A Multilevel Model of the European Entry-Exit Gas Market

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Abstract. In entry-exit gas markets as they are currently implemented in Europe, network constraints do not affect market interaction beyond the technical capacities determined by the TSO that restrict the quantities individual firms can trade at the market. It is an up to now unanswered question to what extent existing network capacity remains unused in an entry-exit design and to what extent feasible adjustments of the market design could alleviate inefficiencies. In this paper, we offer a four-level modeling framework that is capable of analyzing these issues and provide some first results on the model structure. In order to decouple gas trading from network congestion management, the TSO is required to determine technical capacities and corresponding booking fees at every entry and exit node up front. Firms book those capacities, which gives them the right to charge or discharge an amount of gas at a certain node up to this capacity in every scenario. Beyond these technical capacities and the resulting bookings, gas trade is unaffected by network constraints. The technical capacities have to ensure that transportation of traded quantities is always feasible. We assume that the TSO is regulated and determines technical capacities, fees, and transportation costs under a welfare objective. As a first step we moreover assume perfect competition among gas traders and show that the booking and nomination decisions can be analyzed in a single level. We prove that this aggregated model has a unique solution. We also show that the TSO's decisions can be subsumed in one level as well. If so, the model boils down to a mixed-integer nonlinear bilevel problem with robust aspects. In addition, we provide a first-best benchmark that allows to assess welfare losses that occur in an entry-exit system. Our approach provides a generic framework to analyze various aspects in the context of semi-liberalized gas markets. Therefore, we finally discuss and provide guidance on how to include several important aspects into the approach, such as network and production capacity investment, uncertain data, market power, and intra-day trading.

1. Introduction

Energy forecasts predict natural gas to be among the fastest growing energy carriers in Europe; cf., e.g., Commission (2012). The reasons are manyfold. First, natural gas is considered the bridge to a low-carbon future in the electricity sector and beyond. Second, the well established gas infrastructure will play a key role when it comes to coupling of electricity and heating markets. Third, future mobility concepts might directly or indirectly rely on (synthetic) gas or its infrastructure. Fourth and finally, the emergence of Power-to-Gas technologies opens up the opportunity to use gas networks as storage devices for electricity. These recent and upcoming trends will challenge the current gas market designs in Europe, which typically do not lead to an efficient use of the existing network infrastructure; cf., e.g., Vazquez et al. (2012).

The goal of this article is to describe a formal mathematical model of an idealized version of the European gas market. This model's purpose is to give a framework

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that allows to analyze different changes in the regulatory framework with respect to their impact on the market participants and the overall welfare outcomes. As we will discuss throughout the paper, the model itself is highly challenging both from a theoretical as well as from an algorithmic perspective. We show, however, that meaningful simplifications can yield interesting results that give insights for the full model. For instance, we show which simplifications lead to a first-best model that can be used to benchmark different variants of the basic model. Moreover, we prove that certain levels of our multilevel problem can be aggregated yielding a bilevel optimization problem. We also discuss how additional aspects of real-world markets can be addressed using our approach.

In order to get a broader picture, let us briefly review the history of the gas market liberalization in Europe and the current status. For a more detailed overview of the gas market regulation see, e.g., Rövekamp (2015). Gas market liberalization in Europe started in the 1990s. A key milestone was the so-called First Gas Directive (Dir 1998/30/EC) that laid the ground for the main steps to liberalization, such as market opening and non-discriminatory network access. According to several benchmark reports, however, the amount of gas transported due to third party access was still modest in 2001; cf. Commission (2001). To speed up the liberalization, the Commission issued the Second Gas Directive (Dir 2003/55/EC) that required the implementation of rules that would enable all non-household buyers to purchase gas from the supplier of their choice as of July 2007. In order to ensure non-discriminatory third party access based on published tariffs the directive required legal and functional (but not ownership) unbundling as of July 2004. In 2005, the Commission started a sector inquiry that revealed various examples of discriminatory behavior of transmission system operators (TSOs)—a shortcoming that was attributed to the fact that the Second Gas Directive did not eliminate incentives of the national champions to discriminate. As a consequence, stricter rules for unbundling were introduced in the Third Energy Package (Dir 2009/73/EC): either ownership unbundling or ISO (Independent System Operator) or ITO (Independent Transmission Operator) regimes. The "Third Package" reforms of 2009 also required entry-exit pricing systems as the target model for the European Union, where the gas market and the physical system are basically decoupled.

Today, European TSOs typically operate under variants of the entry-exit system of gas transport, which allows users to book capacity rights independently at entry and exit points. Bookings are usually long-term decisions and capacity is priced based on the contracted, i.e., booked, capacities. Since 2011 auctions are used for most of the capacities, where the regulated tariffs serve as reserve prices (i.e., booking fees that have to be paid at a certain access point in any case); cf. Kema for European Commission (2013, p. 96). Trading of gas takes place daily at virtual trading points where users that have booked entry or exit capacities can sell or buy gas. In this setup, gas can easily change its ownership. However, a prerequisite to be eligible at the market is that the user has booked capacity on a long-term basis beforehand. Traded quantities are nominated for gas transport on a daily basis. These nominations are free of charge. For each user, any nomination below the individually booked capacity is feasible if it is in balance with the other nominations. As we will argue later, this requirement imposes a severe restriction on the bookable capacities that the TSO can announce.

In this paper, we lay the ground to analyze questions of gas market design in an equilibrium modeling approach. From a welfare perspective, two issues are important. First, whether the gas market is prone to market power, and, second, whether the decoupling of gas transport and trading leaves network capacity unused and therefore implies inefficiencies.

Much of the literature up to date has focused on the first issue, i.e., on strategic interaction of suppliers at gas markets. One strand of literature—in the tradition of the classical industrial organization literature—analyzes very stylized networks consisting of very few (typically two) nodes. This allows to obtain analytically tractable frameworks and yields some general insights on market power in network based industries, at least in the frameworks considered. Examples are Cremer and Laffont (2002), Hubert and Ikonnikova (2011), Ikonnikova and G. T. Zwart (2014), Jansen et al. (2012), Meran et al. (2010), Nagayama and Horita (2014), Oliver et al. (2014), and Yang et al. (2016). Another strand of literature analyzes strategic interaction in gas markets using complementarity problems that allow to computationally derive equilibrium predictions. Those contributions typically rely on less restrictive assumptions regarding the analyzed network structure but do not provide general analytical solutions of the market interaction. Those contributions include Baltensperger et al. (2016), Boots et al. (2004), Boucher and Smeers (1985), Boucher and Smeers (1987), Chyong and Hobbs (2014), Egging, S. A. Gabriel, et al. (2008), Egging, Holz, et al. (2010), S. A. Gabriel, Kiet, et al. (2005), Holz et al. (2008), Huppmann (2013), Siddiqui and S. A. Gabriel (2017), and G. Zwart and Mulder (2006).

All contributions mentioned above restrict their analysis to classical linear network flow problems where network links exhibit flow capacity constraints. The influence of pressure gradients between nodes of the network is ignored. Exceptions in this context are Midthun, Bjorndal, et al. (2009), Midthun, Fodstad, et al. (2015), and Rømo et al. (2009), who consider approximations of gas flows (e.g., the so-called Weymouth equation; cf. Weymouth (1912)) to computationally analyze market interaction in gas markets. Furthermore, all above mentioned articles typically assume that congestion management of scarce network capacities is organized in an efficient and centralized manner, usually comparable to outcomes obtained under a system of nodal pricing.

This leads to the second issue mentioned above: To what extent does the decoupling of gas transport and trading leave network capacity unused and therefore implies inefficiencies? In an entry-exit system, network capacity is—contrary to a nodal pricing system—not necessarily optimally used, even in the absence of market power. This is due to the requirement that any balanced set of quantities below the published technical capacities can be nominated and has to be transportable, which implies that reported bookable capacities may underestimate the true capacity of the network considerably. In other words, the decoupling of gas trading and transportation may come along with a considerable welfare loss due to suboptimal utilization of the available network capacities. Inefficiencies occur if (i) network capacities exceed the bookable capacities for selected scenarios but cannot be used even if that would be beneficial and if (ii) excessive network expansion is necessary due to the restrictive way bookable capacities have to be determined. Both issues will become considerably more important if gas utilization is increasing and if the gas infrastructure plays an increasingly important role in the context of sector coupling. Several authors have argued along these lines; see, e.g., Smeers (2008). By now, however, a detailed analysis is missing. Contributions are typically based on illustrative examples; cf., e.g., Alonso et al. (2010), Glachant et al. (2013), Hallack and Vazquez (2013), Hirschhausen (2006), Hunt (2008), and Vazquez et al. (2012). Whenever a quantitative analysis is included, this is typically based on a simplified network representation.

Equilibrium frameworks that include an accurate representation of the network infrastructure have not yet been developed in the literature. However, the inefficiencies that go along with the decoupling of trading and transportation cannot be assessed in models that oversimplify the network representation. In this paper we propose a framework that combines market modeling with an adequate representation of physics and provide some initial results. With respect to market modeling we focus on the case of perfect competition as more complex models of market interaction fail to deliver clear-cut results already with less complex physical flow models (cf., e.g., the existence of multiple equilibria in case of strategic interaction). With respect to the network model, we can formulate a model using very detailed physical representation. We, however, expect that we can only obtain both theoretical and computational results for very simple physical models in the near future; see also our discussion in Section 2.4.

The approach we propose permits to address various challenging issues that will increase the understanding of gas market design in the European context. First, the inefficiencies of the pure entry-exit system can be quantified by a comparison of the model outcome with a first-best system. Second, existing measures to increase the flexibility with regard to network utilization under the entry-exit system (as, e.g., interruptable capacities) can be assessed. Third, the approach allows to assess possible future advancements of the entry-exit system, in particular against the background of future energy system scenarios that predict an increasing importance and volume of gas trade and gas transport. In a nutshell, the model proposed in this paper sets the stage for a comprehensive analysis of entry-exit gas markets.

The paper is structured as follows. In Section 2 we propose a multilevel model of the gas market and the interplay with network constraints under the assumption of perfect competition. While we are aware that this model necessarily abstracts from various interesting and realistic features of gas markets, we have to point out that, due to the complexity of the network model, this model is already way beyond what is solvable even with currently most advanced techniques. As a main contribution, the framework presents an integrated way to analyze entry-exit gas markets with a consistent integration of transportation constraints and their impact on possible trading volumes and prices—and thus, market outcomes. In the four-level model, the TSO sets maximal technical capacities at the first level and announces (possibly node-specific) booking fees. Buyers and sellers compete for capacities for a longer term at the second level and nominate feasible quantities day-ahead at the third level. At the fourth level, the TSO minimizes transport cost, which also determine network fees due upon booking (announced at level one and due at level two). The total booking charge consists of the booking fee plus booking prices resulting from competition for capacity at the second level. Nominations are costless. Gas is traded at the market clearing price resulting from competition among buyers and sellers at level three. Analogously to similar problems in electricity market research (cf., e.g., Grimm, Kleinert, et al. (2017), Grimm, Martin, Schmidt, et al. (2016), Grimm, Martin, Weibelzahl, et al. (2016), Grimm, Schewe, et al. (2017), and Kleinert and Schmidt (2018)), we show that under mild conditions the third-level outcome corresponds to the solution of an appropriate welfare optimization problem and is unique. Moreover, we prove that the overall four-level model can be equivalently reduced to a bilevel optimization problem that is more amenable for theoretical analysis and computational techniques. In Section 4 we derive a first-best benchmark model. A comparison of the outcomes of the entry-exit model and the first-best benchmark allows to assess the impact of inefficiencies of decoupling trading and transport. Basically, the comparison allows to evaluate to what extent an economically beneficial use of transport capacities is hampered by the restrictive requirements on the determination of technical capacities. Section 5 contains details on several possible extensions of the model that are of vital interest in gas market design. We comment in some detail on how to integrate network investment (Section 5.1) and supply capacity investment (Section 5.2) into the model. Furthermore, we show how to adapt the model to allow for the analysis of demand uncertainty (Section 5.3) and strategic interaction (Section 5.4). Finally, in Section 5.5 we comment on the issue of intra-day trading. The paper closes in Section 6 with some concluding remarks.

2. The Multilevel Model under Perfect Competition

In this section we introduce the multilevel model that describes the interaction between a regulated transmission system operator (TSO) and gas traders in a booking-based entry-exit system as it is used in Europe. Throughout this section we assume perfect competition, i.e., all agents act as price takers. Later, in Section 5.4, we also comment on how to extend the framework to the case of strategic interaction.

In a market design characterized by an entry-exit system, the TSO has to publish maximal technical capacities at all nodes of its network at which gas traders are allowed to feed in or withdraw gas. In order to be able to feed in or withdraw gas, traders have to book appropriate capacities beforehand. These bookings (that we consider to be long-term contracts) determine the maximum quantity that the traders can afterward nominate on a day-ahead market. Nominations in this market design are quantities that the traders plan to physically feed into or withdraw from the network on a day-to-day basis. Finally, the TSO has to operate the network such that the actual nominations are transported as requested.

The gas network of the TSO is modeled as a directed and connected graph $G = (V, A)$ with a node set V and an arc set A. In this section we make the assumption that all entry customers are located at nodes $V_+ \subset V$ and that all exit customers are located at nodes $V_- \subset V$. The remaining nodes $V_0 = V \setminus (V_+ \cup V_-)$ are called junctions or inner nodes. The arc set is split up such that every arc subset models a specific type of network element like, e.g., pipes, compressor stations, or (control) valves. We give a sketch of the technical details of the specific models of these network devices in Section 2.4.

We use the following notation. Nodes are denoted by $u, v \in V$ and arcs are denoted by $a = (u, v) \in A$. Our setup also allows for multiple entry or exit customers at an entry or exit node. These players are denoted by $i \in \mathcal{P}_u$ for a node $u \in V_+ \cup V_-$. The supply or withdrawal of gas by such a player is then denoted by q_i and the total supplied or discharged gas flow at an entry or exit node is represented by $q_u = \sum_{i \in \mathcal{P}_u} q_i$ for $u \in V_+$ or $u \in V_-$, respectively.

In our model, we consider a finite set of scenarios $t \in T$ of gas trading and transport. By $\varphi^k(\cdot)$ we denote the optimal value function of the kth level of our multilevel model. All quantities of the booking stage are marked with the superindex "book" whereas all nomination specific quantities have the superindex "nom".

Vectors of quantities that are indexed with an index set are denoted by simply omitting the indices. For instance, $q = (q_{u,t})_{u \in V, t \in T}$ denotes the vector of gas flows at all nodes $u \in V$ in all scenarios $t \in T$, $q_t = (q_{u,t})_{u \in V}$ denotes the vector of all node flows in scenario t, and, finally $q_u = (q_{u,t})_{t \in T}$ denotes the set of flows at node u in all scenarios. All remaining notation will be explained where it is first used.

2.1. First Level: Specification of Maximal Technical Capacities and Booking Fees.

The player of the first level is the TSO. The TSO has to set the maximal technical capacity $q_u^{\text{TC}} \geq 0$ of every entry and exit node $u \in V_+ \cup V_-$ in the network such that all possibly resulting bookings q_i^{book}, $i \in \mathcal{P}_u$, always lead to balanced nominations that can actually be transported. Moreover, the TSO sets booking fees $\underline{\pi}_u^{\text{book}} \geq 0$ at all entry and exit nodes. These booking fees apply in addition to the booking prices that realize due to competition of the players at an entry or exit node for the scarce technical capacity q_u^{TC}. We assume that the TSO is fully regulated and that its goal is to achieve maximal nomination-based social welfare (i.e., consumer plus producer rents from gas trade minus the TSO's transportation and, e.g., investment costs). This is in line with the goals outlined in the relevant EU regulation E.C. Reg No 715/2009, Art. 16 2a, which has been incorporated into national regulations. Thus, the TSO's optimization problem in level one reads

$$\max_{q^{\mathrm{TC}}, \underline{\pi}^{\mathrm{book}}} \quad \sum_{t \in T} \left(\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} \int_0^{q_{i,t}^{\mathrm{nom}}} P_{i,t}(s) \,\mathrm{d}s - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} c_i^{\mathrm{var}} q_{i,t}^{\mathrm{nom}} \right) - \varphi^4(q^{\mathrm{nom}}) - C \tag{1a}$$

$$\text{s.t.} \quad 0 \le q_u^{\mathrm{TC}}, \ 0 \le \underline{\pi}_u^{\mathrm{book}}, \quad u \in V_+ \cup V_-, \tag{1b}$$

$$\sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \underline{\pi}_u^{\mathrm{book}} q_i^{\mathrm{book}} = \varphi^4(q^{\mathrm{nom}}) + C, \tag{1c}$$

$$\forall \hat{q}^{\mathrm{nom}} \in \mathcal{N}(q^{\mathrm{TC}}) : \mathcal{F}(\hat{q}^{\mathrm{nom}}) \neq \emptyset, \tag{1d}$$

$$q_i^{\mathrm{book}} \in \arg\max \ \varphi_i^2(q_u^{\mathrm{TC}}, \underline{\pi}_u^{\mathrm{book}}), \quad i \in \mathcal{P}_u, \ u \in V_+ \cup V_-, \tag{1e}$$

$$q_{i,t}^{\mathrm{nom}} \in \arg\max \ \varphi_{i,t}^3(q_i^{\mathrm{book}}), \quad i \in \mathcal{P}_u, \ u \in V_+ \cup V_-, \ t \in T. \tag{1f}$$

Note that the TSO's decision variables are the technical capacities q^{TC} at all nodes and the corresponding booking fees $\underline{\pi}^{\mathrm{book}}$. Constraint (1c) restricts the set of feasible booking fees $\underline{\pi}^{\mathrm{book}}$ in the sense that they need to be high enough to recover the transportation cost $\varphi^4(q^{\mathrm{nom}})$ resulting at level four. However, the TSO's revenue

$$\begin{aligned} R &:= \sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} (\underline{\pi}_u^{\mathrm{book}} + \pi_u^{\mathrm{book}}) q_i^{\mathrm{book}} - \varphi^4(q^{\mathrm{nom}}) - C \\ &= \sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \pi_u^{\mathrm{book}} q_i^{\mathrm{book}} \end{aligned}$$

from selling technical capacity may be higher than the aggregated costs, due to the fact that competition among gas traders for available capacity might drive up booking prices at level two; for more details, see Section 2.2. In Constraint (1c) and in the objective, C denotes any exogenously given costs that the system operator should be reimbursed for from revenues (e.g., money that has to be raised for re-investment in network infrastructure) and c_i^{var} denotes the variable production costs for gas of player $i \in \mathcal{P}_u$ at node $u \in V_+$. Elastic demand is modeled by inverse market demand functions $P_{i,t}$ for all $i \in \mathcal{P}_u$, $u \in V_-$. We assume that $P_{i,t}$ is continuous and strictly decreasing for all $i \in \mathcal{P}_u$, $u \in V_-$, and $t \in T$.

Furthermore, $\mathcal{N}(q^{\mathrm{TC}})$ is the set of possible nominations that are compatible with the technical capacity q^{TC} and $\mathcal{F}(q^{\mathrm{nom}})$ is the set of feasible state-control vectors for the given nomination q^{nom}. The set $\mathcal{F}(q^{\mathrm{nom}})$ of all feasible state-control vectors is determined by the physical and technical modeling of gas transport and is part of the modeling decision in level four. The set of possible nominations $\mathcal{N}(q^{\mathrm{TC}})$ is the set of all balanced in-/outflow vectors that use at most the capacity given by q^{TC}; more formally

$$\mathcal{N}(q^{\mathrm{TC}}) := \left\{ q \in \mathbb{R}^{V_+ \cup V_-} : 0 \le q \le q^{\mathrm{TC}}, \sum_{u \in V_+} q_u = \sum_{u \in V_-} q_u \right\}.$$

Overall, we observe that the first-level model (1) is an adjustable robust mixed-integer optimal control problem. The mixed-integer aspect is added by Constraint (1d) and by the optimal value function φ^4 in the objective and (1c) because they involve mixed-integer nonlinear models for modeling physical and technical feasibility of gas transport. The robustness aspect (in the sense of robust optimization) is added by Constraint (1d).

Before we consider the second level of our problem, let us comment on the determination of the booking fees $\underline{\pi}_u^{\text{book}}$. Note, first, that in our setup booking fees $\underline{\pi}_u^{\text{book}}$ are assumed to be fixed at the first level by the TSO, such that the revenue raised through the fees just recovers the TSO's cost. The TSO sets those fees anticipating the node-specific willingness to pay for capacity at level two (which of course depends on equilibrium outcomes at level three) and furthermore taking into account his zero-profit condition (1c).

Thus, revenues collected through fees cover costs of gas transport and possibly a contribution to investment cost reimbursement—and thereby are a natural lower bound for the sum of revenues to be raised from selling capacity in the entry-exit system. Since the TSO has a welfare objective, fees are primarily set at nodes where the willingness to pay for capacity is high enough, such that aggregate booked capacity (at level two) is not affected. Note that welfare optimal booking fees are not necessarily unique, since the TSO can potentially collect them at all nodes where the willingness to pay for capacity is sufficiently high. Moreover, note that welfare optimal booking fees might not be compatible with common pricing schemes like, e.g., uniform minimum prices or uniform prices per unit of capacity. The TSO might be able to collect high fees at particular nodes without decreasing welfare, if marginal gains from trade of the respective traders are anticipated to be high. At the same time, the welfare objective may restrict booking fees at other nodes. In particular, the optimal booking fee may need to be (close to) zero at nodes where anticipated marginal gains from trade are low. In the latter case, high booking fees would preclude desirable trade. Beyond welfare optimal booking fees, our model could also be used to analyze inefficiencies arising from suboptimal booking fee schemes, e.g., a requirement that the booking fee $\underline{\pi}_u^{\text{book}}$ should be identical at all nodes or at subsets of nodes.

The booking fees set by the TSO are "minimum prices" that constrain competition for capacity among various players at each node at level two. Under the assumption of perfect competition, the resulting market clearing booking prices efficiently ration the customers; for the detailed analysis see Section 2.2. Moreover, the TSO raises the highest possible revenue that is still compatible with implementing the overall efficient allocation.

In summary, our approach allows to characterize a set of desirable booking fees compatible with the maximization of social welfare. The question how the cost of network operation should be distributed among the players via the booking fees is not immediately obvious and cannot be finally answered if only welfare matters. On top of that we analyze competition for capacity at level two in the absence of market power.

In the context of an extension of our framework to the case of strategic booking decisions (see Section 5.4), various further aspects arise. Depending on the specific auction formats used to sell bookable capacities, strategic firms might want to strategically manipulate those prices either downwards (to reduce their booking cost) or upwards (to foreclose competitors). Complementary to our framework, the analysis of explicit booking mechanisms is desirable and would allow conclusions concerning the impact of strategic booking behavior on market outcomes. Also in such a framework, the TSO would set booking fees to guarantee his zero-profit condition at level one, while booking prices would emerge endogenously at level two from market interaction among strategic firms.

2.2. **Second Level: Booking.** The players of the second level are the gas traders $i \in \mathcal{P}_u$, i.e., the gas buyers at nodes $u \in V_-$ and sellers at nodes $u \in V_+$, who book capacity rights that determine the maximum amounts that they later can nominate in the subsequent third level. The objective of every trader is to book an amount $0 \leq q_i^{\text{book}} \leq q_u^{\text{TC}}$ that later leads to surplus maximizing nominations; see the third-level models (8) and (9). By doing so, the traders are only restricted by the technical capacities and the booking fees that are outcome of the first level decision of the TSO. We assume that bookings are not made strategically. This implies that players ignore the possibility that they can exclude competitors from the market by preemptive bookings and also do not consider limiting their access to the network in order to drive up spot-market prices.

At this level, all players anticipate profits from market interaction at level three. Hence, each player maximizes expected level-three revenues minus booking cost, i.e., every player $i \in \mathcal{P}_u$ solves the booking problem

$$\max_{q_i^{\text{book}}} \quad \sum_{t \in T} \varphi_{i,t}^3(q_i^{\text{book}}) - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) q_i^{\text{book}} \tag{2a}$$

$$\text{s.t.} \quad q_i^{\text{book}} \geq 0, \tag{2b}$$

$$\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}}, \tag{2c}$$

where π_u^{book} are booking prices that result from competition of the players $i \in \mathcal{P}_u$ for the scarce technical capacity q_u^{TC}. The latter means that the resulting booking price π_u^{book} is zero if the shared constraint (2c) is not binding. In summary and regarding (2), every player aims to take a profit maximizing booking decision in a setup where the player's feasible set depends on the decisions of the other players as well; cf. Constraint (2c). Thus, we face a generalized Nash equilibrium problem (GNEP) at every entry and exit node of the network. In this setting the shared booking constraint $\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}}$ is equipped with $|\mathcal{P}_u|$ Lagrange multipliers $\alpha_i \geq 0$ and it is well known that these problems typically possess infinitely many solutions; cf., e.g., Facchinei and Kanzow (2007).

As a remedy, we choose the specific GNEP solution that is also a variational equilibrium, which is the case if and only if the shared constraints have the same multipliers for all players, i.e., $\alpha = \alpha_i$ for all $i \in \mathcal{P}_u$; cf., Facchinei, Fischer, et al. (2007), Facchinei and Kanzow (2007), and Harker (1991). This is equivalent to the formulation in which every player solves

$$\varphi_i^2(q_u^{\text{TC}}, \underline{\pi}_u) := \max_{q_i^{\text{book}} \geq 0} \quad \sum_{t \in T} \varphi_{i,t}^3(q_i^{\text{book}}) - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) q_i^{\text{book}} \tag{3}$$

and where the shared booking constraint $\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}}$ is considered as an "external" market clearing condition.

We will later show in Section 2.3 that the third level (nomination) has a unique solution. If we assume that LICQ holds at the optimal points of the third-level problem, all assumptions of Corollary 4.6 of Dempe (2002) are satisfied and we obtain that the optimal value functions $\varphi_{i,t}^3(q_i^{\text{book}})$ are differentiable. In our context, the LICQ holds at all optimal points of the third level if no optimal booking (as outcome of the second level) is zero and if not every nomination in the third level is at its bounds. Both assumptions seem to be rather mild in the context of the application. Thus, under these assumptions, all second-level player's problems are smooth and bound-constrained optimization problems, in which the difference of the optimal value function of the nomination level and the booking costs is maximized.

Moreover, the optimal value functions $\varphi_{i,t}^3(q_i^{\text{book}})$ are concave because the corresponding optimization problems are concave as well; cf., e.g., Corollary 2.2 in Fiacco and Kyparisis (1986). Thus, the necessary and sufficient optimality conditions for the player's booking problems (3) are the KKT conditions

$$\sum_{t \in T} \frac{\mathrm{d}}{\mathrm{d} q_i^{\text{book}}} \varphi_{i,t}^3(q_i^{\text{book}}) - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) + \beta_i^- = 0, \tag{4a}$$

$$q_i^{\text{book}} \geq 0, \; \beta_i^- \geq 0, \tag{4b}$$

$$\beta_i^- q_i^{\text{book}} = 0, \tag{4c}$$

where β_i^- are the dual variables of the non-negativity constraints. In addition to these KKT conditions of all players $i \in \mathcal{P}_u$ we need the market clearing constraint

$$\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}}$$

and the complementarity condition

$$\pi_u^{\text{book}} \left(\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} - q_u^{\text{TC}} \right) = 0$$

that determines the scarcity price. This setting is quite classical; see, e.g., the discussion of the Arrow–Debreu abstract economy model in Facchinei and Kanzow (2007). In summary, this yields the mixed nonlinear complementarity problem (MNCP)

$$\sum_{t \in T} \frac{\mathrm{d}}{\mathrm{d} q_i^{\text{book}}} \varphi_{i,t}^3(q_i^{\text{book}}) - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) + \beta_i^- = 0, \quad i \in \mathcal{P}_u, \tag{5a}$$

$$\pi_u^{\text{book}} \geq 0, \; q_i^{\text{book}} \geq 0, \; \beta_i^- \geq 0, \quad i \in \mathcal{P}_u, \tag{5b}$$

$$\beta_i^- q_i^{\text{book}} = 0, \quad i \in \mathcal{P}_u, \tag{5c}$$

$$\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}}, \; \pi_u^{\text{book}} \left(\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} - q_u^{\text{TC}} \right) = 0. \tag{5d}$$

A solution of this MNCP is a GNEP solution that is also a variational equilibrium and that additionally yields scarcity based booking prices π_u^{book}. Moreover, this MNCP is equivalent to a properly chosen single-level optimization problem.

Theorem 2.1. *Consider the concave maximization problem*

$$\varphi_u^2(q_u^{\text{TC}}, \underline{\pi}_u) := \max_{q_u^{\text{book}}} \quad \sum_{i \in \mathcal{P}_u} \sum_{t \in T} \varphi_{i,t}^3(q_i^{\text{book}}) - \underline{\pi}_u^{\text{book}} q_i^{\text{book}} \tag{6a}$$

$$\text{s.t.} \quad q_i^{\text{book}} \geq 0, \quad i \in \mathcal{P}_u, \tag{6b}$$

$$\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \leq q_u^{\text{TC}} \tag{6c}$$

with $q_u^{\text{book}} = (q_i^{\text{book}})_{i \in \mathcal{P}_u}$. *Let* $\alpha_u \geq 0$ *denote the dual variable of the booking constraint and let* $(q_u^{\text{book}}; \beta_u^-, \alpha_u)$ *be a primal-dual solution of Problem* (6). *Then,* $(q_u^{\text{book}}, \beta_u^-)$ *is also a solution of the equilibrium problem* (5) *for prices* $\pi_u^{\text{book}} = \alpha_u$. *Moreover, a solution* $(q_u^{\text{book}}, \beta_u^-)$ *of the equilibrium problem* (5) *for given prices* π_u^{book} *also yields a primal-dual solution* $(q_u^{\text{book}}; \beta_u^-, \pi_u^{\text{book}})$ *of* (6).

Proof. As discussed above, the optimal value functions $\varphi_{i,t}^3$ are differentiable and concave. Thus, the KKT conditions of Problem (6) are both necessary and sufficient. A comparison of these KKT conditions with the MNCP (5) reveals that they are identical for $\pi_u^{\text{book}} = \alpha_u$. □

Let us now discuss some insights on the equilibrium booking capacity allocation and booking prices under the assumption of perfect competition. First, booking capacity is clearly allocated to those players that value capacity the most, i.e., to those players with the highest equilibrium values of $\sum_{t \in T} \varphi^3_{i,t}(q_i^{\text{book}})$. At a supply node, for constant marginal costs c_i^{var} at level three, capacity is shared among those players with lowest marginal costs—or is entirely allocated to the player with the lowest marginal costs, if the constant marginal costs are pairwise distinct. Thus, under the latter assumption, booked quantities are unique at entry nodes. At demand nodes, due to the assumption of downward sloping demand, capacity is allocated such that the marginal willingness to pay for additional capacity is equal among all players that obtain a positive share. Thus, and since Problem (6) for demand nodes is a strictly concave maximization problem, the booked quantities are also unique. This proves the following theorem.

Theorem 2.2. *Suppose that all variable costs c_i^{var} are pairwise distinct and that all inverse demand functions $P_{i,t}$ are strictly decreasing. Then, the optimal solution of Problem (6) is unique both for entry and exit nodes.*

One can also show that the LICQ is satisfied for the constraint set (6b) and (6c) if $q_u^{\text{TC}} > 0$ holds. Hence, the dual variables, i.e., the booking prices are also unique.

Note that the competitive booking equilibrium at level two shifts revenues from the gas traders to the TSO. In particular, among the welfare optimal solutions, the booking equilibrium determines the one that allocates the highest profits to the TSO and the lowest profits to the gas traders. Obviously, the revenue from competition for capacity on top of the revenue from the fees (that cover the TSO's costs) could be extracted by the regulator through a lump sum payment without affecting the equilibrium outcome. We note that the collection of booking fees reduces the overall welfare as compared to a regime where the TSO recovers transportation cost by charging lump sum payments from the gas traders only in a special case—namely the case in which the TSO collects revenues exclusively through the booking fees, while competitive prices at level two are zero at all nodes; i.e., $\pi_u^{\text{book}} = 0$ for all $u \in V_+ \cup V_-$.

Note, finally, that for every gas seller $i \in \mathcal{P}_u$ with $u \in V_+$ it might be appropriate to add the constraint

$$q_i^{\text{book}} \leq q_i^{\text{cap}} \tag{7}$$

that bounds the booking from above by a corresponding production capacity $q_i^{\text{cap}} > 0$.

2.3. **Third Level: Nomination.** For the nomination level we also assume a perfect competition between the buyers and sellers at all day-ahead markets $t \in T$. That is, both kinds of players act as price takers. The case of market power is discussed in Section 5.4. We denote the gas price in scenario t by π_t^{nom} for all $t \in T$. Both players optimize their surplus: The gas sellers maximize their profits subject to their bookings of the second level. That is, we have the optimization problem

$$\varphi^3_{i,t}(q_i^{\text{book}}) := \max_{q_{i,t}^{\text{nom}}} \quad \pi_t^{\text{nom}} q_{i,t}^{\text{nom}} - c_i(q_{i,t}^{\text{nom}}) \tag{8a}$$

$$\text{s.t.} \quad 0 \leq q_{i,t}^{\text{nom}} \leq q_i^{\text{book}} \tag{8b}$$

for every seller $i \in \mathcal{P}_u$, $u \in V_+$, in every scenario $t \in T$. Typically, the cost function c_i is linear and fully determined by the variable production costs c_i^{var}, i.e., we have

$$c_i(q_{i,t}^{\text{nom}}) = c_i^{\text{var}} q_{i,t}^{\text{nom}}, \quad c_i^{\text{var}} \in \mathbb{R}_{\geq 0}.$$

In this setting, (8) is a linear problem that can be solved analytically. For the ease of presentation we use the linear cost model in what follows.

On the other hand, every gas buyer $i \in \mathcal{P}_u$, $u \in V_-$, maximizes its benefit, i.e., every buyer solves the optimization problem

$$\varphi^3_{i,t}(q^{\text{book}}_i) := \max_{q^{\text{nom}}_{i,t}} \quad \int_0^{q^{\text{nom}}_{i,t}} P_{i,t}(s)\,\mathrm{d}s - \pi^{\text{nom}}_t q^{\text{nom}}_{i,t} \tag{9a}$$

$$\text{s.t.} \quad 0 \le q^{\text{nom}}_{i,t} \le q^{\text{book}}_i \tag{9b}$$

in every scenario $t \in T$. We assume that all demand functions $P_{i,t}$ are continuous and strictly decreasing, which renders (9) a concave maximization problem that can be solved efficiently. In addition to (8) and (9), we have the market clearing conditions

$$\sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} q^{\text{nom}}_{i,t} = \sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} q^{\text{nom}}_{i,t}, \quad t \in T. \tag{10}$$

The optimal solutions of both (8) and (9) are characterized by their first-order optimality conditions. For the sellers, these KKT conditions consist of dual feasibility

$$\pi^{\text{nom}}_t - c^{\text{var}}_i + \beta^-_{i,t} - \beta^+_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T,$$

of primal feasibility (8b), of non-negativity of the dual variables $\beta^-_{i,t}$ and $\beta^+_{i,t}$ of the inequality constraints in (8b),

$$\beta^-_{i,t},\ \beta^+_{i,t} \ge 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T,$$

and of KKT complementarity

$$\beta^-_{i,t} q^{\text{nom}}_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T,$$
$$\beta^+_{i,t}(q^{\text{nom}}_{i,t} - q^{\text{book}}_i) = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T.$$

In analogy, the optimality conditions of the buyers read

$$\begin{aligned} P_{i,t}(q^{\text{nom}}_{i,t}) - \pi^{\text{nom}}_t + \gamma^-_{i,t} - \gamma^+_{i,t} &= 0, \\ q^{\text{book}}_i \ge q^{\text{nom}}_{i,t} &\ge 0, \\ \gamma^-_{i,t}, \gamma^+_{i,t} &\ge 0, \\ \gamma^-_{i,t} q^{\text{nom}}_{i,t} &= 0, \\ \gamma^+_{i,t}(q^{\text{nom}}_{i,t} - q^{\text{book}}_i) &= 0 \end{aligned}$$

for all $i \in \mathcal{P}_u$, $u \in V_-$, and $t \in T$.

If we combine both sets of optimality conditions with the market clearing condition (10) we obtain the MNCP

$$\pi^{\text{nom}}_t - c^{\text{var}}_i + \beta^-_{i,t} - \beta^+_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T, \tag{11a}$$
$$P_{i,t}(q^{\text{nom}}_{i,t}) - \pi^{\text{nom}}_t + \gamma^-_{i,t} - \gamma^+_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_-,\ t \in T, \tag{11b}$$
$$q^{\text{book}}_i \ge q^{\text{nom}}_{i,t} \ge 0, \quad i \in \mathcal{P}_u,\ u \in V_+ \cup V_-,\ t \in T, \tag{11c}$$
$$\beta^-_{i,t} q^{\text{nom}}_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T, \tag{11d}$$
$$\beta^+_{i,t}(q^{\text{nom}}_{i,t} - q^{\text{book}}_i) = 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T, \tag{11e}$$
$$\gamma^-_{i,t} q^{\text{nom}}_{i,t} = 0, \quad i \in \mathcal{P}_u,\ u \in V_-,\ t \in T, \tag{11f}$$
$$\gamma^+_{i,t}(q^{\text{nom}}_{i,t} - q^{\text{book}}_i) = 0, \quad i \in \mathcal{P}_u,\ u \in V_-,\ t \in T, \tag{11g}$$
$$\beta^\pm_{i,t} \ge 0, \quad i \in \mathcal{P}_u,\ u \in V_+,\ t \in T, \tag{11h}$$
$$\gamma^\pm_{i,t} \ge 0, \quad i \in \mathcal{P}_u,\ u \in V_-,\ t \in T, \tag{11i}$$
$$\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} q^{\text{nom}}_{i,t} - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} q^{\text{nom}}_{i,t} = 0, \quad t \in T, \tag{11j}$$

in which all agents act as price takers, i.e., no agent can directly affect the price by his individual choice. Note that the only nonlinearity despite the complementarity constraints are the demand functions $P_{i,t}$. Thus, (11) is a mixed linear complementarity problem (MLCP) if and only if $P_{i,t}$ is linear for all $i \in \mathcal{P}_u$, $u \in V_-$, and $t \in T$.

Under the assumption of perfect competition, we have the following theorem. The proof goes along the lines of the proof of Theorem 2.1.

Theorem 2.3. *Consider the welfare maximization problem*

$$\varphi^3(q^{\text{book}}) := \max \quad \sum_{t \in T} \left(\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} \int_0^{q_{i,t}^{\text{nom}}} P_{i,t}(s) \, \mathrm{d}s - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} c_i^{\text{var}} q_{i,t}^{\text{nom}} \right) \tag{12a}$$

$$\text{s.t.} \quad 0 \le q_{i,t}^{\text{nom}} \le q_i^{\text{book}}, \quad i \in \mathcal{P}_u, \ u \in V_+ \cup V_-, \ t \in T, \tag{12b}$$

$$\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} q_{i,t}^{\text{nom}} - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} q_{i,t}^{\text{nom}} = 0, \quad t \in T, \tag{12c}$$

and a primal-dual solution $(q^{\text{nom}}; \beta^\pm, \gamma^\pm, \alpha)$ *of* (12)*, where* $\beta^\pm$ *and* $\gamma^\pm$ *are the dual variables of the inequality constraints* (12b) *and* α *denotes the dual variables of the market clearing constraints* (12c)*. Then,* $(q^{\text{nom}}; \beta^\pm, \gamma^\pm)$ *is a solution of the MNCP* (11) *for* $\pi_t^{\text{nom}} = \alpha_t$*,* $t \in T$*. Moreover, every solution* $(q^{\text{nom}}; \beta^\pm, \gamma^\pm)$ *of the MNCP* (11) *for given prices* π^{nom} *corresponds to an optimal primal-dual point* $(q^{\text{nom}}; \beta^\pm, \gamma^\pm, \pi^{\text{nom}})$ *of* (12)*.*

We note that the reformulation given in the last theorem of a complementarity problem modeling price takers as a single-level welfare maximization problem is standard in the literature of (energy) economics; cf., e.g., the modeling tutorial in Hobbs and Helman (2004) for electricity markets or the book S. A. Gabriel, Conejo, et al. (2012).

Theorem 2.4. *Suppose that all variable costs* c_i^{var} *are pairwise distinct and that all inverse demand functions* $P_{i,t}$ *are strictly decreasing. Then, the optimal solution of* (12) *is unique.*

Proof. Uniqueness of demands follows from Mangasarian (1988). Thus, it remains to prove that

$$\min_{q^{\text{nom}}} \quad \sum_{t \in T} \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} c_i^{\text{var}} q_{i,t}^{\text{nom}} \tag{13a}$$

$$\text{s.t.} \quad 0 \le q_{i,t}^{\text{nom}} \le q_i^{\text{book}}, \quad i \in \mathcal{P}_u, \ u \in V_+, \ t \in T, \tag{13b}$$

$$Q_t - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} q_{u,t}^{\text{nom}} = 0, \quad t \in T, \tag{13c}$$

has a unique solution $q_{i,t}^{\text{nom}}$ for all $i \in \mathcal{P}_u$, $u \in V_+$, $t \in T$, for every given total demand Q_t. We now apply a merit order argument. Since the network is not relevant at the nomination stage, we sort all gas selling customers $i \in \mathcal{P}_u$ at all entry nodes $u \in V_+$ and obtain the seller's merit order $(i_1, \dots, i_\nu)$, $\nu = \sum_{u \in V_+} |\mathcal{P}_u|$, such that

$$c_{i_j}^{\text{var}} < c_{i_m}^{\text{var}}$$

holds for all $j, m \in \{1, \dots, \nu\}$ with $j < m$. Moreover, we define

$$Q_j^{\text{book}} := \sum_{\ell=1}^{j} q_{i_\ell}^{\text{book}}$$

for $j \leq \nu$. With these preparations it is easy to see that

$$q_{i_\ell,t}^{\mathrm{nom}} = \begin{cases} q_{i_\ell,t}^{\mathrm{book}}, & \text{if } Q_\ell^{\mathrm{book}} \leq Q_t, \\ Q_t - Q_{\ell-1}^{\mathrm{book}}, & \text{if } Q_{\ell-1}^{\mathrm{book}} < Q_t < Q_\ell^{\mathrm{book}}, \\ 0, & \text{if } Q_{\ell-1}^{\mathrm{book}} \geq Q_t, \end{cases}$$

is the unique solution of (13) for all $\ell \in \{1, \ldots, \nu\}$. □

Theorem 2.3 states that the optimal solutions of the welfare maximization problem (12) and the solutions of the MNCP (11) are in a 1:1 correspondence. Since the welfare optimal solution is unique due to Theorem 2.4 the solution of (11) is unique as well. This gives us the following corollary.

Corollary 2.5. *Suppose that all variable costs c_i^{var} are pairwise distinct and that all inverse demand functions $P_{i,t}$ are strictly decreasing. If the MNCP (11) has a solution, then this solution is unique.*

Theorems 2.3, 2.4, and Corollary 2.5 show that the market equilibrium of the third level can be easily computed by solving the welfare optimum and state conditions under which the solution is unique. These results are particularly important in the context of a multilevel analysis since multiplicity of equilibria at lower levels hamper the analysis and the interpretation of results; cf., e.g., Dempe (2002). Let us emphasize that uniqueness hinges on the assumption of perfect competition—strategic interaction typically implies multiplicity of equilibria. These issues have previously been discussed in the literature on electricity market modeling; cf., e.g., Zöttl (2010).

2.4. **Fourth Level: Cost-Minimal Transport.** In this final level, the TSO minimizes transport costs, i.e., solves the problem

$$\varphi^4(q^{\mathrm{nom}}) := \min_{p,q,z,\ldots} \quad \sum_{t \in T} c_t(q^{\mathrm{nom}}) \tag{14a}$$

$$\text{s.t.} \quad (p, q, z, \ldots) \in \mathcal{F}(q^{\mathrm{nom}}), \tag{14b}$$

where $c_t(q^{\mathrm{nom}})$ models transportation costs in dependence of the given nomination q^{nom} to transport.

In this level, there is a large variety of possible models for the concretization of the feasible set $\mathcal{F}(q^{\mathrm{nom}})$ that restrict gas pressures p, gas mass flows q, discrete controls z, etc. First of all, we discuss the aspect of time in these models. Up to now we considered the set T as a finite set of scenarios. These scenarios correspond to day-ahead markets, cf. Section 2.3, that are in fact discrete events. On the other hand, gas flow through pipeline systems depends on partial differential equations (PDEs) in continuous time. These so-called Euler equations for the flow of gas form a system of nonlinear hyperbolic PDEs, which represent the motion of a compressible non-viscous fluid or a gas. They consist of the continuity equation, the balance of moments, and the energy equation. The full set of equations is given by (cf., e.g., Brouwer et al. (2011), Le Veque (1992), and Le Veque (2002))

$$\begin{aligned} \partial_t \rho + \partial_x(\rho v) &= 0, \\ \partial_t(\rho v) + \partial_x(p + \rho v^2) &= -\frac{\lambda}{2D}\rho v|v| - g\rho h', \\ \partial_t\left(\rho(\frac{1}{2}v^2 + e)\right) + \partial_x\left(\rho v(\frac{1}{2}v^2 + e) + pv\right) &= -\frac{k_w}{D}(T - T_w). \end{aligned} \tag{15}$$

Here, ρ denotes gas density, v its velocity, and p its pressure. The quantity e denotes the internal energy of the gas. The required technical parameters of the pipe are its diameter D, the temperature of its wall T_w, and its heat coefficient k_w.

The constant slope of the pipe is denoted by $h' \in [-1, 1]$. A full model of gas physics in pipes additionally consists of an equation of state and empirical models for the compressibility factor and the friction coefficient λ. Finally, g stands for the gravitational acceleration.

From a macroscopic point of view, one of the most important effects modeled by these equations is the pressure drop in pipes that occurs in the direction of gas flow. In other words: Gas flows typically from higher to lower pressure. To overcome this effect, pressure levels are increased in the network by compressor machines in order to be able to transport gas over long distances. These machines can be activated or shut off, which introduces discrete controls and additional finite-dimensional nonlinearities in Model (14). Moreover, the operation of compressor machines typically constitutes the main part of the transportation costs $c_t(q^{\text{nom}})$.

In summary, Model (14) is a nonconvex mixed-integer nonlinear optimal control problem that is governed by a system of nonlinear hyperbolic PDEs on a graph. This model considered standalone is already out of scope with respect to the current state of theoretical knowledge and algorithmic technology. Thus, Model (14) as one part of a four-level optimization problem has to be drastically simplified for practical as well as theoretical purposes. Typical simplifications of System (15) are the following: We only consider the stationary case, i.e., all partial derivatives w.r.t. time vanish, we assume that temperature and gravitational effects due to height differences can be neglected, and compressibility factors can be approximated by suitable constants. Under these assumptions it can be shown (cf., e.g., Fügenschuh et al. (2015) and Schmidt et al. (2016)) that the pressure drop in a pipe can be modeled as

$$p(x) = \sqrt{p(0)^2 - \gamma_a q|q|x}, \quad x \in [0, L], \tag{16}$$

where L is the pipe's length. Plugging in $x = L$ yields the finite-dimensional nonlinear relation

$$p_v^2 = p_u^2 - \gamma_a q_a |q_a| \tag{17}$$

between the pressures p_u, p_v at the in- and outlet as well as the flow q_a through the pipe $a = (u, v) \in A$. Here, γ_a is a collection of technical and physical constants of the pipe. Formula (17) is known as the Weymouth formula. Together with finite-dimensional mixed-integer (non)linear models of compressors and (control) valves, this stationary variant of gas flow leads to algebraic mixed-integer nonlinear models (MINLPs) of gas flow for specifying the abstract feasible set in (14b).

We remark that considering the simplification of stationarity may have an drastic impact on the outcome of the model. Due to the compressibility of gas, pipes themselves may serve as a storage. This phenomenon is called "linepack" and, of course, has a significant impact of what can be shipped or not. A stationary description of gas physics cannot model this behavior.

Before we close this section on the modeling of an entry-exit gas market model we discuss the dependencies between the four levels; see Fig. 1. First, we see that the TSO acting in level one anticipates the outcomes of all subsequent levels, i.e., its decision on maximal technical capacities and optimal booking fees depends on the bookings q^{book} of the second level, the nominations q^{nom} as outcome of the third level, as well as the cost-optimal transport (φ^4) in the fourth level. On the other hand, the only direct impact of the TSO's decision is in the booking level in which both the bookings' upper bound and its fee depend on the first-level decision of the TSO. The second and third level have a bidirectional dependence: The booking is done in anticipation of the subsequent nominations (φ^3), which themselves are bounded by the booked capacities q^{book}. Finally, the nominated amounts q^{nom} enter level four in which they have to be actually transported. Feasibility of the transport of nominations is guaranteed in the first level; cf. Constraint (1d). The

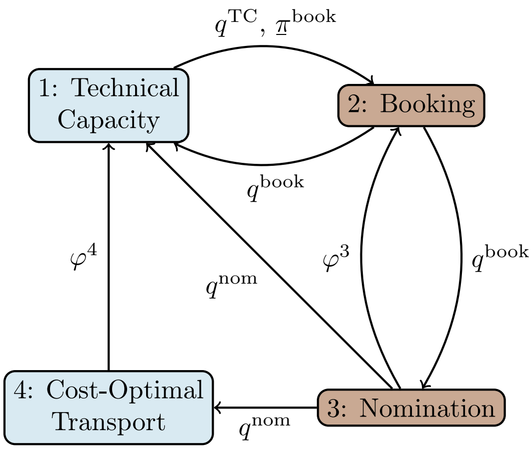

FIGURE 1. Dependencies between the levels of the model

optimal objective value φ^4 of the fourth level, i.e., the minimum transport costs, also appears in the first-level constraints as described above.

As we noted above, there is still the fact to contend that the model leaves it open how the welfare is distributed among the players. The obvious point, where this is the case, is the setting of booking fees through the TSO. We have only stated that the booking fees need to cover the operational costs of the TSO and some form of markup for investment costs; cf. Constraint (1c). We do, however, conjecture that there are situations where different booking fees lead to the same market outcomes on the other levels. Then the different booking fees correspond to different allocations of the surpluses to the players. When considering, e.g., long-term models with investment decisions, these differences would matter as they directly influence these investments. Therefore, it is an interesting open question, how these surpluses should be distributed and whether these distributions are different in the model at hand or the first-best model of Section 4.

3. Aggregated Model

Theorem 2.3 reveals that the genuine complementarity system in the third level can be replaced by the welfare optimization problem (12) because all gas buyers and sellers act welfare-maximizing in our setup of perfect competition. It also holds

$$\varphi^3(q^{book}) = \sum_{t \in T} \sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \varphi^3_{i,t}(q_i^{book}). \tag{18}$$

Moreover, we can aggregate the $|V_+ \cup V_-|$ second-level problems (6) into the single optimization problem

$$\varphi^2(q_u^{TC}, \underline{\pi}^{book}) := \max_{q^{book}} \quad \varphi^3(q^{book}) - \sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \underline{\pi}_u^{book} q_i^{book} \tag{19a}$$

$$\text{s.t.} \quad 0 \le q_i^{book}, \quad i \in \mathcal{P}_u, \; u \in V_+ \cup V_-, \tag{19b}$$

$$\sum_{i \in \mathcal{P}_u} q_i^{book} \le q_u^{TC}, \quad u \in V_+ \cup V_-. \tag{19c}$$

With these insights at hand, we can easily prove the following theorem that states that the second and third level of the problem can be merged. The theorem also reveals that gas traders both decide on welfare optimal nominations and the corresponding booking in the aggregated level.

Theorem 3.1. *The aggregated second-level problem* (19) *and the third-level problems* (8) *and* (9) *together with the market clearing condition* (10) *can be equivalently replaced by the single-level optimization problem*

$$\max_{q^{\text{book}}, q^{\text{nom}}} \quad \sum_{t \in T} \left(\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} \int_0^{q_{i,t}^{\text{nom}}} P_{i,t}(s) \, \mathrm{d}s - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} c_i^{\text{var}} q_{i,t}^{\text{nom}} \right) - \sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \underline{\pi}_u^{\text{book}} q_i^{\text{book}} \tag{20a}$$

$$\text{s.t.} \quad 0 \le q_i^{\text{book}}, \quad i \in \mathcal{P}_u, \ u \in V_+ \cup V_-, \tag{20b}$$

$$\sum_{i \in \mathcal{P}_u} q_i^{\text{book}} \le q_u^{\text{TC}}, \quad u \in V_+ \cup V_-, \tag{20c}$$

$$0 \le q_{i,t}^{\text{nom}} \le q_i^{\text{book}}, \quad i \in \mathcal{P}_u, \ u \in V_+ \cup V_-, \ t \in T, \tag{20d}$$

$$\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} q_{i,t}^{\text{nom}} - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} q_{i,t}^{\text{nom}} = 0, \quad t \in T. \tag{20e}$$

Proof. The vectors q^{book} and q^{nom} are feasible for the original two levels if and only if they are feasible for (20). Moreover, the objective function (20a) equals the one of (19), which completes the proof. □

Model (20) is a standard two-stage program that has the structure of classical peak-load pricing models where firms first choose their capacities under uncertain or fluctuating demand and then choose their final output. This can be easily seen by interpreting the booking variables as capacity investments and the nomination variables as productions.

In the context of the multilevel setting discussed here, we note that Model (20) can be replaced directly by its optimality conditions. Since the problem is convex, the KKT conditions are both necessary and sufficient. Theorem 3.1 also reveals that it is also possible to state the overall four-level model as a three-level model using the intermediate level given in the theorem. There are two main reasons why we choose to present the model initially as a four-level model with separated booking and nomination levels. First, it corresponds to the real timing of the market organization and thus models the entry-exit system in a direct way. Second, it enables us to consider more complicated models like those integrating strategic behavior as we do in Section 5.4. In the latter setting it is not possible anymore to aggregate the original second and third level. Thus, instead of being faced with the single-level optimization problem (20) one then has to solve a multi-leader-multi-follower problem (EPEC). In summary, our opinion is that the four-level version of the model is the more general setting that then allows to address different aspects of market design by using different versions of the model (as we will discuss in Section 5).

We next discuss some theoretical properties of Model (20). For a single firm, optimal capacity and pricing decisions have been thoroughly analyzed in the literature—for a survey see Crew et al. (1995). For a discussion how those results can be readily extended to the case of perfectly competitive firms, see, e.g., Joskow and Tirole (2007). Murphy and Smeers (2005), Zöttl (2010), and Grimm and Zöttl (2013) analyze the case of strategically interacting firms. All those articles abstract from potentially arising network congestion. In the recent articles Grimm, Schewe, et al. (2017), Krebs, Schewe, et al. (2017), and Krebs and Schmidt (2017), the authors focus on the case of perfectly competitive firms and extend the analysis to include issues arising due to network congestion. The latter papers particularly focus on the uniqueness of solutions. Although the setting in Grimm, Schewe, et al. (2017)

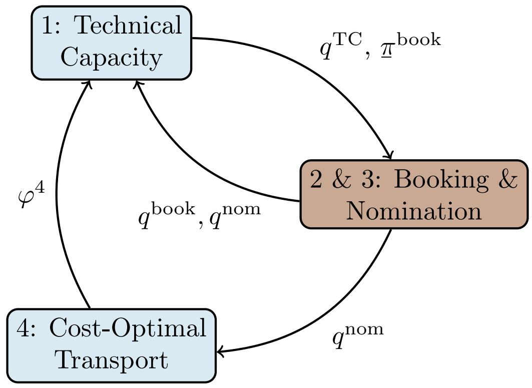

FIGURE 2. Dependencies between the three levels of the reduced model

is almost a generalization of the setting in (20), a constraint of type (20c) is not considered. However, we think that the proof technique can also be used—maybe after slight modifications—to show the uniqueness of the solution of Model (20). Fortunately, in our setting, the uniqueness also follows directly by Theorem 2.2, 2.4, and 3.1.

Theorem 3.2. *Suppose that all variable costs c_i^{var} are pairwise distinct and that all inverse demand functions $P_{i,t}$ are strictly decreasing. Then, Model (20) has a unique solution.*

Moreover, in Grimm, Schewe, et al. (2017), the unique solutions are characterized and a first result towards market power modeling is given using the approach of conjectural variations; cf., e.g., Wogrin et al. (2013). To be more specific, it is shown that Theorem 3.2 also holds if the objective function (20a) is replaced by a convex combination of (20a) and a monopoly objective. We think that similar results can also be achieved in our setting.

The formulation of the aggregated model (20) also reveals that, by optimality, we know that the booked capacity q_i^{book} is always the maximal nomination of that player.

Proposition 3.3. *Let q^{book}, q^{nom} be an optimal solution of Model (20). Then,*

$$q_i^{\mathrm{book}} = \max_{t \in T} \{q_{i,t}^{\mathrm{nom}}\}$$

for all $i \in \mathcal{P}_u$, $u \in V_+ \cup V_-$.

We note that this proposition is obviously true under the assumption of perfect competition, whereas strategic interaction or uncertainty that realizes between the booking and the nomination level would readily yield booking decisions violating the proposition. However, in the case of uncertainty there would hold an analogue proposition stating that there is always one uncertain scenario in which the equation of the proposition holds.

After applying Theorem 3.1 we obtain a three-level model, i.e., we reduced to number of levels in our multilevel setting by one. Figure 2 shows the dependencies between the three levels of the reduced model, in which the original first and fourth level stay unchanged. Up to now, the (in)tractability of the overall model has not changed very much. However, having a new aggregated level (instead of two separate levels) that is theoretically well understood (cf. Theorem 3.2) is a first improvement in the understanding of the overall market model. Moreover, it paves the way for the next theorem: In addition to the reduction of four to three levels it is further possible to reduce one more level by aggregating the original levels one and four.

Theorem 3.4. *The trilevel problem with aggregated booking and nomination level is equivalent to the bilevel problem*

$$\max_{q^{\mathrm{TC}}, \underline{\pi}^{\mathrm{book}}} \quad \sum_{t \in T} \left(\sum_{u \in V_-} \sum_{i \in \mathcal{P}_u} \int_0^{q_{i,t}^{\mathrm{nom}}} P_{i,t}(s) \,\mathrm{d}s - \sum_{u \in V_+} \sum_{i \in \mathcal{P}_u} c_i^{\mathrm{var}} q_{i,t}^{\mathrm{nom}} \right) - \eta - C \tag{21a}$$

$$\text{s.t.} \quad 0 \leq q_u^{\mathrm{TC}}, \; 0 \leq \underline{\pi}_u^{\mathrm{book}}, \quad u \in V_+ \cup V_-, \tag{21b}$$

$$\sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \underline{\pi}_u^{\mathrm{book}} q_i^{\mathrm{book}} = \eta + C, \tag{21c}$$

$$\forall \hat{q}^{\mathrm{nom}} \in \mathcal{N}(q^{\mathrm{TC}}) : \mathcal{F}(\hat{q}^{\mathrm{nom}}) \neq \emptyset, \tag{21d}$$

$$\eta \geq \sum_{t \in T} c_t(q_t^{\mathrm{nom}}), \quad (p, q, z, \ldots) \in \mathcal{F}(q^{\mathrm{nom}}), \tag{21e}$$

$$(q^{\mathrm{book}}, q^{\mathrm{nom}}) \in \arg\max \; \hat{\varphi}^{2,3}(q_u^{\mathrm{TC}}, \underline{\pi}_u^{\mathrm{book}}), \tag{21f}$$

where $\hat{\varphi}^{2,3}$ *is the optimal value function of Model* (20).

Proof. The theorem follows by applying the same arguments as in the proof of Theorem 3.1. □

The economic reasoning behind this theorem is that the player both on the first and the last level is the regulated TSO, who is not acting on the intermediate levels 2 and 3. Since the objectives of the TSO are welfare-driven in level 1 and 4, the optimization direction is the same and the levels can thus be combined. See also Grimm, Kleinert, et al. (2017) and Grimm, Martin, Schmidt, et al. (2016) for a similar argument in the context of multilevel electricity market models.

In order to give more structural insight on the bilevel model (21), we briefly discuss its constraints. First, Constraint (21e) models the original fourth level, i.e., the cost-minimal transport of actually realized nominations. The term η models these minimal transport costs and also appears in the objective function (21a). Constraints (21b)–(21d) are the original first-level constraints that remain on the first level of the bilevel problem. Finally, Constraint (21f) models the bilevel problem's lower level using its optimal value function.

The result of Theorem 3.4 is that the overall four-level model can be reduced to a bilevel model as it is depicted in Figure 3. The upper level model of this bilevel problem is a robust mixed-integer nonlinear optimal control problem (depending on the specific choice of the physics and engineering models). Its lower level is a concave maximization problem subject to linear constraints. The latter reveals that the overall problem can also be replaced with an equivalent single-level problem if one replaces the lower level with its necessary and sufficient optimality conditions. Still, the resulting bilevel model is a hard optimization problem. However, by using the reformulations presented in this section we now have a problem that can be tackled, which is not the case for the general four-level model that we started with.

Moreover, in the setting of Model (21) it is perfectly visible where economic inefficiencies may arise from. The robustness constraint (21d) is still part of the model although it makes a statement regarding potential nominations that may possibly occur but which in fact never occur due to economic reasons.

4. A First-Best Benchmark Model

As we have discussed at the end of the last section it is highly probable that the considered entry-exit system leads to inefficiencies. To obtain a benchmark with which to compare different market outcomes we need to provide a first-best model. For its derivation, we assume that we have an integrated gas company that is fully

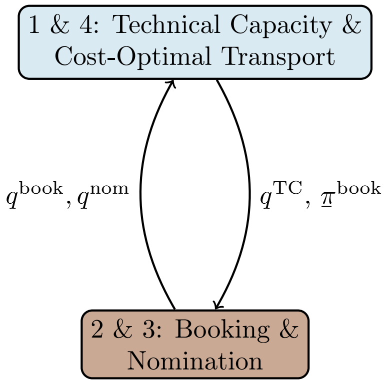

FIGURE 3. Dependencies between the two levels of the reduced bilevel model

regulated and that takes all decisions in a welfare-optimal manner. From this it follows that we can remove the need for a prior determination of technical capacities and bookings. We thus obtain a model that determines the decisions at the third and fourth level of the original model, accounting for welfare optimal production and demand as well as cost-optimal transport simultaneously:

$$\max_{q^{\text{nom}},p,q,z,\dots} \quad \sum_{t\in T}\left(\sum_{u\in V_-}\sum_{i\in\mathcal{P}_u}\int_0^{q_{i,t}^{\text{nom}}} P_{i,t}(s)\,\mathrm{d}s - \sum_{u\in V_+}\sum_{i\in\mathcal{P}_u} c_i^{\text{var}} q_{i,t}^{\text{nom}}\right) \tag{22a}$$

$$- \sum_{t\in T} c_t(q_t^{\text{nom}}) - C \tag{22b}$$

$$\text{s.t.} \quad (p,q,z,\dots) \in \mathcal{F}(q^{\text{nom}}). \tag{22c}$$

Note that due to the absence of the booking level, nominations are only restricted by the true technical capacities for every given scenario. In a strict sense, no nominations take place but the integrated firm decides in each scenario on optimal production and consumption profiles that are constrained by the network. We use similar notation, however, to emphasize the relationship between the four-level model and this model.

It is easy to see that every solution of our full four-level model is feasible for this model. As the objective functions are identical we have the following result.

Theorem 4.1. *Model (22) is a relaxation of the four-level model of Section 2.1, i.e., every solution of the four-level model of Section 4 is feasible and has the same objective value. Furthermore, the optimal value of Model (22) is always greater or equal than the optimal value of four-level model.*

We can therefore use Model (22) as a first-best benchmark that specifies the maximum possible economic welfare in our setup and thereby allows to assess the impact of different market designs. The structure of Model (22) is far simpler than the structure of Model (1): It is "only" a mixed-integer optimal control problem, which, for coarse physical models, can be solved by standard techniques from the literature. For surveys on such models, we refer to Koch et al. (2015), Ríos-Mercado and Borraz-Sánchez (2015), Shaw (1994), and Zheng et al. (2010). Recent publications on solving comparable models are, e.g., Schmidt (2013), Pfetsch et al. (2015), Schmidt et al. (2015), Schmidt et al. (2016), Rose et al. (2016), Geißler et al. (2015), Borraz-Sánchez et al. (2016), Domschke et al. (2011), Moritz (2007), Mahlke et al. (2010), Geißler, Morsi, and Schewe (2013), Mehrmann et al. (2017), Schmidt, Sirvent, et al. (2017), Gugat et al. (2018), and Geißler et al. (2017). The amount of the cited papers indicate that the Model (22) is not new by itself. However, it is important for the evaluation of the outcomes of other market models. For example, another use of Model (22) is as a basis for variants that can be used to estimate the welfare of other designs. It is, for instance, possible to set arbitrary values as "bookings", i.e., upper bounds on the nominated values, to estimate the impact of fixed bookings on the welfare of a system. This variant gives an estimate of the welfare loss that we incur in an entry-exit system, which is comparatively easy to compute.

We note that the relation between the first-best model (22) and a corresponding locational pricing scheme ("nodal pricing") is not a 1-1 correspondence as it is the case for typical models in electricity markets. This and other related results are discussed in detail in Grimm, Grübel, et al. (2017).

5. Model Extensions

Despite the fact that the presented four-level model is quite complicated, it obviously does not incorporate every aspect that one might consider in the context of entry-exit systems. In this section, we highlight some possible and important extensions of the model discussed so far.

5.1. **Investment in Network Infrastructure.** The TSO acts in level one and four of the model presented in Section 2. In level one, maximal technical capacities and booking fees are determined and in level four the TSO is faced with the task of cost-minimal transport of a given nomination. Both levels model short-run decisions in the sense that the gas network infrastructure is assumed to be given and not subject to change. However, in reality the TSOs consider network expansion in order to increase the technical capacity of their network. Network expansion may be realized differently—for instance by building new pipes or by installing new compressor machines or (control) valves. For on overview of the literature on mathematical optimization of gas transport network expansion see the recent publication Borraz-Sánchez et al. (2016) and the references therein.

In a nutshell, expanding the network can be modeled using binary variables $z_j \in \{0, 1\}$, $j \in J$, where J denotes a set of possible expansion candidates like, e.g., new pipes or new compressors. Investment in $j \in J$ then corresponds to $z_j = 1$, which yields additional costs c_j^{inv}. These costs then replace the exogenously given costs C in the TSO's first-level model (1), yielding the modified constraint

$$\sum_{u \in V_+ \cup V_-} \sum_{i \in \mathcal{P}_u} \underline{\pi}_u^{\text{book}} q_i^{\text{book}} = \varphi^4(q^{\text{nom}}) + \sum_{j \in J} c_j^{\text{inv}} z_j$$

instead of (1c). The objective functions of the first-level model (1) needs to be adjusted accordingly as well. Note that this adds further discrete variables to the overall problem.

The second impact on level one is the definition of the set $\mathcal{F}(\hat{q}^{\text{nom}})$ of physical and technical feasible controls and states of the network for a given nomination $\hat{q}^{\text{nom}}$. This feasible set has to be replaced with $\mathcal{F}(\hat{q}^{\text{nom}}, z)$, $z = (z_j)_{j \in J}$, because the newly build pipes or the newly installed equipment changes the controllability and the flow situations in the network. The same replacement regarding the feasible set has to be done in the fourth-level model (14). The rest of the four-level model stays the same.

5.2. **Investment in Supply Capacity.** Besides the long-term investments in network infrastructure by the TSO discussed in the last section, it might be also appropriate to consider long-term investments of gas suppliers in their production capacity. The second-level constraint (7) then becomes

$$q_i^{\text{book}} \le q_i^{\text{cap}} + q_i^{\text{inv}}$$

with the new variable $q_i^{\text{inv}} \ge 0$ and the second-level objective function of the gas seller $i \in \mathcal{P}_u$ at node $u \in V_+$ then reads

$$\sum_{t \in T} \varphi_{i,t}^3(q_i^{\text{book}}) - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) q_i^{\text{book}} - c_i^{\text{inv}} q_i^{\text{inv}},$$

where $c_i^{\text{inv}} > 0$ denotes the corresponding capacity investment costs.

5.3. **Uncertainty.** The basic setting described in Section 2 can also be extended by allowing for data uncertainty. In this section, we sketch a possible model for modeling uncertainty of the demands. To this end, we modify the setting by allowing the inverse demand functions $P_{i,t}$ of the buyers $i \in \mathcal{P}_u$ at exit nodes $u \in V_-$ to also depend on an uncertain parameter ξ. We deliberately choose a very abstract setup, which can accommodate multiple models of uncertainty. One idea could be to assume that the $P_{i,t}$ are linear functions with a fixed slope and that the intercept of the function depends on the uncertain parameter ξ, e.g., $P_{i,t}(s;\xi) = c_i s + (d_{i,t} + \xi_{i,t})$. This type of uncertainty model is widely used in the electricity literature. For gas markets, it might be interesting to consider different models: The uncertain parameter ξ might model the ambient temperature and the functions $P_{i,t}$ then model the demand response to the temperature. As long as we do not consider any inter-temporal effects in the uncertainty and the physical model, and all players are risk-neutral, the theory from a purely deterministic setting carries over quite well, cf., e.g., the discussion in Grimm, Schewe, et al. (2017).

In a first step, one might only consider the case where all players try to optimize their expected profit. The case where the players are risk-averse is far more difficult to analyze as the objective functions would become more complex if players maximize expected utility instead of expected profit. Uncertainty in the third-level optimization problem yields the modified problem

$$\varphi_{i,t}^3(q_i^{\text{book}};\xi) := \max_{q_{i,t}^{\text{nom}}} \quad \int_0^{q_{i,t}^{\text{nom}}} P_{i,t}(s;\xi)\,\mathrm{d}s - \pi_t^{\text{nom}} q_{i,t}^{\text{nom}} \tag{23a}$$

$$\text{s.t.} \quad 0 \le q_{i,t}^{\text{nom}} \le q_i^{\text{book}}. \tag{23b}$$

The objective of the second-level's player problem (3) can be modified accordingly and then reads

$$\max_{q_i^{\text{book}}} \quad \sum_{t \in T} \mathbb{E}_\xi \left[\varphi_{i,t}^3(q_i^{\text{book}};\xi)\right] - (\underline{\pi}_u^{\text{book}} + \pi_u^{\text{book}}) q_i^{\text{book}} \quad \text{s.t.} \quad q_i^{\text{book}} \ge 0. \tag{24}$$

It is an open question whether an analogue of Theorem 2.3 also holds even in the risk-neutral case.

Related articles in the context of uncertainty in gas market modeling are Abada, S. Gabriel, et al. (2013), Abada and Massol (2011), Egging (2013), S. A. Gabriel, Zhuang, et al. (2009), and Zhuang and S. A. Gabriel (2008). Note, however, that this literature focuses on the strategic interaction of producers and that it is typically based on strongly simplified transmission networks. Furthermore, those studies usually focus on an idealized congestion management and disregard problems related to inefficiencies arising due to the specific market design regarding the congestion management regime, such as the entry-exit system. The focus of our approach is the explicit modeling of the potential inefficiencies arising in the entry-exit system.

5.4. **Market Power.** An important aspect of natural gas markets is that suppliers have substantial market power, i.e., they are not price takers. The model in its current formulation (see Section 2) does not accommodate for this. There are several instances of our framework that would require modification to fully allow for the consideration of all potentially arising channels to exercise market power.

First, in the current formulation the objective functions in level two and three would require a modification since strategically acting firms do maximize their overall profits anticipating the impact of their booking decisions (level two) and their nominations (level three) on the respective market clearing prices. Related but conceptually different problems in that context already have been analyzed for liberalized electricity markets, where strategic firms have to choose their production capacities prior to making their production decisions; cf., e.g., Grimm and Zöttl (2013), Zöttl (2010), and Zöttl (2011). The modifications necessary to accomodate for those channels in our approach will be briefly outlined in the present section further below.

In the following we discuss the reformulations of level two and level three that allow to analyze the impact of market power in our framework. Modeling of the third-level interaction is straightforward. We show the change for the seller model (8), which then becomes

$$\varphi^3_{i,t}(q^{\text{book}}_i) := \max_{q^{\text{nom}}_{i,t}} \quad \pi^{\text{nom}}_t(q^{\text{nom}}_t) q^{\text{nom}}_{i,t} - c_i(q^{\text{nom}}_{i,t}) \tag{25a}$$

$$\text{s.t.} \quad 0 \leq q^{\text{nom}}_{i,t} \leq q^{\text{book}}_i \tag{25b}$$

for every seller $i \in \mathcal{P}_u$, $u \in V_+$, in every scenario $t \in T$. In this setup the seller does not act as a price taker, but anticipates that the price depends on the actions of all players, i.e., also on his own nomination. Since this market game at level three is a standard Cournot game with capacity constrained players and since the network constraints do not show up at this stage, it is feasible to account for strategic interaction under appropriate but mild assumptions.

The change in the second level is more involved. In particular, strategic players could try to foreclose rival firms by acquiring excessive capacity in order to reduce competition at level three. Moreover, players might be active at more than one node, which is relevant if players act strategically, but not under perfect competition. Note that we can retain the technical assumption that a node is never used both as an entry and exit point as the technical capacity is allocated separately for entry and exit capacity. This means that in our physical model we can always introduce two artificial nodes for the entry and exit capacity and connect them using an artificial pipe that does not possess any pressure difference.

We introduce a set $\mathcal{P}$ of players and a set

$$\mathcal{K} \subseteq \mathcal{P} \times (V_+ \cup V_-)$$

that contains all pairs of players and entry and exit nodes, such that player p trades at node u if $(p, u) \in \mathcal{K}$. Instead of having to solve the optimization problems (3) as described in Section 2.2, we now need to find an equilibrium where each player $p \in \mathcal{P}$ solves

$$\max_{q^{\text{book}}_p} \quad \sum_{t \in T} \sum_{u:(p,u) \in \mathcal{K}} \varphi^3_{p,t}\left(q^{\text{book}}_p\right) - \left(\underline{\pi}^{\text{book}}_u + \pi^{\text{book}}_u\left(q^{\text{book}}_u\right)\right) q^{\text{book}}_{u,p}, \tag{26}$$

where q^{book}_p denotes the vector of all bookings $q^{\text{book}}_{u,p}$ of player p with u such that $(p, u) \in \mathcal{K}$ and q^{book}_u denotes the vector of all bookings at node u. For each node $u \in V_+ \cup V_-$, the players need to fulfill the additional shared booking constraint

$$\sum_{p:(p,u) \in \mathcal{K}} q^{\text{book}}_{u,p} \leq q^{\text{TC}}_u. \tag{27}$$

The third-level seller problem then needs to be rewritten as

$$\varphi^3_{p,t}(q^{\text{book}}_p) := \max_{q^{\text{nom}}_{p,t}} \quad \sum_{u:(p,u)\in\mathcal{K}} \pi^{\text{nom}}_t(q^{\text{nom}}_t) q^{\text{nom}}_{u,p,t} - \sum_{u:(p,u)\in\mathcal{K}} c_{u,p}(q^{\text{nom}}_{u,p,t}) \tag{28a}$$

$$\text{s.t.} \quad 0 \le q^{\text{nom}}_{u,p,t} \le q^{\text{book}}_{u,p} \quad \text{for all } u \text{ with } (p,u) \in \mathcal{K} \tag{28b}$$

for all $t \in T$. This captures the fact that each seller may now book at multiple nodes and that there is imperfect competition for capacity at each single node.

5.5. **Intra-Day Trading.** System (15) is a model of non-stationary gas flow. However, much of our analysis up to now has focused on the stationary case based on (17). Since booking and nomination of firm capacities takes place on a daily basis we feel comfortable to model the resulting market interaction based on a stationary flow formulation.

As an extension of the present framework we are planning to later consider issues arising for more flexible capacity products or re-nominations, which result to be feasible given the realized operation status of the network. This allows for the adjustments of nominations during the day of delivery by trading on an intra-day market. For a discussion of the incentives of market participants at the intra-day market see, e.g., Keyaerts and D'haeseleer (2014). Such intra-day adjustments are desirable, e.g., when consumption or supply are subject to uncertainty that successively unravels during the course of the day of delivery. When gas flows are to be modified during the course of the day, however, it becomes considerably more relevant to explicitly model problems related to transient gas flows and linepack. For a discussion of the economic potentials of linepack see, e.g., Keyaerts, Hallack, et al. (2011). The specific setup to model transient gas flows and linepack problems in a computationally tractable way in an market environment can, e.g., be based on the work of Domschke et al. (2011).

Clearly, the analysis of a setup that allows to consider equilibrium behavior of agents in a stochastic environment and transient gas flows is highly challenging and, to the best of our knowledge, has not yet been analyzed in the literature. As a first starting point one could refer to the recent work Fodstad et al. (2015). Abstracting from problems arising due to transient gas flows, the authors determine the value of more flexible capacity products in the market environment. They avoid to solve for the market equilibrium resulting from the considered multilevel setting but approximate equilibrium behavior by solving a sequence of stochastic optimization problems.

6. Conclusion

Up to now the literature on European gas markets has widely abstracted from network issues. At a first glance, this is intuitive, since it is the explicit purpose of the current European gas market design to decouple gas trading from congestion management to foster competition in gas markets; cf., e.g., Glachant et al. (2013).

However, as several authors have argued, the current system leaves considerable network capacity unused; cf., e.g., Vazquez et al. (2012) or Hallack and Vazquez (2013). The extent of this inefficiency is unknown up to now due to the lack of models that allow to analyze and quantify the effects. However, in the light of the fact that gas is a fast-growing energy carrier and that sector coupling will increase the utilization of gas infrastructure, missing access to existing network capacities due to the market design implies the necessity of substantial investment in new capacities. This situation calls for the development of models as well as theory and algorithmic knowledge that allow (i) to quantify inefficiencies that occur in the entry-exit system due to unused capacity and (ii) to analyze and evaluate alternative market designs that make capacities usable and thereby help to avoid excessive investment in new capacities. This paper is a first step in this direction.

In this paper, we provide a multilevel equilibrium model that captures key aspects of entry-exit gas markets with an explicit consideration of network issues. An integrated analysis of the TSO's and the firms' decisions requires a four-level model. In order to decouple gas trading from transport, the TSO is required to determine technical capacities and corresponding booking fees at every possible entry and exit node up front (level one). Firms compete for the available capacities at each node as a long-run decision (level two), which gives them the right to trade gas and charge or discharge a quantity of gas at a certain node up to this capacity in every scenario (level three). Beyond the constraints through bookings, gas trade is unaffected by network constraints because technical capacities have to ensure that transportation of traded quantities is always feasible. We assume that the TSO is regulated and determines technical capacities, booking fees, and transportation cost (level four) under a welfare objective. As a first step we assume perfect competition among gas traders.

The main contribution of this paper is to provide a general multilevel entry-exit gas market model and some first results that lay the ground for the application of the model to analyze specific gas markets. We first show that due to the fact that gas trading is basically decoupled from network management issues, booking and nomination decisions are comparatively easy to analyze under the assumption of perfect competition. In particular, we show that the booking and nomination decisions can be analyzed in a single level. We moreover prove that this aggregated market level has a unique equilibrium and that also the TSO's decisions can be subsumed in one level. Thus, the model boils down to a mixed-integer nonlinear bilevel problem with robust aspects. In addition, we provide a first-best benchmark that allows to assess welfare losses that occur in an entry-exit system. Finally, we discuss and provide guidance on how to include several important aspects into our modeling approach, such as network and production capacity investment, uncertainty, market power, and intra-day trading.

Let us finally sketch an agenda that builds on our analysis and would substantially deepen the understanding of market design issues and their consequences in gas markets. First, appropriate algorithmic techniques have to be developed to solve the reduced bilevel problem. Second, while the optimal booking fees are clearly not unique (unless in very special cases), welfare optimal technical capacities might be unique in our setup, which could be investigated. Third, the structure of the booking fees might be of interest. If these fees are determined from a welfare perspective, their structure might violate common normative principles—for instance, since fees at particular nodes have to be set to zero in order to not deter trade. An interesting endeavor would be to quantify the inefficiencies that would arise due to minimum entry/exit fees, as it is common in the European gas market. Fourth, the analysis of market power is easily tractable at the nomination level and could be integrated in the analysis. Market power at the booking level is complex and might induce multiple equilibria.

In the long run, the analysis of investment incentives in gas infrastructure is of key interest. In this respect, our approach has the potential to visualize the trade-off between adjustments of the market design that lead to a better access to the true network capacity on the one hand and network expansion on the other hand.

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