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**Network Effects and Learning Effects
in a Heterogeneous Dyopoly**

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Generally, he who occupies the field of battle first and awaits his enemy is at ease; he who comes later to the scene and rushes into the fight is weary. (Sun Tzu)¹

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I. Introduction

In recent times there has been a renewed emphasis on the historical impact on economic development (David 1985, Arthur 1989, Blankart/Knieps 1991). History matters. Two areas of study where this seems obvious are learning and network effects.

Learning effects (experience curve, progress curves) make today's average costs depend negatively on past sales². Sales today depend positively on past sales when network effects are present³. There-

¹Sun Tzu, The Art of War, (translated by Samuel B. Griffith), Oxford University Press, London, Oxford, New York 1963, S. 96.

²For a survey see Yelle 1979.

³An illuminating survey of ideas and problems related to network effects is Farrell/Saloner (1987).

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fore a small "lead down the experience curve" gives a decisive advantage over rivals, as the management literature points out. Analogously, network effects should present the first mover with ample opportunity to dominate the market.

Since network and learning effects give rise to similar strategic interdependences between firms it may be apt to analyse them in a parallel fashion. Before I go on to present my model some general remarks on modelling elements in network and learning effect duopoly models seem to be in order. For these models, I want to argue that historic asymmetries on the one hand and heterogeneity on the other hand are important modelling features to be included.

Historic small events

Since historic small events can easily topple the market outcome one way or the other I suggest that network and learning effect duopoly models be built to explicitly incorporate historic asymmetries between the firms. In the literature, there are models that are symmetric from the outset and generate symmetric equilibria⁴ (Fudenberg/Tirole 1983; Smiley/Ravid 1983; Ghemawat/ Spence 1985; Chu 1988). Would the equilibria in these models be upset in a big way if small historic asymmetries were introduced? Specifically, instead of symmetric equilibria, would one firm become the monopoly and push out the others, i.e. do the market parameters make for a natural monopoly?

As we will show later, a general answer to this question cannot be obtained but depends, i.a., on the strength of the effect, on the extent to which the effect is firm specific and on whether the products are more or less heterogeneous. However, it can be shown that the model by Fudenberg and Tirole is not robust against small historic asymmetries.⁵

⁴A critique of the assumption of identical firms in the case of learning effects is also brought forward by Dasgupta/Stiglitz (1988, 248).

⁵I solved a simplified version of the Fudenberg/Tirole model with

How then can "historic small events" be included in models dealing with network or learning effects? Two different ways come to mind. First, the arrival time of firms to the market could vary as Bauer (1986), Ross (1986), Spence (1981), Smiley/Ravid (1983) (a limit price model), Lee (1975) (an evolutionary limit entry model) and Conrad/Duchatelet (1987) suggest. Thus the first mover has some time to produce and gain a competitive advantage. Different arrival times could be justified (and maybe even made endogenously) by referring to the R&D procedures and the like involved (Spence 1981, p. 53, FN 7).

Second, historic sales could be different for the companies. Thus, one firm has an advantage of the installed base. This modelling technique has been employed in the simulation work done by Wiese (1990).

Instead of modelling a minor "lead down the curve" some learning effect models include asymmetries by letting firms have different rates of learning, initial cost (Bauer 1986) or discount rates

two periods where I retain their linear demand function (Preis = 1-Quantity sold), set the number of firms to two (I call them x and y) and do away with discounting, but include historic sales for both firms, x_0 resp. y_0 . The average costs read $k-e x_0$ in the first period and $k-e(x_0+x_1)$ in the second period, where k is the basic average cost without learning and e the learning rate. Apart from other interesting results, the precommitment model shows the following: (i) Neither firm produces if $k \geq 1+e \max(x_0, y_0)$. (ii) If e (rate of learning) is bigger than 1 (but smaller than 2 to ensure a negative definite Hessian and best answers to be finite), only those equilibria are stable that have one or the other firm being the monopolist in both periods. (iii) If e is smaller than 1 (which is necessary for a stable equilibrium that has both firms producing in both the Tirole/Fudenberg and this model) than both firms do produce in both periods iff $\frac{ey_0-(1-k)(1-e)}{2e-e^2} < x_0 < (2-e)y_0+(1-k)[1/e-$

1]. This guarantees that historic sales are not too far apart. As a corollary we get that a potential competitor ($y_0=0$) can overcome the entry barrier if the incumbent's initial market share (x_0) is smaller than $(1-k)[1/e-1]$ which may be the case for small basic average cost (k) as compared to the maximal willingness to pay (1) and for a small learning effect rate e .

(Teng/Thompson 1983). Albach (1987) has one firm apply a myopic cost plus policy whereas the other optimizes over the time horizon. With network effects, average cost are modelled as different between firms and periods (Katz/Shapiro 1986a and 1986b), consumers have different exogenous expectations about the technologies final network sizes (Katz/Shapiro 1985), firms or customers prefer one technology (one standard) over the other and contemplate switching to the more preferred one (Farrell/Saloner 1985 resp. Farrell/Saloner 1986).

Heterogeneity of products

A second modelling feature that seems important is heterogeneity of the products. Not only is heterogeneity in one form or the other an economic fact. But also, and more importantly, heterogeneity can be part of a defense strategy against monopolizing behaviour of an opponent. The simulation model by Wiese (1990) is the only one which focuses the installed base of an opponent. Heterogeneity is missing in all models known to me except those by Farrell/Saloner (1986), Albach (1987) and Wiese (1990). Only the last, however, concentrates on heterogeneity as a strategic factor.

According to my knowledge, there is no oligopoly model for network or learning effects where a precommitment equilibrium (open loop) or a perfect equilibrium (closed loop) is applied that incorporates heterogeneity as well as a historic asymmetry. The only exception may be the simulation model by Wiese (1990).⁶ Thus, the purpose of this paper is to present dynamic network effect and learning effect duopoly models including the two modelling features "historic asymmetry" and "heterogeneity". To be more precise, the model incorporates four elements:

⁶The modelling procedure involves some inconsistency. Furthermore, the models presented here are analytic rather than by simulation.

Four parameters

First, historic asymmetry is modelled in the same way as in the simulation model in Wiese (1990). Initial sales of the two firms are a parameter of the model thus reflecting a historic advantage of one of the firms. (Abbreviation x_0, y_0)

Second, the heterogeneity factor reflects the cost to consumers of not being able to buy the particular product characteristics they prefer. Heterogeneity allows for situations where one firm has the historic advantage, but still does not monopolize the whole market. (Abbreviation h)

Third, the strength of the (learning resp. network) effect is measured by a parameter e . In the case of learning, this parameter indicates how much average costs are reduced with every additional unit of past sales. In the case of network effects, the parameter indicates how much consumers value the network sizes of the goods as compared to prices or product placement. (Abbreviation e)

Fourth, a specificity parameter determines whether the effects rest on a company's own sales only or also (partly) on the sales of the other firm, i.e. whether effects are firm specific or more or less industry specific. Less than total specificity arises, e.g., when learning spills from one company to another through the so-called Silicon Valley effect (by employees of one firm transferring knowledge to another firm when changing jobs⁷). With network effects, less than total specificity means partial or total compatibility. If computers are compatible, consumers are not interested in the network size of a special brand but rather in the network size of all compatible computers together. (Abbreviation s)

How these parameters work together will be shown in chapter II. Chapter III will then report some important findings while some

⁷A second example is learning that happens in supplier firms. All firms in the industry profit from learning made possible by larger orders by one of the firms.

concluding remarks will be made in chapter IV.

II. The models

I want to present two models (but will refer to results of variant models sometimes). Both are heterogeneous two period duopoly models with firms competing in price paths, i.e. a precommitment equilibrium instead of a perfect equilibrium is calculated. One of the models incorporates learning effects, the other network effects. Both these effects are modelled in the most simple possible manner, i.e. linearly. Since the general setup is very similar, I will define the relevant terms to be applied to network as well as to learning effects.

A list of abbreviations

Let me first give a list of the abbreviations and definitions. (You can browse through them first and use them for reference later.)

h	heterogeneity factor, $h > 0$
e	strength of (learning resp. network) effect, $e \geq 0$
s	specificity, $0 \leq s \leq 1$ s=0 corresponds to complete learning spillovers in case of learning resp. to compatibility in case of network effects. s=1 corresponds to purely firm specific learning (no spillovers) resp. to incompatibility.
es	effect product
λ	effect-heterogeneity-ratio
	$\lambda = \frac{es}{2h}$
k	average cost (without learning)
t	period, $0 \leq t \leq 2$
p_t	price of good x in period t
q_t	price for good y in period t

b_t modified price of good x

$$b_t = \frac{p_t - k}{2h}$$

g_t modified price for good y

$$g_t = \frac{q_t - k}{2h}$$

x_t demand for good x in period t, $0 \leq x_t \leq 2$, in case of learning

$$x_t = \frac{q_t - p_t + h}{2h} = 1/2 + g_t - b_t, \quad t \geq 1$$

resp. in case of network effect

$$x_t = \frac{q_t - p_t + h + e s (x_{t-1} - y_{t-1})}{2h}$$

$$= 1/2 + g_t - \lambda y_{t-1} - (b_t - \lambda x_{t-1}), \quad t \geq 1$$

x_0 parameter

y_t demand for good y in period t, $0 \leq x_t \leq 2$, in case of learning

$$y_t = \frac{p_t - q_t + h}{2h} = 1/2 + b_t - g_t, \quad t \geq 1$$

resp. in case of network effect

$$y_t = \frac{p_t - q_t + h + e s (y_{t-1} - x_{t-1})}{2h}$$

$$= 1/2 + b_t - \lambda x_{t-1} - (g_t - \lambda y_{t-1}), \quad t \geq 1$$

$$y_0 = 1 - x_0$$

P_t Profit for firm x in period t in case of learning

$$P_t = [p_t - k + e(x_{t-1} + (1-s)y_{t-1})] \frac{q_t - p_t + h}{2h}$$

resp. in case of network effects

$$P_t = [p_t - k] \frac{q_t - p_t + h + e s (x_{t-1} - y_{t-1})}{2h}$$

$$t=1,2$$

P Cumulated profit for firm x, $P = P_1 + P_2$

B_t modified profit for firm x in period t

$$B_t = \frac{P_t}{2h}$$

B modified cumulated profit for firm x

$$B = \frac{P}{2h}$$

Q_t profit for firm y in period t

Q Cumulated profit for firm y

G_t modified profit for firm y in period t

$$G_t = \frac{Q_t}{2h}$$

G modified cumulated profit for firm y

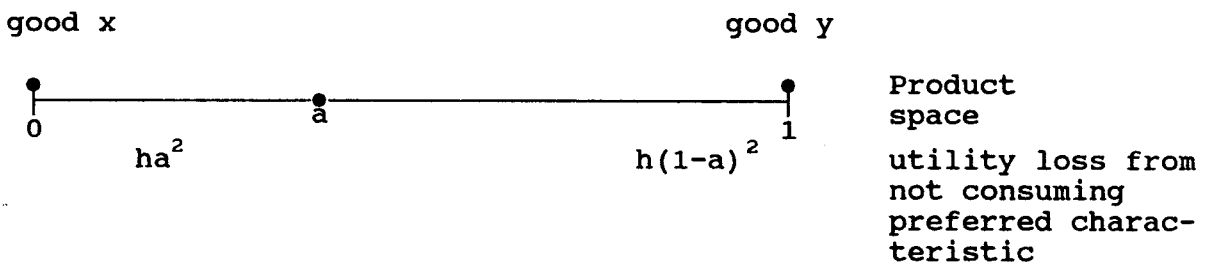
$$G = \frac{Q}{2h}$$

List 1: definitions

Modelling demand

Heterogeneity (abbreviated h) is modelled by using the Hotelling linear unit space in both periods. Consumers, who are distributed uniformly along the product space, buy exactly one of the two products located at the ends of the product space.⁸ This implies that overall demand is totally inelastic. A consumer at point a in the product space suffers a utility loss of ha^2 when consuming good x or of $h(1-a)^2$ when consuming good y from not consuming the particular characteristic he prefers (a). Picture 1 gives a graphic representation of the linear product space.

⁸Alternatively, we could consider firms who are located more or less apart from each other. However, the resulting formulas are much more complicated. I will therefore look on h as a parameter of heterogeneity. Hotelling's linear city model is used to derive a consistent demand curve.



Picture 1

The consumers compare the utility indices of the two goods, i.e.

"- price - utility loss"

in case of learning resp.

"- price - utility loss + network size gain"

in case of network effects

and buy the good that commands a higher utility index.

To calculate demand we have to find the consumer who is just indifferent between buying x and buying y. Doing this yields the demand functions in the above list.

However, we have to be careful to restrict sales to nonnegative amounts. Doing this, the demand function in case of learning, for example, is as follows:

$$x_t = \begin{cases} 1/2 + g_t - b_t & \text{falls } |g_t - b_t| \leq 1/2 & \text{(i)} \\ 0 & \text{falls } b_t > g_t + 1/2 & \text{(ii)} \\ 1 & \text{falls } b_t < g_t - 1/2 & \text{(iii)} \end{cases} \quad t = 1, 2$$

From now on we will always assume (or force) $|g_t - b_t| \leq 1/2$. The justification for this is that we want to solve the model for Nash equilibria. (ii) cannot be part of a Nash equilibrium since firm y could increase its price, up to $b_t - 1/2$ for example, without losing any sales. Analogously, firm x can increase profits in case of (iii). Thus neither (ii) nor (iii) can be part of a Nash equilibrium and demand will always be as put down in list 1.

Profit functions

In each period t , $t=1,2$, profits for firm x are

$$P_t = [p_t^{-k+e}(x_{t-1}+(1-s)y_{t-1})] \frac{q_t - p_t + h}{2h}$$

in case of learning resp.

$$P_t = [p_t^{-k}] \frac{q_t - p_t + h + e s (x_{t-1} - y_{t-1})}{2h}$$

in case of network effects.

Note that learning and network effects are modelled linearly and refer to last period's sales only. In case of learning, this may be interpreted as instant depreciation of the knowledge capital. In case of network effects, one could assume that the prices are not purchase prices but rather rent.

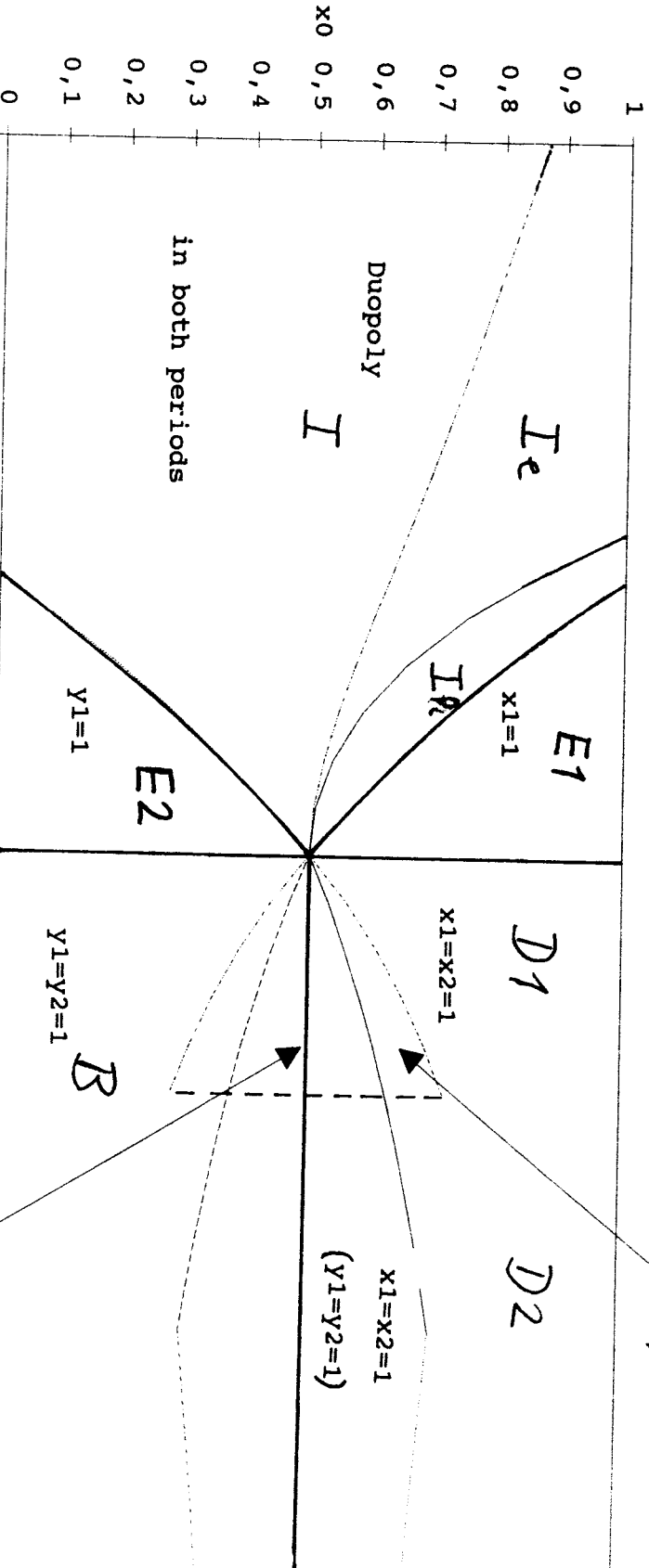
Cumulated profits P are the sum of P_1 and P_2 , disregarding discounting of second period's profits. Since maximizing P with respect to p_1 and p_2 is equivalent to maximizing B with respect to b_1 and b_2 , we will often refer to the modified variables instead of the "real" ones. The appendix shows some steps on the way to solving the model.

III. Results

1. Results with learning effects

As it turns out, the general distribution of Nash equilibria depends only on two parameters, on λ and on x_0 . Picture 2 (next page) tells the distribution of Nash equilibria. Thus, if firm x is the monopolist in both periods we have $x_1=x_2=1$ (picture 2), whereas if firm x is monopolist in period 1 only we get $x_1=1$ (picture 2).

Distribution of Nash equilibria
in a precommitment
learning effect duopoly model



$x_1 = x_2 = 1$
 $(y_2 = 1)$
 $(0 < x_1, x_2 < 1)$

Effect heterogeneity ratio,
i.e. $e s / (2h)$

$2,5$
 $(x_1 = x_2 = 1)$
 $(0 < x_1, x_2 < 1)$

in both periods

For every combination of these parameters, a Nash equilibrium exists. Furthermore, there are constellations in which multiple Nash equilibria occur.

First, in monopolistic situations (one or both periods) we typically encounter many Nash equilibria. Here I consider the non producing player as "unimportant" in the sense of Harsanyi and Selten (1988). Whereas in their monograph on equilibrium selection they insist on strict payoff dominance, they tentatively propose in their closing chapter to put more weight on the payoffs of the "more important" player (Harsanyi/Selten 1988, p. 362). In this case, this amounts to the criterion of weak payoff dominance. Thus in my model, the Nash equilibrium with highest prices in those periods when only one firm produces gets selected.

Second, in the areas with $\lambda > 1.5$ and with the historic sales not too far apart (see picture 2) there are two or three equilibria. Even weak payoff dominance does not render a selection here. Though multiplicity of equilibria is an important result in itself, I will apply the following criterion:

Since the players are asymmetric in x_0 , I propose to give the power of selection to the stronger player, i.e. to x in case of $x_0 > 1/2$ and to y otherwise. This procedure could be rationalized by assuming that a larger historic market share indicates a better knowledge of the market and a better ability to commit on an equilibrium.⁹

This criterion results in the distribution of selected Nash equilibria as they can be seen from picture 2: Firm x is the monopolist in both periods when $\lambda \geq 3/2$ and $x_0 > 1/2$.¹⁰

⁹This ability to commit is much weaker than the one demanded of a leader in a leader-follower model. A model of that variety has also been solved.

¹⁰It can be shown that the selected equilibria are the only ones that make for continuous horizontal cuts through picture 2, i.e. considering outputs as functions of λ for a given $x_0 \neq 1/2$ output changes are continuous only when the selected equilibria apply.

Not only equilibria, also results have to be selected. Since I do not want to bore you with pages of prices and modified prices and corresponding quantities and profits, I will proceed by pointing out interesting results and interpreting them.

Proposition 1:

For firm x, profits in a precommitment equilibrium are

- | | | |
|-----------------|-------------------------------------|-------|
| h | in case of $e=0$ or $s=0$, | (i) |
| $h-es/4$ | in case of $x_0=y_0$, | (ii) |
| $2esx_0-2h$ | in case of a $\lambda < 2$ monopoly | (iii) |
| $2h+es(2x_0-1)$ | in case of a $\lambda > 2$ monopoly | (iv) |

Case (i) is the usual non-effect duopoly where profits depend positively on heterogeneity. This positive dependence follows from the lessened rivalry. If h were zero, we would get the familiar Bertrand-paradox: even in duopoly, firms cannot commit to prices above their average cost.

One may wonder, however, why not only the absence of learning effects but also the absence of specificity (with learning effects) generates this result. In this model, prices can always be written in the form $k + \dots$ so that profits are not depressed by higher average costs for both firms. Learning effects that do spillover totally have the effect of reducing average cost in each period for both firms by e. Since prices are reduced by e also, profits are not affected. This is not a result that can be expected to hold in general. It is generated here because the overall demand is inelastic, so that a reduction in prices does not lead to an increase in demand.

As with case (i), case (ii) also reflects a symmetric situation of the two firms. Here we note that profits are lower than in the non-effect case. Whereas in case (i) the nettoed price¹¹ is $k+h$, in case

¹¹The nettoed price is the price net of cost reductions through learning.

(ii) it is lower, at $k+h-es/2$. The cost reduction possibility leads to a price war that reduces prices more than average prices are lowered. In this case, if $s>0$ the intensity of price competition is higher since undercutting the competitor seems to yield more of a gain than in the case of freely transferable learning. Though we still have to keep the peculiarly inelastic demand in mind, firms can actually suffer from an environment that allows fast learning, especially if they are not in a very strong position. Or, referring to s rather than e , they have an incentive to institutionalize knowledge transfer as a means of taming competition. Cooperation in research and development or patent information systems are just two examples of how this might be effected.

As an aside it is interesting to note that profits are $h-es/4$ in a perfect instead of $h-es/3$ in a precommitment equilibrium. Why are profits lower when firm think in terms of backward induction, i.e. when they are aware of the fact that first period prices influence the competitor's second period choice? Starting from a precommitment equilibrium we can see how this comes about. Firm x considers lowering his first period price. A lower first period price will lower the competitor's first period sales and hence increase y 's second period average cost. Higher average cost will then lead to higher prices by firm y in the second period. This effect is welcome to firm x that will indeed lower his first period price. Since both will proceed along this lines profits for both will suffer.

Proceeding from symmetric situations to monopoly situations, we note that the profits depend on whether λ is smaller or bigger 2. In case (iii) we get $2esx_0-2h$. This formula has a neat interpretation. The monopolizing firm can conquer the market more easily, when the effect product and the historic sales are large and when the competing products are homogeneous. With homogeneous products it is easy to lure away costumers that prefer the rival's product by undercutting the rival marginally. When the effect product (es) is large, cost reductions for past sales (x_0) are large and investment in first period's sales is worthwhile.

Case (iii) seems to be the typical monopoly case. This is confirmed by the profit function in area E1¹², which also exhibits a positive dependence of profits on the effect product and a negative dependence on heterogeneity. However, the profit $2h+es(2x_0-1)$ in the monopoly case (iv) tells quite another story. Here profits depend positively on h. This is astonishing in light of the interpretation given for case (iii). With λ being higher than 2, the market has become very volatile. Now each firm, even the smaller one, sees a chance to undercut the rival in the first period and enjoy the second period's profits. Undercutting the rival and taking the whole market in period 1 is easier the more homogeneous the products are. Thus the monopolist has to lower his first period's price when products are homogeneous. If firm x is the big firm historically, it enjoys the advantage of the installed base $2x_0-1 > 0$. Then it profits from high effect products. If it is the smaller firm and still manages to become the monopolist (which is not a selected equilibrium) it actually suffers from large learning opportunities. Having said all this we still have to keep in mind that the parameters which allow the domination of the market require h to be relatively low and e and s to relatively high.

¹²Profits for firm x are $P = -h + es(2x_0 - 3/2) + (es)^2 / (6h + 2h(1/2 + \lambda/3))^2$.

Proposition 2:

A potential competitor y with $y_0=0$ can enter the learning effects market

in both periods	for $\lambda < 0.93^{13}$,	(i)
in the second period	for $0.93 \leq \lambda < 3/2$ and	(ii)
will keep outside	for $\lambda \geq 3/2$.	(iii)

If the actual competitor is the leader and the potential competitor the follower¹⁴, the leader will

allow entry in both periods	for $\lambda \leq 1.64$	(iv)
prevent entry	for $\lambda > 1.64$	(v)

Cases (i) and (iv) are those cases in which it does not pay the actual competitor to prevent entry in any period. The effect-heterogeneity ratio is too low to hold onto the market monopolistically.

In case (ii), the potential entrant is kept out in order to allow the actual competitor a comfortable lead in the second period. In the second period, however, it does not pay to prevent entry. Here investments do not take place anymore.

Cases (iii) and (v) are the entry preventing cases. The actual competitor does not allow entry in either period. This is worth its while only if the effect-heterogeneity ratio is high, making domination easy.

It may seem counterintuitive that the actual competitor-leader ("Stackelberg case") does seem to behave less aggressively than the actual competitor in the cases (i) to (iii) ("Cournot case"). In fact, by noting the recommendations for aggressive strategies in the management literature and by referring to the Stackelberg model as

The exact number is $3(\sqrt{5}-1)/4$.

¹⁴In the leader-follower model firm y is the leader who takes the reaction price path of firm x into account.

"the classic model of aggressive behavior", Ross (1986) commends the Stackelberg model for the analysis of learning effect markets. Why does aggressive behavior not show here?

The reason is, of course, that I modelled price instead of quantity competition. The Stackelberg model reflects aggressive behavior only when firms compete in quantities. When they compete in prices, in the non-effect case, the leader actually raises the price to let the competitor follow suit. Thus, the idea of the Stackelberg model is not aggressive behavior, as Ross claims, but rather (and just) the fact that the leader takes the reaction of the follower into account. Having said this, it may well be the right modelling approach to attribute the power to commit to "off-reaction-curve" prices to the firm already in the market.

When dealing with learning or network effects, the leader (competing in prices) is torn between two forces. On the one hand, he is tempted to raise prices knowing the other will (partly) follow; on the other hand he is in a position to commit to low prices that will gain him a "lead down the experience curve". In fact, the leader does price more aggressively than the follower in many parameter regions, especially those where the historic market shares are not too far apart. Here he then monopolizes a market in one or both periods that would otherwise not be monopolized or even fall to the competitor.

Proposition 3:

Profits for firm x depend positively on the heterogeneity h when in I (with the exception of a small stripe along $E1$ called I_h) and $D2$ (see picture 2).

If there are no learning effects, increasing heterogeneity is clearly beneficial. The more the markets are separated, the bigger the profits for the competitors. With learning effects, heterogeneity is a mixed blessing. In order to pursue an aggressive strategy it is easier to gain a significant market share when

individuals are not brand loyal.

In terms of picture 2, the profits of firm x depend negatively on h in E1 and I_h and in D1. E1 and D1 are the areas where an aggressive buildup leads to a monopoly in the first period that makes for significant learning effects in the second period. As noted earlier, in D2 the learning opportunities are so prevalent that the monopoly may even seek protection through market niches.

Proposition 4:

Profits for firm x depend positively on the effect product e_s when in I_e, E1 or D.

When e is high two effects can be noted. On the one hand, e reduces costs (cost effect), on the other hand price competition is made tougher by efforts of building installed bases (antagonistic effect). If s is zero these two effects just cancel and profits do not depend on e (proposition 1). It should be noted, however, that this does not hold in general but follows from the inelastic demand curve which stresses the antagonistic effect.

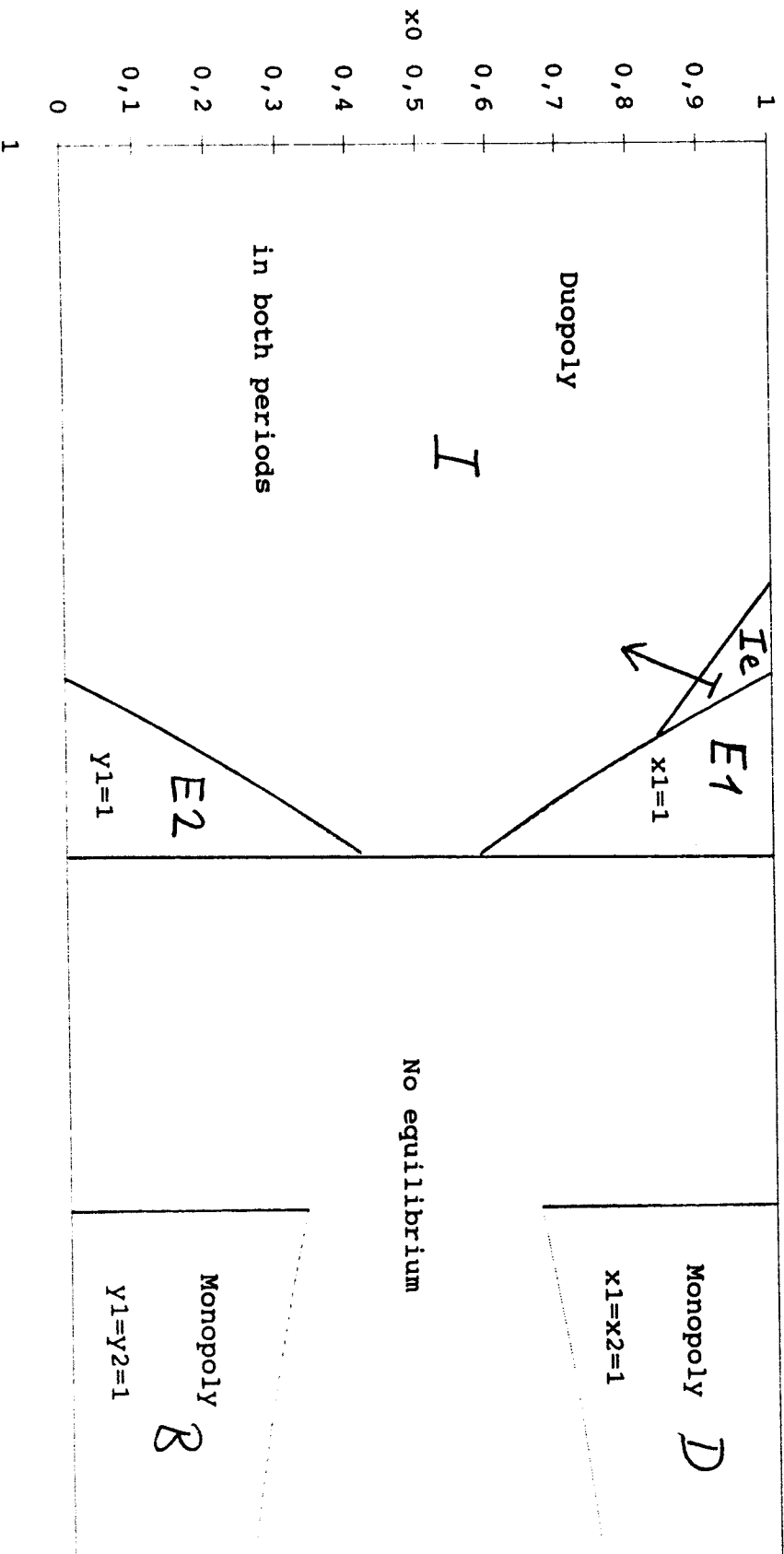
If s is not zero, profits depend positively on e_s in most of the upper part of picture 2. Here firm x is the historically large firm and profits from the historic sales and from investment possibilities in the installed base. However, λ and x_0 have to be big enough so that an investment pays and the other firm gets more or less subdued.

2. Results with network effects

As with learning effects the distribution of equilibria in the depends on λ and x_0 . However, as picture 3 reveals, there are some parameter regions where no Nash equilibrium exists. As in picture 2, in picture 3 I is the region where both firms produce, in E1 firm x is a monopolist in period 1, in E2 firm y is a monopolist in period

1, and D resp. B are the regions where firm x resp. firm y monopolizes the markets in both periods, too. Selection problems do not arise apart from the need of applying the weak dominance payoff criterion in the case of monopolizations in one or two periods.

Distribution of Nash equilibria in a network effect duopoly model



Effect heterogeneity ratio, i.e. $e s / (2h)$

The network effect model is more "dynamic" than the learning effect model. The reason for this may be that learning effects depend on one's own sales (plus some sales of the other firm in case of spill-overs), whereas with network effects, the demand depends on the difference in sales, thereby accentuating the rivalry, especially with the inelastic overall demand. Formally, this shows in the fact that the Hessian matrix of the two firms is indefinite for $\lambda > 1$ (as compared to $\lambda > 2$ with learning effects).

The following propositions report some of the findings.

Proposition 5:

For firm x , profits are

- | | | |
|-----------------|--------------------------------------|------|
| h | in case of $e=0, s=0$ or $x_0=y_0$, | (i) |
| $2h+2esx_0-2es$ | in case of a monopoly | (ii) |

As with proposition 1, case (i) is the usual non-effect duopoly where profits depend positively on the extent to which firms can separate their markets through brand loyalty, etc. Since consumers compare differences in prices with differences in network effects and overall demand is inelastic¹⁵, the non-effect case arises also when historic sales are the same for both firms.

The monopoly profits in case (ii) depend positively on h and negatively on the effect product es . This is the defensive parameter dependence. Relatively high heterogeneity and not too large effects make it more difficult for the rival to try to dominate himself.

Proposition 6:

A potential competitor y with $y_0=0$ can enter the network effects market

- | | | |
|----------------------|---------------------------------|-------|
| in both periods | for $\lambda < 0.75$, | (i) |
| in the second period | for $0.75 \leq \lambda < 1$ and | (ii) |
| will keep outside | for $\lambda \geq 3/2$. | (iii) |

¹⁵Very much as in Katz/Shapiro 1986a and 1986b.

The interpretation of this proposition is much the same as the one in the case of learning effects (proposition 2). Thus the main results between these two markets verify the intuition that the analysis can proceed very much along the same general lines.

Proposition 7:

For the historically large firm, profits depend positively on e in I_e and in E_1 (picture 3).

Whereas this result is not surprising in light of the interpretation given in proposition 4, I would like to venture an explanation of the cooperation between IBM and Apple that is underway now. IBM is not so big anymore (or does not consider itself big enough) that it can hope to profit from increased competition by a high network effect product.¹⁶ Thus it aims at making its product more compatible (lowering s) with one of its main incompatible rivals. This may happen over quite a period of time, and will probably not involve drastic decisions of making the two product lines compatible in every respect. But cooperation will have the effect of making the transition from one product to the other easier for the consumer. In terms of picture 3, the beginning of the arrow is in the parameter region IBM used to be in. Here profits depend positively on the effect product. However, as time proceeded IBM lost some market share and/or the network characteristic is not as important as it used to be¹⁷. IBM is now in a region where the profits depend negatively on the effect product. Reducing specificity (i.e. increasing compatibility) increases profits.

¹⁶Remember that IBM tried to fight its "clones" by producing a new line of computers that are proprietary.

¹⁷One reason for this could be the fact that the installed base for many computer types and hence the software available for these types is rather big already.

IV. Concluding Comments

The modelling approach pursued in this paper rests mainly on two premises. First, a historic asymmetry is allowed for. Second, products are modelled heterogenous. I think that network or learning effects in a duopoly context should be analyzed using these two ingredients whenever possible.

The analysis in this paper can be extended in a number of ways.

First of all, modelling total demand inelastically stresses the rivalrous nature of network and learning effect competition. More general demand functions that allow for demand to react positively to overall low prices should be included (while maintaining heterogeneity).

One could try to solve the models for more than two periods or more than two firms. With more than two firms to analyze the circular city would be the appropriate product space. With two firms the linear and the circular city produce the same results.

Interesting results may also obtain from discounting of second periods profits, including consumer switching costs, allowing for different average costs between firms or different quality of the goods or by allowing for non-instantaneous depreciation of learning or network capital, to name a few.

Appendix: Steps to solving the precommitment learning effect duopoly model:

For firm x, the profit function in period t reads:

$$P_t = [p_t - (k - e (x_{t-1} + (1-s) Y_{t-1}))] x_t$$

$$= 2h[b_t + \lambda(1/s - 1/2 + g_{t-1} - b_{t-1})] [1/2 + g_t - b_t] \quad (t = 1, 2)$$

Since maximizing P with respect to p_1 and p_2 is equivalent to maximizing B with respect to b_1 and b_2 we get the following modified profit function B:

$$[b_1 + \lambda(1/s - 1 + x_0)] [1/2 + g_1 - b_1]$$

$$+ [b_2 + \lambda(1/s - 1/2 + g_1 - b_1)] [1/2 + g_2 - b_2]$$

For $\lambda < 2$ the Hessian of the above program is negative definite. Choosing the best answer to a given price pair of firm y can entail a corner, an edge or an interior point of the price square described by

$$g_1 - 1/2 \leq b_1 \leq g_1 + 1/2 \text{ and}$$

$$g_2 - 1/2 \leq b_2 \leq g_2 + 1/2.$$

The Kuhn-Tucker conditions of this maximization program are necessary and sufficient. The reaction function of firm x is given by the following table.

Case	Condition	best answer
<i>Corners as best answers</i>		
A	$g_1 \geq -\lambda/s + \lambda(1-x_0) + 3/2$ $g_2 \leq -\lambda/s - 1/2$	$(g_1 - 1/2, g_2 + 1/2)$
B	$g_1 \geq -\lambda/s - \lambda x_0 + 3/2$ $g_2 \geq -\lambda/s + 3/2$	$(g_1 - 1/2, g_2 - 1/2)$
C	$g_1 \leq -\lambda/s - \lambda x_0 - 1/2$ $g_2 \geq -\lambda/s + \lambda + 3/2$	$(g_1 + 1/2, g_2 - 1/2)$
D	$g_1 \leq -\lambda/s + \lambda(1-x_0) - 1/2$ $g_2 \leq -\lambda/s + \lambda - 1/2$	$(g_1 + 1/2, g_2 + 1/2)$
<i>Edges as best answers</i>		
E	$g_2 \geq -(\lambda+2)/s - (4x_0-3)/2 - (2g_1-3)/\lambda$ $-1/2 - \lambda/s \leq g_2 \leq 3/2 - \lambda/s$	$(g_1 - 1/2,$ $-\lambda/(2s) + 1/4 + g_2/2)$
F	$g_2 \geq -\lambda(\lambda+2)/(2s) - (2\lambda^2 x_0 + \lambda(2g_1-3) - 6)/4$ $-\lambda/s - 1/2 - \lambda x_0 \leq g_1 \leq -\lambda/s + 3/2 - \lambda x_0$	$(-\lambda/(2s) + 1/4 + g_1/2,$ $-\lambda x_0/2, g_2 - 1/2)$
G	$g_2 \leq -(\lambda+2)/s + (2\lambda^2 + \lambda(3-4x_0) - 4g_1 - 2)/(2\lambda)$ $-1/2 - \lambda/s + \lambda \leq g_2 \leq 3/2 - \lambda/s + \lambda$	$(g_1 + 1/2, \lambda/2,$ $-\lambda/(2s) + 1/4 + g_2/2)$
H	$g_2 \leq -\lambda(\lambda+2)/(2s)$ $-(2\lambda^2(x_0-1) + \lambda(2g_1-3) + 2)/4$ $-\lambda/s - 1/2 + \lambda(1-x_0) \leq g_1 \leq -\lambda/s + 3/2 + \lambda(1-x_0)$	$(-\lambda/(2s) + 1/4 + g_1/2,$ $+\lambda(1-x_0)/2, g_2 + 1/2)$
<i>Inner point as best answer</i>		
I	$g_2 \leq -(\lambda+2)/s - (4x_0-3)/2 - (2g_1-3)/\lambda$ $g_2 \leq -\lambda(\lambda+2)/(2s) - (2\lambda^2 x_0 + \lambda(2g_1-3) - 6)/4$ $g_2 \geq -(\lambda+2)/s + (2\lambda^2 + \lambda(3-4x_0) - 4g_1 - 2)/(2\lambda)$ $g_2 \geq -\lambda(\lambda+2)/(2s) - (2\lambda^2(x_0-1) + \lambda(2g_1-3) + 2)/4$	(b_1^0, b_2^0)

Table 1

The reaction curve of firm y is the analogue of this table 1. The b and g values are interchanged and $y_0=1-x_0$ is substituted for x_0 .

The Nash equilibria are derived in a straightforward manner by considering nine times nine cases.

For $\lambda > 2$ the Hessian of the above program is indefinite. Thus we will not get a solution where both firms produce positive amounts. Getting the reaction functions in this case is much harder. In fact, I

did not bother formulating them. However, table 2 gives the necessary conditions for the "potential answer" to be part of the reaction function. This is not the reaction function since the conditions need not be intersection free with respect to interior points.

Cases	Condition	potential answer
<i>Corners as best answers</i>		
A	$g_1 \geq -\lambda/s + \lambda(1-x_0) + 3/2$ $g_2 \leq -\lambda/s - 1/2$	$(g_1 - 1/2, g_2 + 1/2)$
B	$g_1 \geq -\lambda/s - \lambda x_0 + 3/2$ $g_2 \geq -\lambda/s + 3/2$	$(g_1 - 1/2, g_2 - 1/2)$
C	$g_1 \leq -\lambda/s - \lambda x_0 - 1/2$ $g_2 \geq -\lambda/s + \lambda + 3/2$	$(g_1 + 1/2, g_2 - 1/2)$
D	$g_1 \leq -\lambda/s + \lambda(1-x_0) - 1/2$ $g_2 \leq -\lambda/s + \lambda - 1/2$	$(g_1 + 1/2, g_2 + 1/2)$
<i>Edges as best answers</i>		
E	$g_2 \leq -(\lambda+2)/s - (4x_0-3)/2 - (2g_1-3)/\lambda$ $-1/2 - \lambda/s \leq g_2 \leq 3/2 - \lambda/s$	$(g_1 - 1/2,$ $-\lambda/(2s) + 1/4 + g_2/2)$
F	$g_2 \leq -\lambda(\lambda+2)/(2s) - (2\lambda^2 x_0 + \lambda(2g_1-3) - 6)/4$ $-\lambda/s - 1/2 - \lambda x_0 \leq g_1 \leq -\lambda/s + 3/2 - \lambda x_0$	$(-\lambda/(2s) + 1/4 + g_1/2$ $-\lambda x_0/2, g_2 - 1/2)$
G	$g_2 \geq -(\lambda+2)/s + (2\lambda^2 + \lambda(3-4x_0) - 4g_1 - 2)/(2\lambda)$ $-1/2 - \lambda/s + \lambda \leq g_2 \leq 3/2 - \lambda/s + \lambda$	$(g_1 + 1/2, \lambda/2$ $-\lambda/(2s) + 1/4 + g_2/2)$
H	$g_2 \geq -\lambda(\lambda+2)/(2s)$ $-(2\lambda^2(x_0-1) + \lambda(2g_1-3) + 2)/4$ $-\lambda/s - 1/2 + \lambda(1-x_0) \leq g_1 \leq -\lambda/s + 3/2 + \lambda(1-x_0)$	$(-\lambda/(2s) + 1/4 + g_1/2$ $+ \lambda(1-x_0)/2, g_2 + 1/2)$

Table 2

To arrive at the Nash equilibria is a bit messy in this case. However, formulating the corresponding table for firm y and checking for possible Nash equilibrium candidates reduces the number of cases fast. Only monopoly cases remain. Then we have to check with the help of table 2 whether monopoly cases are indeed Nash equilibria. Details of the procedure can be obtained from the author.

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