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**Crime, Coordination, and Punishment:
An Economic Analysis**

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CRIME, COORDINATION, AND PUNISHMENT:

AN ECONOMIC ANALYSIS*

Abstract

In the standard economic theory of crime and punishment, a risk-neutral individual will commit an offense if and only if his private benefit exceeds the expected sanction for doing so. To the extent that several individuals simultaneously choose whether or not to commit the offense, it is assumed that their decisions are independent of each other.

The purpose of this paper is to investigate a situation in which an individual's propensity to engage in an illegal activity may depend also on the behavior of other individuals. A policeman is in charge of the arrest and conviction of offenders. We assume that he can arrest only one offender. A police authority coordinates his actions. In this situation, interactive behavior arises when the police authority has a limited enforcement budget such that one officer cannot police individuals with certainty: the expected payoff to an individual from committing the offense is greater the larger the number of individuals that also decide to behave illegally.

The following questions are answered: When do individuals respond to others' behavior, when do they influence others' behavior; and finally, what is the optimal enforcement policy to forestall interactive behavior.

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1. Introduction

In Basel, Switzerland, handicrafts are traditionally organized in guilds. Besides representing the interests of their members, the main purpose of guilds is to share business information and to cultivate commercial relations. Commenting on enforcement of environmental regulations, representatives of the environmental agencies in Basel remarked that meetings by guild members are also used to exchange information concerning individual compliance with law. They expect that such information about the behavior of other members influences the propensity of one member to comply with environmental regulations. If, for example, the owner of an environmentally risky plant knows that others are already violating environmental regulations, his incentive to behave illegally increases, simply because the agency cannot detect all violations if its enforcement budget is limited. One of the most important issue confronted by environmental agents thus is to organize their enforcement activities to forestall such interactive behavior.

The standard literature on crime and punishment fails to give a suitable theoretical answer to this issue. According to Becker's [1968] seminal study, a risk-neutral individual will commit an offense if and only if his private benefit exceeds the expected sanction for doing so.¹ To the extent that several individuals simultaneously choose whether or not to commit the offense, it is assumed that their decisions are independent of each other. An individual's compliance decision then is not influenced by the behavior of other individuals. The behavior of the aggregate is merely an extrapolation from the behavior of an individual.

Situations, however, in which an individual's behavior depends on what others are doing usually do not permit simple extrapolation of individual behavior to the aggregate.² To make that connection we then have to look at the system of interaction between individuals: When do individuals respond to others' behavior; when do they influence others' behavior; and (in case of criminal activities) what is the optimal enforcement policy to forestall interactive behavior?

The purpose of this paper is to investigate these questions in a game-theoretical model.³ There are two risk-neutral individuals. Each individual knows his own private benefit from committing an offense, but not the other individual's payoff. The timing of the game has two periods. In each period, individuals decide simultaneously whether to commit the offense. An offender is subject to a sanction when he is caught. Before his second period decision, an individual observes the first period behavior of the other individual. One policeman is in charge of the arrest and conviction of offenders. We assume that he can arrest only one offender per period. A police authority coordinates his actions in both periods.

To see how interactive behavior emerges in this situation, suppose that the police authority has a limited enforcement budget such that the officer does not police individuals with certainty in one of the two periods. In this case, the expected sanction for an offender if both individuals commit the offense is lower than the expected sanction if only one individual commits the offense. As a result, the expected payoff to an individual from committing the offense is lower the more likely it is that the other individual is also an offender.⁴

The first issue with interactive behavior in our model lies on the supply side of offenses. Because of their interdependent payoff functions, an individual must anticipate the behavior of the other. In the two period model, an individual will use the first period behavior of the other individual as a signal of his propensity to engage in the illegal activity at date two. Of course, to be (or not to be) an offender in period one is not a commitment to future behavior. This introduces coordination problems. Since both individuals prefer to coordinate their decision to commit the offense, the following two behavior patterns may arise: In case of active coordination, an individual commits the offense at date one in order to demonstrate high private benefits but switches to legal

behavior at date two if the other individual behaved legally at date one. And, in case of passive coordination, an individual behaves legally at date one and switches to illegal behavior at date two if the other individual committed the offense at date one.

The second issue relates to the demand side of offenses, i.e. the way police authority should allocate resources for policing between the two periods. The way in which police resources are allocated influences the possibility for interactive behavior of individuals and therefore the overall crime rate. We assume that the authority's objective is to minimize the total number of offenses in both periods. To this purpose, it will try to reduce possible coordination between individuals.

In Section 2 we introduce into the basic one-period model. In the next two sections we analyze the two-period model. First, in Section 3, we consider the case in which the police authority has a fixed identical budget for each period. This case captures the features of many enforcement problems in reality and focuses only on interactive behavior under a given institutional framework. Second, in Section 4, we analyze the general case in which the police authority has the possibility to shift part of its budget from one period into another period. We characterize individuals' behavior for any given allocation of resources and then show how the authority should allocate resources optimally in order to minimize the expected number of offenses. Section 5 concludes with some final remarks.

2. The One-Period Model

There are two risk-neutral individuals. Both have to decide simultaneously at date 1 whether or not to commit an offense. Let O , respectively N , denote the decision (Not) to commit the Offense. An individual's benefit from engaging in the illegal activity is denoted by $i \in [0, 1]$; i is his private information and is independently drawn from the uniform distribution on $[0, 1]$. If an individual does not commit the offense, his benefit is normalized to zero.

One policeman is in charge of detecting offenders. We assume that the police authority has a limited enforcement budget such that the officer cannot police individuals with certainty. Let $p \in [0, 1]$ be the corresponding probability of detection. We assume that the policeman can arrest only one offender. If an individual is arrested, he has to pay a sanction F . F is assumed to be fixed and lower than one, $F \in [0, 1]$.

The sanction an offender has to expect then depends on the behavior of the other individual; i.e., individuals' payoff functions are interdependent. To commit an offense is more beneficial to an individual the higher the probability that the other individual behaves illegally. In fact, let $\pi(i, s; s')$ denote the expected payoff to an individual with benefit i if he chooses action $s \in \{N, O\}$ and the other individual chooses action $s' \in \{N, O\}$. Then

$$\begin{aligned}\pi(i, N; s') &= 0 \text{ for } s' \in \{N, O\}, \\ \pi(i, O; N) &= i - pF \\ \pi(i, O; O) &= i - pF/2.\end{aligned}$$

In the following, let f denote the expected sanction for an individual who committed the offense if the other individual also engaged in the illegal activity, i.e. $f = pF/2$.

To analyze individuals' behavior, we define an equilibrium strategy profile as a function

$$\sigma^* : [0, 1] \rightarrow S = \{N, O\}$$

such that $\sigma^*(i)$ maximizes the expected payoff of an individual with private benefit i , $i \in [0, 1]$, given the other individual with benefit j , $j \in [0, 1]$, chooses the strategy $\sigma^*(j)$. That is

$$\sigma^*(i) = \max_{s \in S} \int_0^1 \pi(i, s; \sigma^*(j)) dj.$$

Result 1 : There exists a unique equilibrium strategy profile σ^* with the following properties: Let $i_s^* = f/(1-f)$. Then an individual with benefit i less than i_s^* does not commit the offense, whereas an individual with a benefit higher than this critical value decides to commit the offense. That is,

$$\sigma^*(i) = \begin{cases} N & \text{if } i < i_s^*, \\ O & \text{otherwise.} \end{cases}$$

To prove this result, suppose that an individual with benefit i decides to commit the offense. Then an individual with a higher benefit $j \geq i$ will behave in the same way. Similarly, if an individual with benefit i decides not to commit the offense, an individual with a lower benefit $j \leq i$ will not engage in the illegal activity. Hence, there exists a critical value i_s^* such that individuals with higher (lower) benefits choose (not) to commit the offense. The individual with benefit i_s^* then must be indifferent between the two choices. That is,

$$0 = i_s^*(i_s^* - 2f) + (1 - i_s^*)(i_s^* - f),$$

$$\text{hence } i_s^* = \frac{f}{1-f}.$$

3. The Two-Period Model without Budget Shifting

In the two-period model, individuals decide simultaneously in each period whether or not to commit an offense. If an individual commits the offense, he enjoys his benefit i but may be sanctioned for his behavior. After period 1 each individual observes the behavior of the other individual. At date 0, a police authority coordinates the activities of the officer who is in charge of the arrest and conviction of offenders in both periods. We assume in this section, that the police authority has an identical limited enforcement budget for each period. Budget shifting from one period into another period is not possible. Hence, the probabilities of detection p_1 in period 1 and p_2 in period 2 are identical, $p_1 = p_2 = p$.

Each individual has eight possible strategies in the two-period model:

- $s_1 = (N, N)$: do not commit the offense at date 1 and 2;
- $s_2 = (N, O)$: do not commit the offense at date 1 but do commit the offense at date 2;
- $s_3 = (O, N)$: commit the offense at date 1 but not at date 2;
- $s_4 = (O, O)$: commit the offense at date 1 and at date 2;
- $s_5 = (N, O|O)$: do not commit the offense at date 1 and commit the offense at date 2 only if the other individual behaved illegally at date 1;
- $s_6 = (N, O|N)$: do not commit the offense at date 1 and commit the offense at date 2 only if the other individual behaved illegally at date 1;
- $s_7 = (O, O|O)$: commit the offense at date 1 and commit the offense at date 2 only if the other individual behaved illegally at date 1; and
- $s_8 = (O, O|N)$: commit the offense at date 1 and commit the offense at date 2 only if the other individual behaved illegally at date 1.

Let $S = \{s_1, \dots, s_8\}$. For a given strategy profile $\sigma : [0, 1] \rightarrow S$ let δ_k denote the probability that an individual chooses action s_k . Then an individual with benefit $i \in [0, 1]$ has the following payoff when choosing s_k (with $f = pF/2$):

$$\begin{aligned}
\pi(i, s_1; \sigma) &= 0 \\
\pi(i, s_2; \sigma) &= (\delta_2 + \delta_4 + \delta_6 + \delta_8)(i - f) + (\delta_1 + \delta_3 + \delta_5 + \delta_7)(i - 2f) \\
\pi(i, s_3; \sigma) &= (\delta_3 + \delta_4 + \delta_7 + \delta_8)(i - f) + (\delta_1 + \delta_2 + \delta_5 + \delta_6)(i - 2f) \\
\pi(i, s_4; \sigma) &= \pi(i, s_3; \sigma) + (\delta_2 + \delta_4 + \delta_5 + \delta_7)(i - f) + (\delta_1 + \delta_3 + \delta_6 + \delta_8)(i - 2f) \\
\pi(i, s_5; \sigma) &= (\delta_4 + \delta_8)(i - f) + (\delta_3 + \delta_7)(i - 2f) \\
\pi(i, s_6; \sigma) &= (\delta_2 + \delta_6)(i - f) + (\delta_1 + \delta_5)(i - 2f) \\
\pi(i, s_7; \sigma) &= \pi(i, s_3; \sigma) + (\delta_4 + \delta_7)(i - f) + (\delta_3 + \delta_8)(i - 2f) \\
\pi(i, s_8; \sigma) &= \pi(i, s_3; \sigma) + (\delta_2 + \delta_5)(i - f) + (\delta_1 + \delta_6)(i - 2f)
\end{aligned}$$

An equilibrium for the two-period model without budget shifting then is a function $\sigma^* : [0, 1] \rightarrow S = \{s_1, \dots, s_8\}$ such that for every $i \in [0, 1]$

$$\sigma^*(i) = \max_{s \in S} \int_0^1 \pi(i, s; \sigma^*(j)) dj.$$

Consider an individual with private benefit $i \geq 2f$. His payoff when committing an offense is positive regardless of the decision of the other individual. Since his benefit would be zero if he would decide not to commit the offense, he will choose strategy (O, O) . Thus, in every equilibrium $\delta_4 \geq 1 - 2f$. On the other hand, an individual with a low benefit $i < f$ will never commit the offense. He plays (N, N) since if he failed to do so, his expected payoff would be negative. Thus, $\delta_1 \geq f$ in every equilibrium.

Now consider an individual whose private benefit is in the intermediate range $[f, 2f]$. First, we claim that it can never be optimal for him to commit the offense at date 2 if the other individual behaved legally at date 1. That is, $(N, O|N)$ or $(O, O|N)$ cannot be equilibrium strategies. Intuitively, this result rests on the fact that in our model individuals want to conform and not to be different; both individuals prefer to coordinate their illegal activity. Since a legal behavior in period 1 indicates low private benefits, it is likely that this individual behaves legally also in period 2. Thus, committing the offense in response to a legal behavior cannot be optimal.

Second, we claim that it can never be optimal for an individual to switch to another behavior regardless of what the other individual did. That is, strategy (N, O) or (O, N) cannot be equilibrium strategies. The intuitive argument here is as follows: Since expected penalties are identical in both periods, an individual has an incentive to change his behavior only if his private benefit is not too low (i.e., higher than f) and if it is not too high (i.e., lower than $1 - 2f$). Otherwise, he would stick to his first period decision. However, he can increase his (second period) payoff if he conditions his behavior on the other individual's first period behavior. This is because he knows that with probability f a legal behavior in period 1 indicates a legal behavior in period 2, and that with probability $1 - 2f$ an offender at date 1 will also commit the offense at date 2. In sum, these arguments motivate the following result:

Result 2 : For any equilibrium $\sigma^*(\cdot)$, the strategies $(N, O|N)$, $(O, O|N)$, (N, O) and (O, N) cannot be equilibrium strategies for any individual.

An immediate consequence of this result is that an individual's propensity to engage in the illegal activity is directly correlated with private benefit. The higher the private benefit, the greater the property to engage in the illegal activity. To confirm this, we have simply to compare the expected payoff when choosing different strategies. Because an individual with a sufficiently low, resp. high, benefit decides to choose strategy (N, N) , resp. (O, O) , i.e. δ_1 and δ_4 are strictly positive, simple calculation shows that for an individual with benefit $i \in [f, 2f]$

$$\frac{\partial}{\partial i} (\pi(i, s_k; \sigma^*) - \pi(i, s_l; \sigma^*)) > 0 \text{ for } s_k = (O, O), (O, O|O) \text{ and } s_l = (N, N), (N, O|O).$$

Hence, the incentive to commit the offense in period 1 is greater the higher the private benefit to an individual. To see that this monotonicity property is also true for the second period decision, it remains to be shown that the expected payoff difference between playing strategies (N, N) and $(N, O|O)$, resp. $(O, O|O)$ and (O, O) , is positively correlated with the benefits. Simple calculation and the fact that the probability to commit the offense at date 2 increases from zero (when playing strategy (N, N)) to $\delta_4 + \delta_7 \in (0, 1)$ (when playing strategy $(N, O|O)$ or $(O, O|O)$) up to one (when playing strategy (O, O)) then proves the following result:

Result 3 : Every equilibrium $\sigma^*(\cdot)$ has the following monotonicity properties:

1. If an individual with private benefit $i \in [0, 1]$ decides to commit the offense at date 1, every individual with a higher benefit $i' \geq i$ also commits the offense at date 1.
2. The probability that an individual commits the offense at date 2 is higher the greater his private benefit $i \in [0, 1]$.

We now characterize individuals' behavior in equilibrium. According to our previous results, any equilibrium involves the strategies (N, N) and (O, O) and may involve the strategies $(N, O|O)$ or $(O, O|O)$. Note first that there is always an individual with private benefit in $[f, 2f]$ for whom neither (N, N) nor (O, O) is optimal. For purposes of argument, suppose this is false. Then there exists an individual who is just indifferent between these two strategies (according to Result 1, his private benefit is determined by $f/1 - f$). His expected payoff is zero. But then this individual benefits from deviating to strategy $(N, O|O)$. Because he can be sure that an offender at date 1 will repeat his offense at date 2, his second period payoff is strictly positive when he switches to an illegal behavior in response to the other individual's offense. This, however, contradicts the assumption. As a consequence, any equilibrium consists at least of three different possible strategies. There are three cases to analyze:

Case 1: Active Coordination

Suppose that strategy $(N, O|O)$ is not played by any individual. In this situation an individual either chooses strategy (N, N) , $(O, O|O)$ or (O, O) . We call this situation *active coordination* since an individual who chooses $(O, O|O)$ commits the offense in the first period in order to signal his propensity to engage in the illegal activity in the second period. In fact, as we will see, although his overall expected payoff is positive, his expected first period payoff is negative. The corresponding strategy profile $\sigma(\cdot)$ has the following structure:

figure 1

An individual with benefit i_1 then is indifferent between strategies (N, N) and $(O, O|O)$ if

$$0 = (1 - i_1)(i_1 - f) + i_1(i_1 - 2f) + (1 - i_1)(i_1 - f),$$

which implies $i_1 = 1 - \sqrt{1 - 2f}$. Indifference between strategies $(O, O|O)$ and (O, O) for an individual with benefit i_2 requires

$$(1 - i_1)(i_2 - f) = (1 - i_1)(i_2 - f) + i_1(i_2 - 2f),$$

which implies $i_2 = 2f$. To show that this strategy profile actually forms an equilibrium, it remains to be proven that no individual with benefit $i \in [i_1, i_2]$ has an incentive to deviate from his equilibrium strategy by choosing $(N, O|O)$. According to Result 3, it is sufficient to show that an individual with benefit i_1 cannot benefit by deviating. But this is true, since his expected payoff would be negative, equal to $-(1 - i_2)(i_1 - f)$.

Case 2: Passive Coordination

Suppose that strategy $(O, O|O)$ is not played in equilibrium. Then only the strategies (N, N) , $(N, O|O)$ and (O, O) are part of a potential equilibrium. We call this situation *passive coordination* since an individual who chooses strategy $(N, O|O)$ only engages in an illegal behavior (at date 2) if he is sure that the other individual will do so. Note that if he observes an offender at date 1, he knows with certainty that this individual repeats the offense at date 2. The strategy profile $\sigma(\cdot)$ in this case has the following structure:

figure 2

An individual with benefit i_1 then is indifference between strategies (N, N) and $(N, O|O)$ if

$$0 = (1 - i_2)(i_1 - f),$$

which implies $i_1 = f$. Moreover, an individual with benefit i_2 is indifference between strategies $(N, N|N)$ and (O, O) if

$$(1 - i_2)(i_2 - f) = (1 - i_2)(i_2 - f) + i_2(i_2 - 2f) + (1 - i_1)(i_2 - f) + i_1(i_2 - 2f),$$

which implies $i_2 = f + (1 - \sqrt{1 + 8f^2})/2$. This strategy profile constitutes an equilibrium, if no individual has an incentive to deviate to strategy $(O, O|O)$. Consider an individual with benefit i_2 . If he deviates to strategy $(O, O|O)$, his first period payoff has to be negative. According to Result 1, this is satisfied only if his private benefit i_2 is lower than the critical value i_s^* . In fact, $\pi(i_2, s_7; \sigma)$ is lower than $\pi(i_2, s_5; \sigma)$ if and only if $(1 - i_2)(i_2 - f) + i_2(i_2 - 2f)$ is negative, hence, i_2 has to be lower than $f/1 - f$. Using the characterization of i_2 , his property is satisfied if f is sufficiently high, that is $f > (3 - \sqrt{5})/2$.

Case 3: Active and Passive Coordination

Finally, consider the case in which $(N, O|O)$ as well as $(O, O|O)$ are part of an equilibrium. The strategy profile $\sigma(\cdot)$ then has the following structure:

figure 3

Calculation shows that indifference between the strategies (N, N) and $(N, O|O)$ at $i = i_1$ between $(N, O|O)$ and $(O, O|O)$ at $i = i_2$ and between $(O, O|O)$ and (O, O) at $i = i_3$ imply the following system of equations:

$$0 = (1 - i_3)(i_1 - f) + (i_3 - i_2)(i_1 - 2f) \quad (1)$$

$$0 = (1 - 2i_2 + i_3)(i_2 - f) + (2i_2 - i_3)(i_2 - 2f) \quad (2)$$

$$0 = (i_2 - i_1)(i_3 - f) + i_1(i_3 - 2f) \quad (3)$$

Here, an individual with benefit i_2 must have a negative first period payoff from committing the offense in period 2. This is because his second period payoff when choosing $(O, O|O)$ is higher than when playing $(N, O|O)$. Using strategy $(O, O|O)$ he signals his propensity to engage in the illegal behavior in period 2. This increases the incentive of the other individual to also behave illegally at date 2. A negative first period payoff when choosing $(O, O|O)$ then implies that i_2 has to be lower than the critical value i_2^* .

Moreover, an individual with benefit i_3 must have a positive first period payoff from engaging in the illegal activity. Otherwise, he could benefit from choosing strategy $(N, O|O)$. Therefore, i_3 must be higher than i_3^* .

Similar to the situation studied in case 2, the condition $i_2 < i_2^*$ requires that the penalty f has to be higher than the critical value $f > (3 - \sqrt{5})/2$. This ensures that the expected first period payoff for an offender at date 1 is negative. To see this, consider the limit case in which i_2 tends to i_3 ; the above strategy profile then reduces to the equilibrium profile in case 2 and i_2 becomes $f + (1 - \sqrt{1 + 8f^2})/2$. Moreover, equation (2) then implies that $i_2 = f/1 - f$. Since i_2 is positively correlated with f , i_2 is lower than i_2^* only if $f - (1 - \sqrt{1 + 8f^2})/2 < f/1 - f$ which is equivalent to $f > (3 - \sqrt{5})/2$.

Result 4 : The two-period model without budget shifting has the following equilibria:

- For $f \in [0, \frac{3-\sqrt{5}}{2})$,

$$\sigma_1^*(i) = \begin{cases} (N, N) & \text{if } i < i_1^* \\ (O, O|O) & \text{if } i \in [i_1^*, i_2^*) \\ (O, O) & \text{if } i \geq i_2^* \end{cases} \quad \text{with } i_1^* = 1 - \sqrt{1 - 2f} \text{ and } i_2^* = 2f$$

is the unique equilibrium strategy profile.

- For $f \in [\frac{3-\sqrt{5}}{2}, 1/2]$, σ_1^* ,

$$\sigma_2^*(i) = \begin{cases} (N, N) & \text{if } i < i_1^* \\ (N, O|O) & \text{if } i \in [i_1^*, i_2^*) \\ (O, O) & \text{if } i \geq i_2^* \end{cases} \quad \text{with } i_1^* = f \text{ and } i_2^* = f + \frac{\sqrt{1 + 8f^2} - 1}{2} \text{ and}$$

$$\sigma_3^*(i) = \begin{cases} (N, N) & \text{if } i < i_1^* \\ (N, O|O) & \text{if } i \in [i_1^*, i_2^*) \\ (O, O|O) & \text{if } i \in [i_2^*, i_3^*) \\ (O, O) & \text{if } i \geq i_3^* \end{cases} \quad \text{with } i_1^* > f \text{ and } i_2^* < \frac{f}{1-f} < i_3^* < 2f$$

are equilibrium strategy profiles.

4. The General Two-Period Model with Budget Shifting

In this section, we assume that the police authority is free to decide which part of its budget to use for detection in period 1 or 2. It will choose a partition of its budget in order to minimize the expected number of offenses. As before, the overall enforcement budget is limited such that one officer cannot police individuals with certainty in every period.

Let $(p_1, p_2) \in [0, 1]^2$ be the probabilities of detection in period 1 and 2 such that total budget is used. Define $f_i = p_i F/2$ as the expected sanction for an offender in period i , $i = 1, 2$, if the other individual also commits the offense. Then $(f_1, f_2) \in [0, 1/2]^2$. In the following, we consider (f_1, f_2) as the police authority's policy variables. Limited budget then restricts the police to $f_1 + f_2 \leq B$ for some $B \in [0, 1]$.

As in Section 3, let $S = \{s_1, \dots, s_8\}$ denote the possible strategies for an individual and let δ_k denote the probability that an individual plays strategy s_k . For a given strategy profile $\sigma : [0, 1] \rightarrow S$, an individual with benefit $i \in [0, 1]$ then has the following payoff when choosing strategy s_k :

$$\begin{aligned}\pi(i, s_1; \sigma) &= 0 \\ \pi(i, s_2; \sigma) &= (\delta_2 + \delta_4 + \delta_6 + \delta_8)(i - f_2) + (\delta_1 + \delta_3 + \delta_5 + \delta_7)(i - 2f_2) \\ \pi(i, s_3; \sigma) &= (\delta_3 + \delta_4 + \delta_7 + \delta_8)(i - f_1) + (\delta_1 + \delta_2 + \delta_5 + \delta_6)(i - 2f_1) \\ \pi(i, s_4; \sigma) &= \pi(i, s_3; \sigma) + (\delta_2 + \delta_4 + \delta_5 + \delta_7)(i - f_2) + (\delta_1 + \delta_3 + \delta_6 + \delta_8)(i - 2f_2) \\ \pi(i, s_5; \sigma) &= (\delta_4 + \delta_8)(i - f_2) + (\delta_3 + \delta_7)(i - 2f_2) \\ \pi(i, s_6; \sigma) &= (\delta_2 + \delta_6)(i - f_2) + (\delta_1 + \delta_5)(i - 2f_2) \\ \pi(i, s_7; \sigma) &= \pi(i, s_3; \sigma) + (\delta_4 + \delta_7)(i - f_2) + (\delta_3 + \delta_8)(i - 2f_2) \\ \pi(i, s_8; \sigma) &= \pi(i, s_3; \sigma) + (\delta_2 + \delta_5)(i - f_2) + (\delta_1 + \delta_6)(i - 2f_2)\end{aligned}$$

An equilibrium for the two-period model with budget shifting is a triple (f_1^*, f_2^*, σ^*) with $\sigma^* : [0, 1] \times [0, 1/2]^2 \rightarrow S$ such that

- for a given policy (f_1, f_2)

$$\sigma^*(i; f_1, f_2) = \max_{s \in S} \int_0^1 \pi(i, s; \sigma^*(j)) dj,$$

for every individual with benefit $i \in [0, 1]$ and

- the policy (f_1^*, f_2^*) minimizes the expected number of offenses over all policies (f_1, f_2) with $f_1 + f_2 \leq B$.

To analyze the model, we first consider the behavior of individuals in response to the police authority's policy choice (f_1, f_2) . Similar to the argument in Section 3, it can be shown that it is never optimal for an individual to choose strategy $(N, O|N)$ or $(O, O|N)$. In either case, an individual would commit the offense in response to a legal behavior of the other individual, although this indicates low private benefits and, hence, a low propensity of the other individual to commit the offense in period 2. In contrast to Result 2, however, it may now be optimal for an individual to choose strategy (N, O) , resp. (O, N) ; if, for example, there is no policing at date 1, resp. date 2, an individual commits the offense regardless of his private benefit.

Moreover, as in the two-period model without budget shifting, it is straightforward to show that individuals' incentives to engage in the illegal activity are greater the higher the private benefits. If an individual with benefit i commits the offense at date 1, then an individual with a higher benefit $i' > i$ will do so; and, the probability that an individual will be an offender at date 2 is greater the higher his private benefit.

An immediate consequence from this monotonicity result is that the strategies (O, N) and (N, O) , resp. $(N, O|O)$, cannot be part of the same equilibrium strategy profile σ^* . To see this, suppose for example, that (O, N) and (N, O) is optimal for an individual with benefit i , resp. i' . Monotonicity in the propensity to commit the offense in period 1 then requires $i > i'$. Monotonicity in period 2, however, requires $i < i'$, a contradiction. Similarly, it is easy to show that (N, O) and $(O, O|O)$ are not optimal strategies for individuals in the same equilibrium profile.

With these remarks we are now in the position to characterize the equilibrium strategy profile $\sigma^*(\cdot, f_1, f_2)$.

Result 5 : Suppose that the police authority chooses a policy $(f_1, f_2) \in [0, 1/2]^2$. Then there exist four critical values x_1, x_2, x_3, x_4 with $0 < x_1 < x_2$ and $0 < x_3 < x_4$ such that an equilibrium strategy profile $\sigma^*(\cdot, f_1, f_2)$ is given as follows:

- For $f_1 \leq x_1$

$$s_1^*(i, f_1, f_2) = \begin{cases} (N, N) & \text{if } i < i_1, \\ (O, N) & \text{if } i \in [i_1, i_2], \\ (O, O|O) & \text{if } i \in [i_2, i_3], \\ (O, O) & \text{if } i \geq i_3 \end{cases}$$

- For $f_1 \in [x_1, x_2]$

$$s_2^*(i, f_1, f_2) = \begin{cases} (N, N) & \text{if } i < i_1, \\ (O, O|O) & \text{if } i \in [i_1, i_2] \\ (O, O) & \text{if } i \geq i_2 \end{cases}$$

- For $f_1 \in [\min\{x_2, x_3\}, \max\{x_2, x_3\}]$

$$s_3^*(i, f_1, f_2) = \begin{cases} (N, N) & \text{if } i < i_1, \\ (N, O|O) & \text{if } i \in [i_1, i_2] \\ (O, O|O) & \text{if } i \in [i_2, i_3] \\ (O, O) & \text{if } i \geq i_3 \end{cases}$$

- For $f_1 \in [x_3, \min\{x_4, 1/2\}]$

$$s_4^*(i, f_1, f_2) = \begin{cases} (N, N) & \text{if } i < i_1, \\ (N, O|O) & \text{if } i \in [i_1, i_2] \\ (O, O) & \text{if } i \geq i_2 \end{cases}$$

- For $f_1 \in [\min\{x_4, 1/2\}, 1/2]$

$$s_5^*(i, f_1, f_2) = \begin{cases} (N, N) & \text{if } i < i_1, \\ (N, O|O) & \text{if } i \in [i_1, i_2] \\ (N, O) & \text{if } i \in [i_2, i_3] \\ (O, O) & \text{if } i \geq i_3 \end{cases}$$

The proof of Result 5 can be found in the Appendix as well as the characterization of the critical values x_1, \dots, x_4 and i_1, i_2 , respectively i_3 , for each strategy - they are well defined functions of f_1 and f_2 . Two remarks are worth noting. First, x_2 is lower than x_3 if and only if $f_1 + f_2$ is lower than some critical value around 0.618. In this case, there exists for every $f_1 \in [0, 1/2]$ a unique equilibrium strategy profile. If, however, $x_3 < x_2$ then there are three possible equilibria for $f_1 \in [x_3, x_2]$. Second, x_4 is lower than $1/2$ if $f_1 + f_2$ is lower than $\sqrt{3}/2 = 0.866$. If this is not the case, the strategy profile σ_5^* is never played.

The following figure provides an example of the unique equilibrium strategy profile in the case where $f_1 + f_2 = 1/2$ (that is, one policeman can police individuals with certainty in one of the two periods and the sanction F is one):

Figure 4

Of course, if there is no policing at date 1, i.e. $f_1 = 0$, an individual commits the offense regardless of his private benefit. However, the expected penalty $f_2 = 1/2$ ensures that no individual repeats his offense at date 2 (see Result 1). If the police authority now increases its policing in period 1 and reduces proportionally its policing in period 2, individuals with low benefits will not commit in either period but those with high benefits will be offenders in both periods. Moreover, it becomes profitable for some individuals to coordinate actively; they commit the offense in the first period in order to signal their propensity to engage in the illegal behavior in the second period. If the first period expected sanction f_1 exceeds the critical value x_1 , playing strategy (O, N) is no longer optimal. Since the second period sanction f_2 is sufficiently low, the probability that an offender repeats his offense is sufficiently high. However, if the first period sanction f_1 increases further, active coordination becomes less attractive. If f_1 then is greater than the critical value x_2 , some individuals will switch to passive coordination. They do not commit the offense in the first period, but do so in the second period if the other individual committed the offense in the first period. Passive and active coordination becomes more, resp. less, attractive to individuals the higher the first period sanction f_1 . In fact, if this sanction becomes sufficiently high, $f_1 \geq x_3$, no individual will coordinate actively. When the first period sanction f_1 then becomes higher and, therefore, the second period sanction f_2 becomes lower, the incentive to commit the offense at date 2 becomes higher. If $f_1 \geq x_4$ it is then optimal for some individuals to choose strategy (N, O) . In the limit, if there is no policing in period two, an individual commits the offense at date 2 regardless of his benefit. However, since $f_1 = 1/2$ in this case, no individual will be an offender at date 1 (see Result 1).

Having characterized the individuals' behavior in response to a policy choice (f_1, f_2) of the police authority, we now consider how police actions should be coordinated in both periods when the enforcement budget is limited such that $f_1 + f_2 \leq B$. The example above suggests the following result: whenever one policeman is in action with certainty in one period, there is no offense in this period, but both individuals commit the offense in the other period. In this situation, an individual cannot learn anything about the actual benefit of the other individual. That is, his ex post probability distribution at date 2 on the other's benefit is identical to his ex ante probability distribution at date 1. This, however, is no longer true if the police authority decides to use one policeman in both periods; in this case, there is a positive probability that an individual coordinates his second period decision on the observed behavior of the other individual in the first period. Hence, an individual has better information on the other individual's benefit at date 2 than at date 1. This increases his incentives to behave illegally, either in period 1 by signaling his propensity to commit the offense also in the second period or in period 2 as a response to the illegal behavior of the other individual in period 1. As a consequence, the expected number of offenses will be higher than in a situation in which no coordination is possible.

Result 6 : In the two-period model with budget shifting, it is optimal for the police authority to choose the following policing strategy (f_1^*, f_2^*) :

- If $B \in [0, \sqrt{3}/2]$, then

$$f_1^* = \begin{cases} B & \text{if } B < 1/2, \\ 1/2 & \text{otherwise} \end{cases} \quad f_2^* = \begin{cases} 0 & \text{if } B < 1/2, \\ B - 1/2 & \text{otherwise} \end{cases}$$

as well as

$$f_1^* = \begin{cases} 0 & \text{if } B < 1/2, \\ B - 1/2 & \text{otherwise} \end{cases} \quad f_2^* = \begin{cases} B & \text{if } B < 1/2, \\ 1/2 & \text{otherwise} \end{cases}$$

minimizes the expected number of offenses which are

$$4 - \frac{2B}{1-B} \text{ for } B < 1/2, \quad \text{resp.} \quad 4 - \frac{4}{3-2B} \text{ for } B \geq 1/2.$$

- If $B \in [\sqrt{3}/2, 1]$, then

$$f_1^* = \begin{cases} B & \text{if } B < 1/2, \\ 1/2 & \text{otherwise} \end{cases} \quad f_2^* = \begin{cases} 0 & \text{if } B < 1/2, \\ B - 1/2 & \text{otherwise} \end{cases}$$

minimizes the expected number of offenses which are

$$4 - \frac{4}{3-2B}.$$

The proof of Result 6 can be found in the Appendix. The result can be reconciled with the intuitive argument discussed before: it is optimal for the police authority to minimize coordination between individuals. As a consequence, one policeman should be in action in one of the two periods with maximal probability. It is unimportant whether this is done in period 1 or 2, as long as the enforcement budget is not too high. In fact, the strategy profiles σ_1^* and σ_5^* always lead to the same number of expected offenses. However, as Result 5 shows, if B exceeds the critical value $\sqrt{3}/2$, the strategy profile σ_5^* is no longer an equilibrium strategy profile. Instead, σ_4^* now describes the optimal individuals' behavior for maximal detection in period 1. But σ_4^* allows more coordination than σ_1^* .

5. Conclusion and Extensions

In this article we have analyzed decision-interdependency among potential offenders and characterized the optimal enforcement policy to forestall interactive behavior. We have shown that it is optimal for a police authority to concentrate police actions in order to minimize coordination between individuals.

To the extent that there are more than two periods, this result suggests that it is optimal for the police authority to use the following strategy: concentrate resources in period 1 to forestall criminal behavior, and do so in all subsequent periods as long as the budget allows. This, of course, is a well known strategy in real law enforcement situations. For example, local transport companies often use surprise controls with several ticket inspectors to avoid interactive travelling without a ticket.

Our result could also be extended to different types of offenses. Suppose for example that there are two types of offenses and that the police authority has a limited budget to enforce both offenses. If the authority is free to allocate its resources, our results suggest the following optimal policing strategy: concentrate police actions on one type of offense in period 1 and on the other type of offense in period 2. If, however, the police authority is restricted in its allocation of resources, conditional policing in period 2 is optimal: shift

as many resources as possible to the enforcement of that offense, that is most prevalent in period 1.

We assumed throughout the paper that the police authority can commit itself to a certain policy. Therefore, individuals (when deciding on whether to commit the offense) can condition their behavior on this information. Although this is a standard assumption in the theory of crime and punishment (see Besanko and Spulber [1989] for an exception), it would be of interest to consider a situation in which the police authority decides on policing simultaneously with the individuals' decisions. The literature on tax evasion suggests that the optimal policing crucially differs from the one derived here; see e.g. Reinganum and Wilde [1985, 1986].

The model could be extended to consider the case in which the authority is required partially to self-finance by retaining a share of the sanctions it collects. Further work should also consider more than two individuals. The mechanism discussed here may then serve as an explanation of the formation of riots and mobs.

Footnotes

¹ See e.g. Ehrlicher [1973], Becker and Landes [1974], Heinecke [1978], Andreano and Siegfried [1980], Pyle [1983], Schmidt and Witte [1984] or the symposium on the economics of crime in the *Journal of Economic Perspectives Vol.10(1)* [1996] and the references therein.

² Empirical evidence on tax evasion, for example, suggests that evasion decisions are not independent. Instead, individuals are more likely to evade once they expect others evading; see e.g. Benjamini and Maitail [1985], Schlicht [1985] or Gordon [1989]. This interactive behavior is also common to many other criminal or social activities; see Case and Katz [1991]. For instance, one's incentive to double-park is higher if many others do so and one is less discouraged from smoking marijuana if most of one's friends do. The formation of mobs, riots, or panic behavior are other examples. See Schelling [1978, p.36ff] for a more elaborate discussion.

³ Decision-interdependency among potential offenders is also analyzed also in Sah [1991] and Chu [1993]. Sah analyzes a dynamic relationship in which an individual's current information concerning the probability of punishment is influenced by past experiences of the individual and of his acquaintances. Chu concentrates on a special class of interactions characterized by the model of critical mass.

⁴ These network externalities are analyzed in the industrial organization literature, see e.g. Farrell and Saloner [1985, 1986] or Katz and Shapiro [1985, 1986]. In this literature, however, it is assumed that an individual's decision in period 1 is a commitment for his behavior in period 2.

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Appendix

Proof of Result 2:

Let σ^* be an equilibrium strategy profile. We distinguish four cases:

1. Consider first the case of strategy $s_6 = (N, O|N)$ and suppose that s_6 is part of σ^* . Then there exists an equilibrium in which δ_6 is strictly positive. Hence, there exists a range in $[f, 2f]$ such that an individual with private benefit in this range chooses strategy s_6 . Let $j \in [f, 2f]$ be the individual with the lowest benefit j such that s_6 is his equilibrium strategy profile. Since strategy $s_1 = (N, N)$ yields zero payoff, the individual with benefit j must have at least a non-negative payoff, that is

$$\pi(j, s_6; \sigma^*) = (\delta_2 + \delta_6)(j - f) + (\delta_1 + \delta_5)(j - 2f) \geq 0. \quad (1)$$

We now argue to a contradiction: we show that j must be greater than $2f$, which implies that $\delta_6 = 0$, since an individual with benefit greater than $2f$ prefers to choose strategy (O, O) in equilibrium.

To prove this, let $j = tf$ for $t \in [1, 2]$. Suppose that it is optimal for an individual with benefit $i \in [f, 2f]$ to play strategy $s_2 = (N, O)$. Then we claim that it cannot be optimal for an individual with a higher benefit $i' \geq i$ to play strategy $(N, O|N)$. To see this, note that δ_4 is strictly positive. Hence,

$$\frac{\partial}{\partial i} (\pi(i, s_2; \sigma^*) - \pi(i, s_6; \sigma^*)) > 0,$$

and the incentives to play $s_2 = (N, O)$ instead of $s_6 = (N, O|N)$ are greater the higher the individual's benefit. In particular, $j < i$. As a consequence, the probability that an individual plays strategy s_2 or s_6 is less than $f(2 - t)$. That is, $\delta_2 + \delta_6 < f(2 - t)$.

Since $\delta_1 \geq f$ and $\pi_6(j, s_6; \sigma^*)$ is an increasing function of δ_2 and δ_6 , but is also a decreasing function of δ_1 and δ_5 for $j \in [f, 2f]$, inequality (1) then implies that (with $t = 1$)

$$f(j - f) + f(j - 2f) > 0, \quad \text{that is } j > \frac{3}{2}f.$$

But $j > \frac{3}{2}f$ implies, that $\delta_2 + \delta_6 < \frac{f}{2}$ (with $t = \frac{3}{2}$) and, hence,

$$\frac{f}{2}(j - f) + f(j - 2f) > 0, \quad \text{that is } j > \frac{5}{3}f$$

More generally, starting with $t_1 = 1$ and $\delta_2 + \delta_6 < t_1 f$, repetition of this argument shows that $\delta_2 + \delta_6 < t_n f$ with $t_n = 2 + t_{n-1}/(1 + t_{n-1})$. In the limit,

$$j > \lim_{n \rightarrow \infty} \frac{2 + t_{n-1}}{1 + t_{n-1}} \cdot f = 2f, \quad \text{a contradiction.}$$

2. Consider now the case of strategy $s_8 = (O, O|N)$. As before, suppose that $\delta_8 > 0$. Let $j \in [f, 2f]$ be the lowest benefit to an individual for whom it is optimal to play strategy s_8 . Then $\pi(j, s_8; \sigma^*)$ must be higher than $\pi(j, s_3; \sigma^*)$ which implies that

$$(\delta_2 + \delta_5)(j - f) + \delta_1(j - 2f) \geq 0.$$

Similar to the argument in part one of this proof it follows that $j > 2f$, a contradiction.

3. Suppose that $\delta_2 > 0$. Let $j \in [f, 2f]$ be the lowest benefit to an individual for whom it is optimal to play strategy $s_2 = (N, O)$. Then $\pi(j, s_2; \sigma^*)$ must be higher than $\pi(j, s_5; \sigma^*)$ which implies that

$$\delta_2(j - f) + (\delta_1 + \delta_5)(j - 2f) \geq 0.$$

Since $\delta_2 < f$ and $\delta_1 + \delta_5 \geq f$, an argument similar to part one of this proof shows that $j > 2f$, a contradiction.

4. Suppose that strategy $s_3 = (O, N)$ is played by some individuals, that is $\delta_3 > 0$. We distinguish two cases:

4.1. Suppose that $\delta_5 > 0$. Since δ_1 is strictly positive, the incentives to play strategy $s_3 = (O, N)$ instead of strategy $s_5 = (N, O|O)$ are greater the higher the individual's benefit,

$$\frac{\partial}{\partial i} (\pi(i, s_3; \sigma^*) - \pi(i, s_5; \sigma^*)) > 0.$$

Moreover, since

$$\frac{\partial}{\partial i} (\pi(i, s_7; \sigma^*) - \pi(i, s_5; \sigma^*)) > 0,$$

it follows that there exists a critical value $i_1 \in [f, 2f]$ such that $\pi(i_1, s_3; \sigma^*) = \pi(i_1, s_5; \sigma^*)$. This implies

$$(\delta_3 + \delta_7)f = (\delta_1 + \delta_5)(i_1 - 2f).$$

Since $i_1 < 2f$, the right hand side of this equation is negative, whereas the left hand side is positive, a contradiction.

4.2. Suppose that $\delta_5 = 0$. Then the equilibrium $\sigma^*(\cdot)$ consists of the strategies s_1, s_3, s_7, s_4 . To see that $\delta_7 > 0$, note that $\pi(i, s_3; \sigma^*) - \pi(i, s_2; \sigma^*) = a\delta_7$ is independent of an individual's benefit i . Since $\delta_3 > 0$ and $s_2 = (N, O)$ cannot be part of the equilibrium, δ_7 must be strictly positive. According to part 4.1, if it is optimal for an individual with benefit $i \in [f, 2f]$ to play strategy s_7 , it cannot be optimal for an individual with a higher benefit $i' \geq i$ to play strategy s_3 .

An individual with benefit i_1 then is indifferent between strategies s_1 and s_3 if $i_1 = f/1 - f$ (see Result 1) and an individual with benefit i_2 is indifferent between strategies s_3 and s_7 if $0 = (1 - i_2)(i_2 - f) + (i_2 - i_1)(i_2 - 2f)$. Hence,

$$i_2 = f \cdot \frac{1 - 2i_1}{1 - f - i_1} = f \cdot \frac{1 - 3f}{1 - 3f + f^2}.$$

Simple calculation then shows that $i_2 < 0$ for $f \in (\frac{1}{3}, f')$, $i_2 < i_1$ for $f < \frac{1}{3}$ and $i_2 > 1$ for $f > f'$, where f' solves $1 - 3f + f^2 = 0$. Together, this leads to a contradiction.

Q.E.D.

Proof of Result 5:

Let B be fixed, $B \leq 1$. We have to consider five cases, depending on the policy (f_1, f_2) with $f_1 + f_2 = B$:

Case 1: $(N, N), (O, N), (O, O|O)$ and (O, O) are part of a strategy profile

The monotonicity property ensures that there exist three critical values i_1, i_2, i_3 such that

$$\begin{aligned} \pi(i_1, s_1; \sigma) &= \pi(i_1, s_3; \sigma), \\ \pi(i_2, s_3; \sigma) &= \pi(i_2, s_7; \sigma), \\ \pi(i_3, s_7; \sigma) &= \pi(i_3, s_4; \sigma). \end{aligned}$$

Simple calculation shows that these equations imply

$$i_1 = \frac{f_1}{1 - f_1}, \quad i_2 = f_2 \frac{1 - 2i_1}{1 - f_2 - i_1}, \quad i_3 = 2f_2.$$

Then $i_1 \leq i_2$ if and only if $\frac{f_1}{1-f_1} \leq f_2$ and $i_2 \leq i_3$ if and only if $f_2 \leq \frac{1}{2}$. Whereas the second condition is always satisfied, the first condition holds if the first period expected sanction f_1 is lower than some critical value x_1 , where x_1 is defined by

$$\frac{x_1}{1-x_1} = B - x_1.$$

Note that if there is no first period sanction, $f_1 = 0$, an individual commits the offense at date 1 regardless of his private benefit, i.e. $i_1 = 0$ and does so at date 2 if his benefit is sufficiently high, i.e. $i_2 = f_2/1 - f_2$.

Case 2: (N, N), (O, O|O) and (O, O) are part of a strategy profile

In this case there exist two critical values i_1, i_2 , such that

$$\begin{aligned}\pi(i_1, s_1; \sigma) &= \pi(i_1, s_7; \sigma), \\ \pi(i_2, s_7; \sigma) &= \pi(i_2, s_4; \sigma).\end{aligned}$$

These equations are equivalent to

$$\begin{aligned}0 &= (1-i_1)(i_1-f_1) + i_1(i_1-2f_1) + (1-i_1)(i_1-f_2) \\ 0 &= i_1(i_2-2f_2)\end{aligned}$$

The solutions for i_1 and i_2 can be easily calculated as

$$i_1 = -\frac{2-f_1+f_2}{2} - \frac{\sqrt{(2-f_1+f_2)^2 - 4(f_1+f_2)}}{2}, \quad i_2 = 2f_2.$$

Now, the first equation implies that $i_1 \geq f_2$ for otherwise (O, N) would yield a higher payoff than $(O, O|O)$. But then the expected first period payoff must be negative, that is $i_1 \leq \frac{f_1}{1-f_1}$. Therefore, a necessary condition for (s_1, s_7, s_4) to constitute an equilibrium strategy profile is $f_1 \geq x_1$.

In addition, the strategy $(N, O|O)$ must result in a negative payoff for an individual with benefit i_1 . This condition is equivalent to

$$(1-i_2)(i_1-f_2) + (i_2-i_1)(i_1-2f_2) \leq 0.$$

If it holds, i_1 is lower than $2f_2$ which ensures that $i_1 < i_2$, since the second equation implies $i_2 = 2f_2$. Moreover, this condition is satisfied for $f_1 = x_1$: the left hand side then is identical to $-(f_2)^2$. However, if f_2 is sufficiently small, the left hand side becomes positive. Hence, there exists a critical value $x_2 < 1/2$, $x_2 > x_1$, such that $\pi(i_1, s_5; \sigma) \leq 0$ for all $f_1 \leq x_2$. In fact, x_2 is the smallest solution of the identity $(1-i_2)(i_1-(B-x_2)) + (i_2-i_1)(i_1-2(B-x_2)) = 0$ with

$$i_1 = \frac{2+B-2x_2}{2} - \frac{\sqrt{(2+B-2x_2)^2 - 4B}}{2}, \quad i_2 = 2(B-x_2).$$

Note that if $f_1 = f_2$, the previous condition is always satisfied (see Result 2). Hence, $2x_2 > B$.

Case 3: (N, N), (N, O|O) and (O, O) are part of a strategy profile

In this case two critical values i_1, i_2 , exist such that an individual with benefit i_1 is indifferent between choosing strategies (N, N) and $(N, O|O)$, i.e. $\pi(i_1, s_1; \sigma) = \pi(i_1, s_7; \sigma)$, and an individual with benefit i_2 is indifferent between $(N, O|O)$ and (O, O) , i.e. $\pi(i_2, s_7; \sigma) = \pi(i_2, s_4; \sigma)$. These two values are solutions of the following two equations:

$$\begin{aligned}
0 &= (1 - i_2)(i_1 - f_2) \\
0 &= (1 - i_2)(i_2 - f_1) + i_2(i_2 - 2f_1) + (i_2 - i_1)(i_2 - f_2) + i_1(i_2 - 2f_2)
\end{aligned}$$

Simple calculation shows that

$$i_1 = f_2, \quad i_2 = -\frac{1 - f_1 - f_2}{2} + \frac{\sqrt{(1 - f_1 - f_2)^2 - 4(f_1 + (f_2)^2)}}{2}.$$

Strategy (N, O) yields a lower payoff than $(N, O|O)$ for an individual with benefit i_2 if $i_2(i_2 - 2f_2)$ is negative, i.e. $i_2 < 2f_2$. Moreover, strategy $(N, O|O)$ is more profitable than $(O, O|O)$ if

$$(1 - i_2)(i_2 - f_2) + i_2(i_2 - 2f_1) \leq 0.$$

This inequality is satisfied if and only if $i_2 \leq \frac{f_1}{1 - f_1}$. In addition, the second equation implies

$$(1 - i_2)(i_2 - f_2) + i_2(i_2 - 2f_1) \leq 0,$$

hence $i_2 \geq f_2$. Therefore, $i_1 \leq i_2$, if (s_1, s_5, s_4) is an equilibrium profile. Using the solution for i_2 , the two necessary conditions $i_2 \leq \frac{f_1}{1 - f_1}$ and $i_2 \leq 2f_2$ for (s_1, s_5, s_4) to be an equilibrium profile then reduce to

$$f_1 \geq x_3 \text{ and } f_1 \leq x_4,$$

where x_3 is the smallest solution of the identity

$$\frac{x_3}{1 - x_3} = -\frac{1 - B}{2} + \frac{\sqrt{(1 - B)^2 + 4(x_3 + (F - x_3)^2)}}{2}$$

and x_4 is the smallest solution of the identity

$$2x_4 = -\frac{1 - B}{2} + \frac{\sqrt{(1 - B)^2 + 4(x_4 + (F - x_4)^2)}}{2}.$$

Three remarks are worth noting. First, x_4 is strictly greater than x_3 for all $B \in [0, 1]$. Second, x_3 is strictly lower than $1/2$ for all $B \in [0, 1]$. Third, x_4 is higher than $\frac{1}{2}$ for B higher than $\sqrt{3}/2$.

Case 4: (N, N) , $(N, O|O)$, $(O, O|O)$ and (O, O) are part of a strategy profile

An individual with benefit i_1 is indifferent between strategies (N, N) and $(N, O|O)$, an individual with benefit i_2 is indifferent between $(N, O|O)$ and $(O, O|O)$, and an individual with benefit i_3 is indifferent between $(O, O|O)$ and (O, O) if the following three equations are satisfied:

$$\begin{aligned}
0 &= (1 - i_3)(i_1 - f_2) + (i_3 - i_2)(i_1 - 2f_2) \\
0 &= (1 - i_2)(i_2 - f_1) + i_2(i_2 - 2f_1) + (i_3 - i_2)f_2 \\
0 &= i_1(i_3 - f_2) + (i_2 - i_1)(i_3 - f_2)
\end{aligned}$$

This system of equations requires that $i_1 > f_2$ and $i_3 < 2f_2$. Note that if i_1 is identical to i_2 , the system of equations reduces to the one studied in case 2 and if i_2 is identical to i_3 , it reduces to the one studied in case 3. The range in which the strategy profile (s_1, s_5, s_7, s_4) then constitutes an equilibrium profile depends on the critical values x_2

and x_3 : If $x_2 < x_3$, the profile is an equilibrium strategy profile for $f_1 \in [x_2, x_3]$. If, otherwise, $x_2 > x_3$, it is an equilibrium profile for $f_1 \in [x_3, x_2]$. Note that $x_2 > x_3$ implies multiplicity of equilibrium profiles. This occurs if F is higher than some critical value around 0.617. Note that although 0.617 is lower than $3 - \sqrt{5}$, this result is in line with Result 2: as long as B is lower than $3 - \sqrt{5}$, $3 - \sqrt{5}/2$ is lower than x_3 and, hence, is not within the interval $[x_3, x_2]$. For $B = 3 - \sqrt{5}$ we then have $x_3 = 3 - \sqrt{5}/2$.

Case 5: (N, N) , $(N, O|O)$, (N, O) and (O, O) are part of a strategy profile

In this case there exists an individual with benefit i_1 who is indifferent between choosing strategies (N, N) and $(N, O|O)$, i.e. $\pi(i_1, s_1; \sigma) = \pi(i_1, s_7; \sigma)$, an individual with benefit i_2 who is indifferent between $(N, O|O)$ and (N, O) , i.e. $\pi(i_2, s_7; \sigma) = \pi(i_2, s_2; \sigma)$ and an individual with benefit i_3 who is indifferent between strategies (N, O) and (O, O) , i.e. $\pi(i_2, s_2; \sigma) = \pi(i_2, s_4; \sigma)$. These three critical values are solutions of the following system of equations:

$$\begin{aligned} 0 &= (1 - i_3)(i_1 - f_2) \\ 0 &= (i_3 - i_2)(i_2 - f_1) + i_2(i_2 - 2f_1) \\ 0 &= (1 - i_3)(i_3 - f_1) + i_3(i_3 - 2f_1) + (i_2 - i_1)f_2 \end{aligned}$$

The first and the second equations imply $i_1 = f_2$ and $i_2 = i_3 f_2 / i_3 - f_2$. Hence $i_1 < i_2$. Using the characterization for i_2 , the condition $i_2 \leq i_3$ is satisfied if and only if $i_3 \geq 2f_2$. Since for i_2 identical to i_3 the system of equations reduces to the one studied in case 3, this condition is equivalent to $f_1 \geq x_4$. Note that the third equation also implies that $i_3 \leq f_1 / 1 - f_1$.

Q.E.D.

Proof of Result 6:

Let V_i denote the expected number of offenses if individuals chooses to play the strategy profile σ_i^* (see Result 5). Then the expected number of offenses can be derived as follows:

$$\begin{aligned} V_1 &= 4 - 2i_1(1 + i_3) - 2i_2(1 - i_1) \\ V_2 &= 4 - 2i_1(2 - i_1 + i_2) \\ V_3 &= 4 - 2i_1(1 - i_2) - 2i_2(1 + i_3) \\ V_4 &= 4 - 2i_1(1 - i_2) - 2i_2(1 + i_2) \\ V_5 &= 4 - 2i_1(1 - i_3) - 2i_3(1 + i_2) \end{aligned}$$

Using the characterization of the critical values i_1 , i_2 , resp. i_3 (see the proof of Result 5), calculation shows that V_1 is increasing in f_1 and V_2 , V_3 , V_4 and V_5 are decreasing in f_1 . Since the strategy profile σ_i^* is identical to the strategy profile σ_{i+1}^* for $f_1 = x_i$, x_i as defined in the proof of Result 5, the expected number of offenses then is minimal for $f_1 = 0$ or $f_2 = 0$. The claim of Result 6 then follows immediately.

Q.E.D.

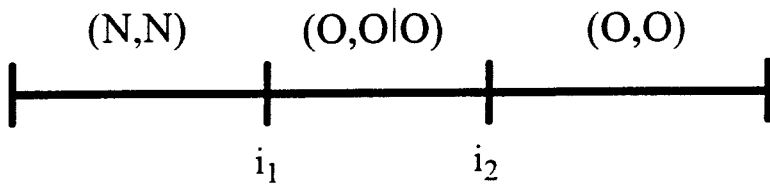


Figure 1: Active Coordination

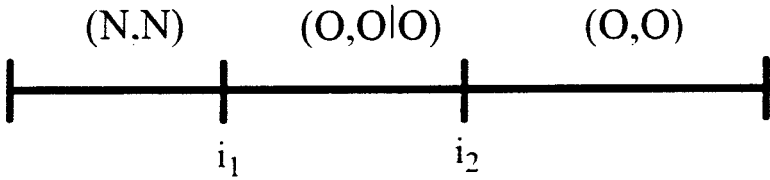


Figure 2: Passive Coordination

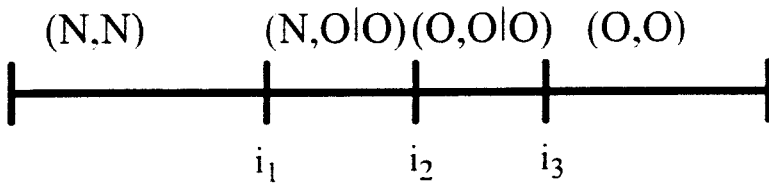
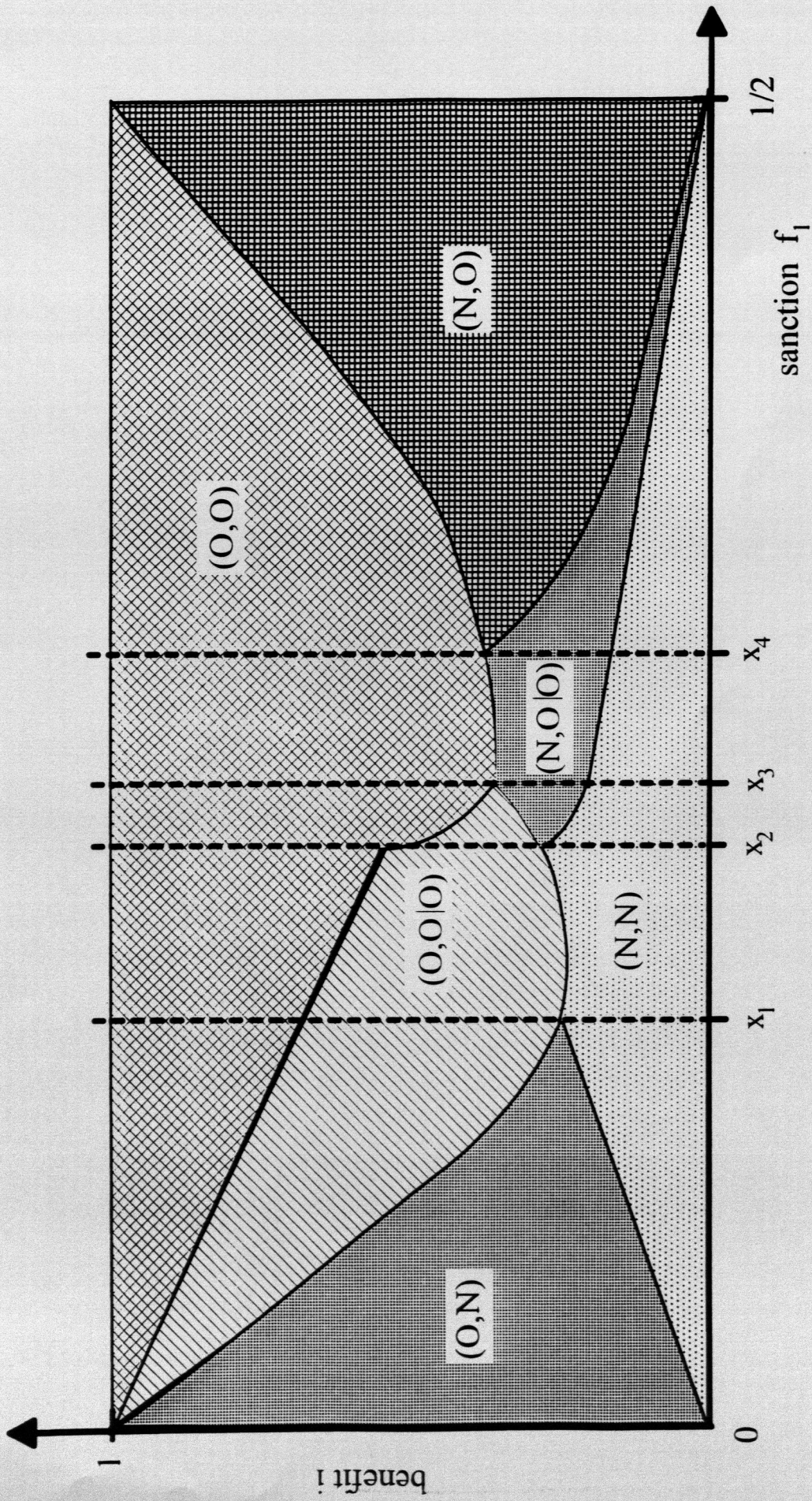


Figure 3: Active and Passive Coordination



$$B=1/2 \quad x_1 = 0.219 \quad x_3 = 0.270$$

$$x_2 = 0.262 \quad x_4 = 0.306$$

Figure 4: Individuals' behavior