# Dissertation Hedging Retail Promotions 

## Zero Out-of-Stock and Zero End-of-Period Coverage with Supply Options

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Für Masha

## Foreword

In today's competitive market situation, efficient promotion management is critical for both manufacturers and retailers to manage customer satisfaction and market share. While the industry has recently been accused of price fixing (to better be able to predict demand) and cartel authorities have issued fines in excess of 100 million Euros, this research proposes supply option contracts to share demand risk between channel partners while maximizing channel turnover.

According to this thesis, manufacturers should exploit two-part tariffs that they can bid into the market which can completely dominate wholesale price arrangements. This certainly reduces the imminent friction in the annual buying negotiations. What is new is that the seller considers the benefit of advance production and only produces short-term - if at all - under the high-demand scenario, e.g., when competitors do not promote. The buyer considers the negative impact of leftover inventory that curtails his ability to promote successor products. Even in the case when there is buying power, i.e., additional orders are placed short-term and trade spent is being requested by the retailer on top (which is going practice according to recent media publications), portfolios of contracts are dominant procurement strategies.

The thesis does not stop at deriving the optimal pricing structure offered by the manufacturer to the retailer. Also, the problem of predicting demand over a planning horizon with multiple promotions is analyzed. The key insights are i) that promotion demand is linked via consumer stockpile inventories and consumption rates and ii) that orders by non-loyal customers is highly predictable (using a twosegment forecasting model). This leads to two scenario planning problems, i.e., one for the retailer and one for the supplier, which can easily be solved using stochastic programming models.

The application case study provides strong evidence that jointly improving forecasts and adding flexible supply option contracts can greatly enhance channel profitability while maximizing the pie. The thesis is very well structured and discusses each aspect separately. Thus, it makes for a very important and accessible reading for both academics and practitioners alike.

Building trust in the FMCG industry is a widely announced paradigm shift by senior executives. However, using a single wholesale price negotiated by isolated functions in organizations (sellers and buyers) creates confusion, misguides effort and destroys trust. When mismatches in demand and supply occur, the value of the product - which consumer care most about - is no longer in the focus of channel partners. Therefore, this research represents a major contribution to effective supply chain coordination in theory and in practice.

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## Preface

This work was written as a doctoral dissertation at the chair of production management at WHU - Otto Beisheim School of Management between 2007 and 2010. Matching supply and demand is one of the key challenges in both retailing and manufacturing of fast moving consumer goods. In my discussions with FMCG managers, I had noticed that there still exists no viable solution to this matching issue among the market players. The broad interest of the FMCG industry in supply contracts had convinced me that theoretical work in this direction could help supply chain managers to find more efficient ways of serving uncertain demand during price promotions.

I would like to thank my advisors, Prof. Dr. Arnd Huchzermeier and Prof. Dr. Martin Fassnacht for their guidance and inspiration throughout my time at WHU - Otto Beisheim School of Management. In my dissertation I learned from their expertise in the areas of supply chain management and pricing, which are the two cornerstones of this dissertation. Moreover, Prof. Huchzermeier provided an involving and international working environment at the chair of production management and my colleagues were always open for discussions and fostered a truly inspiring intellectual environment. This work greatly benefited from a stay as a visiting scholar at Northwestern University - Kellogg School of Management. I am grateful to the Kellogg faculty for providing this opportunity and for extending such a warm welcome to Chicago.

Most importantly, I would like to thank my fiancée Maria and my family for their continuous support during these three years. I could not have met this challenge without them.

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## Chapter 1 Motivation

The fast moving consumer goods ("FMCG") industry is characterized by frequent marketing campaigns. While retailers target to generate store traffic, manufacturers focus on the market share of their brands. More than in other industries, product differentiation is driven by advertising and price signals, as opportunities to gain a sustainable competitive advantage through product design are often limited. The need for promotions generates a major challenge. Eliminating forecast errors during promotion periods is difficult or even impossible. Moreover, contracting for supply to serve demand during promotions is limited by the structure of available contracts.

Mismatches between supply and demand have a significant impact on profitability. Hendricks and Singhal (2009) show that excess inventories, as an indicator of misaligned supply chain activities, trigger major stock market reactions. In their sample, companies lost on average between 6.79 and 6.93 percent in terms of share prices after announcing excess inventories. Controlling for mismatches between supply and demand is a top management issue, because it negatively impacts the value of the firm in the medium- to long-term.

In the FMCG industry, the major retail chains have significant bargaining power, as the retail side of the market is much more consolidated than the supply side. According to GS1 Germany (2005), the five largest retail chains in the German market control 27 percent market share in terms of revenues, while the five largest manufacturers only control six percent of supply. Retailers potentially employ their market power to gain competitive advantage. We see that retailers often require flexible deliveries and confirm actual orders only shortly before promotion prices are announced. Thereby, retailers hedge against demand risk.

This dissertation analyzes FMCG supply chains which are characterized by i) frequent promotions with unpredictable demand and ii) powerful retailers. Consider a simple FMCG supply chain consisting of one manufacturer and one retailer. The manufacturer produces products, which are sold by the retailer to consumers. Products move downstream in the supply chain, while funds move upstream. Consider diapers as a typical FMCG product. Figure 1.1 shows the characteristics of the three parties involved in a diapers supply chain. The retailer frequently runs price promotions and triggers uncertain demand from consumers, although consumption rates


Fig. 1.1 Example of a FMCG supply chain: diapers
are fairly constant. Thus, the retailer requires flexible supply from the manufacturer to ensure high service levels. The manufacturer reacts by aiming for flexibility in his own processes.

The main objective of this dissertation is to propose a mutually beneficial coordination mechanism meant to significantly improve the status quo. We thus introduce a supply chain coordination concept to achieve efficient retail promotions. First, an improved forecasting approach is proposed to reduce forecast risk. Second, forward and option contracts are added to complement the limited existing choice of supply contracts. Thereby, the remaining demand risk can be optimally distributed in the supply chain. As a result, both supply chain profit and service levels increase simultaneously.

In this first chapter, we motivate our analysis. First, we review the impact of price promotions on demand risk in the FMCG industry. We discuss the relevance of such promotions and we show the impact of discounts on service levels and demand risk. Second, we review the status quo of demand risk management in practice. Here, we present the major industry initiatives and discuss current industry practice. Then, we explore several generally accepted myths about the way retailer-manufacturer relationships are supposed to work. Third, we introduce the concept of profitable price promotions and outline the structure of the present manuscript.

### 1.1 The challenge of unpredictable promotional demand

Pricing is one of the major levers used to generate additional sales, e.g., from price sensitive customers. Price promotions increase demand for the product over a short period of time (Gupta 1988; Bell et al. 1999; Ailawadi et al. 2007). In the face of competition, the exact sales effect of price discounts cannot be determined in general, resulting in demand risk (Wiehenbrauk 2010). In this context, demand risk
refers to the challenge that the retailer has to foresee the number of units that will be sold during a promotion. This reflects on the retailer's purchasing decisions, which are consequently made in the presence of risk. Inventory inadequacy, i.e., excess inventories and shortages, have negative effects on profit (Nahmias 2008). As demand risk often results in inventory inadequacy, price promotions reduce expected profitability in the supply chain. In the sequel, we will discuss this issue in depth.

In this section, we first discuss price promotion strategies in the FMCG industry. Second, we review the reasons that retailers need to promote. Third we discuss why forecasts cannot very well predict demand during promotions and fourth, we show that forecast errors are a major threat to profitability. We resume this section with a summary of the origin and effects of the challenge of unpredictable promotions.

### 1.1.1 Price promotion strategies in the FMCG industry

Pricing is one of the key instruments of the marketing mix. Each retailer in the market decides on an overall price level and optionally on a price discount strategy. For some retail chains and products, sales through price promotions represent a significant share of total sales volumes. Huchzermeier et al. (2002) report that the promotional share of diapers sales reaches as much as 75 to 85 percent of total sales. Thus, pricing is important and it can be employed as a lever of a company's competitive strategy. There are two basic retail strategies in the FMCG market: the "Every Day Low Price" (EDLP) strategy and the "High-Low" (price promotions) strategy.

EDLP means that prices stay constantly at the same low level. Retailers pursuing this strategy explicitly abandon the option to trigger demand through price promotions. This competitive disadvantage is balanced by stable demand patterns. More predictable demand allows for a lean supply chain and a high service level. That is, EDLP players limit out-of-stock situations to a minimum without investing in major inventories. The resulting lower cost per unit sold allow for a generally lower retail price than a comparable retail chain relying on price promotions could afford. Known EDLP players are discounters like Wal-Mart in the U.S. and Aldi as well as Lidl in Germany.

Retailers competing on a promotion based strategy offer low promotion prices and high regular prices. This price promotion strategy enables retailers to extract high margins from consumers who are loyal to the store. These consumers purchase even at the high regular price, e.g., because they highly value convenience. During promotion periods, prices often drop to a level below the EDLP price, motivating potential store and brand switchers to purchase. The price promotions strategy thus allows for a tactical price discount option. Retailers employing price promotions pay for this competitive advantage with higher demand risk. As we will see, demand risk translates into additional costs compared to EDLP players.

German EDLP players respond to price promotion strategies by offering implicit discounts, based on variations of the package size. That is, they offer additional


Fig. 1.2 Discounters outperform the market in terms of growth (Source of data: GfK Panel Services 2009)
units at the regular price. Thereby, EDLP players move towards the price promotion approach, following a hybrid strategy. Recently, even dm-Drogeriemarkt, a leading German drugstore, abandoned their EDLP strategy (Hoos et al. 2010).

Promotion-based retailing is under pressure from EDLP players. The segment has lost significant market share for years. Figure 1.2 provides the growth rates of discounters and the FMCG market as a whole in Germany over the last years. Discounters clearly outperform the market and achieve superior growth in terms of market share (GfK Panel Services 2009). Consequently, the promotion-based business model is challenged.

FMCG retailers following a price promotion strategy need to explore unrealized opportunities. Promotions provide a significant advantage over EDLP strategies, when demand risk is successfully controlled. Then, price promotions fulfill their tactical role of attracting switching consumers, e.g., from discounters, without hurting the cost structure with unnecessary demand risk. Thus, to maintain a competitive position in the long term, retailers employing price promotions must learn to more efficiently deal with the inherent demand risk.

### 1.1.2 The need to promote in FMCG retailing

Price promotions have several effects on demand (compare Section 2.2.1). Some consumers purchase more units of the product than usual in order to benefit from the reduced retail price. These customers wait for deals and only purchase at a discount. They are characterized by their willingness to build up inventory of a product and their ability to wait for the next promotion (Blattberg et al. 1981). Mela et al. (1998) state that such a buying pattern is particularly common for non-perishable products with constant consumption rates. This consumer inventory effect is important in the context of competition. Consumers purchasing large quantities at a store running
a promotion will not be willing to purchase additional quantities of the product at other stores in the near future (Bell et al. 1999). There is simply too much inventory left at consumers' homes and consequently, consumers do not shop for the category.

Consumers, who switch away from a competitor to purchase a large quantity of an item on promotion, are lost for the other player over the next several periods. Thus, price promotions are an effective competitive measure against competitors and particularly against EDLP players. However, the resulting purchase pattern of consumers waiting for deals results in greater volatility in demand and consequently reduces profitability of the category (Mela et al. 1998). Thus, promoting retailers face a dilemma. In order to grow market share, they need to offer deals, but at the same time, deals lead to higher demand uncertainty.

### 1.1.3 Forecast errors during promotions cannot be eliminated

Price promotions result in forecast errors. Event based forecasts, which are frequently employed in the industry, do not consider path dependencies of demand risk. Assuming that demand simply depends on the current promotion price is not correct. A two consumer segment model presented by Wiehenbrauk (2010) explains forecast errors by promotion induced stockpiling and random promotion pricing in a competitive environment. Random promotion pricing is necessary to be unpredictable towards competitors. Thus, part of promotional demand risk originates from retailers' own efforts to be unpredictable. Analogously, uncertainty from competitive pricing cannot be eliminated by means of manufacturer-retailer collaboration when complying with antitrust regulation.

Promoting retailers decide on two parameters to achieve the maximum impact on demand. First, the size of the discount determines the impact of the price promotion. Second the frequency of discounts determines the effect of each discount. Manipulating frequency and discount depth allows for influencing demand during promotions. Still, the effect of promotions on demand is uncertain. Advanced forecasting methods can reduce forecast errors; however, eliminating forecast errors during promotions is impossible, because forecast errors result partially from pricing strategies. Residual demand uncertainty remains, even when the manufacturer shares upstream information as proposed by Wiehenbrauk (2010). In the next step, we discuss the harmful effects of forecast errors in a FMCG environment.

### 1.1.4 The value of high service levels and low inventory

Price promotions potentially generate additional demand; however, the product is sold at a lower margin. Whether price promotions are profitable depends on the size of the demand effect and the retailers' ability to serve additional demand. We define service levels throughout as alpha service levels. This type of service level


Fig. 1.3 Scents used as a key promotional feature in a recent advertisement by P\&G (Source: Procter \& Gamble 2010)
reflects the probability that all demand can be served from stock during a promotion, given a certain amount of products ordered. Both high and low service levels have disadvantages. The retailer trades off the individual effects of overly low and overly high service levels.

A low service level may result in lost sales and unsatisfied consumers, some of whom will not return for future price promotions. Thus, the inability to serve consumers during promotions negatively affects future promotions. Stockouts on the retailer level in addition reflect on the brand and thus on the manufacturer's market share. Corsten and Gruen (2003) report that 39 percent of consumers facing a stockouts in the diapers category switch to another store, while 20 percent purchase another brand. Consequently, both retailers and manufacturers have to invest to gain store and brand switchers back as loyal consumers.

Stockouts occur when a consumer cannot purchase an item. A particular challenge in FMCG retailing is refilling shelves to avoid stockouts of items that are in store inventories but not on shelves. Corsten and Gruen (2003) analyzed 40 reports on out-of-stock situations in retailing. They found that the frequency of out-of-stock is on average 8.3 percent. The Swiss supermarket chain Migros reported that the frequency of out-of-stock situations dropped from $4-5$ percent in 2006 to 3 percent in 2009 and dm-Drogeriemarkt has achieved an out-of-stock quota of only 2 percent (Rode 2009).

A high service level prevents stockouts in most cases, but it also increases the probability and severity of excess inventory at the end of the sales period. Costs associated with inventory depend on the product and the promotion campaign. In the worst case, perishable products cannot be kept on stock at all, resulting in severe losses. Analogously, end-of-period coverage for non-perishable products is harmful. Promotion items are often packaged differently from regular items. Frequent minor innovations as, e.g., new scents enable brands to differentiate. According to Krishna et al. (2010) product information associated with scents is longer remembered. Individual flavors are important, as product scents are then mentally connected to the


Fig. 1.4 The doom loop of wasteful promotions
single product instead of a group of items. Figure 1.3 shows a recent advertisement by the detergent brand "DASH". DASH competes on a differentiation strategy, offering seasonal scents and promoting this advantage over generic products in advertisements. Leftover inventories from past promotions reduce the manufacturer's flexibility in promoting his brand. Consequently, end-of-period coverage considerations must be pursued in parallel with service level considerations to derive an optimal risk management policy.

### 1.1.5 The challenge for profitable price promotions

Price promotions trigger demand from consumers, who potentially would otherwise not have purchased this category at this store. Thereby, promotions increase demand and allow for additional sales. In the status quo, these additional sales are paid for with reduced margins: We discussed i) that retailers, other than discounters, need to promote to compete in the market, ii) that price promotions necessarily trigger demand risk and iii) that demand risk hurts service levels and/or results in end-ofperiod coverage. Retailers are challenged to deal with demand risk as efficiently as possible to benefit from price promotions.

Figure 1.4 summarizes the situation in the status quo: Promotions in FMCG retailing trigger demand risk. Forecast errors cannot be eliminated, even when consumer demand is fairly stable, e.g., for diapers as analyzed by Huchzermeier et al. (2002). Procter \& Gamble reports that volatility of demand increases and that busi-
ness from promotions becomes increasingly important (Ochs 2009). Demand risk results in expensive excess inventory, on the one hand, and stockouts, on the other hand. These cost factors threaten profitability of price promotions and create waste in the channel. In the status quo, only 18 percent of brands on promotion directly benefit store profit (Srinivasan et al. 2004). Thus, the waste created by demand risk, that is inventories and stockouts, threatens the viability of the price promotions strategy.

Allocation of profit and inventories in the supply chain is subject to fierce negotiations. Obviously, such an environment is not suited to foster trust and collaboration. We further explore this complication in the next section.

### 1.2 Demand risk as a key source of waste and tension

The relationship between retailers and manufacturers in the FMCG industry is threatened by competition for margins and fierce negotiations. Trust and collaboration are recognized as improvement levers; however, the difficult market environment in FMCG retailing puts pressure on industry initiatives. Then, insufficient communication and a lack of information sharing may lead to coordination issues.

Demand risk management in the FMCG industry relies i) on forecasts and ii) on supply contracts. Over the last years, forecasting has begun to incorporate information sharing to improve accuracy. Supply chain parties share to some degree their knowledge on the market to achieve the best possible forecast. In parallel, supply contracts started developing from basic wholesale price agreements to more collaborative strategies, e.g., buy-back and quantity discount schemes. However, there are still challenges in both areas. Regularly, retailers hedge themselves against demand risk by requiring full flexibility from their suppliers, thus shifting risk completely to manufacturers. Moreover, retailers are not given a chance (or supply contracts) to insure themselves.

We first review recent developments in demand risk management. Second, we shortly discuss flexibility requirements in the status quo. Finally, we challenge persistent myths about retailer-manufacturer relationships in the FMCG industry.

### 1.2.1 Challenges and developments in demand risk management

The FMCG industry knows the critical challenges very well. Three issues are predominantly discussed among FMCG managers. First, retailers and manufacturers experience a lack of trust in collaborative approaches. The classical image of supply relationships in this industry is that of battles for margins in the yearly price negotiations. Margins determine how competitive a retailer can price a product. The focus on wholesale prices creates an environment where collaborative approaches have low priority. Second, information sharing still has a lot of potential benefits to offer.


Fig. 1.5 The ECR collaboration framework (Source: Globalscorecard.net 2010)

In the absence of information sharing, unexpected order patterns result in suboptimal production policies. Sharing information reduces the effect of disruptions on the supply chain. Without information sharing, a disruption, for example caused by a price promotion, impacts the whole supply chain. Upstream parties cannot interpret demand signals correctly and order additional quantities to maintain service levels. As a result, the amplitude of the disruption grows as it moves upstream through the supply chain. This phenomenon is called the "bull-whip effect" (Lee et al. 1997). Chen and Lee (2009) show how order smoothing and evaluation of historic order data can overcome this effect. Third, critical forecast errors result in a struggle for flexibility. Retailers try to avoid commitments, while manufacturers optimize their production processes towards the extreme of flexibility.

There are three major challenges in the status quo of risk management in the FMCG industry. First, trust alone does not maximize supply chain profit, since it does not tackle the issue of double marginalization. Coordination requires more elaborated contracts in addition to trust in the relationship. Second, relationships, based on wholesale price transactions only, do not set incentives for continuous improvement. The costs associated with purchasing decisions are not transparent, as the retailer only sees the wholesale price. Thus, she is not directly affected by her actions. Third, extreme flexibility requirements induce unnecessary waste into the supply chain. Frequent disruptions of production and large inventory requirements further reduce the efficiency of the supply chain.

Retailers and manufacturers are aware of these challenges. Over the last years, collaborative approaches gained in importance. Aviv (2001) models collaborative forecasting in a two-tier supply chain. He finds that collaborative approaches are highly beneficial, when both parties have to contribute information to the forecast. That is, collaboration is most important, when players have diversified capabilities. We present three major initiatives involving information sharing and collaboration: "Efficient Consumer Response" (ECR), "Collaborative Planning, Forecasting and Replenishment" (CPFR) and "Jointly Agreed Growth" (JAG).

## "Efficient Consumer Response"

Efficient Consumer Response (ECR) targets to improve efficiency in the supply chain through coordination. Coordination is achieved by fostering collaboration i) between retailers and manufacturers and ii) among internal functions. According to Seifert (2002), ECR consists of two main components: supply chain management and category management. Table 1.1 shows the six concepts that trigger improvement in the ECR framework: efficient replenishment, efficient administration, efficient operating standards, efficient store assortments, efficient promotions and efficient product introductions (Seifert 2002).

Table 1.1 The Efficient Consumer Response-Concept (Source: Seifert 2002, p.16)

| Supply Chain Management (SCM) | Category Management (CM) |
| :--- | :--- |
| Efficient Replenishment (ER) | Efficient Store Assortment (ESA) |
| Efficient Administration (EA) | Efficient Promotion (EP) |
| Efficient Operating Standards (EOS) | Efficient Product Introduction (EPI) |

The two key ideas underlying ECR are information sharing and rolling out a pull order scheme throughout the supply chain. The overall goal of ECR is to create a win-win-win situation for manufacturers, retailers and consumers (Seifert 2002). Thus, savings from efficiency gains are distributed in the supply chain. Figure 1.5 shows the ECR collaboration framework: Optimal consumer response is reached by improving collaboration between production at the manufacturer level, sales at the manufacturer level, purchasing at the retailer level and distribution at the retailer level.

The ECR scorecard (Figure 1.6) provides a tool to standardize measurement of ECR implementation. The four main groups are demand management, supply management, enablers and integrators. Each group contains a number of subgroups. The ECR framework breaks down the levers of improvement to individual categories, e.g., "Product Flow Techniques" or "Transport Optimization". These are part of the subgroup "Responsive Supply", which is again part of the group "Supply Management".

## "Collaborative Planning, Forecasting and Replenishment"

Collaborative Planning, Forecasting and Replenishment (CPFR) is the next step of ECR on the supply side (Seifert 2002). Tools like Vendor Managed Inventories (VMI) and Continuous Replenishment Policies (CRP) improve cooperation between supply chain parties. CPFR was originally started by Wal-Mart and targets to create value on multiple dimensions: faster response times to consumer demand, better forecasts, improved communication channels, more sales through fewer out-ofstock situations, reduced inventories and reduced costs (Seifert 2002). The Volun-


Fig. 1.6 The ECR scorecard (Source: Hofstetter and Jones 2005)
tary Interindustry Commerce Solutions Association (VICS) developed an extensive framework to describe the drivers of CPFR. This framework is shown in Figure 1.7.

## "Jointly Agreed Growth"

Jointly Agreed Growth (JAG) is a process that fosters sustainable growth in the consumer goods industry. JAG moves the focus of industry from standardization towards cooperation, as Dirk Boer, member of the board of Ahold, notes (Boer 2008). ECR Europe (2009) proposes a three year horizon to lever growth potentials in retailer-manufacturer relations along a detailed business plan. JAG improves the status quo by agreements between the retailer and the manufacturer. These agreements are summarized in Table 1.2. Peter Brabeck-Letmathe, chairman of Nestlé, sees more efficient innovation as the key to converting achieved growth into sustainable growth (Brabeck-Letmathe 2008). JAG thus targets to move the industry from short-term promotion plans to medium-term growth plans, which are regularly evaluated employing key performance indicators (Rode 2008).

### 1.2.2 Excessive flexibility requirements

Relations based approaches improve collaboration in terms of contracting and forecasting, e.g., Procter\&Gamble reports that a CPFR pilot with dm-Drogeriemarkt reduced forecast errors at the retailer level by 50 percent (Rode 2003). Still retailers face residual risk from forecast errors. The CPFR pilot achieved a reduction of forecast errors from 48 percent at the manufacturer level and 47 percent at the retailer level to 24 percent and 28 percent, respectively (Rode 2003). Demand risk results in overage costs, when too much inventory is ordered. Retailers avoid overage costs by
forcing their suppliers to accept extremely flexible order schemes. In addition, demand risk results in underage costs when order volumes are insufficient. Then, sales volumes are lost due to stockouts. In the diapers industry, approximately 3.8 percent of sales are lost due to out-of-stock situations (Corsten and Gruen 2003). Therefore, retailers require high availability of inventory, covering most states of the demand forecast. The combination of high flexibility requirements and high service level requirements creates major costs for the manufacturer.

Retailers require suppliers to have either ample reserves of inventory or very flexible production capabilities. Manufacturers must ensure the ability to deliver even high volumes on short notice. Holding costs, additional labor costs and other costs of flexibility increase production costs and thus the price of the supplier's product. As a result, flexibility and service level requirements reflect on the retailer, who then reacts with pressure on retail prices.


Fig. 1.7 The CPFR framework (Source: Collaborative Planning, Forecasting \& Replenishment Committee 2009)

Table 1.2 The performance levels of JAG (Source: ECR Europe 2009)

| Trade Barriers | Jointly Agreed Growth |
| :--- | :--- |
| Lack of strategic alignment | A jointly agreed growth strategy |
| Lack of long-term visibility | A three-year framework plan |
| Buyer-seller 11th hour agreements impeding | Joint business planning starting with category |
| marketing plans | strategy and role of innovation |
| Fear of own-label match impeding new prod- | An agreed category strategy and set of princi- |
| uct introduction | ples |
| Slow distribution of new product/ general | Long-term integrated process avoiding supply |
| available issues | disruption |
| Lack of KPIs to measure effective returns | A set of agreed pre-defined growth criteria, |
|  | balancing financial, marketing and operational |
|  | KPIs |

Retailers face a dilemma. They need to serve unpredictable demand even in extreme states to avoid losing consumers to competition during promotions. When insufficient stock is available, promotions do not attain the full demand effect. Thus, stockouts could reduce additional sales volumes to a level where the volume effect is outweighed by the effect of lower margins on sales to loyal consumers. In addition, stockouts hurt the image of the retail chain. Both stockouts and waste in the supply chain threaten profitability of the retailer and the manufacturer.

Consequently, retailers and manufacturers try to shift inventory to the other party, while fighting for margins. The image of profit as a "pie" that is distributed among players is generally accepted. This dissertation will show that truly efficient promotions mean growing the pie and focusing more on customers instead of arguing about the division of profit. Thereby, both players' shares increase simultaneously.

### 1.2.3 Four myths about retailer-manufacturer relationships

Waste in the FMCG supply chain is a result of the way the game between retailers and manufacturers is played in the presence of intense competition. Several accepted rules of bargaining must be questioned to achieve improvements in retailermanufacturer relations. We analyze four generally accepted myths that emerged in discussions with industry representatives.

## "Players in the FMCG industry must never commit"

Commitments mean naturally losing flexibility to react to demand risk. In the diapers category, manufacturers ask for a three month planning period. This time span allegedly allows for optimal production processes. However, orders at this point in time are in practice not commitments and are still subject to change. Retailers tend to postpone committing quantities until the very last minute. This provides the
opportunity to adapt orders to promotion prices after discounts have been decided upon. The retail price is only determined at the moment the catalog is printed, i.e., a few days before the promotion starts. E.g., Real, a German retail chain, employs a tool by Comosoft that enables the company to change retail prices in catalogs until five minutes before printing (Rück 2010).

This lack of commitment results in a shift of demand risk from the retailer to the manufacturer. In the status quo, the retailer demands flexible deliveries at a fixed purchase price. We will show that demand risk can be shared without allocating additional physical inventory to the retailer. Thereby, end-of-period coverage considerations are taken care of, while improving supply chain performance in terms of service levels.

## "Value destruction is inherent to retailer-manufacturer relationships in FMCG"

Retailer-manufacturer relationships in the FMCG industry are characterized by competitive price negotiations. The FMCG market features a number of parallel sales channels and substitutive products, which reduce the market power of either party. The harsh negotiation culture limits information exchange, as providing information is a potential disadvantage. This lack of trust and exchange results in suboptimal profitability of the channel from the viewpoint of the whole industry.

Value destruction can be reduced by building trust between parties and by understanding that risk sharing results in additional supply chain profit, which can be shared between parties. Risk sharing enables higher service levels and provides additional value to both manufacturers and retailers.

## "Wholesale pricing achieves an efficient supply chain"

The predominant type of supply contract employed in retailer-manufacturer relationships in the FMCG industry is the wholesale price contract. Both parties meet once a year for negotiations and specify the amount paid per unit delivered. In some cases, the contract is extended to a quantity discount, where a reward is provided to the retailer for exceeding specified target quantities.

The wholesale price contract does not coordinate the channel. That is, the incentives of the retailer and the manufacturer are not aligned as risk is not properly shared. Quantity discounts do not align incentives either, when the retailer orders short-term only. In such a supply chain layout, the retailer orders according to demand and thus, she postpones commitments. As the retailer takes no risk, quantity discounts have no coordinating effect and even worse, they potentially lead to supply chain disruptions at the end of the year, as retailers could try to reach the discount targets by building up inventories.

Wholesale price contracts could be combined with or replaced by more elaborate contracts that split risk between both parties. The resulting elimination of waste


Fig. 1.8 Managing demand risk with supply chain tools to achieve efficient promotions
in the supply chain could outweigh the additional effort for deriving optimal order quantities, given these seemingly more complex contracts. We will follow this line of argument in further chapters and suggest a portfolio of contracts to achieve higher supply chain profit.

## "Manufacturers are helpless against powerful retailers and cannot change the situation"

In discussions, manufacturers claim that they are not in the position to introduce new contracts. They justify this position with the argument that retailers can always threaten to delist a less important stock keeping unit (SKU). Consequently, even manufacturers who possess strong brands, that are difficult to replace, are at a disadvantage when negotiating with retailers.

Although manufacturers do not have the negotiation power to change the situation by force, they can still change the game through other means. The status quo does not achieve the supply chain optimum in terms of profit. Manufacturers should take the initiative by accessing potential benefits from improved forecasting and contracting. These benefits can only be realized and distributed when better supply contracts are implemented. So, the retailer must comply if she is rational. We will show that the manufacturer can bid Pareto improving contract schemes into the market and thereby, he can improve the status quo.


Fig. 1.9 The concept of truly efficient promotions: zero out-of-stock and zero inventories

### 1.3 The solution concept of efficient price promotions

Suboptimal distribution and management of demand risk is a major source of waste in the supply chain. We identify two potential improvement levers (see Figure 1.8). Forecast management is one option to overcome the dilemma between service level requirements and inventory risk. Better forecasts provide additional information and thus reduce the range of the distribution of demand. Consequently, less inventory is required to achieve a given service level. Price promotions mutate stable consumption patterns into volatile demand patterns. According to Huchzermeier and Iyer (2010) forecast errors during price promotions are as important as 30 percent to 140 percent. Improvements of the forecast algorithms lower the overall demand risk associated with purchase and production decisions. Perfect forecasts cannot be achieved, as there are random parameters. These sources of risk include promotion prices. Retailers cannot change this practice, as it is required in order to be unpredictable from the perspectives of competitors and consumers (Wiehenbrauk 2010).

The next step towards an improved relationship between retailers and manufacturers in the FMCG industry should be creating an integrated view of contracting and planning activities. Purchasing activities build on demand signals and forecasts. Consider Figure 1.9, which summarizes the concept of truly efficient promotions. Random promotion pricing creates demand risk. We suggest that retailers and manufacturers should employ i) two-segment forecasts to reduce demand risk and ii) portfolios of contracts to distribute demand risk.

Both players collaborate to develop a two-segment forecast. Then, the retailer orders a portfolio of hedging contracts. Consider a portfolio of inflexible forward
contracts and flexible supply options. Shortly before each promotion, retail prices are chosen randomly. When realized demand is low, the retailer employs her forwards and no or few options. In case of high demand realizations, option contracts provide the necessary flexibility to ensure supply. The option reservation fee remunerates the manufacturer for providing sufficient supply even in high states of demand. Thus, option contracts enable the retailer to provide high service levels at zero inventories. There is no end-of-period coverage and the channel is immediately ready for new promotions and product innovations. The high service level achieved ensures unparalleled customer satisfaction.

Retailers and suppliers must actively optimize the distribution of risk in the channel by employing suitable contracts. Both forward and option contracts assign risk to the retailer, too. When the retailer consciously reduces flexibility embedded in the contractual structure, this can be interpreted as investing in the relationship. This commitment allows for simultaneously increasing service levels and improving profit. Thus, we propose to move from a spot market type of structure to relationships involving more commitment on the retailer side. From the retailer's point of view, the move from spot contracts to (partial) commitments is only reasonable when forecasts are sufficiently accurate and when uncertainty is reflected in contract parameters. Valid forecast information is a core requirement for optimal pricing of supply contracts. Otherwise, incentives of the retailer and the manufacturer cannot be aligned to eliminate waste and to achieve the supply chain optimal profit.

Our central hypothesis is that a portfolio of risk sharing supply contracts generates value in a price promotions environment. The traditional relationship of retailers and manufacturers in the FMCG industry is transformed towards more commitment, resulting in a win-win situation. The key steps in modeling the approach are understanding demand and deriving all players' optimal decisions.

### 1.4 Structure of further analysis

Profitability of price promotions is a key concern of retailers in the fast moving consumer goods industry. We analyze the potential of supply chain management in this context. Forecast accuracy can never be fully achieved. There will always be a need for flexibility on both the retailer and the manufacturer sides. Under a wholesale price scheme, coordination is not achieved. The result can be a lack of trust as well as low growth, profitability and customer satisfaction. We envision a relationship where both the retailer and the manufacturer are highly responsive and provide excellent service levels. Thereby, higher customer satisfaction is achieved. In this contribution, we show how to achieve these targets.

Chapter 2 reviews literature on supply chain coordination with contracts and on retail promotions. Chapter 3 models portfolios of supply chain contracts and derives coordinating supply chain parameters. We explicitly consider flexible production and channel power as key characteristics of the FMCG industry. Chapter 4 analyzes two-segment demand forecasts and provides further insights into the structure of
demand risk in a price promotion environment. We review the household inventory model of demand and derive the two-segment forecast. We employ point-of-sales data to show the performance of the two-segment demand forecast. Chapter 5 combines the insights from our analysis of coordinating contracts with the two-segment forecast and develops a portfolio model, considering interdependencies of multiple price promotions following each other. Chapter 6 contains a numerical study that shows the impact of coordinating contracts and two-segment forecasts in a price promotions environment. Finally, Chapter 7 discusses managerial implications of our work and concludes with suggestions for future research.

## Chapter 2 Literature Review

Our work is related to two fields of literature. First, we review literature on supply contracts. Supply contracts can i) achieve higher supply chain profit and ii) hedge against risk. Second, we discuss contributions in the field of price promotion research. We review i) several studies on the demand effects of price promotions and ii) theory on the stockpiling effect of price promotions.

### 2.1 Review of contributions in the field of supply contracts

The discipline of supply chain management has evolved over the past two decades. Scholars realized that simply considering material flows is not sufficient to grasp the complexity of links among companies in supply chains. Chopra and Meindl (2009) consider flows of funds, information and materials to be the three types of transfers that establish a supply chain. In this context, supply chain management is the discipline of managing these three flows. In the following, we concentrate on supply contracts as one critical means of supply chain management.

Supply contracts formalize the parameters employed when exchanging products and funds between two stages in a supply chain. Parties involved in the contract specify magnitude and timing of payment streams as well as the amounts of goods exchanged. Moreover, players' actions convey valuable information. Thereby, supply contracts are a means of organizing information flows in supply chains. Consequently, supply contracts link the three dimensions of supply chain management.

Consider a supply chain consisting of a manufacturer and a retailer. Figure 2.1 shows a general outline of this simple supply chain. The retailer sells goods to consumers at the retail price. She purchases the goods required from the manufacturer at the wholesale price. A supply contract between the retailer and the manufacturer specifies the terms of this exchange. In the most simple case, both parties negotiate on a wholesale price paid per unit ordered and delivered.

Supply contracts can have a major impact on the performance of the supply chain and of individual players in terms of both profit and risk. First, we discuss how sup-


Fig. 2.1 A simple supply chain layout (Source: adapted from Tsay et al. 1999, p. 304)
ply contracts can align decisions in a supply chain to achieve the first best solution (as defined below). Second, we review supply options as an effective means of hedging risk with contracts.

### 2.1.1 Channel coordination with supply contracts

A major stream of research analyzes contracts in terms of their ability to coordinate the supply chain. Coordination is defined as achieving the first best solution by aligning the objectives of all parties involved in the transaction. In Section 2.1.1.1, we outline the challenges requiring coordination as well as the basic coordination mechanisms. Then, we review literature regarding different coordinating contracts. At this point, we present an overview of coordination literature and later, in Chapter 3 , we analyze coordination in detail from an analytical point of view.

### 2.1.1.1 Double marginalization and channel coordination

Before discussing channel coordination, we would like to elaborate on the challenges involved. By definition, supply chains consist of more than one entity, e.g., a manufacturer and a retailer. Each of these independent entities maximizes their individual profit. All players derive optimal policies for their decisions. A policy in this context specifies the optimal response to other players' actions. The policies derived in a decentralized decision-making process can differ from the centralized
solution. This effect was formalized by Spengler (1950). In the supply chain context, maximizing individual profit at all stages of the supply chain can be a detriment to overall supply chain profit. The inferior performance of decentralized decision making processes is known in this context as "double marginalization". Overcoming this shortcoming of decentralized decision making has been an important issue in operations and marketing literature ever since.

Jeuland and Shugan (1983) are one of the first to discuss the effect of decentralized decision making in an explicit supply chain setting. They consider a channel consisting of a manufacturer and a retailer. Demand is certain and downward sloping in the retail price. Both players always serve demand. The manufacturer decides on product quality, where higher quality leads to increased variable costs. The retailer can analogously pay to improve store service. Jeuland and Shugan (1983) show that each supply chain partner can "free ride" on the other's profit. That is, each player has an incentive to reduce their efforts in order to save on variable costs. As a result, overall supply chain profit decreases. The game resembles a prisoner's dilemma. Jeuland and Shugan (1983) show that profit sharing can achieve coordination. They suggest employing quantity discounts as a means of profit sharing. Quantity discounts reduce the price paid per unit for larger purchase quantities. Thereby, individual incentives are made compatible with the objective of channel profit maximization.

Double marginalization is no longer a problem when both the manufacturer and the retailer are integrated into a single entity. Then, the objective of the combined entity equals the objective of the whole chain. The inefficiencies of double marginalization can be overcome with contracts. Jeuland and Shugan (1988) discuss, whether channel coordination can also be achieved without formal contracts. Their approach is based on all parties' knowledge of the mechanics of double marginalization. Then, all players can derive rational conjectures about reactions to their behavior. Jeuland and Shugan (1988) find that rational conjectures can increase channel profit compared to the uncoordinated case; however, the first best solution is not reached, in general.

These two references show the importance of supply contracts as a means of reaching the first best solution. Rational players will not achieve the supply chain maximum profit in an uncoordinated supply chain, even when they understand the mechanics of double marginalization. Double marginalization results in lower orders than in the optimum and thus in lower sales to consumers. The retailer cannot achieve the channel optimal service level, because her share of supply chain profit is not sufficient to accept the required amount of risk associated with higher inventories (see Cachon 2003 and Nahmias 2008). In the following, we discuss different types of supply contracts with respect to coordination.

### 2.1.1.2 The wholesale price contract

The wholesale price contract specifies a constant per unit price. This price is applied to each unit purchased, independently of the timing and the size of the transaction.

It has been shown that wholesale price contracts alone do not coordinate the channel in general (Lariviere and Porteus 2001; Nahmias 2008). The manufacturer optimally sets a wholesale price that maximizes his profit; however, this wholesale price does not achieve the first best solution. The reason is again double marginalization. The manufacturer can increase his profit by decreasing the retailers profit, that is through increasing the wholesale price. As a result, the retailer purchases a quantity that is below the channel optimal quantity.

Still, there are arguments in favor of the wholesale price contract. Lariviere and Porteus (2001) suggest that the double marginalization reasoning ignores several important factors that influence the wholesale price. First, they analyze the impact of the distribution of demand as the source of risk on channel efficiency. They find that channel efficiency decreases in the coefficient of variation of the demand distribution. Second, the article suggests that there are further relevant factors that determine the wholesale price. The manufacturer should offer a wholesale price that is closer to the coordinating wholesale price than predicted by simpler models. First, the manufacturer can employ the wholesale price to reward the retailer for sharing demand information. Second, a powerful retailer could decline the original offer because she has other outside opportunities. Lariviere and Porteus (2001) show that channel efficiency depends not only on the contract scheme chosen, but also on external constraints. As a result, the actual loss of supply chain efficiency caused by double marginalization can be expected to be overestimated by most models. Still, wholesale price contracts do not achieve the optimal solution in terms of channel profit.

### 2.1.1.3 Advance purchase discounts

Cachon (2004) studies a model, where a single retailer and a single manufacturer employ a wholesale price contract scheme. There are two basic wholesale price layouts that Cachon (2004) names "push" and "pull". Under a push scheme, the manufacturer offers the product at a constant wholesale price and the retailer decides on her order quantity before uncertainty is resolved. This is the classical Newsvendor model, where the buyer sells the product from inventory and cannot react to the state of the world. Under a pull scheme, the manufacturer produces to stock and the retailer orders after uncertainty is resolved. In both cases, one supply chain party can completely avoid inventory risk. Cachon (2004) defines inventory risk as the risk of overestimating demand, which results in excess stock. The article reviews whether there exists a distribution of risk in the supply chain that achieves channel coordination. First, considering both push and pull schemes improves supply chain efficiency and achieves Pareto improvement. Channel power on the retailer side can decrease supply chain efficiency in this setting. Second, Cachon (2004) suggests the use of advance purchase contracts. Advance purchase contracts employ two wholesale prices, a regular price during the selling season and a discounted price for advance purchases. Advance purchase contracts achieve coordination of the supply chain and allow for an arbitrary distribution of profit between supply chain parties.

Finally, the article suggests that a second, more costly, production opportunity could allow for coordination, even when the retailer decides on the pull wholesale price.

The advance purchase contract is still a price-only contract and the logical next step after analyzing the wholesale price contract. Although it is simple in its setup, the advance purchase contract can coordinate the supply chain. We consider further contributions in this area to line out the relevance of advance purchase discounts.

Özer et al. (2007) model the effect of advance purchase discounts when demand is updated. They consider two periods, where the retailer receives an information update after the first period. The manufacturer has a cheap long-term production technology and the opportunity to produce short-term at higher costs. Long-term production takes place under maximal risk before the information update. The retailer has the opportunity to purchase in both periods. Özer et al. (2007) show that the retailer postpones her order decision until the second period under a wholesale price contract. As a result, the manufacturer produces under uncertainty. The advance purchase discount reduces the effect of double marginalization on the retailer's purchase decision. Therefore, the retailer serves a larger fraction of demand and Pareto improvement can be achieved. This holds true when the manufacturer decides on the discount, but not on the wholesale price. Özer et al. (2007) argue that this assumption is reasonable for many industries, as negotiations on wholesale prices take place long in advance. They show that this mechanism holds, even when there is no long-term production opportunity. One should note, that the manufacturer is assumed to always serve demand. This assumption has a major impact on the results.

Dong and Zhu (2007) analyze push, pull and advance purchase discounts with respect to the coordinating effect of inventory ownership. They model two order opportunities, one under uncertainty and one during the selling season. The article shows that appropriate allocations of inventory ownership can achieve Pareto improvement in terms of profit. Dong and Zhu (2007) account for the possibility of an established retail price and show how introducing a second order opportunity can achieve improvement. Their approach is different from this dissertation in that they only allow for a single long-term production opportunity and that they do not consider option contracts and channel power. A related issue is the retailer's optimal timing of the second purchase decision taking place during the selling season, as presented by Li et al. (2009). The article simultaneously considers the two quantity decision and the timing decision from the retailer's perspective. We do not further consider the timing component of the optimization problem, as FMCG promotions are rather short events and as there are no information updates during promotions.

So far, we discussed advance purchase discounts between retailers and manufacturers. Analogously, advance purchase discounts can be offered to consumers. Tang et al. (2004), McCardle et al. (2004) and Prasad et al. (2010) discuss advance purchase discounts in this context. Tang et al. (2004) argue that advance purchase discounts are required in settings with long production lead-times. In this case, information updates during the sales season are of no value to the manufacturer. Advance purchase discounts achieve early commitments. Precommitted orders provide the seller with additional information on demand. The article derives the optimal
discount in an environment without competition. McCardle et al. (2004) consider advance purchase contracts in a model with retail competition. The seller benefits from the advance purchase discount because he receives updated demand information. However, he pays for this reduction of risk with reduced margins. The article suggests that, in equilibrium, all retailers should offer advance purchase discounts to consumers. Again, the advance purchase discount serves as a means of hedging demand risk. Prasad et al. (2010) argue that the coordinating effect of advance purchase discounts depends on market parameters and consumer characteristics. Among other factors, risk aversion and heterogeneity among consumers determine whether transferring inventory risk to consumers is optimal from the retailer's point of view.

Advance purchase discount offer two major benefits to the supply chain. First, they share inventory risk in the supply chain and as a result, they have the ability to coordinate the channel. That is, advance purchase discounts align individual objectives with channel profit maximization. Thereby, Pareto improvement in terms of profit can be achieved. Second, advance purchase contracts convey information regarding demand. When there are information asymmetries, advance purchase contracts can serve as a means to reveal private information on demand.

### 2.1.1.4 Quantity flexibility contracts

Tsay (1999) considers forecasting-related coordination issues. He models a buyer who provides a forecast to the manufacturer. This forecast is not yet a committed order. The buyer has an incentive to induce the manufacturer to provide a high service level. Therefore, overstating demand in the forecast is rational. The manufacturer reacts by adjusting his production plan for the expected bias in the buyer's forecast. The article models a manufacturer who carries all costs of uncertainty of demand and a retailer who commits as late as possible. He proposes to achieve coordination through a quantity-flexibility contract. Under this scheme, the retailer provides a forecast and commits to purchase at least a certain percentage of the forecast quantity. The manufacturer in turn commits to deliver up to a certain level above the forecast quantity. Each unit eventually sold to the retailer results in a fixed transfer payment. This policy provides reliable supply to the retailer while reducing the incentive to overestimate demand. Tsay (1999) shows that appropriate parameters can achieve a state where both parties make system optimal decisions. Anupindi and Bassok (1999) discuss quantity flexibility contracts in a multi-period setting. At the beginning of the planning horizon, the buyer places orders for all future periods. The article determines her optimal decisions, when orders can be adjusted in a rolling horizon mode. Schoenmeyr and Graves (2009) discuss the optimal allocation of inventory under such an evolving forecast; however, this model does not consider flexible supply.

Quantity flexibility contracts are related to advance purchase discounts in that the retailer commits early to a certain quantity; however, there is no discount offered. The incentive for the retailer to commit results from the necessity to transfer infor-
mation in order to assure supply. The major disadvantage of both advance purchase discounts and quantity flexibility contracts is that they require both parties to carry inventories. This is inevitable, as coordination is achieved by distributing inventory risk in the supply chain. In the next step, we review contracts that are designed to achieve coordination without spreading physical inventories throughout the supply network.

### 2.1.1.5 Two-part tariffs

Two-part tariffs work with transfer payments rewarding the manufacturer for taking inventory risk. As a result, all two-part tariffs require two transactions. There are multiple two-part tariff schemes that achieve supply chain coordination, that is the maximum of supply chain profit possible. We consider buy-back, option and revenue-sharing contracts for this review. Like advance purchase discounts, two-part tariffs achieve coordination by sharing risk between the supply chain parties.

Pasternack (1985) discusses return policies as a means of achieving coordination. A retailer selling a perishable commodity under uncertain demand faces the challenge of ordering an optimal quantity. Unsold products are lost, so the retailer is reluctant to purchase and, consequently, she orders less than the optimal quantity. The manufacturer offers a return policy to share risk in the supply chain. The article shows that return policies where the retailer can return all unsold inventory for a full refund are suboptimal. A buy-back contract where the manufacturer offers partial refund for all unsold units can coordinate the channel. This scheme reaches the first best solution in terms of channel profit. The contribution of Pasternack (1985) is to show that risk-sharing return policies can achieve superior channel performance compared to a regular wholesale price contract. Under the buy-back contract, inventory is ultimately pooled at the manufacturer level. The difference between the wholesale price and the return provides the manufacturer with a compensation for taking on risk. Padmanabhan and Png (1997) argue that buy-back contracts have an additional competitive effect. Retail prices fall in the presence of a buy-back contract, as retailers are less constrained by the availability of inventory. Consequently, competition among retailers is more intense. Return opportunities can also be offered to consumers with a similar motivation as under the buy-back contract; however Su (2009) argue that such a scheme might reduce performance of coordinating contracts between retailers and manufacturers.

Besides return policies, flexibility in order schemes can achieve coordination as well. Under option schemes, the retailer reserves quantities in advance and pays the manufacturer a compensation for taking the inventory risk associated with the reservation. The manufacturer commits to being able to deliver the quantity reserved. The ultimate decision on execution of orders is postponed, until additional information is revealed. Thereby, the retailer gains flexibility without holding physical inventories. As in the buy-back contract, the manufacturer is compensated for taking on risk. Consider the scenario of a short selling season that allows for a single demand update. Barnes-Schuster et al. (2002) analyze the performance of a portfolio of flex-
ible option contracts and inflexible forward contracts in such a context. Forward contracts are early commitments of the retailer in push mode. The article considers a single manufacturer and a single retailer, who interact before the selling season and again later, after a demand update occurred. Barnes-Schuster et al. (2002) employ numerical analysis to solve for optimal parameters. They find that options alone do not coordinate the channel in all cases. Their model shows that results from simple models cannot always be generalized to more complex environments.

As we have seen, channel coordination is reached through distributing demand risk among channel parties. Demand risk ultimately affects revenues. Therefore, contracting on revenues is straight forward from a coordination perspective. Revenue sharing contracts define a wholesale price and a share of revenues transfered from the retailer to the manufacturer. Dana and Spier (2001) argue that manufacturers need to achieve that retailers order sufficient inventories, while keeping retail competition low. They suggest that revenue-sharing contracts can coordinate the channel. According to Cachon and Lariviere (2005) revenue-sharing contracts provide better results than buyback and price-discount contracts when the retailer has the ability to choose her retail price.

Most articles on supply chain coordination with supply contracts assume that there is no competition on price and that retail prices are determined exogenously. According to Ingene and Parry (1995), single two-part tariffs with constant per-unit fees cannot achieve coordination when there are non-identical competing retailers. Instead, they suggest to employ a quantity discount scheme. Bernstein and Federgruen (2005) show that linear price discounts can achieve coordination, even when retailers choose retail prices endogenously. Under a price discount, the wholesale price is a function of the retail price. Price discount schemes achieve coordination under retail competition; however, a non-linear component is required to reach the first best solution. Narayanan et al. (2005) model a manufacturer who serves two competing retailers. They find that subsidizing retailer inventories is required to achieve coordinated fill rates. Krishnan and Winter (2010) model multiple retailers competing on price and inventory availability. They suggest that coordination requirements depend on the durability of the product. Retailers tend to excessively engage in price competition, when products are perishable. Coordinating supply contracts must increase retailer service levels in this case. For non-perishable products, retailers tend to hold too much inventory in non-coordinated supply chains. Krishnan and Winter (2010) suggest that an inventory penalty can achieve coordination.

Channel coordination can be reached by several approaches, including two-part tariffs and quantity discounts. Raju and Zhang (2005) compare these two coordination mechanisms in the presence of a dominant retailer. The article is motivated by the increasing importance of so called "power retailers", which take price leadership and have market power. The model considers the fact that manufacturers pay transfers to important retailers, so called "street money". These funds are intended to motivate the retailer to provide additional sales services. First, Raju and Zhang (2005) analyze the quantity discount as presented by Jeuland and Shugan (1983). Here, the dominant retailer pays a lower price per unit than the competitive fringe, as
she purchases higher quantities and receives a compensation for services. Raju and Zhang (2005) characterize this compensation payment to be "street money". The second contract analyzed is a two-part tariff. The manufacturer must offer two different price schedules to separate the dominant retailer from the competitive fringe. The article shows that the manufacturer must distribute a disproportionally larger share of profit to the dominant retailer in order to provide sufficient incentives for services when the costs of services are high. Consequently, a high market share of the dominant retailer favors a two-part tariff, as the schedules intended for the competitive fringe and the dominant retailer can be better differentiated. On the other hand, high costs of providing services support quantity discount schedules. Here, the manufacturer is required to overcompensate the retailer for high service costs when employing the two-part tariff.

There is few empirical research on the Newsvendor problem and coordination. Olivares et al. (2008) empirically analyze the Newsvendor problem, that is an advance order decision under demand uncertainty. They employ data from surgery room reservations at a hospital to estimate the costs of "overbookings" and "idle capacity". In this sample, the cost of excess capacity were valuated much higher than the cost of insufficient capacity. Katok and Wu (2009) tests coordinating contracts in a laboratory setting. They consider a two-staged supply chain, where the retailer faces a classical Newsvendor problem. Wholesale price, buy-back and revenue sharing contracts are tested with respect to their ability to coordinate the channel. Although, both buy-back and revenue sharing contracts achieve the first best solution in theory, both retailers' purchasing and manufacturer's pricing decisions do not reach the optimum in the laboratory setting. Manufacturers are reluctant to take a significant share of risk, which is required for coordination.

We conclude the discussion of two-part tariffs with an article showing that coordinating contracts can have adverse side effects. Krishnan et al. (2004) discuss buy-back contracts in a promotion setting. The retailer makes two decisions in this model. First, she orders a quantity that is ultimately determined by the contract parameters. The manufacturer allows for returns of unsold inventories to induce a supply chain optimal order decision. Second, the retailer decides on her effort level to promote the product. High efforts are costly, but they also increase demand. The return policy reduces the incentive to invest in promotions from the retailer's perspective. As a result, the contract scheme, which is intended to achieve coordination results in moral hazard. The article discusses extensions of the return policy to include the retailer's effort in the scope of the contract. This article shows that more complex supply contracts potentially result in additional coordination problems. In the case of Krishnan et al. (2004), both purchase quantities and retailer effort must be coordinated simultaneously. Taylor (2002) model a related setting and find that a combination of a return contract and a target rebate can achieve coordination under sales effort considerations. Similar to Barnes-Schuster et al. (2002), Taylor (2002) and Krishnan et al. (2004) show that the ability of a contract to coordinate the supply chain depends to a large extent on the environment.

All three types of two-part tariffs presented achieve coordination by sharing demand risk, as did advance purchase discounts, quantity discounts and quantity flex-
ibility contracts. Thus, channel coordination is ultimately a matter of risk management. In the next section, we focus on option contracts and assess their ability to hedge against demand risk.

### 2.1.2 Supply chain risk management with option contracts

Option contracts allow to distribute risk in the supply chain. We first review how option contracts work and then, we discuss literature on supply options as a means of hedging supply chain risks.

### 2.1.2.1 The mechanics of option contracts

Option contracts are supply contracts that require two separate transactions. They are two-part tariffs. In the first transaction, the buyer purchases option contracts at a fixed fee. As we model European type options, the second transaction takes place at the maturity of the contract. At that point, the buyer decides whether to execute the options. An option is always a right but not an obligation, where the effect of execution depends on the type of option purchased. There are two basic types of supply option contracts.

- Call options allow the purchase of the product at a specified price, named execution fee, at a certain point in time, called maturity (Damodaran 2001). In the supply chain context, a call option is a supply option (Burnetas and Ritchken 2005).
- Put options allow the sale of the product at a specified price at maturity (Damodaran 2001). This right allows the holder of the option to return the product. Thus, in a supply chain contest, put options are return options (Burnetas and Ritchken 2005).

Both types of option contracts provide the buyer with flexibility and reduce risk taking. The call option ensures that supply will be available, as an option must always be served. In addition, the purchase price is known in advance. Thus, call options hedge against supply risk and the risk of high spot prices. Call options ensure that the product can be sold, if necessary. Again the price is known in advance. The holder of the option is hedged against insufficient liquidity of the market and against low spot market prices.

The seller of option contracts takes the associated risk. He is remunerated with the reservation fee. The reservation fee is due in every state of the world and it is thus certain. The seller will keep the reservation fee, even if the option is not exercised.

Purchasing an option results in a "long" position, while selling an option results in a "short" position. Given the two types of options, there are four basic positions: long call, short call, long put and short put. The value of a position depends on the value of the underlying. The underlying can be, e.g., the spot price for the product.


Fig. 2.2 Payoffs of call and put positions dependent on the value of the underlying at maturity

A high spot price at maturity results in a high value of long calls, while short calls result in a major loss. Short puts are then slightly positive and long puts slightly negative. A call is only valuable when the price of the underlying exceeds the execution fee at maturity. Analogously, a put is valuable when the price of the underlying is inferior to the strike price at maturity. Figure 2.2 shows the payoffs of the four positions depending on the value of the underlying at maturity. Consider the case of buying a call option with execution fee $x$. At maturity, the option has a positive value, when the value of the underlying exceeds the strike price. When considering the reservation fee, the value of the option is only positive when the value of the underlying covers both, the strike price and the reservation fee. Analogously, a put is valuable, when the value of the underlying does not reach the strike price at maturity.

Positions can be combined to achieve an predetermined risk profile. Consider the example of a combination of a long and a short call with different execution fees. A retailer purchases a call option with strike price $x_{1}$ and simultaneously sells a call option with strike price $x_{2}$, with $x_{1}<x_{2}$. At maturity, call options increase in value as the price of the underlying increases. Therefore, the reservation fee associated with $x_{1}$ must be higher than the one associated with $x_{2}$. The combined position provides a slightly negative return for low states of the underlying and a constant positive return for high states. The combination reduces downside risk and achieves this benefit by truncating the upside potential. Figure 2.3 shows the resulting payoff scheme.

Consider a retailer who faces uncertain demand. Both, low and high states of demand result in additional costs and thus hurt the retailer's profit. Thus, she purchases a number of supply (call) and return (put) options to hedge against extreme states of demand. Figure 2.4 shows the resulting payoff scheme. When demand is not extreme, the retailer does not employ either type of option. As a result, she loses the reservation fees. When demand is very low, the retailer benefits from the right to return products to the manufacturer. Therefore, put options are valuable. Analogously, call options are valuable in high states of demand, as the retailer can serve all demand.

We model option contracts in Chapter 3. There, we provide analytical details regarding the ability of options to share risk and to coordinate the supply chain. In Section 2.1.2.2, we review articles on the use of options for risk management purposes.

### 2.1.2.2 Options as hedging devices in supply chain management

Supply option contracts are related to financial hedging contracts, e.g., Burnetas and Ritchken (2005) apply financial concepts to Operations Management. They consider both call and put contracts. In this model, the manufacturer offers portfolios of option and forward contracts. The manufacturer produces to order and he has both long-term and short-term production opportunities. Burnetas and Ritchken (2005) assume that the retailer has sufficient power to renegotiate. Therefore, the only credible execution fee equals marginal costs. Put and call options are set into relation by applying the put-call-parity, known from finance theory. Thereby, Burne-


Fig. 2.3 Payoff of a combined position of a long and a short call with different execution fees at maturity


Fig. 2.4 Payoff of a combined position consisting of a long call and a long put with different execution fees at maturity
tas and Ritchken (2005) establish a close relationship of option and return policies. They find that option contracts are in general not a zero sum game. The model assumes that the game has a Stackelberg structure, where the manufacturer is the first mover in determining contract parameters. Therefore, when option contracts are introduced, the retailer potentially loses profit compared to a wholesale price scenario. However, the article shows that Pareto improvement can be achieved.

The value of risk-sharing contracts depends on the number, relation and importance of sources of risk. While most articles exclusively consider demand risk, Spinler et al. (2003) take marginal cost risk and spot price risk into account. They employ a portfolio of option and spot contracts. At the point in time of contracting, the spot price and the marginal costs at execution are uncertain. The article finds that the execution fee should be set at marginal production costs. This policy ensures that the uncertain spot price undercuts the execution fee in as few cases as possible. Thereby, the probability that options are actually executed is maximized. The model achieves Pareto improvement compared to the case involving only spot contracts. Given the results of Spinler et al. (2003) and Burnetas and Ritchken (2005), there are two compelling reasons for pricing the execution fee at marginal costs. Spinler and Huchzermeier (2006) extend the model of Spinler et al. (2003) by establishing boundaries on feasible option parameters.

Gan et al. (2004) argue that supply chain parties may not be risk neutral in reality. They develop an approach to find Pareto sets and optimal solutions when one or both agents are risk averse. Kleindorfer and Li (2005) consider the effect of a Value-at-Risk constraint on a portfolio of supply contracts including options. The Value-at-Risk constraint is formulated on a long-term basis; however, portfolio decisions are often rather short-term. The article provides an optimization approach and applies it to data from the energy market. It shows how option contracts can be employed to hedge against risk in order to achieve a Value-at-Risk target. Another portfolio approach is presented by Martinez-de Albéniz and Simchi-Levi (2006), who evaluate options and spot contracts from a mean-variance point of view. The article derives the efficient frontier, that is the set of option contracts purchased in a mean-variance framework, and identifies the Newsvendor solution.

Wu and Kleindorfer (2005) analyze risk-sharing contracts in an auction setting. They aim to analytically integrate a contract market where a buyer purchases from selected suppliers in advance and an open spot market. Consider a scenario where several sellers offer their products to a single buyer. Sellers are different in their cost and capacity structures, such that each seller prefers another set of contract parameters. The buyer purchases option contracts and, in addition, she has the opportunity to purchase on the spot market. Wu and Kleindorfer (2005) employ an auction mechanism to select efficient contracts. The buyer collects all offers simultaneously and chooses a portfolio of contracts. When there is competition among suppliers, options can achieve the maximum of supply chain efficiency.

Martinez-de Albéniz and Simchi-Levi (2005) consider portfolios of option contracts. They propose a multi-period model that provides an optimal purchase policy. The model considers a single buyer and multiple suppliers, which offer a variety of supply contracts with different parameters. First, the article determines which contracts are dominated by other contracts in the portfolio of supply contracts offered. These contracts cannot be part of an efficient portfolio. Second, Martinez-de Albéniz and Simchi-Levi (2005) provide numerical results on the efficiency of forwards, options and portfolios of both types of contracts. They find that inventory risk is an important driver of option value. In the beginning of the selling season, inventory risk is low. As a result, forward contracts are optimal. Towards the end of the selling season, options increase in importance as there is salvage risk. Consequently, a portfolio of both options and forwards is optimal from the buyer's point of view.

Perakis and Zaretsky (2008) go beyond the research of Martinez-de Albéniz and Simchi-Levi (2005) in that they model capacity limitations at the supplier level. Consider a setting with multiple capacitated competing manufacturers, which offer option contracts, as well as an uncapacitated manufacturer selling on the spot market only. The future spot price is uncertain. The retailer hedges against spot price risk by purchasing option contracts. Perakis and Zaretsky (2008) find that the retailer should either purchase no options or all options available from a manufacturer, depending on the contract parameters. The key contribution of the article is to show that option contracts can achieve coordination even under competition.

### 2.2 Review of contributions in the field of price promotion research

Price promotions are one of the key marketing instruments. Discounts constitute a powerful tool for increasing sales and attracting additional customers. In FMCG retailing, a significant fraction of sales occurs during promotions. Hendel and Nevo (2006a) find in a survey of scanner data on the sales of liquid and powder detergents that 43 and 36 percent, respectively, of these products have been sold with a price discount. Beasley (1998) reports that in four categories (ground caffeinated coffee,
paper towels, toilet paper and canned tuna), more than 50 percent of all purchases took place during a price promotion.

First, we review articles that study the relationship between discounts and consumer demand. Price promotions generally increase demand during the time the discount is being offered. There are multiple explanations for this effect. We review a number of empirical studies that try to decompose the demand effect of price promotions. Multi-period models of stockpiling provide insights into the long-term effects of price promotions and stockpiling behavior. Second, we review theory on price discounts as a source of demand. Here, we focus mainly on consumer stockpiling as an explanation of promotional demand patterns.

### 2.2.1 Demand effects of price promotions

Price discounts potentially increase demand. After discussing the classical model of demand, we review articles addressing two important research questions. First, we review the reasons why additional demand is generated during promotions. Empirical studies provide evidence on the importance of the different drivers of demand. Second, we review literature that attempts to answer the question: "Do promotions add long-term value?"

### 2.2.1.1 The classical demand curve

The standard economic model of the price-demand correspondence is the downwardsloping demand function. This model is frequently employed in both, operations and marketing literature. A prominent example from operations literature is Burnetas and Ritchken (2005), who employ a downward-sloping demand curve to price option contracts. From a marketing point of view, Jeuland and Shugan (1983) employ a downward-sloping demand curve to discuss channel coordination.

Price promotions intend to create additional demand through lower prices. Let $D$ be aggregated demand from consumers in a certain period. Let $p$ be the retail price. For most products, consumers' aggregated demand is negatively correlated with retail prices. Thus, when the retail price decreases, aggregated demand increases. This holds when demand in a market is composed of demand from individual consumers. For lower values of $p$, there are simply more consumers, who value the product enough to be willing to pay this retail price $p$. That is, the welfare of individual consumers from consuming the product exceeds the retail price in more cases when retail prices are lower. The term "reservation price" or "willingness-to-pay" is generally employed to describe the maximum amount, a consumer is willing to pay to buy a product. For extremely high retail prices, aggregated demand approaches zero sales since the retail price exceeds the reservation price of all potential consumers.

Knowledge about the shape of the demand function is relevant for monopolies and companies acting in a market where differentiation is possible. These compa-
nies can choose their retail prices and determine demand. However, when companies face perfect competition, retail prices are dictated by the market. Consequently, only the price and the quantity emerging in equilibrium apply. We see that companies in reality have control over their retail prices to a certain degree. This is the case because markets are usually imperfect. Besides other factors, there are inhomogeneous preferences, limited transparency and positive transaction costs. These factors allow for differentiation on the product and by retail outlet levels. Brands and retailers can create additional welfare or lower costs for individual consumers by positioning closer to that consumers' expectations. As a result, companies receive market power. They have the ability to maximize profit and they can employ pricing as a means of competition.

In the next step, we review the effect of competing products, i.e., when consumers face budget restrictions. Consider a market where consumers allocate their limited wealth $W$ on a product $X$ and an alternative $Y$. The set of feasible purchase decisions is given by the budget constraint. Consumers distribute their wealth on the two products, such that

$$
\begin{equation*}
W=p_{x} x+p_{y} y . \tag{2.1}
\end{equation*}
$$

where $p_{x}$ and $p_{y}$ are the prices of products $X$ and $Y$, respectively, and where $x$ and $y$ stand for the quantities purchased of products $X$ and $Y$ during a certain period of time. Refer to Mas-Colell et al. (1995) for further details on this discussion. Consumers maximize their individual welfare. The amount of welfare gained from purchasing a product depends on a consumer's utility function $U$. The utility function determines a consumer's optimal wealth allocation across the product choices available. When varying the price of product $X$, the budget constraint is adjusted, resulting in a new optimal allocation. The mapping of prices $p_{x}$ on optimal purchase quantities $x$ is called the consumer's individual demand function. The individual demand function provides a consumer's optimal purchase quantity $x^{*}$, given the price of product $X, p_{x}$. Figure 2.5 illustrates that price discounts ensure that consumers purchase larger quantities when a product is on sale. The company chooses to lower the price of $X$ from $p_{1}$ to $p_{2}$. As a result, consumers increase their demand of $X$ for certain types of utility functions. The impact on $Y$ depends on the utility function in place. The sum of all individual demand functions is the aggregated demand function.

To summarize, in the case of imperfect markets, companies have control over prices. The aggregated demand function is then usually downward sloping. However, companies' control over demand by pricing is limited. When prices are raised above an upper threshold, the highest reservation price in the market is exceeded. Below a lower threshold, the company receives all demand available in the market. In both cases, the demand function would be inelastic and thus changes in the retail price would not affect demand.


Fig. 2.5 Budget constraints and demand curves

### 2.2.1.2 Decomposing the demand effect of price promotions

Price promotions affect demand in a number of ways. In general, primary and secondary demand effects have to be distinguished. The primary demand effect describes increases in demand related to stockpiling. Stockpiling is a consumer reaction to promotions where consumers build up inventories of the product that is discounted. They thereby expect to save on purchase costs. Ailawadi et al. (2007) propose a comprehensive terminology for effects associated with household inventories. First, stockpiling increases consumption of some categories. This phenomenon is referred to as the "consumption effect". Second, stockpiling decreases future demand by loyal customers. This hurts the retailer since these customers would have purchased the product at a higher price in the future. This pattern is named "loyal acceleration". Third, stockpiling prevents purchases of competitive brands in the near future, which is known as "preemptive switching". Finally, stockpiling might alter customers' repurchase probabilities. This is referred to as the "repeat purchase ef-
fect". Secondary demand effects are related to brand and store switching. We focus on primary demand effects throughout our analysis.

Price promotions have several effects on demand, both direct and indirect. Gupta (1988) provides an early systematic review of the effects of price discounts. He decomposes the demand effect of price promotions according to distinct drivers: brand switching, purchase time acceleration and increases in the quantities purchased. The article analyzes the important question of whether price promotions generate additional sales by merely shifting future demand to the promotion period, or by generating additional demand from brand switchers. The estimation of the brand choice component shows that marketing variables such as promotions and consumer characteristics are significant in explaining the choice of a certain product. However, marketing variables fail to predict the purchase time decision. Gupta (1988) explains this surprising result by suggesting that consumers do not recognize price promotions when they do not intend to purchase this product in the period in question. In the coffee category, 84 percent of the increase in sales is the result of brand switching, while purchase time acceleration ( 14 percent) and stockpiling ( 2 percent) are less relevant. The article suggests that this result could be triggered by characteristics of the product. Coffee loses freshness when stockpiled and has a relatively large volume, given its consumption rate.

Bell et al. (1999) build on the work by Gupta (1988) and analyze 173 brands in 13 product categories to achieve generalizable results. Bell et al. (1999) find that, consistent with Gupta (1988), brand switching is an important effect of price promotions; however, they also find major stockpiling related effects that are underestimated by the single category study of Gupta (1988). The most relevant parameters are category specific, while the characteristics of the brand and its core customer have less explanatory power. Bell et al. (1999) present evidence that promotions increase consumption for certain categories, while other categories show typical forward buying patterns. The ability to hold a product over a longer period of time determines if customers increase purchase frequency or if they stockpile. Evidence for stockpiling behavior could be found in demand patterns shortly before and after price promotions. Macé and Neslin (2004) analyze pre- and post-promotion dips for 10 product categories at 83 stores and find several factors, e.g., household size and car ownership, that indicate stockpiling behavior.

Ailawadi et al. (2007) further explain the effects of stockpiling. Their model considers increases in consumption, switching to and from competitive brands, intertemporal acceleration of purchases and repeated purchases in the future. Ailawadi et al. (2007) quantify the magnitude of the effects of stockpiling and thus produce results on the total demand impact of promotions. They estimate their model for the yogurt and ketchup categories. The article finds that stockpiling increases consumers' brand preferences for the promoted brand in the yogurt category. For the ketchup category, results are ambiguous. In addition, stockpiling increases the probability of repeated purchases of the brand. Customers purchasing yogurt react to a promotion by mainly increasing consumption, followed by brand switching. Purchase acceleration accounts for only 10 percent of the increase in demand. The overall effect on demand, including long-term impacts of the promotion, is highly
positive, ranging from about 50 percent to over 60 percent in terms of increase in demand. Consumers in the ketchup category react differently. Here, brand switching is the largest component, whereas additional consumption is lower than in the yogurt category. Purchase acceleration is more important than in the yogurt category. The total effect on demand is less important than for yogurt and ranges from 16 percent to 32 percent with the exception of 74 percent for Heinz, which benefits from a very important repeat purchase effect. The model provides insight into the importance of the various effects of price promotions, given categories with very different consumption patterns.

The very different results of decomposition studies over different categories show that a model of promotional demand must clearly define assumptions about the relevant effects. Demand effects of price promotions vary strongly between categories.

### 2.2.1.3 The long-term demand effects of price promotions

Price promotions allow for short-term increases in demand. Some researchers argue that this effect is at the cost of long-term demand. The dimensions of short-term gains and long-term losses determine whether price promotions create or destroy value. Mela et al. (1998) analyze the effect of price promotions on consumers' sensitivity to deals. Eight years of data on sales of fast moving consumer products are employed to derive developments of the stockpiling behavior as a reaction to price promotions. The article distinguishes between a short-term demand increase due to stockpiling and a long-term behavioral effect. Frequent price promotions train consumers to wait for deals, which reduces the fraction of high margin purchase incidences. Mela et al. (1998) formulate a model connecting purchase incidences and purchase quantities. A trend regression shows that during the eight years in question, promotions increase while purchase rates decrease. Both results are significant. Thus, while promotions become more frequent, consumers purchase less often. In addition, the difference of average prices when a purchase occurred and when no purchase was recorded increased by a factor of seven. This indicates a "lying-in-wait heuristic". Expected purchase quantities stayed constant. The analysis shows that promotions reduce the likelihood of a purchase incidence in the future and decrease promotion sensitivity. Thus, with an increasing number of promotions, consumers learn to ignore some of the deals in favor of better deals. Thereby, promotions become less effective over time and retailers are required to attract customers with even deeper discounts.

Srinivasan et al. (2004) analyze whether price promotions are beneficial in terms of profit and whether the outcome is Pareto improving for retailers and manufacturers. To derive the marginal effect of price promotions, Srinivasan et al. (2004) compare a model based on data without promotional shocks with a model based on data including price promotions. They find that price promotions have no significant long-term effect in their data set. Manufacturers' revenues increase during price promotions in 73 percent of all cases in the sample, as price promotions increase the volume sold. Retailers slightly benefit from promotions when only direct effects are
considered; however, considering secondary effects results in several cases in a negative net impact of price promotions for retailers in terms of category revenues. In 62 percent of all cases in the sample, price promotions have no significant effect on retailer category revenues. In terms of retail margins, 54 percent of the brands considered experience negative effects. Only 18 percent of brands on promotion increase store profit. The article shows that price promotions often do not generate enough additional sales and traffic to compensate for the reduction in margins.

According to Hendel and Nevo (2006a) durable products require a dynamic analysis of demand patterns, as price promotions lead to important intertemporal demand substitution. Consequently static analyses might derive wrong estimates for long-term price elasticities. The contribution of the article is to consider the relevant history when estimating own and cross-price elasticities. Households are modeled to optimize expected utility under price uncertainty where holding inventory is costly. By analyzing aggregate data on quantities purchased, Hendel and Nevo (2006a) find that demand increases in the time interval since the last promotion. This result holds for both sales and non-sales periods and suggests a consumer stockpiling mechanism as a driver of results. This explanation is supported by the finding that the time interval between purchase incidences is shorter during promotions. Holding costs seem to explain the differences in purchase patterns of different product categories. In their analysis of own and cross-price elasticities, Hendel and Nevo (2006a) derive that consumers that switch to another brand usually choose the same package size they used to purchase from the old brand. However, cross-price elasticities between package sizes of the same brand are usually much higher than cross-price elasticities between different brands. This reflects brand loyalty. Elasticities vary significantly between static and dynamic model formulations, as static formulations fail to acknowledge intertemporal substitution. As a result, the importance of the outside option, which is no purchase at all, is greatly overestimated in static models.

Single period models are inappropriate for describing the effects of stockpiling on future demand and result in suboptimal policies. Stockpiling increases demand during promotions, but at the same time reduces demand in periods of regular pricing, where consumers would have purchased at higher prices. Thus, the net financial impact of price promotions is ambiguous. In addition, price promotions appear to change the behavior of consumers over time. In the next section, we delve deeper into behavioral aspects of price promotions.

### 2.2.2 Behavioral aspects of price promotions

Price promotions attempt to change the behavior of consumers in that consumers are expected to purchase additional quantities. An important aspect of price promotions research is reviewing further behavioral effects associated with discounts. Ailawadi and Neslin (1998) survey the effect of household inventories on the speed of consumption. Promotions lead to stockpiling and thus they increase inventory levels at consumers' homes. Ailawadi and Neslin (1998) choose two extreme categories in
terms of consumption patterns. Yogurt is a typical convenient product which can be consumed as a snack, while ketchup is never eaten independently. The article focuses on the effect of stockpiling on the consumption rate. This greatly affects the value-added of price promotions to industry profit. While the consumption of yogurt continuously increases with inventory, the ketchup consumption rate is fairly stable. This result shows that convenient products are consumed faster when stockpiled. In a second step, a simulation approach is employed in order to decompose the primary and the secondary demand effects. For yogurt, as much as 35 percent of the increase in sales can be traced back to increased consumption. The article shows that stockpiling increases consumption. This effect is stronger for convenient products and can reach a magnitude of more than one third of the total increase in sales. Price promotions not only shift sales between brands and over time, but also can create additional consumption.

Chandon and Wansink (2002) analyze the effect of stockpiling on consumption behavior, combining statistical analysis of scanner data with field experiments. According to Chandon and Wansink (2002), consumption decisions consist of two steps. First, a consumption incidence must occur and then consumers decide on consumption quantities. The article suggests that consumption decisions are influenced by the convenience of the product and its salience, that is, its visibility at home. Stockpiling increases costs of inventory and reduces replacement costs. Thus, from the economic viewpoint, stockpiling should increase consumption. Chandon and Wansink (2002) find that some products, such as fruit juices and cookies, are consumed faster when larger quantities are available at home. For other products, like detergents, the speed of consumption does not alter. Stockpiling increases consumption to a larger extent when a product is convenient to consume. A lab experiment shows that stockpiling increases consumption by increasing salience of the product. Finally, during a second laboratory experiment, convenience is manipulated and the effect of salience on consumption is measured. The experiment confirms that salience triggers consumption only when the product in question is convenient to consume.

Chandon and Wansink (2006) survey how consumers estimate their current inventory when facing a purchasing decision. Potential overstocking and the fear from stockouts particularly affect consumers' purchasing behavior during promotions. Consumers, tending to underestimate their current stock level, are more deal-prone, while those who overestimate their inventories react more reluctantly to price discounts. Consumers are modeled to employ internal and external reference points rather than actual inventory levels. When a product is visible, consumers are likely to adjust their perception according to real inventory levels. A first laboratory experiment shows that consumer inventory estimates follow a compressive function, thus consumers' estimates tend to be neither very low nor very high. Consumers tend to shift their estimates towards external anchors. The second laboratory experiment tests for the effects of the estimated average inventory, which constitutes an internal anchor. Visibility increases the accuracy of the estimate, while internal anchors shift the estimate towards the reference inventory. Chandon and Wansink (2006) find that purchase incidences are more related to estimated inventory levels
than to real inventory levels. In a second step, Chandon and Wansink (2006) extend the sample of products to non-food articles. They find that estimates are particularly biased whenever the product is usually bought on impulse, when it is difficult to stockpile and when it has a low average promotional elasticity. In both field studies, less than half of the estimates are accurate and underestimations are more likely than overestimations.

Stockpiling is a reaction of consumers to discounts. Consequently, both, purchase decisions and consumption decisions, are relevant for the analysis. One of the most important questions is the impact of stockpiling on the consumption rate. If stockpiling increases the consumption rate, retailers will gain a net increase in demand by promoting a product. Moreover, stockpiling depends on consumers' knowledge of household inventory levels. Here, the key insight is that consumers cannot be expected to perfectly know their inventory levels and that underestimations are more probable. This effect strengthens stockpiling.

### 2.2.3 Theory of price promotions from a stockpiling perspective

Some consumers shift purchases over time in order to benefit from price discounts. As a result, demand from this segment increases during the promotion period and decreases thereafter. This effect is particularly relevant in categories that are characterized by stable consumption patterns and long shelf life (Mela et al. 1998). We review three influential models of price promotions developed by Varian (1980), Blattberg et al. (1981) and Salop and Stiglitz (1982). Then, we review empirical evidence and thereafter we consider several articles that apply this approach to demand forecasting.

### 2.2.3.1 Theory of promotional demand

Varian (1980) models promotions as pricing games between retailers. Multiple retailers bid prices into the market. Consumers are divided into two segments. Informed consumers purchase at the cheapest store, while uninformed consumers choose randomly a store. As a result, stores have to trade-off additional sales to the informed segment against lower margins on sales to the uninformed segment. The article finds that retailers should choose retail prices randomly to price discriminate between uninformed and informed consumers. Informed consumers have an advantage, as they always purchase at the lowest price. Varian (1980) calculates the value of being informed from a customer perspective.

Blattberg et al. (1981) propose a model, explaining price promotions as a means of transferring inventory holding costs from retailers to consumers. Consumers are modeled to trade-off holding costs against price discounts. Retailers have the opportunity to reduce their inventories by offering promotions. Thereby, they force consumers to focus their purchases on a limited time span - the promotion; how-
ever, deals also reduce retailers' margins. Blattberg et al. (1981) employ three target parameters: deal magnitude, deal frequency and purchase quantities. The model is estimated for four product categories: aluminum foil, facial tissues, liquid detergents and waxed paper. To identify stockpiling, the quantities purchased during promotions as well as the intervals between purchases are measured. Stockpiling behavior is characterized by longer intervals between incidences or by larger purchase quantities per incident. The empirical study shows that stockpiling in both forms exists for the categories reviewed. The importance of the effects depends on the characteristics of the products. Blattberg et al. (1981) show that deals are more likely for products with important regular sales, than for products, which exhibit low regular sales. The article introduces and formalizes the concept of price promotions as a means of shifting holding costs to consumers. According to Blattberg et al. (1981) price deals are offered to save costs and do not serve the primary purpose of informing consumers about products. In a setting closely related to Blattberg et al. (1981), Jeuland and Narasimhan (1985) find that inducing stockpiling behavior through promotions is particularly efficient from the sellers point of view, when buyers incurring high holding costs are those buyers that purchase large quantities. In contrast to Blattberg et al. (1981), Jeuland and Narasimhan (1985) base their findings on a price discrimination argument. They assume that consumers purchasing during promotions are unlikely to purchase at the regular price.

Salop and Stiglitz (1982) model a number of retailers who choose a wholesale price at random. All consumers are equal and adapt their purchase behavior to the retail price encountered at the point of sale. Consumers have two shopping opportunities. Consumers, who face a high retail price have the opportunity to purchase a small amount and to reenter the market later. Consumers, who face a low retail price purchase a larger amount and leave the market. Salop and Stiglitz (1982) consider both holding and transaction costs. Firms offering low prices receive more demand at lower margins, while high price firms receive less demand, but higher margins. In equilibrium both strategies must be equally profitable. Bell and Hilber (2006) empirically test predictions derived from this equilibrium and find significant effects of storage constraints on consumer behavior.

Pesendorfer (2002) and Hendel and Nevo (2006b) empirically test stockpiling as a key driver of demand during promotions, as predicted by the household inventory model. Pesendorfer (2002) employs a set of data covering daily prices of tomato ketchup for 80 percent of all retail stores in Springfield, Missouri. Moreover, a second data set provides the purchase behavior of 1,500 households. Manufacturers offer infrequent price promotions at a rate of approximately one per quarter. The article finds that demand depends on past prices. Demand during promotions is significantly higher when prices in preceding periods where high. In addition, $\mathrm{Pe}-$ sendorfer (2002) discriminates between store switching and store loyal consumers. Both consumer segments hold inventory.

Hendel and Nevo (2006b) consider both consumption and stockpiling effects. These effects are difficult to separate, as consumption cannot be directly observed. Using aggregated data, Hendel and Nevo (2006b) find that consumers purchase larger quantities when the period since the last sale is long. Moreover, high storage
costs of a category reduce the importance of sales during promotions. The article derives from household level data that households stockpile to take advantage of lower prices during promotions. Consumers are modeled to follow an order-up-to policy. Larger remaining household inventories seem to reduce demand during promotions. Consumers which have potentially lower storage costs react more to promotions. Hendel and Nevo (2006b) employ the size of houses and the existence of dogs in households as instrumental variables in this context. The results presented in the article confirm the predictions of the household inventory model. Consumers are less likely to purchase for a longer period of time after promotions, which is most probably because of stockpiling effects.

There is empirical evidence that stockpiling drives promotional sales for some categories. This knowledge is valuable for forecasting purposes. In the next section, we review articles that apply the modeling approach of Blattberg et al. (1981) to derive forecasts. These articles provide evidence of the performance of the household inventory model in forecasting.

### 2.2.3.2 Forecasting demand from price promotions

The insights gained by employing the household inventory model of promotional demand can be leveraged to improve the quality of forecasts. Iyer and Ye (2000) use a forecast model based on a consumer stockpiling logic with two consumer segments. Consumers in the first group purchase equal amounts of the product in each period, independently of the retail price. The second group of customers is deal-prone and shifts demand over time. Iyer and Ye (2000) calibrate their forecast model with empirical data. Collaboration in forecasting is shown to be beneficial for both the retailer and the manufacturer. Sharing promotion schedules allows for synchronizing production plans and demand peaks. The model allows for returning excess promotion inventory to the supplier. This is a means of redistributing risk between supply chain parties. A 100 percent service level on the supply side is assumed to avoid game theoretic complications, as the supplier would consider the distribution of demand at the retailer level when taking his production decision. Consequently, Iyer and Ye (2000) model independent production and purchasing decisions. They find that promotions can be inefficient when safety stock requirements are excessive. The amount of safety stock required depends on the distribution of demand. Still, predictable promotion campaigns involving forecast sharing are even more profitable than an EDLP strategy from the supplier's point of view.

Huchzermeier et al. (2002) build on the model of Iyer and Ye (2000). They analyze the retail market for diapers. Retailers segment consumers by offering different package sizes, each implying a different per unit price. Promotions are package size specific. Consequently, consumers benefit from promotions by i) switching between package sizes and by ii) holding inventories. Huchzermeier et al. (2002) develop a model explaining demand during promotions. They show that the retail price alone does not explain demand. When adding consumer inventories and package size switching as explanatory effects, forecast errors shrink and adjusted $\mathrm{R}^{2}$
values reach 91 percent. Huchzermeier et al. (2002) show that consumer inventory models attain very good estimates of future demand during promotions for certain categories.

A related issue is the use of information sharing to improve forecasting of demand during price promotions. Wiehenbrauk (2010) analyzes price promotions in a competitive setting and she derives two key insights. First, in equilibrium, retailers should optimally mix promotion prices to be unpredictable. Otherwise, competitors can underbid the promotion price to receive all demand from switching customers. Second, manufacturer's have superior knowledge of the competitive environment, as they see all orders from all retailers in the market. Wiehenbrauk (2010) proposes that manufacturers should publish a competition index to provide signals to retailers. Thereby, Pareto improvement in terms of profit can be achieved.

### 2.3 Summary and next steps

Price promotions have multiple effects on demand. In general, one can distinguish between primary and secondary demand effects. Primary demand effects originate from stockpiling and purchase acceleration, while secondary demand effects are competitive effects. Price promotions affect consumer behavior: First, consumers learn to react to price promotions by waiting for discounts. Price promotions train consumers to switch from regular purchasing patterns to inventory holding. Second, consumption can be altered by promotion induced stockpiling for some categories.

Models of price promotions can be employed to achieve good forecasts; however, a forecast error remains, resulting in demand risk. Supply contracts can be employed to share risk. We presented a number of approaches, including two-part tariffs and advance purchase discounts. These contracts can achieve additional supply chain profit compared to a simple wholesale price contract.

In our view, the long-term benefit of truly efficient promotions stems mainly from continuous innovations with price discounts for product launches. Thus, they are closely related to continuous new (or seasonal) product introductions. Supply contracts can simultaneously provide the required flexibility and ensure supply.

In the next chapter, we formalize the coordination effect of supply contracts and model portfolios of risk-sharing contracts. Then, in Chapter 4, we extend the household inventory model of promotional demand to multiple promotions, considering path dependencies. Thereafter, we employ stochastic programming to integrate insights on risk-sharing contracts and promotional demand in a single model in Chapter 5. We apply this model to FMCG data and show the usefulness of our approach in Chapter 6.

## Chapter 3

## Portfolios of Coordinating Contracts

This chapter develops theory and sets the stage for the analysis of flexible sourcing contracts. The first section presents the model layout. The second section discusses theory on channel coordination and supply contracts. At this point we review the analysis of Cachon (2004) regarding pull and push contracts. We extend this portfolio model in several ways: In the third section, we include option contracts and show that bidding contracts into the market can improve the status quo. In the fourth section, we add costs for short-term production. In the fifth section, we consider the case, where retailers exert power i) through reductions in the wholesale price and ii) through emergency orders during times of promotions. We derive optimal contract parameters and determine which contracts should be part of an efficient portfolio. Finally, we present a numerical study to further explore the dynamics discovered.

### 3.1 Model layout

Consider a supply chain consisting of a single manufacturer supplying a product to a single retailer, who serves uncertain demand. First we present details on the layout and the dynamics of the supply chain modeled and second, we state the key assumptions employed.

### 3.1.1 Supply chain layout and dynamics

Consider the following time line. We model a single order period with a starting date $t_{1}$ and a sales date $t_{2}$. Forward and option contracts are closed at $t_{1}$. At this point in time, decisions must be made under demand risk. The retailer serves uncertain demand $\widetilde{D}$ at $t_{2}$ after uncertainty is resolved. Option executions and instant (spot) orders are placed at $t_{2}$ under certainty. The order schemes employed characterize the transactions involved. Manufacturers require inventories and/or flexible
production opportunities to serve orders. Retailers potentially can enforce service level requirements. Thus, order schemes, inventories, production opportunities and service level requirements characterize the dynamics of the supply chain. We review these four factors in detail before stating the key assumptions underlying the model. This section is intended to provide an overview on the concepts discussed in this chapter and on the notation employed.

### 3.1.1.1 Orders

The retailer has the opportunity to order at $t_{1}$ and at $t_{2}$. At $t_{1}$, she can commit purchases with forward orders and she can reserve quantities with option reservations for delivery at $t_{2}$. At $t_{2}$, the retailer places additional "emergency" instant orders for immediate delivery and she executes options. Let $z$ be the number of forwards ordered and let $y$ be the number of options reserved. Then, $y^{\prime}=z+y$ is the minimum committed supply by the manufacturer. Further, let $w_{1}, w_{2}, r$ and $e$ be the forward, instant order, option reservation and option execution fees payed by the retailer to the manufacturer. Anupindi and Bassok (1999) characterize the degree of flexibility embedded in supply contracts as the maximum magnitude and frequency of adjustments permissible. Forward orders represent long-term commitments, while instant orders and option reservations allow for flexibility. In our model, forward orders are not limited in size by production capacities. Instant orders can be implicitly limited, as the manufacturer can decide, not to serve all instant orders. The amount of instant orders served, depends on the manufacturer's willingness to invest in speculative inventories and on the availability of a short-term production opportunity. We first model supply chain relations based on individual contracts in Section 3.2. Then, we expand our analysis to portfolios of contracts in Section 3.3.

### 3.1.1.2 Inventories

In this thesis, we assume that the product in question is not perishable. Therefore, the manufacturer can hold inventories to serve instant orders even without shortterm production. That is, he produces long-term quantities exceeding the retailer's reservations to potentially serve short-term instant orders. However, speculative inventory is costly, as unsold units must be salvaged at a salvage value $v$ after $t_{2}$, e.g., to clear the channel for a new promotion. Analogously, the retailer must salvage forward orders exceeding demand at the salvage value $v$ after $t_{2}$.

### 3.1.1.3 Production opportunities

The manufacturer can use two types of capacities to produce the product. The longterm production technology is employed to produce a quantity $q$ at $t_{1}$ for delivery at $t_{2}$, with the opportunity to produce to stock, speculating for instant orders at $t_{2}$. Each


Fig. 3.1 Structure of inventories and capacities in the supply chain
unit produced long-term results in production costs $c_{1}$. The short-term production technology allows for flexibility on the manufacturer side of the supply chain. The manufacturer produces at $t_{2}$ for immediate delivery. Short-term production could for example be transactions between regional warehouses or last minute production involving overtime. As a result, production lead-times can be very short and production is possible even after uncertainty is resolved, that is during the selling season. However, this short-term technology is more expensive than long-term production, resulting in costs $c_{2}>c_{1}$. Figure 3.1 shows the layout of the supply chain modeled with respect to inventories and production opportunities. We first model a supply chain, where the manufacturer is restricted to long-term production and then we analyze the impact of a costly short-term production opportunity in Section 3.4.

### 3.1.1.4 Distribution of power in the channel

In a balanced supply chain relationship, the manufacturer will allow for stockouts, if serving instant orders is not profitable. While option and forward orders must be served, the manufacturer can decline to serve all instant orders. Particularly in high states of demand, the cost of speculative inventories and/or short-term production could be excessive. However, stockouts hurt the retailer's profit. A powerful retailer can enforce a required minimum service level to ensure that instant orders are served even in high states of demand. The manufacturer is then required to serve instant orders at minimum up to a quantity $q^{\text {req. }}$. Moreover, a powerful retailer can put pressure on wholesale prices to lower purchasing costs. Both required minimum service levels and fierce price negotiations appear in industry. We first restrict our analysis to supply chains without formal channel power and then allow for both forms of channel power on the retailer side in Section 3.5.

### 3.1.2 Key assumptions

We set several key assumptions. First, demand $\widetilde{D}$ is uncertain and defined in the positive range. The retailer shares demand information with the manufacturer. Both parties have identical believes about the cumulative demand distribution $F(x)$ of de-
mand and a respective density function $f(x)$. We assume that $F(x)$ is continuous, differentiable and invertible. The distribution of demand fulfills the increasing generalized failure rate property (IGFR). Under the IGFR property $g(x)=x f(x) /(1-F(x))$ is increasing in $x$. This property is fulfilled for example by the Normal and the gamma distributions (Cachon 2004). Please refer to Lariviere (2006) for further details on this concept. Moreover, $F(0)=0$ and $\lim _{x \rightarrow \infty} F(x)=1$.
Assumption 3.1 Demand is uncertain. No party has an advantage in terms of information on the distribution of future demand and thus both parties base their decisions on similar beliefs.

Given that the product is durable, both, the manufacturer and the retailer have the ability to hold inventories. As we do not model the time after $t_{2}$ explicitly, we allow for salvaging unused inventory. Both parties receive a salvage value $v<c_{1}$ for every unit not sold at $t_{2}$. Given that the salvage value is lower than long-term production costs, both players are penalized for holding inventory after $t_{2}$. Thus, the manufacturer risks losing profit, when producing with a long lead-time at low costs. Analogously, forward orders at $t_{1}$ result in risk for the retailer. Assuming equal salvage values for both parties ensures that results are not driven by salvage opportunities.
Assumption 3.2 Both the retailer and the manufacturer can salvage left-over inventory and no party has an advantage in doing so.

After modeling the base case in Section 3.2, we extend our model beyond the scope of of the recent literature. First, we model portfolios of contracts in Section 3.3 without additional assumptions. Second, we assume that the manufacturer has the ability to produce short-term in Section 3.4. Here, we assume that manufacturing costs are lead-time dependent, where the long-term technology is cheaper. This assumption models the costs of flexibility, such as overtime payments. A discount on long-term production has been employed by Özer et al. (2007) in a related setting.

Assumption 3.3 Short-term production is more expensive than long-term production, where short-term production costs can exceed the instant order wholesale price price. Lead-times are short enough to enable short-term production under certainty during the selling season.

Third, we extend our model in Section 3.5 by assuming that the retailer has superior bargaining power compared to the manufacturer. The retailer drives sales of branded products by assigning shelf space. She can credibly threaten to delist less important SKUs of the manufacturer. Therefore, even strong brands are subject to unequal negotiations on contract parameters. Lariviere and Porteus (2001) employ a similar assumption to show the effects of low wholesale prices. They argue that retailers must take into consideration the opportunity cost of carrying a product. The retailer forces wholesale prices below that level, as she has otherwise no incentive to purchase the product. The manufacturer can either agree to the lower wholesale price or withdraw from the market. The second form of channel power considered is a required minimum service level.

Table 3.1 Notation employed in Chapter 3

| Symbol | Description | Symbol | Description |
| :--- | :--- | :--- | :--- |
| $D$ | expected demand | $p$ | retail price |
| $\Pi_{r}$ | retailer's objective function | $w_{1}$ | forward price |
| $\Pi_{m}$ | manufacturer's objective function | $w_{2}$ | instant order price |
| $q$ | quantity produced long-term | $r$ | option reservation fee |
| $z$ | quantity contracted using forwards | $e$ | option execution fee |
| $y$ | quantity reserved using options | $c_{1}$ | long-term production cost |
| $y^{\prime}$ | combined forward and option reservations | $c_{2}$ | short-term production cost |
| $S(\cdot)$ | expected sales from a quantity $(\cdot)$ | $v$ | salvage value |
| $F(\cdot)$ | service level achieved with a quantity $(\cdot)$ | $q^{\text {req. }}$ | required service level |
| $q^{\text {limit }}$ | limit on short-term production capacity | $q^{\text {int. }}$ | service level under coordination |

Assumption 3.4 The retailer has superior bargaining power and can put pressure on the manufacturer to lower the wholesale price or to increase service levels. The retailer must consider the manufacturer's participation constraint which is nonnegative profit.

We introduced the channel layout and the structure of our analysis. Table 3.1 summarizes the notation employed throughout this chapter. In the next step, we discuss forward, option and instant order contracts with respect to channel performance.

### 3.2 The impact of supply contracts on supply chain performance

We review the performance of wholesale price contracts and compare them to advance purchase discounts (i.e., portfolios of forward and instant order contracts) and two-part tariffs (i.e., option or buy-back contracts). We determine channel coordinating parameters and compare these contract schemes in terms of profitability, limitations and their usefulness in practice.

### 3.2.1 The wholesale price contract (I.1 and I.2)

The most common contract in practice is the wholesale price contract. Under a wholesale price scheme, the retailer and the manufacturer agree on a single parameter, the wholesale price $w$. Figure 3.2 shows the relationship between manufacturing costs $c$, wholesale price $w$ and retail price $p$. The wholesale price is located between the manufacturing cost and the retail price. If the wholesale price exceeds these limits, one party will receive a negative profit. The distribution of the profit achieved depends on each parties' negotiation power in the supply chain and the mechanism


Fig. 3.2 Parameters of the wholesale price contract
employed to derive contract parameters. For the discussion of wholesale price contracts, we follow Cachon (2004).

The wholesale price contract is the simplest contract possible. This is a major strength, as the contract can easily be communicated. The low cost required to implement such a scheme is one reason for its popularity. There are two contracting schemes possible with a single wholesale price. We first review the "push" scheme, where the manufacturer requires a substantial order lead-time. Then, we analyze the "pull" scheme, where orders are postponed until the selling season has begun and uncertainty is resolved.

### 3.2.1.1 The push scheme (I.1)

Due to production and transportation delays there is a significant period of time between orders and deliveries. Consequently, uncertainty exists regarding future demand at the time when the retailer places orders. Under a push scheme, the retailer must decide at $t_{1}$ how many units of the product she would like to order for future delivery at $t_{2}$. The manufacturer produces to order and delivers at the contracted point in time. The retailer pays a wholesale price per unit. Figure 3.3 shows the sequence of events under the push scheme.

Under the push scheme, the retailer bears all of the inventory risk in the supply chain (see Cachon 2004). Inventory risk is the risk of producing or purchasing more units of the product, than demanded. The retailer bears inventory risk as consumer demand in $t_{2}$ is uncertain at the time of ordering in $t_{1}$. Consequently, the retailer risks purchasing more units than she can sell to consumers. As a result, she may have to salvage some units.

In the next step, we model the push contract and derive the optimal wholesale price. We follow the approach and notation of Cachon (2004). Let $q$ be the manufacturer's production quantity. The manufacturer pays variable production $\operatorname{costs} c_{1}$ at $t_{1}$ and he is compensated by receiving the wholesale price $w_{1}$ for every unit sold to the retailer.

Let $z$ be the retailer's order quantity. Under the push scheme, the manufacturer produces to order, thus it must hold that $z=q$. The retailer sells the product to consumers and receives the regular retail price $p$ for each unit sold. At $t_{1}$, the retailer carries zero initial inventory and at $t_{2}$, left-over inventory is immediately salvaged

## Model I. 1 Wholesale Price Contract (Push Scheme)



Fig. 3.3 Layout of the wholesale price contract model under a push scheme
for $v$ per unit. We define $S(q)$ as expected sales to consumers. The retailer can never sell more units than she purchased from the manufacturer, i.e.,

$$
\begin{equation*}
S(q)=E(\min (\widetilde{D}, q)) \tag{3.1}
\end{equation*}
$$

Let $F(x)$ be the cumulative distribution function of uncertain demand $\widetilde{D}$. Let $f(x)$ be the respective density function. We derive an expression for $S(q)$ and employ integration by parts to rearrange terms (see Cachon 2003). The retailer sells a quantity equaling demand until she has no more units to sell. Thus, in high states of demand, she sells the quantity contacted. Formally,

$$
\begin{equation*}
S(q)=\int_{0}^{q} x f(x) \mathrm{d} x+q(1-F(q))=q-\int_{0}^{q} F(x) \mathrm{d} x . \tag{3.2}
\end{equation*}
$$

The first-order derivative of $S(q)$ with respect to $q$ represents the effect of additional orders on expected sales, namely

$$
\begin{equation*}
\frac{\mathrm{d} S(q)}{\mathrm{d} q}=1-F(q) \tag{3.3}
\end{equation*}
$$



Fig. 3.4 Relation between wholesale prices and service levels for the example of a normal distribution of demand

This expression will play an important role in our analysis. The retailer makes her order decision before uncertainty is resolved. Thus, she makes her decision under demand risk and she is affected by costs associated with risk. There are two cases to be considered. First, if realized demand exceeds the quantity ordered, the retailer will incur stockouts. In this case, she loses an opportunity cost $\left(p-w_{1}\right)$ per unit, which she would have earned, if she had purchased additional units. These costs are often called underage costs $c_{u}$ (Nahmias 2008). Second, if realized demand is less than the quantity ordered, the retailer will salvage excess inventories. We only consider the non-trivial case $v<w_{1}$. Under this assumption, the retailer is punished for purchasing excess quantities. We name these costs $\left(w_{1}-v\right)$ overage costs $c_{o}$ (Nahmias 2008).

First, we formulate the retailer's profit function. The retailer maximizes expected profit $\Pi_{r}$. For every unit purchased $q$, she pays the wholesale price $w_{1}$ and simultaneously gains the opportunity to salvage the product at the salvage value $v$. For every unit eventually sold, the retailer earns the retail price $p$ and loses the salvage opportunity, i.e.,

$$
\begin{equation*}
\Pi_{r}^{\text {push }}\left(q, w_{1}\right)=(p-v) S(q)-\left(w_{1}-v\right) q \tag{3.4}
\end{equation*}
$$

The retailer decides on the order quantity $z=q$ at $t_{1}$. She faces a trade-off between overage and underage costs. We derive the first-order condition of the retailer's profit function with respect to $q$, that is,

$$
\begin{equation*}
\frac{\mathrm{d} \Pi_{r}^{p u s h}}{\mathrm{~d} q}=(p-v)(1-F(q))+\left(v-w_{1}\right)=0 \tag{3.5}
\end{equation*}
$$

The retailer optimizes her profit in that she chooses a service level $F(q)=\alpha$, i.e., the probability to serve all demand. The optimal service level depends on the wholesale price. For example, a 95 percent service level means that the retailer incurs stockouts only in 5 percent of all states of the world. To achieve a service level $\alpha$, she orders a quantity $q=F^{-1}(\alpha)$. This optimization problem is generally referred to as the "newsvendor problem" (Nahmias 2008). The solution to the newsvendor problem depends on the relative importance of overage and underage costs and is referred to as the "critical fractile" $F\left(q^{*}\right)$. In a supply chain operating in push mode, the retailer determines the production quantity. $\Pi_{r}^{\text {push }}$ is concave in $q$. We receive the optimal service level by solving (3.5) for $F(q)$. We find that,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{u}}{c_{u}+c_{o}}=\frac{\left(p-w_{1}\right)}{\left(p-w_{1}\right)+\left(w_{1}-v\right)}=\frac{p-w_{1}}{p-v} . \tag{3.6}
\end{equation*}
$$

The retailer's optimal order policy is a function of the wholesale price $w_{1}$. Higher wholesale prices reduce underage costs and thus decrease the optimal service level. As a result, higher wholesale prices lead to more frequent stockouts at the retailer level. Figure 3.4 depicts the finding that the relationship between wholesale price and service level is linear and downward sloping. The maximum service level is attained for the integrated channel, which we will discuss later.

Next, we assume that the game is sequential and that it has a Stackelberg structure. The manufacturer moves first and decides on the wholesale price. He considers the retailer's best response function (see Equation 3.6). The manufacturer chooses $w_{1}$ such that he receives the maximum possible profit, given the retailer's best response function. We rearrange terms to derive the wholesale price as a function of the order quantity, i.e.,

$$
\begin{equation*}
w_{1}(q)=p-(p-v) F(q) \tag{3.7}
\end{equation*}
$$

We substitute the wholesale price in the retailer's profit function, to respect that the wholesale price is a function of $q$,

$$
\begin{equation*}
\Pi_{r}^{p u s h}\left(q, w_{1}(q)\right)=(p-v)(1-F(q))\left(\frac{S(q)}{1-F(q)}-q\right) \tag{3.8}
\end{equation*}
$$

The first-order derivative with respect to $q$ shows that the retailer's profit increase in $q$,

$$
\begin{equation*}
\frac{\mathrm{d} \Pi_{r}^{p u s h}}{\mathrm{~d} q}=(p-v) f(q) q \geq 0 . \tag{3.9}
\end{equation*}
$$

We formulate the manufacturer's profit function. The manufacturer maximizes expected profit $\Pi_{m}$. He solves a trade-off between generating more profit per unit sold by increasing $w_{1}$ and selling more units by decreasing $w_{1}$. Analogously, increasing $w_{1}$ reduces the quantity sold and reducing $w_{1}$ reduces the profit per unit sold. The manufacturer earns the difference between the wholesale price and pro-
duction costs for each unit of product sold to the retailer,

$$
\begin{equation*}
\Pi_{m}^{p u s h}\left(q, w_{1}\right)=\left(w_{1}-c_{1}\right) q . \tag{3.10}
\end{equation*}
$$

Again we substitute the wholesale price from equation (3.7) to formulate manufacturer profit given the retailer's best response,

$$
\begin{equation*}
\Pi_{m}^{p u s h}\left(q, w_{1}(q)\right)=\left(p-c_{1}\right) q-(p-v) F(q) q \tag{3.11}
\end{equation*}
$$

We derive the first-order derivative with respect to $q$, namely,

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{m}^{p u s h}}{\mathrm{~d} q}=\left(p-c_{1}\right)-(p-v)(F(q)+f(q) q)=0  \tag{3.12}\\
& \quad \Rightarrow(1-F(q))\left(1-\frac{f(q) q}{1-F(q)}\right)=1-\frac{p-c_{1}}{p-v}
\end{align*}
$$

Manufacturer profit is unimodal in $q$ if the demand distribution fulfills the increasing generalized failure rate (IGFR) property. In this case, there is exactly one optimal $q^{*}$ that maximizes manufacturer profit. We follow Lariviere and Porteus (2001) in the following proof to show that there is a unique optimal production quantity.

Proof. $q$ is only defined in the positive range. Consider equation 3.12. On the lefthand side, $1-F(q)$ is positive and strictly decreasing in $q$, because $F(q)$ is a cumulative distribution function. The second term on the left-hand side is positive and weakly decreasing in $q$, given that the IGFR property holds. Then, the left-hand side of equation 3.12 is decreasing in $q$. For $q=0$, the left-hand side equals one. When $q$ approaches infinity, the left-hand side approaches zero. The right-hand side is constant in $q$ and its value is between zero and one. Then, $\Pi_{m}$ must be unimodal and concave.

### 3.2.1.2 The pull scheme (I.2)

Consider a scenario where the manufacturer achieved delivery lead-times close to zero. This allows for serving orders that are placed at $t_{2}$ after uncertainty is resolved. Then, the manufacturer can offer a wholesale price scheme that is based on orders under certainty. Figure 3.5 shows this alternative time-line. The manufacturer makes his production decision at $t_{1}$ when demand is still uncertain. The retailer places orders at $t_{2}$ for immediate delivery. The retailer knows demand at $t_{2}$ and thus, she orders exactly the quantity required by consumers. Consequently, the manufacturer bears all of the inventory risk in this setting. As the retailer orders inventory according to demand, we name this wholesale price scheme a "pull" scheme.

The manufacturer again maximizes expected profit. His profit function in pull mode is closely related to the retailer's profit function in push mode. He pays production costs $c_{1}$ for every unit produced and he receives the opportunity to salvage.


Fig. 3.5 Layout of the wholesale price contract model under a pull scheme

Each unit that is eventually sold, generates revenues $p$ and reduces the salvage opportunity. The manufacturer's profit in this case reads

$$
\begin{equation*}
\Pi_{m}^{\text {pull }}\left(q, w_{2}\right)=\left(w_{2}-v\right) S(q)-\left(c_{1}-v\right) q . \tag{3.13}
\end{equation*}
$$

The first-order condition with respect to $q$ is

$$
\begin{equation*}
\left.\frac{\mathrm{d} \Pi_{m}^{p u l l}}{\mathrm{~d} q}=\left(w_{2}-v\right)\left(1-F\left(q^{*}\right)\right)\right)-\left(c_{1}-v\right)=0 . \tag{3.14}
\end{equation*}
$$

Solving for $F\left(q^{*}\right)$ provides the optimal production service level. That is, the manufacturer produces a quantity $q^{*}$ that is sufficient to serve demand with a probability $F\left(q^{*}\right)$. Again the service level is a Newsvendor solution. Underage costs equal $\left(w_{2}-c_{1}\right)$, while overage costs equal $\left(c_{1}-v\right)$. The optimal service level reads as

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{\left(w_{2}-c_{1}\right)}{\left(w_{2}-c_{1}\right)+\left(c_{1}-v\right)}=\frac{w_{2}-c_{1}}{w_{2}-v} . \tag{3.15}
\end{equation*}
$$

We solve for the wholesale price,

$$
\begin{equation*}
w_{2}(q)=\frac{c_{1}-v F(q)}{1-F(q)} . \tag{3.16}
\end{equation*}
$$

We substitute $w_{2}(q)$ into the manufacturer's profit function (3.13) and obtain

$$
\begin{equation*}
\Pi_{m}^{p u l l}(q)=(p-v)\left(1-\frac{p-c_{1}}{p-v}\right)\left(\frac{S(q)}{1-F(q)}-q\right) \tag{3.17}
\end{equation*}
$$

We calculate the derivative of expected manufacturer profit with respect to $q$,

$$
\begin{equation*}
\frac{\mathrm{d} \Pi_{m}^{p u l l}}{\mathrm{~d} q}=(p-v)\left(1-\frac{p-c_{1}}{p-v}\right) \frac{S(q) f(q)}{(1-F(q))^{2}} \geq 0 \tag{3.18}
\end{equation*}
$$

Manufacturer profit increases in $q$. Please refer to Cachon (2004) for a detailed proof. The retailer earns the retail price $p$ less the wholesale price $w_{2}$ for every unit sold to consumers,

$$
\begin{equation*}
\Pi_{r}^{p u s h}\left(q, w_{2}\right)=\left(p-w_{2}\right) S(q) \tag{3.19}
\end{equation*}
$$

Again, the wholesale price is a function of the production quantity, i.e.,

$$
\begin{equation*}
\Pi_{r}^{p u s h}(q)=(p-v)\left(\frac{p-c_{1}}{p-v}-F(q)\right) \frac{S(q)}{(1-F(q))} \tag{3.20}
\end{equation*}
$$

The retailer's profit function under a pull scheme has a unique maximum, where the derivative of retailer profit with respect to $q$ is decreasing in $q$. Please refer to Cachon (2004) for a detailed proof.

### 3.2.2 The integrated benchmark and double marginalization

In this section, we compare the performance of the two wholesale price contract schemes (push and pull) to the performance of an integrated supply chain. In an integrated supply chain, the manufacturer and the retailer solve a single optimization problem. They act as if they where both part of a single profit maximizing entity. We employ the integrated supply chain as a benchmark to punctuate the weaknesses of the wholesale price contract.

### 3.2.2.1 The service level of an integrated firm

Consider the case where both, the manufacturer and the retailer, are part of a single company. This company maximizes her expected profit, which now equal expected supply chain profit. The integrated company pays production costs $c_{1}$ for each unit of the product produced and earns the retail price $p$ for each unit sold. Demand is again uncertain. The difference from the two wholesale price schemes is that there is no longer a transfer price between the manufacturing unit and the retailing unit. Consequently, there is only a single optimization problem, namely

$$
\begin{equation*}
\Pi^{\text {int }}(q)=(p-v) S(q)-\left(c_{1}-v\right) q . \tag{3.21}
\end{equation*}
$$

The first- and second-order derivatives with respect to $q$ show that supply chain profit is concave in $q$. The term $(v-p)$ is negative and thus, the second-order derivative is decreasing in $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi^{\text {int }}}{\mathrm{d} q}=(p-v)(1-F(q))-\left(c_{1}-v\right)=0 \\
\frac{\mathrm{~d}\left(\Pi^{\text {int }}\right)^{2}}{\mathrm{~d}^{2} q}=(v-p) f(q) \leq 0 . \tag{3.23}
\end{array}
$$

The critical fractile determines the optimal production quantity. The firm pays underage costs $c_{u}=\left(p-c_{1}\right)$ per unit of lost sales. Excess inventory is again punished with a salvage value below production costs $c_{o}=(p-v)$. From rearranging (3.22), we obtain

$$
\begin{equation*}
F\left(q^{i n t .}\right)=\frac{c_{u}}{c_{u}+c_{o}}=\frac{\left(p-c_{1}\right)}{\left(p-c_{1}\right)+(p-v)}=\frac{p-c_{1}}{p-v} . \tag{3.24}
\end{equation*}
$$

As supply chain profit equals the single company profit, the service level associated with $q^{\text {int. }}$ maximizes supply chain profit. In the next step, we compare the wholesale price contract to the integrated firm model.

### 3.2.2.2 The challenge of double marginalization and supply chain coordination

The service level of the integrated firm $F\left(q^{\text {int. }}\right)$ maximizes supply chain profit. We compare this optimal service level with the service levels achieved under push and pull schemes,

$$
\begin{gather*}
F\left(q^{\text {int. }}\right)=\frac{p-c_{1}}{p-v} \geq \frac{p-w_{1}}{p-v}=F\left(q^{\text {push }}\right)  \tag{3.25}\\
F\left(q^{\text {int. }}\right)=\frac{p-c_{1}}{p-v} \geq \frac{w_{2}-c_{1}}{w_{2}-v}=F\left(q^{\text {pull }}\right) . \tag{3.26}
\end{gather*}
$$

Compare the service levels in the integrated case and in the push mode. Obviously, the optimal service level is only reached when $w_{1}=c_{1}$. If this was true, the manufacturer would earn zero profit, as he sells the product at production costs. When the manufacturer has at least some channel power, the outcome of negotiations will be a wholesale price that exceeds production costs. As a result, the retailer orders fewer units than in the optimal state and the supply chain loses sales to consumers.

Analogously, the pull mode only achieves the channel optimal service level when the wholesale price equals the retail price. Then, the retailer earns zero profit. Again, the retailer will not accept such a wholesale price when she has at least some channel power. A lower wholesale price reduces the production quantity by the manufacturer and thus results in lost sales to consumers.


Fig. 3.6 The effect of double marginalization on the service level

Under a wholesale price contract, the supply chain serves less demand than would be optimal in terms of supply chain profit. Thus, supply chain profit decreases (see Cachon 2003 and Nahmias 2008). The service level suffers because the manufacturer (under pull) and the retailer (under push) can increase their own profits by deviating from the optimal service level. Both players optimize their profits independently, without considering the impact of their decision on the other party's profit. Figure 3.6 shows the effect of double marginalization on service levels for the example of the push scheme.

Double marginalization calls for supply chain coordination. In a coordinated supply chain, each player makes decisions such that supply chain profit is maximized. Under coordination, incentives of all players are aligned. The service level of a coordinated supply chain equals the service level of the integrated firm. Then, the players maximize the "pie" that they distribute among each other.

The wholesale price contract clearly does not achieve channel coordination, as lost sales hurt supply chain profit. The root cause is that wholesale price contracts allow one player to increase his or her profit at the expense of supply chain profit. Moreover, consumers experience more frequent stockouts, which reduces their welfare and satisfaction. In the next step, we show that a suitable contract achieves coordination even in the two-firm scenario.

### 3.2.3 Advance purchase discounts (I.3)

Cachon (2004) suggests to combine both, push and pull schemes, to coordinate the supply chain. The retailer purchases a quantity $z$ at time $t_{1}$, as explained for the push mode. Then, the manufacturer makes his production decision. After uncertainty is resolved at $t_{2}$, the retailer has the opportunity to place additional instant orders, as described for the pull mode. Then, the manufacturer delivers all units ordered instantly. Figure 3.7 shows this extended supply chain layout. Again, we analyze advance purchase discounts along the lines of Cachon (2004).


Fig. 3.7 Layout of the advance purchase discount model

The manufacturer offers a discount to the retailer for advance commitments. Forward orders at $t_{1}$ shift inventory risk from the manufacturer to the retailer, while instant orders at $t_{2}$ shift inventory risk from the retailer to the manufacturer. Thus, advance purchase discounts achieve risk sharing between both supply chain parties. As a result, both parties now potentially salvage excess inventories in case of low demand realizations.

The manufacturer pays production $\operatorname{costs} c_{1}$ per unit and gains the opportunity to salvage the product at the salvage value $v$. He sells with certainty all forward orders and receives the push wholesale price $w_{1}$. For each additional unit sold on instant orders, the manufacturer earns the pull wholesale price $w_{2}$. The retailer will only execute instant orders, when demand exceeds her forward orders. Forward orders are sunk and thus, they are always executed first. This reflects in the manufacturer's profit function,

$$
\begin{equation*}
\Pi_{m}^{a p d}(z, q)=\left(w_{1}-v\right) z+\left(w_{2}-v\right)(S(q)-S(z))-\left(c_{1}-v\right) q . \tag{3.27}
\end{equation*}
$$

This formulation of the manufacturer's profit function implicitly assumes that the retailer's forward orders do not exceed the manufacturer's optimal production quantity, such that $z \leq q$. In case that $z>q$, the manufacturer adjusts his production quantity, as forward orders are free of risk from his perspective. Then, production equals forward orders $z=q$ and the model collapses to a pure push scheme.

We calculate the first- and second-order conditions for maximizing the manufacturer's profit. The manufacturer decides on his production quantity $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{m}^{a p d}}{\mathrm{~d} q}=\left(w_{2}-v\right)(1-F(q))-\left(c_{1}-v\right)=0 \\
\frac{\mathrm{~d}\left(\Pi_{m}^{a p d}\right)^{2}}{\mathrm{~d}^{2} q}=\left(v-w_{2}\right) f(q) \leq 0 \tag{3.29}
\end{array}
$$

The first term of the first-order derivative strictly increases in $q$, the second term is a negative constant. Thus, there exists a unique extreme point in the positive range. The second-order derivative shows that the extreme point is a maximum in terms of profit. Thus, there is a unique, positive production quantity, which maximizes manufacturer profit.

We rearrange terms and derive the optimal production service level under an advance purchase discount scheme. Underage costs now equal ( $w_{2}-c_{1}$ ) and overage costs are again $\left(c_{1}-v\right)$, such that

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{\left(w_{2}-c_{1}\right)}{\left(w_{2}-c_{1}\right)+\left(c_{1}-v\right)}=\frac{w_{2}-c_{1}}{w_{2}-v} . \tag{3.30}
\end{equation*}
$$

This result equals the optimal production service level derived for the pull mode. The manufacturer produces the supply chain optimal quantity, when $w_{2}^{\text {int. }}=p$. Then, the retailer earns zero profit on instant orders. Solving for $w_{2}$ provides the instant order wholesale price as a function of the service level reached,

$$
\begin{equation*}
w_{2}=\frac{c_{1}-F\left(q^{*}\right) v}{1-F\left(q^{*}\right)} \tag{3.31}
\end{equation*}
$$

The retailer pays the forward wholesale price $w_{1}$ for every unit ordered at $t_{1}$. Instant orders cost $w_{2}$ per unit. For every unit of the product sold to consumers, the retailer receives the retail price $p$,

$$
\begin{equation*}
\Pi_{r}^{a p d}(z, q)=-\left(w_{1}-v\right) z+(p-v) S(z)+\left(p-w_{2}\right)(S(q)-S(z)) \tag{3.32}
\end{equation*}
$$

The retailer maximizes profit by choosing an optimal forward order quantity $z$. We calculate the first- and second-order conditions and find that the retailer's profit function is concave in $z$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{a p d}}{\mathrm{~d} z}=-\left(w_{1}-v\right)+\left(w_{2}-v\right)(1-F(z))=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{a p d}\right)^{2}}{\mathrm{~d}^{2} z}=\left(v-w_{2}\right) f(z) \leq 0 . \tag{3.34}
\end{array}
$$

There is a unique forward order quantity, which maximizes retailer profit. We rearrange terms to find the associated optimal forward order service level,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{\left(w_{2}-w_{1}\right)}{\left(w_{2}-w_{1}\right)+\left(w_{1}-v\right)}=\frac{w_{2}-w_{1}}{w_{2}-v} . \tag{3.35}
\end{equation*}
$$

We solve for $w_{1}$ and obtain

$$
\begin{equation*}
w_{1}=w_{2}-F\left(z^{*}\right)\left(w_{2}-v\right) . \tag{3.36}
\end{equation*}
$$

The manufacturer only produces the channel coordinating quantity, when the retailer accepts an instant order wholesale price that equals the retail price. Thus, only contracts, where $w_{2}^{\text {int. }}=p$ are part of the channel coordinating set of contract parameters. A profit maximizing retailer will accept such a scheme, when she is rewarded with a discount on forward orders, that improves her expected profit compared to the original wholesale price contract. The manufacturer is willing to offer such a pricing scheme, if he also increases his profit compared to the wholesale price contract. Thus, a feasible pricing scheme must be Pareto improving. Retailer profit increases in $w_{1}$. When $w_{1}=c_{1}$, the manufacturer earns zero profit and the retailer receives the profit of the coordinated supply chain. When $w_{1}=w_{2}$, the retailer's profit decreases to zero and the manufacturer earns the coordinated supply chain profit. Consequently, the advance purchase discount achieves coordination and allows for an arbitrary distribution of profit between both players. Cachon (2004) names the set of parameters satisfying $w_{2}=p$ and $c_{1} \leq w_{1} \leq w_{2}$ the Pareto set. Again, we assume, that the retailer purchases less forward contracts than the manufacturer optimally produces; otherwise the game collapses to Case I. 1 (push mode).

Lemma 3.1. An advance purchase discount scheme (Case I.3) achieves coordination for $w_{2}^{\text {int. }}=p$ and $c_{1} \leq w_{1} \leq w_{2}$. Channel profit can be arbitrarily distributed.

### 3.2.4 Two-part tariffs (I.4)

We have seen that a single wholesale price contract does not coordinate the supply chain and that advance purchase discounts can achieve coordination. In the next step, we show how a two-part tariff can solve the challenge of double marginalization. First, we characterize two-part tariffs. Then we model the buyback and the option contract to show that the optimal service level can be reached.

Two-part tariffs are different from single wholesale price contracts in that they share demand risk between both parties. Under push and pull schemes, one party carries all inventory risk. Under a two-part tariff, the retailer receives flexibility; however, she participates in taking part of the risk. Two-part tariffs can achieve an optimal distribution of risk in the supply chain and thus they align incentives to maximize supply chain profit. Like advance purchase discounts, they are a price mechanism to achieve the first best solution.

Two-part tariffs require two parameters. The retailer pays an initial fee per unit contracted upfront when ordering inventory. When uncertainty is resolved, a second
transfer takes place. Both transfers depend on the type of contract employed. We specify both transfers for the buyback contract and the option contract.

### 3.2.4.1 Buy-back contracts

The buyback contract was introduced by Pasternack (1985). The manufacturer commits to accept returns of unsold inventory at the end of the sales period. He pays the retailer a buyback fee $b$ for each unit returned. To avoid arbitrage, the buyback fee $b$ must be smaller than or equal to the sum of wholesale price $w_{1}$ and transportation costs $t$, so $b \leq\left(w_{1}+t\right)$. The buyback contract shares inventory risk as the manufacturer increases his share of demand risk compared to the push mode. Higher buyback fees shift more risk to the manufacturer's end of the supply chain. As a result, the buyback contract reduces the overage costs of the retailer $c_{o}=\left(w_{1}-b+t\right)$ compared to the push mode,

$$
\begin{equation*}
F\left(q^{\text {buyback }}\right)=\frac{c_{u}}{c_{u}+c_{o}}=\frac{\left(p-w_{1}\right)}{\left(p-w_{1}\right)+\left(w_{1}-b+t\right)} . \tag{3.37}
\end{equation*}
$$

The buyback contract can achieve coordination. When the buyback fee $b$ equals the wholesale price $w_{1}$ plus transportation costs $t$, the manufacturer takes all the risk. Then, service levels rise theoretically to 100 percent; however, this is not optimal for the manufacturer. Still, the buyback contract can achieve every service level between the one achieved by the wholesale price and 100 percent. This includes the channel profit maximizing service level, derived for the integrated case. We calculate the optimal buyback fee, which achieves the service level of the integrated firm, i.e.,

$$
\begin{gather*}
F\left(q^{\text {buyback }}\right)=\frac{p-w_{1}}{p-b+t}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right)  \tag{3.38}\\
\Rightarrow b^{\text {int }}=(p+t)-\frac{\left(p-w_{1}\right)(p-v)}{\left(p-c_{1}\right)} \tag{3.39}
\end{gather*}
$$

The major disadvantage of this scheme is unnecessary shipping of inventory from the manufacturer to the retailer and again back to the manufacturer. These costs reduce supply chain profit, as transportation fees are lost for the supply chain. Moreover, there is no benefit of a short-term production opportunity under the buy-back scheme, as all inventory has to be produced upfront. Thus, we turn our attention to option contracts.

### 3.2.4.2 Option contracts (I.4)

Option contracts work similarly to buyback contracts, but they avoid return shipments. The retailer pays a reservation fee $r$ for every unit of inventory reserved. An option reservation then provides the retailer with the right, but not the obligation to purchase a unit of the product on short notice at a specified point of time. The

## Model I. 4 Option Contracts



Fig. 3.8 Layout of the option contract model
retailer pays the execution fee $e$ for every option exercised. Figure 3.8 shows the layout of the option contract model. The manufacturer grants the retailer flexibility and he is rewarded by receiving the reservation fee for all units reserved, including units that are eventually not executed. The additional flexibility reduces the retailer's overage costs to just $r$. Thereby, she has an incentive to increase her service level, compared to the wholesale price contract,

$$
\begin{equation*}
F\left(q^{\text {option }}\right)=\frac{c_{u}}{c_{u}+c_{o}}=\frac{(p-r-e)}{(p-r-e)+r}=\frac{p-r-e}{p-e} . \tag{3.40}
\end{equation*}
$$

Again, we derive the contract parameters achieving channel coordination,

$$
\begin{align*}
F\left(q^{\text {option }}\right)=\frac{p-r-e}{p-e}= & \frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right)  \tag{3.41}\\
\Rightarrow r^{\text {int }} & =\frac{(p-e)\left(c_{1}-v\right)}{(p-v)} \tag{3.42}
\end{align*}
$$

We find that reservation and execution fees are related in that a higher execution fee requires a lower reservation fee to be feasible. Options achieve coordination and the two contract parameters allow for distributing profit in the supply chain. A buyback contract with a zero buyback fee and an option contract with a zero execution fee resemble the push mode described before. The opposite extreme is
a buyback fee that equals the wholesale price or a zero reservation fee. Here, the retailer receives unlimited flexibility. The model then collapses to the pull mode.

Lemma 3.2. The option scheme (Case I.4) achieves coordination for $r^{\text {int }}$ as defined in (3.42). The contract parameters allow for arbitrarily distributing profit.

### 3.2.5 Implementing supply contracts

We asses wholesale price contracts, advance purchase discounts and two-part tariffs in terms of implementation. First, we discuss the performance of different contracts in terms of service levels and profitability for both parties. The integrated supply chain serves as a benchmark. Then, we review the challenges of implementation associated with these contract schemes.

### 3.2.5.1 Service levels and profitability

Advance purchase discounts and two-part tariffs achieve better service levels than wholesale price contracts, as risk can be distributed between the two supply chain parties. The improvement in service levels depends on the parameter choice. Optimal parameter choices ensure that the first best solution is reached. Both, two-part tariffs and advance purchase discounts allow for distributing the coordinated supply chain profit between supply chain partners.

Pareto improvement in terms of profit is a prerequisite for implementation. No company accepts a new contract, which reduces their profit compared to the status quo. As a result, a coordinating contract must reach the channel optimal service level and it must ensure that the first target can be achieved with a Pareto improving set of contract parameters. Advance purchase discounts, buyback and option contracts fulfill both requirements in the general setting described. Both players improve their situations and there is no incentive not to accept the new contract as opposed to the original wholesale price contract. See, for example, Barnes-Schuster et al. (2002) and Spinler et al. (2003), who achieve Pareto improvement employing option contract. In addition, improved service levels benefit consumers who face fewer stockouts. Therefore, implementation of advance purchase discounts and two-part tariffs is in the best interest of all supply chain partners.

### 3.2.5.2 Challenges of implementation in the case of coordinating contracts

Wholesale price schemes do not coordinate the channel. From a channel coordination perspective, they are inferior to other contracts, which achieve the first best solution in terms of channel profit. Still, we see wholesale price schemes frequently in practice. We review two potential factors in favor of wholesale price contracts as opposed to advance purchase discounts and two-part tariffs.

First, wholesale price contracts are cheap to implement. These contracts are easily understood and they require a minimum of transactions and decisions. Advance purchase discounts and two-part tariffs require one additional transaction and one additional decision compared to wholesale price contracts. These interactions are not costless in reality and they are likely to reduce the attractiveness of more complicated contracts (Cachon 2004). Two-part tariffs are less transparent and their coordinating effect is less obvious in practice. Based on a laboratory experiment Katok and Wu (2009) suggest that the coordinating effect of advanced supply contracts may be severely limited by bounded rationality and preferences. When benefits from coordination are lower than the costs of implementation, wholesale prices can be the optimal choice.

Second, contracts are subject to compliance problems. Compliance determines, whether parties actually fulfill their duties specified in the contract. Cachon and Lariviere (2001) argue that even a legally binding contract does not automatically enforce compliance. The manufacturer can choose not to serve part of an order and claim that factors beyond his force have led to this result. Thus, compliance under supply contracts depends on the legal environment and on the design of the contract. Wholesale price contracts in push mode are resistant to compliance issues. It is always in the best interest of the manufacturer to serve all orders by the retailer. However, in the case of option contracts, the manufacturer could have an incentive to allow for stockouts in high states of demand. This policy can reduce the necessary amount of inventory significantly, while stockouts are rather uncertain. As a result, from a compliance point of view, wholesale price contracts in push mode are superior to option tariffs. Still, the moral hazard described is not credible in many industries and relationships.

Third, wholesale price contracts require only a single shipment. Buy-back contracts allow the retailer to send back unsold inventory, which results in additional shipping costs. These shipping costs reduce supply chain profit. When shipping costs are important, this can be a major disadvantage of buyback contracts as opposed to wholesale price contracts.

### 3.2.6 Bidding coordinating contracts into the market

We reviewed the analysis of Cachon (2004) considering forward, instant order and advance purchase contracts. In addition we discussed supply options as a means of channel coordination. Option contracts achieve coordination and thus, they offer potential for Pareto improvement in terms of profit. Even when the manufacturer cannot change the retail price, he can bid forward and option contracts into the market, inducing the retailer to carry portfolios of option, forward and instant order contracts. In the next step, we model portfolios of supply contracts and we account explicitly for the possibility of an established wholesale price.

Overall, we consider three scenarios. First, we model the base case, where the manufacturer is restricted to long-term production. That is, he makes his production


Fig. 3.9 Overview on the scenarios (I.-III.) and cases (1-7) considered
decision under uncertainty. Second, we allow for short-term production. Then, the manufacturer has to choose an optimal long-term production quantity considering the opportunity to produce short-term at higher costs, after uncertainty is resolved. Third, we consider channel power by the retailer. The retailer can force the manufacturer to offer a required minimum service level. For each scenario, we model portfolios of contracts. We derive i) the optimal composition of portfolios in terms of types and quantities of contracts, ii) service levels and iii) coordinating contract parameters. Figure 3.9 provides an overview on the scenarios and cases considered throughout our analysis in this chapter.

### 3.3 Portfolios of option, forward and instant order contracts

In the first step, we model a portfolio of options and forwards. Then, we review a portfolio of options and instant orders and finally, we analyze a combination of all three contract types. For all three models, we only allow for long-term production at this point (Scenario I).

## Model I. 5 Portfolio of Options and Forwards



Fig. 3.10 Layout of decisions and actions for option and forward contracts

### 3.3.1 A portfolio of option and forward contracts (I.5)

Consider a manufacturer, who offers a portfolio of option and forward contracts. Let $q$ be the manufacturer's production quantity. The manufacturer can only produce long-term. Let $z$ be the number of forward contracts purchased by the retailer. Let $y$ be the number of options ordered by the retailer. Both order decisions occur at $t_{1}$, before the manufacturer makes his production decision. Thus, the manufacturer produces exactly the sum of the forward and option quantities ordered by the retailer, $q=y^{\prime}=z+y$. The manufacturer earns the reservation fee $r$ for all units reserved. He pays production costs $c_{1}$ for all units produced and receives the opportunity to salvage excess inventory at a salvage value $v$. The manufacturer receives for all forward orders the wholesale price $w_{1}$ and for all option executions the execution fee $e$. Figure 3.10 shows the layout of the supply chain in this scenario. The manufacturer earns

$$
\begin{equation*}
\Pi_{m}^{\text {opt.,forw. }}(z, q)=(e-v)(S(q)-S(z))+\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r(q-z) . \tag{3.43}
\end{equation*}
$$

The manufacturer does not actually make a decision, but he simply produces to the retailer's orders. The retailer pays the wholesale price $w_{1}$ for each forward order, $r$ for each unit reserved and $e$ for each unit executed. She receives the retail price $p$ for each unit eventually sold. The retailer prioritizes forwards over options,
when executing contracts at $t_{2}$. Forwards are sunk and thus, there are no additional costs. In the case of options, only the reservation fee is sunk, which is less than the wholesale price. The retailer earns

$$
\begin{align*}
\Pi_{r}^{\text {opt.,forw. }}(z, q)=-\left(w_{1}-v\right) z & +(p-e)(S(q)-S(z)) \\
& +(p-v) S(z)-r(q-z) \tag{3.44}
\end{align*}
$$

We differentiate the retailer's profit function with respect to $z$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{\text {opt.,forw. }}}{\mathrm{d} z}=\left(v+r-w_{1}\right)+(e-v)(1-F(z))=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,forw. }}\right)^{2}}{\mathrm{~d}^{2} z}=(v-e) f(z) \leq 0 . \tag{3.46}
\end{array}
$$

There is an optimal forward service level,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{r+e-w_{1}}{e-v} \tag{3.47}
\end{equation*}
$$

We differentiate the retailer's profit function with respect to $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{\text {opt.,forw. }}}{\mathrm{d} q}=(p-e)(1-F(q))-r=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,forw. }}\right)^{2}}{\mathrm{~d}^{2} q}=(e-p) f(q) \leq 0 \tag{3.49}
\end{array}
$$

and we derive the unique optimal production service level, which is determined by option reservations,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{(p-e-r)}{(p-e-r)+r}=\frac{p-e-r}{p-e} . \tag{3.50}
\end{equation*}
$$

The retailer's total order quantity from options and forwards equals the order quantity derived for the case of options only (Case I.4). The retailer executes forwards first. Consequently, the last units executed only depend on option parameters. Coordination is achieved, when $F(q)=F\left(q^{\text {int. }}\right)$ or when $F(z)=F\left(q^{\text {int. }}\right)$. In the latter case, the model collapses to pure push mode, as no options are ordered.

Rearranging terms provides the reservation fee as a function of the total quantity reserved,

$$
\begin{equation*}
r=(p-e)\left(1-F\left(q^{*}\right)\right) \tag{3.51}
\end{equation*}
$$

and the wholesale price as a function of the forward service level,

$$
\begin{equation*}
w_{1}=(e+r)-(e-v) F\left(z^{*}\right) . \tag{3.52}
\end{equation*}
$$

Integration is achieved, when the manufacturer produces the channel profit maximizing service level $q^{\text {int. }}$, such that

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{p-e-r}{p-e}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) . \tag{3.53}
\end{equation*}
$$

Solving for $r$ provides the reservation fee that achieves coordination. The reservation fee is positive, i.e.,

$$
\begin{equation*}
r^{i n t}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)} \geq 0 \tag{3.54}
\end{equation*}
$$

$w_{1}$ and $e$ can be employed to distribute profit in the supply chain. In the next step, we derive the boundary conditions for $w_{1}$, that is, i) the value of $w_{1}$ for which options are no longer part of the efficient portfolio and ii) the value of $w_{1}$ for which forwards are no longer part of the efficient portfolio. First,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{r+e-w_{1}}{e-v}<\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) . \tag{3.55}
\end{equation*}
$$

We solve for $w_{1}$,

$$
\begin{equation*}
e+r-\frac{\left(p-c_{1}\right)(e-v)}{(p-v)}<w_{1} \tag{3.56}
\end{equation*}
$$

When this condition does not hold, the model collapses to Case I. 1 (push mode), that is, option contracts are ignored. Second,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{r+e-w_{1}}{e-v}>0 . \tag{3.57}
\end{equation*}
$$

It follows immediately that $w_{1}<r+e$, which is straightforward, given that the flexibility embedded in options has nonnegative value. Otherwise the model collapses to Case I. 4 (options only).

Lemma 3.3. A portfolio of forwards and options (Case I.5) achieves coordination for $r^{\text {int }}$ as derived in (3.54), where

$$
(r+e)-\frac{\left(p-c_{1}\right)(e-v)}{(p-v)}<w_{1}<r+e
$$

is required to assure that both, options and forwards, are part of the efficient portfolio. The manufacturer then exactly produces the quantity $q^{\text {int. }}=y+z$ ordered by the retailer.

### 3.3.2 A portfolio of option and instant order contracts (I.6)

In the next step, we combine option and instant order contracts in a portfolio. Again, let $q$ be the manufacturer's production quantity and let $y$ be the number of options ordered by the retailer. The manufacturer produces at least the quantity reserved by the retailer, such that $q \geq y$, as options are binding commitments to deliver the

## Model I. 6 Portfolio of Options and Instant Orders



Fig. 3.11 Layout of decisions and actions for option and instant order contracts
reserved quantity if executed. If the manufacturer decides to produce exactly the quantity reserved, there will be no instant order market and the model simplifies to the pure option model. Figure 3.11 shows the layout of the supply chain. The manufacturer earns

$$
\begin{equation*}
\Pi_{m}^{\text {opt.,inst. }}(y, q)=(e-v) S(y)+\left(w_{2}-v\right)(S(q)-S(y))-\left(c_{1}-v\right) q+r y \tag{3.58}
\end{equation*}
$$

We derive the first- and second-order conditions with respect to $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{m}^{\text {opt.,inst. }}}{\mathrm{d} q}=\left(w_{2}-v\right)(1-F(q))-\left(c_{1}-v\right)=0 \\
\frac{\mathrm{~d}\left(\Pi_{m}^{\text {opt.,inst. }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(w_{2}-v\right) f(q) \leq 0 \tag{3.60}
\end{array}
$$

Note that the optimal manufacturer service level is independent from the option parameters chosen and only depends on the wholesale price. The production service level derived equals the optimal production service level found for advance purchase discounts (Case I.3),

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{w_{2}-c_{1}}{w_{2}-v} \tag{3.61}
\end{equation*}
$$

The retailer never holds inventory. She executes options before she places instant orders, as the reservation fee is sunk at $t_{2}$,

$$
\begin{equation*}
\Pi_{r}^{\text {opt.,inst. }}(y, q)=(p-e) S(y)+\left(p-w_{2}\right)(S(q)-S(y))-r y . \tag{3.62}
\end{equation*}
$$

We calculate the first- and second-order conditions with respect to $y$,

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {opt.,inst. }}}{\mathrm{d} y}=\left(w_{2}-e\right)(1-F(y))-r=0  \tag{3.63}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,inst. }}\right)^{2}}{\mathrm{~d}^{2} y}=-\left(w_{2}-e\right) f(y) \leq 0 \tag{3.64}
\end{align*}
$$

We rearrange terms and derive the optimal option reservation quantity,

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{w_{2}-e-r}{w_{2}-e} \tag{3.65}
\end{equation*}
$$

This result corresponds to a Newsvendor solution with underage costs $c_{u}=\left(w_{2}-\right.$ $e-r)$ and overage costs $c_{o}=r$. Options are executed first; thus, when the retailer has executed all options reserved, she incurs higher costs for ordering on the instant order market. When the retailer orders too many options, she loses the reservation fee. We solve for the reservation fee,

$$
\begin{equation*}
r=\left(w_{2}-e\right)\left(1-F\left(y^{*}\right)\right) . \tag{3.66}
\end{equation*}
$$

Coordination is achieved, when the manufacturer produces the channel coordinating quantity $(q \geq y)$, or when the retailer reserves the channel coordinating quantity and thus forces the manufacturer to comply $(q<y)$. We first consider the case, where the retailer produces the optimal quantity without need for coordination through option contracts,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{w_{2}-c_{1}}{w_{2}-v}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) \tag{3.67}
\end{equation*}
$$

Coordination is only achieved for $w_{2}^{\text {int. }}=p$. The wholesale price thus must be raised to equal the retail price. Note that options allow for redistributing profit in this case, which is why the retailer can still earn a positive profit. We calculate the boundaries for the reservation fee under coordination, finding the Pareto set for $w_{2}^{\text {int. }}=p$, given $e$, e.g., $e=c_{1}$,

$$
\begin{equation*}
F\left(q^{\text {int. }}\right)=\frac{p-c_{1}}{p-v}>\frac{w_{2}-e-r}{w_{2}-e}=F\left(y^{*}\right) \tag{3.68}
\end{equation*}
$$

Solving for $r$ provides the lower boundary, required for instant orders to be part of the portfolio. From (3.67) and (3.68),

$$
\begin{align*}
r & >\left(w_{2}-e\right)-\frac{\left(p-c_{1}\right)\left(w_{2}-e\right)}{(p-v)}  \tag{3.69}\\
& \Rightarrow r>\left(w_{2}-e\right)\left(1-F\left(q^{i n t .}\right)\right) \tag{3.70}
\end{align*}
$$

Next, we calculate the upper boundary for $r$. When $r$ exceeds this value, options are not part of the efficient portfolio,

$$
\begin{align*}
F\left(y^{*}\right)= & \frac{w_{2}-e-r}{p-e}>0  \tag{3.71}\\
& \Rightarrow r<w_{2}-e \tag{3.72}
\end{align*}
$$

Lemma 3.4. A portfolio consisting of options and instant orders (Case I.6) achieves coordination for $w_{2}^{\text {int. }}=p$ and $\left(w_{2}-e\right)\left(1-F\left(q^{\text {int. }}\right)<r<\left(w_{2}-e\right)\right.$. Option contracts redistribute profit within these boundaries and thus can enable coordination under positive profits for both parties in contrast to instant orders only.

We further consider the case where the wholesale price is below the retail price and cannot be renegotiated. Then the coordination approach described in Lemma 3.4 is not viable. Consequently, the retailer must order a sufficient amount of options to achieve the channel optimal long-term production quantity. This requires appropriate reservation and execution fees,

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{p-e-r}{p-e}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int } .}\right) . \tag{3.73}
\end{equation*}
$$

Solving for $r$ provides

$$
\begin{equation*}
r^{i n t}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)} \tag{3.74}
\end{equation*}
$$

The reservation fee is positive, as the execution fee must be smaller than the wholesale price. Thus, channel coordination can be achieved by bidding an option contract into the market. It is no longer necessary that the wholesale price $w_{2}$ equals the retail price $p$ to achieve coordination. This case only holds, when the retailer enforces a higher service level through option purchases than the manufacturer's optimal production service level. Otherwise, Lemma 3.4 applies. We derive the boundary condition in terms of the reservation fee $r$,

$$
\begin{array}{r}
F\left(y^{*}\right)=\frac{p-e-r}{p-e}>\frac{w_{2}-c_{1}}{w_{2}-v}=F\left(q^{*}\right) \\
\Rightarrow 0 \leq r<\frac{(p-e)\left(c_{1}-v\right)}{\left(w_{2}-v\right)} \tag{3.76}
\end{array}
$$

Lemma 3.5. Bidding option contracts into a pull market achieves coordination, even when the wholesale price cannot be altered. The coordinating reservation fee is derived in (3.74). Then, instant order contracts are no longer part of the efficient portfolio. The execution fee allows to distribute profit in the channel within boundaries. For $e=c_{1}$, profit cannot be distributed.


Fig. 3.12 Order and production service levels under a portfolio of contracts at the example of a normal distribution

### 3.3.3 A portfolio of options, forwards and instant orders (I.7)

Finally, we combine option, forward and instant order contracts to derive solutions for mixed portfolios. We analyze the properties of such portfolios and review, whether they have advantages over the contract schemes previously discussed. In the first step, we set up the manufacturer's profit function. Let $y^{\prime}=y+z$ be the total number of forward and option contracts purchased by the retailer. The manufacturer must at least produce $q \geq y^{\prime}$, where $q=y^{\prime}$ results in a portfolio of options and forwards only (Case I.5). Figure 3.12 shows order and production service levels for the example of a normal distribution. Figure 3.13 shows the resulting layout of the supply chain. The manufacturer earns

$$
\begin{align*}
& \Pi_{m}^{\text {opt.,forw.,inst. }}(y, z, q)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+y r \\
& \quad+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right)+\left(w_{2}-v\right)\left(S(q)-S\left(y^{\prime}\right)\right) \tag{3.77}
\end{align*}
$$

We calculate the first- and second-order conditions with respect to $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{m}^{\text {opt.,forw.,inst. }}}{\mathrm{d} q}=\left(v-c_{1}\right)+\left(w_{2}-v\right)(1-F(q))=0 \\
\frac{\mathrm{~d}\left(\Pi_{m}^{\text {opt.,forw.,inst. }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(w_{2}-v\right) f(q) \leq 0 \tag{3.79}
\end{array}
$$

Rearranging terms provides the unique optimal production quantity, which corresponds to the results received for the advance purchase discount (Case I.3) and the portfolio of options and instant orders (Case I.6),

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{w_{2}-c_{1}}{w_{2}-v} \tag{3.80}
\end{equation*}
$$

## Model I. 7 Portfolio of Hedging Contracts



Fig. 3.13 Layout of decisions and actions for option, forward and instant order contracts

We establish the retailer's profit function. The retailer executes first all forward orders, then options and finally in very high states of demand she places instant orders (see Figure 3.12),

$$
\begin{array}{r}
\Pi_{r}^{\text {opt.,forw.,inst. }}(y, z, q)=-\left(w_{1}-v\right) z+(p-e)\left(S\left(y^{\prime}\right)-S(z)\right) \\
+(p-v) S(z)-y r+\left(p-w_{2}\right)\left(S(q)-S\left(y^{\prime}\right)\right) \tag{3.81}
\end{array}
$$

We derive the first- and second-order conditions with respect to $y$ and solve for the optimal service level as well as contract parameters,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{\text {opt. .forw. inst. }}}{\mathrm{d} y}=\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)-r=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt. } . \text { forw. inst. }}\right)^{2}}{\mathrm{~d}^{2} y}=-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 \tag{3.83}
\end{array}
$$

There is a unique optimal purchase quantity for options and forwards, given by

$$
\begin{equation*}
F\left(y^{\prime *}\right)=\frac{w_{2}-e-r}{w_{2}-e} . \tag{3.84}
\end{equation*}
$$

Rearranging terms provides the optimal reservation fee, which is a function of the wholesale price,

$$
\begin{equation*}
r=\left(w_{2}-e\right)\left(1-F\left(y^{*}\right)\right) . \tag{3.85}
\end{equation*}
$$

Next, we differentiate the retailer's profit function with respect to $z$,

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {opt.,forw.,inst. }}}{\mathrm{d} z}=\left(v-w_{1}\right)+(e-v)(1-F(z))+\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)=0  \tag{3.86}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,forw.,inst. }}\right)^{2}}{\mathrm{~d}^{2} z}=-(e-v) f(z)-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 \tag{3.87}
\end{align*}
$$

There is a unique optimal forward quantity,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{\left(w_{2}-w_{1}\right)-\left(w_{2}-e\right) F\left(y^{\prime}\right)}{e-v} \tag{3.88}
\end{equation*}
$$

We can further simplify this equation by substituting (3.85), i.e.,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{\left(e+r-w_{1}\right)}{(e-v)} \tag{3.89}
\end{equation*}
$$

Rearranging terms provides the forward price,

$$
\begin{equation*}
w_{1}=w_{2}-(e-v) F\left(z^{*}\right)-\left(w_{2}-e\right) F\left(y^{\prime}\right)=(e+r)-F\left(z^{*}\right)(e-v) . \tag{3.90}
\end{equation*}
$$

The total number of options and forwards $y^{\prime}$ and the corresponding service level $F\left(y^{\prime}\right)$ equal the results we found for the portfolio of options and instant orders. A mixed portfolio of forwards, options and instant orders is coordinated when $w_{2}^{\text {int. }}=$ $p$, such that

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{w_{2}-c_{1}}{w_{2}-v}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) \tag{3.91}
\end{equation*}
$$

It must hold that $0<F(z)<F\left(y^{\prime}\right)<F\left(q^{\text {int. }}\right)$. This translates into two boundary conditions,

$$
\begin{align*}
F\left(y^{*}\right)= & \frac{w_{2}-e-r}{w_{2}-e}<\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right)  \tag{3.92}\\
& \Rightarrow r>\left(w_{2}-e\right)\left(1-F\left(q^{\text {int. }}\right)\right) \tag{3.93}
\end{align*}
$$

and

$$
\begin{align*}
F\left(y^{\prime *}\right)= & \frac{w_{2}-e-r}{w_{2}-e}>0  \tag{3.94}\\
& \Rightarrow r<w_{2}-e \tag{3.95}
\end{align*}
$$

The first boundary condition equals the one found for options and instant orders (Case I.6),

$$
\begin{equation*}
\left(w_{2}-e\right)\left(1-F\left(q^{\text {int. }}\right)\right)<r<w_{2}-e . \tag{3.96}
\end{equation*}
$$

Still, we need to show that the coordinated portfolio contains both, options and forwards,

$$
\begin{align*}
F\left(z^{*}\right)=\frac{e+r-w_{1}}{e-v} & <\frac{w_{2}-e-r}{w_{2}-e}=F\left(y^{*}\right)  \tag{3.97}\\
& \Rightarrow w_{1}>v+r \cdot \frac{w_{2}-v}{w_{2}-e} . \tag{3.98}
\end{align*}
$$

The wholesale price must exceed the salvage value plus a premium that depends on the wholesale price and the option parameters. Otherwise, forward contracts dominate the other types of contracts and the game collapses to the push mode (Case I.1). Moreover, the forward order quantity must be positive to ensure that the portfolio contains forwards,

$$
\begin{align*}
F\left(z^{*}\right)= & \frac{e+r-w_{1}}{e-v}>0  \tag{3.99}\\
& \Rightarrow w_{1}<e+r . \tag{3.100}
\end{align*}
$$

The second boundary condition follows immediately,

$$
\begin{equation*}
v+r \cdot \frac{w_{2}-v}{w_{2}-e}<w_{1}<e+r . \tag{3.101}
\end{equation*}
$$

Lemma 3.6. A portfolio containing forwards, options and instant orders (Case 1.7) exists when conditions (3.96) and (3.101) hold. The portfolio is coordinated only for $w_{2}^{\text {int. }}=p$. The remaining contract parameters can be employed to distribute i) profit and ii) inventories in the supply chain.

Again, it is possible to reach coordination, even when the wholesale price cannot be raised to equal the retail price. Then, the manufacturer must underbid the wholesale price and force the retailer to order the channel optimal quantity using options and forwards. We discussed this scenario for options only in Case I.6. The optimal reservation fee, as stated in Equation (3.74) applies. When forwards are not part of the portfolio, the boundary conditions stated in Lemma 3.5 hold. The service level $F\left(y^{\prime}\right)$ achieved with forward and option contracts must exceed the manufacturer's production service level, otherwise Lemma 3.6 applies,

$$
\begin{equation*}
F\left(y^{\prime *}\right)=\frac{p-e-r}{p-e}>\frac{w_{2}-c_{1}}{w_{2}-v}=F\left(q^{*}\right) . \tag{3.102}
\end{equation*}
$$

The condition for the domination of instant orders follows from solving for $r$ and equals the one derived for Case I.6,

$$
\begin{equation*}
0 \leq r<\frac{(p-e)\left(c_{1}-v\right)}{\left(w_{2}-v\right)} \tag{3.103}
\end{equation*}
$$

When forwards are part of the efficient portfolio, we require an additional boundary condition, which ensures that forwards are not dominated. We stated this bound-
ary condition already in Lemma 3.6, namely

$$
\begin{equation*}
v+r \cdot \frac{p-v}{p-e}<w_{1}<e+r . \tag{3.104}
\end{equation*}
$$

Lemma 3.7. Coordination can be achieved, even when the wholesale price cannot be employed to coordinate production decisions. Then, bidding options and forwards into the market can achieve coordination. Instant orders are dominated. The results derived in Lemma 3.5 are extended by the additional boundary condition (3.104) to include forward contracts into the efficient portfolio. Because of forward contracts, profit can be distributed within boundaries even for $e=c_{1}$.

### 3.3.4 Summary of results

For the base cases I. 1 to I.7, we discussed the seven possible combinations of option, forward and instant order contracts. For all portfolios, it must hold that $q \geq y^{\prime} \geq z \geq$ 0 , thus the production quantity exceeds the quantity reserved by the retailer plus the number of forward contracts. When two of these parameters are equal, at least one contract type is not part of the portfolio. For example, $y^{\prime}=z$ means that the portfolio does not contain options. As a result, the last, most general model I. 7 contains all other six models. Table 3.2 and Table 3.3 show the coordinating contract parameters as well as the constraints that must hold, when different contract types are part of the portfolio.

Table 3.2 shows that the optimal production quantity depends on the types of contracts that are part of the optimal portfolio. The most flexible contract considered in a portfolio determines marginal revenues from the manufacturer's perspective. For this reason, all portfolios containing pull contracts trigger an equal production quantity.

We showed that a supply chain is exactly then coordinated, when the preferred production quantity of the manufacturer equals the supply chain optimal production quantity. A portfolio of forward, option and instant order contracts thus reaches coordination only for $w_{2}^{\text {int. }}=p$. The manufacturer can compensate the retailer by offering a discount on forward orders and options. Thereby, profit is shared between supply chain partners and Pareto improvement can be achieved. When $w_{2}<p$ cannot be altered, coordination can still be reached by bidding coordinating contracts into the market. However, in this case, instant orders are no longer part of the coordinating portfolio.
Table 3.3 Properties of the portfolios of contracts reviewed (continued)

| Mode | Coordinating Parameters | Conditions | Distribution of Profit | Efficiency |
| :--- | :--- | :--- | :--- | :--- |
| Push Mode | $w_{1}=c_{1}$ | n.a. | $100 \%$ retailer | Newsvendor |
| Pull Mode | $w_{2}=p$ | n.a. | $100 \%$ manufacturer | Newsvendor |
| Push and Pull (APD) | $w_{2}=p$ | arbitrary | Coordination possible |  |
| Options | $r^{\text {int }}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}$ | n.a. | arbitrary | Coordination possible |
| Options and Push | $r^{\text {int }}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}$ | $(e+r)-\frac{\left(p-c_{1}\right)(e-v)}{(p-v)}<w_{1}<(r+e)$ | within boundaries | Coordination possible |
| Options and Pull | $w_{2}=p$ | $\left(w_{2}-e\right)\left(1-F\left(q^{\text {int. })<r<\left(w_{2}-e\right)}\right.\right.$ | within boundaries | Coordination possible |
| " (instant orders dominated) | $r^{\text {int }}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}$ | $r<\frac{(p-e)\left(c_{1}-v\right)}{\left(w_{2}-v\right)}$ | within boundaries | Coordination possible ${ }^{a}$ |
| Options, Pull and Push | $w_{2}=p$ | $\left(w_{2}-e\right)\left(1-F\left(q^{\text {int. })<r<\left(w_{2}-e\right),}\right.\right.$ | within boundaries | Coordination possible |
| (instant orders dominated) | $r^{\text {int }}=\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}$ | $r<\frac{(p-e)\left(c_{1}-v\right)}{\left(w_{2}-v\right)}$, | within boundaries | Coordination possible ${ }^{b}$ |
|  |  | $v+r \cdot \frac{p-v}{p-e}<w_{1}<(e+r)$ |  |  |

[^0]Proposition 3.1. For the basic model setup,
i) the optimal production quantity depends on the contract parameters chosen, as production takes place under uncertainty.
ii) coordination is achieved, when the manufacturer produces the channel optimal quantity, such that $F\left(q^{*}\right)=F\left(q^{\text {int. }}\right)$. A portfolio of option, forward and instant order contracts achieves coordination and allows for distributing profit in the supply chain.
iii) portfolios containing both, forward and flexible contracts, allow for a flexible distribution of inventories in the supply chain, which distinguishes them from regular coordinating contracts.
iv) coordination can furthermore be achieved, when the retailer enforces the optimal production quantity through her orders. A portfolio containing option and forward contracts can provide the necessary incentives. The suboptimally priced instant order contracts are dominated and they are no longer part of the efficient, coordinating portfolio. Thereby, coordination can even be achieved, when the instant order wholesale price cannot be altered.

Proof. We have to consider three cases. In the first case, instant orders are part of the portfolio (Cases I.2, I.3, I.6. (base case), I.7. (base case). Then the manufacturer's optimal production quantity depends on the instant order wholesale price $w_{2}$,

$$
F\left(q^{*}\right)=\frac{w_{2}-c_{1}}{w_{2}-v}
$$

Coordination is achieved, when $F\left(q^{*}\right)=F\left(q^{\text {int. }}\right)$. It follows immediately that coordination is achieved, when $w_{2}^{\text {int. }}=p$. For portfolio solutions, the respective boundary conditions stated in Table 3.3 and in Lemmas 3.1, 3.4 and 3.6 apply.

In the second case, the optimal portfolio of contracts contains options, but it excludes instant orders (Cases I.4, I.5). Then the optimal production quantity is a direct result of the retailer's reservations. The production quantity depends on the execution fee $e$ and the reservation fee $r$,

$$
F\left(q^{*}\right)=\frac{p-e-r}{p-e} .
$$

The channel is coordinated when $F\left(q^{*}\right)=F\left(q^{\text {int. }}\right)$,

$$
\begin{aligned}
F\left(q^{*}\right)=\frac{p-e-r}{p-e} & =\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) \\
r^{\text {int. }} & =\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}
\end{aligned}
$$

The channel coordinating reservation fee is positive. Again, for portfolio solutions, the respective boundary conditions stated in Table 3.3 and in Lemmas 3.2 and 3.3 apply.

Moreover, when the manufacturer cannot change the instant order price, he is forced to underbid it to achieve coordination. This results in domination of the suboptimally priced instant orders (Cases I.6.2, I.7.2). Production policies then depend on the retail price $p$,

$$
F\left(q^{*}\right)=\frac{p-e-r}{p-e}
$$

Coordination is achieved, when $F\left(q^{*}\right)=F\left(q^{\text {int. }}\right)$. The respective boundary conditions stated in Table 3.3 and in Lemmas 3.5 and 3.7 apply,

$$
\begin{aligned}
F\left(q^{*}\right)=\frac{p-e-r}{p-e} & =\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) \\
r^{\text {int. }} & =\frac{(p-e)\left(c_{1}-v\right)}{(p-v)}
\end{aligned}
$$

In the third case, the optimal portfolio of contracts contains forward orders, but it excludes instant orders and options (Case I.1). Then the optimal production quantity depends on the forward price $w_{1}$,

$$
F\left(q^{*}\right)=\frac{p-w_{1}}{p-v}
$$

Coordination is achieved, when $F\left(q^{*}\right)=F\left(q^{\text {int. }}\right)$,

$$
F\left(q^{*}\right)=\frac{p-w_{1}}{p-v}=\frac{p-c_{1}}{p-v}=F\left(q^{\text {int. }}\right) .
$$

It follows immediately that coordination is only achieved, when $w_{1}^{\text {int. }}=c_{1}$.
Contract parameters allow for splitting profit among the two supply chain parties within the boundary conditions. Portfolios containing both forward and option contracts allow in addition for splitting inventory risk among the two supply chain parties. Setting contract parameters such that more forwards are purchased increases the share of inventory risk the retailer carries. Lowering the reservation fee results in a higher share of risk taken by the manufacturer.

### 3.4 Flexible production

Few articles consider short-term production capabilities in a coordination setting. In the next step, we allow for a second production opportunity at $t_{2}$, after uncertainty is resolved, i.e., Scenario II. The manufacturer pays short-term production costs $c_{2}>c_{1}$ for every unit produced last minute. Last minute production is postponed until uncertainty is resolved. The retailer announces instant orders, before the manufacturer finalizes production. The manufacturer produces during the sales period and thus, she has the opportunity to serve all demand. However, the short-term production technology is expensive and may be unprofitable.

Consider a portfolio of forward, option and instant order contracts. The manufacturer can employ expensive short-term production to hedge against demand risk from i) instant orders and ii) option executions. When the costs of short-term production exceed the instant order price, the manufacturer only produces options short-term (Case II.A). When the costs of short-term production are less than the instant order price, all instant orders are served (Case II.B). Then, considering options, the manufacturer has two possibilities. First, the manufacturer can decide to produce all instant orders and part of the options employing the short term technology (Case II.B.1). All forwards and the remainder of the options are produced long-term. This is reasonable, when the savings from long-term production are outbalanced by inventory risk. Second, the manufacturer can decide to produce only part of the instant orders employing short-term production (Case II.B.2). Then, all forwards, all options and part of the instant orders are produced long-term. We analyze both cases.

Figure 3.14 shows the layout of the supply chain. In the first case, long-term production is not sufficient to serve all options and $c_{2}>w_{2}$. Then, short-term production is employed to hedge against inventory risk (Case II.A). In the second case, long-term production exceeds option orders and $c_{2}>w_{2}$. Then, the base case (I.7) applies. In the third and fourth cases, $c_{2} \leq w_{2}$. Consequently, all instant orders are served (Cases II.B. 1 and II.B.2).

### 3.4.1 Short-term production for options only (Case II.A)

In the first case, producing short-term to serve instant orders is not profitable from the manufacturer's point of view, as $c_{2}>w_{2}$. For $q \geq y^{\prime}$, the previous section applies. However, when the manufacturer bids forward and option contracts into the market, he can choose to reduce inventory risk by producing part of option executions shortterm. Forward orders are committed and thus, it is optimal to employ the long-term production technology to serve these orders. Consider Case II.A and $q<y^{\prime}$. Then the manufacturer only serves forward and option orders; however, he employs shortterm production for part of the option executions,

$$
\begin{align*}
& \pi_{m}^{\text {portfolio }}(y, z, q)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+e \cdot\left(S\left(y^{\prime}\right)-S(z)\right)  \tag{3.105}\\
& -v \cdot(S(q)-S(z))-c_{2} \cdot\left(S\left(y^{\prime}\right)-S(q)\right) .
\end{align*}
$$

We derive the first and second order conditions with respect to $q$,

$$
\begin{align*}
\frac{\mathrm{d} \pi_{m}^{\text {portfolio }}}{\mathrm{d} q} & =\left(v-c_{1}\right)+\left(c_{2}-v\right)(1-F(q))=0  \tag{3.106}\\
& \frac{\mathrm{~d}\left(\pi_{m}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.107}
\end{align*}
$$

## Model II.A / II.B. 1 / II.B. 2 Portfolio of Contracts



Fig. 3.14 Layout of decisions and actions for short-term production opportunities in the absence of retail power

The optimal long-term production service level is unique and independent from contract parameters, i.e.,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{2}-c_{1}}{c_{2}-v} \tag{3.108}
\end{equation*}
$$

The optimal production policy merely depends on cost parameters and salvage values. The manufacturer now solves a cost minimization problem. Sales only depend on demand and no longer on production. The retailer's objective function and optimal orders are similar to the results from the previous section.

### 3.4.2 Short-term production for both options and instant orders (Case II.B)

In the second case, the manufacturer can profitably serve instant orders with her short-term production technology, as $c_{2} \leq w_{2}$. Consequently, he will always serve all instant orders and the retailer can reach a 100 percent service level. The manufacturer then simply optimizes his production. He employs short-term production either to produce a part of option executions and all instant orders (Case II.B.1), i.e.,

$$
\begin{array}{r}
\pi_{m}^{\text {portfolio }}(y, z, q)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+e \cdot\left(S\left(y^{\prime}\right)-S(z)\right)  \tag{3.109}\\
-v \cdot(S(q)-S(z))-c_{2} \cdot(D-S(q))+w_{2} \cdot\left(D-S\left(y^{\prime}\right)\right)
\end{array}
$$

or he limits short-term production to instant orders (Case II.B.2), such that

$$
\begin{align*}
\pi_{m}^{p o r t f o l i o}(y, z, q)= & \left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right)  \tag{3.110}\\
& +\left(w_{2}-v\right)\left(S(q)-S\left(y^{\prime}\right)\right)+\left(w_{2}-c_{2}\right)(D-S(q))
\end{align*}
$$

We derive the first and second order conditions with respect to $q$ which are identical for Cases II.B. 1 and II.B. 2 i.e.,

$$
\begin{array}{r}
\frac{\mathrm{d} \pi_{m}^{\text {opt.,forw.,inst. }}}{\mathrm{d} q}=\left(v-c_{1}\right)+\left(c_{2}-v\right)(1-F(q))=0 \\
\frac{\mathrm{~d}\left(\pi_{m}^{\text {opt.,forw.,inst. }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.112}
\end{array}
$$

We solve for the optimal long-term production service level. The manufacturer's objective function is concave in $q$ in both cases and the optimum is unique. We find for both Case II.B. 1 and Case II.B. 2 that the manufacturer should produce $q^{*}$ units long-term, such that

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{2}-c_{1}}{c_{2}-v} \tag{3.113}
\end{equation*}
$$

The optimal long-term production quantity is again independent from contract parameters. The retailer's profit function is identical for both cases,

$$
\begin{align*}
\Pi_{r}^{\text {portfolio }}(y, z, q)=- & \left(w_{1}-v\right) z-r y+(p-e)\left(S\left(y^{\prime}\right)-S(z)\right)  \tag{3.114}\\
& +(p-v) S(z)+\left(p-w_{2}\right)\left(D-S\left(y^{\prime}\right)\right) .
\end{align*}
$$

We derive the first and second order conditions with respect to $y$

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {portfolio }}}{\mathrm{d} y}=\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)-r=0  \tag{3.115}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} y}=-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 \tag{3.116}
\end{align*}
$$

and to $z$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{\text {portfolio }}}{\mathrm{d} z}=\left(v-w_{1}\right)+(e-v)(1-F(z))+\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} z}=-(e-v) f(z)-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 \tag{3.118}
\end{array}
$$

Solving for $y^{\prime}$ and $z$ provides the optimal service level from options and forwards,

$$
\begin{equation*}
F\left(y^{\prime *}\right)=\frac{w_{2}-e-r}{w_{2}-e} \tag{3.119}
\end{equation*}
$$

and forwards

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{e+r-w_{1}}{e-v} \tag{3.120}
\end{equation*}
$$

Both optimal service levels are unique, as the retailer's objective function is again concave in both $y$ and $z$. Optimal contract parameters follow immediately from rearranging terms. Contract parameters are no longer required to achieve coordination, as the 100 percent service level is reached under profitable short-term production without capacity constraints. Therefore, both the reservation fee

$$
\begin{equation*}
r=\left(w_{2}-e\right)\left(1-F\left(y^{\prime *}\right)\right) \tag{3.121}
\end{equation*}
$$

and the forward price

$$
\begin{equation*}
w_{1}=(e+r)-F\left(z^{*}\right)(e-v) \tag{3.122}
\end{equation*}
$$

can be chosen within the boundaries stated, i.e.,

$$
\begin{align*}
& 0<F(z)<F(q)<F\left(y^{\prime}\right) \text { (Case II.A) }  \tag{3.123}\\
& 0<F(z)<F\left(y^{\prime}\right)<F(q) \text { (Case II.B) } \tag{3.124}
\end{align*}
$$

### 3.4.3 Summary of results

We discussed four cases. The manufacturer can employ short-term production to profitably serve all orders (Cases II.B. 1 and II.B.2) or to reduce inventory risk from option reservations without serving instant orders (Case II.A).

Proposition 3.2. The use of short-term production capacities depends on the instant wholesale price.
i) For $w_{2} \geq c_{2}$, the manufacturer serves all orders and he provides a 100 percent service level, when there are no capacity constraints. Then, the manufacturer's long-term production decision is independent from contract parameters.
ii) For $w_{2}<c_{2}$, the manufacturer can employ short-term production to hedge against inventory risk from option contracts. However, he does not use his shortterm production opportunity to serve instant orders.

Proof. For both cases, production of some options and all instant orders short-term,

$$
\begin{array}{r}
\Pi_{m}^{\text {portfolio }}(y, z, q)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+e \cdot\left(S\left(y^{\prime}\right)-S(z)\right) \\
-v \cdot(S(q)-S(z))-c_{2} \cdot(D-S(q))+w_{2} \cdot\left(D-S\left(y^{\prime}\right)\right),
\end{array}
$$

and production of some instant orders short-term,

$$
\begin{aligned}
\Pi_{m}^{p o r t f o l i o}(y, z, q)= & \left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right) \\
& +\left(w_{2}-v\right)\left(S(q)-S\left(y^{\prime}\right)\right)+(D-S(q))\left(w_{2}-c_{2}\right)
\end{aligned}
$$

an additional unit of demand served achieves revenues $\left(w_{2}-c_{2}\right)$. For $w_{2} \geq c_{2}$, the manufacturer earns on marginal units sold as instant orders $\left(w_{2}-c_{2}\right) \geq 0$. For $w_{2}<$ $c_{2}$, the manufacturer loses profit, as $\left(w_{2}-c_{2}\right)<0$. In both cases, the first and second order derivatives with respect to $q$ are

$$
\frac{\mathrm{d} \Pi_{m}^{\text {opt.,forw.,inst. }}}{\mathrm{d} q}=\left(v-c_{1}\right)+\left(c_{2}-v\right)(1-F(q))=0
$$

and

$$
\frac{\mathrm{d}\left(\Pi_{m}^{\text {opt. } . \text { forw.,inst. }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0
$$

The optimal long-term production quantity is unique and independent of the contract parameters chosen. Short-term production is either zero or it equals the difference between demand and long-term production.

Even, when short-term production for serving instant orders is not profitable, it is possible that part of the inventory required to serve option executions should be produced short-term. The manufacturer earns

$$
\begin{array}{r}
\Pi_{m}^{\text {portfolio }}(y, z, q)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+e \cdot\left(S\left(y^{\prime}\right)-S(z)\right) \\
-v \cdot(S(q)-S(z))-c_{2} \cdot\left(S\left(y^{\prime}\right)-S(q)\right) .
\end{array}
$$

The first derivative with respect to $y$ is

$$
\frac{\mathrm{d}\left(\Pi_{m}^{\text {opt.,forw. } \text { inst. } .}\right)}{\mathrm{d} y}=r+\left(e-c_{2}\right)\left(1-F\left(y^{\prime}\right)\right)
$$

For $c_{2} \leq e$, short-term production for options is profitable, while for $c_{2}>e$, shortterm production for options is only profitable up to a service level

$$
F\left(y^{\prime}\right)=\frac{r+e-c_{2}}{e-c_{2}} .
$$

### 3.5 Channel power on the retailer side

Finally, we consider channel power on the retailer side, i.e., Scenario III. First, the retailer has an incentive to employ her channel power to force the manufacturer to increase his service level to $q^{\text {req. }}>q^{*}$. Second, the retailer can force the manufacturer to decrease the wholesale price, e.g., to reward the retailer for sales and marketing efforts. Here, we assume both.

### 3.5.1 Additional assumptions for modeling channel power

Channel power on the retailer side constrains the manufacturer in his production decision. The retailer forces the manufacturer to produce at least a quantity $q^{\text {req. }}$. When $q^{\text {req. }} \leq q^{*}$, this constraint is not binding, as the manufacturer has an incentive to produce additional speculative quantities. Then, channel power has no effect on the supply chain.

Assumption 3.5 The retailer sets her minimum service level requirement such that it constitutes a binding constraint for the retailer, $q^{\text {req. }}>q^{*}$, where $q^{*}$ is the optimal long-term production quantity. Otherwise, Scenarios I and II apply.

Channel power enables the retailer to enforce higher service levels for instant orders, than the manufacturer would provide voluntarily. Service levels for forward and option orders are by definition not subject to channel power, as the retailer determines the production level by her orders. We review two cases: first, shortterm production costs exceed the instant order wholesale price, $c_{2}>w_{2}$, and second, short-term production costs are less than or equal to the instant order wholesale price, $c_{2} \leq w_{2}$.

In the first case, channel power on the retailer side forces the manufacturer to provide additional short-term quantities at a negative margin. Consequently, channel power hurts the manufacturer's profit. The retailer benefits from channel power, when she sells additional quantities to consumers at a positive margin $\left(p-w_{2}\right)>0$. Consumers experience fewer stockouts during promotions and thus, they benefit from channel power on the retailer side, too.

In the second case, the manufacturer earns a positive margin on every unit produced short-term and sold on the instant order market. Therefore, the manufacturer has an incentive to serve all instant orders, as he earns a positive margin even in extreme states of demand. Then, channel power on the retailer side has no effect and the model collapses to Case II.B. As a result, we focus on the first case.

Assumption 3.6 Short-term production costs exceed the instant order wholesale price, $c_{2}>w_{2}$. The manufacturer employs his short-term production technology only, when he is forced by the retailer to do so.

Models III. 1 (forward contracts), III. 4 (option contracts) and III. 5 (portfolio of forward and option contracts) equal the respective cases without channel power, as
the manufacturer produces to order. Consequently, channel power has no impact in these settings and we focus on all portfolios containing instant orders. We first analyze pure instant orders schemes under channel power. Then, we add forwards and options.

### 3.5.2 Instant orders (III.2)

The manufacturer only offers instant order contracts. He is required to serve all demand up to the required service level. In case that the manufacturer did not produce a sufficient long-term quantity, he can produce short-term. The manufacturer faces two challenges: i) the manufacturer incurs a loss for every unit produced short-term for sale on the instant order market and ii) the manufacturer incurs a loss, when excess inventory is salvaged,

$$
\begin{equation*}
\Pi_{m}^{\text {pull }}(q)=\left(w_{2}-v\right) S(q)-\left(c_{1}-v\right) q+\left(w_{2}-c_{2}\right)\left(S\left(q^{\text {req. }}\right)-S(q)\right) \tag{3.125}
\end{equation*}
$$

We calculate the first- and second-order conditions with respect to $q$,

$$
\begin{align*}
\frac{\mathrm{d} \Pi_{m}^{\text {pull }}}{\mathrm{d} q}= & \left(c_{2}-v\right)(1-F(q))-\left(c_{1}-v\right)=0  \tag{3.126}\\
& \frac{\mathrm{~d}\left(\Pi_{m}^{\text {pull }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.127}
\end{align*}
$$

The optimal production quantity is unique and the optimal service level from long-term production follows immediately,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{2}-c_{1}}{c_{2}-v} \tag{3.128}
\end{equation*}
$$

The retailer's profit function does not depend on the manufacturer's production decision, as short-term production is possible,

$$
\begin{equation*}
\Pi_{r}^{p u s h}(q)=\left(p-w_{2}\right) S(q)+\left(p-w_{2}\right)\left(S\left(q^{\text {req. }}\right)-S(q)\right)=\left(p-w_{2}\right) S\left(q^{\text {req. }}\right) \tag{3.129}
\end{equation*}
$$

The retailer makes no decisions under a pull scheme. Orders simply equal demand from consumers. When orders exceed $q^{\text {req. }}$, the manufacturer restricts deliveries to $q^{\text {req. }}$. When the retail price is less than the cost of short-term production, the supply chain optimal short-term production quantity is zero. Then, channel power hurts supply chain profit. However, when the retail price exceeds the costs of short-term production, the supply chain should serve all orders, although this is suboptimal from the manufacturer's point of view. Channel power improves supply chain efficiency in this case. Thus, there can be double marginalization regarding the short-term production decision.

Lemma 3.8. Consider the case that retailers employ channel power to force the manufacturer to provide a minimum service level. Let the required minimum service level exceed the optimal long-term production service level. Let the cost of shortterm production exceed the instant order price. For $w_{2}<c_{2}<p$ (Case III.A), supply chain profit increases in the required service level $q^{\text {req. }}$, as the retailer's profit increases faster than the manufacturer's profit decreases. For $w_{2}<p<c_{2}$ (Case III.B), supply chain profit decreases in the required service level $q^{\text {req. }}$. Here, additional profit at the retailer level cannot offset losses at the manufacturer level.

In Case III.A, the manufacturer reduces supply chain profit by limiting short-term production to the value enforced by the retailer. Thereby, he increases his profit on the expense of supply chain efficiency and consumer welfare. In the next step, we show, how bidding coordinating contracts into the supply relation can achieve coordination in a setting characterized by retailer power. We add i) forward contracts, ii) option contracts and iii) both, forward and option contracts, to the portfolio of supply contracts offered to the retailer. We show that option contracts can coordinate Case III.A, however, Case III.B can only be coordinated, when the retailer abandons the required minimum service level.

### 3.5.3 Advance purchase discounts (III.3)

We analyze, whether advance purchase discounts can achieve coordination under channel power. Again we first derive expected profit of the manufacturer. The manufacturer now offers additionally a discount for early commitments, such that

$$
\begin{array}{r}
\Pi_{m}^{\text {apd }}\left(z, q, q^{\text {req. }}\right)=\left(w_{1}-v\right) z+\left(w_{2}-v\right)(S(q)-S(z))  \tag{3.130}\\
-\left(c_{1}-v\right) q+\left(S\left(q^{\text {req. }}\right)-S(q)\right)\left(w_{2}-c_{2}\right) .
\end{array}
$$

We calculate the first- and second-order conditions for maximizing the manufacturer's profit,

$$
\begin{align*}
\frac{\mathrm{d} \Pi_{m}^{a p d}}{\mathrm{~d} q}= & \left(c_{2}-v\right)(1-F(q))-\left(c_{1}-v\right)=0  \tag{3.131}\\
& \frac{\mathrm{~d}\left(\Pi_{m}^{a p d}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.132}
\end{align*}
$$

We rearrange terms and derive the unique optimal long-term production service level under an advance purchase discount scheme with channel power,

$$
\begin{equation*}
F\left(q^{*}\right)^{a p d}=\frac{c_{2}-c_{1}}{c_{2}-v} . \tag{3.133}
\end{equation*}
$$

This result equals the optimal long-term production service level derived for the pull mode. The retailer decides on her forward purchase quantity. She benefits from
the increased service level. The retailer's profit function is

$$
\begin{equation*}
\Pi_{r}^{a p d}\left(z, q^{r e q .}\right)=-\left(w_{1}-v\right) z+\left(w_{2}-v\right) S(z)+\left(p-w_{2}\right) S\left(q^{\text {req. }}\right) \tag{3.134}
\end{equation*}
$$

Again, we calculate the first- and second-order conditions with respect to $z$ to maximize the retailer's profit,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{a p d}}{\mathrm{~d} z}=-\left(w_{1}-v\right)+\left(w_{2}-v\right)(1-F(z))=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{a p d}\right)^{2}}{\mathrm{~d}^{2} z}=-\left(w_{2}-v\right) f(z) \leq 0 . \tag{3.136}
\end{array}
$$

We rearrange terms to find the unique optimal forward order service level,

$$
\begin{equation*}
F\left(z^{*}\right)^{a p d}=\frac{\left(w_{2}-w_{1}\right)}{\left(w_{2}-w_{1}\right)+\left(w_{1}-v\right)}=\frac{w_{2}-w_{1}}{w_{2}-v} \tag{3.137}
\end{equation*}
$$

We solve for the forward price $w_{1}$. Our result shows that the forward price is a function of the instant order price and the desired weight of forward contracts in the portfolio, that is the fraction of demand served with forwards,

$$
\begin{equation*}
w_{1}=w_{2}-F\left(z^{*}\right)\left(w_{2}-v\right) . \tag{3.138}
\end{equation*}
$$

The retailer cannot enforce a higher short-term production quantity by ordering such that $F(z)>F\left(q^{\text {req. }}\right)$, as the manufacturer would then produce long-term only and the retailer takes large inventory risk. The model collapses to Case I.1. Consequently, advance purchase discounts do not coordinate under retailer power.

Lemma 3.9. Consider the case that the manufacturer fails to produce the optimal short-term production quantity, because of $w_{2}<c_{2}<p$. Advance purchase discounts cannot achieve coordination under channel power.

### 3.5.4 A portfolio of options and instant orders (III.6)

In the next step, we combine option and instant order contracts (pull). The manufacturer's profit function is

$$
\begin{array}{r}
\Pi_{m}^{\text {opt.,inst. }}\left(y, q, q^{\text {req. }}\right)=(e-v) S(y)+\left(w_{2}-v\right)(S(q)-S(y)) \\
-\left(c_{1}-v\right) q+r y+\left(S\left(q^{\text {req. }}\right)-S(q)\right)\left(w_{2}-c_{2}\right) . \tag{3.139}
\end{array}
$$

We derive the first- and second-order conditions with respect to $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{m}^{\text {opt.,inst. }}}{\mathrm{d} q}=\left(c_{2}-v\right)(1-F(q))-\left(c_{1}-v\right)=0 \\
\frac{\mathrm{~d}\left(\Pi_{m}^{\text {opt.,inst. }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.141}
\end{array}
$$

We rearrange terms to derive the unique optimal long-term production service level,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{2}-c_{1}}{c_{2}-v} \tag{3.142}
\end{equation*}
$$

The retailer never holds inventory and her profit does not depend on the manufacturer's decision. Again, the retailer's profit function is independent of $q$,

$$
\begin{equation*}
\Pi_{r}^{\text {opt. inst. }}\left(y, q^{\text {req. }}\right)=(p-e) S(y)+\left(p-w_{2}\right)\left(S\left(q^{\text {req. }}\right)-S(y)\right)-r y . \tag{3.143}
\end{equation*}
$$

We derive the first and second oder conditions with respect to $y$,

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {opt.,inst. }}}{\mathrm{d} y}=\left(w_{2}-e\right)(1-F(y))-r=0  \tag{3.144}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,inst. }}\right)^{2}}{\mathrm{~d}^{2} y}=-\left(w_{2}-e\right) f(y) \leq 0 \tag{3.145}
\end{align*}
$$

We solve for the unique optimal option reservation quantity,

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{w_{2}-e-r}{w_{2}-e} \tag{3.146}
\end{equation*}
$$

Rearranging terms provides the reservation fee depending on the instant order wholesale price. There exists a continuum of optimal parameters, which allow for distributing profit in the supply chain, i.e.,

$$
\begin{equation*}
r=\left(1-F\left(y^{*}\right)\right)\left(w_{2}-e\right) . \tag{3.147}
\end{equation*}
$$

In contrast to forward contracts, options leave inventory risk at the manufacturer level. Consequently, for $w_{2}<c_{2}<p$, the retailer can enforce a higher short-term production quantity by ordering such that $F(y)>F\left(q^{\text {req. }}\right)$. As a result, the model collapses to the case of options only. The optimal production service level stays constant; however, the retailer's profit function changes, i.e.,

$$
\begin{equation*}
\Pi_{r}^{o p t .}(y)=(p-e) S(y)-r y \tag{3.148}
\end{equation*}
$$

where $y>q^{\text {req. }}$. The Newsvendor solution from Case I. 4 applies,

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{p-e-r}{p-e} . \tag{3.149}
\end{equation*}
$$

Lower execution fees and lower reservation fees i) increase the supply chain service level and ii) distribute more profit to the retailer. The supply chain optimally
serves every consumer for $w_{2}<c_{2}<p$, as the retail price exceeds production costs. Thus, the supply chain optimal service level equals 100 percent. Note that although short-term production takes place under certainty, the retailer has to pay the reservation fee upfront under risk. The manufacturer does not serve orders above $F\left(y^{*}\right)$. Therefore, a 100 percent service level can only be reached when the amount of inventory reserved equals the maximum state of demand. Obviously, this can result in infinite reservation payments in general. The target can only be reached, when the distribution of demand has either an upper bound, or when the reservation fee is priced such that $r=0$. In the second case, the optimal reservation service level $y^{*}$ equals one. The execution fee $e$ must then exceed short-term production cost $c_{2}$ to allow for positive manufacturer profit. Then, the model collapses to the case of profitable short-term production, discussed in Case II.B.

Lemma 3.10. The retailer can improve supply chain profit by reserving a larger quantity with options than the manufacturer would provide under the required minimum service level. Bidding option contracts into a instant order based relation thus can increase the service level towards the optimum; however, the optimum is not reached for all distributions of demand.

### 3.5.5 A portfolio of options, forwards and instant orders (III.7)

The manufacturer now offers a portfolio of forward, option and instant order contracts,

$$
\begin{array}{r}
\Pi_{m}^{\text {portfolio }}\left(y, z, q, q^{\text {req. }}\right)=\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y \\
+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right)+\left(w_{2}-v\right)\left(S(q)-S\left(y^{\prime}\right)\right)  \tag{3.150}\\
+\left(S\left(q^{\text {req. }}\right)-S(q)\right)\left(w_{2}-c_{2}\right)
\end{array}
$$

We calculate the first- and second-order conditions with respect to $q$,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{m}^{\text {portfolio }}}{\mathrm{d} q}=\left(v-c_{1}\right)+\left(c_{2}-v\right)(1-F(q))=0 \\
\frac{\mathrm{~d}\left(\Pi_{m}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} q}=-\left(c_{2}-v\right) f(q) \leq 0 \tag{3.152}
\end{array}
$$

The optimal long-term production quantity is unique and equals the results from the previous cases,

$$
\begin{equation*}
F\left(q^{*}\right)=\frac{c_{2}-c_{1}}{c_{2}-v} \tag{3.153}
\end{equation*}
$$

We establish the retailer's profit function, which is again independent from $q$,

$$
\begin{array}{r}
\Pi_{r}^{\text {portfolio }}\left(y, z, q^{\text {req. }}\right)=-\left(w_{1}-v\right) z+(p-e)\left(S\left(y^{\prime}\right)-S(z)\right) \\
+(p-v) S(z)-r y+\left(p-w_{2}\right)\left(S\left(q^{\text {req. }}\right)-S\left(y^{\prime}\right)\right) . \tag{3.154}
\end{array}
$$

We calculate the first- and second-order conditions with respect to $y$,

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {portfolio }}}{\mathrm{d} y}=\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)-r=0  \tag{3.155}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} y}=-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 \tag{3.156}
\end{align*}
$$

and $z$,

$$
\begin{gather*}
\frac{\mathrm{d} \Pi_{r}^{\text {portfolio }}}{\mathrm{d} z}=\left(\begin{array}{c}
\left.v-w_{1}\right)+(e-v)(1-F(z)) \\
\\
+\left(w_{2}-e\right)\left(1-F\left(y^{\prime}\right)\right)=0
\end{array}\right.  \tag{3.157}\\
\frac{\mathrm{d}\left(\Pi_{r}^{\text {portfolio }}\right)^{2}}{\mathrm{~d}^{2} z}=-(e-v) f(z)-\left(w_{2}-e\right) f\left(y^{\prime}\right) \leq 0 .
\end{gather*}
$$

Rearranging terms provides the unique optimal service levels for options and forwards,

$$
\begin{equation*}
F\left(y^{\prime *}\right)=\frac{w_{2}-e-r}{w_{2}-e}, \tag{3.159}
\end{equation*}
$$

as well as for forwards,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{\left(w_{2}-w_{1}\right)-\left(w_{2}-e\right) F\left(y^{\prime}\right)}{(e-v)}=\frac{e+r-w_{1}}{(e-v)} . \tag{3.160}
\end{equation*}
$$

Optimal contract parameters follow immediately,

$$
\begin{equation*}
r=\left(w_{2}-e\right)\left(1-F\left(y^{\prime *}\right)\right), w_{1}=(e+r)-F\left(z^{*}\right)(e-v) . \tag{3.161}
\end{equation*}
$$

We explained the impact of the required minimum service level in Lemma 3.8. Figure 3.15 shows the case of profitable short-term production from a supply chain perspective (Case III.A). In Case III.B, presented in Figure 3.16, supply chain profit can only be improved by abolishing the required minimum service level.

In the next step, we discuss the effect of bidding both, option and forward contracts, into a instant order based relation with channel power on the retailer side. The retailer employs forward and option contracts to reserve a quantity exceeding the required minimum service level. Then, the manufacturer does not serve any additional instant orders. The retailer has to rely on forwards and options, i.e.,

$$
\begin{equation*}
\Pi_{r}^{\text {opt.,forw. }}(z, y)=-\left(w_{1}-v\right) z+(p-e)\left(S\left(y^{\prime}\right)-S(z)\right)+(p-v) S(z)-r y \tag{3.162}
\end{equation*}
$$

where $y^{\prime}>q^{\text {req. }}$. We differentiate the retailer's profit function with respect to $y$ and we derive the optimal amount of option reservations,


Fig. 3.15 Layout of decisions, actions and service levels for short term production opportunities under retail power, when $w_{2}<c_{2}<p$ (Case III.A)


Fig. 3.16 Layout of decisions, actions and service levels for short term production opportunities under retail power, when $w_{2}<p<c_{2}$ (Case III.B)

$$
\begin{align*}
& \frac{\mathrm{d} \Pi_{r}^{\text {opt.,forw. }}}{\mathrm{d} y}=(p-e)\left(1-F\left(y^{\prime}\right)\right)-r=0  \tag{3.163}\\
& \frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,forw. }}\right)^{2}}{\mathrm{~d}^{2} y}=(e-p) f\left(y^{\prime}\right) \leq 0 \tag{3.164}
\end{align*}
$$

The retailer purchases option and forward contracts to ensure supply even in states of demand above the required minimum service level. There is a unique optimal reservation quantity, to be secured using forwards and options. Rearranging terms provides the associated reservation service level,

$$
\begin{equation*}
F\left(y^{\prime}\right)=\frac{(p-e-r)}{(p-e-r)+r}=\frac{p-e-r}{p-e} . \tag{3.165}
\end{equation*}
$$

We differentiate the retailer's profit function with respect to $z$ and we derive the optimal service level from forwards,

$$
\begin{array}{r}
\frac{\mathrm{d} \Pi_{r}^{\text {opt.,forw. }}}{\mathrm{d} z}=\left(p-w_{1}\right)-(e-v) F(z)-(p-e) F\left(y^{\prime}\right)=0 \\
\frac{\mathrm{~d}\left(\Pi_{r}^{\text {opt.,forw. }}\right)^{2}}{\mathrm{~d}^{2} z}=(v-e) f(z)+(e-p) f\left(y^{\prime}\right) \leq 0 \tag{3.167}
\end{array}
$$

The optimal forward purchase quantity follows immediately,

$$
\begin{equation*}
F\left(z^{*}\right)=\frac{e+r-w_{1}}{e-v} \tag{3.168}
\end{equation*}
$$

Rearranging terms provides the optimal reservation fee and the forward price,

$$
\begin{align*}
w_{1} & =(e+r)-(e-v) F\left(z^{*}\right)  \tag{3.169}\\
& r=(p-e)\left(1-F\left(y^{*}\right)\right) . \tag{3.170}
\end{align*}
$$

As in Case III.5, the supply chain service level decreases in the execution fee and the reservation fee. The forward price is limited in that it must not exceed the sum of execution and reservation fee. Otherwise, option contracts dominate forward contracts. Therefore, forward contracts cannot be employed as a means to transfer profit to the manufacturer. Still, forward contracts allow for distributing inventory in the supply chain.

Lemma 3.11. As in the case of bidding options into the market (Lemma 3.10), bidding options and forwards into the instant order based relation can increase the service level towards the optimum, without reaching the optimum in general, when the manufacturer requires positive payoffs. In addition, inventories can be distributed in the supply chain. When $w_{1}$ approaches $e+r$, inventory risk is shifted towards the manufacturer.

Proposition 3.3. Consider an environment, where short-term production costs exceed the instant order wholesale price. Consider a portfolio that contains instant order contracts, such that the manufacturer does not simply produce to order. Then, a minimum service level requirement constitutes a binding constraint. Table 3.4 summarizes our results.
i) A retailer, who can enforce a service level $q^{\text {req. }}>q^{*}$ increases her profit at the expense of the manufacturer's profit.
ii) Introducing channel power does not change a) optimal order quantities and b) optimal long-term production quantities, when the manufacturer has the opportunity to produce short-term.
iii) For $w_{2}<c_{2}<p$, bidding option contracts into a instant order based relation can improve the service level towards the optimum.

Proof. Consider the case of retailer power and short-term production. A constraint $q^{\text {req. is only binding if it is greater than the manufacturer's long-term production }}$ quantity $q^{*}$. Rising $q^{\text {req. above this level impacts the retailer's profit by }\left(p-w_{2}\right) ~}$ per unit and impacts the manufacturer's profit by $\left(w_{2}-c_{2}\right)$ per unit. The retailer gains from increasing the demanded service level, when $\left(p-w_{2}\right)>0$, which is required for instant order purchases to be rational. The manufacturer loses profit, when $\left(w_{2}-c_{2}\right)<0$, which is required to achieve a binding constraint.

The optimal order service levels and the optimal long-term production quantities derived for Scenarios II. and III. are equal. Equations (3.129), (3.134), (3.143) and (3.154) show that the retailer's profit function does not depend on the long-term production quantity.

The supply chain gains $\left(p-c_{2}\right)$ per additional unit produced short-term to serve demand. Therefore, the optimal service level is 100 percent. Ordering $y^{\prime}>q^{\text {req. }}$. thus improves the service level towards the optimum. See Lemmas 3.10 and 3.11.

### 3.5.6 Reducing the retail price by means of channel power

The retailer has an incentive to force the manufacturer to reduce the wholesale price. Then, the retailer experiences lower costs and she receives higher profit. In the previous sections, we showed that the manufacturer's long-term production quantity is independent from contract parameters, when short-term production is employed (Proposition 3.3). As a result, supply chain profit does not change, when the retailer exerts channel power to reduce the wholesale price, as long as the manufacturer participates in the game. We compare the first derivatives of the retailer and manufacturer profit functions with respect to $w_{2}$. For binding constraints, it must hold that the required minimum service level exceeds the reserved quantity. Then, the manufacturer's profit increases in the instant order price,

$$
\begin{equation*}
\frac{\mathrm{d} \Pi_{m}^{\text {portfolio }}}{\mathrm{d} w_{2}}=\left(S\left(q^{\text {req. }}\right)-S\left(y^{\prime}\right)\right) \geq 0 \tag{3.171}
\end{equation*}
$$




|  | $\frac{\partial-d}{d-a-d}=(, K) d$ | $\frac{1-\partial}{\mathrm{I}_{M}-1+\partial}=(z)_{H}$ | $\frac{a-z_{0}}{I_{0}-z_{0}}=(b) d$ | $b=,<>z>0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| uо!̣еu!̣p.rooว on | $\frac{\partial-\tau_{\mathcal{M}}}{1-\partial-\tau_{\mathcal{M}}}=(, К)_{H}$ | $\frac{1-\partial}{\mathrm{I}_{\mathcal{M}}-1+\partial}=(z)_{\boldsymbol{I}}$ | $\frac{a-\tau_{0}}{I_{0}-\tau_{0}}=(b)_{d}$ | $b>, K>z>0$ | II $^{\text {n }}$ d pue Ysn ${ }^{\text {d }}$ 'suoḷd O |
|  | $\frac{a-d}{d-\partial-d}=(, \mathbb{K})_{H}$ | O.ІəZ | $\frac{a-z_{0}}{I_{0}-z_{0}}=(b) d$ | $b=, \prec>z=0$ |  |
| uо!̣eu!̣pıooว on | $\frac{\partial-\tau_{M}}{1-\partial-\tau_{M}}=(\Omega)_{I}$ | O.ІəZ | $\frac{a-\tau_{0}}{I_{0}-\tau_{0}}=(b)_{d}$ | $b>, K>z=0$ | $\mathrm{If}^{\text {d }}$ pue suould ${ }^{\text {a }}$ |
| uо!̣еu!p.rooว on | O.əə | $\frac{1-\tau_{M}}{I_{M}-\tau_{M}}=\left(z_{H}\right.$ | $\frac{a-\tau_{0}}{\jmath_{0}-\tau_{0}}=(b)_{H}$ | $b>, K=z>0$ |  |
| uо!̣eu!p.rooว on | O.əZ | O.ІəZ | $\frac{a-\tau_{0}}{\jmath_{0}-\tau_{0}}=(b)_{H}$ | $b>, K=z=0$ | ${ }^{\text {ppow }}$ II ${ }^{\text {d }}$ |
| иоп̣еи!p.oо入 | saseyo.ind uọd ${ }^{\text {d }}$ | saseyo.ind $^{\text {p.iem.ion }}$ | uọpnpo.id | sə!!uenठ | әрой |

and the retailer's profit equally decreases in the instant order price,

$$
\begin{equation*}
\frac{\mathrm{d} \Pi_{r}^{\text {portfolio }}}{\mathrm{d} w_{2}}=-\left(S\left(q^{\text {req. }}\right)-S\left(y^{\prime}\right)\right) \leq 0 \tag{3.172}
\end{equation*}
$$

In contrast to required minimum service levels, increasing the instant order price has no effect on supply chain efficiency. Consequently, a powerful retailer can reduce manufacturer profit to the participation constraint and earn a maximal possible share of supply chain profit.

So far, we assumed that the manufacturer simply leaves the market, when his minimum participation requirement is no longer fulfilled. However, the manufacturer could also force the retailer to reduce the required service level $q^{\text {req. }}$ by threatening to leave the market. Leaving the market is a credible threat, when the retailer earns a positive profit, while the manufacturer does not receive his minimum requirement. Reducing the required service level decreases costs on the manufacturer side of the supply chain. Then, the manufacturer stays in the market, even when the retailer further reduces the instant order wholesale price. Thus, both forms of channel power are intertwined. In the next step, we calculate the boundaries for the retailer's decision.

First, the manufacturer leaves the market, when the instant order wholesale price is too low. We assume that the manufacturer requires at least zero profit,

$$
\begin{equation*}
\Pi_{m}^{\text {portfolio }}\left(w_{2}\right) \geq 0 \tag{3.173}
\end{equation*}
$$

Solving (3.150) for $w_{2}$ provides the wholesale price that is minimally required by the manufacturer,

$$
\begin{array}{r}
w_{2}^{\min }\left(S\left(q^{r e q .}\right)\right)=\left[\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right)\right.  \tag{3.174}\\
\left.-v\left(S(q)-S\left(y^{\prime}\right)\right)-c_{2}\left(S\left(q^{r e q .}\right)-S(q)\right)\right] /\left[S\left(y^{\prime}\right)-S\left(q^{\text {req. }}\right)\right] .
\end{array}
$$

Second, the manufacturer leaves the market, when the forced service level is so high that he earns a negative profit,

$$
\begin{equation*}
\Pi_{m}^{\text {portfolio }}\left(S\left(q^{\text {req. }}\right)\right) \geq 0 . \tag{3.175}
\end{equation*}
$$

Solving (3.150) for $S\left(q^{\text {req. }}\right)$ provides the maximum required service level that provides zero profit to the manufacturer,

$$
\begin{align*}
S^{\max }\left(q^{r e q}\right)= & -\left[\left(w_{1}-v\right) z-\left(c_{1}-v\right) q+r y+(e-v)\left(S\left(y^{\prime}\right)-S(z)\right)\right. \\
& \left.+\left(w_{2}-v\right)\left(S(q)-S\left(y^{\prime}\right)\right)-S(q)\left(w_{2}-c_{2}\right)\right] /\left[w_{2}-c_{2}\right] . \tag{3.176}
\end{align*}
$$

Proposition 3.4. A retailer in possession of channel power can reduce the manufacturer's profit to his participation constraint. The retailer faces a trade-off between a low instant order wholesale price and a high required service level. The following boundaries constrain her decision:

$$
w_{2}^{\min } \leq w_{2} \text { and } S\left(q^{\text {req. }}\right) \leq S^{\max }\left(q^{\text {req. }}\right)
$$

where $w_{2}^{\text {min }}$ is defined in (3.174) and where $S^{\max }\left(q^{\text {req. }}\right)$ is defined in (3.176).
Proof. The retailer earns $\delta \cdot w_{2} \cdot\left(S\left(q^{\text {req. }}\right)-S\left(y^{\prime}\right)\right)$, when she achieves to reduce the wholesale price by a fraction $\delta$, cf. Equation (3.154). The participation constraint limits her power to reduce $w_{2}$. Rising $q^{\text {req. }}$ increases the retailer's profit by $\left(p-w_{2}\right)$ per additional unit sold. The maximum possible service level that the retailer can enforce on the manufacturer is again limited by the manufacturer's participation constraint.

### 3.5.7 Constraints on short-term production capacities

So far, we assumed that short-term production capacities are unlimited. This is reasonable for some FMCG categories, as for example for detergents. For other categories, the short-term production opportunity may be restricted in capacity by a threshold $q^{\text {limit }}$. This has three effects. First, the ability to hedge with short-term production against uncertain option executions is then limited, $\left(y^{\prime}-q^{*}\right) \leq q^{\text {limit }}$ for $q^{*}<y^{\prime}$. Second, under profitable short-term production, a 100 percent service level is no longer feasible. The service level is restricted by the capacity constraint. Third, the ability of portfolios of contracts to increase service levels is limited by short-term production capacities. Major option orders then result in additional long-term production above the optimum. The supply chain thus incurs additional salvage costs for $y^{\prime}>\left(q^{*}+q^{\text {limit }}\right)$. Obviously, the optimal reservation service level then equals the service level achieved by the long-term production quantity of the integrated channel plus the available short-term production quantity, $F\left(y^{\prime *}\right)=F\left(q^{\text {int. }}+q^{\text {limit }}\right)$. As a result, the supply chain is coordinated, as the benefit from further increasing service levels with option orders is outweighed by the additional salvage costs.

### 3.6 Numerical study

We employ a numerical study to provide further insights into the supply chain setup described in Case III.7. The manufacturer offers a portfolio of forward, option and instant order contracts. He has both, long-term and short-term production opportunities. First, we review the effect of channel power on supply chain profit. Second, we discuss the optimal production policy and third, we review contract pricing.

Table 3.5 The impact of channel power

| $\mathbf{q}^{\text {max }}$ | $\mathbf{w}_{\mathbf{2}}$ | $\Pi_{\mathbf{m}}$ | $\Pi_{r}$ | $\Pi_{\text {chain }}$ | Efficiency (\%) | Participation |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.95 | 20 | $0.5(4 \%)$ | 12.0 (96\%) | 12.5 | $58 \%$ | Yes |
| $"$ | 19 | -2.5 (n.a.) | 15.0 (n.a.) | 12.5 | $58 \%$ | No |
| $"$ | 18 | -6.1 (n.a.) | 18.6 (n.a.) | 12.5 | $58 \%$ | No |
| $"$ | 17 | -10.1 (n.a.) | 22.6 (n.a.) | 12.5 | $58 \%$ | No |
| 0.85 | 20 | $4.0(25 \%)$ | 12.0 (75\%) | 16.0 | $74 \%$ | Yes |
| $"$ | 19 | 1.9 (12\%) | 14.1 (88\%) | 16.0 | $74 \%$ | Yes |
| $"$ | 18 | -0.8 (n.a.) | 16.8 (n.a.) | 16.0 | $74 \%$ | No |
| $"$ | 17 | -3.9 (n.a.) | 19.9 (n.a.) | 16.0 | $74 \%$ | No |
|  |  |  |  |  |  |  |
| 0.75 | 20 | $7.5(38 \%)$ | $12.0(62 \%)$ | 19.5 | $90 \%$ | Yes |
| $"$ | 19 | $6.2(32 \%)$ | $13.3(68 \%)$ | 19.5 | $90 \%$ | Yes |
| $"$ | 18 | $4.4(22 \%)$ | $15.1(78 \%)$ | 19.5 | $90 \%$ | Yes |
| $"$ | 17 | $2.1(10 \%)$ | $17.4(90 \%)$ | 19.5 | $90 \%$ | Yes |

Parameters: $w_{1}=14, r=3, p=20, c_{1}=12, c_{2}=24$
Demand is gamma distributed with mean 10 and a coefficient of variation $2^{-\frac{1}{2}}$.

### 3.6.1 Model setup

For this numerical study, we employ generic contract parameters along the lines of Cachon (2004) and Lariviere and Porteus (2002). Let the retail price equal $p=20.0$. Let short-term production costs be $c_{1}=12.0$ and let long-term production costs be $c_{2}=24.0$. Products can be salvaged at a value $v=8.0$. The execution fee is set at long-term production costs $e=c_{1}=12.0$. We follow Cachon (2004) and employ a gamma distribution with mean 10 and a coefficient of variation $2^{-1 / 2} \approx 0.707$. This function does not produce negative values and is therefore suitable for modeling demand.

### 3.6.2 Channel power and supply chain profit

The retailer maximizes profit. When she has the opportunity to exploit her channel power, she can force the manufacturer to reduce the wholesale price and to offer higher service levels. We modeled both cases analytically in the previous section.

We simultaneously reduce the wholesale price and increase the service level. Table 3.5 provides both parties' profits and profit shares, as well as supply chain profit and efficiency in terms of the service level achieved. Figure 3.17 shows the effect of channel power on profit. We find that reducing the wholesale price shifts a higher share of channel profit to the retailer without reducing channel profit. When the wholesale price is forced down too far, the manufacturer leaves the market, as he earns negative profit. The retailer can employ her channel power to reduce the


Fig. 3.17 The effect of channel power on profit
manufacturer's profit share to zero, given that the manufacturer requires at least non-negative profit. Then, the retailer earns all supply chain profit.

The retailer's profit also increases in the maximum service level $q^{\text {req. }}$, given a constant wholesale price $w_{2}$. Thus, the retailer has an incentive to force the manufacturer to increase the maximum of instant orders served. Again, doing so hurts the manufacturer's profit. When the retailer demands an excessive service level, the manufacturer leaves the market, as soon as he earns a negative profit. Note that short-term production costs exceed the retail price given the data chosen. We thus model the case, where required minimum service levels hurt supply chain profit. For $w_{2}<c_{2}<p$, results would change such that the retailer loses less than the retailer wins when the required minimum service level increases. Then, a required minimum service level would benefit supply chain efficiency.

### 3.6.3 The drivers of the optimal production policy

Reviewing the drivers of production decisions is the next step in our analysis. The manufacturer decides on the fraction of the required quantity $q^{\text {req. }}$ that he produces with his long-term technology. We showed that there is a unique optimal production policy which only depends on costs and salvage values. The manufacturer's production policy is independent from the required service level. Figure 3.18 visualizes this result.

The manufacturer trades off overage and underage costs. His optimal production policy is reached when expected marginal underage costs equal expected marginal overage costs. When the manufacturer produces more than demand, he



Manufacturer Profits ( $q^{\text {req }}=0.75$ ) $\quad \square$ Retailer Profits ( $q^{\text {req }}=0.75$ )
Parameters: $F\left(y^{\prime}\right)=0,66, F(z)=0,33, p=20, c_{1}=12, c_{2}=24$
Demand is gamma distributed with mean 10 and a coefficient of variation $2^{-1 / 2}$
Fig. 3.18 The impact of long-term production service levels on profit


Fig. 3.19 The impact of production costs and salvage values on the optimal long-term production service level
loses $c_{o}=\left(c_{1}-v\right)$ on each unit. When demand exceeds production, he has to produce short term and loses $c_{u}=\left(c_{2}-c_{1}\right)$. As both, underage costs and overage costs do not depend on contract parameters, his production decision is not dependent on the retailer's orders. Figure 3.19 shows how the optimal production policy reacts to changes in short-term production costs, long-term production costs and salvage values.

Further, we see that the retailer's profit is independent from the manufacturer's production policy, when there is a short-term production opportunity and a required service level. The retailer determines the expected production quantity, which is $S\left(q^{\text {req. }}\right)=E\left(\min \left(\widetilde{D}, q^{r e q .}\right)\right)$. The manufacturer's production policy then does no longer influence the amount sold and thus, it is reduced to a cost-minimization problem.

### 3.6.4 Contract pricing

We derived analytical solutions for forward prices and reservation fees, depending on the wholesale price in place. In the next step, we review the effect of contract parameters on the mix of contracts optimally purchased by the retailer to maximize profit.

Table 3.6 The impact of contract pricing on order decisions and profit

| $\mathbf{r}$ | $\mathbf{w}_{\mathbf{1}}$ | $\mathbf{w}_{\mathbf{2}}$ | $\mathbf{y}(\%)$ | $\mathbf{z}(\%)$ | $\mathbf{s} \mathbf{( \% )}$ | $\Pi_{\mathbf{m}}$ | $\Pi_{\mathbf{r}}$ | $\Pi_{\text {chain }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2 | 14.0 | 19.0 | $79 \%$ | $1 \%$ | $20 \%$ | $0.9(5 \%)$ | $18.4(95 \%)$ | 19.3 |
| 2.6 | $"$ | $"$ | $60 \%$ | $6 \%$ | $34 \%$ | $4.1(21 \%)$ | $15.2(79 \%)$ | 19.3 |
| 3.0 | $"$ | $"$ | $41 \%$ | $14 \%$ | $45 \%$ | $6.1(32 \%)$ | $13.2(68 \%)$ | 19.3 |
| 3.4 | $"$ | $"$ | $22 \%$ | $23 \%$ | $55 \%$ | $7.3(38 \%)$ | $12.0(62 \%)$ | 19.3 |
| 3.8 | $"$ | $"$ | $0 \%$ | $36 \%$ | $64 \%$ | $7.8(40 \%)$ | $11.5(60 \%)$ | 19.3 |
|  |  |  |  |  |  |  |  |  |
| 3.0 | 15.0 | 19.0 | $55 \%$ | $0 \%$ | $45 \%$ | $7.3(38 \%)$ | $12.0(62 \%)$ | 19.3 |
| $"$ | 14.5 | $"$ | $50 \%$ | $5 \%$ | $45 \%$ | $6.1(32 \%)$ | $13.2(68 \%)$ | 19.3 |
| $"$ | 14.0 | $"$ | $41 \%$ | $14 \%$ | $45 \%$ | $4.4(23 \%)$ | $14.9(77 \%)$ | 19.3 |
| $"$ | 13.5 | $"$ | $29 \%$ | $26 \%$ | $45 \%$ | $2.0(11 \%)$ | $17.3(80 \%)$ | 19.3 |
| $"$ | 13.0 | $"$ | $12 \%$ | $43 \%$ | $45 \%$ | $-1.1($ n.a. $)$ | $20.5($ n.a. $)$ | 19.3 |
|  |  |  |  |  |  |  |  |  |
| 3.0 | 14.0 | 20.0 | $52 \%$ | $14 \%$ | $34 \%$ | $6.6(34 \%)$ | $12.7(66 \%)$ | 19.3 |
| $"$ | $"$ | 19.0 | $41 \%$ | $14 \%$ | $45 \%$ | $6.5(34 \%)$ | $12.8(66 \%)$ | 19.3 |
| $"$ | $"$ | 18.0 | $29 \%$ | $14 \%$ | $57 \%$ | $6.1(32 \%)$ | $13.2(68 \%)$ | 19.3 |
| $"$ | $"$ | 17.0 | $15 \%$ | $14 \%$ | $71 \%$ | $5.2(27 \%)$ | $14.2(73 \%)$ | 19.3 |
| $"$ | $"$ | 16.0 | $0 \%$ | $14 \%$ | $86 \%$ | $3.5(18 \%)$ | $15.8(82 \%)$ | 19.3 |

Parameters: $p=20, c_{1}=12, c_{2}=24$
Demand is gamma distributed with mean 10 and a coefficient of variation $2^{-\frac{1}{2}}$.

The composition of the portfolio depends on the reservation fee $r$, the forward price $w_{1}$ and the instant order wholesale price $w_{2}$. We vary one of these parameters


Fig. 3.20 The impact of the reservation fee on order decision


Parameters: $w_{2}=19, r=3, p=20, c_{1}=12, c_{2}=24$
Demand is gamma distributed with mean 10 and a coefficient of variation $2^{-1 / 2}$
Fig. 3.21 The impact of the forward price on order decision
at a time and find the optimal portfolio of contracts and the associated split of profit. Table 3.6 shows our results. We find that the choice of contract parameters determines the distribution of profit in the supply chain, but it has no effect on total supply chain profit, for the reasons discussed before.

The number of option contracts purchased decreases in the reservation fee. When options become less attractive, the retailer places more forward orders and instant orders. Figure 3.20 shows these dynamics. Increasing the reservation fee rises the manufacturer's share of profit and decreases the retailer's share.

The number of forward contracts demanded by the retailer decreases in the forward price. Forwards are replaced with option contracts, but not with instant orders. The instant order volume is independent from the forward price, when the portfolio contains option contracts. Again, increasing the forward price benefits the manufacturer and hurts the retailer. Figure 3.21 shows our results.


Fig. 3.22 The impact of the instant order wholesale price on order decision

Increasing the instant order wholesale price reduces the number of instant order contracts purchased. The retailer still requires the associated flexibility and purchases additional options. The number of forwards is independent from the instant order wholesale price. The manufacturer benefits from a higher instant order wholesale price, while the retailer prefers a lower instant order wholesale price. Figure 3.22 visualizes these results.

### 3.7 Contribution to literature and link to the FMCG industry

The chapter concludes with i) a summary of our contributions to literature and ii) a discussion of the application of our results to a FMCG context.

### 3.7.1 Contribution to literature

The work presented in this chapter extends existing literature in two dimensions. First, we consider portfolios of option, forward and instant order contracts with respect to coordination. We derive closed form solutions for the optimal structure of portfolios of supply contracts. In addition, we solve for optimal contract parameters. Second, we consider channel power on the retailer side in an environment with two production opportunities. We find that the ability to produce short-term results in independence of production and purchasing decision. The ability to dictate a minimum service level can either hurt or benefit supply chain profit, depending on the ratio of short-term production cost to retail price. We derive analytical solutions for the boundaries of channel power and we show that bidding option and forward contracts into the market can achieve coordination in some settings.

Our work is closest to Cachon (2004). We extend the analysis of Cachon (2004) to include option contracts as part of the portfolio. In addition, we account for shortterm production opportunities and channel power. The work of Martinez-de Albéniz and Simchi-Levi (2005) is related, in that they review portfolios containing supply options. In contrast to Martinez-de Albéniz and Simchi-Levi (2005), we model the effects of contract portfolios on profits of both, the retailer and the manufacturer. Barnes-Schuster et al. (2002) consider portfolios containing supply options in a coordination setting. Our model is different, in that we consider channel power and focus on the ability to achieve coordination and possibly Pareto improvement by bidding additional contracts into the market. Raju and Zhang (2005) model dominance by a powerful retailer; however, they focus on the issue of providing incentives to such a retailer in the presence of smaller competitors. Our contribution shows in contrast, how the dominant retailer can impact pricing and production processes on the manufacturer level.

Our approach is limited in that we focus on option contracts as the coordinating component of the portfolio of contracts. However, we showed that advance purchase discounts do not achieve coordination of supply decisions. Buyback contracts are likely to achieve equal benefits as option contracts; however, they require additional shipments to the retailer and back to the manufacturer. The associated transportation cost are lost for the supply chain. Moreover, buy-back contracts cannot be hedged with short-term production. All quantity must be shipped at the beginning of the sales period. For these two reasons, option contracts are superior to buy-back contracts in the context discussed.

### 3.7.2 Putting the single period model into a promotional context

This chapter is dedicated to channel coordination with a portfolio of hedging contracts. To conclude, we would like to relate our results to the challenges encountered by the FMCG industry.

Retailers avoid commitments as long as possible in the FMCG industry. Orders are subject to change until uncertainty is resolved. Manufacturers have the ability to produce both, short-term and long-term; however, short-term production is clearly seen as an emergency alternative, not a regular production technology. Retailers have major power in FMCG relations and they are focused on wholesale prices and service levels. Manufacturers can enforce positive participation constraints and thus, channel power alone does not coordinate the supply chain. This supply chain layout is modeled in Scenario III.

In general, the wholesale price is not time-dependent in retailer-manufacturer relations in the FMCG industry. Thus, there is no advance purchase discount in place. Given the uncommitted nature of orders in this industry, contracts can be best characterized as instant order wholesale price contracts. As a result, FMCG supply chains operate in pull mode. We characterize model III. 2 as the status quo
for relationships with channel power on the retailer side and II. 2 for relationships without channel power.

In chapter 5, we review portfolios of contracts in multi-period settings. The single period model developed in this chapter assumes that unsold inventory is salvaged at time $t_{2}$. However, in a multi-period setting, excess inventory can be reused in the following promotion. Holding inventory from one period to the next results in holding costs $h$. It is reasonable that $\left(c_{1}-v\right)>h$ and $\left(w_{1}-v\right)>h$. Thus, both, the retailer and the manufacturer are likely to prefer holding inventory to salvaging. Then, overage costs decrease in the multi-period model, compared to the single period model.

## Chapter 4 <br> The Two-Segment Demand Forecast Model

In this chapter, we present a demand forecasting approach for promotions based on two consumer segments. In the first step, we discuss the mechanisms that determine consumer demand during a promotion. In the second step, we develop a model of promotional demand when pricing decisions are uncertain. Here, we explicitly consider the fact that different groups of consumers react each in their own way to discounts. We restrict our analysis to two consumer segments, following the established literature on demand models developed by Blattberg et al. (1981). Running profitable price promotions requires a profound knowledge of the effects of price discounts in order to optimally order inventory. Key issues are the mechanism of promotions in triggering demand and the role of stockpiling in this context.

### 4.1 Loyal and stockpiling consumers

It is assumed that each consumer segment reacts very differently to price promotions. The two-segment model groups consumers according to their stockpiling preferences. Consequently, demand is modeled for each individual group, resulting in a better prediction model, when compared to an approach assuming a single homogeneous consumer group. Iyer and Ye (2000), Freiheit (2001) and Huchzermeier et al. (2002) develop forecasting methods based on the stockpiling model of Blattberg et al. (1981). All three references contain empirical analyses showing exceptionally good fits. Thus, the consumer inventory model of promotional demand can be considered as a stable basis for further analyses. The two-segment model is based on consumer's willingness to stockpile, which we model first. Then, we derive consumers' purchase decisions and develop the basic two-segment demand forecast model. We extend the model by adding random intervals between consecutive promotions.

### 4.1.1 The demand impact of price promotions

A major stream of marketing research targets to explain why some consumers purchase additional units of a product when it is on promotion. The general microeconomic explanation, that the ratio of utility to cost of the product improves, is a starting point (see Chapter 2). Bell et al. (1999) characterize the demand effects of promotions by distinguishing between primary and secondary demand effects.

Consumers are potentially willing to hold inventory. Thus, price sensitive consumers purchase at a low promotion price in equilibrium, knowing that the savings will exceed the costs required to hold the product until consumption takes place. Therefore, promotions increase the amounts consumers buy at a single purchase incidence. These primary demand effects represent a shift of demand over time. Consumers either purchase more of a product than usual or repurchase earlier (see Gupta 1988, Bell et al. 1999). Consequently, consumers will reduce the portion of their budget spent on the product promoted during the next periods. There are several reasons, why a retailer could be willing to offer a discount to accelerate purchases. First, inventories can trigger additional consumption. According to a study by Chandon and Wansink (2002), additional consumption depends on the convenience of the product and its visibility at consumer's homes. Second, purchase acceleration can help to manage inventories at the retailer and manufacturer level, as Blattberg et al. (1981) supposes. Third, different retail prices over time are a form of price discrimination (Simon and Fassnacht 2009). Consumers who are not willing to wait for discounts are charged a higher retail price. Price sensitive consumers are served at the lower retail price. Anderson and Dana (2009) explore the conditions under which price discrimination is profitable in general. Finally, purchase acceleration can have a competitive effect. When consumers purchase one product in large quantities at a discounted price, they are not likely to search for a competitor's product (Bell et al. 1999). They simply have too much inventory left at home to consider alternative products when going shopping.

Secondary demand effects refer to package size switching (see Gupta 1988, Freiheit 2001, Huchzermeier et al. 2002) or brand switching (see Gupta 1988, Bell et al. 1999). Here, consumers buy another product than they would have done without the discount. According to Blattberg et al. (1995), cross-promotional effects are asymmetric, i.e., the importance of brand switching depends on the promoting brand. Higher quality brands attract more brand switchers, when discounted, than low range products (Blattberg et al. 1995). Thus, budget customers are more likely to react to a discount. Hendel and Nevo (2006a) find that consumers switching to another brand are likely to stay with their usual package size. Moreover, consumers prefer package size switching to brand switching. Hendel and Nevo (2006a) find that cross-price elasticities among different package sizes of one brand are sometimes 20 times higher than those of competing brands. Thus, consumers are rather likely to switch package sizes, when one size is on promotion; however, it is much more difficult to attract additional demand from competitive brands.

It is important to note that the relative importance of the individual demand effects depends on the category and the market. A model of promotional demand is
only valid for categories, where the effects underlying the model are strong. We focus on stockpiling as the foundation of the two-segment forecast. As a result, our forecast is likely to be more reliable for categories that allow for holding inventory.

### 4.1.2 Allocating demand over time

Demand effects are driven by consumers' wealth savings, when buying during promotions. Thereby, they create inventories. As inventories require valuable space and funds, delaying purchases is costly. Consumers purchase additional products at a discount to avoid purchases at the higher regular price in the near future; however, they have to bear additional inventory holding costs. Thus, stockpiling represents a trade-off between discounts and holding costs.

Blattberg et al. (1978) consider household inventory decisions to be determined by transaction costs, holding costs, stockout costs and retail prices. Each shopping trip is costly for consumers as it requires time and transportation. When consumers run out of stock of a specific product, they incur disutility, as they are restricted in consumption. Holding costs result from the limitation of storage space at consumers' homes. Particularly large volume products are negatively affected by storage costs.

Blattberg et al. (1981) provide a model of consumer stockpiling. The approach has been successfully applied to data sets several times (Blattberg et al. 1981; Beasley 1998; Iyer and Ye 2000; Huchzermeier et al. 2002). This line of theory and explanation of promotional demand is the basis for our forecasting model. We follow Blattberg et al. (1981) and Iyer and Ye (2000) in that we focus on intertemporal demand shifts as the predominant driver of promotional demand.

### 4.1.3 Consumers' willingness to stockpile

The first step in developing a model of demand during promotions is characterizing consumers. We consider a category where consumer stockpiling is an important driver of demand during promotions. Consumers minimize purchase costs, where one major factor is holding costs. Retail prices vary over time. For the moment, we assume that retail prices can only take two realizations: a regular retail price and a lower promotion price. Consumers face a trade-off between holding costs and cheaper retail prices during promotions.

Stockpiling can save on total purchase costs. Still, stockpiling has disadvantages and not all consumers consider stockpiling. The willingness to stockpile allows consumers to maximally benefit from price promotions, by, i.e., building up inventory when retail prices are low. When consumers are not willing to stockpile, they still can benefit from lower prices during promotions; however, they may be required to purchase at the regular price between promotions.


Fig. 4.1 Demand generated from loyal and deal-prone consumers

We divide consumers into two groups, as proposed by Blattberg et al. (1981) and Iyer and Ye (2000). The members of the first group are characterized by an unwillingness to keep inventory. They incur disutility when stockpiling, which translates into higher holding costs. The members of the second group are willing to stockpile.

Consumers face a trade-off between holding costs and savings from purchases during price promotions. For some consumers savings outweigh holding costs to some degree. This low holding cost segment shifts future purchases to a promotion period, as long as savings exceed additional holding costs. The stockpiling behavior requires planning and building up of inventories. We name the stockpiling consumer segment according to Blattberg et al. (1978) "deal-prone consumers". Deal-prone consumers incur rather low holding costs and they are willing to stockpile.

Deal-prone consumers do not purchase if there is no promotion. They link demand to retail prices. This segment stockpiles and takes remaining household inventories into consideration when taking purchasing decisions.

Consumers who are unwilling to stockpile cannot benefit from price promotions as much, as they incur high holding costs. Consequently, these consumers purchase only enough to bridge the period of time to the next shopping trip. These consumers are forced to purchase regularly in order to avoid stockouts. Consumers showing these characteristics are referred to as "loyal consumers".

Loyal consumers always purchase an equal amount per period. They do not stockpile and thus they do not react to price promotions.

Demand can be characterized as emerging from two sources, deal-prone and loyal consumers. Loyal consumers demand a constant base load, while deal prone consumers create demand spikes (see Figure 4.1). A retailer must realize that she serves two distinct consumer segments, which require different policies. The loyal
segment purchases regularly, while the deal-prone segment waits for promotions. Promotions have only a major effect, if the deal-prone segment is large enough. We take a single store perspective and do not model switching between stores. A previous analysis by Huchzermeier et al. (2002) along these lines showed high empirical fit. We provide additional empirical evidence that this approach is very well applicable for the diapers category.

### 4.1.4 Consumers' purchase decisions

The trade-off between holding costs and savings from discounts implies an optimal purchase and inventory policy. Consumers minimize the total costs of purchasing $T C\left(Q_{t}\right)$, where $Q_{t}$ is an order up-to-level. We follow Blattberg et al. (1981) for this discussion. Total costs depend on the segment specific holding costs $h^{\text {seg. and }}$ the saving from deals $\left(u-p_{t}\right)$, where $u$ is the reservation price and $p_{t}$ is the time dependent retail price. Table 4.1 summarizes the notation employed in this chapter.

Table 4.1 Notation employed in Chapter 4

| Parameter | Description |
| :--- | :--- |
| $D_{t}^{\text {seg. }}$ | demand from the consumer segment |
| $p_{t}$ | retail price at time $t$ |
| $u$ | reservation price per unit |
| $h^{\text {seg. }}$ | consumers' holding costs per unit |
| $\varepsilon$ | probability of a deep promotion |
| $C^{\text {seg. }}$ | consumption rate of the consumer segment |
| $Q_{t}^{\text {seg. }}$ | consumers' order-up-to level at time $t$ |
| $T C^{\text {seg. }}$ | total cost of purchasing of the consumer segment |
| $I_{t}^{\text {seg. }}$ | consumers' inventory at time $t$ |
| $\tau$ | time interval between promotions |

Assumption 4.1 There are two-segments of consumers, characterized by high and low holding costs respectively, where all consumers in one segment have equal holding costs and equal reservation prices.

Consumers minimize their total costs of purchasing by deciding on the optimal purchase quantity $Q_{t}$. Per period consumption is denoted as $C^{\text {seg. }}$. Both, the optimal purchase quantity and the per period consumption rate are again segment specific. The total cost is

$$
\begin{equation*}
T C\left(Q^{\text {seg. }}\right)=\int_{0}^{Q^{\text {seg. }} / C^{\text {seg. }}} h^{\text {seg. }} \cdot\left(Q^{\text {seg. }}-C^{\text {seg. }} t\right) \mathrm{d} t-Q^{\text {seg. }} \cdot\left(u-p_{t}\right) \tag{4.1}
\end{equation*}
$$

The first term of equation (4.1) describes the additional holding costs resulting from stockpiling. Holding costs decrease over time, as part of the inventory is con-
sumed. Inventory reaches zero after $Q^{\text {seg. }} / C^{\text {seg. periods. The second term represents }}$ the savings consumers gain from purchasing at a discount instead of the regular retail price.

Unlike deal-prone consumers, loyal consumers are unwilling to stockpile, as they experience high holding costs. Their optimal purchase quantity is the quantity required for the period of time until the next shopping trip. Thus, their total cost function does not contain holding costs. Loyal consumers thus cannot influence their total costs of purchasing and will always purchase the product independently of the retail price. For $h_{\text {loyal }} \rightarrow \infty$, (4.1) is finite only for $Q^{\text {loyal }}=C^{\text {loyal }}$. Then, (4.1) converges to (4.2).

$$
\begin{gather*}
T C\left(Q^{\text {loyal }}\right)=\left(p_{t}-u\right) Q^{\text {loyal }}  \tag{4.2}\\
\text { where } Q^{\text {loyal }}=C^{\text {loyal }}
\end{gather*}
$$

Deal-prone consumers shift purchases over time. Again, we derive the optimal purchase decision by minimizing total costs of purchasing. Equation (4.1) can be simplified, i.e.,

$$
\begin{equation*}
T C\left(Q^{d p}\right)=h^{d p} \cdot \frac{Q^{d p}}{C^{d p}} \cdot \frac{Q^{d p}}{2}-\left(u-p_{t}\right) \cdot Q^{d p} \tag{4.3}
\end{equation*}
$$

We differentiate equation (4.3) with respect to $Q^{d p}$ and derive the first and second order conditions,

$$
\begin{array}{r}
\frac{d T C\left(Q^{d p}\right)}{d Q^{d p}}=\frac{h^{d p} Q^{d p}}{C^{d p}}-\left(u-p_{t}\right)=0 \\
\frac{d T C\left(Q^{d p}\right)^{2}}{d^{2} Q^{d p}}=\frac{h^{d p}}{C^{d p}} \geq 0 \tag{4.5}
\end{array}
$$

The optimal purchase quantity $Q^{*}$ of a deal-prone consumer and the optimal time between orders $\tau^{*}$ follow immediately. This multi-period model characterizes a renewal process with fixed time periods between promotions. Please consider again Figure 4.1. The price spikes indicate the order-up-to level of deal-prone consumers. Consumers purchase sufficient inventory to reach an inventory level $Q^{*}$. Then, they reduce inventory through consumption until the next promotion occurs. The model assumes that promotions are offered such that consumers have the opportunity to restock after their optimal interval $\tau^{*}$. In this model, consumers purchase exactly at the point in time, when inventory reaches zero.

Lemma 4.1. Deal-prone consumers react on price discounts. Deeper discounts trigger more demand from this segment and increase the time between purchases. There exists an optimal order-up-to policy and a corresponding optimal time between promotions (see Blattberg et al. 1981),

$$
\begin{gather*}
Q^{*}=\frac{\left(u-p_{t}\right) C^{d p}}{h^{d p}},  \tag{4.6}\\
\tau^{*}=\frac{u-p_{t}}{h^{d p}} \tag{4.7}
\end{gather*}
$$

Proof. Follows immediately from the first order and second order conditions.
These results have been derived by Blattberg et al. (1981). There is an incentive for retailers to adjust their promotion schedules to consumers' needs. Promotions take place exactly after $\tau^{*}$ units of time, providing deal-prone consumers with an opportunity to purchase at the moment, when inventories reach zero. As a result, demand during promotions equals the optimal order quantity derived. Iyer and Ye (2000) and Huchzermeier et al. (2002) further develop this line of research. In the next section, the model is generalized to consider unequal intervals between promotions. Then, we model multiple promotion prices.

### 4.1.5 Modeling unequal intervals between promotions

The model presented in the previous section assumed that promotion schedules perfectly fit deal-prone consumers' needs. That is, promotions take place exactly when household inventories approach zero. However, promotions do not necessarily occur in regular intervals over time. Then, time between promotions is random from a consumer point of view. Thus, it is possible that deal-prone consumers still have left-over inventories when the next promotion occurs. Analogously, it is possible that consumers run out of inventory, as they have to wait longer than expected for the next promotion. As a result, demand during a promotion depends on the retail price and the consumer inventory level at the promotion. The model of promotional demand thus must be extended by an inventory component to account for random times between promotions. We present the analysis along the lines of Iyer and Ye (2000) and Huchzermeier et al. (2002).

Consider the case of unequal intervals between promotions. Let $I_{t}\left(p_{t-1}\right)$ be consumer inventories at the beginning of the promotion at time $t$, given the retail price $p_{t-1}$ at the last promotion. Deal-prone consumers order a sufficient amount to reach their optimal order-up-to quantity in $t-1$, as described in Lemma 4.1. They do not order, when their inventory level at the beginning of the promotion exceeds the optimal order-up-to level. Inventory at the beginning of the current promotion $I_{t}\left(p_{t-1}\right)$ is calculated as inventory after the previous promotion less consumption over $\tau$ units of time. Note that $\tau_{t}$ is defined as a time interval, where $\tau_{t-1}$ is the time interval between the previous and the current promotion. When deal-prone consumers run out of inventory, or when the retailer cannot supply enough products to allow for consumption, we assume that consumers do not backlog. Household inventories of deal-prone consumers equal

$$
\begin{equation*}
I_{t}^{d p}\left(p_{t-1}\right)=\left[Q_{t-1}^{*}\left(p_{t-1}\right)-\tau_{t-1} \cdot C^{d p}\right]^{+} \tag{4.8}
\end{equation*}
$$

Demand from deal-prone consumers equals the difference between the optimal order-up-to level $Q_{t}^{*}\left(p_{t}\right)$ and the consumer inventory level at the beginning of the promotion $I_{t}^{d p}\left(p_{t-1}\right)$,

$$
\begin{equation*}
D_{t}^{d p}\left(p_{t}\right)=\left[Q_{t}^{*}\left(p_{t}\right)-I_{t}^{d p}\left(p_{t-1}\right)\right]^{+} \tag{4.9}
\end{equation*}
$$

Equation (4.9) is formulated such that high consumer inventory levels can result in zero demand during promotions, when discounts are rather shallow. Then, dealprone consumers have no incentive to purchase additional inventory and they wait until the next promotion. This is rational, as expected holding costs of additional inventories outweigh the savings that can be achieved at the current promotion price. This behavior has been perceived in an empirical study by Mela et al. (1998).

The second consumer segment, loyal consumers, do not react to promotions. Thus, demand from loyal customers $C_{\text {loyal }}$ is constant over all points in time. Let $D_{t}\left(p_{t}\right)$ be the cumulated demand from both consumer segments during the period of time between a promotion at time $t$ and the beginning of the next promotion at time $t+1$, i.e.,

$$
\begin{equation*}
D_{t}\left(p_{t}\right)=D_{t}^{d p}\left(p_{t}\right)+D_{t}^{\text {loyal }}=\left[Q_{t}^{*}\left(p_{t}\right)-I_{t}^{d p}\left(p_{t-1}\right)\right]^{+}+\tau_{t} \cdot C_{\text {loyal }} \tag{4.10}
\end{equation*}
$$

Lemma 4.2. Demand experienced by a single retailer depends on the retail price and the level of household inventories. When deal-prone consumers' inventory levels exceed the optimal order-up-to quantity, only loyal consumers purchase (see Huchzermeier et al. 2002, p. 232).

$$
D_{t}\left(p_{t}\right)= \begin{cases}Q_{t}^{*}\left(p_{t}\right)-I_{t}^{d p}\left(p_{t-1}\right)+\tau_{t} \cdot C_{\text {loyal }} & \text { if } Q_{t}^{*}\left(p_{t}\right) \geq I_{t}^{d p}\left(p_{t-1}\right)  \tag{4.11}\\ \tau_{t-1} \cdot C_{\text {loyal }} & \text { if } Q_{t}^{*}\left(p_{t}\right)<I_{t}^{d p}\left(p_{t-1}\right)\end{cases}
$$

Figure 4.2 shows the extended mechanism considering unequal time intervals between promotions. There are two additional scenarios compared to the model by Blattberg et al. (1981) discussed before. When promotions occur faster than after $\tau^{*}$ units of time, consumers still have leftover inventories, resulting in lower demand spikes (Case A). If there is no promotion after $\tau^{*}$ units of time, consumers run out of inventory. However, as there is no backlogging, demand is then lost for the retailer. At the next promotion, deal-prone consumers purchase sufficiently to reach their optimal order-up-to quantity (Case B).


Fig. 4.2 Consumer inventories, when times between promotions are unequal

### 4.1.6 Modeling multiple promotion prices over a planning horizon

In the next step, the model is extended to consider multiple realizations of the promotion price. The retail price determines consumers' purchase behavior. Price discounts result in purchases to reach the associated order-up-to levels and thus, discounts determine household inventories. As a result, each retail price determines demand in the corresponding promotion and influences demand in the following promotions. First of all, we have to consider the number of possible retail prices in the FMCG market.

In markets, regular price levels evolve over time for most products. These price levels are undercut during promotions. Rotemberg (2005) and Wiehenbrauk (2010) provide empirical examples for this dynamic. Prices return to the original level after promotions. Both analyses suggest that retailers either run shallow or deep promotions. This point of view is supported by interviews with FMCG industry representatives. Thus, we model three possible retail prices in the further analysis: a regular price $p_{\text {regular }}$, a shallow discount $p_{\text {shallow }}$ and a deep discount $p_{\text {deep }}$.

We assume two possible promotion prices without loss of generality. Each of these prices is associated with an optimal order-up-to policy, which corresponds to


Fig. 4.3 Optimal order policies
the optimal order quantities $Q_{t}^{*}$, derived for the renewal process described above. Deal-prone consumers check at every promotion, whether it would be optimal to increase their inventories. When there is no promotion, deal-prone consumers' refrain from purchasing. There exist two optimal order-up-to levels, i.e.,

$$
Q_{t}^{*}\left(p_{t}\right)= \begin{cases}Q_{\text {deep }}^{*}=\frac{\left(u-p_{\text {deep }}\right) C^{d p}}{h^{d p}} & \text { if } p_{t}=p_{\text {deep }}  \tag{4.12}\\ Q_{\text {shallow }}^{*}=\frac{\left(u-p_{\text {shallow }}\right) C^{d p}}{h^{d p}} & \text { if } p_{t}=p_{\text {shallow }} \\ 0 & \text { if } p_{t}=p_{\text {regular }}\end{cases}
$$

Figure 4.3 visualizes the optimal order-up-to policies and their connection to the current retail price. The order-up-to policies determine the possible levels of household inventory after the promotion. Combining equation (4.12) with equation (4.8) provides three cases, which must be distinguished.


Fig. 4.4 Linking order-up-to levels, inventories and demand

$$
I_{t}^{d p}\left(p_{t-1}\right)=\left\{\begin{array}{l}
{\left[Q_{\text {deep }}^{*}-\tau_{t-1} \cdot C^{d p}\right]^{+}=\left[\frac{\left(u-p_{\text {deep }}\right) C^{d p}}{h^{d p}}-\tau_{t-1} \cdot C^{d p}\right]^{+}}  \tag{4.13}\\
\text {if } p_{t-1}=p_{\text {deep }} \\
{\left[Q_{\text {shal. }}^{*}-\tau_{t-1} \cdot C^{d p}\right]^{+}=\left[\frac{\left(u-p_{\text {shal. }}\right) C^{d p}}{h^{d p}}-\tau_{t-1} \cdot C^{d p}\right]^{+}} \\
\text {if } p_{t-1}=p_{\text {shallow }} \\
{\left[I_{t-1}^{d p}-\tau_{t-1} \cdot C^{d p}\right]^{+}} \\
\text {if } p_{t-1}=p_{\text {regular }}
\end{array}\right.
$$

Discounts trigger demand from the deal-prone segment; however, the actual size of the spike depends on consumer inventories. The inventory level is the link between price deals and demand peaks. Figure 4.3 shows the relationship between customer inventories, retail prices and demand. Equation (4.13) provides a link from retail prices to optimal inventory levels. Deal-prone consumers replenish their inventories to the optimal level, given the promotion price. Then, they consume inventory at a rate $C^{d p}$.

Figure 4.4 visualizes the impact of two possible promotions prices on demand. Shallow promotion prices create in general lower demand peaks than do deep dis-
counts. A shallow promotion following a deep discount has less impact on demand, because household inventories are still filled to a rather high level (Case C). When a shallow promotion closely follows a deep promotion, the optimal order-up-to level for shallow promotions can even be lower than remaining inventories. Then, the shallow promotion has no effect on demand (Case D).

We have seen that household inventories are the link between price discounts and demand during promotions. The optimal order-up-to levels depend on retail prices. In the next step, we take a closer look at promotion pricing and how pricing decisions are modeled.

### 4.2 Random promotion pricing

The next step in developing a model of demand in a promotional environment is to analyze pricing decisions. As shown in Lemma 4.2, retail prices determine demand. The retailer faces a trade-off between generating demand and receiving high margins. A discount attracts deal-prone consumers; however, at the same time the retailer loses profit on her loyal consumers. How should the retailer optimally behave? We first go back to the downward-sloping demand function and review, how stockpiling fits into this model. Then, we discuss promotions as a means of competition between retailers and review game theoretic modeling approaches linked to this issue. We conclude this section with insights from evolutionary game theory, visualizing the impact of competition on the choice of retail prices.

### 4.2.1 Stockpiling and the downward-sloping demand function

Discounts possibly attract deal-prone consumers and thus increase revenues. We denote this effect as the demand effect of price promotions. However, retailers lose margins on their loyal customers when offering deals, which we refer to as the margin effect. Thus, the profitability of deals is not always ensured and it depends on the elasticity of demand of deal-prone consumers and on the amount of inventory left at consumers' homes. The classical demand curve model discussed above provides further insights. Transforming (4.11) and substituting for $Q^{*}$ employing (4.6) provides the price-demand relation for deal-prone consumers, i.e.,

$$
\begin{gather*}
p_{t}=u-\frac{h^{d p}}{C^{d p}}\left(D_{d p}+I_{t}^{d p}\right) \text { for } Q^{*}-I_{t}^{d p} \geq 0  \tag{4.14}\\
D_{d p}=0 \text { otherwise }
\end{gather*}
$$

Figure 4.5 illustrates the connection between leftover inventories at consumers' homes and the price-demand relation. Leftover inventories shift the price-demand function downwards, thus reducing demand for a given price. Note that the slope


Fig. 4.5 Dependency of the price-demand function on consumer inventories
of the function and thus the elasticity of demand stay constant. The elasticity of demand exclusively depends on holding costs and on consumption rates. It cannot be influenced by the retailer.

Household inventories shift the price-demand relation vertically. Larger amounts of household inventory shift the function downwards, resulting in lower demand from the deal-prone segment for a given retail price. Consequently, the remaining inventory at consumers' homes determines the retailer's pricing opportunities. For high inventory levels, discounts must be deep to achieve a quantity impact.

Lemma 4.3. According to the price-demand relation, price promotions are particularly effective, when consumer inventory levels are low, when holding costs are low and when consumption rates are high.

Proof. Consider the derivatives of equation (4.14) with respect to $I_{t}^{d p}, h^{d p}$ and $C^{d p}$,

$$
\begin{array}{r}
\frac{\mathrm{d} p_{t}}{\mathrm{~d} I_{t}^{d p}}=-\frac{h^{d p}}{C^{d p}} \leq 0 \\
\frac{\mathrm{~d} p_{t}}{\mathrm{~d} h^{d p}}=-\frac{D_{d p}}{C^{d p}}-\frac{I_{t-1}^{d p}}{C^{d p}} \leq 0 \\
\frac{\mathrm{~d} p_{t}}{\mathrm{~d} C^{d p}}=\frac{h^{d p}}{C_{d p}^{2}} D_{d p}+\frac{h^{d p}}{C_{d p}^{2}} I_{t}^{d p} \geq 0
\end{array}
$$

The price-demand function continuously decreases in $I_{t}^{d p}$ and in $h^{d p}$. It continuously increases in $C^{d p}$.


Fig. 4.6 Assumptions concerning the split of revenues among competing retailers depending on the retail prices (Source: adapted from Wiehenbrauk 2010, p. 52)

### 4.2.2 Promotions as a means of competition

Price deals directly influence demand for the product at the store level. Thus, discounts are one of the most important competitive measures in retailing. Notably, the FMCG industry lives in a constant fear of price wars, which permanently destroy margins. In 2009, the German discounter Aldi lowered prices on more than 250 articles, leading competitors to notion of destruction of value (Schulz 2010). Retailers must find the balance between discounts and regular prices to avoid cannibalization of margins earned from loyal consumers.

Wiehenbrauk (2010) analyzes the effects of store switching and stockpiling in a competitive environment. Consider a market with complete information and rational consumers. We assume that consumers have no costs associated with store switching. In order to attract switching consumers, a retailer must offer competitive prices. If another retailer provides lower discounts, this retailer receives all demand from switchers, while the first retailer still suffers from the reduced margin on sales to her loyal customers. Figure 4.6 shows how revenues are split between two competing retailers, depending on the retail prices offered. Let $\alpha$ be the profit, each retailer receives from her respective loyal segment and let $\beta$ be the aggregated demand of switchers in the market. We assume that each retailer receives loyal demand $\alpha$ independently of the decisions made. Further, we assume that the retailer offering the lowest retail price receives all demand from switching consumers $\beta$. When both re-


Fig. 4.7 Effects of price choices on profitability
tailers bid the lowest price, we assume that demand from switchers is equally split and each retailer receives $\beta / 2$.

Profitability of a promotion depends on the relative importance of the demand and margin effects. A profitable promotion creates sufficient demand to outweigh margin losses. The margin effect only depends on the price discount set by the retailer and it is not affected by competitive actions. The demand effect depends on both the retailer's and the competitor's actions, as consumers' purchase decisions depend on both retail prices.

Figure 4.7 shows the combined margin and demand effects. Let $D_{D}$ and $D_{S}$ be the demand effects of deep and shallow promotions, that is the additional profit from higher sales volumes. Let $M_{D}$ and $M_{S}$ be the margin effects of deep and shallow promotions, that is the lost profit on volumes sold to loyal consumers. Whenever a player plays "regular", she incurs no additional costs and does not receive additional demand. Her payoff from promotional activity is always zero, independently of competitors' actions. Consider the set of rationalizable prices. A promotion price (a pure strategy) is only rationalizable, when the maximum demand effect outweighs the margin effect. This can only be the case, when the retailer offers a lower price than her competitor. Thus, whenever a player underbids the competitor's price, she must earn a positive payoff. Otherwise, the price in question is not a rationalizable strategy and will never be played. When a player plays "shallow" and the competitor plays "deep", she receives no additional demand; however, she still has to bear the costs of running a promotion, that is, the margin effect. When both players bid very low prices, this represents a price war. It is likely, that this state decreases prof-


Fig. 4.8 Pure strategies of a retailer (Source: Wiehenbrauk 2010, p. 49)
itability compared to the status quo. The effect when both players choose to run shallow promotions can be negative or positive, depending on the relative size of the switching segment. We show a numerical example after discussing promotional pricing from a game theoretical perspective.

### 4.2.3 A game theoretic model of promotional competition on price

Retailers consider competitors' pricing decisions to derive their own optimal pricing strategies. Game theory provides the toolbox to analyze the pricing game described. In addition, we review the dynamics of the game from the perspective of evolutionary game theory.

### 4.2.3.1 The competitive pricing model of Wiehenbrauk (2010)

Retailers have to decide upon their pricing strategies, considering opponents' actions. Optimal strategy choices depend on the size of the stockpiling segment. Wiehenbrauk (2010) shows that there is a critical size of the switching segment, where the best strategy switches from regular to promotion. When analyzing a data set from diapers retailing, she finds that there are three distinct price levels. When the retailer decides to promote, the promotion price is drawn randomly from the two choices deep and shallow. Figure 4.8 shows the structure of the pricing mechanism. First, the retailer decides, whether there is a promotion or not at a certain point in time. In the second step, the retailer decides on the promotion price. When there is no promotion, the game ends after the first step.

Retailers can in addition choose to randomly mix strategies. Mixed strategies are the optimal response under competition. Wiehenbrauk (2010) finds that retailers should mix randomly between regular prices, shallow and deep discounts. This approach is necessary to be unpredictable towards other retailers. When a strategy becomes predictable, competitors can easily underbid the retail price, thereby gaining all demand. In the Nash equilibrium, all retailers play completely mixed strategies. By definition, no player can gain higher profit through deviation (Mas-Colell
et al. 1995, p. 246). Thus, only a mixed pricing strategy is viable in a promotion environment. Please refer to Wiehenbrauk (2010) for further details and proofs.

### 4.2.3.2 Analysis from the point of view of evolutionary game theory

Evolutionary game theory considers the development of strategies over time and analyzes whether the resulting equilibria are resistant to "mutations". In the pricing game, a mutation could be an entrant offering extremely deep promotions at a very high probability. If the entry of this new player into the market changes the way the game is played, then the old equilibrium was not evolutionary stable.

Consider a market where retailers compete for demand from switching consumers. There is a continuum of agents randomly drawn from a large set of companies to play a symmetric 2-player game $\Gamma=(N, K, u)$, where $N$ is the set of players, where $K$ is the set of available pure strategies $s$ and where $u$ is the payoff function. Let $\Delta$ be the space of all mixed strategies $x$. Agents choose from the three pure pricing strategies discussed: deep promotions, shallow discounts and regular prices. The frequencies of the individual pricing decisions $s$ at time $t$ constitute a mixed strategy $x(t)$. The development of the mix of pure strategies $s$ in the mixed strategy $x(t)$ played in the market is determined by the replicator dynamics. The replicator dynamics are designed to ensure that agents play successful strategies more frequently. Those strategies providing lower payoffs, when played against the market mix, lose importance over time. We focus on switching customers and, in contrast to Wiehenbrauk (2010), we do not consider stockpiling. The replicator dynamics resemble a natural selection process, which leads to the notion of evolutionary game theory. We employ the replicator dynamics as derived by Weibull (1995), p. 72,

$$
\begin{equation*}
\dot{x}_{s}=x_{s}[u(s, x)-u(x, x)] \text { for all } s \in K \text {, where } \dot{x}_{s}=\frac{d x_{s}}{d t} . \tag{4.15}
\end{equation*}
$$

$x_{s}$ denotes the weight given to a pure strategy $s$ in the mixed strategy $x$. Whenever a pure strategy $s$ outperforms a mixed strategy $x$, when played against $x$, the weight given to a pure strategy $s$ in $x$ is increased. $\dot{x}_{s}$ stands for the change in probability of a pure strategy $s$ in a mixed strategy $x$ over time $t$. This game represents a scenario, where players are learning over time and adapt their strategies to optimally compete against the strategy mix played by opponents. The development of strategies in the market depends on two factors. First, the initial state of the market describes the strategies played at the beginning of the analysis. Second, the payoff matrix determines the welfare generated by playing this strategy combination. By existence, there must be at least one Nash equilibrium for all configurations of the payoff matrix and initial states. This Nash equilibrium is reached after a limited number of iterations. It should be noted that the replicator dynamics require a completely mixed strategy profile as a starting point to work properly. Boundary points in the strategy simplex achieve results which can be suboptimal when mutations playing the non-represented strategies enter.


Fig. 4.9 Example of the development of the mix of strategies played towards a stable equilibrium

For a payoff matrix as shown in Figure 4.7, the resulting Nash equilibrium can be completely mixed. We employ the replicator dynamics (Equation 4.15) to gain further insights into the development of pricing strategies over time.

A mutation can only be successful, when it performs better against the current market strategy, than the market strategy itself. We employ the following approach: In the first step, we choose a mixed strategy, which contains all pure strategies available. When the Nash Equilibrium is unique, all interior starting points lead to the same result, as the replicator dynamics ultimately converge all mixed strategy profiles to the equilibrium profile. We simulate three scenarios and choose the following initial mixes of pure strategies:

- Scenario 1: $20 \%$ deep promotions, $30 \%$ shallow promotions and $50 \%$ regular promotions
- Scenario 2: 33\% deep promotions, 33\% shallow promotions and 33\% regular promotions
- Scenario 3: 50\% deep promotions, $30 \%$ shallow promotions and $20 \%$ regular promotions

We choose a payoff matrix, which fulfills the requirements stated in Figure 4.7. That is, no pure strategy is dominated and each pure strategy is a best response against exactly one other pure strategy. We assume the following payoff matrix:

- Playing "regular" always results in a payoff of zero, as no additional switching customers are attracted.
- Playing "shallow" against "regular" results in a payoff of three (strongly positive), as switchers are attracted at minimal costs
- Playing "shallow" against "shallows" results in a payoff of one (weakly positive), as only half of the switchers respond to the own promotion, while the retailer still bears the full cost of the discount to loyal customers
- Playing "shallow" against "deep" results in a payoff of minus three (strongly negative), as the retailer loses margin and does not earn additional customers
- Playing "deep" against "regular" or "shallow" results in a payoff of one (weakly positive), as switchers are attracted at high costs
- Playing "deep" against "deep" results in a payoff of minus two (negative), as only half of the switchers respond to the own, expensive, promotion

This matrix ensures that each strategy is a best response. "Shallow" is a best response to "regular", as a low discount achieves all demand from the switching segment. "Deep" is a best response to "shallow", as it underbids the competitive promotion and achieves again all demand from the switching segment. When both players choose "deep", the resulting price war results in losses. Thus, it is optimal to play "regular" against "deep".

We enter the initial values into the replicator dynamics and iterate several times. Figure 4.9 demonstrates how strategies evolve over time and that the Nash equilibrium is reached after few iterations. In the first scenario, strategy "regular" is initially played rather often, which encourages mutations playing more often strategy "shallow". After "shallow" got strong, there is an incentive to increase the frequency of playing "deep". This again encourages to play more "regular". Players improve their strategy profiles and after some iterations, the equilibrium emerges. Note that this equilibrium is identical in all three scenarios.

The resulting equilibrium is stable. Deviating from the equilibrium never results in a new equilibrium in the example shown, as long as a pure strategy is not eliminated completely. Thus, the equilibrium is asymptotically stable. As a result, new entrants into the retail market cannot change the way the pricing game is played. It is possible that the entrant causes some disturbance of pricing policies; however after some periods, the original equilibrium is reached again. The notion of completely mixed strategy means that players choose prices randomly. They must not be predictable, as a predictable strategy would result in this context in a suboptimal payoff.

Under the assumptions summarized in Figure 4.7, a completely mixed strategy profile can emerge as the unique Nash equilibrium of the pricing game. Players choose retail prices randomly, where probabilities are determined by the payoff scheme. The resulting equilibrium is asymptotically stable. For other payoff ma-
trices, it is possible that no weight is given to one or even two pure strategies in equilibrium.

Wiehenbrauk (2010) showed that a completely mixed pricing strategy is optimal in a promotional context. In interviews with FMCG companies, we were told that retailers print their promotion catalogs only very shortly before the promotion starts. Therefore, they are unpredictable and play a mixed strategy. We employed evolutionary game theory to further explore this issue. The random pricing strategy necessarily results in uncertainty and thus, it must be considered when thinking about demand forecasts.

### 4.3 Forecasting demand based on the two-segment model

So far, we have considered a number of isolated modeling issues that are employed in the next step to derive a demand forecast. According to Blattberg et al. (1981), promotions increase the inventory at deal-prone consumers' homes and allow for shifting demand over time. According to Iyer and Ye (2000), a purchase decision is not only dependent on the current retail price, but also on earlier retail prices, as the stockpiling effect reduces demand after a promotion. Finally, Wiehenbrauk (2010) showed that retail prices are drawn randomly, where probabilities depend on the payoffs received in a competitive game. A demand forecast based on the twosegment model of consumer demand combines these three results. We model several promotions over a planning horizon, since cost-optimal production requires a planning lead time. To predict consumer demand, a stochastic process is constructed, which tracks consumer purchases conditionally on retail prices.

### 4.3.1 Modeling multiple promotions

The household inventory model describes demand during a promotion period as a function of the promotion price and consumer inventories. When forecasting demand, consumer inventories are uncertain, as there are multiple promotions within the forecast horizon. Therefore, intermediate promotions within the forecast horizon are a source of demand risk. Demand depends on the promotion price at the point in time to be forecast, on the history of retail prices within the forecast horizon and on the current consumer inventory level. Each promotion in the forecast horizon adds uncertainty about retail prices, as pricing must be unpredictable for competitive reasons. We do not model competition per se in the two-segment forecast and we will show that this approach is rational, as our model achieves very good predictions with limited information requirements.

We develop a stochastic model of consumer inventory by considering the effects of past retail prices on future demand. In FMCG retailing, promotions are planned by the marketing department in advance. The retailer communicates the promo-

(B) - Regular Retail Price (S) - Shallow Promotion Price (D) - Deep Promotion Price

Fig. 4.10 Model of planned promotions
tion schedule to the manufacturer and thus, timing can be considered to be fixed and known to both the retailer and the manufacturer. Retailers divide the year into promotion periods. In each promotion period, one single promotion is planned. Promotion pricing is randomized and communicated as late as possible (Wiehenbrauk 2010). We discuss the case of randomized promotion timing later and first review the case of regular intervals between promotions. Figure 4.10 shows the layout of our basic promotion model. Here, we assume equal time spans between promotions and a commitment to promote. That is, the retailer never leaves out a planned promotion. We will relax this assumption in the further discussion.

In this basic model, consumption between two promotions is always constant, as intervals between promotions, promotion prices and consumption rates, are all constant. In each unit of time, $C^{d p}$ units of the product are consumed by deal-prone consumers. Thus, deal-prone consumers require $\tau \cdot C^{d p}$ units of the product per promotion period, where $\tau$ is the length of a promotion period.

We allow for shallow and deep promotion prices. Demand uncertainty arises from this price choice, resulting in different purchase quantities by deal-prone consumers, depending on the random discount. Shallow promotions result in lower demand spikes, but they also have a lower impact on future promotions.

This first case is characterized by a commitment to promote and equal intervals between promotions. Under these assumptions, there is a maximum of four possible states of demand, as only the previous promotion influences demand in the next promotion. This is the case, because consumers adjust household inventories either to the shallow or to the deep order-up-to levels at every promotion and as intervals between promotions are constant. Note that this restriction of possible states of demand does no longer hold, when either of the two key assumption is relaxed.

A second case results from the fact that retailers can additionally consider the option to abandon a planned promotion. Again, the optimal planning policy is based on equal intervals of regular pricing between promotions. Compared to the first model, the retailer chooses randomly from three possible retail prices: shallow discount, deep discount and regular price. The retailer makes the decision whether to offer a discount before the scheduled start of the promotion. This additional decision increases demand uncertainty compared to the first model. Now also past promotions before the preceding promotion can have an effect on the promotion to be forecast.

Two more cases have to be considered, when allowing for unequal time spans between promotions. Retailers consider other factors than consumer inventory when planning promotions and thus derive a schedule with regular pricing periods of different lengths. Again, promotion dates can either be committed or only announced. In the second case, the decision whether to promote is made shortly before the promotion is scheduled to start. Figure 4.11 provides an overview on the four cases described. The left hand models allow for skipping promotions and the upper models allow for unequal periods between promotions.

Random intervals between promotions result in additional possible states of demand. When an interval is rather short and when the previous discount was deep enough, consumers still have left-over inventories. Then, demand at the next promotion is reduced. Longer intervals between promotions reduce household inventories and thus, they are less likely to hurt demand at the following promotions. The combination of random discounts and random intervals between promotions allows for a larger number of reachable states of demand.

In the next step, we derive a stochastic model of demand. We employ the case with random intervals and a commitment to promote. This approach reflects best the situation we see in diapers retailing. Promotions are planned in advance; however, retail prices are decided short-term. Consequently, the retailer randomly draws at each promotion date either the shallow or the deep discount.

### 4.3.2 A two-segment forecast of promotional demand

Consider a supply chain consisting of a single manufacturer, a single retailer and two consumer segments, loyal and deal-prone. Consumers in the deal-prone segment are willing to build up inventories and behave according to the stockpiling model of consumer demand (Blattberg et al. 1981; Iyer and Ye 2000). The retailer purchases inventory to serve demand during the planning period. There are two promotions scheduled in the planning period. Time between the contracting phase and the first promotion is $\tau_{0}$ units of time. After an additional $\tau_{1}$ units of time, the second promotion takes place. There is a remaining length of $\tau_{2}$ units of time until the end of the planning period. Figure 4.12 shows the development of consumer inventory over time.

Loyal consumers purchase at all points in time, while deal-prone consumers only consider the promotion offers at $t_{1}$ and $t_{2}$. Timing of all promotions during the cy-


Fig. 4.11 Four possible promotion dynamics
cle is planned in advance. This information is shared with the manufacturer at $t_{0}$. Thus $\tau_{0}, \tau_{1}$ and $\tau_{2}$ are known to both the retailer and the manufacturer, but not to consumers. Prices are set randomly at the beginning of the respective promotions. The next step is deriving a stochastic model of demand as a function of consumer inventory and retail prices. Wiehenbrauk (2010) showed that a retailer competing in a market will adopt a mixed pricing strategy. According to Lemma 4.1, demand depends, besides on retail prices, on consumption rates, reservation prices, holding costs and consumer inventories. When consumption rates are constant, there is a direct relationship between consumer inventories and demand, given the retail price.

Assumption 4.2 For the two-segment forecast, holding costs, reservation prices and consumption rates of consumers are independent and remain constant over time.

Let $I_{t}$ be the aggregated inventory position of consumers at the beginning of a promotion at a point in time $t . I_{t}^{d p}$ is an element of the set $I^{d p}$, which consists of a finite number of elements for finite planning horizons. There are two possible states of inventory before each promotion, corresponding to the two possible promotion prices at the last promotion. Remember that we assumed a commitment to promote, which best reflects promotion plans as seen in practice. Thus, the development of $I_{t}^{d p}$ over time depends on retail prices $p_{t}$. We construct a stochastic process of household inventories (compare Taylor and Karlin 1993 and Karr 1990 for background on stochastic processes).

The possible states of inventory given two scheduled promotions follow immediately. Periods with no promotions are incorporated by multiplying the consumption rate with the time between promotions $\tau$. Initial inventory or beginning-of-period inventory $I_{0}^{d p}$ is assumed to be known. Inventory at the beginning of the first promotion scheduled is certain; however, this promotion has an uncertain effect on inventory after the first promotion and thus at the beginning of the second promotion. In general, household inventories equal,

$$
I_{t}^{d p}\left(p_{t-1}\right)= \begin{cases}{\left[\max \left[I_{t-1}^{d p} ; Q_{\text {deep }}^{*}\right]-\tau_{t-1} C^{d p}\right]^{+}} & \text {if } p_{t-1}=p_{\text {deep }}  \tag{4.16}\\ {\left[\max \left[I_{t-1}^{d p} ; Q_{\text {shal. }}^{*}\right]-\tau_{t-1} C^{d p}\right]^{+}} & \text {if } p_{t-1}=p_{\text {shal }}\end{cases}
$$

In particular, for $t \in\{1,2\}$,

$$
\begin{align*}
& I_{1}^{d p}\left(p_{0}\right)= \begin{cases}{\left[\max \left[I_{0}^{d p} ; Q_{\text {deep }}^{*}\right]-\tau_{0} C^{d p}\right]^{+}} & \text {if } p_{0}=p_{\text {deep }} \\
{\left[\max \left[I_{0}^{d p} ; Q_{\text {shal. }}^{*}\right]-\tau_{0} C^{d p}\right]^{+}} & \text {if } p_{0}=p_{\text {shal. }}\end{cases} \\
& I_{2}^{d p}\left(p_{1}\right)= \begin{cases}{\left[\max \left[I_{1}^{d p} ; Q_{\text {deep }}^{*}\right]-\tau_{1} C^{d p}\right]^{+}} & \text {if } p_{1}=p_{\text {deep }} \\
{\left[\max \left[I_{1}^{d p} ; Q_{\text {shal. }}^{*}\right]-\tau_{1} C^{d p}\right]^{+}} & \text {if } p_{1}=p_{\text {shal. }}\end{cases} \tag{4.17}
\end{align*}
$$

Let $D_{p, t}$ be the set of all possible states of demand from the deal-prone segment for all retail prices $p$ over all promotion periods $t$. When no promotion is planned, demand from the deal-prone segment equals zero. The set $D_{p, t}$ is characterized by the combinations of promotion periods $t$ and possible retail prices $p_{t}$,


Fig. 4.12 Example of a 2-promotion plan over a planning horizon

$$
\begin{equation*}
D_{p, t}=\left\{\left(D_{p_{\text {dee } p}, 1}, D_{p_{\text {shallow }, 1}}\right), \ldots,\left(D_{p_{\text {deep }, T}}, D_{p_{\text {shallow }, T}}\right)\right\} \tag{4.18}
\end{equation*}
$$

where $p=\left\{\left(p_{\text {deep }}, p_{\text {regular }}\right)\right\}$ is the set of prices and where $t=\left\{t_{1}, t_{2}, \ldots, T\right\}$ is the set of periods.
Demand from the deal-prone segment equals the optimal order order-up-to quantity, given the retail price $p_{t}$, less leftover inventories from the last period $I_{1}^{d p}$. Demand remains positive and is therefore capped at zero. There is no backlogging. In the case of two promotion periods within the planning cycle, we find a limited number of realizations of demand,

$$
\begin{equation*}
D_{t}=\left\{D_{p_{\text {dee }, t}}, D_{p_{\text {shal. }, t}}\right\}=\left\{\left[Q_{\text {deep }}^{*}-I_{t}^{d p}\right]^{+},\left[Q_{\text {shal. }}^{*}-I_{t}^{d p}\right]^{+}\right\} \tag{4.19}
\end{equation*}
$$

In particular, for $t \in\{1,2\}$,

$$
D_{1}= \begin{cases}{\left[Q_{\text {deep }}^{*}-I_{1}^{d p}\right]^{+}} & \text {if } p_{1}=p_{\text {deep }}  \tag{4.20}\\ {\left[Q_{\text {shal. }}^{*}-I_{1}^{d p}\right]^{+}} & \text {if } p_{1}=p_{\text {shal }} .\end{cases}
$$



Fig. 4.13 Structure of a planning period with two promotions

$$
D_{2}= \begin{cases}{\left[Q_{\text {deep }}^{*}-I_{2}^{d p}\left(p_{\text {deep }}\right)\right]^{+}} & \text {if } p_{1}=p_{\text {deep }} \text { and } p_{2}=p_{\text {deep }}  \tag{4.21}\\ {\left[Q_{\text {shal. }}^{*}-I_{2}^{d p}\left(p_{\text {deep }}\right)\right]^{+}} & \text {if } p_{1}=p_{\text {deep }} \text { and } p_{2}=p_{\text {shal }} \\ {\left[Q_{\text {deep }}^{*}-I_{2}^{d p}\left(p_{\text {shal. }}\right)\right]^{+}} & \text {if } p_{1}=p_{\text {shal } .} \text { and } p_{2}=p_{\text {deep }} \\ {\left[Q_{\text {shal. }}^{*}-I_{2}^{d p}\left(p_{\text {shal. }}\right)\right]^{+}} & \text {if } p_{1}=p_{\text {shal } .} \text { and } p_{2}=p_{\text {shal }} .\end{cases}
$$

Total demand over both periods is calculated as the sum of $D_{1}$ and $D_{2}$. Demand is a function of historic retail prices and an initial inventory level. The probability that demand equals $x$ at a point in time $t$ is $P\left\{D_{t}=x\right\}$. We denote the probability of a deep promotion with $\varepsilon$. The probability of a shallow promotion is written as $1-\varepsilon$. As promotions are scheduled and communicated to the supplier, the probability, that there is no promotion in a promotion period $t$ is zero.

$$
\begin{array}{r}
P\left\{D_{t}=\left[Q_{\text {deep }}^{*}-I_{t}^{d p}\right]^{+}\right\}=\varepsilon  \tag{4.22}\\
P\left\{D_{t}=\left[Q_{\text {shallow }}^{*}-I_{t}^{d p}\right]^{+}\right\}=1-\varepsilon
\end{array}
$$

The distributions of demand in both promotion periods follow immediately. In $t=1$, demand exclusively depends on the retail price. The random promotion pricing leads to a distribution of demand. In $t=2$, four possible states of demand can be reached. These represent all possible combinations of the promotions prices in $t=1$ and $t=2$. The price in $t=1$ has an impact on demand in $t=2$, as it affects consumers' inventory levels in $t=2$. Figure 4.13 shows the structure of a planning period with two promotions.


Fig. 4.14 Source and description of the data set (Source: adapted from Wiehenbrauk 2010, p. 118)

$$
\begin{gather*}
P\left\{D_{1}=\left[Q_{\text {deep }}^{*}-I_{1}^{d p}\right]^{+}\right\}=P\left(p_{1}=p_{\text {deep }}\right)=\varepsilon  \tag{4.23}\\
P\left\{D_{1}=\left[Q_{\text {shal. }}^{*}-I_{1}^{d p}\right]^{+}\right\}=P\left(p_{1}=p_{\text {shal. }}\right)=1-\varepsilon \\
P\left\{D_{2}=\left[Q_{\text {deep }}^{*}-I_{2}^{d p}\left(p_{\text {deep }}\right)\right]^{+}\right\}=P\left(p_{1}=p_{\text {deep }}, p_{2}=p_{\text {deep }}\right)=\varepsilon^{2} \\
P\left\{D_{2}=\left[Q_{\text {shal. }}^{*}-I_{2}^{d p}\left(p_{\text {deep }}\right)\right]^{+}\right\}=P\left(p_{1}=p_{\text {deep }}, p_{2}=p_{\text {shal. }}\right)=\varepsilon(1-\varepsilon)  \tag{4.24}\\
P\left\{D_{2}=\left[Q_{\text {deep }}^{*}-I_{2}^{d p}\left(p_{\text {shal. }}\right)\right]^{+}\right\}=P\left(p_{1}=p_{\text {shal. }}, p_{2}=p_{\text {deep }}\right)=\varepsilon(1-\varepsilon) \\
P\left\{D_{2}=\left[Q_{\text {shal. }}^{*}-I_{2}^{d p}\left(p_{\text {shal. }}\right)\right]^{+}\right\}=P\left(p_{1}=p_{\text {shal. }}, p_{2}=p_{\text {shal. }}\right)=(1-\varepsilon)^{2}
\end{gather*}
$$

### 4.4 Empirical analysis of price promotions in the diapers market

Consumers' purchase behavior during price promotions depends on their willingness to stockpile. We support the demand model derived with an empirical analysis based on point-of-sales data from the diapers category. The data set provides consolidated sales and price information of the Pampers Baby Dry category for a major supermarket chain in Germany. Consumers can be expected to make well-informed decisions about shopping for diapers, as this category requires a significant portion of the budget spent on consumer goods. Therefore, we expect to see stockpiling behavior. We show that the forecast model provides good fit on chain level data. In the first step, we describe the data set and review the promotion patterns of the retail chain in question. Further, we estimate the necessary parameters and combine those with industry information to generate a forecast according to the two-segment model of promotional demand. We review the accuracy of the forecast.

### 4.4.1 The data set

The data was first published by Wiehenbrauk (2010). It provides insights into consumers' reactions to price promotions. Point-of-sales data allows a direct view on consumers' purchase decisions. We describe the data set and review the characteristics of the category analyzed. The key information contained in the data set is retail prices and sales levels. We review these parameters and search for patterns related to promotions.

### 4.4.1.1 Description of the data set and the category analyzed

The data set includes weekly aggregated sales information for diapers at all locations of a major German supermarket chain. The data set covers the time interval from the middle of 2002 to the end of 2004. It provides information on sales volumes, sales values and pricing. The product reviewed is Pampers Baby Dry, a known diaper brand. In total, the data set contains 121 data points, where each data point is a combination of a retail price and an aggregated sales volume over all stores in one week. Figure 4.14 provides an overview of the dataset.

The retailer sells the product in three package sizes: Value, Jumbo and Mega. In addition, diapers are sold in a number of variants for children of different ages. As variants are differently sized, the number of diapers in one pack is not constant over all variants. Consequently, the number of diapers per pack varies from 40-52 units in case of the Value Pack to 160-208 units in case of the Mega Pack (Wiehenbrauk 2010). We employ statistical units (SU) consisting of 180 diapers each to uniformly base volumes. We do not consider the impact of switching between package sizes in our analysis. Please consider Huchzermeier et al. (2002) for a discussion of the effect of package sizes on stockpiling.

Demand for diapers depends on the number of children requiring diapers in the market. The time span a child requires diapers varies and so does the seasonal birth rate. Still, the category is characterized by rather stable demand.

Diapers and Pampers Baby Dry in particular are a suitable product for the empirical validation of the model for several reasons. First, diapers are non-perishable and can be stored. This is a prerequisite for consumer stockpiling. Household inventories of diapers are reasonable, as the product is wrapped in plastic and can be easily stored.

Second, consumers need to purchase diapers frequently. Some consumers are willing to align their shopping plans to promotion schedules of diapers. Therefore, diapers are generally recognized to be a traffic generating product. Consumers recognize that diapers require a relevant portion of their budget and thus behave strategically. As a result, a willingness to stockpile can be expected from a significant number of consumers.

Third, diapers sales should not be affected by consumption effects. Larger household inventories should not increase the number of diapers used per day. The absence of a consumption effect ensures that deal-prone consumers' consumption rates are independent of household inventories. Consequently, consumption rates can be estimated as a constant parameter.

Forth, Pampers diapers have an important market share of 73 percent on average at the retail chain in question (Wiehenbrauk 2010, p. 120). Pampers Baby Dry is again the most important product in the Pampers family. This limits the effects of brand and product switching effects, which are not considered in the two-segment model derived.


Fig. 4.15 Frequency of demand (in units sold)

### 4.4.1.2 Patterns of retail prices and sales

Diapers are one of the more expensive fast moving consumer goods. The regular retail price is about 0.23 Euro per diaper, where large package sizes contain up to 208 diapers. Retail prices change frequently over time. Usually, the retail chain offers discounts every third week; however, retail prices sometimes diverge from this pattern. Shallow discounts are regularly about 0.04 Euro per diaper, resulting in a shallow promotion price of 0.19 Euro per diaper (34.18 Euro per statistical unit). Deep promotions are less frequent than shallow promotions. Diapers cost only about 0.18 Euro per diaper during deep promotions (32.04 Euro per statistical unit). In addition, there are two extremely deep promotions in the period of time covered by our data set. Retail prices drop to 0.16 and 0.10 Euro per diaper respectively during these promotions. The latter was a special two-for one offer (Skarka 2003). In our analysis, we focus on shallow and deep price promotions.

Sales in terms of both, units and revenues, show a characteristic promotion pattern. Over all weeks, base sales are rather constant. In addition, there are frequent sales spikes, lasting each for one week with few exceptions. We analyze the relationship of sales and discounts in the next section. A frequency chart of sales, as provided in Figure 4.15 shows that the retailer sells the base load in about 65 percent of all weeks. There is a clear separation between regular and promotion sales levels. The probability of selling a quantity in-between the two levels is very low.

### 4.4.2 Estimating the two-segment model of promotional demand

Several researchers successfully applied the two-segment model of promotional demand to data sets (Blattberg et al. 1981; Iyer and Ye 2000; Freiheit 2001; Huchz-


Fig. 4.16 Regular, subsidized and additional sales volumes
ermeier et al. 2002). Contract design requires order-up-to levels derived from the two-segment model. Therefore, we estimate the model parameters and generate a forecast. We compare the forecast to realized sales and show, that the two-segment model provides a good foundation for further analyses.

### 4.4.2.1 Fitting the model

We employ the data set to estimate the required model parameters. The two-segment model of promotional demand requires sizes and consumption rates of both consumer segments, as well as holding costs and reservation prices of the deal-prone segment. We multiply the number of consumes in each segment with the respective consumption rate to derive the overall consumption of each segment. The overall consumption rate of the loyal segment, is part of the data set, as sales to loyal consumers equal the amount sold at the regular retail price. The loyal segment consumes on average 4,773 units per period. This quantity is quite stable over time, with a standard deviation of 23 percent of mean sales at prices close to the regular price. Deal prone consumers' per period consumption, their holding costs and their reservation prices are subject to estimation in the next step.

Retail prices for each period constitute the input of the two-segment model. The output of the model is a forecast of demand for each period, depending on retail prices. The fitting procedure employed minimizes the quadratic differences between the calculated value of sales given a retail price and the actual outcome. The quadratic weight of error terms requires the fitting algorithm to prefer solutions, that follow demand spikes closely. Large deviations between the output of the model and the realization of demand are punished disproportionaly.

In weeks 50/2002 to 51/2002 an extremely deep promotion over two weeks occurs (promotion A in Figure 4.17). In weeks 19/2003 to 24/2003, an extreme discount potentially influences consumers' behavior (promotion B in Figure 4.17). During these extreme promotions, brand and store switching effects are likely to
be important in scale and may lead to suboptimal parameter estimations. In order to calibrate the model to the usual retail price space, we leave out these two promotions for purposes of fitting. That is, we assign zero value to deviations of the forecasted values from realized sales for these weeks. In total we employ 114 out of 121 weeks of data to determine the required model parameters. We present the performance of the forecast derived for i) all data points and ii) the limited set of data excluding extreme promotions.

We derive a consumption rate of 13,784 units per period for the deal-prone segment. Comparing deal-prone and loyal consumption rates provides an idea of the relative importance of deal-prone consumers. The estimation of deal-prone consumers' holding costs is 2.06 Euro per statistical unit. This equals 5 percent of their reservation price for this product, which is estimated to be 38.90 Euro per statistical unit per week. Prices above this threshold will not trigger additional demand in forecasts generated by the model. The fitted model explains $R^{2}=0.891$ of all variability of sales in the data set. When leaving out the two extreme promotions mentioned, we receive a very good explanatory power of $R^{2}=0.951$.

### 4.4.2.2 Evaluation of results

Price promotions are important for the diapers category at the retail chain in question. In total 84 percent of all units are sold during promotions. Only 16 percent of total volumes are sold at a price above 38.90 Euro per statistical unit ( $\approx 0.22$ Euro per diaper), which is the price level estimated to be the reservation price of the deal-prone segment. Moreover, at least 9 percentage points out of the 84 percent of volume, which are sold during promotions, can be characterized as subsidized sales to loyal consumers (Figure 4.16). Given an average weighted regular price of 41.08 Euro per statistical unit ( $\approx 0.23$ Euro per diaper) and an average weighted promotion price of 32.06 Euro per statistical unit ( $\approx 0.18$ Euro per diaper), the retail chain incurs an average loss of 9.02 Euro on every statistical unit sold to loyal consumers during promotions. This loss is due to being unable to discriminate between loyal and deal-prone consumers, when offering discounts.

In the next step, we review the data set for evidences of stockpiling. We expect that deep promotions and short intervals between promotions negatively affect demand at the next promotion. All else being equal, including the retail price, demand spikes are expected to be lower, when one of these two factors applies. Figure 4.17 shows retail prices, forecasts and realized demand over the full two years of data available. With few exceptions, which we discuss later, the forecast predicts realized demand very well.

## Impact of deep promotions on stockpiling

There are five equally deep discounts in weeks 02/2003 (promotion D in Figure 4.17), $05 / 2003,08 / 2003,11 / 2003$ and 14/2003. Time intervals between promotions


Fig. 4.17 Forecast and realized sales volumes of Pampers Baby Dry at the retail chain (Source: sales volumes and prices based on Wiehenbrauk 2010; forecast based on own calculations)
are constantly two weeks of regular pricing before each of the five promotions. However, the promotion before the first in the row of five, was a deep promotion. We see that consumers build up more inventory than usually and still have left-over inventories in week $02 / 2003$. As a result, consumers purchase less at promotion D than at each of the other four promotions, although the retail price is the same. We see further evidences of stockpiling in week 22/2004 (promotion F) and in week 32/2004 (promotion G).

## Impact of short intervals between promotions on stockpiling

Two promotions follow each other directly in weeks 44/2002 and 45/2002 (promotion C). As there is no period of regular pricing between these two promotions, consumers still hold large inventories and do not react to the second promotion. In week 49/2003 (promotion E), two deep promotions follow each other with only one week of regular pricing in between. As expected, demand in the second promotion is lower than in the first promotion. In week 48/2004 (promotion H) a deep promotion follows tightly after a shallow promotion. Here, consumers purchased an unexpectedly high quantity during the promotion in week $46 / 2004$ and thus they have more inventory at hand than calculated by the forecast. As a result, we see a stronger stockpiling effect than expected for this promotion.

## Evaluation of model performance

To evaluate the value-added of i) the two-segment and ii) the consumer inventory component of our model, we consider two reference forecasts as benchmarks. First consider a simple forecast of the form $D=\beta_{0}+\beta_{1} \cdot p_{t}$, where $\beta_{0}$ and $\beta_{1}$ are fitted employing the data set. The forecast explains $R^{2}=0.82$ of variation in demand for $\beta_{0}=178,796$ and $\beta_{1}=-4,233$. When we do not consider the extreme promotions B and C, we receive an explanatory value of $R^{2}=0.86$. The simplified model is worse in explaining demand than the two-segment forecast.

Second, we model a two-segment forecast without carry over of inventories. In contrast to our final model, leftover inventories are not considered, i.e.,

$$
\begin{equation*}
D=\left[\frac{\left(u-p_{t}\right) C^{d p}}{h^{d p}}\right]^{+}+D_{\text {loyal }} \tag{4.25}
\end{equation*}
$$

Again, we leave out the two extreme promotions A and B and we receive $R^{2}=$ 0.922 for the simplified model (as compared to $R^{2}=0.951$ for the inventory model). The simplified model is worse in explaining sales. Thus, the household inventory perspective adds value and reduces forecast errors by more than one third compared to the two-segment model without inventory carry-overs in the case of Pampers Baby Dry at the chain reviewed.

In the next step, we consider all data points including the two-for-one special promotion. The simplified model explains $R^{2}=0.904$ of all variability of sales in the data set (as compared to the inventory model with $R^{2}=0.891$ ). The model without inventory is minimally better in explaining demand. Important store switching effects could be an explanation for this phenomenon.

In general, the stockpiling component of the forecast provides a value-added to the accuracy of the forecast. Comparing the three forecast models shows that a two-segment forecast with inventory holding reduces forecast errors by 64 percent compared to the simple regression of demand on retail prices for the data set considered (Figure 4.18). Given that a promotion period contains multiple promotions, this improvement of forecast accuracy has major effect on inventory requirements.

### 4.4.2 3 Implementation

Forecasting is a trade-off between the costs of gathering information and the accuracy achieved. Incorporating brand and store switching behavior would require collecting real-time data on pricing of all competitors of the retailer and on all substitutes of the brand. Given the excellent fit of the model presented, this effort is not justified for the diapers category at the retail chain in question. The data requirement of the forecast presented is limited to the knowledge of the retailer's own past pricing policy for the product.


Fig. 4.18 The value of the two-segment forecast in terms of errors

Our analysis is based on chain level data. Moreover, Huchzermeier et al. (2002) showed for a store level data set from the diapers category that the two-segment model provides very good fit.

The two-segment forecast still shows a minor or in many cases even negligible forecast error. This forecast error is measurable and can be controlled. The retailer has two choices. First, she can add safety stock in the amount of the remaining forecast error, possibly resulting in end-of-period coverage. Second, she can hedge against residual forecast errors employing additional option contracts and thus avoid any end-of-period coverage. When the retailer does not account for residual forecast errors, she will incur limited stockouts in some unexpectedly high cases of demand.

## Chapter 5 <br> Hedging Retail Promotions

We analyzed coordinating forward and option contracts and we modeled demand forecasts in a price promotion environment. We found that even with a two-segment forecast, there is residual demand risk, which cannot be eliminated. In this chapter, we derive a portfolio of coordinating contracts that hedges against demand risk resulting from price promotions. First, we set up the model layout. Second, we establish the retailer's optimization problem consisting of i) the execution policy and ii) the reservation policy. Third, we discuss the manufacturer's optimization problem, which determines the production policy.

### 5.1 Model layout

We model a supply chain consisting of a retailer and a manufacturer. The setup is closely related to Chapter 3; however, here, we consider multiple promotions within one planning period.

### 5.1.1 Consumer and market dynamics

Consider a market for a single nonperishable FMCG product. The product has no close substitutes and is consumed frequently. We follow the lead of Huchzermeier et al. (2002) and choose diapers as a representative category. We consider two companies in a supply chain relationship. The supplier produces a single product. The retailer serves the market for this product. She runs price promotions to compete in the market. The retailer chooses prices randomly to be unpredictable as described by Wiehenbrauk (2010).

There is a large number of consumers in the market. Promotional demand is assumed to be triggered by consumers' stockpiling behavior. Taking a single product and store perspective, the model does not consider brand and store switching.

Huchzermeier et al. (2002) showed very good fit applying the two-segment model with a single brand and store perspective on diapers retailing. In Chapter 4 we presented additional empirical evidence generated with a FMCG point-of-sales data set at the chain level. Consumers make purchasing decisions according to i) their willingness to hold inventory and ii) the retail price offered by the retailer. We divide consumers into two segments according to their willingness to stockpile, as described in Chapter 4. Deal-prone consumers hold household inventories and trade-off holding costs with price discounts. This consumer segment triggers demand spikes during price promotions. The magnitude of these demand spikes depends on the level of household inventory and the retail price. In contrast, loyal consumers are not willing to hold inventories and purchase equal amounts of the product at each point in time.

Assumption 5.1 There are two segments of consumers that differ in their willingness to hold inventory. Stockpiling is a major source of promotional demand for the diapers category at the retail chain modeled.

### 5.1.2 Timing of promotions

The retailer's marketing department creates a (year long) promotion plan jointly with the manufacturer's key account managers. This plan specifies the timing of promotions within the planning horizon, but leaves open promotion prices to ensure that the promotion strategy is not predictable by competitors (see Chapter 4). For purchasing purposes, the year-long plan is divided into shorter planning periods. Each of these planning periods contains $N$ promotions in $T$ units of time. Figure 5.1 shows the time line of a planning period. Each point in time $t_{n}$ represents one promotion, such that promotions take place at time $t_{n} \in\left\{t_{1}, t_{2}, \ldots t_{N}\right\}$. Let $t_{0}$ be the starting date of the planning period and let $T$ be its end. Intervals between promotions are denoted $\tau$. $\tau_{n}$ is the interval of time between the beginning of the $n^{t h}$ promotion in week $t_{n}$ and the beginning of the $(n+1)^{s t}$ promotion in week $t_{n+1}$, thus $\tau_{n}=t_{n+1}-t_{n}$. Time intervals between promotions can be different from promotion to promotion. All $\tau_{i}$ contain a random number of units of time and their lengths are always a multiple of weeks. In the diapers category, a promotion lasts typically one week and there are multiple promotions each quarter. The average time between promotions is two to three weeks of regular pricing (compare Figure 4.17 in Chapter 4).

Promotion items change periodically to retain consumer's attention. The retailer does not sell excess "outdated" products after time $T$. Therefore, end-of-period coverage is an important issue for FMCG retailers. Selling "outdated" items must be avoided and thus, any left-over inventory in the supply chain must be salvaged at time $T$.

Demand forecasts are jointly developed by the retailer and the manufacturer before $t_{0}$. The forecast is updated at each promotion $t_{n}$ when the promotion price is


Fig. 5.1 The planning horizon
realized. Consequently, we assume that both parties share equal believes about demand at all time.

### 5.1.3 Contracting and fulfillment processes

In the diapers industry, manufacturers ask for preliminary orders, for example, three months in advance, as we learned in an interview with a senior manager at a leading FMCG company. They do so, as longer lead-times reduce the costs of production, because fewer disruptions disturb production processes. Thus, the manufacturer's production costs are lead-time dependent.

The retailer maximizes profit. She earns revenues from sales to consumers and she incurs purchasing costs. In addition, she pays holding costs when carrying inventory. Thus, she has an incentive to require flexibility from the manufacturer. Promotion plans convey information on the timing of promotions, but not on the quantities required by the retailer. As demand is uncertain, the retailer requires that orders can still be adapted at no charge until the products are delivered. We model this relationship as instant order contracts, where orders are confirmed only shortly before a promotion starts.

The manufacturer maximizes profit from selling products to the retailer. He pays holding costs for unsold inventories and he incurs additional costs for short-time production (production with short lead-time, compare Chapter 3). Thus, the manufacturer has an incentive to provide less flexibility. Under an instant order (pull) contract scheme, the manufacturer incurs a maximum of production and holding
cost per unit sold to the retailer, given that a high service level is required by the retailer. Under a forward contract, the manufacturer serves demand at the minimal cost and never holds leftover inventory.

Clearly, positions on the optimal distribution of risk diverge. Each party would like to minimize their risk exposure. The actual distribution of risk depends on the bargaining power of both parties. In the FMCG industry, the retailer has more bargaining power in general, as she can threaten to delist products of the manufacturer. She exploits this advantage by demanding maximum flexibility while maintaining a high required minimum service level. Thus, the retailer enforces a wholesale price scheme in pull mode with retailer power (see Chapter 3). Uncertainty regarding the retailer's orders is resolved only shortly before delivery is due. The pull scheme assigns all inventory risk and all cost of short-term production to the manufacturer.

Assumption 5.2 The retailer has an advantageous bargaining position. The manufacturer must accept serving last minute spot orders and must maintain a high predefined service level for all types of contracts including instant order contracts.

The retailer's power in the supply chain is limited by the minimum margin requirement of the manufacturer. The manufacturer will exercise his option to leave the market, if the retailer does not provide him with the minimum of profit demanded. Both the manufacturer and the retailer will never accept a contract which does not improve his or her own position compared to the status quo. Consequently, only Pareto improving contracts are feasible in practice.

### 5.1.4 Supply contracts

The choice of supply contracts offered determines the ability to reach channel coordination, as shown in Chapter 3. In a multi-period setting, additionally the timing of contracts is relevant.

### 5.1.4.1 Contract schemes available

The manufacturer offers three types of contracts, namely

- Instant orders / spot contracts - last minute orders placed shortly before the beginning of the targeted promotion
- Forward contracts - long-term commitments placed in advance
- Option contracts - flexible contracts reserved one or more periods ahead for execution shortly before the beginning of the targeted promotion

The retailer employs the different contract types to construct an optimal portfolio of supply contracts. Each type of contract has different characteristics in terms of flexibility (see Chapter 3). In the multi-period model discussed in this section, different lead-times and maturities allow for additional degrees of freedom.


Fig. 5.2 Three different contract schemes for forwards and options

### 5.1.4.2 Timing of contract schemes

The retailer purchases options and forwards for delivery at a future point in time. One can imagine different contract schemes: First, contracts can be restricted to be purchased only upfront at time $t_{-1}$, where $t_{-1}$ is the date of the last promotion before the planning horizon. Contracting at this point in time is reasonable, as no more information is revealed until the first promotion within the planning horizon. Second, contracts can be restricted to execution at the next promotion at time $t_{n+1}$. Then, contracts are short-term hedging instruments. Third, contracts can be purchased at any point in time for any execution date. Different schemes may apply to forward and option contracts. There could also be different schemes within one contract class. Let $q_{i j}$ be purchasing quantities ordered at time $i$ for delivery at time $j$. Figure 5.2 provides an overview of the three different approaches to time both order and execution dates. The unrestricted scheme allows for additional reservation and execution combinations compared to the other two schemes. We model the unrestricted scheme to achieve generalizability.


Fig. 5.3 Structure of risk in the two-period game - realizations of the retail price

### 5.1.5 Modeling demand forecasts

We derived a two-segment model for promotional demand in Chapter 4. We found that demand during promotions depends on both retail prices and left-over inventories at consumers' homes. In practice, retailers and manufacturers employ regressions on price and scenario models to forecast demand. We compare these approaches to the multi-segment forecast.

### 5.1.5.1 The single promotion view of myopic retailers and manufacturers: regressions on price

A common practice to forecast demand is employing a simple regression on retail prices. The estimation provides a continuous distribution of demand, e.g., a normal distribution as shown in Figure 5.3, Forecast A. The forecast predicts demand at a single point in time and does not consider interdependence between price promotions. It is purely based on historic data ("historic forecast") and does not take into consideration any promotion specific information. Given that promotion pricing is random and that there is no advance price information, a regression on retail prices provides a distribution of demand that is equal for all promotions.

### 5.1.5.2 The demand scenario view of price promotions: price based scenarios

Developing price based scenarios provides further insights into consumer demand. When isolating individual historic promotions according to the retail price, a discrete demand function can be derived, as the number of demand realizations per retail price is limited. However, without knowledge of the mediating function of inventories, promotions within the planning horizon do not add additional information to the forecast. Thus, given a random pricing policy, Forecast B is not updated during the planning period.

### 5.1.5.3 The multi-segment view of price promotions: household inventory model

In reality, demand is path dependent as discussed in Chapter 4. Household inventories explain, why a deep promotion can result in less demand than a shallow promotion at different points in time. Under a multi-segment forecast, knowledge of the retail prices at past promotions provides information on consumer inventories and thus on the distribution of demand at the next promotion (Forecast C).

Without a two-segment forecast this update does not take place. Consequently, in the status quo, both players face a maximum of uncertainty until uncertainty is resolved at the beginning of the promotion to be forecasted. Let a retail price path be the sequence of realized retail prices in a planning period. Consider the case that there are two promotions within the forecast horizon. Then, there are four possible retail price paths: two deep or two shallow promotions could follow each other or one deep and one shallow promotion could occur in either order. We map the discrete cumulative distribution function in Figure 5.4 and contrast it to a continuous cumulative distribution function, as employed in the historic forecast. The forecast shown bases on the point-of-sales data discussed in Chapter 4. It predicts sales of the Pampers Baby Dry diapers brand at a major German supermarket chain. Note that the discrete function explains as much as 95 percent of variance in demand


Fig. 5.4 Actual and perceived cumulative distributions of demand
to perfection (see Chapter 4). Consequently, we focus on the discrete two-segment forecast for the further analysis.

### 5.1.6 Notation

Let $t=i$ be the point in time of the current promotion, let $t=k$ be the date of a past promotion and let $t=j$ be the date of a future promotion, where $i, j, k \in$ $\left\{t_{-1}, t_{0}, t_{1}, t_{2}, \ldots, t_{N}, T\right\}$. Contracts are either initiated at time $t=i$ for delivery at a future point in time $t=j$, where $i \leq j$, or have been initiated at a past point in time $t=k$, where $k<i$ and are delivered at $t=i$ or a future point in time $t=j$.

In the previous section, we defined a retail price path as the set of realized retail prices over the planning period. At the time of forecasting, a number of retail price paths is possible. Let $z_{i}$ be the state of the world reached at time $i$. The state of the world contains information on the current and on all past retail prices given the node in the retail price path reached. We elaborate on the set of reachable states of the world in Section 5.2.1, when discussing the structure of the retailer's optimization problem. In this notation, e.g., $q_{i j}^{f}\left(z_{i}\right)$ denotes the amount of forward purchases contracted in the current node $z_{i}$ in $t=i$ for delivery at a future point in time $t=j$.

Let $\delta$ be the period of time between order and maturity of a contract. In the case of forward contracts, maturity is the point in time when the manufacturer delivers the products ordered. In the case of options, maturity is the point in time when the retailer decides on the execution of the quantities reserved. The manufacturer then delivers instantly. For spot orders, maturity equals the point in time when the order


Channel Coordinating
Contract Parameters
Fig. 5.5 Sequel of decisions
is placed. We define $\delta_{i j}$ as the period of time between $t=i$ and $t=j$, where $j$ is a point in time in the future. Analogously, $\delta_{k i}$ stands for the period of time between $t=k$ and $t=i$, where $k$ is a point in time in the past.

$$
\begin{equation*}
\delta_{i j}=j-i \quad \delta_{k i}=i-k \tag{5.1}
\end{equation*}
$$

Table 5.1 summarizes the notation employed in this chapter. Note that demand, all purchase quantities, lost sales and inventories are state dependent. All contract parameters are not state dependent, as they are negotiated in advance. Still, they can be time dependent.

### 5.1.7 Direction of analysis

Contracting and fulfillment processes start with a negotiation on contract parameters or a contract offer by the manufacturer. This step is unique in that it is performed upfront and only once. The following steps are repeated for every promotion in the forecast horizon: 1) The retailer orders inflexible forward inventory and reserves option contracts in advance with the required lead-time. 2) Shortly before the promotion begins, the retailer randomly determines the retail price and decides on her optimal order and execution quantities of flexible inventories. 3) The manufacturer anticipates instant orders as well as option executions. She produces and delivers the orders to the retailer. Instant orders and option executions are served using flexible capacities and speculative inventories. The manufacturer will not serve all instant orders, if he is not required to do so and if stockouts are profit maximizing. 4) During the promotion, consumers purchase the product. Purchase quantities from the deal-prone segment depend on both the retail price and remaining household inventories.

Figure 5.5 shows the layout of decisions and the respective programs employed to solve for optimal policies. The retailer orders optimal quantities, given the contract parameters negotiated. Then, the manufacturer receives orders and decides on his optimal production policy. Contract pricing is based on the closed form solutions derived in Chapter 3. Salvage costs are reduced to holding costs for all but the last

Table 5.1 Notation employed in Chapter 5

| Market environment |  |
| :---: | :---: |
| $D_{i}\left(z_{i}\right)$ | demand in state $z_{i}$ |
| $z_{i}$ | state of the world defined by current and past retail prices |
| $p_{i}\left(z_{i}\right)$ | retail price in state $z_{i}$ |
| $\pi_{z_{i}}$ | probability to reach state $z_{i}$ |
| $\delta_{k i}$ | time interval between date $k$ and date $i$ |
| $\delta_{i j}$ | time interval between date $i$ and date $j$ |
| $\tau_{i}$ | time interval after time $i$ until the beginning of the next promotion |
| Contract pricing and production costs |  |
| $r\left(\delta_{i j}\right)$ | reservation fee at time $i$ for options with maturity at time $j$ |
| $e\left(\delta_{k i}\right)$ | execution fee at time $i$ for options purchased at time $k$ |
| $f\left(\delta_{i j}\right)$ | forward price at time $i$ for forwards with maturity at time $j$ |
| $s_{i}$ | spot price at time $i$ |
| $c\left(\delta_{i j}\right)$ | production costs for one unit of the product with lead-time $\delta_{i j}$ |
| $c\left(\delta_{i i=0}\right)$ | production costs for last minute-production with zero lead-time |
| Contracting and production quantities (decision variables) |  |
| $q_{i}^{s}\left(z_{i}\right)$ | quantity of spot contracts purchased in state $z_{i}$ |
| $q_{i j}^{r}\left(z_{i}\right)$ | quantity of options reserved in state $z_{i}$ with maturity at time $j$ |
| $q_{k i}^{e}\left(z_{i}\right)$ | quantity of options purchased at time $k$ and executed in state $z_{i}$ |
| $q_{i j}^{f}\left(z_{i}\right)$ | quantity of forwards purchased in state $z_{i}$ with maturity at time $j$ |
| $q_{i p}^{p}\left(z_{i}\right)$ | amount of production in state $z_{i}$ for immediate delivery |
| $q_{i j}^{p}\left(z_{i}\right)$ | amount of production started in state $z_{i}$ for delivery at time $j$ |
| $G_{i}\left(z_{i}\right)$ | lost sales at the retailer level in state $z_{i}$ |
| $K_{i}\left(z_{i}\right)$ | instant orders not served in state $z_{i}$ |
| Inventory positions and associated costs |  |
| $I_{i}\left(z_{i}\right)$ | retailer inventory at time $i$ in state $z_{i}$ |
| $J_{i}\left(z_{i}\right)$ | manufacturer inventory at time $i$ in state $z_{i}$ |
| $g$ | the retailer's cost of lost goodwill per unit of lost sales |
| $h$ | holding costs per unit of excess inventory for both parties |
| $v_{T}$ | salvage value per unit of inventory at time $T$ |
| $q^{\text {req }}$. | minimum required total quantity that must be served by the manufacturer |
| Objective functions |  |
| $\Pi_{m}$ | expected manufacturer profit |
| $\Pi_{r}$ | expected retailer profit |
| Other functions |  |
| $R(x>y)$ | indicator function that equals 1 for $x>y$ and 0 otherwise |

promotion as the game is repeated several times. The closed form solutions can be employed to derive coordinating contract parameters for the cases described. For general cases, a numerical optimization approach can be required.

The aim of the following model is to derive a portfolio of hedging contracts that redistributes risk and increases profit of both the retailer and the manufacturer. Then, players are no longer in a wholesale price world, but they can improve their situation by reducing inefficiencies in the supply chain. We follow the order of events in the discussion of both players' optimization problems. First, we discuss the retailer's purchasing and execution problems. Orders are required before the manufacturer can derive her optimal production policy in the second step.

### 5.2 The retailer's optimization problem

The retailer purchases products to serve uncertain consumer demand. She makes i) reservation and ii) execution decisions to maximize her profit. We review the structure of her optimization problem, derive a general objective function and present a stochastic program. This program can be implemented by retailers to optimize purchasing decisions, based on the two-segment forecast. We conclude this section with a discussion of the structure of the solution to the stochastic program.

### 5.2.1 Structure of the retailer's optimization problem

The retailer's objective is profit maximization, where profit depends on both her decisions and uncertain demand. The retailer faces an optimization problem with two components:

- execution decision - which types of available contracts should be employed to serve demand
- reservation policy - how many contracts should be reserved and/or committed in advance

The execution decision constitutes a profit maximization problem, where the cheapest contracts are employed first to serve demand. The reservation decision is a trade-off between lower purchase costs and the risk of excess inventory after the sales period.

The retailer purchases supply contracts for all $N$ promotions within the planning horizon. Shortly before each promotion, the retail price is determined randomly. Given history and the current promotion price, the retailer observes demand and makes contracting decisions to serve consumers. She employs a two-segment model of consumer demand. Promotion leaflets are printed shortly before demand realizes. Thus, there is sufficient time for the retailer to take advantage of her knowledge of the retail price. Spot purchases and option executions occur after the information


Fig. 5.6 The possible retail price paths over a long forecast horizon
update, but before the beginning of the promotion. The following list provides an overview of important steps in the time line:

- $t_{-1}$ - upfront contracting
- $t_{0}$ - beginning of the planning horizon
- $t_{1}$ - first promotion
- $t_{n}-n^{\text {th }}$ promotion in the planning horizon
- $t_{N}$ - final promotion in the planning horizon
- $T$ - end of the planning horizon

Initial forward and option contracts are closed at time $t_{-1}$, before the planning period starts. Additional option and forward contracts can be purchased after every information update in the planning horizon. Spot orders can be placed before every promotion, after uncertainty is resolved.

At each point in time, where a promotion is scheduled, two promotion prices are possible. Promotions are committed. Thus, the regular price is only charged during the intervals $\tau_{n}$, but never at scheduled promotion dates. Figure 5.6 shows the resulting pricing tree. The pricing tree shows the possible retail price paths, that are generated by random promotion prices. Each promotion can be a deep or a shallow promotion. Thus, each promotion doubles the number of possible retail price paths.

Retail pricing decisions are made randomly. Thus, the retailer cannot influence demand and has to deal with uncertainty. We described the mechanics of promotion
pricing in detail in Chapter 4. Each node represents a random draw of a promotion price, but not a decision by the retailer. Each node is characterized by a state index $z_{t} \in Z_{t}$. The set of possible states $Z_{t}$ is time dependent. Each $z_{t}$ contains the complete path information up to that state at that point in time. In $t_{1}$, the set of possible realizations of the state index $z_{1}$ contains two elements,

$$
Z_{1}=\{\text { deep }, \text { shallow }\}
$$

For $t_{2}$ the set expands, as the second promotion increases the number of possible paths,

$$
Z_{2}=\{\text { deep } / \text { deep }, \text { deep } / \text { shallow }, \text { shallow } / \text { deep }, \text { shallow } / \text { shallow }\}
$$

The retail price in each node can thus be specified as $p_{t}\left(z_{t}\right)$. Given the consumer demand model developed, there is a direct relationship between retail prices and demand in all nodes, given consumers' inventory positions at that point in time. The retailer's problem is to find optimal contracting and execution policies for each node, including $t_{-1}$. As pricing is uncertain, this optimization problem has a stochastic and dynamic form. We go on in our analysis with the retailer's objective function.

### 5.2.2 The retailer's objective function

The retailer targets to be as profitable as possible, given demand and supply restrictions. Profit depends on revenues, purchase costs, holding costs and costs of lost sales. Stochastic demand results in uncertainty about the amount of products required. Therefore, the retailer maximizes expected profit. The retailer considers several sources of revenues and costs:

- Revenues - income from selling products to consumers
- Spot purchase costs - costs of purchasing products through instant orders
- Reservation fees - costs of reserving products
- Execution fees - costs of actually ordering products reserved with option contracts
- Forward purchase costs - costs of purchasing products in advance
- Holding costs - costs of carrying over excess inventory to the next promotion
- Costs of lost sales - costs of winning back consumers who have not been served due to stockouts
- Salvage value - income from salvaging inventory at the end of the promotion period

Revenues depend on retail prices, demand and the amount of products available, where demand again depends on current and past retail prices. Insufficient inventories hurt revenues and result in costs of lost sales. These costs represent the effort required to win back a consumer who has not been served due to a stockout. To
the contrary case of excess inventory, the retailer incurs holding costs. These represent the costs of warehouse space and additional handling. The majority of costs result from purchasing products for resale. Instant orders are generally more expensive than forward contracts, as all inventory risk is shifted to the manufacturer. The manufacturer naturally requires compensation. Analogously, forward orders are less expensive, as the retailer commits early to purchase a certain amount of the product for delivery at a certain point in time. Production can be scheduled more efficiently and inventory risk is allocated solely at the retailer's end of the supply chain. Option contracts combine features of both, spot and forward transactions. The retailer pays a reservation fee upfront to reserve the right but not the obligation to purchase a unit of the product at a certain point in time at a fixed execution fee. Option contracts divide inventory risk between the retailer and the manufacturer.

### 5.2.2.1 Specifying a general retailer objective function

Let $\Pi_{r}$ be expected retailer profit. The retailer earns a retail price $p_{i}\left(z_{i}\right)$ for each unit sold. Let $D_{i}\left(z_{i}\right)$ be the total demand in terms of units at time $i$ in state $z_{i}$. Then, expected revenues equal $\sum_{z_{i}} \pi_{z_{i}} \cdot\left[\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) \cdot p_{i}\left(z_{i}\right)\right]$ in $t=i$, where $\pi_{i}\left(z_{i}\right)$ is the probability of state $z_{i}$ occurring at time $i$. Costs are limited to purchasing, holding and goodwill costs in our analysis. The retailer pays a spot price $s_{i}$ for every unit purchased on the spot market, a reservation fee $r_{i j}$ for every unit reserved on the contract market and an execution fee $e_{k i}$ for every unit eventually executed. Let the respective spot purchase, forward, reservation and execution quantities be $q_{i}^{S}\left(z_{i}\right)$, $q_{i j}^{f}\left(z_{i}\right), q_{i j}^{r}\left(z_{i}\right)$ and $q_{k i}^{e}\left(z_{i}\right)$. Let $h$ be the retailer's holding costs per unit per period. Holding costs are the costs required to store a product until the next promotion. Let $g$ be the advertising costs required to repair the damage to goodwill, when a product is out of stock and let $G_{i}\left(z_{i}\right)$ be the amount of lost sales due to stockouts. Let $v_{T}$ be the salvage value of one unit of the product at time $T$.

The retailer maximizes expected profit over all periods. Given the structure of available supply contracts, the retailer must determine a profit maximizing initial portfolio of contracts and an optimal policy to update this portfolio, when new information is available. The retailer's objective function over all periods is specified as the sum of profits in all nodes, weighted by the probability of reaching the respective nodes. The retailer earns expected revenues that depend on demand in the node reached $D_{i}\left(z_{i}\right)$ and on the quantity not served $G_{i}\left(z_{i}\right)$,

$$
\begin{equation*}
\text { revenues }=\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) p_{i}\left(z_{i}\right)\right] \tag{5.2}
\end{equation*}
$$

Spot purchases and option executions are flexible sources of supply. Spot purchases only depend on the current point in time $i$ and on the state reached $z_{i}$,

$$
\begin{equation*}
\text { spot costs }=\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i}^{s}\left(z_{i}\right) s_{i}\right] . \tag{5.3}
\end{equation*}
$$

Option executions depend in addition on past option reservations at time $k$,

$$
\begin{equation*}
\text { execution costs }=\sum_{i=1}^{N} \sum_{k=-1}^{i-1} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot \cdot\left[q_{k i}^{e}\left(z_{i}\right) e\left(\delta_{k i}\right)\right] \tag{5.4}
\end{equation*}
$$

Forward contracts are inflexible. They are closed for a future date $j$ and cannot be altered, i.e.,

$$
\begin{equation*}
\text { forward costs }=\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{f}\left(z_{i}\right) f\left(\delta_{i j}\right)\right] \tag{5.5}
\end{equation*}
$$

Option reservations take place in advance like forward purchases,

$$
\begin{equation*}
\text { reservation costs }=\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{r}\left(z_{i}\right) r\left(\delta_{i j}\right)\right] \tag{5.6}
\end{equation*}
$$

Insufficient supply results in lost sales $G_{i}\left(z_{i}\right)$ and goodwill costs, while excess orders lead to inventories $I_{i}\left(z_{i}\right)$ on the retailer level,

$$
\begin{align*}
\text { goodwill costs } & =\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[G_{i}\left(z_{i}\right) g\right]  \tag{5.7}\\
\text { holding costs } & =\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[I_{i}\left(z_{i}\right) h\right] \tag{5.8}
\end{align*}
$$

Excess inventories can be salvaged at the end of the promotion period; however, the retailer only receives the salvage value $v_{T}$,

$$
\begin{equation*}
\text { salvage value }=\sum_{z_{T} \in Z_{T}} \pi_{z_{T}} \cdot\left[I_{T}\left(z_{T}\right) v_{T}\right] \tag{5.9}
\end{equation*}
$$

The retailer's expected profit $\Pi_{r}$ is the sum of these factors. The retailer maximizes profit by choosing $q_{i j}^{f}\left(z_{i}\right), q_{i j}^{s}\left(z_{i}\right), q_{i j}^{r}\left(z_{i}\right), q_{k i}^{e}\left(z_{i}\right)$ and $G_{i}\left(z_{i}\right)$. Before we establish the complete optimization program with constraints, we briefly discuss retailer inventories.

### 5.2.3 Retailer inventories

Diapers are not perishable, which provides the retailer with the opportunity to hold inventories. The retailer incurs holding costs per unit and per period. Inventory must be built up when forward orders exceed realized demand. In addition to inventories induced by forward purchases, the retailer may choose to execute options or to purchase products with spot orders to build up inventory. This approach is only rational when, in the following periods, flexible sources are limited or expensive enough to
outweigh the additional holding costs. Thus, inventory after a promotion is calculated as inventory after the last promotion in $t=i-1$, plus any contracts executed in period $i$, less demand served, i.e.,

$$
\begin{equation*}
I_{i}\left(z_{i}\right)=I_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+\sum_{k=-1}^{i-1} q_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{e}-\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) \tag{5.10}
\end{equation*}
$$

The retailer trades-off purchasing cheap forward contracts and stocking excess inventories against purchasing additional flexible contracts. These are the two available strategies that can be used to hedge against demand risk:

- Hedging with inventories - purchasing forwards and stocking excess inventories
- Hedging with flexible contracts - purchasing option contracts or placing spot orders

The retailer chooses the cheaper of the two hedging opportunities. Over all states, a mixture of both hedging strategies may be optimal. Key parameters are holding costs, as well as forward and option contract parameters. The retailer must additionally consider the possibility that spot orders are not always served. This reflects in the requirement of a minimum service level $q^{\text {req. }}$ in the constraints.

### 5.2.4 A stochastic program solving the retailer's optimization problem

The next step is deriving the retailer's best response to the portfolio of supply contracts offered. The set of the retailer's best responses in all possible states of the world constitutes the retailer's optimal purchasing policy. Both execution and reservation decisions are a part of this overall policy. We employ stochastic programming to derive the retailer's set of best responses for all states of the world. First, we model the base case. Second, we provide a general model.

### 5.2.4.1 Optimization problem in the status quo

We characterized the status quo in Chapter 3. Transactions are based on spot orders in pull mode. Then, the general objective function derived earlier in Section 5.2.2.1 collapses to a simple model. The retailer waits for demand to be realized and then decides on her optimal spot orders. She may decide to not serve all demand, if it is optimal to do so (see Section 5.2.5.1). The retailer maximizes profit subject to several constraints. This results in the following stochastic program:

$$
\max _{q_{i}^{s}\left(z_{i}\right), G_{i}\left(z_{i}\right)} \Pi_{r}=\left(\begin{array}{c}
\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) p_{i}\left(z_{i}\right)\right] \\
-\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i}^{s}\left(z_{i}\right) s_{i}\right] \\
-\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[G_{i}\left(z_{i}\right) g+I_{i}\left(z_{i}\right) h\right] \\
+\sum_{z_{T} \in Z_{T}} \pi_{z_{T}} \cdot\left[I_{T}\left(z_{T}\right) v_{T}\right]
\end{array}\right)
$$

subject to:

$$
\begin{array}{ll}
\text { I. } \quad q_{i}^{s}\left(z_{i}\right) \geq 0, G_{i}\left(z_{i}\right) \geq 0 \forall i \\
\text { II. } & D_{i}\left(z_{i}\right) \leq I_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+G_{i}\left(z_{i}\right) \forall i \\
\text { III. } I_{i}\left(z_{i}\right)=I_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{i}^{s}\left(z_{i}\right)-\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) \forall i \\
\text { IV. } & G_{i}\left(z_{i}\right) \leq D_{i}\left(z_{i}\right)-q_{i}^{s}\left(z_{i}\right) \forall i \\
\text { V. } & R(c(\delta=0)>s) \cdot q_{i}^{s}\left(z_{i}\right) \leq q^{\text {req. }} \forall i
\end{array}
$$

Note that the second constraint could be replaced by $I_{i}\left(z_{i}\right) \geq 0$. Omitting this simplification helps to stress the logic of the set of constraints. The first constraint requires the decision variables to be nonnegative. The second constraint ensures that demand in every state is either served or lost. Excess purchases are converted to inventories. The third constraint determines inventory after a promotion, given the inventory after the previous promotion. The fourth constraint defines the upper boundary for lost sales $G_{i}\left(z_{i}\right)$. The fifth constraint limits retailer orders to the required service level. $R$ is an indicator function, which equals one for expensive short term production. Therefore, this constraint is only active when the manufacturer cannot profitably serve instant orders with short-term production, as analytically derived in Chapter 3.

### 5.2.4.2 General program with risk sharing contracts

Now, we include forward and option contracts in our formulation. We model the most general execution scheme, allowing for purchases and executions at every point in time. Adding restrictions will allow us to formulate our general model. Again, the resulting program is stochastic:

$$
q_{i}^{s}\left(z_{i}\right), q_{k i}^{e}\left(z_{i}\right), q_{i j}^{f}\left(z_{i}\right), \quad \Pi_{r}=\left(\begin{array}{c}
\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) p_{i}\left(z_{i}\right)\right] \\
-\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i}^{s}\left(z_{i}\right) s_{i}\right] \\
-\sum_{i=1}^{N} \sum_{k=-1}^{i-1} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{k i}^{e}\left(z_{i}\right) e\left(\delta_{k i}\right)\right] \\
-\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{f}\left(z_{i}\right) f\left(\delta_{i j}\right)\right] \\
-\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{r}\left(z_{i}\right) r\left(\delta_{i j}\right)\right] \\
-\sum_{i=1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[G_{i}\left(z_{i}\right) g+I_{i}\left(z_{i}\right) h\right] \\
\\
\\
\sum_{z_{T} \in Z_{T}} \pi_{z_{T}} \cdot\left[I_{T}\left(z_{T}\right) v_{T}\right]
\end{array}\right.
$$

subject to:
I. $\quad q_{i j}^{s}\left(z_{i}\right) \geq 0, q_{i j}^{f}\left(z_{i}\right) \geq 0, q_{i j}^{r}\left(z_{i}\right) \geq 0, q_{k i}^{e}\left(z_{i}\right) \geq 0, G_{i}\left(z_{i}\right) \geq 0 \forall i$
II. $\quad D_{i}\left(z_{i}\right) \leq I_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+\sum_{k=-1}^{i-1} \bar{q}_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{e}\left(z_{i}\right)+G_{i}\left(z_{i}\right) \forall i$
III. $I_{i}\left(z_{i}\right)=I_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{i}^{S}\left(z_{i}\right)+\sum_{k=-1}^{i-1} \bar{q}_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{e}\left(z_{i}\right)-\left(D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)\right) \forall i$
IV. $G_{i}\left(z_{i}\right) \leq D_{i}\left(z_{i}\right)-q_{i}^{s}\left(z_{i}\right)-\sum_{k=-1}^{i-1} \bar{q}_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{e}\left(z_{i}\right) \forall i$
V. $\quad R(c(\boldsymbol{\delta}=0)>s) \cdot q_{i}^{s}\left(z_{i}\right) \leq \max \left[q^{r e q .}-\sum_{k=-1}^{i-1} \bar{q}_{k i}^{f}-\sum_{k=-1}^{i-1} q_{k i}^{e}\left(z_{i}\right) ; 0\right] \forall i$
VI. $q_{k i}^{e}\left(z_{i}\right) \leq q_{k i}^{r} \forall i$

Linearity of the program is given, as instant orders are either part of the portfolio or there is no required minimum service level, as instant orders are dominated by options and forwards. Whether there are instant orders is determined by contract parameters in advance, as shown in Chapter 3. The fifth constraint sets spot orders to zero for the case of domination and limits spot orders to the required minimum service level otherwise. The existence of options requires an additional constraint. The retailer cannot execute more options than she reserved for that point in time. There is no additional constraint for forwards required. Note that $\bar{q}_{k i}^{f}$ is not a decision variable, as forwards must always be executed.

### 5.2.5 Structure of the solution to the retailer's problem

The optimization program established can be solved by means of standard optimization software. We analytically explore the solution space to derive more about the structure of optimal purchasing decisions. For this purpose, the optimization prob-


Fig. 5.7 Executing option versus spot contracts under forced compliance for both contract types
lem is divided into two subproblems. The execution policy provides optimal spot orders and option executions, while the reservation policy determines forward orders and option reservations for all states of the world and all points in time.

### 5.2.5.1 The retailer's execution policy

The retailer's execution policy characterizes all actions that are taken in a node to serve demand in that node (and possibly for following nodes). At this point in time, the state reached is known and demand in the current node is certain. To find the basic structure of the solution, we assume there are no spot purchases or option executions for future points in time, only for the present promotion.

A profit maximizing retailer executes the cheapest source of products first. Three types of contracts are available: spot, forward and option contracts. As forward fees are sunk, there is no additional cost for serving demand with forwards. Consequently, forwards are free of charge at the time of execution. As a result, all forwards available for the promotion are always employed to serve demand until either demand is fully served or forward inventories are exhausted. As reservation fees are sunk, too, the cost of option capacity depends on the execution fee at the time of
execution. Whether options are executed first or spot capacity is purchased depends on the spot price. The retailer chooses the cheaper of the two flexible contracts, option or spot, to be executed first (see Spinler et al. 2003). In the context of known fixed spot prices and no limit on spot supply, options are dominated when spot is executed first.

For every unit of demand served, the retailer receives the retail price and she avoids cost of lost goodwill. Thus, the sum of the retail price and the cost of lost goodwill constitute a threshold. The retailer does not consider options or instant orders, when purchase costs are above this threshold. Then, the retailer prefers not to serve demand, employing the respective contracts. Equations 5.11 and 5.12 describe the amounts of option and spot contracts executed, when state $z_{i}$ is reached. For this analysis, we assume that the manufacturer serves all spot orders.

$$
\begin{align*}
& \sum_{k=-1}^{i-1} q_{k i}^{e}\left(z_{i}\right)=\left\{\begin{array}{l}
{\left[\begin{array}{l}
\min \left(\sum_{k=-1}^{i-1} q_{k i}^{r} ; D_{i}\left(z_{i}\right)-I_{i-1}\left(z_{i}\right)-\sum_{k=-1}^{i-1} q_{k i}^{f}\right)
\end{array}\right]^{+}} \\
\text {for } e_{i} \leq s_{i} \leq p_{i}\left(z_{i}\right)+g \\
0 \text { for } e_{i}>s_{i} \text { and/or } e_{i}>p_{i}\left(z_{i}\right)+g
\end{array}\right.  \tag{5.11}\\
& q_{i}^{s}\left(z_{i}\right)=\left\{\begin{array}{l}
{\left[D_{i}\left(z_{i}\right)-I_{i-1}\left(z_{i}\right)-\sum_{k=-1}^{i-1} q_{k i}^{f}\right]^{+}} \\
\text {for } e_{i}>s_{i} \text { and } s_{i} \leq p_{i}\left(z_{i}\right)+g \\
{\left[D_{i}\left(z_{i}\right)-I_{i-1}\left(z_{i}\right)-\sum_{k=-1}^{i-1} q_{k i}^{f}-\sum_{k=-1}^{i-1} q_{k i}^{e}\right]^{+}} \\
\text {for } e_{i} \leq s_{i} \text { and } s_{i} \leq p_{i}\left(z_{i}\right)+g \\
0 \text { for } s>p+g
\end{array}\right. \tag{5.12}
\end{align*}
$$

When options are the cheaper source of supply and when not enough options have been reserved, the retailer can serve demand by purchasing additional instant order contracts (Figure 5.7, left column). Moreover, the retailer may choose the option to incur a stockout, if winning back consumers is cheaper than purchasing expensive instant orders and/or executing options. In this case the instant order price and/or execution fee would exceed the retail price by more than the cost of lost sales. This case could be relevant for the promotion environment, as deep promotion prices trigger high demand and increase the probability that cheaper inventory sources have insufficient capacities. Thus, expensive sources of products will be mainly employed if the retail price is particularly low.

When instant orders are cheaper than executing options, the retailer only considers options, if supply of instant orders were too risky (Figure 5.7, right column). Wholesale prices are not state dependent and thus, the retailer knows the execution fee and the spot price in advance, at the point in time, when she reserves options.

The maximum amount of demand served depends on the instant order price, the execution fee, the goodwill cost and the retail price in the state reached. Equation 5.13 describes the resulting four cases, showing the amount of demand served in
each case. Table 5.2 summarizes the insights on priorities of contract types in the execution decision.

$$
D_{i}\left(z_{i}\right)-G_{i}\left(z_{i}\right)=\left\{\begin{array}{l}
\min \left[D_{i}\left(z_{i}\right) ; I_{i-1}\left(z_{i}\right)+\sum_{k=-1}^{i-1} q_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{r}\right] \\
\text { for } s_{i} \geq p_{i}\left(z_{i}\right)+g \text { and } e_{i}<p_{i}\left(z_{i}\right)+g \\
\min \left[D_{i}\left(z_{i}\right) ; I_{i-1}\left(z_{i}\right)+\sum_{k=-1}^{i-1} q_{k i}^{f}+q_{i}^{s}\right] \\
\text { for } s_{i}<p_{i}\left(z_{i}\right)+g \text { and } e_{i} \geq p_{i}\left(z_{i}\right)+g \\
\min \left[D_{i}\left(z_{i}\right) ; I_{i-1}\left(z_{i}\right)+\sum_{k=-1}^{i-1} q_{k i}^{f}+\sum_{k=-1}^{i-1} q_{k i}^{r}+q_{i}^{s}\right] \\
\text { for } s_{i}<p_{i}\left(z_{i}\right)+g \text { and } e_{i}<p_{i}\left(z_{i}\right)+g \\
\min \left[D_{i}\left(z_{i}\right) ; I_{i-1}\left(z_{i}\right)+\sum_{k=-1}^{i-1} q_{k i}^{f}\right] \\
\operatorname{for} s_{i} \geq p_{i}\left(z_{i}\right)+g \text { and } e_{i} \geq p_{i}\left(z_{i}\right)+g
\end{array}\right.
$$

Table 5.2 Priorities of inventory types for execution

|  | $s-p \leq g ; e-p \leq g$ |  | $\begin{gathered} s-p>g ; e-p>g \\ e>s ; e \leq s \end{gathered}$ |
| :---: | :---: | :---: | :---: |
|  | $e>s$ | $e \leq s$ |  |
| forward | $1^{\text {st }}$ priority | $1^{\text {st }}$ priority | $1{ }^{\text {st }}$ priority |
| instant order | $2^{\text {nd }}$ priority | $3^{\text {rd }}$ priority | no purchases |
| option | no execution | $2^{\text {nd }}$ priority | no execution |
| lost sales | no lost sales | no lost sales | $2^{\text {nd }}$ priority |
|  | $s-p \leq g ; e-p>g$ |  | $s-p>g ; e-p \leq g$ |
|  | $e>s ; e \leq s$ |  | $e>s ; e \leq s$ |
| forward | $1^{\text {st }}$ priority |  | $1{ }^{\text {st }}$ priority |
| instant order | $2^{\text {nd }}$ priority |  | no purchases |
| option | no execution |  | $2^{\text {nd }}$ priority |
| lost sales | no lost sales |  | $3{ }^{\text {rd }}$ priority |

The retailer's service level is limited by the availability of supply. Spot orders are not always served, as the manufacturer maximizes profit. A powerful retailer can request a required minimum service level, as discussed in Chapter 3. When the constraint is binding and when forward and option orders do not exceed the required service level, our analysis in this section is adapted, in that demand $D_{i}\left(z_{i}\right)$ is replaced with $\min \left(D_{i}\left(z_{i}\right) ; q^{\text {req. }}\right)$, where $q^{\text {req. }}$ is again the required total quantity corresponding to the required minimum service level $F\left(q^{\text {req. }}\right)$.

### 5.2.5.2 The retailer's reservation policy

Reservation decisions determine the availability of forward and option inventories in future periods. Technically, the difference compared to execution decisions is that reservation decisions are always made under risk. The retailer knows all possible future states of demand and the respective probabilities of reaching all of these nodes in the pricing tree. The retailer makes two decisions that are characterized by this information structure:

- Forward purchases - obligation to purchase the reserved quantity at a future point in time for a known forward price
- Option reservations - right but not obligation to execute the reserved quantity at a future point in time for a known execution fee

The reservation policy addresses two challenges. First, reservations ensure supply. The manufacturer is required to serve both forward and option contracts in all states of the world. Thereby, the retailer can hedge against the risk of insufficient supply. Second, reservations can coordinate and eliminate waste in the channel. Thus, they allow for reduced cost of inventory. However, these advantages are not free. The retailer must pay by taking a higher share of supply chain risk.

The retailer decides on the service level attained by contracting a corresponding portfolio of supply contracts. Early commitments ensure supply. Moreover, the retailer trades-off paying costs of lost sales against paying for more expensive spot orders. We described this trade-off in detail in the previous section. There are two cases to be considered for the analysis of service levels. For $s_{i}>\left(p_{i}\left(z_{i}\right)+g\right)$, the retailer does not consider spot orders. For this case, her service level is determined by her option and forward orders as well as by remaining inventories from the last promotion. For $s_{i} \leq\left(p_{i}\left(z_{i}\right)+g\right)$, the retailer serves as much demand as possible, subject to the service level enforced by the retailer towards the manufacturer. The retailer fills the gap in supply with spot orders to achieve at least $F\left(q^{\text {req. }}\right)$. Moreover, the retailer can ensure a higher service level by placing option and forward orders that exceed the required minimum service level, as presented in Chapter 3.

### 5.3 The manufacturer's optimization problem

The manufacturer produces products for sale to the retailer. We first describe the structure of his optimization problem. Time dependent manufacturing costs are a key driver of the dynamics of the production process. We discuss this issue as well as manufacturer service levels in the second step. Third, we review manufacturer inventories and associated costs. In the fourth step, we formulate the manufacturer's objective function. In the fifth and last step, we derive the manufacturer's optimal production policy.

### 5.3.1 Structure of the manufacturer's optimization problem

The manufacturer solves a production optimization problem in which he has different production opportunities that are associated with different costs and lead-times. The manufacturer anticipates retailer orders as outlined in the previous section and determines the profit maximizing production policy that fulfills all constraints. The manufacturer decides on production schedules. Producing with longer lead-times for future periods reduces production costs and simultaneously results in an additional share of inventory risk on the manufacturer side of the supply chain. Thus, the manufacturer must find the optimal balance between taking risk and producing short-term. He will choose to not serve all spot orders if doing so is optimal. Spot orders are not committed and therefore, the manufacturer may increase profit by allowing for stockouts in high states of demand.

The manufacturer's optimal policy specifies production given the state of the world and given the retailer's orders. That is, for each possible state, the policy provides an optimal production schedule. This schedule specifies production quantities and due-dates. We determine the policy via a stochastic program. The program provides the best response to the retailer's orders.

### 5.3.2 Service levels and lead-time dependent production costs

The manufacturer incurs variable costs for each unit of the product produced. As in Chapter 3, we normalize fixed costs to zero without loss of generality. Variable costs are likely to be dependent on the time span between the order and the scheduled delivery date. In contrast to Chapter 3, we now consider more than one promotion. Consequently, it is no longer sufficient to model a short-term and a single long-term production cost.

The more time the manufacturer has for production, the easier he can implement a lean and smooth production schedule. In contrast, last minute production create additional costs. Short-term production disturbs production plans and requires adapting optimized schedules, e.g.,

- Overtime charges - extending production to an additional shift or paying for overtime
- High set-up times - producing smaller batches resulting in additional set-ups
- Reduced average machine utilization - smoothing of orders over time is not possible

All of these cost factors can be avoided by increasing average lead-times. Thus, the manufacturer's production cost function is time dependent. Production cost $c\left(\delta_{i j}\right)$ depend on the production lead-time $\delta_{i j}$. Longer production lead-times reduce variable costs, as production processes become smother.

Assumption 5.3 The manufacturer incurs lead-time dependent variable production costs. Variable production costs decrease in the lead-time available. The manufacturer's fixed production costs are assumed to be zero without loss of generality.

As in the single period model (Chapter 3, Scenarios II and III), the manufacturer can produce an unlimited quantity of the product on short notice; however, it is expensive to do so. Thus, the manufacturer produces in advance for promotions and solves a dynamic stochastic program. He faces two types of risk: producing excess quantities for a promotion results in costly inventories and insufficient production leads to costly short-term production.

### 5.3.3 Manufacturer inventories

The manufacturer has the ability to store unused inventory. Holding inventory is costly. Holding a unit of inventory until the next promotion results in holding costs $h$. Note that holding costs are assumed to be equal at both the manufacturer and the retailer level. This is reasonable, as both parties often hold inventory under approximately equal conditions, e.g., in consignment warehouses or vendor-managedinventory policies.

Longer production lead-times reduce manufacturing cost. As a result, the manufacturer faces a trade-off between producing early under uncertainty at low cost and waiting until uncertainty is resolved, which results in higher production cost. Under the status quo, the retailer exclusively purchases instant order contracts. Still, the manufacturer faces the production related trade-off described. High short-term production cost penalties force him to consider producing in advance, even under a wholesale price scheme. Thereby, he is likely to invest in speculative inventories.

Options are risky in terms of holding costs, too, as the retailer can decide not to exercise all options reserved. This again results in inventory at the manufacturer level. However, compared to instant orders, option contracts share risk between the manufacturer and the retailer. When the retailer decides not to exercise an option, she loses the reservation fee. The manufacturer receives the reservation fee as a remuneration for the costs associated with the inventory risk taken.

Forward contracts never result in holding costs at the manufacturer level, as quantities are fully committed. Thus, there is no trade-off between production cost and inventory risk. Still, longer production lead-times result in lower production cost. Therefore, the manufacturer produces quantities for serving forward contracts at the latest, when the retailer commits.

Inventory at the manufacturer level depends on production decisions. Inventory is built up when production at time $i$ exceeds orders less remaining inventory from the last promotion. When the manufacturer does not produce sufficient quantities of the product, he incurs lost sales $K_{i}$ on instant orders. Options and forwards must be served.

$$
\begin{equation*}
J_{i}\left(z_{i}\right)=J_{i-1}\left(z_{i-1} \mid z_{i}\right)+\sum_{k=-1}^{i} q_{k i}^{p}\left(z_{i}\right)-\left(q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f}-K_{i}\left(z_{i}\right)\right) \tag{5.14}
\end{equation*}
$$

Inventories are calculated as inventories after the previous promotion plus production for the period in question less orders served. Inventories in state $z_{i}$ depend on the state reached in the previous period. A deeper discount results in higher sales and thus lower inventories after the previous promotion. The model setup determines that each state has exactly one predecessor state and that each state is predecessor of two successor states (see Figure 5.6). Thus, the information of the state $z_{i}$ reached at time $i$ is sufficient to determine the predecessor state $z_{i-1}$ reached at time $i-1$.

### 5.3.4 The manufacturer's objective function

The next step is deriving a formulation of manufacturer profit. The manufacturer earns revenues for every unit sold to the retailer. The amount earned for each unit of the product depends on the type of contract employed and the contract parameters specified. The number of products and the mix of contracts purchased by the retailer to serve demand depend again on contract parameters.

The manufacturer's expected profit $\Pi_{m}$ calculates as the profit from selling the product to the retailer, less costs of producing the product and less costs of holding inventory. The manufacturer can salvage inventory at time $T$ at the minimum production costs. There is no arbitrage possible. The manufacturer thus considers several sources of costs and revenues:

- Revenues from forwards - certain revenues from forward sales
- Revenues from instant order contracts - risky revenues from instant orders
- Revenues from reservation fees - certain revenues from option reservations
- Revenues from execution fees - risky revenues from option executions
- Production costs - lead-time dependent expenses for producing products
- Holding costs - expenses for holding an unit on inventory until the next promotion
- Salvage value - remuneration for salvaging products at the end of the game

The profit function calculates expected profit for all future points in time within the forecast period. States of the world are weighted with the probability that they are actually reached. The manufacturer earns revenues from selling products to the retailer. Forward contracts are sold for a future point in time $j$,

$$
\begin{equation*}
\text { revenues from forwards }=\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{f}\left(z_{i}\right) f\left(\delta_{i j}\right)\right] . \tag{5.15}
\end{equation*}
$$

Spot contracts are forward contracts with zero lead-time. We avoid to merge spot and forward formulations to model all three types of contracts separately, i.e.,

$$
\begin{equation*}
\text { revenues from spot contracts }=\sum_{i=-1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\left(q_{i}^{s}\left(z_{i}\right)-K_{i}\left(z_{i}\right)\right) s_{i}\right] \tag{5.16}
\end{equation*}
$$

Reservation fees are paid to ensure availability of products at a future point in time. A lead-time dependent discount is possible,

$$
\begin{equation*}
\text { revenues from reservation fees }=\sum_{i=-1}^{N} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{r}\left(z_{i}\right) r\left(\delta_{i j}\right)\right] \tag{5.17}
\end{equation*}
$$

The manufacturer receives execution fees for option contracts that have been closed in the past and that are now executed. Again a lead-time dependent discount is possible,

$$
\begin{equation*}
\text { revenues from execution fees }=\sum_{i=1}^{N} \sum_{k=-1}^{i-1} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{k i}^{e}\left(z_{i}\right) e\left(\delta_{k i}\right)\right] . \tag{5.18}
\end{equation*}
$$

Production costs are lead-time dependent. The manufacturer thus has the opportunity to reduce her costs by producing earlier, i.e.,

$$
\begin{equation*}
\text { production costs }=\sum_{i=-1}^{N} \sum_{j=i}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[q_{i j}^{p}\left(z_{i}\right) c\left(\delta_{i j}\right)\right] \tag{5.19}
\end{equation*}
$$

Speculative long-term production can result in manufacturer inventories,

$$
\begin{equation*}
\text { holding costs }=\sum_{i=-1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[J_{i}\left(z_{i}\right) h\right] \tag{5.20}
\end{equation*}
$$

Excess inventories can be salvaged at the end of the promotion period; however, the manufacturer only receives the salvage value $v_{T}$,

$$
\begin{equation*}
\text { salvage value }=\sum_{z_{T} \in Z_{T}} \pi_{z_{T}} \cdot\left[J_{T}\left(z_{T}\right) v_{T}\right] \tag{5.21}
\end{equation*}
$$

Combining these terms provides the manufacturer's objective function. The decision variables of the manufacturer are $q_{i j}^{p}\left(z_{i}\right)$ and $K_{i}\left(z_{i}\right)$. The retailer's decision variables $\bar{q}_{i j}^{f}\left(z_{i}\right), \bar{q}_{i j}^{s}\left(z_{i}\right), \bar{q}_{i j}^{r}\left(z_{i}\right)$ and $\bar{q}_{k i}^{e}\left(z_{i}\right)$ are determined by the retailer's optimization problem and thus they are exogenous.

### 5.3.5 The manufacturer's production policy

The lead-time dependent component of variable costs requires the manufacturer to incur higher production costs for shorter production lead-times. The manufacturer produces quantities purchased on forward contracts as early as possible. Option
and spot orders are covered partially by speculative inventories and partially by expensive short-term production.

The manufacturer produces such that he maximizes expected profit. His production problem depends on the retailer's purchasing decision and thus indirectly on contract parameters. The manufacturer solves his production problem for all reachable states of the world in all periods within the planning period, resulting in the following stochastic program:

$$
\underset{q_{i j}^{p}\left(z_{i}\right), K_{i}\left(z_{i}\right)}{\max } \Pi_{m}=\left(\begin{array}{c}
\sum_{i=-1}^{N-1} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\bar{q}_{i j}^{f}\left(z_{i}\right) f\left(\delta_{i j}\right)\right] \\
+\sum_{i=-1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}}\left[\left(\bar{q}_{i}^{S}\left(z_{i}\right)-K_{i}\left(z_{i}\right)\right) s_{i}\right] \\
+\sum_{i=-1}^{N} \sum_{j=i+1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\bar{q}_{i j}^{r}\left(z_{i}\right) r\left(\delta_{i j}\right)\right] \\
+\sum_{i=1}^{N} \sum_{k=-1}^{i-1} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[\bar{q}_{k i}^{e}\left(z_{i}\right) e\left(\delta_{k i}\right)\right] \\
-\sum_{i=-1}^{N} \sum_{j=i z_{i} \in Z_{i}}^{N} \sum_{z_{i}} \cdot\left[q_{i j}^{p}\left(z_{i}\right) c\left(\delta_{i j}\right)\right] \\
-\sum_{i=-1}^{N} \sum_{z_{i} \in Z_{i}} \pi_{z_{i}} \cdot\left[J_{i}\left(z_{i}\right) h\right] \\
-\sum_{z_{T} \in Z_{T}} \pi_{z_{T}} \cdot\left[J_{T}\left(z_{T}\right) v_{T}\right]
\end{array}\right)
$$

subject to:
I. $\quad q_{i j}^{p} \geq 0, K_{i} \geq 0 \forall i$
II. $\quad J_{i}\left(z_{i}\right)=J_{i-1}\left(z_{i-1} \mid z_{i}\right)+q_{k i}^{p}-\left(q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f}-K_{i}\left(z_{i}\right)\right) \forall i$
III. $\sum_{k=-1}^{i} q_{k i}^{p}+J_{i-1}\left(z_{i-1} \mid z_{i}\right) \geq q_{i}^{e}\left(z_{i}\right)+q_{i}^{f} \forall i$
IV. $\quad K_{i}\left(z_{i}\right) \leq R(c(\delta=0)>s) \cdot\left[q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f}-q^{\text {req. }}\right] \forall i$
V. $\quad K_{i}\left(z_{i}\right) \leq q_{i}^{s}\left(z_{i}\right) \forall i$

VIa. $\sum_{k=-1}^{i} q_{k i}^{p}+J_{i-1}\left(z_{i-1} \mid z_{i}\right) \geq q^{\text {req. }} \forall i$
for $q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f} \geq q^{r e q}$.
VIb. $\sum_{k=-1}^{i} q_{k i}^{p}+J_{i-1}\left(z_{i-1} \mid z_{i}\right) \geq q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f} \forall i$
for $q_{i}^{e}\left(z_{i}\right)+q_{i}^{s}\left(z_{i}\right)+q_{i}^{f} \leq q^{\text {req }}$.
The first constraint limits production quantities to the positive range. The second constraint defines manufacturer inventories. The third constraint ensures that forward and option orders are always served. The fourth constraint is required to make $q^{\text {req. }}$ binding. It is only active, when the manufacturer cannot profitably produce short-term to serve instant orders. Note that the fifth constraint in the retailer's optimization problem ensures that the right hand side of this constraint is greater or equal to zero. The manufacturer's fifth constraint ensures that only spot contracts
are potentially not served. The sixth constraint forces the manufacturer to serve demand at least a) up to the required service level or b) up to the quantity demanded by the retailer. In each node, either a) or b) apply, depending on whether orders exceed the required service level or not. The manufacturer can decide to serve more than the required minimum, when doing so is profitable. In combination with the third constraint, case a) is extended, such that option and forward contracts are served, even when they exceed the required minimum service level.

A reduced program, describing the base case of spot orders only, follows immediately when setting $\bar{q}_{i j}^{f}\left(z_{i}\right)=0, \bar{q}_{i j}^{r}\left(z_{i}\right)=0$ and $\bar{q}_{k i}^{e}\left(z_{i}\right)=0$. The manufacturer's decision variables do not change.

### 5.4 Summary: channel coordination with contract portfolios

We provided two programs to derive optimal purchasing and production policies in a price promotion environment. Combining options, forwards and spot contracts allows for a set of coordinating contract parameters and respective portfolios.

In the status quo, the manufacturer may allow for stockouts when orders exceed the required minimum service level. Portfolios of option, forward and spot contracts can achieve channel coordination in that purchasing and production decisions are aligned. Thereby Pareto improvement can be achieved.
i) The retailer has an incentive to take more supply chain risk and to commit earlier
ii) The manufacturer has an incentive to improve his service level

Portfolios of option, forward and spot contract can achieve a continuum of inventory allocations. Inventory can be directed to the retailer side of the channel early in a promotion campaign. End-of-period coverage can be avoided towards the end of a promotion campaign.
i) Option contracts allocate inventories to the manufacturer level
ii) Forward contracts allocate inventories to the retailer level

Thereby, zero out-of-stock (channel coordination) and zero end-of-period coverage (optimal amount of inventory at the retailer level at each point in time) can be achieved simultaneously, when short-term production is profitable for the supply chain.

We showed the mechanics assuming that all variability in demand is explained by random promotion pricing and stockpiling. We have shown in Chapter 4 that there is a minor forecast error. However, it can be "simply" hedged with additional supply contracts. 95 percent of variability in demand is covered to perfection. Note that option contracts ensure supply in all cases but the case of very high demand above the maximum level derived by the two-segment forecast. That is, only part of the unexplained variation in demand must be hedged. Adding additional option contracts to the optimal order policy ensures supply in these extreme states of the world. Otherwise, there can be stockouts in very rare cases. Analogously, substituting some forward contracts with option contracts hedges against end-of-period
coverage from demand levels below the minimum demand level derived by the twosegment forecast. Consequently, zero out-of-stock and zero end-of-period coverage can be achieved even when there is a minor residual forecast error that cannot be explained by the two-segment forecast.

The manufacturer allows for stockouts whenever serving spot orders with shortterm production is not profitable (Proposition 3.2). Channel power on the retailer side results in a required minimum service level (Proposition 3.4). Still there will be stockouts on the manufacturer level if the required minimum service level is not raised to 100 percent (follows directly from Proposition 3.2).

Introducing option contracts allows for aligning decisions. The manufacturer is now required to provide the service level enforced by option purchases, as defined in the constraints for the production policy. For $s<c(\delta=0)<p_{\text {deep }}$, offering well priced option contracts is profitable from the manufacturer's point of view, because i) additional sales volumes increase supply chain profit and ii) the reservation fee is certainly received in all states of the world. As a result, the manufacturer can profitably offer a higher service level than in the status quo by bidding options into the market. From the retailer's point of view, introducing options is profitable, because i) total sales volumes increase and ii) goodwill costs decrease. Additional supply chain profit from increased sales can be distributed among both parties to achieve Pareto improvement. We show these effects in detail in a numerical study in the next chapter.

Combining forward contracts with option contracts allows for arbitrarily allocating inventories in the supply chain. The number of feasible allocations is limited by the coordination requirement. Granting a discount on forward contracts ensures that the supply chain is filled with promotion items at the beginning of a campaign. Option contracts provide the retailer with the necessary flexibility to avoid end-ofperiod coverage.

## Chapter 6 Numerical Study

A portfolio of risk sharing contracts can achieve coordination. We provide a stochastic program in Chapter 5. In the next step, we apply this program to data from the diapers industry. We develop a business case to show the effect of the hedging approach developed. Applying the insights generated throughout the previous chapters allows for calculating the expected benefits in a realistic setting. First, we discuss the setup of the business case and we model the status quo. Second, we consider several steps of improvement: Our target is to show that the manufacturer and the retailer can jointly improve their profit by i) employing a two-segment forecast and ii) adding forward and option contracts to their portfolio of supply contracts.

### 6.1 Setup of the numerical study

The numerical study is based on the market conditions of diapers retailing in Germany. We study the dynamics of FMCG markets in detail in Chapter 4. Consider a manufacturer of fast moving consumer goods, who serves a single retailer. This retailer employs frequent price promotions to attract deal-prone consumers. Price promotions regularly take a single week and there are two weeks of regular pricing between promotions.

Consider a planning period that contains two promotions. The timing of these two promotions has been agreed on in a promotion plan, which is known to both, the retailer and the manufacturer. We name these promotions the first and the second promotion within the forecast horizon. The retailer and the manufacturer negotiate on contract prices in a yearly negotiation. Then, time passes, until the planning period begins. In accordance with the time line developed in Section 5.1.2, we name this point in time $t_{0}$. The first promotion takes place at time $t_{1}$ and the second promotion at time $t_{2}$.

The retailer places her initial orders at the beginning of the planning period. At this point in time, demand at both promotions is uncertain. Demand depends on retail prices; however, the retailer decides on promotion prices only shortly before the


Fig. 6.1 Time line of decisions considered in the numerical study
beginning of a promotion. This behavior is optimal to remain unpredictable towards competitors and consumers, as discussed in Chapter 4. The retailer has the opportunity to place additional orders after the beginning of each promotion, when the promotion price and thus demand during this promotion is known. To summarize, the retailer has the opportunity to order at three points in time: i) at the beginning of the planning period, ii) after the promotion price of the first promotion is known and iii) after the promotion price of the second promotion is known. Ordering at any other points in time is not rational, as an equally well informed decision could have been made earlier.

The manufacturer observes the retailer's orders and then he decides on his optimal production policy. The manufacturer has access to the retailer's forecast (collaborative forecasting). The manufacturer realizes savings on production costs with longer production lead-times, as discussed in Chapter 5. Please consider Figure 6.1 for the order of decisions.

The data set analyzed in Chapter 4 provides retail prices and associated order-up-to levels for a diapers brand at a supermarket chain. Furthermore, we estimated consumption rates, reservation prices and holding costs of the deal-prone segment. We only consider the deal-prone consumer segment, as the loyal segment is predictable and thus it is not interesting from a hedging perspective.

Further, we assume reasonable values for variable production and holding costs. In order to account for the importance of end-of-period coverage, we assume zero salvage value at the retailer level. Thereby the channel is cleared after the end of the promotion period. Transportation costs and other costs associated with serving the retailer's orders are included in variable production costs. We take a marginal costs perspective and do not consider fixed costs. Please refer to Table 6.1 for a summary of the parameter estimates and choices employed. We employ again statistical units ( SU ) as the basic unit of measuring volumes. One SU equals 180 diapers.

Let the period of time between the starting dates of promotions be three weeks, such that $\tau_{0}=3$ weeks and $\tau_{1}=3$ weeks. Production costs are lead-time dependent. We assume that producing with only three weeks lead-time is 10 percent more expensive than producing with six weeks lead-time. Short-term production is 20 percent more expensive than long-term production. That is, we assume a linear cost function. Left-over inventories after the second promotion are salvaged at a loss. Before salvaging, the manufacturer pays holding costs. Salvaging inventory is always negative in terms of profit. The promotion items modeled are "outdated" after the six week promotion cycle. For example, a new packaging is applied or a new toy included in promotion items. We have seen the importance of end-of-period coverage considerations in practice and therefore, we explicitly take different salvage opportunities into consideration.

Table 6.1 Parameter choices for the numerical study

| Parameter | Description |
| :--- | :--- |
| $p_{\text {shallow }}=34.18$ Euro/SU | retail price for shallow discounts, $\approx 0.19$ Euro/diaper |
| $p_{\text {deep }}=32.04$ Euro/SU | retail price for deep discounts, $\approx 0.18$ Euro/diaper |
| $\varepsilon_{\text {shallow }}=0.75$ | probability of a shallow promotion |
| $\varepsilon_{\text {deep }}=0.25$ | probability of a deep promotion |
| $C^{d p}=13,800$ SU/week | consumption rate of the deal-prone consumer segment |
| $h^{d p}=2.06$ Euro/SU | holding costs of deal-prone consumers |
| $u=38.90$ Euro/SU | reservation price of deal prone consumers |
| $\tau_{0}=3$ weeks | time between initial orders and the first promotion |
| $\tau_{1}=3$ weeks | time between starting dates of both promotions |
| $c_{\text {two periods }}=25.0$ Euro/SU | long-term production costs (six weeks lead-time) |
| $c_{\text {one period }}=27.5$ Euro/SU | medium-term production costs (three weeks lead-time) |
| $c_{\text {zero lead-time }}=30.0$ Euro/SU | short-term production costs (no lead-time) |
| $h=5$ Euro/SU | holding and handling costs per SU of inventory |
| $g_{r}^{r}=10$ Euro/SU | costs of goodwill for every SU of demand not served |
| $v_{T}^{m}=16$ Euro/SU | salvage value for the manufacturer |
| $v_{T}^{r}=0$ Euro/SU | salvage value for the retailer |

### 6.2 Forecasting

In the status quo, both players employ the same historic forecast, which is the aggregated distribution of demand over a longer period of time. To derive the long-term distribution of demand, we consider steady time intervals between starting dates of promotions. At each promotion, either a deep or a shallow discount is offered to consumers. Please compare our results in Chapter 4 for the regular intervals between promotions and realized promotion prices.

Demand at a promotion is determined by the discount at the previous promotion and the current discount. Each promotion determines household inventories at the beginning of the following promotion. As promotions occur in regular time intervals, inventories at the beginning of each promotion can only have two values: a high value for deep discounts or a low value for shallow discounts during the last promotion. Given that each promotion has two possible states of the retail price, four states of demand can occur. The major gap in volume between the highest and the lowest possible states of demand shows why flexibility is so important in the fast moving consumer goods industry.

- A deep discount after a shallow discount: 45,900 statistical units of demand at the targeted promotion
- A deep discount after a deep discount: 41,400 statistical units of demand at the targeted promotion
- A shallow discount after a shallow discount: 31,600 statistical units of demand at the targeted promotion
- A shallow discount after a deep discount: 27,100 statistical units of demand at the targeted promotion


Fig. 6.2 The distribution of demand over the planning period

Knowing the probabilities of deep ( 25 percent) and shallow ( 75 percent) promotions in the data set, we derive the long-term distribution of promotion prices. Two shallow discounts after each other occur in 56.25 percent of all cases. Two deep promotions after each other are rather improbable and occur in only 6.25 percent of all cases. The two possible combinations of deep and shallow discounts occur with a probability of 18.75 percent each. Figure 6.2 shows the resulting discrete distribution of demand.

When employing historic forecasts, supply chain partners consider promotions to be independent. They do not derive any information on future demand from past promotion prices. This information structure has the form shown in the upper half of Figure 6.3. Consequently, demand planners do not realize that their knowledge on the previous promotion price reduces the number of possible states of demand to two states at each point in time (compare our analysis in Chapter 4). Compare the real structure of risk, as shown in the lower half of Figure 6.3. Demand depends on the history of retail prices.

### 6.3 Status quo

Consider the status quo of a manufacturer-retailer relationship based on wholesale price contracts. The supply chain operates in "pull" mode in the notation of Cachon (2004). We consider two cases: i) a powerful retailer requires a minimum service level and ii) a powerful manufacturer optimizes his production policy, without service level requirements. In this second case, the manufacturer avoids short-term production to achieve higher profit. These two scenarios show the range of reasonable outcomes of negotiations between retailers and manufacturers for the diapers category.


Fig. 6.3 Structure of risk under the historic and two-segment forecasts

Shortly before each promotion, the retailer observes the retail price and orders an optimal quantity in terms of profit. She pays a fixed wholesale price for each unit. As the retailer orders under certainty, she never holds inventories. Consequently, she successfully hedges against end-of-period coverage by requiring full flexibility on the demand side.

The manufacturer produces a portion of the expected orders in advance to save production costs. Long-term production decisions always take place under uncertainty. Consequently, the manufacturer holds inventories in low states of demand. These inventories can be employed to serve demand at the next promotion. Remaining orders are either served with the expensive short-term production technology or the manufacturer allows for stockouts.

### 6.3.1 The challenge of channel coordination

In the status quo, there is a single fixed retail price, which is independent from the point in time, when the order is placed. A rational retailer will postpone her
commitments as long as possible under these conditions, which is what we actually see in practice.

The retailer and the manufacturer agree on the wholesale price in a yearly negotiation. The wholesale price is fixed and cannot be adapted on short notice. In our study, the retailer would like to achieve a 100 percent service level, as it is always profitable to serve all demand. The retail price exceeds the wholesale price in all states of the world. This is reasonable, as retailers can be expected not to offer retail prices, which do not even cover variable costs.

Short-term production costs are below the deep promotion price plus goodwill costs and thus, the supply chain gains additional profit from producing short-term to serve demand. Still, short-term production costs can be above the wholesale price. Then, from the manufacturer's point of view, short-term production is too expensive to produce for instant orders (compare Proposition 3.2). The manufacturer only relies on long-term production with three and six weeks lead-time to serve demand. As a result, the 100 percent service level requested by the retailer is not reached for all promotion prices. The supply chain fails to achieve coordination due to double marginalization. Note that in contrast to other coordination models, the production decision must be coordinated, while the purchasing decision is optimal.

### 6.3.2 Channel power on the retailer side

In the first scenario, the retailer has sufficient channel power to request a minimum service level from the manufacturer. He optimizes the required minimum service level under the manufacturer's participation constraint. We assume that the manufacturer requires at least zero profit. The retailer extracts as much profit as possible, given the retail price. She earns a positive margin on every unit sold, independently of the state of demand reached. Therefore, she has an incentive to increase the required minimum service level as far as possible. The setting described is a multiperiod version of Scenario III. 2 in Chapter 3. We assume that the wholesale price equals 26.73 EUR/statistical unit ( $\approx 0.15 \mathrm{EUR} /$ diaper). This wholesale price reduces manufacturer profit to zero under a 81 percent required service level. That is, the manufacturer serves retailer orders in all cases but a deep discount after a shallow discount. Even in the case of a stockout, the loss is only 4,500 units. That is, 90 percent of consumers are served even in the highest state of demand. The corresponding out-of-stock rate of 1.9 percent during promotions fulfills the requirement of high availability in the FMCG industry.

The manufacturer chooses his optimal long-term and short-term production quantities in $t_{0}, t_{1}$ and $t_{2}$. We employ the stochastic program developed in Chapter 5 and find the optimal production policy as stated in Table 6.2. If the game would go on after $t_{2}$, the manufacturer would place additional long-term production; however, the direction of results would not change. The manufacturer splits production between short-term and long-term technologies. As a historic forecast is employed, long-term production quantities are equal in both periods. The manufacturer em-

Table 6.2 Optimal production policy under retailer power

| Time | Case | Action |
| :--- | :--- | :--- |
| $t_{0}$ |  | Produce 31,600 units for $t_{2}, 31,600$ units for $t_{1}$ |
| $t_{1}$ | $p_{1}=$ deep | Produce 9,800 units for $t_{1}$, stockout of 4,500 units |
| $t_{1}$ | $p_{1}=$ shallow | Serve all demand |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ deep | Produce 14,300 units for $t_{2}$, serve all demand |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | Serve all demand, salvage 4,500 units |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | Produce 9,800 units for $t_{2}$, stockout of 4,500 units |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | Serve all demand |

ploys his short-term technology to serve all orders until the required service level is exhausted. Table 6.3 shows revenues, costs and profit of both parties in the status quo. The supply chain earns a profit of 437,000 Euro, which is exclusively allocated to the powerful retailer.

Table 6.3 Revenues, costs and profit under retailer power

| Retailer | (in tEUR) | Manufacturer | (in tEUR) |
| :--- | ---: | :--- | ---: |
| Revenues | 2,255 | Spot Sales | 1,798 |
| Spot Costs | $-1,798$ | Production Costs | $-1,806$ |
| Holding Costs | -0 | Holding Costs | -4 |
| Goodwill Costs | -20 | Salvage Opportunity | 12 |
| Profit Retailer | $\mathbf{4 3 7}$ | Profit Manufacturer | $\mathbf{0}^{*}$ |

* The wholesale price is fitted to zero manufacturer profit under retailer power.


### 6.3.3 Channel power on the manufacturer side

Now consider the case of a powerful manufacturer. Although we do not see this scenario in the diapers industry, we find it worthwhile to be analyzed in order to build up an intuition for outcomes in cases of power on both sides. In the scenario analyzed, short-term production costs exceed the wholesale price. Therefore, shortterm production is not profitable under a wholesale price scheme. The manufacturer employs long-term production to reduce his costs. Long-term production constitutes a commitment on the manufacturer side. The inventory is physically at the manufacturer's warehouse at the targeted point in time. When demand is not sufficiently high, the manufacturer has to store inventory until the next promotion and he incurs holding costs. Thereby, he possibly earns negative profit in low states of demand. As a result, the manufacturer does not serve all demand in some states of the world. We analyzed this trade-off in detail in Chapter 3.

Table 6.4 Optimal production policy under manufacturer power

| Time | Case | Action |
| :--- | :--- | :--- |
| $t_{0}$ |  | Produce 31,600 units for $t_{2}, 31,600$ units for $t_{1}$ |
| $t_{1}$ | $p_{1}=$ deep | Allow for a stockout of 14,300 units |
| $t_{1}$ | $p_{1}=$ shallow | Serve all demand |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ deep | Allow for a stockout of 9,800 units |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | Serve all demand, salvage 4,500 units |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | Allow for a stockout of 14,300 units |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | Serve all demand |

A powerful manufacturer can refuse to employ unprofitable short-term production at all. Thereby, the model collapses to Scenario I.2, the pull-Newsvendor model. The wholesale price determines the manufacturer's service level, where only the long-term production technology is employed. However, there are two promotions in a row. To achieve comparable results, we assume that the retail price is again at $26.73 \mathrm{EUR} /$ statistical unit and that the retailer serves at least demand in shallow promotions. We find that the manufacturer allows for major stockouts to maximize his individual profit (Table 6.4). Deep promotions now always lead to stockouts. Thereby the manufacturer hurts supply chain profit. Table 6.5 shows the resulting revenues, costs and profit under manufacturer power.

Table 6.5 Revenues, costs and profit under manufacturer power

| Retailer | (in tEUR) | Manufacturer | (in tEUR) |
| :--- | ---: | :--- | ---: |
| Revenues | 2,098 | Spot Sales | 1,667 |
| Spot Costs | $-1,667$ | Production Costs | $-1,659$ |
| Holding Costs | -0 | Holding Costs | -4 |
| Goodwill Costs | -69 | Salvage Opportunity | 12 |
| Profit Retailer | $\mathbf{3 6 2}$ | Profit Manufacturer | $\mathbf{1 6}$ |
| $*$ |  |  |  |

* Given the wholesale price as under retailer power.


### 6.3.4 Supply chain efficiency in the status quo

An integrated firm serves all demand in all states given the data provided. Therefore, both scenarios do not reach the integrated channel profit of 461 tEUR. Under manufacturer power, the supply chain reaches only 82 percent of the possible channel profit. Retailer power increases channel profit to 95 percent of the profit in the integrated case; however, manufacturer profit is reduced to zero. There is still a gap of 5 percent of channel profit that is lost due to double marginalization. Figure 6.4


Fig. 6.4 Profit under retailer and manufacturer power
provides the respective profits of both parties as well as potential improvements through coordination under a historic forecast.

The wholesale price contract alone cannot coordinate the supply chain. The retailer's hedging strategy which is solely based on requiring flexibility from the manufacturer is not optimal.

### 6.4 Employing a two-segment forecast

In the next step, we consider the information provided by the two segment forecast. The retailer and the manufacturer realize that demand in the second promotion actually depends on the retail price in the first promotion. As a result, the manufacturer can adjust his production policy, after he perceives the first discount in $t_{1}$. We find that the two-segment forecast is beneficial in that the manufacturer can postpone part of his long-term production decision in $t_{0}$ until he receives additional information in $t_{1}$. He produces a lower amount with his long-term technology in $t_{0}$ for $t_{2}$ and produces an additional quantity in $t_{1}$ for $t_{2}$, when the discount in $t_{1}$ is only shallow. Thereby, he can eliminate end-of-period coverage in $t_{2}$.

Consider again the case of retailer power. The manufacturer's production policy changes, as shown in Table 6.6. We find that adding a two-segment forecast increases manufacturer profit without hurting retailer profit. The size of the effect depends on the form of the lead-time dependent discount. We analyze two cases, i) linear and ii) nonlinear production cost functions.

Table 6.6 Optimal production policy under a two-segment forecast

| Time Case | Action |  |
| :--- | :--- | :--- |
| $t_{0}$ |  | Produce 31,600 units for $t_{2}, 27,100$ units for $t_{1}(-4,500$ <br> units $)$ |
| $t_{1}$ | $p_{1}=$ deep | Produce 9,800 units for $t_{1}$, stockout of 4,500 units |
| $t_{1}$ | $p_{1}=$ shallow | Produce 4,500 units for $t_{2}(+4,500$ units $)$ |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ deep | Stockout of 9,800 units |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | Produce 14,300 units for $t_{2}$ |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | No production, salvage 0 units $(-4,500$ units $)$ |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | Produce 9,800 units for $t_{2}$, stockout of 4,500 units |

### 6.4.1 Production costs decrease linearly in lead-times

Under linear production cost discounts, supply chain profit increases by 3.0 tEUR. The reason for the limited impact of two-segment forecasts on profit in the setup described is straightforward: The manufacturer postpones part of his long-term production decision. Thereby, he saves on holding costs, as he only produces the full optimal advance production quantity, when the first discount is shallow. At the same time, he loses profit, as postponing part of his production decision increases production costs. These two effects are contrary and therefore, direct net impact on profit can be rather low.

### 6.4.2 Production costs decrease nonlinearly in lead-times

Now consider the case of a non-linear dependency of production costs on lead-times. It is reasonable to assume that the gain in efficiency between zero and three weeks lead-time exceeds the gain in efficiency between three and six weeks lead-time. We set $c_{\text {two periods }}=26.25 \mathrm{EUR} /$ statistical unit, $c_{\text {one period }}=26.25 \mathrm{EUR} /$ statistical unit and $c_{\text {zero lead-time }}=30.00 \mathrm{EUR} /$ statistical unit. These costs are chosen such that supply chain profit in the base case (retailer power) does not change. However, the impact of introducing the two-segment forecast increases by 300 percent to 12.5 tEUR. This result shows that the benefit from two-segment forecasts strongly depends on the relative importance of holding costs and production cost discounts.

### 6.5 Sharing risk with supply contracts

Consider the case with retailer power to be the status quo. The manufacturer is in an unfortunate situation, receiving zero profit. In the next step, we show that the manufacturer can i) coordinate the supply chain and ii) achieve positive profit, by bidding option contracts into the market.

In the status quo, demand risk is very unevenly distributed in the supply chain. The two-segment forecast reduces risk; however, it does not eliminate risk. The key challenge is that the manufacturer receives negative profit from serving wholesale price contracts with short-term production. Long-term production is, however, too risky to serve high states of demand. Risk sharing contracts can solve this challenge. We add option contracts to the portfolio of supply contracts offered to the retailer.

### 6.5.1 Achieving coordination

The manufacturer does not serve spot orders, when total orders for a promotion exceed the required minimum service level. To achieve an optimal service level that exceeds the required minimum service level, option contracts are needed. Option contracts are always served and thus, the supply chain optimal service level can be reached. Contract pricing assures that both players achieve Pareto improvement.

We employ option contracts with maturity one period after the purchase date. That is, the retailer can purchase options in $t_{0}$ for execution in $t_{1}$ and in $t_{1}$ for execution in $t_{2}$. We show that this limited offer of option contracts is sufficient to achieve coordination under the Pareto improvement constraint in the setting described.

The retailer reserves a sufficient number of options to be able to serve demand in all states. Now, the retailer benefits from the two-segment forecast, too, as she can adjust option reservations. When the last discount was deep, consumers have still leftover household inventories. Therefore, the retailer orders 4,500 options less than in case of a previous shallow promotion. The number of options executed equals demand, as it is profitable to serve all consumers. Demand is served in all states of the world and consumers do no longer experience stockouts. Coordination is achieved and supply chain profit increases by 26 tEUR to 464 tEUR. Note that profit under coordination now includes benefits from the two-segment forecast.

Supply chain profit increases by five percent to the amount earned by the integrated supply chain. This additional profit comes from two sources (Figure 6.5). First, the retailer saves goodwill costs, as a higher service level is achieved. The size of this effect depends i) on the gap between the service level in the status quo and the coordinating service level and ii) on the importance of the goodwill costs. Second, the supply chain earns profit from the additional units sold to consumers. The amount of supply chain profit earned per incremental unit depends on production costs and on the deep promotion price.

### 6.5.2 Pricing option contracts

We set the execution fee at marginal production costs to avoid renegotiation: $e=25$ EUR/statistical unit ( $\approx 0.14 \mathrm{EUR} /$ diaper). Then, the reservation fee determines the allocation of both, risk and profit, on the two supply chain parties. The minimum

Table 6.7 Optimal reservation, execution and production policies with options

| Time Case | Action |  |
| :--- | :--- | :--- |
| $t_{0}$ |  | Reserve 45,900 options for $t_{1}$ |
| $t_{1}$ | $p_{1}=$ deep | Reserve 41,400 options for $t_{2}$, execute 45,900 options |
| $t_{1}$ | $p_{1}=$ shallow | Reserve 45,900 options for $t_{2}$, execute 31,600 options |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ deep | Execute 41,400 options |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | Execute 27,100 options |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | Execute 45,600 options |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | Execute 31,600 options |
| $t_{0}$ |  | Produce 31,600 units for $t_{2}, 27,100$ units for $t_{1}$ |
| $t_{1}$ | $p_{1}=$ deep | Produce 14,300 units for $t_{1}$ |
| $t_{1}$ | $p_{1}=$ shallow | Produce 4,500 units for $t_{2}$ |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ deep | Produce 14,300 units for $t_{2}$ |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | No production |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | Produce 14,300 units for $t_{2}$ |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | No production |



Fig. 6.5 The origin of additional supply chain profit under coordination
reservation fee depends on the manufacturer's optimization problem. The manufacturer receives zero profit when the reservation fee is lowered to $1,40 \mathrm{EUR} /$ statistical unit, given that all demand is served in all states of demand. This reservation fee represents the lower bound of the set of Pareto-improving prices. The upper bound of the Pareto set is driven by the retailer's profit function. The retailer requires at least the profit earned in the status quo. We find that the retailer demands a reservation price of $1,55 \mathrm{EUR} /$ statistical unit or lower to participate. Otherwise, she simply ignores the manufacturer's offer and returns to the status quo.

Table 6.8 Revenues, costs and profit under option contracts

| Retailer | (in tEUR) | Manufacturer | (in tEUR) |
| :--- | ---: | :--- | ---: |
| Revenues | 2,317 | Spot Sales | 0 |
| Spot Costs | -0 | Reservation Fees | 134 |
| Reservation Costs | -134 | Execution Fees | 1,731 |
| Execution Costs | -1731 | Production Costs | $-1,854$ |
| Holding Costs | -0 | Holding Costs | -0 |
| Goodwill Costs | -0 | Salvage Opportunity | 0 |
| Profit Retailer | $\mathbf{4 5 2}$ | Profit Manufacturer | $\mathbf{1 1}$ |

Contract pricing allows for arbitrarily distributing additional profit from coordination on the two supply chain parties. A reservation fee of 1.48 EUR/statistical units equally shares benefits from coordination. Table 6.8 shows the associated revenues, costs and profit.

### 6.6 Considering the effect of delivery lead-times on supply chain profit

So far, we assumed that the manufacturer's costs of serving an order depend only on the lead-time between production and delivery. However, it is reasonable to assume in addition that the transportation cost component of the manufacturer's costs of serving an order depends on the lead-time between commitment and latest delivery (compare Cachon 2004). Spot orders and option executions must be served immediately, while forward contracts allow for more time to deliver the products to the retailer.

Consider a scenario, where forward contracts allow reducing transportation costs. Then, the supply chain can earn additional profit, given that the retailer commits early for a fraction of her purchases. We first discuss pricing forward contracts and then, we quantify the benefits achievable.

### 6.6.1 Pricing forward contracts

The manufacturer always serves forward orders. Thereby, the retailer hedges supply risk; however, forwards transfer inventory risk to the retailer. Again we find a Pareto set of contract parameters achieving an increase in profits of both players.

The retailer does not accept forward contracts, when the forward price exceeds the sum of the reservation and the execution fees. For lower forward prices, the retailer purchases at least the certain fraction of demand employing forward contracts. Thus, the upper boundary equal 26.48 EUR/statistical unit given an execution fee of 25.00 EUR/statistical unit and a reservation fee of 1.48 EUR/statistical unit.

The lower boundary is defined by the manufacturer's Pareto improvement constraint. We assume that each unit ordered under a forward scheme saves 0.50 EUR/statistical unit in transportation costs, which is less than 2 percent of the costs of serving an order. Then, the manufacturer requires at least a forward price of 25.98 EUR/statistical unit. Note that the retailer does not order more than the certain fraction of demand even at the lower bound of the Pareto set, in the setting described. The discount offered is not sufficient to outbalance expected holding costs for higher forward orders.

### 6.6.2 Benefits achieved with forward contracts

Consider a forward price of 26.23 EUR/statistical unit. This forward price lays within the Pareto set and shares benefits from introducing forwards evenly between both parties. The manufacturer's production policy is independent from forward orders, as long as forward orders do not exceed optimal long-term production quantities. The retailer's optimal order and execution policies are shown in Table 6.9. Both supply chain parties improve profit. Table 6.10 provides revenues, costs and profit, given a portfolio of option and forward contracts.

Table 6.9 Optimal purchasing policy under a portfolio of forward and option contracts

| Time Case | Action |  |
| :--- | :--- | :--- |
| $t_{0}$ |  | Reserve 14,300 options for $t_{1}$, buy 31,600 forwards for $t_{1}$ <br> $t_{1}$ |
| $p_{1}=$ deep | Reserve 14,300 options for $t_{2}$, buy 27,100 forwards for $t_{2}$, <br> execute 14,300 options |  |
| $t_{1}$ | $p_{1}=$ shallow | Reserve 14,300 options for $t_{2}$, buy 31,600 forwards for $t_{2}$, <br>  <br> $t_{2}$ |
| $p_{1}=$ deep and $p_{2}=$ deep | execute 0 options |  |
| $t_{2}$ | $p_{1}=$ deep and $p_{2}=$ shallow | Execute 14,300 options |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ deep | Execute 14,300 options |
| $t_{2}$ | $p_{1}=$ shallow and $p_{2}=$ shallow | Execute 0 options |

### 6.7 Summary

We found that wholesale price contracts do not achieve coordination. The resulting loss in supply chain profitability depends on the distribution of power in the supply chain. Retailer power improves supply chain efficiency; however, manufacturer profit can be reduced to zero. We discussed three levers of improvement. i) Two-segment forecasts reduce inventory requirements and end-of-period coverage at the manufacturer level. ii) Option contracts achieve coordination and Pareto improvement. iii) Forward contracts reduce costs to serve an order and thus provide

Table 6.10 Revenues, costs and profit under portfolios of hedging contracts

| Retailer | (in tEUR) | Manufacturer | (in tEUR) |
| :--- | ---: | :--- | ---: |
| Revenues | 2,317 | Spot Sales | 0 |
| Spot Costs | -0 | Reservation Fees | 42 |
| Reservation Costs | -42 | Execution Fees | 179 |
| Execution Costs | -179 | Forward Sales | 1,628 |
| Forward Costs | $-1,628$ | Production Costs | $-1,853$ |
| Holding Costs | -0 | Holding Costs | -0 |
| Goodwill Costs | -0 | Transportation Savings | 31 |
| Profit Retailer | $\mathbf{4 6 8}$ | Profit Manufacturer | $\mathbf{2 7}$ |

additional profit to both parties. We assume without loss of generality that the manufacturer earns zero profit under retailer power. Figure 6.6 shows that supply chain profit can be increased by 13 percent in the conservative case of a 81 percent initial service level (retailer power) and a linear discount on production costs. Therefore, hedging price promotions using options and forwards can be highly profitable.


Fig. 6.6 Profit in the status quo and under a portfolio of hedging contracts

## Chapter 7 <br> Conclusion and Suggestions for Future Research

In this chapter, we summarize the key insights derived throughout the analysis. Nine statements point out the key takeaways. Then we provide directions for future research in this area. The dissertation ends with a conclusion.

### 7.1 Key insights

The major contribution of this dissertation is to integrate research on risk sharing supply contracts into models of price promotions. In the first step, we extended the model by Cachon (2004) to consider portfolios of forward, option and spot contracts. We derived closed-form solutions for optimal contract parameters and considered the cases of short-term production and a dominant retailer. In the second step, we built upon the work of Blattberg et al. (1981) and Iyer and Ye (2000) to derive a stochastic process of demand in an environment that is characterized by frequent promotion activities. This stochastic process enabled us to describe the structure of demand risk in such a setting. We developed profound understanding of causality between promotional pricing and demand fluctuations. Finally, we developed a stochastic dynamic model, which links portfolios of hedging contracts to the stochastic promotion process. We calibrated the model of demand using a point-of-sales data set. In a subsequent simulation the model matched data very well. We employed the parameters estimated to assess a business case and we proved that our approach to hedging price promotions is Pareto-efficient. We summarize our key insights in nine statements:

Key Insight 1 There exist contract parameters, such that portfolios of options, forwards and spot contracts are the optimal choice of the retailer (see Lemma 3.6).

We derived analytical solutions for contract parameters such that mixed portfolios of risk sharing contracts are reached. Thereby, we showed that portfolios of forward, option and spot contracts are feasible. We derived boundary conditions
for optimal contract parameters and found that a range of distributions of risk and physical inventories can be reached under supply chain coordination.

Key Insight 2 Assuming that the manufacturer is risk-neutral, the manufacturer's long-term production decision can be independent from the retailer's order decision when profitable short-term production is possible (see Proposition 3.2) or when a high service level is required (see Proposition 3.3). This result is different from the standard model without short-term production and retailer power (see Proposition 3.1).

We found that the manufacturer's production policy can be independent from the retailer's orders under short-term production, when production costs are cheap enough to produce short-term to serve instant orders. Under channel power, this result additionally holds for a required minimum manufacturer service level, which exceeds i) the manufacturer's optimal service level from long-term production and ii) the retailer's option and forward orders. Then, the manufacturer's profit maximization problem collapses to a cost minimization problem. He derives an optimal production policy specifying the amount of long-term production.

Key Insight 3 Flexible production does not achieve coordination in all cases. Thus, investing in short-term production capacities without considering appropriate supply contracts may not be the optimal solution (see Proposition 3.2).

When a manufacturer hedges demand risk with flexible production and inventories, the supply chain can fail to coordinate. This is the case, when short-term production cost restrict its use to serving instant orders up to the required minimum service level. Portfolios of coordinating contracts can achieve the optimal service level.

Key Insight 4 Bidding portfolios containing options and forwards into the market can achieve Pareto improvement in terms of profit in a pure pull setting with a dominant retailer. Both profit and inventories can be distributed in the supply chain (see Propositions 3.1 and 3.3).

We showed that employing portfolios of hedging contracts can achieve Pareto improvement. Retailer profit stays constant or increases, when a new contract is introduced, as the retailer can always insist on keeping the status quo. The manufacturer bids contracts into the market. He must choose the contract parameters such that he does not lose profit compared to the status quo. Option contracts share risk and achieve coordination. Forward contracts distribute inventories in the supply chain.

Key Insight 5 The channel should avoid excess inventory at the retailer level to avoid hurting future promotions. Therefore, a portfolio of hedging contracts must contain option contracts. Forward contracts are not sufficient in this setting (see Lemma 3.9).

A portfolio of forward and spot contracts can achieve risk sharing, as shown by Cachon (2004). We showed that this portfolio is not sufficient to hedge risk arising from FMCG promotions. Promotion items change frequently. Therefore, the retailer cannot risk holding major inventories. Retailer inventories are also not in the interest of the manufacturer, as they block the channel from future promotions. Adding option contracts to the portfolio results in independence of the allocation of risk and the location of physical inventory. Thereby coordination can even be achieved in a setting, where the retailer cannot salvage inventory.

Key Insight 6 Price promotions are interdependent. Past promotions have an important impact on demand during future promotions. A stochastic process can be employed to forecast demand during promotions (see Chapter 4).

Price promotions increase household inventories. Demand during promotions depends on inventory positions. Therefore, each price promotion has an effect on future promotions. Consumers' consumption rates limit the time horizon of the effect. A model of price promotions must take into consideration the interdependent structure of promotions. A simple event-based adjustment mechanism is not sufficient to represent the path-dependent mechanisms determining demand.

Key Insight 7 Optimal purchasing and production policies can be implemented as stochastic programs as shown in Chapter 5.

Finding optimal purchasing and production policies is possible with few effort. The numerical study in Chapter 6 shows that a limited number of maturities is sufficient to achieve a coordinating portfolio.

Key Insight 8 It is possible that a portfolio containing option contracts achieves zero out-of-stock situations and zero end-of-period coverage at the retailer level simultaneously, when short-term production is profitable for the supply chain and when the residual forecast error of the two-segment forecast is small. Existing wholesale price contracts can be dominated by option and forward contracts. Zero end-of-period coverage at the manufacturer level can be achieved employing a twosegment forecast (see Chapter 6).

Achieving perfect service levels at zero retailer inventories at the end of the sales period is not impossible as generally assumed. Consequently, the trade-off discussed in industry may actually be overcome by means of supply options in many cases. Targeting zero manufacturer inventories at the end of the sales period is achievable, although it may not be optimal for any parameter choice.
Key Insight 9 A portfolio of hedging contracts must contain forward contracts, when longer transportation lead-times reduce supply chain costs. Then, a portfolio of option and spot contracts alone is not sufficient (see Chapter 6).

A portfolio must contain forward contracts, when delivery costs are lead-time dependent. Then, the manufacturer can offer discounts to achieve long-term commitments from the retailer. Thereby, he can realize savings on transportation costs that can be distributed between both players. Option and spot contracts cannot achieve these savings, as deliveries only occur last minute.

### 7.2 Suggestions for future research

This section provides suggestions for further investigations in how to optimally hedge retail promotions. We propose five approaches for future research.

## Experimental evidence for domination of wholesale prices by option contracts

There are established non-coordinating wholesale price contracts in FMCG relationships. We showed that bidding option contracts into the market can coordinate the channel, as optimally price option contracts dominate non-coordinating instant order contracts. Thus, a rational retailer will choose options over instant orders. However, it is unclear, whether managers would actually make the rational order decision. We suggest to run laboratory experiments with FMCG managers to develop insights into the acceptance of option schemes. Buy-back contracts are widely spread in the FMCG industry; however, these contracts do not support flexible production. Option contracts are of similar complexity and achieve superior results. It would be interesting to survey, under what conditions managers are willing to switch from wholesale price schemes and/or buy-back schemes to option schemes.

## Perishable products

Consumer react to promotions by stockpiling. However, household inventories are only possible, when the product is non-perishable. This was one of our key assumptions. Restricting the model to perishable products results in a major simplification. Promotions are then no longer connected and can be analyzed as being independent events. The analysis shown in Chapter 3 applies. This simplified model could be employed as a base case for considering, e.g., moral hazard issues.

## Moral hazard with respect to sales effort

Flexible contracts may result in a need to induce the retailer to exert sales effort early in a promotion campaign. Forward contracts could provide those incentives. Compare Krishnan et al. (2004) for the benefits of non-refundable inventories in terms of retailer sales effort. The retailer knows that end-of-period coverage hurts her profitability and thus, she focuses on selling inflexible inventories of one product, instead of execution option contract to sell more of another product. Moral hazard issues could thus be analyzed in a setup considering retail competition.


Fig. 7.1 From wasteful promotions to truly efficient promotions

## Independent flexible and inflexible suppliers

We assumed that option and forward contracts are sold by the same supplier. Sting and Huchzermeier (2010) discusses supply risk, considering a flexible local and an inflexible cheap supplier who is located abroad. Proximity allows for shorter leadtimes and better responsiveness at higher cost. Consider a supply chain where one retailer sources from two manufacturers. Competing manufacturers with differentiated capabilities could achieve an equilibrium that is different from our results in Chapter 3.

## Competitive promotions

Stockpiling has a strong competitive effect. Consumers, who hold sufficient inventory, are not on the market, which is why promotions have a preemptive effect. The role of competition is complex, as both competitive brands and alternative retailer locations have to be considered. Every promotion by every brand in the category and by every retailer in the region can potentially increase household inventories of a subgroup of consumers. In addition, Ailawadi et al. (2007) show that promotions can trigger sales of competitive brands, which are not on promotion, as behavioral effects come into play. Still, our simplified approach provides excellent empirical fit on the data set employed.

An important question for retailers is, how to optimally behave in markets with a large number of switching customers. The hedging strategies developed can serve as a starting point; however, our model neglects competitive effects of hedging portfolios. It would be interesting to see, whether risk sharing could have impact on pricing strategies under retail competition. On the one hand, retailers take some demand risk in our model as option contracts share risk between both parties. Consequently, it could be possible that retailers reduce their exposure to demand risk by reducing their promotion frequency and/or adapting their pricing strategy. On the other hand, additional margins achieved by adopting the concept of efficient retail promotions allow for more aggressive promotions. Thus, the effect of more efficient promotions on the competitive equilibrium is a fruitful field for future research.

### 7.3 Conclusion

This dissertation is about portfolios of forward, option and instant order contracts and their ability to coordinate supply chains. Most literature on channel coordinating considers only a single supply contract. We show that combining coordinating contracts with wholesale price contracts in portfolios achieves independence of inventory risk from the location of physical inventories. Thereby, the supply chain can be coordinated while physical inventories can be distributed at will. A portfolio of hedging supply contracts can achieve efficient promotions. Very high service levels at zero inventories result in a strong competitive position. In the context of FMCG promotion campaigns, end-of-period coverage at the retailer level can be avoided to achieve a flexible supply chain that sustains frequent innovations. Adapting contract parameters allows to fill the channel with inventory, or to achieve more efficient shipping. Figure 7.1 shows how emergency orders and stockouts disappear, when option contracts are added to the portfolio of supply contracts offered. Optimal contract parameters achieve coordination and potentially result in a 100 percent service level.

Supply chain parties in the FMCG industry complain that the way, the game is presently played does not allow for coordination. This dissertation uncovers that myth and shows that coordination can be achieved even when an established instant order price is in place, as option contracts can be priced such that instant orders are dominated. The additional supply chain revenues can be employed to achieve Pareto improvement. In the FMCG industry, lengthy discussions about short-term ordering at the cost of low service-levels are obsolete in this way. Frictions between supply chain partners are not a necessity, but incentives can be aligned. This would be a huge step forward for retailer-manufacturer relations in the FMCG industry and could be a first step towards successful implementation of more collaborative improvement initiatives. Consequently, retailers and manufacturers can focus on serving consumers.

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[^0]:    ${ }^{a}$ Bidding option contracts into a market with a fixed suboptimal wholesale price
    ${ }^{b}$ Bidding a coordinating portfolio of option and forward contracts into a market with a fixed suboptimal wholesale price

