

V65  
040  
019

WHU-Forschungspapier Nr. 5 / November 1991

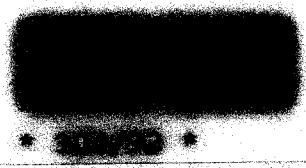
**Income Tax Progression and Redistributive  
Effect: The Influence of Changes  
in the Pre-Tax Income Distribution**

VON

*Peter Lambert und Wilhelm Pfähler*

- erscheint in: PUBLIC FINANCE/FINANCES PUBLIQUES 1992 -

Peter Lambert, Ph.D.  
Senior Lecturer  
University of York  
Heslington  
GB-York YO1 5 DD



Prof. Dr. Wilhelm Pfähler  
Lehrstuhl für Volkswirtschaftslehre  
Wissenschaftliche Hochschule für  
Unternehmensführung Koblenz  
Haus d'Estes - Heerstraße 52  
D-5414 Vallendar

# INCOME TAX PROGRESSION AND REDISTRIBUTIVE EFFECT: THE INFLUENCE OF CHANGES IN THE PRE-TAX INCOME DISTRIBUTION

by

PETER J. LAMBERT\* AND WILHELM PFÄHLER\*\*

## I. Introduction

Since the seminal article "Income Tax Progression, 1929-1948" by Musgrave and Thin, published more than forty years ago (in 1948), the redistributive impact of progressive income taxation has been visualized and measured by comparing the Lorenz curves of post-tax and pre-tax income. Interpretation of the empirical research has always rested upon an intuitive perception that the distance or area between these two Lorenz curves depends on the distribution of pre-tax income and the level and graduation of taxes. For instance, Musgrave and Thin themselves speculated that "...the less equal the distribution of income before tax, the more potent will be a (given) progressive tax structure in equalizing income" (*op.cit.* p. 510). But it was not until Kakwani's (1977a) contribution "Measurement of Tax Progressivity: An International Comparison" that the exact character of this interaction started to be analysed in more detail. Kakwani himself showed how, in the aggregate, the redistributive effect can be decomposed into tax level and progressivity components. Of course, the components of this decomposition are interdependent and themselves depend, in turn, on the two ultimate ingredients: the income tax schedule and the distribution of pre-tax income.

Indeed, any measure of the redistributive effect of progressive taxation is necessarily defined in terms of the tax schedule and the pre-tax income distribution upon which this tax schedule operates. Thus, at the formal level, each such measure can be

viewed as a different mapping of the <tax schedule, pre-tax income distribution> pair. The question then arises, how do these mappings respond when one or the other, or indeed both, of the ingredients change. That is, what happens to the redistributive effect if

- (1) the tax schedule changes while the pre-tax income distribution remains fixed; or
- (2) the pre-tax income distribution changes while the tax schedule is held fixed; or
- (3) the tax schedule and the pre-tax income distribution change together.

A solid understanding of the way the tax schedule and income distribution interact in determining redistributive effect is needed for various purposes. For the purpose of *tax policy formation* we want to know, for instance, if tax policies intended to bring about a change in the post-tax income distribution indeed work in the desired direction and, in addition, are not counteracted by (exogenous) shifts in the distribution of pre-tax income. Or we want to know if a general tax hike, needed either to meet increased public needs or to close a budget deficit, will produce an unwanted redistributive effect. Moreover, the redistributive effect might politically be judged more or less desirable depending on the general level of taxation and/or the disparity of pre-tax incomes.

From a *public choice perspective* of tax policy formation one might ask whether manipulating the pre-tax income distribution is a more promising way to serve vested interests than manipulating the tax schedule. There is far more expertise and public awareness for the tax side and its redistributive effects and, hence, far more control and/or resistance to tax policy changes. Repercussions of the pre-tax income distribution on the distribution of post-tax income, on the other hand, are not well understood. This lack of public knowledge can be exploited for partisan tax policy. Likewise, if the public is committed to "no more tax changes", politicians might want to pursue redistributive objectives by, for example, redistributing earning opportunities, influencing

wage policies, changing industrial policies or any other determinant of the pre-tax income distribution.

Finally, and most important, for the purpose of *empirical research* we want to understand why in international or intertemporal comparisons the redistributive effects of progressive taxation differ between countries and/or over time. Is it because of differences in the tax base or tax rate structure, or because of differences in the pre-tax income distribution, or because of differences in both? Likewise it is important to know whether similar redistributive effects are due to similar pre-tax income distributions and tax schedules or whether they result from vastly different constellations.

Until now, a more detailed analysis of these interaction patterns has primarily been concerned with just the first aforementioned aspect of the whole picture, namely how a *ceteris paribus* change in the tax schedule, holding pre-tax income distribution fixed, affects the redistributive effect of taxation and, more specifically, how this can in turn be traced back to tax progressivity effects. The literature has revealed that, except in very specific cases, we cannot predict the outcome of such *ceteris paribus* changes by theoretical reasoning alone (Jakobsson, 1976; Kakwani, 1977b; Hemming and Keen, 1983; Hutton and Lambert, 1983; Pfähler, 1983, 1984). These specific cases are extensively reviewed in Lambert (1989a, chaps. 7 and 9).

Another aspect, as yet neglected in the literature, relates to the second aforementioned question, of how a *ceteris paribus* change in the pre-tax income distribution, now holding the tax schedule constant, would influence the redistributive effect of progressive taxation and how this, in turn, can be traced back to tax progressivity. The primary purpose of this paper is to address this as yet neglected aspect. After a technical introduction in section II, this will be done in section III.

Our strategy in analyzing this problem is to investigate changes in the pre-tax income distribution resulting from growth in all

incomes and from transfers. The basic conclusion is negative again: except in specific albeit empirically relevant cases to be revealed by our analysis, one cannot predict the outcome of such *ceteris paribus* changes by theoretical reasoning alone. A *fortiori*, it is even more difficult, though not always impossible (as we shall show), to predict robustly the effects of simultaneous changes in the tax schedule and pre-tax income distribution upon redistributive effect and aggregate progressivity. This will become clear in section IV.

Given these largely inconclusive results of purely theoretical reasoning, a case is made for empirical (econometric) research with actual sets of empirical data or for simulations in order to reveal systematic and determinate forces at work. This not only could throw light on the outcome of empirical studies done so far, but also would offer insights into their political assessment. Thus, in the concluding section V we call for further work, both theoretical and empirical, in this area.

Since in this paper we are primarily concerned with the influence of pre-tax income changes on redistributive effect and global progressivity, the entire analysis is conducted in the Lorenz curve framework as initiated by Musgrave and Thin (1948) and Kakwani (1977a) - rather than in the alternative framework of relative concentration curves proposed by Hainsworth (1964) and Suits (1977). It is only in the former framework that redistributive effect/progressivity is measured *explicitly* by distances or areas with respect to the *Lorenz curve of pre-tax income*. Nevertheless, all theorems derived here have their equivalents in the Hainsworth-Suits framework.

## II. The Measurement of Redistributive Effect and Progressivity: Technical Preliminaries

The ultimate determinants of the redistributive effect of progressive taxation are the distribution of pre-tax income and the tax schedule operating on this distribution. Thus, let  $F(x)$  denote the distribution function,  $f(x)$  the density function and the  $\mu_x$  the arithmetic mean of pre-tax income  $x$ . For  $0 < p < 1$ , let the 100p percent poorest income units be those with income equal or less than  $y$ ; then  $p = F(y) = \int_0^y f(x)dx$ . In these terms,  $L_x(p) = \int_0^y xf(x)dx/\mu_x$  is the Lorenz curve and  $G_x = 1 - 2\int_0^1 L_x(p)dp$  is the Gini coefficient of pre-tax income.

The income tax schedule, representing the tax liability of an income unit with pre-tax income  $x$ , shall be written as  $T(x)$ . We assume a progressive schedule, i.e. the average tax rate,  $T(x)/x$ , (weakly) increases with income. Thus, liability progression  $LP(x) := xT'(x)/T(x) \geq 1$  and residual progression  $RP(x) := x[1-T'(x)]/[x-T(x)] \leq 1$  for all  $x$ . For non-taxpayers we define  $LP(x) = RP(x) = 1$ .

$T'(x)$  is the marginal tax rate and we further assume  $0 \leq T'(x) \leq 1$  and  $T''(x) \geq 0$  for all  $x$ , so that no income recipient experiences either a negative marginal tax rate or a marginal tax rate in excess of 100% and moreover the marginal rate increases with pre-tax income. This means that there is no reranking of income units in the transition from pre-tax income  $x$  to post-tax income  $x-T(x)$ .<sup>1</sup>

Now, let this tax schedule operate on the given distribution of pre-tax income (fig.1a). The arithmetic means of taxes and post-tax incomes may be denoted  $\mu_T$  and  $\mu_{x-T}$ , respectively. The Lorenz curves for the distributions of taxes and post-tax incomes are  $L_T(p) = \int_0^y T(x)f(x)dx/\mu_T$  and  $L_{x-T} = \int_0^y (x-T(x))f(x)dx/\mu_{x-T}$ , respectively (see fig.1b). Let  $G_T$  and  $G_{x-T}$  be their respective Gini coefficients. The total tax ratio is  $t := \mu_T/\mu_x$ .

FIGURE 1a AND 1b ABOUT HERE

The redistributive effect of progressive taxation can be measured either in multi-valued terms by the distance  $L_{X-T}(p) - L_X(p)$  between the Lorenz curves at each  $0 \leq p \leq 1$ , or in scalar terms by the index  $G_X - G_{X-T}$ , measuring (twice) area between these Lorenz curves (see fig.1b). Likewise, global progressivity is measured either in multi-valued terms by the distance  $L_X(p) - L_T(p)$  between Lorenz curves, or in scalar terms by the Kakwani (1977a) index  $G_T - G_X$ , measuring (twice) the area between these curves (again see fig. 1b). The distance and area measures of redistributive effect and progressivity are related as  $L_{X-T}(p) - L_X(p) = [(t/(1-t))] \cdot [L_X(p) - L_T(p)]$  and  $G_X - G_{X-T} = [t/(1-t)][G_T - G_X]$ .<sup>2</sup>

### III. The Effects of Ceteris Paribus Changes in Pre-Tax Income Distribution

What, now, is the influence upon redistributive effect and progressivity of changes in pre-tax income distribution with the tax schedule held fixed? Musgrave and Thin's suggestion that "...the less equal the distribution of income before taxes, the more potent will be a (given) progressive tax structure in equalizing income" has stood for more than 40 years without receiving analytical attention. In fact, as will emerge, their assertion is impossible to validate except in very special (though empirically relevant) cases.

For a detailed analysis, we may distinguish three scenarios of a *ceteris paribus* change in pre-tax income distribution. The first would simply entail an equiproportionate growth of all incomes. This increases mean income  $\mu_X$  but does not affect the Lorenz curve  $L_X(p)$ . The second scenario would do the opposite, i.e. keep mean income fixed while inducing an unambiguous shift, up or down, of the Lorenz curve. This is the case of (dis-)equalizing pre-tax

income transfers. The third scenario envisages changes in both the mean income and the Lorenz curve. This empirically most relevant scenario combines both income growth, equiproportionate or not, with transfers.

**a. Equiproportionate growth of pre-tax incomes**

There are certain definitive results to be found in the literature concerning the influence upon redistributive effect and global progressivity of equiproportionate pre-tax income growth when the tax schedule is held fixed. These results, demonstrated by Jakobsson (1976) and Moyes (1989), are summarized in the following

**THEOREM 1**

1.1. (Jacobsson) *The necessary and sufficient condition for the redistributive effect/global progressivity to be unaltered by equiproportionate growth of pre-tax incomes, whatever the shape of the pre-tax income distribution, is that the (given) tax schedule has everywhere constant residual/liability progression.*

1.2. (Moyes) *The necessary and sufficient condition for the redistributive effect/global progressivity to be enhanced by equiproportionate growth of pre-tax incomes, whatever the shape of the pre-tax income distribution, is that the (given) tax schedule has everywhere increasing residual/liability progression.<sup>3</sup>*

Unfortunately, in the real world we find tax schedules for which residual/liability progression increases in some ranges and decreases in others. Just look at fig. 2, which depicts in stylized form what pertains in Britain and Germany. The tax schedules of both countries (as indeed elsewhere) specify a tax exempt income causing a jump in progression at that income level. There are further jumps if, as in Britain, the tax is piecewise linear. A common feature in both countries (as indeed elsewhere) is that,



due to a maximum marginal tax rate incorporated in the income tax schedules (currently 40% in Britain and 53% in Germany), residual/liability progression approaches, but is still everywhere smaller/larger than, unity at the highest income levels.

FIGURE 2 ABOUT HERE

Thus, the implication of theorem 1 for real world taxes is that the influence of equiproportionate growth of pre-tax incomes on redistributive effect/aggregate progressivity is *inherently ambiguous*. Indeed, provided that there is a tax exempt income, taking some people out of tax, ambiguity is inevitable, as the following theorem demonstrates (see the appendix for the proof):

#### THEOREM 2

2.1. If with equiproportionate pre-tax income growth the richest person's post-tax income grows proportionately more than the average post-tax income, then redistributive effect, measured by the distance  $L_{X-T}(p) - L_X(p)$ , is reduced for low  $p$  and increased for high  $p$ . Hence the change in  $G_{X-T} - G_X$  is of indeterminate sign.

2.2 If with equiproportionate pre-tax income growth the richest person's tax liability grows proportionately less than the average tax liability, then global progressivity, as measured by  $L_X(p) - L_T(p)$ , is increased for low  $p$  and reduced for high  $p$ . Hence the change in  $G_T - G_X$  is indeterminate in sign.

In the real world, the preconditions of this theorem are met. This is because real world tax schedules approach unit progression at the highest incomes (see again fig. 2), whereas the percentage change of the average post-tax income/tax liability is always well below/above unity (see Lambert, 1989a, chapt. 8.5).

### b. Pure income transfers

Poor-to-rich transfers (or vice versa) keep mean income fixed while inducing an unambiguous shift down (or up) of the Lorenz curve, provided the transfers preserve the rank of the income units in the distribution. This, of course, is Dalton's principle of transfers. The analysis of how such transfers influence redistributive effect and progressivity can be simplified by just looking at a simple transfer between a poor man and a rich man (or woman, for that matter). The implications of this simplified analysis carry over to all transfer mechanisms inducing an unambiguous shift of the Lorenz curve of pre-tax income.

Thus, consider a small rank-preserving transfer from a poor man with income  $x_p$  to a rich man with income  $x_r > x_p$ . Let  $p_p = F(x_p)$  and  $p_r = F(x_r)$  be the ranks of the donor and recipient respectively, and assume, for the moment, that the poor man is not the poorest (i.e.  $p_p > 0$ ) and the rich man is not the richest (i.e.  $p_r < 1$ ). It is clear that the Lorenz curve for pre-tax income is shifted downwards between  $p = p_p$  and  $p = p_r$  (see fig.3). Moreover, for a given progressive tax schedule, the poor-to-rich transfer causes total taxes to rise and total after-tax income to fall, since the tax saving of the poor man with a low marginal tax rate is less than the tax increase of the rich man with a high marginal tax rate.

FIGURE 3 ABOUT HERE

Now looking at the Lorenz curve of tax payments, the transfer results in a new Lorenz curve which lies below the old one for  $p < p_p$  and above it for  $p > p_r$ . This is because the bottom  $100p_p$  percent and top  $100(1-p_r)$  percent of income units pay the same tax as before; but total tax has risen so their shares have fallen. Thus, the new Lorenz curve of tax payments intersects the old from below. The opposite property holds for Lorenz curves of post-tax incomes: the new will intersect the old from above (see fig. 3 again).

Hence, still assuming that the donor is not the worst-off income unit, and the recipient is not the best-off, the effect of the small poor-to-rich transfer is *ambiguous* on both the redistributive effect, as measured by the distance  $L_{X-T}(p) - L_X(p)$ , and global progressivity, as measured by the distance  $L_X(p) - L_T(p)$ . This is due to Lorenz curve intersections and can be seen in fig. 3. *A fortiori*, the effect on the area measures  $G_X - G_{X-T}$  and  $G_T - G_X$  is ambiguous, too.

When the transfer is, instead, from rich to poor, the arguments above and the Lorenz shifts in fig. 3 are reversed, but the essential ambiguity remains. It is only in the empirically uninteresting case of transfers between the poorest and the richest that the effects are, in fact, straightforward.

It should thus be clear that, since we cannot make a definitive prediction for the case of a simple transfer, the general case of mean-preserving inequality changes, involving sequences of transfers (see Rothschild and Stiglitz, 1973), is even more fraught. This essential ambiguity means that the Musgrave and Thin remark: "... the less equal is the distribution of income before tax, the more potent will be a progressive tax structure in equalizing income" is indeed highly speculative.

However, there do exist special conditions under which for the area measures this ambiguity arising from a transfer in either direction can indeed be resolved. These are stated in the following theorem (see the appendix for a proof):

## THEOREM 3

## 3.1. Poor-to-rich transfers:

a. Redistributive effect  $G_X - G_{X-T}$  is reduced/increased if the marginal tax rate  $T'(x_p)$  of the poor person is lower/higher than the total tax ratio  $t$ , and the rank  $p_r$  of the rich person is lower/higher than  $(1+G_{X-T})/2$ .

b. Aggregate progressivity  $G_T - G_X$  is reduced/increased if the marginal tax rate  $T'(x_p)$  of the poor person is lower/higher than the total tax ratio  $t$ , and the rank  $p_r$  of the rich person is lower/higher than  $(1+G_T)/2$ .

## 3.2. Rich-to-poor transfers:

a. Redistributive effect  $G_X - G_{X-T}$  is increased/reduced if the marginal tax rate  $T'(x_p)$  of the poor person is lower/higher than the total tax ratio  $t$ , and the rank  $p_r$  of the rich person is lower/higher than  $(1+G_{X-T})/2$ .

b. Aggregate progressivity  $G_T - G_X$  is increased/reduced if the marginal tax rate  $T'(x_p)$  of the poor person is lower/higher than the total tax ratio  $t$ , and the rank  $p_r$  of the rich person is lower/higher than  $(1+G_T)/2$ .

In Britain in 1985, for example, the parameters were circa  $t = 0.15$ ,  $G_{X-T} = 0.35$  and the lowest marginal tax rate was 29 percent. Thus, the theorem is applicable to all transfers between non-taxpayers and better-off people in the bottom 2/3 of the income pyramid, and all transfers between taxpayers and better-off people in the top 1/3 of the 1985 UK income pyramid. (see fig. 4)

FIGURE 4 ABOUT HERE

This theorem has at least two general and empirically significant interpretations: First, in real-world intertemporal comparisons,

observed trends in  $G_X - G_{X-T}$  have a bias whose sign can be identified from perceived transfer patterns. Second, for international comparisons of redistributive trends, we must expect (pre-tax) transfer patterns to play differing roles depending on the respective values of (i) the lowest marginal tax rate in relation to the total tax ratio and (ii) the post-tax Gini coefficient.

### c. Growth and transfers

Typically, real world changes of the pre-tax income distribution involve changes in both the mean and the Lorenz curve. Then either everybody gains, albeit in differing proportions; or some lose, but less than the others gain. The latter case can be conceived as a combination of growth for everybody, whether equiproportionate or not, with pure transfers. Since we have already dealt with equiproportionate growth and pure transfers in the two preceding subsections, the final new element to be grappled with is non-equiproportionate growth in all pre-tax incomes.

Thus, assume that for each person the new pre-tax income  $x_2$  is some given monotonic transformation of his previous income  $x_1$ , i.e.  $x_2 = k(x_1)$  where  $k'(x) > 0$  for all  $x$ . Crucially,  $k'(x) > 0$  for all  $x$  means that everyone maintains his or her rank in the distribution, an assumption that carries right through our analysis. Let  $g(x)$  denote the elasticity of  $k(x)$ , which is to be interpreted as the multiplicative increase in relative income differentials of neighbouring income units along the income scale due to growth. Thus, if  $g(x) = 1$  for all  $x$ , income growth is distributionally neutral (equiproportionate);  $g(x) > 1$  for all  $x$  means disequalizing growth due to expanded relative income differentials; whereas  $g(x) < 1$  for all  $x$  causes equalizing growth due to contracted relative income differentials.<sup>4</sup>

Then, for the pre-tax income growth pattern  $g(x)$ , the following theorem can be proved [see Pfähler(1982) and the appendix]:

**THEOREM 4**

4.1. Redistributive effect  $L_{X-T} - L_X$  is unambiguously enhanced (reduced) by  $g(x)$  if  $g(x_1) \geq (\leq) 1$  and  $RP(x_2) < (>) RP(x_1)/g(x_1)$  for all  $x_1$ .

4.2. Global progressivity  $L_X - L_T$  is unambiguously enhanced (reduced) by  $g(x)$  if  $g(x_1) \leq (\geq) 1$   $LP(x_2) > (<) LP(x_1)/g(x_1)$  for all  $x_1$ .

Obviously, theorem 1 on equiproportionate growth (for which  $g(x) = 1$  for all  $x$ ) is but a corollary of this theorem.<sup>5</sup> By careful inspection, at least two inferences can be drawn from this theorem.

First, since in the real world residual progression  $RP(x)$  approaches unity at the highest incomes levels, the inequalities in theorem 4.1 cannot hold at the highest income levels, either if income growth is disequalizing or if it is equalizing. Therefore, both disequalizing and equalizing growth in the real world must inevitably have *ambiguous* influences on the redistributive effect. Second, the same qualifications exactly hold also for global progressivity, namely both disequalizing and equalizing growth have *ambiguous* influences.

**IV. Simultaneous Change of Pre-Tax Distribution and Tax Schedule**

The dynamics of a market economy working in conjunction with the dynamics of the budgetary process assure that, except for very short periods, we observe in the real world continuous shifts in pre-tax income distribution, accompanied by perpetual smaller or larger revisions of the income tax schedule. This real world

scenario poses the true challenge to a meaningful interpretation of 'empirical data as well as to the formation of a rational tax policy. If we can say little that is clear-cut about the consequences of *ceteris paribus* changes, be it of the tax schedule or of the pre-tax income distribution, for redistributive effect and progressivity, then still less successfully may we expect to track the effects of simultaneous change in tax schedule and income distribution. Nevertheless, there are some definitive results, obtained by extending the approach of theorem 4, which at the least reveal the forces at work. Specifically, we obtain (see the appendix for the proof):

#### THEOREM 5

5.1 Redistributive effect  $L_{X-T} - L_X$  is unambiguously enhanced (reduced) by the change in pre-tax income distribution specified by  $g(x)$  and the change in tax schedule from  $T_1(x)$  to  $T_2(x)$  if  $g(x_1) \geq (\leq) 1$  and  $RP_2(x_2) < (>) RP_1(x_1)/g(x_1)$  for all  $x_1$  [where  $RP_i(x)$  is the residual progression of  $T_i(x)$  ( $i = 1,2$ )].

5.2 Global progressivity  $L_X - L_T$  is unambiguously enhanced (reduced) by the change in pre-tax income distribution specified by  $g(x)$  and the change in tax schedule from  $T_1(x)$  to  $T_2(x)$  if  $g(x_1) \leq (\geq) 1$  and  $LP_2(x_2) > (<) LP_1(x_1)/g(x_1)$  for all  $x_1$  [where  $LP_i(x)$  is the liability progression of  $T_i(x)$  ( $i = 1,2$ )].

This result simply generalizes theorem 4. At least two new inferences can be drawn out for the situation of simultaneous change.

First, given disequalizing income growth  $g(x) > 1$ , and any particular real world tax schedule  $T_1(x)$ , it is possible for the inequality in theorem 5.1 to hold at the highest income levels provided that the tax schedule is made more progressive: viz. by appropriate choice of  $T_2(x)$ . The intuition here is clear. In order

to enhance or maintain redistributive effect, disequalizing pre-tax income growth can be counteracted by a discretionary bolstering of residual progression. Second, and correspondingly, in order to enhance global progressivity, equalizing pre-tax income growth (if such occurs) can be counteracted by a discretionary bolstering of liability progression.

Theorem 5 also sheds light on a proposal of Kakwani (1977a), implemented by himself and others such as Tachibanaki (1981) in empirical work, to trace back post-tax distributional changes to progressivity and tax level. Starting with the general relationship  $G_{X-T} = G_X - [t/(1-t)] [G_T - G_X]$ , Kakwani showed that the percentage change in post-tax income inequality  $[dG_{X-T}/G_{X-T}]$  could be decomposed into percentage changes in pre-tax income inequality  $[dG_X/G_X]$ , aggregate progressivity  $[d(G_T - G_X)/(G_T - G_X)]$  and the total tax rate  $[dt/t]$ . As theorem 5 suggests, this decomposition procedure is unlikely to be meaningful, except in a purely accounting sense, because all components are *interdependent*.<sup>6</sup>

## V. Conclusions

Our analysis in this paper has demonstrated why it is not possible generally to predict, for real world scenarios, the influence of *ceteris paribus* changes of the pre-tax income distribution on redistributive effect and aggregate tax progressivity. Nevertheless, some definitive predictions can be made. As we have demonstrated, for some specific, potentially relevant real world constellations, helpful insights as to the interactive pattern of the forces at work do emerge from the analysis. These cast doubt on the significance of the casual interpretations to be found in many existing simulation and empirical studies (see, for example, Berglas (1971), Reynold and Smolensky (1977), Lecaillon et al. (1984), Kiefer (1985), Morris and Preston (1986) among others. Intuition cannot safely be relied upon; the subject matter lends itself to plenty of pitfalls.



Moreover, the intricate way in which income tax and income distribution interact in determining redistributive effects, as exposed in this paper, likewise must cast doubt on the scope of piecemeal tax or income policies which predominantly aim at achieving redistributive objectives. Politicians might easily find themselves confronted with the opposite result of what they have intended.

Thus, more work needs to be done in this area, both theoretical and empirical. The theoretical work should aim at an even deeper analysis of redistributive effects of composite transfers in combination with realistic pre-tax income growth patterns; it should also be directed towards a more fundamental study of the redistributive consequences of indexing devices designed to avoid "bracket-creep". Empirical work should, if it is of an econometric nature, bear in mind the interdependencies and approach the problem with techniques designed to cope with these. Simple regression analysis along the lines of the decomposition procedure of Kakwani will not serve the purpose. As things stand, the most promising approach seems to be through simulation. Here, the insights we have gained in this paper should help in identifying the appropriate variables, or combinations of variables, in terms of which to conduct the analysis.

## NOTES

\* Lecturer in Economics at the University of York and Research Associate at the Institute for Fiscal Studies, UK.

\*\* Professor of Economics at the Wissenschaftliche Hochschule für Unternehmensführung Koblenz (The Koblenz School of Corporate Management), FRG.

1 Of course, in the real world we have multiple tax schedules, for income units of different types (e.g. single/married), and reranking between income unit types is common. But this problem will not be incorporated here since it would only obscure the analysis; to seek to incorporate influences of demographic trends and social policy motivated tax changes would obfuscate the very mechanisms we are seeking to investigate. For more on this see Lambert (1989b).

2 These measures are two special cases in a more general class introduced in this journal by Pfähler (1987).

3 By increasing residual and liability progression we mean, respectively, that  $RP(x)$  decreases and  $LP(x)$  increases with income  $x$ .

4 As an example, if income is Pareto distributed, and the Pareto parameter is changed from  $\alpha_1$  to  $\alpha_2$  due to income growth, then  $g(x) = \alpha_2/\alpha_1$  for all  $x$ . This is not the only case in which  $g(x)$  would be constant for all  $x$ . If incomes are unit exponentially distributed before income growth, and Weibull distributed after growth,  $g(x)$  is also constant for all  $x$ . In fact, all possible constant values of  $g(x)$  can be obtained, by appropriately varying the Weibull parameters. See Johnson and Kotz (1970) on this.

5 The equivalent result for the Hainsworth-Suits framework is to be found in Pfähler (1983), the main difference being that the elasticities of the residual income ratio (rather than residual income) and of the average tax (rather than tax liability) come into the inequalities.

6 Separability would hold, however, thus providing for a meaningful decomposition, if the taxing authority followed a very specific indexing rule, namely to respond to any growth pattern  $g(x)$  by changing liability progression for all income units from  $LP_1(x_1)$  to  $LP_2(x_2) = LP_1(x_1)/g(x_1)$ . In the special case of equi-proportionate growth, combined with constant liability progression, separability is thus assured (see Pfähler, 1982, for more on this).

Appendix: the proofs of theorems 2, 3 and 5

We work in terms of a general attribute  $W(x) \geq 0$  of income  $x$ , substituting  $W(x) = x - T(x)$  and  $W(x) = T(x)$  for the results on redistributive effect and global progressivity respectively. Thus let  $\mu_w$  be the arithmetic mean of  $W(x)$  across income units, and  $L_w$  its concentration curve.

For each  $p \in (0,1)$  let  $j_w(p)$  be the elasticity of  $L_w(p)$  with respect to mean income  $\mu_x$  when all incomes grow equiproportionately. If  $p = F(y)$ , we have:

$$(1) \quad j_w(p) = h_w(y) - h_w(b)$$

where:

$$(2) \quad h_w(y) = \int_a^y x W'(x) f(x) dx / \int_a^y W(x) f(x) dx$$

and  $[a,b]$  is the support of  $f(x)$ . See Lambert (1989a), pages 205-209, on this, where it is also shown that  $h_w(b)$  is the elasticity, say  $e_w$ , of  $\mu_w$  with respect to  $\mu_x$ :

$$(3) \quad h_w(b) = e_w$$

Now let the elasticity of attribute  $W(x)$  with respect to income  $x$  be  $e_w(x)$ :

$$(4) \quad e_w(x) = x W'(x) / W(x)$$

The value of  $j_w(p)$  is  $e_w(a) - e_w$  at  $p=0$  and zero at  $p=1$ , and as a function of  $p$  its slope at  $p=1$  has the same sign as  $e_w(b) - e_w$ . Hence a sufficient condition for  $j_w(p)$  to change sign on the interval  $0 < p < 1$ , is:

$$(5) \quad \min \{e_w(a), e_w(b)\} > e_w$$

and another is:

$$(6) \quad \max \{e_w(a), e_w(b)\} < e_w$$

If  $W(x) = x - T(x)$ ,  $e_w(x) = RP(x)$  and if  $W(x) = T(x)$ ,  $e_w(x) = LP(x)$ . For theorem 2 we assume  $RP(a) = LP(a) = 1$ , whilst  $RP(b) < 1$  and  $LP(b) > 1$  (e.g. recall fig. 2). Hence (5) proves part 1 of this theorem and (6) proves part 2.

Now consider the effect of a transfer of small amount  $dx$  from income  $x_p$  to income  $x_r > x_p$ . Let  $\mu_w^*$  be the new mean, and  $L_w^*(p)$  the new concentration curve, for attribute  $W(x)$ . Since  $\mu_w \cdot L_w(p)$  and  $\mu_w^* \cdot L_w^*(p)$  measure the per capita amounts going to the bottom 100% before and after the transfer, we have:

$$(7) \quad \mu_w^* \cdot L_w^*(p) = \mu_w \cdot L_w(p) + \theta_w(p) \cdot \delta x / N$$

where  $N$  is the total number of income units and:

$$(8) \quad \theta_w(p) = \begin{cases} 0 & 0 < p < p_p \\ -W'(x_p) & p_p < p < p_r \\ W'(x_r) - W'(x_p) & p_r < p < 1 \end{cases}$$

In particular, set  $W(x) = x$  in (7):

$$(9) \quad [L_{x^*}(p) - L_x(p)] = \theta_x(p) \cdot \delta x / N\mu_x$$

Combining (7) and (9), we find:

$$(10) \quad \mu_{\omega^*} \cdot \Delta [L_x(p) - L_{\omega}(p)] = \Delta \mu_{\omega} \cdot L_{\omega}(p) + [\mu_{\omega^*} \cdot \theta_x(p) - \mu_x \theta_{\omega}(p)] \cdot \delta x / N\mu_x$$

where  $\Delta$  is the increase due to the transfer (e.g.  $\Delta \mu_{\omega} = \mu_{\omega^*} - \mu_{\omega}$  etc.). Now set  $W(x) = T(x)$  and integrate with respect to  $p$  in (10), using

$$(11) \quad \Delta \mu_T = \mu_x \cdot \Delta t = \mu_x (t^* - t) = [t'(x_r) - t'(x_p)] \cdot \delta x / N,$$

to obtain the effect of the transfer on aggregate progressivity:

$$(12) \quad t^* \cdot \Delta [G_T - G_x] = \Delta t \cdot [1 - G_T - 2(1 - p_r)] + 2(p_r - p_p) [t'(x_p) - t^*] \cdot \delta x / N\mu_x$$

Using the Kakwani (1977a) relationship  $[G_x - G_{x-T}] = [t/(1-t)] \cdot [G_T - G_x]$ , we can also deduce the influence on redistributive effect:

$$(13) \quad (1 - t^*) \cdot \Delta [G_x - G_{x-T}] = \Delta t \cdot [1 - G_{x-T} - 2(1 - p_r)] + 2(p_r - p_p) [t'(x_p) - t^*] \cdot \delta x / N\mu_x$$

Theorem 3 follows, noting that  $\Delta t > 0$  and  $\delta x > 0$  for the poor-to-rich transfer, and vice versa in the rich-to-poor case.

When the attribute changes from  $W_1(x)$  to  $W_2(x)$ , and income growth is specified by  $g(x)$ , define a mapping  $\psi$  from the old to the new concentration curve by:

$$(14) \quad L_{w_2}(p) = \psi [L_{w_1}(p)]$$

for each  $p = F_1(x_1) = F_2(x_2)$ . Now differentiate with respect to  $p$ :

$$(15) \quad \psi' [.] = L_{w_2}'(p) / L_{w_1}'(p) = \frac{\mu_{w_2,1} \cdot W_2(x_2)}{\mu_{w_2,2} \cdot W_1(x_1)}$$

then take logarithms:

$$(16) \quad \ln \psi' [.] = \ln W_2(x_2) - \ln W_1(x_1) + \text{constant}$$

and finally differentiate again, noting that for all  $x$ ,  $d \ln W_i(x) / dx = e_{w_i}(x) / x$ .

It follows that:

$$(17) \quad \text{sign } \psi'' [.] = \text{sign} [e_{w_2}(x_2) - e_{w_1}(x_1) / g(x_1)]$$

Given that  $\psi [.]$  is increasing, and defined on the unit square, this function lies entirely above/below the main diagonal if it is strictly concave/convex. In the cases  $W(x) = x$ ,  $W(x) = x - T(x)$  and  $W(x) = T(x)$  we may conclude:

$$(18) \quad g(x_1) \geq 1 \text{ for all } x_1 \Rightarrow L_{x,2}(p) \leq L_{x,1}(p) \text{ for all } p \in (0,1)$$

$$(19) \quad RP_2(x_2) \leq RP_1(x_1) / g(x_1) \text{ for all } x_1 \Rightarrow L_{x-T,2}(p) \geq L_{x-T,1}(p) \text{ for all } p \in (0,1)$$

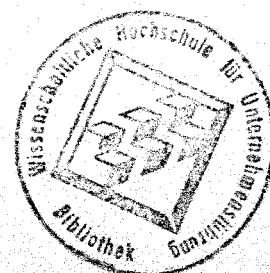
$$(20) \quad LP_2(x_2) \geq LP_1(x_1) / g(x_1) \text{ for all } x_1 \Rightarrow L_{T,2}(p) \leq L_{T,1}(p) \text{ for all } p \in (0,1)$$

and theorem 5 follows.

## REFERENCES

- Berglas, E. (1971), "Income tax and the distribution of income: An international comparison", *Public Finance/Finances Publiques*, Vol. 26, pp. 532-545.
- Hainsworth, G.B. (1964), "The Lorenz curve as a general tool of economic analysis", *Economic Record*, Vol. 40, pp. 426-441.
- Hemming R. and M.J. Keen (1983), "Single crossing conditions in comparisons of tax progressivity", *Journal of Public Economics*, Vol. 20, pp. 373-380.
- Hutton, J.P. and P.J. Lambert (1980), "Evaluating income tax revenue elasticities", *Economic Journal*, Vol. 90, pp. 901-906.
- Hutton, J.P. and P.J. Lambert (1983), "Inequality and revenue elasticity in tax reform", *Scottish Journal of Political Economy*, Vol. 30, pp. 221-234.
- Jakobsson, U. (1976), "On the measurement of the degree of progression", *Journal of Public Economics*, Vol. 5, pp. 161-168.
- Johnson, N.L. and S. Kotz (1970), *Distributions in Statistics: Continuous Univariate Distributions*, New York: Wiley.
- Kakwani, N.C. (1977a), "Measurement of tax progressivity: an international comparison", *Economic Journal*, Vol. 87, pp. 71-80.
- Kakwani, N.C. (1977b), "Applications of Lorenz curves in economic analysis", *Econometrica*, Vol. 45, pp. 719-727.
- Kiefer, D.W. (1985), "Distributional tax progressivity indexes", *National Tax Journal*, Vol. 37, pp. 497-513.
- Lambert, P.J. (1989a), *The Distribution and Redistribution of Income: A Mathematical Analysis*, Oxford and Cambridge MA: Blackwell.
- Lambert, P.J. (1989b), "Progression and differences in needs: can we justify our income taxes?", *Institute for Fiscal Studies Working Paper No. W89/7*.
- Lecaillon, J., F. Paukert, C. Morrison and D. Germidis (1985), *Income Distribution and Economic Development: An Analytical Survey*, Geneva: ILO.
- Morris, C.N. and I.P. Preston (1986), "Inequality, poverty and the redistribution of income", *Bulletin of Economic Research*, Vol. 38, pp. 277-344.

- Moyes, P. (1989), "Equiproportionate growth of incomes and after-tax inequality", *Bulletin of Economic Research*, Vol. 41, pp. 287-294.
- Musgrave, R.A. and T. Thin (1948), "Income tax progression, 1929-1948", *Journal of Political Economy*, Vol. 56, pp. 498-514.
- Pfähler, W. (1982), "Zur Messung der Progression und Umverteilungswirkung der Steuern", *Zeitschrift für Wirtschafts- und Sozialwissenschaften*, Vol. 102, pp. 77-96.
- Pfähler, W. (1983), "Measuring redistributive effects of tax progressivity by Lorenz curves", *Jahrbücher für Nationalökonomie und Statistik*, Vol. 198, pp. 237-249.
- Pfähler, W. (1984), "'Linear' income tax cuts: distributional effects, social preferences and revenue elasticities", *Journal of Public Economics*, Vol. 24, pp. 381-388.
- Pfähler, W. (1987), "Redistributive effects of tax progressivity: Evaluating a general class of aggregate measures", *Public Finance/Finances Publiques*, Vol. 37, pp. 1-31.
- Reynolds, M. and E. Smolensky (1977), *Public Expenditure, Taxes, and the Distribution of Income: The United States, 1950, 1961, 1970*. New York: Academic Press.
- Rothschild, M. and J.E. Stiglitz (1973), "Some further results on the measurement of inequality", *Journal of Economic Theory*, Vol. 6, pp. 188-204.
- Suits, D. (1977), "Measurement of tax progressivity", *American Economic Review*, Vol. 67, pp. 747-752.
- Tachibanaki, T. (1981), "A note on the impact of tax on income redistribution", *Review of Income and Wealth*, Vol. 37, pp. 327-332.



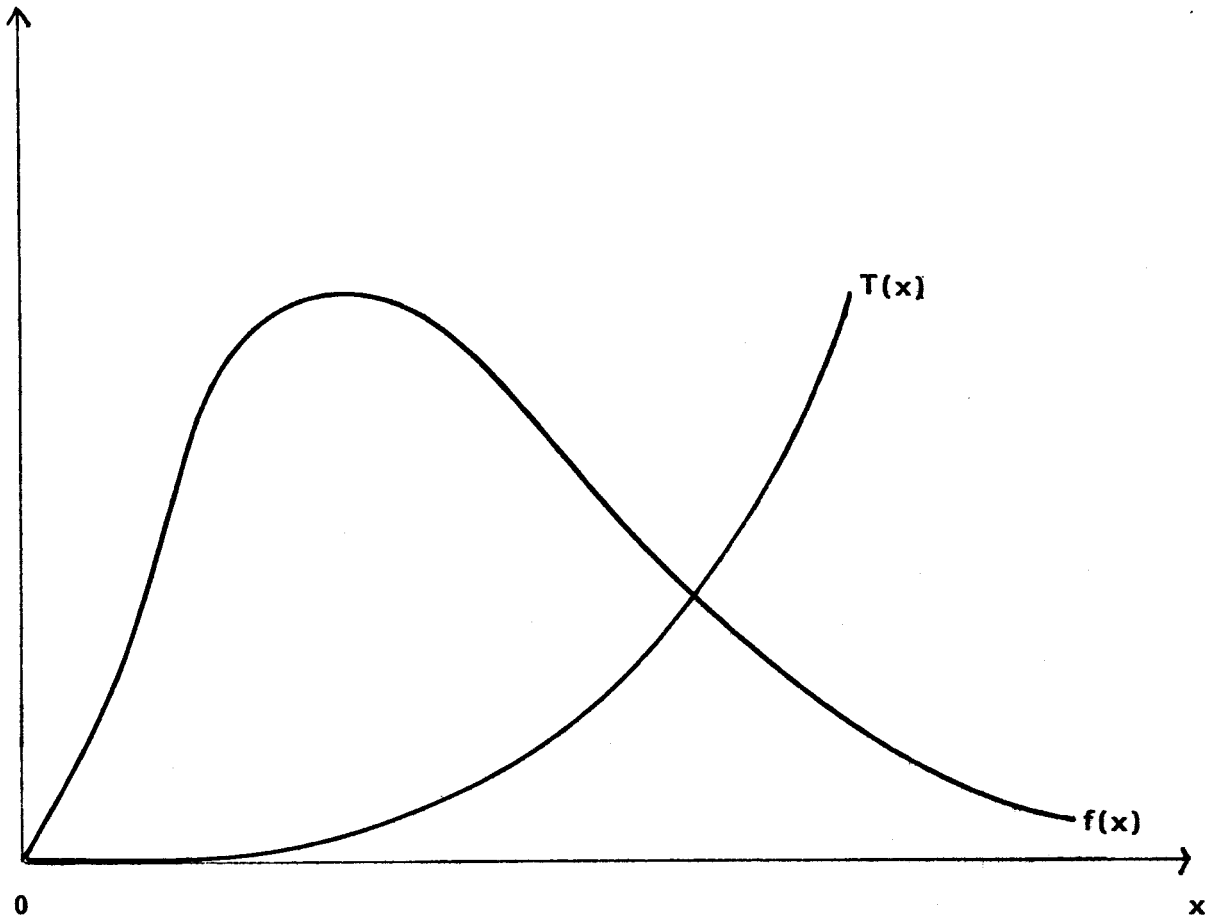


FIGURE 1 a

The pre-tax income distribution (its density function) and tax schedule

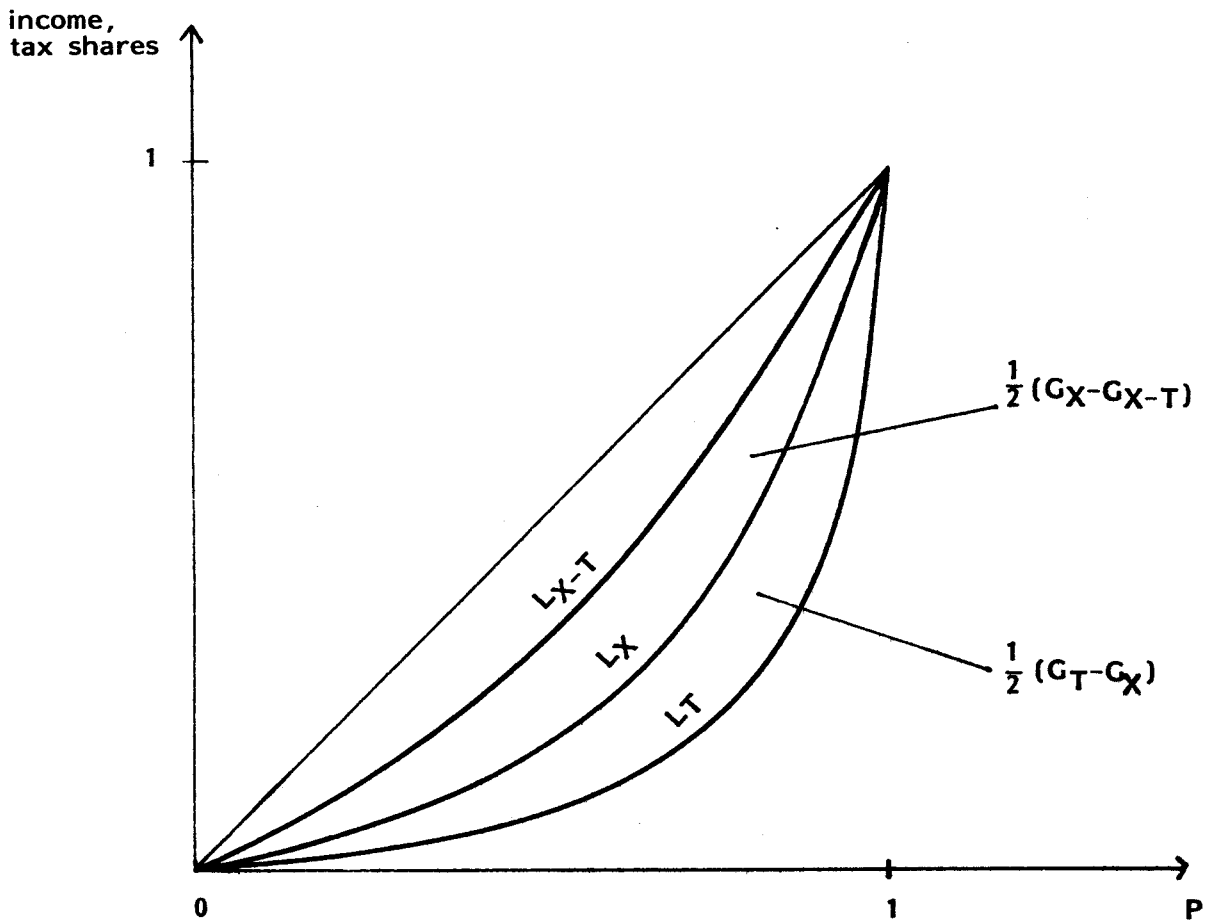


FIGURE 1 b

Typical configuration of Lorenz curves for a progressive income tax



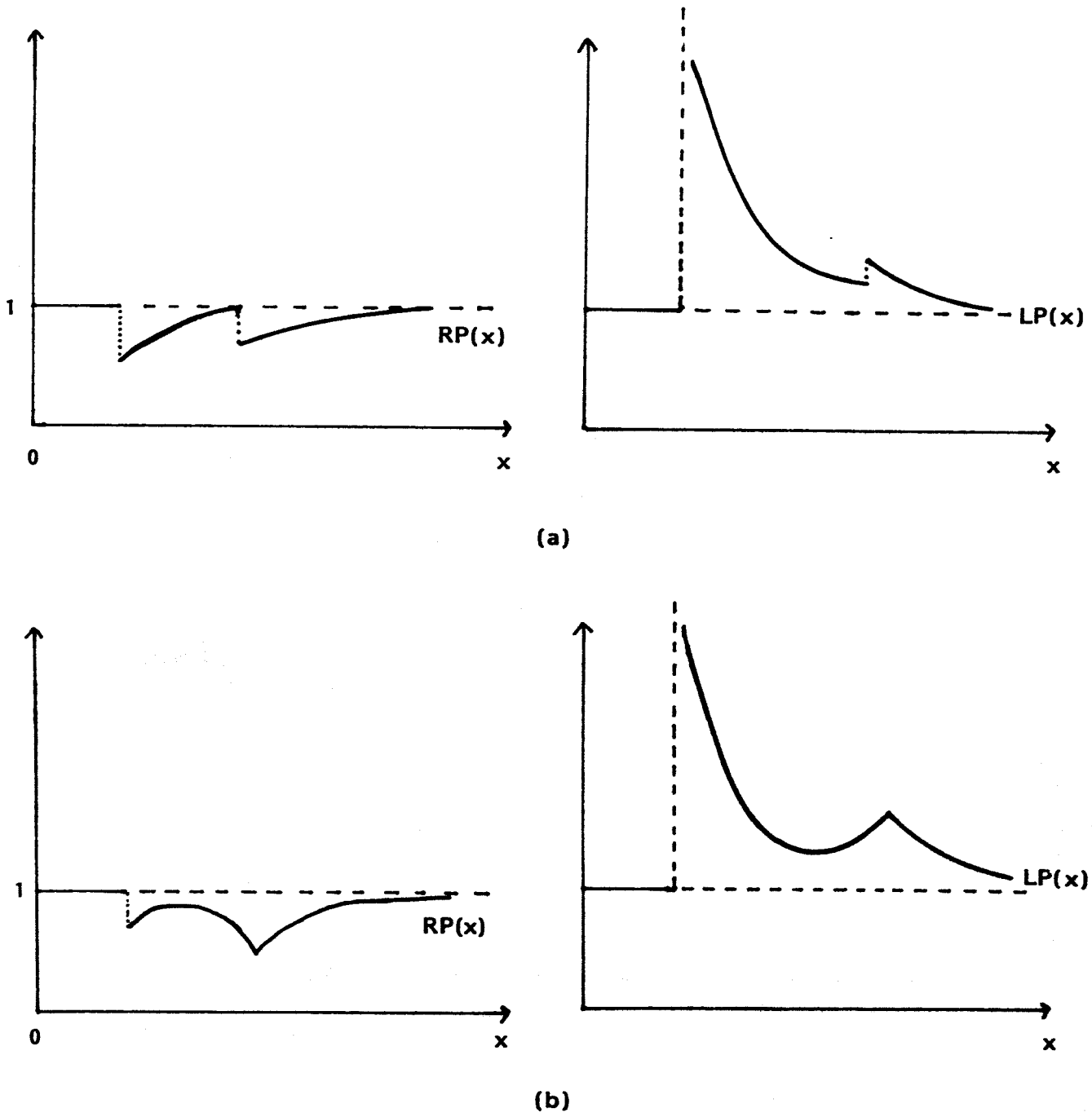
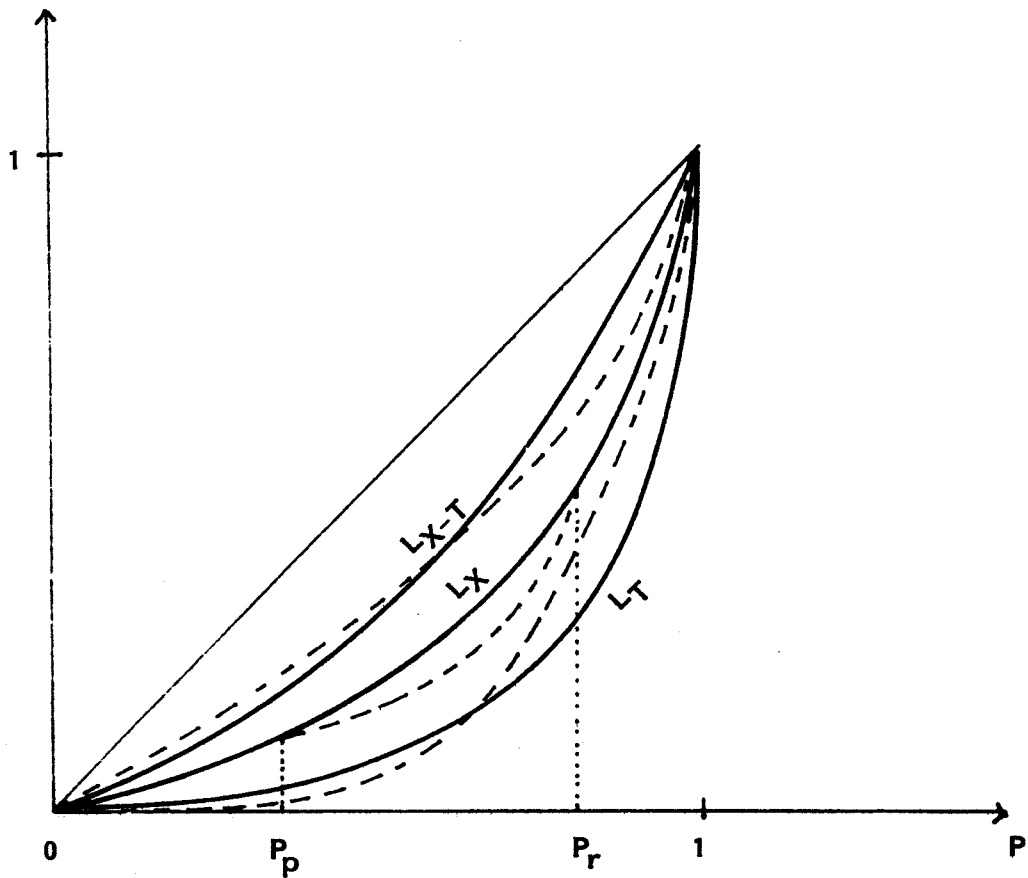


FIGURE 2

Residual progression  $RP(x)$  and liability progression  $LP(x)$  for (a) UK and (b) German type of income taxes



**FIGURE 3**

The effect of a poor-to-rich pre-tax income transfer on Lorenz curves  
 (dotted = after the transfer)

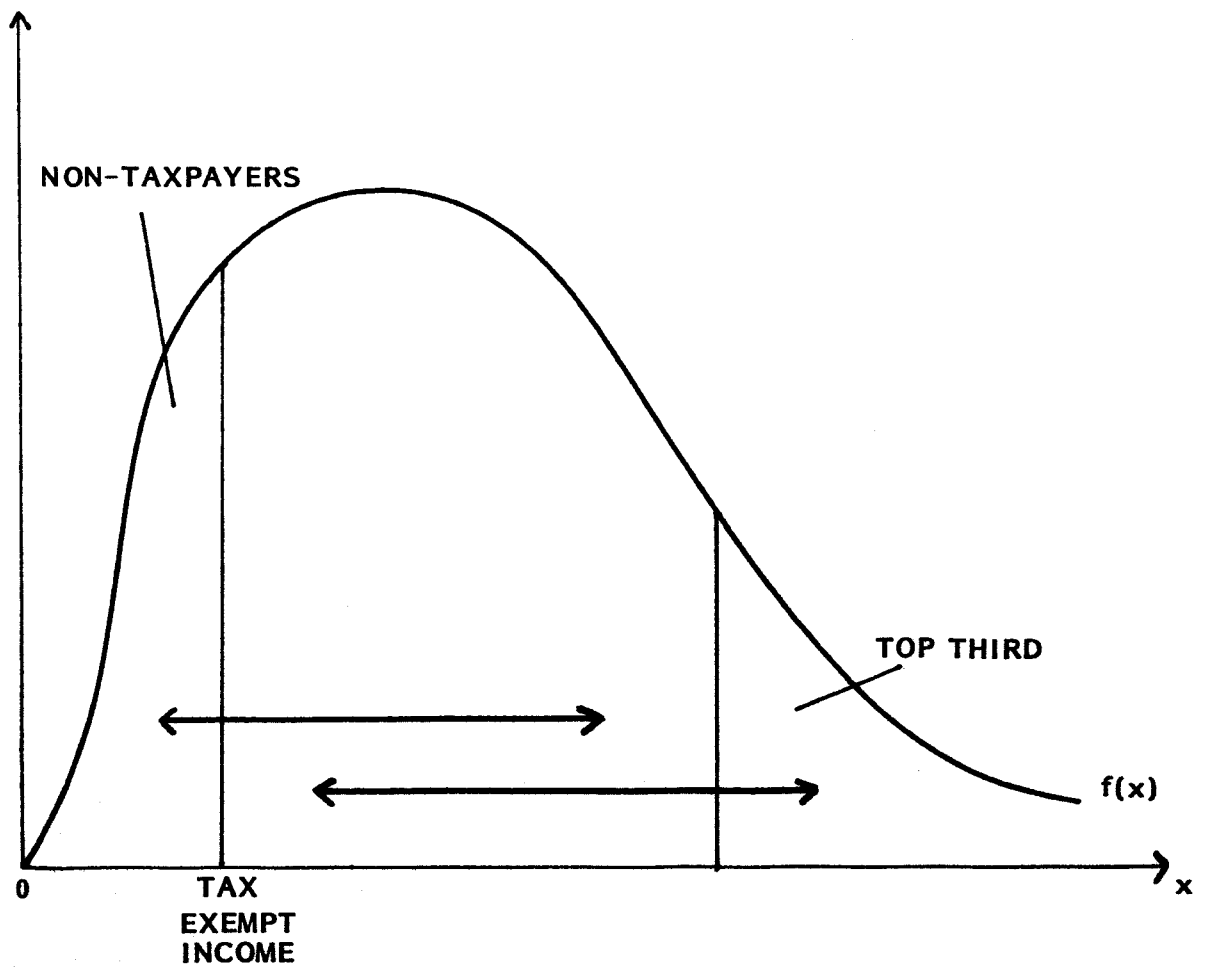


FIGURE 4

The UK pre-tax income distribution 1985:  
 transfers to which theorem 3 applies