Situational decomposition in controlling complex objects

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The method of optimum control of the complex multivariate objects, based on the account of situations during the moment of acceptance of operating decisions is offered. The method allows to reduce the initial problem of management complicated for the decision, to set of more simple consistently solved problems representing the modified initial problem. The method is realized in a control system of the variable structure combining principles of centralized and decentralized management.

To a creation of automatic system for the optimum control by major industrial objects often prevents a high complexity appeared in the task of control caused by a big dimensionality and presence of numerous functional links between the parameters of the condition, which obstructs its effective decision. In similar situations we can use the methods of decomposition, letting to reduce the complex and multivariate task of control to the set of more private subtasks, joint decision of which can lead to the decision of an initial task.

In this article the author suggests some aspect of realization of one of these methods, which is called decomposition on situations.

The well-known methods of decomposition, which are applied to the tasks of control by the object, allowing a division to some numbers N-structural elements—subsystems, for each of that we can formulate own purpose of operation (for). So these tasks must have a property of separity, that is to allow a division to the subtasks. They can be formulated as follows:

\[ \sum_{i=1}^{N} f_i(x_i, u_i) \rightarrow \max \]

\[ g_i(x_i, u_i) = 0, \quad i = 1, 2, \ldots, N \]

\[ h_i(x_i, u_i) \geq 0, \quad i = 1, 2, \ldots, N \quad (1) \]

\[ x_i = \sum_{j=1}^{N} C_{ij} y_j, \quad i = 1, 2, \ldots, N \]

where \( x_i, u_i \) — are vectors of entrances and controls of \( i \)-subsystem, \( g_i, h_i \) — are given vector-marked functions in the mathematic models of subsystems and considered restrictions; \( C_{ij} \) — is a matrix of zeroes and units, describing the connection of \( i \) subsystem with the exits of \( j \) subsystem.

In result of decomposition the task is reduced to \( N \) local task of control:

\[ f_i(x_i(s_i), u_i(s_i)) \rightarrow \max \quad u_i \in U_i, \quad i = 1, 2, \ldots, N \quad (2) \]

and to the task of coordination:

\[ \sum_{i=1}^{N} f_i(s_i) \rightarrow \max \quad s \in S \quad (3) \]

\[ s=(s_1, s_2, \ldots, s_N), \quad S=S_1 \times S_2 \times \ldots \times S_N \]

where \( s_i, U_i \) — is a parameter of coordination and a set of allowable decisions to \( i \)-local tasks, \( s \) — is a set of allowable meanings of parameters of coordination; \( S_i \) — is a subset of \( S \) to \( i \)-local tasks.

The difference of the method of decomposition on situation is that it does not have any special requirements to the structure of the task of control. Thus, it suggest an allocation of only one local task, that in different situation of adoption of controlling decision can change its structure. The property of the method is concluded in simplification of the task of control, on account of the account of situation of decision making, in which only a part of the variable from the full structure can be efficient.

It means that the other variables does not have an observable influence on condition of the object of control. In this case the initial task of control is formulated as follows:

\[ F(x, u, y) \rightarrow \max \quad u \in U \]

\[ U = \{u : g(x, u, y) = 0; \quad h(x, u, y) \geq 0\} \quad (4) \]
where \( x, u, y \) – are vectors of entrances, control and outputs of the object of control; \( U \) – a set of allowable control; \( g, h \) – are given vector-marked functions in a model of object and restrictions.

And the task is reduced to a set of modified private tasks, considering the situation of decision making, as:

\[
F_i(x_i, u_i, y_i) \rightarrow \max_{u_i \in \bar{U}_i \subset U} \quad \sum_{i=1}^{N} \sum_{D_i} R(x, u, y) = 0
\]

\[
U_i = \left\{ u_i : g_i(x_i, u_i, y_i) = 0, h_i(x_i, u_i, y_i) \geq 0 \right\}
\]

\[
\bigcup_{i=1}^{N} U_i = U
\]

and to the task of coordination as:

\[
(x, u, y) : \rightarrow i, \bar{D}_i
\]

where \( i \) – is a number of current situation, \( N \) – is a number of possible situations, \( x_i, u_i, y_i \) – are modified vectors of entrances, controls, and outputs. \( F_i \) – is a modified criterion function, \( U_i \) – are modified set of \( U \), given with account of modified functions \( g, h, g_i, h_i, x_i, u_i, y_i \) – are meanings of variables \( x, u, y \) in the moment of decision making. \( \bar{D}_i \) – is a set of variables of the task of control, considering in \( i \)-situations. \( \bar{D}_i \subset D = \bigcup_{i=1}^{N} \bigcup_{D_i} \) \( x, u, y \) – are set of the structure of component vectors \( x, u, y \) as well. \( R \) – is an operator of reflection of the vector in pare with \( i, \bar{D}_i \).

The task will evidently be less complicated than the task. But together with that, parts of them can still be left complicated. Their use in the system of control will be complicated on the point of provision of the time of decision, setting in the periods between the appeared situations.

In this case the possible variant is when the task is solved earlier and its result is entered in the memory of the computer. In necessity it is reproduced without the immediate decision of the tasks of control. By the similar way we can also act with the repeating situations. Besides if a structure of the task allows, that is, if the task is separable then it can be decomposed in accordance with one of the known methods.

The basic problem, connected with the realization of the considered approach is consisted in the building of the operator of the reflection \( R \) for identification of the current situation. Its task, in an analytical form, in particular cannot be possible in the form of function \( R(x, u, y) \) as a rule, because it is difficult to reveal the regularities, connecting the constant changing meanings of the variables of the task with discrete meanings of the numbers of situation. By this reason as the basic method of the task of operator \( R \) becomes the use of numerical procedures of selective selection of signs of situations. In general the number of possible situations are exceedingly big and a full specification of each situation requires the account of big numbers of signs, that leads to the complex calculating schemas or to the necessity of building of the expert systems.

Another problem consists in revealing the important variables of \( \bar{D}_i \), in the modified task to the each number of situation \( i \). Such separation suggests making the estimation of sensibility of the output variables \( y \), to the allowable in- \( i \)-situations, changes of controlling variables, on the given meaning , or the decision of additional tasks, an identification of the model of object of control with the definition of optimum structure of a model.

To the decision of the pointed problem we suggest to consider not all possible situations but only the standards which are significantly less.

 Appearing current situations are correlated with standards and in saving the definite conditions is equated to the concrete typical situations. In condition of real production usually a systematization of standard situations is possible, connected with taking of controlling decision to the concrete object.

Thus, for each standard situation there may be formulated the modified tasks of control in which only the efficient variables and conditions in restrictions are considered. At the same time the systems of signs may be built on which a concrete variables of standard situation is identified. In this case the consistent excess of a system of signs of standard situation guarantees the identification of current standard situation.

The quality of control may be exceeded on account of simultaneous account of two or more standard situations when the appearing current situation cannot be uniquely referred to the one of standard situations. On this event a mechanism of discovery of a general variable must be provided for crossing situations, from which is formed a composition of variable of the modified task of control, with addition of private variables of considered situation.

In the real systems of control the production of controlling decision is connected with the appearance of revolting influences caused by changes of meanings of the factors of quality of processed raw material or
changes of loading on the technological equipment. So the estimation of situation is realized on meanings of the variable $x$.

On this method of estimation their possible number $N$ in general is enough great as it is defined by the number of possible combinations of the component vector $x$, but also by the number of possible discrete meaning to each component of $x$. But if the dimensionality of vector $x$ is equaled to $n$, and the number of possible discrete meaning to each of components are equaled to $m$, the quality of situation on the account is defined by correlation

$$N = \sum_{i=1}^{n} i \cdot m \cdot \frac{n!}{i!(n-i)!}$$  \hspace{1cm} (7)

The identification of such number of possible situations is changed to a difficult problem, which complicates a practical use of the considered method. In this connection we take into account only the standard situations. The standard situation applicable to the considered situation is characterized with its efficiency in the definite part of the component vector of control $u$, that is only these components have an influence on the condition of the object of control and are considered as given constants.

For the standard situations a task can be simplified on the account of reduction of the numbers of considered variables, corresponding the simplification of the criterion function and mathematic models and restrictions.

In result a decision of the given task and control of the object is simplified as a whole.

The control with the use of standard situations is approached to the principles of control by productive and technological processes realized by a person. The use of a full composition of parameters of control is here observed in exclusive situations than in usual practice. Herewith, coming to a conclusion a person acts by his gained experience and systemized standard situations with which he correlates the appearing current situations.

Normal move of the processes is valued as collection of staff, that is the standard situation for which known and worked out an efficient algorithm of the reaction and decision making. Supernumerary situations are considered as exclusive, requiring to find the corresponding algorithm of the reaction and production of the controlling decision. Such algorithms are usually built on the base of a combined account of the standard situation. If this built algorithm of decision making turns out to be efficient it is remembered and but the corresponding situation is translated in the category of staff. Thereby in the course of time the number of possible supernumerary situations is reduced and the control is approached to the actions in a condition of only staff situation. The last circumstance may be referred to a certain dignity of the situational principle of control and it can be characterized as the control with accumulation of information about a behavior of the object of control.

The system of control realizing the given principle gains a property of self-training.

Now we can formulate the considered principle of control with decomposition on situations.

Let us mark through $D$-set of situations considered in the task (5) for each of which corresponds their own structure of the efficient variables.

Let us suppose that the given set of $D$ allows a division on $L$ subsets $D_{jk}$, $k=1,2,...,L$ corresponding to the standard situation.

All current situations are valued on attribute to determine the set of $D_{k}$, $k=1,2,...,L$, but the task (5) is changed by equivalent task for standard situation $D_{j}$.

Division of the set $D$ into subsets $D_{jk}$, $k=1,2,...,L$ can be realized on a base of formatting the system of distinguishers of the standard situation. As such signs besides the meaning of component vector $x$, there may be used the number component, receiving increment, the size and mark of this increment and other quantitative estimations.

In the most simple event the selected standard situations will not have an intersection that is:

$$D_{jk} \cap D_{ij} = \emptyset, k=1,2,...,L; j=1,2,...,L; k \neq j$$  \hspace{1cm} (8)

It means that only separate standard situation will exist in the process of control in its clean type. However such a delimitation presents an exception than a rule. In the most general case the condition is not executed, t.e the standard situation can be crossed on separate signs. Then in the task must be taken into account all standard situations on which exists an intersection.

Reference of the current situation to the standard is realized on a base of calculation of all distinguishers of the standard situation with their following collation with systems of signs for separate standard situation. It will correspond to such standard situation for which ex-
ists coincidence on all signs. The absence of such standard situation will mean a property of current situation to the crossing standard situations that is the presence of distinguishers of the current situation which belongs simultaneously to the different standard situations.

With the account of given circumstance the task of coordination for most general cases, may be formulated as the task of definition of distinguishers of the current situation, belonging to the different standard situations with following association of crossing situation. The given task can be formulated as follows:

\[ d = 0; a_j = 0, j = 1,2,\ldots,L; \]
\[ \exists k = 1,2,\ldots,L : p_k \in P_k, s = 1,2,\ldots,S \Rightarrow \]
\[ \Rightarrow d = 1; b_k = 1; a_k = 1 \]
\[ \bar{D} = \bigcup_{j=1,2,\ldots,L} \Rightarrow \]
\[ (9) \]

where \( d, b_k, a_k \) are auxiliary variables used as the indicators, \( p_k \) - s a sign of situation, \( P_k \) - is a set of distinguishers of L-standard situation, \( D \) - is a set of variables considered in the modified task of control, \( \bar{D} \) - is a crossing set of standard situations for which \( a_j \neq 0 \).

In the absence of crossing signs the task is reduced to the following excess of system of the signs of standard situations \( P_k \) with the purpose of finding the system, which completely coincides with the signs of standard situation.

The given task may be formulated in the form of:

\[ d = 1 \Rightarrow \sum_{i=1}^{s} b_{i} \rightarrow \max, k = 1,2,\ldots,L; \bar{D} = D_k, \]
\[ (10) \]

A decision of the given task is \( k=k^* \), to which the sum of meaningful signs of standard situation \( \sum_{i=1}^{s} b_{i} \) - is maximum.

Accordingly the set of considered variables of the modified task of control is \( D_k \).

Local task of the second level in this case may be presented as:

\[ F_i(x_i, u_k, y_i) \rightarrow \max_{u_{k \in L_k}} \]
\[ U_k = \left\{ u_k : g_k(x_i, u_k, y_i) = 0 \right\}, \]
\[ \left\{ h_k(x_i, u_k, y_i) \geq 0 \right\} \]
\[ \bigcup U_k = U \]
\[ \bigcup U_k = U \]
\[ i = 1,2,\ldots, N \]

Where \( i \) - is a number of current situation, \( k \) - is a number of standard situation. \( i \in \{1,2,\ldots,N\}; k \in \{1,2,\ldots,L\} \).

The suggested method of decision of a task may be characterized as a method of decomposition on standard situation. Accordingly, the system of control, realizing the given method - will be decentralized system of situate control.

The method is also may be considered as a method of projection of an initial task \( U \), the subsets \( U_k, k=1,2,\ldots,L \).

The considered principle of control is considered quite perspective as it is approached to the principles of natural control, realized by a person. It promotes increasing a level of intellect of system of control, giving to it more high flexibility and increasing the number of the degrees of the liberty in, that it does not present some special requirements to the structure of the problem of control, in connection with the area of its using can be broader in contrast with all known methods of decomposition.

**Literature**


Ven-Tsen, Khu; Umbetov, U. (2007): Decentralized control of the multivariate objects with decomposition on situation ISSN 0002-3191. Intelligence of the National academy of Science of the Republic of Kazakhstan, physico-mathematical series, No 1, p. 82-85.


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