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**Doctoral thesis**

# Delegated Portfolio Management

## Optimal Portfolio Policies under Compensation, Capital Flow and Price Influence

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# Contents

**List of Figures** IV

**List of Tables** VI

**List of Symbols** VII

**List of Abbreviations** XI

**Acknowledgements** XII

**1. Introduction** 1

1.1. Research Question . . . . . 1
1.2. Research Approach and Related Research . . . . . 7
1.3. Structure of Work . . . . . 10

**2. Research Context of Delegated Investment Decisions** 13

2.1. Involvement in Investment Decisions . . . . . 13
2.2. Products for Delegated Portfolio Management . . . . . 15
2.3. Historical Development . . . . . 18
2.4. Focus on Mutual Funds and Institutional Accounts . . . . . 20

**3. Conditions of Delegated Portfolio Management** 23

3.1. Compensation as Explicit Incentive . . . . . 24
3.1.1. General Compensation Form . . . . . 25
3.1.2. Level of Management Fees . . . . . 26
3.1.3. Payment Arrangement . . . . . 27
3.1.4. Results for Model Design . . . . . 27
3.2. Capital Flow as Implicit Incentive . . . . . 28
3.2.1. Factors Affecting Capital Flow . . . . . 29
3.2.2. Nonlinear Performance-Flow Relationship for Mutual Funds 33

3.2.3. Linear Relationship for Institutional Accounts . . . . . 34
3.2.4. Results for Model Design . . . . . 36
3.3. Price Effects from Trades . . . . . 37
3.3.1. Evidence for Price Effects . . . . . 38
3.3.2. Mutual Fund Flow and Price Effects . . . . . 41
3.3.3. Permanent and Temporary Price Effects . . . . . 42
3.3.4. Form of Price Impact Function . . . . . 43
3.3.5. Results for Model Design . . . . . 46

**4. Methodology** **47**
4.1. Dynamic Programming and Portfolio Optimization . . . . . 47
4.1.1. Intertemporal Portfolio Optimization . . . . . 48
4.1.2. Discussion of Standard Results . . . . . 50
4.2. Numerical Approximations with Markov Chains . . . . . 52
4.2.1. Markov Chain Approximation Method . . . . . 53
4.2.2. Explicit Approximation for Finite Horizon Problems . . . . 55
4.2.3. Finite Horizon Portfolio Optimization . . . . . 57
4.3. Discrete Time Models . . . . . 60
4.3.1. Multinomial Portfolio Models . . . . . 60
4.3.2. Discussion of Numerical Results . . . . . 64

**5. Portfolio Policies under Compensation and Capital Flow** **69**
5.1. Related Research . . . . . 69
5.1.1. Compensation and Portfolio Optimization . . . . . 69
5.1.2. Capital Flows and Portfolio Optimization . . . . . 73
5.2. The Situation of Delegation . . . . . 74
5.3. Closed-Form Solution for Linear Model . . . . . 76
5.3.1. Model Setup . . . . . 76
5.3.2. Optimal Portfolio Policy . . . . . 79
5.3.3. Influence of Management Fee . . . . . 82
5.3.4. Effects of Market Performance-Related Capital Flows . . . 85
5.3.5. Effects of a Benchmark Portfolio . . . . . 91
5.4. Numerical Solution for Nonlinear Performance-Flow Relation . . . 98
5.4.1. Model Setup . . . . . 98
5.4.2. Markov Chain Approximation . . . . . 102
5.4.3. Influence of Nonlinear Performance Flow . . . . . 104

5.5. Concluding Remarks . . . . . 111

**6. Portfolio Policies under Price Influence 113**

6.1. Related Research . . . . . 113
6.2. Model Setup with Price Effects . . . . . 116
6.2.1. Model Processes . . . . . 116
6.2.2. Optimization Problem . . . . . 122
6.2.3. Summary of Model Setup . . . . . 125
6.3. Numerical Solutions for Portfolio Optimization with Price Effects . 126
6.3.1. Single-Period Model . . . . . 128
6.3.2. Multiperiod Model with Cash Endowment . . . . . 131
6.3.3. Multiperiod Model Without Initial Price Effect . . . . . 135
6.3.4. Multiperiod Model with Capital Flow . . . . . 137
6.4. Concluding Remarks . . . . . 139

**7. Conclusion 143**

**A. Portfolio Policies under Compensation and Capital Flow 151**

A.1. Optimal Control Candidate . . . . . 151
A.2. Solution to the HJB Equation . . . . . 152
A.3. Checking Assumptions . . . . . 159

**B. Numerical Solution Approach for Trinomial Portfolio Models 163**

B.1. Details of Numerical Solutions . . . . . 163
B.2. Implementation of Trinomial Portfolio Model . . . . . 168

**Bibliography 175**

# List of Figures


1.1. Overview of related literature. . . . . 10

2.1. Involvement in investment decisions. . . . . 14
2.2. Development of equity ownership in the US. . . . . 19
2.3. Portfolio management products in Germany. . . . . 20

3.1. Conditions of mutual fund and institutional account portfolio management. . . . . 24
3.2. Performance-flow relationship for mutual funds. . . . . 35
3.3. Excess return in relation to order imbalance. . . . . 40
3.4. Nonlinear price impact functions. . . . . 45

4.1. Efficient investment of self-acting primary investor. . . . . 52
4.2. Probability condition and optimal portfolio policy in Markov chain approximation. . . . . 59
4.3. Trinomial tree model with two periods. . . . . 62
4.4. Portfolio optimization problem in discrete time framework with two periods. . . . . 65
4.5. Results for the standard portfolio optimization problem in discrete time framework with one and two periods. . . . . 67

5.1. Efficient frontier and optimal portfolio for a tracking error minimizing portfolio manager. . . . . 71
5.2. General model setup of delegated portfolio management under compensation and capital flows. . . . . 74
5.3. Schematic illustration of capital flows. . . . . 76
5.4. Efficiency of delegated portfolios with management fee. . . . . 83
5.5. Adjusted investment strategy to increase efficiency of delegated portfolios with management fee. . . . . 85

5.6. Efficiency of delegated portfolios with capital flow related to general market performance. . . . . 86
5.7. Numerical example 1 of delegated portfolios with capital flow related to general market performance in a $\mu - \sigma-$diagram. . . . . 89
5.8. Numerical example 2 of delegated portfolios with capital flow related to general market performance in a $\mu - \sigma-$diagram. . . . . 90
5.9. Efficiency of delegated portfolios with performance-related capital flow. . . . . 92
5.10. Numerical example 1 of delegated portfolios under performance-related capital flow in a $\mu - \sigma-$diagram. . . . . 93
5.11. Distance of portfolios to tangency portfolio. . . . . 96
5.12. Numerical example 2 of delegated portfolios with performance-related capital flow in a $\mu - \sigma-$diagram. . . . . 97
5.13. Exponential approximation of performance-flow relationship. . . . . 99
5.14. Optimal portfolio policies with nonlinear performance-flow relationship with Markov chain approximation. . . . . 105
5.15. Sensitivity of portfolio policy for convexity of nonlinear performance-flow relationship. . . . . 108
5.16. Influence of risk aversion on portfolio policy with nonlinear performance flow. . . . . 109

6.1. Standard model vs. model with initial price effect and reversal. . . . 117
6.2. Price impact functions. . . . . 121
6.3. General single-period model setup with price effects. . . . . 125
6.4. Overview of numerical models with price effects and modifications. 127
6.5. Results for two-period model with initial cash endowment and linear price impact function. . . . . 132
6.6. Results for two-period model with initial cash endowment and concave price impact function. . . . . 134
6.7. Two-period model without price effect of initial transactions and linear price impact function. . . . . 136
6.8. Results for the two-period model with performance-related capital flow but without price effects. . . . . 138
6.9. Results for the two-period model without price effect of initial transactions and performance-related capital flow. . . . . 139

# List of Tables

4.1. Parameter values of the standard trinomial model. . . . . 66

6.1. Parameter values and results for the single-period model with price effects. . . . . 129
6.2. Parameter values for the two-period model with initial cash endowment and linear price impact function. . . . . 131
6.3. Parameter values for the two-period model with initial cash endowment and concave price impact function. . . . . 134
6.4. Parameter values for the two-period model without price effect of initial transactions and linear price impact function. . . . . 136
6.5. Parameter values for the two-period model without price effect of initial transactions and performance-related capital flow. . . . . 137

B.1. Parameter values and results for the two-period model with initial cash endowment and linear price impact function. . . . . 164
B.2. Parameter values and results for the two-period model with initial cash endowment and concave price impact function. . . . . 165
B.3. Parameter values and results for the two-period model without price effect of initial transactions and linear price impact function. . . . . 166
B.4. Parameter values and results for the two-period model without price effect of initial transactions and performance-related capital flow. . 167

# List of Symbols

| | |
|---|---|
| $T$ | Time horizon |
| $t$, $t_0$, $t_1$ | Points in time |
| $u(t)$ | Continuous time $d$-dimensional stochastic control process |
| $U$ | Control space |
| $\mathcal{A}(t_0, x)$ | Set of admissible controls $u$ that start in $(t_0, x)$ |
| $dt$ | Instantaneous time period |
| $dB(t)$ | Vector of Brownian motions, $m$ dimensions |
| $\mu(\cdot)$ | Drift function with $n$ dimensions |
| $\sigma(\cdot)$ | Standard deviation function with $n \times m$ dimensions |
| $Q_0$ | State space set $Q_0$ |
| $C^{1,2}(S)$ | Set of once continuously differentiable functions with regard to the first argument, twice continuously differentiable with regard to the second argument on $S$ |
| $f_x$ | First partial derivative of function $f$ with regard to the argument $x$ |
| $f_{xy}$ | Second partial derivative of function $f$ with regard to $x$ and $y$ |
| $\Omega$ | Sample space |
| $E(x)$ | Expectation of $x$ |
| $E_{t,x_0}(x)$ | Expectation at $t$ of $x$ with initial condition $x_0$ |
| $L(t, x, v)$ | Running cost/utility function with arguments time $t$, stochastic process $x$, and control $v$ |
| $\Psi(t, x)$ | Terminal cost/utility function (Bequest utility function) with arguments time $t$ and stochastic process $x$ |
| $J(t, x; u)$ | Minimization/maximization criterion |
| $V(t, x)$ | Value function |
| $P_i(t)$ | Continuous time price process of the $i$-th asset, $i = 1, \ldots, n$ |
| $P_0(t)$ | Continuous time price process of the riskless asset |
| $\alpha_i$ | Drift of the $i$-th asset, $i = 1, \ldots, n$ |
| $\sigma_i$ | Volatility of the $i$-th asset, $i = 1, \ldots, n$ |

| | |
|---|---|
| $r$ | Interest rate |
| $X(t)$ | Total wealth, portfolio value |
| $N_i(t)$, $N(t)$ | Holdings of the $i$-th asset in $t$ in terms of number of shares, vector of $N_i(t)$ |
| $w_i(t)$, $w(t)$ | Share of portfolio value invested in the $i$-th asset, vector of $w_i(t)$ |
| $w_i^*(t)$, $w^*(t)$ | Optimal share of portfolio value invested in the $i$-th asset, vector of $w_i^*(t)$ |
| $c(t)$ | Consumption in $t$ |
| $c^*(t)$ | Optimal consumption in $t$ |
| $\mathbf{1}$ | Vector with dimension $n$ of ones |
| $\Sigma$, $\Sigma^{-1}$ | Covariance matrix of $n$ Brownian motions, inverse of covariance matrix |
| $x^{'}$ | Transposition of vector $x$ |
| $\gamma$ | Risk aversion factor in CRRA utility function |
| $\beta$ | Time preference factor in utility function |
| $MV$ | Minimum-variance sub-portfolio |
| $g_M(t)$ | Capital flow related to general market performance |
| $a$ | Size of capital flow related to general market performance |
| $w_M$ | Market portfolio in terms of relative holdings |
| $g_B(t)$ | Capital flow related to individual performance compared to benchmark portfolio |
| $c$ | Size of performance-related capital flow |
| $w_B$ | Benchmark portfolio in terms of relative holdings |
| $g(t)$ | Total capital flow function in continuous time model |
| $\text{Diff}_\text{B}$ | Difference of weighting factor for return-oriented portfolio with benchmark portfolio |
| $\text{Diff}_\text{D}$ | Difference of weighting factor for return-oriented portfolio without benchmark portfolio |
| $\xi^h$, $\xi$ | Markov chain |
| $\mathcal{R}_h^n$, $\mathcal{R}^n$ | Uniform $h$-grid as state space |
| $h$ | State space increment |
| $e_i$ | $i$-th unit coordinate vector |
| $y, z$ | States |
| $Z(y)$ | Set of states reachable from the state $y$ |
| $p(y, z\|u)$ | Probability to traverse from state $y$ to $z$ given the control $u$ |

| | |
|---|---|
| $u_i$ | Control for Markov chain at time step $i$ |
| $\tau$ | Continuous time models: exit time of stochastic process; Markov chain models: time interpolation constant |
| $N_\tau$ | Number of time steps |
| $B^+$ | Maximum of $B$ and $0$ |
| $B^-$ | Maximum of $-B$ and $0$ |
| $H_i(t)$, $H(t)$ | Holdings in the $i$-th asset in terms of absolute value, vector of $H_i(t)$s |
| $y_1$, $y_2$ | State space of current absolute holdings in two assets |
| $D_i(t)$, $D_i'(t)$ | Absolute/relative contribution of $i$-th asset to performance difference |
| $G_3(t)$, $G_3'(t)$ | Convex capital flow due to absolute/relative performance difference |
| $\phi$ | Size of convex capital flow |
| $H_{B_i}(t)$,$H_B(t)$ | Absolute holdings of the benchmark portfolio in the $i$-th asset, vector of $H_{B_i}(t)$ |
| $K$ | Number of states in finite sample space |
| $p(k)$ | Probability for $k$-th state |
| $P_i(t, \kappa_t)$ | Price of $i$-th asset in $t$ and state $k$ for discrete time models |
| $P_{i,0}$ | Initial price of the $i$-th asset |
| $s_i(t, \kappa_t)$ | Return of the $i$-th asset in time $t$ and state $\kappa_t$ |
| $N_i(t, \kappa_t)$ | Holdings in terms of number of shares in the $i$-th asset in $t$ and state $\kappa_t$ |
| $w_i(t, \kappa_t)$ | Relative share of portfolio value invested in the $i$-th asset in $t$ and state $\kappa_t$ |
| $X_0$ | Initial endowment in discrete time models |
| $X^p(t, \kappa_t)$ | Portfolio value after transactions in $t$ and $\kappa_t$, no price effects |
| $X^a(t, \kappa_t)$ | Portfolio value before transactions in $t$ and $\kappa_t$, no price effects |
| $J(X_0)$ | Maximization criterion given initial endowment $X_0$ for discrete time models |
| $E_{X_0}$ | Expectation in $t = 1$, given initial endowment $X_0$ |
| $P_i^a(t, \kappa_t)$ | Price before transactions, i.e., without price effect |
| $P_i^p(t, \kappa_t)$ | Price after transactions, i.e., with price effect |
| $P_i^r(t, \kappa_t)$ | Price after price reversal |
| $U_i(t, \kappa_t)$ | Transaction in the $i$-th asset in $t$ and state $\kappa_t$ |
| $r_i$ | Price impact function |

| | |
|---|---|
| $q_i$ | Price reversal function |
| $l_i$ | Size of price effect of $i$-th asset in linear price impact function |
| $\eta_i$, $\theta_i$ | Size of price effect of $i$-th asset in concave price impact function |
| $\rho_i$ | Size of price reversal for all price impact functions |
| $X^{a,a}(t, \kappa_t)$ | Portfolio value before transactions and before price effects in $t$ and state $\kappa_t$ |
| $X^{a,p}(t, \kappa_t)$ | Portfolio value before transactions and after price effects in $t$ and state $\kappa_t$ |
| $X^{p,p}(t, \kappa_t)$ | Portfolio value after transactions and after price effects in $t$ and state $\kappa_t$ |
| $c(t, \kappa_t)$ | Management fee in $t$ and state $\kappa_t$ |
| $f(t, \kappa_t)$ | Capital flow in $t$ and in state $\kappa_t$ |
| $b_i$ | Benchmark portfolio in terms of relative holdings |
| $\delta$ | Size of capital flow in discrete time model |

# List of Abbreviations

| | |
|---|---|
| AR | Autoregression process |
| BVI | Bundesverband Investment und Asset Management e.V. (German investment and asset management industry association) |
| CAPM | Capital asset pricing model |
| cf. | Confer |
| CRRA | Constant relative risk aversion |
| e.g. | For example |
| etc. | Et cetera |
| EUR | Euro, currency |
| f., ff. | And following page, and following pages |
| high-cap | Highly capitalized |
| HJB | Hamilton-Jacobi-Bellman |
| i.e. | That is |
| ICI & SIA | Investment Company Institute and Securities Industry Association |
| NASDAQ | National Association of Securities Dealers Automated Quotient |
| p., pp. | Page, pages |
| PDE | Partial differential equation |
| S&P 500 | Standard and Poor's 500, stock index |
| small-cap | Small capitalized |
| US | United States |
| USD | US dollar |
| vs. | Versus |

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# 1. Introduction

## 1.1. Research Question

This work investigates the intertemporal portfolio optimization of professional portfolio managers. It analyzes whether the special conditions of delegation in which portfolio managers make investment decisions can explain the observable investment patterns of portfolio managers. It further evaluates optimal portfolio policies from the primary investor's perspective. The focus is restricted to equity portfolios.

The classical models of portfolio optimization introduced by Markowitz (1952) and developed in a multiperiod continuous time context by Merton (1969), Merton (1971) and Cox/Huang (1989) assume investors who act on behalf of their own account. These investors invest their own capital, maximize utility from their own consumption, and consider prices on the capital market as given.[1] This setup reflects the dominant situation on the capital markets in the past. In 1950 for example, about 90% of corporate equity was held by private households.[2] But since then, more and more households have delegated the management of their portfolios to professional portfolio managers. In 2000, less than 40% of private households managed their equity portfolios on their own. In contrast, the market for delegated investment decisions has experienced long-term, continuous growth over the last 50 years. In 2000, equity holdings of mutual funds, the most prominent form of delegated investment decisions, accounted for approximately 20% in the US. Together with private pension funds and state and local pension funds, both forms for delegated investment decisions, they reached over 40% of total equities outstanding. As a result, direct equity investments represent only a minority of all equity investments today - the delegation of equity investments has become the rule.

Primary investors, such as private households and corporations, have good rea-

[1] Cf. Merton (1990), chapters 2 & 3.

[2] Cf. for this and the remaining paragraph Allen (2001), p. 1167. For example, Fortune (1997) provides a very detailed overview of the American mutual fund industry.

sons to delegate their investment decisions to portfolio managers.[3] Managing an equity portfolio is time-consuming. Investors need to conduct research to estimate risk and expected return, derive the optimal portfolio, execute the trades, and monitor each position.[4] Moreover, execution of trades is costly and difficult. Small investors pay trading fees that are usually higher than those paid by large professional investors.[5] In addition, with limited wealth it might be hard or impossible to trade the fractions of assets required to realize the optimal portfolio. Professional portfolio managers, on the other hand, can realize scale effects because they usually offer investment services to more than one customer. They benefit from smaller transaction fees and can, at least theoretically, invest in the optimal portfolio with, if any, only limited distortion.

Even though it is attractive for primary investors to delegate investment decisions, the drawback is that investment strategies of portfolio managers might differ from those of primary investors.[6] This potential deviation is rooted in the agency relation between portfolio managers and their clients.[7] After primary investors have delegated investment decisions to portfolio managers, they have no or only limited power to influence the portfolio manager's decisions.[8] This limitation of control allows portfolio managers to follow special investment strategies taking into account their own conditions, ultimately leading to different demand patterns of portfolio managers and primary investors.[9]

Various empirical studies provide evidence that professional portfolio managers follow special investment strategies:[10] they focus on large stocks with high liquidity,[11] and they prefer stocks that are listed in wide market indices (like the S&P

[3] Cf. e.g., Gruber (1996), p. 784f. , Cornell/Roll (2005), p. 58.

[4] Zhu (2005), for example, finds that households with higher costs of time are more likely to delegate their investment decisions.

[5] Brennan/Hughes (1991) report that trading fees decline with increasing size of orders. Cf. Keim/Madhavan (1997) and Keim/Madhavan (1998) for analysis of equity trading costs of institutional investors.

[6] Cf. Gompers/Metrick (2001), p. 237.

[7] Cf. Chevalier/Ellison (1997), p. 1168, Gompers/Metrick (2001), p. 237.

[8] Cf. Gompers/Metrick (2001), p. 237f.

[9] Cf. Gompers/Metrick (2001), p. 238.

[10] Beside the sources quoted in the text cf., e.g., del Guercio (1996). Elton/Gruber/Blake (2003) investigate the influence of incentive fees. In an interesting study Gaspar/Massa/Matos (2006) reveal distortions within a mutual fund family. They find that performance differences within a fund family are higher compared to similar funds that are not in one family.

[11] Cf. Falkenstein (1996), p. 116f, Gompers/Metrick (2001), p. 242, Lakonishok et al. (1991), p. 228.

500).[12] However, it is still under debate whether they also show preference for a particular risk-return profile.[13] The following paragraphs provide an overview of the investment strategies of portfolio managers.

Falkenstein (1996) shows that mutual funds in general have a preference for high-cap stocks and stocks with high liquidity, except for mutual funds that explicitly focus on small-cap stocks.[14] Using simple sorting and various regression techniques, he analyzes the influence of certain characteristics, including liquidity and size, on the equity holdings by mutual funds.[15] The basis for comparison is the mean share of equity holdings for all mutual funds.[16] Across all methodologies, he finds clear evidence that mutual funds prefer stocks with high liquidity, measured as "monthly volume divided by shares outstanding[17]". In addition, there was a strong preference for stocks of large size, measured as market equity,[18] though not as strong as liquidity. Falkenstein (1996) can confirm the preference for high liquidity across multiple mutual fund categories.[19] Only mutual funds that specialize in small-cap stocks show no preference for high liquidity.[20] Preference for size is not as dominant across different mutual fund categories.[21] Gompers/Metrick (2001) confirm these investment preferences in a comprehensive study.[22] They analyze with multivariate regression the power of various variables to explain institutional ownership. Across different measures such as size, share price, and turnover[23] they find that in-

[12] Cf. Gompers/Metrick (2001), p. 242, Chan/Chen/Lakonishok (2002), pp. 1418-1422.

[13] Cf. Falkenstein (1996), p. 129, Gompers/Metrick (2001), p. 243, Xu/Malkiel (2003), pp. 636-638.

[14] Falkenstein (1996) investigates the equity holdings of over 1000 US mutual funds in 1991 and 1992. He includes only those mutual funds that hold more than 50% of their assets in equity. Cf. Falkenstein (1996), p. 113.

[15] Cf. Falkenstein (1996), p. 116f. He also discusses the influence of size, price, beta, age, and news stories. However, in this work I will focus on liquidity and size.

[16] Cf. Falkenstein (1996), p. 115.

[17] Falkenstein (1996), p. 117.

[18] Cf. Falkenstein (1996), p. 120.

[19] Cf. Falkenstein (1996), p. 130f.

[20] Cf. Falkenstein (1996), p. 130f.

[21] Cf. Falkenstein (1996), p. 130f.

[22] They use holding data from all portfolio managers that manage more than USD 100 million. These managers are required to file their holdings exceeding 10,000 shares or USD 200,000 (so-called "13F" filings). Portfolio managers include banks, insurance companies, investment companies (mutual funds), investment advisers (including large brokerage firms), and others (including pension funds and university endowments). (Gompers/Metrick (2001), p. 232.) The market holdings of these types in 1996 were 21.6%, 9.4%, 25.3%, 37.2%, and 6.5% amounting to 51.6% of total market capitalization of all publicly traded stocks. (Gompers/Metrick (2001), p. 235f.)

[23] Size is measured in terms of market capitalization, and turnover is measured in terms of monthly volume divided by shares outstanding. Cf. Gompers/Metrick (2001), p. 240.

stitutional investors prefer large stocks and stocks that provide sufficient liquidity.[24] Lakonishok et al. (1991) report similar strategies for pension funds, another prominent form for delegated investment decisions.[25] They report that pension funds focus on large stocks and hold only a few small stocks.[26]

Regarding preference for stocks listed in market indices Gompers/Metrick (2001) report that institutional investors prefer stocks that are listed in the S&P500 index.[27] This is consistent with findings of Chan/Chen/Lakonishok (2002). They investigate the clustering of mutual funds investment strategies around the S&P500 as the most widely used reference portfolio.[28] Chan/Chen/Lakonishok (2002) find that most mutual funds prefer to structure their portfolio with regard to stock size similar to the S&P500.[29]

Regarding risk-return profiles of stocks, research results are still mixed. Chan/Chen/Lakonishok (2002) report that, regarding past performance and book-to-market value, mutual funds cluster around the center and avoid extreme performance and book-to-market values.[30] This is confirmed by Lakonishok et al. (1991). They find that pension funds avoid stocks with extreme performances, they "hold less extreme losers and winners[31]". Falkenstein (1996) reports, that mutual fund managers generally prefer stocks with higher volatility, in particular they avoid stocks with low idiosyncratic volatility.[32] Gompers/Metrick (2001) report similar findings. They mention mostly positive and significant regression coefficients

[24] Cf. Gompers/Metrick (2001), p. 242

[25] Lakonishok et al. (1991) use a data sample of 769 equity pension funds over the period 1985 to 1989. These funds managed USD 124 billion, about 18% of actively managed holdings of pension funds in 1989.

[26] Cf. Lakonishok et al. (1991), p. 228.

[27] Cf. Gompers/Metrick (2001), p. 242. Gompers/Metrick (2001) put this measure in the context of "prudent-man rule" investment style. In addition to the factors mentioned in the text, they investigate book-to-market value, yield, volatility, age, momentum over past three months, and momentum past nine months. However, these factors - beside volatility - will not be considered here as they provide mixed results. (Cf. Gompers/Metrick (2001), pp. 242-244.

[28] Cf. Chan/Chen/Lakonishok (2002), p. 1418f. Chan/Chen/Lakonishok (2002) used a data sample of 3336 US funds with information on fund holdings from 1983 till 1997. (Cf. Chan/Chen/Lakonishok (2002), p. 1411f.)

[29] Cf. Chan/Chen/Lakonishok (2002), pp. 1418, 1420, 1422. Though there is a significant distortion towards small-cap stock holdings with the mutual funds, Chan/Chen/Lakonishok (2002) attribute this to the significant portfolio of mutual funds in the sample that focus on small-cap stocks. (Cf. Chan/Chen/Lakonishok (2002), p. 1420.)

[30] Cf. Chan/Chen/Lakonishok (2002), pp. 1418, 1420-1423. However, if mutual funds consider extreme values, they prefer stocks with overperformance and lower book-to-market values. (Cf. Chan/Chen/Lakonishok (2002), pp. 1418, 1420-1423.)

[31] Lakonishok et al. (1991), p. 228.

[32] Cf. Falkenstein (1996), p. 129.

for volatility.[33] Xu/Malkiel (2003) also report a strong link between institutional ownership - pension funds, mutual funds, etc. - and the idiosyncratic volatility of stocks.[34] However, it remains contingent whether institutional investors prefer stocks with higher volatility or whether they are the reason for higher volatility. Xu/Malkiel (2003) provide evidence that institutional investors could cause this increased volatility.[35] They argue that the volatility of stocks with high institutional ownership could be higher because institutions receive similar information at the same time and react similarly.[36]

This work seeks to explain the undebated empirical results about the investment strategies of portfolio managers, i.e., their focus on stocks with high liquidity and stocks that are listed in market indices. It considers the different conditions under which portfolio managers and primary investors pursue investment decisions to explain the deviation in their investment decisions. I will focus on three conditions that have a potential impact on optimal portfolio policies and are empirically sound.[37] First, portfolio managers cannot derive utility from consumption or from terminal wealth as primary investors do. Instead, a portfolio manager's incentive system usually consists of a *compensation* in the form of a fixed management fee based on total assets under management.[38] They will usually derive utility from this compensation. Therefore, their optimization criterion is different from the one of the primary investor.

Second, portfolio managers face the allocation behavior of the primary investor. The contribution of capital of primary investors to portfolio managers is, even though with a long-term perspective, changeable at short notice. Indeed, in particular mutual fund investors systematically allocate capital to portfolio managers with

[33] Cf. Gompers/Metrick (2001), p. 243.
[34] Cf. Xu/Malkiel (2003), pp. 636-638.
[35] Cf. Xu/Malkiel (2003), p. 638.
[36] Cf. Xu/Malkiel (2003), p. 635. Some authors observe that portfolio managers increase the risk of their portfolios towards the end of the financial year if they did not meet a benchmark requirement. Cf. Brown/Harlow/Starks (1996), Koski/Pontiff (1999), and Chevalier/Ellison (1997). However, this phenomenon proved spurious when higher frequency data was used. Cf. Busse (2001).
[37] Beside the conditions mentioned here, others could be relevant as well. For example, del Guercio (1996) and Gompers/Metrick (2001) count the "prudent-man rule" as one factor. (Cf. Gompers/Metrick (2001), p. 238.) Gompers/Metrick (2001) also name historical return patterns of stocks as a potential factor to explain institutional investment patterns. (Cf. Gompers/Metrick (2001), p. 239.)
[38] Cf. e.g., Golec (1992), Golec (2003), Halpern/Fowler (1991), and Lakonishok/Shleifer/Vishny (1992). Section 3.1 provides more empirical details on compensation forms.

superior past performance.[39] The resulting *capital flow,* sometimes called "return-chasing behavior"[40], can be considered as an implicit incentive, additionally altering the optimization problem of the portfolio manager.

Third, portfolio managers often trade significant amounts of capital. Unlike small private investors who take prices on the capital market as given, their trades can have an effect on prices.[41] Hence, the small investor assumption,[42] common in portfolio optimization models, does not hold. It should be expected that the optimal portfolio of a portfolio manager reflects this *price effect from trades*.[43]

Given the special characteristics of portfolio manager's investment strategies and the general conditions of their work, the objective of this thesis is to answer the following questions:

1. In which way is the situation of a portfolio manager different from a primary investor? What are the consequences for the setup of portfolio optimization models for delegated portfolio management? Three aspects are pivotal:
   a) What is the structure of the compensation for a portfolio manager?
   b) How do primary investors allocate their capital to portfolio managers? How important is past performance for reallocation of capital?
   c) What impact do portfolio managers have on prices if they trade large portions of stocks?
2. What is the optimal portfolio policy for portfolio managers concerning the special conditions of delegated portfolio management, in particular
   a) What is the effect of the compensation?
   b) How does the capital allocation of primary investors change the optimal portfolio policy?
   c) What is the structure of the optimal portfolio, given price effects from trades?

[39] Cf. e.g., Ippolito (1992), Chevalier/Ellison (1997), and Sirri/Tufano (1998). Section 3.2 provides more details on primary investor behavior.
[40] Cf. Karceski (2002).
[41] Cf. e.g., Warther (1995) and Edelen/Warner (2001). Section 3.3 provides more details on price effects from trades.
[42] Cf. e.g., Merton (1990), p. 17f.
[43] Cf. Gompers/Metrick (2001), p. 238f.

3. Can the conditions of delegated portfolio management explain observed investment patterns regarding liquidity preference and preference for stocks listed in market indices? What are the implications for primary investors in terms of the efficiency of their delegated investments?

The next section will provide an overview of the research approach to address the research questions above and highlight important related research. More details on related research will be given in chapters 5 and 6 in conjunction with the respective models.

## 1.2. Research Approach and Related Research

Question 1 will be addressed with a review of existing research literature. Questions 2 and 3 represent the core of this work. Both are studied with a theoretical modeling approach, investigating the influence of management fees, capital flows, and price effects from trades on the optimal portfolio policy of portfolio managers.

The models used are based on different methodologies depending on their complexity. All models are laid out in an intertemporal finite horizon context. The intertemporal, multiperiod character of the models reflects the typical delegation situation that is intended to be a long-term relationship with potential actions during the relationship. A finite horizon situation is typical for long-term savings, e.g., for retirement purposes. As far as possible the method of stochastic control, i.e., dynamic programming in continuous time, is used to derive closed-form solutions of optimal portfolio policies. This methodology was pioneered by Merton (1969) and Merton (1971) in the context of portfolio theory. Comprehensive treatments of the methodology are numerous, examples are Fleming/Soner (1993) and Korn/Korn (2001). In cases where a closed-form solution is not available, Markov chain approximations will be used to solve the models numerically. The Markov chain approximation approach was initiated by Kushner (1977). It is extensively described in Kushner/Dupuis (1992). Munk (1997) and Munk (2003) present its application to portfolio theory. Certain complex models with price effects will directly be set up in discrete time models using multinomial tree price processes. Binomial trees were introduced by Cox/Ross/Rubinstein (1979) and extended by Boyle (1988), Boyle/Evine/Gibbs (1989) and He (1991) to multinomial versions. Pliska (1997) provides an excellent overview over their use in portfolio optimization.

The relationship of primary investors and portfolio managers is an agency relationship similar to the relationship of equity owners and managers.[44] Hence, agency theory appears to be a natural theoretical fit for delegated portfolio management.[45] In fact, the setup of the models in this work can very well be classified as hidden action models with moral hazard,[46] although agency theory is not used here. The objective of this work is to derive optimal portfolio policies in order to predict portfolio managers' behavior and explain the empirically observed investment strategies of portfolio managers. It is not the purpose of this work to solve the agency conflict between the portfolio manager and the primary investor. Nevertheless, the behavior of portfolio managers is also discussed from the primary investor's perspective.

The influence of compensation and incentive systems in general on optimal portfolio policies has been studied before.[47] Starks (1987) and Roll (1992) investigate the impact of benchmark-related performance fees. Starks (1987) focuses on symmetric and asymmetric performance fees, whereas Roll (1992) analyzes the impact of fees that induce the portfolio manager to minimize the tracking error. Stoughton (1993) and Admati/Pfleiderer (1997) investigate the incentive for the portfolio manager to reveal private information under various incentive schemes. Nietert (1996) analyzes the portfolio structures of managers who maximize utility derived from a management fee calculated on terminal wealth. Ou-Yang (2003), Kraft/Korn (2008), Cvitanic/Wan/Zhang (2006), and Cadenillas/ Cvitanic/Zapatero (2007) present continuous time models investigating various incentive systems. Research about general option-like compensation schemes as in Grinblatt/Titman (1989), Carpenter (2000) and Ross (2004) is only remotely related. Cornell/Roll (2005) present an asset pricing model that incorporates delegated portfolio management.

The influence of capital flow on the portfolio manager's decisions has been rarely analyzed. Karceski (2002) uses a single-period model with linear capital reallocation by private investors. Hugonnier/Kaniel (2010) implement a continuous time model in which the capital managed by the portfolio manager is proportional to the Sharpe ratio. Huberman/Kandel (1993) study the behavior of portfolio managers in a signaling framework with reputation building. Huddart (1999) investigates the

[44] Allen (2001), p. 1166f, Gompers/Metrick (2001), p. 237.

[45] Stracca (2006) provides a recent overview of principal-agent related literature and delegated portfolio management

[46] For such a setup cf. Kraft/Korn (2008).

[47] For a detailed discussion of some of the research mentioned in this and the following paragraphs, cf. section 5.1.

influence of reputation effects in the context of the private information of portfolio managers. General reputation effects, unrelated to portfolio management, have been studied e.g., by Scharfstein/Stein (1990) and Farnsworth (2003).

Only little research is available on portfolio optimization under price effects from trades.[48] Cuoco/Cvitanic (1998) present a model in which the parameters of the price process depend on the portfolio policy but not on transactions. Longstaff (2001) and Matsumoto (2006) analyze portfolio optimization if portfolio managers face restrictions of executing orders, either because not all orders are executed (Matsumoto (2006)) or because they can only trade a certain amount of stocks (Longstaff (2001)). Vath/Mnif/Pham (2007) analyze the price influence from trades in conjunction with transaction costs in an impulsive control framework. Isaenko (2010) investigate the implications of transitory price impact on trading behavior. Optimal portfolio choices under liquidity constraints are also analyzed by Ghysels/Pereira (2008) and Pereira/Zhang (2010) using numerical methodolgies. The implications of price effects from trades have been developed dominantly in the context of hedging and option pricing. Jarrow (1992) and Jarrow (1994) present early discrete time models. Frey/Stremme (1997), Platen/Schweizer (1998), Sircar/Papanicolaou (1998), Jonsson/Keppo (2002), Cetin/Jarrow/Protter (2004), and Bank/Baum (2004) follow with continuous time implementations. Research about optimal liquidation or purchasing strategies of single stocks is related with distance, e.g., Bertsimas/Lo (1998), Almgren (2003), and Huberman/Stanzl (2005) for discrete time, He/Mamaysky (2005) and Mönch (2005) for continuous time, and Subramanian/Jarrow (2001) for impulse control models.

Figure 1.1 presents an overview of other research. It also highlights the contribution of this work to existing research:

- It investigates the effect of the predominantly used proportional management fee based on assets under management that is continuously withdrawn from portfolio wealth

- It provides optimal portfolio policies in situations with linear and nonlinear capital flows induced by primary investors

- It demonstrates the influence of price effects from trades on optimal portfolio policies in a discrete time context using several alternative setups.

[48] A detailed discussion of these models is provided in chapter 6.

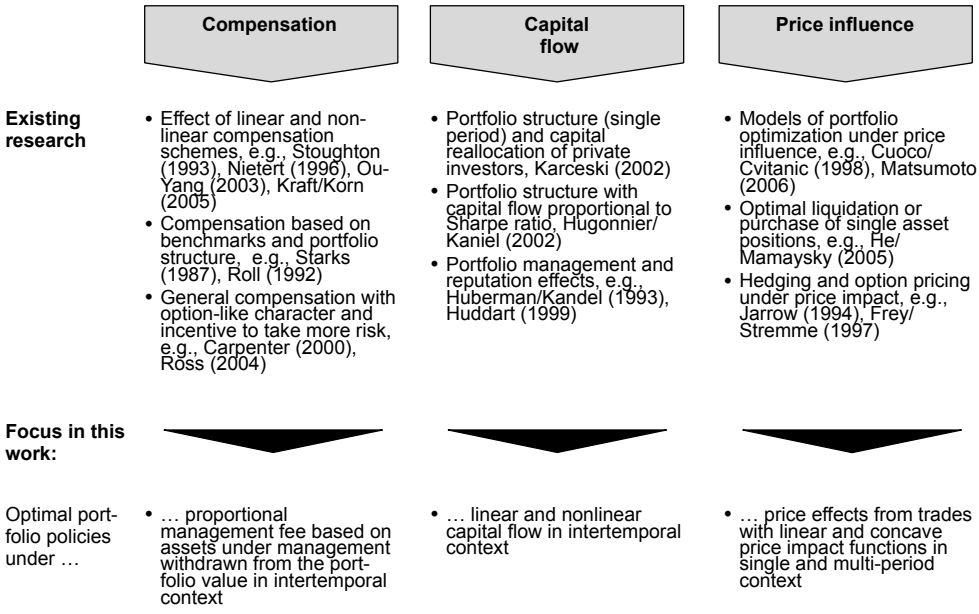



Figure 1.1.: Overview of related literature. (Source: Own illustration)

## 1.3. Structure of Work

This thesis is structured as follows. Chapter 2 defines delegated portfolio management and provides an overview of the historical development and existing products in this area. Chapter 3 contains a detailed review of empirical literature on the general conditions of delegated portfolio management through mutual funds and institutional accounts: common compensation schemes, the behavior of primary investors and how they allocate capital to portfolio managers, and price effects from trades. These empirical findings build the foundation for the theoretical models in chapters 5 and 6. Chapter 4 very briefly lays out the methodology used in the following chapters. It summarizes stochastic control and its application to portfolio optimization. Markov chain approximations are presented as a numerical approach and its application to portfolio optimization is demonstrated. Multinomial, in particular trinomial, portfolio models are outlined as a purely discrete time framework. Chapter 5 presents two models of delegated portfolio management both incorporating management fees and capital flows. Using the stochastic programming approach a closed-form solution is presented for a linear model. A second model implements nonlinear capital flow and numerical solutions will be given. After a general discussion of model results, they will be reflected on from the primary investor perspective

and their ability to explain investment patterns will be probed. Chapter 6 investigates the effect of price influence from trades on the optimal portfolio policy. It starts by formulating a portfolio model based on trinomial price processes and provides results on the influence of price effects on the optimal portfolio structure in various alternative model setups. Chapter 7 concludes the thesis. Various mathematical calculations and a description of the numerical solution approach are in the Appendix.

# 2. Research Context of Delegated Investment Decisions

Before addressing the research questions, this chapter will provide the necessary definition of delegated portfolio management and will lay out different forms of investment decisions and their realization as products and services. It also explains the focus of this work on mutual funds and institutional accounts. If not mentioned differently, this work is restricted to the US equity market, because the most data and information are available for this market.

## 2.1. Involvement in Investment Decisions

A primary investor, private or corporate, can invest in equity stocks with three different degrees of involvement:[49] they can directly invest, call someone for support, or delegate their investments. Figure 2.1 provides an overview of these different forms.

*Direct investments* are those investments that are made without support from third parties. The investor bases their decisions solely on information that is publicly available for all market participants (historical data on stock prices, company news, etc.). They might also use recommendations from publicly available analyst reports or expert magazines. The investors typically use direct brokers or their normal banking relationships for their transactions.

[49] The American Investment Company Institute and the Securities Industry Association use a similar classification, cf. Investment Company Institute/Securities Industry Association (2005b), p. 3 and p. 6. They use two dimensions: depending on the investment vehicle, they discriminate between individual stocks and stock mutual funds; depending on the source, they discriminate between direct investment and investments through professional financial advisors. Cf. Investment Company Institute/Securities Industry Association (2005a). Even though these institutes only consider holdings of corporate equity, the same structure applies for all kinds of financial instruments such as bills, money market certificates, etc.

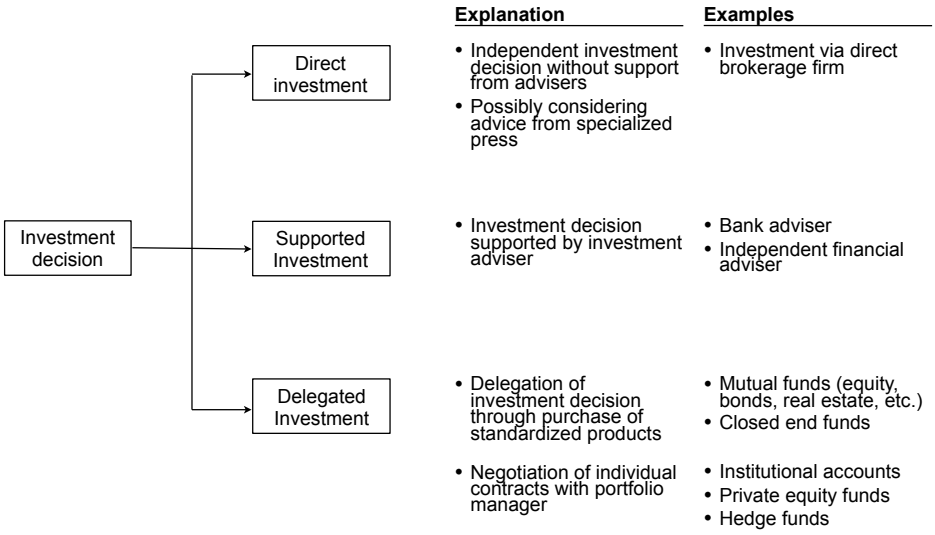



Figure 2.1.: Forms of investment with different degrees of involvement. (Source: Own illustration)

*Supported investments* are investments for which the investor turns to professional advisers. The investor still makes the final decision but relies to a certain extent on the recommendations of a financial adviser. The financial adviser can be a full-service broker, an independent financial planner, a bank and savings institution representative, an insurance agent, or an accountant.[50]

In the case of *delegated investments,* the primary investor delegates the responsibility for individual portfolio decisions, i.e., which assets to purchase, hold, and sell. They transfer a certain amount of capital to a portfolio manager for delegated portfolio management and define the principles for the investment strategy. These principles can comprise return targets, accepted levels of risk, certain investment constraints, etc. The portfolio manager then makes their own decisions on particular stocks, framed by the principles, and reports investment results to the investor.

This setup of delegated investments with only a primary investor and portfolio manager is rather simplified. In reality, the circumstances of delegation might be much more complex, both on the side of the primary investor and the portfolio manager. For example, if the primary investor is an institutional investor (e.g., a corporation), there will most probably also be delegation within the institutional investor subject (e.g., the corporation has its own treasury department that delegates

[50]Cf. Investment Company Institute/Securities Industry Association (2005b), p. 6, for the definition of professional financial advisors.

portfolio management).[51]

Within the portfolio manager subject, there might also be some delegation, e.g., the portfolio manager is an employee of a portfolio management company.[52] In this case, there is another level of delegation from the management and ownership of the portfolio management company to the individual portfolio manager. It is also possible that the portfolio manager company itself, e.g., a small insurance company without its own treasury department, delegates the investment decision to another portfolio manager.

In this work, the simplified setup with delegation only between the portfolio management function and the primary investor subject will be considered. Additional layers of delegation within the investor subject and the portfolio manager subject will be disregarded.[53]

## 2.2. Products for Delegated Portfolio Management

Portfolio managers offer primarily mutual funds and institutional accounts to provide delegated portfolio management. These products are explicitly offered as delegated portfolio management products. In addition to these, there are implicit portfolio management products such as insurance policies and pension funds. They are offered to serve a special purpose, e.g., insurance against risk or accumulation of pension contributions. However, by accumulating significant capital, they also incorporate a portfolio management component. The following paragraphs will describe these major products.[54]

[51] Cf. as an example, the delegation in the context of pension funds in Lakonishok/Shleifer/Vishny (1992), p. 341ff.

[52] Cf. Pozen (1998), p. 22f, Qiu (2003), p. 6.

[53] For an analysis of other layers of delegation cf. e.g, Lakonishok/Shleifer/Vishny (1992) and Ferris/Yan (2007).

[54] The primary investor delegates their investment decision with mutual funds and insurance policies by purchasing standardized products. With institutional accounts and pension funds, the primary investor delegates their investment decision by agreeing an indivual contral with the portfolio manager. The conditions of the contract are freely negotiable but tend to be similar.

### Mutual Funds

Mutual funds pool capital from their shareholders and invest it in stocks, bonds, or in the money market.[55] In this work, I will focus on equity mutual funds. Their legal structure varies from country to country. In the US, for example, mutual funds are legally a single investment company.[56] Mutual funds are *standardized* products for delegated portfolio management. The delegation happens through the investor's purchase of a share of the fund. There is no individual bargaining about the conditions of delegation, goals, or constraints. All this is predetermined by the portfolio manager's offering. The investor can only accept, i.e., purchase, or not accept, i.e., not purchase, the portfolio manager's offer.

Mutual funds are predominantly targeted at private investors. Private investors usually invest in mutual funds because they want to participate in the equity market but do not want to or cannot manage their own portfolio. Investors can buy and sell shares in mutual funds daily at their current price, which is the price of their underlying investments at the end of the day, making mutual funds a very liquid investment.[57]

### Institutional Accounts

For institutional customers such as corporates, insurance companies, and pension funds, portfolio managers offer institutional accounts.[58] Institutional accounts are individual services, their conditions are negotiated between the investor and the portfolio manager. The contract regulates the principles of investment and the terms and conditions (e.g., costs, reporting, etc.). It might be based on standardized forms but it is generally open for bargaining. Within such an institutional account, the portfolio manager will manage the capital of one institutional customer according to the contract.

[55] Similar to mutual funds are closed-end funds, exchange-traded funds, and unit investment trusts. However, these products are small in size compared to mutual funds and therefore not considered here. Cf. Investment Company Institute (2006) for information and the market development of these products. Cf. Investment Company Institute (2006), p. 118f., for a detailed definition of mutual funds and exchange-traded funds and unit investment funds.

[56] Cf. Pozen (1998), p. 16.

[57] Cf. Pozen (1998), p. 18.

[58] Cf. Maurer (2004) for an overview of institutional investors in Germany, in particular insurance companies and investment funds.

### Insurance Policies

Insurance companies offer capital accumulating insurance policies. For the German market as an example, the cash-value life insurance is the most important policy and demonstrates an exceptional market share, due to tax advantages granted by the government. Cash-value life insurance policies are highly standardized products. They provide a combination of a pure life-insurance and a special sort of savings account. In addition to the risk premium, the policy holder contributes to the savings account. After a fixed period of time (usually several years), the investor will receive a guaranteed capital return plus an additional capital gain that depends on the investment performance of the insurance company. There are some legal investment restrictions for insurance companies regarding their investments in the stock market and other financial instruments. However, the investor cannot observe the detailed investments of the insurer. Cash-value life insurance policies are marketed as rather riskless investments with a potential excess return for private investors. A typical private investor would only invest part of their wealth in cash-value life insurance policies.

### Pension Funds

Pension funds offer corporates and public plan sponsors and endowments the management of their pension plans. They manage the retirement contributions of corporations to fulfill their obligations for retirement plans for their employees.[59] In 1995, the pension fund industry in the US served around 45,000 sponsors.[60] The average pension fund size is about USD 67 million.[61] Sponsors regularly contribute to the pension fund with payments for retirement contribution plans. Pension funds are very common in the US, due to the local accounting rules, but play no important role in Germany.

Pension funds can represent a complicated system of delegation.[62] First, a corporation delivers on its obligations originating in defined-benefit pension plans and contributes into a pension fund. This fund can be managed by a corporate-owned

[59] Cf. Lakonishok/Shleifer/Vishny (1992), p. 340.

[60] Cf. del Guercio/Tkac (2002), p. 527.

[61] Cf. del Guercio/Tkac (2002), p. 527, there cited from McGraw-Hill's "1995 Money Market Directory" and the "1995 Directory of Pension Funds and their Investment Managers" (McGraw Hill).

[62] Cf. Lakonishok/Shleifer/Vishny (1992), p. 340ff. For a detailed analysis of potential legal structures of pension funds, cf. Clark (2000), pp. 76-93.

treasury department.[63] However, the treasury department usually does not manage the contributions by itself. Instead, it most frequently delegates the management of the contributions to professional portfolio managers.[64] The corporation can found its own pension fund or participate in a joint pension fund that the portfolio manager handles for many investors.[65] The portfolio management can be done by insurance companies, banks and trust companies, and investment counselors.[66] In a survey among 139 state, county, and city retirement systems with 55 replies, Halpern/Fowler (1991) report that only 14 funds do not use portfolio managers.[67] A majority of more than two thirds of pension funds delegated the management of their funds to portfolio managers. Portfolio managers usually offer an institutional account for the pension fund.

## 2.3. Historical Development

Over the past 50 years, there has been a clear shift towards delegation of investment decisions. Figure 2.2 shows the development of equity ownership in the US over the last 50 years.

In 1950, 90% of households invested in corporate equity. This share declined roughly 4% per year to drop to a total of 39% in 2000. At the same time, investments of mutual funds, pension funds, and insurance companies into corporate equity have experienced the opposite trend. Its share has steadily grown from 6% in 1950 to 49% in 2000. In 2000, pension funds represented the biggest portion with 23%, slightly ahead of mutual funds with 19% and far ahead of insurance companies with 7%.

Pension funds experienced the largest increase from 1970 with 9% to 1990 with 24%. Afterwards, its share slightly decreased from 24% in 1990 to 23% in 2000. From 1990 to 2000, the amount of state and local pension funds further in-

[63] I will focus on defined-benefit plans as that is the most common plan underlying pension funds in 1990. Cf. Lakonishok/Shleifer/Vishny (1992), p. 340. In a defined-benefit plan, the corporation benefits from the performance of the fund exceeding its obligations towards employees, but also leaving it as a residual claimant in the case of insufficient performance of the fund. Cf. Newman/ Milgate/Eatwell (1992), p. 131, and Lakonishok/Shleifer/Vishny (1992), p. 340.

[64] Cf. Lakonishok/Shleifer/Vishny (1992),p. 227.

[65] Cf. Lakonishok/Shleifer/Vishny (1992), p. 343.

[66] Cf. Lakonishok/Shleifer/Vishny (1992), p. 340.

[67] Cf. Halpern/Fowler (1991), p. 75. The survey covered 426 institutional accounts and 190 portfolio managers. Of the largest 100 portfolio managers, the survey comprises 76. Cf. Halpern/ Fowler (1991), p. 75.

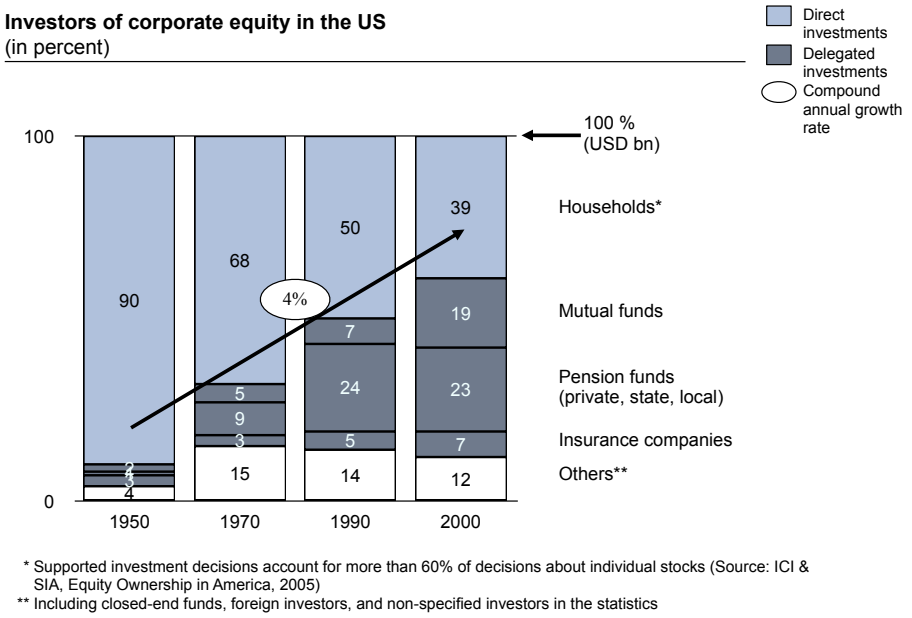



Figure 2.2.: Development of equity ownership in the US. (Source: Federal Reserve Board „Flow of funds“, Allen (2001), p. 1167.)

creased, however the amount of capital invested in private pension funds decreased by roughly the same amount.[68] Mutual funds showed the largest growth from 7% in 1990 to 19% in 2000. Before 1990, they played only a marginal role with 2% and 5% in 1950 and 1970 respectively. The overall growth in pension funds and mutual funds is even more astonishing regarding the general growth of corporate equity from USD 143 billion in 1950 to USD 19,047 billion in 2000. Life insurances also grew from 3% in 1950 to 7% in 2000, but only slightly.

The amount of capital held by institutional investors such as pension funds, mutual funds, and insurance companies is usually entrusted to them as delegated investments. In that sense, about half of equity holdings, 49%, are delegated investments. Holdings by households are either direct investments or supported investments through professional advisers only. In that sense, delegation really matters.

[68] These details are not displayed in figure 2.2.

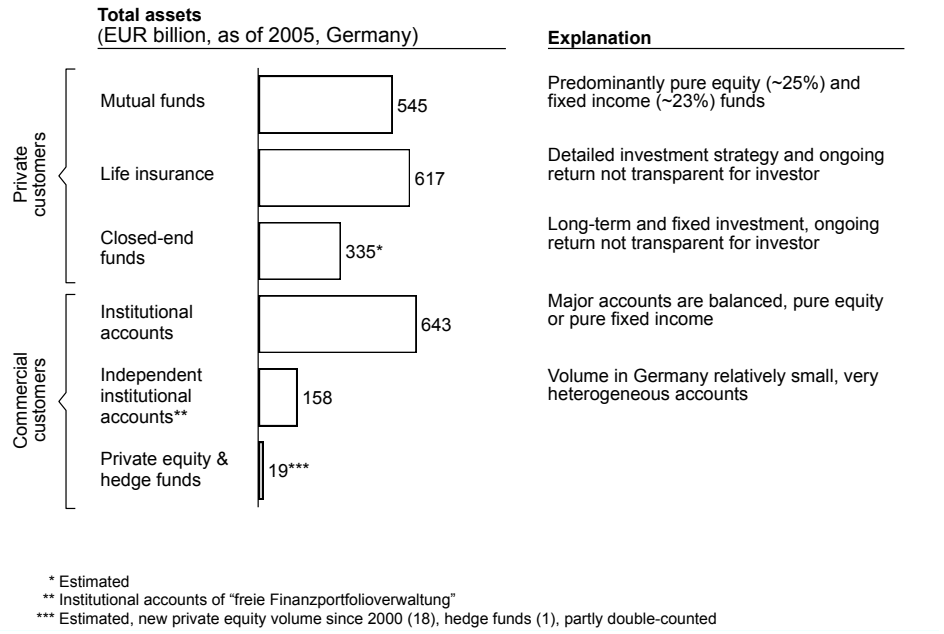



Figure 2.3.: Portfolio management products in Germany. (Source: BVI, Deutsche Bundesbank.)

## 2.4. Focus on Mutual Funds and Institutional Accounts

The empirical basis for models in chapters 5 and 6 will be based on mutual funds and institutional accounts. Both are explicit *portfolio management products* offering the investor delegation of management of the whole or a large portion of their portfolio. This management is usually done by the portfolio manager and not outsourced to other professionals. Mutual funds in particular have the advantage that data on product details is freely available. Mutual funds usually provide extensive information on their investment principles, terms, and conditions. In contrast, institutional accounts are generally not transparent. However, typical contractual details of contracts are publicly known.

Pension funds and insurance policies are not considered. Pension funds are generally a portfolio management product, but they very often outsource their portfolio management to professionals via institutional accounts.[69] Hence, it appears more

[69] Cf. section 2.2.

realistic to base the models on institutional accounts directly. Insurance policies, in particular cash-value life insurance policies, include portfolio management only as a minor component. With a guaranteed interest rate, they are closer to a savings account than a portfolio management product. However, their relevance in the German market should not be underestimated. Figures 2.3 illustrates that life-insurance companies manage more assets than mutual funds. Additionally, information from insurance companies is very limited and their investment strategy is not publicly available.

Overall, the characteristics of mutual funds and institutional accounts appear to be best suited to form the basis for the empirical modeling of delegated portfolio management. The following chapter will describe in detail the special conditions of delegated portfolio management associated with mutual funds and institutional accounts. It lays the foundations for the theoretical models in chapters 5 and 6.

# 3. Conditions of Delegated Portfolio Management

Portfolio managers follow special investment strategies:[70] they focus on large stocks with high liquidity,[71] and they prefer stocks that are listed in wide market indices (such as the S&P 500).[72] A potential explanation could be the different conditions under which portfolio managers and primary investors pursue investment decisions. I will focus on three special aspects that are characteristic of delegated portfolio management, in particular mutual funds and institutional accounts, and potentially relevant for optimal portfolio policies:

1. Optimization criteria: the optimization criteria for a portfolio manager is to maximize their *compensation*. This will, therefore, determine the structure of the optimal portfolio.[73]

2. External changes of assets under management: the volume of assets under management of a portfolio manager is determined by contributions and withdrawals of the primary investor. Hence, it is critical for the portfolio manager to understand how the primary investor allocates their capital. They face a *capital flow* from and to their primary investors that they should consider in their optimization problem.[74]

3. Influence on price process: portfolio managers very often manage huge amounts of capital in one fund. Changing investment positions, therefore, involves transactions of significant size. With such trades, portfolio managers experience *price effects* and the small investor assumption does not hold for them. This should have an influence on their optimization problem.

[70] Cf. section 1.1.
[71] Cf. Falkenstein (1996), p. 116f, Gompers/Metrick (2001), p. 242, Lakonishok et al. (1991), p. 228.
[72] Cf. Gompers/Metrick (2001), p. 242, Chan/Chen/Lakonishok (2002), pp. 1418-1422.
[73] The compensation is considered as an explicit incentive.
[74] This capital flow is considered as an implicit incentive.

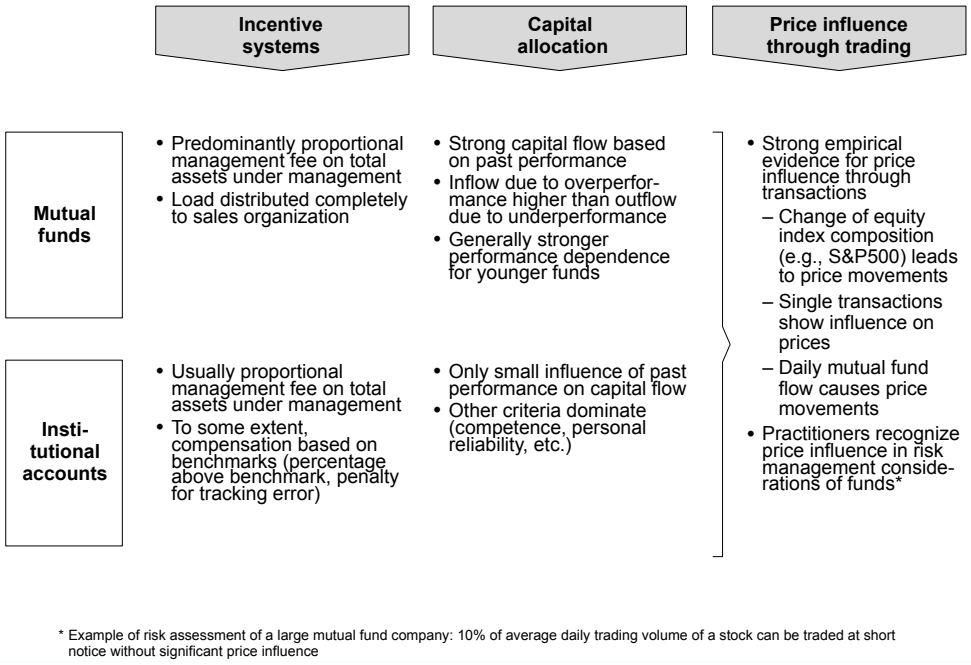



Figure 3.1.: Conditions of mutual fund and institutional account portfolio management. (Source: Own illustration)

The first two conditions are typical for delegated portfolio management. The third condition is not restricted to delegated portfolio management, and affects all investors who execute large transactions on the capital market. This chapter summarizes investment patterns of portfolio managers and presents empirical results from existing research about the three aspects above. Figure 3.1 summarizes the main findings regarding the conditions of delegated portfolio management.

## 3.1. Compensation as Explicit Incentive

An incentive system can consist of explicit and implicit incentives. Explicit incentives are those that are fixed in the contract between the portfolio manager and the investor. In contrast, implicit incentives are not made explicit in the contract, they emerge from the behavior of the contract partners.[75] For the portfolio manager, the explicit incentive is their compensation, usually a management fee.[76] Their implicit

[75] Fama (1980) first mentioned this type of incentive in the context of labor markets.

[76] For mutual funds, the sales commission that is paid by the investor upon purchase of shares of funds (usually as "front load") is not a reward for the portfolio manager. The sales commission is usually a remuneration for the sales force and varies depending on fund type and sales channel

incentive arises from the behavior of the primary investor. In the context of portfolio management, implicit incentives can be the contribution and withdrawal of assets under management ("capital flow")[77] or the dismissal of the portfolio manager ("career concerns").[78] This section will cover the explicit incentives, implicit incentives will be discussed in the next section. The following paragraphs provide the essential characteristics of the compensation for mutual fund and institutional account portfolio managers. It highlights the general form, its level, and the payment conditions. A brief summary picks up these characteristics that form the basis for all model designs in chapters 5 and 6.

### 3.1.1. General Compensation Form

Asset-based fees are the dominant compensation form. Only a minority of funds and accounts use performance-based fees. For mutual funds, this result is provided by Golec (1992) and Golec (2003).[79] In the latter study, out of a sample of overall 2351 mutual funds, a large majority of 2190 funds charged only a management fee as a fraction of assets under management. Only a minority of 146 funds charged a performance-based fee in addition to an asset-based fee. 39 funds charged a bonus or penalties if the fund's performance exceeds or falls short of an index's performance. 109 funds charged management fees as a fraction of the income of the fund. The results of Golec (1992) are along these lines.[80]

Regarding the fee structure of institutional accounts, only limited information is available.[81] Standard fee schedules are available but the actual fees differ significantly due to discounts offered by portfolio managers and other individual adjustments in contracts.[82] In a survey, Halpern/Fowler (1991) conclude that the majority of institutional equity accounts use asset-based fees, performance-based fees can only be found in a few equity accounts.[83] Among 271 surveyed accounts, 254 were

from 3 to 6%. Cf. Fortune (1997), p. 53.

[77] Cf. e.g., Stracca (2006), p. 836f. and Karceski (2002).

[78] Cf. e.g., Heinkel/Stoughton (1994) and Holmstrom (1999)

[79] Cf. also Lesseig/Long/Smythe (2002) and Deli (2002) for mutual fund fee structure in the US. Studies about the fee structure outside the US include Gil-Bazo/Sedano (2004) and Korkeamaki/Thomas I. Smythe (2004).

[80] Heinkel/Stoughton (1994) argue that the lack of performance-based fees could be caused by the higher relative risk aversion of the portfolio managers compared to primary, in particular private, investors.

[81] Cf. Halpern/Fowler (1991), p. 74, del Guercio/Tkac (2002), p. 524.

[82] Cf. Halpern/Fowler (1991), p. 74.

[83] Other accounts than traditional equity funds such as venture capital, real estate, private equity, or

actively managed.[84] For the majority of these 254 accounts, portfolio managers charged asset-based fees.[85] Only for 29 accounts, did portfolio managers charge performance-based fees.[86]

### 3.1.2. Level of Management Fees

Both mutual funds and institutional accounts charge around 50 basis points or more as a management fee. A slight majority of mutual funds have fees that decline with growing assets under management. Regarding the level of management fees, Golec (2003) reports that the majority of about 80% of mutual funds charge more than 50 basis points (0.5%). The remaining 20% charge 50 basis points or less.[87] Out of those funds that charge asset-based fees, about 60% scale their fees, meaning that the fund charges less if the fund grows. This results in a concave form of management fees with regard to size.[88]

Lakonishok/Shleifer/Vishny (1992) report management fees of around 50 basis points for institutional accounts, calculated on the assets under management in the portfolio. For accounts of USD 50 million and above, 50% of investors paid between 43 and 56 basis points.[89] Halpern/Fowler (1991) provide similar results. Asset-based fees range from 33 to 68 basis points for small US equity accounts with USD 10 million.[90] The gap narrows for larger US equity accounts of USD

hedge funds use different fee structures. They generally prefer performance-based fees. Halpern/Fowler (1991) report that the base fee of performance-based accounts are lower than for asset-based accounts. For US equities, it is 30 basis points. This base fee is also much less dependent on the size of the accounts. For call-option type performance compensation that is typical in the hedge-fund industry, cf. Arnott (2005), p. 10, and Goetzmann/Ingersoll/Ross (2003)

[84] Cf. Halpern/Fowler (1991), pp. 75-77. The remaining 17 accounts were passively managed accounts that do not require portfolio optimization by the portfolio manager.

[85] Cf. Halpern/Fowler (1991), pp. 75-77.

[86] Cf. Halpern/Fowler (1991), pp. 75-77.

[87] According to Golec (2003), there was a significant increase in fees by 1995. In earlier years, the distribution of fees was in favor of cheaper funds. Between 1984 and 1995, the share of funds charging more than 0.5% exploded.

[88] Heinkel/Stoughton (1994) suspect that declining fees reflect the existence of fixed costs on the side of the portfolio manager. According to Golec (2003), for the few funds that use performance-based fees, the majority (22 funds) charge 0.5% as asset-based fees. None of these funds used scaling of fees, i.e., the fees remain constant regardless of the size of the fund. It is noteworthy that the bonus and penalty fees are required to be equal for each fund resulting from new regulations starting in 1971. Cf. Section 205 of the Investment Advisers Act. Cf. Garcia (2001) who regards symmetric contracts as typical for the mutual fund industry.

[89] Cf. Lakonishok/Shleifer/Vishny (1992).

[90] Cf. Halpern/Fowler (1991), p. 76. Halpern/Fowler (1991) make their fee calculations consistent across different accounts through standardization on the three account sizes USD 10, 50, and 250 million. Cf. Halpern/Fowler (1991), p. 75.

250 million that charge 23 to 40 basis points.[91] Similar results are available for international and global equity accounts.[92] Along the lines of mutual funds fees, there appears to be a concave functional form with regard to the size of the fund. However, this concave form is only visible aggregating different accounts in the statistics.[93] Individual accounts can be assumed to have constant fees. The dependence of the fee on size might vary for individual accounts.

### 3.1.3. Payment Arrangement

For mutual funds and institutional accounts, the management fee is deducted directly from the fund's assets. It is calculated once a year based on the average of the fund's assets under management.[94] In addition to the management fee that compensates the portfolio manager for managing the portfolio, there are other costs. The so-called 12b-1 fee is charged for the distribution and marketing costs of the portfolio manager.[95] It can go up to 1%. Other expenses include "custodial, legal, transfer agent and subtransfer agent/recordkeeping payments, and various other expenses."[96] All costs are deducted from the fund's assets.

### 3.1.4. Results for Model Design

Given these empirical results, the theoretical models will incorporate an asset-based fee with a fixed fraction of the assets under management as the compensation for the portfolio manager. The scaling of fees will be neglected because the effect in total is small and it is only used by a small majority.[97] Additional fees will be disregarded because their purpose is to compensate the portfolio manager for their own costs and, therefore, do not represent an incentive.

[91] Cf. Halpern/Fowler (1991), p. 76.
[92] Cf. Halpern/Fowler (1991), p. 77.
[93] Cf. Halpern/Fowler (1991), p. 75f.
[94] Cf. e.g., the fund prospectus of American Funds "The Growth Fund of America," Funds (2006), p. 11. "The Growth Fund of America" is one of the largest mutual funds in the US with almost USD 150 billion assets under management as of November 2006.
[95] Cf. e.g., Funds (2006), p. 5.
[96] Cf. Funds (2006).
[97] As will turn out later in chapter 5, the level of management fees won't have an influence on the portfolio anyway.

## 3.2. Capital Flow as Implicit Incentive

Capital flows, i.e., the contributions and withdrawals of capital by primary investors are a very important incentive for the portfolio manager. Stracca (2006) cites: “The real business of money management is not managing money, it is getting money to manage."[98] These capital flows are considered as implicit incentives.[99] They are implicit because they function as an incentive only through an explicit incentive. Because portfolio managers are compensated with an asset-based fee, the volume of assets under management determine the level of their compensation. Hence, capital flows affect the compensation of portfolio managers. The understanding of primary investors' behavior, how they allocate their capital, how they shift their investments between different funds and accounts, is essential for portfolio managers in order to maximize their compensation.

For mutual funds, capital flow[100] appears as the purchase or sale of fund shares, i.e., primary investors purchase or sell shares of the portfolio manager's fund. According to del Guercio/Tkac (2002), in 1995 29% of mutual fund investors changed their funds and 14% closed an account.[101] Empirical research reveals that the flow of funds represents a significant incentive for portfolio managers, in particular for mutual fund portfolio managers.

For institutional accounts, capital flow appears primarily as gaining or losing single customers. In 1995, 22% of institutional account investors terminated a portfolio manager, 28% hired, and 15% terminated and hired a portfolio manager within a year.[102] Gaining or losing a single institutional customer is particularly important because the amount of capital managed within institutional accounts is much larger than the individual investments of primary investors in mutual funds.

The following paragraphs provides empirical evidence on the capital flow. It

[98] Stracca (2006), p. 836.
[99] Cf. Chevalier/Ellison (1997), p. 1168. Early applications of the concept of implicit incentives in the context of portfolio management have been contributed by Heinkel/Stoughton (1994), Chevalier/Ellison (1997), and Holmstrom (1999).
[100] The capital flow is sometimes termed “flow of funds”, cf. e.g., Berk/Green (2004).
[101] Cf. del Guercio/Tkac (2002), p. 526.
[102] Cf. del Guercio/Tkac (2002), p. 526.

draws on capital market[103] as well as consumer and marketing research.[104] After discussing factors that affect mutual fund capital flow, such as fund performance, general market condition, risk, and costs, the section presents details on the performance-flow relationship for mutual funds as the most important factor. The influence of performance and risk on institutional account flow is discussed in a separate section. Concluding remarks summarize the relevant characteristics for the theoretical models.

## 3.2.1. Factors Affecting Capital Flow

Potential factors for capital flows are past fund performance, general market condition, risk, management fee, and load. These factors are discussed in this section.

### Mutual Fund Performance

It had long been conjectured that the performance of mutual funds influences primary investors' decisions to contribute or withdraw capital. Though early studies of the 60s and 70s were not able to prove this relationship.[105] Spitz (1970) conjectures that "inflows are more a function of the effectiveness of the fund's sales operation than its performance."[106] Ippolito (1992) was among the first to show that primary investors base their allocation decision to a large extent on past performance. He studies the performance-flow relationship of 143 mutual funds over the period of 1965 to 1984. Ippolito (1992) uses a CAPM-based performance estimate but also a simple, non-risk adjusted measurement of performance.[107] For both

[103] In addition to those fundamental studies in this section, others focus on detailed questions. For example, Fant/O'Neal (2000) were able to show the persistence of a nonlinear performance-flow relationship over the period from 1978 to 1997. Bergstresser/Poterba (2002) confirm the performance-flow relationship for after-tax performance measures. Guercio/Tkac (2008) analyze private investors' reaction to Morningstar ratings. Other general studies on performance flow include Patel/Zeckhauser/Hendricks (Cambridge, 1992), Sapp/Tiwari (2004), Decker (2005), and Bu/Lacey (2008).

[104] Cf. Dechow (2003) for a recent and comprehensive overview of the literature of private investor's mutual fund selection criteria. I will only focus on descriptive research of investors' behavior. For a discussion about broader implications of mutual fund investors' behavior, cf. e.g., Bhattacharya/Pfleiderer (1985) and Greene/Hodges (2002). Cf. Kaplan/Schoar (2005) for private equity fund investor's behavior.

[105] of Finance/Commerce (1962) and Spitz (1970) find no or only a small link between performance and capital flow. Smith (1978) was able to provide slightly stronger relationships, but the relationship was still limited.

[106] Spitz (1970), p. 145.

[107] Cf. Ippolito (1992), pp. 49-51, 59f.

performance measurements he finds significant and strong performance-flow relationships for mutual funds. The positive relative return in excess of the market portfolio of 2% translates into a higher growth of 1.4 percent points.[108] Ippolito (1992) points out that investors react more strongly to recent fund performance than long-term performance. This relationship is unquestioned today and supported by several subsequent studies for different data sets, e.g., Sirri/Tufano (1998), Chevalier/ Ellison (1997), and Karceski (2002).

### General Market Condition

Beside the performance-flow relationship reported for mutual funds, there is a considerable flow due to general market performance. Edelen/Warner (2001) show with a data sample on a daily basis for the period from February 1998 to June 1999 that lagged market returns explain almost half of the variation in daily flow.[109] For example, two-thirds of one standard deviation of daily flow can be caused by a one standard deviation shock to one-day lagged market returns.[110] Karceski (2002) shows with a monthly sample of mutual funds over the period from 1984 to 1996 that there is an influence both from current and lagged performance of the mutual fund.[111] He can show that total asset value as well as net new capital flow is positively influenced by lagged returns.[112] Unexpected net new capital flow is only influenced by concurrent market return.[113] "The echo of these return-related shocks propagate through time via the AR(3) process," hence current flow depends on past return.[114]

### Risk

Regarding the influence of risk on capital flow, the research results are ambiguous. Sirri/Tufano (1998) report that "there is mixed evidence that consumers are sensitive

[108] Cf. Ippolito (1992), p. 59f.
[109] Cf. Edelen/Warner (2001), p. 201f.
[110] Cf. Edelen/Warner (2001), p. 201.
[111] Cf. Karceski (2002), pp. 569-574.
[112] Cf. Karceski (2002), p. 571.
[113] Karceski (2002), p. 571.
[114] The finding of a positive relationship between concurrent market return and unexpected net new capital flow is consistent with Warther (1995). However, Warther (1995) interprets this as a hint for price influence from mutual fund transactions which is confirmed by Edelen/Warner (2001) later on a daily basis. (Cf. section 3.3.2.) I will not use general market condition-related capital flow in chapter 6, where price effects will be modeled explicitly.

to the ex-post riskiness of fund investments."[115] Using several separate measures of performance and risk, regression coefficients on the risk measure are marginally significant only in one analysis. In all other cases, pure risk measures were not significant.[116] There is even a significant influence of simple performance measures if they are considered together with risk-adjusted measures.[117] Krahnen/Schmidt/Theissen (1997) report for the German mutual fund market that the influence of risk depends on the time window considered.[118] For short periods (below 18 months), the risk-adjusted measure provides a better explanation of flow-of-funds, whereas for longer periods (24 months) there is no preference for either measurement. del Guercio/Tkac (2002) suggest that the appearing relationship between risk-adjusted performance measures and flow of funds might be due to a high correlation between the risk-adjusted measures and publicly available performance assessments such as the Morningstar rating.[119] They find no explanatory power of Jensen's alpha if they include the Morningstar rating into their regression analysis.[120] In that sense, investors do not actively consider the risk of mutual funds, but indirectly by referring to publicly available performance measures when choosing their fund investments.[121]

### Management Fee

The level of management fees, in particular the change of management fees, has an influence on capital flow. [122] Sirri/Tufano (1998) report that overall costs are negatively related to capital flows as expected. In particular, he reports on the influence of management fees that an increase of fees has almost no effect, but a decrease of fees leads to an inflow of capital.[123] One potential reason for this asymmetry, he

[115] Sirri/Tufano (1998), p. 1590.
[116] Cf. Sirri/Tufano (1998), p. 1598, pp. 1600-1604.
[117] Cf. Sirri/Tufano (1998), p. 1601.
[118] Cf. Krahnen/Schmidt/Theissen (1997), p. 10.
[119] Cf. del Guercio/Tkac (2002), pp. 525, 544f.
[120] Cf. del Guercio/Tkac (2002), p. 544f.
[121] Cf. del Guercio/Tkac (2002), p. 528.
[122] Other factors that might influence the flow-of-fund relation but that can either not be influenced through portfolio management or that are more a matter of marketing strategy, and therefore, lie outside the effects of portfolio management will not be considered. For the influence of age refer to e.g., Chevalier/Ellison (1997), for the type of fund cf. Ippolito (1992), for the influence of media attention refer to e.g., Sirri/Tufano (1998), for the influence of name changes refer to Cooper/Gulen/Rau (2005), for the influence of other asset prices on mutual fund flow cf. Goetzmann/Massa/Rouwenhorst (2000), and for the role of performance relative to fund family cf. Kempf/Ruenzi (2008).
[123] Cf. Sirri/Tufano (1998), p. 1606, 1612.

argues, could be that mutual funds advertise a decrease in fees much more than they would advertise an increase. Being largely unaware of this increase, investors do not react.

### Load

The influence of fund load is mixed. Sirri/Tufano (1998) find that both an increase as well as a decrease in loads have a positive influence on flow-of-funds.[124] He conjectures that an increase in loads is compensated by the higher motivation of the sales representative, because the load is effectively a sales commission.[125] Ippolito (1992) identifies differences of performance-flow relationship for load vs. non-load funds.[126] The performance-flow relationship is stronger for non-load funds than for load funds. He conjectures that "the up-front charge at the time of purchase discourages the flow of monies in and out of these funds compared to no-load funds."[127] However, Sirri/Tufano (1998), in contrast, find that high-load funds show a stronger performance sensitivity than low-load funds. They attribute this to the higher marketing expenses of high-load funds that enable them to advertise high-performing funds.[128]

### Consumer and Market Research

Studies from market and consumer research confirm the general importance of performance for capital flows: mutual fund performance is the primary selection criteria of private investors. Next are criteria related to costs, such as management fees and loads. Risk plays a marginal role only.

In telephone surveys conducted by Capon/Fitzsimons/Prince (1996), a clear majority of participants reported that "Investment Performance Track Record" is the most important selection criterion out of nine different criteria.[129] "Fund manager reputation" is rated as the second most important criterion.[130] Because reputation

[124] Cf. Sirri/Tufano (1998), pp. 1607, 1612.
[125] Cf. Sirri/Tufano (1998), p. 1612.
[126] Cf. Ippolito (1992), p. 62f.
[127] Ippolito (1992), p. 63.
[128] Cf. Sirri/Tufano (1998), p. 1612f.
[129] These criteria were: investment performance track record, fund manager reputation, scope (number of funds in family), responsiveness to inquiries, management fees, investment management style, additional features, confidentiality, and community service/charity record. Cf. Capon/Fitzsimons/Prince (1996), p. 66.
[130] Cf. Capon/Fitzsimons/Prince (1996), pp. 65-70.

is very much influenced by past fund performance, these criteria are very closely related.[131] With conjoint experiments, Wilcox (2003) shows that customers rank 10-year average annual performance and previous year's performance as the most important decision criteria.[132]

Regarding the influence of risk, Capon/Fitzsimons/Prince (1996) report that only 14% of private investors used standard deviation to measure risk, 10% use beta and 4% use alpha or the Sharpe measure.[133] Fees and loads are relevant criteria but generally of less importance than performance. Capon/Fitzsimons/Prince (1996) revealed that criteria such as management fees, scope (number of funds in family), and responsiveness to inquiries are relevant. With only very limited influence or almost no influence were criteria such as investment management style, additional features (checking, brokerage), confidentiality, and community service/charity record. Capon/Fitzsimons/Prince (1996) identified advertising and commission-based financial advisers as important information sources.[134] The importance of management fees is confirmed by Wilcox (2003).

### 3.2.2. Nonlinear Performance-Flow Relationship for Mutual Funds

More detailed research is available about the relationship between fund performance and capital flow. Ippolito (1992) reports asymmetries regarding the reaction of private investors to positive and negative performance compared to a benchmark.[135] Mutual fund investors contribute more capital in the case of positive relative performance than they withdraw in the case of negative relative performance.[136] The positive effect is about three times higher than the negative effect. Among others,

[131] Nanda (2004) confirms the relevance of the reputation effect. Nanda (2004) shows that there is a positive spillover effect from high performing mutual funds to other funds of the same asset management company.

[132] Cf. Wilcox (2003), pp. 648-650. Other criteria have been: including load, management fee, company name, previous year's performance, 10-year average annual performance and fund's beta rating.

[133] del Guercio/Tkac (2002), p. 528.

[134] The importance of advertisements is underpinned by Barber/Odean/Zheng (2005). The relevance of commission-based financial advisers for mutual funds is similar to advice regarding the investment in individual stocks. Financial advisers add another layer of delegation, which, however, I will not consider here.

[135] Goetzmann/Peles (1997) aim to explain the nonlinearity of the performance-fund relationship with the phenomenon of "cognitive dissonance."

[136] Cf. Ippolito (1992), p. 61.

Sirri/Tufano (1998) and Chevalier/Ellison (1997) investigate more thoroughly the asymmetry of the performance-flow relationship.[137] Sirri/Tufano (1998) use a sample of 690 mutual equity funds over the period from 1971 to 1990. They measure performance and risk with simple measures such as historical returns, relative returns, and standard deviation of historical returns because these are the measures that are available to mutual fund investors through purveyors of mutual funds.[138] Sirri/Tufano (1998) are able to report a clear nonlinear performance-flow relationship resembling the return profile of an option. High relative performance (compared to other mutual funds) allow mutual funds to grow at high rates, whereas low relative performance is followed by only a modest outflow of capital.[139] These results partially hold with other performance measures,[140] including risk-adjusted measures and long-term performance measurements.[141]

Chevalier/Ellison (1997) were able to reproduce these results. Figure 3.2 shows their very clear result based on a sample of 449 mutual equity funds over the period of 1983 to 1993. Chevalier/Ellison (1997) measure excess performance as the difference between the performance of the fund and a value-weighted market index. The performance-flow relationship has the shape of a call option. The magnitude of the performance flow is rather high. For example, a fund with a performance of 10 percentage points above the market return grows by 55%.

### 3.2.3. Linear Relationship for Institutional Accounts

The literature about the behavior of institutional account investors is more limited. Lakonishok/Shleifer/Vishny (1992) find evidence that the number of clients of institutional portfolio managers is related to the performance of the portfolio manager. Using a data sample that covers about 50% of the equity pension funds under delegation in 1990,[142] they show that the number of accounts gained over the period

[137] Other research studying the nonlinear character include Carhart (1997), Goetzmann/Peles (1997), and Gruber (1996). Lynch/Musto (2003) explain the nonlinearity in a model where funds with bad performance change their management. Berk/Green (2004) explains the nonlinear relationship by the rational behavior of investors under the assumption that portfolio management skills are a scarce resource and dissipate with growing funds.

[138] Cf. Sirri/Tufano (1998), p. 1594.

[139] Cf. Sirri/Tufano (1998), pp. 1595f., 1598-1600.

[140] It is interesting to note that Sirri/Tufano (1998) report a significant influence of simple performance measures even together with risk-adjusted measures. Cf. Sirri/Tufano (1998), p. 1601.

[141] Cf. Sirri/Tufano (1998), p. 1601.

[142] Cf. Lakonishok/Shleifer/Vishny (1992), p. 344ff.

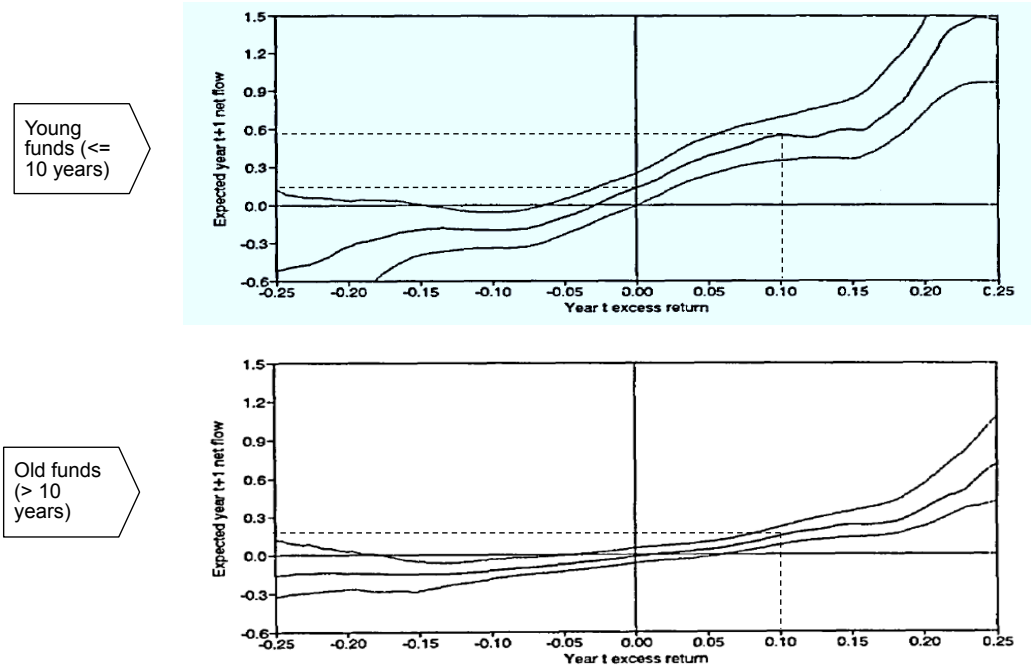



Figure 3.2.: Performance-flow relationship for young ($\leq$10 years) and old ($>$10 years) mutual funds. (Source: Chevalier/Ellison (1997), p. 1177)

1987 to 1990 is sensitive to past performance.[143] They estimate that for every 100 basis points the portfolio management company beats the average portfolio manager, it will gain an additional 2.6% accounts over the following three years. For a high performing portfolio management company in the 90th percentile achieving an annual over-performance of 450 basis points over the past three years, this would mean an increase in accounts of 11.7% in the following years. Lakonishok/Shleifer/Vishny (1992) estimate that the true figures on a single fund level are even higher, as their analysis is based on aggregated data on the portfolio management company level.

del Guercio/Tkac (2002) confirm these results. Using a sample of 562 portfolio managers from 388 management companies over the years 1987-1994,[144] they find that institutional accounts use quantitative performance measures to initially assess the quality of their investments in institutional accounts and reallocate funds accordingly.[145] del Guercio/Tkac (2002) report that it is more likely that institutional

[143]Cf. Lakonishok/Shleifer/Vishny (1992), p. 367f.
[144]Cf. del Guercio/Tkac (2002), p. 532.
[145]Cf. del Guercio/Tkac (2002), p. 524.

investors use risk-adjusted measures such as Jensen's alpha.[146] The relationship between pure performance and flow is much weaker. However, unlike for mutual funds, there is a relationship between Jensen's alpha and the flow of funds which is linear.[147] Institutional investors punish portfolio managers for poor performance by withdrawing capital.[148] Measurement against a benchmark and the related tracking errors also plays a prominent role. The fact that institutional investors unlike private investors consider risk-adjusted performance measurements appears reasonable. Institutional investors are usually better informed than mutual fund investors.[149] Additionally, they increasingly use support from consultants who monitor portfolio managers and recommend appropriate selections.[150] However, the evaluation process of institutional account portfolio managers goes beyond quantitative analysis. After such a quantitative analysis, institutional investors evaluate their portfolio managers "in face-to-face meetings on non-performance characteristics such as reputation and credibility."[151] As a result, the explanatory power of quantitative measures for the flow of funds is lower than for the mutual fund industry.[152] They include portfolio managers' characteristics such as personality, credibility, reputation, and attentiveness.[153]

### 3.2.4. Results for Model Design

From the empirical results regarding capital flow, I will focus on implementing the most important phenomena in the theoretical models in chapters 5 and 6. Given the vast evidence, the models will include a linear (for institutional accounts) and a nonlinear (for mutual funds) performance-flow relationship. In addition, I will implement the concurrent flow based on general market condition. Not implemented will be risk-adjusted measures, management fee, and load, due to their limited importance.

[146] Cf. del Guercio/Tkac (2002), p. 538f.
[147] Cf. del Guercio/Tkac (2002), pp. 541, 543f.
[148] Cf. del Guercio/Tkac (2002), p. 550ff.
[149] Cf. del Guercio/Tkac (2002), p. 527.
[150] Cf. del Guercio/Tkac (2002), p. 528.
[151] del Guercio/Tkac (2002), p. 524. In fact, institutional investors introduce special consultants to support their assessment of portfolio managers. Cf. del Guercio/Tkac (2002), p. 529.
[152] Cf. del Guercio/Tkac (2002), p. 525.
[153] Cf. del Guercio/Tkac (2002), p. 529.

## 3.3. Price Effects from Trades

Traditional portfolio theory assumes that investors consider prices on the capital market as given, i.e., their transactions have no influence on prices.[154] However, empirical research indicates that, in contrast, transactions have an influence on asset prices.[155] Therefore traders, in particular large traders, should incorporate their influence on prices into the decision-making process.

Mutual fund and institutional account portfolio managers can be considered as large traders, because they manage funds with a significant amount of capital.[156] They also face capital flows as documented in the previous section. This requires, in particular for mutual fund portfolio managers, trading regularly. However, because they do not receive information about recent capital flow in real time, mutual fund portfolio managers cannot respond to every single small transaction of fund shares.[157] Instead, they receive collected transactions in a single batch early in the afternoon of each trading day.[158] Only then are they able to execute accumulated trades to adjust the capital of the mutual fund for the new flow.

The following section provides results from empirical research about price effects. It focuses on price effects caused by single large trades and mutual fund flows.[159] A differentiated view on price effects for purchases and sales is provided as well as specifications of price impact functions. Concluding remarks highlight those specialties of price effects that will be picked up in the theoretical model in chapter 6.

[154] Cf. e.g., Merton (1990), chapter 2, p. 17f.

[155] I will focus on empirical results on price influence. For a discussion about three supporting hypotheses to explain such price effects, i.e. price pressure hypothesis, imperfect substitutes hypothesis, and information hypothesis, cf. Scholes (1972), pp. 180-184, Harris/Gurel (1986), p. 815f, Holthausen/Leftwich/Mayers (1987), p. 240f, Chan/Lakonishok (1993), p. 176, and Cochrane (2004), p. 6. Cf. also Kraus/Stoll (1972), Hess/Frost (1982), and Mikkelson/Partch (1985).

[156] In 1997, the average value of equity and mixed mutual funds in the US was about 650 million USD. Cf. Fortune (1997), p. 59.

[157] Cf. for this and the remaining paragraph Edelen/Warner (2001), p. 198.

[158] In some special cases, e.g., for large institutional investors, they receive information about flows informally in advance. Edelen/Warner (2001) report that portfolio managers claim that this flow data has some predictive power.

[159] I will not discuss research on the topic of price effect from index (e.g., S&P 500) adjustments. For price effects from index adjustments cf. e.g., Shleifer (1986), Harris/Gurel (1986), Goetzmann/Garry (1986), Dhillon/Johnson (1991), Collins/Wansley/Robinson (1995), Beneish/Whaley (1996), Lynch/Mendenhall (1997), Kaul/Mehrotra/Morck (2000), Liu (2000), Peterson (2004), Chen/Noronha/Singal (2004), Biktimirov/Cowan/Jordan (2004), Biktimirov (2004), Chakrabarti et al. (2005), and Greenwood (2005). Mitchell/Pulvion/Stafford (2004) analyze price reactions in the context of mergers.

It should be noted that market participants will most probably anticipate the effect of their transactions on prices and, therefore, take countermeasures. They might e.g., split up large block orders to avoid price effects. Hence, empirical research can only capture the remaining price effect after individuals have taken measures to preclude price effects.[160]

### 3.3.1. Evidence for Price Effects

Research that provides evidence of price effects from trades is vast. Therefore, I will focus on some important contributions in this section.[161] Kraus/Stoll (1972) are among the first to analyze the influence of block trades on asset prices. They analyze 7009 block trades with more than 10,000 shares on the New York Stock Exchange.[162] For seller-initiated trades, they find an average price decrease of 0.43% over the day.[163] For buyer-initiated trades, they find an average price increase of 0.66%. Kraus/Stoll (1972) can also show that the size of the price effect depends on the size of the order.[164] Holthausen/Leftwich/Mayers (1987) report very similar results. They find that a sale of 1% of equity or 1% of normal daily trading volume temporarily decreases the asset price by 0.371% or 0.357% respectively.[165] For buyer-initiated trades, a sale of 1% of equity is related to an increase of 0.287%.[166] Chan/Lakonishok (1993) find an increase of prices of 0.34% from opening to closing quote after a buyer-initiated trade and a modest decrease of -0.04% after a seller-

[160] Cf. Holthausen/Leftwich/Mayers (1987), p. 239.

[161] Beside the research quoted here, other authors have investigated price effects from trades: Scholes (1972), Loeb (1983), Glosten/Milgrom (1985), Mikkelson/Partch (1985), Burdette/O'Hara (1987), Easley/O'Hara (1987), Glosten (1989), Ball/Finn (1989), Barclay/Litzenberger/Warner (1990), Holthausen/Leftwich/Mayers (1990), Seppi (1990), Hasbrouck (1991a), Hasbrouck (1991b), Madhavan/Smidt (1991), Easley/O'Hara (1992), Easely/O'Hara (1992), Grossman (1992), Hausman/Lo/MacKinlay (1992), Barclay/Warner (1993), Madhavan/Smidt (1993), Huang/Stoll (1994), Keim/Madhavan (1996), Chakravarty (2001), Levin/Wright (2002), Chordia/Roll/Subrahmanyam (2005), and Dey/Kazemi (2008). O'Hara (1995) and Madhavan (2000) provide an extensive review of recent research of the literature of market microstructure, including empirical studies about price effects from trades. Campbell/Ramadorai/Vuolteenaho (2009) provide general recent research references. Some special issues are discussed e.g., by Sias (1997) (price effects with closed end funds) and Avramov/Chordia/Goyal (2006) (discuss the relation between illiquidity and short-run reversal).

[162] Kraus/Stoll (1972) identify a trade as buyer-initiated if the tick immediately before the block trade was lower, and as seller-initiated if the tick before the block trade was higher than the tick of the block trade, trades with a zero tick are excluded.

[163] Cf. Kraus/Stoll (1972), p. 575f.

[164] Cf. Kraus/Stoll (1972), p. 582.

[165] Cf. Holthausen/Leftwich/Mayers (1987), pp. 249, 264.

[166] Cf. Holthausen/Leftwich/Mayers (1987), pp. 249, 264.

initiated trade.[167] Overall, the reported price effects are of the same magnitude.

Chan/Lakonishok (1995) were able to consider the fact that many large orders are split up into smaller transactions that would otherwise be large block transactions.[168] They are able to "treat the entire sequence ('package') of trades as the basic unit of analysis [...]."[169] Chan/Lakonishok (1995) report that 53.2% of all trades (dollar-valued) span a period of four or more days.[170] For such packages, they report an increase of 0.98% for buyer-initiated trades and a decrease of 0.35% for seller-initiated trades.

Griffin/Harris/Topaloglu (2003), unlike most authors before, find no evidence for price influence. Instead, they interpret their results as evidence that much of the price increase associated with institutional trading is due to institutional investors following intraday trends.[171] Their data sample consists of all NASDAQ trades and quotes for NASDAQ 100 stocks within the period from May 1, 2000 till February 28, 2001.[172] On an intraday basis using 5-minute intervals and selecting data with the largest purchases and sales,[173] Griffin/Harris/Topaloglu (2003) find that prices during the 30 minutes before an institutional large block purchase show an abnormal return of around 0.70% and no further abnormal return after the institutional purchase.[174] Prices show negative abnormal returns of around -0.60% before a large institutional sale, followed by no more abnormal returns after the sale.[175] Figure 3.3 shows the price movement and order imbalance.

However, the interpretations of Griffin/Harris/Topaloglu (2003) are not plausible. First, it appears at least odd that institutional buyers realize a trade just in the

[167] Cf. Chan/Lakonishok (1993), pp. 179ff., 183f. They use over 1 million individual transactions from 37 large money managers from July 1986 to the end of 1988, covering 5% of the dollar value traded during that period. (p. 177) They are also able to correctly identify buyer-and seller-initiated trades without using the tick classification. (p. 175)

[168] Cf. Chan/Lakonishok (1995), p. 1147. For a more detailed view of the trading behavior of institutional investors cf. Conrad/Johnson/Wahal (2001), Conrad/Johnson/Wahal (2002), and Conrad/Johnson/Wahal (2003).

[169] Chan/Lakonishok (1995), p. 1148. Chan/Lakonishok (1995) combine orders of the same direction of one portfolio manager if they are not interrupted by more than 5 days. Cf. Chan/Lakonishok (1995), p. 1149.

[170] Cf. Chan/Lakonishok (1995), p. 1152. Their sample consists of 155789 packages. Cf. Chan/Lakonishok (1995), p. 1150.

[171] Cf. Griffin/Harris/Topaloglu (2003), p. 2285.

[172] Cf. Griffin/Harris/Topaloglu (2003), p. 2289ff.

[173] Griffin/Harris/Topaloglu (2003) use the buy-sell imbalance, the difference between buy and sell volume, to identify large purchases and sales. Cf. Griffin/Harris/Topaloglu (2003), p. 2292.

[174] Cf. Griffin/Harris/Topaloglu (2003), p. 2301ff.

[175] Cf. Griffin/Harris/Topaloglu (2003), p. 2301ff.

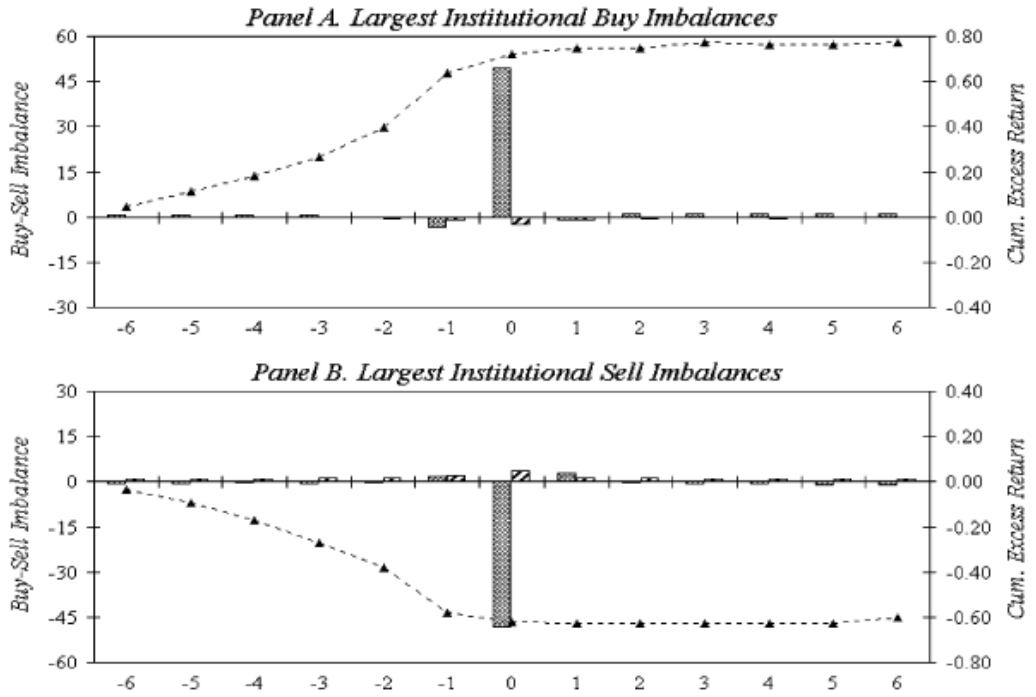



Figure 3.3.: Excess return in relation to order imbalance. The horizontal axis shows 5-minutes intervals before and after the transaction. The grey bar depicts the institutional imbalance, the diagonal shaded bar the individual imbalance. The dashed line represents the return relative to the NASDAQ 100 index. (Source: Griffin/Harris/Topaloglu (2003), p. 2302, figure 3, panels A and B)

moment when the cumulative abnormal return reaches its peak.[176] Second, block trading very often involves a different trading mechanism.[177] "Many block trades use the service of a block trader, or 'upstairs market maker', who forms a syndicate of buyers to take the other side of the trade."[178] These syndicates are set up in advance of the actual trade which could explain the abnormal returns prior to the actual trading of the block.[179] And third, selected trade data using the largest positive and negative excess return, instead of the largest purchase and sale, provides different results. Griffin/Harris/Topaloglu (2003) show that based on this selection, stock prices show an abnormal return of around 1.75% for the largest buys and around -2.00% for the largest sales.[180] But unlike in the previous selection of trades, the

[176]Cf. Griffin/Harris/Topaloglu (2003), p. 2301ff.
[177]Cf. O'Hara (1995), p. 233.
[178]O'Hara (1995), p. 233.
[179]Cf. , p. 32.
[180]Cf. Griffin/Harris/Topaloglu (2003), p. 2304.

prices here rise and fall just in the 5-minutes interval when the institutional trades buy and sell, i.e., their imbalance rises significantly.[181] After the initial rise, prices show a small reversal of abnormal returns of about 0.25% for purchases and about 0.5% for sales.[182] Overall, it is more plausible that the results of Griffin/Harris/ Topaloglu (2003) support price effects from trades than trend-following behavior of institutional investors.

### 3.3.2. Mutual Fund Flow and Price Effects

There is also extensive research supporting the hypothesis that mutual fund flow causes price effects. From the vast literature, I will report on two representative studies.[183] Warther (1995) finds that unexpected mutual fund flows are concurrent with a significant increase in stock prices on a monthly basis over the period from January 1984 till December 1992.[184] He separates the expected from the unexpected flow of funds using an AR(3) model on the lagged net sales.[185] It turns out that the unexpected net sales of mutual funds explain largely concurrent returns with an $R^2$ of 53%.[186] A 1% sudden net sale which is roughly two times the standard deviation of the unexpected net sales results in a stock return of 5.7%.

Edelen/Warner (2001) find evidence for the impact of mutual fund capital flow on asset prices on a daily basis. They find for the period from February 1998 till June 1999 that unexpected flows on a daily basis affect stock prices with the same magnitude but slightly lower than individual institutions' trades.[187] An unexpected flow of one standard deviation results in an abnormal return of 0.37%.[188] Using intraday return figures, Edelen/Warner (2001) find that returns in the late afternoon

[181] Cf. Griffin/Harris/Topaloglu (2003), p. 2304.
[182] Cf. Griffin/Harris/Topaloglu (2003), p. 2304.
[183] The literature on this topic is again vast, I will, therefore. focus on some important contributions. Beside the below mentioned research please cf. Ferson/Warther (1996), Sias (1996), Remolona/Kleiman/Gruenstein (1997), Adler/Yi (1998), Boyer/Zheng (2002), Fortune (1998), Nofsinger/Sias (1999), Edelen/Warner (1999), Edelen (1999), Wermers (1999), Zheng (1999), Engen/Lehnert (2000), Potter (2000), Chen/Jegadeesh/Wermers (2000), Cha/Lee (2001), Goetzmann/Massa (2003), Pastor/Stambaugh (2003), Chen et al. (2004), Kadiyala (2004), and Philippas (2005).
[184] Cf. Warther (1995), pp. 210, 213.
[185] Cf. Warther (1995), p. 217f.
[186] Warther (1995) reports that there is no significant difference between using the flow of funds vs. net purchases of funds as both are highly correlated with 89%. Cf. Warther (1995), p. 219f.
[187] Cf. Edelen/Warner (2001), p. 196f.
[188] Cf. Edelen/Warner (2001), p. 203. In particular, Edelen/Warner (2001), p. 203f., find no evidence for a relation between return and lagged flow.

are positively affected by the unexpected flow of the same day.[189] They explain this with preliminary daily flow calculations and executions of mutual funds.[190] They argue that if the calculation of the daily flow is costly, it is best for mutual funds to calculate a rough estimate if the majority of flows are available and they can still act on it. To execute flow adjustments before the closing quote is important for mutual funds because the mutual fund investor always executes trades in mutual funds at the closing quote.[191]

### 3.3.3. Permanent and Temporary Price Effects

Early research provides evidence for permanent price effects from purchases and largely temporary price effects from sales. Kraus/Stoll (1972) report for purchases an initial price increase of 0.75% followed by only a small decrease of 0.09% resulting in an overall increase of 0.66%.[192] In contrast sales cause an initial decrease of 1.14% followed by an increase of 0.71% resulting in a net decrease of 0.43%. Holthausen/Leftwich/Mayers (1987) find that their reported increase for buyer-initiated trades of 0.287% is permanent, whereas the decrease for seller-initiated trades of 0.371% is temporary.[193] Chan/Lakonishok (1993) report the overall price increase associated with buyer-initiated trades can be separated into a 0.22% increase from opening quote to block quote and a further increase of 0.12% from block to closing quote.[194] In contrast, price effects from seller-initiated trades show a reversal on the same day. After an initial decrease of -0.14% from opening quote to block quote, an increase of 0.10% follows from block quote to closing quote.[195]

Kraus/Stoll (1972) attribute the phenomenon of the different duration of price effects to the fact that large purchases are quite rare and liquidity is not provided by market makers or other investors, because market makers and investors seldom go short.[196] Chan/Lakonishok (1993) conclude that large purchases reveal private information because institutional investors usually hold a limited portfolio and the

[189] Cf. Edelen/Warner (2001), p. 211.
[190] Cf. Edelen/Warner (2001), p. 207f.
[191] Cf. Goetzmann/Massa (2003), p. 10.
[192] Cf. Kraus/Stoll (1972), p. 575f.
[193] Cf. Holthausen/Leftwich/Mayers (1987), pp. 249, 264.
[194] Cf. Chan/Lakonishok (1993), p. 179ff.
[195] Cf. Chan/Lakonishok (1993), p. 175.
[196] They cite market professionals with the sentence "blocks are sold, not bought." (Kraus/Stoll (1972), p. 573) They also remark that their sample includes stocks involved in mergers and takeovers which might be, in part, responsible for the discrepancy. Cf. also Cochrane (2004), p. 16, for short-selling constraints, in particular regarding small stocks.

selection of a particular stock out of the universe of potential investments might reflect some private information.[197] Large sales, on the other hand are more likely to be driven by the liquidity demand of the portfolio manager.[198] Another reason could be that brokers and market makers are more likely to take long positions as counterparties for sales for lower price concessions, compared to the more risky and, therefore, expensive short positions as counterparties to purchases.[199]

Chiyachantana et al. (2004) revisit the asymmetry of price effects for purchases and sales as reported by previous authors.[200] They suggest that the general market condition is responsible for the differences in price effects. In a positive market environment with rising stock prices, purchases cause a larger effect, in a negative market environment with falling stock prices sales cause a larger effect.[201] They explain this dependence by lower levels of liquidity for purchases in positive market environments and for sales in negative market environments: "Institutions pay for consuming liquidity (immediacy) when buying in rising markets and selling in falling markets. On the contrary, when trading against the market, institutions effectively provide liquidity and, therefore, face lower price impact."[202]

In contrast, research by Potters/Bouchaud (2003), whose results will be presented below, indicates that price effects are permanent. Overall, whether and under which conditions price effects are permanent and temporary appears to still be under debate.

### 3.3.4. Form of Price Impact Function

The proprietary nature of data on institutional trading has made empirical research on the detailed form of price impact functions difficult.[203] However, recent research

[197] Cf. Chan/Lakonishok (1993), p. 184f.
[198] Cf. Chan/Lakonishok (1993), p. 184f.
[199] Cf. Chan/Lakonishok (1993), p. 184f.
[200] Chiyachantana et al. (2004) use a sample of institutional trades in international stocks from 36 countries covering the years 1997, 1998 (upward trend), and 2001 (downward trend) with over one million transactions. They also investigate effects that are beyond the scope of this research, e.g., a positive relationship of price effect with volatility and order complexity, higher trading costs in "emerging markets, nonliberalized markets, and markets with poor shareholder rights." (Chiyachantana et al. (2004), p. 871.)
[201] Cf. also Saar (2001) for comprehensive literature on this issue.
[202] Chiyachantana et al. (2004), p. 885.
[203] Cf. Chiyachantana et al. (2004),p. 872. Early studies attempted to estimate the elasticity of the demand curve using IPO data or repurchases of stocks (cf. e.g., Scholes (1972), Loderer/Cooney/ van Drunen (1991), and Field/Hanka (2001)), but prices around IPOs are probably influenced by additional information. Cf. e.g., Levin/Wright (2002) to overcome this information effect.

has been able to report detailed characteristics of the price impact function. The authors consistently report monotone increasing and concave price impact function. According to Kritzman/Myrgren/Page (2006), a concave price impact function is also commonly used by practitioners.[204] This section summarizes the results from recent research.[205]

In order to reveal the functional relation between order size and price effect, Plerou et al. (2002) use the trades and quotes of the 116 most frequently traded stocks at the New York Stock Exchange over the period from 1994 to 1995.[206] To approximate the price effect they define $\Omega$ as the difference between the number of buyer-initiated and seller-initiated trades. They find a nonlinear, concave relationship between the trade volume $\Omega$ and the change in price, the conditional expectations.[207] More specific, Plerou et al. (2002) fit a hyperbolic tangens function $B_0 \tanh(B_1\Omega)$, with parameters $B_0$ and $B_1$ and the imbalance volume $\Omega$, to the observed price effects. This fit is suitable for trade effects measured after a longer time period as can be seen from figure 3.4b.[208] For price effects shortly after trades, a power law function $\Omega^{1/\delta}$ approximates the price effects better. Optimal parameters are $1/\delta = 0.66 \pm 0.02$ after 15 minutes and $1/\delta = 0.34 \pm 0.03$ after 5 minutes.[209] Figure 3.4b shows the short-term fit.

Lillo/Farmer/Mantegna (2002) and Lillo/Farmer/Mantegna (2003) confirm the findings of Plerou et al. (2002) and are able to reconstruct a single price impact curve for many stocks with different capitalization. They use a sample of the largest 1000 stocks at the New York Stock Exchange over the period of 1995 to 1998, covering about 113 million transactions and 173 million quotes.[210] As price effect, they

[204] Cf. Kritzman/Myrgren/Page (2006), p. 27.

[205] Cf. also Kempf/Korn (1999), Gabaix et al. (2003), Plerou et al. (2004), and Coggins/Lim (2005) for additional research. Iori et al. (2003) present a model that predicts the concave price impact function in an order-driven market. Perold/Salomon (1991) explain concave price impact functions with transaction costs and convex price impact functions with information-motivated trading. Dey/Kazemi (2008) derive price impact curves theoretically for a market with pure information, pure liquidity, and information-liquidity traders. Keim/Madhavan (1995) investigate the duration of trades. Keim/Madhavan (1996) find in an early study that the price effect has a concave form and attribute this to the fact that the "upstairs market" is involved for larger transactions. Cf. Keim/Madhavan (1996), pp. 26-31, as another source for a concave price impact function.

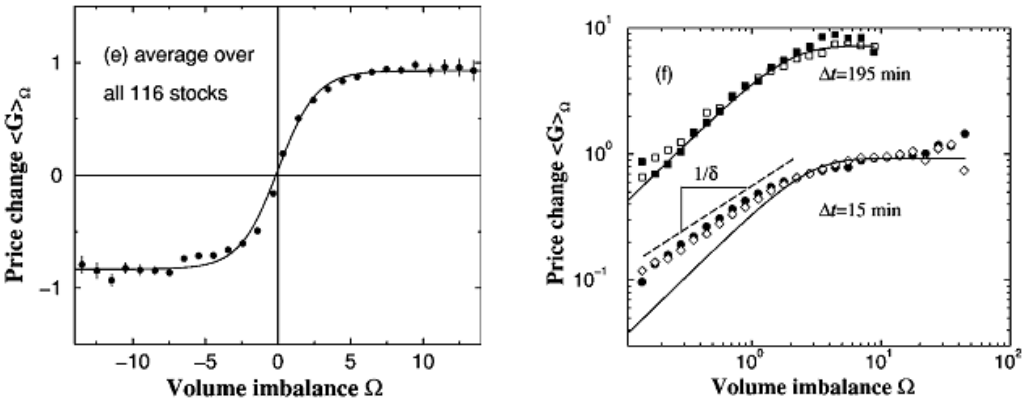
[206] Cf. Plerou et al. (2002), p. 027104-0.

[207] Cf. Plerou et al. (2002), p. 027104-2.

[208] Cf. Plerou et al. (2002), p. 027104-3.

[209] Cf. Plerou et al. (2002), p. 027104-3.

[210] Cf. Lillo/Farmer/Mantegna (2002), p. 1. Coggins/Lim (2005) apply the same methodology for a different sample with similar results for the Australian stock market.




(a) Hyperbolic price impact function for seperate positive and negative imbalances.

(b) Concave price impact function for merged positive and negative imbalances.

Figure 3.4.: Nonlinear price impact functions. $\Omega$ is defined as the difference between the number of buyer-initiated and seller-initiated trades. $\langle G \rangle_\Omega$ are conditional expectations of price changes. (Source: Plerou et al. (2002), p. 027104-2, figure 2.)

define the change in price from one transaction to the next quote revision.[211] They find that the price effect has a concave form and grows with transaction size.[212] The price impact function can be approximated by a power low function $\omega^\gamma$, where $\omega$ is the normalized transaction size[213] and $\gamma$ depends on the market capitalization, ranging from 0.5 for small transactions in larger stocks to 0.2 for larger transactions in smaller stocks.[214] These values are in line with those reported by Plerou et al. (2002) for short period price effects. Adjusting for market capitalization and liquidity, they are able to reconstruct a single price impact function.[215] Therefore, market capitalization appears to be the most important driver for the scaling of the price effect-trading size relationship.

Chan/Lakonishok (1993) already indicated the influence of firm and order size on the price effect.[216] They find that the price effect of purchases is bigger for small companies and small for bigger companies, ranging from 0.29% for the largest firm

[211] Cf. Lillo/Farmer/Mantegna (2002), p. 2.
[212] Cf. Lillo/Farmer/Mantegna (2002), p. 1.
[213] Lillo/Farmer/Mantegna (2002) normalize the transaction size by dividing through "the average value for each stock in each year." (Lillo/Farmer/Mantegna (2002), p. 2.)
[214] Generally a smaller exponent means a larger price effect relative to the size of the transaction. Cf. Lillo/Farmer/Mantegna (2002), p. 1.
[215] Cf. Lillo/Farmer/Mantegna (2002), p. 3.
[216] Cf. Chan/Lakonishok (1993), p. 186f, for this and the following paragraph. Chan/ Lakonishok (1995) largely confirm these results for packaged orders.

to 0.49% for the smallest firm. For sales, this relationship holds as well, however the return reversal is also stronger for smaller companies.

Potters/Bouchaud (2003) identify a concave price impact function of logarithmic form. They estimate the price impact function being of the form $r(\tau)\ln(V)$ with $V$ as the trading volume, $\tau$ as the time distance from the order.[217] They find that the time-dependence of $r(\tau)$ is rather limited. $r(\tau)$ increases during the first 100 seconds and then decreases to a finite value. The total variation of $r(\tau)$ does not exceed 50%. The influence of the trading volume appears to be logarithmic.[218] Potters/Bouchaud (2003) conclude that the price effect is quasi-permanent. They conjecture that market participants interpret each new trade as new information and that orders rearrange around the latest trading price.[219]

### 3.3.5. Results for Model Design

Given the extensive research on price effects from transactions and mutual fund flow alone, it appears reasonable to incorporate price effects into portfolio optimization models. Because there is strong evidence for general price reversal, this phenomenon will be implemented. Regarding a differentiation between purchases and sales, the literature provides an ambiguous picture. I will, therefore, not include this differentiation in the model. As price impact functions, I will consider a linear form to generally understand price effects and a more realistic concave function as suggested by Plerou et al. (2002), Lillo/Farmer/Mantegna (2002), Lillo/Farmer/Mantegna (2003), and Potters/Bouchaud (2003).

After having discussed the empirical characteristics of the situation of portfolio managers that will form the basis for the theoretical models in chapter 5 and 6, the following chapter presents an overview of the methodology that will be used to solve the theoretical models.

[217] Cf. Potters/Bouchaud (2003), p. 138f.
[218] Cf. Potters/Bouchaud (2003), p. 139.
[219] Cf. Potters/Bouchaud (2003), p. 139.

# 4. Methodology

This chapter outlines three methodologies to solve intertemporal portfolio problems: continuous time dynamic programming as an algebraic approach, Markov chain approximations as a numerical methodology for continuous time models, and purely discrete time approaches using the lattice multinomial approach. Continuous time dynamic programming will allow the derivation of algebraic closed-form solutions for multiperiod but rather "simple" models. Markov chain approximations will be used for more complicated models where closed-form solutions are not attainable. Purely discrete time models are applied to provide solutions for optimal portfolio policies with price effects. Since continuous time dynamic programming and the lattice multinomial approach are well known, this chapter will only briefly outline its application to standard portfolio optimization problems. For the Markov chain approximation method, this chapter will additionally outline the general approach, because this methodology is not as widely used in the finance literature. The results presented from standard portfolio optimization problems using all three methods will build a basis for further discussion in chapters 5 and 6.

## 4.1. Dynamic Programming and Portfolio Optimization

This section outlines the standard portfolio optimization problem in a multiperiod, continuous time context and its solution using dynamic programming.[220] It then discusses the standard's results.

[220] For a general introduction to continuous time dynamic programming, please refer to the standard literature, e.g., Merton (1969), Merton (1971), Fleming/Soner (1993), chapter 4, Korn/Korn (2001), chapter 5. Other standard references are Duffie (1992) and Björk (Oxford: 2004). For multiperiod discrete time models, cf. Duffie (1992), chapters 1-4, and Pliska (1997). The dynamic programming technique was initially developed by Bellman (1957). An alternative approach to solve multiperiod problems in continuous time is the martingale approach, pioneered by Cox/Huang (1989). However, even though the martingale approach is mathematically more elegant, the dynamic approach remains more intuitive.

### 4.1.1. Intertemporal Portfolio Optimization

Portfolio optimization addresses the problem of optimal investment-consumption decisions. Merton initiated the application of dynamic programming to solve the portfolio optimization problem in continuous time.[221] Let there be $i = 1 \dots n$ risky assets whose prices $P_i$ follow the stochastic differential equations

$$dP_i(t) = \alpha_i P_i(t)dt + \sigma_i P_i(t)dB_i(t) \qquad i = 1 \dots n \tag{4.1}$$

with constant market coefficients, the drift $\alpha_i$ and the standard deviation $\sigma_i$, and the Brownian motion $dB_i(t)$.

The investor consumes $c(t)$ at time $t$. Let $N_i(t)$ be the number of the i-th risky asset held by the investor in $t$. Aggregating all $n$ asset together, the total wealth $X(t)$ of the investor develops according to

$$dX(t) = \sum_{i=1}^{n} N_i(t)dP_i(t) - c(t)dt.$$

The portfolio policy $w_i(t) = \frac{N_i(t)P_i(t)}{X(t)}$ represents the share of wealth $X(t)$ invested in the $i$-th asset. Substituting $w_i(t)$ into the wealth equation above gives

$$dX(t) = \sum_{i=1}^{n} w_i(t)X(t)\frac{dP_i(t)}{P_i(t)} - c(t)dt. \tag{4.2}$$

Using the definition of the price process in equation 4.1 this is equivalent to

$$dX(t) = \left(\sum_{i=1}^{n} w_i(t)\alpha_i\right) X(t)dt - c(t)dt + \sum_{i=1}^{n} w_i(t)\sigma_i X(t)dB_i. \tag{4.3}$$

The constraint $\sum_{i=1}^{n} w_i(t) = 1$ ensures that the total wealth is invested. If there is a riskless asset governed by the differential equation

$$dP_0(t) = rP_0(t)dt,$$

[221] Cf. Merton (1969) and Merton (1971).The derivation of the fundamental wealth process in this section closely follows Merton (1990), chapter 5, pp. 124-127. The solution approach presented afterwards follows Korn/Korn (2001), chapter 5.4, p. 236ff.

the wealth equation becomes

$$dX(t) = \left( r + \sum_{i=1}^{n} w_i(t)(\alpha_i - r) \right) X(t)dt - c(t)dt + \sum_{i=1}^{n} w_i(t)\sigma_i X(t)dB_i \quad (4.4)$$

The investor seeks to maximize expected utility from consumption over a finite period and from terminal wealth by choosing their optimal rate of consumption $c(t)$ and the optimal portfolio policy $w_i(t)$, $i = 0, \ldots, n$. Technically, the investor maximizes

$$J(t, X; u) = E_{t,X}\left[ \int_t^T L(s, c(s))ds + \Psi(T, X(T)) \right] \quad (4.5)$$

with $L(\cdot)$ as the utility and $\Psi(\cdot)$ as the Bequest utility function of terminal wealth of the investor. $E_{t,X}$ is the expectation at time $t$ and given the value $X$ of the stochastic process at time $t$.

To provide an explicit solution to the maximization problem, a CRRA utility function is used. Let

$$L(t, c(t)) := \frac{1}{\gamma} e^{-\beta t} c(t)^{\gamma} \quad (4.6)$$

with $\beta > 0$, $\gamma \in (0, 1)$. $\gamma$ is the parameter of risk aversion. The Bequest utility function in analogy is

$$\Psi(T, X(T)) = \frac{1}{\gamma} X(T)^{\gamma}$$

The maximization problem can be solved using dynamic programming. The optimal portfolio policy with riskless asset results in

$$w^*(t) = \frac{1}{1-\gamma} \Sigma^{-1}(\alpha - \mathbf{1}r), \quad (4.7)$$

with $\mathbf{1}$ as $n$-dimensional vector of ones, $\Sigma^{-1}$ as the inverse of the covariance matrix of the $n$ Brownian motions. The optimal consumption is given by

$$c^*(t) = \left( e^{\beta t} \cdot f(t) \right)^{\frac{1}{\gamma - 1}} \cdot X(t) \quad (4.8)$$

with

$$f(t)^{\frac{1}{1-\gamma}} = e^{\frac{a}{1-\gamma}(T-t)} + \frac{1-\gamma}{\gamma(a-\beta)} \left( e^{\frac{a-\beta}{1-\gamma}T} - e^{\frac{a-\beta}{1-\gamma}t} \right) e^{\frac{a}{1-\gamma}(T-t)}$$

and

$$a := -\frac{1}{2}(\alpha - \mathbf{1}r)' \Sigma^{-1}(\alpha - \mathbf{1}r) \frac{1}{\gamma - 1} + r.$$

For the special case that there are only risky assets for investment, the optimal portfolio policy changes to

$$w^*(t) = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} + \frac{1}{1-\gamma}\left[\Sigma^{-1}\alpha - \frac{\mathbf{1}'\Sigma^{-1}\alpha\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right] \tag{4.9}$$

Two things are noteworthy in the standard setup. First, the model determines separately the optimal share $w_i(t)$ of wealth $W(t)$ invested in each particular asset and the optimal consumption $c(t)$. This is also reflected in the optimal portfolio policy 4.7 and 4.9 which is independent of optimal consumption 4.8. In fact, the setup is such that both are independent of each other at each point in time $t$. Looking at the wealth equation 4.3 and 4.4, consumption influences the change in wealth $dX(t)$. But the portfolio policy is unaffected by it because the wealth at $t$ is not affected by consumption, i.e., the whole wealth is invested which is assured by $\sum_{i=1}^{n} w_i(t) = 1$.

And second, the transactions necessary to stay at the optimal portfolio policy $w_i(t)$ are not made explicit in this approach. However, they do happen and are necessary: the investor must trade at every "infinitesimal small" time $t$ to keep the actual shares $w_i(t)$ at their optimal value. In a frictionless market as it is assumed here, in particular without feedback processes on prices induced by trades or without transaction costs, this approach appears appropriate. However, in order to introduce price influence or transaction costs, these transactions must be modeled explicitly. This constraint will be addressed in chapter 6.

The following section briefly discusses optimal portfolio structures 4.7 and 4.9 from an economic perspective.

### 4.1.2. Discussion of Standard Results

The classical solutions for the optimal portfolio in equations 4.7 and 4.9 have a well-known structure.[222] The optimal portfolio policy for the case without a riskless asset is given by[223]

$$w^*(t) = \underbrace{\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1}}_{(a)} + \underbrace{\frac{1}{1-\gamma}}_{(c)}\underbrace{\left[\Sigma^{-1}\alpha - \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1}\right]}_{(b)}.$$

[222] Cf. e.g., Merton (1990), p. 105f. and Ingersoll (1987), p. 287f.
[223] Cf. Ingersoll (1987), p. 287f.

The solution term consists of a minimum-variance sub-portfolio (a) and a return-oriented sub-portfolio (b). The weights of both parts are determined by the factor (c), the reciprocal of the constant relative risk aversion score $1-\gamma$ of the CRRA-utility function. In order to ease notation, I will note the minimum-variance term $\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1}$with the vector $MV$, resulting in

$$w^*(t) = MV + \frac{1}{1-\gamma}\left[\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right]. \tag{4.10}$$

The structure of the return-oriented sub-portfolio contains the return-oriented element $\Sigma^{-1}\alpha$ itself, as well as a volume adjustment of the minimum-variance sub-portfolio, $\mathbf{1}'\Sigma^{-1}\alpha \cdot MV$. This latter adjustment simply decreases the minimum-variance sub-portfolio for the same volume as it is increased by the return-oriented portfolio such that the constraint $\mathbf{1}'w = 1$ still holds. The optimal portfolio policy remains the same with different maximization objectives, e.g., maximization of expected utility from consumption till $T$.[224]

In the case with a riskless asset that provides a constant yield $r$, the solution simplifies to[225]

$$w^*(t) = \frac{1}{1-\gamma}\Sigma^{-1}(\alpha - r\mathbf{1}).$$

The minimum-variance and the return-oriented portfolio are combined together to the so-called tangency portfolio $\Sigma^{-1}(\alpha - r\mathbf{1})$. The demand for assets is structurally independent from the risk aversion of the investor, the so-called Tobin separation.[226] Only the amount of wealth invested is determined by the reciprocal of the measure of relative risk aversion. In addition to their investments in risky assets, the investor will invest the fraction $1 - \mathbf{1}'w^*$ of their wealth in the riskless asset.

The optimal consumption

$$c^*(t) = \left(e^{\beta t} \cdot f(t)\right)^{\frac{1}{\gamma-1}} \cdot X(t)$$

is proportional to the current wealth with the proportionality factor depending on time.

Figure 4.1 illustrates the optimal solution in the typical return-variance diagram.[227] The $x$-axis displays the standard deviation $\sigma$ of the depicted portfolios, the

[224]Cf. e.g., Merton (1969).
[225]Cf. e.g., Merton (1990), p. 105.
[226]Cf. Tobin (1957).
[227]Cf. e.g.,Merton (1990), p. 136.

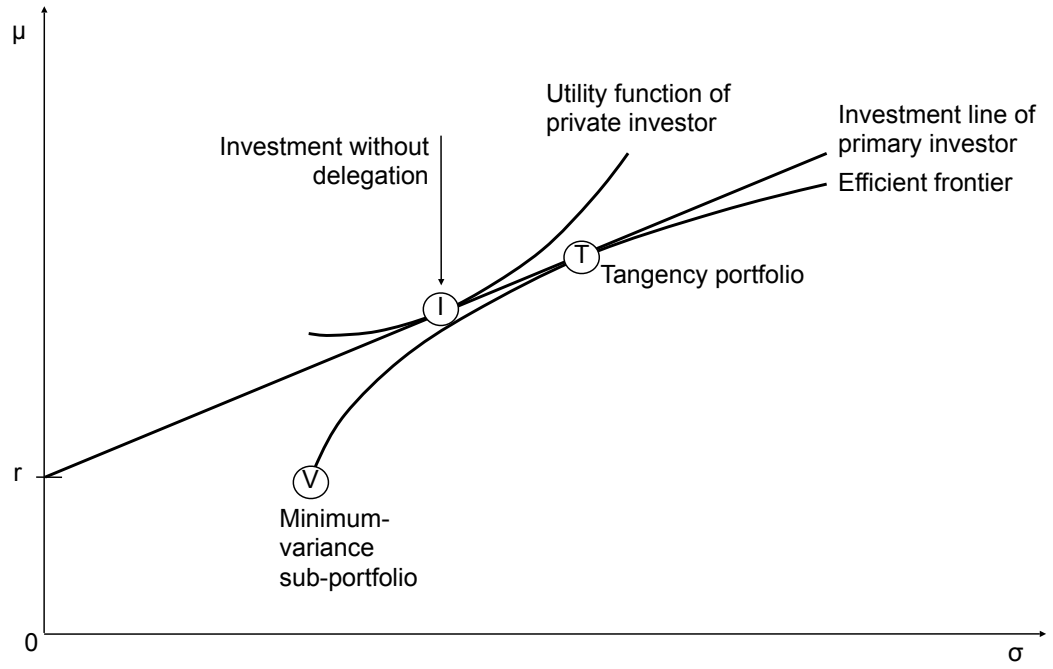



Figure 4.1.: Efficient investment of self-acting primary investor displayed in $\mu - \sigma-$diagram. The horizontal axis represents the standard deviation, the vertical axis represents the expected return of portfolios. $r$ is the risk-less rate. (Source: Own illustration)

$y$-axis the expected return $\mu$. The minimum-variance and the return-oriented sub-portfolios span the efficient frontier. The efficient frontier starts at the minimum-variance portfolio V. The tangency portfolio T is located where the investment line of the primary investor through the $\mu$-axis at point $r$ (risk-free rate) touches the efficient frontier. This line represents the risk-return profile of efficient portfolios for the investor. They realize a portfolio with the risk-return profile at $I$ consisting of the risk-free asset and the tangency portfolio. This portfolio is located where the utility function of the investor touches the investment line.

## 4.2. Numerical Approximations with Markov Chains

The dynamic programming approach outlined in the previous section provides closed-form solutions for stochastic control problems in continuous time. But de-

pending on the complexity of the dynamic problem, a closed-form solution may not be available. In fact, in many cases such a solution cannot be found.[228] For these cases, numerical solution methods are available. For stochastic control problems, two approaches can be distinguished, the finite difference/finite element method to solve the HJB-equation and the Markov chain approximation of the underlying stochastic system.[229]

The first approach, the finite difference/finite element method, requires the solution of the optimal control in terms of the value function. Substituting this solution into the HJB results - for controlled Markov diffusions - in a non-linear partial differential equation (PDE). This equation can then be solved for the value function using the finite difference/finite element method. However, it requires a symbolical solution of the optimal controls. In addition, in some cases the PDE will not have a solution in the classical sense.[230]

The second approach, the Markov chain approximation, was introduced by Kushner (1977) and is described in detail in Kushner/Dupuis (1992).[231] Its basic idea is to discretize the underlying state- and time-continuous stochastic system using a discrete state and discrete time Markov chain. The transition probabilities of the Markov chain are chosen so that the resulting discrete value function approximates the continuous value function well. From the comprehensive toolbox of Markov chain approximation, I will only focus on the relevant aspects for this work.

## 4.2.1. Markov Chain Approximation Method

The outline of the Markov chain approximation method in this and the following section 4.2.2 follows Munk (1997), Munk (2003), and Kushner/Dupuis (1992).[232]

### Discretization of State Space and Time

The objective is to approximate a continuous state and continuous time stochastic process, e.g. as in equation 4.3, with a controlled discrete Markov chain. Let $\xi^h =$

[228] Cf. Munk (2003), p. 48.

[229] Cf. Zakamouline (2005), p. 334, Munk (1997), chapter 5. Tapiero/Sulem (1994) provide a comprehensive overview of numerical solution methods for stochastic control problems. Cf. also Chow (1993) and Chow (1996) for a Lagrange-based method which is only applicable for infinite horizon problems and will, therefore, not be considered.

[230] Cf. Munk (1997), chapter 5. For an application of the finite element method in the context of portfolio optimization cf. Muthuraman/Kumar (2006) and Muthuraman (2007).

[231] For the application in portfolio/consumption problems cf. Munk (1997) and Munk (2003).

[232] Cf. Munk (1997), chapter 5, Munk (2003), pp. 51-57, and Kushner/Dupuis (1992), chapter 5.

$\left\{\xi_i^h | i \in \mathbb{N}_0\right\}$ be such a controlled discrete Markov chain on a discrete state space $\mathcal{R}_h^n$ with time steps $i \in \mathbb{N}_0$. Throughout this text, finite difference type approximation of the continuous stochastic process will be used.[233] For that type of approximation, the state space $\mathcal{R}_h^n$ is a uniform $h$-grid on $\mathbb{R}^n$, i.e.

$$\mathcal{R}_h^n := \left\{ x \in \mathbb{R}^n | x = \sum_{i=1}^{n} k_i e_i h, k_i \in \mathbb{Z} \right\},$$

where $e_i$ is the $i$-th unit coordinate vector and the parameter $h$ is the discretization parameter that determines the discrete steps of the state space.[234] To simplify notation, I will drop the super- and subscript $h$ where possible. The set of states $Z(y)$ that can evolve from a given state $y$ is defined as

$$Z(y) := \{z \in \mathcal{R}^n | \exists i : z = y \pm e_i h \text{ or } \exists i, j \neq i : z = y \pm e_i h \pm e_j h\}$$

The stochastic evolution of the Markov chain $\xi$ will be governed by transition probabilities $p(y, z|u)$ with $y, z \in \mathcal{R}^n$ and $u$ as control. $p(y, z|u)$ defines the probability of the Markov chain to evolve from the state $y$ to the state $z$ over the next time step, given the control $u$ is applied in state $y$.

Let $u_i = u(\xi_i)$ be the control at time step $i$. The sequence $u = (u_i)_{i \in \mathbb{N}_0}$ is called an admissible control for $\xi$ if $u \in U \subset \mathbb{R}^n$, if $\xi$ is a Markov chain and if $\xi$ is controlled by $u$. Denote the set of admissible controls for $\xi$ with the initial condition $\xi_0 = y$ as $\mathcal{A}(0, y)$.

For the discretization of time, I assume, for simplicity, fixed time steps $\tau$ with number of time steps $N_\tau = \frac{T}{\tau}$, where $T$ is the finite time horizon. $N_\tau$ is then the number of time steps. The point in time $t_i$ is then given by $t_i = i \cdot \tau$, $i \in \mathbf{N_0}$.

[233] Cf. Munk (1997), chapter 5.5, for details on the finite difference type approximation.

[234] The state space is uniform because the distance between two states is equal for all dimensions. I restrict the state space to the uniform case for the ease of computation. Cf. Munk (1997), chapter 5, for a non-uniform definition.

### Value Function and Convergence

In the finite horizon case with discrete space and discrete time approximation the criterion to minimize (the discrete analogy to equation 4.5) is

$$J(t_i, y; u) = E\left[\sum_{j=i}^{N_\tau - 1} \exp\left\{-\sum_{k=i}^{j-1} \beta(t_k, \xi_k, u_k)\tau\right\} L(\xi_j, u_j)\tau \right. \\ \left. + \exp\left\{-\sum_{k=i}^{N_\tau - 1} \beta(t_k, \xi_k, u_k)\tau\right\} \Psi(\xi_{N_\tau}) \middle| \xi_i = y\right]$$

The approximation to the continuous time value function is then given by

$$V(t_i, y) = \sup_{u \in \mathcal{A}(t_i, y)} J(t_i, y; u) \tag{4.11}$$

The objective is to choose the Markov chain approximation, i.e., the transition probabilities, in a way that the discrete value function 4.11 converges to the analogue continuous value function if $h$ approaches 0 for all states $y \in \mathcal{R}^n$. This is the case if the conditional mean and variances of the Markov chain approximation converge to the drift and the variance term of the continuous process.[235] A definition of a Markov chain that converges to the price process in equation 4.1 is given in the next section.

## 4.2.2. Explicit Approximation for Finite Horizon Problems

For a finite horizon stochastic control problem, this section provides the sufficient transition probabilities. The probabilities will be specified such that the time appears as an explicit variable.[236] In this case, the discrete dynamic programming equation for the value function in equation 4.11 becomes

$$V(t_i, y) = \sup_{u \in \mathcal{A}(t_i, y)} \left\{ e^{-\beta(t_i, y, u)\tau} \sum_{z \in Z} p(y, z|u) V(t_{i+1}, z) + \tau L(t_i, y, u) \right\} \tag{4.12}$$

with boundary condition $V(T, y) = \Psi(y)$. Equation 4.12 is quite intuitive. The value of the current state $y$ contains two parts: the first part consists of the dis-

[235] For more details on the convergence cf. Munk (1997), chapter 5.3, and Kushner/Dupuis (1992), chapters 9, 10 for finite horizon problems 12.

[236] Cf. Munk (1997), chapter 5.7. Munk (1997), chapter 5.7, also provides sufficient probabilities if time is considered as an implicit variable.

counted sum of all values $V(z)$ of all potential next states $z$, multiplied with the probabilities $p(y, z|u)$ conditional under the action $u$ with which these states will be attained. The second part represents the utility or cost $L(t_i, y, u)$ of the current state $y$ and action $u$ for the duration $\tau$. The optimal solution is the one that maximizes equation 4.12.

The problem represented in this value function can be solved with a backward iteration algorithm.[237] With that algorithm, the solution starts at the finite horizon $T$ to find the optimal control and value function of the finite horizon state. Using these solutions, one can then move backwards to $T - \tau$ to find the optimal control and value function again. This continues until $t_i = 0$.

Let $\mu$ and $\sigma$ be the drift and variance terms in the continuous time state process

$$dy = \mu(t, y(t), u(t))dt + \sigma(t, y(t), u(t))dB(t).$$

$\mu$ and $\sigma$ depend on time $t$, the current state $y(t)$, and the control $u(t)$. To simplify the notation, I will drop the explicit time dependence of the state $y$ and the control $u$. Let

$$A_i(t_k, y, u) := \mu_i(t_k, y, u)$$

$$B_{ij}(t_k, y, u) := \rho_{ij}\sigma_i(t_k, y, u)\sigma_j(t_k, y, u)$$

Then the transition probabilities $p(y, z|u)$ are the following:

$$p(y, y|u, t_k) = 1 - \tau \sum_{i=1}^{n} \frac{1}{h^2} B_{ii}(t_k, y, u) + \tau \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \frac{1}{2h^2} |B_{ij}(t_k, y, u)| - \tau \sum_{i=1}^{n} \frac{1}{h} |A_i(t_k, y, u)| \quad (4.13)$$

$$p(y, y \pm e_i h|u, t_k) = \tau \frac{1}{2h^2} B_{ii}(t_k, y, u) - \tau \sum_{j=1, j\neq i}^{n} \frac{1}{2h^2} |B_{ij}(t_k, y, u)| + \tau \frac{1}{h} A_i^{\pm}(t_k, y, u) \quad (4.14)$$

[237] Cf. Munk (1997), chapter 5.7.1.

$$p(y, y + e_i h + e_j h|u, t_k) = p(y, y - e_i h - e_j h|u, t_k) = \tau \frac{1}{2h^2} B_{ij}^+(t_k, y, u), \quad j \neq i \tag{4.15}$$

$$p(y, y - e_i h + e_j h|u, t_k) = p(y, y + e_i h - e_j h|u, t_k) = \tau \frac{1}{2h^2} B_{ij}^-(t_k, y, u), \quad j \neq i \tag{4.16}$$

$$p(y, z|u, t_k) = 0 \quad \text{for } z \notin Z(y) \cup \{y\} \tag{4.17}$$

Here $B^+ = \max(B, 0)$ and $B^- = \max(-B, 0)$. To ensure that equations 4.13 to 4.17 define a probability measure, the relation

$$\tau \left( \sum_{i=1}^{n} \frac{1}{h^2} B_{ii}(t_k, y, u) + \sum_{i=1}^{n} \frac{1}{h} |A_i(t_k, y, u)| - \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \frac{1}{2h^2} |B_{ij}(t_k, y, u)| \right) < 1 \tag{4.18}$$

has to hold for all $k$, $y$, and $u$. This ensures that the probability 4.13 remains positive. One way to achieve this is to select $\tau$ small enough so that condition 4.18 holds. However, this can be very disadvantageous from a numerical standpoint because it might imply a large number of time steps. It might also be necessary to restrict the control space $U$ so that for a given $\tau$ condition 4.18 holds.

## 4.2.3. Finite Horizon Portfolio Optimization

This section will describe the solution of the standard portfolio problem outlined in section 4.1.1 using the Markov chain approximation method outlined above.[238]

### Model Setup

I will consider the portfolio problem without consumption where utility is derived from terminal wealth only with a power utility function. Therefore $L(\xi_j, u_j) = 0$ and $\Psi(\xi_{N_\tau}) = \frac{1}{\gamma} (\xi_{N_\tau})^\gamma$. The time preference factor $\beta(t_k, \xi_k, u_k)$ is set to 0 to avoid time-dependence of the solution.

Let

$$dX(t) = \left( \sum_{i=1}^{n} H_i(t) \alpha_i \right) dt + \sum_{i=1}^{n} H_i(t) \sigma_i dB_i$$

be the continuous time wealth process, where $H_i(t)$ is the amount of money held in Asset $i$ at time $t$, with $\sum_{i=1}^{n} H_i(t) = X(t)$ and time-independent parameters $\alpha_i$ and

[238] Cf. Munk (1997), chapter 6, and Munk (2003) for another application of Markov chains for portfolio optimization.

$\sigma_i$.[239] As the model parameters are not time-dependent, I will restrict the portfolio optimization to the last period, i.e., from $N_\tau - 1$ to $N_\tau$.

### Parameter Values

For computational simplicity, the portfolio consists of two assets only. For the remainder of this section, the following numerical values will be used. Asset 1 has a yield of $\alpha_1 = 0.1$ and asset 2 has a yield of $\alpha_2 = 0.05$. The covariance matrix between both assets is given as

$$\begin{pmatrix} 2.25 & 0.825 \\ 0.825 & 1.21 \end{pmatrix}.$$

The risk aversion factor is $\gamma = 0.5$.

The state space consists of two variables $y_1$ and $y_2$ which represent the current holdings in each asset. It is by definition restricted to $[0, 1] \times [0, 1]$ with $h = 1/30$ for both assets. Hence, in total there are 900 states. The total assets are, therefore, $X = y_1 + y_2$. The finite horizon is set to $T = 1$, the number of time steps is set to $N_\tau = 10000$. The control space for the control $u = (H_1, H_2)$ is restricted to

$$U := \{u \in [\underline{H}, \overline{H}]^2 : \ H_1(t) + H_2(t) = X(t)\} \tag{4.19}$$

with $\underline{H} = -1.5$ and $\bar{H} = 1.5$.

In order to simplify numerical computations, the problem in two controls can be reduced to a problem in one control. The control space $U$ in equation 4.19 allows for the substitution of one control as

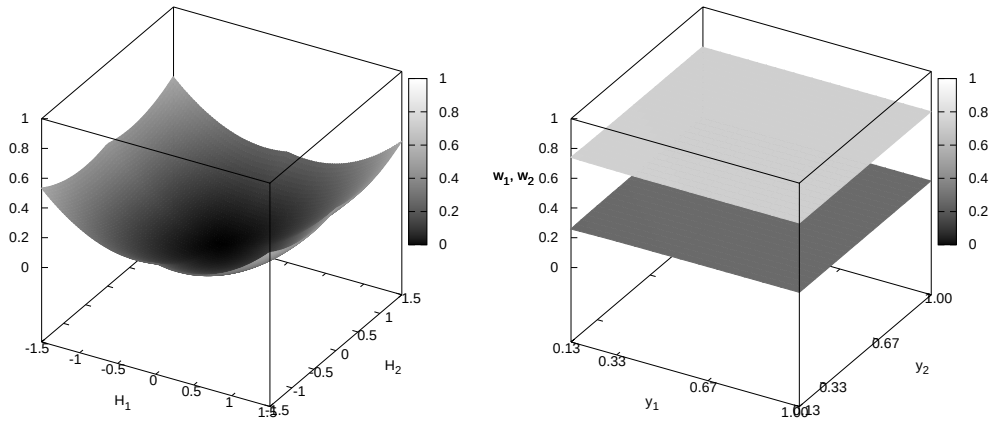
$$H_2 = X - H_1.$$

With this substitution, the problem must only be solved for $H_1$.

### Numerical Solution

Figure 4.2a depicts the visual inspection of the probability condition in equation 4.18. It displays the value of the left-hand side of equation 4.18 for all admissible values of the two controls. In this case, as outlined above, the controls are restricted

[239] $w_i(t) = \frac{H_i(t)}{X(t)}$ represents the share of wealth invested in the $i$-th asset at time $t$. With this, the numerical solutions can easily be compared to the closed-form solution in equation 4.10.




(a) Value of probability condition given in equation 4.18

(b) Optimal portfolio policy for two assets. Relative portfolio holdings $w_1$ and $w_2$ depending on state variables $y_1$ and $y_2$.

Figure 4.2.: Probability condition and optimal portfolio policy in Markov chain approximation. (Source: own calculation)

to the interval $[-1.5, 1.5]$. For the combination of $N_\tau$ and the control space the figure indicates that condition 4.18 holds. The value function in equation 4.12 has a convex shape for the set of admissible controls which allows for robust numerical solutions.

Figure 4.2b depicts the typical solution of the outlined standard portfolio problem. The graph shows that the share of total wealth $w_i(t)$ invested in each stock remains constant for all values of total assets.[240] In absolute terms, this would mean an increasing investment in both assets for increasing assets under management.

The numerical solution is sufficiently close to the analytical solution. For the $20$-th state in both variables, which is approximately at a state value of $0.63$, the analytical solution gives for $H_1$ and $H_2$ the values $0.2680$ and $0.7320$. The numerical solution at that state is very close with $0.2673$ and $0.7327$.

[240] The numerical solution entails a distortion for very small total assets close to 0. Therefore, the graphs shows the optimal shares only from the fifth state on which is roughly $0.13$. A similar distortion is visible in Munk (2003), e.g., p. 79, figure 1.

## 4.3. Discrete Time Models

For rather complex portfolio optimization problems, neither the continuous time framework nor Markov chain approximations might be feasible to find solutions. It then might be appropriate to revert to single or multiple period discrete modeling. Even though such models, in particular single-period models, are not very realistic, they are often suited to understanding basic interrelationships.[241] Moreover, they are usually mathematically simpler and multiperiod discrete models are widely used in the financial industry.[242] An approach that is intuitive and easy to implement[243] is the lattice binomial approach for a single risky asset developed by Cox/Ross/Rubinstein (1979) and later extended by Boyle (1988) and Boyle/Evine/Gibbs (1989) for multiple risky assets. This approach was initially developed in the field of option pricing.[244] He (1991) demonstrates the use of trinomial and general multinomial processes in the context of portfolio optimization.[245]

This section presents portfolio solutions using binomial and trinomial models. It omits a more general outline of the theoretical concepts.[246] In particular, the latter will be used in chapter 6 as a benchmark for comparison.

### 4.3.1. Multinomial Portfolio Models

Let there be $i = 1, \ldots, n$ risky assets whose multninomial discrete price process $P_i(t)$ is recursively defined as

$$P_i(t) = \begin{cases} P_{i,0} & \text{if } t = 1 \\ P_i(t-1) \cdot s_i & \text{if } t > 1. \end{cases} \tag{4.20}$$

where $P_{i,0}$ is the initially given price in $t = 0$ of the $i$-th asset. $s_i$ is a discrete random variable with $K$ different states $k = 1, \ldots, K$ representing the return of the $i$-th asset from $t-1$ to $t$. Every state will be reached with the probability $p(k), k = 1, \ldots, K$. For $K = 3$ this price process is said to be trinomial. The realization of a particular price processes is marked in each $t$ with the argument $k = 1, \ldots, K$

[241] Cf. Pliska (1997), p. 1.
[242] Cf. Pliska (1997), pp. 1, 72 and 100.
[243] Cf. Boyle (1988), p. 2f.
[244] Cf. e.g., Korn/Korn (2001), pp. 205-215.
[245] Cf. He (1991), pp. 345-355.
[246] For an outline of the general theory please refer to e.g., Pliska (1997), chapter 3. Other references are Cox/Ross/Rubinstein (1979), He (1991), and Korn/Korn (2001), pp. 205-215.

for the realization of the stochastic variable, $s_i(k)$, and $\kappa_t = 1, \ldots, K^{t-1}$ for the realization of the price, $P_i(t, \kappa_t)$, and dependent variables, e.g., asset holdings. A riskless asset integrates well into this framework. If the riskless asset $i$ provides a riskless rate of $r$, the return factors $s_i(k)$ change to

$$s_i(k) = 1 + r \qquad k = 1, \ldots, K.$$

This price process has the form of a tree.[247] At each point in time, the discrete randam variable $s_i$ determines which path of the tree to follow to the next period. Figure 4.3 depicts the functional values of a trinomial process over two periods, i.e., with $t \in \{1, 2, 3\}$ using the common tree picture. After three periods the price can have one out of 9 possible final states. At each point in time, one state of nature is realized in the price tree as depicted in figure 4.3 and some other states of nature become unreachable. In the trinomial model in figure 4.3, at time $t = 1$ all prices in $t = 2$ and $t = 3$ are possible. If in $t = 2$, for example, the second state realizes, i.e., $P_i(2, 2) = P_i(1, 1) \cdot s_i(2)$ is the current price, only the prices $P_i(3, 4)$, $P_i(3, 5)$, and $P_i(3, 6)$ become reachable at $t = 3$, until finally in $t = 3$ the state 4 is realized and the price reaches $P_i(3, 4)$.

The investor holds $N_i(t)$ number of shares of the $i$-th asset over the period $t$ to $t+1$ for all $t = 1, \ldots, T - 1$.[248] Negative values of $N_i(t)$ correspond to short positions in that particular asset. $N_i(t)$, respectively the vector $N(t) = (N_1(t), \ldots, N_n(t))$, is the control to the state process representing total wealth. The investor arrives in $t$ with a total wealth before transactions and consumption, indicated with the superscript $a$,[249] of

$$X^a(t) = \begin{cases} X_0 & \text{if } t = 1 \\ \sum_{i=1}^{n} N_i(t-1) \cdot P_i(t) & \text{if } t > 1. \end{cases} \tag{4.21}$$

with previous portfolio holdings $N_i(t - 1)$ and current prices $P_i(t)$. The initial portfolio value is given externally as $X_0$.[250] In $t$, the investor consumes the amount

[247] Cf. for a trinomial example He (1991), pp. 345-347. Cf. Shreve (2004), pp. 2 and 8 for binomial models.

[248] This notation and the following definitions of total wealth and consumption are consistent with e.g., Merton (1990), p. 125. Cf. Pliska (1997) for a different notation where the investor decides at $t - 1$ about the investment for the period $t - 1$ till $t$.

[249] The superscript $a$ stands for $ante$.

[250] For simplicity, it is assumed here that this value is given without specifying whether it contains assets or cash. This will be relevant in the model with price impact in chapter 6 and will,

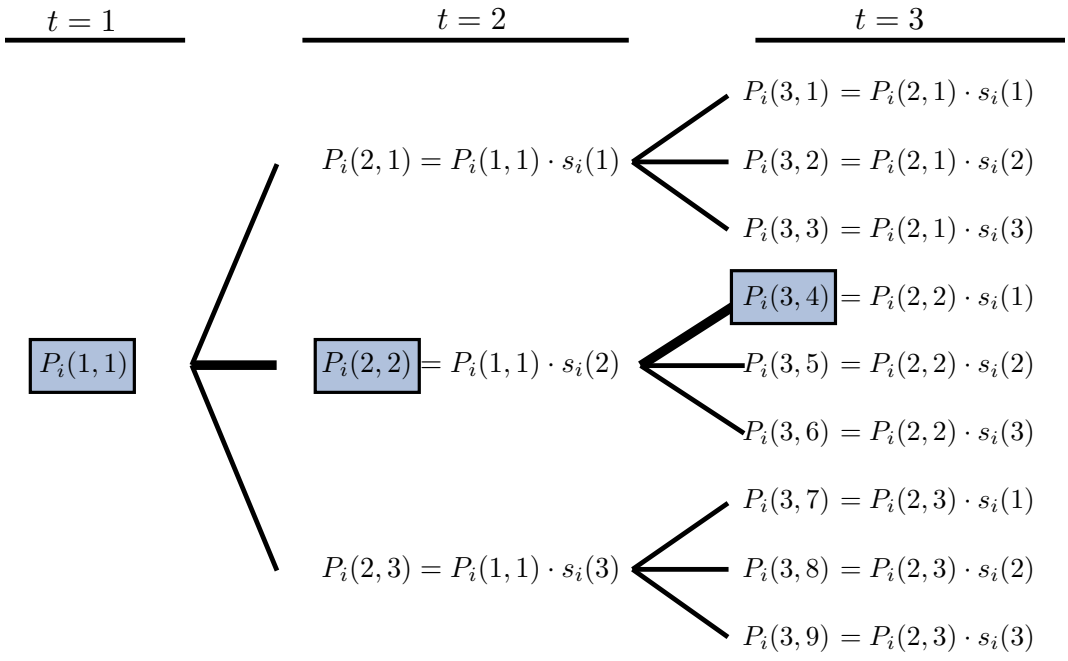



Figure 4.3.: Trinomial tree model with two periods. Bold lines and shaded boxes indicate the realization of an example price process. (Source: Own illustration. For a similar tree with two assets cf. He (1991), p. 347)

$c(t)$ financed by transactions which is expressed in the budget constraint

$$\sum_{i=1}^{n} N_i(t-1) \cdot P_i(t) - c(t) = \sum_{i=1}^{n} N_i(t) \cdot P_i(t).$$

Their wealth after consumption and transaction, indicated by the superscript $p$,[251] is given by

$$X^p(t) = \sum_{i=1}^{n} N_i(t) \cdot P_i(t).$$

Of particular interest in the following analysis will be the share of wealth invested in a certain asset. This share is defined as

$$w_i(t) = \frac{N_i(t) \cdot P_i(t)}{X^p(t)}.$$

therefore, be discussed there.

[251] The superscript $p$ stands for $post$.

In analogy to the continuous time case, the criterion to maximize is[252]

$$J(X_0) = E_{X_0}\left\{\sum_{t=1}^{T-1} L(t, c(t)) + \Psi(T, X^a(T))\right\} \tag{4.22}$$

subjected to the self-financing constraint[253]

$$X^a(t) - c(t) = X^p(t) \qquad t = 1, \ldots, T-1. \tag{4.23}$$

According to equation 4.22, the investor consumes until $T-1$ and derives utility in $T$ only from terminal wealth, which can then be measured as $X^a(T)$. The self-financing constraint 4.23 ensures that the same amount of wealth is invested with the transaction that is available before the transactions in $t$.[254]

Given the definition of the price process as a multinomial process, the criterion to maximize in equation 4.22 can be specified to

$$J(X_0) = \sum_{t=1}^{T}\sum_{\kappa_t=1}^{K^{t-1}} \tilde{p}(\kappa_t) L(t, c(t, \kappa_t)) + \sum_{\kappa_T=1}^{K^{T-1}} \tilde{p}(\kappa_T)\Psi(T, X^a(T, \kappa_T)). \tag{4.24}$$

s.t.

$$X^a(t, \kappa_t) - c(t, \kappa_t) = X^p(t, \kappa_t) \qquad t = 1, \ldots, T-1,\ \kappa_t = 1, \ldots, K^{t-1}. \tag{4.25}$$

with $\tilde{p}(\kappa_t)$ as probability of state $\kappa_t$ given by multiplying all probabilities along the path in the tree till $t$.

Several approaches are available to arrive at closed-form solutions of the model in equations 4.24 and 4.25.[255] First, to arrive at algebraic solutions dynamic programming in discrete time could be used.[256] In a multinomial model as outlined above, the value function can be calculated with a backward iteration algorithm. With multiple periods and complex model equations, this will become increasingly difficult.[257] Another option to solve the maximization problem is to use a Lagrange

[252] For simplicity, I omit the time preference factor, cf. e.g., Pliska (1997), p. 163, for a criterion to maximize with time preference factor.
[253] Unlike in Pliska (1997), p. 149, there is no need for an additional constraint $X^{pre}(1, \cdot) = X_0$ because that is already part of the definition of $X^{pre}$ in equation 4.21.
[254] Cf. Pliska (1997), p. 82.
[255] Cf. Pliska (1997), pp. 149-162.
[256] Cf. Pliska (1997), pp. 153-155.
[257] Cf. Pliska (1997), p. 155.

approach with equations 4.24 and 4.25.[258] However, depending on the structure of the problem, the number of periods and assets, this can even be more difficult.[259] The martingale approach is another solution approach.[260]

Because the model with price effects is rather complex, I will use numerical solution methods directly on equations 4.24 and 4.25.[261] The optimization problem in equations 4.24 will be solved for a given set of parameters using the constraints in 4.25. The disadvantage of such numerical optimizations is that it is often impossible to ensure global optimality of a specific solution unless the maximization criterion satisfies certain conditions, for example, convexity or concavity in all variables. However, to provide reliable solution values, I have probed the optimization problems with several parameter values not reported in this text, in particular artificial constraints to force the algorithm outside the initial solution space and small changes in parameter values to monitor solution values. Even though this cannot exclude local optima, it can at least ensure plausible values and reduce the danger of tapping into a local optimum.

### 4.3.2. Discussion of Numerical Results

After describing the general methodology of multinomial portfolio models in the previous section, this section will provide results of standard portfolio problems using trinomial specifications. I will focus on pure portfolio problems without consumption because these models will be the focus of analysis in chapter 6.

In analogy to section 4.2.2, I will consider a portfolio problem without consumption where utility is derived from terminal wealth only with a CRRA utility function.[262] Hence, let $L(t, c(t, \kappa_t)) = 0$ and

$$\Psi(T, X^a(T, \kappa_T)) = \frac{1}{\gamma} \left(X^a(T, \kappa_T)\right)^\gamma .$$

Let $K = 3$ and $T = 3$, i.e. a trinomial two-period portfolio optimization problem.

[258] Cf. Pliska (1997), pp. 150-151.
[259] Cf. Pliska (1997), p. 152.
[260] Cf. Pliska (1997), section 5.2.
[261] Cf. to Appendix B for the implementation of the numerical solution method.
[262] The CRRA utility function has the advantage of a constant relative risk aversion, i.e. the risk aversion does not depend on the size of the portfolio. This is especially helpful for the numerical analysis in chapter 6. If the portfolio structure with capital flows and price effects deviates from the standard deviation, this deviation is then due to these characteristics and not due to a different risk aversion triggered by altering portfolio sizes.

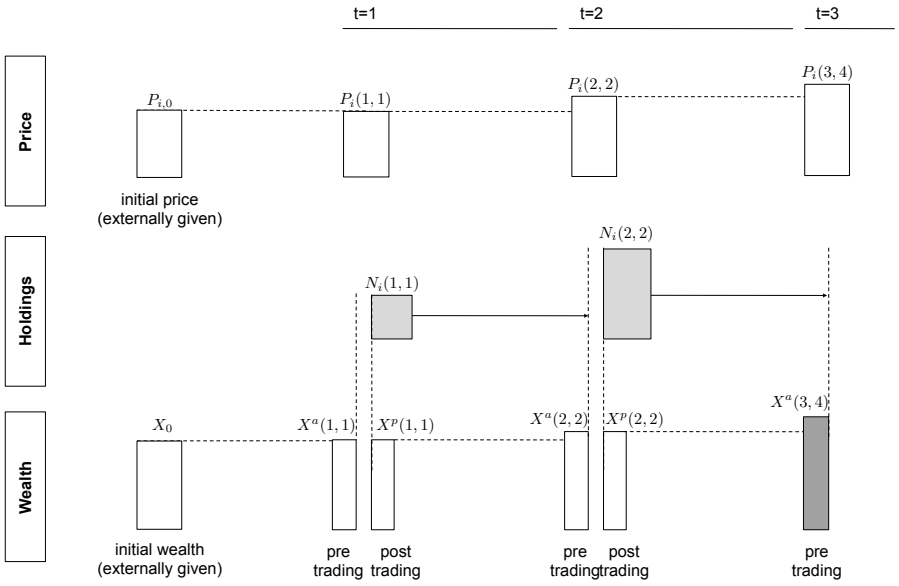



Figure 4.4.: Portfolio optimization problem in discrete time framework with two periods. The dark grey bar, the terminal wealth, represents the target of the maximization. The light grey bars represent the controls that achieve the maximum terminal wealth. (Source: Own illustration)

The maximization problem can now be specified to

$$\max_{N(1),N(2)} \sum_{\kappa_3=1}^{9} \tilde{p}(\kappa_3) \frac{1}{\gamma} \left(X^a(3,\kappa_3)\right)^\gamma$$

subject to

$$X^a(t,\kappa_t) = X^p(t,\kappa_t) \qquad t = 1,2,\ \kappa_t = 1,\ldots,9.$$

Figure 4.4 illustrates this optimization problem. It shows the three components price process, control, and the wealth process. For price process the example from figure 4.3 is used. The holdings represent the control. The last row shows the two definitions of portfolio value, $X^a$ and $X^p$. The horizontal alignment of prices and holdings indicate that the portfolio value $X^a(t,\kappa_t)$ is calculated based on holdings $N_i(t-1,\kappa_t)$ and prices $P_i(t,\kappa_t)$ whereas the portfolio value $X^p(t,\kappa_t)$ is calculated based on $N_i(t,\kappa_t)$ and the same prices $P_i(t,\kappa_t)$. Note that the initial wealth is given

| *Parameter* | *Description* | *Value* |
|---|---|---|
| $X_0$ | Initial assets under management | 1,000,000.00 |
| $P_{1,0}$ | First asset, initial price | 100.00 |
| $P_{2,0}$ | Second asset, initial price | 100.00 |
| $s_1(k)$ | First asset, multiplying factor | 1.07, 1.01, 0.96 |
| $s_2(k)$ | Second asset, multiplying factor | 0.98, 1.01, 1.05 |
| $p(k)$ | Probability for first state | $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$ |
| $\gamma$ | Risk aversion factor for utility function | 0.5 |

Table 4.1.: Parameter values of the standard trinomial model. (Source: Own illustration)

as $X_0$ and no initial number of shares is specified because by definition $X^a(1,1) = X_0$.

### Results for the Trinomial Model Without Riskless Asset

Numerical solutions will be provided for a single- and two-period model. Models with more periods do not add additional insight as the solutions are identical to those of the single- and two-period models. A trinomial model will be used in order to investigate the investment strategy with two risky assets. The parameter values are summarized in table 4.1. The standard deviation of the first risky asset is higher than that of the second risky asset. The assets are almost perfectly negatively correlated with a correlation coefficient of $\approx 0.99$. Such an extreme parameter was selected because it allowed rather stable numerical optimizations, in particular in conjunction with price effects in chapter 6.[263] The first asset performs well in states in which the second asset performs poorly and vice versa. All states are reached with the same probability. The risk aversion parameter is arbitrarily set to $0.5$. With these parameter settings, the model is arbitrage-free.[264]

Figure 4.5 reports the results for the single-period as well as the two-period model. Given the parameter settings above, the investor invests less in the asset with the higher standard deviation. They also invest the same share of wealth in all the periods of the two-period models. In the two-period model, the number of

[263] For a derivation of return factors in a trinomial case from explicit values for return, standard deviation and correlation, cf. He (1991), pp. 345-347.

[264] Regarding arbitrage-free multiperiod models cf. Pliska (1997), pp. 92-96.

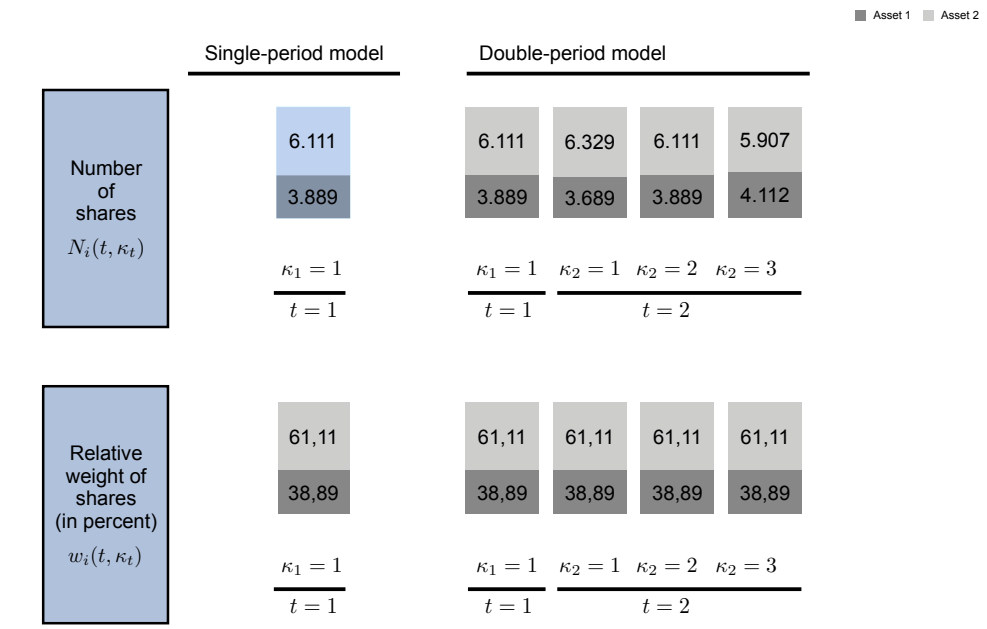



Figure 4.5.: Results for the standard portfolio optimization problem in discrete time framework with one and two periods. (Source: Own calculations)

shares invested in the second period changes, due to the changed wealth at the end of the first period depending on the realized state $k$.

Discrete time models will be considered in chapter 6 to analyze the influence of price effects on the optimal portfolio policies. Prior to this, the following chapter 5 will investigate the effect of compensation and capital flows in continuous time models.

# 5. Portfolio Policies under Compensation and Capital Flow

After an outline of the methodological basis to solve portfolio problems in the previous chapter, this chapter will address the research questions in a continuous time model using dynamic programming and Markov chain approximations. After a discussion of related literature and a presentation of the general setup of the portfolio optimization models used, section 5.3 addresses the problem of optimal portfolio policies under management fee and capital flow. It contains a linear model and closed-form solutions are compared with results from standard portfolio problems presented in the previous chapter. Section 5.4 provides solutions for optimal portfolio policies in more complex, nonlinear setups. Results are also discussed from the primary investor perspective and are probed whether they can explain observed investment strategies of institutional investors.

## 5.1. Related Research

### 5.1.1. Compensation and Portfolio Optimization

Starks (1987) investigates the effect of symmetric and asymmetric performance fees on investment decisions of portfolio managers. A symmetric performance fee rewards the portfolio manager if the performance of their portfolio exceeds that of a benchmark portfolio. The symmetric performance fee penalizes the portfolio manager if their portfolio falls behind the benchmark. An asymmetric performance fee rewards the good performance of the portfolio manager, however does not penalize them for bad performance. In his single-period setup with general utility functions, the portfolio manager's fund is perfectly correlated with the market portfolio, which

also functions as the benchmark portfolio. He finds that in such a setup a symmetric performance fee does not induce the portfolio manager to increase the level of risk (the $\beta$ of the fund). They will invest so that their utility curve is tangential to the security market line. With an asymmetric performance fee however, the portfolio manager will always invest more at higher risk (choose a higher $\beta$) than the primary investor.[265]

Stoughton (1993) analyzes the effect of linear and nonlinear compensation schemes as an incentive for the portfolio manager to extend their efforts. Compensation is based on current wealth, analogous to the linear management fee used in this section. Stoughton (1993) uses a single-period agency-theoretic model in which the agent can influence the quality of their expectations about future returns by expanding or restricting their efforts. He finds that with a linear contract the agent does not expend their full efforts. Even worse, the efforts of the portfolio manager cannot be extended with a higher linear share of benefits for the portfolio manager (unlike in typical agency frameworks). A quadratic contract, on the other hand, can reduce the agency conflict. The portfolio manager is compensated based on the difference between the actual outcome of the state of nature and a signal that the agent should disclose to the primary investor. These quadratic contracts are even asymptotically optimal for very risk-tolerant primary investors.

Roll (1992) analyzes portfolio decisions of a portfolio manager who is compensated based on their portfolio's tracking error with regard to a benchmark portfolio. Roll (1992) argues that a portfolio with minimal tracking error indicates persistent outperformance of the portfolio manager over the benchmark, given that the tracking target is set above the benchmark. With such compensation, the portfolio manager will try to minimize the variance of the tracking error. Figure 5.1 illustrates the main results of this single-period model in the common $\mu - \sigma$-diagram. Unless the benchmark portfolio is a globally efficient portfolio, i.e., it lies on the efficient frontier, the managed portfolio will not be efficient. The benchmark portfolio defines a new efficient frontier to the right of the globally efficient frontier. Naturally, if all portfolios on this efficient frontier are dominant, the investment of the portfolio manager is inefficient. The same applies if additional restrictions are imposed on the $\beta$-factor of the managed portfolio. His results are very similar to those presented in this work, however the underlying models are different.

[265] Starks (1987) extends the model such that the actions of the portfolio manager are accompanied by costs. In such a situation, their effort level is suboptimal from the perspective of the primary investor. This situation is aggravated in the case of asymmetric performance fees.

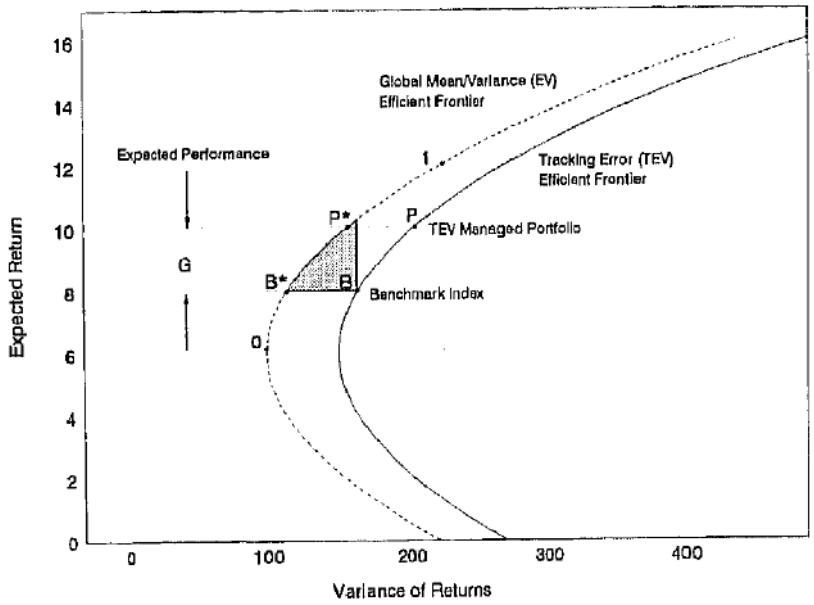



Figure 5.1.: Efficient frontier and optimal portfolio for a tracking error minimizing portfolio manager. (Source: Roll (1992), p. 16.)

Admati/Pfleiderer (1997) extend the model of Roll (1992) and include private information on the side of the professional portfolio manager. Assuming, similar to Roll (1992), a benchmark-oriented compensation, they show that this will lead to an inefficient portfolio in most circumstances. This is due to two effects: first, the level of risk aversion can be different for the portfolio manager and for the primary investor.[266] Second, due to the benchmark-adjusted compensation element the portfolio manager partly realizes the benchmark portfolio. In addition to the analysis of the portfolio structure, Admati/Pfleiderer (1997) also study whether benchmark-adjusted compensation can reduce moral hazard and adverse selection arising from the private information of the portfolio manager. Their results are overall negative. Benchmark-adjusted compensation even reduces the level of effort of the portfolio manager and is not helpful to discriminate between the informed portfolio manager and the uninformed.

Nietert (1996) uses a continuous time setup similar to the setup in this work. In his models, a portfolio manager maximizes expected utility from compensation.[267] This compensation is defined as a fixed fraction of terminal wealth and alternatively

[266] Admati/Pfleiderer (1997) model both as risk-averse with an exponential utility function but different risk aversion factors. Cf. Admati/Pfleiderer (1997), p. 328 f.

[267] Cf. Nietert (1996), p. 31.

as a fixed fraction of continuous wealth.[268] The compensation is distributed to the portfolio manager but not deduced from the portfolio value. Nietert (1996) provides closed-form solutions for the finite horizon as well as the infinite horizon case. In all cases, the optimal portfolio policy is identical to the solution in Merton (1971) presented in section 4.1.2.[269] Unlike in Nietert (1996), the models in this work will implement compensation that is deducted from the portfolio value.

A continuous time analysis of delegated portfolio management in the context of symmetric information is presented by Ou-Yang (2003).[270] He finds that an optimal contract is symmetric, i.e., it penalizes and rewards the portfolio manager in a symmetric way. The contract should depend on the current wealth as well as on the portfolio's excess return compared to an active, i.e., time-varying, benchmark index and reward the portfolio manager at the end of the period. Ou-Yang (2003) develops this solution in a framework in which the portfolio manager incurs quadratic costs based on the portfolio structure and linear costs based on the wealth level using an exponential utility function. This cost structure implies that the portfolio manager faces larger costs the more they invest in certain stocks. Even though this cost structure is usual in principal-agent frameworks, its viability in this particular context is not empirically grounded.

Kraft/Korn (2008) provide a similar analysis to Ou-Yang (2003), but they use a contract including a quadratic deviation from a benchmark to solve an agency conflict for a general class of utility functions. Kraft/Korn (2008) show that a quadratic or option-like contract that compensates the portfolio manager at the end of the contract can solve the agency conflict and induce the portfolio manager to seek the first best solution, i.e., as if the primary investor could control their actions. Such a quadratic contract includes a linear component, the typical risk-sharing compensation of principal agent models. The quadratic term, that penalizes deviations from a benchmark portfolio, forces the portfolio manager not to deviate from the linear compensation part. Generally, the growth optimal portfolio will serve as a sufficient benchmark portfolio.

[268] Cf. Nietert (1996), p. 31f.

[269] Cf. Nietert (1996), pp. 37, 45, and 47.

[270] Other continuous time models of delegated portfolio management are Cvitanic/Wan/Zhang (2006) and Cadenillas/Cvitanic/Zapatero (2007). However, in their models, the portfolio manager can influence the drift of the underlying stochastic process with their level of effort. This is not a situation, I will consider here.

### 5.1.2. Capital Flows and Portfolio Optimization

Karceski (2002) provides a model that incorporates capital flows into the investment decision of the portfolio manager. In his single-period model with actively and passively managed funds, the portfolio manager faces linear flow depending on the general market performance. Additionally, primary investors shift capital from a passive fund towards an active fund based on its outperformance, while penalizing a high variance of the tracking error. To simplify the model, the portfolio manager can only invest in two types of stocks with low- and high-beta characteristics. The active portfolio manager maximizes the expected assets under management, hence they are risk-neutral.[271] As a result, the portfolio manager tilts their investments in high-beta, i.e., riskier stocks. Karceski (2002) explains this behavior by the fact that the portfolio investor has the chance to outperform the market in the case of a positive general market development. In such a situation, they would benefit from higher aggregate equity inflows.

In a working paper Hugonnier/Kaniel (2010) develop a continuous time multi-period model of a single agent and a single principal without private information.[272] They incorporate capital flow generated from the assumption that the primary investor is unable to invest directly in risky assets but only through the professional portfolio manager. They choose a setup in which the amount invested at higher risk is equal to the Sharpe ratio.[273] Hugonnier/Kaniel (2010) distinguish two cases with the parameters driving the stochastic price process constant or time-dependent. In the case of constant parameters, the amount invested in the risky asset is proportional and the flow is reciprocal to the fee level. In the case of varying parameters, the portfolio manager hedges against unfavorable shifts in the flow. They invest in a way that the wealth of the investor is correlated with the Sharpe ratio: because the portfolio manager will receive more capital if the Sharpe ratio is high, they will get even more if at the same time the overall wealth of the primary investor is high. Hugonnier/Kaniel (2010) do not investigate the efficiency of the portfolio.

[271] Karceski (2002) does not use a typical utility function for the portfolio manager. Cf. Karceski (2002), p. 564.

[272] Hugonnier/Kaniel (2010) use backward stochastic differential equations to solve their model.

[273] Hugonnier/Kaniel (2010) model a myopic investor with log- utility. Cf. Hugonnier/ Kaniel (2010), p. 188.

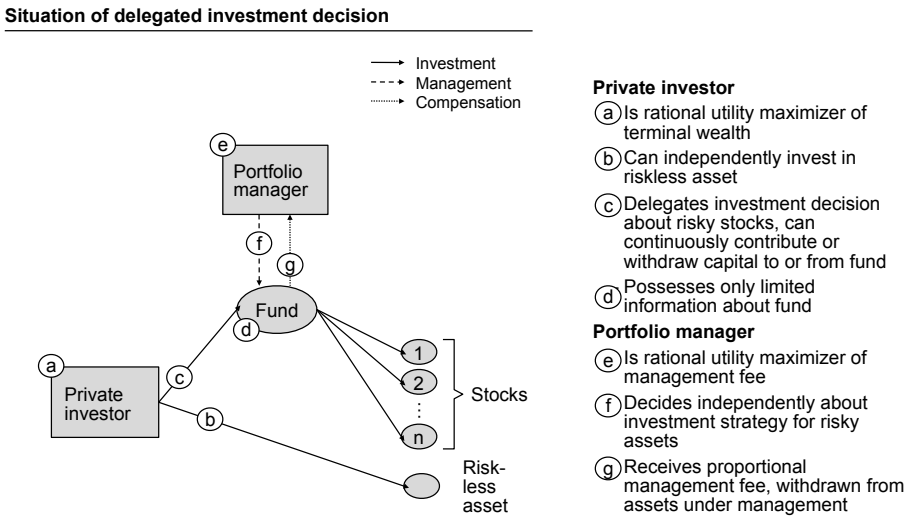



Figure 5.2.: General model setup of delegated portfolio management under compensation and capital flows. (Source: Own illustration)

## 5.2. The Situation of Delegation

The general model setup, illustrated in figure 5.2, is the following: the primary investor decides to delegate their investment decisions concerning the risky assets to a portfolio manager. The investment possibilities of the portfolio manager are restricted to these risky assets only, they cannot invest in the risk-free asset. Within the set of risky assets, their investments are not restricted, they can invest in the same $n$ risky assets as the primary investor could do, both short and long. However, the primary investor might invest in the riskless asset by themselves.

The primary investor and the portfolio investor have the same information about the market.[274] Both have the same information about the expected drift of asset prices $\alpha$, its standard deviations $\sigma$, and the correlations $\rho$ between them. However, the primary investor cannot control the exact investment of the professional investor. They can only infer from the resulting performance which includes the realization of a random event. They do not use any bonding or monitoring to influence the

[274] This is a homogeneous expectations setting, cf. Kraft/Korn (2008).

professional investor's behavior.[275]

The contract between the primary investor and the portfolio manager is not renegotiable and lasts for the whole investment period $[0, T]$. The primary investor transfers their wealth at the beginning of the period to the professional investor and receives the resulting wealth back at the end of the period. The primary investor compensates the professional investor for their efforts with a fixed fraction $\lambda$ of the portfolio wealth over the course of the investment until the end of the fixed investment period. There is no additional terminal compensation. Both share the same utility function but with potentially different risk aversion parameter $\gamma$.

The contract allows the primary investor to reallocate their capital, i.e., to contribute more capital to the portfolio manager or to withdraw capital. The decision about the volume of capital allocated to the portfolio manager is influenced by the general market performance and the investment performance of the portfolio manager compared to a benchmark portfolio.

1. *Capital market performance*: depending on the overall performance of the capital market, the primary investor adjusts their capital under management. If the overall capital market is doing well above the expectations measured with a - still to be defined - general market portfolio, the primary investor will contribute new capital to the professional portfolio manager. If the overall capital market is doing poorly below expectations, they will withdraw capital.

2. *Investment performance*: the primary investor delegates more or fewer of their investment decisions to the portfolio manager depending on the past performance of the portfolio manager. If the portfolio manager has achieved an outperformance compared to a - still to be defined - benchmark portfolio, they will delegate more of their capital. If the portfolio manager has underperformed compared to the benchmark portfolio, they will delegate less. A linear dependence on investment performance will be used in section 5.3. Section 5.4 will provide solutions for nonlinear dependencies.

The model in section 5.3 comprises compensation and capital flow in a single setup, due to methodological similarities. However, the solution for both conditions will be discussed separately. The model in section 5.4, even though it contains compensations, will focus on the effects of capital on the optimal portfolio policy.

[275] In an agency context, this is the classical setting of hidden action derived from hidden information after contracting (moral hazard). There will be no hidden information before contracting, i.e., no adverse selection problem.

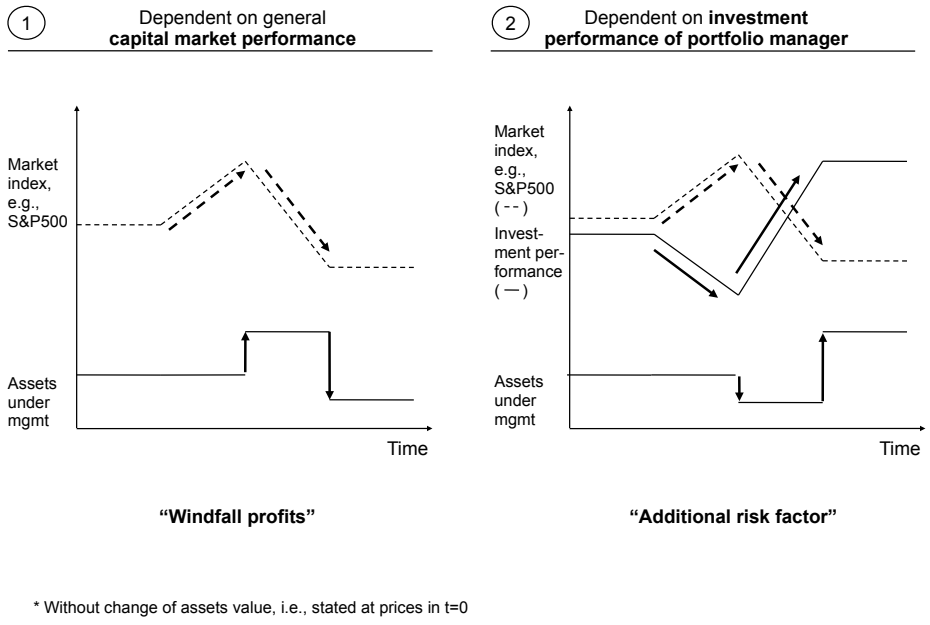



Figure 5.3.: Schematic illustration of capital flows. (Source: Own illustration)

## 5.3. Closed-Form Solution for Linear Model

### 5.3.1. Model Setup

#### Management Fee

The combined model is based on the wealth process 4.3 of the standard portfolio optimization model. The wealth process in this case represents the total of all assets under management of the portfolio manager. Consumption in the standard model is replaced with the compensation of the portfolio manager. The portfolio manager is allowed to withdraw the management fee $\lambda X(t)$. This management fee is calculated as a fixed fraction $\lambda$ based on the current assets under management $X(t)$, i.e., the value of the portfolio. The management fee is withdrawn from the portfolio value.

#### Capital Flow

The capital flow that influences the volume of assets under management of the portfolio manager is modeled with two functions, see figure 5.3. The capital market performance is represented by the development of a general market portfolio. Note

that this general market portfolio is not necessarily the market portfolio in the sense of classical portfolio optimization as the portfolio with only systematic risk. Let the $n$-dimensional vector $w_M$ be this general market portfolio with elements $w_{M_i}$, $i = 1, \ldots, n$, as the percentage of total assets invested in asset $i$ with $\Sigma_{i=1}^{n} w_{M_i} = 1$. $dP(t)$ is the vector of price processes of all $i = 1, \ldots$,n risky assets according to equation 4.1. The portfolio manager is affected by this portfolio because their assets under management will increase or decrease depending on the performance of this portfolio. The function

$$g_M(t) \quad = \quad a \cdot \Sigma_{i=1}^{n} w_{M_i} \frac{dP_i(t)}{P_i(t)} X(t)$$

models this influence. The structure of this equation is similar to Merton's wealth equation 4.2.

The past performance of the portfolio manager will be measured against a benchmark portfolio $w_B$, an $n$-dimensional vector of elements $w_{B_i}$ representing the percentage of total assets under management invested in the $i$-th asset with $\Sigma_{i=1}^{n} w_{B_i} = 1$. Using a linear relationship, the flow of funds related to the portfolio manager's performance can be expressed as

$$g_B(t) \quad = \quad c \cdot \Sigma_{i=1}^{n} \left[ w_i(t) - w_{B_i} \right] \frac{dP_i(t)}{P_i(t)} X(t) \tag{5.1}$$

The total flow of capital $g(t) = g_M(t) + g_B(t)$therefore becomes

$$g(t) = a \cdot \Sigma_{i=1}^{n} w_{M_i} \frac{dP_i(t)}{P_i(t)} X(t) + c \cdot \Sigma_{i=1}^{n} \left[ w_i(t) - w_{B_i} \right] \frac{dP_i(t)}{P_i(t)} X(t)$$

With the price process from equation 4.1 this results in

$$\begin{aligned} g(t) = a \cdot \sum_{i=1}^{n} w_{M_i} \alpha_i X(t) dt + c \cdot \sum_{i=1}^{n} \left[ w_i(t) - w_{B_i} \right] \alpha_i X(t) dt \\ + a \cdot \sum_{i=1}^{n} w_{M_i} \sigma_i X(t) dB_i + c \cdot \sum_{i=1}^{n} \left[ w_i(t) - w_{B_i} \right] \sigma_i X(t) dB_i \end{aligned}$$

which can be simplified to

$$g(t) = \sum_{i=1}^{n} \left(a \cdot w_{M_i} + c \cdot [w_i(t) - w_{B_i}]\right) \alpha_i X(t) dt + \sum_{i=1}^{n} \left(a \cdot w_{M_i} + c \cdot [w_i(t) - w_{B_i}]\right) \sigma_i X(t) dB_i \quad (5.2)$$

### Assets Under Management Process

Taking together the management fee and the flow of funds, the assets under management develop according to

$$dX(t) = \left(\sum_{i=1}^{n} w_i(t)\alpha_i\right) X(t)dt - \lambda X(t)dt + g(t) + \sum_{i=1}^{n} w_i(t)\sigma_i X(t) dB_i \quad (5.3)$$

with $g(t)$ according to equation 5.2.[276] Note that the consumption in the standard portfolio model is replaced with a fixed management fee $\lambda X(t)$ that is deducted from the assets under management. Using the definition of $g(t)$, equation 5.3 can be rearranged to

$$dX(t) = \left[\sum_{i=1}^{n} \left((1+c)w_i(t) + a \cdot w_{M_i} - c \cdot w_{B_i}\right) \alpha_i X(t) - \lambda X(t)\right] dt + \sum_{i=1}^{n} \left((1+c)w_i(t) + a \cdot w_{M_i} - c \cdot w_{B_i}\right) \sigma_i X(t) dB_i$$

It should be noted that the management fee only has influence on the drift of the overall process, whereas the flow of capital has influence both on the drift and the stochastic factor of the assets under management process. Compared with the standard problem in 4.3, two things have changed. First, consumption is replaced with a fixed management fee. Second, not only the portfolio policy has an influence on the drift and stochastic factor but also the general market portfolio $w_M$ and the benchmark portfolio $w_B$. It should also be noted that the factor $c$ of the benchmark-related flow scales the portfolio policies influence on drift and stochastic element.

[276] The state space is given by $Q_0 = [0, T) \times \mathbb{R}$. The state process is not restricted to $[0, T) \times \mathbb{R}_+$ to remain positive, however the solution will be such that the process is always positive.

### Minimization Criterion and HJB Equation

Let $U = \{u \in [\underline{w}, \overline{w}]^n : \mathbf{1}'u = 1\}$ be the control space for the control $u(t) = (w_1(t), \ldots, w_n(t))$. The additional restriction $\mathbf{1}'u = 1$ ensures that the total wealth is invested. The criterion to maximize in this case is

$$J(t, X; u) = E_{t,X}\left\{\int_t^T L[s, X(t)]ds\right\}$$

The professional investor maximizes the utility derived from management fees that depends on current wealth $X(t)$ over the period of investment $[t, T]$. There is no utility from terminal wealth for the portfolio manager. They are only rewarded on a permanent basis with management fee and no extra remuneration at the end of the investment horizon.

Let

$$V(t, X) = \sup_{u \in \mathcal{A}(t,X)} J(t, X; u)$$

be the value function associated with the optimization problem 5.29. The HJB equation for this problem can then be stated as

$$\begin{aligned}\max_{w \in U}\Big\{\Big(((1+c)w(t) + a \cdot w_M - c \cdot w_B)' \alpha X(t) - \lambda X(t)\Big) \cdot V_X(t, X) \\ + \frac{1}{2}((1+c)w(t) + a \cdot w_M - c \cdot w_B)' \Sigma ((1+c)w(t) + a \cdot w_M - c \cdot w_B) \\ \cdot X^2(t) \cdot V_{XX}(t, X) + L(t, X) + V_t(t, X)\Big\} = 0. \quad (5.4)\end{aligned}$$

## 5.3.2. Optimal Portfolio Policy

To derive the optimal portfolio policy, I will follow a general three-step approach proposed by Korn/Korn (2001) and Korn/Kraft (2004):[277]

1. Solve the optimization problem given by the HJB equation resulting in a candidate $v^*$ for the optimal control.

2. Insert this candidate $v^*$ into the HJB and develop a solution $W$ to the nonlinear partial differential equation (HJB), i.e., a specification of the value function. The verification theorem ensures that this value function also solves the original optimization problem.

[277] Cf. Korn/Korn (2001), p. 232 and Korn/Kraft (2004), p. 405.

3. Check all assumptions made, in particular those necessary to apply the verification theorem.

### Step 1: Optimal Control Candidates

Let the utility function in analogy to equation 4.6 be

$$L(t, \lambda X(t)) := \frac{1}{\gamma} e^{-\beta t} \left[\lambda X(t)\right]^{\gamma}. \tag{5.5}$$

Compared to the standard utility function, the consumption argument is replaced with the management fee for the investor.

Let the value function be strictly concave and the wealth process strictly positive. I use the Lagrange approach to search for optimal control candidates. The Lagrange approach will allow to incorporate the investment restriction $\sum_{i=1}^{n} w_i(t) = 1$ into the optimization problem. The Lagrange function for equation 5.4 is

$$\begin{aligned} L := L[t, X(t)] + V_t + V_X \left[\left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right)' \alpha - \lambda\right] X(t) + \frac{1}{2} V_{XX} \\ \cdot \left[\left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right)' \Sigma \left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right) X(t)^2\right] \\ - \lambda_0 (1 - \mathbf{1}' w(t)) \end{aligned}$$

including the budget constraint $1 - \mathbf{1}' w(t) = 0$ with the Lagrange factor $\lambda_0$. The necessary conditions for optimality are therefore

$$\begin{aligned} 0 = L_w(w) = -\lambda_0 \mathbf{1} + V_X (1+c) \alpha X(t) \\ + V_{XX} (1+c) \Sigma \left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right) X(t)^2 \end{aligned} \tag{5.6}$$

$$0 = L_{\lambda_0}(w) = 1 - \mathbf{1}' w \tag{5.7}$$

$$\begin{aligned} 0 = \Big(\left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right)'(t) \alpha X(t) - \lambda X(t)\Big) \cdot V_X(t, X) \\ + \frac{1}{2} \left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right)' \Sigma \left((1+c)w(t) + a \cdot w_M - c \cdot w_B\right) X^2(t) \\ \cdot V_{XX}(t, X) + L(t, X) + V_t(t, X). \end{aligned} \tag{5.8}$$

Assuming a concave value function and a strictly positive assets under management process, these conditions are also sufficient for the optimal solution. The optimal

control candidate depending on the value function $V$ develops to (see Appendix A)

$$w^*(t) = MV + \frac{1}{1+c}\left(\frac{V_X}{V_{XX}X(t)}\left[\mathbf{1}'\Sigma^{-1}\alpha \cdot MV - \Sigma^{-1}\alpha\right] - (a \cdot w_M - c \cdot w_B) + \mathbf{1}'(a \cdot w_M - c \cdot w_B) \cdot MV\right)$$

### Solution to the HJB Equation

Using the approach

$$V(t,X) = f(t) \cdot \frac{1}{\gamma}\left[\lambda X(t)\right]^\gamma \tag{5.9}$$

the optimal control results in (see Appendix A)

$$w^*(t) = MV + \frac{1}{1+c}\left(\frac{1}{1-\gamma}\left[\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right] - (a \cdot w_M - c \cdot w_B) + \mathbf{1}'(a \cdot w_M - c \cdot w_B) \cdot MV\right)$$

Using $\mathbf{1}'w_M = 1$ and $\mathbf{1}'w_B = 1$ the above equation reduces to

$$w^*(t) = MV + \frac{1}{1+c}\left(\frac{1}{1-\gamma}\left[\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right] - (a \cdot w_M - c \cdot w_B) + (a-c) \cdot MV\right) \tag{5.10}$$

The approach in 5.9 is a solution to the optimization problem with the factor

$$f(t) = \frac{1}{-\beta + a} \cdot \left(e^{(-\beta+a)T} - e^{(-\beta+a)t}\right) e^{-at} + e^{a(T-t)}$$

and

$$a = \gamma\left[(1+c)\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{(a-c)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha - \lambda + \frac{1}{2}(\gamma-1)\left((1+c)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{(a-c)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{(a-c)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) + \frac{1}{2}\frac{1}{(\gamma-1)}\left(\frac{\left(\mathbf{1}'\Sigma^{-1}\alpha\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right)\right].$$

### Checking Assumptions

All assumptions made are satisfied:[278]

- The wealth process $X^*(t)$ with the optimal portfolio policy $w^*(t)$ is the unique strictly positive solution to the wealth process from equation 5.3 with the optimal portfolio policy in A.8. The differential equation satisfies the conditions on the coefficients of the stochastic process.[279]

- The optimal control $w^*(t)$ is constant and, therefore, bounded in $U$.

- The value function, as the product of the strictly positive function $f(t)$ and the concave utility function, is strictly concave as well. It is, like the standard solution, a classical $C^{1,2}$ solution and satisfies the polynomial growth condition.[280]

For details on these assumptions cf. Appendix A.

## 5.3.3. Influence of Management Fee

This solution will be discussed in detail in this and the following two sections. I will start to discuss the effect of the management fee without flow of capital. Then I will discuss separately the influence of both capital flow effects in the following two sections.

If no flow of capital is present, i.e., $a = c = 0$, then the general solution from equation 5.10 becomes

$$w^*(t) = MV + \frac{1}{1-\gamma}\left[\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right] \tag{5.11}$$

This is essentially the same as the solution to the standard portfolio problem without riskless asset in equation 4.9. Hence, the structure of the optimal portfolio for the self-investing investor in equation 4.9 does not change if the investor is compensated with a proportional management fee which is deducted from the portfolio value.[281]

[278] Cf. Korn/Korn (2001), p. 234, 239f.
[279] Regarding the conditions on the coefficients cf. Korn/Korn (2001), p. 224.
[280] Cf. Korn/Korn (2001), p. 239.
[281] Nietert (1996), pp. 37, 45, and 47, reports the same result if compensation is not deducted from the portfolio in a framework as Merton (1971). Cf. also Wilhelm (1987), p. 197, for optimal proportional compensation in an agency-theoretic framework.

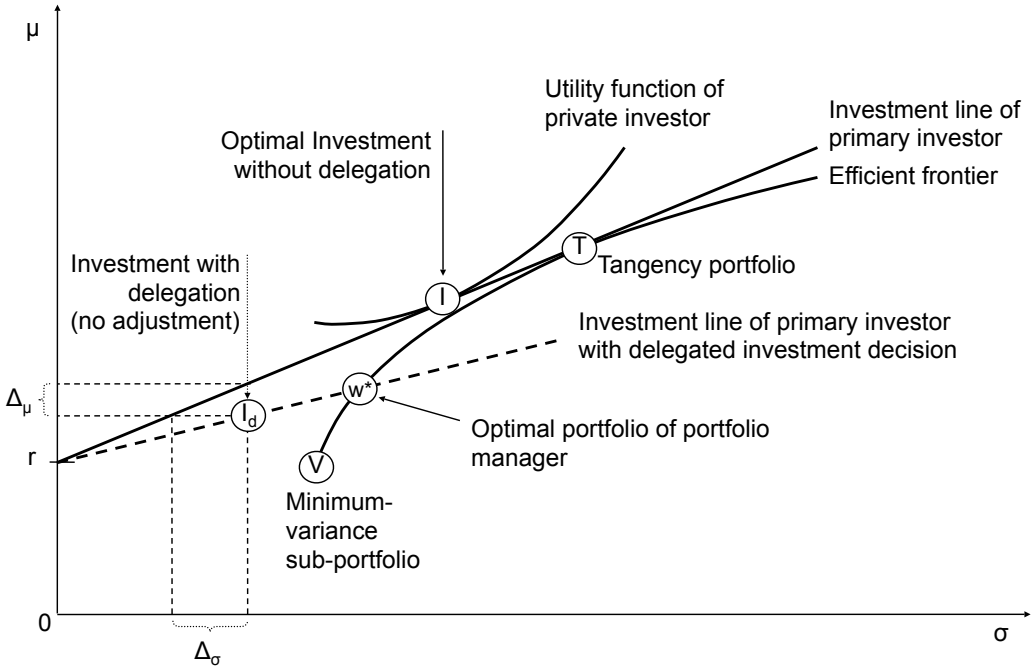



Figure 5.4.: Efficiency of delegated portfolios with management fee. Legend as in figure 4.1. (Source: Own illustration)

Given the structural similarity of the situation with riskless asset, the same result can be expected for this case.

However, from the perspective of the primary investor there are two caveats. First, depending on the risk aversion factor $\gamma$, the portfolio manager will most likely not invest in the same optimal portfolio as the primary investor would. In most cases, the risk aversion of a portfolio manager will not be the same as that of the primary investor.

Second, the potential identity of the optimal portfolios only holds if both can invest in the same set of assets. In a typical situation, the primary investor will delegate their investments in risky assets only and manage the investment in the riskless asset by themselves. In such a situation the portfolio manager will most likely not invest in the tangency portfolio. Figure 5.4 depicts the general situation of the primary investor if the portfolio manager invests in risky assets only, whereas the primary investor manages their investment in the riskless asset.

Assume that the portfolio manager realizes portfolio $w^*$ based on their risk aversion factor $\gamma$. The dashed line indicates the risk-return results of all available portfolios for the primary investor by combining the risk-free asset with the del-

egated portfolio. Clearly all these portfolios are inefficient compared to those on the straight line above. If the primary investor does not adjust the amount of risky investment, they realize the risk-return position $I_d$ in figure 5.4. The underlying portfolio is inefficient as they could increase the return by $\Delta_\mu$ or decrease the standard deviation by $\Delta_\sigma$ if they realized the corresponding portfolio on their original investment line.

The portfolio manager will invest in the tangency portfolio only if their relative risk aversion factor satisfies

$$\frac{1}{1-\gamma} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})}, \tag{5.12}$$

or equivalently if

$$\gamma = 1 - \mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})$$

The risk aversion parameter $\gamma$ is equal to one minus the total amount of the risky investment in case that the riskless asset exists.[282]

In the more general situation in which the portfolio manager does not invest in the tangency portfolio, the primary investor could at least increase their utility by adjusting the amount of risky investment if they knew the risk aversion or the investment strategy of the portfolio manager. Figure 5.5 illustrates this situation. They could then maximize their utility by moving their utility curve downwards until the dashed investment line is the tangency to the utility curve. This would lead to the optimal portfolio $I'$ in the situation of delegated investment. Compared to the portfolio $I_d$, the primary investor increases their derived utility. However, in the particular situation depicted in figure 5.5, this would imply investing more than $100\%$

[282] A short calculation verifies this. Substitute 5.12 into equation 5.11 and adjust the portfolio for the amount $\mathbf{1}'\left[\Sigma^{-1}(\alpha - r\mathbf{1})\right]$ invested in risky assets. Then 5.11 becomes

$$\mathbf{1}'\left[\Sigma^{-1}(\alpha - r\mathbf{1})\right] \cdot \left( MV + \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})}\left[\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right]\right)$$

This term can be simplified (using the symmetry of the covariance matrix $\Sigma$) to

$$\begin{aligned}
=&\mathbf{1}'\left[\Sigma^{-1}(\alpha - r\mathbf{1})\right] \cdot MV + \Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV \\
=&\mathbf{1}'\Sigma^{-1}\alpha \cdot MV - r\frac{\mathbf{1}'\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} + \Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV \\
=&-r\Sigma^{-1}\mathbf{1} + \Sigma^{-1}\alpha \\
=&\Sigma^{-1}(\alpha - r\mathbf{1})
\end{aligned}$$

This is the tangency portfolio structure from equation 4.7 of the primary investor.

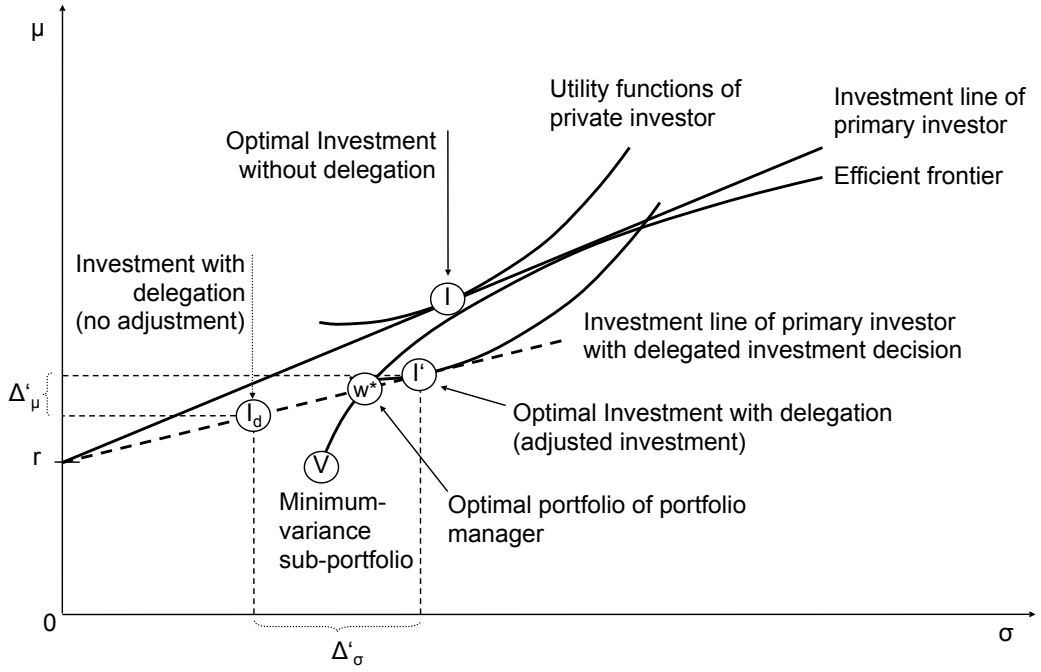



Figure 5.5.: Adjusted investment strategy to increase efficiency of delegated portfolios with management fee. Legend as in figure 4.1. (Source: Own illustration)

of their wealth in the delegated portfolio and finance the exceeding investment with a credit. In such a situation, the primary investor would prefer to increase the riskiness of their investment by $\Delta'_\sigma$ and, therefore, receive an increase in the expected return of $\Delta'_\mu$.

### 5.3.4. Effects of Market Performance-Related Capital Flows

A rearrangement of equation 5.10 separates the different capital flow effects:

$$w^*(t) = MV - \frac{1}{1+c}\left(aw_M - a \cdot MV\right) + \frac{1}{1+c}\left(cw_B - c \cdot MV\right) + \frac{1}{(1+c)}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \quad (5.13)$$

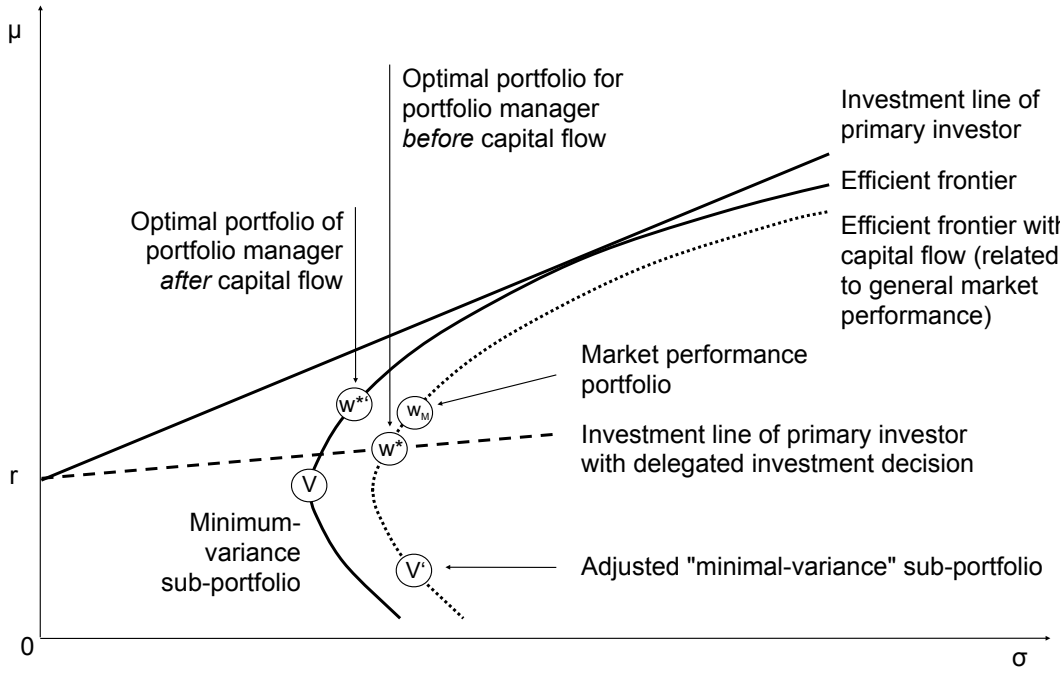



Figure 5.6.: Efficiency of delegated portfolios with capital flow related to general market performance. Legend as in figure 4.1. The parameter $a$ is assumed to be 1, therefore the adjusted efficient frontier crosses the portfolio $w_M$. (Source: Own illustration)

Let now $a > 0$ and $c = 0$, i.e. there is only capital flow related to the general market condition. Equation 5.13 then reduces to

$$w^*(t) = MV - (aw_M - aMV) + \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \tag{5.14}$$

Compared to the solution without market performance-related capital flow, an additional sub-portfolio incorporating the market portfolio is added. This portfolio is, like the minimum-variance portfolio, independent of the investor's risk aversion.

Figure 5.6 illustrates the structure of the optimal portfolio in a $\mu$-$\sigma-$diagram. For illustration purposes, the portfolio $w_M$ is assumed to be non-efficient, i.e., it is located below the efficient frontier of the primary investor. In addition, the parameter $a$ is assumed to be 1 in this illustration.[283] The optimal portfolio $w^*(t)$ of the portfolio manager is then located on an efficient frontier below the efficient frontier of

[283] As result, the portfolio $w_M$ lies on the adjusted efficient frontier, displayed as the dotted line.

the primary investor. This adjusted efficient frontier is determined by the term

$$MV - (aw_M(t) - a \cdot MV)\,,$$

which is independent of the risk aversion $\gamma$ and the standard return-oriented sub-portfolio. This base portfolio has the same functionality as the minimum-variance portfolio in the standard case. It differs from the minimum-variance portfolio in the standard solution by the adjustments induced by the flow of capital regarding the overall market performance. This adjustment subtracts the market portfolio $w_M(t)$ with the magnitude $a$ from the minimum-variance portfolio, just as the return-oriented portfolio is subtracted from the minimum-variance portfolio in the standard solution.

In effect, the professional investor 'disinvests' the market portfolio $w_M(t)$ in order to hold the efficient portfolio after market-related capital flow. The professional investor realizes an efficient portfolio after market-related flows as can be shown with a short calculation:

$$\begin{aligned} w^*(t) + aw_M &= MV - (aw_M - a \cdot MV) \\ &\quad + \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) + aw_M \\ &= MV + a \cdot MV + \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\ &= (1+a) \cdot MV + \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \end{aligned} \quad (5.15)$$

Of course, the flow of capital leads to a sum of the weights of this portfolio that is not equal to one:

$$\mathbf{1}' \cdot \left[(1+a) \cdot MV + \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)\right] = 1 + a$$

Normalizing the portfolio in equation 5.15 with the factor $\frac{1}{1+a}$ to one leads to

$$MV + \frac{1}{1+a}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)$$

Obviously, this portfolio is located on the efficient frontier.

From the portfolio manager's perspective, they realize an efficient portfolio after portfolio flows. The weight of the return-oriented portfolio is smaller because

$\frac{1}{1+a} < 1$ for $a > 0$. Hence, the portfolio manager always invests in the same portfolio after capital flows, regardless of the structure of the market portfolio. In effect, the portfolio manager invests as if they were more risk-averse. This tendency depends on the factor $a$ that determines the level of capital flow related to market performance. The higher the level of capital flow related to general market performance, the more risk-averse the portfolio manager will invest.

The structure of this optimal portfolio policy is similar to that of non-marketable income in the static portfolio cases.[284] In the case of nonmarketable income, the optimal portfolio structure consisting of the minimum-variance and the return-oriented sub-portfolios is extended by an additional sub-portfolio that hedges the unsystematic risks of the additional income.[285] It is equal to the risk-weighted (i.e., with the inverse of the market covariance matrix) covariance matrix of the non-marketable income with the marketable assets.[286]

The solution above has the same structure: $aw_M$ is the hedging portfolio. Because the hedging portfolio is perfectly correlated with the market portfolio (the stochastic drivers are the same Brownian motions), the covariance matrix of the market benchmark portfolio multiplied by the market covariance matrix results in 1.

The situation for the primary investor is worse compared to the situation without capital adjustments. Their investment line (the lower dashed line in figure 5.6) is located below the investment line without capital adjustments meaning that positions on that line are more inefficient. They could reduce the risk by $\Delta_\sigma$ associated with their investment by simply sticking with the risk-free asset. Compared to their original investment line, they lose $\Delta_\mu$ in expected return. The losses in efficiency are even higher compared to the standard delegation case without capital adjustments.

Like the compensation scheme, the capital flow related to market performance cannot explain the observed investment patterns of institutional investors. If one assumes that a realistic market portfolio consists of assets of larger companies or companies listed on highly visible indices, such as the S&P500, this capital flow even counteracts the investment patterns.

The following numerical examples illustrate the findings above. There are three

[284] Cf. Mayers (1973) and Brito (1977).

[285] Cf. Brito (1977), pp. 1110, 1112. Brito (1977) names this hedging portfolio $\theta$.

[286] Brito (1977) calculates the hedging portfolio as $\theta = C^{-1}C_0$, where $C^{-1}$ is the inverse of the market covariance matrix and $C_0$ is the covariance vector of the non-marketable asset.

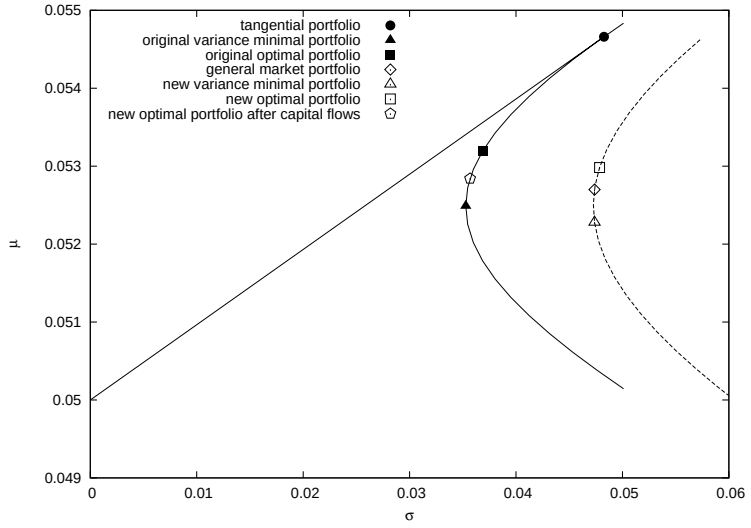



Figure 5.7.: Numerical example 1 of delegated portfolios with capital flow related to general market performance in a $\mu - \sigma-$diagram. The market portfolio $w_M$ is located away from the efficient frontier. Thje horizontal axis represents the standard deviation, the vertical axis represents the expected return of portfolios. The riskless rate is fixed at 5%, the parameter $a$ is set to 1. Legend is given in the figure. (Source: Own calculations)

risky assets with expected returns

$$\alpha = \begin{pmatrix} 0.10 \\ 0.12 \\ 0.08 \end{pmatrix}$$

and the covariance matrix

$$\Sigma = \begin{pmatrix} 1.0 & 0.2 & 0.5 \\ 0.2 & 2/3 & 0.3 \\ 0.5 & 0.3 & 0.5 \end{pmatrix}.$$

The riskless rate is $0.05$. The market portfolio is arbitrarily given by

$$w_M = \begin{pmatrix} 0.8 \\ 0.1 \\ 0.1 \end{pmatrix}$$

Figure 5.7 shows the results.

The market portfolio $w_M$ (empty diamond) together with the minimum-variance

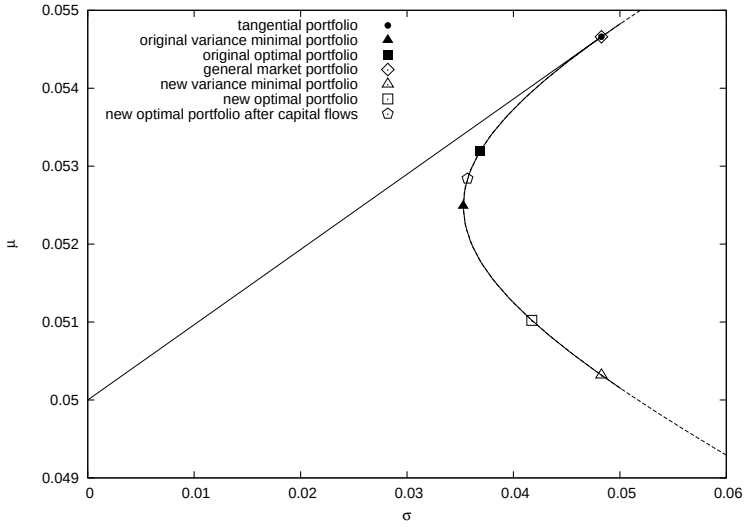



Figure 5.8.: Numerical example 2 of delegated portfolios with capital flow related to general market performance in a $\mu - \sigma-$diagram. The market portfolio $w_M$ is defined to be the tangency portfolio. Legend as in figure 5.7. (Source: Own calculations)

portfolio (filled triangle) defines the basis point for the new efficient frontier of the portfolio manager. The new frontier lies below the efficient frontier of the primary investor. The portfolio manager realizes a portfolio on this new efficient frontier (empty square). Taking the capital adjustment into account, they realize the portfolio indicated on the standard efficient frontier of the primary investor (empty pentagon). This portfolio still has a lower risk than the original portfolio marked on the same line.

The special case in which the tangency portfolio of the primary investor (filled circle) is chosen as the market portfolio (empty diamond) is depicted in figure 5.8. The situation is similar to that in the first numerical example. However in this example the original and adjusted efficient frontier are identical. The investor realizes a portfolio on the lower portion of the efficient frontier (empty square), the inefficient area. Despite this, they realize the same portfolio after capital adjustments (empty pentagon) as in the previous example above.

## 5.3.5. Effects of a Benchmark Portfolio

### General Benchmark Portfolio

Let now $c > 0$ and $a = 0$, i.e., there is only capital flow related to the benchmark portfolio. The optimal portfolio in equation 5.13 then reduces to

$$w^*(t) = MV + \frac{1}{1+c}\left(cw_B(t) - c \cdot MV\right) + \frac{1}{(1+c)}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \tag{5.16}$$

Beside the factor $\frac{1}{1+c}$ this portfolio is structurally similar to the portfolio if only a general market condition-related flow occurs, cf. equation 5.14, only the algebraic sign is reversed. In this case, the portfolio manager additionally invests in the benchmark portfolio $w_B(t)$ and reduces the share invested in the minimum-variance portfolio by $c \cdot MV$ in order to keep the sum of all weights at 1. The reason is similar: regardless of how good the portfolio manager's performance is, they will lose capital of the size $c \cdot \Sigma_{i=1}^{n} w_{B_i} \frac{dP_i(t)}{P_i(t)} X(t)$ as defined in equation 5.1. However, there is no equivalent structure for the capital they receive of the size $c \cdot \Sigma_{i=1}^{n} w(t) \frac{dP_i(t)}{P_i(t)} X(t)$.

The reason for this is that the capital they receive, which directly depends on their optimal portfolio policy, works as a scaling factor for the underlying stochastic process. It increases the return vector $\alpha$ of their optimal portfolio and, at the same time, the standard deviation $\sigma$. Because the standard deviation enters the optimal portfolio policy squared as covariance matrix $\Sigma$, only the factor $\frac{1}{1+c}$ remains. In essence, the risk associated with the benchmark-related capital flow outweighs the additional potential return. As a result, the factor $\frac{1}{1+c}$, that occurs in all three sums, reduces the amount invested in all portfolios but the minimum-variance portfolio for $c > 0$.

The solution in equation 5.16 is also different to the static case with non-marketable income. The additional sub-portfolio $cw_B(t)$ is equivalent, however the factor $\frac{1}{1+c}$ changes the weight of the return-oriented sub-portfolio. This return-oriented sub-portfolio is unaffected in the case of non-marketable income. The change is due to the structure of the performance-related capital flow. This capital flow depends on the structure of the optimal portfolio as defined in equation 5.1. The non-marketable income in the static case as defined in Brito (1977) is independent of the optimal portfolio policy. Hence, the influence of the performance-related capital flow on the return-oriented sub-portfolio.

Figure 5.9 illustrates the effect on the efficiency of the portfolio choice. In this

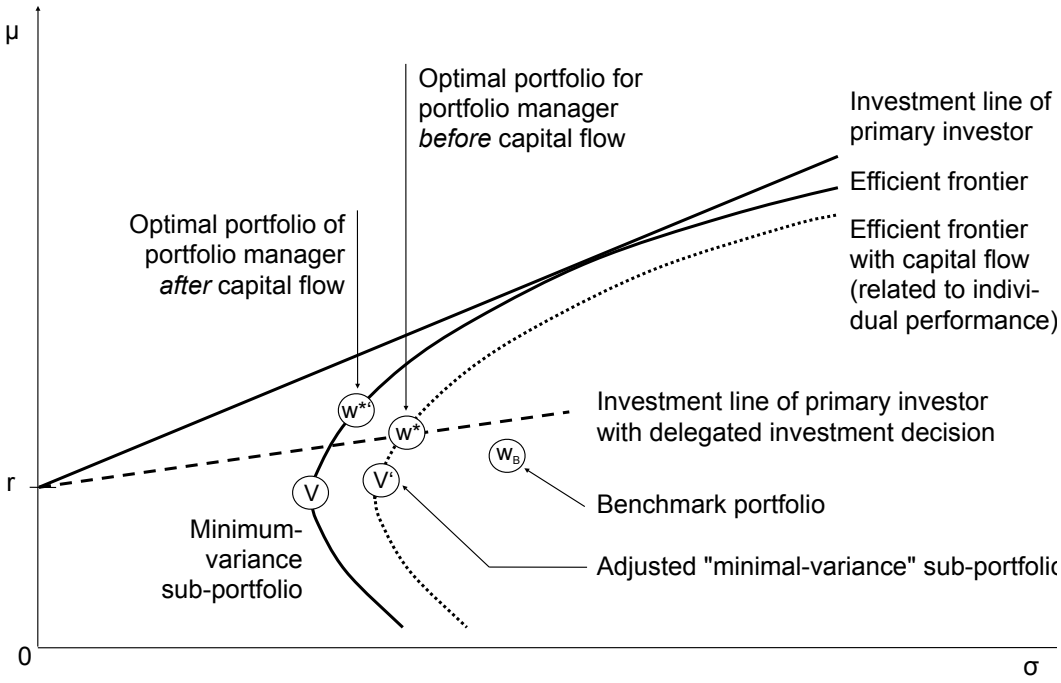



Figure 5.9.: Efficiency of delegated portfolios with performance-related capital flow. Legend as in figure 4.1. (Source: Own illustration)

illustration, the benchmark portfolio $w_B$ is assumed to be non-efficient, i.e., it is not located on the efficient frontier. The sub-portfolio that is independent of risk preference is shown as the basis for the new efficient frontier. Even though the factor $c$ that determines the magnitude of the capital adjustment is set to $1$, the benchmark portfolio is not located on the new efficient frontier due to the additional factor $\frac{1}{1+c}$.

As with market-related capital flow, it can be shown in this case that the portfolio manager realizes a portfolio that provides an efficient position after capital flow:

$$\begin{aligned} w^*(t) + c(w^* - w_B) &= (1+c)w^*(t) - cw_B \\ &= (1+c)\left[MV + \frac{1}{1+c}\left(cw_B(t) - c \cdot MV\right)\right. \\ &\quad \left. + \frac{1}{(1+c)}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)\right] - cw_B \end{aligned}$$

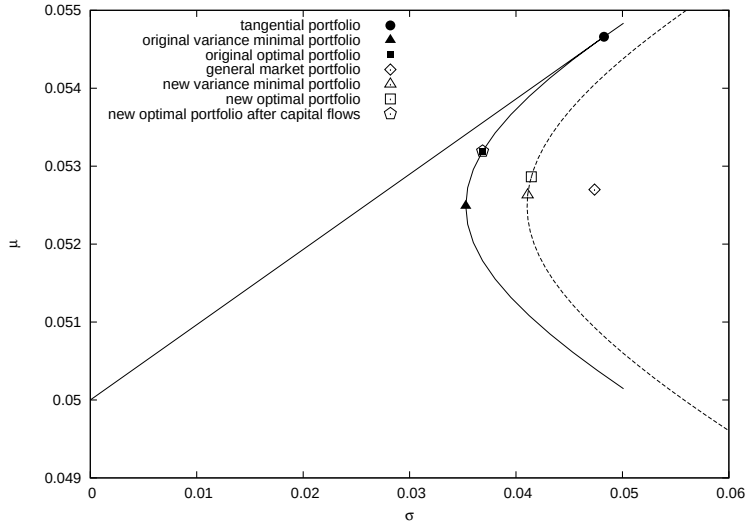



Figure 5.10.: Numerical example 1 of delegated portfolios with performance-related capital flow in a $\mu - \sigma-$diagram. The benchmark portfolio $w_B$ is located away from the efficient frontier. The riskless rate is set to 5%, the parameter $c$ set to 1. Legend as in figure 5.7. (Source: Own calculations)

$$
\begin{aligned}
&= (1+c) \cdot MV + (cw_B(t) - c \cdot MV) \\
&\quad + \frac{1}{1-\gamma} \left( \Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV \right) - cw_B \\
&= MV + \frac{1}{1-\gamma} \left( \Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV \right)
\end{aligned}
$$

Obviously, this is the solution without any capital adjustments in 4.9. Here the portfolio manager realizes the standard optimal portfolio after capital flows. From the perspective of the portfolio manager, the benchmark-related flow does not change their optimal portfolio policy after capital flow. They can completely hedge the effects of the benchmark-related flow.

This situation is suboptimal from the perspective of the primary investor. If they set the benchmark portfolio away from the efficient frontier, their new investment is located below the original investment line. Compared to their original investment line, they lose efficiency. The losses in efficiency are even higher compared to the standard delegation case without capital adjustments.

Figure 5.10 depicts a numerical example of the situation with capital adjustments due to a benchmark portfolio. The example uses the same data as in the previous example. The benchmark portfolio $w_B$ has been chosen to be the same as the market

portfolio $w_M$ in the previous example,

$$w_B = \begin{pmatrix} 0.8 \\ 0.1 \\ 0.1 \end{pmatrix}$$

The efficient frontier of the professional investor (dotted-dashed line) moves towards the benchmark portfolio $w_B$ (empty diamond), however the benchmark portfolio is not located on this efficient frontier. This is due to the factor $\frac{1}{1+c}$ associated with the risk aversion of the portfolio investor.

### Tangency Portfolio as Benchmark Portfolio

The typical primary investor that invests in the riskless asset by themselves can improve their efficiency situation, if they use the tangency portfolio as the benchmark portfolio. Let $a = 0$, so the optimal portfolio policy is given by equation 5.16. Let the benchmark portfolio be structurally equal to the tangency portfolio 4.7 and scaled so that $\mathbf{1}^{'} w_B = 1$, i.e.,

$$w_B := \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)} \Sigma^{-1}(\alpha - \mathbf{1}r)$$

The optimal portfolio from equation 5.16 then develops to

$$\begin{aligned}
w^*(t) &= MV + \frac{1}{1+c}\left(c \cdot \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\Sigma^{-1}(\alpha - \mathbf{1}r) - c \cdot MV\right) \\
&\quad + \frac{1}{1+c}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\
&= MV + \frac{c}{1+c}\left(\frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\Sigma^{-1}(\alpha - \mathbf{1}r) - \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1}\right) \\
&\quad + \frac{1}{1+c}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\
&= MV + \frac{c}{1+c}\frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)\mathbf{1}'\Sigma^{-1}\mathbf{1}} \\
&\quad \cdot \left(\mathbf{1}'\Sigma^{-1}\mathbf{1}\Sigma^{-1}(\alpha - \mathbf{1}r) - \mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)\Sigma^{-1}\mathbf{1}\right) \\
&\quad + \frac{1}{1+c}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)
\end{aligned}$$

$$
\begin{aligned}
&= MV + \frac{c}{1+c}\frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\mathbf{1}'\Sigma^{-1}\mathbf{1}\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha\Sigma^{-1}\mathbf{1}\right) \\
&\quad + \frac{1}{1+c}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\
&= MV + \frac{c}{1+c}\frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\
&\quad + \frac{1}{1+c}\frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\
&= MV + \left[\frac{1}{1+c}\frac{1}{1-\gamma} + \frac{c}{1+c}\frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\right]\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)
\end{aligned}
$$

Obviously, this portfolio is located on the original efficient frontier.

If equation 5.12 $\frac{1}{1-\gamma} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}$ holds, then the portfolio manager invests in the tangency portfolio regardless of the factor $c$.[287] Now, let $\frac{1}{1-\gamma} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)} + \epsilon$ with $\epsilon \in \mathbb{R}$. In this case, the portfolio manager does not invest in the tangency portfolio but realizes the portfolio

$$
w^*(t) = MV + \left[\frac{1}{1+c} \cdot \epsilon + \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\right]\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)
$$

As can easily being seen, the distance $\frac{1}{1+c} \cdot \epsilon$ of this portfolio from the tangency portfolio decreases with increasing $c$. Hence, the selection of the tangency portfolio as the benchmark portfolio induces the portfolio manager to invest closer to the tangency portfolio and the stronger the benchmark-related capital flow, the stronger the tilt of the portfolio towards the tangency portfolio.

Figure 5.11 illustrates the tilt towards the tangency portfolio. Let

$$
\text{Diff}_\text{B} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})} - \left[\frac{1}{1+c} \cdot \epsilon + \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - \mathbf{1}r)}\right] = \frac{1}{1+c} \cdot \epsilon
$$

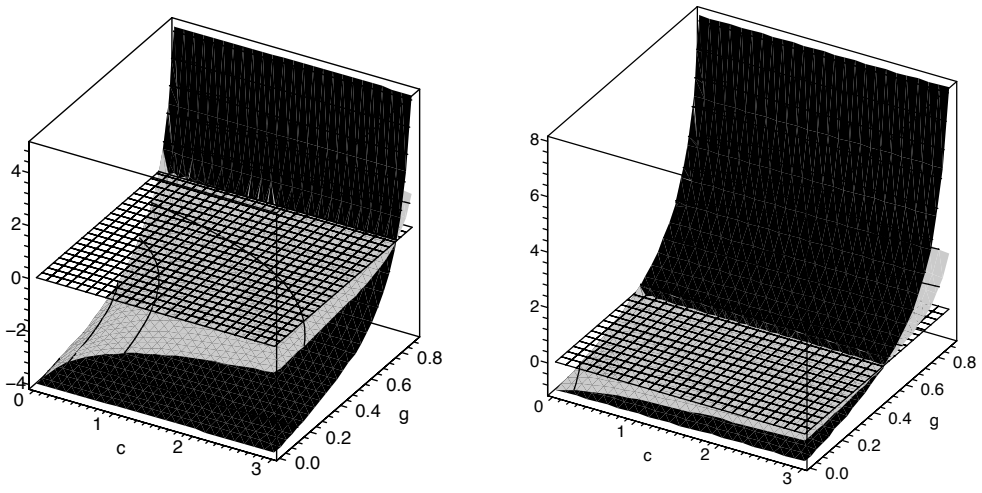
be the distance of the portfolio with benchmark-related capital flow from the tangency portfolio and

$$
\text{Diff}_\text{D} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})} - \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})} + \epsilon = \epsilon
$$

be the distance of the portfolio without benchmark-related capital flow from the tangency portfolio.

Figure 5.11 shows numerical results on the differences $\text{Diff}_\text{B}$ and $\text{Diff}_\text{D}$ for differ-

[287] Cf. footnote 282 in section 5.3.3.




(a) Differences for tangential portfolio of size 0.2 (b) Differences for tangential portfolio of size 0.5

Figure 5.11.: Distance of portfolios to tangency portfolio. The dark grey area represents the difference $\text{Diff}_\text{D}$ for portfolios without performance-related capital flow. The light grey area represents the difference $\text{Diff}_\text{B}$ for portfolios with performance-related flow. The straight black grid represents zero difference. (Source: Own calculations)

ent values of $\gamma$ with $\frac{1}{1-\gamma} = \frac{1}{\mathbf{1}'\Sigma^{-1}(\alpha - r\mathbf{1})} + \epsilon$ and $c$. The light grey areas represent the difference $\text{Diff}_\text{B}$, the dark grey areas the difference $\text{Diff}_\text{D}$. The grid indicates a zero distance. The area for the difference $\text{Diff}_\text{D}$ is bent only in the $\gamma$ direction because it does not depend on the parameter $c$. Figure 5.11a depicts the differences for a size of the tangency portfolio of $0.2$, figure 5.11b for a size of the tangency portfolio of $0.5$. The parameters $c$ and $\gamma$ range from $0$ to $3$ and $0$ to $0.9$, respectively.

Both figures clearly show that the benchmark-related capital flow pulls the portfolio towards the tangency portfolio. For negative differences as well as for positive differences, the absolute distance of the portfolio benchmark portfolio is always smaller than the distance of the portfolio without benchmark portfolio. Clearly, the parameter $c$ increases this effect. For larger values of $c$, the absolute difference of the portfolio with benchmark portfolio becomes even smaller. The marginal effect of an increasing $c$ is rather strong for small values of $c$ and decreases for larger values. In figure 5.11a, a value of $c$ close to $3$ pulls the portfolio only slightly closer to the benchmark portfolio. An increasing parameter $\gamma$ leads generally to a more risky investment. However, this effect is smaller for the portfolio with benchmark portfolio.

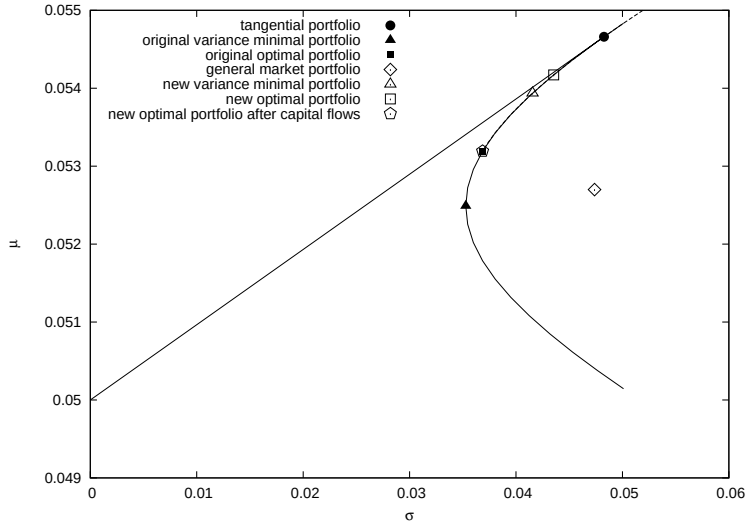



Figure 5.12.: Numerical example 2 of delegated portfolios with performance-related capital flow in a $\mu - \sigma-$diagram. The benchmark portfolio $w_B$ is defined to be the tangency portfolio. Parameter values as in figure 5.10. Legend as in figure 5.7. (Source: Own calculations)

Figure 5.12 depicts the optimal portfolio in the numerical example above if the benchmark portfolio is equal to the tangency portfolio. It clearly shows that with the given parameters the investment of the portfolio manager (empty square) is tilted towards the tangency portfolio (filled circle). By setting the tangency portfolio as the benchmark portfolio, the primary investor can induce the portfolio manager to invest more closely in the tangency portfolio. Hence, the efficiency loss reduces for the primary investor. The stronger their capital flow reaction, the stronger the pull towards the tangency portfolio and the fewer efficiency losses they incur.

The capital flow related to performance relative to a benchmark portfolio also has the potential to explain the observed investment patterns of institutional investors. Assuming that primary investors pick benchmark portfolios consisting of large companies or companies that are members of a popular index such as the S&P500, their capital flow reaction induces portfolio managers to tilt their portfolio towards these large companies or index members. In particular, given the size of the performance-related capital flow, the parameter $c$ can be expected to have a significant size.

## 5.4. Numerical Solution for Nonlinear Performance-Flow Relation

The previous section demonstrated a linear modeling approach to incorporate capital flow into the portfolio optimization problem. The linear model allowed for a closed-form solution in equation 5.10. However, empirical research suggests a nonlinear relationship between past performance and capital flow.[288] In order to analyze the influence of such a nonlinear relationship, numerical approximations will be used in this section.

### 5.4.1. Model Setup

#### Performance-Flow Relationship

Chevalier/Ellison (1997) identify a nonlinear relationship between past performance and future capital flow with an option-like shape. The exponential function

$$\text{Netflow}(t+1) = \exp\left\{\phi^{'} \cdot \text{ExcessReturn}(t)\right\} + \psi^{'} \tag{5.17}$$

fits the observed flow rather well. Figure 5.13 displays the approximation with parameter values $\phi^{'} = 3.5$ and $\psi^{'} = -0.85$. Most importantly, equation 5.17 reflects the convex option-like character of the performance-flow relationship.

In the continuous time framework, this empirical relationship could be modelled with the exponential expression

$$\begin{aligned} g_3^{'}(t) &= \exp\left\{\phi \cdot \left[w^{'}(t) - w_B^{'}\right] \cdot \frac{dP(t)}{P(t)} X(t)\right\} \\ &= \exp\left\{\phi \cdot \left(\sum_{i=1}^{n}[w_i(t) - w_{B_i}]\alpha_i X(t)dt + \sum_{i=1}^{n}[w_i(t) - w_{B_i}]\sigma_i X(t) dB_i\right)\right\} \end{aligned} \tag{5.18}$$

with $\mathbf{1}'w(t) = \mathbf{1}'w_B = 1$. However, the time difference of one year in the empirical analysis of Chevalier/Ellison (1997) cannot be implemented in a continuous time model such as equation 5.18. Also, for simplicity the parameter $\psi^{'}$ will be dropped in the continuous time model because it simply shifts the curve up or down. Overall, the nonlinear convex character of the relationship remains.

With this definition of the performance-flow relationship, the stochastic process

[288] Cf. section 3.2 for more details.

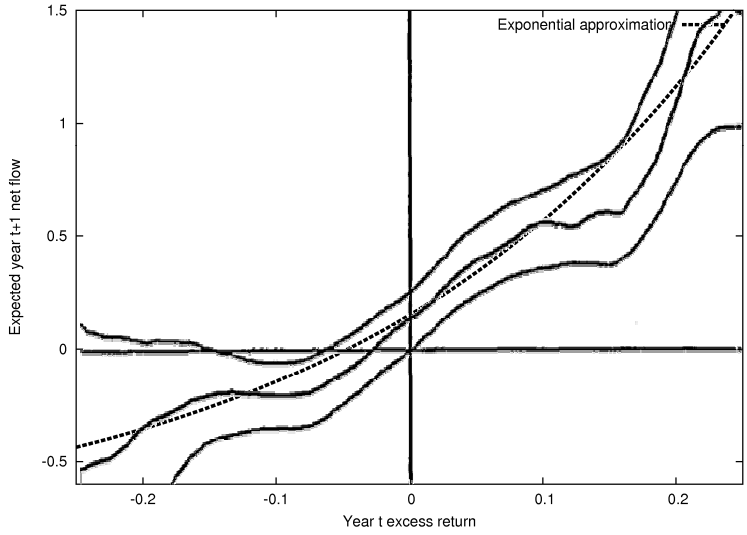



Figure 5.13.: Exponential approximation of performance-flow relationship through equation 5.17 with parameter values $\phi' = 3.5$ and $\psi' = -0.85$. (Source: Chevalier/Ellison (1997), p. 1177, o^wn illustration)

in differential form appears as a parameter in the exponential function. That makes it intractable with a Markov chain approximation, because the stochastic process is non-separable into drift and variance term.

A slight change of the performance-flow relationship in equation 5.18 overcomes this difficulty. The idea is that the performance flow at time $t$ depends on the accumulated performance difference of the portfolio manager's portfolio and the benchmark portfolio. This can be interpreted as if at time $t$ the primary investor compares their actual wealth with the fictional value of their wealth if they had invested their capital in the benchmark portfolio from the beginning. The primary investor then contributes or withdraws capital according to the development of this difference.

Let

$$D_i(t) = D_i(0) + \int_0^t [w_i(s) - w_{B_i}]\alpha_i X(s) ds + \int_0^t [w_i(s) - w_{B_i}]\sigma_i X(s) dB_i, \quad (5.19)$$

with $D_i(0) = 0$ be the individual asset's absolute contribution to the performance difference between the portfolio manager's portfolio and the benchmark portfolio. One could argue that in this specification the volume of flow depends on the level of the assets under management, which might not necessarily be the case. The

alternative formulation

$$D_i^{'}(t) = D_i^{'}(0) + \int_0^t [w_i(s) - w_{B_i}]\alpha_i ds + \int_0^t [w_i(s) - w_{B_i}]\sigma_i dB_i \tag{5.20}$$

with $D_i^{'}(t) = 0$, defines a flow that depends only on relative performance. Both expressions will be considered.

The performance-flow reaction is then given by the exponential functions

$$G_3(t, D(t)) = \exp\left(\phi \cdot \sum_{i=1}^{n} D_i(t)\right) \tag{5.21}$$

for absolute performance differences and

$$G_3^{'}(t, D(t)) = \exp\left(\phi \cdot \sum_{i=1}^{n} D_i^{'}(t)\right) \tag{5.22}$$

for relative performance differences. Now the application of Itô's lemma for multi-dimensional processes allows the evaluation of equations 5.21 and 5.22.

**Theorem.** (Itô's lemma)[289]. Let $S(t) = (S_1(t), \ldots, S_n(t))$be an $n$-dimensional Ito-process with

$$S_i(t) = S_i(0) + \int_0^t a_i(s)ds + \int_0^t b_i(s)dB_i(s), \qquad i = 1, \ldots, n,$$

with the $n$-dimensional Brownian motion $B(t) = (B_1(t), \ldots, B_n(t))$. Let $f : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a $C^{1,2}$ function (i.e. $f(t, S_1(t), \ldots, S_n(t))$ is continuous, once continuously differentiable with regard to the first argument, and twice differentiable with regard to the second argument). Then:

$$f(t) = f(0) + \int_0^t f_t(s)ds + \sum_{i=1}^{n} \int_0^t f_{S_i}(s)dS_i(s) + \frac{1}{2}\sum_{i=0}^{n}\sum_{j=0}^{n} \int_0^t f_{S_iS_j}(s)d\left\langle S_i, S_j\right\rangle_s \tag{5.23}$$

[289]Cf. e.g., Merton (1990), chapter 5, p. 122f., Korn/Korn (2001), p. 59.

Equation 5.23 evaluates to

$$f(t) = f(0) + \int_0^t f_t(s)ds + \left[\sum_{i=1}^{n}\int_0^t f_{S_i}(s)a_i(s) + \frac{1}{2}\sum_{i=0}^{n}\sum_{j=0}^{n}\int_0^t f_{S_iS_j}(s)\rho_{ij}b_{ij}^2(s)\right]ds + \sum_{i=1}^{n}\int_0^t f_{S_i}(s)b_i(s)dB_i$$

Given equations 5.19 and 5.20, the parameters for Itô's lemma are $a_i(s) = [w_i(s) - w_{B_i}]\alpha_i X(s)$ and $b_i(s) = [w_i(s) - w_{B_i}]\sigma_i X(s)$ for the absolute performance difference and $a_i(s) = [w_i(s) - w_{B_i}]\alpha_i$ and $b_i(s) = [w_i(s) - w_{B_i}]\sigma_i$ for the relative performance difference. In the former case, the lemma allows the development of the function $G_3(t, D(t))$ to

$$G_3(t) = \phi \cdot \left\{\sum_{i=1}^{n}\int_0^t G_3(s)[w_i(s) - w_{B_i}]\alpha_i X(s)ds + \frac{1}{2}\sum_{i=0}^{n}\sum_{j=0}^{n}\int_0^t G_3(s)\rho_{ij}[w_i(s) - w_{B_i}]\sigma_i X(s)[w_j(s) - w_{B_j}]\sigma_j X(s)ds + \sum_{i=1}^{n}\int_0^t G_3(s)[w_i(s) - w_{B_i}]\sigma_i X(s)dB_i\right\}$$

which can be formulated in differential notation as

$$dG_3 = \phi \cdot \left\{\sum_{i=1}^{n} G_3(t)[w_i(t) - w_{B_i}]\alpha_i X(t)dt + \frac{1}{2}\sum_{i=0}^{n}\sum_{j=0}^{n} G_3(t)\rho_{ij}[w_i(t) - w_{B_i}]\sigma_i X(t)[w_j(t) - w_{B_j}]\sigma_j X(t)dt + \sum_{i=1}^{n} G_3(t)[w_i(t) - w_{B_i}]\sigma_i X(t)dB_i\right\} \quad (5.24)$$

$$G_3(0) = 0. \quad (5.25)$$

In analogy, equation 5.22 with the relative performance difference from equation

5.20 develops to

$$dG_3^{'} = \phi \cdot \left\{ \sum_{i=1}^{n} G_3^{'}(t)[w_i(t) - w_{B_i}]\alpha_i dt \right. \\ + \frac{1}{2} \sum_{i=0}^{n} \sum_{j=0}^{n} G_3^{'}(t)\rho_{ij}[w_i(t) - w_{B_i}]\sigma_i[w_j(t) - w_{B_j}]\sigma_j dt \\ \left. + \sum_{i=1}^{n} G_3^{'}(t)[w_i(t) - w_{B_i}]\sigma_i dB_i \right\} \quad (5.26)$$

$$G_3^{'}(0) = 0. \quad (5.27)$$

### Assets Under Management Process and Minimization Criterion

The assets under management process will be restricted to performance-related flow only. Therefore, equation 5.3 becomes

$$dX(t) = \left( \sum_{i=1}^{n} w_i(t)\alpha_i \right) X(t)dt - \lambda X(t)dt + dG_3(t) + \sum_{i=1}^{n} w_i(t)\sigma_i X(t)dB_i \quad (5.28)$$

with the performance flow $dG_3$ given in equation 5.24 and 5.25 or in analogy with the performance flow $dG_3^{'}$ given in equation 5.26 and 5.27.[290]

The minimization criterion is the same as in the previous model in section 5.3. Let $U = \{u \in [\underline{w}, \overline{w}]^n : \mathbf{1}'u = 1\}$ be the control space for the control $u(t) = (w_1(t), \ldots, w_n(t))$ with $\sum_{i=1}^{n} w_i(t) = 1$. The criterion to maximize is

$$J(t, X; u) = E_{t,X} \left\{ \int_t^T L[s, X(t)]ds \right\}. \quad (5.29)$$

## 5.4.2. Markov Chain Approximation

### State Process

The basic idea of the Markov chain approximation is the discretization of state space and time of a continuous time stochastic dynamic problem. In this case, the discretization incorporates not only the assets under management process in equation 5.28, but also the process of performance-flow reaction in equation 5.24 and 5.25 respectively 5.26 and 5.27. This additional process increases the dimensionality of

[290] As in section 5.3, $Q_0 = [0, T) \times \mathbb{R}$ is the state space of this stochastic process.

the discrete optimization problem. Therefore, the implementation will consider the current performance differences $G_3(t)$ and $G_3'(t)$ of the portfolio manager's portfolio and the benchmark portfolio as external variable. Otherwise, the numerical implementation is adapted similar to the standard portfolio problem in 4.2.3. The control will be the holdings in each asset $w_i(t)X(t)$. Therefore, the substitutes $H_i(t) := w_i(t)X(t)$, with $\sum_{i=1}^{n} H_i(t) = X(t)$ and $H_{B_i}(t) := w_{B_i}X(t)$ will be used.

With this adaption, the model can be expressed in a single equation. With performance flow based on absolute performance difference, the assets under management process develops according to

$$\begin{aligned} dX(t) = &\left(\sum_{i=1}^{n} H_i(t)\alpha_i\right) dt - \lambda X(t)dt + \phi \cdot \sum_{i=1}^{n} G_3(t)[H_i(t) - H_{B_i}(t)]\alpha_i dt \\ &+ \phi \cdot \frac{1}{2} \sum_{i=0}^{n}\sum_{j=0}^{n} G_3(t)\rho_{ij}[H_i(t) - H_{B_i}(t)]\sigma_i[H_j(t) - H_{B_j}(t)]\sigma_j dt \\ &+ \sum_{i=1}^{n} H_i(t)\sigma_i dB_i + \phi \cdot \sum_{i=1}^{n} G_3(t)[H_i(t) - H_{B_i}(t)]\sigma_i dB_i \end{aligned}$$

This can be simplified to

$$\begin{aligned} dX(t) = &\sum_{i=1}^{n} \left(H_i(t) + \phi \cdot G_3(t)[H_i(t) - H_{B_i}(t)]\right) \alpha_i dt - \lambda X(t)dt \\ &+ \phi \cdot \frac{1}{2} \sum_{i=0}^{n}\sum_{j=0}^{n} G_3(t)\rho_{ij}[H_i(t) - H_{B_i}(t)]\sigma_i[H_j(t) - H_{B_j}(t)]\sigma_j dt \\ &+ \sum_{i=1}^{n} \left(H_i(t) + \phi \cdot G_3(t)[H_i(t) - H_{B_i}(t)]\right) \sigma_i dB_i \end{aligned} \tag{5.30}$$

where $G_3(t)$ is externally given.

For the performance flow with relative performance difference, the process in equation 5.30 is adopted such that absolute performance difference $H_i(t) - H_{B_i}(t)$ is divided by the current assets under management $X(t)$. Equation 5.30, therefore,

becomes for the relative performance difference

$$dX(t) = \sum_{i=1}^{n} \left( H_i(t) + \phi \cdot G_3'(t) \cdot \frac{1}{X(t)} \left[ H_i(t) - H_{B_i}(t) \right] \right) \alpha_i dt - \lambda X(t) dt$$
$$+ \phi \cdot \frac{1}{2} \sum_{i=0}^{n} \sum_{j=0}^{n} G_3(t) \rho_{ij} \cdot \frac{1}{X(t)} \left[ H_i(t) - H_{B_i}(t) \right] \sigma_i \cdot \frac{1}{X(t)} \left[ H_i(t) - H_{B_i}(t) \right] \sigma_j dt$$
$$+ \sum_{i=1}^{n} \left( H_i(t) + \phi \cdot G_3(t) \cdot \frac{1}{X(t)} \left[ H_i(t) - H_{B_i}(t) \right] \right) \sigma_i dB_i \quad (5.31)$$

### Transition Probabilities

The process for absolute performance differences above allows for the following substitutions:

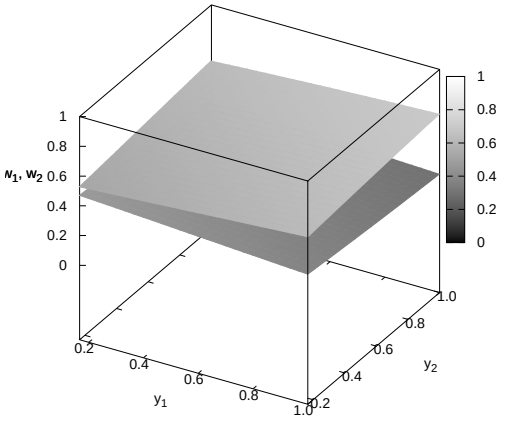
$$A_i(t_k, X, H_i) := \left( H_i(t_k) + \phi \cdot G_3(t_k) [H_i(t_k) - H_{B_i}(t_k)] \right) \alpha_i - \lambda X(t_k)$$
$$+ \phi \cdot \frac{1}{2} \sum_{j=0}^{n} G_3(t_k) \rho_{ij} [H_i(t_k) - H_{B_i}(t_k)] \sigma_i [H_j(t_k) - H_{B_j}(t_k)] \sigma_j$$

$$B_{ij}(t_k, X, H_i) := \left( H_i(t_k) + \phi \cdot G_3(t_k) [H_i(t_k) - H_{B_i}(t_k)] \right) \sigma_i \cdot \rho_{ij}$$
$$\cdot \left( H_j(t_k) + \phi \cdot G_3(t_k) [H_j(t_k) - H_{B_j}(t_k)] \right) \sigma_j$$

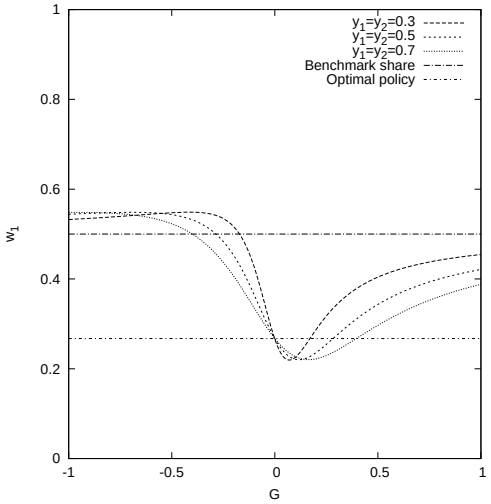
The substitutions for relative performance differences are adopted accordingly. With these definitions, the probabilities given in equations 4.13-4.17 provide a Markov chain approximation for the processes 5.30 and 5.31, respectively.

## 5.4.3. Influence of Nonlinear Performance Flow

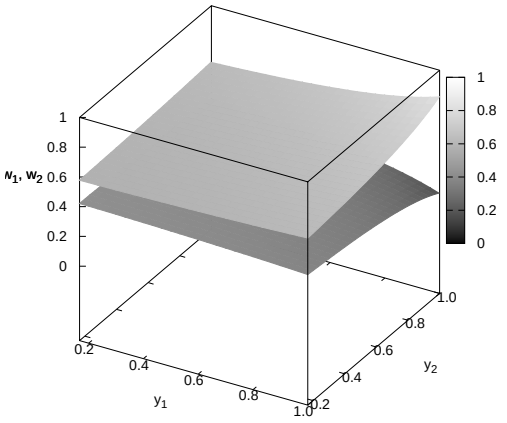
In order to investigate the influence of the exponential performance-flow relationship, the benchmark portfolio is set to $(0.5, 0.5)$. If not stated otherwise, the investment decision is considered with current holdings in both assets of $(0.63, 0.63)$ which is arbitrarily chosen. The parameters $\phi$, $G$ and $\gamma$ will vary depending on the analysis. Other parameter values (drift and standard deviation) are taken from the example in section 4.2.3. The discussion starts with the inspection of the optimal portfolio policy. Then, the dependence on the parameter $\phi$ is investigated. A third paragraph focuses on the influence of the risk aversion factor $\gamma$. In all discussions,



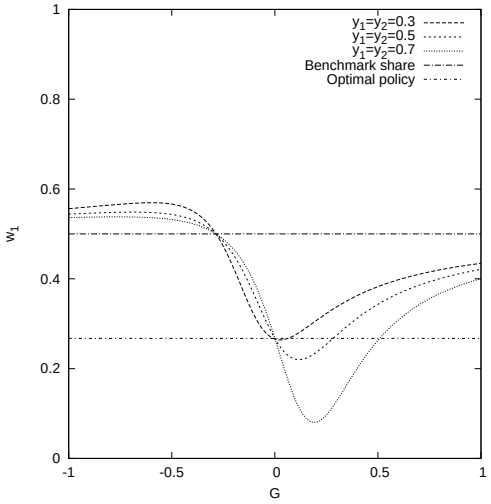

(a) Relative performance difference: optimal portfolio policies for two assets and performance difference at $G = 0.5$.




(b) Relative performance difference: optimal portfolio policy for first asset depending on $G$ for different states of asset holdings.




(c) Absolute performance difference: optimal portfolio policies for two assets and performance difference at $G = 0.5$.




(d) Absolute performance difference: optimal portfolio policy for first asset depending on $G$ for different states of asset holdings.

Figure 5.14.: Optimal portfolio policies with nonlinear performance-flow relationship with Markov chain approximation. (Source: Own calculations)

the two models with relative and with absolute performance difference will be examined in conjunction.

### Optimal Portfolio Policy

The parameter $\phi$ is set to $3.5$. Figure 5.14 displays the optimal portfolio policy for one and two assets with relative performance difference (above) and absolute performance difference (below) for various parameter settings. The left-hand side shows the policies for both assets for various states with fixed $G$. The right-hand side displays the optimal policy for the first asset depending on $G$ but with fixed states. On the right-hand side, the lower and upper straight dashed dotted lines are the optimal portfolio policy without performance flow (at $w_1 = 0.2673$) and the benchmark portfolio share of the first asset (at $w_1 = 0.5$) respectively. The relative performance case will be discussed first.

Figure 5.14a depicts the optimal portfolio policy with fixed $G = 0.5$. Differently from the standard solution in figure 4.2b, the optimal policy is not constant with regard to the state of the asset holdings. The portfolio policy for the first asset linearly increases with regard to both states and decreases for the second asset. It is interesting to note that with the given parameters the portfolio policy is rather close to the benchmark portfolio of $(0.5, 0.5)$ for small asset holdings. It drifts away towards the standard solution for larger asset holdings. A reason for this behavior could be the independence of the performance-flow relationship of the current state in the case of relative performance differences. In that case, even a small performance difference has a relatively large impact for small asset holdings. However with big asset holdings, a small performance difference is less severe.

Figure 5.14b shows the optimal portfolio policy for the first asset depending on the parameter $G$ for three states of asset holdings. The graph exhibits a nonlinear structure of the optimal portfolio policy's dependence on $G$. For negative performance differences, i.e., if the portfolio manager's portfolio has been less successful than the benchmark portfolio, the optimal portfolio policy is tilted towards the benchmark portfolio's share of $0.5$. The optimal policy first increases towards the benchmark portfolio, then even exceeds the benchmark portfolio to be pulled back downwards for large negative performance differences. For positive performance differences, the optimal portfolio policy exhibits two phases as well. In the first phase with relatively small performance differences, the optimal portfolio is tilted further away from the benchmark portfolio, below the optimal portfolio policy without performance-related flow. From a certain performance difference on, the "critical" performance difference, the portfolio policy increases again and is tilted towards the benchmark portfolio. For smaller states, the nonlinearity is

more contracted, whereas for larger states the tilt towards the benchmark portfolio is weaker. This contraction with smaller asset holdings confirms the conjecture that the relative performance difference is more important in such cases.

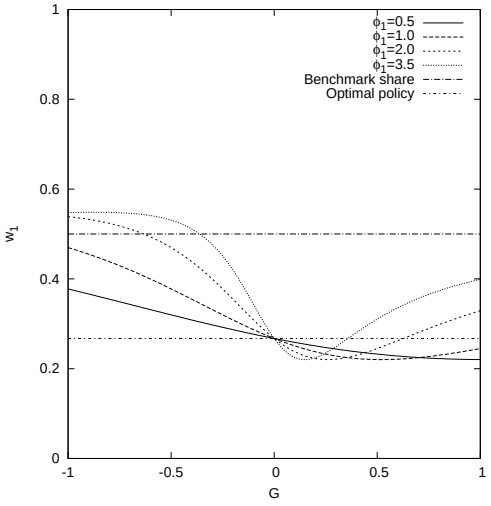
An explanation for the unequal behavior for positive and negative performance differences could be the following: negative performance differences imply a portfolio policy that deviates from the benchmark portfolio. This bears the risk of further negative performance differences in the future. Moreover, the portfolio manager is currently already experiencing an outflow of capital. Both together could drive the portfolio manager to invest even more than the benchmark in the first asset in the case of negative performance differences. Positive performance differences are ambiguous. They also indicate different portfolio policy with chances and risks of future negative performance differences. On the other hand, the current positive performance difference provides the portfolio manager with a bolster to counterbalance contingent adverse developments. This could explain their slower convergence to the benchmark portfolio.

In contrast to the solution for relative performance differences, the portfolio policy for absolute performance differences exhibits a nonlinear structure as indicated in figure 5.14c. Note in analogy with relative performance difference, that the optimal portfolio policy for small assets is more distant from the benchmark portfolio and then moves further away. Because the absolute performance difference depends on the current asset holdings, it is smaller for small asset holdings. In relation to the assets under management, the resulting capital flow is smaller than with relative performance difference. Additionally, figure 5.14c indicates another difference to the relative performance case: the overall variance increases and, in particular, the minimum of the portfolio policy is not constant but decreases for larger asset holdings.

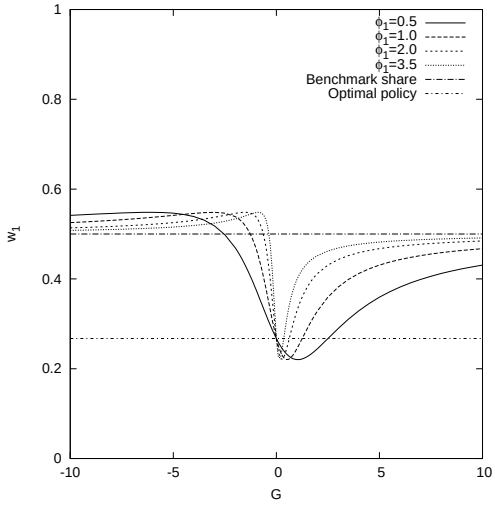
### Influence of Exponential Parameter $\phi$

Figure 5.15 displays the optimal portfolio policy of the first asset depending on the performance difference for various parameter values $\phi$ and fixed asset holdings. The two figures above represent relative performance differences, the two figures below represent absolute performance differences. The left-hand side enlarges a certain range of the variable $G$ from the right-hand side. Naturally, the optimal policy is identical for all parameter values of $\phi$ if the performance difference is zero.

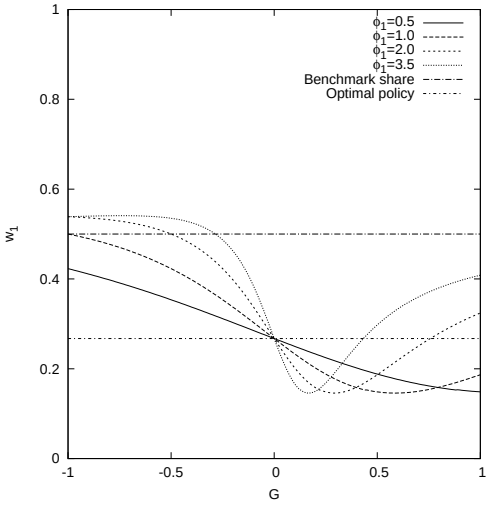
The general structure of nonlinearities in all figures are identical to those in figure



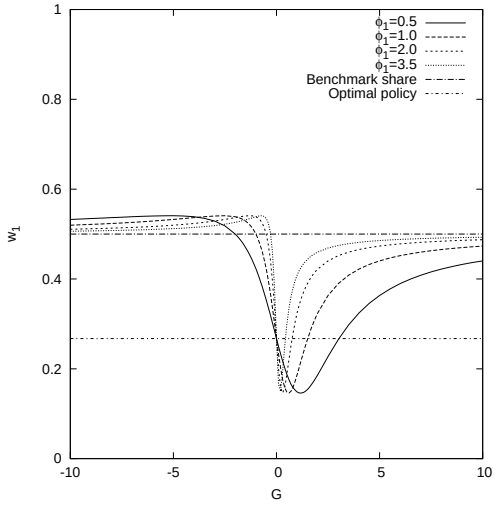

(a) Relative performance difference. $G$ between $-1$ and $1$




(b) Relative performance difference. $G$ between $-10$ and $10$

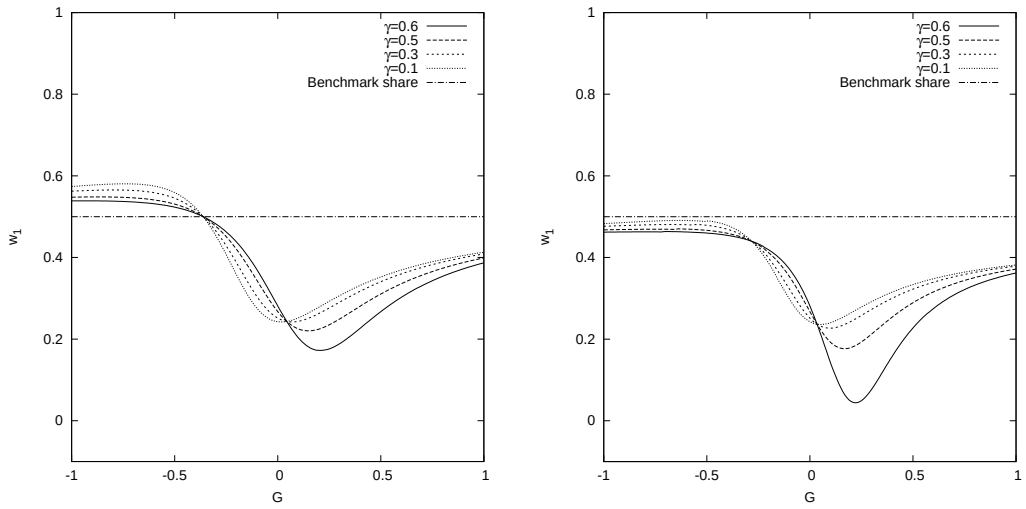



(c) Absolute performance difference. $G$ between $-1$ and $1$




(d) Absolute performance difference. $G$ between $-10$ and $10$

Figure 5.15.: Sensitivity of portfolio policy for convexity of nonlinear performance-flow relationship. Relative investment in first asset displayed, i.e., $w_1 = H_1/X$. Risk aversion parameter $\gamma$ set at $0.5$. (Source: Own calculation)




(a) Relative performance difference.

(b) Absolute performance difference.

Figure 5.16.: Influence of risk aversion on portfolio policy with nonlinear performance flow. Relative investment in first asset displayed, i.e., $w_1 = H_1/X$. Parameter $\phi$ set to $3.5$. (Source: Own calculation)

5.14. The parameter $\phi$ determines how fast the portfolio policies converge towards the benchmark portfolio. For higher values of $\phi$, the portfolio policy is contracted around the origin and converges faster towards the benchmark portfolio. This behavior is expected as the parameter $\phi$ defines the strength of the convexity of the capital function.

The difference between relative and absolute performance difference is limited. The overall shape of the curves is identical. The convergence is slightly stronger for absolute performance differences. For the case of absolute performance differences, the minimum of the portfolio policy is also lower leading to an overall larger variance of portfolio policies.

### Influence of Risk Aversion

Figure 5.16 highlights the role of the risk aversion parameter $\gamma$. The left-hand figure depicts the optimal portfolio policy depending on the performance difference $G$ for various values of $\gamma$ for relative performance differences, the right-hand side for absolute performance differences. The asset holdings and the parameter $\phi$ are fixed.

Given the utility function from equation 4.6, a high $\gamma$ represents a low risk aversion and a low $\gamma$ a high risk aversion. From both figures 5.16a and 5.16b, one can see that the overall variation of the optimal portfolio policy is lower for higher risk

aversion. For a very high risk aversion, the minimum of the portfolio policy is close to the portfolio policy without performance-related flow at $G = 0$. It converges for negative as well as for positive performance differences towards the benchmark portfolio, however with a stronger tilt for negative performance differences. Only a decreasing risk aversion then allows the optimal portfolio policy to even move below the optimal policy without performance-related flow. The convergence towards the benchmark portfolio is slower for investors with low risk aversion.

### Evaluation of Portfolio Policy

For the primary investor, the results are overall positive, though the performance-flow relationship does not resolve the conflict of interest totally. Conditional on the model setup in equations 5.24 and 5.25 or 5.26 and 5.27 respectively, the influence of the benchmark portfolio strongly depends on the current absolute performance difference between the portfolio manager's portfolio and the benchmark portfolio. If both portfolios show the same performance then the portfolio policy of the portfolio manager is not influenced by the benchmark portfolio. The primary investor suffers efficiency losses as indicated in section 5.3.3.

But most likely, if their portfolio policy is not equal to the benchmark portfolio, then there will be a performance difference. If the absolute or relative performance difference is negative, the portfolio manager is tilted towards the benchmark portfolio relatively fast. Even small performance differences bring them close to the benchmark portfolio. The larger the exponential parameter $\phi$, the risk factor $\gamma$ (i.e., the smaller the risk aversion), and the current asset holdings in absolute terms, the more decided the tilt towards the benchmark portfolio is.

If the absolute or relative performance difference is positive, the portfolio manager's reaction is not necessarily tilted towards the benchmark portfolio. For smaller performance differences, they might even deviate more strongly from the benchmark portfolio than they would without performance-related flow. This effect becomes stronger for larger $\gamma$ (i.e., lower risk aversion) and bigger current asset holdings. But if the performance difference becomes bigger, they will eventually tilt their portfolio towards the benchmark portfolio.

A clear recommendation regarding the absolute or relative performance difference seems impossible. The intensity of the convergence depends on all three parameters $G$, $\phi$, and $\gamma$. Ceteris paribus, the relative performance difference is advantageous for very small asset holdings. However, a bigger parameter $\phi$ causes a

greater convergence for absolute performance differences.

Considering the dynamic nature of the problem, the portfolio manager might continuously change their portfolio policy depending on the current performance difference. Generally speaking they will tilt their portfolio towards the benchmark portfolio if "all goes bad" and will be reluctant to tilt it towards the benchmark portfolio if "all goes well." Overall, they will only invest in the benchmark portfolio almost by chance.

Quite generally, the nonlinear capital flow can also explain the observed investment patterns of institutional investors. Given a suitable benchmark, usually a portfolio of large stocks or a well-known index, the portfolio manager will tilt their portfolio towards the benchmark if their past performance has led them towards a situation where they deviate from it. However, the numerical analysis above was not able to identify the equilibrium for the portfolio manager, i.e., a portfolio policy where they balance the risk of underperforming the benchmark, hence being punished by the primary investor, and their own investment preferences according to their utility function and risk aversion.

## 5.5. Concluding Remarks

This chapter has presented two models of compensation and capital flows to analyze optimal portfolio policies from the primary investor's perspective and to probe for their explanatory power regarding observed investment patterns of institutional investors.

The first model with a proportional management fee and linear capital flow regarding both general market performance and portfolio manager performance showed ambiguous results. A management fee alone does not change the optimal portfolio policy compared to a primary investor who invests on their own. However, the primary investor might experience an efficiency loss if the portfolio manager can only invest in risky assets and, hence, does not realize the tangency portfolio. Capital flow due to general market performance does not improve the situation. In that case, the portfolio manager will disinvest the market portfolio leading to an increased efficiency loss for the primary investor. But capital flow due to the portfolio manager's performance compared to a benchmark portfolio improves the situation for the primary investor. Their reallocation induces the portfolio manager to tilt their portfolio towards the benchmark portfolio. This has been demonstrated for the

tangency portfolio as benchmark portfolio.

Regarding the explanatory power of the linear model, only capital flow-related to the portfolio manager's performance can explain why institutional investors prefer stocks that are listed in important market indices such as the S&P500. It appears highly likely that primary investors use such indices as benchmarks in the real world. Given the relatively large size of the performance-related flow reported by e.g., Chevalier/Ellison (1997) and Sirri/Tufano (1998), the tilt towards these benchmarks appears reasonable.

The second model implemented a nonlinear capital flow with regard to the portfolio manager's performance. Using a Markov chain approximation, it could be shown that with such a capital flow the portfolio policy of the portfolio manager is tilted towards the benchmark portfolio. In particular, the tilt is stronger for larger current deviations from the benchmark and for a stronger convex capital flow. This analysis also supports the hypothesis that capital flow related to portfolio manager's performance is at least partly responsible for the observed investment patterns of institutional investors.

Next to compensation and capital flow, portfolio managers, in particular if they manage large portfolios, face price effects from their trades. The next chapter demonstrates how this can be incorporated into their optimization problem and what the effect on the optimal portfolio policy is.

# 6. Portfolio Policies under Price Influence

This chapter investigates the influence of price effects from trades on optimal portfolio policies in a single and multiperiod context. It accounts for the special characteristics of portfolio management as outlined in chapter 3, i.e., compensation, capital flows, and price effects. After a review of related research, I present a trinomial portfolio model incorporating price effects that is based on a standard trinomial model as outlined in section 4.3. This discrete time model will allow the main implications of price effects from transactions to be understood. I will then present numerical solutions for several single and multiperiod models. The solutions start with optimal portfolio policies under compensation and price effects in a single-period context. The model is then extended to multiple periods. Finally, the joint influence of compensation, capital flows, and price effects is investigated. Results are discussed with regard to their explanatory power for observed investment strategies as well as from the perspective of the primary investor. Concluding remarks terminate this chapter.

## 6.1. Related Research

Incorporating price effects into investment decisions was initially developed in the context of hedging strategies.[291] Dynamical aspects of price effects are usually discussed in the context of optimal liquidation strategies of large stock holdings or portfolios.[292] Results from research on portfolio optimization under price effects in

[291] Cf. Jarrow (1992) and Jarrow (1994) for discrete time models. For continuous time models cf. Frey/Stremme (1997), Platen/Schweizer (1998), Sircar/Papanicolaou (1998), Jonsson/Keppo (2002), Cetin/Jarrow/Protter (2004), and Bank/Baum (2004).

[292] For discrete time approaches, cf. Bertsimas/Lo (1998), Almgren (2003), and Huberman/Stanzl (2005). Impulse control approach is pursued by Subramanian/Jarrow (2001). Vath/Mnif/Pham (2007) use an impulse control framework to analyze simultaneous price impact and transaction costs. Cf. He/Mamaysky (2005) and Mönch (2005) for continuous time liquidation strate-

a multiperiod context are summarized below.

Cuoco/Cvitanic (1998) are among the first to address the large investor problem in portfolio optimization in a multiperiod context.[293] They consider a situation where the *portfolio policy*, i.e., the optimal weights $w_i(t)$, of the investor affects the drift of asset prices. Cuoco/Cvitanic (1998) add an additional drift term to the stochastic price process in order to make the price dependent on the portfolio policy. The price process of stochastic assets becomes

$$dP_i(t) = [\alpha_i + \hat{\alpha}_i(w_i(t))]P_i(t)dt + P_i dt + \sigma_i P_i(t) dB_i(t)$$

where $\hat{\alpha}_i(w_i(t))$ is the drift that depends on the portfolio policy $w_i(t)$. Using the martingale technique together with a duality approach, Cuoco/Cvitanic (1998) provide closed-form solutions for investors with a logarithmic utility function. For a logarithmic investor, they find that consumption does not change under price pressure. But the share of wealth in absolute terms invested in the risky asset is always lower than without price effects. However, the model of Cuoco/Cvitanic (1998) incorporates price effects not based on transactions but based on the actual share of wealth invested in the particular assets. This might be relevant for very large-scale investments where the capital market assumes an influence of the investor on the corporate's performance. However, it does not cover the price effect investors face when they try to purchase or sell large portions of stocks on a day-to-day basis as outlined in section 3.3.

Matsumoto (2006) investigates the situation in which investors cannot be 100% sure that purchases or sales are executed.[294] He introduces a random success function that indicates whether a desired transaction has successfully been executed or not. Using the standard price process $dP(t)$ for the risky asset, Matsumoto (2006) models the development of the holdings in one risky asset as

$$S(t) = S_0 + \int_0^t S(s-)\frac{dP(s)}{P(s)}ds + \int_0^t V(s)dO(s)$$

where $S_0$ is the initial asset position in $t = 0$, $S(t-)$ represent the asset holding before the transaction, and $\frac{dP(t)}{P(t)}$ is the relative change in price. The success rate is

gies.

[293] Cf. also Karoui/Peng/Quenez (1997). They use a recursive utility function.

[294] Gârleanu (2009) proposes a similar setup in which investors can only trade a certain portions of the time line, represented by Poisson arrival times.

modeled with a Poisson-Process $dO(t)$ that enables the agent to trade an amount of $V(t)$ at its jump times. He provides a solution for the case of an investor who maximizes expected utility from terminal wealth under a logarithmic utility function. In that case, the optimal transactions converge to the standard Merton solution when liquidity increases.

Longstaff (2001) provides a similar portfolio selection model in which he models illiquidity as "the risk that a trader might not be able to extricate himself from a position quickly when need arises."[295] The investor is allowed to trade $\gamma(t)dt$ assets at $t$, where $\gamma(t)$ is a process bounded from below and above. He conjectures that the investor trades as much as possible at any time. However, Longstaff (2001) does not provide a formal solution to the value function.

Vath/Mnif/Pham (2007) use a transaction-based approach to analyze the simultaneous influence of price impact function and proportional transaction costs. They allow the investor to trade at discrete times only. They model this situation as an impulse control problem. Let $Y(t_n)$ be the number of shares of an asset hold by the investor at $t_n$ and $\zeta(t_n)$ the number of shares purchased or sold at $t_n$. Then $Y(t_{n+1}) = Y(t_n) + \zeta(t_n)$. They introduce an exponential (i.e., convex) price impact function $Q(\zeta, P) := P \cdot e^{\lambda\zeta}$ with $\lambda > 0$ and $P$ as the pre-trading price. However, Vath/Mnif/Pham (2007) do focus their analysis on the form of the value function. They do not, however, provide an explicit solution for the optimal portfolio.

In a similar setup, Isaenko (2010) analyzes the effect of transitory price impact. The price impact is modeled in a convex form using the sum of a linear cost term $\alpha_1 S\,|u|$ and a nonlinear cost term $\alpha_2 S\,|u|^{1+\varepsilon}$ with $\varepsilon > 0$, where $\alpha_1$ and $\alpha_2$ are cost parameters, $S$ is the current share price, and $u$ the number of shares traded.[296] Isaenko (2010) shows that the linear cost term implies a non-trading area around the traditional optimal portfolio choice. The nonlinear cost term leads to a limitation of overall tradeing volume.

Ghysels/Pereira (2008) choose a non-parametric approach to investigate the influence of liquidity defined by three different measures on optimal portfolio choices. Based on empirical data from the New York Stock Exchange, they estimate optimal portfolio choices for small and large stocks. Ghysels/Pereira (2008) show that the

[295] Longstaff (2001), p. 408. Diesinger/Kraft/Seifried (2010) investigate in a related article the implications of potential complete market breakdowns on optimal portfolio choices. However, Diesinger/Kraft/Seifried (2010) use a different mathematical setup applying catlag-processes to simulate the market state.

[296] Cf. Isaenko (2010), p. 2377-2379.

optimal portfolio choices depend on the liquidity of the underlying assets, however with different specifications depending on the sample of assets and the investment and trading horizon.

In a numerical analysis Pereira/Zhang (2010) investigate the influence of constant and stochastic liquidity, in particular its volatility, on optimal portfolio policies with numerical methods (open-loop feedback control and closed-loop control). They find that investors trade according to the current liquidity state with high volumes at times of high liquidity and low volumes at times of low liquidity. Pereira/ Zhang (2010) also demonstrate with their model that assets with high liquidity volatility require a lower liquidity premium.

## 6.2. Model Setup with Price Effects

The approaches presented above do not incorporate price effects either as price changes depending on the transaction size or with a concave price impact function as outlined in section 3.3 or both. Hence, a new modeling approach is developed based on the discrete time trinomial model presented in section 4.3. This model will differ from that in section 4.3 in two aspects: first, the price process will depend on the transactions in each period. Second, instead of maximizing expected utility of terminal wealth, the investor will optimize expected utility from the management fee that is withdrawn from the portfolio.

### 6.2.1. Model Processes

#### Price Process

According to empirical research, price effects from trades can be separated into two parts: the execution price initially changes due to the transaction.[297] After this initial change, the price effect reverses to a certain extent. This separation results in three different prices at each time $t$ compared to a single price in the standard model:

1. A price $P_i^a$ before the transaction that is comparable to the price in the standard model.[298]

[297] Cf. Pereira/Zhang (2010), p. 1081f., for a similar setup of price changes.
[298] $a$ is meant as abbreviation for $ante$.

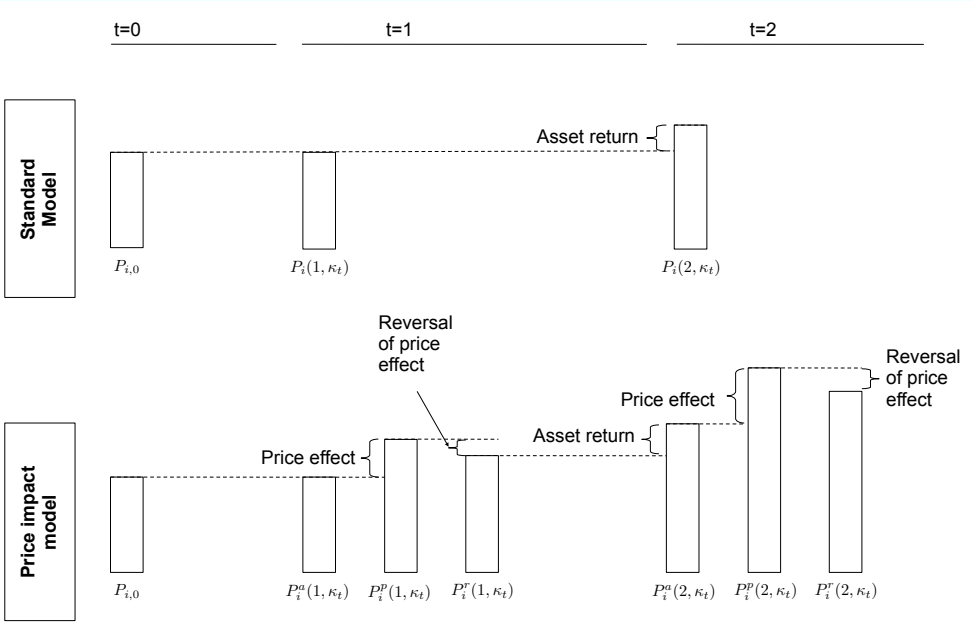



Figure 6.1.: Standard model vs. model with initial price effect and reversal. (Source: Own illustration)

2. A price $P_i^p$ at which the transaction is executed that incorporates the price effect due to the transaction.[299]

3. A price $P_i^r$ after the transaction has been executed and the price effect represented in $P_i^p$ has reversed.[300]

Figure 6.1 accompanies the definition and discussion of the following price process. It displays the single-period case but can easily be extended to the multiperiod case as well. The upper part of the illustration shows the standard case without price effects. The price $P_i(1,\kappa_t)$ in $t=1$ is equal to an initial price $P_{i,0}$ given as external value. The price in $t=2$ develops from $P_i(1,\kappa_t)$ according to equation 4.20 as $P_i(2,\kappa_t) = P_i(1,\kappa_t) \cdot s_i(k)$. The difference between these prices is considered as asset return. The lower part of this figure describes the price process with price effects from transactions and will be discussed in the remaining section.

The prices $P_i^a$, $P_i^p$, and $P_i^r$ can be defined as follows. Let $P_i(t-1,\kappa_t)$ be the - later to be defined - price in $t-1$ from which the price in $t$ develops. The investor

[299] $p$ is meant as abbreviation for $post$.
[300] $r$ is meant as abbreviation for $reversed$.

then arrives in $t$ with a price

$$P_i^a(t,\kappa_t) = \begin{cases} P_{i,0} & \text{if } t=1 \\ P_i(t-1,\kappa_t)\cdot s_i(k) & \text{if } t>1 \end{cases} \qquad i=1,\ldots,n.$$

This is the price before transactions have been executed. It is equal to $P_{i,0}$ in $t=0$, which is given as an external value and indicated in figure 6.1. The price $P_i(t-1,\kappa_t)$ for $t>1$ is the price of the previous period which changes due to the return factor $s_i(k)$. This price will be specified below as the price $P_i^r(t-1,\kappa_t)$ after price reversal for $t>1$. Generally, the calculation of the price $P_i^a$ follows the one in the standard model without price effects.

Now let

$$U_i(t,\kappa_t) = \begin{cases} N_i(1,\kappa_t) - N_{i,0} & \text{if } t=1 \\ N_i(t,\kappa_t) - N_i(t-1,\kappa_t) & \text{if } t>1 \end{cases}$$

be the transaction necessary at time $t$ to hold $N_i(t,\kappa_t)$ number of shares of asset $i$ if the holding in $t-1$ is $N_i(t-1,\kappa_t)$. For $t=1$ the holding $N_{i,0}$ represent the initial holdings given as the external parameter. These holdings are provided to the investor.

Let the function $q_i(U_i(t,\kappa_t), P_i^a(t,\kappa_t))$ be the price impact function that defines the effect of a transaction on the asset price. With this price impact function, the new price immediately after transactions can be defined as

$$P_i^p(t,\kappa_t) = q_i\left(U_i(t,\kappa_t), P_i^a(t,\kappa_t)\right) \qquad i=1,\ldots,n$$

In figure 6.1, the prices $P_i^p(1,\kappa_t)$ and $P_i^p(2,\kappa_t)$ follow the prices before transactions $P_i^a(1,\kappa_t)$ and $P_i^a(2,\kappa_t)$. The indicated difference is due to the price effect from, in this example, purchasing transactions $U_i(1,\kappa_t)$ and $U_i(2,\kappa_t)$ respectively.

After the transaction is executed at this price, the price reverses according to a price reversal function $r_i\left(U_i(t,\kappa_t), P_i^p(t,\kappa_t)\right)$ and results in the new price

$$P_i^r(t,\kappa_t) = r_i\left(U_i(t,\kappa_t), P_i^p(t,\kappa_t)\right) \qquad i=1,\ldots,n$$

Both, the price impact function $q_i$ as well as the reversal function $r_i$ are specific for each asset and can, therefore, differ. This price $P_i^r(t,\kappa_t)$ after price reversal in each period provides the basis for the price $P_i^a(t+1,\kappa_t)$ before price effects in the following period. In figure 6.1, the prices after price reversal follow the

prices $P_i^p(1, \kappa_t)$ and $P_i^p(2, \kappa_t)$. The difference indicates the reversal of price effects. Because the transactions $U_i(1, \kappa_t)$ and $U_i(2, \kappa_t)$ are assumed to be purchases, the price reversal is negative. The price $P_i^r(1, \kappa_t)$ builds the basis for the price $P_i^a(2, \kappa_t)$ in the following period.

In the later analysis, three cases of price reversals will be considered: no price reversal, partial price reversal, and total price reversal. In the first case, the prices after transaction $P_i^p(t, \kappa_t)$ and after price reversal $P_i^r(t, \kappa_t)$ are equal. The price effect is permanent and not lost in future periods but is added to the return of the asset. In case of total price reversal, the price after price reversal $P_i^r(t, \kappa_t)$ is equal to the price before transaction $P_i^a(t, \kappa_t)$ of the same period. In this case, the price effect is fully temporary and comparable to additional transaction costs. The case of partial price reversal in which the price after price reversal $P_i^r(t, \kappa_t)$ is located somewhere in between $P_i^a(t, \kappa_t)$ and $P_i^p(t, \kappa_t)$ contains both components. This price effect is partly temporary and partly permanent.

It should be noted that the price process above might not necessarily be arbitrage-free.[301] Using feasible transactions, it might be possible to create profit without initial capital. However, the conditions of an arbitrage-free market and the role of price effects on arbitrage opportunities are not the focus of this work.[302]

### Price Impact Functions

Two alternative price impact functions will be considered. In order to study the general influence of price effects on the portfolio policy, a linear price impact function will be used. The linear price impact function is defined as

$$q_i^{lin}\left(U_i(t, \kappa_t), P_i^a(t, \kappa_t)\right) = \left(1 + l_i \cdot U_i(t, \kappa_t)\right) \cdot P_i^a(t, \kappa_t)$$

with $l_i \in \mathbb{R}^+$. The positivity of $l_i$ implies the realistic scenario that purchases lead to an increase of prices and sales to a decrease of prices. The new price $P_i^p(t, \kappa_t)$ depends linearly on the size of the transaction. The factor $l_i$ determines the level of influence of the transaction. The corresponding reversal function $r_i^{lin}$ is defined as

$$r_i^{lin}\left(U_i(t, \kappa_t), P_i^p(t, \kappa_t)\right) = \left(\frac{1}{1 + \rho_i \cdot l_i \cdot U_i(t, \kappa_t)}\right) \cdot P_i^p(t, \kappa_t)$$

[301] Cf. Pliska (1997), p. 92, for a definition of arbitrage opportunities in a multiperiod securities market.

[302] Cf. e.g., Jarrow (1994) and Jarrow/Protter (2005). Parameter settings in numerical solutions are set to avoid arbitrage opportunities.

with $\rho_i \in [0,1]$. For $\rho_i = 1$ this leads to a complete reversal of the price effect, $\rho_i = 0$ implies no reversal at all. The factor is constrained on the interval $[0,1]$ in order to ensure that the price reversal does not exceed the original price effect and that the reversal does not further increase the price effect.

As an alternative price impact function a concave hyperbolic tangent function will be used as indicated by Plerou et al. (2002).[303] It is defined as

$$q_i^{tanh}\left(U_i(t,\kappa_t), P_i^a(t,\kappa_t)\right) = \left(1 + \eta_i \cdot \tanh(\theta_i \cdot U_i(t,\kappa_t))\right) \cdot P_i^a(t,\kappa_t)$$

with two parameters $\eta_i \in \mathbb{R}^+$ and $\theta_i \in \mathbb{R}^+$. Correspondingly the reversal function is defined as

$$r_i^{tanh}\left(U_i(t,\kappa_t), P_i^p(t,\kappa_t)\right) = \left(\frac{1}{1 + \rho_i \cdot \eta_i \cdot \tanh(\theta_i \cdot U_i(t,\kappa_t))}\right) \cdot P_i^p(t,\kappa_t).$$

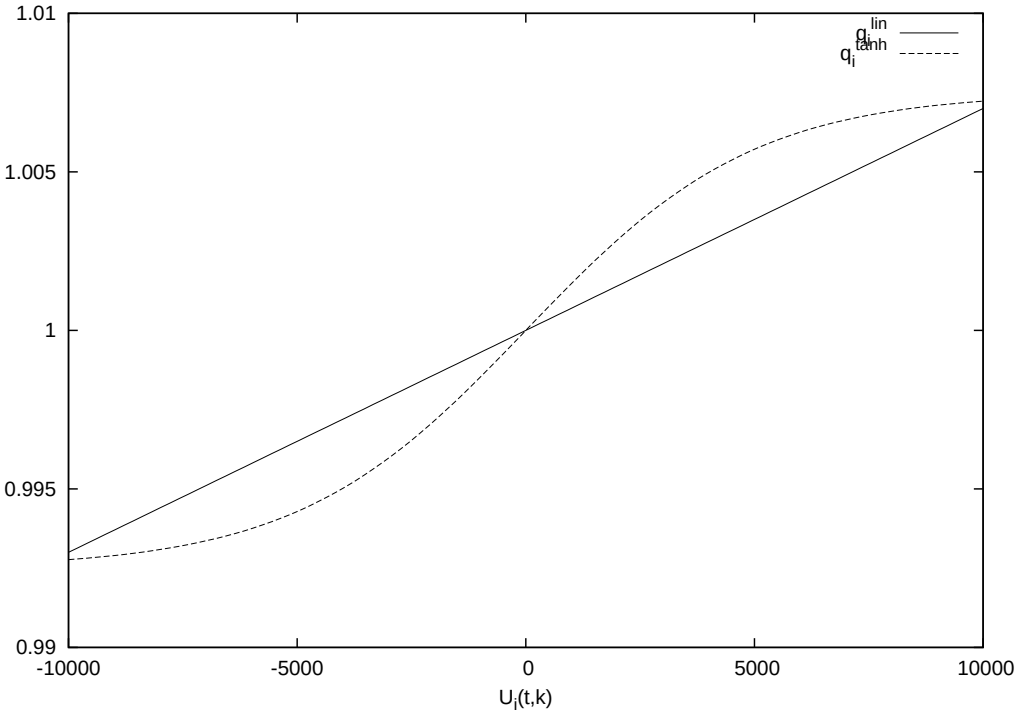
Figure 6.2 illustrates the price impact functions with parameter values $l_i = 7 \cdot 10^{-7}$, $\eta_i = 7.5 \cdot 10^{-3}$ and $\theta_i = 2 \cdot 10^{-4}$. Kraus/Stoll (1972) reported for transactions with more than 10,000 shares an average price decrease of $0.43\%$ for sales and an average increase of $0.66\%$ for purchases.[304] With these values in mind, parameter values are set so that a transaction of 10,000 shares leads to a price increase of about $0.6\%$ to $0.7\%$. This is only an approximation of the real price effect and can be considered as the upper bar for price effects because Kraus/Stoll (1972) use all transactions with more than 10,000 shares. However, this determination of parameter values is sufficient to allow a general understanding of price effects. In specific models below, these parameter values and variations will be used.

### State Process

The standard model in 4.3 used two state process definitions for portfolio values before transactions $X^a(t,\kappa_t)$ and after transactions $X^p(t,\kappa_t)$. These definitions need to be redefined considering price effects. Let the investor be given an initial portfolio $\sum_{i=1}^{n} N_{i,0} \cdot P_{i,0}$. The equivalent to $X^a(t,\kappa_t)$in the context with price effects

[303] Cf. section 3.3.4. The description of price effects with such a function might not fit the price effect in the short term precisely. However, the hyperbolic tangent function has the advantage of not being discontinuous in zero which is helpful for the numerical optimization algorithm used.

[304] Cf. Kraus/Stoll (1972), p. 575f.




(a) Price impact functions for $q_i^{lin}$ and $q_i^{tanh}$.

Figure 6.2.: Price impact functions. (Source: Own illustration)

is

$$X_c^{a,p}(t,\kappa_t) = \begin{cases} X_0 & \text{if } t = 1 \\ \sum_{i=1}^n N_i(t-1,\kappa_t) \cdot P_i^p(t,\kappa_t) & \text{if } t > 1. \end{cases}$$

This is the value of the portfolio value using previous portfolio holdings and current prices after transactions. In this case the investor is given a fixed amount $X_0$ of capital in cash. If the investor is given a portfolio of value $N_{i,0} \cdot P_{i,0}$ as an endowment, the portfolio value is defined as

$$X_{pf}^{a,p}(t,\kappa_t) = \begin{cases} \sum_{i=1}^n N_{i,0} \cdot P_i^p(t,\kappa_t) & \text{if } t = 1 \\ \sum_{i=1}^n N_i(t-1,\kappa_t) \cdot P_i^p(t,\kappa_t) & \text{if } t > 1. \end{cases}$$

In this case, the investor has to make initial transactions $U_i(1,\kappa_t) = N_i(1,\kappa_t) - N_{i,0}$ in $t = 1$ which will influence the price. Note that the portfolio value $X^{a,p}$ is purely calculatory, a portfolio with such value never exists.

The equivalent to $X^p$ in the context with price effects is

$$X^{p,p}(t, \kappa_t) = \sum_{i=1}^{n} N_i(t, \kappa_t) \cdot P_i^p(t, \kappa_t).$$

It is calculated with the new holdings in $t$ and prices after the transaction. $X^{a,p}$ and $X^{p,p}$ will form the basis for the self-financing constraint.

In addition to these definitions, the portfolio value before transactions and before price effects will be used later. If the investor is endowed with capital $X_0$ in cash, this portfolio value can be defined as

$$X_c^{a,a}(t, \kappa_t) = \begin{cases} X_0 & \text{if } t = 1 \\ \sum_{i=1}^{n} N_i(t-1, \kappa_t) \cdot P_i^a(t, \kappa_t) & \text{if } t > 1. \end{cases} \tag{6.1}$$

In case of endowment with an initial portfolio this changes to

$$X_{pf}^{a,a}(t, \kappa_t) = \begin{cases} \sum_{i=1}^{n} N_{i,0} \cdot P_{i,0} & \text{if } t = 1 \\ \sum_{i=1}^{n} N_i(t-1, \kappa_t) \cdot P_i^a(t, \kappa_t) & \text{if } t > 1. \end{cases}$$

## 6.2.2. Optimization Problem

### Compensation and Maximization Criterion

Unlike in the portfolio model outlined in section 4.3 the criterion to maximize in the context of delegated portfolio management is the expected utility of a periodical management fee. Following the continuous time model in section 5.3, the management fee is calculated as a fraction of total assets under management

$$c(t, \kappa_t) = \begin{cases} 0 & \text{if } t = 1 \\ \lambda X(t, \kappa_t) & \text{if } t > 1 \end{cases}$$

where $X(t)$ are the assets under management. The management fee is calculated at the end of each period and deducted from the portfolio value.[305] The investor does not receive management fee in $t = 1$ because they initially choose an optimal portfolio at that time. With price effects and three different definitions of portfo-

[305] This is comparable to the financing of consumption in Merton (1990), p. 125. The consumption is financed with the difference of old and new portfolio holdings, i.e., $-C(t)h = \sum_{i=1}^{n} [N_i(t) - N_i(t-1)] \cdot P_i(t)$. Cf. Merton (1990), p. 125, eq. 5.8.

lio values above, it must be considered which of the portfolio values is relevant for the calculation of management fees. If $X^{a,p}$ or $X^{p,p}$ are used as the basis then the transaction to finance the management fee influences the portfolio value which then again influences the compensation and so forth. The investor would try to maximize expected utility from compensation but this solution will affect the compensation through the price effect, which does not appear to be a realistic scenario. In contrast, it appears reasonable to require a separation between the calculation of the management fee and its deduction from the portfolio value. Two options are available.

First, the management fee could be based on the previous period's portfolio value $X^{a,p}(t-1,\kappa_t)$ before transaction and after price effects. It would be financed one period later in $t$ which introduces a time gap of one period.

Second, the management fee could be based on the current period's portfolio value $X^{a,a}(t,\kappa_t)$ before transaction and before price effects. This portfolio value represents what would be considered as the success of the portfolio strategy from $t$ to $t+1$. It contains the price effect and price reversal from the previous period's transactions and the development of the portfolio value resulting from the portfolio structure. The management fee would be financed in $t$, which would be reflected in the portfolio value $X^{p,p}(t,\kappa_t)$ after transaction and after price effects. But this financing transaction would not influence the level of compensation.

I will generally follow the second option because it allows the influence of the management fee in a single-period model to be investigated. For the first option, a single-period model would not be possible. Hence

$$c(t,\kappa_t) = \begin{cases} 0 & \text{if } t = 1 \\ \lambda X^{a,a}(t,\kappa_t) & \text{if } t > 1 \end{cases}$$

With the CRRA utility function

$$L(t, X^{a,a}(t,\kappa_t), N(t,\kappa_t)) = \frac{1}{\gamma}\left(c(t,\kappa_t)\right)^{\gamma},$$

the criterion to maximize is

$$J(X_0) = \sum_{t=1}^{T} \sum_{\kappa_t=1}^{K^{T-1}} \left[ \tilde{p}(\kappa_t) \frac{1}{\gamma}\left(c(t,\kappa_t)\right)^{\gamma} \right].$$

### Performance-Related Capital Flow

For the calculation of capital flow, I will use the same portfolio value $X^{a,a}(t,k)$.[306]

Performance-based capital flow can be defined as

$$f(t,\kappa_t) = \begin{cases} 0 & \text{if } t = 1 \\ \delta\left(X^{a,a}(t,\kappa_t) - \sum_{i=1}^{n} \frac{b_i \cdot X^{a,a}(t-1,\kappa_t)}{P_i^a(t-1,\kappa_t)} \cdot P_i^a(t,\kappa_t)\right) & \text{if } t > 1. \end{cases} \tag{6.2}$$

Here capital flow starts in $t = 2$ similar to management fee. From that point in time on, the portfolio manager faces capital flow as a linear function of their performance compared to a benchmark portfolio $b = (b_1, \ldots, b_n)'$. The benchmark portfolio invests fixed percentages $b_i, i = 1, \ldots, n$ in $n$ assets. The factor $\frac{b_i \cdot X^{a,a}(t-1,\kappa_t)}{P_i^a(t-1,\kappa_t)}$ represents the number of shares invested in the $i$-th asset. The value of the benchmark portfolio in $t$ at prices $P_i^a(t,\kappa_t)$ before transactions is compared to the portfolio value $X^{a,a}(t,\kappa_t)$.

With performance-related capital flow, compensation changes to

$$c(t,\kappa_t) = \begin{cases} 0 & \text{if } t = 1 \\ \lambda\left(X^{a,a}(t,\kappa_t) + f(t,\kappa_t)\right) & \text{if } t > 1. \end{cases} \tag{6.3}$$

This definition incorporates compensation for current performance-based capital flow in $t$.

### Maximization Criterion

Hence, the criterion to maximize is

$$J(X_0) = \sum_{t=1}^{T} \sum_{\kappa_t=1}^{K^{T-1}} \left[\tilde{p}(\kappa_t) \frac{1}{\gamma} \left(c(t,\kappa_t)\right)^{\gamma}\right]$$

using the definition of compensation from equation 6.3 and the definition of capital flow from equation 6.2. The self-financing constraint in the situation with price effects is

$$X^{a,p}(t,\kappa_t) + f(t,\kappa_t) = X^{p,p}(t,\kappa_t) + c(t,\kappa_t) \qquad t = 1, \ldots, T-1,\ \kappa_t = 1, \ldots, K^{t-1}.$$

[306] I will restrict capital flow to performance-based flow considering the different explanatory power of both phenomena regarding the investment strategies of professional portfolio managers in the continuous time models in chapter 5.

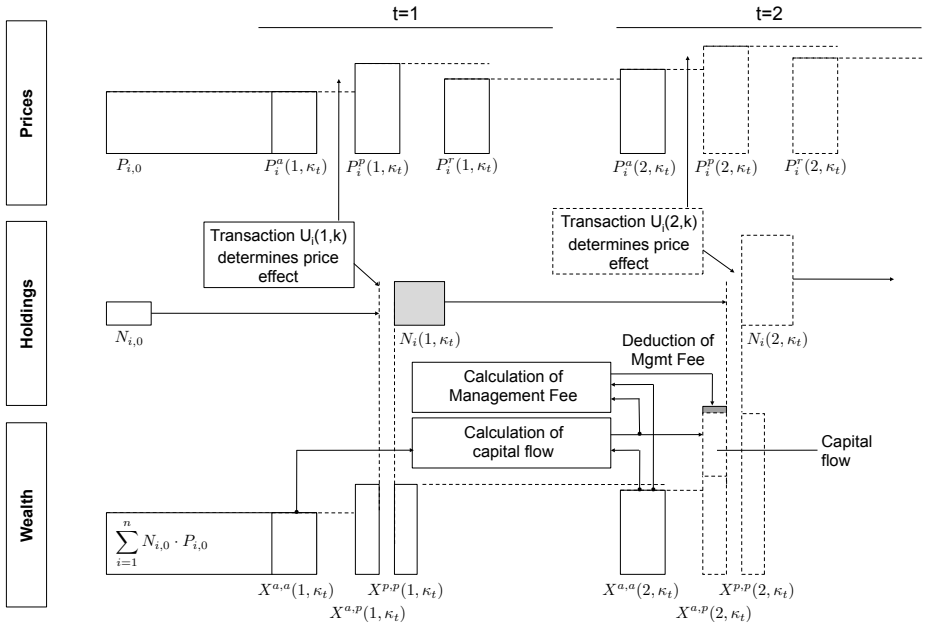



Figure 6.3.: General single-period model setup with price effects. The dashed boxes become relevant in a multiperiod model. The investor optimizes the dark shaded box, the management fee, by choosing the optimal holdings represented by the light grey shaded box. (Source: Own illustration)

The left-hand side, the portfolio value $X^{a,p}(t,\kappa_t) = \sum_{i=1}^{n} N_i(t-1,\kappa_t) \cdot P_i^p(t,\kappa_t)$ (for $t > 1$) before transactions and after price effects and the capital flow $f(t,\kappa_t)$, represent the available capital for refinancing. The available capital must be equal to the sum of the new portfolio value $X^{p,p}(t,\kappa_t) = \sum_{i=1}^{n} N_i(t,\kappa_t) \cdot P_i^p(t,\kappa_t)$ (for $t > 1$) and the withdrawn management fee $c(t,\kappa_t)$. Both portfolio values are based on asset prices after price effects because it is assumed that restructuring of the portfolio, i.e., the purchases and sales of assets, is done simultaneously.

### 6.2.3. Summary of Model Setup

Figure 6.3 summarizes the single-period model setup. In the upper third, it shows the price process, below in the middle the holdings process. The lower third displays the state process, the portfolio value. It is simplified by depicting only one price and one holding process representing the processes for all assets contained in the

portfolio. The setup easily extends to several periods by extrapolating the $t = 2$ prices, holdings, and portfolio values into future periods.

The investor is endowed with an initial wealth consisting of the holdings $N_{i,0}$ with price $P_{i,0}$. The price $P_i^a(1, \kappa_t)$ is equal to $P_{i,0}$ and $X^{a,a}(1, \kappa_t)$ is equal to $\sum_{i=1}^{n} N_{i,0} \cdot P_{i,0}$. At the end of the period, the portfolio manager receives additional capital flow based on their performance compared to a benchmark. This is added to the available capital in $t = 2$. The objective is to maximize the expected utility of the management fee that is calculated based on $X^{a,a}(2, \kappa_t) + c(2, \kappa_t)$ by choosing the optimal holdings $N_i(1, \kappa_t)$. These holdings together with the initial holdings $N_{i,0}$ determine the transactions necessary to carry out in $t = 1$. The transactions $U_i(1, \kappa_t)$ determine the price effect that is reflected in prices $P_i^p(1, \kappa_t)$.

The figure illustrates that the price in $t = 1$ after the transaction $P_i^p(1, \kappa_t)$ is relevant to calculate the portfolio values before and after the transaction, $X^{a,p}(1, \kappa_t)$ and $X^{p,p}(1, \kappa_t)$, which must be equal in the first period because compensation and capital flow are $0$. For the single-period model, there is no self-financing constraint $X^{a,p}(2, \kappa_t) + f(t, \kappa_t) = X^{p,p}(2, \kappa_t) - c(t, \kappa_t)$ in $t = 2$, because the optimization objective is already in $t = 2$ being based on $X^{a,a}(2, \kappa_t)$.

This implies that the investor ends with an undetermined portfolio structure because there is no requirement to finance the management fee or invest the capital flow in the last period with any transaction. In principle, the self-financing requirement for the last period could easily be added. This would require including the financing of the management fee and investment of capital flow into the optimization problem by optimizing the holdings $N_i(2, \kappa_t)$ as well. However, this optimization would not target a risk-return optimal portfolio structure but only the optimal financing of the management fee and investment of capital flow. Unreported results show that the portfolio structure in $t = 2$ will show extreme and hence unrealistic positions. Therefore, I omit the self-financing constraint for the last period.

## 6.3. Numerical Solutions for Portfolio Optimization with Price Effects

After the definition of the general model in the previous section, this section will present numerical results from various alternative model setups. The general setup follows the trinomial model with two risky assets in section 4.3.2 and uses the same

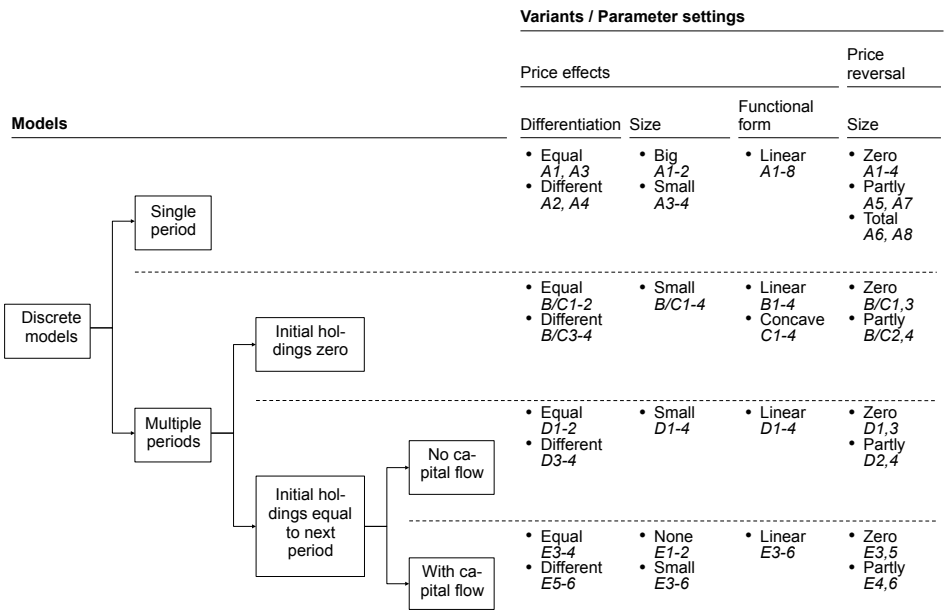



Figure 6.4.: Overview of numerical models with price effects and modifications. (Source: Own illustration)

parameter values. An overview over the different models and their modifications is provided in figure 6.4. In total, four different kinds of models will be considered. The first three models will have no capital flow. This will be added in the fourth model.

First, a single-period model will provide initial insight into the influence of price effects (section 6.3.1, model A). Then, three different multiperiod models will be analyzed. I will start with a two-period model without initial holdings where the investor is endowed with cash (section 6.3.2, models B and C). This model will provide insights about the allocation of transactions over time. The next model assumes that the investor can choose their optimal portfolio in $t = 1$ without price effects (section 6.3.3, model D). This model allows the understanding of the pure price effect from the portfolio adjustments due to the stochastic development of prices. The last model incorporates capital flow into the previous model (section 6.3.4, model E). Results for this model reflect the joint influence of compensation, capital flows and price effects.

Different modifications in terms of parameter setups of these basic model types

will be investigated. Regarding price effects, I will analyze the effect of equal or different price effects for both assets. In reality, assets of small companies with smaller capitalization often show a higher standard deviation. Therefore, the price effect of the first asset with a higher standard deviation is modeled with higher price effects. The size of the price effect will be varied to be big or small. Regarding the functional form, I will consider linear and concave price impact functions in the second model to analyze the dynamic effect of these different functional forms. For all other models, only the linear function will be considered. Regarding price reversal, models will be simulated with zero, partial, and total price reversal. The fourth model will additionally incorporate capital flow as a linear function of past performance compared to a benchmark.

The discussion in this chapter will focus on the influence of price effects. Hence, the comparison of solutions presented here with optimal portfolio holdings of primary investors as outlined in the continuous time case in chapter 5, including efficiency portraits, will be very limited. In this regard, results for the discrete models in this chapter are expected to be very similar to those in chapter 5.

### 6.3.1. Single-Period Model

The single-period model setup follows the general model structure outlined above in section 6.2.3. Instead of an initial portfolio, the portfolio manager is endowed with a certain amount of cash $X_0$ that they need to invest for a single-period. A linear price impact function will be used. In a single-period context, a linear price impact function is sufficient to understand the general implications of price effects. The nonlinear price impact function becomes interesting in a multiperiod setup where the form of the price impact function can potentially alter the allocation of transactions over time. This issue will be discussed in section 6.3.2. The general parameter settings are identical to the standard portfolio model in table 4.1.

The analyzed models vary in terms of the differentiation of the price effect for each asset, the size of the price effect, and the reversal of the price effect. Table 6.1 summarizes the characteristics and parameter settings for the models A1-A8. The characteristics have the following meaning:

- *Size:* Refers to the size of the price effect. Big size is equal to the values described in section 6.2.1 and represents an almost realistic scenario. The small price effect is a magnitude smaller, i.e., a tenth of the big.

| | $S$ | $A1$ | $A2$ | $A3$ | $A4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | big | big | small | small |
| Differentiation | n.a. | equal | different | equal | different |
| Reversal | n.a. | none | none | none | none |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $7 \cdot 10^{-7}$ | $3.5 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0 | 0 | 0 |
| *V*ariable | | | | | |
| $w_1(1,1)$ | 38.89% | 38.89% | 38.89% | 38.89% | 38.89% |
| $w_2(1,1)$ | 61.11% | 61.11% | 61.11% | 61.11% | 61.11% |
| $N_1(1,1)$ | 3889.27 | 3878.36 | 3878.36 | 3887.83 | 3887.83 |
| $N_2(1,1)$ | 6110.73 | 6085.19 | 6098.10 | 6108.50 | 6109.80 |
| $U_1(1,1)$ | 3889.27 | 3878.36 | 3878.36 | 3887.83 | 3887.83 |
| $U_2(1,1)$ | 6110.73 | 6085.19 | 6098.10 | 6108.50 | 6109.80 |
| $P_1^p(1,1)$ | 100.00 | 100.27 | 100.27 | 100.03 | 100.03 |
| $P_1^r(1,1)$ | 100.00 | 100.27 | 100.27 | 100.03 | 100.03 |
| $P_2^p(1,1)$ | 100.00 | 100.43 | 100.21 | 100.04 | 100.02 |
| $P_2^r(1,1)$ | 100.00 | 100.43 | 100.21 | 100.04 | 100.02 |

| | $S$ | $A5$ | $A6$ | $A7$ | $A8$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | big | big | small | small |
| Differentiation | n.a. | different | different | different | different |
| Reversal | n.a. | partial | total | partial | total |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $3.5 \cdot 10^{-7}$ | $3.5 \cdot 10^{-7}$ | $3.5 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0.2 | 1 | 0.2 | 1 |
| *V*ariable | | | | | |
| $w_1(1,1)$ | 38.89% | 35.48% | 33.96% | 38.12% | 36.42% |
| $w_2(1,1)$ | 61.11% | 64.52% | 66.04% | 61.88% | 63.58% |
| $N_1(1,1)$ | 3889.27 | 3539.17 | 3387.52 | 3811.26 | 3641.11 |
| $N_2(1,1)$ | 6110.73 | 6437.56 | 6589.26 | 6186.38 | 6356.55 |
| $U_1(1,1)$ | 3889.27 | 3539.17 | 3387.52 | 3811.26 | 3641.11 |
| $U_2(1,1)$ | 6110.73 | 6437.56 | 6589.26 | 6186.38 | 6356.55 |
| $P_1^p(1,1)$ | 100.00 | 100.25 | 100.24 | 100.03 | 100.03 |
| $P_1^r(1,1)$ | 100.00 | 100.20 | 100.00 | 100.02 | 100.00 |
| $P_2^p(1,1)$ | 100.00 | 100.23 | 100.23 | 100.02 | 100.02 |
| $P_2^r(1,1)$ | 100.00 | 100.18 | 100.00 | 100.02 | 100.00 |

Table 6.1.: Parameter values and results for the single-period model with price effects. The model $S$ refers to the standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

- *Differentiation:* Indicates different price effects for both assets. Equal price effect uses the same parameters for both assets. Different price effects reduce the price effect for the second asset to one half.
- *Reversal:* Marks the kind of reversal of price effects. Models with "none" reversal show no reversal of price effects. "Partial" price reversal uses a reversal parameter of $0.2$, "total" reversal a parameter of $1$.

Table 6.1 provides the numerical solutions of single-period models with price ef-

fects. The upper part shows the numerical solutions with permanent price effect, the lower part with price reversal. These and the following tables share an identical structure: at the top, various characteristics of the models are listed. Below these characteristics, important parameter settings related to these characteristics are displayed.[307] The remaining part of the table contains solution values for various variables, usually the optimal share of wealth, optimal holdings, resulting transactions, prices after transaction, and prices after reversal. The first column with numerical values repeats the solutions of the standard model as a reference.

Models A1 to A4, all models with permanent price effects, highlight the first finding: a permanent price effect has no influence on the portfolio structure in the single-period context. This is plausible because the investor maximizes their portfolio by considering the return on asset holdings in terms of absolute value and not in terms of number of assets.

The investor maximizes the expected utility of their compensation which is equivalent to maximizing the expected terminal wealth due to the monotony of the utility and compensation function. The portfolio value in $t = 2$ which is relevant for compensation from equation 6.1 is $X^{a,a}(2, \kappa_t) = \sum_{i=1}^{n} N_i(1, \kappa_t) \cdot P_i^a(2, \kappa_t)$. Dissolving the price yields $\sum_{i=1}^{n} N_i(1, \kappa_t) \cdot P_i^r(1, \kappa_t) \cdot s_i(2, k)$. Because price effects are assumed to be permanent, this is equal to $\sum_{i=1}^{n} N_i(1, \kappa_t) \cdot P_i^p(1, \kappa_t) \cdot s_i(2, k)$. From this equation, it becomes clear that the investor chooses the share of wealth $N_i(1, \kappa_t) \cdot P_i^p(1, \kappa_t)$ invested in the $i$-th asset by selecting the appropriate number of stocks $N_i(1, \kappa_t)$. Because the investor is endowed with cash holdings, the portfolio value after transactions $X^{p,p}(1, \kappa_t)$ is equal to the portfolio value before transactions $X^{a,p}(1, \kappa_t)$. The investor invests the same amount of capital and receives the return based on the absolute amount invested in each asset. Hence, the central conditions of the optimization problem have not changed compared to the standard problem without price effects. Under these conditions, the solution must be equal.

The situation is different if price effects are not permanent and price reversal occurs. Given the parameter values, the investor purchases assets, hence $P_i^r(1, \kappa_t) < P_i^p(1, \kappa_t)$. In that case, the value of the portfolio after price reversals $\sum_{i=1}^{n} N_i(1, \kappa_t) \cdot P_i^r(1, \kappa_t) \cdot s_i(2, k)$ is smaller than after price effects $\sum_{i=1}^{n} N_i(1, \kappa_t) \cdot P_i^p(1, \kappa_t) \cdot s_i(2, k)$. Compared to the situation without price effects or with permanent price effects, the investor incurs a loss of $\sum_{i=1}^{n} N_i(1, \kappa_t) \cdot (P_i^p(1, \kappa_t) - P_i^r(1, \kappa_t)) \cdot s_i(2, k)$ at the end of the period. Hence, it is rational to

[307] For other parameter settings, cf. table 4.1.

| | $S$ | $B1$ | $B2$ | $B3$ | $B4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |

Table 6.2.: Parameter values for the two-period model with initial cash endowment and linear price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

reduce the amount invested in the asset that shows the bigger price effect and reversal.

This is reflected in the results of the models A5 to A8 in table 6.1. The larger the price effect and the larger the price reversal, the less capital is invested in the first asset that has the bigger price effect. For example, in model A6 the investor reduces their investment in the first asset by 5 percentage points compared to the solution without price effects and price reversal. However, the situation changes completely in a multiperiod setting as is described in the following section.

### 6.3.2. Multiperiod Model with Cash Endowment

The multiperiod model with cash endowment extends the single-period model to two periods. The investor receives a fixed amount of capital $X_0$ as endowment which they can only invest in two assets. Their objective is to maximize the expected utility from the periodical compensation that is calculated based on the portfolio values $X^{a,a}(1, \kappa_t)$ and $X^{a,a}(2, \kappa_t)$. I will present results for models similar to the single-period case (models B1-B4). In addition, the effect of a concave price impact function will be analyzed (models C1-C4).

Table 6.2 provides the parameter values and figure 6.5 the results for different models B1-B4 with a linear price impact function.[308] Generally, price effects lead to a time dependency of the optimal portfolio structure. In addition, price reversals introduce a path dependency.

With permanent and equal price effects for both assets (model B1), the investor

[308] More detailed numerical results are shown in table B.1 in section B.

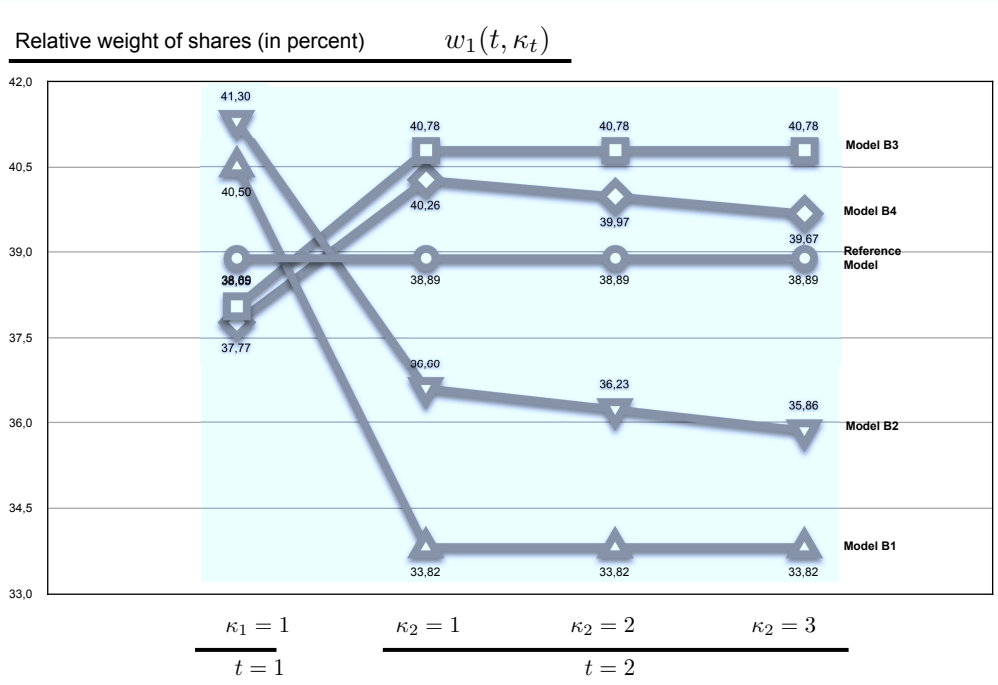



Figure 6.5.: Results for two-period model with initial cash endowment and linear price impact function. (Source: Own illustration)

initially invests more in the first assets only to disinvest from that asset in $t = 2$ and hold much less than in the standard solution. This investment strategy turns around in case of permanent but different price effects for both assets (model B3). If the price effect associated with the first asset is higher, the investor initially invests less in the first asset only to increase their holdings above the standard solution in $t = 2$. The numerical solutions supply no information about the underlying logic and structure of this solution. Therefore, I can only conjecture the reasons.

Generally in a situation without price effects, the investor prefers the second asset, because it has the more appealing risk-return profile. The multiperiod context with price effects allows the investor to influence the risk-return profile. By selling the first asset in $t = 2$, they reduce the overall return over the period from $t = 2$ to $t = 3$ via the price effect because the return is calculated based on the portfolio value before transactions in $t = 2$, i.e. $X^{a,a}(2, \kappa_t)$, and the portfolio value before transactions in $t = 3$, i.e. $X^{a,a}(3, \kappa_t)$, including price effects in $t = 2$. Because the return in $t = 3$ is stochastic from the perspective of time $t = 2$, any transaction affects the return in all three states of nature, hence it decreases the standard deviation as well. On the other hand, by purchasing the second asset in $t = 2$, the investor increases the return and risk of that asset.

From standard portfolio theory, it is well-known that the return enters the optimal portfolio equation linearly whereas the risk, the standard deviation, enters the equation in squared form as variance, respectively covariance.[309] Hence, if both, risk and return are scaled up linearly the risk-return profile becomes less preferable for purchases and more preferable for sales. Therefore, by selling the first asset the portfolio manager can improve the risk-return profile of the first asset relative to the second asset which they purchase.

The model B3 gives results for a different situation. Here, the permanent price effect of the second, generally preferred, asset is smaller than the price effect of the first asset. It then depends on the set of parameters regarding risk, return, and covariance in the unknown structure of the optimal solution whether one or the other asset is preferred. With a price effect for the second asset that is half the size of the price effect for the first asset, it is preferable to invest in the first asset.

In the case of price reversal visible in models B2 and B4, the investor moves away from the optimal solution with permanent price effects towards the standard solution. As discussed in the previous section, price reversals convert price effects into additional cost. Because of this, in model B2 the investor increases the holdings in the first asset in $t = 2$ as its risk-return profile becomes less preferable. In analogy, in model B4 the investor decreases their investments to come closer to the solution without price effects.

Price reversals introduce a path dependency of the optimal portfolio structure. The investor invests more in "good" states and less in "bad" states, independently of the time dependency of the strategy. However, it remains unclear why the investor in $t = 1$ invests above the standard solution at equal price effects and below at different price effects. This might be due to a complex mix of risk-return profile of assets and the total risk of the portfolio over both periods.

Unreported results show that this overall investment strategy holds also in three and more period models. In all cases, the initial investment in model B1 is slightly above in $t = 1$, to fall below the standard solution without price effects in $t = 2$, and then further reduce till the final transaction. The more periods the model has, the lower the holdings are in the last period.

The two-period model represents a suitable basis to analyze the allocation of portfolio decisions over time with non-linear price impact functions. Table 6.3 provides the parameter values and figure 6.6 an overview of the model results with a concave

[309] Cf., for example, the solutions provided in sections 4.1.2 and 5.3.

| | $S$ | $C1$ | $C2$ | $C3$ | $C4$ |
|---|---|---|---|---|---|
| Characteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| Parameter | | | | | |
| $\eta_1$ | 0 | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ |
| $\theta_1$ | 0 | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ |
| $\eta_2$ | 0 | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $3.25 \cdot 10^{-3}$ | $3.25 \cdot 10^{-3}$ |
| $\theta_2$ | 0 | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |

Table 6.3.: Parameter values for the two-period model with initial cash endowment and concave price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

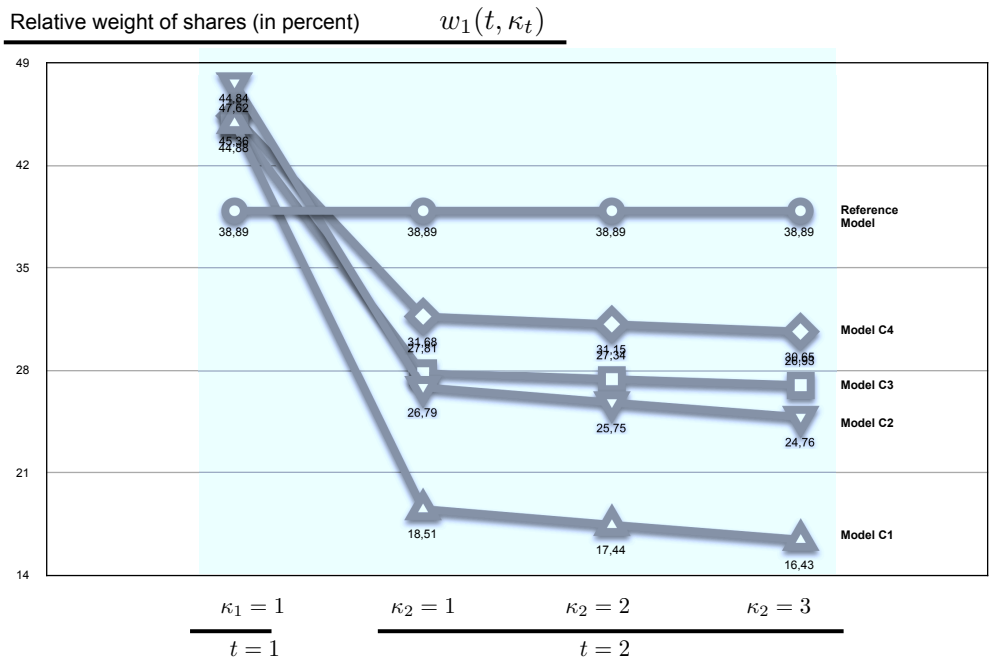



Figure 6.6.: Results for two-period model with initial cash endowment and concave price impact function. (Source: Own illustration)

utility function.[310]

Overall, the results are comparable to the results with a linear price impact function. Models C1 and C2 are similar to models B1 and B2. In both cases, the investor starts with an initially higher investment in the first asset to sell the asset in $t = 2$ and hold a much smaller portion of their portfolio in that asset. Though the absolute values are much more extreme with a concave price impact function.

Models C3 and C4 differ from B3 and B4 and are similar to C1 and C2. Here the difference of price effects between both assets was apparently not sufficient to make the first asset more attractive relative to the second. In both cases the investor initially invests more in the first asset to sell large portions of their holdings and keep a small position in $t = 2$. This effect is slightly reduced with partial price reversal.

These results are not surprising. A concave price impact function does not make it preferable to stretch large investments over a longer time period. This is due to the decreasing marginal costs associated with such types of functions. The price effect of large trades is relatively smaller compared to the effect of many small trades. Hence, it is hard to tell whether the more extreme positions that imply more extreme trades in models C1 to C4 are due to generally higher price effects or to a lower relative price effect of larger transactions. A convex price impact function might reverse this result. With a convex price impact function and its increasing marginal costs, it could be profitable to stretch transactions over a longer time period. However, a convex price impact function is not reported in empirical research.

The following section will present results with price effects only after the initial trade to investigate the implications of price effects from trades that are necessary to dynamically follow the optimal portfolio policy.

### 6.3.3. Multiperiod Model Without Initial Price Effect

The results in the previous section 6.3.2 above combined price effects from two sources: the initial purchase of assets and the transactions triggered by the stochastic development of prices. The first effect has been analyzed in the single-period model in section 6.3.1. In order to investigate the second effect, the two-period model will now be changed. At the first trading time $t = 1$, the investor can build their

[310] More detailed numerical results are shown in table B.2 in section B.

| | $S$ | $D1$ | $D2$ | $D3$ | $D4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |

Table 6.4.: Parameter values for the two-period model without price effect of initial transactions and linear price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

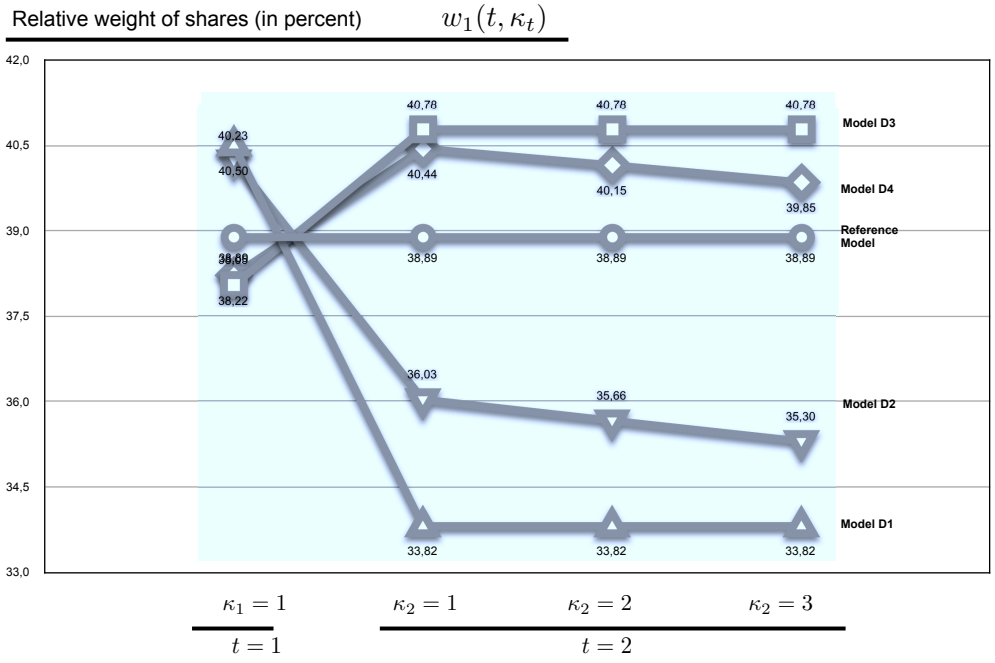



Figure 6.7.: Results for the two-period model without price effect of initial transactions and linear price impact function. (Source: Own illustration)

portfolio without any price effects. This is equivalent to as if the investor were given a portfolio whose structure they can determine. Due to technical reasons, the problem is realized in this second way. Because the given portfolio is equal to the one chosen in $t = 1$ no transaction is necessary in $t = 1$. Hence, it is $P_{i,0} = P_i^a(1,1) = P_i^p(1,1) = P_i^r(1,1)$ and $X^{a,a}(1,1) = X^{a,p}(1,1) = X^{p,p}(1,1)$.

Table 6.4 presents the parameter values and figure 6.7 the results of this model.[311] The model modifications considered here are the same as in the two-period model

[311] More detailed numerical results are shown in table B.3 in section B.

| | *S* | *E*1 | *E*2 | *E*3 | *E*4 | *E*5 | *E*6 |
|---|---|---|---|---|---|---|---|
| *C*haracteristic | | | | | | | |
| Size | n.a. | n.a. | n.a. | small | small | small | small |
| Differentiation | n.a. | n.a. | n.a. | equal | equal | different | different |
| Reversal | n.a. | n.a. | n.a. | none | partial | none | partial |
| *P*arameter | | | | | | | |
| $l_1$ | 0 | 0 | 0 | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | 0 | 0 | $7 \cdot 10^{-7}$ | $3.5 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0 | 0 | 0.2 | 0 | 0.2 |
| $\delta$ | 0 | 0.2 | 0.6 | 0.2 | 0.2 | 0.2 | 0.2 |

Table 6.5.: Parameter values for the two-period model without price effect of initial transactions and performance-related capital flow. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

with linear transaction costs and cash endowment. The results are almost identical to models B1 to B4. Depending on the difference of price effects, the investor initially purchases more in $t = 1$ to sell in $t = 2$ or vice versa.

While the results for permanent price effects are identical, the results with price reversal are slightly different. First, the effect of price reversal is smaller in $t = 2$. The tilt towards the standard portfolio is less distinctive without transaction costs in $t = 1$. This is valid for equal as well as for different price effects. Second, the effect of price reversal in $t = 1$ has the opposite direction. Instead of increasing the deviation from the standard solution, the holdings are slightly tilted back towards the standard solution.

The principle consequences of price effect and reversals have been discussed. The next section incorporates performance-related capital flow into the model.

### 6.3.4. Multiperiod Model with Capital Flow

This last section will present models that include performance-related capital flow and combine all three characteristics of portfolio managers described in chapter 3: compensation as a proportional management fee, performance-related capital flow, and price effects and reversals. There will be no price effects from transactions in $t = 1$ in order to analyze the dynamic effect of capital flow. However, it is not expected that results would considerably deviate from those presented here if the investor was endowed with cash as indicated in the previous section.

Table 6.5 provides parameter values and figure 6.8 and 6.9 numerical results.[312]

[312]More detailed numerical results are shown in table B.4 in section B.

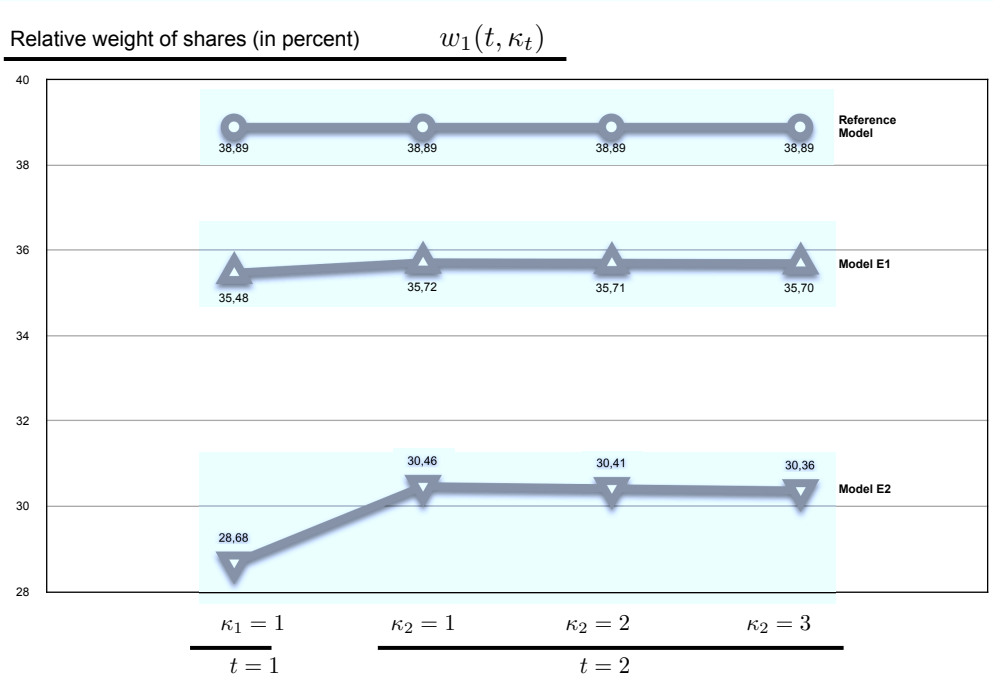



Figure 6.8.: Results for the two-period model with performance-related capital flow but without price effects. (Source: Own illustration)

The benchmark portfolio will be fixed throughout this section to be $b_1 = 0.2$ and $b_2 = 0.8$. The size of the capital flow may vary to be either $\delta = 0.2$ or $\delta = 0.6$. Models E1 and E2 investigate the isolated effect of performance-related capital flow. Models E3 to E6 then also include price effects and price reversal.

As evident from model E1, the effect of the benchmark portfolio is comparable to the continuous time model in chapter 5. Clearly, the holdings are tilted towards the benchmark portfolio from time $t = 1$ on. In addition, this effect becomes stronger for bigger capital flows as shown with model E2. However unlike in the continuous time case with linear performance-related flow, the portfolio holdings become time- and slightly state-dependent. The numerical results indicate that the benchmark effect is stronger at $t = 1$ than at the last trading chance in $t = 2$. This means that the portfolio deviates more strongly from the benchmark portfolio towards the end of the investment horizon.

I suspect that this is due to compound effects. If the investor deviates strongly from the benchmark portfolio in the beginning, they potentially suffer first from immediate outflow and second from smaller absolute returns in future periods. This effect becomes stronger for bigger capital flows. Unreported results from a four period model of type E1 show that the initial holdings further slightly decrease,

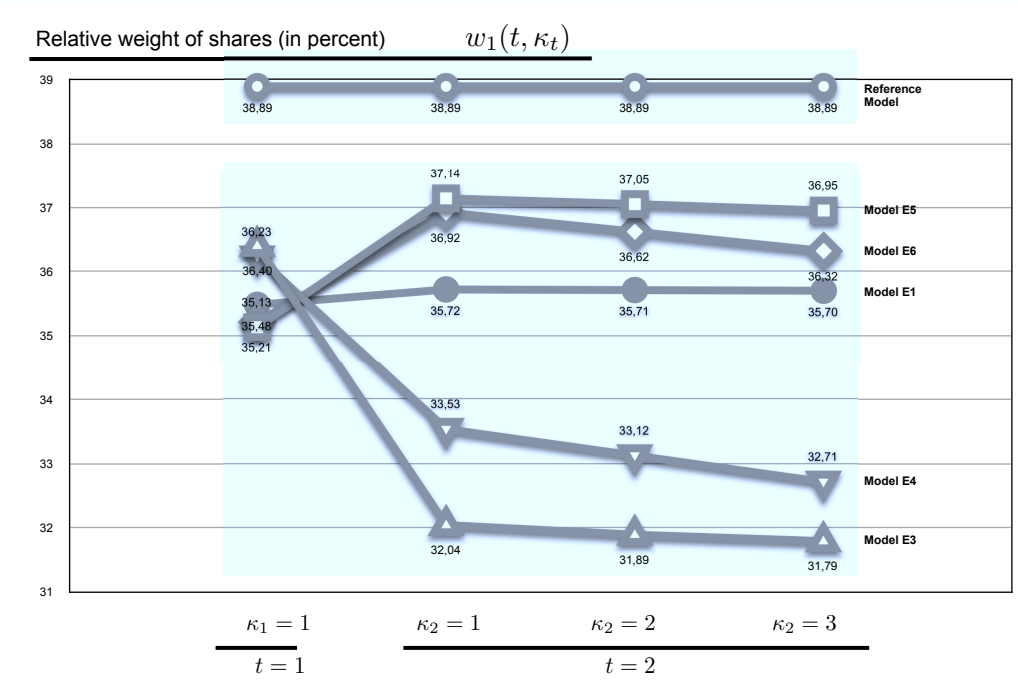



Figure 6.9.: Results for the two-period model without price effect of initial transactions and performance-related capital flow. (Source: Own illustration)

whereas the holdings at the end do not exceed those reported for model E1. This further supports the argumentation with compound effects.

The introduction of price effects leads to results similar to those in models D1 to D4 with E1, not S, as the reference portfolio. In model E3 with equal price effects, the portfolio manager invests slightly more in $t = 1$ only to disinvest in $t = 2$ significantly. In model E5 with different price effects, the strategy is the opposite. The introduction of price reversal tilts both results towards the reference portfolio E1.

## 6.4. Concluding Remarks

This chapter investigated optimal portfolio policies under the conditions of professional portfolio managers in single and multiple period contexts using a trinomial model setup. The central findings are summarized in the following paragraphs.

The use of periodical management fee deducted from the portfolio value does not alter the optimal portfolio structure compared to a terminal wealth maximizing agent.

The investigation of price effects in a trinomial model of one and two periods led

to interesting results. In a single-period setup, permanent price effects have no influence on the optimal portfolio structure expressed as relative portfolio holdings. The transactions are adjusted so that the share of wealth invested in each asset remains constant across several modifications. The effect of price reversals is comparable to additional costs associated with transactions. They reduce the returns of the affected assets resulting in portfolio structures that are tilted towards assets with lower price effects and price reversals.

In a two-period setup, these results do not hold. Price effects introduce a time dependency, and a complex interaction of the risk return profile of the assets and the size of the price effect determines the portfolio structure. The details of the strategy, hence, depend on the actual parameter settings. Generally, the share of the less preferred asset is bigger, the larger its price effect relative to the preferred asset. This effect is visible in situations with and without price effects from initial trades. Price reversals generally tilt the portfolio structures towards the structure of the standard solution without price effects. Only in the case with cash endowment, does the initial trade further deviate from the standard solution. The introduction of a concave price impact function does not change these general results.

Performance-related flow has similar effects as in the continuous time case. It tilts the investment decision of the portfolio manager towards the benchmark portfolio. This effect increases with larger capital flows. Unlike in the continuous time case, the portfolio structure becomes time-dependent in that the structure is tilted towards the benchmark portfolio more strongly at the beginning than at the end of the overall investment period. This is most likely due to compound effects of capital flows. The joint occurrence of capital flow and price effects is comparable to the situation without capital flow. The portfolio structure shows the same characteristics with the capital-flow-only portfolio as the reference portfolio.

From the perspective of the primary investor, these results demonstrate that price effects can lead to an efficiency loss because the portfolio manager generally does not invest in the standard portfolio solution. Only in the one period model without price reversal, does the portfolio manager follow the standard solution. In all other situations, they deviate significantly. However, it depends on the specific parameter constellation whether the actual portfolio holdings are tilted towards the tangency portfolio or not.

Only in the case of performance-related capital flow, is the primary investor able to influence the portfolio manager reliably to invest in their interests. By selecting

the right benchmark portfolio, for example the tangency portfolio, the portfolio is tilted towards this benchmark. The extent of this tilt depends on the size of the capital flow. Additional price effects do not improve the situation for the primary investor. Instead, they could make it worse depending on the parameter setup.

The explanatory power of price effects regarding the portfolio manager's preference for highly capitalized assets with high liquidity was limited. Only price effects with reversal in a single-period model can explain the tilt towards highly capitalized assets. Results regarding the influence of performance-related capital flow were comparable to those of the continuous time models in chapter 5. Price effects could not explain this preference in a multiperiod context.

This is most likely due to the concavity of the price impact function that makes large transactions appear preferable. A convex price impact function might change these results. However, the empirical research outlined in chapter 3.3 very strongly indicated that the price impact function is indeed concave. Another explanation could be the focus of practitioners on single-period models. If they applied models similar to the single-period model in this work, they would tilt their portfolios towards highly liquid assets.[313]

However, it appears very unlikely that participants in the capital market continuously forgo utility using simplified models. In essence, the general preference for liquidity reported in empirical studies cannot be explained with the multiperiod models presented here.

[313] As an example, Sofianos/Takriti/Tierens (2007) present a model by Goldman Sachs that provides optimal portfolio structures taking "shortfall risk" (similar to price risk effect with reversal) into account. Even though they do not disclose detailed information about this model, in a reported example the holdings in the asset with lower liquidity is reduced compared with the case of no "shortfall risk".

# 7. Conclusion

The objective of this work was to investigate the optimal portfolio policies of professional portfolio managers. Accounting for the conditions of delegated portfolio management, i.e., compensation, capital flow, and price effects, I probed whether these conditions can explain empirically observed investment strategies of portfolio managers. Empirical research provides evidence that institutional investors prefer large stocks and stocks that are listed in a prominent index, e.g., the S&P500. Furthermore, I discussed optimal portfolio structures from the primary investor's perspective. This work focused on equity portfolio management and, in particular, mutual funds and institutional accounts. Empirical details were presented for the US market, but similar conditions should apply to other regions as well.

I summarized empirical research about compensation, capital flow, and price effects that form the conditions for delegated portfolio management in chapter 3. The predominant compensation for portfolio managers of mutual funds as well as portfolio managers of institutional accounts is a proportional management fee based on assets under management. Because compensation is based on the level of assets under management, it is important for mutual fund and institutional account portfolio managers to understand why primary investors will contribute or withdraw capital. In that sense, mutual funds face capital flow that largely depends on the past performance of the fund as well as some concurrent flow that is influenced by the current general market situation. The performance-related flow has a nonlinear form with high inflows for overperformance and only marginal outflows for underperformance. For institutional accounts, capital flow plays a much less important role and is overall linear. Because portfolio managers very often maneuver large volumes it is important for them to understand the effect of their trades on prices. It turns out that price effects have a concave form with regard to trading volume. Price effects are generally larger for small-cap companies with low liquidity, measured e.g., as normal trading volume.

In chapter 5, I presented two models that incorporate compensation and capital

flow following empirical research from chapter 3. The models were based on the seminal work of Merton (1969) and Merton (1971). In contrast to Merton (1971), the portfolio manager in both models could not choose their optimal consumption but maximized expected utility from a fixed management fee. The first model extended the model of Merton (1971) with capital flows with regard to general market performance and fund performance in a linear way. For this model, I was able to derive closed-form solutions using the method of stochastic control. It turns out that by maximizing expected utility from management fee, the optimal portfolio policy does not change compared to the standard solution.[314] However, if the portfolio manager is restricted to risky assets only, they will most likely not invest in the tangency portfolio which creates an efficiency loss for the primary investor whose investment line then becomes inefficient. Concurrent flow increases the inefficiency further. With a market portfolio that is not located on the efficient frontier, the portfolio manager will chose an optimal portfolio that is located away from the efficient frontier. They will disinvest from the market portfolio. With linear performance-related flow, the situation can improve from the primary investor's perspective. Similarly to capital flow related to general market condition, the portfolio manager will invest away from the efficient frontier if the benchmark portfolio is not located on the efficient frontier.[315] However, if the primary investor chooses the tangency portfolio as the benchmark portfolio, they can induce the portfolio manager to invest closer to the tangency portfolio. This tilt becomes stronger with increasing capital flow.

With regard to empirically observed investment patterns of institutional investors, the performance-related flow could in principle explain the preference of portfolio managers to invest in assets that are listed in popular indices assuming that this index is used as a benchmark portfolio. However, the general market-related capital flow was not able to explain this preference, in fact it acted in the opposite direction and pulled the portfolio manager away from the benchmark portfolio.

In a second model, I incorporated nonlinear performance-related capital flow. I approximated the convex character of the capital flow with an exponential function. Using Markov chain approximations, numerical solutions could be provided. Unlike in the linear model, the investment strategy of the portfolio manager with exponential capital flow depended on the current level of performance difference

[314] This is in line with e.g., Wilhelm (1987).
[315] This result is very similar to Roll (1992).

between their and a benchmark portfolio. This model predicted that portfolio managers would tilt their portfolios towards the benchmark portfolio if they deviate from the benchmark. Quite generally, the tilt towards the benchmark portfolio was stronger for larger performance differences and more convex capital flows. But due to the dependence of the portfolio policy on the current level of performance difference, a general statement about the tilt towards the benchmark portfolio was not possible.

In effect, the nonlinear capital flow could also improve the situation for the primary investor as it induces the portfolio manager to tilt their portfolio towards the benchmark if their performance deviates significantly. Hence, nonlinear capital flow can, similarly to linear capital flow, also explain observed investment patterns of institutional investors.

A third model was set up in discrete time to analyze price effects from trades in chapter 6. This model also incorporated compensation and capital flow effects. I provided numerical solutions for a trinomial specification with two risky assets. Many modifications of the basic model have been solved including linear and concave price impact functions, permanent and temporary price effects, i.e., price effects without and with reversal, and single and multiperiod setups.

In a single-period context, temporary price effects lead to portfolios with a smaller portion invested in the asset with higher price effects and reversal. Permanent price effects did not change the portfolio structure. The multiperiod models with price effects lead to different results. Depending on the size of the relative price effect of the first asset, its share in the portfolio increased or decreased compared to the standard solution. Hence, a general preference for assets with high liquidity was not visible. The introduction of price reversals pulled the portfolio structure towards the standard solution but generally left it above or below, depending on the parameter setup. In consequence, a price effect can only explain the preference of investors for highly liquid assets in a single-period context, not in a multiperiod context.

The discrete model including capital flow provided similar results to the continuous time models. In models with performance-related capital flow, the portfolio was tilted towards the benchmark portfolio. This pull was stronger for higher capital flows. A joint model of capital flow and price effect showed a slightly reverted effect. The portfolio structure in such models was pulled back to the standard solution leading to a counter effect. Hence, performance-related capital flow can explain the

preference for larger stocks assuming that they are part of the benchmark, but price effects do not generally increase this capital flow effect.

Overall, the use of benchmark portfolios with regard to capital flow is an effective behavior of primary investors to induce portfolio managers to invest in their interests. This has been shown in a continuous time and discrete time setup. However, one should keep in mind that the analysis presented here was restricted to the portfolio manager. In particular, I did not model explicitly an equilibrium behavior of the primary investor with regard to capital flows. On an aggregate level, an unconstrained capital flow as modeled here is reasonable. But this is not necessarily the case on the level of individual primary investors, because the amount a primary investor can delegate is limited to their total wealth.

Similar results to the effect of capital flows can be attained by introducing a compensation based on benchmarks as proposed by e.g., Ou-Yang (2003) and Kraft/ Korn (2008). Because the majority of portfolio managers do not offer benchmark-related compensation schemes, the observed performance-related capital flow can be understood as reaction of primary investors to this lack. With performance-related capital flow, they can at least incite portfolio managers to invest in their interests.

To avoid efficiency loss, primary investors could alternatively - at least from a theoretical viewpoint - invest in mixed mutual funds or balanced institutional accounts, i.e., portfolios that consist of risky and riskless assets to improve their efficiency situation. Portfolio theory indicates that a portfolio manager would then invest in the tangency portfolio.[316] However, it might be unlikely from a practical viewpoint that portfolio managers do indeed invest in the tangency portfolio. First, they must differentiate themselves from other portfolio managers which would require an investment at least partly away from the tangency portfolio. Secondly, they must provide a value proposition against index certificates that simply mimic the performance of indices.[317] This can only be done by investing at least partly away

[316] Even though I do not present optimal portfolio policies for portfolio managers who can also invest in the riskless asset, it is very likely that the portfolio structure would be equal to the standard solution. I conjecture this given the equality of the solutions for the case of risky assets only. The introduction of the riskless asset would not affect the structure of the optimization problem.

[317] Index certificates also beg the question why active portfolio management actually exists. In particular in light of their largely debated performance results documented in the ever growing research literature about mutual fund performance, cf. Bollen (2005) and Wermers (2000) as two recent references with differing results. However, this question could not be addressed in this thesis.

from the index. Given these practical constraints, the focus on benchmarks appears to be the more suitable strategy.

The analysis of price effects provided interesting insights into an important modification of portfolio optimization. There is still much to be understood regarding the complex influence of risk, return, and price effects on the portfolio structure in a multiperiod context. Results presented in this work showed that the optimal structure largely depends on the set of parameter values. An analytical solution, even of a short two-period model, would be helpful to understand their interrelation. In addition, other variants with different maximization criteria and framing conditions, such as initial and terminal holdings or even a convex price impact function, could be analyzed. Practitioners should reconsider the usage of single-period portfolio models. The numerical results indicate that the structure of portfolios with price effects is clearly different in a single and a multiperiod context. However, further research is necessary to allow for general recommendations.

# Appendix

# A. Portfolio Policies under Compensation and Capital Flow

This chapter provides detailed steps for the solution of the portfolio optimization problem defined in chapter 5.3

## A.1. Optimal Control Candidate

To simplify the notation, let $w_\Delta = a \cdot w_M - c \cdot w_B$. The conditions of optimality 5.6-5.8 allow to calculate an optimal control candidate. Assuming that $\Sigma$ has full rank, equation 5.6 can be solved for $w^*(t)$:

$$\begin{aligned}
0 &= -\lambda_0 \mathbf{1} + V_X(1+c)\alpha X(t) \\
&\quad + V_{XX}(1+c)\Sigma\left[(1+c)w^*(t) + w_\Delta\right] X(t)^2 \\
\Leftrightarrow 0 &= -\lambda_0 \mathbf{1} + V_X(1+c)\alpha X(t) \\
&\quad + V_{XX}(1+c)\Sigma w_\Delta X(t)^2 \\
&\quad + V_{XX}(1+c)\Sigma(1+c)w^*(t)X(t)^2 \\
\Leftrightarrow V_{XX}(1+c)^2\Sigma w^*(t)X(t)^2 &= \lambda_0 \mathbf{1} - V_X(1+c)\alpha X(t) \\
&\quad - V_{XX}(1+c)\Sigma w_\Delta(t) X(t)^2 \\
\Leftrightarrow w^*(t) &= \frac{\lambda_0}{(1+c)^2 V_{XX} X(t)^2}\Sigma^{-1}\mathbf{1} \\
&\quad - \frac{V_X}{(1+c)V_{XX}X(t)}\Sigma^{-1}\alpha - \frac{1}{1+c}w_\Delta \quad \text{(A.1)}
\end{aligned}$$

Substituting equation A.1 into 5.7 one can solve for $\lambda_0$

$$\begin{aligned}
0 &= 1 - \mathbf{1}' w^*(t) \\
\Leftrightarrow 0 &= 1 - \mathbf{1}' \frac{\lambda_0}{(1+c)^2 V_{XX} X(t)^2} \Sigma^{-1}\mathbf{1} \\
&\quad + \mathbf{1}' \frac{V_X}{(1+c) V_{XX} X(t)} \Sigma^{-1}\alpha + \mathbf{1}' \frac{1}{1+c} w_\Delta \\
\Leftrightarrow \frac{\lambda_0}{(1+c)^2 V_{XX} X(t)^2} \mathbf{1}'\Sigma^{-1}\mathbf{1} &= 1 + \frac{V_X}{(1+c) V_{XX} X(t)} \mathbf{1}'\Sigma^{-1}\alpha + \frac{1}{1+c}\mathbf{1}' w_\Delta \\
\Leftrightarrow \lambda_0 &= \frac{(1+c)^2 V_{XX} X(t)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{(1+c) V_X X(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha \\
&\quad + \frac{(1+c) V_{XX} X(t)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}' w_\Delta \\
\Leftrightarrow \lambda_0 &= \frac{(1+c) V_{XX} X(t)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \left[(1+c) + \mathbf{1}' w_\Delta\right] \\
&\quad + \frac{(1+c) V_X X(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha
\end{aligned}$$

Replacing $\lambda_0$ in equation A.1 leads to the specific equation for $w^*(t)$

$$\begin{aligned}
w^*(t) &= \left( \frac{(1+c) V_{XX} X(t)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \left[(1+c) + \mathbf{1}' w_\Delta\right] + \frac{(1+c) V_X X(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha \right) \\
&\quad \cdot \frac{1}{(1+c)^2 V_{XX} X(t)^2} \Sigma^{-1}\mathbf{1} - \frac{V_X}{(1+c) V_{XX} X(t)} \Sigma^{-1}\alpha - \frac{1}{1+c} w_\Delta \\
&= \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} + \frac{1}{1+c} \left[ \frac{\mathbf{1}' w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} - w_\Delta \right] \\
&\quad + \frac{V_X}{(1+c) V_{XX} X(t)} \left[ \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} - \Sigma^{-1}\alpha \right] \qquad \text{(A.2)}
\end{aligned}$$

## A.2. Solution to the HJB Equation

To find a solution for the value function $V(t, X)$ I substitute equation A.2 into equation 5.8. The resulting equation

$$0 = L[t, X(t)] + V_t + \overbrace{V_X \left[ w_\Delta^{'} \alpha + (1+c) w^*(t)' \alpha - \lambda \right] X(t)}^{(1)} + \frac{1}{2} V_{XX} \overbrace{\left[ w_\Delta + (1+c) w^*(t) \right]' \overbrace{\Sigma \left[ w_\Delta + (1+c) w^*(t) \right] X(t)^2}^{(2)}}^{(3)} \quad \text{(A.3)}$$

can be simplified. The first term $(1)$ develops as follows. Using the fact that $\Sigma^{-1}$ is symmetric[318] and some further expansion lead to

$$\begin{aligned} & V_X w_\Delta^{'} \alpha X(t) - V_X \lambda X(t) \\ & \quad + V_X X(t)(1+c) \left[ \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} + \frac{1}{1+c} \left( \frac{\mathbf{1}' w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} - w_\Delta \right) \right. \\ & \qquad \left. + \frac{V_X}{(1+c) V_{XX} X(t)} \left( \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \Sigma^{-1}\mathbf{1} - \Sigma^{-1}\alpha \right) \right]' \alpha \end{aligned}$$

$$\begin{aligned} & = V_X w_\Delta^{'} \alpha X(t) - V_X \lambda X(t) \\ & + (1+c) V_X X(t) \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \left( \Sigma^{-1}\mathbf{1} \right)' \alpha + V_X X(t) \left( \frac{\mathbf{1}' w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \left( \Sigma^{-1}\mathbf{1} \right)' \alpha - w_\Delta^{'} \alpha \right) \\ & \qquad + \frac{{V_X}^2}{V_{XX}} \left( \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \left( \Sigma^{-1}\mathbf{1} \right)' \alpha - \left( \Sigma^{-1}\alpha \right)' \alpha \right) \end{aligned}$$

$$\begin{aligned} & = (1+c) V_X X(t) \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + V_X X(t) \left( \frac{\mathbf{1}' w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha \right) \\ & \qquad + \frac{{V_X}^2}{V_{XX}} \left( \frac{\left( \mathbf{1}'\Sigma^{-1}\alpha \right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha \right) - V_X \lambda X(t) \quad \text{(A.4)} \end{aligned}$$

[318] The symmetry of $\Sigma^{-1}$ implies $\mathbf{1}'\Sigma^{-1}\alpha = \alpha^{'}\Sigma^{-1}\mathbf{1}$.

The second term $(2)$ can be transformed as follows

$$\Sigma w_\Delta X(t)^2 + \Sigma(1+c)X(t)^2 \left[\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} + \frac{1}{1+c}\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - w_\Delta\right) + \frac{V_X}{(1+c)V_{XX}X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - \Sigma^{-1}\alpha\right)\right]$$

$$= (1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} + X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) + \frac{V_X X(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} - \alpha\right)$$

Using this result, the third term $(3)$ can be transformed as follows

$$V_{XX}\left[w_\Delta + (1+c)w^*(t)\right]' \cdot \left[(1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} + X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) + \frac{V_X X(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} - \alpha\right)\right]$$

$$= \left[V_{XX}(1+c)\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)' + V_{XX}\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\right) + \frac{V_X}{X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)' - \left(\Sigma^{-1}\alpha\right)'\right)\right] \cdot \left[(1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} + X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) + \frac{V_X X(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} - \alpha\right)\right]$$

$$
\begin{aligned}
= V_{XX}(1+c)\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\cdot(1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} \\
+ V_{XX}\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\right)\cdot(1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} \\
+ \frac{V_X}{X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'-\left(\Sigma^{-1}\alpha\right)'\right)\cdot(1+c)X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1} \\
+ V_{XX}(1+c)\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\cdot X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) \\
+ V_{XX}\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\right)\cdot X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) \\
+ \frac{V_X}{X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'-\left(\Sigma^{-1}\alpha\right)'\right)\cdot X(t)^2\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) \\
+ V_{XX}(1+c)\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\cdot\frac{V_XX(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}-\alpha\right) \\
+ V_{XX}\left(\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'\right)\cdot\frac{V_XX(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}-\alpha\right) \\
+ \frac{V_X}{X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\left(\Sigma^{-1}\mathbf{1}\right)'-\left(\Sigma^{-1}\alpha\right)'\right)\cdot\frac{V_XX(t)}{V_{XX}}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}-\alpha\right)
\end{aligned}
$$

$$
\begin{aligned}
= V_{XX}(1+c)^2X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}+V_{XX}(1+c)X(t)^2\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}+0 \\
+ V_{XX}(1+c)X(t)^2\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}+V_{XX}\left(\frac{\left(\mathbf{1}'w_\Delta\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)X(t)^2+0 \\
+0+0-\frac{{V_X}^2}{V_{XX}}\left(\frac{\left(\mathbf{1}'\Sigma^{-1}\alpha\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}-\alpha'\Sigma^{-1}\alpha\right)
\end{aligned}
$$

$$
\begin{aligned}
= V_{XX}(1+c)^2X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}-\frac{{V_X}^2}{V_{XX}}\left(\frac{\left(\mathbf{1}'\Sigma^{-1}\alpha\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}-\alpha'\Sigma^{-1}\alpha\right) \\
+ V_{XX}X(t)^2\left(\frac{\left(\mathbf{1}'w_\Delta\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}+2(1+c)\frac{\mathbf{1}'w_\Delta}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) \quad \text{(A.5)}
\end{aligned}
$$

Substituting A.4 and A.5 back into A.3 leads to

$$0 = L(t, X(t)] + V_t + V_X \left[ w_\Delta^{'} \alpha + (1+c) w^*(t)' \alpha - \lambda \right] X(t) + \frac{1}{2} V_{XX} \left[ w_\Delta^{'} + (1+c) w^*(t)' \right] \Sigma \left[ w_\Delta + (1+c) w^*(t) \right] X(t)^2$$

$$\Leftrightarrow 0 = L[t, X(t)] + V_t + (1+c) V_X X(t) \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{{V_X}^2}{V_{XX}} \left( \frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha \right) + V_X X(t) \left( \frac{\mathbf{1}' w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha \right) - V_X \lambda X(t) + \frac{1}{2} \left[ V_{XX} (1+c)^2 X(t)^2 \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \frac{{V_X}^2}{V_{XX}} \left( \frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha \right) + V_{XX} X(t)^2 \left( \frac{(\mathbf{1}' w_\Delta(t))^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c) \frac{\mathbf{1}' w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \right) \right]$$

$$\Leftrightarrow 0 = L[t, X(t)] + V_t + (1+c) V_X X(t) \frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + V_X X(t) \left( \frac{\mathbf{1}' w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \mathbf{1}'\Sigma^{-1}\alpha \right) - V_X \lambda X(t) + \frac{1}{2} V_{XX} (1+c)^2 X(t)^2 \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{1}{2} \frac{{V_X}^2}{V_{XX}} \left( \frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha \right) + \frac{1}{2} V_{XX} X(t)^2 \left( \frac{(\mathbf{1}' w_\Delta(t))^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c) \frac{\mathbf{1}' w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \right) \quad \text{(A.6)}$$

Condition 5.8 is fulfilled if there exists a $V$ that solves equation A.6. Let $L(t, X(t))$ be the utility function in equation 5.5. To find a solution for the value function, let $V(t, X(t))$ be defined as in equation 5.9

$$V(t, X) = f(t) \cdot \frac{1}{\gamma} \left[ \lambda X(t) \right]^\gamma, \quad f(T) = 1. \quad \text{(A.7)}$$

The relevant derivatives are[319]

$$\begin{aligned} V_t &= \dot{f}(t) \cdot \frac{1}{\gamma}[\lambda X(t)]^\gamma \\ V_X &= f(t) \cdot \lambda^\gamma X(t)^{\gamma-1} \\ V_{XX} &= f(t) \cdot (\gamma-1)\lambda^\gamma X(t)^{\gamma-2} \end{aligned}$$

Replacing the derivatives of $V$ in equation A.6 leads to

$$\begin{aligned} 0 = \frac{1}{\gamma}e^{-\beta t}[\lambda X(t)]^\gamma + \dot{f}(t) \cdot \frac{1}{\gamma}[\lambda X(t)]^\gamma + (1+c)f(t) \cdot \lambda^\gamma X(t)^{\gamma-1}X(t)\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \\ + f(t) \cdot \lambda^\gamma X(t)^{\gamma-1}X(t)\left(\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha\right) - f(t) \cdot \lambda^\gamma X(t)^{\gamma-1}\lambda X(t) \\ + \frac{1}{2}f(t) \cdot (\gamma-1)\lambda^\gamma X(t)^{\gamma-2}(1+c)^2X(t)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \\ + \frac{1}{2}\frac{\left(f(t) \cdot \lambda^\gamma X(t)^{\gamma-1}\right)^2}{f(t) \cdot (\gamma-1)\lambda^\gamma X(t)^{\gamma-2}}\left(\frac{\left(\mathbf{1}'\Sigma^{-1}\alpha\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right) \\ + \frac{1}{2}f(t) \cdot (\gamma-1)\lambda^\gamma X(t)^{\gamma-2}X(t)^2\left(\frac{\left(\mathbf{1}'w_\Delta(t)\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) \end{aligned}$$

which can further be simplified to

$$\begin{aligned} 0 = \frac{1}{\gamma}e^{-\beta t}[\lambda X(t)]^\gamma + \dot{f}(t) \cdot \frac{1}{\gamma}[\lambda X(t)]^\gamma + (1+c)f(t) \cdot [\lambda X(t)]^\gamma\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \\ + f(t) \cdot [\lambda X(t)]^\gamma\left(\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha - \lambda\right) \\ + \frac{1}{2}f(t) \cdot (\gamma-1)[\lambda X(t)]^\gamma(1+c)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \\ + \frac{1}{2}\frac{f(t) \cdot [\lambda X(t)]^\gamma}{(\gamma-1)}\left(\frac{\left(\mathbf{1}'\Sigma^{-1}\alpha\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right) \\ + \frac{1}{2}f(t) \cdot (\gamma-1)[\lambda X(t)]^\gamma\left(\frac{\left(\mathbf{1}'w_\Delta(t)\right)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) \end{aligned}$$

[319] Note that $X(t)$ is known at $t$, it has no functional form that could be derived with regard to time.

Dividing by $[\lambda X(t)]^\gamma$ and rearrangements give the ordinary differential equation

$$\begin{aligned}0 = \frac{1}{\gamma}e^{-\beta t} + \dot{f}(t)\frac{1}{\gamma} + (1+c)f(t)\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + f(t)\left(\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha - \lambda\right) \\ + \frac{1}{2}f(t)(\gamma-1)(1+c)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{1}{2}\frac{f(t)}{(\gamma-1)}\left(\frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right) \\ + \frac{1}{2}f(t)(\gamma-1)\left(\frac{(\mathbf{1}'w_\Delta(t))^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)\end{aligned}$$

$$\begin{aligned}0 = e^{-\beta t} + \dot{f}(t) + \gamma f(t)\Bigg[(1+c)\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha - \lambda \\ + \frac{1}{2}(\gamma-1)\left((1+c)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{(\mathbf{1}'w_\Delta(t))^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) \\ + \frac{1}{2}\frac{1}{(\gamma-1)}\left(\frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right)\Bigg]\end{aligned}$$

Now let

$$\begin{aligned}a := \gamma\Bigg[(1+c)\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}'\Sigma^{-1}\alpha - \lambda \\ + \frac{1}{2}(\gamma-1)\left((1+c)^2\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + \frac{(\mathbf{1}'w_\Delta(t))^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} + 2(1+c)\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right) \\ + \frac{1}{2}\frac{1}{(\gamma-1)}\left(\frac{(\mathbf{1}'\Sigma^{-1}\alpha)^2}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} - \alpha'\Sigma^{-1}\alpha\right)\Bigg]\end{aligned}$$

Then the ordinary differential equation has the form

$$\dot{f}(t) = -e^{-\beta t} - f(t)a$$

with final condition $f(T) = 1$. A solution to this equation is given by

$$f(t) = \frac{1}{-\beta + a}\cdot\left(e^{(-\beta+a)T} - e^{(-\beta+a)t}\right)e^{-at} + e^{a(T-t)}$$

This solution is strictly positive, because for $T > t$ the sign of the term in brackets is the same as the sign of the factor $\frac{1}{-\beta+a}$. If $a > \beta$ then the term in brackets as well as the factor is positive. For $a < \beta$ the exponential $e^{(-\beta+a)T}$ is smaller than

$e^{(-\beta+a)t}$, leading to a negative term, same sign as the factor.

Explicit solutions for the optimal controls can be found by substituting the value function from equation A.7 into equation A.2

$$w^*(t) = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} + \frac{1}{1+c}\left(\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - w_\Delta(t)\right) + \frac{f(t)\cdot\lambda^\gamma X(t)^{\gamma-1}}{(1+c)f(t)\cdot(\gamma-1)\lambda^\gamma X(t)^{\gamma-2}X(t)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - \Sigma^{-1}\alpha\right)$$

This can be simplified to

$$w^*(t) = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} + \frac{1}{1+c}\left(\frac{\mathbf{1}'w_\Delta(t)}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - w_\Delta(t)\right) + \frac{1}{(1+c)(\gamma-1)}\left(\frac{\mathbf{1}'\Sigma^{-1}\alpha}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\Sigma^{-1}\mathbf{1} - \Sigma^{-1}\alpha\right)$$

Denoting the minimum-variance sub-portfolio with $MV = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\cdot\Sigma^{-1}\mathbf{1}$ gives

$$w^*(t) = MV - \frac{1}{1+c}\left(w_\Delta(t) - \mathbf{1}'w_\Delta(t)\cdot MV\right) + \frac{1}{(1+c)(1-\gamma)}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha\cdot MV\right) \quad \text{(A.8)}$$

## A.3. Checking Assumptions

All assumptions have been satisfied:

### Wealth Process

The wealth process $X^*(t)$ with the optimal portfolio policy $w^*(t)$ is the unique strictly positive solution to the wealth process from equation 5.3 with the optimal portfolio policy in A.8,

$$dX(t) = \left(\sum_{i=1}^{n} w_i^*(t)\alpha_i\right)X(t)dt - \lambda X(t)dt + g(t) + \sum_{i=1}^{n} w_i^*(t)\sigma_i X(t)dB_i$$

$$X(0) = x.$$

This will be shown below. In addition, the coefficients of the stochastic differential equation satisfy the conditions on the coefficients of the stochastic process.[320]

Now denote the return-oriented portfolio as $R := \frac{1}{1-\gamma}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right)$, let $MV_i$ and $R_i$ be the $i$-th element of the vectors $MV$ and $R$. Using the fact that $\mathbf{1}'w_B = \mathbf{1}'w_M = 1$, equation A.8 becomes

$$\begin{aligned} w^*(t) &= MV - \frac{1}{1+c}\left(w_\Delta(t) - (a-c)\cdot MV\right) \\ &\quad + \frac{1}{(1+c)(1-\gamma)}\left(\Sigma^{-1}\alpha - \mathbf{1}'\Sigma^{-1}\alpha \cdot MV\right) \\ &= MV + \frac{1}{1+c}\left((a-c)\cdot MV - w_\Delta\right) + \frac{1}{1+c}R \\ &= \frac{1}{1+c}\left[(1+a)MV + R - w_\Delta\right] \end{aligned}$$

or

$$w_i^*(t) = \frac{1}{1+c}\left[(1+a)MV_i + R_i - w_{\Delta_i}\right] \tag{A.9}$$

Using the substitute $w_\Delta$ equation $g(t)$ (equation 5.2) can be written as

$$\begin{aligned} g(t) &= \sum_{i=1}^{n}\left(a\cdot w_{M_i} + c\cdot\left[w_i^*(t) - w_{B_i}\right]\right)\alpha_i X(t)dt \\ &\quad + \sum_{i=1}^{n}\left(a\cdot w_{M_i} + c\cdot\left[w_i^*(t) - w_{B_i}\right]\right)\sigma_i X(t)dB_i \\ &= \sum_{i=1}^{n}\left(c\cdot w_i^*(t) + w_{\Delta_i}\right)\alpha_i X(t)dt + \sum_{i=1}^{n}\left(c\cdot w_i^*(t) + w_{\Delta_i}\right)\sigma_i X(t)dB_i \end{aligned}$$

With equation A.9 this becomes

$$\begin{aligned} g(t) &= \sum_{i=1}^{n}\left(c\cdot\frac{1}{1+c}\left[(1+a)MV_i + R_i - w_{\Delta_i}\right] + w_{\Delta_i}\right)\alpha_i X(t)dt \\ &\quad + \sum_{i=1}^{n}\left(c\cdot\frac{1}{1+c}\left[(1+a)MV_i + R_i - w_{\Delta_i}\right] + w_{\Delta_i}\right)\sigma_i X(t)dB_i \\ &= \sum_{i=1}^{n}\frac{1}{1+c}\left(c\left[(1+a)MV_i + R_i\right] + w_{\Delta_i}\right)\alpha_i X(t)dt \\ &\quad + \sum_{i=1}^{n}\frac{1}{1+c}\left(c\left[(1+a)MV_i + R_i\right] + w_{\Delta_i}\right)\sigma_i X(t)dB_i \end{aligned}$$

[320] Regarding the conditions on the coefficients cf. Korn/Korn (2001), p. 224.

With this equation the stochastic differential equation

$$dX(t) \;=\; \left(\sum_{i=1}^{n} w_i^*(t)\alpha_i\right) X(t)dt \;-\; \lambda X(t)dt \;+\; g(t) \;+\; \sum_{i=1}^{n} w_i^*(t)\sigma_i X(t)dB_i$$

becomes

$$\begin{aligned} dX(t) \;=\; & \left(\sum_{i=1}^{n} \frac{1}{1+c}\left[(1+a)MV_i + R_i - w_{\Delta_i}\right]\alpha_i\right) X(t)dt - \lambda X(t)dt \\ & + \sum_{i=1}^{n} \frac{1}{1+c}\left(c\left[(1+a)MV_i + R_i\right] + w_{\Delta_i}\right)\alpha_i X(t)dt \\ & + \sum_{i=1}^{n} \frac{1}{1+c}\left(c\left[(1+a)MV_i + R_i\right] + w_{\Delta_i}\right)\sigma_i X(t)dB_i \\ & + \sum_{i=1}^{n} \frac{1}{1+c}\left[(1+a)MV_i + R_i - w_{\Delta_i}\right]\sigma_i X(t)dB_i \\ =\; & \sum_{i=1}^{n}\left((1+a)MV_i + R_i\right)\alpha_i X(t)dt - \lambda X(t)dt \\ & + \sum_{i=1}^{n}\left((1+a)MV_i + R_i\right)\sigma_i X(t)dB_i \end{aligned} \tag{A.10}$$

This process equation differs from the standard process with regard to the management fee parameter $\lambda$ and the term $1+a$. The term $1+a$ represents the additional capital due to general market performance. Interestingly, the parameter $c$ for performance-related flow does not appear in the process of assets under management. This means that the process is independent of the performance-related flow because the portfolio is perfectly hedged against this flow.

Now substitute $\mu(t, X(t)) = \sum_{i=1}^{n}\left((1+a)MV_i + R_i\right)\alpha_i X(t) - \lambda X(t)$ and $\sigma(t, X(t)) = \sum_{i=1}^{n}\left((1+a)MV_i + R_i\right)\sigma_i X(t)$. With time-independent $MV_i$ and $R_i$, the coefficients $\mu(t, X(t))$ and $\sigma(t, X(t))$ satisfy the conditions on the coefficients of the stochastic process.[321] With the explicit form in equation A.10, the stochastic differential process has a unique strictly positive solution that can be obtained using the "variation of constants"-theorem.[322] Let $A(t) = \sum_{i=1}^{n}\left((1+a)MV_i + R_i\right)\alpha_i - \lambda$ and $S_i(t) = \left((1+a)MV_i + R_i\right)\sigma_i$. Then with the

[321] Regarding the conditions on the coefficients cf. Korn/Korn (2001), p. 224.
[322] Cf. Korn/Korn (2001), p. 54f., theorem 2.54.

"variation of constants"-theorem equation A.10 has the unique solution[323]

$$X(t) = x \cdot \exp\left\{ \int_0^t \left( A(s) - \frac{1}{2} \sum_{i=1}^{n} S_i^2(s) \right) ds + \sum_{i=1}^{n} \int_0^t S_i(s) dB_i(s) \right\}.$$

with initial positive value $X(0) = x > 0$. Due to the exponential function the process of assets under management is always positive.

### Optimal Control

The optimal control $w^*(t)$ is constant and therefore bounded in $U$.

### Value Function

The value function, as the product of the strictly positive function $f(t)$ and the concave utility function, is strictly concave as well. It is, like the standard solution, a classical $C^{1,2}$ solution and satisfies the polynomial growth condition.[324]

[323] Cf. also in analogy the stock price solution Korn/Korn (2001), p. 57.
[324] Cf. Korn/Korn (2001), p. 239.

# B. Numerical Solution Approach for Trinomial Portfolio Models

## B.1. Details of Numerical Solutions

This section shows the detailed numerical solutions for models B to E in chapter 6.3.2, 6.3.3, and 6.3.4.

| | $S$ | $B1$ | $B2$ | $B3$ | $B4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |
| *V*ariable | | | | | |
| $w_1(1,1)$ | 38.89% | 40.50% | 41.30% | 38.05% | 37.77% |
| $w_1(2,1)$ | 38.89% | 33.82% | 36.60% | 40.78% | 40.26% |
| $w_1(2,2)$ | 38.89% | 33.82% | 36.23% | 40.78% | 39.97% |
| $w_1(2,3)$ | 38.89% | 33.82% | 35.86% | 40.78% | 39.67% |
| $w_2(1,1)$ | 61.11% | 59.50% | 58.70% | 61.95% | 62.23% |
| $w_2(2,1)$ | 61.11% | 66.18% | 63.40% | 59.22% | 59.74% |
| $w_2(2,2)$ | 61.11% | 66.18% | 63.77% | 59.22% | 60.03% |
| $w_2(2,3)$ | 61.11% | 66.18% | 64.14% | 59.22% | 60.33% |
| $N_1(1,1)$ | 3888.89 | 4048.45 | 4128.67 | 3804.32 | 3776.49 |
| $N_1(2,1)$ | 3652.10 | 3180.35 | 3443.43 | 3825.54 | 3775.79 |
| $N_1(2,2)$ | 3850.00 | 3347.78 | 3585.85 | 4035.73 | 3955.90 |
| $N_1(2,3)$ | 4070.57 | 3534.40 | 3744.46 | 4269.99 | 4155.17 |
| $N_2(1,1)$ | 6111.11 | 5947.93 | 5867.73 | 6193.32 | 6221.16 |
| $N_2(2,1)$ | 6266.07 | 6791.84 | 6512.01 | 6066.68 | 6118.47 |
| $N_2(2,2)$ | 6050.00 | 6548.48 | 6310.50 | 5861.92 | 5941.76 |
| $N_2(2,3)$ | 5848.33 | 6321.33 | 6122.53 | 5670.81 | 5778.17 |
| $U_1(1,1)$ | 3888.89 | 4048.45 | 4128.67 | 3804.32 | 3776.49 |
| $U_1(2,1)$ | -236.79 | -868.10 | -685.24 | 21.21 | -0.69 |
| $U_1(2,2)$ | -38.89 | -700.66 | -542.82 | 231.40 | 179.41 |
| $U_1(2,3)$ | 181.68 | -514.04 | -384.20 | 465.67 | 378.68 |
| $U_2(1,1)$ | 6111.11 | 5947.93 | 5867.73 | 6193.32 | 6221.16 |
| $U_2(2,1)$ | 154.96 | 843.91 | 644.28 | -126.64 | -102.69 |
| $U_2(2,2)$ | -61.11 | 600.55 | 442.77 | -331.40 | -279.40 |
| $U_2(2,3)$ | -262.78 | 373.40 | 254.80 | -522.51 | -442.99 |
| $P_1^p(1,1)$ | 100.00 | 100.03 | 100.03 | 100.03 | 100.03 |
| $P_1^p(2,1)$ | 107.00 | 107.02 | 107.02 | 107.03 | 107.02 |
| $P_1^p(2,2)$ | 101.00 | 101.02 | 101.02 | 101.03 | 101.02 |
| $P_1^p(2,3)$ | 96.00 | 96.02 | 96.02 | 96.03 | 96.02 |
| $P_1^r(1,1)$ | 100.00 | 100.03 | 100.02 | 100.03 | 100.02 |
| $P_1^r(2,1)$ | 107.00 | 107.02 | 107.02 | 107.03 | 107.02 |
| $P_1^r(2,2)$ | 101.00 | 101.02 | 101.02 | 101.03 | 101.02 |
| $P_1^r(2,3)$ | 96.00 | 96.02 | 96.02 | 96.03 | 96.02 |
| $P_2^p(1,1)$ | 100.00 | 100.04 | 100.04 | 100.02 | 100.02 |
| $P_2^p(2,1)$ | 98.00 | 98.05 | 98.04 | 98.02 | 98.02 |
| $P_2^p(2,2)$ | 101.00 | 101.05 | 101.04 | 101.02 | 101.02 |
| $P_2^p(2,3)$ | 105.00 | 105.05 | 105.04 | 105.02 | 105.02 |
| $P_2^r(1,1)$ | 100.00 | 100.04 | 100.03 | 100.02 | 100.02 |
| $P_2^r(2,1)$ | 98.00 | 98.05 | 98.04 | 98.02 | 98.02 |
| $P_2^r(2,2)$ | 101.00 | 101.05 | 101.04 | 101.02 | 101.02 |
| $P_2^r(2,3)$ | 105.00 | 105.05 | 105.04 | 105.02 | 105.02 |

Table B.1.: Parameter values and results for the two-period model with initial cash endowment and linear price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

| | $S$ | $C1$ | $C2$ | $C3$ | $C4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| *P*arameter | | | | | |
| $\eta_1$ | 0 | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ |
| $\theta_1$ | 0 | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ |
| $\eta_2$ | 0 | $7.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-3}$ | $3.25 \cdot 10^{-3}$ | $3.25 \cdot 10^{-3}$ |
| $\theta_2$ | 0 | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ | $2 \cdot 10^{-4}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |
| *V*ariable | | | | | |
| $w_1(1,1)$ | 38.89% | 44.88% | 47.62% | 44.84% | 45.36% |
| $w_1(2,1)$ | 38.89% | 18.51% | 26.79% | 27.81% | 31.68% |
| $w_1(2,2)$ | 38.89% | 17.44% | 25.75% | 27.34% | 31.15% |
| $w_1(2,3)$ | 38.89% | 16.43% | 24.76% | 26.93% | 30.65% |
| $w_2(1,1)$ | 61.11% | 55.12% | 52.38% | 55.16% | 54.64% |
| $w_2(2,1)$ | 61.11% | 81.49% | 73.21% | 72.19% | 68.32% |
| $w_2(2,2)$ | 61.11% | 82.56% | 74.25% | 72.66% | 68.85% |
| $w_2(2,3)$ | 61.11% | 83.57% | 75.24% | 73.07% | 69.35% |
| $N_1(1,1)$ | 3888.89 | 4487.99 | 4761.74 | 4483.81 | 4535.24 |
| $N_1(2,1)$ | 3652.10 | 1747.46 | 2535.49 | 2625.56 | 2992.49 |
| $N_1(2,2)$ | 3850.00 | 1726.41 | 2549.57 | 2706.39 | 3083.29 |
| $N_1(2,3)$ | 4070.57 | 1710.82 | 2572.08 | 2803.62 | 3189.84 |
| $N_2(1,1)$ | 6111.11 | 5508.40 | 5234.86 | 5514.23 | 5462.82 |
| $N_2(2,1)$ | 6266.07 | 8393.71 | 7559.40 | 7438.25 | 7042.58 |
| $N_2(2,2)$ | 6050.00 | 8167.48 | 7345.47 | 7191.01 | 6814.36 |
| $N_2(2,3)$ | 5848.33 | 7949.15 | 7139.55 | 6953.73 | 6596.47 |
| $U_1(1,1)$ | 3888.89 | 4487.99 | 4761.74 | 4483.81 | 4535.24 |
| $U_1(2,1)$ | -236.79 | -2740.53 | -2226.25 | -1858.25 | -1542.75 |
| $U_1(2,2)$ | -38.89 | -2761.58 | -2212.17 | -1777.42 | -1451.95 |
| $U_1(2,3)$ | 181.68 | -2777.17 | -2189.66 | -1680.20 | -1345.40 |
| $U_2(1,1)$ | 6111.11 | 5508.40 | 5234.86 | 5514.23 | 5462.82 |
| $U_2(2,1)$ | 154.96 | 2885.30 | 2324.54 | 1924.02 | 1579.76 |
| $U_2(2,2)$ | -61.11 | 2659.08 | 2110.61 | 1676.78 | 1351.54 |
| $U_2(2,3)$ | -262.78 | 2440.74 | 1904.70 | 1439.50 | 1133.65 |
| $P_1^p(1,1)$ | 100.00 | 100.01 | 100.01 | 100.01 | 100.01 |
| $P_1^p(2,1)$ | 107.00 | 107.00 | 107.00 | 107.00 | 107.00 |
| $P_1^p(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_1^p(2,3)$ | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| $P_1^r(1,1)$ | 100.00 | 100.01 | 100.01 | 100.01 | 100.01 |
| $P_1^r(2,1)$ | 107.00 | 107.00 | 107.00 | 107.00 | 107.00 |
| $P_1^r(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_1^r(2,3)$ | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| $P_2^p(1,1)$ | 100.00 | 100.06 | 100.06 | 100.03 | 100.03 |
| $P_2^p(2,1)$ | 98.00 | 98.10 | 98.08 | 98.04 | 98.03 |
| $P_2^p(2,2)$ | 101.00 | 101.10 | 101.08 | 101.04 | 101.03 |
| $P_2^p(2,3)$ | 105.00 | 105.10 | 105.08 | 105.04 | 105.03 |
| $P_2^r(1,1)$ | 100.00 | 100.06 | 100.05 | 100.03 | 100.02 |
| $P_2^r(2,1)$ | 98.00 | 98.10 | 98.07 | 98.04 | 98.03 |
| $P_2^r(2,2)$ | 101.00 | 101.10 | 101.07 | 101.04 | 101.03 |
| $P_2^r(2,3)$ | 105.00 | 105.10 | 105.07 | 105.04 | 105.03 |

Table B.2.: Parameter values and results for the two-period model with initial cash endowment and concave price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

| | $S$ | $D1$ | $D2$ | $D3$ | $D4$ |
|---|---|---|---|---|---|
| *C*haracteristic | | | | | |
| Size | n.a. | small | small | small | small |
| Differentiation | n.a. | equal | equal | different | different |
| Reversal | n.a. | none | partial | none | partial |
| Funcational form | n.a. | linear | linear | linear | linear |
| *P*arameter | | | | | |
| $l_1$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0.2 | 0 | 0.2 |
| *V*ariable | | | | | |
| $w_1(1,1)$ | 38.89% | 40.50% | 40.23% | 38.05% | 38.22% |
| $w_1(2,1)$ | 38.89% | 33.82% | 36.03% | 40.78% | 40.44% |
| $w_1(2,2)$ | 38.89% | 33.82% | 35.66% | 40.78% | 40.15% |
| $w_1(2,3)$ | 38.89% | 33.82% | 35.30% | 40.78% | 39.85% |
| $w_2(1,1)$ | 61.11% | 59.50% | 59.77% | 61.95% | 61.78% |
| $w_2(2,1)$ | 61.11% | 66.18% | 63.97% | 59.22% | 59.56% |
| $w_2(2,2)$ | 61.11% | 66.18% | 64.34% | 59.22% | 59.85% |
| $w_2(2,3)$ | 61.11% | 66.18% | 64.70% | 59.22% | 60.15% |
| $N_1(1,1)$ | 3888.89 | 4049.55 | 4023.18 | 3805.34 | 3821.63 |
| $N_1(2,1)$ | 3652.10 | 3180.90 | 3387.70 | 3826.64 | 3795.52 |
| $N_1(2,2)$ | 3850.00 | 3348.36 | 3530.89 | 4036.89 | 3975.10 |
| $N_1(2,3)$ | 4070.57 | 3535.01 | 3690.27 | 4271.22 | 4173.82 |
| $N_2(1,1)$ | 6111.11 | 5950.45 | 5976.82 | 6194.66 | 6178.37 |
| $N_2(2,1)$ | 6266.07 | 6795.05 | 6566.91 | 6067.91 | 6103.37 |
| $N_2(2,2)$ | 6050.00 | 6551.58 | 6369.08 | 5863.10 | 5924.89 |
| $N_2(2,3)$ | 5848.33 | 6324.33 | 6184.63 | 5671.95 | 5759.63 |
| $U_1(1,1)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $U_1(2,1)$ | -236.79 | -868.66 | -635.48 | 21.31 | -26.11 |
| $U_1(2,2)$ | -38.89 | -701.19 | -492.28 | 231.56 | 153.47 |
| $U_1(2,3)$ | 181.68 | -514.54 | -332.91 | 465.89 | 352.19 |
| $U_2(1,1)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $U_2(2,1)$ | 154.96 | 844.61 | 590.09 | -126.76 | -75.00 |
| $U_2(2,2)$ | -61.11 | 601.13 | 392.26 | -331.56 | -253.48 |
| $U_2(2,3)$ | -262.78 | 373.88 | 207.81 | -522.71 | -418.74 |
| $P_1^p(1,1)$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $P_1^p(2,1)$ | 107.00 | 106.99 | 107.00 | 107.00 | 107.00 |
| $P_1^p(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_1^p(2,3)$ | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| $P_1^r(1,1)$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $P_1^r(2,1)$ | 107.00 | 106.99 | 107.00 | 107.00 | 107.00 |
| $P_1^r(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_1^r(2,3)$ | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| $P_2^p(1,1)$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $P_2^p(2,1)$ | 98.00 | 98.01 | 98.00 | 98.00 | 98.00 |
| $P_2^p(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_2^p(2,3)$ | 105.00 | 105.00 | 105.00 | 105.00 | 105.00 |
| $P_2^r(1,1)$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| $P_2^r(2,1)$ | 98.00 | 98.01 | 98.00 | 98.00 | 98.00 |
| $P_2^r(2,2)$ | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| $P_2^r(2,3)$ | 105.00 | 105.00 | 105.00 | 105.00 | 105.00 |

Table B.3.: Parameter values and results for the two-period model without price effect of initial transactions and linear price impact function. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

| | *S* | *E*1 | *E*2 | *E*3 | *E*4 | *E*5 | *E*6 |
|---|---|---|---|---|---|---|---|
| *C*haracteristic | | | | | | | |
| Size | n.a. | n.a. | n.a. | small | small | small | small |
| Differentiation | n.a. | n.a. | n.a. | equal | equal | different | different |
| Reversal | n.a. | n.a. | n.a. | none | partial | none | partial |
| *P*arameter | | | | | | | |
| $l_1$ | 0 | 0 | 0 | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $7 \cdot 10^{-8}$ |
| $l_2$ | 0 | 0 | 0 | $7 \cdot 10^{-7}$ | $3.5 \cdot 10^{-7}$ | $7 \cdot 10^{-8}$ | $3.5 \cdot 10^{-8}$ |
| $\rho_1, \rho_2$ | 0 | 0 | 0 | 0 | 0.2 | 0 | 0.2 |
| $\delta$ | 0 | 0.2 | 0.6 | 0.2 | 0.2 | 0.2 | 0.2 |
| *V*ariable | | | | | | | |
| $w_1(1,1)$ | 38.89% | 35.48% | 28.68% | 36.40% | 36.23% | 35.13% | 35.21% |
| $w_1(2,1)$ | 38.89% | 35.72% | 30.46% | 32.04% | 33.53% | 37.14% | 36.92% |
| $w_1(2,2)$ | 38.89% | 35.71% | 30.41% | 31.89% | 33.12% | 37.05% | 36.62% |
| $w_1(2,3)$ | 38.89% | 35.70% | 30.36% | 31.79% | 32.71% | 36.95% | 36.32% |
| $w_2(1,1)$ | 61.11% | 64.52% | 71.32% | 63.60% | 63.77% | 64.87% | 64.79% |
| $w_2(2,1)$ | 61.11% | 64.28% | 69.54% | 67.96% | 66.47% | 62.86% | 63.08% |
| $w_2(2,2)$ | 61.11% | 64.29% | 69.59% | 68.11% | 66.88% | 62.95% | 63.38% |
| $w_2(2,3)$ | 61.11% | 64.30% | 69.64% | 68.21% | 67.29% | 63.05% | 63.68% |
| $N_1(1,1)$ | 3,888.89 | 3,547.53 | 2,868.29 | 3,640.22 | 3,622.80 | 3,512.63 | 3,520.96 |
| $N_1(2,1)$ | 3,652.10 | 3,353.34 | 2,852.40 | 3,011.23 | 3,150.18 | 3,485.85 | 3,465.50 |
| $N_1(2,2)$ | 3,850.00 | 3,535.19 | 3,010.49 | 3,157.67 | 3,278.68 | 3,667.43 | 3,625.07 |
| $N_1(2,3)$ | 4,070.57 | 3,737.86 | 3,186.70 | 3,325.44 | 3,422.02 | 3,869.68 | 3,804.17 |
| $N_2(1,1)$ | 6,111.11 | 6,452.47 | 7,131.71 | 6,359.78 | 6,377.20 | 6,487.37 | 6,479.04 |
| $N_2(2,1)$ | 6,266.07 | 6,589.37 | 7,109.58 | 6,972.96 | 6,819.37 | 6,440.89 | 6,464.02 |
| $N_2(2,2)$ | 6,050.00 | 6,364.81 | 6,889.51 | 6,742.31 | 6,621.31 | 6,232.57 | 6,274.93 |
| $N_2(2,3)$ | 5,848.33 | 6,155.23 | 6,684.10 | 6,522.85 | 6,436.33 | 6,038.25 | 6,097.30 |
| $U_1(1,1)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $U_1(2,1)$ | -236.79 | -194.19 | -15.89 | -628.99 | -472.62 | -26.78 | -55.46 |
| $U_1(2,2)$ | -38.89 | -12.34 | 142.21 | -482.55 | -344.12 | 154.80 | 104.12 |
| $U_1(2,3)$ | 181.68 | 190.34 | 318.42 | -314.78 | -200.78 | 357.05 | 283.21 |
| $U_2(1,1)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $U_2(2,1)$ | 154.96 | 136.90 | -22.13 | 613.18 | 442.17 | -46.48 | -15.03 |
| $U_2(2,2)$ | -61.11 | -87.66 | -242.21 | 382.53 | 244.11 | -254.80 | -204.12 |
| $U_2(2,3)$ | -262.78 | -297.25 | -447.61 | 163.07 | 59.13 | -449.12 | -381.74 |
| $CF^{2,1}$ | 0.00 | 2,785.55 | 6,251.65 | 2,952.39 | 2,921.03 | 2,722.73 | 2,737.72 |
| $CF^{2,2}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $CF^{2,3}$ | 0.00 | -2,785.55 | -6,251.65 | -2,952.39 | -2,921.03 | -2,722.73 | -2,737.72 |
| $CF^{3,1}$ | 0.00 | 1,372.73 | 4,457.85 | 738.27 | 1,001.92 | 1,617.47 | 1,580.69 |
| $CF^{3,2}$ | 0.00 | -1,487.04 | -3,126.16 | -1,458.64 | -1,463.35 | -1,498.98 | -1,496.22 |
| $CF^{3,3}$ | 0.00 | -4,361.53 | -10,741.12 | -3,670.00 | -3,943.11 | -4,630.26 | -4,587.94 |
| $CF^{3,4}$ | 0.00 | 811.37 | -570.09 | 123.00 | 343.88 | 1,052.41 | 975.14 |
| $CF^{3,5}$ | 0.00 | -2,040.20 | -8,160.80 | -2,042.23 | -2,041.34 | -2,039.59 | -2,039.85 |
| $CF^{3,6}$ | 0.00 | -4,911.97 | -15,832.31 | -4,227.67 | -4,446.78 | -5,151.78 | -5,075.04 |
| $CF^{3,7}$ | 0.00 | 258.04 | -5,603.68 | -483.36 | -310.72 | 497.40 | 381.26 |
| $CF^{3,8}$ | 0.00 | -2,613.56 | -13,276.24 | -2,646.23 | -2,639.72 | -2,600.53 | -2,603.80 |
| $CF^{3,9}$ | 0.00 | -5,511.04 | -21,080.25 | -4,835.29 | -4,994.86 | -5,724.21 | -5,614.65 |

Table B.4.: Parameter values and results for the two-period model without price effect of initial transactions and performance-related capital flow. The model $S$ refers to the two-period standard model in figure 4.5 for comparison. N.a. indicates a non-relevant feature. (Source: Own calculation)

## B.2. Implementation of Trinomial Portfolio Model

The numerical solutions in section 4.3 and in chapter 6 have been calculated using the GAMS software package.[325] GAMS requires to define the problem in a special modeling language. Using a set of Python scripts,[326] various definition files have been created implementing the characteristics of models in the text. The program listing below represents a comprehensive model (model E4 from section 6.3.4) including management fee and price effects.[327] A linear price impact function is used in that case. Other trinomial models with a linear price impact function can be derived as special cases from this model.[328] For other forms of price impact functions, a different number of assets or states, a new file must be created.

The listing contains additional definitions in lines starting with "`*py`" that will be substituted in the definition of equations. These substitutions make the source file more readable and make it closely follow the mathematical formulation in chapter 6. Another Python script performs the substitution resulting in a very long (and unreadable) file that can then be used with GAMS.

Lines 1-41 define parameter values. Solution variables are declared in lines 42-69. Lines 70-85 determine lower and upper bounds of variables for technical reasons. Initial and terminal holdings are given in 105. Lines 106-183 define the prices $P_i^a(t,k)$, $P_i^p(t,k)$, and $P_i^r(t,k)$ in altering order. Terms for portfolio values $X^{a,a}(t,k)$, $X^{a,p}(t,k)$, and $X^{p,p}(t,k)$ are given in lines 184-222. Lines 223-235 define the value of the benchmark portfolio, lines 236-248 the capital flow, and lines 249-261 the management fee. The target variable is declared in lines 262 and 263. Lines 264-277 represent the maximization criterion. Financing constraints are defined in lines 278-289. Line 290 defines the total model and line 291 calls the "discontinuous nonlinear programming" solver to provide numerical solutions that maximize the target variable.

```
1  Scalar
2          s1 price impact scaling /0.1/
3          s2 price impact scaling /0.1/
```

[325] Cf. `www.gams.com`. A version, that is sufficient for all models presented in this work, is freely available for download.

[326] These scripts are available from the author upon request. Python is available from `www.python.org`.

[327] Excluded from the program listing are those technical but lengthy parts that only perform the output of solution values.

[328] The models are available from the author upon request.

```
4           r1 prive reversal /0.2/
5           r2 price reversal /0.2/
6           f linear capital flow factor /0.2/
7           g risk aversion parameter /0.5/
8           XT1S1 initial wealth /1.0E6/
9           p1 probability 1 /0.33/
10          p2 probability 2 /0.33/
11          p3 probability 2 /0.34/
12          P1T0i  /100/
13          P2T0i  /100/
14          tA1S1  /1.07/
15          tA1S2  /1.01/
16          tA1S3  /0.96/
17          tA2S1  /0.98/
18          tA2S2  /1.01/
19          tA2S3  /1.05/
20          ce1 price impact variable /7.5E-3/
21          cf1  /2.0E-5/
22          l1  /7.0E-7/
23          vexe1  /7/
24          vexf1  /1.0E-5/
25          revpos1 positive price reverseal /0/
26          revneg1 negative price reverseal /0/
27          ce2 price impact variable /7.5E-3/
28          cf2  /2.0E-4/
29          l2  /7.0E-7/
30          vexe2  /7/
31          vexf2  /1.0E-5/
32          revpos2 positive price reverseal /0/
33          revneg2 negative price reverseal /0/
34          b1 first benchmark asset /0.2/
35          b2 second benchmark asset /0.8/
36          lambda management fee /0.01/
37  ;
38  * Set probabilities to 1/3 and use maximal precision of GAMS
39  p1 = 1/3;
40  p2 = 1/3;
41  p3 = 1/3;
42  Positive Variables
43  N1T1S1
44  N1T2S1
45  N1T2S2
46  N1T2S3
47  N1T3S1
48  N1T3S2
49  N1T3S3
50  N1T3S4
51  N1T3S5
52  N1T3S6
53  N1T3S7
54  N1T3S8
55  N1T3S9
56  N2T1S1
```

```
57  N2T2S1
58  N2T2S2
59  N2T2S3
60  N2T3S1
61  N2T3S2
62  N2T3S3
63  N2T3S4
64  N2T3S5
65  N2T3S6
66  N2T3S7
67  N2T3S8
68  N2T3S9
69  ;
70          N1T1S1.lo = 1;
71          N1T2S1.lo = 1;
72          N1T2S2.lo = 1;
73          N1T2S3.lo = 1;
74          N2T1S1.lo = 1;
75          N2T2S1.lo = 1;
76          N2T2S2.lo = 1;
77          N2T2S3.lo = 1;
78          N1T1S1.up = 9999;
79          N1T2S1.up = 9999;
80          N1T2S2.up = 9999;
81          N1T2S3.up = 9999;
82          N2T1S1.up = 9999;
83          N2T2S1.up = 9999;
84          N2T2S2.up = 9999;
85          N2T2S3.up = 9999;
86  *py _N1T0S1i_ == (N1T1S1);
87  *py _N2T0S1i_ == (N2T1S1);
88  *py _N1T3S1t_ == (N1T2S1);
89  *py _N1T3S2t_ == (N1T2S1);
90  *py _N1T3S3t_ == (N1T2S1);
91  *py _N1T3S4t_ == (N1T2S2);
92  *py _N1T3S5t_ == (N1T2S2);
93  *py _N1T3S6t_ == (N1T2S2);
94  *py _N1T3S7t_ == (N1T2S3);
95  *py _N1T3S8t_ == (N1T2S3);
96  *py _N1T3S9t_ == (N1T2S3);
97  *py _N2T3S1t_ == (N2T2S1);
98  *py _N2T3S2t_ == (N2T2S1);
99  *py _N2T3S3t_ == (N2T2S1);
100 *py _N2T3S4t_ == (N2T2S2);
101 *py _N2T3S5t_ == (N2T2S2);
102 *py _N2T3S6t_ == (N2T2S2);
103 *py _N2T3S7t_ == (N2T2S3);
104 *py _N2T3S8t_ == (N2T2S3);
105 *py _N2T3S9t_ == (N2T2S3);
106 *py _Pa1T1S1_ == (P1T0i);
107 *py _Pp1T1S1_ == (((1+s1*l1*(N1T1S1-_N1T0S1i_))*_Pa1T1S1_));
108 *py _Pr1T1S1_ == ((1/(1+r1*s1*l1*(N1T1S1-_N1T0S1i_))*_Pp1T1S1_));
109 *py _Pa1T2S1_ == (_Pr1T1S1_*tA1S1);
```

```
110 *py _Pp1T2S1_ == (((1+s1*l1*(N1T2S1-N1T1S1))*_Pa1T2S1_));
111 *py _Pr1T2S1_ == ((1/(1+r1*s1*l1*(N1T2S1-N1T1S1))*_Pp1T2S1_));
112 *py _Pa1T2S2_ == (_Pr1T1S1_*tA1S2);
113 *py _Pp1T2S2_ == (((1+s1*l1*(N1T2S2-N1T1S1))*_Pa1T2S2_));
114 *py _Pr1T2S2_ == ((1/(1+r1*s1*l1*(N1T2S2-N1T1S1))*_Pp1T2S2_));
115 *py _Pa1T2S3_ == (_Pr1T1S1_*tA1S3);
116 *py _Pp1T2S3_ == (((1+s1*l1*(N1T2S3-N1T1S1))*_Pa1T2S3_));
117 *py _Pr1T2S3_ == ((1/(1+r1*s1*l1*(N1T2S3-N1T1S1))*_Pp1T2S3_));
118 *py _Pa1T3S1_ == (_Pr1T2S1_*tA1S1);
119 *py _Pp1T3S1_ == (((1+s1*l1*(_N1T3S1t_-N1T2S1))*_Pa1T3S1_));
120 *py _Pr1T3S1_ == ((1/(1+r1*s1*l1*(_N1T3S1t_-N1T2S1))*_Pp1T3S1_));
121 *py _Pa1T3S2_ == (_Pr1T2S1_*tA1S2);
122 *py _Pp1T3S2_ == (((1+s1*l1*(_N1T3S2t_-N1T2S1))*_Pa1T3S2_));
123 *py _Pr1T3S2_ == ((1/(1+r1*s1*l1*(_N1T3S2t_-N1T2S1))*_Pp1T3S2_));
124 *py _Pa1T3S3_ == (_Pr1T2S1_*tA1S3);
125 *py _Pp1T3S3_ == (((1+s1*l1*(_N1T3S3t_-N1T2S1))*_Pa1T3S3_));
126 *py _Pr1T3S3_ == ((1/(1+r1*s1*l1*(_N1T3S3t_-N1T2S1))*_Pp1T3S3_));
127 *py _Pa1T3S4_ == (_Pr1T2S2_*tA1S1);
128 *py _Pp1T3S4_ == (((1+s1*l1*(_N1T3S4t_-N1T2S2))*_Pa1T3S4_));
129 *py _Pr1T3S4_ == ((1/(1+r1*s1*l1*(_N1T3S4t_-N1T2S2))*_Pp1T3S4_));
130 *py _Pa1T3S5_ == (_Pr1T2S2_*tA1S2);
131 *py _Pp1T3S5_ == (((1+s1*l1*(_N1T3S5t_-N1T2S2))*_Pa1T3S5_));
132 *py _Pr1T3S5_ == ((1/(1+r1*s1*l1*(_N1T3S5t_-N1T2S2))*_Pp1T3S5_));
133 *py _Pa1T3S6_ == (_Pr1T2S2_*tA1S3);
134 *py _Pp1T3S6_ == (((1+s1*l1*(_N1T3S6t_-N1T2S2))*_Pa1T3S6_));
135 *py _Pr1T3S6_ == ((1/(1+r1*s1*l1*(_N1T3S6t_-N1T2S2))*_Pp1T3S6_));
136 *py _Pa1T3S7_ == (_Pr1T2S3_*tA1S1);
137 *py _Pp1T3S7_ == (((1+s1*l1*(_N1T3S7t_-N1T2S3))*_Pa1T3S7_));
138 *py _Pr1T3S7_ == ((1/(1+r1*s1*l1*(_N1T3S7t_-N1T2S3))*_Pp1T3S7_));
139 *py _Pa1T3S8_ == (_Pr1T2S3_*tA1S2);
140 *py _Pp1T3S8_ == (((1+s1*l1*(_N1T3S8t_-N1T2S3))*_Pa1T3S8_));
141 *py _Pr1T3S8_ == ((1/(1+r1*s1*l1*(_N1T3S8t_-N1T2S3))*_Pp1T3S8_));
142 *py _Pa1T3S9_ == (_Pr1T2S3_*tA1S3);
143 *py _Pp1T3S9_ == (((1+s1*l1*(_N1T3S9t_-N1T2S3))*_Pa1T3S9_));
144 *py _Pr1T3S9_ == ((1/(1+r1*s1*l1*(_N1T3S9t_-N1T2S3))*_Pp1T3S9_));
145 *py _Pa2T1S1_ == (P2T0i);
146 *py _Pp2T1S1_ == (((1+s2*l2*(N2T1S1-_N2T0S1i_))*_Pa2T1S1_));
147 *py _Pr2T1S1_ == ((1/(1+r2*s2*l2*(N2T1S1-_N2T0S1i_))*_Pp2T1S1_));
148 *py _Pa2T2S1_ == (_Pr2T1S1_*tA2S1);
149 *py _Pp2T2S1_ == (((1+s2*l2*(N2T2S1-N2T1S1))*_Pa2T2S1_));
150 *py _Pr2T2S1_ == ((1/(1+r2*s2*l2*(N2T2S1-N2T1S1))*_Pp2T2S1_));
151 *py _Pa2T2S2_ == (_Pr2T1S1_*tA2S2);
152 *py _Pp2T2S2_ == (((1+s2*l2*(N2T2S2-N2T1S1))*_Pa2T2S2_));
153 *py _Pr2T2S2_ == ((1/(1+r2*s2*l2*(N2T2S2-N2T1S1))*_Pp2T2S2_));
154 *py _Pa2T2S3_ == (_Pr2T1S1_*tA2S3);
155 *py _Pp2T2S3_ == (((1+s2*l2*(N2T2S3-N2T1S1))*_Pa2T2S3_));
156 *py _Pr2T2S3_ == ((1/(1+r2*s2*l2*(N2T2S3-N2T1S1))*_Pp2T2S3_));
157 *py _Pa2T3S1_ == (_Pr2T2S1_*tA2S1);
158 *py _Pp2T3S1_ == (((1+s2*l2*(_N2T3S1t_-N2T2S1))*_Pa2T3S1_));
159 *py _Pr2T3S1_ == ((1/(1+r2*s2*l2*(_N2T3S1t_-N2T2S1))*_Pp2T3S1_));
160 *py _Pa2T3S2_ == (_Pr2T2S1_*tA2S2);
161 *py _Pp2T3S2_ == (((1+s2*l2*(_N2T3S2t_-N2T2S1))*_Pa2T3S2_));
162 *py _Pr2T3S2_ == ((1/(1+r2*s2*l2*(_N2T3S2t_-N2T2S1))*_Pp2T3S2_));
```

```
163 *py _Pa2T3S3_ == (_Pr2T2S1_*tA2S3);
164 *py _Pp2T3S3_ == (((1+s2*l2*(_N2T3S3t_-N2T2S1))*_Pa2T3S3_));
165 *py _Pr2T3S3_ == ((1/(1+r2*s2*l2*(_N2T3S3t_-N2T2S1))*_Pp2T3S3_));
166 *py _Pa2T3S4_ == (_Pr2T2S2_*tA2S1);
167 *py _Pp2T3S4_ == (((1+s2*l2*(_N2T3S4t_-N2T2S2))*_Pa2T3S4_));
168 *py _Pr2T3S4_ == ((1/(1+r2*s2*l2*(_N2T3S4t_-N2T2S2))*_Pp2T3S4_));
169 *py _Pa2T3S5_ == (_Pr2T2S2_*tA2S2);
170 *py _Pp2T3S5_ == (((1+s2*l2*(_N2T3S5t_-N2T2S2))*_Pa2T3S5_));
171 *py _Pr2T3S5_ == ((1/(1+r2*s2*l2*(_N2T3S5t_-N2T2S2))*_Pp2T3S5_));
172 *py _Pa2T3S6_ == (_Pr2T2S2_*tA2S3);
173 *py _Pp2T3S6_ == (((1+s2*l2*(_N2T3S6t_-N2T2S2))*_Pa2T3S6_));
174 *py _Pr2T3S6_ == ((1/(1+r2*s2*l2*(_N2T3S6t_-N2T2S2))*_Pp2T3S6_));
175 *py _Pa2T3S7_ == (_Pr2T2S3_*tA2S1);
176 *py _Pp2T3S7_ == (((1+s2*l2*(_N2T3S7t_-N2T2S3))*_Pa2T3S7_));
177 *py _Pr2T3S7_ == ((1/(1+r2*s2*l2*(_N2T3S7t_-N2T2S3))*_Pp2T3S7_));
178 *py _Pa2T3S8_ == (_Pr2T2S3_*tA2S2);
179 *py _Pp2T3S8_ == (((1+s2*l2*(_N2T3S8t_-N2T2S3))*_Pa2T3S8_));
180 *py _Pr2T3S8_ == ((1/(1+r2*s2*l2*(_N2T3S8t_-N2T2S3))*_Pp2T3S8_));
181 *py _Pa2T3S9_ == (_Pr2T2S3_*tA2S3);
182 *py _Pp2T3S9_ == (((1+s2*l2*(_N2T3S9t_-N2T2S3))*_Pa2T3S9_));
183 *py _Pr2T3S9_ == ((1/(1+r2*s2*l2*(_N2T3S9t_-N2T2S3))*_Pp2T3S9_));
184 *py _XaPaT1S1_ == (XT1S1);
185 *py _XaPaT2S1_ == (_Pa1T2S1_*N1T1S1+_Pa2T2S1_*N2T1S1);
186 *py _XaPaT2S2_ == (_Pa1T2S2_*N1T1S1+_Pa2T2S2_*N2T1S1);
187 *py _XaPaT2S3_ == (_Pa1T2S3_*N1T1S1+_Pa2T2S3_*N2T1S1);
188 *py _XaPaT3S1_ == (_Pa1T3S1_*N1T2S1+_Pa2T3S1_*N2T2S1);
189 *py _XaPaT3S2_ == (_Pa1T3S2_*N1T2S1+_Pa2T3S2_*N2T2S1);
190 *py _XaPaT3S3_ == (_Pa1T3S3_*N1T2S1+_Pa2T3S3_*N2T2S1);
191 *py _XaPaT3S4_ == (_Pa1T3S4_*N1T2S2+_Pa2T3S4_*N2T2S2);
192 *py _XaPaT3S5_ == (_Pa1T3S5_*N1T2S2+_Pa2T3S5_*N2T2S2);
193 *py _XaPaT3S6_ == (_Pa1T3S6_*N1T2S2+_Pa2T3S6_*N2T2S2);
194 *py _XaPaT3S7_ == (_Pa1T3S7_*N1T2S3+_Pa2T3S7_*N2T2S3);
195 *py _XaPaT3S8_ == (_Pa1T3S8_*N1T2S3+_Pa2T3S8_*N2T2S3);
196 *py _XaPaT3S9_ == (_Pa1T3S9_*N1T2S3+_Pa2T3S9_*N2T2S3);
197 *py _XaPpT1S1_ == (XT1S1);
198 *py _XaPpT2S1_ == (_Pp1T2S1_*N1T1S1+_Pp2T2S1_*N2T1S1);
199 *py _XaPpT2S2_ == (_Pp1T2S2_*N1T1S1+_Pp2T2S2_*N2T1S1);
200 *py _XaPpT2S3_ == (_Pp1T2S3_*N1T1S1+_Pp2T2S3_*N2T1S1);
201 *py _XaPpT3S1_ == (_Pp1T3S1_*N1T2S1+_Pp2T3S1_*N2T2S1);
202 *py _XaPpT3S2_ == (_Pp1T3S2_*N1T2S1+_Pp2T3S2_*N2T2S1);
203 *py _XaPpT3S3_ == (_Pp1T3S3_*N1T2S1+_Pp2T3S3_*N2T2S1);
204 *py _XaPpT3S4_ == (_Pp1T3S4_*N1T2S2+_Pp2T3S4_*N2T2S2);
205 *py _XaPpT3S5_ == (_Pp1T3S5_*N1T2S2+_Pp2T3S5_*N2T2S2);
206 *py _XaPpT3S6_ == (_Pp1T3S6_*N1T2S2+_Pp2T3S6_*N2T2S2);
207 *py _XaPpT3S7_ == (_Pp1T3S7_*N1T2S3+_Pp2T3S7_*N2T2S3);
208 *py _XaPpT3S8_ == (_Pp1T3S8_*N1T2S3+_Pp2T3S8_*N2T2S3);
209 *py _XaPpT3S9_ == (_Pp1T3S9_*N1T2S3+_Pp2T3S9_*N2T2S3);
210 *py _XpPpT1S1_ == (_Pp1T1S1_*N1T1S1+_Pp2T1S1_*N2T1S1);
211 *py _XpPpT2S1_ == (_Pp1T2S1_*N1T2S1+_Pp2T2S1_*N2T2S1);
212 *py _XpPpT2S2_ == (_Pp1T2S2_*N1T2S2+_Pp2T2S2_*N2T2S2);
213 *py _XpPpT2S3_ == (_Pp1T2S3_*N1T2S3+_Pp2T2S3_*N2T2S3);
214 *py _XpPpT3S1_ == (_Pp1T3S1_*N1T3S1t+_Pp2T3S1_*N2T3S1t);
215 *py _XpPpT3S2_ == (_Pp1T3S2_*N1T3S2t+_Pp2T3S2_*N2T3S2t);
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216 *py _XpPpT3S3_ == (_Pp1T3S3_*N1T3S3t+_Pp2T3S3_*N2T3S3t);
217 *py _XpPpT3S4_ == (_Pp1T3S4_*N1T3S4t+_Pp2T3S4_*N2T3S4t);
218 *py _XpPpT3S5_ == (_Pp1T3S5_*N1T3S5t+_Pp2T3S5_*N2T3S5t);
219 *py _XpPpT3S6_ == (_Pp1T3S6_*N1T3S6t+_Pp2T3S6_*N2T3S6t);
220 *py _XpPpT3S7_ == (_Pp1T3S7_*N1T3S7t+_Pp2T3S7_*N2T3S7t);
221 *py _XpPpT3S8_ == (_Pp1T3S8_*N1T3S8t+_Pp2T3S8_*N2T3S8t);
222 *py _XpPpT3S9_ == (_Pp1T3S9_*N1T3S9t+_Pp2T3S9_*N2T3S9t);
223 *py _BPaT1S1_ == (_XaPaT1S1_);
224 *py _BPaT2S1_ == (b1*(_XaPaT1S1_)/_Pa1T1S1_*_Pa1T2S1_+b2*(_XaPaT1S1_)/_Pa2T1S1_*
        _Pa2T2S1_);
225 *py _BPaT2S2_ == (b1*(_XaPaT1S1_)/_Pa1T1S1_*_Pa1T2S2_+b2*(_XaPaT1S1_)/_Pa2T1S1_*
        _Pa2T2S2_);
226 *py _BPaT2S3_ == (b1*(_XaPaT1S1_)/_Pa1T1S1_*_Pa1T2S3_+b2*(_XaPaT1S1_)/_Pa2T1S1_*
        _Pa2T2S3_);
227 *py _BPaT3S1_ == (b1*(_XaPaT2S1_)/_Pa1T2S1_*_Pa1T3S1_+b2*(_XaPaT2S1_)/_Pa2T2S1_*
        _Pa2T3S1_);
228 *py _BPaT3S2_ == (b1*(_XaPaT2S1_)/_Pa1T2S1_*_Pa1T3S2_+b2*(_XaPaT2S1_)/_Pa2T2S1_*
        _Pa2T3S2_);
229 *py _BPaT3S3_ == (b1*(_XaPaT2S1_)/_Pa1T2S1_*_Pa1T3S3_+b2*(_XaPaT2S1_)/_Pa2T2S1_*
        _Pa2T3S3_);
230 *py _BPaT3S4_ == (b1*(_XaPaT2S2_)/_Pa1T2S2_*_Pa1T3S4_+b2*(_XaPaT2S2_)/_Pa2T2S2_*
        _Pa2T3S4_);
231 *py _BPaT3S5_ == (b1*(_XaPaT2S2_)/_Pa1T2S2_*_Pa1T3S5_+b2*(_XaPaT2S2_)/_Pa2T2S2_*
        _Pa2T3S5_);
232 *py _BPaT3S6_ == (b1*(_XaPaT2S2_)/_Pa1T2S2_*_Pa1T3S6_+b2*(_XaPaT2S2_)/_Pa2T2S2_*
        _Pa2T3S6_);
233 *py _BPaT3S7_ == (b1*(_XaPaT2S3_)/_Pa1T2S3_*_Pa1T3S7_+b2*(_XaPaT2S3_)/_Pa2T2S3_*
        _Pa2T3S7_);
234 *py _BPaT3S8_ == (b1*(_XaPaT2S3_)/_Pa1T2S3_*_Pa1T3S8_+b2*(_XaPaT2S3_)/_Pa2T2S3_*
        _Pa2T3S8_);
235 *py _BPaT3S9_ == (b1*(_XaPaT2S3_)/_Pa1T2S3_*_Pa1T3S9_+b2*(_XaPaT2S3_)/_Pa2T2S3_*
        _Pa2T3S9_);
236 *py _CFT1S1_ == (0);
237 *py _CFT2S1_ == ((f*(_XaPaT2S1_-_BPaT2S1_)));
238 *py _CFT2S2_ == ((f*(_XaPaT2S2_-_BPaT2S2_)));
239 *py _CFT2S3_ == ((f*(_XaPaT2S3_-_BPaT2S3_)));
240 *py _CFT3S1_ == ((f*(_XaPaT3S1_-_BPaT3S1_)));
241 *py _CFT3S2_ == ((f*(_XaPaT3S2_-_BPaT3S2_)));
242 *py _CFT3S3_ == ((f*(_XaPaT3S3_-_BPaT3S3_)));
243 *py _CFT3S4_ == ((f*(_XaPaT3S4_-_BPaT3S4_)));
244 *py _CFT3S5_ == ((f*(_XaPaT3S5_-_BPaT3S5_)));
245 *py _CFT3S6_ == ((f*(_XaPaT3S6_-_BPaT3S6_)));
246 *py _CFT3S7_ == ((f*(_XaPaT3S7_-_BPaT3S7_)));
247 *py _CFT3S8_ == ((f*(_XaPaT3S8_-_BPaT3S8_)));
248 *py _CFT3S9_ == ((f*(_XaPaT3S9_-_BPaT3S9_)));
249 *py _MFT1S1_ == (0);
250 *py _MFT2S1_ == ((lambda * (_XaPaT2S1_+_CFT2S1_)));
251 *py _MFT2S2_ == ((lambda * (_XaPaT2S2_+_CFT2S2_)));
252 *py _MFT2S3_ == ((lambda * (_XaPaT2S3_+_CFT2S3_)));
253 *py _MFT3S1_ == ((lambda * (_XaPaT3S1_+_CFT3S1_)));
254 *py _MFT3S2_ == ((lambda * (_XaPaT3S2_+_CFT3S2_)));
255 *py _MFT3S3_ == ((lambda * (_XaPaT3S3_+_CFT3S3_)));
256 *py _MFT3S4_ == ((lambda * (_XaPaT3S4_+_CFT3S4_)));
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257 *py _MFT3S5_ == ((lambda * (_XaPaT3S5_+_CFT3S5_)));
258 *py _MFT3S6_ == ((lambda * (_XaPaT3S6_+_CFT3S6_)));
259 *py _MFT3S7_ == ((lambda * (_XaPaT3S7_+_CFT3S7_)));
260 *py _MFT3S8_ == ((lambda * (_XaPaT3S8_+_CFT3S8_)));
261 *py _MFT3S9_ == ((lambda * (_XaPaT3S9_+_CFT3S9_)));
262 Variables
263         value;
264 Equations
265         crit;
266 crit..  value=e=p1*1/g*((max(_MFT2S1_,0))**g)
267                 + p2*1/g*((max(_MFT2S2_,0))**g)
268                 + p3*1/g*((max(_MFT2S3_,0))**g)
269                 + p1*p1*1/g*((max(_MFT3S1_,0))**g)
270                 + p1*p2*1/g*((max(_MFT3S2_,0))**g)
271                 + p1*p3*1/g*((max(_MFT3S3_,0))**g)
272                 + p2*p1*1/g*((max(_MFT3S4_,0))**g)
273                 + p2*p2*1/g*((max(_MFT3S5_,0))**g)
274                 + p2*p3*1/g*((max(_MFT3S6_,0))**g)
275                 + p3*p1*1/g*((max(_MFT3S7_,0))**g)
276                 + p3*p2*1/g*((max(_MFT3S8_,0))**g)
277                 + p3*p3*1/g*((max(_MFT3S9_,0))**g);
278 Equations
279         XfinT1S1;
280 XfinT1S1..      _XaPpT1S1_ + _CFT1S1_ - _MFT1S1_=e=_XpPpT1S1_;
281 Equations
282         XfinT2S1;
283 XfinT2S1..      _XaPpT2S1_ + _CFT2S1_ - _MFT2S1_=e=_XpPpT2S1_;
284 Equations
285         XfinT2S2;
286 XfinT2S2..      _XaPpT2S2_ + _CFT2S2_ - _MFT2S2_=e=_XpPpT2S2_;
287 Equations
288         XfinT2S3;
289 XfinT2S3..      _XaPpT2S3_ + _CFT2S3_ - _MFT2S3_=e=_XpPpT2S3_;
290 model optpf /all/;
291 solve optpf using dnlp maximizing value;
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