effect in futures markets. *Journal of Futures Markets*, 6:443–460, 1986.
- N. T. Milonas. Measuring seasonalities in commodity markets and the half-month effect. *Journal of Futures Markets*, 11:331–345, 1991.
- K. R. Miltersen. Commodity price modelling that matches current observables: A new approach. *Quantitative Finance*, 3:51–58, 2003.
- K. R. Miltersen and E. S. Schwartz. Pricing of options on commodity futures with stochastic term structures of convenience yield and interest rates. *Journal of Financial and Quantitative Analysis*, 33:33–59, 1998.
- N. Movassagh and B. Modjtahedi. Bias and backwardation in natural gas futures prices. *Journal of Futures Markets*, 25:281–308, 2005.
- X. Mu. Weather, storage, and natural gas price dynamics: Fundamentals and volatility. *Energy Economics*, 29:46–63, 2007.
- M. Muck and M. Rudolf. The pricing of electricity forwards. In J. F. Fabozzi, D. G. Kaiser, and R. Füss, editors, *The Handbook of Commodity Investing*. John Wiley, Hoboken, 2008.

- J. M. Mulvey, S. S. N. Kaul, and K. D. Simsek. Evaluating a trend-following commodity index for multi-period asset allocation. *Journal of Alternative Investments*, 7:54–69, 2004.
- V. K. Ng and S. C. Pirrong. Fundamentals and volatility: Storage, spreads, and the dynamics of metals prices. *Journal of Business*, 67:203–230, 1994.
- M. J. Nielsen and E. S. Schwartz. Theory of storage and the pricing of commodity claims. *Review of Derivatives Research*, 7:5–24, 2004.
- N. K. Nomikos and O. Soldatos. Using affine jump diffusion models for modelling and pricing electricity derivatives. *Applied Mathematical Finance*, 15:41–41, 2008.
- R. Paschke and M. Prokopczuk. Integrating multiple commodities in a model of stochastic price dynamics. *Journal of Energy Markets*, 2:47–82, 2009.
- R. Paschke and M. Prokopczuk. Commodity derivatives valuation with autoregressive and moving average components in the price dynamics. *Journal of Banking & Finance*, 34:2742–2752, 2010.
- C. Pirrong and M. Jermakyan. The price of power: The valuation of power and weather derivatives. *Journal of Banking & Finance*, 32:2520–2529, 2008.
- H. Reisman. Movements of the term structure of commodity futures and the pricing of commodity claims. Working Paper, 1992.
- D. R. Ribeiro and S. D. Hodges. A contango-constrained model for storable commodity prices. *Journal of Futures Markets*, 25:1025–1044, 2005.
- M. Richter and C. Sørensen. Stochastic volatility and seasonality in commodity futures and options: The case of soybeans. Working Paper, 2002.
- E. I. Ronn and J. Wimschulte. Intra-day risk premia in European electricity forward markets. Working Paper, 2008.
- S. A. Ross. Hedging long run commitments: Exercises in incomplete market pricing. *Banca Monte Economic Notes*, 26:99–132, 1997.

- B. R. Routledge, D. J. Seppi, and C. S. Spatt. Equilibrium forward curves for commodities. *Journal of Finance*, 55:1297–1338, 2000.
- M. Rudolf, H. Zimmermann, and C. Zogg-Wetter. Anlage- und Portfolioeigenschaften von Commodities am Beispiel des GSCI. *Finanzmarkt und Portfolio Management*, 7:339–359, 1993.
- P. A. Samuelson. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6:41–49, 1965.
- E. S. Schwartz. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance*, 52:923–973, 1997.
- E. S. Schwartz and J. E. Smith. Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46:893–911, 2000.
- J. Seifert and M. Uhrig-Homburg. Modelling jumps in electricity prices: Theory and empirical evidence. *Review of Derivatives Research*, 10:59–85, 2007.
- C. Sørensen. Modeling seasonality in agricultural commodity futures. *Journal of Futures Markets*, 22:393–426, 2002.
- H. R. Stoll. Commodity futures and spot price determination and hedging in capital market equilibrium. *Journal of Financial and Quantitative Analysis*, 14:873–894, 1979.
- D. H. Streeter and W. G. Tomek. Variability in soybean futures prices: An integrated framework. *Journal of Futures Markets*, 12:705–728, 1992.
- H. Suenaga, A. Smith, and J. Williams. Volatility dynamics of NYMEX natural gas futures prices. *Journal of Futures Markets*, 28:438–463, 2008.
- K. Tang. Time-varying long-run mean of commodity prices and the modeling of futures term structures. *Quantitative Finance*, forthcoming (published online), 2010.
- L. G. Telser. Futures trading and the storage of cotton and wheat. *Journal of Political Economy*, 66:233–255, 1958.

- M. I. Todorova. Modeling energy commodity futures: Is seasonality part of it? *Journal of Alternative Investments*, 7:10–32, 2004.
- A. B. Trolle and E. S. Schwartz. Unspanned stochastic volatility and the pricing of commodity derivatives. *Review of Financial Studies*, 22:4423–4461, 2009.
- A. B. Trolle and E. S. Schwartz. Variance risk premia in energy commodities. *Journal of Derivatives*, 17:15–32, 2010.
- M. T. Vo. Regime-switching stochastic volatility: Evidence from the crude oil market. *Energy Economics*, 31:779–788, 2009.
- M. S. Williams and A. Hoffman. *Fundamentals of the Options market*. McGraw-Hill, 2001.
- H. Working. The theory of the price of storage. *American Economic Review*, 39:1254–1262, 1949.
- X. Yan. Valuation of commodity derivatives in a new multi-factor model. *Review of Derivatives Research*, 5:251–271, 2002.