

Pricing in Overlapping Transport Networks

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Abstract

This paper considers a transport network with two firms that operate a parallel service on a hub-to-hub connection and monopoly services on spoke-to-hub connections under increasing returns to scale. We find the following: A symmetric equilibrium cannot occur under independent (non-cooperative) pricing when the number of spoke-to-spoke passengers becomes positive. The effect of cooperative pricing on mark-ups in spoke-to-hub, hub-to-hub and spoke-to-spoke markets (where double marginalization can occur) can be positive or negative. Cooperation can reduce total welfare though hub-to-hub markets are small.

JEL Classification: D43, L13, L40, L52, L91.

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1 Introduction

Markets where firms operate different transport networks with overlapping parts are common. For example, many airlines operate hub-and-spoke networks that connect a number of cities in their home country (through their own hub) with a hub airport of a second airline that also operates on the hub-to-hub market and connects a number of cities in its own home country. In this scenario, networks are overlapping on the hub-to-hub part and complementary on spoke-to-spoke connections. Similar constellations occur in sea transport markets where liners operate hub-and-spoke networks, in urban transport markets where providers of bus, tram and metro services operate overlapping networks, or in logistics markets where firms operate dense networks in their home countries that are loosely connected.

Two aspects that are particularly of interest in the context of overlapping transport networks. First, double marginalization may be exercised by firms resulting in excessive prices for complementary parts of the network. Second, transport operations exhibit increasing returns to scale due to economies in vehicle size or due to fixed network costs. To remedy the problem of double marginalization and to take full advantage of the increasing returns to scale, firms often consider cooperation and apply for exemptions from antitrust law. This paper investigates the social benefits of cooperative pricing in transport markets. The major contributions are to provide a better understanding of the role of spoke-to-hub markets for prices in overlapping transport networks and to identify general conditions under which cooperative pricing is preferred from the social viewpoint.¹

¹Czerny (2009) considers the role of spoke-to-hub markets in a linear model with complementary networks where one hub city and n spoke cities exist. However, he abstracts away from overlapping network parts and costs.

We consider a transport network that connects four cities. Two cities are spoke cities (A and B) and two are hub cities (H and J) each of which is connected to the other hub city and to one spoke city. This network serves six markets (four spoke-to-hub, one hub-to-hub and one spoke-to-spoke). On the supply side, there are two firms with each firm serving three markets (two spoke-to-hub markets and the hub-to-hub market). Thus, the firms operate a parallel service on the hub-to-hub part and monopoly services on the spoke-to-hub connections. Transport costs exhibit increasing returns to scale. We consider two pricing scenarios.

In the first scenario, to which we refer as *independent pricing*, firms simultaneously and independently choose prices for the spoke-to-hub markets and quantities on the hub-to-hub market (i.e. firms are in Cournot competition on the hub-to-hub part). In this scenario, spoke-to-spoke passengers need to buy two separate spoke-to-hub tickets to complete their travel, as the firms' networks are complementary. Since the hub-to-hub part is served by both firms, spoke-to-spoke passengers may consider two alternative routings: traveling with one firm from spoke city A to hub city H and with the other firm from hub city H to spoke city B or traveling with one firm from spoke city A to hub city J and with the other firm from hub city J to spoke city B.

In the second scenario, to which we refer as *cooperative pricing*, firms jointly maximize total profit. This is similar to the case of a merger or to the case of a single joint-venture alliance.² Cooperative pricing considers joint minimization of operating costs, collusive pricing in the hub-to-hub market, and price discrimination between spoke-to-hub and spoke-to-spoke

²For the analysis of competing alliances see Park and Zhang (2000), Brueckner (2003), Bilotkach (2005), Brueckner and Pels (2005), Zhang and Zhang (2006), and Flores-Fillol and Moner-Colonques (2007).

passengers in the following sense.³ Together firms can bundle spoke-to-hub services to offer the spoke-to-spoke connection as one service and charge a single price for the whole tripe. This single price must however be lower than the sum of two separate spoke-to-hub tickets, which is called the fare-arbitrage constraint (Brueckner, 2001). Note that bundling can be considered as a special form of price discrimination (for example, Adams and Yellen, 1976 and Armstrong, 2006).⁴ Moreover, firms can jointly minimize costs on the hub-to-hub part when they cooperate (that is, together they can take full advantage of returns to scale).

In the case of independent pricing, we find that a symmetric equilibrium where each firm carries half of the spoke-to-spoke passengers on the hub-to-hub part cannot exist when the costs of transporting passengers are positive and economies of scale exist. The economic intuition is the following. Spoke-to-spoke passengers choose the route with the cheapest total fare, and thus a small deviation in one price can shift spoke-to-spoke demand from one route to the other. Suppose that the two routes are priced equally and that both firms carry spoke-to-spoke passengers on the hub-to-hub part. Then, if one price changes, all spoke-to-spoke passengers shift to only one route. In this situation, one firm saves the costs of transporting spoke-to-spoke passengers on the hub-to-hub part; suppose that prices are such that this firm is better off by saving these costs. Then, sharing spoke-to-spoke passengers on the hub-to-hub part is not an equilibrium. By contrast, suppose that prices are such that this firm (which does not carry spoke-to-spoke passengers on the hub-to-hub part anymore) is worse off because the decrease in revenues is greater than the cost savings on the hub-to-hub part. However, this firm could then

³Bilotkach (2005), Flores-Fillol and Moner-Colonques (2007) and Czerny (2009) also consider price discrimination between spoke-to-hub and spoke-to-spoke passengers.

⁴Following Adams and Yellen's diction, firms adopt a mixed bundling strategy because they offer spoke-to-hub services separately as well as in packages.

increase its profit by changing prices such that it carries all spoke-to-spoke passengers on the hub-to-hub part by taking full advantage of economies of scale. Again, sharing spoke-to-spoke passengers on the hub-to-hub part is not an equilibrium.

Note that the asymmetry of equilibria is of special importance because the literature on overlapping transport networks has, so far, concentrated on symmetric solutions. For example, the network and market structure considered by Brueckner (2001) is closely related to our framework but considers four spoke cities. Moreover, Brueckner assumes that firms can always price discriminate between spoke-to-hub and spoke-to-spoke passengers (i.e. even in a non-cooperative situation, firms can bundle services provided by different firms). A second example is the network structure considered in the paper by Brueckner and Proost (2010). Their setting exactly coincides with the one considered in this paper with the exception that our framework is more general and includes spoke-to-hub markets, while Brueckner and Proost abstract away from spoke-to-hub markets (they concentrate on hub-to-hub and spoke-to-spoke passengers). Both papers, Brueckner (2001) and Brueckner and Proost (2010), take symmetric solutions as given.

Furthermore, we find that spoke-to-hub markets can limit or even eliminate double marginalization in spoke-to-spoke markets. This is because, under independent pricing, spoke-to-hub prices are relevant for both spoke-to-hub passengers and spoke-to-spoke passengers, and therefore the demand elasticities of both passenger groups determine mark-ups. From a social point of view, this result has important implications for the desirability of cooperative pricing because joint profit maximization may no longer provide an argument to avoid the negative impacts of double marginalization under such circumstances. For example, if demand elasticities are such that spoke-

to-hub markets increase mark-ups in the spoke-to-spoke segment, the effect cooperative pricing that reduces mark-ups in the spoke-to-spoke market can actually be undesirable from the social viewpoint.⁵ The reason is that, in this situation, cooperative pricing can increase prices in spoke-to-hub markets due to price discrimination (and in the hub-to-hub market due to collusion).⁶

This line of reasoning is consistent with the empirical results of Armantier and Richard (2006, 2008), who investigated the effects of domestic airline code-share agreements on non-stop and interline passengers in the US. With code-share agreements an airline can market seats of its partners' flights, and hence bundle the services of different firms and price discriminate between non-stop and interline passengers. Armantier and Richard found that code-share agreements between Continental Airlines and Northwest Airlines reduced average prices for interline passengers but increased the average price paid by non-stop passengers.

Numerical instances indicate that, from the social viewpoint, cooperative pricing is beneficial, rather intuitively, if spoke-to-spoke markets are large relative to hub-to-hub markets. There are however intermediate parameter ranges of significant size where cooperation increases total surplus (i.e. the sum of consumer surplus and firms' profits) but not consumer surplus, while the reverse is not possible. This suggests that the approval of cooperation often depends on the social weights attached to profits. The numerical simulations also reveal a clear positive relationship between returns to scale and the benefits of cooperative pricing, which is intuitive as well.

⁵Brueckner and Whalen (2000), Brueckner (2003), and Whalen (2007) provide some empirical evidence that collaboration reduces ticket prices for interline passengers in airline markets.

⁶Chen and Ross (2000) found that, from the social viewpoint, cooperation can have further negative effects when market entry and capacity investments are considered.

The plan of the paper is as follows. In Section 2 the model is presented. In Section 3 we show that, under independent pricing, symmetric solutions cannot exist when the number of spoke-to-spoke passengers is positive. Furthermore, we elaborate on mark-ups under independent pricing and demonstrate that multiple equilibria can exist. Section 4 considers cooperative pricing behavior and elaborates on the issue of double marginalization. Section 5 evaluates the social benefits of independent pricing behavior and cooperative pricing behavior. Therein, numerical simulations are presented to illustrate the effect of pricing scenarios on total welfare and consumer surplus. Section 6 provides conclusions and avenues for future research.

2 The Model

We first describe the supply side. There are four cities A, H, J and B and two firms 1 and 2. Firms operate different networks. Firm 1's network connects A and H as well as H and J; hence, H is firm 1's hub. Firm 2's network connects H and J as well as J and B; hence, J is firm 2's hub. Both firms' networks connect hubs H and J. Networks are thus overlapping but not identical because firm 1 offers a connection between A and H that is not offered by firm 2, and firm 2 offers a connection between J and B that is not offered by firm 1.

Denote the total number of passengers who travel on the AH part by $Q_{ah} \geq 0$ and the total number passengers who travel on the JB part by $Q_{jb} \geq 0$.⁷ Furthermore, denote the total number passengers who travel on the HJ part by Q_{hj} . Passengers can choose between firms 1 and 2 on

⁷We consider passenger traffic, but our setting could easily be adapted to freight traffic as well.

this connection and letting $Q_{i,hj} \geq 0$ with $i = 1, 2$ denote the number of passengers that travel with firm i on the HJ connection, $Q_{hj} = Q_{1,hj} + Q_{2,hj}$.

The number of passengers at each connection is composed of passengers belonging to different origin-destination markets (round trips). We distinguish six markets. First, there are four spoke-to-hub markets where firm 1 carries passengers between A and H as well as between A and J and where firm 2 carries passengers between J and B as well as H and B. These markets are indicated by subscripts ah, aj, jb and hb . Denote the number of passengers in these markets by $q_x \geq 0$ with $x \in \{ah, aj, jb, hb\}$. Second, there is one hub-to-hub market where firms carry passengers between H and J, which we indicate by subscript hj . Denote the number of passengers that firm i , $i = 1, 2$, carries in the hub-to-hub market by $q_{i,hj} \geq 0$ and the total number of HJ passengers by $q_{hj} \geq 0$ with $q_{hj} = q_{1,hj} + q_{2,hj}$. Finally, there is one spoke-to-spoke market where firms carry passengers between A and B, which we indicate by subscript ab . Denote the passenger number in the spoke-to-spoke market by $q_{ab} \geq 0$.

Firms are monopolies in spoke-to-hub markets ah and jb . Furthermore, passengers in market aj are entirely carried by firm 1 and passengers in market hb are entirely carried by firm 2. This is reasonable as long as the single ticket for the entire trip is cheaper than the two separate tickets for the hub-to-hub part and the spoke part, which is known as the fare-arbitrage constraint (Brueckner, 2001). We assume that firms are in Cournot competition and provide homogenous services in the hub-to-hub market. By contrast, passengers in the spoke-to-spoke market are always served by two firms, since networks are complementary. Then, a share of spoke-to-spoke passengers, denoted by $\alpha \in [0, 1]$, uses firm 1 on the hub-to-hub part and the

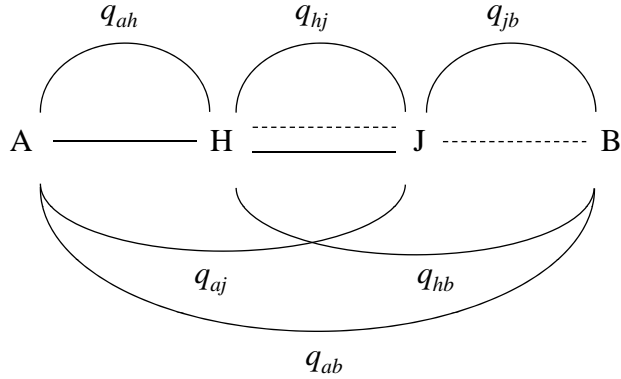


Figure 1: Networks of firm 1 (solid straight lines) and firm 2 (dashed straight lines), and passenger numbers in the six markets (curved lines).

corresponding share of spoke-to-spoke passengers, $(1 - \alpha)$, uses firm 2 on the hub-to-hub part. It now holds that

$$Q_{ah} = q_{ah} + q_{aj} + q_{ab}, \quad Q_{1,hj} = q_{aj} + q_{1,hj} + \alpha q_{ab},$$

$$Q_{2,hj} = q_{hb} + q_{2,hj} + (1 - \alpha)q_{ab} \quad \text{and} \quad Q_{jb} = q_{jb} + q_{hb} + q_{ab}. \quad (1)$$

Figure 1 illustrates the networks (straight lines) and markets (curved lines).

The costs depend on the number of passengers carried by firms. Denote the costs of firm 1 on the AH part by $C_{ah}(Q_{ah}) \geq 0$, the costs of firm i , $i = 1, 2$, on the HJ part by $C_{i,hj}(Q_{i,hj}) \geq 0$, and the costs of firm 2 on the JB part by $C_{jb}(Q_{jb}) \geq 0$. Transport costs exhibit increasing returns to scale due to economies in the vehicle size or due to fixed network costs. Therefore, all

costs are strictly increasing and strictly concave, that is, $C'_y > 0$ and $C''_y < 0$ for all $y \in \{ah; 1, hj; 2, hj; jb\}$.

We now proceed with the demand side. Let $P_x(q_x) \geq 0$ denote the inverse demand in market $x \in \{ah, aj, hj, hb, jb, ab\}$ with $P'_x < 0$, and let $D_x(p_x)$ denote the respective demands with $D'_y < 0$ and $(D_x/D'_x)' < 0$ where $p_x \geq 0$ is the price charged in market x . Spoke-to-spoke passengers can choose between two alternative routes. If they use firm 1 to travel on the AH part and firm 2 to travel on the HJ part and the JB part, they pay a (total) price $p_{ah} + p_{hb}$. If they use firm 1 to travel on the AH part and the JB part and firm 2 to travel on the HJ part, they pay a price $p_{aj} + p_{jb}$. Of these two options, passengers choose the cheaper total ticket price. Furthermore, assuming that the fare-arbitrage conditions

$$p_{hb} \leq p_{hj} + p_{jb} \quad \text{and} \quad p_{aj} \leq p_{ah} + p_{hj} \quad (2)$$

are satisfied, passengers cannot be better off by buying two tickets for the AH part and the HJ part or the HJ part and the JB part. This implies

$$\alpha = \begin{cases} 0 & \text{if} \\ 1/2 & \text{if } p_{ah} + p_{hb} \\ 1 & \text{if} \end{cases} \begin{cases} > \\ = \\ < \end{cases} p_{aj} + p_{jb}. \quad (3)$$

This setting is designed to investigate the equilibrium pricing outcome when firms simultaneously and independently choose prices or quantities in a scenario with overlapping transport networks. We will also investigate the role of spoke-to-hub markets for the evaluation of cooperative pricing from the policy viewpoint. In our case, cooperative pricing subsumes three elements. First, firms collude in the hub-to-hub market. Second, firms together

minimize costs such that the total costs of carrying passengers on the HJ part becomes $C_{hj}(Q_{hj})$. Third, firms 1 and 2 bundle spoke-to-hub connections and charge a single price p_{ab} for the entire trip with $p_{ab} < \min\{p_{ah} + p_{hb}, p_{aj} + p_{jb}\}$, which is a special form of price discrimination by bundling.

3 Independent Pricing

In this section, we study the firms' behavior under independent pricing. We first develop the profit expressions and prove that an asymmetric equilibrium emerges as the only solution when spoke-to-spoke passengers are served. We then derive the first-order conditions and provide closed-form expressions for the mark-ups. We conclude this section with a further characterization of the equilibria.

3.1 The asymmetry of equilibria

We first develop the firms' profit expressions. Firm 1's revenues in market ah are $p_{ah}D_{ah}$, in market aj revenues are $p_{aj}D_{aj}$, in market hj revenues are $q_{1,hj}P_{hj}$, and in market ab revenues are $[\alpha p_{ah} + (1 - \alpha)p_{aj}]D_{ab}$. Firm 1's costs of operating services on the AH part and the HJ part are C_{ah} and $C_{1,hj}$, respectively. Firm 2's revenues in market jb are $p_{jb}D_{jb}$, in market hb revenues are $p_{hb}D_{hb}$, in market hj revenues are $q_{2,hj}P_{hj}$, and in market ab revenues are $[(1 - \alpha)p_{jb} + \alpha p_{hb}]D_{ab}$. Firm 2's costs of operating services on the JB part and the HJ part are C_{jb} and $C_{2,hj}$, respectively. Thus, firm 1's and firm 2's profits can be written as

$$\Pi_1 = p_{ah} [D_{ah} + \alpha D_{ab}] + p_{aj} [D_{aj} + (1 - \alpha) D_{ab}] + q_{1,hj} P_{hj}$$

$$- [C_{ah} (D_{ah} + D_{ab} + D_{aj}) + C_{1,hj} (D_{ab} + D_{aj} + q_{1,hj})] \quad (4)$$

and

$$\begin{aligned} \Pi_2 = & p_{jb} [D_{jb} + (1 - \alpha) D_{ab}] + p_{hb} [D_{hb} + \alpha D_{ab}] + q_{2,hj} P_{hj} \\ & - [C_{jb} (D_{jb} + D_{ab} + D_{hb}) + C_{2,hj} (D_{hb} + q_{2,hj})], \quad (5) \end{aligned}$$

respectively. Then, an equilibrium is characterized by a situation where each firm chooses its prices or quantities to maximize its profit in (4) or (5) given the other firm's prices and quantities. Let superscript N indicate the equilibrium under independent pricing (N standing for no cooperation).

Note that profits in (4) and (5) are not smooth in prices because spoke-to-spoke passengers choose the route with the cheapest total fare. More specifically, if $D_{ab} > 0$ and $p_{ah} + p_{hb} = p_{aj} + p_{jb}$, $\alpha = 1/2$ and firm 1's profit is

$$\begin{aligned} \Pi_1 = & p_{ah} [D_{ah} + D_{ab}/2] + p_{aj} [D_{aj} + D_{ab}/2] + q_{1,hj} P_{hj} \\ & - (C_{ah} + C_{1,hj} (D_{ab}/2 + D_{aj} + q_{1,hj})). \quad (6) \end{aligned}$$

But, if p_{aj} is reduced by ε , α becomes 0 and firm 1's profit changes to

$$\begin{aligned} \Pi_1(p_{aj} - \varepsilon) = & p_{ah} D_{ah} + p_{aj} [D_{aj} + D_{ab}] + q_{1,hj} P_{hj} \\ & - (C_{ah} + C_{1,hj} (D_{ab} + D_{aj} + q_{1,hj})), \quad (7) \end{aligned}$$

which has two effects. First, revenues from spoke-to-spoke passengers change from $(p_{aj} + p_{ah}) D_{ab}/2$ to $p_{aj} D_{ab}$, and the difference is $(p_{aj} - p_{ah}) D_{ab}/2$. Second, firm 1's costs on the hub-to-hub part increase by

$$\phi_0 \equiv C_{1,hj}(D_{ab} + D_{aj} + q_{1,hj}) - C_{1,hj}(D_{ab}/2 + D_{aj} + q_{1,hj}). \quad (8)$$

So, the net profit gain achieved by reducing p_{aj} is $(p_{aj} - p_{ah}) D_{ab}/2 - \phi_0$. On the other hand, if p_{aj} increases by ε , α becomes 1 and firm 1's profit becomes

$$\begin{aligned} \Pi_1(p_{aj} + \varepsilon) = & p_{ah} [D_{ah} + D_{ab}] + p_{aj} D_{aj} + q_{1,hj} P_{hj} \\ & - (C_{ah} + C_{1,hj}(D_{aj} + q_{1,hj})). \end{aligned} \quad (9)$$

This change in p_{aj} has, again, two effects. First, revenues from spoke-to-spoke passengers change from $(p_{aj} + p_{ah}) D_{ab}/2$ to $p_{ah} D_{ab}$, so that the revenue changes by $(p_{ah} - p_{aj}) D_{ab}/2$. Second, firm 1's costs on the hub-to-hub part fall by

$$\phi_1 \equiv C_{1,hj}(D_{ab}/2 + D_{aj} + q_{1,hj}) - C_{1,hj}(D_{aj} + q_{1,hj}) \quad (10)$$

with

$$\phi_1 > \phi_0, \quad (11)$$

since $C''_{i,hj} < 0$ for all $i = 1, 2$. So, the net gain of increasing p_{aj} by ε is $(p_{ah} - p_{aj}) D_{ab}/2 + \phi_1$.

This leads to the following equilibrium result, which states that whenever spoke-to-spoke passengers are served under independent pricing, an asymmetric equilibrium arises. This result is independent of the existence of spoke-to-hub markets.⁸

⁸If the demand in all spoke-to-hub markets is zero (that is, $D_{ah} = D_{aj} = D_{jb} = D_{hb} = 0$), prices p_{ah}, p_{aj}, p_{jb} and p_{hb} are similar to subfares charged by firms 1 and 2 for carrying spoke-to-spoke passengers (where the sum of complementary subfares gives the

Lemma 1 *If $D_{ab}^N > 0$, the total ticket prices in the spoke-to-spoke market ab are unequal under independent pricing, that is $p_{ah}^N + p_{hb}^N \neq p_{aj}^N + p_{jb}^N$.*

Proof We proof this lemma by contradiction. Suppose the equilibrium prices are such that $p_{ah}^N + p_{hb}^N = p_{aj}^N + p_{jb}^N$. Then, no deviation in prices by either firm 1 or firm 2 is profitable. Consider firm 1. In equilibrium, $\Pi_1^N \geq \Pi_1(p_{aj}^N - \varepsilon) \Leftrightarrow (p_{aj}^N - p_{ah}^N) D_{ab}^N/2 - \phi_0 \leq 0$ and $\Pi_1^N \geq \Pi_1(p_{aj}^N + \varepsilon) \Leftrightarrow (p_{ah}^N - p_{aj}^N) D_{ab}^N/2 + \phi_1 \leq 0$, which leads to

$$\phi_0 \geq (p_{aj}^N - p_{ah}^N) \frac{D_{ab}^N}{2} \geq \phi_1 \quad (12)$$

and implies

$$\phi_0 \geq \phi_1. \quad (13)$$

This is a contradiction, since $\phi_0 < \phi_1$ by (11). Similarly, the same holds for firm 2. Therefore, $\alpha \neq 1/2$ in equilibrium, and $p_{ah}^N + p_{hb}^N \neq p_{aj}^N + p_{jb}^N$. ■

The intuition for this result is that whenever a firm is better off by serving half the number of spoke-to-spoke passengers on the hub-to-hub part, it can further increase profits by serving all spoke-to-spoke passengers on the hub-to-hub market. This is because costs exhibit economies of scale (i.e. $C_x'' < 0$). Or, by contrast, if firms are not better off by serving all spoke-to-spoke passengers on the hub-to-hub market, then they will not serve spoke-to-spoke passengers on this connection at all. However, if there are no costs, $C_{i,hj}(\cdot) = 0$ for all $i = 1, 2$, then firm 1 has no incentive to deviate if and only if $p_{ah} = p_{aj}$ because this would imply a change in revenues equal to

price for a single ticket charged to spoke-to-spoke passengers). Subfares are relevant when firms can bundle services and price discriminate between spoke-to-hub and spoke-to-spoke services. Firms may choose subfares independently or cooperatively. Brueckner (2001) and Brueckner and Proost (2010) analyze subfares in overlapping airline networks. The equilibrium result in Lemma 1 is thus relevant for their analysis.

$(p_{ah} - p_{aj}) D_{ab}/2 = 0$. Hence, $p_{ah}^N + p_{hb}^N = p_{aj}^N + p_{jb}^N$ implies $p_{ah}^N = p_{aj}^N$ and $p_{hb}^N = p_{jb}^N$ in this situation. Note that the literature on overlapping networks has thus far concentrated on symmetric solutions.⁹

3.2 First-order conditions and mark-ups

Assume, without loss of generality, that $\alpha = 0$. Firm 1's first-order conditions are

$$\frac{\partial}{\partial p_{ah}} \Pi_1 = (D_{ah} + p_{ah} D'_{ah}) - D'_{ah} C'_{ah} (D_{ah} + D_{ab} + D_{aj}) = 0, \quad (14)$$

$$\begin{aligned} \frac{\partial}{\partial p_{aj}} \Pi_1 &= (D_{ab} + p_{aj} D'_{ab}) + (D_{aj} + p_{aj} D'_{aj}) - \\ &[(D'_{ab} + D'_{aj}) \{C'_{ah} (D_{ah} + D_{ab} + D_{aj}) + C'_{1,hj} (D_{ab} + D_{aj} + q_{1,hj})\}] = 0, \end{aligned} \quad (15)$$

and

$$\frac{\partial}{\partial q_{1,hj}} \Pi_1 = (P_{hj} + q_{1,hj} P'_{hj}) - C'_{1,hj} (D_{ab} + D_{aj} + q_{1,hj}) = 0, \quad (16)$$

and firm 2's first-order conditions are

$$\begin{aligned} \frac{\partial}{\partial p_{jb}} \Pi_2 &= (D_{jb} + p_{jb} D'_{jb}) + (D_{ab} + p_{jb} D'_{ab}) \\ &- (D'_{jb} + D'_{ab}) C'_{jb} (D_{jb} + D_{ab} + D_{hb}) = 0, \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial}{\partial p_{hb}} \Pi_2 &= (D_{hb} + p_{hb} D'_{hb}) \\ &- [D'_{hb} \{C'_{jb} (D_{jb} + D_{ab} + D_{hb}) + C'_{2,hj} (D_{hb} + q_{2,hj})\}] = 0, \end{aligned} \quad (18)$$

and

$$\frac{\partial}{\partial q_{2,hj}} \Pi_2 = (P_{hj} + q_{2,hj} P'_{hj}) - C'_{2,hj} (D_{hb} + q_{2,hj}) = 0. \quad (19)$$

⁹For example, Brueckner (2001) and Brueckner and Proost (2010).

Observe that prices p_{aj} and p_{jb} are related to demand D_{ab} due to the first-order conditions in (15) and (17). This is because $\alpha = 0$, which implies that spoke-to-spoke passengers travel with firm 1 on the AJ part and they travel with firm 2 on the JB part. The first-order conditions that hold for $D_{ab} = 0$ can be obtained by substituting 0 for D_{ab} in the first-order conditions in (14)-(19). Note that the lack of spoke-to-spoke passengers in these expressions would affect revenues but also marginal costs.

Since mark-ups are an indicator for profits as well as for the gains in total surplus and consumer surplus that could (eventually) be reached under regulation, we concentrate on mark-ups in the following. Denote mark-ups (Lerner indices) by

$$\begin{aligned}\Omega_{ah} &= \frac{p_{ah} - C'_{ah}}{p_{ah}}, \quad \Omega_{hb} = \frac{p_{hb} - (C'_{jb} + C'_{2,hj})}{p_{hb}}, \quad \Omega_{aj} = \frac{p_{aj} - (C'_{ah} + C'_{1,hj})}{p_{aj}}, \\ \Omega_{jb} &= \frac{p_{jb} - C'_{jb}}{p_{jb}}, \quad \Omega_{i,hj} = \frac{P_{hj} - C'_{i,hj}}{P_{hj}}, \quad \Omega_{ab} = \frac{p_{aj} + p_{jb} - (C'_{ah} + C'_{1,hj} + C'_{jb})}{p_{aj} + p_{jb}}.\end{aligned}\tag{20}$$

Furthermore, let η_x denote the elasticity of demand D_x with respect to p_x and let $\eta_{x,z}$ denote the elasticity of the composed demand ($D_x + D_z$) with respect to p_x with

$$\eta_x = -D'_x \frac{p_x}{D_x} \quad \text{and} \quad \eta_{x,z} = -(D'_x + D'_z) \frac{p_x}{D_x + D_z}\tag{21}$$

where $\partial\eta_x/\partial p_x, \partial\eta_{x,z}/\partial p_x > 0$ due to $(D_x/D'_x)' < 0$. Moreover, note that $\eta_x < \eta_{x,z}$ implies $\eta_z > \eta_{z,x}$.

Then, the first-order conditions in (14)-(19) can be transformed into mark-up expressions

$$\begin{aligned}\Omega_{ah} &= \frac{1}{\eta_{ah}}, \quad \Omega_{hb} = \frac{1}{\eta_{hb}}, \quad \Omega_{aj} = \frac{1}{\eta_{aj,ab}}, \quad \Omega_{jb} = \frac{1}{\eta_{jb,ab}}, \\ \Omega_{ab} &= \frac{1}{\eta_{ab,aj}} + \frac{1}{\eta_{ab,jb}} \quad \text{and} \quad \Omega_{i,hj} = \frac{q_{i,hj}}{q_{hj}} \frac{1}{\eta_{hj}}.\end{aligned}\tag{22}$$

Thus, the mark-ups in spoke-to-hub markets ah and hb depend only on marginal costs and the elasticities of own demands η_{ah} or, respectively, η_{hb} . By contrast, mark-ups in spoke-to-hub markets aj and jb depend on marginal costs and the elasticity of composed demands $\eta_{aj,ab}$ or, respectively, $\eta_{jb,ab}$. Furthermore, Ω_{ab} depends on the elasticities of composed demand $\eta_{ab,aj}$ and $\eta_{ab,jb}$. Since $q_{i,hj}/q_{hj} < 1$, $\Omega_{i,hj}$ is reduced by competition in the hub-to-hub market. If $D_{ab} = 0$, mark-ups reduce to

$$\Omega_x = \frac{1}{\eta_x},\tag{23}$$

for all $x \in \{ah, aj, hb, jb\}$, while the structure of mark-ups in market hj remains unchanged, that is $\Omega_{i,hj} = q_{i,hj}/(q_{hj}\eta_{hj})$. Hence, the relationship between markets is only based on costs when $D_{ab} = 0$.

3.3 Further characterization of equilibria

Constellations can occur where not all markets are served (that is, $D_x^N = 0$ for some $x \in \{ah, aj, hb, jb, hj, ab\}$). For example, letting a_{ab} denote the maximum reservation price of spoke-to-spoke passengers with $\partial D_{ab}/\partial a_{ab} > 0$, $D_{ab} = 0$ if $p_{aj} + p_{jb} \geq a_{ab}$ and $p_{ah} + p_{hb} \geq a_{ab}$. In such a situation, profits are obtained by substituting 0 for D_{ab} into profits in (4) and (5). For the sake of simplicity, we assume that maximum reservation prices in markets ah, aj, jb

and hb are sufficiently large so that $D_{ah}^N, D_{aj}^N, D_{hb}^N, D_{jb}^N, D_{hj}^N > 0$ always holds (that is, only the demand of spoke-to-spoke passengers may become zero in equilibrium).

To further characterize equilibria, we derive the critical conditions that have to be satisfied such that $D_{ab} > 0$ or $D_{ab} = 0$ in equilibrium. We also derive critical conditions that have to be satisfied such that $\alpha = 0$ in equilibrium. More specifically, in what follows, we derive three threshold values with respect to a_{ab} . The first, which we denote by \bar{a}_{ab} , provides a threshold value for the firms' incentives to deviate such that the outcome changes from a situation where $D_{ab} > 0$ towards a situation where $D_{ab} = 0$. The second, which we denote by \hat{a}_{ab} , provides a threshold value for the firms' incentives to deviate such that the outcome changes from a situation where $D_{ab} = 0$ towards a situation where $D_{ab} > 0$. The third, which we denote by \tilde{a}_{ab} , provides a threshold value for the firms' incentives to deviate from a situation where $\alpha = 0$ towards a situation where $\alpha = 1$.

Suppose that $p_{aj}^N + p_{jb}^N \leq a_{ab}$ (i.e. $D_{ab}^N > 0$). If firm 1 deviates and chooses $(\bar{p}_{ah}, \bar{p}_{aj}, \bar{q}_{1,hj})$ with $\bar{p}_{ah} > a_{ab} - p_{hb}^N$ and $\bar{p}_{aj} > a_{ab} - p_{jb}^N$ (i.e. D_{ab} becomes zero) that satisfy the first-order conditions in (14)-(16), we denote the profit of firm 1 under deviation by $\bar{\Pi}_1$.¹⁰ If firm 2 deviates and chooses $(\bar{p}_{hb}, \bar{p}_{jb}, \bar{q}_{2,hj})$ with $\bar{p}_{hb} > a_{ab} - p_{ah}^N$ and $\bar{p}_{jb} > a_{ab} - p_{aj}^N$ that satisfy the first-order conditions in (17)-(19) for $D_{ab} = 0$, we denote the profit of firm 2 under deviation by $\bar{\Pi}_2$. Note that the revenue gains from serving spoke-to-spoke passengers becomes small for low values a_{ab} , while the loss of revenues in spoke-to-hub markets

¹⁰ $\bar{\Pi}_1$ can be obtained by substituting $(\bar{p}_{ah}, \bar{p}_{aj}, \bar{q}_{1,hj})$ for $(p_{ah}, p_{aj}, q_{1,hj})$, $(p_{hb}^N, p_{jb}^N, q_{2,hj}^N)$ for $(p_{hb}, p_{jb}, q_{2,hj})$ and 0 for D_{ab} in (4).

aj and jb becomes great, since $D_x^N > 0$ always holds for all ah, aj, hb, jb by assumption. For this reason, there is a unique lower limit of a_{ab} ,

$$\bar{a}_{ab} = \min\{a_{ab} : \Pi_1^N \geq \bar{\Pi}_1 \text{ and } \Pi_2^N \geq \bar{\Pi}_2\}, \quad (24)$$

where firms will deviate. Then, $D_{ab} > 0$ does not hold in equilibrium if $a_{ab} < \bar{a}_{ab}$ holds true.

Suppose now that $p_{ah}^N + p_{hb}^N > a_{ab}$ and $p_{aj}^N + p_{jb}^N > a_{ab}$ (i.e. $D_{ab}^N = 0$). If firm 1 chooses $(\hat{p}_{ah}, \hat{p}_{aj}, \hat{q}_{1,hj})$ with $\hat{p}_{aj} < a_{ab} - p_{jb}^N$ (i.e. D_{ab} becomes positive) that satisfy the first-order conditions in (14)-(16), we denote the profit of firm 1 under deviation by $\hat{\Pi}_1$. If firm 2 deviates and chooses prices $(\hat{p}_{hb}, \hat{p}_{jb}, \hat{q}_{2,hj})$ with $\hat{p}_{jb} < a_{ab} - p_{aj}^N$ that satisfy the first-order conditions in (17)-(19), we denote the profit of firm 2 by $\hat{\Pi}_2$. There is a unique upper limit of a_{ab} ,

$$\hat{a}_{ab} = \max\{a_{ab} : \Pi_1^N \geq \hat{\Pi}_1 \text{ and } \Pi_2^N \geq \hat{\Pi}_2\}, \quad (25)$$

where firms deviate. Then, $D_{ab} = 0$ does not hold in equilibrium if $a_{ab} > \hat{a}_{ab}$ holds true.

Finally, suppose that $D_{ab}^N > 0$ and that $p_{aj}^N + p_{jb}^N < p_{ah}^N + p_{hb}^N$ (that is, $\alpha = 0$). If firm 1 deviates and chooses $(\tilde{p}_{ah}, \tilde{p}_{aj}, \tilde{q}_{1,hj})$ with $\tilde{p}_{aj} + p_{jb}^N > \tilde{p}_{ah} + p_{hb}^N$ (that is, $\alpha = 1$) that satisfy the first-order conditions in (17)-(19) where indices $(2, hb, jb)$ are substituted by indices $(1, ah, aj)$, we denote the profit of firm 1 under deviation by $\tilde{\Pi}_1$.¹¹ Observe that firm 1 can save operating costs if α changes to 1 because, on the HJ part, spoke-to-spoke passengers are all carried by firm 2 in this case and that these costs are increasing in

¹¹ $\tilde{\Pi}_1$ can be obtained by first substituting $(2, hb, jb)$ for $(1, ah, aj)$ in (5) and thereafter substituting $(\tilde{p}_{ah}, \tilde{p}_{aj}, \tilde{q}_{1,hj})$ for $(p_{ah}, p_{aj}, q_{1,hj})$ and $(p_{hb}^N, p_{jb}^N, q_{2,hj}^N)$ for $(p_{hb}, p_{jb}, q_{2,hj})$.

the demand of spoke-to-spoke passengers. For this reason, there is a unique upper limit of a_{ab} ,

$$\tilde{a}_{ab} = \max\{a_{ab} : \Pi_1 \geq \tilde{\Pi}_1\}, \quad (26)$$

where firm 1 will just deviate. Then, $\alpha = 0$ cannot hold in equilibrium if $a_{ab} > \tilde{a}_{ab}$. Altogether, these three threshold values for a_{ab} imply that, under independent pricing, equilibria can be unique with $D_{ab}^N = 0$ or $D_{ab}^N > 0$, or two equilibria can exist in parallel with $D_{ab}^N = 0$ or $D_{ab}^N > 0$, which depends on the size of spoke-to-spoke passenger demand determined by a_{ab} . More specifically:

Lemma 2 *Under independent pricing, it holds:*

- (i) *If $a_{ab} < \min\{\bar{a}_{ab}, \hat{a}_{ab}\}$, a unique equilibrium exists where $D_{ab}^N = 0$.*
- (ii) *If $a_{ab} \in [\bar{a}_{ab}, \hat{a}_{ab}]$ with $\hat{a}_{ab} \leq \tilde{a}_{ab}$, two equilibria exist where either $D_{ab}^N = 0$ or $D_{ab}^N > 0$.*
- (iii) *If $a_{ab} \in (\max\{\bar{a}_{ab}, \hat{a}_{ab}\}, \tilde{a}_{ab}]$, a unique equilibrium exists where $D_{ab}^N > 0$.*

Proof See Appendix A. ■

Note that an equilibrium in pure pricing and quantity strategies may not exist at all, which can occur if $a_{ab} > \tilde{a}_{ab}$.

Figure 2 illustrates the areas with a unique equilibrium or multiple equilibria in the a_{hj} - a_{ab} -space.¹² The values are based on the specifications presented later in Subsection 5.2 where demands in markets ah , aj , hb and jb are symmetric.¹³ Since markets are considered as symmetric in this instance, an equilibrium in pure pricing and quantity strategies does not exist if $a_{ab} > \tilde{a}_{ab}$.

¹²The investigation of mixed strategies is beyond the scope of this paper, however.

¹³Specifications are $C_x = Q_x(1/2 - Q_x/50)$ for all $x \in \{ah; 1, hj; 2, hj; hj; jb\}$ and $P_y = a_y - q_y$ for all $y \in \{ah, aj, hb, jb, hj, ab\}$ with $a_{ah} = a_{aj} = a_{hb} = a_{jb} = 4$.

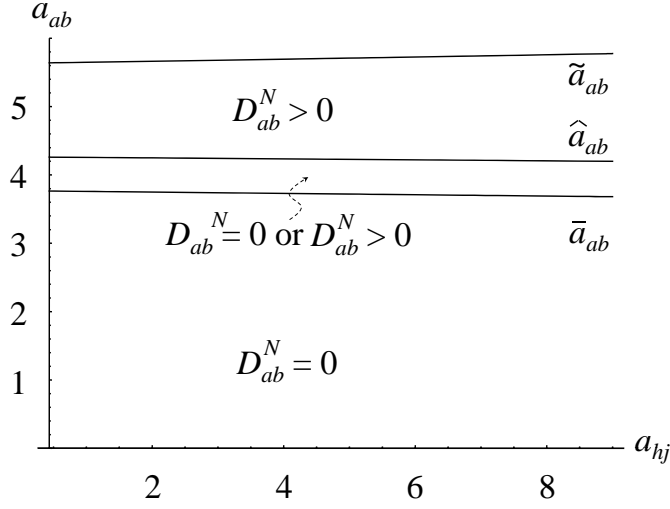


Figure 2: Areas with a unique equilibrium or two equilibria in the a_{hj} - a_{ab} -space. The values are based on the specifications presented in Subsection 5.2 (also see Footnote 12 and Footnote 17).

4 Cooperative Pricing

If firms 1 and 2 cooperate, they collude in the hj market. Furthermore, they can bundle services, offer the spoke-to-spoke connection as one service and charge a single price p_{ab} with $p_{ab} < p_{aj} + p_{jb}$ to spoke-to-spoke passengers.¹⁴ A further effect of cooperation is that firms can together optimize operations and reduce costs on the hub-to-hub part. Letting superscript C indicate the cooperation case, the total profit under cooperation can be written as

$$\Pi^C = \sum_{x \in \{ah, hj, jb, aj, hb, ab\}} p_x D_x - \sum_{y \in \{ah, hj, jb\}} C_y. \quad (27)$$

¹⁴The bundling of services can provide additional benefits to spoke-to-spoke passengers. For example, cooperation can lead to more convenient scheduling, improved luggage handling, and greater coordination in the case of congestion. Therefore, the passengers' maximum reservation price can be greater in the case of cooperative pricing (Carlton et al., 1980). For simplicity, we abstract away from additional benefits that can be created by coordination.

This leads to the first-order conditions

$$\frac{\partial}{\partial p_{ah}} \Pi^C = D_{ah} + p_{ah} D'_{ah} - D'_{ah} C'_{ah} = 0, \quad (28)$$

$$\frac{\partial}{\partial p_{aj}} \Pi^C = D_{aj} + p_{aj} D'_{aj} - D'_{aj} (C'_{ah} + C'_{hj}) = 0, \quad (29)$$

$$\frac{\partial}{\partial q_{hj}} \Pi^C = P_{hj} + q_{hj} P'_{hj} - C'_{hj} = 0, \quad (30)$$

$$\frac{\partial}{\partial p_{jb}} \Pi^C = D_{jb} + p_{jb} D'_{jb} - D'_{jb} C'_{jb} = 0, \quad (31)$$

$$\frac{\partial}{\partial p_{hb}} \Pi^C = D_{hb} + p_{hb} D'_{hb} - D'_{hb} (C'_{jb} + C'_{hj}) = 0 \quad \text{and} \quad (32)$$

$$\frac{\partial}{\partial p_{ab}} \Pi^C = D_{ab} + p_{ab} D'_{ab} - D'_{ab} (C'_{ah} + C'_{hj} + C'_{jb}) = 0. \quad (33)$$

Based on these first-order conditions, we can calculate the mark-ups that exist when firms cooperate. Letting Ω_{hj} denote $(P_{hj} - C'_{hj}) / P_{hj}$ these are given by

$$\Omega_x^C = \frac{1}{\eta_x} \quad (34)$$

for all $x \in \{ah, aj, hj, hb, jb, ab\}$. Thus, prices depend on marginal costs and the elasticity of own demand only (that is, the elasticities of composed demands are not relevant anymore). As a consequence, under cooperative pricing, the number of spoke-to-spoke passengers is strictly positive if $a_{ab} > C'_{ah}(Q_{ah}) + C'_{hj}(Q_{hj}) + C'_{jb}(Q_{jb})$ is satisfied. We concentrate on this case by assuming that $a_{ab} > C'_{ah}(0) + C'_{hj}(0) + C'_{jb}(0)$. Thus, $D_{ab}^C > 0$ is always true in our setting.

We now further elaborate on the relevance of spoke-to-hub markets aj and jb for the ticket prices paid by spoke-to-spoke passengers. Suppose that $\eta_{ab,aj}^N = \eta_{ab,jb}^N = \eta_{ab}^N$ holds true. In this situation, the mark-up paid

by spoke-to-spoke passengers in the case of independent pricing is equal to $2/\eta_{ab}^N$, while it equals $1/\eta_{ab}^C$ in the case of cooperative pricing, which follows from (22) and (34).¹⁵ Thus, cooperation can exhibit downward pressure on prices; the reason is that double marginalization can be avoided (Brueckner, 2001). If, however, $\eta_{aj,ab}^N \gtrless \eta_{jb,ab}^N \gtrless \eta_{ab}^N$ the relationship between mark-ups under independent pricing and under cooperative pricing is not clear-cut. We say that double marginalization is amplified if $\Omega_{ab}^N \geq 2/\eta_{ab}^N$, that double marginalization is limited if $\Omega_{ab}^N \in (1/\eta_{ab}^N, 2/\eta_{ab}^N)$, and that double marginalization is eliminated if $\Omega_{ab}^N \leq 1/\eta_{ab}^N$. We can now establish the following relationships about mark-ups in the spoke-to-spoke market.¹⁶

Proposition 1 *If $a_{ab} \in [\bar{a}_{ab}, \tilde{a}_{ab}]$ and $D_{ab}^N > 0$, double marginalization is amplified when*

$$\frac{1}{\eta_{ab,aj}} + \frac{1}{\eta_{ab,jb}} \geq \frac{2}{\eta_{ab}}, \quad (35)$$

limited when

$$\frac{1}{\eta_{ab,aj}} + \frac{1}{\eta_{ab,jb}} \in \left[\frac{1}{\eta_{ab}}, \frac{2}{\eta_{ab}} \right), \quad (36)$$

or eliminated otherwise.

Proof This follows from the mark up conditions in (22). ■

The effect of cooperative pricing on mark-ups in markets ah, aj, jb, hb and hj is also unclear and depends on the values of Q_x^C , which determine marginal costs. Recall that greater values of Q_x reduce marginal costs C'_x due to $C''_x < 0$. Therefore, greater passenger numbers are associated with a lower

¹⁵Observe that the mark-up paid by spoke-to-spoke passengers in the case of independent pricing would also be equal to $2/\eta_{ab}^N$ if $D_{aj} = D_{jb} = 0$, which is the standard case.

¹⁶The relationship between spoke-to-hub markets and spoke-to-spoke markets and double marginalization has also been considered by Czerny (2009) for the linear case of complementary and non-overlapping airline networks.

price, lower elasticities η_x (this is because $(D_x/D'_x)' < 0$), and greater mark-ups Ω_x . The relationship between cooperative pricing and mark-ups is more complicated in the hub-to-hub market hj because of collusion. Comparing mark-ups $\Omega_{i,hj}$ in (22) or (23) with the mark-up Ω_{hj}^C in (34) shows that collusion increases the mark-up there because $1 > q_{i,hj}/q_{hj}$. However, since prices also depend on marginal costs, which can be lower under cooperative pricing, the overall effect of cooperative pricing on the hub-to-hub market is difficult to predict. The ambiguous effect of cooperation on prices in the hub-to-hub market has previously been pointed out by Brueckner (2001) and by Brueckner and Proost (2010). Altogether, the effect of cooperative pricing on mark-ups, prices and hence quantities is hard to predict, which is true for all markets.

5 Social Evaluation of Cooperative Pricing

5.1 General considerations

In our setting with two firms, a move from independent pricing towards cooperative pricing alters prices and cost structures, and colluding firms are always better off under cooperative pricing. This is because they can price discriminate between spoke-to-spoke and spoke-to-hub passengers, set prices collusively on the hub-to-hub part, and jointly minimize costs on the hub-to-hub part.¹⁷ Turning to the social viewpoint, consumer surplus is

$$CS = \sum_y \left(\int_{p_y}^{\infty} D_y(z) dz \right), \quad (37)$$

¹⁷Note that the number of firms that exist in the market is crucial for this result. For example, Salant et al. (1983) found that collusion between two firms is unprofitable in a standard Cournot oligopoly with constant marginal costs when more than two firms exist.

and, the regulator's objective function is

$$W = \begin{cases} CS + \Psi (\Pi_1 + \Pi_2) & \text{under independent pricing} \\ CS + \Psi \Pi^C & \text{under cooperative pricing} \end{cases} \quad (38)$$

with $\Psi \in \{0, 1\}$. If $\Psi = 0$, the regulator concentrates on consumer surplus, while she concentrates on total welfare if $\Psi = 1$. Observe that the objective depends on the pricing regime, since costs on the hub-to-hub depend on whether firms cooperate or not. This leads to the following result.

Proposition 2 *The social benefits of cooperative pricing are smaller when the regulator concentrates only on consumer surplus (i.e. $\Psi = 0$) as compared to a regulator who concentrates on total surplus (i.e. $\Psi = 1$), that is*

$$(W^C - W^N) |_{\Psi=0} \leq (W^C - W^N) |_{\Psi=1}. \quad (39)$$

Proof $W^C = CS^C + \Psi \Pi^C$ and $W^N = CS^N + \Psi (\Pi_1^N + \Pi_2^N)$ leads to

$$W^C - W^N = CS^C - CS^N + \Psi [\Pi^C - (\Pi_1^N + \Pi_2^N)]. \quad (40)$$

Note that the last term on the right-hand side is always positive. Thus the benefits of cooperative pricing are always greater if the regulator concentrates on total surplus (that is, $\Psi = 1$), rather than on consumer surplus (that is, $\Psi = 0$). ■

This result is consistent with the numerical findings of Brueckner (2001). A direct consequence of Proposition 2 is that regulators should be more dismissive with respect to cooperative pricing if they are concerned about consumer surplus in the first place. The question is, however, under which conditions will cooperative pricing be beneficial from the social perspective.

Proposition 3 *A move from independent pricing towards cooperative pricing increases the value of the regulator's objective function, that is $W^C > W^N$ for all $\Psi \in \{0, 1\}$, if $Q_{ah}^C \geq Q_{ah}^N$, $Q_{jb}^C \geq Q_{jb}^N$, and $(C_{hj}^C)' \leq P_{hj}^N (1 - 1/\eta_{hj}^N)$, and if in addition*

$$(i) \ a_{ab} < \min \{\bar{a}_{ab}, \hat{a}_{ab}\}, \text{ or}$$

$$(ii) \ a_{ab} \geq \min \{\bar{a}_{ab}, \hat{a}_{ab}\}, \ \eta_{x,ab} \leq \eta_x \text{ for all } x \in \{aj, jb\} \text{ and } 1/\eta_{ab,aj} + 1/\eta_{ab,jb} \geq 1/\eta_{ab}.$$

Otherwise, the relative benefits of cooperative pricing are unclear from the social viewpoint.

Proof See Appendix B. ■

The next section illustrates outcomes under independent pricing and cooperative pricing from the social viewpoint.

5.2 Numerical Illustration

In this subsection, we look at a more structured model to identify parameter ranges in the a_{hj} - a_{ab} -space where the social evaluation of cooperation depends on the weight regulators attach to firms' profits (that is, whether $\Psi = 0$ or $\Psi = 1$). Assume that

$$C_y = Q_y \left(\frac{1}{2} - \theta Q_y \right) \quad (41)$$

for all $y \in \{ah; 1, hj; 2, hj; hj; jb\}$ and that demands follow a linear relationship given by

$$P_x = a_x - q_x \quad (42)$$

for all $x \in \{ah, aj, hb, jb, hj, ab\}$ with $a_{ah} = a_{aj} = a_{hb} = a_{jb} = 4$. Figure 2 is based on these specification with $\theta = 1/50$; hence, in this instance, parameter constellations exist where two equilibria exist (this is, when $a_{ab} \in [\bar{a}_{ab}, \hat{a}_{ab}]$).

In the case of two equilibria, one of these equilibria is Pareto-dominant if both firms strictly prefer this equilibrium over the other equilibrium. Thus, if $\Pi_i^N|_{D_{ab}=0} > \Pi_i^N|_{D_{ab}>0}$ for all $i = 1, 2$, $D_{ab}^N = 0$ in the Pareto-dominant equilibrium. By contrast, if $\Pi_i^N|_{D_{ab}=0} < \Pi_i^N|_{D_{ab}>0}$ for all $i = 1, 2$, $D_{ab}^N > 0$ in the Pareto-dominant equilibrium. Denote the level of a_{ab} where $D_{ab}^N > 0$ just holds true in the Pareto-dominant equilibrium by a_{ab}^p , that is

$$a_{ab}^p = \min \{a_{ab} : \Pi_1^N|_{D_{ab}>0} \geq \Pi_1^N|_{D_{ab}=0} \text{ and } \Pi_2^N|_{D_{ab}>0} \geq \Pi_2^N|_{D_{ab}=0}\}. \quad (43)$$

Recall that $a_{ab} \geq \bar{a}_{ab}$ must hold such that $D_{ab}^N > 0$; hence, a_{ab}^p is not defined if $a_{ab} < \bar{a}_{ab}$. Now, in the case of two equilibria, we concentrate on the Pareto-dominant equilibrium. Thus, if $a_{ab} > a_{ab}^p$, we take $D_{ab}^N > 0$ as given and vice-versa.

The regulator prefers cooperative pricing over independent pricing if $W^C > W^N$. Denote the critical level of a_{ab} where the regulator is indifferent with respect to the pricing regime when she concentrates on consumer surplus only by a_{ab}^0 . Furthermore, denote the critical level of a_{ab} where the regulator is indifferent between pricing regimes when she concentrates on total welfare by a_{ab}^1 . The superscripts hence indicate whether $\Psi = 0$ or $\Psi = 1$, and

$$a_{ab}^0 = \{a_{ab} : W^C|_{\Psi=0} = W^N|_{\Psi=0}\} \text{ and } a_{ab}^1 = \{a_{ab} : W^C|_{\Psi=1} = W^N|_{\Psi=1}\}. \quad (44)$$

Figure 3 shows the areas in the a_{hj} - a_{ab} -space where depending on Ψ cooperative pricing increases the regulator's objective function. Recall that $a_{ab} \leq \tilde{a}_{ab}$ ensures that under independent pricing firms do not deviate from a situation where $\alpha = 0$ towards prices and quantities that imply $\alpha = 1$. The figure depicts a_{ab}^0 and a_{ab}^1 depending on a_{hj} . Observe that a_{ab}^0 and a_{ab}^1 are not represented by continuous functions and that they depend on whether

$a_{ab} > a_{ab}^p$ or $a_{ab} < a_{ab}^p$. Furthermore, a_{ab} must exceed the dashed horizontal line to ensure that $D_{ab}^C > 0$ under cooperative pricing.¹⁸

Consider area ABCDE where $a_{ab} \in (a_{ab}^p, \tilde{a}_{ab})$. To the left of this area (i.e. for great values of a_{ab} and low values of a_{hj}), cooperative pricing is always preferred over independent pricing (that is, the social evaluation of cooperative pricing is independent of Ψ). This is because, in this area, double marginalization can be avoided by cooperative pricing, the detrimental effect of collusion in the hub-to-hub market is limited by the small size of the market, and because firms can take full advantage of economies of scale. Altogether, firms and passengers benefit from cooperative pricing in these parameter ranges. By contrast, to the right of area ABCDE (i.e. for great values of both a_{ab} and a_{hj}), independent pricing is always preferred over cooperative pricing. That is, regardless of Ψ the regulator's objective is always higher under independent pricing than under cooperative pricing. This is driven by the large size of the hub-to-hub market such that the detrimental effect of collusive pricing on consumers dominates. Now, consider the area within ABCDE. The approval of cooperative pricing depends on Ψ . In this area the total welfare is greater under cooperative pricing, while consumer surplus is greater under independent pricing; hence, the relative social value of independent pricing or cooperative pricing depends on the social weight attached to firms' profits, Ψ , in this area of the a_{hj} - a_{ab} -space.

Observe that $A < B$ in Figure 3; thus, even if the hub-to-hub passenger demand is close to zero (i.e. $a_{hj} \rightarrow 0$) and therefore the detrimental effect of collusion on consumers is limited, a move from independent pricing towards cooperative pricing can lead to total welfare losses. The reason is that spoke-to-spoke passengers exert downward pressure on prices in spoke-to-hub

¹⁸Note that, in Figure 3, we do not start with $a_{hj} = 0$ to ensure that $D_{hj} > 0$ always holds.

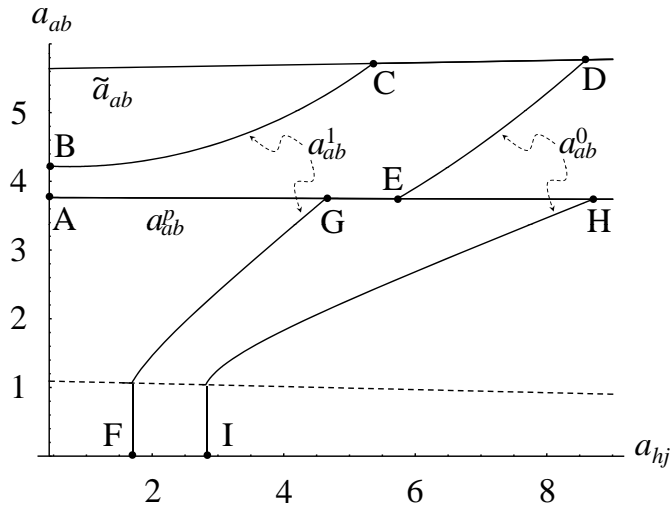


Figure 3: Areas ABCDE and FGHI determine the areas where a conflict between the interests of firms and passengers exist. Parameters are $a_{ah} = a_{aj} = a_{hb} = a_{jb} = 4$ and $\theta = 1/50$.

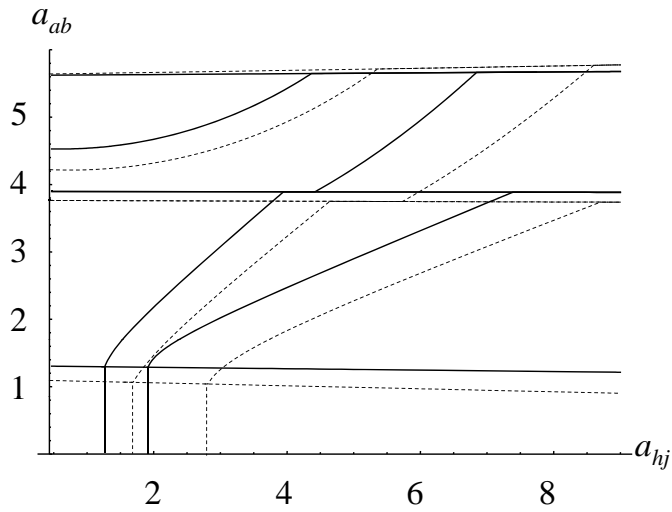


Figure 4: The effect of θ on the areas where a conflict between the interests of firms and passengers exist. Parameters are $a_{ah} = a_{aj} = a_{hb} = a_{jb} = 4$ and $\theta = 1/50$ (dashed lines) or $\theta = 1/100$ (solid lines).

markets aj and jb if $\eta_{aj} < \eta_{aj,ab}$ and $\eta_{jb} < \eta_{jb,ab}$. Then, a move towards cooperative pricing and price discrimination raises prices in spoke-to-hub markets aj and jb , which reduces total welfare.¹⁹ This reveals the importance of considering a complete coverage of the different markets that can exist in a given network. Indeed, otherwise the social benefits of cooperative pricing can be overestimated or underestimated. For example, Armantier and Richard (2006, 2008) found that domestic airline code-share agreements increased the average price paid by non-stop passengers in the US. Therefore, an empirical evaluation of code-share agreements that concentrates on inter-line passengers would overestimate the social benefits of cooperation in this situation.

Now, consider the area FGHI where $a_{ab} < a_{ab}^p$, which implies $D_{ab} = 0$ under independent pricing. Following the same arguments provided earlier, to the left of FGHI cooperative pricing is preferred for any Ψ , while independent pricing is preferred to the right of FGHI for any Ψ , and the approval of cooperative pricing within the area of FGHI depends on the regulator's value of Ψ .

Finally, Figure 4 illustrates the role of economies of scale on the social evaluation of cooperative pricing relative to independent pricing by varying the value of θ . This figure reproduces Figure 3 where $\theta = 1/50$ (dashed lines) and replicates the same graphs for $\theta = 1/100$ (solid lines). The reduction of θ from $1/50$ towards $1/100$ implies that the downward pressure of passenger numbers on marginal costs is reduced or economies of density are of less importance. Observe that this change in the value of θ shifts the graphs upwards and to the left. This suggests that if economies of scale are limited, cooperative pricing is less likely to be preferred from the social viewpoint.

¹⁹This is consistent with the theoretical findings of Czerny (2009).

6 Conclusions

This paper considered a simple network model with two hub and two spoke cities giving rise to six markets (four spoke-to-hub, one hub-to-hub and one spoke-to-spoke) operated by two firms that offer a parallel service on the hub-to-hub part but monopoly services on all spoke-to-hub connections. Transport costs exhibit increasing returns to scale. Based on this setting, we investigated the social benefits of independent pricing behavior relative to the benefits of cooperative pricing behavior (that is, pricing behavior under a merger).

We found that a symmetric equilibrium where each firm carries half of the total number of spoke-to-spoke passengers on the hub-to-hub part cannot occur. We also found that it is important to consider a complete coverage of markets (i.e. all spoke-to-hub, hub-to-hub and spoke-to-spoke markets), otherwise the estimation of the social benefits of cooperative pricing can be biased. For example, ignoring spoke-to-hub markets can lead to erroneous results, since the effect of double marginalization in spoke-to-spoke markets depends on the demand elasticities in spoke-to-hub markets. Moreover, we found that, in many cases, the approval of cooperative pricing depends on the weight attached to profits and that a regulator who concentrates on consumer surplus is more dismissive with respect to cooperative pricing than a regulator who concentrates on total welfare.

There are however limitations to our analysis that should be addressed by future research to provide more exact predictions of the social benefits of cooperative pricing behavior. First, we considered simple and given networks, while in practice networks are more complex, include a great number of spoke cities and hub cities, and they are endogenously formed. Second, multiple service providers could compete on all the different parts of the network, while

we only considered competition between two firms on the hub-to-hub part. Since competition may resolve the problem of excessive pricing and double marginalization in some cases, the benefits of cooperative pricing could be overestimated in this paper. By contrast to this latter point, there also is a tendency to underestimate the social benefits arising from cooperation in our analysis as we have abstracted away from the benefits that can be generated by the bundling of services (for example, more convenient scheduling and improved luggage handling). Overall, to obtain a more realistic picture it would be useful to consider a more general network setting with market entry, multiple firms on the different parts of the whole network, and a more general demand of spoke-to-spoke passengers that depends on the pricing regime.

A Proof of Lemma 2

To establish part (i), suppose that $D_{ab}^N = 0$. In this situation, prices and quantities are chosen to satisfy the first-order conditions in (14)-(19) for $D_{ab} = 0$ given. Since $a_{ab} < \min\{\bar{a}_{ab}, \hat{a}_{ab}\}$ implies $a_{ab} < \hat{a}_{ab}$, firms have no incentive to deviate towards a situation with $D_{ab} > 0$. Hence, $D_{ab} = 0$ in equilibrium. Now, suppose that $D_{ab} > 0$. In this situation, prices and quantities are chosen to satisfy the first-order conditions in (14)-(19) for $D_{ab} > 0$ given. Since $a_{ab} < \min\{\bar{a}_{ab}, \hat{a}_{ab}\}$ implies $a_{ab} < \bar{a}_{ab}$, firms have an incentive to deviate towards a situation with $D_{ab} = 0$. Hence, $D_{ab} > 0$ cannot hold in equilibrium, and therefore it exists a unique equilibrium with $D_{ab} = 0$.

To establish part (ii), note that $a_{ab} \in [\bar{a}_{ab}, \hat{a}_{ab}]$ implies $\bar{a}_{ab} \leq \hat{a}_{ab}$. Starting from a situation where $D_{ab} = 0$ and the first-order conditions in (14)-(19) are

satisfied, firms have no incentive to deviate towards a situation with $D_{ab} > 0$ if $a_{ab} \leq \hat{a}_{ab}$. On the other hand, starting from a situation where $D_{ab} > 0$ and the first-order conditions in (14)-(19) are satisfied, firms have no incentive to deviate towards a situation with $D_{ab} = 0$ if $a_{ab} \geq \bar{a}_{ab}$. Moreover, firms have no incentive to deviate towards a situation with $\alpha = 1$ because $\hat{a}_{ab} \leq \tilde{a}_{ab}$. Thus, under the conditions specified in part (ii), two equilibria exist where either $D_{ab}^N = 0$ or $D_{ab}^N > 0$.

To establish part (iii), suppose that $D_{ab}^N = 0$. In this situation, prices and quantities are chosen to satisfy the first-order conditions in (14)-(19) for $D_{ab} = 0$. Since $a_{ab} > \max\{\bar{a}_{ab}, \hat{a}_{ab}\}$ implies $a_{ab} > \hat{a}_{ab}$, firms have an incentive to deviate towards a situation with $D_{ab} > 0$, which is a contradiction. Now, suppose that $D_{ab}^N > 0$. In this situation, prices and quantities are chosen to satisfy the first-order conditions in (14)-(19) for $D_{ab} > 0$ given. Since $a_{ab} > \max\{\bar{a}_{ab}, \hat{a}_{ab}\}$ implies $a_{ab} > \bar{a}_{ab}$, firms have no incentive to deviate towards a situation with $D_{ab} = 0$. Moreover, firms have no incentive to deviate towards a situation with $\alpha = 1$ if $a_{ab} \leq \tilde{a}_{ab}$. Thus, under the conditions specified in part (iii), a unique equilibrium exists with $D_{ab}^N > 0$.

B Proof of Proposition 3

Note that $W^C > W^N$ for $\Psi = 0$ (i.e. $CS^C > CS^N$) directly implies that $W^C > W^N$ for $\Psi = 1$. In a first step, we therefore show that consumer surplus increases if the conditions mentioned in parts (i) and (ii) are satisfied. In a second step, we show that the effect of cooperative pricing on consumer surplus is unclear if only one of these conditions is not satisfied.

To establish part (i), we demonstrate that passengers in all markets benefit from cooperative pricing under the specified conditions. Note that

$a_{ab} < \min \{\bar{a}_{ab}, \hat{a}_{ab}\}$ implies $D_{ab}^N = 0$ due to Lemma 2. By contrast, $D_{ab}^C > 0$ is always true by assumption and therefore a move towards cooperative pricing is always to the benefit of spoke-to-spoke passengers in this situation. Furthermore, note that $(C_{hj}^C)' \leq P_{hj}^N (1 - 1/\eta_{hj}^N)$ implies $(C_{hj}^C)' < (C_{i,hj}^N)'$ for all $i = 1, 2$. To see this rearrange $\Omega_{i,hj}$ in (23), which gives

$$(C_{i,hj}^N)' = P_{hj}^N \left(1 - \frac{q_{i,hj}^N}{q_{hj}^N} \frac{1}{\eta_{hj}^N} \right). \quad (45)$$

Hence, the conditions in part (i) imply lower marginal costs of spoke-to-hub passengers (markets ah, aj, hb and jb) under cooperative pricing due to $C_x'' < 0$. Moreover, reductions in marginal costs are associated with reductions in prices due to $((D_x/D_x)') < 0$, which increases consumer surplus in all spoke-to-hub markets. Turning to the hub-to-hub market hj , note that $(C_{hj}^C)' \leq P_{hj}^N (1 - 1/\eta_{hj}^N)$ implies $P_{hj}^C \leq P_{hj}^N$; hence, passengers in this market are also better off by cooperative pricing when the conditions in part (i) are satisfied.

Turning to part (ii), note that the line of reasoning that we used to establish part (i) can be used to show that passengers in spoke-to-hub markets ah, hb and hj are better off under the conditions specified in (ii). This is because prices in these markets are not directly related to the demand elasticity of spoke-to-spoke passengers. By contrast, pricing in spoke-to-hub markets aj and jb and the spoke-to-spoke market ab are interrelated by demand elasticities and, since $\eta_{x,ab} \leq \eta_x$ for all $x \in \{aj, jb\}$ passengers in spoke-to-hub markets aj and jb will benefit from a move towards cooperative pricing because this reduces mark-ups and prices. The same is true for spoke-to-spoke passengers. This is because $\eta_{x,ab} \leq \eta_x$ for all $x \in \{aj, jb\}$ implies $1/\eta_{ab,aj} + 1/\eta_{ab,jb} \leq 2/\eta_{ab}$ (because $\eta_x > \eta_{x,ab}$ implies $\eta_{ab} < \eta_{ab,x}$). Hence, double marginalization is limited under the conditions in (ii) but not

eliminated, since $1/\eta_{ab,aj} + 1/\eta_{ab,jb} \geq 1/\eta_{ab}$. Therefore a move towards cooperative pricing would completely eliminate double marginalization to the benefit of spoke-to-spoke passengers. Moreover, passengers in markets aj , jb and ab also benefit from lower prices due to lower marginal costs.

Note that an increase of marginal costs for one group of passengers would increase the price in their market or that only a small reduction of marginal costs in the hub-to-hub market would not be sufficient to prevent greater prices by collusion. The overall increase of consumer surplus would therefore be unclear in these situations. Furthermore, note that $\eta_{x,ab} > \eta_x$ with $x \in \{aj, jb\}$ may imply that passengers in spoke-to-hub markets aj and jb will pay a greater price under cooperative pricing. Finally, if $1/\eta_{ab,aj} + 1/\eta_{ab,jb} < 1/\eta_{ab}$ would be true, spoke-to-spoke passengers could experience greater prices under cooperative pricing.

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