

Transient Modeling and Simulation of Gas Pipe Networks with Characteristic Diagram Models for Compressors

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One challenge for the simulation and optimization of real gas pipe networks is the treatment of compressors. Their behavior is usually described by characteristic diagrams reflecting the connection of the volumetric flow and the enthalpy change or shaft torque. Such models are commonly used for an optimal control of compressors and compressor stations [4, 7] using stationary models for the gas flow through the pipes. For transient simulations of gas networks, simplified compressor models have been studied in [1–3]. Here, we present a transient simulation of gas pipe networks with characteristic diagram models of compressors using a stable network formulation as (partial) differential-algebraic system.

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1 Modeling

1.1 Network Topology

Let $G = (V, E)$ a directed graph with vertices $V = V_+ \cup V_-$ and edges $E = E_{\mathcal{P}} \cup E_{\mathcal{C}}$ where V_- and V_+ are the nodes where gas can enter and exit the network respectively and $E_{\mathcal{P}}$ and $E_{\mathcal{C}}$ being the set of pipes and compressors respectively. In addition we define the sets $\delta^-(u)$ and $\delta^+(u)$ as the set of edges that are directed towards and away from node $u \in V$ respectively.

Theorem 1.1 *Let $G = (V, E)$ be a connected, directed graph that describes a gas network with pipes and compressors and let G fulfill $V_+ \neq \emptyset$, $\nexists e \in E$ such that $e \in \delta^-(V_+) \cap \delta^+(V_+)$ and $|\delta^-(u) \cap E_{\mathcal{C}}| \leq 1 \forall u \in V$. Then it holds, that the pipes can be directed in a way that*

$$\forall u \in V_+ : \delta^-(u) = \emptyset \quad \text{and} \quad \forall u \in V_- : \delta^-(u) \neq \emptyset \quad \text{and} \quad \forall u \in V_- : \delta^-(u) \cap E_{\mathcal{P}} = \emptyset \quad \text{if} \quad \delta^-(u) \cap E_{\mathcal{C}} \neq \emptyset.$$

1.2 Pipe Modeling

We model pipes by a simplification of the isothermal Euler equations [2]

$$\partial_t p_e + \frac{c^2}{a_e} \partial_x q_e = 0, \quad \partial_t q_e + a_e \partial_x p_e = -\frac{\lambda_e c^2}{2D_e a_e} \frac{q_e |q_e|}{p_e} - \frac{g a_e h'_e}{c^2} p_e, \quad e \in E_{\mathcal{P}} \quad (1)$$

on $[0, T] \times [0, \ell_e]$ where p and q are the pressure and mass flow along pipe e , a_e is the cross-sectional area, D_e the diameter, λ_e the friction factor, g the gravitational acceleration, h'_e the elevation of the pipe, c is the speed of sound, ℓ_e the length of the pipe. Also we identify the point $x = 0$ with the position at the node u and $x = \ell_e$ with node v for $e = (u, v)$. Such a modeling is known to describe the gas flow through a pipe sufficiently well if the velocity of the gas is much less than the speed of sound which is usually the case for real gas transport networks.

1.3 Compressor Modeling

For the compressor model we consider the characteristic diagram model for turbo-compressors describing the relation between the adiabatic enthalpy and the volumetric flow rate [4, 7].

$$H_e = T_m R_s \frac{\kappa}{\kappa - 1} \left[\left(\frac{p_v}{p_u} \right)^{\frac{\kappa - 1}{\kappa}} - 1 \right], \quad H_e = \Psi_e(Q_e, n_e), \quad Q_e = c^2 \frac{q_e}{p_u}, \quad p_v = p_e^c, \quad e = (u, v) \in E_{\mathcal{C}}. \quad (2)$$

Here H_e is the adiabatic enthalpy, Q the volumetric flow rate, q_e the massflow and n_e the speed. $\Psi_e(Q, n) = \bar{Q}_e^T A_e \bar{n}_e$ with $A_e \in \mathbb{R}^{3 \times 3}$, $\bar{Q}_e^T = (1, Q_e, Q_e^2)$ and $\bar{n}_e^T = (1, n_e, n_e^2)^T$. The fourth equation of (2) determines the control of the compressor. We model the entry-nodes as a boundary condition for the pressure, leading to $p_e(t, 0) = p_u^T(t)$ for $e \in \delta^+(u)$, $u \in V_+$. For the nodes $u \in V_-$ we model the coupling by balance equations for the massflows

$$\sum_{e_{\mathcal{P}} \in \delta^-(u)} q_{e_{\mathcal{P}}}(t, \ell_{e_{\mathcal{P}}}) - \sum_{e_{\mathcal{P}} \in \delta^+(u)} q_{e_{\mathcal{P}}}(t, 0) + \sum_{e_{\mathcal{C}} \in \delta^-(u)} q_{e_{\mathcal{C}}}(t) - \sum_{e_{\mathcal{C}} \in \delta^+(u)} q_{e_{\mathcal{C}}}(t) = q_u^T(t) \quad (3)$$

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and a mapping of pressures

$$p_{e_{\mathcal{P}}}(t, \ell_{e_{\mathcal{P}}}) = p_v(t) \quad p_{e_{\mathcal{P}}}(t, 0) = p_u(t) \quad e_{\mathcal{P}} = (u, v) \in E_{\mathcal{P}} \quad (4)$$

The coupled system (1)-(4) represents a partial differential-algebraic system (PDAE) including a hyperbolic partial differential equation system for the gas pipes.

2 Spatial Discretization of the PDAE System

The following spatial discretization (1) leads to a well-posed differential-algebraic system (DAE)

$$p'_v(t) + \frac{c^2}{a_e \ell_e} (q_e(t, \ell_e) - q_e(t, 0)) = 0 \quad (5a)$$

$$q'_e(t, 0) + \frac{a_e}{\ell_e} (p_v(t) - p_u(t)) = -\frac{\lambda_e c^2}{2D_e a_e} \frac{q_e(t, 0) |q_e(t, 0)|}{p_v(t)} - \frac{g a_e h'_e}{c^2} p_v(t) \quad (5b)$$

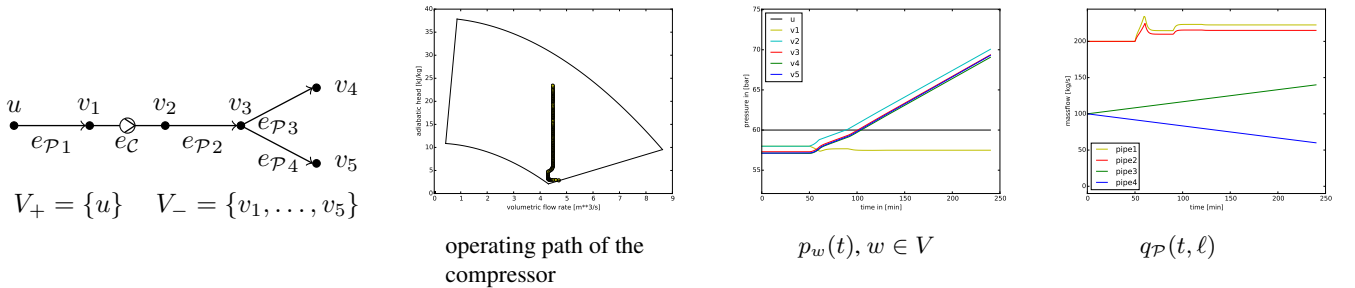
Together with Equations (2) - (4) we get a semi-linear DAE

$$A(Dy)' + b(y, t) = 0 \quad \text{with} \quad y = (p_+, p_-, q_{\mathcal{P}}(\cdot, 0), q_{\mathcal{P}}(\cdot, \ell), q_c, H, Q, n)$$

where p_+ and p_- correspond to the pressures at the entry and exit-nodes respectively. The flows, adiabatic enthalpies, the volumetric flow rates and the speeds of all compressors are collected in q_c , H , Q and n . In comparison with well-established discretization schemes as the implicit box scheme which lead to DAEs of index 2 [1,6] the discretization here has the advantage that it leads to DAE system of index 1 if the assumptions of Theorem 1.1 are satisfied, the pipes are directed as described in Theorem 1.1 and the pipe arcs always direct from the spatial position 0 to the spatial position ℓ_e [5].

3 Simulation Results

We present the simulation results for a simple benchmark (see network below) with constant pressure of 80 bar at u and zero demand at nodes v_1 - v_3 . The control at v_3 and the demands at nodes v_4 and v_5 can be taken from the figures below. Note that $q_{e_{\mathcal{P}_1}}(t, \ell_1) = q_c(t)$.



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