

**Three Essays on  
Individual Investors' Early Exercise Behavior  
in the Fixed-Income Market**

Dissertation

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# Einleitung

Zahlreiche Finanzprodukte für Privatinvestoren enthalten heutzutage vorzeitige Kündigungs- oder Rückzahlungsoptionen, welche dem Investor das Recht geben, bereits vor Ablauf der vereinbarten Laufzeit seine Anlage zurückzufordern oder seine Schulden vorzeitig (zumindest teilweise) zu tilgen. Klassische Beispiele für solche Produkte sind Bundesschatzbriefe, Hypothekenkredite oder auch Festgeldkonten mit Sonderkündigungsrechten. Ein derartiges vertraglich vereinbartes Recht erhöht offensichtlich die Flexibilität und damit die Sicherheit des Investors, erschwert jedoch gleichzeitig die Beurteilung und Bewertung solcher Finanzanlagen auf Seiten der Gegenpartei. Die klassische Optionspreistheorie hat in diesem Zusammenhang verschiedene Modelle entwickelt, um vorzeitige Kündigungsrechte aus finanzmathematischer Sicht fair – also ohne Arbitragemöglichkeiten – zu bewerten. Eine wesentliche Annahme in diesen Modellen ist jedoch grundsätzlich, dass die beteiligten Parteien das Kündigungsrecht an dem theoretisch optimalen Zeitpunkt in Anspruch nehmen.

Aus empirischer Sicht scheint diese Annahme schwer haltbar zu sein. So weisen Studien auf Aktienmärkten darauf hin,<sup>1</sup> dass Investoren vorzeitige Kündigungsrechte überwiegend “suboptimal” einsetzen. Insbesondere zeigt sich, dass Investoren regelmäßig finanziell attraktive vorzeitige Ausübungsmöglichkeiten ungenutzt verstreichen lassen. Vielmehr nehmen sie ihr Optionsrecht zumeist an einem aus finanzmathematischer Sicht zu späten Zeitpunkt in

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<sup>1</sup>Untersuchungen über die Ausübung von Optionsrechten im amerikanischen Aktienmarkt finden sich beispielsweise bei Diz und Finucane (1993), Overdahl und Martin (1994), Dawson (1996), Finucane (1997), Poteshman und Serbin (2003), Pool et al. (2008) und Barraclough und Whaley (2012). Ähnliche Untersuchungen für den schwedischen Markt finden sich bei Engström et al. (2002), für den chinesischen Markt bei Liao et al. (2013).

Anspruch oder vernachlässigen es sogar bis zum Ende der Laufzeit komplett, obwohl eine vorzeitige Ausübung ihnen einen finanziellen Vorteil gebracht hätte. Weitere suboptimale Entscheidungen sind offensichtlich auch zu frühe Ausstiege von Investoren, also an Zeitpunkten, an denen der Fortführungswert des jeweiligen Produktes den Ausübungswert (noch) übersteigt.

Es ist jedoch festzustellen, dass sich die skizzierte Forschung nahezu ausschließlich auf den US-amerikanischen Markt und auf wenige ausgewählte Aktienderivate konzentriert. Die Nutzung vergleichbarer Kündigungsrechte im sehr viel größeren Markt der strukturierten Zinsprodukte ist – mit Ausnahme des Hypothekengeschäfts<sup>2</sup> – kaum erforscht. Gleichzeitig liefert die aktuelle Literatur auch nur vereinzelt Erkenntnisse über das Ausübungsverhalten bestimmter Investorengruppen. So ist beispielsweise nur wenig über die Strategien der spezifischen Gruppe von Privatinvestoren bekannt, obwohl für diese – unter Berücksichtigung der vermeintlich weniger fundierten Kapitalmarktkenntnisse und des vermeintlich geringeren Zeit- und Ressourceneinsatzes im Vergleich zu professionellen Großinvestoren – ein suboptimales Anlageverhalten besonders naheliegend erscheint.

An dieser Stelle setzt die vorliegende Arbeit an. Drei Essays untersuchen, wie Privatinvestoren vorzeitige Kündigungsrechte in strukturierten Zinsprodukten nutzen. Als Grundlage für die Analyse dient hierbei ein neuartiger, nicht öffentlich verfügbarer Datensatz, der über einen Zeitraum von circa 13 Jahren Entscheidungen von mehr als 800.000 Privatinvestoren über ein weiteres Halten oder eine vorzeitige Kündigung von Puttable Bonds (Bundesschatzbriefen) abbildet. Das Ziel der Arbeit ist es, das Verständnis von finanziellen Entscheidungsstrategien von Privatinvestoren theoretisch und empirisch zu erweitern sowie mögliche Unterschiede innerhalb dieser Investorengruppe und im Vergleich zu anderen Kapitalmarktakteuren zu

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<sup>2</sup>Untersuchungen zur Nutzung von impliziten Rückgaberechten im Hypothekengeschäft finden sich beispielsweise bei Schwartz und Toruos (1989), Stanton (1995) und Kalotay (2004).

identifizieren. Darüber hinaus soll die Arbeit mögliche Handlungsfelder für Emittenten und Banken, welche vergleichbare Finanzprodukte anbieten, aufzeigen.

Der inhaltliche Schwerpunkt der drei Essays orientiert sich an dieser Zielsetzung. Zu Beginn wird in einem ersten Beitrag ein neues Modell zur Abbildung und Bewertung von Entscheidungsstrategien für Optionsrechte aus Sicht eines Investors entwickelt und an empirische Daten kalibriert.<sup>3</sup> Als wesentlicher neuer Beitrag zur Literatur wird in diesem Modell die Annahme einer kontinuierlichen Entscheidungsfindung durch den Investor aufgegeben und stattdessen endogen eine optimale Entscheidungsstrategie ermittelt, die potentiell auftretende Transaktions- und Entscheidungskosten berücksichtigt. Der Schwerpunkt des zweiten Beitrags liegt auf einer detaillierten empirischen Analyse der Nutzung von vorzeitigen Rückgaberechten in Putable Bonds durch Privatinvestoren.<sup>4</sup> Erstmals in der Literatur werden hierbei auch Unterschiede in der ökonomischen Nutzung solcher Kündigungsrechte in Abhängigkeit von persönlichen Eigenschaften des Investors beleuchtet – beispielsweise differenziert nach Alter, Geschlecht, Wohnort und vorheriger Investitionserfahrung. Schließlich untersucht der dritte Beitrag den Einfluss der Kuponstruktur von Putable Bonds. Auf Basis von Simulationsrechnungen und Szenarioanalysen wird diskutiert, wie Emittenten durch spezifische Produktgestaltungen finanzielle Vorteile in Bezug auf die empirisch zu beobachtenden Ausübungsmuster von Privatinvestoren generieren können.

Wie bereits angedeutet, befasst sich der erste Essay dieser Arbeit (*“Individual Investors and Suboptimal Early Exercises in the Fixed-Income Market”*) mit einem neuen theoretischen Modell zur Bewertung von Optionsrechten aus Sicht eines Investors. Aufbauend auf

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<sup>3</sup>Dieses Kapitel entspricht nach Absprache mit dem Betreuer dem Working Paper “Individual Investors and Suboptimal Early Exercises in the Fixed-Income Market”, welches in Ko-Autorenschaft mit Oliver Entrop und Marco Wilkens entstanden ist.

<sup>4</sup>Dieses Kapitel entspricht nach Absprache mit dem Betreuer dem Working Paper “What Makes Individual Investors Exercise Early? Empirical Evidence from the Fixed-Income Market”, welches in Ko-Autorenschaft mit Oliver Entrop und Marco Wilkens entstanden ist.



dem klassischen risiko-neutralen Bewertungsansatz in der Optionspreistheorie steht im Mittelpunkt dieses neuen Ansatzes die endogene Bestimmung einer zeitpunktspezifischen optimalen Entscheidungs- und Ausübungsstrategie unter Berücksichtigung von Transaktions- und Entscheidungskosten. Die ökonomische Intuition hinter der modellierten Entscheidungsfindung ist hierbei, dass ein Investor, der aktuell ein Finanzprodukt mit einem vorzeitigen Kündigungsrecht hält, dessen Fortführungswert weit oberhalb des aktuellen Ausübungswertes liegt, typischerweise nicht sofort im nächsten Zeitpunkt eine erneute Entscheidung über ein weiteres Halten oder eine vorzeitige Ausübung des Produktes treffen wird. Vielmehr wird er eine gewisse Zeit verstreichen lassen, bevor er in eine solche neue Entscheidung "investiert". Im Gegensatz ist zu erwarten, dass der gleiche Investor in der Regel schon nach sehr viel kürzerer Zeit sein Investment erneut bewerten wird, falls der aktuelle Produktwert sehr nahe an dem Ausübungswert liegt.

In der Folge legt dieser erste Beitrag dar, dass derartige nicht-kontinuierliche Entscheidungsstrategien zu einem verminderten Wert des Ausübungsrechtes aus Sicht des Investors führen. Komparative statische Analysen verdeutlichen, dass die Gründe für den sinkenden Optionswert in einer geringeren optimalen Entscheidungsfrequenz und einer längeren durchschnittlichen Zeitspanne zwischen zwei Entscheidungen über ein weiteres Halten oder Ausüben einer Position liegen. Es besteht das Risiko, potentiell ökonomisch attraktive Ausübungsmöglichkeiten zu "überspringen" und somit zu verpassen. Eine empirische Kalibrierung des Modells offenbart dementsprechend auch deutliche Unterschiede zwischen einer Bewertung mit Standardmodellen aus der Literatur und einer Bewertung anhand des neu vorgestellten Modellansatzes. Die geschätzten Modellparameter implizieren dabei, dass viele der untersuchten Privatinvestoren tatsächlich nicht-kontinuierlichen Entscheidungsstrategien folgen. Weiterhin deuten die Kalibrierungsergebnisse darauf hin, dass ein Großteil der Privatinves-

toren nur sehr träge auf geänderte Zinsumgebungen und damit auf neue Bewertungssituationen reagiert und dass es einen hohen Anteil von weitgehend passiven Investoren gibt, die grundsätzlich attraktive Ausübungsentscheidungen auslassen – was mit sehr hohen Entscheidungskosten gleichzusetzen ist.

Der zweite Essay (*“What Makes Individual Investors Exercise Early? Empirical Evidence from the Fixed-Income Market”*) ergänzt den modelltheoretisch geprägten ersten Teil um eine detaillierte empirische Untersuchung des Ausübungsverhaltens von Privatinvestoren für Puttable Bonds. Mit circa 31 Millionen analysierten monatlichen Entscheidungen über ein weiteres Halten oder vorzeitiges Ausüben solcher Bonds bietet diese Studie eine größere Detailschärfe als die meisten vergleichbaren Studien in der Literatur. Die Untersuchung setzt zwei Schwerpunkte: Der erste Abschnitt identifiziert Determinanten der vorzeitigen Rückgabe von Puttable Bonds. In einem zweiten Abschnitt steht die ökonomische Bewertung der empirischen Entscheidungsstrategien von Privatinvestoren im Vordergrund.

Die Ergebnisse zeigen, dass nicht nur Bewertungsänderungen, sondern auch zahlreiche weitere Einflussfaktoren, wie beispielsweise Wachstumsphasen im Aktienmarkt, die Dauer bis zur nächsten Kuponauszahlung oder auch die Investmenthistorie die Wahrscheinlichkeit einer vorzeitigen Kündigung von Puttable Bonds determinieren. Zudem dokumentiert die Untersuchung deutliche Unterschiede im Verhalten von Privatinvestoren in Abhängigkeit von persönlichen Merkmalen. So nehmen zum Beispiel Investoren aus einer mittleren Altersklasse von 20 bis 40 Jahren das vorzeitige Kündigungsrecht sehr viel öfter in Anspruch als Investoren anderen Alters. Insgesamt lässt sich allerdings festhalten, dass – analog zu den angeführten Untersuchungen für Aktienderivate – Privatinvestoren nur einen Bruchteil der aufkommenden ökonomisch attraktiven Ausübungsmöglichkeiten nutzen. Zudem ist die Mehrheit der beobachteten Rückgaben von Puttable Bonds aus theoretischer Sicht nicht sinnvoll. Der Um-

fang der verpassten Chancen und suboptimalen Ausübungsentscheidungen ist hierbei sogar deutlich höher als im Falle der eingangs erwähnten Aktienderivate, was in der Konsequenz zu einer im Durchschnitt negativen Überschussrendite für viele Investoren führt.

In Summe deutet die Analyse des empirischen Ausübungsverhaltens darauf hin, dass eine Steigerung der Investmentperformance nicht das alleinige Motiv hinter der Entscheidung von Privatinvestoren für ein weiteres Halten oder Ausüben ihrer Positionen ist. Stattdessen scheinen der Bedarf nach Liquidität und der Bedarf nach finanzieller Flexibilität weitere treibende Faktoren zu sein. Tatsächlich könnten Liquiditätsargumente einige ungewöhnliche Muster in den empirischen Daten erklären. So liegt es beispielsweise nahe, dass bei Investoren mittleren Alters ein Hauskauf oder eine Familiengründung zu einem höheren Liquiditätsbedarf führt. Auch eine explorative Faktoranalyse, welche die konsolidierte Anlagestrategie von Investoren auf einer individuellen Basis näher untersucht, impliziert, dass im Rahmen der Anlage- und Ausübungsentscheidungen für Puttable Bonds Liquiditäts- und Flexibilitätsargumente für viele Privatinvestoren eine höhere Relevanz aufweisen als eine mögliche Verbesserung der Investmentperformance.

Der dritte Essay (*“Behavioral Financial Engineering in the Fixed-Income Market: The Influence of the Coupon Structure”*) untersucht, aufbauend auf den beiden vorherigen Beiträgen, den Einfluss der Kuponstruktur auf die Anlage- und Ausübungsstrategie von Privatinvestoren in Puttable Bonds. Obwohl der Produktstruktur aus theoretischer Sicht regelmäßig eine entscheidende Rolle bei der Entscheidungsfindung von Investoren zugebilligt wird, finden sich in der Literatur bisher nur ausgesprochen vereinzelt Studien zur empirischen Bedeutung von solch unterschiedlichen Produktmerkmalen. Die vorliegende Untersuchung führt jedoch nicht nur zu neuen Erkenntnissen bezüglich einer bisher wenig erforschten Fragestellung, sondern bietet auch praktische Implikationen für Emittenten und andere Finanzmarktintermediäre.

So verdeutlichen die Ergebnisse, dass “Behavioral Financial Engineering”, also ein zielgruppenspezifisches Angebot bestimmter Produktstrukturen, das auf ein spezielles antizipierbares Verhalten abzielt, zu deutlichen finanziellen Vorteilen führen kann.

Im Fokus der Analyse stehen drei Entscheidungssituationen von Privatinvestoren. Neben dem Einfluss der Kuponstruktur auf die erstmaligen Investitionsentscheidungen wird der Einfluss auf vorzeitige Ausübungsentscheidungen sowie auf Portfolioentscheidungen von Privatinvestoren untersucht. Als wesentliches Ergebnis zeigt sich, dass Produkte mit einer hohen finalen Kuponzahlung und mit einer vergleichsweise kurzen gewichteten Kapitalbindungsdauer (Duration) stärker nachgefragt und länger gehalten werden als finanzmathematisch gleichwertige Produkte, die lediglich eine andere, meist flachere, Kuponstruktur aufweisen. Vergleichbare Zusammenhänge lassen sich im Rahmen von Portfolioentscheidungen beobachten. Diese Präferenz der Investoren für optisch vermeintlich attraktivere Angebote steht offensichtlich nicht im Einklang mit der klassischen Finanztheorie, nach der nur der faire Wert eines Produktes für eine Ausübungs- oder Halteentscheidung relevant ist. Stattdessen deutet das empirische Verhalten von Privatinvestoren auf einen “Behavioral Bias” hin. Offenbar wird Zinsprodukten mit einer steilen Kuponstruktur regelmäßig ein höherer Wert zugeschrieben als vergleichbaren Produktangeboten.

In der Folge erörtert der dritte Beitrag, welche angebotenen Kuponstrukturen Vorteile aus Sicht von Emittenten und anderen Finanzmarktakteuren bieten können. Hierzu wird im Rahmen von Szenarioanalysen für eine Vielzahl von beispielhaft konstruierten Puttable Bonds und Zinsszenarien der faire Wert des jeweiligen Produktes mit einem geschätzten empirischen Wert verglichen, welcher sich an den beobachteten Ausübungsstrategien von Privatinvestoren orientiert. Die Ergebnisse zeigen, dass Emittenten grundsätzlich auf zwei Arten von einer gezielten Gestaltung der Kuponstruktur profitieren können. Zum einen ist für Bonds, die

zum Laufzeitende steil ansteigende Kuponzahlungen und eine kurze durchschnittliche Kapitalbindungsdauer bieten, ein im Vergleich geringerer Mittelabfluss während der Laufzeit zu erwarten, was dem Emittenten eine Reduzierung seiner Liquiditätsreserven ermöglicht. Zum anderen implizieren die Simulationsergebnisse, dass sich die größte Differenz zwischen fairem und empirischem Wert bei Puttable Bonds mit einer konkaven oder flachen Kuponstruktur ergibt. Der Grund ist, dass bei derartigen Produktstrukturen der theoretische Wert des Optionsrechts in der Regel deutlich höher ist als bei Produkten, die über die Laufzeit ansteigende Kuponzahlungen anbieten. Da Privatinvestoren – wie zuvor erläutert – ökonomisch optimale Ausübungsgelegenheiten nur selten wahrnehmen, profitiert der Emittent von einer solchen hohen Bewertung des vorzeitigen Kündigungsrechts.

Zusammenfassend verdeutlichen die Ergebnisse dieser Arbeit, dass Privatinvestoren, die strukturierte Zinsprodukte mit vorzeitigen Kündigungsrechten halten, empirisch regelmäßig anderen Ausübungsstrategien folgen als in der klassischen Finanztheorie modelliert. Drei Besonderheiten werden im Rahmen der folgenden Essays im Detail diskutiert. Erstens, durch Entscheidungskosten induzierte nicht-kontinuierliche Entscheidungsstrategien können zu einer aus theoretischer Sicht suboptimalen Nutzung des vorzeitigen Kündigungsrechts führen. Zweitens, neben der Steigerung der Investmentperformance scheinen weitere Motive, wie beispielsweise die Nachfrage nach Liquidität und finanzieller Flexibilität, eine wichtige Rolle in der Entscheidungsfindung von Privatinvestoren einzunehmen. Drittens, die Kuponstruktur eines Puttable Bonds kann die Entscheidung über ein weiteres Halten oder eine vorzeitige Rückgabe beeinflussen. Als eine wesentliche Schlussfolgerung dieser Arbeit ergibt sich, dass Anbieter von strukturierten Zinsprodukten für Privatinvestoren deutliche Vorteile aus einem erweiterten Verständnis des Verhaltens dieser spezifischen Investorengruppe ziehen können. Insbesondere können sie durch eine gezielte Produktgestaltung das überwiegend suboptimale

Ausübungsverhalten von Privatinvestoren zu ihrem Vorteil nutzen und somit (unter anderem) potentiell höhere Renditen realisieren.

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# 1 | Individual Investors and Suboptimal Early Exercises in the Fixed-Income Market\*

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## Abstract

This paper is the first to analyze and value early exercises of Individual Investors in fixed-income investment products. Assuming decision and transaction costs we consider that a continuous decision-making on holding or exercising is not optimal anymore and propose a new approach to modeling exercise decisions, which endogenously determines the optimal decision strategy. Calibrating our model to a unique data set of about 880,000 early exercises in non-tradable puttable bonds over a time period of 13 years indicates that Individual Investors (i) act very heterogeneously, (ii) behave as if they face significant individual transaction and decision costs, (iii) react sluggishly, and that (iv) exogenous effects such as taxes or investors' desire for liquidity additionally influence early exercise behavior.

**Keywords:** decision strategy; decision costs; transaction costs; puttable bond

**JEL classification:** G10, G12, G13

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## 1 Introduction

The right to exercise or redeem early before maturity, is a common feature of many investment and credit products for Individual Investors, such as savings bonds, stock and index options, mortgages or other financial innovations. However, while Individual Investors' trading and exercise behavior has been researched extensively with regard to stocks, equity index options and mortgages, surprisingly little is known about Individual Investors' use of options and derivatives on fixed-income markets.<sup>1</sup> This is particularly remarkable as Individual Investors' portfolios comprise interest-earning assets much more often than equity products (for Germany: Statistisches Bundesamt, 2010; for the U.S.: United States Census Bureau, 2010; for the U.K.: Office for National Statistics, 2012).

With this paper we aim to fill this gap in the current literature and present a theoretical and empirical analysis of Individual Investors' early exercise decisions in standard fixed-income products. In doing so we make two major contributions. First, we develop a new rational model to determine Individual Investors' early exercise decisions within the standard framework of risk-neutral derivatives valuation. Second, we present, to the best of our knowledge, the first comprehensive overview of Individual Investors' empirical early exercise decisions in puttable bonds (German Federal Saving Notes) over a period of 13 years. Based on this large and unique data set we calibrate and apply our model. Our results provide new insights in Individual Investors' exercise behavior and are—among others—highly relevant for the risk and liquidity management of issuers of fixed-income products.

In general, it is well known in the literature that investors use early exercise rights “sub-

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<sup>1</sup>As mentioned, a notable exception is the broad strand of literature about prepayment rights in mortgages and mortgage-backed securities such as, e.g., Schwartz and Toruos (1989), Schwartz and Torous (1992) and Deng et al. (2000), who propose reduced form models, and, e.g., Dunn and McConnel (1981a,b), Stanton (1995) and Kalotay et al. (2004), who focus on structural models.

optimally”. They commonly fail to follow the optimal strategy and forgo the theoretically optimal exercise (e.g., Overdahl and Martin, 1994; Dawson, 1996; Pool et al., 2008; Barraclough and Whaley, 2012). Instead, a substantial number of investors exercise early but at a time when it is not rational to do so, or they completely neglect the exercise right (e.g., Gay et al., 1989; Diz and Finucane, 1993; Finucane, 1997; Engström et al., 2000; Poteshman and Serbin, 2003; Liao et al., 2013). In addition, the literature provides some empirical evidence that non-professional investors perform even more poorly in exploiting the option component, compared to other market participants (e.g., Poteshman and Serbin, 2003; Barraclough and Whaley, 2012). The main arguments—besides irrationality—for this suboptimal exercise behavior are individual transaction costs (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006; Pool et al., 2008), the costs of learning early exercise rules (e.g., Barraclough and Whaley, 2012; Liao et al., 2013), and the effort required to continuously monitor the investment (e.g., Gay et al., 1989; Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2013). Regarding transaction costs, it is argued that the premium for exercising early, i.e. the difference between the exercise and the continuation value, must not only be positive for an investor but actually exceed the transaction costs incurred. Consequently, an exercise right might be used more seldom or differently than predicted by standard models with no frictions. Learning and monitoring costs are assumed to lead to non-continuous decision-making, which can result in the neglect of ex-post attractive exercise opportunities and thus in suboptimal early exercises.

Intuitively, in view of such costs, it is a more rational strategy for an investor holding a far out of the money early exercise right to let some time pass by before he “invests” in the next decision regarding holding or exercising, since the currently almost worthless option is not expected to be in the money very soon. On the other hand, if the early exercise right is

currently close to the money, the same investor will most likely decide to make the next decision on holding or exercising his option within a short time, because it is more probable that a profitable early exercise opportunity will soon occur. Still, such rational decision strategies have so far rarely been considered in the literature on early exercises of derivatives. Abel et al. (2007), Abel et al. (2009) and Alvarez et al. (2012) investigate “rational inattention” and the implications of “observation costs”, but focus on optimal portfolio choices and consumption.<sup>2</sup> Most related to our approach, Stanton (1995) proposes a rational prepayment model for mortgage-backed securities, where mortgage holders do not re-evaluate their portfolio at every possible time but only at random, discrete intervals. However, as a key innovation we replace in our model Stanton’s random component with an endogenously determined, optimal strategy. This optimal “decision strategy” defines when to make a decision on continuing to hold or on exercising a financial derivative with an incorporated early exercise right. As part of this strategy, we consider also potential transaction costs if the derivative is exercised and additionally that costs may be incurred for each decision (“decision costs”) on holding or exercising.

As a second contribution to the literature, we apply and calibrate our model on a large and unique data set. Over a sample period from July 1996 to February 2009 we examine about 880,000 early exercises of German Federal Saving Notes (GFSN), a simple fixed-income product comparable to a puttable bond. Focusing on GFSN allows us to analyze the isolated early exercise behavior of Individual Investors nearly free of any distortions, since these products are almost exclusively sold to Individual Investors and cannot be traded on a secondary market. This means that Individual Investors in GFSN only face the problem of deciding whether to hold or exercise rather than holding, exercising or selling their position as in the

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<sup>2</sup>A newer strand of literature on executive stock options such as Carpenter et al. (2010) concentrates also on optimal exercise strategies that differ from standard models. Still, modeling a rational decision strategy is not the focus of these studies.

case of, e.g., stock options. Hence, in contrast to similar studies such as those by Poteshman and Serbin (2003) or Barraclough and Whaley (2012), who focus on early exercises of exchange-traded options, we do not have to distinguish between different market participants like institutional or retail investors, and can base our calibration on a more comprehensive overview of Individual Investors' early exercise decisions.

We observe a very good fit for our model, which we attribute to the endogenous determination of the optimal exercise strategy that—in contrast to pure empirical approaches (such as the reduced form models of Schwartz and Toruós, 1989; or Schwartz and Torous, 1992)—allows also to make rational predictions in changing economic environments, as also pointed out by Stanton (1995). Moreover, we stress four findings from the calibration: first, our results suggest that Individual Investors act heterogeneously, which is in line with the observations of, e.g., Stanton (1995) for mortgage holders or Koziol (2006) for warrant holders, and with the general characterization of Individual Investors' behavior by Barber and Odean (2011). Second, a large number of Individual Investors act as if they face significant transaction and decision costs. Third, many investors behave sluggishly, i.e. they do not exercise at the optimal exercise opportunity, but with a delay. Yet another significant group of “passive investors” never make use of their early exercise option, even when it seems strongly advantageous to do so, which is equivalent to very high transaction or decision costs. Fourth, we identify a stable base exercise rate, independent of any market movements, and observe some unusual peaks in the empirical data, which we attribute to both exogenous effects such as need for liquidity or tax-induced early exercises. Based on our findings we conclude, like, e.g., Poteshman and Serbin (2003) or Barraclough and Whaley (2012) in the case of equity options, that Individual Investors' early exercise decisions in the fixed-income market can clearly differ from the decisions of institutional investors, which results in a present value

advantage due to lower option costs for issuers. For our sample we estimate this difference between the standard model value and the empirical value at more than 2%, which is in line with the studies of Stanton (1995) or Koziol (2006), among others.

The remainder of the paper is organized as follows. In Section 2, we present our model and investigate optimal decision and exercise strategies as related to transaction and decision costs. Section 3 introduces our data set and presents summary statistics on the empirical early exercises of Individual Investors. In Section 4, we calibrate the model, test the robustness of our results and estimate the financial advantage for issuers derived from the diverging exercise strategies of Individual Investors. Section 5 concludes and discusses policy implications.

## 2 Model

In the following, we consider a derivative with an American early exercise right, for which payoff at early exercise is reduced by transaction costs and costs arise with every decision to hold or exercise the derivative (“decision costs”). Since the latter implies that decision-making is costly for an investor, a continuous decision-making strategy for holding or exercising is no longer worthwhile. Instead, a rational investor defines after each decision based on all current information at which upcoming point in time it would be best to make the next decision on the derivative. This endogenous determination of the optimal “decision strategy” is a key component of our model.

### 2.1 Model setup

We consider a continuous-time economy over the time period  $[0, T]$ . Let  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, Q)$  be a probability space equipped with a filtration  $(\mathcal{F}_t)_t$  fulfilling the usual conditions, i.e. it

is right-continuous and complete.  $Q$  is the equivalent spot martingale measure where the money market account serves as numéraire. The adapted short rate process is denoted by  $r(t)$ . We discretize time into equidistant time steps  $0 = t_0, \dots, t_N = T$  with  $t_{i+1} - t_i = \Delta t$ . Let  $\mathbb{T} = \{t_0, \dots, t_N\}$ .

The investor holds a derivative  $X$  whose payoff at maturity  $T$  is given by  $X_T$ .<sup>3</sup> Additionally, the derivative can be exercised early in  $t_1, \dots, t_{N-1}$  with a respective payoff  $X_{t_i}$  in  $t_i$ . Note that we do not have to specify the type of underlying (e.g., stock or bond) and its dynamics. The only condition we require is that the payoff of the derivative  $X_{t_i}$  in  $t_i$  is  $\mathcal{F}_{t_i}$ -measurable, i.e. given the information up to  $t_i$  the early exercise payoff or final payoff on the respective date is known, and integrable. Transaction and decision costs in  $t_i$  are denoted by  $TC_{t_i}$  and  $DC_{t_i}$ , respectively, and are also  $\mathcal{F}_{t_i}$ -measurable and integrable.<sup>4</sup>

### *Decision strategy*

Our first step is defining the points in time at which the investor decides to make a decision on the exercise. We call a non-decreasing sequence  $\Gamma = (\Gamma_n)_{n=0, \dots, N}$  of  $(\mathcal{F}_{t_i})$ -stopping times  $\Omega \rightarrow \mathbb{T}$  a *decision strategy* and each  $\Gamma_n$  a *decision point*, if the sequence fulfills the following conditions:

$$\Gamma_0 = 0, \tag{1.1}$$

$$\Gamma_n < \Gamma_{n+1} \text{ if } \Gamma_n \neq t_N, \tag{1.2}$$

$$\Gamma_{n+1} \text{ is } \mathcal{F}_{\Gamma_n}\text{-measurable.} \tag{1.3}$$

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<sup>3</sup>For the sake of brevity and ease of notation, we do not consider payoffs from the derivative here, such as coupon payments from bonds, before exercise or maturity. However, these can easily be integrated.

<sup>4</sup>Transaction and decision costs can be expressed as a fixed sum or as a relative value. Throughout this paper we work with absolute cost levels, however, our methodology can easily be applied with relative cost definitions.

Condition (1.1) simply means that we start today. Condition (1.2) says if a decision is made at decision point  $\Gamma_n$  the next decision point lies in the future, given that  $\Gamma_n$  is not yet the end of the considered period.<sup>5</sup> In condition (1.3),  $\mathcal{F}_{\Gamma_n}$  is the  $\sigma$ -algebra associated with the stopping time  $\Gamma_n$  representing the information available at  $\Gamma_n$ .<sup>6</sup> This condition means that the next decision point  $\Gamma_{n+1}$  is known at the decision point  $\Gamma_n$ . For example, today the investor determines the next decision point  $\Gamma_1$ . Next, in  $\Gamma_1$  based on the available information at that point in time, he determines  $\Gamma_2$ , i.e. when to make the next decision, etc.

### *Exercise strategy*

Given a decision strategy  $\Gamma$ , the derivative can only be exercised at the decision points  $\Gamma_n$  based on the information associated with  $\Gamma_n$ , yielding a payoff of  $X_{\Gamma_n}$ . Exercising the derivative therefore means stopping the process  $(X_{\Gamma_n})_n$ . Based on the decision strategy  $\Gamma$ , we call a function  $\tau^\Gamma : \Omega \rightarrow \{1, \dots, N\}$  an *exercise strategy* if it is a  $(\mathcal{F}_{\Gamma_n})$ -stopping time.

### *Strategy*

The investor has to jointly choose both the decision and the exercise strategy. Therefore, we call a pair  $(\Gamma, \tau^\Gamma)$  a *strategy* that consist of a decision strategy  $\Gamma$  and a respective exercise strategy  $\tau^\Gamma$ . The point in time  $\tau$  where the investor exercises the derivative is then given by:

$$\tau = \Gamma_{\tau^\Gamma}. \tag{1.4}$$

Note that  $\tau$  is an  $(\mathcal{F}_{t_i})$ -stopping time.<sup>7</sup>

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<sup>5</sup>As  $\Gamma = (\Gamma_n)$  is a non-decreasing sequence, condition 1.2 implies that if  $\Gamma_n = t_N$  the following decision points  $\Gamma_{n+k}$  also have the value  $t_N$ . Note that these  $\Gamma_{n+k}$  do not provide any new information and could be omitted. However, allowing the sequence  $\Gamma$  to have always the same length simplifies notation here.

<sup>6</sup>I.e.  $\mathcal{F}_{\Gamma_n} = \{A \in \mathcal{F} : A \cap \{\Gamma_n \leq t_i\} \in \mathcal{F}_{t_i} \text{ for all } t_i \in \mathbb{T}\}$ .

<sup>7</sup>Note that any strategy determines via (1.4) a unique  $(\mathcal{F}_{t_i})$ -stopping time (up to null-sets). On the other

*Optimal strategy and value*

Given a strategy  $(\Gamma, \tau^\Gamma)$ , the investor's cash flow consists of the exercise or final payoff  $X_\tau$ , the transaction costs  $TC_\tau$  and the decision costs  $DC_{\Gamma_n}$  occurring at the decision points until exercise or maturity, i.e. in  $\Gamma_1, \dots, \Gamma_{\tau^\Gamma}$ . We define the sum of the discounted cash flows as  $X_{(\Gamma, \tau^\Gamma)}$ :

$$X_{(\Gamma, \tau^\Gamma)} = \exp\left(-\int_0^\tau r(s)ds\right) (X_\tau - TC_\tau) - \sum_{i=1}^{\tau^\Gamma} \exp\left(-\int_0^{\Gamma_i} r(s)ds\right) DC_{\Gamma_i}, \quad (1.5)$$

where the first summand of (1.5) is the discounted payoff from the derivative minus the transaction costs occurring from the exercise in  $\tau = \Gamma_{\tau^\Gamma}$ . The second summand represents the discounted decision costs that result from the  $\tau^\Gamma$  decisions made until the early exercise or maturity date in  $\tau$ . Given a specific strategy  $(\Gamma, \tau^\Gamma)$ , standard pricing theory implies that the "value" of the position equals  $E^Q\left(X_{(\Gamma, \tau^\Gamma)}\right)$ .

The rational investor aims at choosing a strategy that maximizes his wealth position. This means he chooses a strategy  $(\Gamma, \tau^\Gamma)^{opt}$  such that:

$$E^Q\left(X_{(\Gamma, \tau^\Gamma)^{opt}}\right) = \sup_{(\Gamma, \tau^\Gamma)} E^Q\left(X_{(\Gamma, \tau^\Gamma)}\right) \quad (1.6)$$

holds. Accordingly, we get  $V_0 = E^Q\left(X_{(\Gamma, \tau^\Gamma)^{opt}}\right)$  as the value of the derivative in  $t = 0$ .

In Appendix A we prove the existence of an optimal strategy  $(\Gamma, \tau^\Gamma)^{opt}$ , show how it can be constructed and show that the value of the derivative can be calculated via backward induction. We continue with a solution for the optimal strategy. First, we provide the results on the construction of the value process  $V_{t_i}$  via backward induction:

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hand, any  $(\mathcal{F}_{t_i})$ -stopping time  $\tau$  can be represented via (1.4), for example by setting  $\Gamma_n = t_n$  and  $\tau^\Gamma = \tau$ . However, this representation is not necessarily unique as there may be different strategies inducing a specific  $\tau$  via (1.4).



- In  $t_N$ , i.e. at maturity, set:

$$V_{t_N} = X_{t_N} - TC_{t_N} - DC_{t_N}. \quad (1.7)$$

- Let the value of the derivative be known for each  $t_k > t_n$ . Then the value in  $t_n$  is:

$$V_{t_n} = \max(X_{t_n} - TC_{t_n}, CV_{t_n}) - DC_{t_n} \quad (1.8)$$

$$\text{with } CV_{t_n} = \max_{t_k > t_n} E^Q \left( \exp \left( - \int_{t_n}^{t_k} r(s) ds \right) V_{t_k} \middle| \mathcal{F}_{t_n} \right), \quad (1.9)$$

where  $CV_{t_n}$  is the “continuation value” in  $t_n$ .

- In  $t_0 = 0$  set:

$$V_0 = CV_0. \quad (1.10)$$

Based on this, the optimal strategy  $(\Gamma, \tau^\Gamma)^{opt}$  can be constructed as follows:

- In  $t_0 = 0$  set:

$$\Gamma_0 = 0. \quad (1.11)$$

- Assume that the decision point  $\Gamma_n$  has been constructed. Then the next decision point is:

$$\Gamma_{n+1} = \begin{cases} \min \left( t_i > \Gamma_n : CV_{\Gamma_n} = E^Q \left( \exp \left( - \int_{\Gamma_n}^{t_i} r(s) ds \right) V_{t_i} \middle| \mathcal{F}_{\Gamma_n} \right) \right) & \text{for } \Gamma_n < t_N \\ t_N & \text{for } \Gamma_n = t_N. \end{cases} \quad (1.12)$$

- The optimal exercise strategy is given by:

$$\tau^\Gamma = \min(\{i : X_{\Gamma_i} - TC_{\Gamma_i} \geq CV_{\Gamma_i}, \Gamma_i < t_N\} \cup \{i : \Gamma_i = t_N\}). \quad (1.13)$$

As already stated, a formal proof can be found in Appendix A. In the following, we briefly describe the procedure and provide economic intuition for the results.

Starting at maturity (1.7), the value of the derivative equals its payoff minus transaction and decision costs. In our following analyses, we usually set these costs at zero, i.e.  $TC_{t_N} = DC_{t_N} \equiv 0$ , which we find most plausible, as the payoff at maturity is usually not an investor's decision and as it is usually cost-free. For all other points in time  $t_n$ , equation (1.8) shows that the value equals the maximum of the immediate exercise payoff less transaction costs  $X_{t_n} - TC_{t_n}$  and the continuation value  $CV_{t_n}$ , i.e. the derivative value given it is not exercised in  $t_n$ , minus decision costs. This is a standard procedure in rational decision-making, also applied in standard American option pricing theory: investors exercise a derivative if—and only if—the immediate exercise value minus transaction costs is larger than the continuation value. Note that while the transaction costs in  $t_n$  influence the exercise decision in  $t_n$ , the decision costs in  $t_n$  do not, as they occur anyway since a decision on the exercise is made.

The key difference of our model to standard American option pricing is the structure of the continuation value  $CV_{t_n}$  (see 1.9) that equals the maximum of the conditional expectations of discounted values in  $t_k > t_n$ :  $E^Q \left( \exp \left( - \int_{t_n}^{t_k} r(s) ds \right) V_{t_k} \middle| \mathcal{F}_{t_n} \right)$ . This is highly intuitive for the following reason: let us assume the investor is in  $t_n$  and he is not exercising the derivative, which implies that he has to make another decision at the next decision point. As he aims at maximizing his wealth, he will choose that  $t_k$  as the next decision point that maximizes the value of his position in  $t_n$ . Note that the value of the derivative, given  $t_k$  is the next

decision point, is  $E^Q \left( \exp \left( - \int_{t_n}^{t_k} r(s) ds \right) V_{t_k} \middle| \mathcal{F}_{t_n} \right)$ , so that the investor will choose that  $t_k$  that maximizes this expression. This can also be seen in the structure of the optimal decision strategy (1.12): the optimal next decision point is the first upcoming point in time when the respective expected discounted value equals the continuation value. Finally, equation (1.13) expresses that the derivative should be exercised at the first decision point at which the exercise value minus transaction costs is not smaller than the continuation value or at maturity.

It is clear that the optimal decision strategy strongly depends on decision costs. For example, if the decision costs are very high at any point in time before maturity, it would be optimal to forgo the early exercise opportunity and to choose maturity as the next decision point. In this case the value of the derivative would equal the value of its European version. If all decision costs are zero, i.e. decision-making is cost-free, it would be optimal for an investor to make a decision at each point in time, as he might otherwise miss favorable exercise opportunities by “skipping over” certain dates. This is also implied by our model, as it is straightforward to show that in this case the maximum in (1.9) is always reached in  $t_{n+1}$  so that the continuation value becomes  $CV_{t_n} = E^Q \left( \exp \left( - \int_{t_n}^{t_{n+1}} r(s) ds \right) V_{t_{n+1}} \middle| \mathcal{F}_{t_n} \right)$  implying  $\Gamma_{n+1} = t_{n+1}$  in (1.12). Here our procedure coincides with standard American option pricing models.

We note that our methodology is not restricted to valuing and investigating fixed-income investments of Individual Investors, as is the focus in the following. Based on the endogenous determination of an optimal decision and exercise strategy, it is also possible to thoroughly assess a wide range of derivatives with early exercise rights with relation to transaction and decision costs, or other financial products where an endogenously modeled decision strategy is required.

## 2.2 Numerical example and comparative-static analyses

For illustration, we apply our model in the following to analyze a simple fixed-income derivative under transaction and decision costs. We are mainly interested in the influence of both costs on (i) the value of the derivative, on (ii) the optimal decision strategy, where we investigate the average number of decisions and the average duration between two decision points, and on (iii) the timing of early exercises. Furthermore, we examine the influence of changes in the market environment (volatility) on these results.

We consider an exemplary puttable bond with a maturity of 7 years and a notional amount of 1 that promises a yearly coupon of 5%, where coupons are accrued and paid at maturity. In addition, an investor has an early exercise right. In case of early exercise  $x.y$  years after issuance, he receives the notional amount plus accrued interest, i.e. the exercise value is given by  $1.05^x + 0.05y$ .<sup>8</sup>

The short rate process  $r(t)$  in this numerical exercise is given by the 1-factor-model of Hull and White (1990), i.e.  $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$ , where  $\kappa$  is the mean reversion speed,  $\sigma$  the short rate volatility,  $W(t)$  a standard Brownian motion and  $\theta(t)$  defined according to Hull and White (1994) to fit the current term structure (see, e.g., Brigo and Mercurio (2006) for implementation details). In our standard scenario we set  $\kappa = 20\%$ ,  $\sigma = 2.5\%$  and assume the term structure to be flat at 5%. The latter implies that the hold-to-maturity value of the bond, i.e. the value of the bond without option, equals par, so the value of the derivative can be no less than 1. We consider possible transaction costs between 0% and 12% and decision costs between 0% and 2.5%, but assume that both costs are constant for an investor over time and that these costs do not occur at maturity.

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<sup>8</sup>This profile is a simplified version of the type of bonds (Type B GFSN) we analyze empirically in Section 3.

The valuation according to (1.7) to (1.10) is carried out via Monte-Carlo simulation with 10,000 runs for each cost combination and a step size of  $\Delta t = 1/12$ , i.e. we allow for 84 time steps until maturity using Euler discretization of the short rate process. To compute the conditional risk-neutral expectation values in (1.9), we basically follow the least square Monte-Carlo approach proposed by Longstaff and Schwartz (2001).<sup>9</sup> However, in contrast to Longstaff and Schwartz (2001) we determine at each step not only the conditional expectation values for the next point in time, but perform regressions for all upcoming steps until maturity, so that we can identify the time point with the respective highest conditional expectation value.<sup>10</sup> To reduce a potential upward bias in our results (see for details Broadie and Glasserman, 1997), we apply the interleaving estimator as introduced by Longstaff and Schwartz (2001) and described in detail in Glasserman (2004).

Figure 1 presents the valuation results dependent on transaction and decision costs.

[Figure 1 about here.]

As expected, we find the highest value for our derivative with approximately 1.06 in case of zero transaction and decision costs. Hence, the early exercise right has a value of  $1.06 - 1 = 0.06$ , which coincides with the results from standard valuation models. On the other end, with very high transaction and decision costs, the bond value falls to 1, which means the early exercise right is worthless. For transaction and decision costs between these extremes we recognize in general a strong negative impact of both costs on the valuation, whereby the results are more sensitive regarding changes in decision costs. The first economic intuition is that under transaction and decision costs an investor receives only a reduced payoff at an early exercise and must furthermore account for the cumulated decision costs arising at

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<sup>9</sup>A detailed analysis of this method is provided in Clément et al. (2002).

<sup>10</sup>As basis function for the regression we choose the first four monomials and a constant. Testing different forms of basis function, we find that this choice performs best regarding efficiency and robustness.

each decision. Consequently, there is a trade-off after every decision between “investing” very soon in another decision on further holding or exercising the bond and alternatively waiting some time but risking missing out on ex-post attractive exercise opportunities. Naturally, an investor chooses here the strategy, as described in (1.12), that maximizes the value of the early exercise right. To make the effect of transaction and decision costs on this strategy and thus eventually also on the valuation more obvious, we compare the average number of decisions (Figure 2) and the average duration between two decision points (“decision interval”, Figure 3) on the optimal decision strategy.<sup>11</sup>

[Figure 2 about here.]

[Figure 3 about here.]

Beginning again with the case of zero transaction and decision costs, we observe very frequent decisions until the bond is exercised or redeemed<sup>12</sup> and an average decision interval that lies at approximately 1.0 step. Highly plausible, this implies a de facto continuous decision strategy. It is most to the advantage of an investor to make a decision at every possible point in time, which results in the highest value of the exercise right, as seen before. Higher transaction and, in particular, decision costs induce a more selective optimal decision strategy and a lower average number of decisions. Accordingly, the mean decision interval widens with higher costs, i.e. the investor allows on average more time to pass by before he makes a new decision on his investment. This is reasonable, because if, for example, the early exercise right is currently out of the money, the next potentially attractive exercise point—i.e. the point in time where the option is so deep in the money that it can also account for incurring

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<sup>11</sup>The respective averages are calculated under the risk-neutral measure  $Q$  over all simulated paths.

<sup>12</sup>Compared to the theoretical maximum of 84 decisions, the average number of decisions is relatively low here, which is mainly due to the assumed significant volatility combined with a long maturity of 7 years, a situation that facilitates many early exercises.

transaction and decision costs—tends to be at a later point in time than without these costs. Overall, the lower average number of decisions and the wider decision interval comprehend and amplify the negative effect of transaction and decision costs on the value, since possibly lucrative exercise opportunities might be skipped over and thus the advantage of the option is reduced. Finally, in the case of very high transaction and decision costs, the average number of decisions approaches 1 and the decision interval converges to the bond's maturity (84 months). The possible premium of early exercising is not expected to exceed the incurring costs at any time and it is optimal for the investor to fully neglect his exercise right and to simply hold the bond until maturity. Hence, the early exercise right has no value for the investor.

Still, an investor's optimal strategy and the valuation depend also on further factors, such as the market environment or product characteristics. In the following, we exemplarily examine the sensitivities of our results regarding changes in the volatility.<sup>13</sup> Figure 4 compares the valuation, the average number of decisions and the decision interval for a low (1.0%), a normal (2.5%) and a high volatility (4.0%) scenario, whereby the middle charts correspond to our standard scenario.

[Figure 4 about here.]

Starting with the low volatility scenario, we observe consistently lower values for all combinations of transaction and decision costs compared to our former results. Clearly, this is due to the one-sided opportunity of the early exercise right, which suffers (gains) from a reduced (enhanced) volatility in the market. Correspondingly, the average optimal number of decisions declines, since the probability of an early exercise opportunity that compensates

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<sup>13</sup>Separate analyses not reported here show that the economic relations regarding changes in the product characteristics are similar to the results of the volatility analysis. Again, the results of our model are very reasonable.

also for the cumulated transaction and decision costs is lower than in a moderate or high volatility environment. We observe a strong sensitivity regarding changes in transaction and decision costs, so already with moderate costs typically no exercise is any longer feasible and the average decision interval approaches maturity. On the other hand, in the high volatility scenario, the value of the early exercise right increases substantially and even in the case of high transaction and decision costs the option is not forfeited. Accordingly, our model suggests here a denser average decision frequency with shorter decision intervals, which is reasonable since in a high volatility environment more opportunities for early exercises come up.

While the sensitivity analyses above focus on value and the optimal decision strategy, it is also interesting to examine the timing of early exercises in relation to transaction and decision costs, i.e. at what point in time the exercise right is used. Accordingly, Figure 5 provides an overview of the modeled cumulative exercise distributions over all simulated 10,000 paths over time for our standard scenario and selected cost parameters.

[Figure 5 about here.]

In the case of zero transaction and decision costs (upper-left sub-figure) we recognize an approximately linear cumulative distribution of exercises that approaches almost 80% at the last time step. This means that for only 20% of all paths, the investor does not use the early exercise right and holds the bond until maturity. Observing steady and constant exercises over time is plausible if there are no costs, since with both flat coupon payments and a flat interest term structure an early exercise tends to be equally attractive throughout the bond's lifetime. Next, under transaction costs (lower-left sub-figures) the cumulative distribution curve is pulled down but remains a similar shape, which implies steady but overall fewer early exercises. This is also reasonable, since transaction costs do linearly



reduce the attractiveness of early exercises for each time step and therefore have a consistent influence on exercise rates over time. Taking decision costs into account (right sub-figures), we recognize two effects on the timing of early exercises. First, decision costs lead to a less smooth distribution. As described, with rising decision costs an investor's optimal decision strategy is more selective and focuses on fewer decision points in time. Hence, the formation of clusters and the increasingly stepped exercise distribution corresponds to the mentioned decrease in the average optimal number of decisions. Second, we find a growing period of no exercise activity at the beginning. With increasing transaction and decision costs, the first optimal decision point at which the potential gain from early exercise can also compensate for the incurring costs recedes further towards maturity, since the potential advantage of a one-sided early exercise option naturally grows over time. In summary, transaction and decision costs have a strong impact on the timing of early exercises and the valuation of the respective option. The main reason is that both costs influence an investor's optimal decision strategy and thus also give rise to different exercise decisions.

### 3 Data

#### 3.1 German Federal Saving Notes

We base our empirical analysis on the early exercise behavior of Individual Investors in German Federal Saving Notes ("Bundesschatzbriefe", henceforth GFSN). GFSN are puttable bonds issued by the Federal Republic of Germany for governmental financing.<sup>14</sup> These prod-

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<sup>14</sup>GFSN accounted for about 11.5% of Germany's overall borrowing in July 1996 with outstanding GFSN of approximately €46.3 billion according to the Deutsche Bundesbank. However, the relevance of GFSN for governmental financing decreased with strongly increased debt levels over the years, and the GFSN share of the overall borrowing sank to about 1.0% in February 2009 with an outstanding volume of about €9.5 billion. In 2012, the German government decided to stop offering products exclusively for Individual Investors and stopped issuing new GFSN due to disproportionately high costs.

ucts are very well suited for our analysis for the following reasons. First, GFSN are offered exclusively to Individual Investors and resident institutions serving the public benefit, charitable or religious purposes.<sup>15</sup> Because the latter account for only a very small share of the overall investor group, GFSN are de facto a pure Individual Investor product. Thus focusing on GFSN allows us to observe the isolated early exercise behavior of Individual Investors. We do not have to distinguish between different market participants like institutional or retail investors, as in similar studies such as Poteshman and Serbin (2003) or Barraclough and Whaley (2012), who analyze early exercises of exchange-traded options. Second, there is no secondary market, i.e. GFSN cannot be traded. Individual Investors must only decide whether to hold or exercise, rather than to hold, exercise or sell their position. This gives us a much more comprehensive picture of Individual Investors' early exercise activities compared to studies based on tradable financial products such as warrants or call options, where selling is always an alternative to exercising (e.g., Koziol, 2006; Pool et al., 2008). Third, GFSN are simple, easily comprehensible and standardized mid-term fixed-income products, whose structure and general product features have been unchanged since 1969. New issuances have been offered several times a year, which provides us with sufficient cross-sectional and time-series variation to analyze the general structure of Individual Investors' early exercises. Fourth, issuer's credit risk can be neglected for GFSN due to the high creditworthiness of the Federal Republic of Germany.

Two types of GFSN exist: Type A, a yearly coupon-paying step-up bond with a maturity of 6 years and Type B, an accrued-coupon bond with rising yearly coupons and a maturity of 7 years.<sup>16</sup> GFSN are offered at nominal value plus accrued interests, whereby at new launches

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<sup>15</sup>All product details are described in German Finance Agency (2012).

<sup>16</sup>This means Type B has a zero-bond structure. At early exercising or maturity, an investor receives the nominal value plus all accrued coupons.

all current issuances are closed. Both GFSN types are equipped with an early exercise right—a specific American put option—which allows the investor to reclaim his investment plus the accrued interests and for Type B compounded interests at any time after an initial blocking period of one year.<sup>17</sup>

The main distribution channel for GFSN is direct sale by the German Finance Agency (“Bundesrepublik Deutschland Finanzagentur GmbH”), a state-owned central service provider for Germany’s governmental borrowing and debt management. The German Finance Agency offers cost-free debt accounts for Individual Investors and does not charge fees for purchasing and administration nor for redemption of GFSN. Alternatively, GFSN can also be acquired through banks, which however typically charge custody fees for administration. It is possible to transfer GFSN ordered through an intermediary to a cost-free account at the German Finance Agency at any time.

For our study on Individual Investors’ early exercise behavior we are able to utilize a large, non-publicly available data set from the German Finance Agency. This data set contains—on a daily and single account basis—all Individual Investor investments and early exercises of GFSN from July 1996 to February 2009 that were booked in the German Finance Agency’s debt register accounts (approximately 64% of the overall outstanding volume).

### **3.2 Summary statistics**

There are 102 GFSN issuance dates in our sample period with an average time between issuances of 45 days. At each issuance both Type A and Type B are offered with an identical coupon structure for the first 6 years. Table 1 provides an overview of GFSN coupon structures in comparison to prevailing spot rates at issuance.

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<sup>17</sup>For both types an additional restriction applies: investors are allowed to reclaim a maximal nominal value of €5,000 within 30 interest days, which we ignore in the following.

[Table 1 about here.]

Typically the offered coupons lie below the corresponding spot rates for the first years but rise above market rates towards maturity. Yet in accordance with changes in market conditions, the coupon structure of new issuances is regularly adjusted, implying a significant variance in coupon offerings over time for both Type A and B.

Table 2 presents further summary statistics on our data set along two dimensions. The left sub-tables review the GFSN issuances in our sample. The right sub-tables focus on Individual Investor accounts with funds placed in GFSN.

[Table 2 about here.]

Regarding GFSN issuances, we find that Type A (Type B) GFSN are held on average in 39,719 (26,192) accounts. Still, as indicated by the lower median of only 29,175 (19,155) accounts and the comparatively high standard deviations, the distribution is positively skewed. Similar variations can be observed in the investment volume per GFSN. While the mean volume per GFSN amounts to approximately €103.220 million (€31.666 million), the median volume lies at €77.778 million (€25.703 million). In a more detailed analysis not reported here, we note that both the number of accounts as well as the tendered volume per GFSN tends to decrease over time. Nevertheless, we find that all GFSN are of significant size and sufficiently similar for an overall comparison. Even the least considered GFSN in our sample period (November 2008) is held in more than 4,000 (4,600) accounts at the German Finance Agency.

Over the sample period 812,750 different Individual Investor accounts held GFSN in the German Finance Agency's debt register. About 13% of these accounts acquired both product types, so that we have in total 558,122 Individual Investor accounts for Type A and

361,141 accounts for Type B GFSN. On average each account invested in 3.617 (4.005) GFSN issuances, but we recognize again a distribution skewed strongly to the right, with a median of 1.000 (1.000) investment per account. In other words, the majority of Individual Investor accounts selected only one GFSN issuance throughout the whole sample period. Remarkably, the corresponding mean and median investment volume per account is very small, with only €21,087 (€9,721) or €9,746 (€3,067) per account respectively, which we attribute to the restriction of GFSN to non-institutional investors.

Finally, we observe that Individual Investors in GFSN make frequent use of their early exercise rights. The lower part of Table 2 shows that more than one-fourth of all accounts in our sample exercised early at least once, which equates to an absolute number of 148,812 (95,604) accounts. On average there occurred 6,156 (3,744) early exercises per GFSN issuance, accounting for a mean early exercise volume per GFSN of €17.868 million (€6.740 million). On a single account level, this corresponds to a mean exercise volume per account and GFSN of approximately €2,849 (€1,661).

In summary, the data set provides us with a large number of GFSN issuances, which consistently attracted a wide range of Individual Investors. We find a very low number of investments per account, rather small investment volumes and significant early exercise activities. Since in this study we are most interested in the latter, the following section focuses on analyzing the early exercise rates of GFSN.

### 3.3 Early exercise rates

To further examine Individual Investors' exercise activities in GFSN, we start with calculating the early exercise volume and the remaining exercisable volume per day for each GFSN. Subsequently, we aggregate these empirical data points to an *early exercise rate per*

month  $EE R_{g,t}^{\text{Obs}}$  per GFSN by:

$$EE R_{g,t}^{\text{Obs}} = 1 - \left( \prod_{d=1}^{D_t} \left[ 1 - \frac{\text{Early Exercise Volume}_{g,t,d}}{\text{Exercisable Volume}_{g,t,d}} \right] \right), \quad (1.14)$$

where  $g$  is the product index,  $t$  the month index,  $d$  the day index and  $D_t$  stands for the number of days in the respective month.<sup>18</sup>

As outlined in the lower part of Table 2, the mean monthly exercise rate is 0.570% (0.536%) over all exercisable<sup>19</sup> Type A (Type B) GFSN, whereby the high standard deviation of 1.028% (0.810%) points towards a notable variance among products and months. The upper charts in Figure 6, where we plot the observed early exercise rates for all GFSN against time, also illustrate this. We register strong fluctuations with a maximum early exercise rate of about 16.8% (14.1%) for an individual GFSN and a minimum rate close to 0% for some Type A and B GFSN in selected months. Still, we can identify some patterns. For instance, on an individual bond level we find an increased exercise activity in the first month after the blocking period. Moreover, there are two general peaks in the exercise rates around the years 2000 and 2007, but also a longer period of less exercise activity between 2002 and 2006.

[Figure 6 about here.]

Searching for rationales behind these patterns, we investigate the relationship between the empirical exercise rates and the economic advantage of an exercise in the respective months. To proxy the latter we calculate a “hold-to-exercise ratio” for each time step and GFSN, which we define as the fraction of the present value of a GFSN without early exercise right

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<sup>18</sup>As outlined in Section 3.1, Individual Investors can transfer (or deduct) GFSN investments from their bank accounts to a cost-free debt account at the German Finance Agency at any time. Thus we have to consider possible daily changes in the exercisable volume in our calculation of monthly early exercise rates.

<sup>19</sup>178 GFSN (89 Type A, 89 Type B) are exercisable in our sample, since they have left the initial blocking period.

to its exercise value, i.e.:<sup>20</sup>

$$\text{Hold-to-exercise ratio}_{g,t} = \frac{\text{Present Value of GFSN without option}_{g,t}}{\text{Exercise Value}_{g,t}}. \quad (1.15)$$

We use this ratio as a first simple indicator for the potential benefit of an early exercise, since a hold-to-exercise ratio above 100% implies that it is not reasonable for Individual Investors to early exercise their GFSN—except for possible exogenous reasons such as, e.g., the need for liquidity. In contrast, an exercise might be beneficial if the calculated ratio lies below 100%, depending on the option value. The middle charts of Figure 6 depict the development of the hold-to-exercise ratios for all GFSN over time. Like in our analysis of exercise rates, we notice a broad variance throughout the sample period but again some general patterns. We find that the hold-to-exercise ratios stay clearly above 100% for most outstanding GFSN in most months, which indicates a low attractiveness of early exercises, although there are also several months where an early exercise seems to be highly advantageous.

The lower charts in Figure 6 combine both upper analyses and show the relationship between the monthly exercise rates and the corresponding hold-to-exercise ratios. Even though Individual Investors obviously do not act homogeneously, we can observe a clear negative correlation here. The higher the hold-to-exercise ratio, the lower the average monthly early exercise rates tend to be. Nevertheless, even in highly unattractive months with substantially enhanced hold-to-exercise ratios, some investors still opt to reclaim their investments. As outlined later in more detail, we attribute these unexpected early exercises to exogenous reasons such as investors' liquidity constraints or tax optimizations. On the other hand, early exercise rates tend to grow exponentially with decreasing hold-to-exercise ratios. In

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<sup>20</sup>To keep the analysis simple, we neglect the exercise option in determining the GFSN holding strategy here. However, we find that the economic relations are very similar for more complex ratios.

particular, we find a strong increase as soon as the hold-to-exercise ratio drops below 100%.

At a first glance the described empirical patterns might be surprising, since according to standard option theory, one would expect a significant, uniform jump in the exercise rates as soon as the economic advantage of an early exercise is higher than the present value of holding the GFSN. However, the empirical findings coincide very well with the reasoning behind our model in Section 2.1 regarding transaction and decision costs. With transaction costs, early exercises are only attractive for Individual Investors if the hold-to-exercise ratio falls significantly below 100%, since the economic advantage of an early exercise must compensate also for these additionally incurring costs. Moreover, as pointed out in the comparative-static analysis, transaction and decision costs induce a non-continuous decision strategy and lead to clustered and temporarily offset early exercise points, which might be linked to hold-to-exercise ratios that (clearly) undercut 100%.

To further investigate Individual Investors' early exercises and to avoid misleading inferences based on idiosyncratic patterns of selected GFSN issuances, we move from the single bond perspective to a more consolidated overview. Figure 7 exhibits the *average monthly exercise rate* over all exercisable GFSN of both types over time. Additionally, the corresponding German 1- and 10-year spot rates are depicted, indicating the level and shape of the interest term structure at the respective time in Germany.

[Figure 7 about here.]

We stress five observations here. First, we find that the average monthly rates are in large part very similar for Type A and Type B GFSN. The highest rates for both types are observed around July 2000, where on average about 2.1% (1.9%) of the outstanding volume is returned for Type A (Type B) GFSN. Least activity occurs in both cases at the end of 2005 with average exercise rates as low as 0.2% (0.1%).



Second, similar to the analysis on a single bond base, we identify two periods of elevated exercise rates during our sample period which both coincide with rising short- and middle-term interest rates. In general, Individual Investors' exercise rates appear highly sensitive to spot rate movements and valuation changes. We note a distinct trend among Individual Investors towards more (less) early exercise activity in rising (falling) interest environments. As described, this is reasonable, because with increased interest rates the respective hold-to-exercise ratio falls and it might be advantageous for Individual Investors to deduct their funds or just roll their investments into a newer GFSN issuance with higher coupons.

Third, the aggregated view underlines once again that the investor base acts heterogeneously. We find comparably small mean empirical exercise rates, which also differ among months and products. In the case of homogeneous investors with an identical basis for decision-making, we would instead expect exercise rates of either 100% or close to 0% on a single product level.

Fourth, regardless of any interest rate movements, we identify a base exercise rate of about 0.160% over all GFSN per month. Even in times of broadly appreciating GFSN and high hold-to-exercise ratios there are always some Individual Investors, who still decide to recall their investments before maturity. As already mentioned, we consider this continuous base exercise rate to be exogenously triggered. We argue that, for example, individual liquidity requirements might be a reason for steady early exercises. Indeed, since GFSN can be terminated comparatively easily and without any extra fees, a GFSN exercise might be one of the first choices for an Individual Investor with liquidity constraints.

Fifth, we observe small but regular peaks in the exercise rates of Type B products at the end of a year, especially in 1999 and 2006. We attribute these effects to a second exogenous factor, namely the tax optimization of Individual Investors at the end of a year, a well known

effect described in the literature (e.g., Badrinath and Lewellen, 1991; Sias and Starks, 1997; Ivkovic et al., 2005; who find that Individual Investors show frequent tax-motivated trading before the year is over). In Germany interest payments to Individual Investors are generally taxed when they are distributed, whereby only the share of interest payments is considered that exceeds a saver's tax allowance. At the beginning of both years 2000 and 2007 this tax threshold was significantly reduced in Germany, which put in particular Type B GFSN at a disadvantage, since due to the zero-bond structure the tax allowance for such products can be utilized only once, whereas for Type A GFSN it is considered for each year's interest payment. Hence, it could have been advantageous for Individual Investors to exercise Type B GFSN early with hold-to-exercise ratios above 100% shortly before the new tax regulations in 2000 and 2007 became effective, so as to exploit tax advantages. We also ascribe the small but regular peaks around December (year-end effect) for Type B GFSN to taxation rules. The same argument applies: since the taxation is not spread over several years as for Type A GFSN, year-end optimizations of personal tax allowances are much more relevant and can be a good reasons for early exercises. In the following, we account for both exogenous effects—base exercise rate and taxation—by introducing control variables.

## 4 Empirical analysis

### 4.1 Calibration procedure

In this section we calibrate our model to the data set and derive conclusions about the empirical early exercise behavior of Individual Investors. Assuming that Individual Investors' exercise strategies of GFSN are driven by individual transaction and decision costs, we aim to determine the empirically best-fitting cost levels for our model. We structure the calibration

in four steps. First, we define and estimate an interest rate term structure model. Second, we consider heterogeneity of transaction and decision costs among Individual Investors by forming different investor clusters. Third, we account for increased exercise activities after the initial one-year blocking period and for exogenous effects that are likely to be found in the empirical data—as suggested by the analysis in Section 3.3—but which are not incorporated in our model. Eventually, we specify the calibration via an optimization algorithm.

#### *Interest rate model*

For the valuation of interest rate derivatives, typically no-arbitrage term structure models, such as one- or two-factor HJM models or LIBOR market models, are applied (see, e.g., Hull, 2006). While these models have the advantage of being perfectly consistent with the current interest term structure, they prove to be less suitable for our study due to possible logical violations in comparing the empirical and modeled optimal exercise strategies. With a continuous re-calibration it might happen that the optimal exercise point determined by the model is empirically never feasible, a circumstance that would distort the calibration quality and reduce its reliability. Therefore, we choose to apply a more sophisticated model of the class of essential affine term structure models according to Dai and Singleton (2000), which determines the interest term structure throughout the whole sample period based on a time-invariant function of only a small set of common state variables.

Egorov et al. (2006) and Tang and Xia (2007) show that a three-factor essentially affine model, with one factor affecting the conditional variance matrix, provides empirically the best performance for different countries including Germany, while it also allows for very flexible specifications. Accordingly, we utilize for the calibration a three-factor essentially affine model  $EA_1(3)$ . We calibrate the model to monthly German term structures over the period

of July 1996 to February 2009 via a Kalman filtering algorithm together with quasi-maximum likelihood, under the assumption that the rates of all yearly maturities 1 to 10 years are not perfectly observed. This is basically the same procedure as used by, e.g., Hördahl and Vestin (2005). The estimated model parameters are presented in Table 3.

[Table 3 about here.]

### *Heterogeneity of Individual Investors*

As already discussed, it is likely that transaction and decision costs vary among Individual Investors. To incorporate this heterogeneity in the calibration while also preserving a low number of estimation parameters and numerical efficiency, we make three assumptions. First, each investor has constant transaction and decision costs, i.e. a specific Individual Investor features the same cost profile over the whole sample period and for each acquired GFSN. Second, we do not estimate individual costs for each single account but group Individual Investors in discrete clusters, which is a common approach in the literature (e.g., Stanton, 1995; Koziol, 2006) to capture heterogeneity. We allow for 9 different clusters of transaction costs from 0% to 7.5% and for 9 different clusters of decision costs from 0% to 0.75%,<sup>21</sup> which gives us overall 81 clusters representing a wide choice of different combinations of transaction and decision costs. In addition, we consider that there might be a group of “passive investors”, who under no circumstances make use of their exercise rights, and form an extra cluster with “infinite” or very high transaction and decision costs. Third, the pool of Individual Investors is identical for each GFSN of the same type with regard to the distribution of transaction and decision costs, which seems plausible in view of the broad range of investors in each GFSN and the unchanged product structure throughout the sample period.

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<sup>21</sup>Obviously, there is a broad variety of possible cost ranges we can assign to the clusters. Intensively testing different approaches, we find that the following cost parameters are most suitable for our calibration: decision costs of [0%, 0.010%, 0.025%, 0.050%, 0.0750%, 0.100%, 0.250%, 0.500%, 0.750%] and transaction costs of [0%, 0.100%, 0.250%, 0.500%, 0.750%, 1.000%, 2.500%, 5.000%, 7.500%].

The key challenge of our approach is the determination of appropriate cluster weights. Avoiding estimating each proportion separately, we assume here—extending the approach of Stanton (1995)—that all cluster weights can be described by a combination of two discretized beta distributions. We choose the beta distribution because it can adapt to a multitude of different shapes based on the estimation of only two parameters  $\alpha$  and  $\beta$ .<sup>22</sup> Utilizing the cumulative distribution function of the beta distribution, which is given by:

$$F(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} \int_0^x t^{\alpha-1}(1-t)^{\beta-1} dt, \quad (1.16)$$

where  $B(\alpha, \beta)$  stands for the standard beta distribution with parameters  $\alpha$  and  $\beta$ , we estimate all individual cluster weights  $p$  at issuance through:

$$p_{g,0,i,j} = (1 - \omega) \times \left[ F\left(\frac{i}{9} | \alpha_{TC}, \beta_{TC}\right) - F\left(\frac{i-1}{9} | \alpha_{TC}, \beta_{TC}\right) \right] \\ \times \left[ F\left(\frac{j}{9} | \alpha_{DC}, \beta_{DC}\right) - F\left(\frac{j-1}{9} | \alpha_{DC}, \beta_{DC}\right) \right], \quad (1.17)$$

where  $\omega$  is the proportion of passive investors,  $g$  is the GFSN index,  $i$  and  $j$  are the cluster indices for the respective transaction and decision costs, and the time index 0 denotes that we start at issuance.

We know from our model that there is always only one individual optimal strategy  $(\Gamma, \tau^\Gamma)$  for each combination of transaction and decision costs. This means we get diverging optimal exercise strategies among our clusters. For instance, we can presume that clusters with comparatively low transaction and decision costs use their exercise rights more often and earlier than others. Conversely, clusters associated with high costs might take later or no

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<sup>22</sup>To reduce complexity we limit in the following the interval of possible values for  $\alpha$  and  $\beta$  to  $[0,8]$ . Testing several possible interval ranges we find that this restriction does not have a significant influence on the results.

advantage of attractive exercise opportunities. Such different exercise strategies result in shifts in the relative cluster proportions over time and induce a phenomena known as burnout (e.g., Stanton, 1995). Burnout basically refers to the dependency of early exercise rates on former exercise activities, which leads in our case to a declining sensitivity over time of all GFSN regarding interest rate movements due to the shrinking proportion of clusters with low transaction and decision costs, which have not exercised yet.

Besides the burnout feature we also take into consideration that Individual Investors can act sluggishly, which means that not all investors in a cluster exercise at the optimal exercise month.<sup>23</sup> More precisely, we allow that at the optimal exercise point a sluggish proportion  $s$  of investors delay the exercise (e.g., because they are lazy or hesitant) and instead defers the transaction to an upcoming month. We assume a latest sluggish reaction after 6 months and thus restrict  $s$  to an upper bound of 65%.<sup>24</sup> Consequently, the overall *endogenous* exercise rate  $EER^r$  per month and GFSN is given by:

$$EER_{g,t,i,j}^r = \begin{cases} (1-s) \times p_{g,t,i,j}, & t = \Gamma_{\tau_{g,i,j}}^{\Gamma} \\ (1-s) \times p_{g,t,i,j}, & EER_{g,t-1,i,j}^r > 0 \\ 0, & \text{otherwise,} \end{cases} \quad (1.18)$$

where  $t$  stands for the month index and  $s$  for the sluggish proportion of investors. Equation (1.18) says that a cluster exercises at (or shortly after) an endogenously defined optimal point in time, which is determined by the decision strategy  $\Gamma$  and the respective exercise strategy  $\tau^{\Gamma}$ . At all other months the endogenous exercise rate equals zero. We determine  $\Gamma_{\tau^{\Gamma}}$  for

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<sup>23</sup>Such sluggish reactions are described in the literature on momentum effects, among others. For example Hvidkjaer (2011) finds extremely sluggish reactions of Individual Investors to past returns.

<sup>24</sup>After 6 months, the maximum remaining investor share in a cluster equals  $0.65^6 \approx 0.075$ . We assume that these remaining investors also exercise 6 months after the optimal exercise point.

all GFSN and clusters according to (1.11) to (1.13), whereby we follow the same calculation approach as outlined in Section 2.2 for the comparative-static analysis—except that we now replace the one-factor short rate model with the described essentially affine interest term structure model.<sup>25</sup>

### *Blocking period*

The right to exercise a GFSN early cannot be used in the first year after its purchase. As a consequence, we observe increased exercise activities in the first month after this initial blocking period, which might, e.g., be attributed to investors' deferred liquidity demand over the previous twelve months or could simply be a reaction to earlier market changes. We account for these extraordinary exercises after the blocking period by introducing a dummy variable  $EER^b$  and define  $EER_{g,t,i,j}^b = p_{g,t,i,j} \times EER^b$ .

### *Exogenous effects*

In accordance with our empirical observations and the study of Stanton (1995), we control for three different exogenous effects in the calibration. First, we incorporate a base exercise rate, which we set to the empirically observed rate of 0.160% per GFSN per month. Additionally, we consider for Type B GFSN a possible year-end effect in December and taxation effects in December 1999 and December 2006. Thus the *exogenous* early exercise rate  $EER^e$  per month and GFSN is given by:

$$EER_{g,t,i,j}^e = p_{g,t,i,j} \times (0.0016 + DYE + DT99 + DT06), \quad (1.19)$$

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<sup>25</sup>Since we have to compute for each GFSN and each month not only the respective exercise value but also all possible continuation values, these calculations are still a challenge even for modern computers. Here, there are up to 300 billion path calculations and up to 30 million regressions needed.

where  $DYE$  is the the year-end dummy and  $DT99$  and  $DT06$  are the respective dummies for changes in the tax legislations. Both endogenous (1.18) and exogenous exercises (1.19) as well as the increased demand after the initial blocking period  $EER^b$  lead to shrinking cluster weights over time, since less Individual Investors stay invested in the respective GFSN. Hence, as last step in our calibration approach, we model the development of each cluster weight through:

$$p_{g,t+1,i,j} = p_{g,t,i,j} - EER_{g,t,i,j}^r - EER_{g,t,i,j}^e - EER_{g,t,i,j}^b. \quad (1.20)$$

#### *Overall calibration*

Combining equations (1.16) to (1.20), the overall early exercise rate  $EER$  of a single GFSN in a selected month is given by:

$$EER_{g,t} = \frac{\sum_{i=1}^9 \sum_{j=1}^9 EER_{g,t,i,j}^r + EER_{g,t,i,j}^e + EER_{g,t,i,j}^b}{\sum_{i=1}^9 \sum_{j=1}^9 p_{g,t,i,j} + \omega}, \quad (1.21)$$

where  $g$  is the GFSN index,  $t$  the month index,  $i$  and  $j$  are the cluster indices and  $\omega$  stands for the group of passive investors. Aiming at minimizing the difference between the modeled exercise rates and the empirical observations, we apply a sequential programming approach that searches for best fitting distribution parameters ( $\alpha_{TC}, \beta_{TC}, \alpha_{DC}, \beta_{DC}$ ) and model factors ( $\omega, s, EER^b, DYE, DT99, DT06$ ) for each Type A and B GFSN through a multidimensional non-linear minimization algorithm based on the interior-point method. As a measure of the goodness-of-fit we use the mean squared error (MSE) on a single product base, which we



define over all 151 months and all investigated GFSN per type as:

$$MSE = \frac{1}{151} \sum_{t=1}^{151} \frac{1}{N_t} \sum_{g=1}^{N_t} \left( EER_{g,t-d(g)+1} - EER_{g,t-d(g)+1}^{\text{Obs}} \right)^2, \quad (1.22)$$

where  $EER^{\text{Obs}}$  is the empirical exercise rate for a selected GFSN and month as determined in (1.14),  $EER$  the model-implied exercise rate according to (1.19),  $N$  stands for the number of outstanding and exercisable GFSN at each month and  $d(g)$  is a function determining the issuance month of the respective GFSN.<sup>26</sup> We re-run the calibration algorithm until the measured change in the MSE falls below 0.000001%. Finally, to reduce potential biases in the results due to local minima and to test the robustness of our results, we repeat the entire calibration approach 250 times with random start parameters.

## 4.2 Calibration results

We note that our calibration is not a full general equilibrium specification, since transaction and decision costs—occurring at the decision points and at exercise in our model—may in reality also occur when the hedge portfolio is adjusted over time, affecting the arbitrage reasoning of standard option theory. However, for example (implicitly) assuming equivalence of real and risk-neutral probability measures is a standard approach in calibrating similar models, such as mortgage prepayment models (see, e.g., Stanton, 1995; Stanton and Wallace, 1998). Table 4 provides the calibration results.

[Table 4 about here.]

Focusing first on the full-sample results (left column), we receive a very good fit of the modeled early exercise rates to the empirical data with a mean squared error (MSE) on a single GFSN

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<sup>26</sup>We test several measures of the goodness-of-fit (e.g., MSE of the average exercise rate per month per type), but find that an optimization using the MSE on a single product base provides best results.

and month level of approximately 0.0036% for Type A GFSN (0.0025% for Type B), which equates to a mean absolute error of circa 0.2827% (0.2627%). To verify the model fit, we calculate also the mean  $R^2$  value between modeled and empirical exercise rates over all  $N$  GFSN according to  $R^2 = \frac{1}{N} \sum_{g=1}^N 1 - \frac{\text{Variance of monthly prediction error}_g}{\text{Variance of empirical monthly exercise rate}_g}$  and a consolidated  $R^2$  value between the modeled and empirical average monthly exercise rate according to  $R^2 = 1 - \frac{\text{Variance of average monthly prediction error}}{\text{Variance of empirical average monthly exercise rate}}$ . Both values are remarkably high with about 50% and 79% for Type A and approximately 47% and 73% for Type B GFSN, whereby the second  $R^2$  value is higher due to consolidation effects. For a further analysis of the model fit, the upper part of Figure 8 depicts the consolidated calibration results compared to the mean empirical rates over time.

[Figure 8 about here.]

We note that our model captures different market phases equally well, as the mean modeled and observed rates differ by less than 0.1% for both types for most months. Only at the beginning of the sample period, between 1997 and 1999, does our model output clearly underestimate the empirical data, which we ascribe at least partly to the launch of a new stock market segment in Germany in 1997. This “New Market” broadly attracted Individual Investors and thus presumably also enhanced the exogenous demand for liquidity.<sup>27</sup>

The next part of Table 4 presents the estimated beta distribution factors. The resulting distributions are diagrammed graphically in the lower part of Figure 8. We get mean decision costs of about 0.062% for Type A GFSN (0.138% for Type B) and remarkably higher mean transaction costs of circa 3.993% (3.791%) over all clusters during our sample period, which implies that empirically Individual Investors do not use their exercise rights as predicted by standard models. Instead, they act as if they face significant transaction and decision

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<sup>27</sup>According to the Deutsche Bundesbank, the share of stock investments of Individual Investors’ overall capital increased in Germany from circa 8.0% in 1997 to circa 14.5% in 2000.

costs. It is not surprising that decision costs are estimated significantly lower here since these costs can accrue several times and have—as seen—a stronger impact on an investor’s decision strategy, whereas transaction costs are due only once, at exercise.

Moreover, we find that the estimated beta distributions for decision costs are clearly bent left towards very low or zero costs, whereas the distributions for transaction costs are more even but somewhat skewed to the right. Decision costs are slightly more important for Type B GFSN, which we attribute to the generally higher nominal value of Type GFSN due to the zero-bond structure with accumulated interest payments and to our definition of absolute rather than relative transaction and decision costs. The broad standard deviations of estimated transaction and decision costs imply strongly diverging optimal strategies among investors and over time.

We estimate the share of passive investors at about 56% (53%) of the overall investor group, which expresses that a considerable proportion of investors in GFSN completely neglect the incorporated early exercise right. In addition, the calibration results indicate that approximately 65% (65%) of the investors per cluster respond sluggishly, which is very close to our upper bound of half a year response time and means that a large share of investors show some kind of delay between the optimal decision and the actual exercise.

Regarding exogenous effects, we find that in particular the adjustments in the tax legislation for the year 2000 had a substantial influence on Individual Investors’ early exercise decisions. We estimate that these changes account for additional exercises of circa 1.984% of the outstanding volume of Type B GFSN in December 1999. The year-end effect (0.568%) and the change in taxation in 2006 (1.167%) had, according to our calibration, a smaller but still notable impact.

### 4.3 Robustness

To verify the results, we re-calibrate our model for different sub-samples. The right part (two columns) of Table 4 shows exemplary results for two calibrations, each based on roughly half of the sample period. We find that the estimated parameters for both the transaction and decision cost distributions and for the share of passive or sluggish investors are relatively robust and stay in similar ranges, which validates our former findings. Moreover, while the calibration performs better and is more stable if we incorporate both exercise peaks around 2000 and 2007 and a longer time horizon, the MSE for our exemplary sub-samples remains at a low level of only circa 0.0036% for Type A GFSN (0.0025% for Type B GFSN) and of about 0.0069% (0.0035%) respectively. Accordingly, we compute again convincing sub-sample  $R^2$  values of 86.005% (77.338%) and 70.105% (58.351%) respectively between the average modeled monthly exercise rates and the average empirical rates.

### 4.4 Valuation

The comparative-static analysis in Section 2.2 revealed that transaction and decision costs result in diverging optimal exercise strategies and timings compared to the standard case. Obviously, issuers gain a financial advantage when Individual Investors exercise at other points in time than would, e.g., institutional investors, who follow standard financial rationality. Similarly, issuers benefit when Individual Investors completely forfeit the exercise right or exercise “randomly”.

To quantify the advantage for the issuer, we compare the valuation of the option component of a GFSN at issuance according to standard valuation,<sup>28</sup>—assuming all investors

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<sup>28</sup>As described, standard market valuation gives the same results as our model with no transaction and decision costs.

exercise optimally with no transaction and decision costs—(*model value*) with the *empirical value* which incorporates the actual exercise behavior of Individual Investors. We calculate the empirical value in two steps. First, for a given GFSN we simulate 10,000 paths and calculate the optimal exercise point for each combination of transaction and decision costs. Second, we utilize the full-sample calibration results of Section 4.2 to weight all exercise values according to our estimated cost distributions, whereby we also account for the base exercise rate, the share of passive investors, sluggish reactions and additional exercises for the first month after the initial blocking period and—in the case of Type B GFSN—at the end of the year. We exclude only the one-time tax effects due to changes in the tax legislation because such events cannot be foreseen in a valuation ex-ante. The overall weighted value is our empirical value.

The first two columns of Table 5 show statistics on the model and the empirical value of the option component for all GFSN in our sample. Additionally, the third column exhibits the respective advantage for the issuer, which we calculate simply as the difference between both values.

[Table 5 about here.]

We find that the value of the early exercise right on the issuance date according to standard market valuation amount on average to 2.2433% of the nominal value for Type A and to 2.9859% for Type B GFSN, whereby the higher option value for Type B GFSN can mainly be attributed to the longer maturity. For both types we note a significant variation over time as indicated by option values of 1.1071% (1.3672%) for the 5 percent quantile and 4.0102% (5.2208%) for the 95 percent quantile. However, the empirical value of the early exercise right at issuance is much lower for most products. On average we compute empirical option values of only approximately 0.2009% for Type A GFSN (1.1411% for Type B), whereby the

95 percent quantile lies at only 0.8194% (2.8956%). These significantly lower empirical values are due to a combination of three factors: first, we have a high proportion of passive investors and the empirical value is a weighted average over all investor clusters. Second, transaction and decision costs can lead to missed optimal exercise opportunities. Third, liquidity-driven exercises (base exercise rate) and exogenously motivated exercises (tax effects) can occur at points in time where it is not optimal to exercise according to standard models and where early exercises can actually have a negative effect on the valuation. In fact, for some GFSN this negative effect can even overcompensate the positive influence of early exercises at attractive points in time, which results in negative option costs for the issuer (see the 5 percent quantile in Table 5). Generally, we observe a significant and consistent financial advantage over time for issuers due to Individual Investors' diverging empirical exercise strategies. We determine an average advantage of circa 2.0424% of the nominal value for Type A and circa 1.8449% for Type B GFSN. Similarly, the median advantage accrues to approximately 1.8559% (1.7604%).

## 5 Conclusions

In this paper, we analyzed Individual Investors' early exercise behavior in fixed-income derivatives. We showed that given transaction and decision costs, continuous decision-making is no longer worthwhile and developed a new approach to model early exercises that endogenously determines a rational decision and exercise strategy in the face of such costs. Based on a comprehensive empirical data set, we applied our model and found a convincing fit between the modeled early exercise rates and the empirical observations. Our results suggest that a large proportion of Individual Investors act as if they face significant but heterogeneous transaction and decision costs, that there is also a notable share of passive investors and that

investors' reaction to exercise opportunities is often sluggish. All these findings imply that the optimal timing of early exercises for Individual Investors can clearly differ from that of, e.g., institutional investors.

Following, we derive three policy implications. First, considering transaction and decision costs, the value of a fixed-income derivative with an early exercise right is lower than suggested by standard valuation models. As a result, issuers gain a significant financial advantage in pricing such derivatives without accounting for investors' individual exercise strategies, which we estimate at around 2% of the nominal value for our sample. Second, we reckon that issuers can influence investors' strategies by designing specific derivative structures. In some separate analyses, we find for example that a very steep coupon structure leads to a higher sensitivity towards decision costs and makes decision-making less attractive during much of the GFSN lifespan until maturity. This implies that investors tend to more frequently forgo possible exercise opportunities. Third, issuers or financial intermediaries dealing with Individual Investors must—besides allowing for rational early exercise decisions—also account for a base exercise rate and for potential, unexpected, early exercise peaks in their risk and liquidity management due to exogenous effects such as changes in tax laws.

Finally, we note that our model need not be restricted to analyzing fixed-income derivatives and Individual Investor's exercise decisions as in this study. Our approach can easily be applied to other derivatives, investor groups or overall research questions, where an endogenously determined decision strategy is relevant. For instance, mortgages might be a possible field of application, as it seems reasonable to assume that homeowners also do not continuously make decisions on refinancing or prepaying, but follow similar rational decision and exercise strategies as described in this paper.

## Appendix A — Model proof

The main idea of the proof is to transform the problem into an equivalent optimal stopping problem in a process that is indexed by a partially ordered set. Roughly speaking, we replace the question “When shall I make the next decision?” in the ordered set  $\mathbb{T}$  with the question “In which direction shall I go next?” in a partially ordered set  $\mathbb{I}$ . Re-indexing the (discounted) payoff process accordingly allows us to adopt techniques from the theory of sequential stochastic optimization for multi-dimensionally-indexed processes as, for example, presented by Cairoli and Dalang (1996).

To provide some intuition, let us assume that the derivative can be exercised at the points in time 1, 2 and 3, i.e.  $\mathbb{T} = \{0, 1, 2, 3\}$ . Then, given a realization  $\omega$ , there is a one-to-one relationship between a decision strategy and a sub-sequence of the set  $\mathbb{I} = \{(0), (0, 1), (0, 2), (0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 3), (0, 1, 2, 3)\}$ . Today, in 0, the investor has to decide on the next decision point 1, 2 or 3. This decision is represented by (0,1), (0,2) or (0,3), respectively. If he, for example, decides on 1, i.e. he chooses (0,1), he can make the next decision in 2 or 3, represented by (0,1,2) and (0,1,3). Figure 9 shows the resulting possible transformed strategies. For example, making a decision in 1 and 3 means going from (0) to (0,1) and afterwards to (0,1,3). Thus, making a decision regarding the next decision point is equivalent to choosing a path through the decision tree representing all possible realization of a decision strategy. Assigning to each knot  $\gamma \in \mathbb{I}$  of the decision tree the (discounted) payoff of the derivative  $X_\gamma$ —including the transaction and decision costs occurring when going from (0) to  $\gamma$ —, finding an optimal strategy is equivalent to finding an optimal path through the tree and then finding the best time to stop the path, i.e. to exercise the derivative.

[Figure 9 about here.]

### *Definitions and re-indexing*

Let  $\mathbb{I} = \{(0, \gamma_1, \dots, \gamma_m) : 0 < \gamma_m < \gamma_{m+1}, 0 \leq m \leq N\}$  be the set of all increasing sub-sequences of  $\mathbb{T}$  whose first element is 0. For  $\gamma \in \mathbb{I}$ , define  $l(\gamma) = \text{card}(\gamma) - 1$ ,  $L(\gamma) = \gamma_{l(\gamma)}$ , and  $\gamma^{[m]} = (0, \gamma_1, \dots, \gamma_m)$  for  $m \leq l(\gamma)$  and  $\gamma^{[m]} = \gamma$  for  $m \geq l(\gamma)$ , i.e.  $l(\gamma)$  is the length of the sequence  $\gamma$  excluding the first component,  $L(\gamma)$  is the last component of  $\gamma$ , and  $\gamma^{[m]}$  is



the element of  $\mathbb{I}$  that is formed by the first  $m + 1$  components of  $\gamma$  or equals  $\gamma$ . We define a partial order “ $\prec$ ” on  $\mathbb{I}$  via:

$$\gamma \prec \delta \text{ if } \gamma = \delta^{[l(\gamma)]}, \quad (\text{A.1})$$

i.e.  $\gamma$  is smaller than  $\delta$  if it forms the first components of  $\delta$  that is called a *successor* of  $\gamma$ . Define  $\mathbb{D}_\gamma = \{\delta \in \mathbb{I} : \gamma \prec \delta \text{ and } l(\delta) = l(\gamma) + 1\}$ . Elements of  $\mathbb{D}_\gamma$  are the smallest elements of  $\mathbb{I}$  that are larger than  $\gamma$  with respect to the order  $\prec$  and are therefore called *direct successors* of  $\gamma$ . Note that the set of direct successors of  $\gamma$  is empty if and only if the last component of  $\gamma$ ,  $L(\gamma)$ , equals  $t_N$ .

We set for each  $\gamma \in \mathbb{I}$ :

$$X_\gamma = \exp\left(-\int_0^{L(\gamma)} r(s)ds\right) (X_{L(\gamma)} - TC_{L(\gamma)}) - \sum_{i=1}^{l(\gamma)} \exp\left(-\int_0^{\gamma_i} r(s)ds\right) DC_{\gamma_i}, \quad (\text{A.2})$$

$$\mathcal{G}_\gamma = \mathcal{F}_{L(\gamma)}. \quad (\text{A.3})$$

$X_\gamma$  represents the discounted payoff if the deterministic decision strategy  $\gamma$  is applied and stopped in  $L(\gamma)$ , and the process  $(X_\gamma)_\gamma$ , indexed by the partially ordered set  $\mathbb{I}$ , is adapted to the filtration  $(\mathcal{G}_\gamma)_\gamma$ .

#### *Transformed strategies and equivalent optimization problem*

We call a sequence  $\Gamma' = (\Gamma'_n)_{n=0, \dots, N}$ , of  $(\mathcal{G}_\gamma)$ -stopping points<sup>29</sup>  $\Omega \rightarrow \mathbb{I}$  a *transformed strategy*, if the sequence fulfills the following conditions:

$$\Gamma'_0 = (0), \quad (\text{A.4})$$

$$\Gamma'_{n+1} \in \mathbb{D}_{\Gamma'_n} \text{ if } \Gamma'_{n+1} \neq \Gamma'_n, \quad (\text{A.5})$$

$$\Gamma'_{n+k} = \Gamma'_n \text{ for all } k \geq 1 \text{ if } \Gamma'_{n+1} = \Gamma'_n, \quad (\text{A.6})$$

$$\Gamma'_{n+1} \text{ is } \mathcal{G}_{\Gamma'_n}\text{-measurable.} \quad (\text{A.7})$$

Define  $l(\Gamma') = \inf\{n : \Gamma'_{n+1} = \Gamma'_n\}$  that is a  $(\mathcal{G}_{\Gamma'_n})$ -stopping time. Intuitively, a transformed strategy is a rule for choosing a path through  $\mathbb{I}$  that is stopped in  $\Gamma'_{l(\Gamma')}$ .  $A$  denotes the set

<sup>29</sup>A stopping point is the vector-valued analogon of a stopping time, i.e.  $\{\Gamma'_n \prec \gamma\} \in \mathcal{G}_\gamma$  for all  $\gamma$ .

of all transformed strategies and  $A^\gamma$  denotes for  $\gamma \in \mathbb{I}$  the set of transformed strategies that stop in  $\gamma$  or one of its successors, i.e.  $A^\gamma = \{\Gamma' \in A : \gamma \prec \Gamma'_{l(\Gamma')} \text{ a.s.}\}$ . This implies that the first components of  $\Gamma'$  are deterministic and coincide with  $(\gamma^{[0]}, \gamma^{[1]}, \dots, \gamma^{[l(\gamma)]} = \gamma)$  which means that the path through  $\mathbb{I}$  described by  $\Gamma'$  goes through  $\gamma$ .

There is a one-to-one correspondence between strategies defined in Section 2.1 and transformed strategies defined above. Given a strategy  $(\Gamma = (\Gamma_n)_n, \tau^\Gamma)$ , set

$$\Gamma'_n = \begin{cases} (\Gamma_0, \dots, \Gamma_n) & \text{on } \{\tau^\Gamma \geq n\} \\ (\Gamma_0, \dots, \Gamma_{\tau^\Gamma-1}, \Gamma_{\tau^\Gamma}, \dots, \Gamma_{\tau^\Gamma}) & \text{on } \{\tau^\Gamma < n\}. \end{cases} \quad (\text{A.8})$$

It is straightforward to show that  $\Gamma' = (\Gamma'_n)_n$  is a transformed strategy. On the other hand, given a transformed strategy  $\Gamma'$  set,

$$\Gamma_n = \begin{cases} L(\Gamma'_n) & \text{on } \{l(\Gamma') \geq n\} \\ t_N & \text{on } \{l(\Gamma') < n\}, \end{cases} \quad (\text{A.9})$$

$$\tau^\Gamma = l(\Gamma'). \quad (\text{A.10})$$

Then,  $(\Gamma = (\Gamma_n)_n, \tau^\Gamma)$  is a strategy.

We define for a transformed strategy  $\Gamma'$ :

$$X_{\Gamma'} = \exp\left(-\int_0^{L(\Gamma'_{l(\Gamma')})} r(s) ds\right) \left( X_{L(\Gamma'_{l(\Gamma')})} - TC_{L(\Gamma'_{l(\Gamma')})} \right) - \sum_{i=1}^{l(\Gamma')} \exp\left(-\int_0^{L(\Gamma'_i)} r(s) ds\right) DC_{\gamma_i}. \quad (\text{A.11})$$

Note that each  $\gamma \in \mathbb{I}$  defines a unique deterministic transformed strategy by setting  $\Gamma' = (0, \gamma^{[1]}, \dots, \gamma^{[N]})$  that stops in  $\gamma$ . Thus, (A.2) and (A.11) coincide in this case. Due to the one-to-one correspondence between strategies and transformed strategies the optimization problem (1.6) from Section 2.1 is equivalent to finding an optimal transformed strategy  $\Gamma'^{opt}$ :

$$E^Q(X_{\Gamma'^{opt}}) = \sup_{\Gamma'} E^Q(X_{\Gamma'}). \quad (\text{A.12})$$

*Solving the problem*

We solve this problem via backward induction:<sup>30</sup>

- For all  $\gamma$  with  $\mathbb{D}_\gamma = \emptyset$  set:<sup>31</sup>

$$Z_\gamma = X_\gamma. \quad (\text{A.13})$$

- If  $l(\gamma) < N$ ,  $\mathbb{D}_\gamma \neq \emptyset$  and  $Z_\delta$  has been defined for all  $\delta \in \mathbb{D}_\gamma$ :

$$Z_\gamma = \sup \left( X_\gamma, \sup_{\delta \in \mathbb{D}_\gamma} E^Q(Z_\delta | \mathcal{G}_\gamma) \right). \quad (\text{A.14})$$

$(Z_\gamma)_\gamma$  is adapted to  $(\mathcal{G}_\gamma)_\gamma$  and a supermartingale with respect to  $\prec$ .<sup>32</sup> For each  $\gamma$  we equip  $\mathbb{D}_\gamma$  with an order according to the size of the last element  $L(\delta)$ ,  $\delta \in \mathbb{D}_\gamma$ .

Define:

$$D(\gamma) = \inf \left\{ \delta \in \mathbb{D}_\gamma : E^Q(Z_\delta | \mathcal{G}_\gamma) = \sup_{\delta' \in \mathbb{D}_\gamma} E^Q(Z_{\delta'} | \mathcal{G}_\gamma) \right\} \quad (\text{A.15})$$

and let  $(\Gamma(\gamma))_{\gamma \in \mathbb{I}}$  be a family of random variables defined backward in the following way:

- For all  $\gamma$  with  $\mathbb{D}_\gamma = \emptyset$  set:

$$\Gamma'(\gamma) = (\gamma^{[0]}, \gamma^{[1]}, \dots, \gamma^{[N]}). \quad (\text{A.16})$$

- If  $l(\gamma) < N$ ,  $\mathbb{D}_\gamma \neq \emptyset$  and  $\Gamma'(\delta)$  has been defined for all  $\delta \in \mathbb{D}_\gamma$ , set:

$$\Gamma'(\gamma) = \begin{cases} (\gamma^{[0]}, \gamma^{[1]}, \dots, \gamma^{[N]}) & \text{on } \{Z_\gamma = X_\gamma\} \\ \Gamma'(D(\gamma)) & \text{on } \{Z_\gamma > X_\gamma\}. \end{cases} \quad (\text{A.17})$$

A simple exercise shows that  $\Gamma'(\gamma)$  is a transformed strategy for each  $\gamma$  with  $\Gamma'(\gamma) \in A^\gamma$ . In the following we show that  $\Gamma'((0))$  is optimal in the sense of (A.12). The following theorem is the key.

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<sup>30</sup>The procedure and the following proof of Theorem 1 are analogous to Cairoli and Dalang (1996), pp. 210–212, for constructing so-called optimal “accessible stopping points” when a process is indexed by specific finite subsets of  $\mathbb{N}^d$ .

<sup>31</sup>Note that  $l(\gamma) = N$  implies  $\mathbb{D}_\gamma = \emptyset$ .

<sup>32</sup>It can be shown that  $(Z_\gamma)_\gamma$  is Snell’s envelope, i.e. the smallest supermartingale dominating  $(X_\gamma)_\gamma$ .

**Theorem 1** For all  $\gamma \in \mathbb{I}$  and  $\Delta' \in A^\gamma$

$$Z_\gamma = E^Q(X_{\Gamma'(\gamma)}|\mathcal{G}_\gamma) \geq E^Q(X'_{\Delta'}|\mathcal{G}_\gamma) \quad (\text{A.18})$$

holds.

Proof. (A.18) is trivial for  $\gamma$  with  $\mathbb{D}_\gamma = \emptyset$ . Let  $\gamma \in \mathbb{I}$  with  $\mathbb{D}_\gamma \neq \emptyset$  and assume (A.18) has been shown for all  $\delta \in \mathbb{D}_\gamma$ . We first consider the equality in (A.18). By definition, we have for  $F \in \mathcal{G}_\gamma$ :

$$\int_F Z_\gamma = \int_{F \cap \{Z_\gamma = X_\gamma\}} X_{\Gamma'(\gamma)} + \int_{F \cap \{Z_\gamma > X_\gamma\}} Z_\gamma. \quad (\text{A.19})$$

By hypothesis,  $Z_\delta = E^Q(X_{\Gamma'(\delta)}|\mathcal{G}_\delta)$  holds for all  $\delta \in \mathbb{D}_\gamma$ , and by definition  $Z_\gamma = E^Q(Z_\delta|\mathcal{G}_\gamma)$  on  $\{Z_\gamma > X_\gamma\} \cap \{D(\gamma) = \delta\} \in \mathcal{G}_\gamma$ . This implies  $Z_\gamma = E^Q(X_{\Gamma'(\delta)}|\mathcal{G}_\gamma)$  on  $\{Z_\gamma > X_\gamma\} \cap \{D(\gamma) = \delta\}$  and therefore  $Z_\gamma = E^Q(X_{\Gamma'(D(\gamma))}|\mathcal{G}_\gamma)$  on  $\{Z_\gamma > X_\gamma\}$ . Consequently, the equality in (A.18) holds.

The next step is proving the inequality. Let  $\Delta' \in A^\gamma$  with  $\Delta' \neq \Gamma'(\gamma)$ . For all  $\delta \in \mathbb{D}_\gamma$ , define

$$\Delta'(\delta) = \begin{cases} \Delta' & \text{on } \{\Delta'_{l(\gamma)+1} = \delta\} \\ (\delta^{[0]}, \delta^{[1]}, \dots, \delta^{[M]}) & \text{on } \{\Delta'_{l(\gamma)+1} \neq \delta\}. \end{cases} \quad (\text{A.20})$$

$\Delta'(\delta)$  is a transformed strategy with  $\Delta'(\delta) \in A^\delta$ . As  $\Delta' \in A^\gamma$ ,  $\Delta'$  goes through  $\gamma$  which implies  $\Delta'_{l(\gamma)} = \gamma$ . This implies that we either have  $\Delta'_{l(\gamma)+1} = \gamma$ , i.e.  $\Delta'$  stops in  $\gamma$ , or  $\Delta'_{l(\gamma)+1} \in \mathbb{D}_\gamma$ . Also note that  $\{\Delta'_{l(\gamma)+1} = \delta\} \in \mathcal{G}_\gamma$  for all  $\delta \in \mathbb{D}_\gamma$ . Thus, we have for  $F \in \mathcal{G}_\gamma$ :

$$\int_F Z_\gamma \geq \int_{F \cap \{\Delta'_{l(\gamma)+1} = \gamma\}} Z_{\Delta'} + \sum_{\delta \in \mathbb{D}_\gamma} \int_{F \cap \{\Delta'_{l(\gamma)+1} = \delta\}} Z_\delta \quad (\text{A.21})$$

$$\geq \int_{F \cap \{\Delta'_{l(\gamma)+1} = \gamma\}} X_{\Delta'} + \sum_{\delta \in \mathbb{D}_\gamma} \int_{F \cap \{\Delta'_{l(\gamma)+1} = \delta\}} X_{\Delta'(\delta)} \quad (\text{A.22})$$

$$= \int_{F \cap \{\Delta'_{l(\gamma)+1} = \gamma\}} X_{\Delta'} + \int_{F \cap \{\Delta'_{l(\gamma)+1} \neq \gamma\}} X_{\Delta'} \quad (\text{A.23})$$

$$= \int_F X_{\Delta'}, \quad (\text{A.24})$$

where the first inequality results from the supermartingale property of  $Z_\gamma$  and the second holds by hypothesis. Consequently, the inequality in (A.18) holds.  $\square$

Theorem 1 implies that the transformed strategy  $\Gamma'((0))$  defined by (A.17) is an optimal solution of (A.12). Thus,  $Z_{(0)} = E^Q(X_{\Gamma'((0))} | \mathcal{G}_{(0)})$  equals  $V_0$ , i.e. the value of the derivative, and—due to the one-to-one correspondence—an optimal strategy exists.

The final step is to show (via induction) that the optimal transformed strategy  $\Gamma'((0))$  is equivalent via (A.9) and (A.10) to the strategy defined by (1.11) to (1.13). The equivalence is trivial for  $n = 0$ . Assume it holds for  $n$ . Let  $H = \{(\Gamma_0, \dots, \Gamma_n) = \gamma\} \cap \{D(\gamma) = \delta^*\} \cap \{\tau^\Gamma \geq n\} \in \mathcal{G}_\gamma$ . As  $\delta^*$  is a direct successor of  $\gamma$ , its first  $n$  elements form  $\gamma$ . By (A.9) we have  $\Gamma_{n+1} = L(\Gamma'_{n+1}) = \delta^*_{n+1}$  on  $H$ . It suffices to show that  $\delta^*_{n+1}$  equals the smallest element of  $\mathbb{I}$  larger than  $\gamma_n$  that maximizes  $E^Q\left(\exp\left(-\int_{\gamma_n}^{\delta_{n+1}} r(s)ds\right) V_{\delta_{n+1}} \middle| \mathcal{F}_{\Gamma_n}\right)$  on  $H$  with  $\delta \in \mathbb{D}_\gamma$ .

First, define for each  $\Delta' \in A^\gamma$ :

$$X_{\Delta'}^n = \exp\left(-\int_{\gamma_n}^{L(\Delta'_i)} r(s)ds\right) \left( X_{L(\Delta'_i)} - TC_{L(\Delta'_i)} \right) - \sum_{i=n}^{l(\Delta')} \exp\left(-\int_{\gamma_n}^{L(\Delta'_i)} r(s)ds\right) DC_{L(\Delta'_i)}. \quad (\text{A.25})$$

$E(X_{\Delta'}^n | \mathcal{G}_\gamma)$  equals the “value” of the derivative in  $\gamma_n$  if the strategy  $\Delta'$  is applied. By definition, we have  $E^Q(Z_{\delta^*} | \mathcal{G}_\gamma) \geq E^Q(Z_\delta | \mathcal{G}_\gamma)$  for all  $\delta \in \mathbb{D}_\gamma$  on  $H$ . Given (A.18),  $Z_\delta = E^Q(X_{\Gamma'(\delta)} | \mathcal{G}_\delta)$  holds, and, thus,

$$E^Q(X_{\Gamma'(\delta^*)} | \mathcal{G}_\gamma) \geq E^Q(X_{\Gamma'(\delta)} | \mathcal{G}_\gamma) \text{ for all } \delta \in \mathbb{D}_\gamma \text{ on } H. \quad (\text{A.26})$$

As the  $\Gamma'(\delta)$  and  $\Gamma'(\delta^*)$  coincide in their first  $n$  elements, this implies:

$$E^Q(X_{\Gamma'(\delta^*)}^n | \mathcal{G}_\gamma) \geq E^Q(X_{\Gamma'(\delta)}^n | \mathcal{G}_\gamma) \text{ for all } \delta \in \mathbb{D}_\gamma \text{ on } H, \quad (\text{A.27})$$

which implies that  $\delta^*_{n+1}$  fulfills the above required condition. Analogue considerations hold for  $\tau^\Gamma$ .

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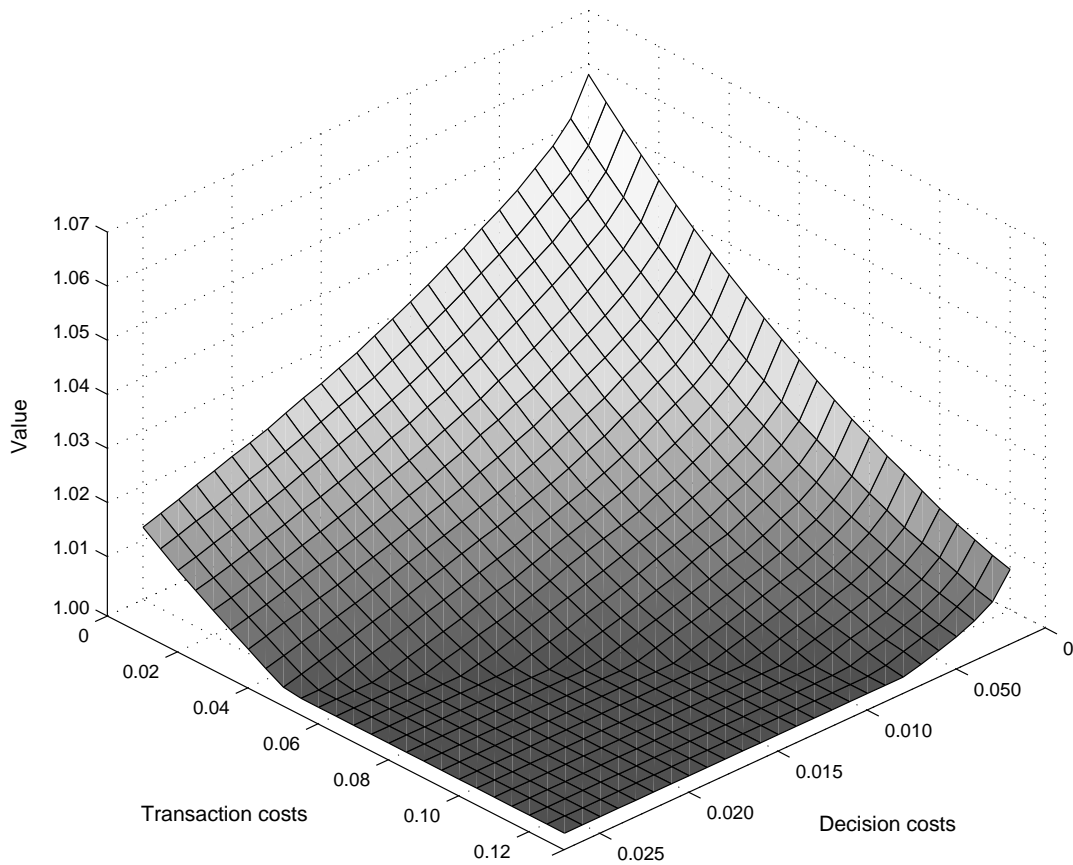
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## Figures

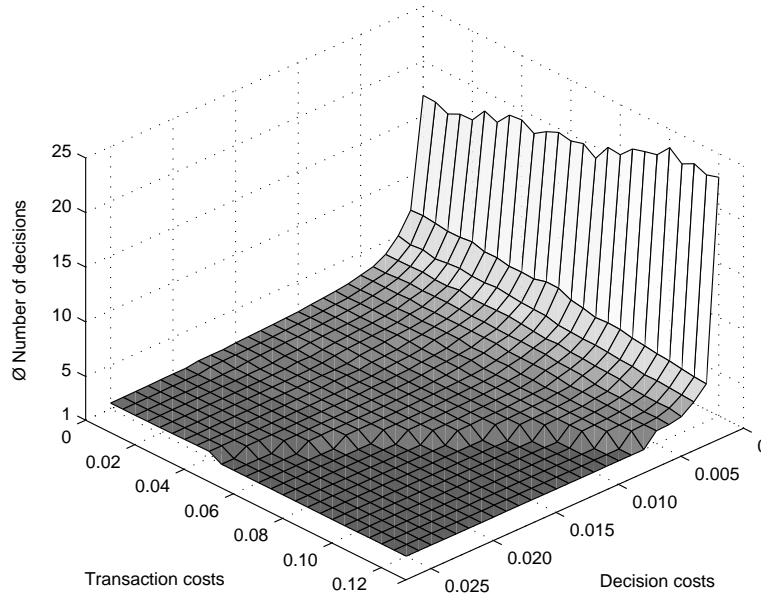
**Figure 1:** Valuation of a puttable bond dependent on transaction and decision costs

This figure shows the exemplary valuation of a puttable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with  $\kappa = 20\%$ ,  $\sigma = 2.5\%$  and  $\theta$  adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and least squares regression methods are applied.



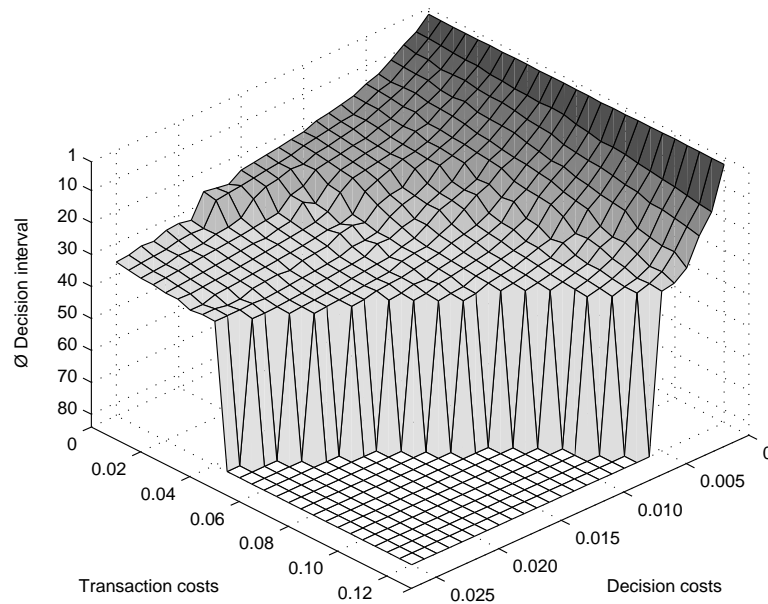
**Figure 2:** Average number of decisions dependent on transaction and decision costs

This figure shows the average optimal number of decisions according to the optimal decision strategy for an exemplary puttable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with  $\kappa = 20\%$ ,  $\sigma = 2.5\%$  and  $\theta$  adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure  $Q$  over all simulated paths.

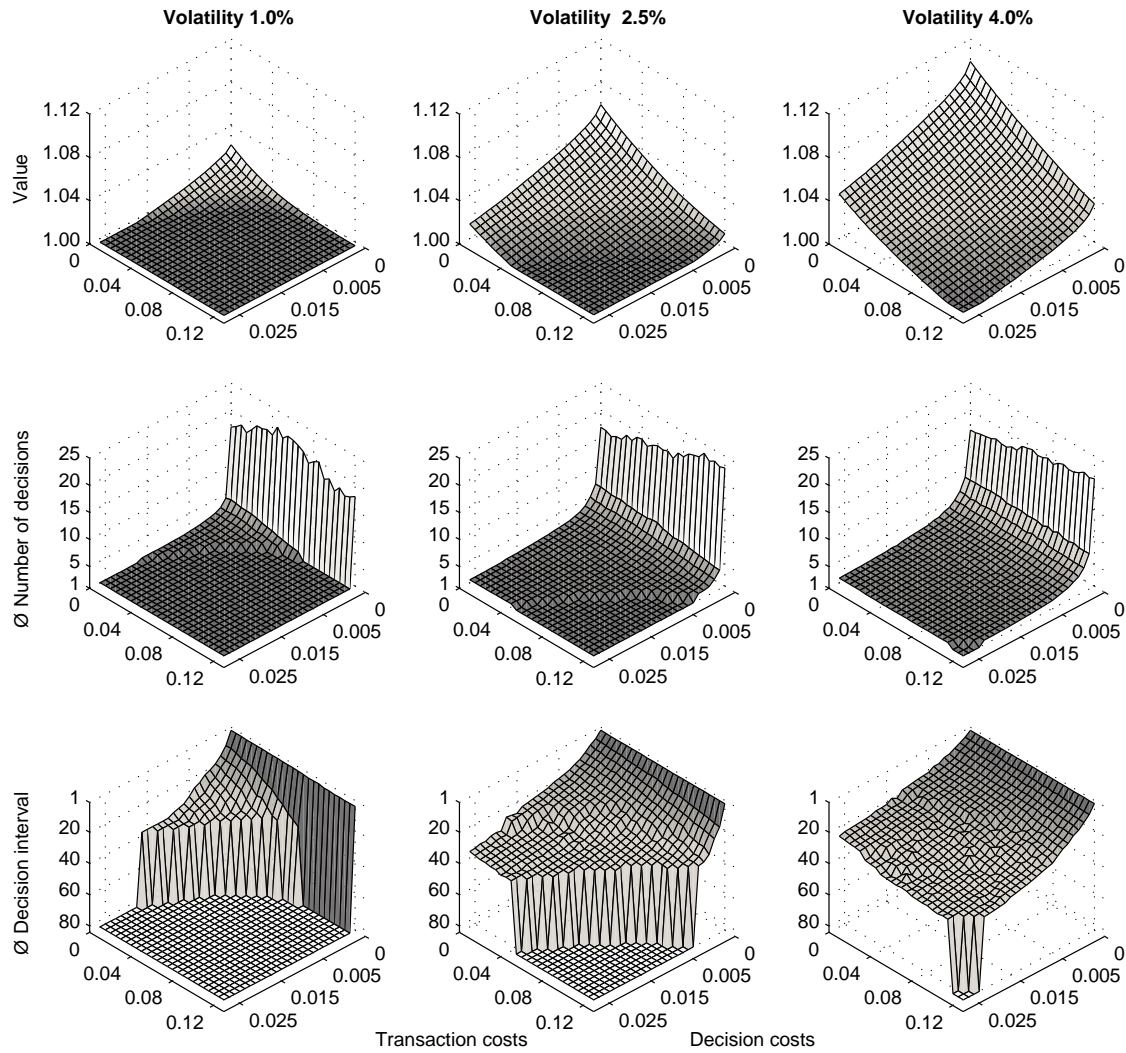


**Figure 3:** Average decision interval dependent on transaction and decision costs

This figure shows the average duration between two decision points ("decision interval") in steps according to the optimal decision strategy for an exemplary puttable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with  $\kappa = 20\%$ ,  $\sigma = 2.5\%$  and  $\theta$  adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure  $Q$  over all simulated paths.

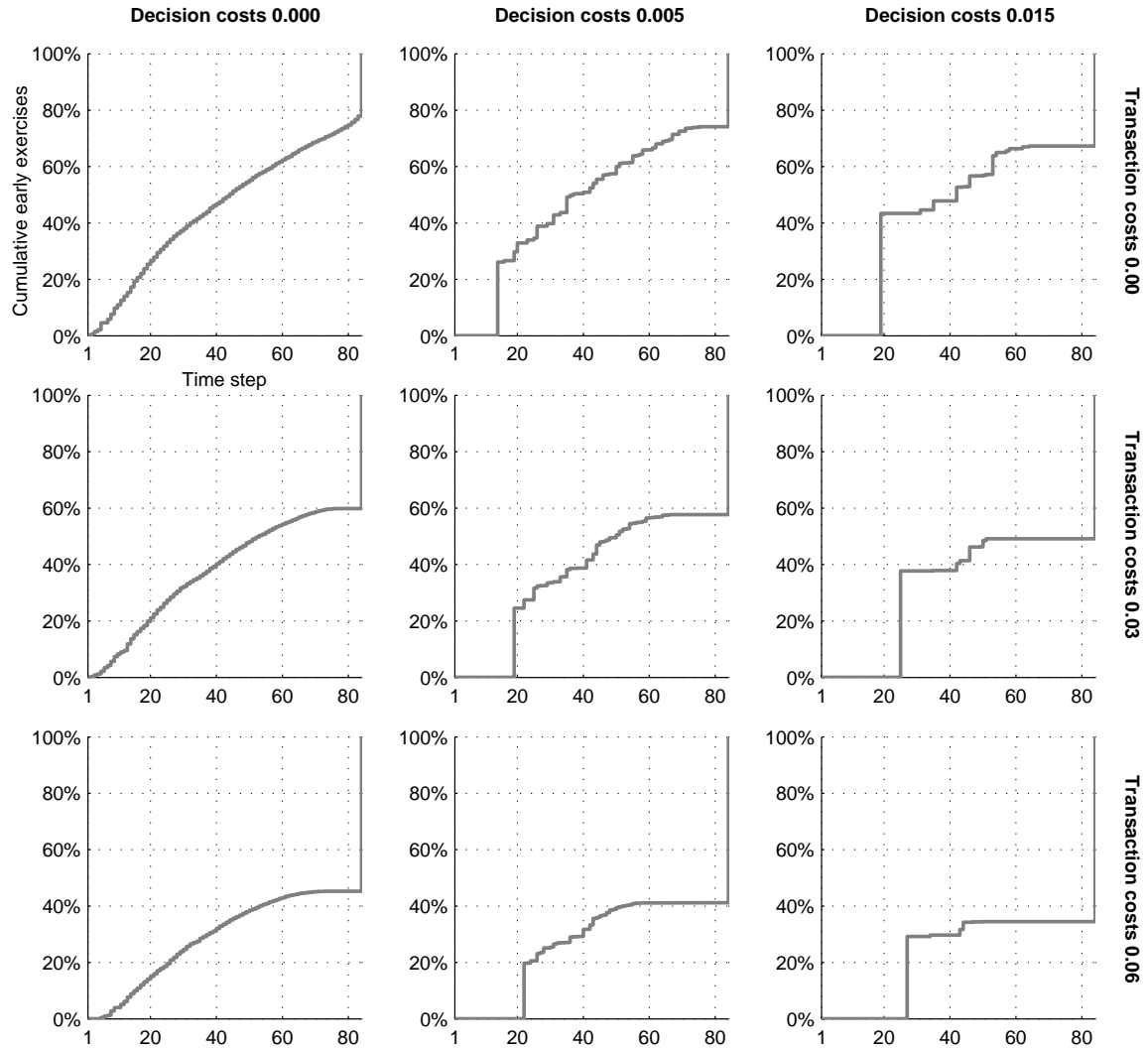


**Figure 4:** Sensitivities of value, number of decisions and decision interval regarding volatility. This figure presents a sensitivity analysis of the value and the optimal decision strategy, i.e. the average optimal number of decisions and the average decision interval (denoted in steps, minimum 1 step to maximum 84 steps), regarding changes in the interest rate volatility. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with  $\kappa = 20\%$ ,  $\sigma$  as shown and  $\theta$  adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure  $Q$  over all simulated paths.



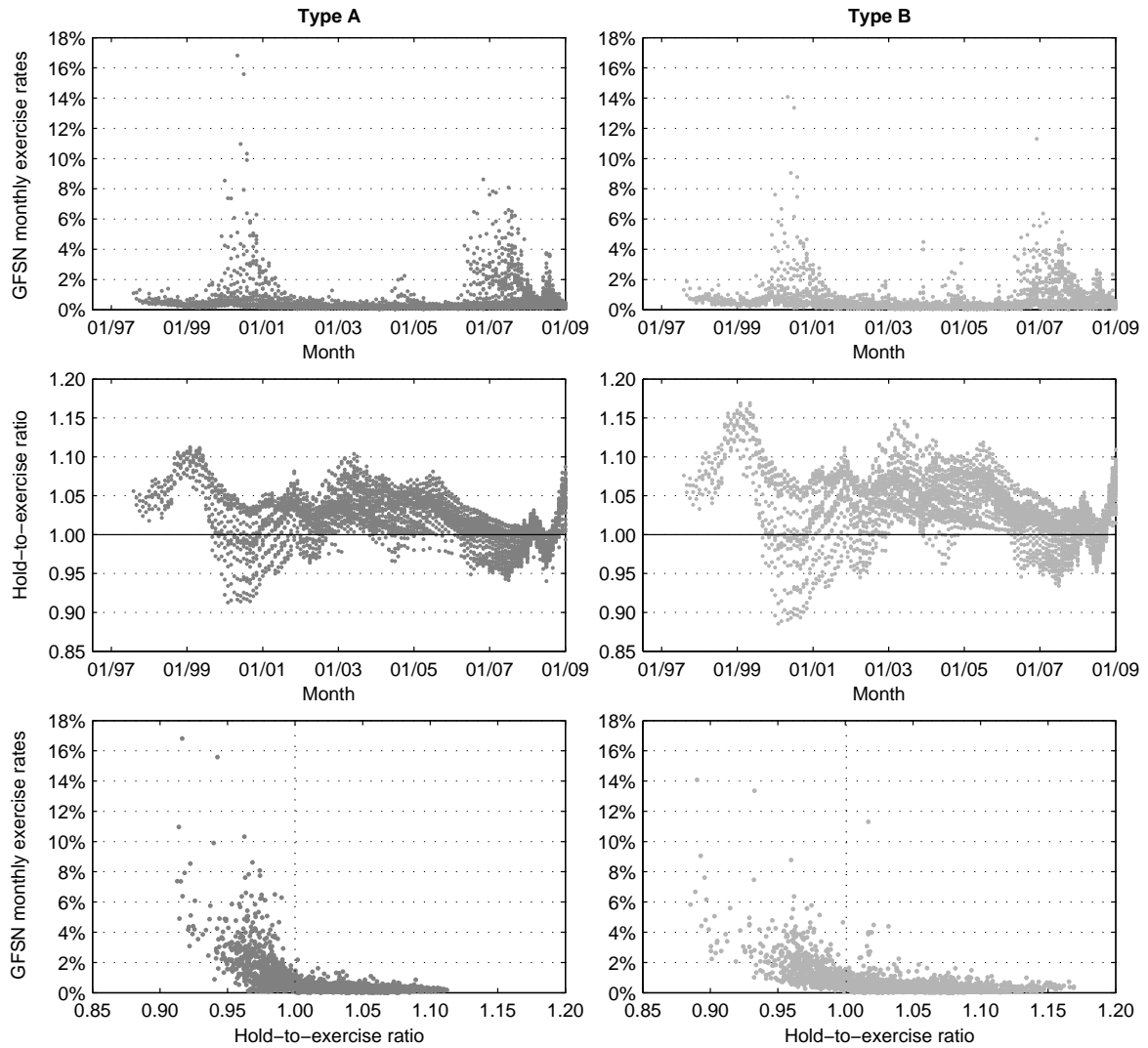
**Figure 5:** Cumulative exercise distributions over time dependent on selected transaction and decision costs

This figure shows cumulative early exercise distributions for an exemplary puttable bond over time dependent on selected transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with  $\kappa = 20\%$ ,  $\sigma = 2.5\%$  and  $\theta$  adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and least squares regression methods are applied.



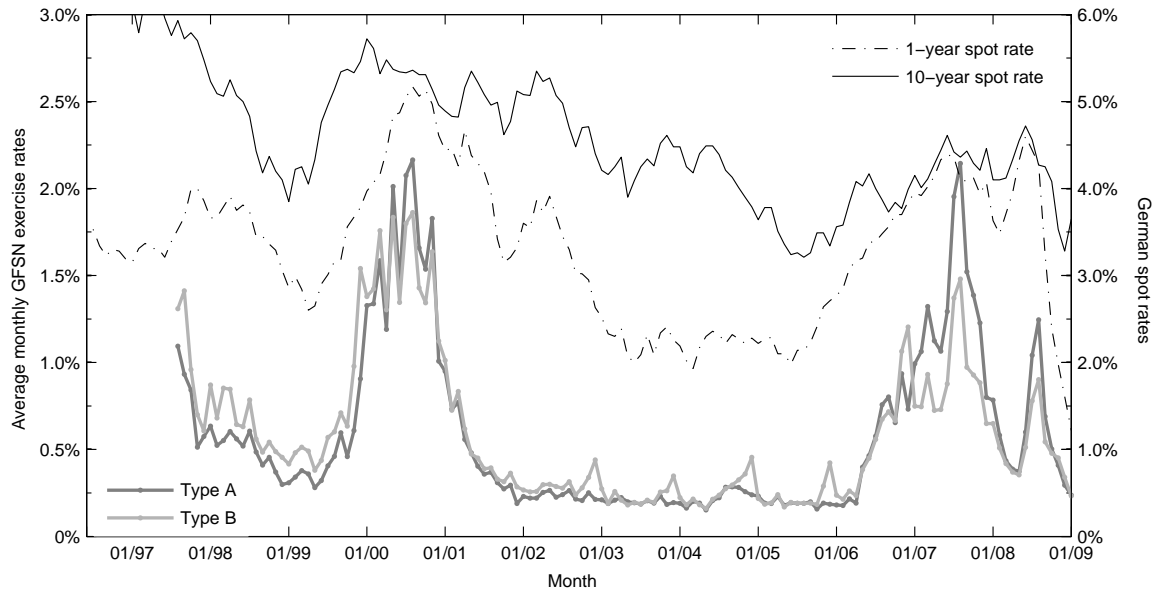
**Figure 6:** GFSN early exercise rates per month in relation to hold-to-exercise ratio

This figure shows the early exercise rates per month for Type A and B GFSN over time and in relation to the respective hold-to-exercise ratio, which is defined as the fraction of the present value of a GFSN without option to its exercise value. The upper charts present the development of the early exercise rates over time (except for the initial one-year blocking period) for all 178 exercisable GFSN issuances. The middle charts exhibit the development of the hold-to-exercise ratios for all GFSN over time. The lower charts plot the economic relation between the hold-to-exercise ratio and the early exercise rates per month. There are no exercises before 1997 due to the initial blocking period.



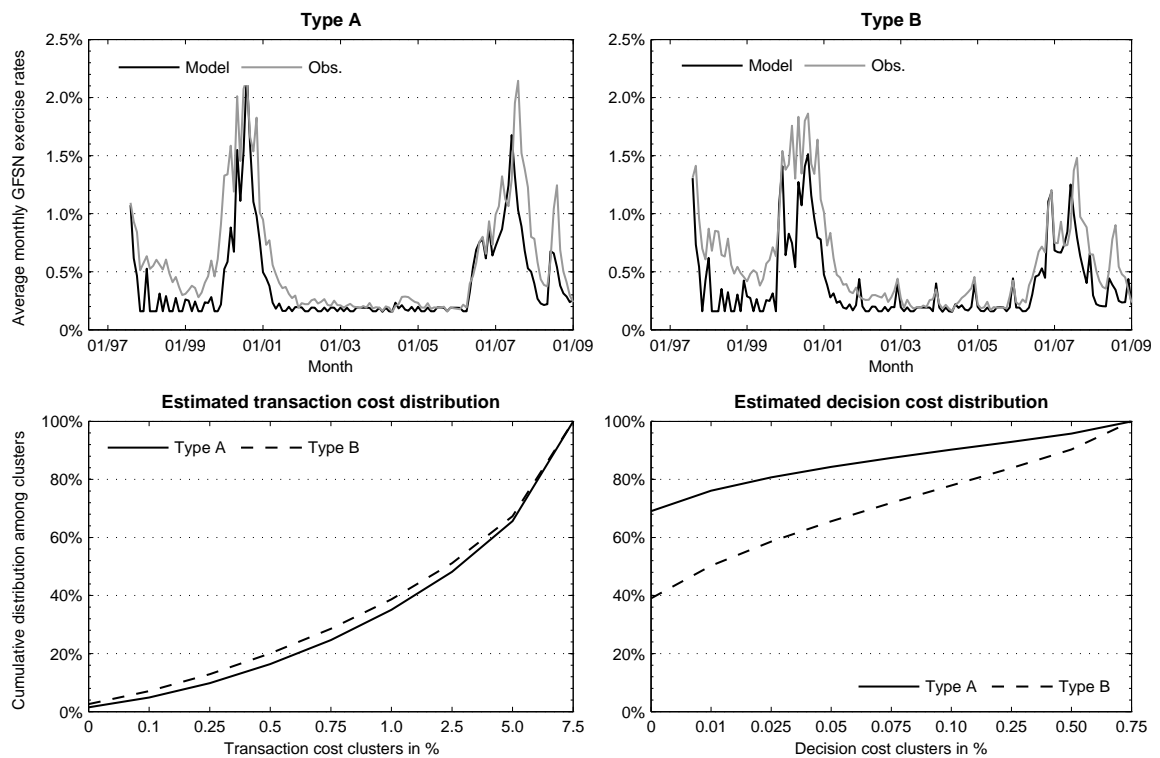
**Figure 7: Average monthly GFSN exercise rates**

This figure presents the average monthly early exercise rates over all exercisable GFSN of Type A and B from July 1996 to February 2009. There are no exercises before 1997 due to the initial one-year blocking period. Additionally, the development of the respective German 1-year and 10-year spot rates (according to Deutsche Bundesbank) are depicted.

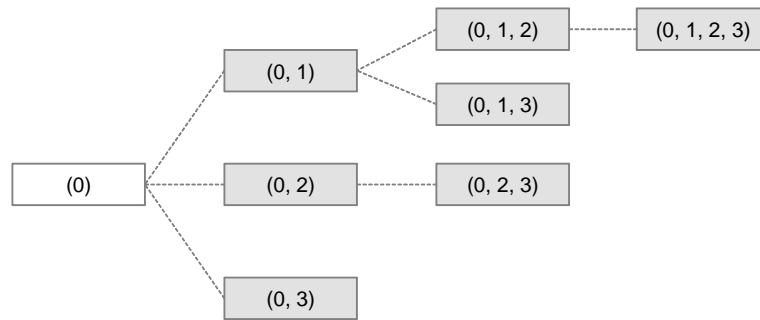


**Figure 8: Calibration results and model fit**

This figure shows the fit of the overall calibration and the estimated beta distributions for transaction and decision costs for both Type A and B GFSN based on the results in the first column of Table 4. The upper charts present a comparison between the mean modeled early exercise rates per month and the empirical observations. The lower charts exhibit the estimated cost distributions over the defined clusters.



**Figure 9:** Exemplary decision tree



## Tables

**Table 1:** GFSN coupon structure and spot rates

		Type A and Type B GFSN						Type B GFSN
		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Yearly coupon in %	Mean	2.745	3.199	3.627	4.034	4.409	4.821	4.944
	Median	2.750	3.250	3.625	4.000	4.250	4.500	4.750
	Min.	1.000	1.500	2.000	2.500	3.250	3.500	3.500
	Max.	4.500	4.750	5.500	6.250	6.750	7.500	7.500
	Std. dev.	0.810	0.811	0.805	0.819	0.856	0.927	0.924
Spot rate at issuance in %	Mean	3.387	3.540	3.700	3.853	3.994	4.122	4.235
	Median	3.429	3.599	3.779	3.898	3.936	4.021	4.117
	Min.	1.909	2.061	2.207	2.383	2.562	2.730	2.882
	Max.	5.119	5.216	5.267	5.293	5.608	5.940	6.202
	Std. dev.	0.821	0.766	0.727	0.709	0.707	0.713	0.724

The table exhibits statistics on the coupon offerings for Type A and B GFSN for all 204 issuances (102 Type A, 102 Type B) in our sample period from July 1996 to February 2009 and on the corresponding German spot rates. For year 1 to 6 identical coupons are offered for Type A and B at each issuance date, whereas the coupons in year 7 are only applicable for Type B. The spot rates represent the term structure of interest rates on listed Federal securities (method by Svensson) at the respective issuance date according to Deutsche Bundesbank.



**Table 2:** Summary statistics for GFSN data set

German Federal Saving Notes (GFSN)				Individual Investor accounts			
		Type A	Type B			Type A	Type B
Total issuances		102	102	Total		558,122	361,141
Number of investors per GFSN	Mean	39,719	26,192	Number of different GFSN investments per account	Mean	3.617	4.005
	Median	29,175	19,155		Median	1.000	1.000
	p5	8,711	7,639		p5	1.000	1.000
	p95	100,068	54,037		p95	13.000	16.000
		Std. dev.	17,385			Std. dev.	8.559
Volume in €m per GFSN	Mean	103.220	31.666	Investment volume in € per GFSN per account	Mean	21,087	9,721
	Median	77.778	25.703		Median	9,746	3,067
	p5	17.139	8.140		p5	511	255
	p95	254.559	66.864		p95	79,897	37,807
		Std. dev.	22.664			Std. dev.	27,702
Number of early exercises		547,864	333,232	Number of accounts with early exercises		148,812 (27%)	95,604 (26%)
Number of early exercises per GFSN	Mean	6,156	3,744	Number of early exercises per account	Mean	0.982	0.923
	Median	5,508	3,718		Median	0.000	0.000
	p5	514	444		p5	0.000	0.000
	p95	13,286	8,275		p95	5.000	4.000
		Std. dev.	2,355			Std. dev.	3.116
Early exercise volume in €m per GFSN	Mean	17.868	6.740	Early exercise volume in € per GFSN per account	Mean	2,849	1,661
	Median	13.989	5.053		Median	0	0
	p5	0.780	0.332		p5	0	0
	p95	46.914	17.164		p95	15,850	9,203
		Std. dev.	6.401			Std. dev.	8,669
Monthly early exercise rates in % per GFSN	Mean	0.570	0.536				
	Median	0.240	0.283				
	p5	0.100	0.096				
	p95	2.397	1.892				
		Std. dev.	1.028				

The table shows summary statistics on the GFSN data set for our sample period from July 1996 to February 2009. Overall 204 GFSN and 812,750 accounts with 881,096 early exercises are considered. The statistics on early exercises are based only on the 178 exercisable GFSN (after the initial one-year blocking period) from July 1996 to February 2008. €m stands for €million.

**Table 3:** Parameters for essentially affine three-factor model

	$\delta$	$\beta$	$\kappa$	$\theta$		
Estimated parameters	0.0106	1.0000	0.0365	0.0000	0.0000	15.0218
	0.0002	4.0189	-0.4965	-0.1798	0.3223	40.8385
	0.0000	2.5195	0.4256	-2.2507	1.6917	50.3077
	0.0013					

The table shows the estimated parameters for an essentially affine three-factor interest term structure model  $EA_1(3)$  on a weekly basis with one factor affecting the conditional variance matrix for the German market from July 1996 to February 2009. The model is defined as  $r_t = \delta_0 + \delta' Y_t$ , whereby the dynamics of  $Y$  are modeled by  $dY_t = K^Q(\theta^Q - Y_t)dt + \Sigma\sqrt{S_t}d\bar{W}_t$  and  $S$  is given by  $S_{t,j,j} = \alpha_j + \beta_j' Y(t)$ .

**Table 4:** Calibration results

		Overall 1996-2009		Sub-sample 1996-2002		Sub-sample 2003-2009	
		Type A	Type B	Type A	Type B	Type A	Type B
Mean squared error in %		0.0036	0.0025	0.0036	0.0025	0.0069	0.0035
Mean absolute error in %		0.2827	0.2627	0.3061	0.3276	0.5059	0.3860
Mean $R^2$ (on single GFSN level) in %		49.946	46.606	56.989	52.701	52.519	50.091
Consolidated $R^2$ (mean exercise rates) in %		78.915	72.715	86.005	77.338	70.105	58.351
Beta distribution of transaction costs	$\alpha_{TC}$	1.652	1.397	0.666	1.410	1.771	1.746
	$\beta_{TC}$	0.637	0.607	0.506	0.929	0.754	1.002
Std. dev.		0.247	0.265	0.336	0.268	0.244	0.249
Skewness		-0.838	-0.742	-0.257	-0.354	-0.731	-0.459
Mean transaction costs in %	$TC$	3.993	3.791	2.911	2.707	3.693	2.909
Median transaction costs in %	$TC$	2.500	2.500	1.000	1.000	2.500	1.000
Beta distribution of decision costs	$\alpha_{DC}$	0.132	0.350	0.123	0.523	0.275	0.512
	$\beta_{DC}$	0.679	0.687	0.825	1.642	1.421	0.796
Std. dev.		0.275	0.331	0.240	0.241	0.224	0.321
Skewness		1.745	0.647	2.093	1.031	1.629	0.408
Mean decision costs in %	$DC$	0.062	0.138	0.043	0.054	0.036	0.155
Median decision costs in %	$DC$	0.000	0.010	0.000	0.010	0.000	0.025
Passive investors in %	$\omega$	56.340	52.741	58.403	58.803	61.786	62.655
Sluggish investors in %	$s$	65.000	64.955	62.911	64.890	64.509	63.960
Additional exercises first month after blocking period in %	$EER^b$	2.092	2.426	3.572	3.175	1.947	3.175
Exogenous early exercises in % (dummies)	$DT99$		1.984		2.258		
	$DT06$		1.167				1.271
	$DYE$		0.568		0.804		0.792

The table shows calibration results for different data samples. The results in the first column are based on a calibration on the whole sample period from July 1996 to February 2009, while for the second and third column the calibration is based only on a sub-sample from July 1996 to December 2002 respectively on a sub-sample from January 2003 to February 2009. For the calibration all exercisable GFSN are valued based on the model of Section 2.1 using Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  (84 steps) and applying least square regression methods. The interest structure is modeled based on an essentially affine interest term structure model on a weekly basis. Accordingly, the calibration parameters are estimated via a multidimensional non-linear minimization algorithm based on the interior-point method with a MSE change tolerance of 0.000001% as stopping criterion. Additionally, the results are checked for local minima by re-running the calibration 250 times with random start parameters. The base exercise rate is fixed to the empirical observed value of 0.160% to ensure robustness of the calibration. The tax-related dummies are only considered for Type B GFSN ( $DT$  stands for tax changes,  $DYE$  for the year-end-effect).

**Table 5:** Comparison of standard model and empirical valuation based on calibration

	Value of early exercise right in GFSN in %				Advantage for issuer	
	Standard model (no transaction and decision costs)		Empirical (according to calibration)		Standard model minus empirical valuation	
	Type A	Type B	Type A	Type B	Type A	Type B
Mean	2.2433	2.9859	0.2009	1.1411	2.0424	1.8449
Median	1.9486	2.7005	0.1165	0.9372	1.8559	1.7604
p5	1.1071	1.3672	-0.1469	-0.1680	1.1741	1.3136
p95	4.0102	5.2208	0.8194	2.8956	3.2183	2.4394
Std. dev.	0.9704	1.2373	0.3134	1.0210	0.6791	0.4108

The table shows statistics on the valuation of the option component of all 178 exercisable GFSN at issuance in the sample period. The left part compares the valuation according to standard market models without transaction and decision costs and the empirical valuation based on the calibration results of Table 4 except the tax dummies. The right part shows statistics on the spread between the standard model and the empirical valuation.

## 2 | What Makes Individual Investors Exercise Early? Empirical Evidence from the Fixed-Income Market\*

M. Eickholt<sup>†</sup>, O. Entrop<sup>‡</sup>, M. Wilkens<sup>††</sup>

### Abstract

This paper studies the empirical early exercise behavior of Individual Investors in non-tradable puttable bonds. Analyzing circa 31 million holding and exercise decisions of more than 220,000 Individual Investors over 13 years, our major findings are: (i) Individual Investors use their early exercise right predominantly at points in time that are not economically advisable, which results on average in negative excess returns from exercising. (ii) Only a small fraction of attractive exercise opportunities are exploited over time. (iii) Exercise behavior differs significantly among investor groups and is related to personal characteristics. (iv) The demand for liquidity and financial flexibility is apparently a more important investment and exercise motive than performance seeking.

**Keywords:** early exercise; failure to exercise; liquidity demand; puttable bond

**JEL classification:** G10, G11

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## 1 Introduction

Individual Investors' use of stocks and equity derivatives has been a major field of research over the last two decades. A variety of papers have investigated Individual Investors' trading and exercise behavior (e.g., Shefrin and Stratman, 1985; Odean, 1999; Grinblatt and Keloharju, 2001; Poteshman and Serbin, 2003; Barber and Odean, 2011; Barraclough and Whaley, 2012), have analyzed investors' performance (e.g., Barber and Odean, 2000; Barber et al., 2009; Bauer et al., 2009), have studied differences dependent on personal characteristics (e.g., Barber and Odean, 2001; Dorn and Huberman, 2005) and examined the influence of exogenous effects, such as taxes (e.g., Badrinath and Lewellen, 1991; Ivkovic et al., 2005). In contrast, surprisingly little is known about Individual Investors' empirical behavior with regard to fixed-income products and derivatives,<sup>1</sup> even though interest-earning products, such as bonds, redeemable long-term deposits or savings bonds, are a common part of many Individual Investors' portfolios (see Bricker et al., 2012).

This paper therefore contributes to the literature by providing a comprehensive empirical analysis of Individual Investors' use of early exercise rights in German governmental non-tradable puttable bonds. Moreover, as we provide several new insights into the general exercise behavior of Individual Investors, we expect our results also to be of considerable interest for, e.g., banks and other issuers that offer similar products with embedded options to this specific investor class.

We basically cover three research questions. First, we analyze determinants of early exercises. In this context we focus on the following dimensions: economic benefit of exercising, investment history, environmental circumstances, product characteristics and investors'

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<sup>1</sup>A notable exception is the broad strand of papers on mortgages (e.g., Green and Shoven, 1986; Schwartz and Toruos, 1989; Stanton, 1995; Deng et al., 2005), which however focus mostly on rather specific questions related to prepayments, defaults or refinancing.

personal characteristics. Second, we examine the economic reasonableness of Individual Investors' exercise behavior and its determinants. More specifically, we analyze how often the empirically observed exercise decisions are financially reasonable according to theory, how often investors fail to exploit attractive opportunities and what excess returns investors achieve through early exercising. Third, we aim at discovering general investment and exercise motives of Individual Investors in puttable bonds.

Our study is based on a not-publicly available data set that comprises circa 31 million decisions of more than 220,000 Individual Investors to hold or exercise puttable bonds (German Federal Saving Notes, GFSN) over a time horizon of 13 years.<sup>2</sup> Besides the immense size and the long sample period our data set offers additional advantages, which allow us to draw a very comprehensive picture of Individual Investors' financial behavior. First of all, the fixed-income products we analyze (GFSN) are sold only to Individual Investors. Therefore, unlike many studies on the equity market (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012) we do not have to distinguish between different market participants, but can exclusively analyze Individual Investors' decisions. Moreover, GFSN cannot be traded on a secondary market. Hence, as an investor's only possibility to monetize a GFSN investment is to use the early exercise right, we obtain a more comprehensive picture of Individual Investors' exercise behavior than in the case of tradable products, such as tradable equity options. Finally, due to the detailed structure of the data set we can conduct analyses on an individual account level, whereas adjacent studies (e.g., Overdahl and Martin, 1994; Finucane, 1997; Lakonishok et al., 2007; Barraclough and Whaley, 2012) must typically rely on aggregated data sets or netted volumes. Consequently, this paper is, to the best of our knowledge, also first in studying the relations of individual exercise decisions

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<sup>2</sup>Our data set is similar to the data analyzed in Eickholt et al. (2014). Yet, in this paper we focus on a subsample of holding and exercise decisions on an individual account level, whereas the study of Eickholt et al. (2014) uses consolidated exercise ratios per product.

to, e.g., an investor's experience, former decisions on the investment position and personal characteristics.

As indicated, we start our analysis with examining the determinants of Individual Investors' early exercises of puttable bonds. Using pooled and random-effects logit regressions, our main findings are as follows: first, the economic benefit is an important determinant. The probability of an exercise increases significantly with a diminishing ratio of present value to exercise value. Second, the exercise probability depends on the investment and decision history. We observe delayed reactions to market changes within the last 6 months. Third, strong movements in the equity market are accompanied by enhanced exercise activities in GFSN, which indicates that Individual Investors use the early exercise right to liquidate investments so as to participate in attractive growth phases of other markets. Similarly, our results suggest—in an analogy to the equity market (e.g., Badrinath and Lewellen, 1991; Grinblatt and Keloharju, 2001; Ivkovic et al., 2005; Liedtka and Nayar, 2012)—that the exercise option is broadly used to optimize tax payments. Fourth, we find differences in the exercise probability related to product characteristics. Besides a product's maturity, the steepness of the coupon structure and the time until the next coupon payment also seem to have an influence on Individual Investors' decision-making. Our analysis shows that investors prefer—valuation, maturity and all else being equal—products with a high final coupon payment to products with a flatter coupon structure, which we attribute to psychological reasons. Moreover, it becomes obvious that investors hesitate to exercise a GFSN shortly before a coupon payment date, which—referring to the mental accounting concept (e.g., Shefrin and Statman, 1984; Szymanowska et al., 2009)—implies that investors differentiate between accrued interests and cash payouts. Fifth, the exercise frequency varies depending on an investor's portfolio and personal characteristics. For instance, we note that experience in early exercis-

ing a GFSN investment leads to an increased probability of another exercise, which might be due to lower information costs or learning effects (compare Nicolosi et al., 2009). As another example, we find that investors between 20 and 40 years of age have a significantly higher exercise probability than other investors groups. We ascribe this to a potentially increased liquidity demand at that stage of life due to, e.g., the purchase of a house.

Next, we study the economic reasonableness of Individual Investors' exercise behavior. Evaluating all decisions to hold or exercise early for each month and each product, our major result is that the broad majority of early exercises (between 76% and 86%) occur at times where it is not beneficial, i.e. where the exercise value lies (significantly) below the continuation value. In addition, we find that Individual Investors regularly fail to exercise (more) attractive exercise opportunities. Throughout our sample period more than 98% of exercise rights in GFSN that should have been used remain unexercised. This extent of "suboptimal" decisions in both directions is remarkably larger than in comparable studies for the equity market (e.g., Potesman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012). Not surprisingly, we calculate an on average negative excess return of exercising of circa -0.117% to -0.300% per year for our sample and conclude—consistent with the study of, e.g., Bauer et al. (2009) for equity options—that only a small number of Individual Investors achieves a better performance through early exercising puttable bonds. While this holds true for almost all investor subgroups, we again find differences depending on personal characteristics. For instance, investors between the ages of 20 to 40 show an even poorer exercise performance than other investor groups. In contrast, investors who use the direct distribution channel stand out with a significantly higher share of economically reasonable exercise decisions, which we attribute to a presumably higher financial literacy.

To sum it up, our main empirical findings in the first two analyses are that a broad range

of factors determines early exercises of GFSN and that Individual Investors use their option right predominantly when it is not economically advisable while they frequently forfeit (more) attractive exercise opportunities. All this indicates that maximizing profit is not the main motive of exercising. Literature typically relates such a non-performance-oriented behavior to irrationality (e.g., Potoshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012), a non-continuous monitoring of the investment (e.g., Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2013) and transaction costs (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006). Yet, several patterns in the data let us assume that—besides these arguments—a further factor influences Individual Investors’ decisions in the fixed-income market. Our results suggest that the demand for liquidity and financial flexibility, driven by, e.g., exogenous events or personal needs, is an important driver of early exercises.

Consequently, our third research interest is to further explore the general motives of Individual Investors’ for holding and exercising puttable bonds. For this, we run an exploratory factor analysis based on consolidated information on investors’ investment and exercise strategies in GFSN. Passing several robustness checks, the factor analysis isolates four to five latent factors, which underlines our former argument that performance is not the sole decision criterion. We interpret the latent factors as follows: the first and most important factor comprises investors’ wish for liquidity and financial flexibility. The second factor represents the importance of the mid- and long-term value of an investment. The third factor isolates the desire for performance and is associated with positive excess returns. In contrast, a fourth factor summarizes an active “trading” strategy that does not result in higher investment yields, for which reason we label this factor “activism”. Finally, the fifth factor represents an investor’s sensitivity to changes in taxation. Not surprisingly, the relevance of these identified latent factors differs among the investor base. Still, the overall results of the factor analysis suggest



again that Individual Investors in putable bonds interpret their early exercise right more as a financial flexibility feature than as an option to improve the investment performance.

The paper proceeds as follows. In Section 2 we introduce the data set. Section 3 analyzes the determinants of early exercising (Section 3.1), evaluates the economic reasonableness of Individual Investors' exercise behavior (Section 3.2), examines investors' performance (Section 3.3) and discusses implications (Section 3.4). Following these, Section 4 runs a factor analysis to gain further insights into Individual Investors' motives and decision-making on putable bonds. Section 5 concludes.

## 2 Data

### 2.1 Product description

For our analyses we are able to utilize a large and unique data set from the German Finance Agency (“Bundesrepublik Deutschland Finanzagentur GmbH”)<sup>3</sup>, which covers investment and early exercise decisions of Individual Investors in German Federal Saving Notes (“Bundesschatzbriefe”, in the following GFSN). GFSN are basically standard putable bonds issued by the Federal Republic of Germany for financing the government,<sup>4</sup> which—most interesting for our study—incorporate an early exercise option that gives the investor the right to reclaim his investment plus accrued interests at any time during maturity after an initial one-year blocking period.<sup>5</sup> No fees or penalty payments are charged in the case of an early

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<sup>3</sup>The German Finance Agency is a state-owned central service agency for Germany's governmental borrowing.

<sup>4</sup>While GFSN accounted for a significant share of Germany borrowing until the 1990s, the relevance has decreased since then. Finally, due to high administrative costs, the German government decided in 2012 to stop offering products exclusively for Individual Investors and stopped issuing new GFSN. Nevertheless, similar products are still sold by, e.g., banks.

<sup>5</sup>An additional restriction applies: the maximal exercise value per investor is capped at €5,000 within 30 interest days, which we ignore in the following.

exercise. At each issuance date two types of GFSN are offered: Type A, a step-up bond with a maturity of 6 years and yearly coupon payments and Type B, a step-up bond with a maturity of 7 years where all coupons are accrued and paid at maturity.

Investors can purchase the current issuance of a GFSN at any time at nominal value plus accrued interests. Regularly, following significant changes in market conditions, all open issuances are closed and new GFSN of both Type A and Type B are issued with identical coupon structures for the first six years. GFSN can be acquired in two ways. First, it is possible to invest directly through the German Finance Agency via telephone, postal order or online order. Second, GFSN can be purchased at banks, which however typically claim custody fees for administering GFSN positions. Investors can shift their investments from an account at a bank to a cost-free account at the German Finance Agency at any time.

Several advantages make GFSN a natural choice to analyze Individual Investors' exercise decisions in the fixed-income market. Most importantly, GFSN are sold exclusively to Individual Investors,<sup>6</sup> hence we do not have to distinguish in our analysis between different investor types as in many adjacent studies (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012). Next, GFSN cannot be traded on a secondary market. The only possibility of monetizing an investment is to use the exercise right and return the product to the issuer, which gives us a very comprehensive picture of investors' exercise activities. Finally, GFSN are simple and standardized fixed-income products that have been continuously offered in the same structure since the 1970s, which allows us to conduct consistent cross-sectional and time-series analyses.

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<sup>6</sup>Additionally, GFSN can also be acquired by resident institutions serving public benefit, charitable or religious purposes, which we neglect in the following due to the very small share of overall investments.

## 2.2 Summary statistics

Our data set comprehends—on a daily and single-account basis—all GFSN transactions and early exercise decisions of 223,017 Individual Investors booked between July 1996 and February 2009 in the German Finance Agency’s debt register account.<sup>78</sup> Overall, these transactions are spread over 204 issuances of Type A and B GFSN in the observation period on 102 issuances dates. In addition, we have some information on each investor’s personal characteristics. This detailed data structure on an individual account and daily level allows us to link individual transactions to, e.g., an investor’s age, gender or investment history, which is not possible in studies that must rely on aggregated data sets or ratios (e.g., Overdahl and Martin, 1994; Finucane, 1997; Lakonishok et al., 2007; Barraclough and Whaley, 2012). Table 1 presents selected summary statistics for the investor sample.

[Table 1 about here.]

In general, we have a broad and relatively balanced investor base that is very similar for both Type A and Type B GFSN structures. For instance, there is an almost equal representation of male (33.469% for Type A, 38.406% for Type B) and female (38.362%, 37.995%) investors (no information for 28.169%, 23.598%). Similarly, we have a wide range of investor ages (on average 43.551 years, 32.711 years) with a slightly positively skewed distribution, i.e. there are somewhat more older than younger investors in our data set. There is also a significant share of very young investors (in particular for the zero-bond structure of Type B GFSN), which we mainly interpret as savings accounts in a child’s name for, e.g., education costs. Concerning investors’ educational background, the data provides us with information

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<sup>7</sup>Our data set is a randomly drawn sample of the original data that represents circa 25% of all accounts at the German Finance Agency.

<sup>8</sup>We exclude accounts with an average transaction volume below €300 since for such small volumes the early exercise right is of comparatively low importance.

on doctoral degrees and professorships, which taken together form 4.150% (3.499%) of the investors. Geographically, the investor base is widespread over Germany. We classify all investors into four clusters of residence areas<sup>9</sup> based on the first two digits of an investor's zip code, which we have in the data, and conclude that the majority of investors in GFSN live in less populated areas (51.371%, 51.417%), while a smaller number (15.380%, 15.089%) lives in highly populated areas or cities. Finally, we note that more than half of the investors prefer to acquire GFSN indirectly (59.181%, 54.011%) via banks and transfer their accounts later to the German Finance Agency, even though typically custody and administration are only cost-free at the German Finance Agency.<sup>10</sup> Next, Table 2 summarizes the activities of our investor sample.

[Table 2 about here.]

The statistics show that most investors invested rarely or only once in GFSN during the sample period. The average number of investments is comparatively small with only 3.147 investments per investor for Type A and 2.790 for the less popular structure of Type B GFSN. Furthermore, we find small median overall investment volumes of only € 10,226 and € 4,090 respectively, whereby the high variances for both product types indicate that there are also investors with significantly higher investment volumes and trading frequencies. In general, we attribute the low activity level in our sample to the strict restriction of GFSN to Individual Investors. With institutional investors we would expect considerably more transactions and

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<sup>9</sup>As a proxy we use the average population density per sq. km according to German Federal Statistics Office (2009) and define the following clusters: <250 = sparsely, 250-750 = moderately, >750 = highly populated area. In a fourth cluster we group investors living in large cities, which we identify by their short double-digit zip codes that are allocated only to major cities in Germany.

<sup>10</sup>Indirect distribution means in this paper that an investor purchases a GFSN at a bank and transfers his investment later to the German Finance Agency. In contrast, we define the preferred distribution channel as direct if an investor executes at least one direct transaction in the overall observation period. We are aware that this classification might produce a bias towards direct distribution since some investors use both channels. On the other hand we do not have direct transactions in our data set from 1996 to 1999 as these were not possible during these times.

higher investment sums.

The right to redeem early a GFSN investment is frequently used (fourth to sixth row). On average each investor in Type A GFSN exercised early 0.526 times (0.517 for Type B) throughout the sample period resulting in a mean overall exercise volume of € 1,812 (€ 1,284). Again the standard deviation is high with 1.491 (1.483) exercises per investor, which is also emphasized by the median of 0 (0) and the 95 percent quantile of 2.000 (2.000) exercises per investor account. The high variance here is a first indicator that Individual Investors use early exercise rights very heterogeneously. We further discuss this issue in a later section.

Finally, the last two rows of Table 2 outline investors' use of the exercise option over time. As proxy we calculate a monthly early exercise rate per GFSN, defined as the ratio of number of exercises to the number of overall investments ( $\frac{\text{Number of exercises}}{\text{Number of investments}}$ ). On average this rate amounts to circa 0.626% (0.518%) per month and GFSN, whereby the median value is smaller due to several exercise peaks throughout our observation period. Interestingly, a quite constant base exercise rate seems to exist over all GFSN. Even the 5 percent quantile of the exercise rates still lies at around 0.081% (0.081%), which implies that exercises occur independently of the market environment.

### 2.3 Variables

To investigate Individual Investors' decisions in the course of time and to identify determinants of early exercises we convert the described transaction data to a longitudinal structure on a monthly basis. The resulting data panel comprises circa 31 million decisions by the investor base to hold or exercise a GFSN, which is a noticeably larger sample than that in most studies on early exercises so far. Moreover, we comprehend the data set with several variables and ratios (Table 3 contains a detailed overview) in the following categories to ac-

count for potential influences on investors' decisions: economic benefit, investment history, environmental circumstances, product characteristics, portfolio and personal characteristics.

[Table 3 about here.]

*Economic benefit*

We use the ratio of present value of a GFSN to its exercise value (PVEV) to measure the economic advantage of a potential early exercise:

$$PVEV = \frac{\text{Present value of GFSN}}{\text{Exercise value of GFSN}}. \quad (2.1)$$

According to standard theory it is optimal to use the exercise right as soon as the present value equals the exercise value. This means only an early exercise at a PVEV-ratio of 1 can be classified as economically reasonable,<sup>11</sup> whereas an exercise is the more disadvantageous the higher the PVEV-ratio rises above 1.

The exercise value of a GFSN is simply the notional amount plus accrued interests. For the calculation of the present value, we have to determine the value of the respective early exercise right, which basically equals an American put option. Our first step in this valuation is to model the underlying interest rate dynamics, applying an essentially affine 3-factor term structure model EA<sub>1</sub>(3) on a weekly basis according to Dai and Singleton (2000). We utilize here the parameterization by Eickholt et al. (2014), who calibrate this model to German term structures over the period from 1996 to 2009. Subsequently, we refer to standard option theory and apply the least-squares simulation approach as suggested by Longstaff and Schwartz (2001), carrying out Monte-Carlo simulations with 10,000 paths, whereby we use

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<sup>11</sup>For simplicity, we define an exercise decision as reasonable if the PVEV-ratio equals 1 at the exercise date regardless of the investment history, even though the exercise decision is theoretically only "optimal" if the exercise happens at the first feasible opportunity. This means, the estimated numbers in this paper represent a lower bound for the share of economically disadvantageous exercise decisions.

Euler discretization on a monthly basis and take the first four monomials as basis function. Moreover, we apply the interleaving estimator according to Glasserman (2004) to mitigate the effects of a potential high bias due to the backward induction approach and of a potential low bias due to a suboptimal stopping rule.

#### *Investment history*

Considering the investment history allows us to set Individual Investors' behavior in relation to, e.g., former valuation movements and former decisions. Accordingly, our first variable MISSEDALL measures for each GFSN and month how many economically attractive exercise opportunities an investor has ignored so far, quantified by the number of months when the PVEV-ratio equaled 1. The second variable MISSED6, defined as the number of economically reasonable exercise months omitted within the last half year, controls for short-term effects and examines whether investors react sluggishly to former exercise opportunities, which is a well known pattern for small traders in equity products (e.g., Hvidkjaer, 2011). Finally, we also incorporate the investment volume (VOLUME).

#### *Environmental circumstances*

We control for four environmental factors. First, we consider the development of the German stock index CDAX as proxy for potential investment yields in the equity class. Based on the assumption that mainly large changes of the CDAX may have an influence on early exercises in the fixed-income market, the dummy variables CDAX10+ and CDAX10- signal increases or decreases of the CDAX of more than 10% within one month. Focusing on longer trends, the dummy variables CDAX25+ and CDAX25- cover index changes of more than 25% within 6 months. Second, VOLDAX and VOLINT measure the option-implied 45-day volatility of

the German stock market (using the VDAX index) and the volatility of the German 5-year spot rate estimated from weekly returns over a 6-month window to encompass investors' reaction to uncertainties in the markets. Third, the dummy variable *NEWMARKET* controls for abnormal capital outflows to the "New Market" between 1998 and 2000, a new stock segment in Germany which attracted considerable attention among Individual Investors during this period. Fourth, we account for tax effects, whereby we focus on Type B GFSN since changes in the tax legislation have a much higher relevance for products with a zero-bond structure—as all coupon payments are taxed at the same time—than for coupon-paying products for which personal tax allowances can be utilized each year. During our observation period two major changes occurred in the German tax legislation. In both years 2000 and 2006 Individual Investors' tax allowances were severely reduced, which might have made it attractive for investors to exercise GFSN positions early shortly before the new regulations became effective—even when a valuation was clearly above the exercise value—to optimize personal tax debts. Accordingly, our variables *TAX99* and *TAX06* are designed to capture potential extraordinary exercise activities in November and December 1999 and 2005, respectively. In addition, the dummy *YEAREND* controls for tax-motivated early exercises at the end of the year when Individual Investors typically review their tax burdens and allowances.

#### *Product characteristics*

While the general structure of GFSN remains constant over all issuances in our sample period, the offered coupons are typically adapted to current market circumstances with every issuance. Accordingly, we have a broad range of coupon structures among the 204 GFSN offerings in our sample. To determine if visual or psychological factors, such as the shape of the coupon structure or shortly awaited coupon payments affect an investor's behavior, we



consider three variables. First, we calculate the average yearly growth of coupon payments until maturity as an indicator for the steepness of the coupon structure (UPSTEEP). Second, we compute for each GFSN and month the duration in years (DURATION) of the bond component, which we interpret as the weighted average time until an investor receives all coupon payments and the initial investment. For the calculation we refer to the definition of Fisher and Weil (1971). Third, the dummy variable COUPON, which is only relevant for Type A GFSN, signals if a coupon payment occurs within the next 60 days.

Lastly, we incorporate the dummy variables BLOCK and LIFETIME (for Type A GFSN). BLOCK controls for potential increased exercise activities in the first month after the initial one-year blocking period. LIFETIME is the elapsed time since issuance.<sup>12</sup>

#### *Portfolio characteristics*

Besides personal attributes, such as age or gender, we also account for an investor's experience in GFSN. We define four proxies: the number of former personal investments in GFSN (INVESTS), the sum of overall investment volumes so far (INVESTSUM), a dummy variable indicating whether the investor has made use of the early exercise right once before for a GFSN investment (EXERCISED) and a dummy variable signaling if he has exercised early at an economically opportune time once before (PERFORMED) as indicated by a PVEV-ratio at exercise of 1.

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<sup>12</sup>Due to the zero-bond structure of Type B GFSN the variables LIFETIME and DURATION are for this type perfectly correlated, wherefore we neglect the LIFETIME variable for Type B in the following regressions. For Type A GFSN the lifetime is also correlated to the duration, which might potentially result in inflated standard errors. However, running the following regressions with orthogonalized regressors leads to similar findings, so that we decided to continue with the described variables.

### 3 Analysis of early exercise decisions

#### 3.1 Determinants

In this section we use the described panel data set to examine determinants of Individual Investors' early exercise decisions. We apply pooled and random-effects logit regressions to estimate the influence of our above-mentioned variables (see Table 3) on the probability of an exercise. For the panel regression we consider that each GFSN per investor represents an own group of observations, whereby the number of observations ranges from 1 for GFSN that are exercised early in the first month after the blocking period, to a maximum of 60 (72 for Type B) for investments held until maturity.

Our analyses are positioned at the very low end of the logit distribution due to the small number of exercise events compared to non-exercise events (as shown in Table 2, the exercise rate for most GFSN lies below 1% per month), which could lead to biased coefficient estimates and standard errors (see King and Zeng, 2001). Yet, rare-event regressions as introduced by King and Zeng (2001) result in only minor deviations in the estimates, wherefore we choose to continue with standard estimators due to the better handling and improved comparability. We also run several robustness checks—not reported here—based on time and investor subsamples, which lead to consistent findings. Table 4 contains the regression results on our overall data set and the respective marginal effects ( $dy/dx$ ) at means indicating the *ceteris paribus* effect of changes of the respective variable (fourth and seventh column). We use robust standard errors in this and all following regressions (see Huber, 1967; White, 1980, 1982).

[Table 4 about here.]

Beginning with the overall model output, we get a pseudo- $R^2$  of 17.00% for the pooled regression and 19.78% for the panel analysis (15.17% and 19.27%). We emphasize six regression results: first, the probability of the option being exercised is related to the economic benefit of an early exercise. A decreasing PVEV-ratio goes hand-in-hand with an increasing exercise probability, which implies that the average investor is sensitive to changes in the value of his investment. Second, the investment history has a statistically significant influence. The positive coefficients for MISSED6 and MISSEDALL indicate that the probability of an exercise increases with the number of missed economically advantageous exercise opportunities. While the influence of MISSED6 might be attributed to delayed or sluggish responses to earlier market or value changes, the positive loading for MISSEDALL is a more surprising result, as we would expect Individual Investors who aim to maximize their investment performance to utilize one of the first economically reasonable exercise opportunities to shift investments or re-arrange their portfolio. We further investigate this issue and how frequently investors exploit arising exercise chances in Section 3.2.

Third, environmental influences have a strong impact. We find that significant movements in the equity segment (CDAX) and the launch of the “New Market” in Germany are accompanied by increasing exercise activity, which suggests that investors use the early exercise right to liquidate investments so as to participate in attractive growth phases of other markets. On the other hand, the exercise probability diminishes in times of severe equity market drops (CDAX25–) and high volatility in the interest term structure (VOLINT). We trace this to a higher attractivenesses of fixed-income investments in times of bear equity markets and in times of higher uncertainty. Finally, the significantly positive coefficient for the tax variables indicates that—in an analogy to the equity market (e.g., Badrinath and Lewellen, 1991; Grinblatt and Keloharju, 2001; Ivkovic et al., 2005; Liedtka and Nayar, 2012)—Individual

Investors use the exercise right broadly to react to tax changes and to optimize their tax debt.

Fourth, the probability of an early exercise depends on product characteristics. We find a in general positive influence of the duration (DURATION) and a (in most cases) negative loading regarding the steepness of the upcoming coupon structure (UPSTEEP). Additionally, we note that investors hesitate to exercise early shortly before a coupon payment date (COUPON). Such behavior cannot be justified with standard economic arguments as, according to theory, an exercise decision should only be based on the current valuation regardless of the upcoming coupon structure. Similarly, there is no structural advantage to waiting until a coupon payment, as accrued interests are considered at an early exercise. Hence, we interpret the empirical patterns related to the product structure as psychological effects. Apparently, Individual Investors value GFSN with increasing coupon payments more highly than almost identical products that offer a flatter coupon structure. Moreover, the reduced exercise probability before coupon payment dates implies that investors differentiate between accrued interests and cash payouts and thus use “mental accounting” (e.g., Shefrin and Statman, 1984; Szymanowska et al., 2009). Lastly, we note that there is a peak in exercise rates in the first month after the blocking period.

Fifth, the early exercise frequency differs among investor groups. The regressions reveal that experience in exercising is associated with a higher early exercise probability (EXERCISED). This effect is even stronger if the investor has exercised at an economically reasonable point in time (PERFORMED). We attribute both effects to experienced investors having greater financial literacy, lower information costs and less inhibition in using the right to exercise. On the other hand, our two more general portfolio variables—the number of investments and the overall investment volume so far—show negative coefficients, which might

be due to less exercise activity of the part of wealthy investors (see also Dhar and Zhu, 2006).

Sixth, male investors and investors between 20 and 40 years of age use the exercise right more often, while investors holding a doctoral degree and investors who acquire directly through the German Finance Agency are slightly less likely to exercise. An investor's geographical location has no statistical influence. We further discuss the apparent differences in exercise probability among the investor base at the end of this section.

Our main conclusion in this first analysis is that a broad range of factors cause Individual Investors to exercise early. As economic arguments are only one determinant among several other influence factors, it seems natural to assume that not all early exercises of GFSN are financial advantageous. Hence, the next section focuses on examining the economic reasonableness of investors' decision-making.

### **3.2 Economic reasonableness**

In this section we investigate the economic reasonableness of Individual Investors' exercise behavior. According to standard theory a putable bond should be exercised as soon as its present value equals the exercise value, which implies a PVEV-ratio of 1. In contrast, there should occur no early exercises if the PVEV-ratio is greater than 1. We use this definition to determine how many early exercise decisions in our data set are economically reasonable and how many attractive exercise opportunities investors exploit or forgo over time. Table 5 contains the results of this economic analysis.

[Table 5 about here.]

The left part of Panel A shows that the majority of early exercises take place when it is not economically reasonable. For Type A GFSN, about 75.674% of early exercises are financially disadvantageous. For Type B, this share even amounts to 86.061%. In other words, only

24.326% (13.939%) of the empirical exercise decisions in our data set comply with the considerations in standard option pricing theory, namely that exercises should occur only at a PVEV-ratio of 1. Instead, we observe exercises at a broad range of valuations. In fact, a more detailed analysis—not reported here—reveals that the largest share of exercises (45.778% and 50.743%) can be construed as clearly economically disadvantageous, with a PVEV-ratio of greater than 1.03. This extent of “suboptimal” decisions of Individual Investors in GFSN is markedly larger than in comparable studies for equity derivatives (e.g., Poteshman and Serbin, 2003). It is also another indicator that enhancing the investment performance is not the primary motive of Individual Investors’ exercise decisions.

Next, we use the calculated PVEV-ratio per month and GFSN to determine how frequently Individual Investors take advantage of upcoming attractive exercise opportunities (right part of Panel A). This means we extend our analysis beyond the investor’s final early exercise and also consider all previous decisions to continue holding the investment. In line with our former findings the results reveal that Individual Investors act with much less financial sophistication than we would expect from, for example, institutional investors. In fact, overall only 1.856% (1.829%) of the attractive exercise opportunities that arise are utilized by Individual Investors. In other words, this implies that for both GFSN types investors waive more than 98% of their chances to increase the investment yield through re-arranging their portfolio, resulting in significant opportunity costs. This finding is also supported by the above-defined variable MISSEDALL (see Table 3), which counts for each investment at maturity or at an early exercise how many economically reasonable exercise months the investor has let pass and instead decided to continue holding the investment. In a separate analysis not shown here, we find that at an early exercise of a Type A GFSN an investor has already missed on average 3.920 economically reasonable exercise opportunities (2.425

for Type B), thereof 1.609 (1.000) within the last half year. While such a general “failure to exercise” is also well documented for retail investors in equity derivatives (Pool et al., 2008; Barraclough and Whaley, 2012), the incidence of missed exercises in our study is again—as for the share of economically disadvantageous exercise decisions—significantly higher. A possible reason might be the more conservative character of fixed-income products compared to other investment classes, as these are presumably preferred by a different type of investor with other investment horizons and motives.

Panel B of Table 5 again analyzes what if any differences exist in the reasonableness of exercise behavior among selected investor groups. For this, we run two logit regressions. First, we regress a dummy variable indicating whether or not an early exercise is reasonable on portfolio and personal characteristics of investors (left part), whereby we control also for the investment volume, for tax influences and for a potentially increased demand in the first month after the blocking period. In the second regression we use a dummy variable that denotes if an investor fails to exercise at an economically attractive opportunity or not (right part). In this case we also consider the investment history as dependent variable (MISSEDALL, MISSED6) to determine whether earlier decisions to forgo similar opportunities have an influence. Moreover, we control for the effect of a shortly upcoming coupon payment date (COUPON), account for different product characteristics (UPSTEEP, DURATION) and incorporate the lifetime of a GFSN.

The pseudo- $R^2$  for the first regression is 19.31% (12.90% for Type B GFSN) and 20.49% (18.79%) for the second regression. In general, the results demonstrate that the economic reasonableness of Individual Investors’ holding and exercise decisions in our sample differs broadly depending on the investment history, environmental influences, product characteristics and on portfolio and personal characteristics. Beginning with the investment history, the

regression output supports our former finding that investors tend to respond sluggishly to market changes. The likelihood of an investor missing a good opportunity to exercise declines if that investor has recently left out some attractive exercise chances (MISSED6). Regarding the product variables, we observe that the probability that an economically attractive exercise opportunity will be ignored is higher if the product characteristics of the respective GFSN match investors' preferred payment profile (see Section 3.1), i.e. if the GFSN has a steeply rising coupon structure and a low capital duration. This is consistent with our results in the last subsection. Similarly, the positive coefficient for COUPON corresponds to the already discussed (irrational) tendency among Individual Investors to ignore (attractive) early exercise opportunities shortly before coupon payment dates. Finally, the regression returns negative loadings for the last two product variables BLOCK and LIFETIME. The negative coefficient for BLOCK in the first regression implies that in the first month after the blocking period a significantly higher number of exercises than usual are economically unreasonable, which we attribute to accumulated requests to exercise, due perhaps to liquidity constraints, that pile up until the first exercise possibility. The negative loading for the LIFETIME variable for Type A GFSN in the second regression indicates that the probability of a failure to exercise increases over time, which might perhaps be due to a lack of attention to products that mature in the near future.

Moreover, we note that the economic reasonableness of an early exercise decision depends on an investor's portfolio and personal characteristics. One reason for this is that experience apparently plays an important role in investors' decision-making as shown by the high coefficients for the portfolio variables PERFORMED and EXERCISED in both analyses in Panel B. The positive influence of PERFORMED in the first regression shows that investors who have already exercised at an economically beneficial point in time once before are much



more likely to make further sophisticated exercise decisions, whereas the negative loading for EXERCISED in the first regression implies that a previous poor decision to exercise at an inopportune time (i.e. the PERFORMED dummy is 0) is regularly followed by a decreasing probability of an economically rational exercise decision on other investments. Similar relationships appear in the second regression, where we observe that the probability of an investor missing an exercise opportunity drops significantly if he has exercised a GFSN before and decreases even more if his earlier exercise decision was economically reasonable (i.e. both the EXERCISED and the PERFORMED dummy equal 1). Overall, the patterns in both regressions indicate that investors' exercise behavior is in principle consistent over time.

Looking at personal characteristics, we summarize that female investors tend to act more cautiously and thus miss more potential chances than men, but achieve economically better results if they do decide to exercise. This finding is in line with several studies on Individual Investors' trading behavior in stocks (e.g., Barber and Odean, 2001; Grinblatt and Keloharju, 2001). Differences also exist related to the age of an investor. Most striking, we detect that investors between 20 and 40 years of age employ their exercise rights considerably more often than other investor groups but also more often at points in time where it is not economically advantageous. A possible reason might be a higher demand for liquidity in that stage of life due to, e.g., costs for purchasing a house, increased expenses of raising a family or repayment of education debts.

We do not find that investors' geographical location has any statistical influence on the economic reasonableness of the exercise behavior. However, differences appear regarding the preferred distribution channel. Investors who prefer to acquire directly through the German Finance Agency make markedly better exercise decisions. This is consistent with the usually higher financial literacy of investors who skip intermediaries. Still, direct investors show also

an increased probability to fail to utilize upcoming economically attractive opportunities. As our last finding, we note that investors holding a doctoral degree have a similar investment profile as the just described direct investors, which we also ascribe to a higher financial literacy of this investor group.

### 3.3 Excess returns of exercising

To quantify the observed differences in investors' exercise strategies and to check the robustness of our results, we determine in this section the performance of each GFSN investment and each respective exercise in our data set. As performance measure we use the difference (“excess return”) between the yearly internal rate of return of a buy-and-hold strategy and the yearly internal rate of return of the empirically observed investment behavior. We assume in the calculation of the latter that at an early exercise the whole investment is directly reinvested in a fixed-income product with the same (remaining) maturity that pays the market par-yield.<sup>13</sup> Moreover, we ignore any potential transaction costs for buying bonds. Table 6 provides the results.

[Table 6 about here.]

As already indicated by the low number of economically reasonable exercises and the high number of missed beneficial opportunities (see Panel A of Table 5), the results show that only few investors achieve a positive yearly excess return through exercising GFSN. This finding is similar to the study of Bauer et al. (2009), who analyze Individual Investors' trading returns for equity options. On average, exercises in our data sample lead to negative excess returns against a buy-and-hold strategy of circa -0.117% per year for Type A and circa -0.300% for Type B GFSN. Not surprisingly, a separate analysis—not reported here—

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<sup>13</sup>This implies that the fictive re-investment product has a market value of 1.

again finds differences in investors' exercise performance related to personal and investment characteristics, whereby the general patterns are in line with our former discussions. For example, investors between 20 and 40 years of age show an even poorer exercise performance (mean excess returns of -0.302% and -0.438%) than, e.g., investors between the ages of 0 and 20 (-0.029% and -0.243%). Furthermore, investors who prefer direct distribution (0.119% and -0.096%) markedly outperform those who prefer to invest at banks (-0.372% and -0.498%). Finally, we note that the best exercise performance for both GFSN types tends to be achieved by investors that exercise early only selected investments, whereas investors that have a high ratio of early exercises to investments regularly realize poor excess returns.

### 3.4 Discussion

Summing up the empirical and economic results of our last three subsections, two key insights emerge. The first is that Individual Investors in GFSN most frequently use their early exercise rights when it is not economically advisable. The second is that they typically forgo numerous attractive exercise opportunities throughout the lifetime of their investments. The equity derivatives literature typically classifies such a non-performance-oriented behavior as “suboptimal” and refers to irrationality (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012), non-continuous monitoring of the investment (e.g., Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2013) and transaction costs (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006).<sup>14</sup>

However, two further findings in our analyses are that investors react very sensitively to environmental influences and that exercise behavior significantly differs among the investor

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<sup>14</sup>Further examples of motives for non-performance oriented exercises are typically gambling and entertainment (e.g., Lakonishok et al., 2007; Bauer et al., 2009), which we ignore here since GFSN are quite conservative medium-term fixed-income products that are obviously less suitable for short-term gambling purposes.

base. For instance, we observe strongly increased exercise activity during the introduction of the “New Market” in Germany and during bullish periods in the equity market. As an example for differences that depend on personal characteristics, we find for investors aged between 20 and 40 years suspiciously high exercise activity at times that are frequently unreasonable from an economic point of view and that result in markedly poorer average excess returns from exercising, compared to other investor groups. Such reactions to environmental changes and such age-related behavior are difficult to reconcile with the standard reasoning in the literature. In fact, the literature seem to offer only weak explanations why the degree of irrationality, the monitoring behavior or transaction costs should differ so substantially dependent on, e.g., the development of the equity market or the affiliation to a specific age cohort. Hence, we suggest that—besides the common factors in the literature—a further factor drives Individual Investors’ exercise decisions in fixed-income products. This factor is the desire for liquidity and financial flexibility.

Indeed, the liquidity and flexibility argument fits well to the described empirical patterns in the data. We link it to five observations in our former analyses. First, the significantly higher probability of early exercise of a GFSN by investors between the ages of 20 to 40 might be due to their higher liquidity requirements at that stage of life, due, for example, to raising a family or purchasing a house. Second, the higher early exercise activity in the first month after the initial blocking period can be attributed to investors’ demand for liquidity that has accrued over the first twelve months of maturity. Third, the increased exercise rates during strong growth phases in the equity markets can be interpreted as a direct reaction of investors in GFSN who use the early exercise right to liquidate their investments so as to benefit from bullish trends in other markets.

Fourth, only very few early exercise decisions in our data seem to be driven by economic or

performance reasons, as most GFSN are exercised early at economically inopportune times. Moreover, we observe a relatively constant base exercise rate, independent of market movements or valuation changes. Besides irrationality, both patterns—the high number of exercise decisions at a PVEV-ratio above 1 and the base exercise rate—can obviously also be attributed to investors' demand for liquidity, in which case the current value of a product is of secondary importance. Fifth, consistency in taking advantage of attractive exercise opportunities is low. Investors have typically already missed several (better) opportunities when they exercise. As the literature states (see, e.g., Barraclough and Whaley, 2012), this might be due to time-dependent transaction costs, non-continuous monitoring or irrationality. On the other hand, the low rate of realizing exercise chances combined with the tendency to exercising early at inopportune times rather than somewhat later when the investment would have worth more also fits very well to investors who follow a conservative buy-and-hold strategy and only exercise to meet liquidity needs or due to environmental influences such as tax changes. The latter argument seems to be particularly relevant as GFSN are medium-term government bonds with a low risk profile and hence presumably attract mainly conservative investors.

#### **4 Investment and exercise motives**

Our analyses so far show that investors' use of the exercise right deviates from the theoretically optimal exercise strategy—even to a higher extent than in the equity market—and that the demand for liquidity and financial flexibility might be important reasons for early exercising. Consequently, in this section we aim to cast further light on the motives of Individual Investors for holding and exercising GFSN. We perform our analysis in two steps.

First, we compile a new data set that describes the average investment and exercise strategy of each Individual Investor in our data set. We then conduct an exploratory factor analysis to extract the main dimensions of variation in the data and to isolate latent factors that drive investors' investment and exercise behavior. Second, we examine the relevance of the identified latent factors for each investor and determine whether there are any statistically significant relationships to personal and financial attributes.

#### 4.1 Exploratory factor analysis

While we have concentrated in this paper so far on decisions and transactions at an individual account and product level, we now focus on the general behavior of Individual Investors. Hence, our first step is to create a new data set that consolidates the individual transaction data for each investor into a few explicit characteristics of behavior. Table 7 provides an overview of the variables we calculate for each account, whereby the selection adheres closely to the identified determinants of early exercising (see Table 4).

[Table 7 about here.]

In short, we consider three kinds of variable. First, we compute economic indicators and ratios that describe an investor's average investment and exercise strategy. For instance, we determine what percentage of his exercises occurs at times when the PVEV-ratio equals 1 (PVEVLOW), which implies—as discussed—an economically rational exercise. Similarly, we calculate the variables PVEVMED and PVEVHIGH, which represent the share of early exercises at medium ( $1 < \text{PVEV} \leq 1.03$ ) or high PVEV-ratios ( $\text{PVEV} > 1.03$ ). Second, we account for the average reaction to environmental influences, such as the percentage of exercise opportunities used when the CDAX has moved by more than 25% during the previous 6 months or the percentage of investments an investor liquidates in the months before changes

in the tax environment become effective. Third, we subsume information on the products an investor chooses to purchase. Here, we consider, for instance, the average steepness of the coupon structure over all chosen investments or the average value of the bond component without the option right. Finally, we also incorporate an investor's number of investments and early exercises.

In this analysis we have to cope with missing data, because some of the defined variables in Table 7 depend on economic or environmental circumstances that may not have arisen during an individual's investment period. For example, we cannot calculate the percentage of exercised investments in the months before changes in the tax legislation become effective if an investor did not hold a GFSN at such a time. According to the classification of Rubin (1976), such data is missing at random (MAR), since its absence is not related to the value of the respective variable but only to other variables, such as the market or tax environment. The literature suggests several methods of handling data missing at random (see, e.g., Allison, 2002; Enders, 2010), yet there seems to be no best commonly accepted approach. We apply multiple imputation algorithms and the full-information maximum likelihood method here, but find only small differences in the results. We proceed with the likelihood estimator for efficiency reasons, whereby we implicitly assume that Individual Investors, for whom we may lack some data, do not systematically make decisions that are widely different from the other investors in our data set, which seems reasonable considering the broad and large investor base.

An exploratory factor analysis, using the maximum-likelihood extraction method and applying robust standard errors to correct for non-multivariate normally distributed data (see Yuan and Bentler, 2000), isolates four factors that primarily drive the investment and exercise behavior of Type A GFSN investors, while five factors seem most suitable for Type B GFSN

investors (see Table 8).<sup>15</sup> For both analyses the model fit is satisfying. The standardized root mean square residual (SRMR) lies at 5.2% for Type A GFSN (3.7% for Type B), the root mean square error of approximation (RMSEA) lies at circa 6.5% (5.0%) and the comparative fit index (CFI) approaches 88% (92%). Overall, the identified factors account cumulatively for circa 55.227% (59.000%) of the overall variance. Table 8 contains details on the factor selection and on the rotated standardized factor loadings, which we estimate via the direct geomin oblique rotation algorithm.

[Table 8 about here.]

The factor analysis returns distinct and strong loading patterns for all identified factors. It is remarkable that the estimated factors and factor structures are very similar for the independently conducted analyses of Type A and Type B GFSN investors, which we take to indicate the economic robustness of our results. The first and most important factor, which in the case of Type A GFSN accounts for about 19.153% of the variance in all variables (17.819% for Type B), shows positive loadings for all three PVEV variables. This indicates that the factor is related to both economically reasonable and disadvantageous exercise decisions, which is plausible if exercises are triggered by exogenous factors such as the introduction of new market segments or strong movements in the equity market. In fact, for this factor we observe statistical positive coefficients for all market variables (CDAX10, CDAX25 and NEWMARKET) and for the BLOCK variable, which indicates an increased exercise rate in the first month after the blocking period. Based on these observations and on our discussion above, we interpret the first factor as being due to an investor's need for liquidity and financial flexibility.

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<sup>15</sup>To determine the number of latent factors, we consider the Kaiser-criterion (Kaiser, 1960), a parallel analysis and a scree plot. For a clear presentation of the results we decided to keep factors with an eigenvalue larger than 1.0, although the results do not differ significantly in the case of other selection criteria.



The second factor represents the importance of the mid- and long-term value of an investment and thus stands for a conservative strategy. It is associated with an investment behavior focusing on steep coupon structures and a high valuation at issuance. The third latent factor consolidates the desire for high investment performance. It loads strongly on the ratio of early exercises at a PVEV-ratio of 1 (PVEVLOW) and accordingly on positive excess returns of exercising (EXCESSRETURN). In contrast, the fourth factor summarizes a highly active investment behavior with a high number of early exercises that, however, do not result in positive excess returns—hence we label this factor “activism”. Finally, the fifth factor, which appears only for Type B GFSN, captures the sensitivity to changes in the tax regulation. It loads mainly on the tax (TAX9906) and the year-end variable (YEAREND).

Overall, we note that the results of the factor analysis coincide with our previous finding that the desire for a better yield performance is only one of several factors in the decision-making of Individual Investors. In fact, the factor analysis suggests again that other motives, such as the need for financial flexibility, play a more important role.

## 4.2 Importance of latent factors

Following the general analysis, we also estimate personal factor scores for each Individual Investor in our data set. To determine whether any differences exist in the relevance of latent factors related to personal characteristics, we regress the computed scores on selected personal and financial characteristics (average investment volume per investment). Table 9 shows the results.

[Table 9 about here.]

We emphasize five regression results relevant to points discussed in this paper. First, the relevance of the financial flexibility factor is most strongly pronounced for investors between 20

and 40 years of age. Additionally, this factor appears to be more important for male investors and for higher average investment volumes. Second, the desire for value mainly drives the decisions of investors with larger investments, who prefer to acquire GFSN indirectly at banks. We also observe that this factor is of higher relevance for investors younger than 20, compared to other age groups, which we attribute to GFSN accounts that are established as savings accounts in a child's name. Third, the performance factor is strongly marked for direct investors who omit any intermediary and presumably have a higher average financial literacy. In contrast, performance seems to be a less important motive for many investors between the ages of 20 and 40, which corresponds to the high relevance of the financial flexibility factor for this investor group. Fourth, activism is stronger associated with male direct investors and is closely related to low average investment volumes. As for all other factors, we find no clear relevance attached to an investor's residence area. Fifth, the tax factor is more relevant for investors with higher investment volumes, which seems very reasonable.

## 5 Conclusions

In this paper we analyzed Individual Investors empirical use of early exercise rights in the fixed-income market. In short, we find that a broad range of environmental and economic factors determine investors' exercise decisions, whereby distinct differences exist among the investor base that depend on portfolio and personal characteristics. Still, most investors have in common the fact that they use the exercise right at times that are not economically reasonable and that they frequently fail to exploit more favorable exercise opportunities. Consequently, a broad majority of exercises in our sample result in negative excess returns.

The observed behavior and several empirical patterns let us infer that performance seeking

is not the sole or main driver of Individual Investors' exercise decisions in puttable bonds. In fact, our results suggest that for Individual Investors performance is in general a less important motivator of exercise decisions than it presumably is for, e.g., professional or institutional traders. Instead of performance, the wish for financial flexibility, for example in the case of substantial changes in the equity market or liquidity constraints, seems to be a major motive for early exercising. The results of an exploratory factor analysis support this hypothesis. We identify five latent factors that mainly drive Individual Investors' investment and exercise strategy, of which the most important factor can in fact be interpreted as an investor's desire for flexibility.

In view of this, we derive three policy implications. First, sophisticated liquidity management is highly important for the issuers of fixed-income derivatives for Individual Investors. Issuers must anticipate that investors will use their exercise rights not mainly to optimize their investment yields, but also to react to things like environmental changes or liquidity constraints. This means early exercises frequently occur at times not predicted by standard theory. Second, information on personal characteristics of the investor base can be used to refine predictions of early exercise activity. For instance, the exercise behavior in our sample differs particularly regarding gender, age and the preferred distribution channel of an investor. Third, as Individual Investors fail to exercise the broad majority of attractive early exercise opportunities and use the option right—as noted—frequently at economically suboptimal points in time, the empirical value of the early exercise right is in general lower than its financial fair value. Hence, issuers gain a (significant) financial margin in pricing puttable bonds for Individual Investors with standard financial models. Moreover, our study shows that issuers can increase this margin by offering specific product designs and exploiting the behavioral biases of Individual Investors. For instance, there are some indications

that Individual Investors prefer bonds with a high final coupon payment to financially fully equivalent products with a flatter coupon structure. Accordingly, our results suggest that investors more often fail to use attractive exercise opportunities that arise for bonds with a steep coupon structure, which is obviously advantageous for the issuer.

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## Tables

**Table 1:** Statistics on investor base

	Type A GFSN		Type B GFSN	
	Percent	Absolute	Percent	Absolute
<b>Personal characteristics</b>				
<i>Gender</i>				
Male	33.469	52,717	38.406	35,595
Female	38.362	60,425	37.995	35,214
n/a	28.169	40,370	23.598	21,871
<i>Age</i>				
0 to 20 years	18.978	29,892	38.515	35,696
21 to 40 years	23.665	37,276	20.986	19,450
41 to 60 years	28.127	44,304	24.455	22,665
61 to 100 years	25.960	40,890	13.166	12,202
n/a	3.270	5,150	2.878	2,667
<i>Doctoral degree</i>				
Doctorate or professorship	4.150	6,536	3.499	3,243
No doctoral degree	95.850	150,976	96.501	89,437
<i>Geographical location</i>				
City	4.165	6,560	3.835	3,554
Highly populated	11.215	17,665	11.254	10,430
Moderately populated	28.547	44,965	29.252	27,111
Sparsely populated	51.371	80,916	51.417	47,653
n/a	4.702	7,406	4.243	3,932
<i>Preferred distribution<sup>(1)</sup></i>				
Indirect (at banks)	59.181	93,217	54.011	50,057
Direct (via the German Finance Agency)	40.819	64,295	45.989	42,623
<b>Overall</b>	<b>100.000</b>	<b>157,512</b>	<b>100.000</b>	<b>92,680</b>

The table exhibits information on personal characteristics of all 223,017 Individual Investors in our data sample, whereof 27,175 investors hold both Type A and Type B GFSN. Investors are clustered in residence areas according to the first two digits of their zip code using an urbanization index and population density figures from German Federal Statistics Office (2009). We classify the population density per sqkm as follows: <250 = sparsely, 250-750 = moderately, >750 = highly populated. (1) We define the preferred distribution channel as direct if an investor executes at least one direct transaction. Indirect means that an investor purchases a GFSN at a bank and later transfers his investment to an account at the German Finance Agency.



**Table 2:** Statistics on investors' financial activities

	Type A GFSN					Type B GFSN				
	Mean	Med.	p5	p95	St.dev.	Mean	Med.	p5	p95	St.dev.
<b>Investments per investor</b>										
Number of investments	3.147	1.000	1.000	11.000	5.200	2.790	1.000	1.000	9.000	5.139
Overall investment volume in €	23,053	10,226	1,019	85,569	45,614	11,526	4,090	511	44,945	30,593
Volume per investment in €	8,818	5,266	639	25,565	14,789	5,085	2,505	460	17,663	6,693
<b>Early exercises per investor</b>										
Number of exercises	0.526	0	0	2.000	1.491	0.517	0	0	2.000	1.483
Overall exercise volume in €	1,812	0	0	10,178	4,761	1,284	0	0	6,647	3,547
Volume per exercise in €	4,094	4,090	511	10,000	2,742	3,017	2,540	511	7,669	2,610
<b>Early exercises over time</b>										
Exercise rate per month and GFSN in %	0.626	0.309	0.081	2.418	1.058	0.518	0.327	0.081	1.693	0.671
Exercise volume per month and GFSN in €	160,356	80,323	2,711	570,448	296,062	55,242	26,212	2,228	209,374	296,062

The table exhibits statistics on the investments and early exercises of all 223,017 Individual Investors in 204 issued GFSN (102 Type A, 102 Type B) during our sample period from July 1996 to February 2009. The early exercise rate is defined as the monthly ratio of number of exercises to the number of current investments in this GFSN.

**Table 3:** Variables considered for analysis of determinants

Abbr.	Variable description	Type A GFSN		Type B GFSN	
		Mean	St.dev.	Mean	St.dev.
<b>Economic benefit</b>					
PVEV	Ratio of present value to exercise value	1.034	0.027	1.0511	0.048
<b>Investment history</b>					
MISSEDALL	Number of economically reasonable exercise months (PVEV=1) since issuance	2.869	6.044	1.726	5.022
MISSED6	Number of economically reasonable months (PVEV=1) within the last 6 months	0.769	1.649	0.427	1.291
VOLUME	Investment volume in €	7,513	11,165	3,972	8,025
<b>Environmental circumstances</b>					
CDAX10+	Dummy, changes in CDAX > +10% points within a month	0.033	0.177	0.031	0.174
CDAX10-	Dummy, changes in CDAX < -10% points within a month	0.080	0.271	0.081	0.273
CDAX25+	Dummy, changes in CDAX > +25% points within 6 months	0.094	0.292	0.095	0.293
CDAX25-	Dummy, changes in CDAX < -25% points within 6 months	0.085	0.278	0.087	0.281
VOLDAX	45-day option-implied volatility of DAX measured by VDAX	24.045	9.846	23.888	9.877
VOLINT	Volatility of German 5-year spot rate estimated from weekly returns over a 6-month window	0.012	0.002	0.012	0.002
NEWMARKET	Dummy, introduction of new stock market “New Market” in Germany (January 1998 to December 2000)	0.197	0.398	0.176	0.381
TAX99	Dummy, change in tax legislation 2000 (November-December 1999)			0.012	0.109
TAX06	Dummy, change in tax legislation 2007 (November-December 2006)			0.017	0.129
YEAREND	Dummy, end of the year (each December)			0.086	0.280
<b>Product characteristics</b>					
UPSTEEP	Average yearly growth of coupon payments until maturity (last coupon minus current coupon divided by years to maturity)	0.005	0.002	0.004	0.002
DURATION	Fisher-Weil duration of bond component in years	2.585	1.315	3.300	1.711
COUPON	Dummy, coupon payment upcoming within the next 60 days	0.090	0.285		
BLOCK	Dummy, first month after blocking period	0.023	0.151	0.021	0.142
LIFETIME	Lifetime since issuance in years	2.315	1.450	2.672	1.711
<b>Portfolio characteristics</b>					
INVESTS	Number of former investments in GFSN	7.397	10.127	8.231	11.668
INVESTSUM	Sum of overall investments in € so far	40,087	73,561	27,156	64,055
EXERCISED	Dummy, signaling if investor has exercised early a GFSN once before	0.104	0.306	0.108	0.311
PERFORMED	Dummy flagging if investor has exercised early a GFSN once before when it was economically reasonable (PVEV=1)	0.037	0.189	0.032	0.176
<i>N</i>		19.089m		11.467m	

The table shows the variables and ratios we consider in our analysis of determinants of Individual Investors' exercise behavior. Overall 19.089m monthly observations are considered for Type A GFSN (11.476m for Type B). The one-year blocking period at beginning of a GFSN is excluded. Data sources are Deutsche Bundesbank for interest term structures, Thomson One Banker for environmental and equity market variables and the German Finance Agency for GFSN data. The descriptive statistics are calculated based on all observations of the data set, wherefore the figures differ from, e.g., the analysis on an individual investor level (see Table 2).

**Table 4:** Determinants of early exercise

	Type A GFSN			Type B GFSN		
	Observations per Investment Mean	Min.	Max.	Observations per Investment Mean	Min.	Max.
	42.370	1.000	60.000	48.770	1.000	72.000
	<i>Logit</i> Coefficient	<i>RE logit</i> Coefficient	Margin <sup>(1)</sup>	<i>Logit</i> Coefficient	<i>RE logit</i> Coefficient	Margin <sup>(1)</sup>
<b>Economic benefit</b>						
PVEV	-13.446*	-12.851*	-18.158*	-7.650*	-3.296*	-11.930*
<b>Investment history</b>						
MISSEDALL	0.022*	0.049*	0.029*	0.001	0.039*	0.002
MISSED6	0.059*	0.171*	0.079*	0.144*	0.224*	0.224*
VOLUME	0.011*	0.023*	0.014*	0.014*	0.031*	0.023*
<b>Environmental circumstances</b>						
CDAX10+	0.261*	0.323*	0.399*	0.289*	0.358*	0.518*
CDAX10-	0.157*	0.124*	0.228*	0.242*	0.194*	0.420*
CDAX25+	0.131*	0.122*	0.187*	0.318*	0.408*	0.568*
CDAX25-	-0.377*	-0.510*	-0.438*	-0.154*	-0.268*	-0.226*
VOLDAX	0.005*	0.007*	0.008*	-0.003*	-0.003*	-0.004*
VOLINT	-86.190*	-84.767*	-116.392*	-49.856*	-70.138*	-77.748*
NEWMARKET	1.010*	1.299*	1.924*	0.991*	1.357*	2.218*
TAX99				0.482*	0.584*	0.960*
TAX06				0.999*	1.118*	2.624*
YEAREND				0.349*	0.405*	0.633*
<b>Product characteristics</b>						
UPSTEEP	-4.943*	59.881*	-6.675*	-78.250*	-68.645*	-122.029*
DURATION	4.346*	12.028*	5.869*	0.204*	-0.177*	0.318*
COUPON	-0.140*	-0.089*	-0.179*			
BLOCK	0.368*	-0.259*	0.595*	0.407*	-0.617*	0.777*
LIFETIME	3.537*	11.025*	4.777*			
<b>Portfolio characteristics</b>						
INVESTS	-0.046*	-0.113*	-0.063*	-0.046*	-0.107*	-0.072*
INVESTSUM	-0.005*	-0.009*	-0.006*	-0.006*	-0.010*	-0.010*
EXERCISED	2.979*	6.445*	18.137*	2.977*	6.497*	20.612*
PERFORMED	0.428*	1.047*	0.709*	0.235*	0.618*	0.410*
<b>Personal characteristics</b>						
<i>Gender</i>						
Female	-0.059*	-0.144*	-0.078*	-0.105*	-0.169*	-0.160*
n/a	0.046*	0.048*	0.064*	0.041*	0.068*	0.067*
<i>Age</i>						
21 to 40 years	0.484*	0.870*	0.759*	0.641*	1.014*	1.139*
41 to 60 years	0.240*	0.362*	0.330*	0.427*	0.652*	0.677*
61 to 100 years	-0.008	-0.126*	-0.009	0.294*	0.348*	0.435*
n/a	-4.228*	-6.579*	-1.206*	-4.004*	-6.726*	-1.248*
<i>Doctoral degree</i>						
Doctorate or professorship	-0.125*	-0.244*	-0.160*	-0.122*	-0.327*	-0.181*
<i>Geographical location</i>						
City	-0.002	-0.118*	-0.003	-0.022	0.010	-0.035
Highly populated	-0.045*	-0.016	-0.061*	0.025	0.025	0.041
Moderately populated	-0.018*	-0.014	-0.025*	-0.054*	-0.053*	-0.084*
n/a	0.016	0.003	0.023	-0.028	-0.109*	-0.044
<i>Preferred distribution</i>						
Direct	-0.221*	-0.526*	-0.292*	-0.114*	-0.290*	-0.177*
<i>Constant</i>						
	-11.204*	-51.235*		1.599*	-3.465*	
<i>Groups (Investments)</i>		450,526			235,307	
N	19.089m	19.089m		11.467m	11.467m	
<i>Pseudo-R<sup>2</sup> in %</i>	17.00	19.78		15.17	19.27	

The table exhibits the results of a pooled logit and a random-effects logit regression on Individual Investors' exercise decisions in Type A and Type B GFSN. Only investments and decisions after the one-year blocking period are considered. For the pooled regressions robust standard errors are used. For the panel estimation we apply the Gauss-Hermite algorithm with 4 integration points. The lifetime variable is not considered for Type B GFSN as it is perfectly correlated to the respective product's duration. The marginal effects (dy/dx) are calculated at means based on the pooled logit regression. Pseudo-R<sup>2</sup> is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. (1) Displayed at 1e+4. \* signals statistical significance at the 5% level.

**Table 5:** Economic reasonableness of early exercise behavior

Panel A	Distribution of early exercises in %				Attractive exercise opportunities in %			
	Type A GFSN		Type B GFSN		Type A GFSN		Type B GFSN	
	PVEV=1	PVEV>1	PVEV=1	PVEV>1	Exploited	Missed	Exploited	Missed
	24.326	75.674	13.939	86.061	1.856	98.144	1.829	98.171
Panel B	<i>Logit:</i> Early exercise economically reasonable (PVEV=1, yes/no)				<i>Logit:</i> Failed to exercise when PVEV=1 (yes/no)			
	Type A GFSN		Type B GFSN		Type A GFSN		Type B GFSN	
	Coefficient	Margin <sup>(1)</sup>	Coefficient	Margin <sup>(1)</sup>	Coefficient	Margin <sup>(1)</sup>	Coefficient	Margin <sup>(1)</sup>
<b>Investment history</b>								
MISSEDALL					0.002	0.074	0.030*	0.128*
MISSED6					-0.068*	-0.202*	-0.033*	-0.143*
VOLUME	-0.011*	-0.002*	-0.011*	-0.002*	-0.006*	-0.018*	-0.011*	-0.048*
<b>Environmental circumstances</b>								
TAX99			-0.784*	-0.111*			0.387*	1.399*
TAX06			0.425*	0.082*			-0.508*	-2.828*
YEAREND			0.378*	0.072*			-1.050*	-7.426*
<b>Product characteristics</b>								
UPSTEEP					26.743*	78.972*	150.609*	654.423*
DURATION					-2.398*	-7.083*	-0.395*	-1.718*
COUPON					0.102*	0.291*		
BLOCK	-1.702*	-0.296*	-1.041*	-0.148*				
LIFETIME					-1.546*	-4.567*		
<b>Portfolio characteristics</b>								
INVESTS	0.012*	0.003*	0.012*	0.002*	0.043*	0.128*	0.043*	0.189*
INVESTSUM	0.003*	0.001*	0.003*	0.000*	0.002*	0.074*	0.002*	0.010*
EXERCISED	-0.624*	-0.137*	-0.562*	-0.100*	-2.403*	-20.308*	-2.455*	-31.188*
PERFORMED	1.873*	0.436*	1.778*	0.394*	-1.240*	-6.641*	-1.257*	-10.137*
<b>Personal characteristics</b>								
<i>Gender</i>								
Female	0.050*	0.011*	0.050	0.009	0.045*	0.130*	0.118*	0.495*
n/a	0.008	0.002	0.030	0.005	-0.121*	-0.375*	-0.119*	-0.563*
<i>Age</i>								
21 to 40 years	-0.641*	-0.138*	-0.650*	-0.115*	-0.149*	-0.487*	-0.241*	-1.143*
41 to 60 years	-0.310*	-0.071*	-0.389*	-0.074*	-0.048*	-0.150*	-0.098*	-0.432*
61 to 100 years	-0.006	-0.001	-0.182*	-0.036*	0.109*	0.314*	-0.027	-0.115
n/a	0.196	0.047	0.231	0.050	3.960*	2.968*	3.620*	4.091*
<i>Doctoral degree</i>								
Doctorate or professorship	0.069	0.015	0.157*	0.029*	0.077*	0.219*	0.062	0.265
<i>Geographical location</i>								
City	-0.012	-0.003	0.019	0.003	0.070*	0.202*	0.094	0.398
Highly populated	0.020	0.004	-0.009	-0.002	0.032	0.096	0.035	0.153
Moderately populated	0.013	0.003	0.037	0.007	0.010	0.031	0.040	0.175
n/a	0.083	0.018	-0.093	-0.016	-0.026	-0.079	0.090	0.384
<i>Preferred distribution</i>								
Direct	0.999*	0.215*	0.698*	0.123*	0.414*	1.274*	0.335*	1.507*
<i>Constant</i>								
Constant	-0.767*		-1.228*		15.105*		5.803*	
<i>N</i>	82,787	82,787	47,900	47,900	3.028m	3.028m	1.114m	1.114m
<i>Pseudo-R<sup>2</sup> in %</i>	19.31		12.90		20.49		18.79	

The table shows four economic analyses of Individual Investors' exercise behavior in GFSN. In the left part of Panel A we compute the share of early exercises in our data set that are economically reasonable and the share of theoretically not beneficial exercises. Second, in the right part of Panel A we calculate the percentage of exploited and missed economically attractive exercise opportunities (PVEV=1). Third, the left part of Panel B presents the results of a pooled logit regression on a dummy variable that indicates if an exercise is economically beneficial. The second column displays the corresponding marginal effects (dy/dx) at means. Fourth, the right part of Panel B exhibits the results of a pooled logit regression on a dummy variable that indicates if an investor fails to use an attractive exercise opportunity. Again, the next column shows the marginal effects (dy/dx) at means. Robust standard errors are used. Pseudo-R<sup>2</sup> is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. (1) Displayed at 1e+4.  
\* signals statistical significance at the 5% level.

**Table 6:** Excess returns of early exercising

Individual Investors' excess return of early exercising in % p.a.		
	Type A GFSN	Type B GFSN
Mean	-0.117	-0.300
Median	-0.098	-0.289
p5	-1.306	-1.429
p10	-1.017	-1.124
p90	0.709	0.572
p95	1.023	0.868
St.dev	0.704	0.711
<i>N</i>	82,787	47,900

The table shows statistics on the average yearly excess return of early exercising based on the exercise decisions of all 223,017 Individual Investors in 204 issued GFSN (102 Type A, 102 Type B) during our sample period from July 1996 to February 2009. For the calculation we assume that an investment exercised early is directly reinvested in a product with an identical remaining maturity paying the market yield. The excess return is then calculated as the difference between the internal rate of return of the exercise strategy and a buy-and-hold strategy.

**Table 7:** Variables calculated to summarize an investor's investment and exercise behavior

Abbr.	Variable description	Type A GFSN		Type B GFSN	
		Mean	St.dev	Mean	St.dev
<b>Economic benefit</b>					
PVEVHIGH	Percentage of early exercises employed where PVEV>1.03	0.147	0.334	0.163	0.348
PVEVMED	Percentage of early exercises employed where 1<PVEV≤1.03	0.056	0.205	0.066	0.221
PVEVLOW	Percentage of early exercises employed where PVEV=1	0.087	0.260	0.082	0.250
Ø EXCESSRETURN	Average excess return of early exercising p.a.	-0.001	0.004	-0.001	0.004
Ø MISSEDALL	Average number of missed attractive exercise opportunities before early exercise / maturity	5.790	5.823	4.383	5.086
<b>Environmental circumstances</b>					
CDAX10	Percentage of exercise opportunities used when CDAX moved by more than +/-10% in last month	0.008	0.047	0.007	0.042
CDAX25	Percentage of exercise opportunities used when CDAX moved by more than +/-25% over last 6 months	0.008	0.042	0.007	0.038
NEWMARKET	Percentage of exercise opportunities used in phase of "New Market" in Germany (January 1998 to December 2000)	0.009	0.060	0.008	0.052
TAX9906	Percentage of Type B GFSN exercise opportunities used in November / December 1999 and 2006			0.012	0.089
YEAREND	Percentage of Type B GFSN exercise opportunities used in December			0.007	0.042
<b>Product characteristics</b>					
Ø VALUE	Average value of GFSN investments at issuance	1.012	0.008	1.019	0.011
Ø BOND	Average value of GFSN bond component at issuance	0.987	0.010	0.986	0.014
Ø STEEPNESS	Average steepness of coupon structure of GFSN investments at issuance	0.027	0.010	0.027	0.010
Ø DURATION	Average Fisher-Weil duration of GFSN investments at issuance in years	5.544	0.188		
BLOCK	Percentage of exercise opportunities used in first month after the blocking period	0.022	0.116	0.018	0.099
<b>Portfolio characteristics</b>					
INVESTS	Number of investments in GFSN	3.755	6.210	4.039	6.961
EXERCISES	Number of early exercises	0.643	1.773	0.735	1.982
<i>N</i>		157,512		92,680	

The table gives an overview of the calculated variables and ratios we use to describe an Individual Investor's average investment and exercise strategy. The excess return is calculated according to the definition in Table 6. The descriptive statistics summarize an investor's average investment and exercise strategy over his whole portfolio, wherefore the figures differ from, e.g., the analysis on an individual investor level (see Table 2). The duration is not considered for Type B GFSN (zero-bond structure) as it always equals maturity.

**Table 8:** Exploratory factor analysis (EFA) on consolidated investment and exercise behavior

	Factor 1		Factor 2		Factor 3		Factor 4		Factor 5
	<i>Liquidity / financial flexibility</i>		<i>Value</i>		<i>Performance</i>		<i>Activism</i>		<i>Tax</i>
	Type A	Type B	Type A	Type B	Type A	Type B	Type A	Type B	Type B
<b>Economic benefit</b>									
PVEVHIGH	0.563*	0.519*	0.036*	0.028*	-0.415*	-0.491*	0.105*	0.100*	0.160*
PVEVMED	0.022*	0.017*	-0.028*	-0.056*	-0.104*	-0.081*	0.182*	0.217*	0.131*
PVEVLOW	0.111*	0.099*	-0.063*	-0.082*	0.285*	0.285*	0.347*	0.318*	0.130*
Ø EXCESSRETURN	-0.068*	-0.051*	-0.050*	-0.056*	1.402*	1.353*	0.042*	0.024*	-0.024*
Ø MISSEDALL	-0.313*	-0.231*	-0.122*	-0.159*	0.072*	0.084*	0.032*	0.012*	-0.113*
<b>Environmental circumstances</b>									
CDAX10	0.389*	0.398*	0.022*	0.016*	-0.124*	-0.104*	0.043*	0.073*	-0.027*
CDAX25	0.449*	0.478*	0.020*	0.026*	-0.024*	-0.051*	0.024*	0.043*	-0.023*
NEWMARKET	0.556*	0.508*	0.070*	0.072*	-0.138*	-0.135*	-0.041*	-0.041*	0.038*
TAX9906		0.051*		0.019*		-0.122*		0.031*	0.695*
YEAREND		0.073*		0.026*		-0.100*		0.050*	0.588*
<b>Product characteristics</b>									
Ø VALUE	0.020*	0.021*	1.666*	1.336*	-0.051*	-0.070*	-0.032*	-0.092*	0.000
Ø BOND	0.065*	0.007*	0.462*	0.564*	-0.029*	-0.021*	-0.102*	0.023*	0.020*
Ø STEEPNESS	0.089*	0.076*	0.311*	0.424*	-0.104*	-0.113*	-0.227*	-0.164*	0.027*
Ø DURATION	-0.080*		0.062*		0.031*		0.382*		
BLOCK	0.678*	0.673*	0.023*	0.002	-0.007*	0.029*	-0.013*	-0.013*	-0.009
<b>Portfolio characteristics</b>									
INVESTS	0.004*	-0.003	-0.093*	-0.129*	0.039*	0.053*	0.768*	0.652*	-0.040*
EXERCISES	0.248*	0.232*	-0.060*	-0.045*	-0.044*	-0.041*	0.734*	0.808*	0.073*
<hr/>									
<i>Eigenvalue</i>	2.873	2.851	2.424	2.373	1.627	1.546	1.360	1.206	1.464
<i>Proportion in %</i>	19.153	17.819	16.160	14.831	10.847	9.663	9.067	7.583	9.150
<i>Cumulative in %</i>	19.153	17.819	35.313	32.650	46.160	42.313	55.227	49.850	59.000
<hr/>									
<i>Indicators for model fit</i>									
SRMR	0.052	0.037							
RMSEA	0.065	0.050							
CFI	0.879	0.923							

The table shows the results of an exploratory factor analysis of Individual Investors' investment and exercise strategies in Type A and Type B GFSN. The results are based on the maximum-likelihood factor extraction method, whereby we keep factors with an eigenvalue larger than 1.0 (Kaiser-criterion). The geomin oblique algorithm is applied to rotate factors. RMSEA stands for root mean square error of approximation, SRMR for standardized root mean square residual and CFI for comparative fit index. \* signals statistical significance at the 5% level.

**Table 9:** Relation of factor scores to investors' personal characteristics

	Factor 1		Factor 2		Factor 3		Factor 4		Factor 5
	<i>Liquidity / financial flexibility</i>		<i>Value</i>		<i>Performance</i>		<i>Activism</i>		<i>Tax</i>
	Type A	Type B	Type A	Type B	Type A	Type B	Type A	Type B	Type B
<b>Personal characteristics</b>									
<i>Gender</i>									
Female	-0.050*	-0.061*	-0.096*	-0.076*	0.006	0.031*	-0.045*	-0.036*	-0.012*
n/a	0.244*	0.008	0.071*	0.037*	-0.008	-0.030	0.019*	0.036*	0.024*
<i>Age</i>									
21 to 40 years	0.181*	0.208*	0.009	-0.042*	-0.345*	-0.305*	0.040*	0.097*	0.093*
41 to 60 years	0.086*	0.114*	-0.211*	-0.249*	-0.167*	-0.115*	0.100*	0.150*	0.047*
61 to 80 years	0.022*	0.079*	-0.462*	-0.517*	0.002	0.030	0.040*	0.119*	0.047*
n/a	-0.214*	-0.228*	-0.246*	0.351*	0.163*	0.303*	-0.186*	-0.024*	-0.130*
<i>Doctoral degree</i>									
Doctorate of professorship	-0.061*	-0.076*	-0.019	-0.164*	0.056*	0.085*	0.039*	0.021	-0.076*
<i>Geographical location</i>									
City	-0.032*	-0.019	-0.046	-0.193*	-0.027	0.046	-0.005	-0.035*	-0.028*
Highly populated	-0.001	0.017	0.062*	-0.037*	0.030*	0.028	0.028*	0.005	-0.001
Moderately populated	0.011*	0.016*	0.049*	-0.006	0.017	0.025	0.029*	0.021*	-0.013*
n/a	-0.002	-0.001	0.518*	0.297*	0.092*	0.077*	0.036*	0.005	-0.023
<i>Preferred distribution</i>									
Direct	-0.033*	-0.049*	-1.019*	-1.395*	0.149*	0.103*	0.473*	0.236*	-0.011*
<i>Average volume per investment</i>									
€ 1,000-3,000	0.044*	0.039*	0.198*	0.034*	-0.005	-0.068*	-0.125*	-0.055*	0.047*
€ 3,000-10,000	0.087*	0.095*	0.258*	0.036*	0.060*	-0.138*	-0.182*	-0.089*	0.131*
> € 10,0000	0.077*	0.064*	0.230*	-0.077	0.003	-0.136*	-0.283*	-0.177*	0.115*
<i>Constant</i>									
	-0.108*	-0.078*	0.254*	0.173*	0.048*	0.100*	-0.069*	-0.178*	-0.070*
<i>N</i>	157,512	92,680	157,512	92,680	157,512	92,680	157,512	92,680	92,680
<i>R<sup>2</sup> in %</i>	1.13	1.71	3.67	15.01	0.97	0.96	12.39	6.63	1.02

The table exhibits regression results for the estimated four (five) latent factor scores on the personal characteristics (as detailed in Table 1) of all 157,512 Individual Investors in Type A GFSN and of all 92,680 Individual Investors in Type B GFSN. The average investment volume is defined in four clusters. Robust standard errors are used. \* signals statistical significance at the 5% level.

### 3 | Behavioral Financial Engineering in the Fixed-Income Market: The Influence of the Coupon Structure\*

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#### Abstract

In this paper we investigate the influence of coupon structure on the financial behavior of Individual Investors in the fixed-income market. Examining circa 26 million decisions on 204 standard puttable bonds with different coupon offerings our major findings are: (i) Products with a flat coupon structure and a high duration attract fewer investors and are significantly more often exercised early, whereas financially equal products with a steeply rising coupon structure arouse more interest among investors and are less often redeemed early. (ii) The shape of the upcoming coupon structure is an important basis on which investors decide which puttable bond in a portfolio to exercise early. (iii) Issuers who exploit empirical patterns related to a bond's coupon structure through "behavioral financial engineering" can benefit from a lower liquidity demand and a diminishing empirical option value.

**Keywords:** Individual Investors; product design; puttable bond

**JEL classification:** G02, G11

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## 1 Introduction

The influence of psychological factors on investors' trading and exercise decisions is attracting more and more attention in those researching market phenomena and investigating Individual Investors' financial behavior (e.g., Odean, 1998; Barber and Odean, 2011). Such psychological arguments imply that investors behave differently than ways that are assumed in standard models and that they do not act fully rationally according to theory. The literature commonly refers to this as the “cognitive biases” or “behavioral biases” of investors (see, e.g., Thaler, 2005). Prominent biases in the literature comprise among others overconfidence (e.g., Odean, 1998), mental accounting (e.g., Thaler, 1985), representativeness (e.g., Kahneman and Tversky, 1979; Shefrin and Statman, 1994) and inconsistent time preferences (e.g., Loewenstein and Prelec, 1992).<sup>1</sup>

While most of the mentioned concepts have been empirically researched for several investor classes and investment forms, few studies have explicitly investigated how the product design itself influences decision-making, that is how investors' decisions depend on, e.g., a product's duration, its coupon structure or the timing of coupon payments. For instance, most recent studies on exercise decisions in equity derivatives (see, e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012) do not incorporate the product structure as a dependent variable. Hence, the motivation of this paper is to extend the current behavioral finance literature by examining how the different coupon designs of puttable bonds affect the decision-making of Individual Investors. Our results are of particular interest to the wide range of issuers and intermediaries who deal with fixed-income derivatives for Individual Investors, as our analyses suggest that proficient “behavioral financial engineering”, i.e.

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<sup>1</sup>A comprehensive overview of behavioral biases can be found in Hirshleifer (2001) and—with a focus on Individual Investors—in De Bondt (1998).

optimizing and adapting the coupon structures of newly issued bonds, can lead to a higher demand and can imply financial benefits for the issuer.

Several papers on investors' behavior in structured products have argued that the product structure influences investment and exercise decisions. As one of the first Hersh and Statman (1993) discuss behavioral aspects in relation to the product design and find that the attractiveness of products varies due to a different framing of cash flows by investors. More recently, Branger and Breuer (2007) derive an optimal demand for retail derivatives and argue that not only the product type but also the contract structure is of importance. Similarly, Hens and Rieger (2009) demonstrate that complicated payoff structures lead to mis-estimations by investors resulting in extra margins for banks. Changing sides, Bernard et al. (2007) develop a theoretical framework for optimally designing structured products from an issuer's perspective. They argue that products with capital protection are advantageous for issuers. Additionally, in an analysis of the optimal product design for specific discount certificates, Breuer and Perst (2007) find that issuers should prefer large stocks as underlying and aim at less competent customers to maximize profit.

However, empirical research following these mainly theoretical works is—as noted—scarce. A remarkable exception is the paper of Henderson and Pearson (2007), who examine patterns in the payoffs of publicly traded structured equity derivatives and show that investors tend to demand concave payoffs for products linked to stocks and convex payoff profiles for index products. In this paper we investigate a similar question, but focus on the fixed-income market. We analyze how the coupon structure of German governmental non-tradable puttable bonds influences Individual Investors' investment, exercise and portfolio decisions and examine the financial implications of different coupon offerings. In particular, we are interested in the potential benefits to issuers who exploit empirical exercise patterns related to the coupon

design of fixed-income products.

Our analyses build on first findings in the studies of Eickholt et al. (2014a) and Eickholt et al. (2014b) regarding Individual Investors' behavior in putable bonds, which provide first indications that the product structure influences the probability of an early exercise. In this paper, we analyze a data set similar to that in the study of Eickholt et al. (2014b), but focus solely on empirical patterns related to the coupon structure. Our data set comprises, over a sample period of almost 13 years, circa 26 million financial decisions by Individual Investors to acquire, hold or exercise early German Federal Saving Notes (GFSN). GFSN are putable government bonds, which are very well suited for our study since they have a simple and stable structure which does not change during the whole observation period, are consistently issued every few weeks or months, cannot be traded and—most interestingly—because their coupon offerings differ steadily over time. This means that we are able to conduct consistent analyses of investors' behavior for a wide range of putable bonds with different degrees of steepness, kurtosis and levels of coupon structure.

Our main findings are threefold: first, the coupon structure influences the attractiveness of an investment (“*investment decision*”). Products with a steeply rising coupon structure and a low duration, i.e. with a short weighted average time until cash flows are paid out, attract more investors and higher overall volumes. Second, controlling for environmental changes and exogenous influences, we observe that such products are also less frequently exercised early (“*exercise decision*”), which is in line with the findings of Eickholt et al. (2014b). This implies that the average Individual Investor values bonds which promise a high final coupon in the future more highly than financially fully equivalent bonds, i.e. GFSN with the same maturity and the same financial fair value, that offer steady yearly coupons but a lower final coupon payment. Our results are robust for different time sub-samples, valuation scenarios

and investor subgroups. For instance, we notice that the preference for steeper coupon structures is independent of an investor's financial sophistication or experience. Investors who very frequently invest and exercise show exercise biases similar to those of investors who invest only once.

Third, the coupon structure is also an important basis on which investors decide which bond in a portfolio of several GFSN to exercise early ("*portfolio decision*"). Consistent with our previous results, we find that the exercise probability is higher for portfolio items with a low valuation, a comparatively flat coupon structure and a high duration. A comparison of different investor groups shows again that these empirical patterns are robust and appear for almost all subsamples. Overall, we attribute the diverging decisions related to the coupon structure here and in our first analysis to a cognitive and behavioral bias of Individual Investors. Apparently, products with a higher final coupon are (irrationally) perceived as more attractive and more valuable.

Finally, we discuss which kind of coupon structure is most promising for an issuer. For this, we run several scenario analyses and examine exemplary coupon designs and interest rate environments for the difference between the financial fair value at issuance and the "empirical value". We estimate these based on the empirically observed exercise behavior of Individual Investors in our sample. Moreover, we use Monte-Carlo simulations to estimate the average empirical and theoretically optimal exercise frequency. The results suggest that issuers who exploit empirical exercise patterns related to the coupon structure can benefit in two ways. First, issuers might reduce their liquidity reserves for bonds with a steeply ascending coupon structure as the early exercise volume tends to decrease, due (among other reasons) to a preference of Individual Investors to continue holding this kind of bond. Additionally, such a lower exercise probability results in an advantage to the issuer at times when an exercise

is theoretically reasonable. Second, our simulations indicate that the difference between the empirical value and the fair value widens in the case of flat or concave coupon structures, which is mainly due to the higher theoretical value placed by Individual Investors on the early exercise right for these products—a right however that they only exploit to a small extent. Obviously, such a poor utilization of a highly valuable option provides a financial advantage to issuers. Following, our main conclusion in this paper is that there is some potential for the issuers of structured fixed-income products that target Individual Investors to increase their financial benefit through proficient “behavioral financial engineering”, i.e. by offering product structures designed to fit their needs.

The remainder of this paper is organized as follows. Section 2 introduces our data set. Section 3 investigates what impact upcoming coupon structure and product duration have on Individual Investors’ investment (Section 3.2), on exercise (Section 3.3) and on portfolio decisions (Section 3.4). In Section 4 we investigate what kind of coupon structures can be advantageous for issuers. Section 5 concludes.

## 2 Data

This paper aims to study the influence of different coupon structures of putable bonds on the financial behavior of Individual Investors. For this analysis we are able to utilize a unique and very detailed data set from the German Finance Agency (“Bundesrepublik Deutschland Finanzagentur GmbH”)<sup>2</sup> that covers all investment, holding and exercise decisions of 223,117 Individual Investors in German Federal Saving Notes (“Bundesschatzbriefe”, GFSN in the following), that were booked through the German Finance Agency between July 1996 and

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<sup>2</sup>The German Finance Agency is a state-owned central service agency for Germany’s governmental borrowing.

February 2009.<sup>3</sup>

GFSN are basically standard puttable bonds issued by the German government, which are very well suited for our study for following reasons. First, GFSN have a standardized and simple product structure but differ over time regarding their coupon offerings, i.e. the steepness, kurtosis and extent of coupons vary with every new issuance. Second, new GFSN are issued every few weeks or months—whereby the former issuance is closed—after significant changes in the market interest rates, which gives us a large sample panel of investors and products. Third, commissions or transaction fees are negligible for GFSN. Fourth, GFSN are sold exclusively to Individual Investors<sup>4</sup> and cannot be traded on a secondary market. This restriction allows us to observe the influence of different coupon designs on investors' exercise behavior more comprehensively than in the case of products that are publicly traded and where a sale to a third party is always an alternative to early exercising. Finally, GFSN issuances have a minimum nominal value of only €50 and regularly attract a high number of Individual Investors in Germany. Even for the single least popular GFSN in our data set, we find transactions of more than 4,000 individual accounts in the German Finance Agency's debt register, which ensures some robustness of our results to, e.g., outliers.

## 2.1 Product statistics

Two types of GFSN (Type A and Type B) are offered at each issuance date. A Type A GFSN is a step-up bond with a maturity of 6 years that pays a yearly rising coupon. Type B products also offer yearly rising coupons over a maturity of 7 years but are issued

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<sup>3</sup>This data set is a randomly drawn subset that covers the transactions of circa 25% of the accounts at the German Finance Agency. Further on, we reduce the level of detail from daily to monthly observations.

<sup>4</sup>Additionally, resident institutions serving public benefit, charitable or religious purposes may also invest in GFSN. We ignore these investors in the following, as they account only for a very small share of overall investments.

in a zero-bond structure and pay the nominal value plus coupons and accrued interest at maturity. In addition, both Type A and Type B GFSN contain an early exercise right that gives an investor the right to redeem early his investment plus accrued interests at any point in time after the initial one-year blocking period. There are no charges for exercising early.

Investors can acquire the current issuance of a GFSN at nominal value plus accrued interest directly from the German Finance Agency via telephone or postal order or indirectly at banks. Banks, however, typically charge custody fees for GFSN that are held with them, whereas the German Finance Agency offers cost-free accounts for all Individual Investors, so that a direct acquisition is in general financially more advantageous for investors. Still, investors can shift their investments between banks and the German Finance Agency at any time without fees.

Our data set comprises transactions for all 204 issued GFSN in the sample period from 1996 to 2009. Table 1 shows summary statistics on the issuances and provides an overview of the offered coupon structures over time. For comparison, we also calculate the difference between the yearly coupons and the respective spot rate at issuance.

[Table 1 about here.]

As noted above, the statistics show that the offered coupons of issued GFSN vary significantly over time with a standard deviation of about 0.8% for each year of maturity. Thus for instance, the coupons for the first year range from 1.500% (5th percentile) to 4.000% (95th percentile), with a mean payment of 2.745% and with a standard deviation of 0.812%. Similarly, we observe a wide range of offered coupons at maturity, from 3.750% (5th percentile) to 7.000% (95th percentile) for Type A GFSN (4.000% to 7.000% for Type B). Typically, the first three coupon payments of a GFSN lie below the spot rates and rise thereafter above the market rates, which indicates an non-linearly increasing coupon structure.

Going further, Table 1 provides also some consolidated information on the shape of the coupon structure at issuance. As a first proxy we compute the steepness of the coupon structure, defined as the difference between the coupon in the sixth (Type A) or seventh year (Type B) and the first year coupon. We find again a remarkable variance among the issued GFSN with an average difference of 0.02076 (0.02199), which corresponds to a relative increase of 92.061% (98.581%). There are also some products with almost flat coupon structures, where the difference between the first and last coupon amounts to only 0.00750 (0.00750) and 18.750% (18.750%) respectively in relative terms. In contrast, this difference reaches 0.04000 (0.04250) for GFSN with the steepest rising coupon offerings.

As a second proxy for the shape of the coupon structure we calculate for Type A GFSN the Fisher-Weil duration (see Fisher and Weil, 1971) based on the coupon offerings and spot rates at issuance, whereby we focus on the duration of the bond component and omit the option right. For Type B GFSN, the duration always equals maturity due to the zero-bond structure, hence we do not further analyze the values for this product type at this stage. In general, we interpret the duration as the weighted average time until all payouts (coupons and nominal value) have been made to the investor. The statistics reveal that the average value over all bonds is circa 5.539 years. The 5th percentile is 5.403 years for a bond with a comparatively flat coupon structure, whereas the longest (95th percentile) weighted average time until payout lies closer to the respective bond's maturity at 5.712 years.

## **2.2 Investor statistics**

Our data set contains circa 31 million decisions by 223,017 Individual Investors to acquire, hold or exercise early one of the above-mentioned 204 GFSN on an individual account and monthly basis. However, we exclude in our following analyses all decisions in the last year of



a product's maturity, as we suspect that at this time investors' decision criteria on holding or exercising a GFSN differ from the criteria for products with a longer remaining lifetime.<sup>5</sup> For example, the question of coupon structure is presumably negligible if the product matures in one month, whereas coupon structure is probably more important if the investment is still bound for several years. This leaves us with circa 26 million decisions, which is—compared to most studies on Individual Investors' behavior in equity derivatives such as Bauer et al. (2009), Schmitz and Weber (2012) or Liao et al. (2013)—still a comparatively large sample. Table 2 provides summary statistics on the selected investor base and on their activities in GFSN.

[Table 2 about here.]

Overall, we note that the investor base in GFSN is quite heterogeneous. Male (33.47% for Type A, 38.41% for Type B GFSN) and female (38.36%, 38.00%) investors of any age invest in GFSN (we have no information for the remaining share of investors). The average investor is 44.09 years (31.15 years) old. Further, the data shows that about 4.15% (3.50%) of investors hold a doctoral degree or professorship. Regarding the distribution channel, we note that Individual Investors slightly prefer to acquire GFSN at a bank and to transfer their accounts later to the German Finance Agency (59.18%, 54.01%). A smaller proportion (40.82%, 45.99%) chooses the direct distribution channel.

Looking at the financial statistics, we find that a large share of the investor base invests only rarely, i.e. has a low investment frequency. Obviously, this is due to the restriction of our sample to Individual Investors, who are typically less active in financial markets than professional investors or funds. An investor in our sample acquires on average only 3.147 (2.790) GFSN throughout the whole observation period of almost 13 years. Similarly, the

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<sup>5</sup>In addition, we exclude accounts with an average transaction volume below €300 since for such small volumes the early exercise right seems to be of comparatively low importance.

mean investment volume is small, with circa €8,818 (€5,085). Nevertheless, Individual Investors make regular use of the exercise option. Almost every second investor redeems an investment early at least once before maturity, which totals overall circa 82,787 (47,900) early exercise decisions in our data set. A separate analysis—not reported here—shows that these early exercises occur for all GFSN in our sample, the share of investments exercise early per GFSN ranging from 9% to 52% (13% to 48%).<sup>6</sup>

We note only a few differences in the average investment behavior of the presented investor subgroups (see Table 2), which we mainly attribute to personal characteristics or environmental circumstances. For instance, the results reveal that older investors and investors holding a doctoral degree put higher volumes in GFSN, which is presumably due to greater personal wealth. In addition, investors who prefer to invest in GFSN via direct distribution show greater investment activity than Individual Investors, who mainly invest through banks, which might be due, for example, to a generally more conservative strategy of indirect investors. A third example is the lower (absolute and relative) exercise activity of female investors. This pattern seems to be an analogy to Individual Investors' behavior on the equity market, where the increased trading activity of men is typically ascribed to a higher confidence of male investors in their financial competency (see, e.g., Barber and Odean, 2001).

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<sup>6</sup>For comparability, we consider in this analysis only GFSN that mature before the end of our sample period.

### 3 Influence of coupon structure on investors' decisions

#### 3.1 Variables

Based on the described data set, this section investigates how the coupon structure of a GFSN influences an investor's investment, exercise and portfolio decisions. For this, we introduce two variables that—similar to our previous analysis in Table 1—describe the shape of the coupon structure at issuance and then at each later point in time.<sup>7</sup> First, we measure the relative steepness of the upcoming coupon structure (*UPSTEEP*), defined as the average linear increase in coupon payments until maturity:

$$UPSTEEP_t = \frac{C_T - C_t}{T - t}, \quad (3.1)$$

where  $C$  is the coupon payment for the current year,  $t$  is the time index and  $T$  stands for the last year of maturity. As the second variable we use the Fisher-Weil duration (see Fisher and Weil, 1971) of the bond component (*DURATION*). As noted before, we ignore the value of the option in this calculation. In general, this duration value consolidates both information on the steepness and kurtosis of the underlying coupon structure. Still, it might also be interpreted as the sensitivity of the respective bond against interest curve shifts, or as the weighted average time of a GFSN until all coupons and the nominal value are paid back. In this paper we will typically refer to the latter interpretation.

We then take into account that besides the coupon structure, the current valuation of a GFSN might play an important role in investors' early exercise decision. To consider the economic benefit of exercising a GFSN, we calculate the following PVEV-ratio for each point

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<sup>7</sup>This variable definition is in line with the study of Eickholt et al. (2014b).

in time:

$$PVEV_t = \frac{PV_t}{EV_t}, \quad (3.2)$$

where  $PV$  stands for the present value and  $EV$  for the exercise value which is paid immediately by the issuer if an investor makes use of his early exercise right. According to theory, an early exercise is only economically rational for an investor if this PVEV-ratio equals 1. In all other cases, it is optimal to keep holding the GFSN since the expected discounted value of all upcoming payments (continuation value) exceeds the amount of the exercise payment.

The calculation of the exercise value for our sample products is straightforward, as it simply equals the notional value plus accrued or compounded interests for the respective GFSN. However, determining the present value proves to be more complicated due to the early exercise right that is granted to an investor. For this valuation we first apply an essentially affine interest rate model  $EA_1(3)$  on a weekly basis (see Dai and Singleton, 2000) to model the dynamics of the interest rates (the parameterization and further explanations are outlined in Eickholt et al., 2014a). Following this, we estimate the value of the early exercise right via Monte-Carlo simulations with 10,000 paths using Euler discretization on a monthly basis, i.e. with a step width of  $\Delta = 1/12$ . To determine the continuation values in the simulation we utilize the least-square regression approach as proposed by Longstaff and Schwartz (2001), with the first four monomials as basis function.

### 3.2 Investment decision

Our first analysis investigates the influence of the product structure on the attractiveness of a newly issued GFSN (“*investment decision*”). We measure the attractiveness by the number of investors and the volume of overall investments.<sup>8</sup> As GFSN are issued in a standard

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<sup>8</sup>We also test the logarithm of the volume as independent variable, which leads to similar results.

structure every few weeks or months with varying coupon offerings, we do not have to account in this analysis for, e.g., new product features, underlying characteristics or diverging credit ratings. Yet, a major assumption in our analysis is that the share of investments booked in the German Finance Agency's debt register account (in contrast to investments held at banks) remains constant for all GFSN in our sample. This means we assume a comparatively stable distribution between direct and indirect investments in GFSN over time.

Moreover, we control in our analysis for several potential environmental influence factors. First, we incorporate the time (quarter of the year) of an issuance, the value of the bond component (BOND) and the value of the option component (OPTION) of the respective GFSN. Splitting the overall value of a GFSN here into two parts allows us to investigate the influence of both factors independently. This can be helpful, for example, in determining the influence of the bond value on decisions of investors with a lower financial literacy, who might not be able to evaluate the value of the early exercise right appropriately. Next, we consider the long-term trend and the overall demand for governmental fixed-income products at the time of issuance. As proxy for the latter we define the variables LAST6INV, that consolidates the number of all investors in the last six issuances of the respective GFSN type, and LAST6VOL, which comprises the cumulated investment volume of all these investors. In some further analyses—not reported here—we substitute these trend variables by yearly dummies to control for further timing and trend influences but find few differences in the results, for which reason we decided to proceed with the described variables LAST6INV and LAST6VOL.

Table 3 shows the results of a regression of an issuance's attractiveness on the coupon structure and the control variables. As noted, we refer to the number of investors (left part) and the overall volume (right part) as indicators of attractiveness. Robust standard errors

(see Huber, 1967; White, 1980, 1982) are used in this and all upcoming regression analyses.

[Table 3 about here.]

The regressions explain in all four models around 50% of the response variable variation. Focusing on the estimated coefficients, we note at first that a high valuation of a newly issued GFSN elicits a stronger interest among investors. Both the bond component and the option value have positive loadings (however, these are not statistically significant for the bond variable) indicating that they are positively correlated to the attractiveness of an issuance, which seems very reasonable. In fact, the high elasticities of these variables suggest that the economic value is an important determinant for investors in deciding on new investments. We then observe some time effects in the regression. There seems to be a higher investment demand in the first quarter of the year which diminishes over the year as expressed by the negative coefficients for the variables Q2 to Q4.

Most interesting for our study, the regressions reveal that the coupon structure has a statistically significant influence on the attractiveness of an issuance. The variable UPSTEEP, which represents the steepness of the coupon structure, receives significant positive loadings throughout all models. This suggests that investors prefer rising coupon structures and a high final coupon payment in their investment selection. On the other hand, the coefficients for our second proxy, the duration (DURATION), are consistently negative. Hence, Individual Investors apparently look for a short weighted average time until the investment and coupons are paid back. Separate regressions show that the described relations are robust for investor subsamples and different time (investment) periods. They are also robust even when we focus only on either UPSTEEP or DURATION to describe a product's coupon structure.

Summing up, our main finding in the regression analysis is that the coupon structure seems to influence Individual Investors' decisions to invest in a GFSN. In a more general context,

the results point to a potential for issuers of fixed-income products to place more successful issuances, i.e. to extend the investor base and to achieve higher investment volumes, through offering specific investment profiles. For our sample, steep coupon structures combined with a low capital duration draw most interest among Individual Investors. The next sections focus on investigating whether such a coupon structure is also advantageous in the case of investors' early exercise and portfolio decisions.

### 3.3 Exercise decision

In this section we use—similar to Eickholt et al. (2014b)—pooled logit regressions to find whether the probability of an early exercise (“*exercise decision*”) differs depending on the (upcoming) coupon structure of a GFSN. As discussed before, we emphasize that according to theory an investor's exercise decision should be linked only to the current valuation of the GFSN.

Besides the defined PVEV-ratio, which indicates if an exercise is economically reasonable, and the proxies for the coupon structure, we control in this analysis for three environmental factors. First, we control for an extraordinary early exercise effect in the first month after the initial one-year blocking period (BLOCK) of a GFSN. In this month, exercises might occur more frequently due to the long-deferred need to exercise that had developed during the first twelve months of maturity when investors could not liquidate their GFSN investments. Second, we account for two major changes in the German tax system during our observation period. Both in 1999 and 2006, the tax allowances for Individual Investors were significantly reduced in Germany, which might have induced increased exercise activities in the months shortly before the tax changes became effective by investors who wished to optimize their tax debts. We introduce two dummy variables (TAX99 and TAX06) to cover this potential effect

in November and December 1998 and 2005. Third, the dummy variable NEWMARKET is designed to capture potential exercise peaks related to the introduction and growth phase of the “New Market” index NEMAX50 between 1998 and 2000 in Germany. This stock index listed new economy companies and gained a high popularity among Individual Investors in the last years of the 1990s. Hence, some Individual Investors might have exercised early their GFSN at this time so as to shift investments from the fixed-income market to the growing equity market. Finally, we incorporate for Type A GFSN the lifetime of a product (LIFETIME), i.e. the time since issuance.<sup>9</sup>

Compared to the number of decisions to continue holding a product, early exercise decisions are comparatively rare in our data set (see also the investor statistics in Table 2). Thus, to ensure that we have no (underestimation) bias in the estimated probabilities, we run rare event regressions according to King and Zeng (2001) on the data set. However, a comparison with the results of a pooled logit regressions does not show substantial differences, so that we decided to continue with standard regressions for efficiency and comparability reasons. Accordingly, Table 4 shows the results of a pooled logit regression of investors’ holding and exercise decisions on the defined variables for Type A and Type B GFSN. In addition, we estimate—if applicable—the respective elasticities at means indicating the proportional change in the exercise probability for a proportional change in the respective dependent variable.

[Table 4 about here.]

Beginning with the overall model output, we note that the regression is based on 15.924 million decisions for Type A and 9.991 million decisions for Type B GFSN. The pseudo- $R^2$

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<sup>9</sup>We note, that the lifetime of a product is correlated to the duration variable and that this collinearity might potentially result in inflated standard errors. However, because running the logit regression with orthogonalized regressors and running the regression without accounting for the lifetime variable leads to similar findings, we decided to continue with the described variables.



lies at 5.34% and 3.90%. We highlight three regression results. First, all four environmental variables have—as expected—a statistically significant positive coefficient, which suggests that the first month after the initial blocking period, tax effects and growth phases in the equity market are all associated with higher exercise rates. Following the argumentation in Eickholt et al. (2014b), this is not surprising as the demand for financial flexibility and liquidity seems to play an important role in the decision-making of Individual Investors with regard to fixed-income investments. Second, the coefficient for the valuation variable (PVEV) is strongly negative implying that the exercise probability decreases with a higher valuation of the respective product. As noted before, this is economically reasonable, since only at the lowest possible PVEV-ratio of 1 is an exercise profitable from a theoretical point of view. Moreover, the high elasticity for this variable indicates that Individual Investors are indeed very sensitive to valuation changes in their investments, which is in line with our observations on investment decisions in the previous subsection. Our third finding is that the coupon structure of a GFSN has (again) a significant influence on the probability of exercise. Both regressions for Type A and Type B GFSN in Table 4 exhibit negative coefficients for the upcoming steepness of the coupon structure (UPSTEEP) and positive loadings for the duration variable (DURATION). This suggests that Individual Investors hesitate to exercise early investments which promise strongly rising coupons in the future, whereas they more quickly redeem products with only slightly sloping coupon structures. Further, investors apparently value more highly products with a comparatively short weighted average time until coupons and investments are paid back, than financially fully equivalent products with a longer duration. Separate analyses show that these relations are robust even when we focus only on either UPSTEEP or DURATION to describe a product's coupon structure.

Overall, the empirical patterns in Individual Investors' exercise decisions are consistent

with our above-described analysis on investment decisions, where we show that issuances with increasing coupon payments until maturity entice more investors and attract higher investment volumes. In summary, we interpret the economically irrational—and contrary to standard theory—penchant for high future coupons in both decisions as a “behavioral bias” of Individual Investors, which may be regarded based on psychological reasons.

Next, we aim to determine whether this bias in exercise behavior also applies to subsamples. We again regress investors’ decisions on the defined product and environmental variables, but restrict the analysis in each case to selected time frames and specific investor groups that differ with regard to personal or investment characteristics. In addition, we investigate two special valuation cases and two different investment history scenarios. Regarding the valuation cases, we consider in a first regression only exercise decisions made at times when the PVEV-ratio is close to 1 ( $PVEV \leq 1.025$ ). In contrast, we focus in another regression on exercise decisions that occur at—from a theoretical point of view—clearly suboptimal points in time ( $PVEV > 1.025$ ). Regarding the investment history cases, we incorporate in one regression all decisions to hold or exercise a GFSN that are made until maximal three feasible exercise opportunities have occurred. Another regression considers only decisions on GFSN that are made after the respective investor has already left out several attractive exercise opportunities. Table 5 presents the results for the described subsamples.

[Table 5 about here.]

In short, we observe that our results are quite robust for the subsample analyses. Beginning again with the environmental variables, the coefficients for BLOCK and NEWMARKET point in the same direction as in our last regression for all subsample regressions and for both Type A and Type B GFSN. The coefficients for the tax variable are again mostly positive. Furthermore, the results show a very consistent influence of the coupon structure. In general,

the probability of an exercise decreases for GFSN that offer a significant growth of coupon payments towards maturity (negative coefficients for UPSTEEP) and increases in almost all samples with a higher duration (positive coefficient for DURATION), which seems to support our hypothesis of a behavioral bias in Individual Investors' exercise behavior. Few exceptions appear. For investors with a high number of early exercises, for products with a high ratio of present to exercise value (PVEV) and for products for which an exercise would have been advantageous several times beforehand the estimated influence of the UPSTEEP variable differs from the other regressions. Still, this must not contradict our former findings regarding a behavioral bias among investors as it seems reasonable to assume that investors who, e.g., regularly omit profitable exercise chances and who use their option right at clearly suboptimal points in time, assign a lower relevance to the coupon design than other investors. A possible motive behind these exercise decisions—even though the respective GFSN currently has a high PVEV-ratio and thus an exercise is not economically advantageous—is the need for liquidity (see also the discussion in Eickholt et al., 2014b), in which case the upcoming coupon structure does naturally matter less.

Besides indicating robustness, the results in Table 5 also suggest that the penchant for specific coupon structures persists over different investor subgroups and applies for investors with varying personal characteristics. Even investors, who might be ascribed a higher financial literacy—such as investors holding a doctoral degree or investors who very frequently exercise GFSN early—seem to prefer investments with higher final coupon payments and short durations.

### 3.4 Portfolio decision

In this section we focus on investors who hold a portfolio of several GFSN. Following our former results, we aim to examine if the coupon structure is also a relevant decision criteria for Individual Investors in selecting a specific portfolio item for early exercise.

As the first step in this analysis, we create a new data set that comprises only “*portfolio decisions*” of the investor base. We define an Individual Investor’s decision as a portfolio decision if he holds at least two exercisable GFSN at the time of decision. This means that the investor has a choice among two or more items when he contemplates the early exercise of a product. As there are only a few differences between both GFSN types we consider the whole portfolio of an investor at this stage, i.e. we no longer differentiate between decisions on Type A and Type B GFSN. Moreover, we simplify our analysis at two points for a clearer presentation of the results. First, we focus on exercise decisions, i.e. we exclude decisions to continue holding the complete portfolio without any changes. Second, we consider a maximum of six items per portfolio and month. If an Individual Investor’s portfolio comprises more than six GFSN investments at the time of exercise we ignore the items with the highest PVEV-ratios, as these products seem to be—as discussed in the last section—less likely to be selected for exercise.

Next, for each month and each portfolio we determine a ranking (from the lowest to the highest value) of all portfolio items according to each of the following product characteristics: valuation (PVEV), duration (DURATION), yearly growth of upcoming coupon payments (UPSTEEP) and investment volume (VOLUME). We also test if exogenous effects, such as tax changes, or the lifetime of a product, have an impact on portfolio decisions. However, separate analyses—not reported here—show that incorporating exogenous effect variables

and running the regressions with the lifetime variable does not change the overall results, thus we decided to continue with the described approach.

As a final step, we use random effects logit regressions to estimate the influence of the ranking position of each considered product characteristic on an investor's portfolio decision, i.e. on the selection of a portfolio item to be exercised early. Technically, we assume for the panel regression that each portfolio represents its own group of observations. Table 6 presents the results of a regression on the overall data set (second column). To examine the robustness of our results we also conduct panel regressions for several investor subsamples (third-to-last columns).

[Table 6 about here.]

Focusing first on the overall statistics, we observe that Individual Investors made overall 179,820 portfolio decisions for 48,590 different portfolios during our sample period, which implies that the average portfolio comprehends circa 3.7 investments in Type A or Type B GFSN.<sup>10</sup> The number of examined portfolio observations is significantly lower here compared to our previous analysis of exercise decisions (see Table 4) due to the exclusion of pure holding decisions and due to a high share of investors holding only one product in their portfolio for most of the time (see investor statistics in Table 2). Nevertheless, we note that the regression on this smaller subsample of portfolio decisions generally resembles the findings in our previous discussion. Three observations are most striking: first, the PVEV-ratio remains an important basis for portfolio decisions. The negative coefficients for this variable for the lower ranking positions imply that investors prefer to hold items in their portfolio with a comparatively high PVEV-ratio. On the other hand, the probability of an early exercise increases for products that have a comparatively low valuation, which seems

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<sup>10</sup>In fact, the average portfolio is slightly larger since we restrict—as outlined—our analysis to the first six items in an investor's portfolio.

very reasonable.

Second, we observe a tendency among investors to select portfolio items with higher investment volumes (positive coefficients for volume). Additionally, the results suggest that Type B GFSN are more often exercised early. Third, the statistically significant coefficients for UPSTEEP and DURATION indicate that an investor's decision as to which item in a portfolio to exercise early also depends on the coupon structure of the respective portfolio items. This is in line with our previous discussion. While we find a negative influence of the UPSTEEP variable, the loadings for DURATION are mostly positive and tend to increase with later ranking positions in the portfolio. This means that the probability of an early exercise decreases for portfolio items with a relatively steep coupon structure and thus with a comparatively high final coupon payment. On the other hand, the exercise probability increases in the case of a long weighted average time until the investment plus coupons is paid back, i.e. for portfolio items with a lower ranking position regarding the duration.

In the following, the right part of Table 6 (third-to-last column) reflects—as noted—the robustness of our results. For this, we rerun the described random effects logit regression but restrict the analysis to portfolio decisions of specific investor groups. In short, our main finding is that the regressions on these subsamples produce coefficients very similar to the results for the overall sample. Interestingly, we also note that only a few differences occur in the decision-making of investors with different personal characteristics. For instance, investors with considerable experience (more than 10 investments in GFSN) and high average investment volumes (more than €10,000) seem to use decision criteria in the selection of portfolio items for exercise that are very similar to those of investors who invest only rarely and make small investments. For both investor groups, the subsample regressions reveal that not only the current valuation level (PVEV-ratio) but also the coupon structure has some

influence. As in our analysis above, we conclude that portfolio holders seem to rate items with slowly rising coupon payments and—less strongly pronounced—a high duration less favorably than portfolio items with different coupon structures.

## 4 Financial relevance of coupon structure

Following our empirical investigation, we use in this section scenario analyses<sup>11</sup> to investigate the financial relevance of the coupon structure of a puttable bond. Furthermore, we seek to determine how issuers of puttable bonds can potentially exploit the apparent penchant of Individual Investors for specific coupon structures.

Our approach can be divided into three steps. First, we design several exemplary bonds with varying coupon structures that are financially fully equivalent in four different interest rate environments (Section 4.1). Second, we use Monte-Carlo simulations to quantify for each of these bonds an issuer’s potential advantage from the often suboptimal exercise behavior of Individual Investors (Section 4.2). For this purpose, we examine the “spread” between the financial fair value at issuance and the “empirical value” of a product, which we estimate based on the empirically observed exercise strategies of Individual Investors in our sample. Moreover, we compute for each product the theoretically optimal and the empirical exercise frequency over all simulation paths to gain a better understanding of the potential demand for liquidity during the years to maturity. Third, we discuss the results and derive implications for issuers (Section 4.3).

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<sup>11</sup>We refer here to scenario analyses instead of using the empirical data to ensure comparability of the results and to exclude exogenous influences on the data, such as diverging bond and option values at issuance, changes in the tax laws or time effects.

#### 4.1 Scenario description

We construct five hypothetical Type A and Type B GFSN with different coupon structures, whereby each bond resembles one of the issued GFSN in the empirical sample. The exemplary bonds cover a representative variety of degrees of steepness and kurtosis. First, we consider a GFSN that pays constant coupons over the respective six or seven years to maturity ( $C_t = CC$ , where  $C$  is the yearly coupon payment,  $t$  the year and  $CC$  the constant coupon rate). Second, we examine a product with a concave coupon structure, which implies a decreasing growth of coupon payments over time. For simplicity, we assume for this GFSN that the coupon payments grow each year by a fixed value divided by the current year of maturity ( $C_t = C_{t-1} + \frac{CF}{t}$ , where  $CF$  is a fixed value). Our third bond offers linearly increasing coupon payments over time ( $C_t = t \times CL$ , where  $CL$  is the linear multiplier). Fourth and fifth, we consider bonds with convex coupon structures and thus with exponentially growing payments over time ( $C_t = C_{t-1} \times CX$ , where  $CX$  is the convexity degree). Because such convex coupon structures seem to be most interesting for our analysis, we design two similar products here that only differ as to the degree of convexity (1.50 and 1.65) and thus as to their kurtosis.

We analyze the exercise strategies and valuations for these bonds in four different interest rate environments. Figure 1 shows the exemplary spot rate curves we assume for the simulation, which we construct based on the well-known function of Nelson and Siegel (1987), i.e.:

$$R(t) = \beta_0 + \beta_1 \frac{1 - e^{(-t/\tau)}}{t/\tau} + \beta_2 \left( \frac{1 - e^{(-t/\tau)}}{t/\tau} - e^{(-t/\tau)} \right), \quad (4.1)$$

where  $r$  is the spot rate at time  $t$ . We use diverging parameters for  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  and  $\tau$ .

[Figure 1 about here.]



Our first scenario is a normal or positive spot rate curve that slopes upwards (“normal”). In the second scenario we assume a similar term structure, where, however, the spot rates increase more steeply over time (“steep”). The third scenario represents an interest rate environment with a humped term structure (“humped”), which implies that spot rates grow at the beginning but decrease in the midterm, so that later spot rates again approach the initial level. In our last scenario we assume constant spot rates over time (“flat”).

## 4.2 Valuation

Our next step is to quantify for each of the described five hypothetical GFSN and for each interest rate environment the potential financial advantage for an issuer. For this, we compare the financial fair value of all products at issuance with an “empirical value” that considers Individual Investors’ empirically observed exercise behavior.

To determine the fair value of a GFSN, we follow an approach similar to that outlined in Section 3. First, we model the dynamics of the interest rates using a 1-factor short rate model according to Hull and White (1990), i.e.  $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$ .<sup>12</sup> For all bond valuations we use the same parameters (mean reversion speed  $\kappa = 30\%$ , short rate volatility  $\sigma = 1.0\%$ ,  $2.5\%$  and  $4.0\%$ , whereby the time-dependent drift  $\theta$  is adjusted to the initial forward rates of the respective scenario following the approach of Brigo and Mercurio (2006). Next, we determine the value of all considered GFSN at each time step via a Monte-Carlo simulation with 10,000 paths and a step size of  $\Delta t = 1/12$  using Euler discretization and applying the least square regression method as proposed by Longstaff and Schwartz (2001).

For result comparability, we assume that all GFSN in all interest rate environments are issued at a financial fair value of 1.020 and with a nominal value of the bond component of

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<sup>12</sup> $W(t)$  stands for a standard Brownian motion.

1.000.<sup>13</sup> This means that from a theoretical point of view all considered products should be equally attractive for investors. To construct a coupon structure for each exemplary bonds that leads to the same fair value, we refer to a non-linear optimization algorithm based on the interior-point method. With this algorithm we estimate the best fitting values for the parameters  $CC$ ,  $CF$ ,  $CL$ ,  $CX_{1.5}$  and  $CX_{1.65}$ , using the mean squared error as goodness-of-fit criteria. We rerun the optimization until the distance from the previous best estimate to the current best estimate is less than  $1 \times 10^{-16}$ . As a short summary of the optimization approach, Table 7 presents the estimated first and last coupon for each product in each interest rate scenario.

[Table 7 about here.]

In general, the statistics show that—as expected—the estimated final coupon payment increases from the first bond with a flat coupon structure to the last example, with a strongly convex coupon structure. In contrast, it generally decreases with a rising volatility level, which is due to a higher value of the early exercise right in more volatile environments. We will discuss this effect in detail at a later stage.

In contrast to the financial fair values, we expect that the empirical values of our exemplary designed GFSN differ among the scenarios due to the preferences of Individual Investors for specific coupon structures and payment profiles. To estimate the empirical values we basically apply a reduced-form model and refer to the estimated determinants of early exercising from Table 4. We ignore the influence of changes in the tax legislation and changes in the market structure, as these cannot be anticipated by investors. This means that the probability

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<sup>13</sup>We also run the simulations with fair values of 1.000 for all constructed GFSN and obtain similar results. The assumed fair value of 1.020 represents an approximate average of the valuations of Type A and B GFSN at issuance during our sample period.

of an exercise is given by:

$$\begin{aligned}
 Pr(ex_t) = & \exp[\hat{\beta}_1 PVEV_t + \hat{\beta}_2 UPSTEEP_t + \hat{\beta}_3 DURATION_t \\
 & + \hat{\beta}_4 BLOCK_t + \hat{\beta}_5 LIFETIME_t + Constant] / \\
 & [1 + \exp(\hat{\beta}_1 PVEV_t + \hat{\beta}_2 UPSTEEP_t + \hat{\beta}_3 DURATION_t \\
 & + \hat{\beta}_4 BLOCK_t + \hat{\beta}_5 LIFETIME_t + Constant)]
 \end{aligned} \tag{4.2}$$

where  $ex$  stands for an early exercise event,  $t$  for the current time step and  $\hat{\beta}$  for the previously estimated regression coefficients. The variables UPSTEEP, DURATION, BLOCK and LIFETIME (for Type A GFSN) are defined as in our former analysis. For the PVEV variable we utilize the results from the theoretical valuation.

Besides considering the influence of the coupon structure, the modeled empirical values also represent Individual Investors' generally suboptimal exercise behavior. As outlined in the study by Eickholt et al. (2014b), Individual Investors often tend to omit attractive exercise opportunities and use their early exercise right at times that are not economically advantageous. The reduced-form model implicitly accounts for both effects.

### 4.3 Scenario results

Table 8 compares the computed "spread" between the financial fair value and the empirical value at issuance for all products and for all interest rate environments.

[Table 8 about here.]

In addition, Table 9 presents for each scenario the simulated exercise frequency over all Monte-Carlo paths for the theoretical optimal strategy (Panel A) and for the empirical valuation approach based on the reduced-form model (Panel B).

[Table 9 about here.]

Beginning with the valuation results in Table 8, we observe clear differences over the presented scenarios. The lowest spreads occur in the simulations for GFSN with strongly convex coupon structures in the case of a steep spot rate curve (0.116% points to 4.766% points for Type A and 0.243% points to 5.477% points for Type B GFSN), whereas the highest spreads are linked to bonds with flat coupon payments (7.728% points to 13.156% points and 8.892% points to 15.604% points). More generally, we find that the spread is positively correlated to a higher volatility and that it varies depending on the assumed interest rate environment. We concentrate in the following on discussing the influence of the volatility and the coupon structure.

Regarding the impact of the volatility, we recap that the option component in a GFSN represents an unilateral right of the investor to reclaim his investment. This means the number of potentially attractive exercise opportunities and hence the value of the early exercise right increase in a high volatility environment. Accordingly, the results in Panel A of Table 9 show in general a rising percentage of exercised paths over the volatility scenarios. For instance, for our exemplary GFSN with a strongly convex coupon structure, the optimal exercise frequency grows from 16.480% for the low volatility scenario to 82.690% for the high volatility scenario (normal term structure, from 16.650% to 84.350% for Type B GFSN). Yet this increasing number of economically reasonable exercise points also means that the optimal exercise strategy acquires a greater financial relevance. Missing attractive exercise opportunities and exercising too early or too late is more disadvantageous in a more volatile environment. In fact, this is the reason why the spread between the fair and the empirical value—which incorporates the frequently suboptimal decision-making of the investor base—widens over the volatility scenarios.

Moreover, we find that the estimated spread typically diminishes along the five designed products. In short, the decreasing values are due to the declining relevance of the early exercise right and thus to the declining importance of the negative influence of Individual Investors' often suboptimal exercise behavior. To explain this effect in detail, we compare, as an example, the simulated exercise frequencies and valuations for our first and last constructed GFSN. First and most importantly, we note that for a GFSN with a flat coupon structure significantly more attractive exercise opportunities arise until maturity than for the last case, a GFSN with a strongly convex coupon structure, where the highest coupon payments are deferred towards maturity. The statistics in Panel A of Table 9 show that the percentage of optimally exercised simulation paths decreases for the low volatility scenario and a normal spot rate curve from 99.070% for the first GFSN to the above-mentioned 16.480% for the last analyzed product (from 99.590% to 16.650% for Type B GFSN). Similarly, the optimal exercise timing differs. A separate analysis—not reported here—indicates that the average optimal holding times over all simulated paths for these two bonds are respectively 28 months and 63 months (28 months and 73 months). Both the lower exercise frequency and the longer holding period, i.e. the later exercise time, are due to on average higher continuation values and thus to an on average higher PVEV-ratio of products that offer high final coupon payments. Recalling our former argument that an early exercise is economically only reasonable if the PVEV-ratio reaches 1, it becomes obvious that the theoretical and empirical option value decreases along the ordered product scenarios. Thus we can infer the main reason for the narrowing spread is the smaller number of economically advantageous exercise opportunities that Individual Investors can potentially omit due to suboptimal exercise decisions. In other words, the risk of missing attractive exercise points is reduced for products with steeply rising coupon structures.

However, in addition to the development of the PVEV-ratio, a second factor also drives—as discussed earlier—differences between the product scenarios, namely the penchant of Individual Investors for specific coupon structures. The reduced-form model for the estimation of the empirical values considers that investors more often hesitate to exercise their investments early in the case of coupon payments that grow more strongly towards maturity (negative coefficient on UPSTEEP in the model), and that they generally prefer to hold products with a short duration (positive coefficient on DURATION). Taken together, this “behavioral bias” leads to a reduced exercise probability in products with steeply rising coupon payments as the influence of UPSTEEP outbalances the influence of the DURATION variable. Compared to the last argument this bias acts in the opposite direction. It has a positive and increasing impact on the spread along the five exemplary GFSN.

Nevertheless, looking at the spread statistics, we find that this positive second effect is widely overcompensated by the negative influence of the above-described diminishing relevance of the option component. However, it is more obvious in the statistics on the modeled empirical exercise strategies shown in Panel B of Table 9. Here, the tendency among investors to continue holding allegedly more attractive bonds further reduces the average exercise frequency for GFSN with high final coupon payments, and contributes—on the other hand—to an increasing number of early exercises for GFSN that offer flat coupon payments.

In summary, the scenario analyses imply that issuers of puttable bonds for Individual Investors can gain a significant financial advantage through “behavioral financial engineering”. In particular, issuers might benefit from two empirical patterns. First, our results suggest that offering products with a (strongly) convex coupon structure allows issuers to reduce their liquidity reserves as Individual Investors prefer to continue holding such bonds and instead tend to exercise early products with visually less attractive coupon offerings. Second, flat

or concave coupon structures seem to be most suitable for issuers that aim to maximize the spread between the empirical and the theoretical value of their issuances. The reason is that for such products, the exercise right has a comparatively high theoretical value, which is advantageous for issuers since Individual Investors regularly forfeit attractive exercise opportunities and typically exploit even very valuable exercise rights to only a small extent. This is also expressed by the difference between the statistics in Panel A and B in Table 9.

## 5 Conclusions

In this paper we investigated the influence of different product designs on Individual Investors' decision-making in puttable bonds. Our main finding is that the coupon structure plays an important role in how investors make investment, exercise and portfolio decisions. Bonds with a steep coupon structure and a short weighted average time until all payouts have been made tend to attract more investors and have a lower exercise probability than products with the same financial fair value that offer a flatter coupon structure. This empirical observation, which appears to be robust for almost all subsamples in our data set, cannot be explained by economic arguments. Hence, we interpret our results as indicating a behavioral bias among Individual Investors. Apparently, Individual Investors value puttable bonds differently, depending on the psychological or visual attractiveness of the remaining coupon payments.

A main conclusion of our study is that there is some potential for issuers of fixed-income products with early exercise rights for Individual Investors to increase the attractiveness of their products and to steer investors' exercise behavior through proficient "behavioral financial engineering". Our analyses indicate that issuers who are looking for high investment

volumes and a low capital outflow throughout maturity should prefer puttable bonds with significantly increasing coupon payments until maturity. Issuers benefit in this case in two ways. First, they must have less liquidity available, as the early exercise volume tends to diminish as Individual Investors prefer to continue holding this kind of bonds. Second, the share of exploited attractive exercise opportunities tends to be lower than in the case of other coupon structures.

On the other hand, for issuers who seek to maximize the spread between the theoretical and empirical value of a newly issued puttable bond, flat or concave coupon structures might be more suitable. The reason is that—as discussed in detail in the study of Eickholt et al. (2014b)—Individual Investors typically exploit even very valuable option rights to only a small extent. Consequently, the spread between theoretical and empirical value is—measured in percent points—larger for products for which the probability of an early exercise, and thus the theoretical option value, is comparatively high, such as puttable bonds with flat coupon offerings.



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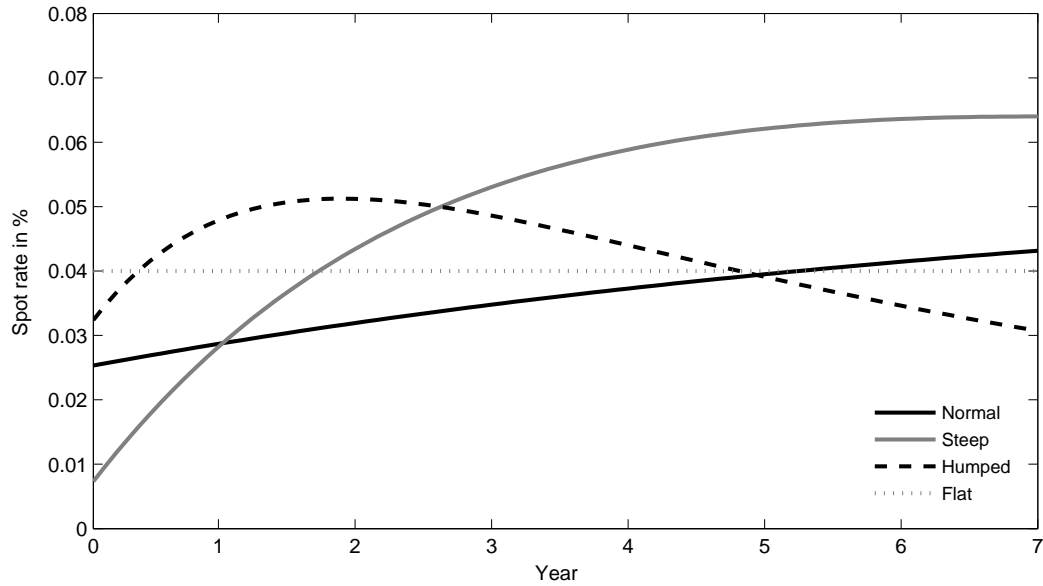
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## Figures

**Figure 1:** Interest rate scenarios for scenario analyses

This figure shows the exemplary spot rate curves at issuance for the scenario analyses. The spot rates are calculated using the formula of Nelson and Siegel (1987), i.e.  $R(t) = \beta_0 + \beta_1 \frac{1 - e^{-t/\tau}}{t/\tau} + \beta_2 \left( \frac{1 - e^{-t/\tau}}{t/\tau} - e^{-t/\tau} \right)$ , with the following parameters. Normal:  $\beta_0 = 0.06$ ,  $\beta_1 = -0.035$ ,  $\beta_2 = 0.020$ ,  $\tau = 7.000$ . Steep:  $\beta_0 = 0.035$ ,  $\beta_1 = -0.030$ ,  $\beta_2 = 0.140$ ,  $\tau = 3.000$ . Humped:  $\beta_0 = 0.000$ ,  $\beta_1 = 0.030$ ,  $\beta_2 = 0.120$ ,  $\tau = 1.500$ . Flat:  $\beta_0 = 0.040$ ,  $\beta_1 = 0.000$ ,  $\beta_2 = 0.000$ ,  $\tau = 6.000$ .



## Tables

Table 1: Summary statistics on GFSN

	Type A GFSN						Type B GFSN					
	Mean	Med.	p5	p95	Std.dev.	$\Delta$ SR	Mean	Med.	p5	p95	Std.dev.	$\Delta$ SR
<b>Coupon p.a. in %</b>												
Year 1	2.745	2.750	1.500	4.000	0.812	-0.641	2.745	2.750	1.500	4.000	0.812	-0.641
Year 2	3.199	3.250	1.750	4.250	0.813	-0.342	3.199	3.250	1.750	4.250	0.813	-0.342
Year 3	3.627	3.625	2.500	5.000	0.807	-0.072	3.627	3.625	2.500	5.000	0.807	-0.072
Year 4	4.034	4.000	3.000	5.750	0.821	0.181	4.034	4.000	3.000	5.750	0.821	0.181
Year 5	4.409	4.250	3.250	6.250	0.858	0.415	4.250	4.250	3.250	6.250	0.858	0.415
Year 6	4.821	4.500	3.750	7.000	0.929	0.700	4.821	4.500	3.750	7.000	0.929	0.700
Year 7							4.944	4.750	4.000	7.000	0.924	0.709
<b>Steepness of coupon structure</b>												
Diff. in %	2.076	2.000	0.750	4.000	1.066		2.199	2.250	0.750	4.250	1.128	
points												
Rel. in %	92.061	80.000	18.750	171.429	69.220		98.581	81.818	18.750	216.667	77.121	
<b>Fisher-Weil duration</b>												
Years	5.539	5.529	5.403	5.712	0.098							

The table exhibits statistics on the coupon offerings for Type A and Type B GFSN for all 204 issuances (102 Type A, 102 Type B) in our sample period from July 1996 to February 2009 and on the corresponding spot rates in Germany. For year 1 to 6 identical coupons are offered for Type A and Type B GFSN at each issuance date, whereas the coupons in year 7 are only applicable for Type B.  $\Delta$ SR stands for the difference between the coupon rate and the respective spot rate for the respective year, whereby the spot rates represent the term structure of interest rates on listed Federal securities (method by Svensson) at the respective issuance date according to Deutsche Bundesbank. Absolute steepness of the coupon structure describes the difference between the last and the first coupon, the relative steepness is the ratio of the last to the first coupon payment.

Table 2: Summary statistics on investor base

	Type A GFSN					Type B GFSN				
	Percent	Abs.	$\bar{\phi}$ Invest-ments	$\bar{\phi}$ Volume in €	$\bar{\phi}$ Early exercises	Percent	Abs.	$\bar{\phi}$ Invest-ments	$\bar{\phi}$ Volume in €	$\bar{\phi}$ Early exercises
<b>Gender</b>										
Male	33.47	52,717	3.282	5,670	0.582	38.41	35,595	2.948	2,858	0.563
Female	38.36	60,425	2.938	11,212	0.450	38.00	35,214	2.561	8,610	0.424
n/a	28.17	40,370	3.273	10,885	0.561	23.60	21,871	2.901	5,053	0.590
<b>Age</b>										
0 to 40 years	42.64	67,168	2.785	6,910	0.534	59.50	55,146	2.543	4,308	0.470
41 to 100 years	54.09	85,194	3.560	9,996	0.550	37.62	34,867	3.314	7,838	0.630
n/a	3.27	5,150	1.042	10,319	0.002	2.88	2,667	1.037	5,053	0.003
<b>Doctoral degree</b>										
Doctorate or professorship	4.15	6,536	3.911	12,382	0.604	3.50	3,243	3.896	11,160	0.594
No doctoral degree	95.85	150,976	3.114	8,664	0.522	96.50	89,473	2.750	4,865	0.514
<b>Preferred distribution<sup>(1)</sup></b>										
Indirect (via banks)	59.18	93,217	1.607	9,016	0.323	54.01	50,057	1.489	4,531	0.356
Direct (at German Finance Agency)	40.82	64,295	5.380	8,533	0.819	45.99	42,263	4.318	5,734	0.706
<b>Overall</b>	100.00	157,512	3.147	8,818	0.526	100.00	92,680	2.790	5,085	0.517

The table exhibits information on personal and investment characteristics of all 223,017 Individual Investors in our data sample, whereof 27,175 investors hold both Type A and Type B GFSN. The first and second column show the relative distribution and absolute numbers of investors with the respective personal characteristics, the third column lists the average number of investments, the fourth column shows the mean investment volume, while the last column shows the average number of early exercises for each cluster. (1) We define the preferred distribution channel as direct if an investor executes at least one direct transaction. Indirect means that an investor purchases a GFSN at a bank and later transfers his investment to the German Finance Agency.

**Table 3:** Influence of coupon structure on investment attractiveness

	Number of investments				Investment volume			
	Type A GFSN		Type B GFSN		Type A GFSN		Type B GFSN	
	Coefficient	Elasticity	Coefficient	Elasticity	Coefficient	Elasticity	Coefficient	Elasticity
<b>Valuation</b>								
BOND	25.316	5.168	23.620*	9.204*	276.223	7.734	124.353*	11.735*
OPTION	100.107*	0.448*	62.421*	0.740*	857.074*	0.526*	283.416*	0.814*
<b>Coupon structure</b>								
UPSTEEP	1,163.491*	1.932*	660.747*	1.841*	11,050.160*	2.516*	3,016.935*	2.036*
DURATION	-6.358*	-7.320*			-53.675*	-8.476*		
<b>Trend</b>								
LAST6INV	0.027	0.164	0.046	0.277				
LAST6VOL					0.014	0.082	0.023	0.165
<b>Timing</b>								
Q2	-1.850*		-1.168*		-16.231*		-5.895*	
Q3	-1.605		-0.986*		-13.566		-4.306*	
Q4	-1.048		-0.283		-10.011		-1.933	
<i>Constant</i>	4.081		-27.258*		-38.339		-140.023*	
<i>N</i>	102		102		102		102	
<i>R</i> <sup>2</sup> in %	50.30		55.76		48.86		50.85	

The table exhibits the results of standard regressions on Individual Investors' investment decisions in Type A and Type B GFSN. In the left part of the table the dependent variable is the overall number of investments, whereas in the right part the cumulated volume per issuance is used. Only investments and decisions after the initial one-year blocking period are considered. The variables LAST6INV and LAST6VOL represent the cumulated number of investors respectively the cumulated GFSN volume over the last 6 issuances and thus represent a mid-term trend. The elasticities (ey/ex) are calculated at means and are not determined for dummy variables. Robust standard errors are used. \* signals statistical significance at the 5% level.

**Table 4:** Influence of coupon structure on early exercise probability

	Type A GFSN		Type B GFSN	
	Coefficient	Elasticity	Coefficient	Elasticity
<b>Economic benefit</b>				
PVEV	-19.291*	-19.982*	-14.277*	-15.003*
<b>Coupon structure</b>				
UPSTEEP	-23.006*	-0.118*	-31.232*	-0.132*
DURATION	4.557*	13.592*	0.305*	1.139*
<b>Environmental circumstances</b>				
BLOCK	0.220*		0.150*	
NEWMARKET	0.732*		0.648*	
TAX99			0.225*	
TAX06			0.585*	
<b>Lifetime in years</b>				
LIFETIME	3.682*	6.783*		
<i>Constant</i>	-6.281*		8.058*	
<i>N</i>	15.924m	15.924m	9.991m	9.991m
<i>Pseudo-R</i> <sup>2</sup> in %		5.34		3.90

The table exhibits the results of a pooled logit regression on Individual Investors' early exercise decisions for Type A and Type B GFSN. Only decisions to hold or exercise after the initial one-year blocking period and before the last year of maturity are considered. The lifetime variable is not considered for Type B GFSN as it is perfectly correlated to the duration. The elasticities (ey/ex) are calculated at means and are not determined for dummy variables. Robust standard errors are used. *Pseudo-R*<sup>2</sup> is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. \* signals statistical significance at the 5% level.

**Table 5:** Influence of coupon structure on early exercise probability for subsamples

Type A GFSN	PVEV	UP- STEEP	DURA- TION	BLOCK	NEW- MARKET	TAX99	TAX06	LIFE- TIME	Const.	N	Pseudo- $R^2$ in %
<b>Issuance date</b>											
2002 or earlier	-12.981*	-15.166*	5.693*	0.274*	0.895*			4.758*	-18.343*	12.460m	4.31
After 2002	-46.038*	-120.364*	2.830*	0.167*				2.148*	29.383*	3.464m	5.96
<b>Gender</b>											
Male	-18.948*	-16.233*	4.548*	0.246*	0.731*			3.684*	-6.573*	5.498m	5.23
Female	-18.101*	-33.709*	4.688*	0.236*	0.734*			3.846*	-8.225*	5.854m	4.73
<b>Age</b>											
≤ 40 years	-15.722*	-15.342*	4.293*	0.236*	0.735*			3.498*	-8.689*	6.072m	4.24
> 40 years	-22.548*	-32.116*	4.772*	0.204*	0.698*			0.137	-3.938*	9.851m	6.35
<b>Doctoral degree</b>											
Doctorate or professorship	-19.72*	-20.939*	5.075*	0.310*	0.900*			4.147*	-8.388*	0.794m	5.78
No doctoral degree	-19.273*	-23.082*	4.537*	0.216*	0.724*			3.664*	-6.194*	15.130m	5.32
<b>Preferred distribution channel</b>											
Direct	-28.761*	-68.844*	3.818*	0.228*	0.486*			2.972*	7.126*	6.546m	6.71
Indirect	-14.258*	-4.784	4.037*	0.157*	0.904*			3.285*	-9.34*	9.377m	4.20
<b>Number of investments</b>											
≤ 3	-15.94*	-2.649	3.986*	0.208*	0.767*			3.228*	-7.278*	10.437m	4.06
> 3	-27.734*	-60.584*	4.553*	0.197*	0.630*			3.592*	2.638*	5.487m	7.44
<b>Ø Investment volume</b>											
≤ €10,000	-19.199*	-25.043*	4.579*	0.207*	0.720*			3.703*	-6.430*	11.891m	5.37
> €10,000	-19.401*	-14.103*	4.256*	0.271*	0.776*			3.410*	-4.928*	4.033m	5.12
<b>Number of early exercises</b>											
≤ 2	-16.809*	-5.861*	3.339*	0.219*	0.894*			2.653*	-3.689*	15.137m	4.09
> 2	-19.309*	-21.672*	5.536*	0.133*	0.249*			4.679*	-9.277*	0.786m	5.69
<b>Valuation scenarios</b>											
PVEV ≤ 1.025	-67.256*	-54.345*	3.538*	0.225*	0.422*			2.735*	47.009*	5.023m	6.24
PVEV > 1.025	-6.582*	37.571*	0.276	0.196*	0.452*			-0.039	-0.217	10.901m	1.44
<b>Investment history</b>											
≤ 3 missed chances	-15.744*	-26.358*	3.557*	0.359*	0.763*			2.781*	-5.359*	12.760m	4.43
> 3 missed chances	-31.94*	-28.759*	4.995*		0.472*			4.048*	4.652*	3.164m	5.66
<b>Type B GFSN</b>											
<b>Issuance date</b>											
2002 or earlier	-9.832*	-52.418*	0.006	0.191*	0.873*	0.268*	1.112*		3.614*	7.692m	3.50
After 2002	-27.745*	-78.055*	0.350*	0.113*			0.227*		21.874*	2.299m	4.29
<b>Gender</b>											
Male	-14.583*	-40.286*	0.318*	0.158*	0.642*	0.220*	0.576*		8.403*	4.022m	4.13
Female	-14.219*	-28.031*	0.280*	0.177*	0.638*	0.229*	0.475*		7.935*	3.664m	3.57
<b>Age</b>											
≤ 40 years	-12.933*	-29.823*	0.286*	0.160*	0.673*	0.192*	0.617*		6.681*	5.730m	3.57
> 40 years	-15.936*	-35.348*	0.320*	0.136*	0.657*	0.284*	0.550*		9.786*	4.261m	4.33
<b>Doctoral degree</b>											
Doctorate or professorship	-16.042*	-86.969*	0.371*	0.265*	0.817*	-0.097	0.455*		9.733*	0.409m	5.09
No doctoral degree	-14.233*	-28.537*	0.303*	0.142*	0.642*	0.234*	0.593*		8.016*	9.582m	3.86
<b>Preferred distribution channel</b>											
Direct	-21.171*	-50.274*	0.354*	0.150*	0.394*	0.041	0.429*		15.053*	4.568m	4.90
Indirect	-8.863*	-52.309*	0.208*	0.160*	0.976*	0.337*	0.875*		2.705*	5.422m	3.53
<b>Number of investments</b>											
≤ 3	-11.347*	-24.923*	0.238*	0.167*	0.720*	0.339*	0.701*		5.207*	6.825m	3.02
> 3	-20.921*	-38.88*	0.410*	0.111*	0.566*	-0.048	0.412*		14.491*	3.166m	5.86
<b>Ø Investment volume</b>											
≤ 10.000	-14.403*	-30.529*	0.308*	0.138*	0.633*	0.175*	0.607*		8.171*	9.031m	3.92
> 10.000	-13.505*	-40.409*	0.281*	0.250*	0.805*	0.601*	0.349*		7.431*	0.960m	3.94
<b>Number of early exercises</b>											
≤ 2	-11.023*	-36.744*	0.217*	0.190*	0.921*	0.309*	0.635*		4.655*	9.482m	3.10
> 2	-16.833*	22.010*	0.241*	0.054	0.204*	-0.034	0.449*		12.51*	0.509m	4.17
<b>Valuation scenarios</b>											
PVEV ≤ 1.025	-53.159*	-33.131*	0.335*	0.162*	0.474*	-0.014	0.492*		47.273*	2.357m	5.26
PVEV > 1.025	-4.335*	19.051*	0.166*	0.202*	0.520*	0.589*	0.684*		-2.212*	7.634m	1.33
<b>Investment history</b>											
≤ 3 missed chances	-11.601*	-32.164*	0.265*	0.311*	0.661*	0.313*	0.714*		5.369*	8.796m	3.06
> 3 missed chances	-21.08*	16.466	0.486*		0.449*		-0.008		14.169*	1.194m	5.57

The table exhibits the results of several pooled logit regressions (each row shows a specific regression result) on Individual Investors' early exercise decisions in Type A and Type B GFSN. The regressions differ according to the issuance time of the underlying GFSN, investors' personal and investment characteristics, valuation ranges and the investment history. The lifetime variable is not considered for Type B GFSN as it is perfectly correlated to the duration. Only investments and decisions after the initial one-year blocking period and before the last year of maturity are considered. Robust standard errors are used. Pseudo- $R^2$  is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. \* signals statistical significance at the 5% level.

**Table 6:** Influence of coupon structure on early exercise probability of portfolio items

Portfolio items ranked from low to high	All	Gender		Age in years		Doctoral degree		Preferred distribution		Number of investments		Ø Investment volume in €	
		Male	Female	≤40	>40	Yes	No	Direct	Indirect	≤10	>10	≤10,000	>10,000
<b>Economic benefit (PVEV)</b>													
Rank 2	-0.905*	-0.897*	-0.873*	-0.952*	-0.874*	-0.863*	-0.907*	-0.907*	-0.899*	-0.923*	-0.782*	-0.857*	-0.918*
Rank 3	-1.432*	-1.387*	-1.456*	-1.521*	-1.368*	-1.542*	-1.425*	-1.431*	-1.412*	-1.467*	-1.206*	-1.313*	-1.461*
Rank 4	-1.739*	-1.716*	-1.792*	-1.814*	-1.683*	-1.786*	-1.736*	-1.722*	-1.744*	-1.767*	-1.538*	-1.64*	-1.764*
Rank 5	-1.962*	-1.938*	-1.964*	-2.059*	-1.883*	-1.944*	-1.962*	-1.961*	-1.924*	-1.973*	-1.785*	-1.881*	-1.979*
Rank 6	-2.053*	-2.072*	-2.111*	-2.192*	-1.947*	-2.138*	-2.048*	-2.033*	-2.062*	-2.085*	-1.838*	-2.051*	-2.043*
<b>Steepness of coupon structure (UPSTEEP)</b>													
Rank 2	-0.268*	-0.293*	-0.275*	-0.28*	-0.261*	-0.205*	-0.272*	-0.299*	-0.224*	-0.250*	-0.332*	-0.285*	-0.263*
Rank 3	-0.497*	-0.506*	-0.516*	-0.500*	-0.494*	-0.284*	-0.510*	-0.496*	-0.484*	-0.490*	-0.429*	-0.498*	-0.492*
Rank 4	-0.602*	-0.599*	-0.633*	-0.590*	-0.603*	-0.521*	-0.607*	-0.601*	-0.576*	-0.567*	-0.562*	-0.636*	-0.584*
Rank 5	-0.669*	-0.669*	-0.711*	-0.651*	-0.67*	-0.574*	-0.674*	-0.665*	-0.645*	-0.610*	-0.628*	-0.720*	-0.644*
Rank 6	-0.733*	-0.785*	-0.753*	-0.709*	-0.736*	-0.743*	-0.732*	-0.747*	-0.666*	-0.578*	-0.800*	-0.840*	-0.689*
<b>Duration ratio (DURATION)</b>													
Rank 2	0.427*	0.437*	0.408*	0.491*	0.391*	0.560*	0.421*	0.418*	0.440*	0.415*	0.495*	0.391*	0.435*
Rank 3	0.494*	0.533*	0.466*	0.617*	0.428*	0.576*	0.491*	0.506*	0.499*	0.470*	0.712*	0.505*	0.495*
Rank 4	0.608*	0.641*	0.536*	0.720*	0.557*	0.570*	0.612*	0.680*	0.511*	0.585*	0.878*	0.669*	0.594*
Rank 5	0.719*	0.747*	0.669*	0.887*	0.646*	0.806*	0.716*	0.767*	0.692*	0.698*	1.054*	0.760*	0.716*
Rank 6	0.864*	0.864*	0.834*	1.063*	0.783*	0.989*	0.86*	0.906*	0.865*	0.817*	1.270*	0.903*	0.863*
<b>Investment volume</b>													
Rank 2	0.206*	0.180*	0.185*	0.282*	0.154*	0.163*	0.209*	0.194*	0.225*	0.224*	0.103*	0.261*	0.195*
Rank 3	0.055*	0.043	0.048	0.102*	0.025	-0.106	0.064*	0.075*	0.046	0.073*	0.056	0.185*	0.025
Rank 4	0.047*	0.039	0.089*	0.143*	-0.011	-0.010	0.051*	0.100*	-0.018	0.060*	0.098*	0.151*	0.027
Rank 5	0.071*	0.030	0.076	0.172*	0.018	0.021	0.075*	0.146*	-0.048	0.094*	0.128*	0.157*	0.057*
Rank 6	0.117*	0.058	0.155*	0.257*	0.044	0.069	0.120*	0.182*	0.026	0.099*	0.256*	0.175*	0.115*
<b>Product type</b>													
Type B GFSN	0.054*	0.082*	0.025	0.015	0.014	-0.050	0.059*	0.054*	0.054*	0.023	0.095*	0.018	0.092*
<i>Constant</i>	0.067*	0.057	0.127*	0.216*	-0.017	-0.066	0.074*	-0.058*	0.199*	0.173*	-0.660*	-0.179*	0.116*
<i>N</i>	179,820	71,304	54,230	71,261	108,559	9,718	170,102	113,698	66,0122	130,907	48,913	41,749	138,071
<i>Portfolios</i>	48,590	18,993	15,117	20,074	28,516	2,482	46,108	28,531	20,059	38,759	9,831	10,204	38,387
<b>Portfolio observations</b>													
<i>Min.</i>	2	2	2	2	2	2	2	2	2	2	2	2	2
<i>Mean</i>	3.7	3.8	3.6	3.5	3.8	3.7	3.9	4.0	3.3	3.4	5.0	4.1	3.6
<i>Max.</i>	6	6	6	6	6	6	6	6	6	6	6	6	6
<i>Pseudo-R<sup>2</sup> in %</i>	10.58	10.43	11.01	11.23	10.19	10.53	11.62	10.49	10.52	10.42	10.59	10.26	10.64

The table shows the results of several random effects logit regressions (each column shows a specific regression result) on Individual Investors' exercise decision depending on the ranking positions of the product in a portfolio. To determine the respective rank per criteria, all products in an investor's portfolio are ordered at each point in time from the smallest to the highest value. The first column shows the results of an overall analysis. The following columns show the results of panel regressions on subsamples that differ regarding personal and investment characteristics. The number of investments describes the cumulated amount of investments of an investor in GFSN over the whole sample period. Only exercise decisions after the initial one-year blocking period and exercise decisions that occur at times when the respective investor holds at least two GFSN are considered. For portfolios that comprise more than six GFSN, we consider only the first six products with the lowest valuation ratio (PVEV). Pseudo- $R^2$  is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. \* signals statistical significance at the 5% level.

**Table 7:** Scenario analyses – exemplary coupon structures

Panel A	Type A GFSN				Type B GFSN			
	Normal	Interest rate scenario		Flat	Normal	Interest rate scenario		Flat
<i>First coupon in %</i>		Steep	Humped			Steep	Humped	
<b>Volatility 1.0%</b>								
Flat	2.752	5.042	2.741	2.314	2.734	5.037	2.353	2.280
Concave	1.849	3.674	1.521	1.466	1.797	3.417	1.222	1.396
Linear	1.071	2.055	0.818	0.835	0.967	1.721	0.604	0.738
Convex	1.103	2.114	0.830	0.857	0.858	1.515	0.528	0.654
Strongly convex	0.788	1.514	0.587	0.611	0.561	0.990	0.343	0.427
<b>Volatility 2.5%</b>								
Flat	1.961	4.512	2.138	1.457	1.897	4.442	1.754	1.366
Concave	1.334	3.214	1.272	0.963	1.257	2.974	0.981	0.873
Linear	0.832	1.924	0.714	0.591	0.740	1.612	0.509	0.507
Convex	0.888	1.998	0.732	0.633	0.703	1.443	0.456	0.486
Strongly convex	0.657	1.446	0.525	0.468	0.476	0.950	0.307	0.330
<b>Volatility 4.0%</b>								
Flat	1.206	3.696	1.423	0.616	1.106	3.537	1.043	0.492
Concave	0.825	2.623	0.880	0.411	0.739	2.387	0.610	0.317
Linear	0.533	1.649	0.516	0.263	0.457	1.378	0.333	0.195
Convex	0.583	1.738	0.539	0.287	0.459	1.274	0.310	0.196
Strongly convex	0.446	1.284	0.396	0.221	0.326	0.854	0.209	0.142
<hr/>								
<b>Panel B</b>								
<i>Final coupon at maturity in %</i>	Normal	Interest rate scenario		Flat	Normal	Interest rate scenario		Flat
		Steep	Humped			Steep	Humped	
<b>Volatility 1.0%</b>								
Flat	2.752	5.042	2.741	2.314	2.734	5.037	2.353	2.280
Concave	4.529	9.001	3.727	3.592	4.660	8.860	3.168	3.619
Linear	6.425	12.329	4.909	5.007	6.769	12.049	4.226	5.162
Convex	8.372	16.054	6.302	6.511	9.773	17.256	6.011	7.446
Strongly convex	9.633	18.514	7.182	7.473	11.316	19.979	6.923	8.618
<b>Volatility 2.5%</b>								
Flat	1.961	4.512	2.138	1.457	1.897	4.442	1.754	1.366
Concave	3.269	7.875	3.117	2.359	3.258	7.710	2.544	2.263
Linear	4.990	11.542	4.283	3.548	5.183	11.280	3.564	3.550
Convex	6.745	15.171	5.561	4.804	8.004	16.438	5.190	5.530
Strongly convex	8.033	17.689	6.414	5.727	9.601	19.168	6.202	6.665
<b>Volatility 4.0%</b>								
Flat	1.206	3.696	1.423	0.616	1.106	3.537	1.043	0.492
Concave	2.022	6.427	2.155	1.006	1.916	6.190	1.582	0.821
Linear	3.199	9.896	3.098	1.581	3.197	9.648	2.333	1.362
Convex	4.425	13.196	4.095	2.179	5.230	14.508	3.527	2.231
Strongly convex	5.456	15.708	4.843	2.705	6.584	17.240	4.225	2.855

The table shows the estimated first (Panel A) and final coupon (Panel B) at maturity for 5 hypothetical GFSN. All GFSN have a financial fair value of 1.020. The respective coupon structure parameters for each bond ( $CC$ ,  $CF$ ,  $CL$ ,  $CX_{1.5}$  and  $CX_{1.65}$ ) are estimated via a non-linear optimization algorithm based on the interior-point method and using the mean squared error as goodness-of-fit criteria. The dynamics of the interest rates are modeled using a 1-factor short rate model according to Hull and White (1990), i.e.  $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$ , with  $\kappa = 30\%$ ,  $\sigma$  as shown and  $\theta$  being adjusted to the initial forward rates of the respective scenario following the approach of Brigo and Mercurio (2006). The fair value of a product is calculated via a Monte-Carlo simulation with 10,000 paths and a step size of  $\Delta t = 1/12$  using Euler discretization and the least square regression method as proposed by Longstaff and Schwartz (2001).



**Table 8:** Scenario analyses – differences between theoretical and empirical value

<i>Value difference in % points</i>	Type A GFSN				Type B GFSN			
	Normal	<i>Interest rate scenario</i>			Normal	<i>Interest rate scenario</i>		
		Steep	Humped	Flat		Steep	Humped	Flat
<b>Volatility 1.0%</b>								
Flat	4.286	7.728	0.751	2.812	6.096	8.892	0.946	4.006
Concave	1.766	0.946	0.491	1.391	2.536	1.318	0.787	1.953
Linear	0.705	0.216	0.408	0.721	1.014	0.591	0.710	1.017
Convex	0.537	0.191	0.426	0.611	0.739	0.353	0.588	0.772
Strongly convex	0.447	0.116	0.358	0.549	0.635	0.243	0.555	0.649
<b>Volatility 2.5%</b>								
Flat	7.424	10.018	3.289	6.098	10.239	11.702	4.024	8.512
Concave	5.862	4.769	2.473	5.214	7.902	5.875	3.344	7.123
Linear	4.350	2.261	2.031	4.266	5.756	2.991	2.705	5.758
Convex	3.811	1.960	1.858	3.842	4.525	2.310	2.408	4.833
Strongly convex	3.255	1.567	1.693	3.465	3.900	1.864	2.008	4.428
<b>Volatility 4.0%</b>								
Flat	10.238	13.156	6.016	8.664	14.479	15.604	7.781	12.859
Concave	9.593	9.306	5.467	8.439	13.155	11.274	7.206	12.285
Linear	8.811	6.242	4.891	8.292	11.712	7.721	6.457	11.807
Convex	8.341	5.640	4.794	7.963	10.752	6.345	6.126	11.276
Strongly convex	7.865	4.766	4.408	7.813	9.945	5.477	5.832	10.941

The table shows the estimated difference at issuance between financial fair value and the empirical value for exemplary Type A and B GFSN with different coupon structures in varying interest rate environments (see Figure 1 for the corresponding spot rate curves). The coupon structures of all bonds are designed for a fair value of 1.020. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990), i.e.  $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$ , with  $\kappa = 30\%$ ,  $\sigma$  as shown and  $\theta$  being adjusted to the initial forward rates of the respective scenario following the approach of Brigo and Mercurio (2006). For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  and least squares regression methods are applied. The estimation of the empirical value is based on a reduced-form model that considers the observed exercise behavior of Individual Investors (see regression results in Table 4).

**Table 9:** Scenario analyses – simulated exercise frequency in theoretical and empirical valuation

Panel A	Type A GFSN				Type B GFSN			
	Normal	Interest rate scenario			Normal	Interest rate scenario		
<i>Optimal exercise frequency in %</i>		Steep	Humped	Flat		Steep	Humped	Flat
<b>Volatility 1.0%</b>								
Flat	99.070	99.840	29.220	93.520	99.590	99.870	29.210	95.910
Concave	80.970	50.740	9.960	70.220	86.280	47.960	11.830	76.470
Linear	39.730	9.310	3.880	37.440	43.960	10.630	4.920	42.130
Convex	23.880	8.230	3.550	23.550	23.110	7.540	3.810	23.240
Strongly convex	16.480	4.390	2.750	17.190	16.650	4.340	3.150	17.900
<b>Volatility 2.5%</b>								
Flat	95.290	97.070	68.680	91.770	97.110	97.440	71.180	94.170
Concave	89.070	79.740	57.600	86.130	92.570	80.700	62.040	89.780
Linear	78.360	52.480	46.390	78.400	82.220	55.050	52.260	82.590
Convex	70.370	44.300	41.060	72.820	70.560	42.580	44.330	74.440
Strongly convex	63.360	36.050	37.030	67.740	63.140	35.860	41.920	68.990
<b>Volatility 4.0%</b>								
Flat	94.520	95.770	79.400	91.770	96.370	96.710	81.840	94.210
Concave	92.190	87.700	74.910	90.360	94.660	89.530	78.840	93.140
Linear	88.520	74.260	70.560	88.600	91.680	77.070	75.120	91.730
Convex	85.690	67.530	67.210	87.150	87.640	66.890	71.190	89.910
Strongly convex	82.690	59.610	64.540	85.610	84.350	59.890	68.870	88.580
<b>Panel B</b>								
Modeled empirical exercise frequency in %	Normal	Interest rate scenario			Normal	Interest rate scenario		
		Steep	Humped	Flat		Steep	Humped	Flat
<b>Volatility 1.0%</b>								
Flat	25.150	16.800	17.820	27.120	31.430	32.220	20.950	30.800
Concave	16.930	7.230	11.890	20.760	25.610	20.000	16.400	25.550
Linear	11.180	2.730	8.030	14.530	18.100	9.580	12.530	20.340
Convex	9.480	2.020	8.330	13.310	11.670	4.540	9.630	14.040
Strongly convex	8.150	1.360	6.760	11.640	9.910	2.960	8.310	10.870
<b>Volatility 2.5%</b>								
Flat	28.550	17.210	21.470	30.460	30.410	31.290	22.860	29.080
Concave	20.690	8.730	16.310	24.430	25.640	21.750	19.140	26.530
Linear	15.980	4.530	13.320	21.180	21.820	13.170	16.980	22.510
Convex	13.520	3.150	11.820	19.100	15.800	6.650	12.990	18.000
Strongly convex	11.840	2.130	11.430	16.710	12.490	4.640	11.190	15.560
<b>Volatility 4.0%</b>								
Flat	32.300	18.760	25.700	36.540	28.470	30.380	23.570	27.530
Concave	27.480	11.320	21.350	33.050	27.110	23.310	21.190	27.530
Linear	22.100	6.200	17.690	29.960	23.940	15.470	19.070	25.320
Convex	20.750	4.430	16.880	29.820	20.190	8.830	16.850	24.150
Strongly convex	19.160	3.520	16.620	28.030	17.090	6.760	15.370	22.080

The table shows the simulated exercise frequency (percentage of exercised Monte-Carlo simulation paths) for exemplary Type A and Type B GFSN with different coupon structures and in varying interest rate environments (see Figure 1 for the corresponding spot rate curves). Panel A shows the results for the fair valuation approach, whereas Panel B shows the results for the estimated empirical value based on a reduced-form model that refers to the estimated determinants of early exercising from Table 4. The coupon structures of all bonds are designed for a fair value of 1.020. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990), i.e.  $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$ , with  $\kappa = 30\%$ ,  $\sigma$  as shown and  $\theta$  being adjusted to the initial forward rates of the respective scenario following the approach of Brigo and Mercurio (2006). For the valuation, we apply a Monte-Carlo simulation with 10,000 paths with a step size of  $\Delta t = 1/12$  and least squares regression methods.

# List of Symbols

$\alpha$	First parameter for beta distribution
$\beta$	Second parameter for beta distribution
$\beta_i$	Parameters for the Nelson-Siegel function (Paper 3)
$\hat{\beta}$	Empirically estimated parameters for reduced-form model
$\Delta t$	Step width for Monte Carlo simulation
$\Delta SR$	Difference of spot rate to coupon rate
$\Gamma$	Decision strategy
$\Gamma_n$	Decision point
$\omega$	Proportion of passive investors
$\Omega$	Sample space
$(\Omega, \mathcal{F}, Q)$	Probability space
$\mathcal{F}$	$\sigma$ -algebra
$\kappa$	Mean reversion speed
$\sigma$	Short rate volatility
$\tau$	Strategy (pair of decision and exercise strategy)
$\tau$	Decay factor in Nelson-Siegel function (Paper 3)
$\theta$	Time dependent drift of short rate
$B$	Standard beta distribution
$C$	Coupon
$CC$	Constant coupon payment for scenario bond
$CF$	Fixed value for concave scenario bond
$CL$	Linear multiplier for scenario bond
$CV$	Continuation value
$CX$	Convexity degree for scenario bond
$DC$	Decision costs
$DT99$	Dummy for change in German tax legislation 1999
$DT99$	Dummy for change in German tax legislation 2006
$DYE$	Year-end dummy
$EER$	Early exercise rate per month
$EV$	Exercise value
$p$	Cluster weight

$p5$	5th percentile
$p95$	95th percentile
$PV$	Present value
$Med.$	Median
$N$	Number of observations
$NV$	Nominal value
$Q$	Equivalent spot martingale measure
$r$	Short rate
$R$	Spot rate
$R^2$	Coefficient of determination
$s$	Share of sluggish investors
$Std.dev.$	Standard deviation
$t$	Time index
$T$	Maturity
$TC$	Transaction costs
$V$	Product value
$W$	Standard Brownian motion
$X$	Payoff

# Glossary

BLOCK	Dummy, first month after blocking period
CDAX10+	Dummy, changes in CDAX $> +10\%$ points within a month
CDAX10-	Dummy, changes in CDAX $< -10\%$ points within a month
CDAX25+	Dummy, changes in CDAX $> +25\%$ points within 6 months
CDAX25-	Dummy, changes in CDAX $< -25\%$ points within 6 months
CFI	Comparative fit index
COUPON	Dummy, coupon payment upcoming within the next 60 days
EA	Essentially affine interest term structure model
EXERCISED	Dummy, signaling if investor has already early exercised before
GFSN	German Federal Saving Notes
GmbH	Gesellschaft mit beschränkter Haftung
HJM	Heath, Jarrow, and Morton model
INVESTS	Number of former personal investments
INVESTSUM	Sum of overall investments in € so far
LAST6INV	Number of all investors in the last six GFSN issuances of the respective Type A or Type B
LAST6VOL	Cumulated investment volume in the last six GFSN issuances of the respective Type A or Type B
LIFETIME	Lifetime since issuance in years
MAR	Missing at random
MISSED6	Number of economically reasonable months (PVEV=1) within the last 6 months
MISSEDALL	Number of economically reasonable exercise months (PVEV=1) since issuance
MSE	Mean squared error
NEWMARKET	Dummy, introduction of new stock market “New Market” in Germany (January 1998 to December 2000)
PERFORMED	Dummy flagging if investor has already early exercised when economically reasonable (PVEV=1)
PVEV	Ratio of present value to exercise value
Q1-Q4	Quarter
RELDUR	Fisher-Weil duration of bond component in years

RMSEA	Root mean square error of approximation
SRMR	Standardized root mean square residual
TAX06	Dummy, change in tax legislation 2007 (November-December 2006)
U.K.	United Kingdom
U.S.	United States
UPSTEEP	Average yearly growth of upcoming coupon structure (last coupon minus current coupon divided by years to maturity)
VOLDAX	45-day option-implied volatility of DAX measured by VDAX
VOLINT	Volatility of German 5-year spot rate estimated from weekly returns over a 6-month window
YEAREND	Dummy, end of the year (each December)