Portfolio Optimization with Irreversible Long-Term Investments in Renewable Energy under Policy Risk: A Mixed-Integer Multistage Stochastic Model and a Moving-Horizon Approach

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Abstract. Portfolio optimization is an ongoing hot topic of mathematical optimization and management science. Due to the current financial market environment with low interest rates and volatile stock markets, it is getting more and more important to extend portfolio optimization models by other types of investments than classical assets. In this paper, we present a mixed-integer multistage stochastic model that includes investment opportunities in irreversible and long-term infrastructure projects in the context of renewable energies, which are also subject to policy risk. On realistic time scales for investment problems of this type, the resulting instances are by far too large to be solved with today's most evolved optimization software. Thus, we present a tailored moving-horizon approach together with suitable approximations and simplifications of the model. We evaluate these approximations and simplifications in a computational sensitivity analysis and derive a final model that can be tackled on a realistic instance by our moving-horizon approach.

1. Introduction

Against the background of low interest rates and volatile stock markets, investments in infrastructure and renewable energy[1] are increasingly relevant for institutional investors looking for a stable income; see, e.g., Gatzert and Kosub (2014). Moreover, renewables become even more important against the background of a stronger focus on sustainable investments, driven by regulatory developments such as the EU Directive 2014/95/EU on Corporate Social Responsibility (CSR) or the introduction of the UN Principles for Responsible Investment (PRI). In this context, renewable energy investments such as wind parks become increasingly relevant.[2] However, such investments often suffer from irreversibility, long investment periods, and policy or regulatory risk, implying considerable challenges with respect to portfolio optimization (Gatzert and Kosub 2014, 2016). In this paper, we extend classic portfolio optimization problems by adding investment opportunities in irreversible and long-term investments in renewable energy projects.[3] In particular,

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[1]Note that in what follows, we refer to infrastructure and renewable energy investments interchangeably.

[2]As one current industry initiative, on September 23, 2019 the Net-Zero Alliance was founded at the UN Secretary-General's Climate Summit in New York. The Net-Zero Alliance is an international group of institutional investors with more than USD 2 trillion in assets under management, aiming for Article 2.1c of the Paris Agreement by committing to a carbon-neutral investment portfolio by 2050; see UNEP Finance Initiative (2019).

[3]For example, Allianz SE reports that "as part of our climate change strategy, we are investing in renewable energy assets. In total, we have financed 6.8 billion Euro with debt and equity

the investor has the "real option" to realize irreversible investments in wind farms in different countries with varying levels of policy risk and cross-country diversification potential (Gatzert and Vogl 2016).

Starting from Markowitz (1952), who considers mean-variance optimal portfolios, there is a wide literature on portfolio optimization and selection. For example, Mansini et al. (2014) provide a comprehensive overview of portfolio optimization problems solvable by linear optimization. However, the properties of the investments considered in this paper lead to very hard mixed-integer multistage stochastic optimization problems, which are also treated, e.g., in Fang et al. (2008), Mansini and Speranza (1999), or Gustafsson et al. (2005). Fang et al. (2008) consider investments in research and development (R&D) projects and extend the work of Gustafsson et al. (2005) by focusing on long investment periods and the inadjustability of R&D projects. Minimum transaction lots are considered by Mansini and Speranza (1999) and Lin and Liu (2008), who tackle the related mixed-integer problems using heuristics. New algorithmic developments for the stochastic mixed-integer linear programming problem are given in the very recent paper by Zou et al. (2019), where also a comprehensive literature survey on this topic can be found. For the respective continuous case see, e.g., Dantzig and Infanger (1993) and the references therein.

Policy risk in the context of investment decisions is also considered by Reuter et al. (2012) in a renewable energy context. In their model, however, the investor maximizes the expected net asset value (and thus does not explicitly consider risk in their investment decision) and has the opportunity to build one plant per year, i.e., either one coal plant, one wind farm, or nothing. As the importance of the consideration of risk and return has already been pointed out by Markowitz (1952), in contrast to Reuter et al. (2012), we explicitly consider risk by maximizing expected utility—which is also done by, e.g., Gennotte and Jung (1994) and Yu et al. (2009)—and by additionally allowing investment opportunities in tradable assets. Finally, for a very recent study on the impact of regulatory uncertainty in general electricity markets, we refer to Ambrosius et al. (2019).

Irreversible investments in general have been treated by Longstaff (2001), who points out that irreversibility is usually related to bid-ask spreads, i.e., transaction costs, or the impossibility of trading (referred to as "thin markets"). Transaction costs in portfolio optimization are, e.g., considered in Lobo et al. (2007), who describe a relaxation method yielding an upper bound via convex optimization and a heuristic method for computing a suboptimal portfolio.

We focus on the investor's perspective and optimize the portfolio consisting of tradable assets and irreversible investment opportunities in renewables, which are subject to policy risk. To this end, we build a multi-period model with discrete investment opportunities in wind farms using model components from Gatzert and Vogl (2016). The latter article builds the main basis for the modeling carried out in the present paper. However, in Gatzert and Vogl (2016), the authors only use Monte-Carlo simulations for the cash flows, whereas we here take their model, extend and modify it, and use it for multistage mixed-integer stochastic optimization. In contrast to Fang et al. (2008), who apply a single-stage model for the R&D projects, we allow for investments in the renewable projects not only at the beginning of the time horizon but also monthly, and we further allow for more than one investment. Contrary to Reuter et al. (2012), we consider risk in the decision-making process, add investment opportunities in traded assets, and investments in multiple countries

investments, this includes 86 wind parks and 9 solar farms (as of 31 December, 2018)."; see Allianz (2019). Therefore, since wind parks are highly relevant, they will serve to illustrate an irreversible (renewable) investment opportunity, while other types of renewables can be modeled as well using our approach.

to investigate cross-country diversification. Finally, we integrate shortfall constraints, which are especially important for regulated institutional investors. For example, insurance companies in the European Union are subject to the Solvency II directive and have to assure that their probability of default is less than 0.5 % within a one-year time horizon, which has to be taken into account in their asset portfolio decisions.

From a mathematical point of view, the described problem leads to extremely challenging mixed-integer multistage stochastic optimization problems; see, e.g., Vigerske (2013) or Römisch and Schultz (2001) as well as the references therein. These problems are defined on scenario trees that result from the discrete time setting and a proper discretization of the underlying stochastic processes. After such discretizations, the resulting finite-dimensional problem is usually by far too large to be solved in practice. This is especially the case in our application since we face multiple stochastic processes. As a remedy, we develop a problem-tailored moving-horizon strategy; see, e.g., Allgöwer et al. (1999) or Grüne and Pannek (2017) as well as the references therein. A similar approach also has been followed in Brown and Smith (2011), where a related approach is applied in the context of dynamic portfolio optimization with transaction costs. In such an algorithm, an optimization problem of the same type as the original one is solved but on a reduced time horizon. Afterward, the resulting solution is applied and the reduced time horizon is shifted forward. This strategy has also been used for other stochastic optimization problems; see, e.g., Drouven et al. (2017) for an application in shale well development or Cui and Engell (2010) where a similar approach is used for the planning of multi-product batch plants. Other applications of the moving-horizon technique applied to multistage stochastic models can be found, e.g., in Silvente et al. (2015) in the context of demand-side management in microgrids, in Guigues and Sagastizábal (2012) for the optimization of hydro-thermal power system planning, or in Möller et al. (2008), where it is applied to airline network revenue management. Another recently published technique for considering multistage stochastic optimization problems with strategic and operational investments is discussed in Kaut et al. (2014), where the authors propose a so-called multi-horizon approach. However, although the underlying problem has similar characteristics, the approach is different to the moving-horizon strategy used in this paper.

The remainder of the paper is structured as follows. After presenting the problem description in Section 2, we discuss the construction of the scenario tree by discretizing the stochastic processes in Section 3. The tailored moving-horizon strategy for the fully discretized model is then presented in Section 4. Numerical results are provided in Section 5, followed by a real-world example and a summary in Section 6.

2. Problem Description

In this section, we introduce the optimization problem to be considered in this paper. In Section 2.1, we first briefly introduce the main model aspects, which are then explained in detail in Sections 2.2 and 2.3. Finally, we state the entire stochastic optimization problem in Section 2.4.

2.1. **General Setup and Overview of the Problem.** We consider an investor who dynamically optimizes her portfolio over a given planning horizon $[t_s, t_e]$. In order to obtain a time-discrete setting, we discretize the planning horizon and obtain the ordered set of trading time points $T := \{t_0, \dots, t_I\}$ with $t_0 = t_s$ and $t_I = t_e$ for some $I \in \mathbb{N}$. Moreover, we denote the equidistant length of the time steps by $\Delta := \Delta_i := t_{i+1} - t_i$ for $i \in [I-1] := \{0, 1, \dots, I-1\}$. For the ease of presentation, we always consider Δ to be one month, which can be extended to any other time discretization as well.

The considered investment opportunities at each time point include traded assets and irreversible investments in renewables. Following Longstaff (2001), we model irreversibility by not allowing any trading in once conducted investments in renewables. We consider $J+1 \in \mathbb{N}$ traded assets with corresponding stochastic price processes S_t^j, $t \in [t_s, t_e]$, $j \in [J]$, as well as a bank account with constant risk-free interest rate r. The stochastic processes will be described in detail below. The amount of traded asset $j \in [J]$ bought at time t_i, $i \in [I-1]$, is denoted by $\alpha_{t_i}^j$ and the amount of traded asset $j \in [J]$ sold at time t_i, $i \in [I-1]$, is denoted by $\beta_{t_i}^j$. Note that we do not allow trading at t_I, since it would not have any effect on the terminal wealth. Moreover, $x_{t_i}^j$ is the amount of traded asset $j \in [J]$ held at time t_i, $i \in [I]$. This notation is based on the one used in Fang et al. (2008) and also allows to easily incorporate transaction costs if desired.

We take $K+1 \in \mathbb{N}$ different long-term investment opportunities in infrastructure into account. In general, this can be different types of investments in different countries, where the latter enables us to consider infrastructure investment in different countries and to analyze cross-country diversification; see, e.g., Gatzert and Vogl (2016). To simplify notation, we only consider one type of investment, namely in wind farms, in $K+1$ different countries. The cash flows of the infrastructure investments depend on the point in time when the project is initiated as well as on several stochastic processes. By $n_{t_i}^k \in \{0, \dots, n_{\max}^k\} \subset \mathbb{N}_0$, $i \in [I-1]$, and $k \in [K]$, we denote the bounded number of investments in country $k \in [K]$ purchased at time t_i, and $C_{t_i}^k$ denotes the corresponding infrastructure investment costs. At this point, the irreversibility is explicitly modeled by excluding negative $n_{t_i}^k$, i.e., we do not allow any selling. Note that we only take discrete amounts of investments in infrastructure into account since this is genuinely the case for the specific application of wind farms (or other large-scale renewable energy projects) that we consider. In particular, this is in contrast to continuous investments in standard securities like it is described above. The value of the infrastructure investments, which is considered in the portfolio optimization as the purchase price, depends on the future cash flows that are subject to uncertainty. Therefore, we extend the set of discrete time points T and define stochastic processes related to the infrastructure investments for all

$$t \in T \cup \{t_{I+1}, \dots, t_{I^{\max}}\} = \{t_0, \dots, t_I, t_{I+1}, \dots, t_{I^{\max}}\}. \tag{1}$$

Here,

$$I^{\max} := I - 1 + \max\left\{T^k : k \in [K]\right\}$$

holds, where $T^k \in \mathbb{N}$ is the length of the investment period in which the infrastructure investment in country $k \in [K]$ generates cash flows. As a consequence, cash flows are zero after this investment period. The future cash flows generated by the infrastructure investments of type k made at time $t_{i'}$, $i' \in [I-1]$, received by the investor at time t_i, $i \in [I^{\max}]$, $i > i'$, are denoted by $\gamma_{t_i, t_{i'}}^k$. All cash flows received at time t_i, $i \in \{i'+1, \dots, i'+T^k\}$, from infrastructure investments made until time $t_{i'}$, $i' \in [I-1]$, in country $k \in [K]$ are given by

$$\Gamma_{t_i, t_{i'}}^k := \sum_{\ell=0}^{\min\{i', i-1\}} n_{t_\ell}^k \gamma_{t_i, t_\ell}^k$$

and for $m \geq I$ we define

$$\Gamma_{t_m, t_I}^k := \sum_{\ell=0}^{I-1} n_{t_\ell}^k \gamma_{t_m, t_\ell}^k.$$

The explicit consideration of $t_{i'}$ is necessary to define the investor's wealth at every time point of the planning horizon, where future cash flows of all already made investments (without possible investments in the future) are taken into account. Given the initial wealth $w_{t_{-1}} > 0$, we can define the wealth at time t_i, $i \in [I]$, depending on the investment strategy by

$$w_{t_i} = L_{t_i} + \sum_{j\in[J]} x^j_{t_i} + \sum_{k\in[K]} \sum_{m=i+1}^{I^{\max}} \frac{\mathbb{E}_{t_i}[\Gamma^k_{t_m,t_i}]}{(1+r+\delta_k)^{t_m-t_i}}, \quad i \in [I], \tag{2}$$

with

$$\begin{aligned}
x^j_{t_i} &= x^j_{t_{i-1}} \frac{S^j_{t_i}}{S^j_{t_{i-1}}} + \alpha^j_{t_i} - \beta^j_{t_i}, \quad i \in [I-1],\ j \in [J],\\
x^j_{t_I} &= x^j_{t_{I-1}} \frac{S^j_{t_I}}{S^j_{t_{I-1}}}, \quad j \in [J],\\
x^j_{t_{-1}} &= 0, \quad j \in [J],\\
L_{t_i} &= L_{t_{i-1}}(1+r) - \sum_{j\in[J]} \alpha^j_{t_i} + \sum_{j\in[J]} \beta^j_{t_i} - \sum_{k\in[K]} n^k_{t_i} C^k_{t_i} + \sum_{k\in[K]} \Gamma^k_{t_i,t_i}, \quad i \in [I-1],\\
L_{t_I} &= L_{t_{I-1}}(1+r) + \sum_{k\in[K]} \Gamma^k_{t_I,t_I},\\
L_{t_{-1}} &= w_{t_{-1}}(1+r)^{-1}.
\end{aligned}$$

Following Fang et al. (2008), the amount of the traded asset $x^j_{t_i}$ is composed of the amount at time t_{i-1} considering the change of the price process and the difference of the amounts of assets bought and sold at time t_i, i.e., $\alpha^j_{t_i} - \beta^j_{t_i}$. The variable L models the liquid cash dedicated to the bank account. In Equation (2),

$$\sum_{k\in[K]} \sum_{m=i+1}^{I^{\max}} \frac{\mathbb{E}_{t_i}[\Gamma^k_{t_m,t_i}]}{(1+r+\delta_k)^{t_m-t_i}}$$

is the present value at time $t_i \in T$ of future cash flows from the infrastructure investments that have been purchased until time $t_i \in T$. It is based on a discounted cash flow model with (monthly) risk-free interest rate r and risk premium δ_k of the corresponding infrastructure investment. Finally, $\mathbb{E}_{t_i}$ denotes the corresponding expected value given all the information until t_i.

The investor's objective is to maximize the expected utility of her terminal wealth w_{t_I} at time t_I, i.e., the objective function reads

$$\max\ \mathbb{E}_{t_s}[u(w_{t_I})].$$

Here, given the initial wealth $w_{t_{-1}} > 0$ and the investment possibilities described above, we let $u : \mathbb{R} \to \mathbb{R}$ be a monotonically increasing utility function. There are various utility functions in the literature on portfolio optimization; see, e.g., Yu et al. (2009). We make use of the quadratic utility function

$$u(w) = w - \frac{w^2}{\alpha}, \quad 0 < \alpha,$$

which is widely used in the literature (see, e.g., Bodnar et al. (2015)) and which can be considered as a second-order approximation for more general utility functions; see Brandt and Santa-Clara (2006). Furthermore, a quadratic utility function is consistent to the mean-variance framework, i.e., instead of maximizing quadratic utility we could maximize a function depending on expectation and variance; see, e.g., Markowitz (2014). Since it is a concave-quadratic objective to be maximized, this utility function is also suitable because it alone does not render the problem intractable. Following Brandt and Santa-Clara (2006), we replace

$$\max\ \mathbb{E}\left[w_{t_I} - \frac{w_{t_I}^2}{\alpha}\right] \tag{3}$$

with

$$\max\ \mathbb{E}\left[\left(\frac{w_{t_I}}{w_{t_{-1}}} - 1\right) - \frac{\rho}{2}\left(\frac{w_{t_I}}{w_{t_{-1}}} - 1\right)^2\right],$$

where $\rho > 0$ is a risk-aversion parameter. Note that the second utility function is only increasing for returns below ρ and that enables us to work with relative values, i.e., return on investment, instead of absolute values, which are not as easy to interpret. Finally, the two utility functions are "equivalent" in the following sense: For a given $\alpha > 0$ there exists a $\rho > 0$ such that the difference of the two utility functions is constant for every $w_{t_I} > 0$.

In the following section, we describe the components of the model sketched above in detail.

2.2. **Traded Assets.** We model the price processes of the traded assets with geometric Brownian motions, as commonly done in financial economics; see, e.g., Merton (1973), given by

$$\frac{\mathrm{d}S_t^j}{S_t^j} = \mu^{S,j}\,\mathrm{d}t + \sigma^{S,j}\,\mathrm{d}W_t^{S,j}, \quad j \in [J],\ t \in [t_s, t_e],$$

where $W_t^{S,j}$, $j \in [J]$, are correlated standard Brownian motions. Using a forward Euler discretization yields

$$\frac{S_{t_{i+1}}^j - S_{t_i}^j}{S_{t_i}^j} = \mu^{S,j} + \sigma^{S,j}\varepsilon_{t_i}^{S,j}, \quad j \in [J],\ i \in [I-1],$$

with normally distributed $\varepsilon_{t_i}^{S,j}$, i.e.,

$$\left(\varepsilon_{t_i}^{S,1}, \dots, \varepsilon_{t_i}^{S,J}\right) \sim \mathcal{N}\left(0, \Sigma^S\right), \quad i \in [I-1],$$

where Σ^S is the respective covariance matrix.

2.3. **Infrastructure Investments.** We consider investment opportunities in wind farms in $K+1$ different countries and model the related cash flows following Gatzert and Vogl (2016) via

$$\gamma_{t_i,t_{i'}}^k = \bar{E}_k \ell_{t_i}^k P_{t_i,t_{i'}} - O^k \hat{P}_{t_i}^k, \quad i \in [I^{\max}],\ i' \in [I-1],\ i > i',\ i - i' \le T^k,\ k \in [K],$$

where $\bar{E}_k$ denotes the installed capacity of a single wind farm and $\ell_{t_i}^k$ is its load factor. Hence, the produced electricity at time $i \in [I^{\max}]$ is given by $\bar{E}_k \ell_{t_i}^k$. Moreover, $P_{t_i,t_{i'}}$ (note that possible subsidies depend on the time $t_{i'}$ the infrastructure investment is made) denotes the price of a unit of electricity, O^k are the operation, maintenance, staffing, and insurance (OMSI) costs, and $\hat{P}_{t_i}^k$ is the index modeling the price development of OMSI costs. Following Gatzert and Vogl (2016) as well as Abadie and Chamorro (2014), the load factor is given by

$$\ell_{t_i}^k = \max\left\{0, \ell_{\mathrm{av}}^k + g_{t_i}^k + \varepsilon_{t_i}^{\ell,k}\right\}, \quad i \in [I^{\max}],\ k \in [K],$$

where ℓ_{av}^k is the long-term average load factor and $g_{t_i}^k$ accounts for the seasonality at the respective location. Thus, $g_{t_i}^k = g_{t_{i+12}}^k$ holds for a monthly discretization. Finally, uncertainty of the load factor is modeled by

$$\left(\varepsilon_{t_i}^{\ell,1}, \dots, \varepsilon_{t_i}^{\ell,K}\right) \sim \mathcal{N}\left(0, \Sigma^\ell\right), \quad i \in [I^{\max}].$$

Again, Σ^{ℓ} is the respective covariance matrix. Note that, in contrast to Gatzert and Vogl (2016) as well as to Abadie and Chamorro (2014), we exclude negative load factors. The price of electricity $P_{t_i,t_{i'}}$ depends on the time $t_{i'}$ the wind farm was installed. Applied wind farm investments have a support period T_{S}^k. During this support period, we assume that the investor receives a feed-in tariff as minimum compensation leading to

$$P_{t_i,t_{i'}} = \max\left\{F_{t_i}^k, P_{t_i}^{\mathrm{ex}}\right\}, \quad i \in \left\{i', \dots, i' + T_{\mathrm{S}}^k\right\},$$

where $F_{t_i}^k$ denotes the guaranteed price of the feed-in tariff and $P_{t_i}^{\mathrm{ex}}$ is the spot-market price of energy. After the support period, the investor receives $P_{t_i}^{\mathrm{ex}}$ as compensation. This results in

$$\begin{aligned}\Gamma_{t_i,t_{i'}}^k = \bar{E}_k \ell_{t_i}^k &\left(\sum_{j=i-T^k}^{\min\{i',i-T_{\mathrm{S}}^k-1\}} n_{t_j}^k P_{t_i}^{\mathrm{ex}} + \sum_{j=i-T_{\mathrm{S}}^k}^{\min\{i',i-1,I-1\}} n_{t_j}^k \max\{F_{t_i}^k, P_{t_i}^{\mathrm{ex}}\}\right) \\ &- O^k \left(\sum_{j=i-T^k}^{\min\{i',i-1,I-1\}} n_{t_j}^k \hat{P}_{t_i}^k\right), \quad i \in [I^{\max}],\ k \in [K],\end{aligned}$$

for $i' \in [I], i \geq i'$, with

$$n_{t_i}^k = 0, \quad i \in \{1 - \max\{T^k, T_{\mathrm{S}}^k\}, \dots, 0\},\ k \in [K],$$

for the total cash flows at time t_i, $i \in [I^{\max}]$, generated by the infrastructure investments made until time $t_{i'}$, $i' \in [I-1]$. The first sum refers to investments outside of the support period whereas the second sum refers to investments within the support period. The last sum refers to the OMSI costs, which do not depend on the support period. For an economic analysis of renewable energy investments under feed-in tariffs we refer to Boomsma et al. (2012), where the investor's strategy is also considered if the support scheme is affected by regulatory uncertainty as it is the case in our model.

We use a Vasicek model for inflation, which is used for the development of the OMSI costs (Gatzert and Vogl 2016; Vasicek 1977):

$$\begin{aligned} \mathrm{d}r_t^k &= \kappa^{\hat{P},k}\left(b^{\hat{P},k} - r_t^k\right)\mathrm{d}t + \sigma^{\hat{P},k}\,\mathrm{d}W_t^{\hat{P},k}, && t \in [t_s, t_e], \\ \hat{P}_t^k &= \hat{P}_{t_s}^k \exp\left(\int_{t_s}^t r_s^k\,\mathrm{d}s\right), && t \in [t_s, t_e]. \end{aligned}$$

Without loss of generality, we choose $\hat{P}_0^k = 1$. Discretization using a forward Euler scheme yields

$$r_{t_{i+1}}^k = r_{t_i}^k + \kappa^{\hat{P},k}(b^{\hat{P},k} - r_{t_i}^k) + \sigma^{\hat{P},k}\varepsilon_{t_i}^{\hat{P},k}, \quad i \in [I^{\max} - 1],$$

with

$$\left(\varepsilon_{t_i}^{\hat{P},1}, \dots, \varepsilon_{t_i}^{\hat{P},K}\right) \sim \mathcal{N}\left(0, \Sigma^{\hat{P}}\right), \quad \hat{P}_{t_i}^k = \exp\left(\sum_{l=1}^{i} r_{t_l}^k\right), \quad i \in [I^{\max} - 1].$$

The OMSI costs $O^k > 0$ are assumed to be constant but indexed with inflation; see Gatzert and Vogl (2016). Following Monjas-Barroso and Balibrea-Iniesta (2013), the spot-market energy prices develop according to the mean-reverting process

$$\mathrm{d}P_t^{\mathrm{ex}} = \kappa^{\mathrm{ex}}\left((a^{\mathrm{ex}}t + c^{\mathrm{ex}}) - P_t^{\mathrm{ex}}\right)\mathrm{d}t + \sigma^{\mathrm{ex}}\,\mathrm{d}W_t^{\mathrm{ex}}, \quad t \in [t_s, t_e],$$

where $a^{\mathrm{ex}}t + c^{\mathrm{ex}}$ with $c^{\mathrm{ex}} > 0$ and $a^{\mathrm{ex}} \in \mathbb{R}$ is the mean-reversion level, $\kappa^{\mathrm{ex}} > 0$ denotes the speed of mean-reversion, σ^{ex} its volatility, and W_t^{ex} is a standard Brownian motion. Note that we assume that P_t^{ex} does not depend on k. Again discretizing using a forward Euler scheme yields

$$P_{t_{i+1}}^{\text{ex}} = P_{t_i}^{\text{ex}} + \kappa^{\text{ex}} \left((a^{\text{ex}} t_i + c^{\text{ex}}) - P_{t_i}^{\text{ex}} \right) + \sigma^{\text{ex}} \varepsilon_{t_i}^{\text{ex}}, \quad \varepsilon_{t_i}^{\text{ex}} \sim \mathcal{N}(0,1)$$

for $i \in [I^{\max} - 1]$.

The feed-in tariff depends on regulations in the specific country. We assume that it is constant, i.e., without policy risk:

$$F_{t_i}^k = F_0^k > 0, \quad k \in [K], \ i \in [I^{\max}].$$

Policy risk is integrated by the (uncertain) time τ^k denoting the time the policy risk scenario materializes. Following Gatzert and Vogl (2016), the scenario is a decrease of the guaranteed feed-in tariff leading to

$$F_{t_i}^k = \begin{cases} F_0^k, & \text{for } \tau^k > t_i, \\ F_0^k \left(1 - d_k\right), & \text{for } \tau^k \leq t_i, \end{cases}$$

with feed-in reduction $d_k \in [0,1]$, $k \in [K]$, $i \in [I^{\max}]$, and τ^k is drawn from a geometric distribution, i.e.,

$$\text{Prob}(\tau^k = t_i) = p_k (1 - p_k)^i, \quad i \in [I^{\max}].$$

Using a geometric distribution is a rather natural choice since we assume that the reduction can occur every month with the same probability. The geometric distribution then models the waiting time for the policy risk. The probability $p_k \in [0,1]$ can be calibrated according to Gatzert and Vogl (2016). Hence, the Markov process for the feed-in tariff is given by

$$\begin{aligned} \text{Prob}\left(F_{t_i}^k = F_0^k \mid F_{t_{i-1}}^k = F_0^k \right) &= \left(1 - p_k\right), \\ \text{Prob}\left(F_{t_i}^k = \left(1 - d_k\right) F_0^k \mid F_{t_{i-1}}^k = F_0^k \right) &= p_k, \\ \text{Prob}\left(F_{t_i}^k = \left(1 - d_k\right) F_0^k \mid F_{t_{i-1}}^k = \left(1 - d_k\right) F_0^k \right) &= 1, \\ i \in [I^{\max}] \setminus \{0\}, \ p_k \in [0,1], \ k \in [K]&. \end{aligned}$$

As described in the previous section, the investor's objective is to maximize expected utility of terminal wealth w_{t_I}. All introduced variables, deterministic parameters, and stochastic processes are summarized in Table 1. We further assume that short-selling is not allowed, i.e., $x_{t_i}^j \geq 0$, $i \in [I]$, $j \in [J]$, and that the investor must guarantee $L_{t_i} \geq 0$, $i \in [I]$, i.e., no debt is allowed. Moreover, regulated investors like insurance companies face shortfall constraints, i.e., their probability of default is required to remain below a certain level; see e.g., Eckert and Gatzert (2018) for the influence of such constraints on shareholder value of a non-life insurance company. Therefore, we introduce the chance constraint

$$\text{Prob}(w_{t_i} < b) \leq p, \quad b \in \mathbb{N}_0, \ p \in [0,1], \ i \in [I],$$

where w_{t_i} denotes the wealth of the investor at time t_i, $i \in [I]$. For general information on chance constraints in stochastic optimization see, e.g., Prékopa (2013) or Nemirovski and Shapiro (2006).

2.4. **Model Summary.** In summary, we obtain the stochastic optimization problem

$$\max \quad \mathbb{E}_{t_s} \left[u \left(w_{t_I} \right) \right] \tag{4a}$$

$$\text{s.t.} \quad w_{t_i} = L_{t_i} + \sum_{j \in [J]} x_{t_i}^j + \sum_{k \in [K]} \sum_{m=i+1}^{I^{\max}} \frac{\mathbb{E}_{t_i}[\Gamma_{t_m,t_i}^k]}{(1 + r + \delta_k)^{t_m - t_i}} \tag{4b}$$

$$\text{for all } i \in [I],$$

Table 1. Variables (top), deterministic parameters (middle), and stochastic processes (bottom) of the model

Variable	Description	Index sets	
$n^k_{t_i}$	Number of newly realized infrastructure investments of type k at time t_i	$[I-1]$	$[K]$
$\alpha^j_{t_i}$	Amount of traded asset j bought at time t_i (€)	$[I-1]$	$[J]$
$\beta^j_{t_i}$	Amount of traded asset j sold at time t_i (€)	$[I-1]$	$[J]$
$x^j_{t_i}$	Amount of traded asset j at time t_i (€)	$[I]$	$[J]$
L_{t_i}	(Liquid) cash at time t_i (€)	$[I]$	—
w_{t_i}	Wealth at time t_i (€)	$[I]$	—
$\Gamma^k_{t_i,t_{i'}}$	Cash flows (€)	$[I^{\max}] \times [I]$	$[K]$
$w_{t_{-1}}$	Initial wealth (€)	—	—
$x^j_{t_{-1}}$	Initial portfolio (typically $= 0$)	—	$[J]$
r	(Monthly) risk-free rate	—	—
δ_k	Spread or risk premium	—	$[K]$
$L_{t_{-1}}$	Initial liquid cash (€); $L_{t_{-1}} = w_{t_{-1}}/(1+r)$	—	—
$C^k_{t_i}$	Costs of infrastructure investment (€)	$[I-1]$	$[K]$
O^k	Operation, maintenance, staffing, and insurance costs (€)	—	$[K]$
F^k_0	Initial feed-in tariff (€/MWh)	—	$[K]$
d_k	Relative reduction in feed-in tariff in case of policy risk	—	$[K]$
T^k	Length of investment period	—	$[K]$
T^k_{S}	Length of support period for FIT (typically, $T^k \geq T^S_k$)	—	$[K]$
$\bar{E}_k$	Maximum production in each time step (in MWh)	—	$[K]$
b	Minimum wealth (€)	—	—
p	Shortfall probability	—	—
ρ	Utility function parameter	—	—
$n^k_{\max}$	Maximum infrastructure investments of type k	—	$[K]$
$S^j_{t_i}$	Price process of traded asset (€)	$[I]$	$[J]$
$P^{\mathrm{ex}}_{t_i}$	Spot market price of electricity (€/MWh)	$[I^{\max}]$	—
$F^k_{t_i}$	Feed-in tariff with included policy risk (€/MWh)	$[I^{\max}]$	$[K]$
$r^k_{t_i}$	Inflation rate	$[I^{\max}]$	$[K]$
$\hat{P}^k_{t_i}$	Price index	$[I^{\max}]$	$[K]$
$\ell^k_{t_i}$	Load factor	$[I^{\max}]$	$[K]$

$$x^j_{t_i} = x^j_{t_{i-1}} \left(1 + \frac{S^j_{t_i} - S^j_{t_{i-1}}}{S^j_{t_{i-1}}}\right) + \alpha^j_{t_i} - \beta^j_{t_i} \tag{4c}$$

for all $i \in [I],\ j \in [J]$,

$$L_{t_i} = L_{t_{i-1}}(1+r) - \sum_{j\in[J]} \alpha^j_{t_i} + \sum_{j\in[J]} \beta^j_{t_i} - \sum_{k\in[K]} n^k_{t_i} C^k_{t_i} + \sum_{k\in[K]} \Gamma^k_{t_i,t_i} \tag{4d}$$

for all $i \in [I]$,

$$\Gamma^k_{t_i,t_{i'}} = \bar{E}_k \ell^k_{t_i} \left(\sum_{j=i-T^k}^{\min\{i', i-T^k_{\mathrm{S}}-1\}} n^k_{t_j} P^{\mathrm{ex}}_{t_i} + \sum_{j=i-T^k_{\mathrm{S}}}^{\min\{i', i-1\}} n^k_{t_j} \max\{F^k_{t_i}, P^{\mathrm{ex}}_{t_i}\} \right) - O^k \left(\sum_{j=i-T^k}^{\min\{i', i-1\}} n^k_{t_j} \hat{P}^k_{t_i} \right) \tag{4e}$$

for all $i \in [I^{\max}],\ i \geq i' \in [I],\ k \in [K]$,

$$\mathrm{Prob}\,(w_{t_i} < b) \leq p \quad \text{for all } i \in [I], \tag{4f}$$

$$\alpha_{t_i}^j, \beta_{t_i}^j \geq 0 \quad \text{for all } i \in [I-1],\ j \in [J], \tag{4g}$$

$$\alpha_{t_I}^j = \beta_{t_I}^j = 0 \quad \text{for all } j \in [J], \tag{4h}$$

$$x_{t_i}^j \geq 0 \quad \text{for all } i \in [I],\ j \in [J], \tag{4i}$$

$$x_{t_{-1}}^j = 0 \quad \text{for all } \ j \in [J], \tag{4j}$$

$$L_{t_i} \geq 0 \quad \text{for all } i \in [I], \tag{4k}$$

$$L_{t_{-1}} = w_{t_{-1}}(1+r)^{-1}, \tag{4l}$$

$$n_{t_i}^k \in \{0, \dots, n_{\max}^k\} \quad \text{for all } i \in [I-1],\ k \in [K], \tag{4m}$$

$$n_{t_i}^k = 0 \quad \text{for all } i \in \{1 - T^k, \dots, -1\} \cup \{I\},\ k \in [K]. \tag{4n}$$

Note that in (4b), the expectation is calculated given all the information up to time t_i, $i \in [I]$.

3. Discretized Model

Problem (4) is a time-discrete stochastic optimization problem. In order to obtain a finite-dimensional optimization problem, we still need to discretize the continuous stochastic processes. This discretization yields an optimization problem over a scenario tree, which models a discrete time filtration on a finite probability space. The interpretation of such a tree is that the root node represents the starting time ("here-and-now") and the other nodes model all possibly occurring future events. In standard terms of stochastic optimization (see, e.g., Birge and Louveaux (2011)), the solution of the problem on a tree is called a policy since it contains optimal decisions for all possible events.

We apply a straightforward discretization of the stochastic processes by drawing a pre-specified number of samples from the corresponding random distributions. This involves using the starting values, sampled residuals, and the equations from the last section to generate the processes one step at a time. This discretization assigns probabilities to tree nodes in a natural way and the pre-specified number of samples thus defines the branching in the tree w.r.t. the discretized process. The implementation details are discussed later in Section 5.

Before we state the fully discretized problem on a scenario tree $\mathcal{T}$, we need to introduce some notation. The set of nodes of the tree is denoted by $\mathcal{V}$ and the root node is denoted by $0 \in \mathcal{V}$. For a given node $v \in \mathcal{V}$, we denote its predecessor by $\pi(v)$ and $\Pi(v, u)$ is the path from the node u back to node v.[4] Moreover, we set $\Pi(v) = \Pi(0, v)$ as the path back to the root node, i.e., $\Pi(v) = \{v, \pi(v), \pi(\pi(v)), \dots, 0\}$. The level of node v is called $t(v)$ and the distance $d(u, v)$ between two nodes u, v is given by $|t(u) - t(v)|$. The set of all direct successors of node v is denoted by $\Phi(v)$. Hence, for $u \in \Phi(v)$ we have $t(u) = t(v) + 1$. The sub-tree rooted at v is denoted by $\mathcal{T}_v$ with node set $\mathcal{V}_v$. Thus, $\mathcal{V}_0 = \mathcal{V}$ holds. Finally, for $v \in \mathcal{V}_u$, we denote by p_v^u the occurrence probability for node v if we are in node u. An illustration of this tree notation is given in Figure 1. With this notation at hand we can now state the fully discretized version of Problem (4) on a scenario tree with depth $I^{\max}$:

$$\max \quad \sum_{v \in \mathcal{V}: t(v) = I} p_v^0\, u(w_v) \tag{5a}$$

$$\text{s.t.} \quad w_v = L_v + \sum_{j \in [J]} x_v^j + \sum_{k \in [K]} \sum_{u \in \mathcal{V}_v \setminus \{v\}} \frac{p_u^v \Gamma_{u,v}^k}{(1 + r + \delta_k)^{d(u,v)}} \tag{5b}$$
$$\text{for all } v \in \mathcal{V} \text{ with } 0 \leq t(v) \leq I,$$

[4]The distinction between the utility function u and a node u of the scenario tree should always be clear from the context.

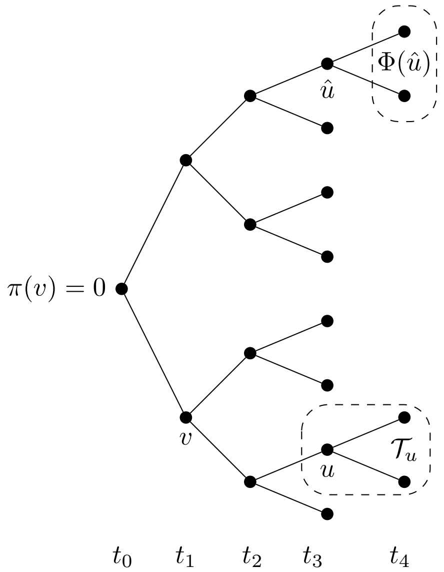

Figure 1. Scenario tree notation

$$x_v^j = x_{\pi(v)}^j \left(1 + \frac{S_v^j - S_{\pi(v)}^j}{S_{\pi(v)}^j}\right) + \alpha_v^j - \beta_v^j \tag{5c}$$
for all $j \in [J]$ and $v \in \mathcal{V}$ with $1 \leq t(v) \leq I$,

$$x_0^j = \alpha_0^j - \beta_0^j \quad \text{for all } j \in [J], \tag{5d}$$

$$L_v = L_{\pi(v)}(1+r) - \sum_{j\in[J]} \alpha_v^j + \sum_{j\in[J]} \beta_v^j - \sum_{k\in[K]} n_v^k C_v^k + \sum_{k\in[K]} \Gamma_{v,v}^k \tag{5e}$$
for all $v \in \mathcal{V}$ with $1 \leq t(v) \leq I$,

$$L_0 = w_{t_{-1}} - \sum_{j\in[J]} \alpha_0^j + \sum_{j\in[J]} \beta_0^j - \sum_{k\in[K]} n_0^k C_0^k, \tag{5f}$$

$$\Gamma_{v,u}^k = \bar{E}_k l_v^k \left(\sum_{\substack{y\in\Pi(u)\setminus\{v\}:\\ T_S^k < d(y,v) \leq T^k}} n_y^k P_v^{\text{ex}} + \sum_{\substack{y\in\Pi(u)\setminus\{v\}:\\ d(y,v)\leq T_S^k}} n_y^k \max\{F_v^k, P_v^{\text{ex}}\} \right) \tag{5g}$$
$$- O^k \left(\sum_{\substack{y\in\Pi(u)\setminus\{v\}:\\ d(y,v)\leq T^k}} n_y^k \hat{P}_v^k \right)$$
for all $u, v \in \mathcal{V},\ u \in \Pi(v),\ t(u) \leq I,\ k \in [K]$,

$$-Mz_v \leq w_v - b \leq M(1 - z_v) \quad \text{for all } v \in \mathcal{V} \text{ with } 0 \leq t(v) \leq I, \tag{5h}$$

$$\sum_{v\in\mathcal{V}:t(v)=i} p_v^0 z_u \leq p \quad \text{for all } i \in [I], \tag{5i}$$

$$z_v \in \{0,1\} \quad \text{for all } v \in \mathcal{V} \text{ with } 0 \leq t(v) \leq I, \tag{5j}$$

$$\alpha_v^j, \beta_v^j \geq 0 \quad \text{for all } j \in [J] \text{ and } v \in \mathcal{V} \text{ with } 0 \leq t(v) < I, \tag{5k}$$

$$\alpha_v^j = \beta_v^j = 0 \quad \text{for all } j \in [J] \text{ and } v \in \mathcal{V} \text{ with } t(v) = I, \tag{5l}$$

$$x_v^j \geq 0 \quad \text{for all } j \in [J] \text{ and } v \in \mathcal{V} \text{ with } 0 \leq t(v) \leq I, \tag{5m}$$

$$L_v \geq 0 \quad \text{for all } v \in \mathcal{V} \text{ with } 0 \leq t(v) \leq I, \tag{5n}$$

$$n_v^k \in \{0, \dots, n_{\max}^k\} \quad \text{for all } k \in [K] \text{ and } v \in \mathcal{V} \text{ with } 0 \leq t(v) < I, \tag{5o}$$

$$n_v^k = 0 \quad \text{for all } k \in [K] \text{ and } v \in \mathcal{V} \text{ with } I \leq t(v) \leq I^{\max}, \tag{5p}$$

where M is a sufficiently large number. Note that the initial wealth $w_{t_{-1}}$ in Constraint (5f) is the same as the one used in Constraint (4l).

The discretized stochastic processes on the scenario tree are given as follows. The load factor is modeled via

$$l_v^k = \max\{0, l_{\text{av}}^k + g_{t(v)}^k + \varepsilon_v^{l,k}\}$$

with $(\varepsilon_v^{l,1}, \dots, \varepsilon_v^{l,K}) \sim \mathcal{N}(0, \Sigma^l)$ for all $v \in \mathcal{V}$ with $1 \leq t(v) \leq I^{\max}$. Additionally, we have $(\varepsilon_v^{l,1}, \dots, \varepsilon_v^{l,K}) = (\varepsilon_u^{l,1}, \dots, \varepsilon_u^{l,K})$ for all nodes v, u with $t(v) = t(u) \bmod 12$. The discretized price processes of the traded assets are given by

$$\frac{S_v^j - S_{\pi(v)}^j}{S_{\pi(v)}^j} = \mu^{S,j} + \sigma^{S,j} \varepsilon_{\pi(v)}^{S,j}$$

with $(\varepsilon_v^{S,1}, \dots, \varepsilon_v^{S,J}) \sim \mathcal{N}(0, \Sigma^S)$ for all $v \in \mathcal{V}$ with $1 \leq t(v) \leq I$ and $j \in [J]$. Moreover, we set $S_0^j = 1$ for all $j \in [J]$. Analogously, spot-market prices are modeled via

$$P_v^{\text{ex}} = P_{\pi(v)}^{\text{ex}} + \kappa^{\text{ex}} \left((a^{\text{ex}} t\,(\pi(v)) + c^{\text{ex}}) - P_{\pi(v)}^{\text{ex}} \right) + \sigma^{\text{ex}} \varepsilon_{\pi(v)}^{\text{ex}}$$

with $\varepsilon_v^{\text{ex}} \sim \mathcal{N}(0, 1)$ for all $v \in \mathcal{V}$ with $1 \leq t(v) \leq I^{\max}$. Finally, the discretized inflation model reads

$$r_v^k = r_{\pi(v)}^k + \kappa^{\hat{P},k} \left(b^{\hat{P},k} - r_{\pi(v)}^k \right) + \sigma^{\hat{P},k} \varepsilon_{\pi(v)}^{\hat{P},k}$$

with $(\varepsilon_v^{\hat{P},1}, \dots, \varepsilon_v^{\hat{P},K}) \sim \mathcal{N}(0, \Sigma^{\hat{P}})$ for all $v \in \mathcal{V}$ with $1 \leq t(v) \leq I^{\max}$ and $k \in [K]$. Again, we set $r_0^k = 0$ for all $k \in [K]$. The whole price index development up to node $v \in \mathcal{V}$ is thus given by

$$\hat{P}_v^k = \exp\left(\sum_{u \in \Pi(v)} r_u^k \right) \quad \text{for } v \in \mathcal{V} \text{ with } 1 \leq t(v) \leq I^{\max} \text{ and } k \in [K].$$

We note that the Constraints (5h)–(5j) are a mixed-integer linear approximation of the shortfall chance constraint (4f). The assignment $z_v = 1$ indicates a node in which the wealth is below its prescribed lower bound b. Constraint (5i) then yields the desired mixed-integer model of (4f).

There is one major difference between the discrete-time stochastic optimization problem (4) and its fully discretized counterpart (5), namely that, in a first step, we abstract from integrating policy risks into the discretized model (5). Preliminary numerical tests revealed that the integration of policy risks into the discretized problem yields numerically unstable models, since the probabilities of occurring policy risks are quite low and thus lead to highly ill-conditioned problems. We discuss in Section 4 how we address policy risk directly in the moving-horizon solution approach for Model (5).

Finally note that all values of C_v^k can be chosen arbitrarily for nodes v with $I < t(v) \leq I^{\max}$ and all $k \in [K]$; see Constraint (5p).

4. A Moving-Horizon Algorithm

After the discretizations of the stochastic processes we obtain the fully discretized problem (5) in Section 3. Since this is a finite-dimensional model it can, in principle, be solved. However, it is well-known that multistage stochastic optimization

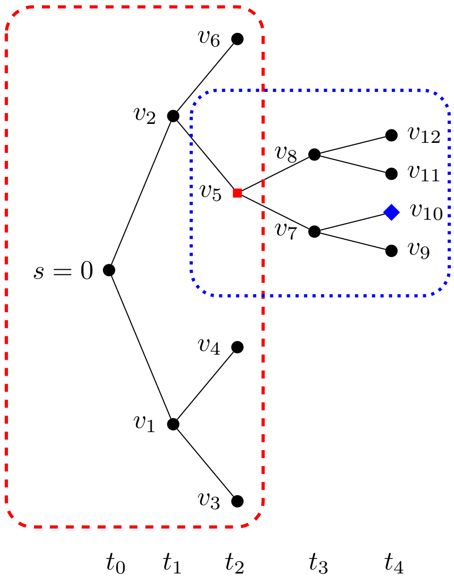

FIGURE 2. Illustration of the moving-horizon algorithm applied with $I^{\text{opt}} = I^{\text{sim}} = 2$

problems become prohibitively large after discretization of the planning horizon and the stochastic processes—even for moderate numbers of samples drawn for the discretization.

In our setting, this aspect is even more drastic, because we have many stochastic processes so that the cross-product of the discretized processes needs to be considered. Even for the smallest possible branching of $b = 2$ for every stochastic process (load factors, prices of traded assets, spot-market prices, and inflation rate) and a practically reasonable planning horizon of $I = 60$, i.e., five years, we obtain a scenario tree with more than 1.8×10^{72} nodes. Obviously, the corresponding mixed-integer optimization problem defined on this tree is highly intractable. Additionally note that, regarding this number, we already neglected the investment period until T^k that extends the planning horizon of the optimization problem even further; see (1).

This is the reason why we are not able to compute a policy on the entire scenario tree for the considered problem. Instead, we develop a tailored moving-horizon algorithm for computing a single investment plan of good quality that is, for every point in time, hedged against the uncertainty of the stochastic processes for a specific future planning horizon I^{opt}, which is smaller than the entire considered planning horizon and thus yields a tractable (sub)problem.

The basic moving-horizon procedure is described in Section 4.1. Afterward, in Section 4.2, we describe different possibilities for handling the additional investment period (up to time $t_{I^{\max}}$) that extends the planning horizon up to time t_I.

4.1. **The Basic Moving-Horizon Procedure.** The main idea of the basic moving-horizon procedure can be described as follows. Instead of considering the fully discretized problem (5) on the entire tree ranging from the initial time point t_0 to the final time point t_I, we first consider a significantly smaller discretized planning horizon $\{t_0, t_1, \ldots, t_{I^{\text{opt}}}\}$, i.e., $I^{\text{opt}} < I$. On this reduced planning horizon, we solve Problem (5) with the originally specified branching for the respective stochastic processes. For $I = 4$ and $I^{\text{opt}} = 2$, the situation is illustrated in Figure 2. In the first iteration, the sub-tree of the problem to be solved is outlined in dashed lines. The solution of the discretized stochastic optimization problem on this reduced

Algorithm 1 The basic moving-horizon algorithm

Require: Problem (5) on the entire scenario tree $\mathcal{T}$ and optimization as well as simulation period lengths I^{opt}, I^{sim}.

Set $\ell \leftarrow 0$, $v^{\ell} \leftarrow 0$, and initialize the empty investment plan $x \in \mathbb{R}^0$.
while $\ell\, I^{\text{sim}} \leq I$ **do**
    Set up and solve the optimization problem (5) on the sub-tree $\mathcal{T}(v^{\ell}, I^{\text{opt}})$ rooted at v^{ℓ} with tree depth $\min\{I^{\text{opt}}, I - \ell\, I^{\text{sim}}\}$.
    Simulate the stochastic processes to obtain the node $v_{I^{\text{sim}}}$ and the path $\Pi(v^{\ell}, v_{I^{\text{sim}}})$ from $v_{I^{\text{sim}}}$ back to the current root v^{ℓ}.
    For all nodes $v \in \Pi(v^{\ell}, v_{I^{\text{sim}}})$, let x_v denote the nodal optimal solution and extend the investment plan via $x \leftarrow (x^{\top}, (x_v^{\top})_{v \in \Pi'(v^{\ell}, v_{I^{\text{sim}}})})^{\top}$, where Π' denotes the reverse of path of path Π.
    Simulate the stochastic process of the policy risks for all $k \in [K]$ along the path $\Pi'(v^{\ell}, v_{I^{\text{sim}}})$. If some policy risk appears, update the data of Constraint (5g) for the next iteration.
    Set $\ell \leftarrow \ell + 1$ and $v^{\ell} \leftarrow v_{I^{\text{sim}}}$.
end while
return investment plan x.

planning horizon gives us a policy for the corresponding sub-tree. In practice, one would now apply the investment plan of the initial time point t_0, wait one time step, observe the realized uncertainty—i.e., whether v_1 or v_2 is attained—apply the respective investment decision, etc. The length of this simulation period is called $I^{\text{sim}} \leq I^{\text{opt}}$. Thus, the simulation is carried out until the time point $t_{I^{\text{sim}}}$ is reached so that we know which node on level I^{sim} is attained. In the example shown in Figure 2, we have $I^{\text{sim}} = I^{\text{opt}} = 2$ and attain the rectangular node v_5 in $t_{I^{\text{sim}}}$. Let us call this node $v_{I^{\text{sim}}}$. At this point, a new reduced variant of Problem (5) is set up that corresponds to the sub-tree $\mathcal{V}(v_{I^{\text{sim}}}, I^{\text{opt}})$ rooted at $v_{I^{\text{sim}}}$ and with tree depth I^{opt}—see the subtree outlined in dotted lines in Figure 2. From now on, we iterate the sketched procedure.

Note that we abstracted from explicitly integrating a policy risk model in the fully discretized problem (5). Instead, we handle policy risk directly in the moving-horizon algorithm in the following way. After the time point $t_{I^{\text{sim}}}$ has been reached, we observe along the obtained path through the tree whether policy risk occurred for some $k \in [K]$ by simulating the respective stochastic process. If this is the case, we update the corresponding data in Constraint (5g) for the next sub-problem that is to be solved.

A formal listing of this basic moving-horizon algorithm is given in Algorithm 1. While the stochastic nature of the problem is better addressed for larger I^{opt} and smaller I^{sim}, larger I^{opt} lead to larger and smaller I^{sim} lead to more optimization problems to solve.

4.2. **Handling of the Investment Period.** Besides the computational challenge due to the huge size of the scenario tree that we addressed in the last section, the investment period poses an additional computational burden. In case of infrastructure investments one is interested in investment steps of, e.g., 20 years—i.e., of 240 additional time steps in our model. If one would consider the moving-horizon subproblems of length I^{opt} plus the respective temporal extension as well as the corresponding branchings due to the discretization of the stochastic processes, this would still lead to moving-horizon subproblems of intractable size. Unfortunately, the moving-horizon approach introduced in the last section cannot resolve this problem since the infrastructure investment decisions in a moving-horizon period

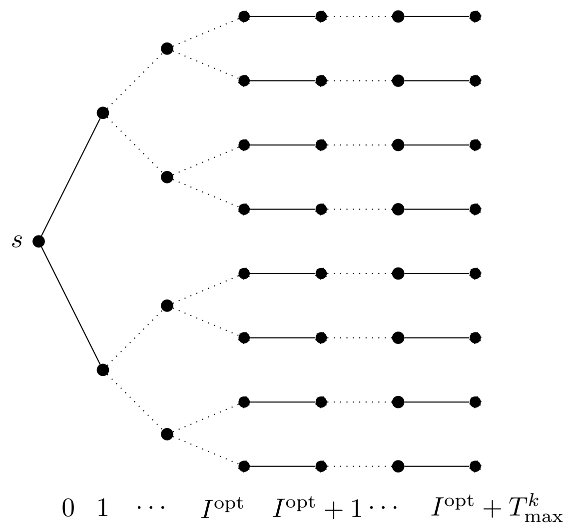

FIGURE 3. Illustration of the maximum-nodes approximation

strongly depend on future cash flows of the respective investment. Hence, considering a moving-horizon of size I^{opt} only (as it is described in the last section) would cut off future cash flows. Since the investment costs stay as they are, this would lead to unprofitable and thus less infrastructure investments.

As a consequence, we need to find an approximation of the investment period's cash flows of reasonable quality that is computational tractable. In the following sections, we describe three such approximations. They all have in common that cash flows after the optimization period (of size I^{opt}) are aggregated in surrogate sub-tree models with a much lower number of nodes than the original formulation.

4.2.1. *Maximum-Nodes Approximation.* Let v^ℓ be the root node of the moving-horizon subproblem that needs to be solved in Line 3 of Algorithm 1. As before, the length of the optimization period is $I^{\text{opt}} < I$. Thus, we consider Problem (5) on the sub-tree $\mathcal{T}(v^\ell, I^{\text{opt}})$ as in Algorithm 1.

In the *maximum-nodes approximation* of the investment period, we consider for every leaf node v of the sub-tree $\mathcal{T}(v^\ell, I^{\text{opt}})$ the number of

$$T^k_{\max} = \max\{T^k \colon k \in [K]\}$$

artificial successor nodes. The structure of the resulting tree is illustrated in Figure 3. Let us consider a leaf node v of the sub-tree $\mathcal{T}(v^\ell, I^{\text{opt}})$ and let us denote its $T^k_{\max}$ successor nodes by u_0, u_1, $\ldots$, $u_{T^k_{\max}-1}$. Then, the discretized stochastic processes for the load factors $l^k_{u_i}$, the spot-market prices $P^{\text{ex}}_{u_i}$, and the inflation rates $r^k_{u_i}$ for $i = 0, 1, \ldots, T^k_{\max} - 1$ are replaced with the approximations

$$\begin{aligned}
l^k_{u_i} &= l^k_{\text{av}} + g^k_{t(u_i)},\\
P^{\text{ex}}_{u_i} &= P^{\text{ex}}_{\pi(u_i)} + \kappa^{\text{ex}}\left((a^{\text{ex}} t(\pi(u_i)) + c^{\text{ex}}) - P^{\text{ex}}_{\pi(u_i)}\right),\\
r^k_{u_i} &= r^k_{\pi(u_i)} + \kappa^{\hat{P},k}\left(b^{\hat{P},k} - r^k_{\pi(u_i)}\right),
\end{aligned}$$

where $\pi(u_0) = v$ is used. Thus, we abstracted from the influence of random variables of the corresponding stochastic processes; see Section 2.3.

4.2.2. *12-Nodes Approximation.* We use the same notation as in the last section 4.2.1. For every leaf node of the sub-tree $\mathcal{T}(v^\ell, I^{\text{opt}})$, we now consider 12 artificial nodes—one for every month in a year. The idea is to aggregate all cash flows obtained in future (i.e., after I^{opt}) Januaries in the first artificial node u_0, all cash flows obtained in future Februaries in the second artificial node u_1, etc. Thus, in the artificial node u_{11} we aggregate all future cash flows that are realized in future December months.

The resulting structure of the tree is the same as in Figure 3 but with $T^k_{\max}$ replaced by 12. Note that the monthly aggregated values for the spot-market prices $P^{\text{ex}}_{u_i}$ and the inflation rate $r^k_{u_i}$ cannot be re-assigned to specific years ex post. Instead, they are directly addressed in the computation of the cash flows and the resulting month-wise aggregated cash flow is stored in the corresponding node. The values for the load factors genuinely depend on yearly seasons, i.e., months, and are still stored at the nodes as it was the case for the maximum-nodes approximation.

4.2.3. *1-Node Approximation.* If we consider the 12-node approximation of the last section and also abstract from the seasonality of the load factor, i.e., we approximate the future load factors by simply using their constant averages l_{av}, we obtain the *1-node approximation* of the investment period. The structure of the resulting tree is the same as in Figure 3 but with $T^k_{\max}$ replaced by 1. The nodes after I^{opt} again collect all aggregated cash flows, where we directly included the spot-market prices and inflation rates; see the 12-nodes approximation. The main rationale of this approach is that the seasonality of the load factors may not be the main driving factor if long investment period lengths, e.g., 30 years are considered.

5. Computational Study

We apply the solution techniques described in Section 4 to the discretized model presented in Section 3. In Section 5.1, we start with describing the calibration of the model in order to obtain a real-world setting. The computational setup and some implementation details are discussed in Section 5.2. In the following Sections 5.3 and 5.4, we analyze the impact of the different approximations of the investment period model (see Section 4.2) and the sensitivity of runtimes and the obtained solutions in dependence of the stochastic processes for the load factors and the inflation rate. Afterward, in Section 6, we use the results of this section to study a real-world example.

5.1. **Calibration of the Model.** We consider the described optimization problem with an investment period of 5 years with monthly discretization, i.e., $I = 60$, and wind farm investment opportunities in two countries, i.e., $K = 1$. Furthermore, we consider two traded assets, i.e., $J = 1$, as well.

The traded assets and the risk-free interest rate are calibrated to indices with different regional focus and asset classes (stocks and bonds) institutional investors like insurers typically invest in (Eckert and Gatzert 2018), i.e., MSCI World ex EMU and JPM GBI Germany All Mats. The parameters are based on data from November 2005 to November 2015 from the Thomson Reuters Datastream database. Each index measures the total returns for its assets on a Euro basis including coupons and dividends where applicable. The empirical (annualized) expected returns, standard deviations, and the associated variance-covariance matrix for the two considered assets are given in Table 2.

The parameters of the wind park in two countries are calibrated to Germany and France following Gatzert and Vogl (2016). Like Abadie and Chamorro (2014), we assume an installed capacity of 50 MW for both countries leading to $\bar{E}_k = 36\,000$, $k \in [K]$. The load factor $\ell^k_{t_i}$, $i \in [I^{\max}]$, $k \in [K]$, is calibrated using least-squares fits

TABLE 2. Project assumptions and input parameters.

Description	Value
Risk aversion parameter in utility function	$\rho = 1$
Number of traded assets	2, i.e., $J = 1$
Expected monthly returns of traded assets	$\mu^{S,1} = 0.006123$
	$\mu^{S,2} = 0.003737$
Covariance matrix of returns of traded assets	$\Sigma^S = 10^{-3} \times \begin{bmatrix} 1.717 & -0.126 \\ -0.126 & 0.162 \end{bmatrix}$
Planning horizon length (in months)	$I = 60$
Initial wealth (€)	$w_{t_{-1}} = 10^9$
(Monthly) risk-free interest rate	$r = 0.166\,\%$
Spread or risk premium	$\delta_k = 0.214\,\%$
Costs of infrastructure investment (€)	$C^k_{t_i} = 80 \times 10^6$
OMSI costs (€)	$O^k = 177\,000$
Initial feed-in tariff (€/MWh)	$F^k_0 = 89.3$
Investment period length	$T^k = 360$
Support period length	$T^k_{\mathrm{S}} = 240$
Installed infrastructure capacity	$\bar{E}_k = 36\,000$
Maximum infrastructure investments	$n^k_{\max} = 10$
Mean-reversion speed of spot-market price	$\kappa^{\mathrm{ex}} = 0.1973$
Mean-reversion level of spot-market price	$a^{\mathrm{ex}} = 0.0190$
	$c^{\mathrm{ex}} = 41.6986$
Volatility of spot-market price	$\sigma^{\mathrm{ex}} = 7.749982$
Average load factor	$\ell^k_{av} = 0.2097$
Covariance matrix of load factors	$\Sigma^{\ell} = 10^{-3} \times \begin{bmatrix} 3.592 & 0 \\ 0 & 3.592 \end{bmatrix}$
Mean-reversion speed of inflation rate	$\kappa^{\hat{P},1} = 0.7572$
	$\kappa^{\hat{P},2} = 1.0098$
Mean-reversion level of inflation rate	$b^{\hat{P},1} = 0.0009679$
	$b^{\hat{P},2} = 0.0011137$
Covariance matrix of inflation rates	$\Sigma^{\hat{P}} = 10^{-06} \times \begin{bmatrix} 2.3859 & 1.9284 \\ 1.9284 & 4.0468 \end{bmatrix}$
FIT reduction size	$d_1 = 0.135417$, $d_2 = 0.130417$
FIT reduction probability (per month)	$p_1 = 0.001197$, $p_2 = 0.000759$

and monthly production data of the German Hochfeld windfarm; see also Abadie and Chamorro (2014). Following Gatzert and Vogl (2016), we assume no (spatial) correlation between the load factors. The annual OMSI costs are assumed to be 42 500 € per installed MW resulting in monthly OMSI costs of 3541.66 € and the costs of infrastructure investment are assumed to be 80×10^6 € (see Gatzert and Vogl (2016) and Wekken (2007)). The inflation rates $r^k_{t_i}$, $i \in [I^{\max}]$, $k \in [K]$, are based on monthly inflation data of Germany and France and the electricity prices are calibrated based on German EEX Phelix Month Base values using the method proposed by Yoshida (1992). The FIT reduction size and probability are taken from Gatzert and Vogl (2016) using expert opinions. For the length of the investment and support period and the initial FIT, we also refer to Gatzert and Vogl (2016).

5.2. **Computational Setup and Implementation Details.** The mixed-integer quadratic optimization models (MIQPs) as well as the moving-horizon algorithm are implemented in Python 2.7.14. We used the NetworkX library for modeling

TABLE 3. Seasonal components $g_{t_i}^k$ for all $k \in [K]$

Month	Value
January	0.0981
February	0.0240
March	0.0489
April	-0.0251
May	-0.0313
June	-0.0662
July	-0.0651
August	-0.0661
September	-0.0318
October	-0.0019
November	0.0411
December	0.0755

the scenario tree; see the website *Software for complex networks* (2018) for further details. All MIQPs are solved using Gurobi 7.5.2 with an optimality gap of 2 %. The sampling of the respective stochastic processes has been realized using R 3.4.1 (in Rstudio 1.0.143) and the given means and variances. All computations have been carried out on a machine with Intel(R) Xeon(R) CPU E5-2699 v4 processors at 2.20 GHz (with 44 cores and hyper threading) and 755 GiB.

The results of our solution approach strongly depend on two stochastic components: the scenario tree and the simulated node in Algorithm 1. Therefore, we base our results in the following section on different sample paths with new scenario trees and simulated nodes each time. Here, we use 100 sample paths. We are aware that one would typically like to use a larger number for the simulations. However, both the number of sample paths and the optimality gap are chosen to achieve a reasonable compromise between the accuracy of the results and acceptable runtimes.

For the stochastic processes of the load factor, the spot-market prices, and the inflation rate, our code uses user-specified branchings that determine the number of scenario tree nodes and, thus, the accuracy of the approximation of the stochastic processes. Our finite-dimensional approximation of the price processes is based on the method presented in Fang et al. (2008). To this end, we consider the interval $[\mu^{S,j} - 3\sqrt{\sigma_j}, \mu^{S,j} + 3\sqrt{\sigma_j}]$ that is split in b_S many equidistant sub-intervals, where the number b_S denotes the number of discrete realizations used for the approximation of the process. In what follows, we set $b_S = 2$ due to the computational burden of larger b_S. Thus, we obtain the sub-intervals $[\mu^{S,j} - 3\sqrt{\sigma_j}, 0]$ and $[0, \mu^{S,j} + 3\sqrt{\sigma_j}]$ and take the corresponding mean values of these intervals as realizations. Note that this yields 2^J possible scenarios per time step. The rationale here is as follows. If one would simply draw random numbers from the respective distributions we could obtain one or multiple assets with only positive performance—which could even be above the risk-free interest rate. This would, however, give the unrealistic result in which everything would be invested in these assets. Since we observed this behavior during our preliminary computational experiments (using the setting discussed above), we decided to follow the described strategy.

The simulation step in Line 6 of Algorithm 1 is realized by always choosing the successor node $u \in \Phi(v)$ with highest probabilities and by breaking ties arbitrarily.

5.3. **The Impact of Approximating the Investment Period.** In Section 4.2, we propose three alternatives for handling the cash flows within the investment period that involve different degrees of simplification. Here, we study the impact

TABLE 4. Total runtime of the moving-horizon approach and mean runtimes for solving the moving-horizon sub-problems using the maximum-nodes, 12-nodes, and 1-node approximation (in minutes).

Approximation	Total runtime	Mean sub-problem runtime
Maximum-nodes	128.26	10.69
12-nodes	25.03	2.09
1-node	16.39	1.37

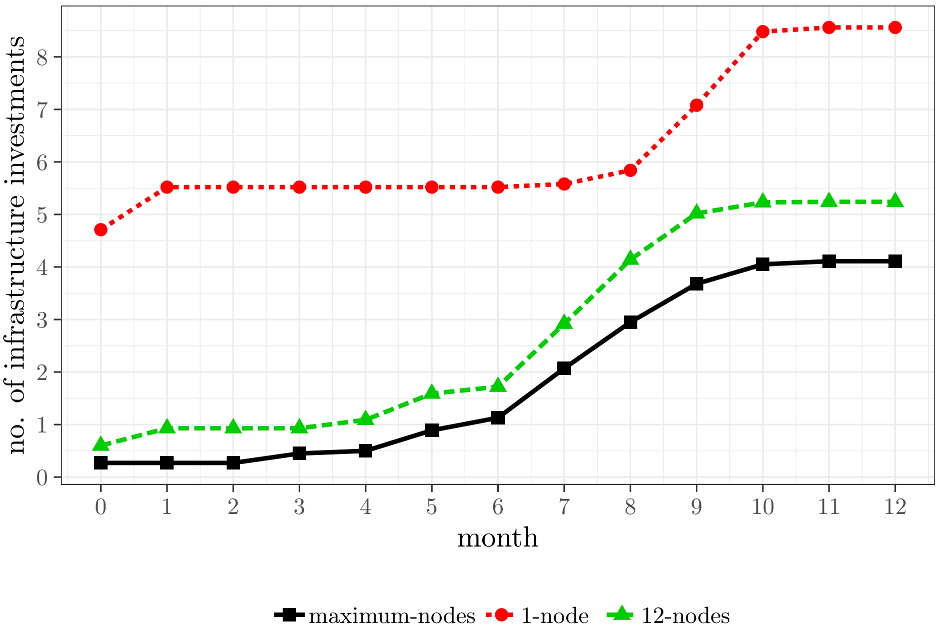

FIGURE 4. Average accumulated number of active infrastructure investments using the maximum-nodes, 12-nodes, and 1-node approximation.

of these three approximations on the result of the optimization problem. To this end, we choose $I = 12$ (one year) as well as $t_{\text{opt}} = 2$ and $t_{\text{sim}} = 1$. These parameters are chosen such that the maximum-node approximation model, which is the computationally most challenging one, can also be solved in reasonable time. For this approximation and $t_{\text{opt}} = 2$ we obtain a scenario tree with 369 697 nodes, whereas $t_{\text{opt}} = 3$ already results in 11 797 537 nodes. The solutions and runtimes depend on the scenario trees and the paths simulated in the moving-horizon algorithm. Table 4 shows the average runtime for the three presented approximation methods. As expected, the maximum-nodes approximation yields the highest runtime, followed by the 12-nodes and the 1-node approximation. The approximation also influences the decision variables and the resulting wealth. Figure 4 shows the average number of infrastructure investments made until the month stated on the x-axis. The results for the 12-nodes and the maximum-nodes approximation are comparable, but the 1-node approximation leads to considerably different results. This observation is also confirmed by Table 5 showing the mean value of infrastructure investments built during the investment horizon and the mean value given that there is at least one infrastructure investment. The latter conditional mean is comparable for all

TABLE 5. Statistical characteristics for the total number of infrastructure investments built during the planning horizon using the maximum-nodes, 12-nodes, and 1-node approximation: mean value (∅) and mean value given that the number of infrastructure investments is larger than zero ($\varnothing_{>0}$).

Approximation	$\varnothing_{>0}$	$\varnothing$
Maximum-nodes	8.74	4.11
12-nodes	9.03	5.24
1-node	9.62	8.56

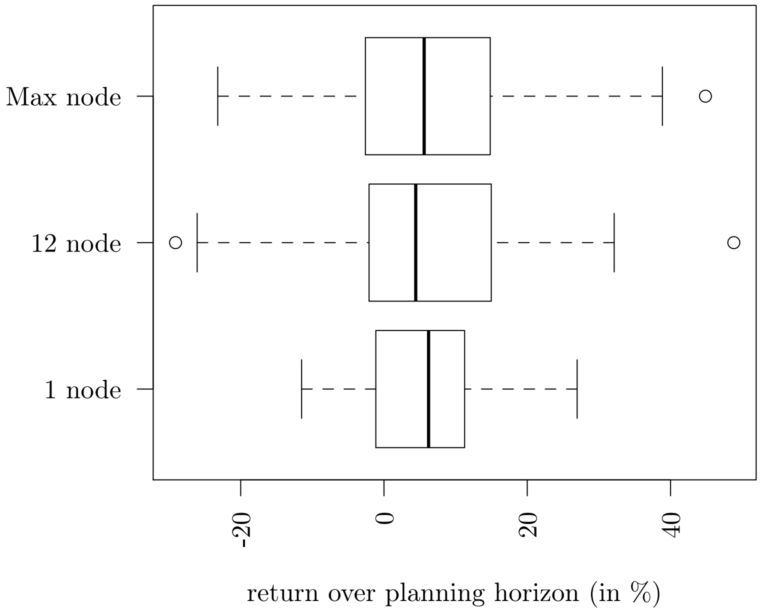

FIGURE 5. Boxplot of the return over the planning horizon using the maximum-nodes, 12-nodes, and 1-node approximation.

three models, but the difference to the overall mean is very small for the 1-node approximation. This indicates that the probabilities of investment are comparable for the 12-nodes approximation and the maximum-nodes approximation, whereas the 1-node approximation seems to be too crude. A similar observation holds for the distribution of the terminal wealth. Figure 5 shows boxplots of the return over the planning horizon for the three proposed models leading to basic characteristics that are similar for the maximum-nodes, and the 12-nodes approximation, whereas the 1-node approximation shows a qualitatively different behavior.

As a result, we will use the 12-nodes approximation in the following sections, leading to reduced runtimes without heavily influencing the outcome.

5.4. **Sensitivity Analysis for the Stochastic Processes.** The size of the scenario tree and, thus, the runtime for the considered instances heavily depend on the number of considered stochastic processes. Table 6 shows the total runtime for the basic model and its approximations, in which either the load factor, the inflation, or both stochastic processes are kept constant. Here, we already consider the setting on the full time horizon as described in Section 5.1. The results are as

TABLE 6. Total runtime of the moving-horizon approach and mean runtimes for solving the moving-horizon sub-problems for models with or without constant stochastic processes (both in min)

Approximation	Total runtime	Mean sub-problem runtime
basic model	222.01	3.70
const. load factor	55.70	0.93
const. inflation	57.66	0.96
const. load factor and inflation	14.59	0.24

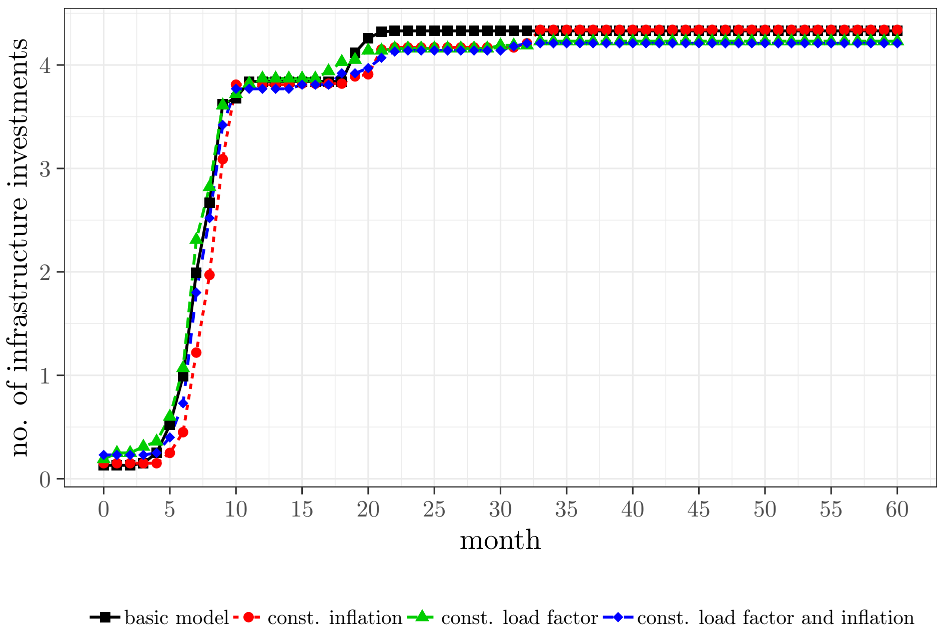

FIGURE 6. Average accumulated number of active infrastructure investments for solving the moving-horizon sub-problems for models with or without constant stochastic processes

expected: abstracting from one or more stochastic processes significantly reduces the runtimes. However, not considering these stochastic processes can of course change the result of our model drastically. Figure 6 shows the effect on the average number of infrastructure investment. Keeping the processes constant has only a small influence on the shown average. This allows us to refrain from including these two stochastic processes as such in the model but replace them using suitable deterministic approximations in the real-world example of the next section.

6. A Real-World Example and Conclusion

So far, we have set up an extremely challenging mixed-integer multistage stochastic model with different stochastic processes. We proposed a tailored moving-horizon approach that also addresses the integration of policy risk and presented different approximations in order to further reduce the size of the problems to solve. In the last section, we carried out some sensitivity analyses with respect to these approximations as well as with respect to the different stochastic processes. The corresponding discussions show that it is reasonable to simplify the presented model

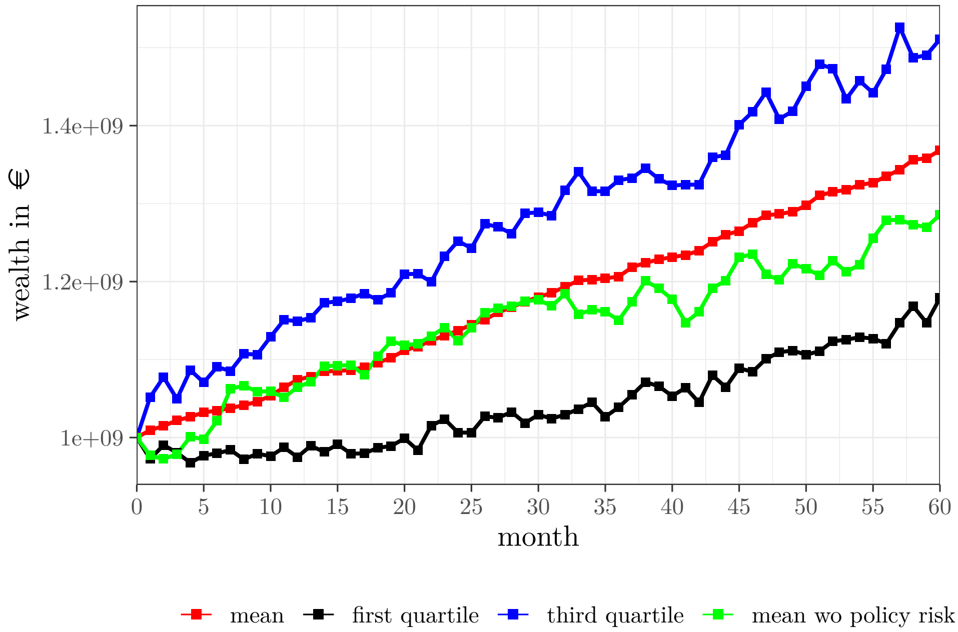

FIGURE 7. Average wealth and quartiles for 100 stochastic sample paths (of w_t) over the planning horizon for the real-world case study. For the input parameters see Table 2. The green line refers to the mean value for sample paths with occurring policy risk.

because it allows to drastically reduce the running time without loosing too much of quality in the solutions. Using these results, we now deal with a real-world example to illustrate the applicability of our approach. The instance consists of a planning horizon of 5 years, i.e., $I = 60$, as well as $I^{\max} = 360$ and we use the simplifications presented and analyzed in Sections 5.3 and 5.4. This heavily reduces the runtime and enables us to choose $t_{\text{opt}} = 3$ and $t_{\text{sim}} = 1$. All other input parameters are given in Table 2. These parameters result in an average runtime for the considered approach of 157.64 minutes. Since we use 100 randomly drawn sample paths, this leads to a total computation time of almost 11 days. Again, the results differ for each simulation run in terms of infrastructure investments and terminal wealth. Figure 7 shows the average wealth and its first and third quartile over the planning horizon. The average terminal wealth is 1368.46×10^6, the minimum value is 753.22×10^6, and the maximum value is 2555.72×10^6, resulting in an average (annual) return of 6.47 % (fluctuating from −5.51 % to 20.64 %). Additionally, the negative effect of occurring policy risk is visible. The mean value is significantly reduced for those sample paths with materializing policy risk. Additionally, Figure 8 shows that the average return is higher if all policy risk is excluded. Hence, this illustrates that the consideration of policy risk—which is covered by our model—is important for the investment problem studied in this paper. Figure 9 shows the average number of infrastructure investments. The average number of infrastructure investments in the planning horizon over the 100 considered simulation runs is 6.03 and most infrastructure investments are conducted in the first year of the planning horizon. With increasing stable returns from the early conducted infrastructure investments, the average share of traded asset 1 in the portfolio is increasing; see Figure 10. Thus, most likely, the volatile returns of this traded asset are well diversified by the infrastructure investments.

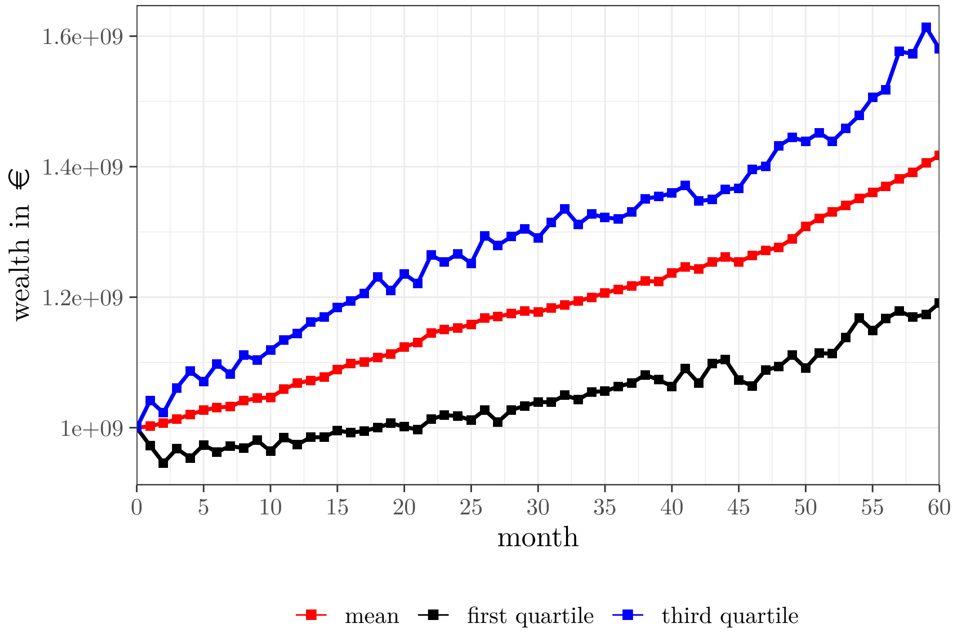

FIGURE 8. Average wealth and quartiles for 100 stochastic sample paths (of w_t) over the planning horizon for the real-world case study without any policy risk.

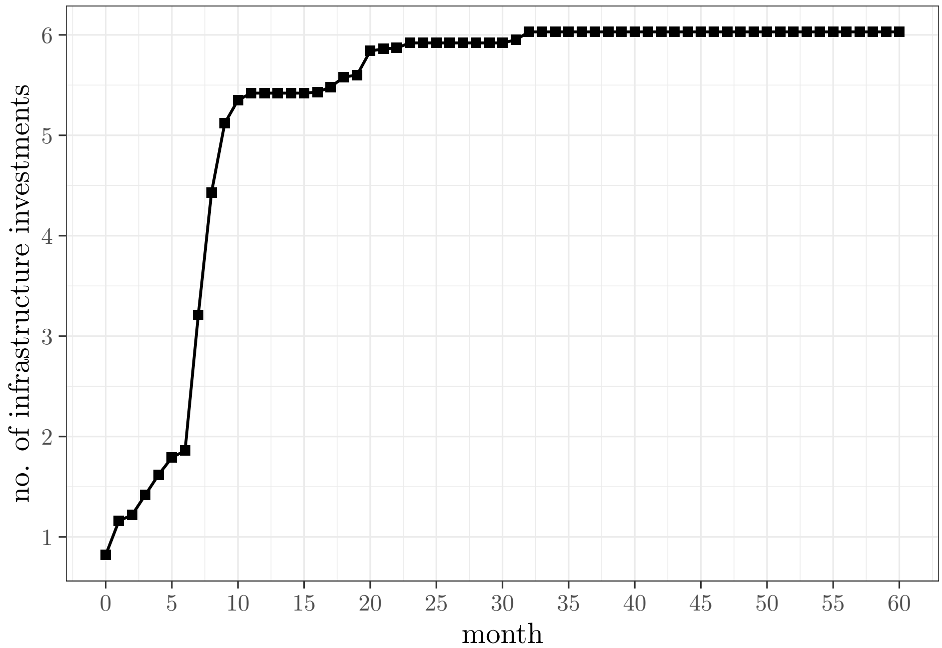

FIGURE 9. Average accumulated number of infrastructure investments for the real-world case study. For the input parameters see Table 2.

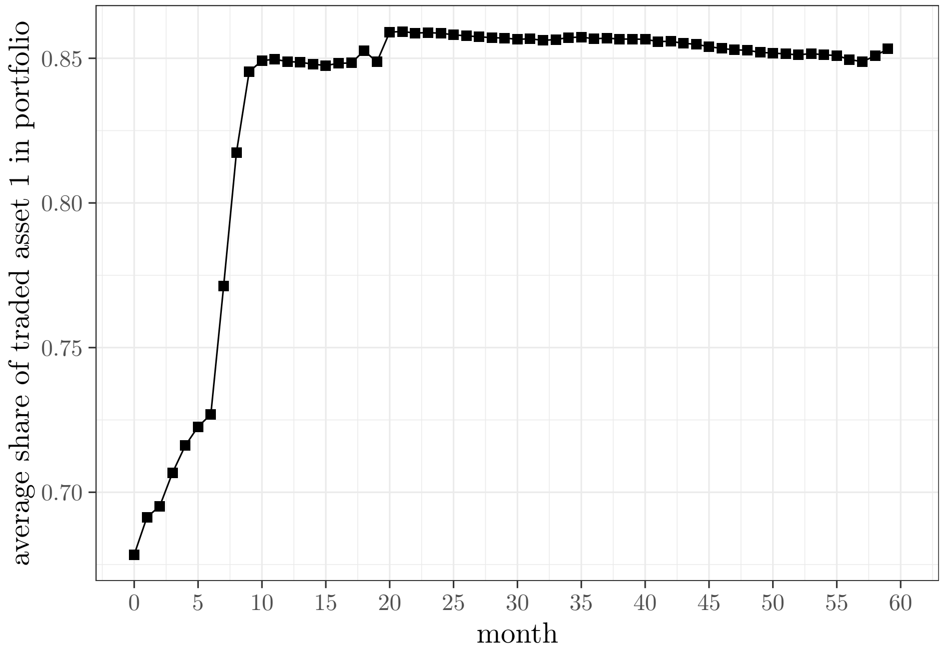

FIGURE 10. Average share of traded asset 1 (with price process S_1) for the real-world case study. For the input parameters see Table 2.

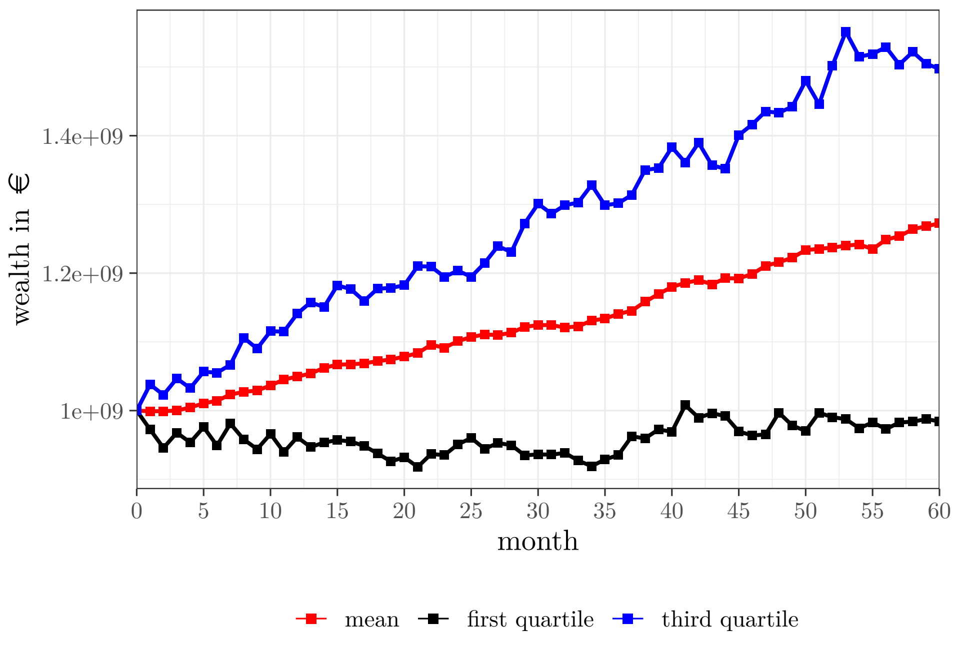

FIGURE 11. Average wealth and quartiles for 100 stochastic sample paths (of w_t) over the planning horizon for the real-world case study if no infrastructure investment are allowed. For the input parameters see Table 2.

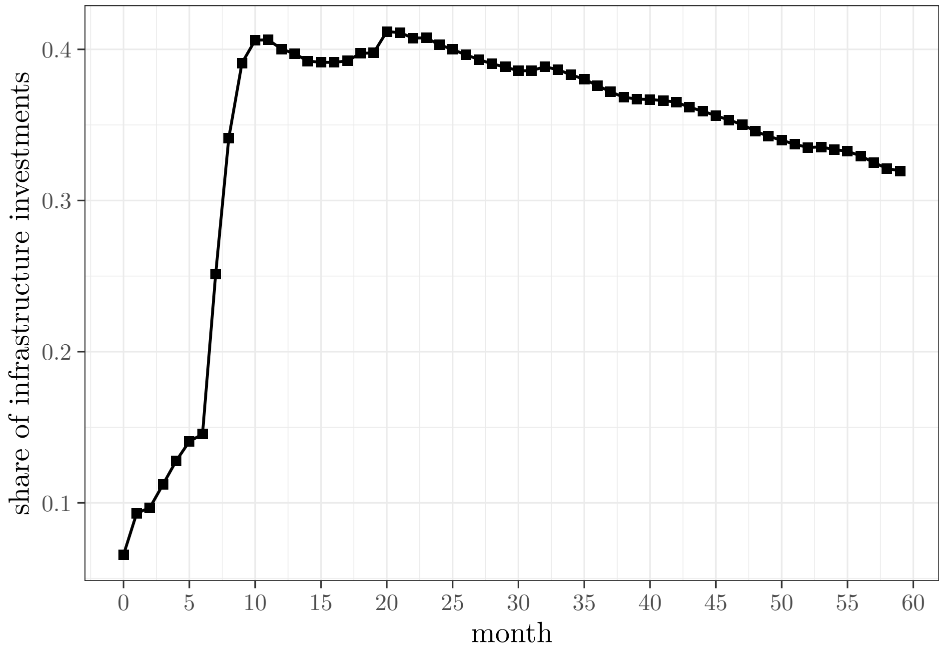

FIGURE 12. Average share invested in infrastructure for the real-world case study. For the input parameters see Table 2.

To emphasize the effect of possible infrastructure investments, we also conduct an analysis in which the investor is not able to carry out infrastructure investments. Figure 11 shows the average wealth and quartiles over the planning horizon for this case. There is a rather large effect on risk and return: the inter-quartile range is increasing by 54.55 %, while the average terminal wealth is decreasing by 7.01 %.

The benefit of diversification with infrastructure investments is reflected both by a higher expected utility of terminal wealth and by decreasing risk. For the comparison of terminal wealth (and thus, implicitly, also of the corresponding utility; see Equation (3)) for the case with and without infrastructure investments we refer to Figure 7 and 11. The volatility (i.e., the standard deviation) of terminal wealth is 350.22 (for the case including infrastructure investments) compared to 369.36 without infrastructure investments. Figure 12 also illustrates that the investor makes siginificant use of the additional investment opportunity in infrastructure: On average up to 41.18 % of the total wealth is invested in infrastructure.

This example shows that the considered problems can be tackled using the tailored moving-horizon approach together with suitable approximations and simplifications of the model. Of course, further sensitivity analyses would be interesting, e.g., with respect to the risk aversion parameter. These analyses are out of scope of the present paper and part of our future work. Moreover, there are still many open directions of future research. A very general question to answer is if and how mixed-integer multistage problems like the one considered in this paper can be solved more effectively. In the literature, there exist techniques that can, in principle, be used to enhance our solution process. This is part of our future work. However, due to their complexity and size, there is not much hope for exact methods. On the other hand, problem-specific scenario reduction techniques might be developed that allow to solve realistic instances with less simplifications. Moreover, tailored scenario reduction techniques might also make it possible to address further model aspects that we abstracted from—e.g., trading of renewable energy investments with transaction costs is a very interesting (but mathematically extremely challenging) topic.

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