Hardware Design and RF Performance Evaluation of a Long Range 2.4 GHz Radio Module

Bachelor Thesis
by
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Hardware Design and RF Performance Evaluation of a Long Range 2.4 GHz Radio Module

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by

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ABSTRACT

Low–power, long–range wireless solutions are needed for transmission of environmental and industrial monitoring data from remote, battery–driven sensors. Chirp spread spectrum (CSS) modulation techniques are able to meet these requirements. A CSS–based radio module was designed to operate in the $2.4 \text{ GHz}$ frequency band, whose advantages are worldwide availability, lack of duty–cycle restrictions and a broad spectrum. The module’s power consumption is satisfactory, but its range has to be improved. The accuracy of time–of–flight (ToF) distance estimation, an extra feature of the module, has not been evaluated. Impedance matching, transmission lines and printed circuit board design are among the practical aspects discussed, while ToF ranging and orthogonality of CSS signals are considered from a theoretical point of view.

Keywords: long–range radio module, printed circuit board design, transmission lines and impedance matching, chirp spread spectrum, time–of–flight ranging.
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<table>
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<th>Abbrev.</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>AC</td>
<td>alternating current</td>
</tr>
<tr>
<td>ADC</td>
<td>analog-to-digital converter</td>
</tr>
<tr>
<td>BOM</td>
<td>bill of materials</td>
</tr>
<tr>
<td>BPSK</td>
<td>binary phase-shift keying</td>
</tr>
<tr>
<td>BW</td>
<td>bandwidth</td>
</tr>
<tr>
<td>CMOS</td>
<td>complementary metal-oxide-semiconductor</td>
</tr>
<tr>
<td>CSS</td>
<td>chirp spread spectrum</td>
</tr>
<tr>
<td>CW</td>
<td>continuous wave</td>
</tr>
<tr>
<td>DAC</td>
<td>digital-to-analog converter</td>
</tr>
<tr>
<td>DBPSK</td>
<td>differential binary phase-shift keying</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>ECC</td>
<td>Electronic Communications Committee</td>
</tr>
<tr>
<td>EDA</td>
<td>electronic design automation</td>
</tr>
<tr>
<td>EEPROM</td>
<td>electrically erasable programmable read-only memory</td>
</tr>
<tr>
<td>EM</td>
<td>electromagnetic</td>
</tr>
<tr>
<td>ESL</td>
<td>equivalent series inductance</td>
</tr>
<tr>
<td>ESR</td>
<td>equivalent series resistance</td>
</tr>
<tr>
<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
</tr>
<tr>
<td>EU</td>
<td>European Union</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>FSK</td>
<td>frequency-shift keying</td>
</tr>
<tr>
<td>FSPL</td>
<td>free space path loss</td>
</tr>
<tr>
<td>GCPWG</td>
<td>grounded coplanar waveguide</td>
</tr>
<tr>
<td>GFSK</td>
<td>Gaussian frequency-shift keying</td>
</tr>
<tr>
<td>GmbH</td>
<td>Gesellschaft mit beschränkter Haftung &quot;company with limited liability&quot;</td>
</tr>
<tr>
<td>GND</td>
<td>ground</td>
</tr>
<tr>
<td>GPIO</td>
<td>general purpose input-output</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>I²C</td>
<td>inter-integrated circuit</td>
</tr>
<tr>
<td>IC</td>
<td>integrated circuit</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IMST</td>
<td>Informations-, Mobilfunk- und Satellitenfunk-Technik (Information, Mobile and Satellite Communication Techniques)</td>
</tr>
<tr>
<td>IMU</td>
<td>inertial measurement unit</td>
</tr>
<tr>
<td>IO</td>
<td>input-output</td>
</tr>
<tr>
<td>IoT</td>
<td>internet of things</td>
</tr>
<tr>
<td>IRQ</td>
<td>interrupt request</td>
</tr>
<tr>
<td>ISM</td>
<td>industrial, scientific and medical</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
</tr>
<tr>
<td>JTAG</td>
<td>Joint Test Action Group</td>
</tr>
<tr>
<td>LDO</td>
<td>low drop-out linear voltage regulator</td>
</tr>
<tr>
<td>LOS</td>
<td>line-of-sight</td>
</tr>
<tr>
<td>MCO</td>
<td>microcontroller clock output</td>
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<table>
<thead>
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<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$dBm$</td>
<td>power ratio in decibels, relative to 1 milliwatt</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>relative permittivity</td>
</tr>
<tr>
<td>$f_c$</td>
<td>central frequency</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>reflection coefficient</td>
</tr>
<tr>
<td>$I$</td>
<td>electrical current</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$mA$</td>
<td>milliampere</td>
</tr>
<tr>
<td>$mA$</td>
<td>milliampere-hour</td>
</tr>
<tr>
<td>$mW$</td>
<td>milliwatt</td>
</tr>
<tr>
<td>$P$</td>
<td>power</td>
</tr>
<tr>
<td>$T$</td>
<td>period</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$V$</td>
<td>voltage</td>
</tr>
<tr>
<td>$W$</td>
<td>work</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>characteristic impedance</td>
</tr>
<tr>
<td>$Z$</td>
<td>impedance</td>
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1. INTRODUCTION

1.1. CURRENT SITUATION

The IoT\(^1\) is a concept in which smart devices, such as sensors, actuators, home appliances, vehicles, etc. are interconnected and accessible through the internet (Said & Masud, 2013, p. 2; Mattern & Floerkemeier, 2010, p. 109). The term "internet of things" has been around for 15 years (Uckelmann, Harrison, & Michahelles, 2011, p. 2) and although there has been consistent growth in the sector, there is still room for improvement (Sun, Bie, Thomas, & Cheng, 2016, p. 653). One particular aspect that needs to be developed further is the last mile of the link to these devices.

The requirements for the last mile vary greatly between systems. However, some common needs can be identified. Usually, this portion of the link has to be wireless, secure, energy-frugal and far reaching (Centenaro, Vangelista, Zanella, & Zorzi, 2016, p. 60; DaCosta, 2013, p. 45; Kushalnagar, Montenegro, & Schumacher, 2007, pp. 2–3). This is especially true in environmental monitoring systems, where acquired data has to be transmitted for long distances in places which normally lack communications infrastructure, thus rendering wired connections infeasible (Lazarescu, 2014, p. 170).

A number of companies started offering solutions targeting precisely the requirements described earlier. Semtech Corporation provides LoRa\(^2\)-enabled TRXs\(^2\) (Semtech Corporation, 2015b, p. 1; 2016d, p. 1). Thanks to LoRa\(^3\) – a CSS\(^3\)-modulation technique, radio modules using these TRXs achieve great ranges (12 km\(^4\)) (IMST GmbH\(^6\), 2016b; 2013b, p. 7) using but a little power (38 mA @ 3.0 V) (IMST GmbH, 2016f, p. 9).

ATIM Radiocommunications promises ranges higher than 25 km by using modulation techniques such as GFSK\(^7\) and BPSK\(^8\), while keeping the power needs low (50 mA @ 3.3 V) (ATIM Radiocommunications, n.d., p. 1). NWave provides radio modules which, by using DBPSK\(^9\), provide 10 km range (NWave, 2015) and yet manage to consume little power (54 mA @ 3.3 V) (NWave, 2016, p. 1).

Besides using smart modulation techniques, these TRXs have some more features in common. They are all operating in ISM\(^10\) / SRD\(^11\) radio bands. According to Loy,

---

1. internet of things
2. transceivers
3. chirp spread spectrum
4. All the range estimates presented in the current section are always referring to line-of-sight conditions.
5. Gesellschaft mit beschränkter Haftung "company with limited liability"
6. Information, Mobile and Satellite Communication Techniques
7. Gaussian frequency-shift keying
8. binary phase-shift keying
9. differential binary phase-shift keying
10. industrial, scientific and medical
11. short range device
Karingattil, and Williams (2005, p. 2), ISM is the name used in the USA\textsuperscript{12}, while SRD is preferred in the EU\textsuperscript{13}. Nonetheless, they represent the same: radio bands where the end user does not need a license to operate a radio device (ECC\textsuperscript{14}, 1997, p. 4). From this point on, the present thesis will use the EU nomenclature: SRD.

Specifically, the TRXs mentioned so far as well as their competing devices can operate in one or more of the following SRD radio bands: 169 MHz, 315 MHz, 433 MHz, 470 MHz, 780 MHz, 868 MHz, 915 MHz. It is worth mentioning that each of these bands is restricted only to specific regions. Furthermore, there are limitations on the duty cycle and the permitted transmit power in the given frequency bands (ECC, 2005, p. 6; 1997, pp. 6–8; ETSI\textsuperscript{15}, 2012a, pp. 28–29; 2012b, p. 10; Circuit Design, Inc., n.d.; ITU\textsuperscript{16}, 2012, p. 60; Loy et al., 2005, pp. 7,11; TI\textsuperscript{17}, 2010, p. 27).

The sub-GHz radio bands are also limited in the data throughput because of the restrictions on the BWs\textsuperscript{18} of the channels operating in these bands. In the light of the given limitations other radio bands become attractive. The 2.4 GHz SRD band has a number of benefits when compared to the bands mentioned so far. For instance, it is available worldwide, allows operation in channels of greater BW and has no restrictions regarding the duty cycle. Nonetheless, there are still limitations on the transmit power in the 2.4 GHz band (ECC, 1997, p. 9).

\section*{1.2. Present Thesis Purpose Statement}

The topic of the present bachelor thesis is the design of a 2.4 GHz long range radio module (iM282A). The module is being developed at IMST GmbH in close cooperation with Semtech Corporation. It is based on Semtech’s SX1280 TRX (Semtech Corporation, 2016e).

IMST GmbH was established in 1992 in Kamp-Lintfort as an affiliated institute of the University of Duisburg-Essen, "in order to allow direct transfer of leading-edge of science and technology" (IMST GmbH, 2016a; n.d.). Nowadays, the company has a variety of core competences: Automotive and Radar, Medical applications, Avionics and Space, etc. One of IMST’s areas of expertise is the development of wireless solutions: radio modules, concentrators, gateways, etc. (IMST GmbH, 2016d).

"Semtech Corporation is a leading supplier of high-quality analog and mixed-signal semiconductor products" (Semtech Corporation, 2016a). Among the numerous fields of activity (Timing & Synchronization, High Reliability Discrete Semiconductors, Touch & Proximity Sensing, etc.), Semtech also specializes in Wireless & RF\textsuperscript{19} (Semtech Corporation, 2016c). Since the acquisition of Cycleo (Semtech Corporation, 2012), who was the

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{12}United States of America
\item \textsuperscript{13}European Union
\item \textsuperscript{14}Electronic Communications Committee
\item \textsuperscript{15}European Telecommunications Standards Institute
\item \textsuperscript{16}International Telecommunication Union
\item \textsuperscript{17}Texas Instruments
\item \textsuperscript{18}bandwidths
\item \textsuperscript{19}radio frequency
\end{itemize}
\end{footnotesize}
de-facto inventor of LoRa™ (Design & Reuse, 2009), Semtech has been one of the key players in the development of long-range radio.

In 2013, IMST GmbH released the iM880A – a low-power, long-range radio module (IMST GmbH, 2013a). Later, it was upgraded to iM880B (IMST GmbH, 2016e). Recently, an even more energy-frugal module was released to the market (iM881A: IMST GmbH, 2016f). The mentioned modules are operating in the 868 MHz frequency band and are based on Semtech’s SX1272 TRX (Semtech Corporation, 2015b). These modules as well as a selection of other LoRa™-based products (IMST GmbH, 2016c) demonstrate IMST Wireless Solutions’ extensive experience in the field of long-range radio and SRDs.

The iM282A module was designed in order to utilize the benefits of the 2.4 GHz SRD band, i.e. lack of duty-cycle limitations, worldwide usage and increased BW as compared to sub-GHz SRD frequency bands (ECC, 1997, pp. 6–9). Unfortunately, these benefits also come with a number of disadvantages: the permitted transmit power is lower than in some sub-GHz bands and the achievable ranges are smaller due to the nature of propagation of radio waves - a topic covered in Section 2.2.2.

Besides long-range, low-power wireless communication, a key feature of the iM282A module is the ToF-based ranging, the principles of which are covered in Section 1.3.2.

In the following Section, an overview of the relevant theoretical background is presented. The methods used during the development and testing of the module are introduced in Chapter 2. The results are presented and discussed in Chapter 3 and finally the summary and an outlook are offered in Chapter 4.

### 1.3. THEORETICAL BACKGROUND

#### 1.3.1. CHIRP SPREAD SPECTRUM

The Shannon-Hartley theorem states that a channel’s maximum capacity $C$ is limited by its bandwidth $B$ and its acceptable SNR$^{21}$ ($\frac{S}{N}$). It is expressed (Shannon, 1949, p. 452, Theorem 2) as follows:

$$C = B \log_2 \left( 1 + \frac{S}{N} \right)$$  \hspace{1cm} (1.1)

Spread-spectrum systems are able to operate at very low SNRs by deliberately using much larger bandwidths than the minimum necessary (Cook & Marsh, 1983, p. 8). Chirp spread spectrum (CSS) is such a technique that uses "chirps" for data encoding.

A chirp, in the scope of the present bachelor thesis, is a signal whose frequency changes over time. Figure 1.1 illustrates some examples of chirps. A chirp's frequency can be increasing (up-chirp, (a) & (c)), or decreasing (down-chirp, (b) & (d)). The frequency change can be linear over time (linear chirp, (a) & (b)), or it can be exponential.

---

$^{20}$time-of-flight

$^{21}$signal-to-noise ratio
(geometric or exponential chirp, (c) & (d)). This classification is not complete and is presented only for a general introduction.

![Figure 1.1: Different kinds of chirps. The time, frequency and amplitude axes are for illustrative purposes only and therefore arbitrary.](image)

Chirps are often found in nature. In fact, the term "chirp" is usually used to describe bird calls. While the calls of most birds are extremely varied and most often not "chirps" in the sense of a frequency changing over time, there are indeed birds that do employ up- and down-chirps as part of their repertoire (Suthers, 1997, p. 646). Additionally, some bats species are using linear down-chirps for echolocation (Simmons & Stein, 1980, p. 62).

Regarding technology, the chirp was first used in the field of radars, as a means to increase the spatial range and resolution while keeping the power consumption constant (Klauder, Price, Darlington, & Albersheim, 1960, p. 745). According to Klauder et al. (1960, p. 747), during the Second World War radars’ range and resolution were mainly improved by increasing the transmitted power levels. The use of chirps led to significant performance improvements over previous radar systems (Klauder et al., 1960, p. 748). Klauder et al. (1960, p. 747) also reported that B. M. Oliver was the first to refer to a linear frequency sweep as "chirp" in 1951.

Shortly after starting to use the chirp in radar technology, researchers recognized its potential for telecommunications. Hirt & Pasupathy (1981, p. 836) claimed that M. R. Winkler was the first to propose using chirps for wireless data transmission as early as 1961. Winkler’s main motivation was the chirp’s "inherent interference rejection capability" (Hirt &
Pasupathy, 1981, p. 836). This property arises because a chirp does not depend on a specific single frequency or set of frequencies, therefore the receiver can ignore single spikes and detect only the specific pattern of interest, i.e. a frequency changing over time in a particular manner.

Even though the relationship between frequency and time could in principle be of any form (Dayton, 1968, p. 296; Khan, Rao, & Wang, 2013, p. 1), this bachelor thesis will only consider linear chirps (Figure 1.1 (a) & (b)) from this point on.

In order to convey useful information, chirps have to employ some kind of keying. One of the simplest possible techniques is the representation of ones and zeros by presence “1”, or absence “0” of signal (Lampe & Ianelli, 2003, p. 10). This is the so-called OOK22, depicted in Figure 1.2 (a).

Another means of transmitting useful information with chirps is to represent “1” by an up-chirp and “0” by a down-chirp. This type of keying is discussed at length in literature (Hirt & Pasupathy, 1981, p. 836; Cook, 1974, p. 472) but usually these authors do not assign any name to it. Still, it seems to be called NRZ23 keying in other contexts. The present thesis will adhere to this nomenclature. Figure 1.2 (b) shows an example of NRZ Keying.

Other more complex keying techniques can also be employed in conjunction with chirps, which are rather versatile and not bound to any specific kind of keying. Consequently, designers of CSS-systems have the ability to implement any keying technique that seems promising for a specific application.

The SX1280 TRX is capable of a number of modulation schemes (Semtech Corporation, 2016e, pp. 1, 24–25). One of them is LR24TM - a CSS technique from Semtech. The technique is not covered in detail, as it is not the merit of the present thesis, neither are all the implementation details important for understanding the iM282A radio module. Nonetheless, the LR24TM, just like other CSS techniques, has a number of interesting properties worth discussing.

Figure 1.2: Two possible keying techniques using chirps. The time and frequency axes are for illustrative purposes only and therefore arbitrary.

---

22 on-off keying
23 non-return-to-zero
First, and most importantly, the LR24™ is supposed to achieve “ultra long range” (Semtech Corporation, 2016e, p. 1). This is a direct consequence of the CSS: by using more bandwidth than the minimum required by Equation 1.1, the transmitter is spreading the energy over the entire bandwidth and the receiver can pick up the signals from below the noise level, thus achieving high sensitivity. This leads to a large link budget which means long distances between the TRXs are possible. Section 2.2.2 goes into more detail regarding path loss and the attainable ranges and then describes some real-life range measurements. The results are presented in Section 3.2.

Second, the LR24™ is employing a range of SFs²⁴ and BWs which are used to trade-off between range and data rate. It is claimed (Semtech Corporation, 2016e, p. 30) that the different SFs are orthogonal to each other. Appendix C discusses the orthogonality of CSS signals.

Next, even though not a claim made about LR24™, it is often said that CSS systems are immune to frequency mismatches occurring as a result of the Doppler shift (Cook, 1974, p. 474; Yanjun, Huali, & Lei, 2012, p. 621; Semtech Corporation, 2015a, p. 9). Appendix D attempts to establish an upper bound for the modules’ resilience to the Doppler shift.

It is worth mentioning that the IEEE²⁵ (2016, pp. 430–445) also defines a CSS physical layer for the 2.4 GHz frequency band. Nonetheless, no indication was found to suggest that the specification of LR24™ is a subset of the CSS layer from IEEE, therefore the 802.15.4-2015 Standard is left out of the scope of the present thesis.

1.3.2. Time-of-Flight for Distance Estimation

Given the position (\( \vec{r} \)) of a particle, its velocity (\( \vec{v} \)) is defined as the rate of change of the position over time (\( t \)):

\[
\vec{v} = \frac{d\vec{r}}{dt}
\]  
(1.2)

Therefore the displacement (\( \Delta \vec{r} \)) of the said particle for a duration \( t_f \) will be:

\[
\Delta \vec{r} = \int_{t_0}^{t_f} \vec{v} \, dt
\]

(1.3)

In one dimension, velocity can simply be called speed (\( v_x \)) and the speed’s sign tells us about the direction of movement. Consequently, the above equation is expressed as:

\[
s_x = \int_{t_0}^{t_f} v_x \, dt
\]

(1.4)

It can readily be seen that for a constant speed the displacement is directly proportional to the time interval. This is a very useful idea, since it means that if there is a particle

---

²⁴Spreading factors
²⁵Institute of Electrical and Electronics Engineers
(or a wave) and its propagation speed is known, one can measure the time it takes to go a certain distance and easily calculate that distance. This is the basic idea behind ToF distance estimation / ranging.

ToF distance estimation can be accomplished with a number of phenomena that are known to propagate at a constant speed. For instance, the sonar is relying on a constant speed of sound in water\textsuperscript{26} for detection of various features like sea bottom or fish.

Alternatively, ToF ranging can be performed with EM\textsuperscript{27}-waves. They propagate at a constant speed \(c\), which is called "the speed of light" and is medium dependent. An example is the GPS\textsuperscript{28}, where the time-stamped signals from at least four satellites with predefined orbits can be used by a receiving device to calculate the distances to these satellites and the error in the local clock. Thus, the position of the receiver is calculated very accurately. In addition, the receiver can calibrate its internal clock in reference to the very "correct" atomic clocks aboard the GPS satellites.

In the context of the present bachelor thesis, the ToF is discussed as a means to estimate the distance between two radio modules. Once the ToF is known, the distance \(d\) between the modules (nodes) is calculated as follows:

\[
d = c \ \text{ToF}
\]

There are already numerous systems that make use of the ToF of EM-waves to estimate distances between radio modules. To date, a rather popular approach is the SDS-TWR\textsuperscript{29} (Sahinoglu & Gezici, 2006, p. 4; Röhrig & Müller, 2009, p. 553; Kirsch & Röhrig, 2010, pp. 194–195; Liu, Wu, & Ben, 2013, pp. 278–279). SDS-TWR was first proposed by Hach (2005, p. 7) as a means to combat the ranging inaccuracy arising from the XTAL\textsuperscript{30} frequency drift and ageing.

Figure 1.3 represents a typical SDS-TWR work-flow and Equation 1.6 describes the calculation of ToF for the given setup. The propagation delays \(T_1\) and \(T_4\) are measured and the processing delays \(T_2\) and \(T_3\) have to be fixed and known.

\[
\text{ToF} = \frac{(T_1 - T_2) + (T_4 - T_3)}{4}
\]

There has been extensive research in ToF ranging and some promising results have been found.

Hur and Ahn (2010) took a ToF ranging-enabled TRX and implemented the manufacturer's reference design (Nanotron Technologies GmbH, 2007, pp. 29–35). Then they found a XTAL with specifications similar to those of the XTAL recommended by the

\textsuperscript{26}The speed of sound in water is not entirely constant, but there are various models which adjust it for temperature, salinity, depth, etc.
\textsuperscript{27}electromagnetic
\textsuperscript{28}Global Positioning System
\textsuperscript{29}symmetric double-sided two-way ranging
\textsuperscript{30}crystal oscillator
manufacturer of the TRX but produced by another company. Swapping the XTALs and experimenting with the loading capacitance values of the said XTALs led to a significant improvement in the TRX’s ranging accuracy (Hur & Ahn, 2010, pp. 1775–1777).

Liu, Li, and Sun (2015) used SDS-TWR to improve the accuracy of an IMU\(^{31}\), which is accurate in the short term but very prone to error accumulation over time. They found that the location estimate based on IMU alone can be drastically improved by combining it with SDS-TWR (Liu et al., 2015, p. 394).

The SX1280 TRXs are also capable of ToF ranging. However, they use a simpler scheme than SDS-TWR. It consists of one ranging packet from the master and one corresponding reply from the slave device. In this configuration only the master gets the ranging result.

The ranging feature in the SX1280 TRX is implemented with chirps. As explained by Seller (2014, p. 4), the procedure starts when the master sends a ranging request. Besides some protocol overhead, this request consists of a number of unmodulated up-chirps. While receiving them, the slave aligns itself precisely to the received time and frequency. Then the slave sends back a ranging response that corresponds exactly to the time and frequency found earlier and consists also of unmodulated up-chirps. The master receives the ranging response, which is, from its perspective, time-shifted by \(2T_{ToF}\). The exact value of ToF can then be calculated by multiplying the response with a locally generated complex conjugate chirp. The result is a constant frequency signal (Equation 1.7\(^{32}\)), whose FFT\(^{33}\) yields \(\Delta f\). The time-of-flight is related to this \(\Delta f\) by

\[\Delta f = \frac{2T_{ToF}}{2} = T_{ToF}\]

\(^{31}\)inertial measurement unit  
\(^{32}\)For details regarding representation and notation of chirp signals, see Appendix B.  
\(^{33}\)fast Fourier transform
Equation 1.8. Figure 1.4 helps visualising the described procedure.

\[
x = x_r x^*_t \\
= e^{2\pi jt (f_c + BW ((t/T_s \text{ mod } 1) - 1/2) + \Delta f)} \\
\times e^{2\pi jt(-f_c - BW ((t/T_s \text{ mod } 1) - 1/2))} \\
= e^{2\pi jt \Delta f}
\]

\[
ToF = \Delta f / 2\alpha
\]

Figure 1.4: Measurement of time-of-flight using chirps. "Master’s reference for ToF calculation" is shown just for illustrative purposes. In fact, the master multiplies the response it gets from the slave by the conjugate of the illustrated signal. Only two instances of the Master’s request and one instance of Slave’s response are shown, but there can be more of them.

The alignment of the slave’s transmitted signal to the received one is an essential step, since it makes the slave response analogous to the master chirp bouncing of an object, thus making the rest of the analysis similar, if not identical, to that of a chirp radar.

Seller (2014, p. 2) presents the method in more detail and further argues that this scheme is inherently more secure because no ranging information is transmitted over the air, in contrast to SDS-TWR. Moreover, Seller (2014, p. 2) lists other advantages of the present system when compared to existing ranging systems: longer distances, simpler and less energy-intensive operation.

Regarding the earlier quoted resilience of SDS-TWR to TRX XTAL’s possible errors, Seller (2014, p. 5) provides a mechanism to combat the relative frequency error of the
two TRXs: he offers using the fact that all the symbols start with the same phase, thereby making a fine measurement of frequency error possible. Moreover, Seller (2014, p. 6) argues that “the system is robust to small errors in frequency offset estimation”, since the likely errors are much smaller than the carrier frequency. Still, should the XTALs’ tolerances prove to be indeed a bottleneck, a SDS-TWR-like approach could be implemented in software.

Among the possible shortcomings of the SX1280’s ranging model, Semtech Corporation (2016e, p. 119) stresses multipath and proposes antenna and/or frequency diversity for combating its effects. While antenna diversity would have to be implemented both in hardware and software, frequency diversity can be achieved using just software. The iM282A module provides only one RF-IO\textsuperscript{34}, thus meeting the specification set forth in Section 2.1. However, the end user will be free to implement antenna diversity and a number of IOs of the module (Table 2.2) could be used for switching among the antennas.

### 1.3.3. Impedance Matching and Transmission Lines

Impedance ($Z$) is a key concept in the analysis of AC\textsuperscript{35}-circuits. It is a complex number, measured in Ohms ($Ω$), which describes the relationship between the amplitudes and the phases of an applied voltage ($V$) and the resulting current ($I$). It is therefore playing a key role in determining the power ($P$) transfer in AC-circuits. In RF-design, the goal can be either delivery of maximal possible power to the antenna and therefore to air, or minimization of reflection into the transmitter. Both of these goals depend on the impedance of the TRX pin and on that of the antenna.

To illustrate the importance of impedance, a simple circuit can be considered. It consists of a source with its impedance $Z_S$ and a load with the corresponding impedance $Z_L$. The sample circuit is illustrated in Figure 1.5. When the two impedances are not equal, part of the signal will bounce back into the source. The ratio of the complex amplitude of the reflected wave to that of the incident wave is the reflection coefficient $Γ$. Its value can be calculated as shown in Equation 1.9.

$$Γ = \frac{Z_L - Z_S}{Z_L + Z_S}$$  \hspace{1cm} (1.9)

Due to the conservation of energy, it is to be understood that if part of a wave is reflected, so is part of its power. When the impedances $Z_L$ and $Z_S$ are purely resistive, maximum power is delivered to the load when $Γ = 0$, which occurs when the impedances are equal ($Z_S = Z_L$). However, when the impedances also have a reactive component, maximum power transfer takes place when $Z_L = Z_S^*$, where $^*$ indicates the complex conjugation. This is a consequence of the maximum power transfer theorem and is explained in numerous textbooks, for example (Zinke & Brunswig, 2000, pp. 99–102).

Since usually the TRX pins’ impedance is neither equal to that of the antenna, nor to its complex conjugate, impedance matching is necessary. It can be achieved by matching

---

\textsuperscript{34}input-output

\textsuperscript{35}alternating current
circuits consisting of lumped passive components, i.e. capacitors and inductors. Figure 1.6 shows an example of such a circuit: the TRX RFIO pin is the source and its impedance is $Z_S$. The rest of the circuit, consisting of transmission lines, a matching network and the antenna itself – represents the load, whose impedance is $Z_L$. Maximum power is extracted from the TRX when $Z_L = Z_S$.

Assuming a given antenna impedance ($Z_A = (50 + 0j)\,\Omega$), then depending on the TRX RFIO pin’s impedance, matching can seldom be achieved with one ideal capacitor or inductor. With two elements, a so-called L-network, one can match to the desired impedance for any given source impedance. However, the frequency range where the impedances match will be very narrow. A better approach, when board space is available and the BOM\(^{36}\) allows it, is to use 3 elements (a $\pi$ or T-network) or more, this allowing for finer tuning of the matched impedance and a greater frequency range. The process of impedance matching is greatly facilitated by a Smith chart, where one can “travel” along circles of constant resistance\(^ {37}\) or conductance\(^ {38}\) by adding matching elements either in series or in parallel to the antenna. A number of Smith chart–based tools exist for aiding the design of impedance matching circuits (Dellsperger & Baud, 1995; Telestrian Limited, 2017).

---

\(^{36}\) bill of materials

\(^{37}\) The real part of the impedance.

\(^{38}\) The real part of the admittance, which is the inverse of impedance.
Due to PCB size and manufacturing constraints, the TRX (source), the matching network and the antenna (load) are situated some distance apart (Figure 1.6), therefore transmission lines are necessary for carrying the RF signals between these parts.

Transmission lines, while extremely important in RF-design, can also introduce impedance discontinuities, which in turn lead to wave reflections and suboptimal power delivery to the load. It can be seen from Figure 1.6 that the impedance of the first transmission line ($Z_{0S}$) must be as close as possible to the TRX’s impedance ($Z_S$) and the impedance of the second line ($Z_{0A}$) must be as close as possible to the antenna’s impedance ($Z_A$) in order to deliver most of the power to the antenna. This establishes the importance of impedance–defined transmission lines.

The effect of a transmission line on the EM-wave that it is carrying can be negligible when its physical length ($L$) is much smaller than the wavelength ($\lambda$), i.e. $L \ll \lambda$. But it becomes increasingly important as higher frequencies are used and therefore the wavelengths of the respective signals are decreasing. For instance, the wavelength in free space of a 2.4 GHz EM-wave is $\lambda_0 \approx 12$ cm and taking into account the PCB materials’ relative permittivity ($\varepsilon_r$), the wavelength in the material ($\varepsilon_r \approx 4$) is $\lambda \approx 6$ cm.

These considerations stress the importance of careful design of the transmission lines in RF-design in general and in this bachelor thesis in particular.

It can be shown that the characteristic impedance ($Z_0$) of a variety of transmission lines can be found from the geometry of the line and the properties of the materials involved (Wadell, 1991, pp. 11–15, Equations 2.2.1–2.2.26). In the context of PCB-design, there is a great variety of feasible transmission line structures, each with their advantages and disadvantages (Wadell, 1991). Due to the large number of possibilities and because talking about them all would shift the focus of the present thesis too much, only three types of transmission lines are considered for further discussion: a stripline, a microstrip and a GCPWG.

Figure 1.7 represents the cross-section of a microstrip - a conducting trace separated from a GND plane by a dielectric. The parameters that are necessary for a microstrip characterization are shown in Figure 1.7: trace width ($w$), trace thickness ($t$), separation from GND plane ($h$) and the relative permittivity of the dielectric ($\varepsilon_r$).

Microstrips are easy to implement and can be used to achieve a range of useful impedances (Wadell, 1991, pp. 93–94). Their disadvantage is that there is no GND plane required around microstrips. On one hand, this implies less GND-mass on the PCB and therefore more interference between various conductors, as well as more spurious emissions. On the other hand, it means more copper etching resulting in more waste products during the PCB production.

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39printed circuit board
40grounded coplanar waveguide
41ground
42etch (v.) - “Engrave (metal, glass, or stone) by coating it with a protective layer, drawing on it with a needle, and then covering it with acid to attack the parts the needle has exposed, especially in order to produce prints from it.” Oxford dictionaries
A more complex transmission line which lacks the microstrip's drawbacks is the stripline - depicted in Figure 1.8. It consists of a signal trace, embedded between two GND planes. The analysis of a stripline can be done by using the same parameters as in the case of a microstrip. While a microstrip can be easily manufactured on a two-layer PCB, the minimum number of required layers for a stripline is three.

An advantage of the stripline is the better containment of electric fields between trace and GND when compared to microstrips. However, it can easily be noticed that the signal trace in a stripline cannot be on the top or bottom layer. Therefore, whenever the signal is generated by some component on the outside of the PCB, it needs to change layers at least once and this is accomplished with vias\textsuperscript{43}. Unfortunately, vias introduce additional impedance discontinuities, leading to signal reflections.

There are alternatives to the transmission lines introduced so far. Among these, there is the GCPWG - essentially a microstrip, but with GND area also on the trace's layer. Alternatively, it can be regarded as a coplanar waveguide, but with an extra layer of GND below and the corresponding vias to stitch the GND together. See Figure 1.9. Besides the parameters used in microstrip analysis, another decisive factor in a GCPWG’s $Z_0$ is the separation of the trace from the GND area on the same layer (s).

\textsuperscript{43}In the context of PCB-design, a via is an electrical connection between different copper layers. On each of the layers that it is connecting, there is a pad. A hole is drilled through the pad and the dielectric material between the layers. The hole is subsequently electroplated to ensure electrical conductivity.
It can be shown that a constant impedance can be obtained even when changing the trace’s width \( w \), just by varying the separation \( s \) accordingly. This is especially useful for creating constant-impedance traces going from the pin of one component to the pin of another, these pins often having different widths.

Another advantage of the GCPWG is that it is located on an external layer, therefore no vias or other impedance discontinuities are necessary. Also, a GCPWG can be fabricated on a two-layer design. Compared to the microstrip, the GCPWG has more GND-mass resulting in better antenna performance and less cross-talk among various traces on the PCB. Furthermore, less waste materials are generated during its production.

In the context of the present thesis, the GCPWG was used for the transmission of RF signals. Section 2.1.1.2 describes the implementation of the given waveguide and motivates the need for a sensitivity analysis. Section 3.3 offers the sensitivity analysis implementation and results and discusses possible shortcomings of the chosen design. Appendix E.1 presents the theoretical model that was used to design the GCPWG.
2. Methods and Materials

2.1. Module Design

The design of the iM282A radio module constitutes the central part of the present bachelor thesis. The iM282A is a long-range radio module for the 2.4 GHz SRD band. Figure 2.1 offers a block overview of the module.

![Block Diagram of iM282A Radio Module](source: Modified from IMST GmbH, 2016e)

**Figure 2.1**: Block diagram of the iM282A radio module. At the module’s pads, the user gets access to the MCU’s interfaces and to the TRX’s RF pin.

While Figure 2.1 represents the final result, the module’s design started with the following requirements:

- carry the SX1280 transceiver (TRX) and make reasonable use of its capabilities;
- feature an STM32L151 microcontroller unit (MCU);
- possess the following interfaces:
  - Communication:
    - universal asynchronous receiver/transmitter (UART),
    - inter-integrated circuit (I²C),
    - serial peripheral interface (SPI),
  - Programming:
    - Joint Test Action Group (JTAG),
    - Serial Wire Debug (SWD).
- have the same form factor as iM880B and iM881A (see Figure 2.3);
- be pin-compatible with iM880B and iM881A (see Figure 2.2);
- have "good" RF performance.

44 "good" meaning reasonable, with minimal emissions in undesired RF-bands and maximal power delivery to the antenna in the intended RF-bands.
Once the basic specification for the module has been established, the design process continued with cyclic alternations between schematic capture and PCB design. These stages have been accomplished in the EDA\textsuperscript{45} tool Altium Designer (v. 16.0.6, Build 282).

2.1.1. PHYSICAL DESCRIPTION

The module has thirty-two pads, one of which is used for positive power supply (VDD\textsuperscript{46}), eight - for negative power supply (GND), one - for RF-IO and the other twenty-two can either be GPIOs\textsuperscript{47} or they are assigned a unique function within one of the interfaces the module provides. Each pin’s function is later detailed in Table 2.2. Figure 2.2 presents the logical layout of the module, while Figure 2.3 demonstrates its footprint.

Figure 2.2: iM282A schematic symbol.

Figure 2.3: iM282A drawing: dimensions and pad numbering.

Structurally, the module is a 4-layer PCB. A photograph of the module is presented in Figure 2.4. The top layer is used for component placement and routing. It is illustrated in Figure 2.5. The next layer is a solid GND plane, interrupted only by non-GND vias between the top and the other layers. The third layer is the signal layer, where the routing of critical high-speed signals is accomplished (e.g. UART\textsuperscript{48}, SPI\textsuperscript{49} and I\textsuperscript{2}C\textsuperscript{50}). The bottom layer is used for routing of a few non-critical signals which could not fit in the other layers.

On all three non-GND layers there are large areas free of any traces. These were filled with copper and connected to GND in order to enhance the RF performance of the module. Another benefit of having extensive GND is reduced interference between

\textsuperscript{45} electronic design automation
\textsuperscript{46} drain supply voltage
\textsuperscript{47} general purpose input-outputs
\textsuperscript{48} universal asynchronous receiver/transmitter
\textsuperscript{49} serial peripheral interface
\textsuperscript{50} inter-integrated circuit
different signal traces. In addition, most of the field lines generated by the components on the module are terminated on these GND areas, so fewer undesirable emissions occur.

The PCB’s internal structure is detailed in Table 2.1. The materials used, along with their thickness, are among the standard choices for multi-layered designs (Multi Circuit Boards Ltd., 2016).

**Table 2.1:** iM282A PCB layers description. The presented information is incomplete, in order to protect IMST’s intellectual property.

<table>
<thead>
<tr>
<th>Layer Name</th>
<th>Type</th>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Dielectric Material</th>
<th>Dielectric Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Solder</td>
<td>Solder Mask</td>
<td>–</td>
<td>$h_1$</td>
<td>yes</td>
<td>$\varepsilon_{r1}$</td>
</tr>
<tr>
<td>Top Layer</td>
<td>Signal Copper</td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>–</td>
<td>Dielectric</td>
<td>–</td>
<td>$h_3$</td>
<td>yes</td>
<td>$\varepsilon_{r2}$</td>
</tr>
<tr>
<td>GND Plane</td>
<td>Internal Plane Copper</td>
<td>$h_4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>–</td>
<td>Dielectric</td>
<td>–</td>
<td>$h_5$</td>
<td>yes</td>
<td>$\varepsilon_{r3}$</td>
</tr>
<tr>
<td>Signal</td>
<td>Signal Copper</td>
<td>$h_4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>–</td>
<td>Dielectric</td>
<td>–</td>
<td>$h_3$</td>
<td>yes</td>
<td>$\varepsilon_{r2}$</td>
</tr>
<tr>
<td>Bottom Layer</td>
<td>Signal Copper</td>
<td>$h_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom Solder</td>
<td>Solder Mask</td>
<td>–</td>
<td>$h_1$</td>
<td>yes</td>
<td>$\varepsilon_{r1}$</td>
</tr>
</tbody>
</table>

Table 2.1 shows that the PCB’s layers are symmetric about the middle. Different materials have different thermal expansion coefficients, so a symmetric design ensures equal expansion on both sides of the board and therefore no bending (IPC, 2003, p. 29). This leads to better mechanical stability and allows for the module to be later soldered onto other systems without compromising performance or mechanical specification.
Another technique to combat bending under thermal stress is distributing the copper in similar proportions on layers symmetric about the middle of the PCB. That is, etching most of the copper from any two layers symmetric about the middle, some copper or as little copper as possible. The last option was chosen for this project, as it allowed to leave most of the copper intact and expand the GND mass, this being essential for enhancing the antenna performance and minimizing crosstalk.\(^{51}\)

A further advantage of leaving most of the copper intact has to do with the production process of PCBs: the copper starts out as a thin layer on cores and prepregs and then it is etched to produce the patterns which become the electrical circuit. By etching little copper, only little solvent has to be used in the process, leading to less intensive subsequent processes of copper recovery, solvent neutralization and disposal. Even if the copper can be fully recovered, there is always a cost associated with it and less etching is more economical, being less material and energy intensive.

A compact design is essential for the iM282A radio module. In order to accomplish it, all the signal traces, except the RF line and the USB data lines, have a defined width which is small enough to save space and large enough to be easily manufactured on a PCB.

### 2.1.1.1. Vias

There are numerous vias in the module. While \(\approx 18\%\) of these are used for routing signals and \(\approx 7\%\) for routing the VDD network, the other \(\approx 75\%\) are binding the GND together. This is rather important, as it ensures the existence of low-impedance return paths for currents and signals. It is also a paramount aspect of RF-design, a continuous GND being essential for good RF performance (Cypress, 2016, p. 21). Moreover, it enhances the operation of the other components as well: it minimizes cross-talk between different traces by providing a path to terminate the field-lines generated by these.

The placement of vias was done with a great deal of care. Whenever a change of layer could be avoided without compromising performance, it was circumvented. Nonetheless, when it was inevitable, two or more traces had their vias placed next to each other. This way, the effect on the GND mass was minimized, i.e. instead of many GND breaches spread across the whole board, only a few localized ones were necessary.

Moreover, on portions of the VDD network where considerable current flow is expected, i.e. the branches going to more than one pin, 3–4 vias were placed in parallel when changing layers. This was done in order to decrease the series resistance and inductance. Traces’ and vias’ inductance is a paramount consideration for voltage supply lines, as discussed in Section 2.1.4.1.

Regarding the GND-vias - even more care was taken. For both ICs\(^{53}\), every GND pin had its own low-impedance via placed as close as possible to the pin itself. All the decoupling capacitors have at least one and when feasible up to three vias placed as close as possible.

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\(^{51}\)Crosstalk (n.) – “Unwanted transfer of signals between communication channels.” *Oxford dictionaries*

\(^{52}\)Universal Serial Bus

\(^{53}\)integrated circuits
possible to their GND pad. The same was applied to pull-down resistors and crystal oscillators. This ensures that no big current loops occur as a result of misplaced vias.

All the vias in the PCB are through-holes (Figure 2.6). It is also noteworthy that only two hole dimensions $\varnothing = 0.2 \text{ mm}$ & $\varnothing = 0.3 \text{ mm}$ have been used for the whole module, both of which are standard values for PCB manufacture. The reduced variety of via types and sizes leads to reduced complexity of fabrication and consequently lower production costs.

![Figure 2.6: Types of vias (1 - through hole, 2 - blind via, 3 - buried via).](https://en.wikipedia.org/wiki/File:Via_Types.svg)

When vias are not covered with solder mask (tented), there is the risk of solder flowing into them during fabrication, leading to a lack of solder in places where it is necessary. Also, there is the possibility of creating unintentional electrical connections. In order to avoid these potential problems, all the vias are tented on top and bottom, even those connected to the exposed bottom pads of the ICs.

### 2.1.1.2. Grounded coplanar waveguide

A transmission line was needed for carrying RF-signals between the RFIO pin of the TRX and the RF pad of the iM282A radio module. Section 1.3.3 discussed the importance of transmission lines in RF-design. It also introduced three kinds of transmission lines and argued that the grounded coplanar waveguide (GCPWG) is the most promising type for the line necessary in the current project.

A defining parameter of the GCPWG is its characteristic impedance ($Z_0$). The module is targeting antennas with $50 \ \Omega$ impedance, so for maximum power delivery to the antenna and therefore to air, the GCPWG should have the same $50 \ \Omega$ impedance.

Appendix E.1 presents the relevant equations for the calculation of $Z_0$. These equations were used for choosing the geometry of the GCPWG, given the dielectric properties and the height of the PCB's dielectric material.

A GCPWG was implemented for carrying the RF signals between the TRX and the module pad through the matching network. For the whole length of this transmission line, it has a calculated characteristic impedance of $50 \ \Omega$. 
A variety of tools exists for estimation of a GCPWG’s characteristic impedance ($Z_0$). Among these, the “PCB Toolkit V7.2” software, as well as an online calculator. Both these solutions were used for a first rough estimation and it was found that their results agree very well. However, since some degree of automation was desirable for these calculations, an own function was written in Python which does just that: calculate the characteristic impedance of a GCPWG based on its geometry and the relative dielectric permittivity of the substrate. Its results also agree very well with those of the previously mentioned tools. The function is presented in Appendix E.2.

Even though calculating $Z_0$ is straightforward, the manufacturing of a PCB includes a variety of processes, among which are etching, cutting and heating. The processing of a PCB is done with instruments of limited precision, while the thermal treatment leads to changes in the dielectric properties of the non-conducting layers of a PCB. As a consequence, each of the factors affecting characteristic impedance are subject to some variability. In order to assess the impact of each parameter’s change on the impedance of the GCPWG, a sensitivity analysis was carried out. The function presented in Appendix E.2 was used for performing this analysis. The results are presented in Section 3.3.

2.1.2. Components selection

At the core of the module is Semtech’s SX1280 transceiver (TRX). It performs the radio communication and is controlled by an STM32L151 MCU. The TRX is the first IC to feature LR24™ — a CSS-modulation scheme for 2.4 GHz Long Range radio. It also supports FLRC (a higher bitrate, lower range version of the LR24™) and FSK/GFSK (Semtech Corporation, 2016e, p. 1). A key feature of the SX1280 is its ability to perform time-of-flight (ToF)–based range estimation. Regarding the analog front-end, the TRX features a high-efficiency transmitter and a high linearity receiver. With respect to power management, it has both an LDO and a high-efficiency DC-DC step-down converter.

The STM32L151 is an ultra-low-power ARM® Cortex®-M3 based 32-bit MCU. It features 128 Kbytes of flash memory, 4 Kbytes of EEPROM and 16/32 Kbytes of RAM. The MCU runs at 32 kHz–32 MHz and has numerous peripherals. Among these are ADC, DAC and USB (STM, 2016, p. 1).

The choice to use the given TRX and MCU is outside the scope of the current thesis and should be regarded as part of the specification. The rest of the components were chosen such that the specification is met and the module makes reasonable use of these two ICs’

54 A freeware offered without licensing by Saturn PCB Design, Inc., url: www.saturnpcb.com
56 microcontroller unit
57 frequency-shift keying
58 low drop-out linear voltage regulator
59 direct current
60 electrically erasable programmable read-only memory
61 random access memory
62 analog-to-digital converter
63 digital-to-analog converter
64 STMicroelectronics
capabilities. While the principles used when choosing these components are discussed, their precise nomenclature and specification are left out as a measure to protect IMST’s intellectual property.

2.1.2.1. Crystal oscillators

In addition to the two ICs, the suitable XTALs had to be chosen. The MCU requires two XTALs: a high-speed oscillator in the range 1 to 24 MHz and a low speed oscillator with a typical frequency of 32.768 kHz (STM, 2016, p. 73).

In order to choose an appropriate high-speed XTAL, STM’s guidelines (2015, pp. 13–14, Section 3.4) were followed. The MCU’s transconductance as specified by STM (2016, p. 74) was compared to the minimal transconductance required by the XTAL, which was calculated according to the formula given by STM (2015, p. 14). Since the gain margin\(^{65}\) was larger than five, as recommended by STM (2015, pp. 10, 13), it was concluded that the chosen XTAL is appropriate for the given MCU. Next, the loading capacitors were chosen such that STM’s recommendations (2015, p. 13, Section 3.3) are satisfied.

Regarding the 32.768 kHz oscillator, the gain margin had to be at least three (STM, 2015, p. 10). The same criteria applied for the loading capacitors as in the case of the high-speed XTAL. All of the named requirements were satisfied by the chosen components. With respect to the XTAL required for TRX’s operation, the one specified by Semtech Corporation (2016e, p. 124) was used.

The drive level\(^{66}\) is a critical parameter for choosing a XTAL: if it is too high, the XTAL can be damaged or its lifetime shortened (STM, 2015, p. 15). However the drive levels were not measured and compared to the XTALs’ specification in the context of this thesis.

2.1.2.2. Other components

A number of passive components are needed for the operation of the module: capacitors, resistors and inductors. Regarding those required by the TRX, the exact specification given by Semtech Corporation (2016e, p. 124) was followed. For the rest of passive components, the choice to use one or another was influenced by component size, specified performance, tolerance and price.

The matching network’s importance is partially covered in Section 1.3.3. In this paragraph it shall only be stated that high-quality components were used for matching the module antenna pad’s impedance to that of the TRX’s RFIO pin.

The criteria for choosing decoupling capacitors are discussed in Section 2.1.4.1.

\(^{65}\)The gain margin is the ratio of the MCU’s transconductance to the minimal transconductance required for a certain XTAL.

\(^{66}\)The drive level is the power dissipated in a XTAL.
2.1.3. **INTERFACES / BACKWARD COMPATIBILITY**

The module operates at $2.1 \leq VDD \leq 3.6$ V. Even though the TRX can in principle operate with $1.8 \leq VDD \leq 3.7$ V (Semtech Corporation, 2016e, p. 16) and the MCU with $1.65 \leq VDD \leq 3.6$ V (STM, 2016, pp. 1, 56), they are both supplied from the same VDD line, so an overlapping range has to be chosen. Furthermore, the MCU's ability to function at $VDD < 2.1$ V has not been implemented in software. Therefore, the module must only be supplied with $2.1 \leq VDD \leq 3.6$ V.

Table 2.2 provides details about each of the module pin's function, its internal connection and eventually - 5 V tolerance.

The 5th column in Table 2.2 is called "IO Structure". There are four categories in this column, each of them explained in the Table's heading. The important thing here is that some of the module's pins can handle 5 V input voltage, while others can only accept voltages as high as VDD. The 5 V tolerant inputs cannot output more than VDD. More information is provided in the MCU's datasheet (STM, 2016, pp. 84–85).

The 6th column in Table 2.2 is called "Main Function". The information in this column emphasizes the iM282A modules' pinout compatibility with other modules from IMST, i.e. iM880B and iM881A. The similarities between these two modules and iM282A are the following:

- The GND, VDD, nReset, BOOT, and RF pads are in the exactly same positions;
- The JTAG\(^{67}\) interface is available at the pads 2-5;
- The SPI interface is available at the pads 12–15;
- The UART interface is provided at the pads 18 & 19. The optional CTS & RTS signals are also available at the pads 8 & 9;
- The I\(^2\)C interface is available at the pads 21 & 23.

A change from the other modules is the addition of the second UART port at the pads 28 and 29. This offers the user the freedom to access the module through the main UART and use the secondary one to connect some peripheral device. The expanded connectivity does not come at the expense of other functions on these pads. Should the user not require a secondary UART, the pads can still be simple Digital IOs or ADCs.

Speaking of ADCs, the 7th column in Table 2.2 specifies which module pads are capable of sampling analog signals. There are seven of them, but of course the same pads can have other functions so the user will be free to trade one functionality for another.

Regarding the additional functions of the iM282A's pads (column 8 in Table 2.2), the following can be highlighted:

- The SWD\(^{68}\) interface is available at the pads 2 and 3 - the same as in the case of iM880B and iM881A;
- There is an USB 2.0 interface available at the pads 8 and 9. Before using it however, one should comply with STM's recommendations (2016, p. 94);

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\(^{67}\)Joint Test Action Group

\(^{68}\)Serial Wire Debug
Table 2.2: Pinout of the iM282A module. Pin types meaning: D = Digital, A = Analog, IO = input-output, in = Input; IO structure meaning: FT = 5 V tolerant IO (see STM, 2016, pp. 56–57, Note (3) under Table 14 for details), TC = Standard IO, 5 V non-tolerant, B = Dedicated BOOT0 pin, 5 V tolerant, RST = reset pin with weak pull-up resistor, 5 V non-tolerant.

<table>
<thead>
<tr>
<th>PIN Name Type (number)</th>
<th>IO Structure</th>
<th>Main Function</th>
<th>ADC (Y/N)</th>
<th>Additional Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 P1 DIO PA_14 (P37)</td>
<td>FT</td>
<td>JCTK</td>
<td>N</td>
<td>SWCLK</td>
</tr>
<tr>
<td>3 P2 DIO PA_13 (P34)</td>
<td>FT</td>
<td>JTMS</td>
<td>N</td>
<td>SWDIO</td>
</tr>
<tr>
<td>4 P3 DIO PB_3 (P39)</td>
<td>FT</td>
<td>JTD0</td>
<td>N</td>
<td>TIM2_CH2, SPI1_SCK, COMP2_INM</td>
</tr>
<tr>
<td>5 P4 DIO PA_15 (P38)</td>
<td>FT</td>
<td>JTDI</td>
<td>N</td>
<td>TIM2_CH1_ETR, SPI1_NSS</td>
</tr>
<tr>
<td>6 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7 nReset Din NRST (P7)</td>
<td>RST</td>
<td>NRST</td>
<td>N</td>
<td>-</td>
</tr>
<tr>
<td>8 P5 DIO PA_11 (P32)</td>
<td>FT</td>
<td>DIO</td>
<td>N</td>
<td>USART1_CTS, SPI1_MISO, USB_DM</td>
</tr>
<tr>
<td>9 P6 DIO PA_12 (P33)</td>
<td>FT</td>
<td>DIO</td>
<td>N</td>
<td>USART1_CTS, SPI1_MOSI, USB_DP</td>
</tr>
<tr>
<td>10 P6a DIO PB_7 (P43)</td>
<td>FT</td>
<td>DIO</td>
<td>N</td>
<td>I2C1_SDA, TIM4_CH2, USART1_RX, PVD_IN</td>
</tr>
<tr>
<td>11 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12 P7 DIO, Ain PB_14 (P27)</td>
<td>FT</td>
<td>DIO, SPI2_MISO</td>
<td>Y</td>
<td>USART3_RTS, TIM9_CH2, COMP1_INP</td>
</tr>
<tr>
<td>13 P8 DIO, Ain PB_15 (P28)</td>
<td>FT</td>
<td>DIO, SPI2_MOSI</td>
<td>Y</td>
<td>TIM11_CH1, COMP1_INP, RTC_REFIN</td>
</tr>
<tr>
<td>14 P9 DIO, Ain PB_13 (P26)</td>
<td>FT</td>
<td>DIO, SPI2_SCK</td>
<td>Y</td>
<td>USART3_CTS, TIM9_CH1, COMP1_INP</td>
</tr>
<tr>
<td>15 P10 DIO, Ain PB_12 (P25)</td>
<td>FT</td>
<td>DIO, SPI2_NSS</td>
<td>Y</td>
<td>I2C2_SMBA, USART3_CK, TIM10_CH1, COMP1_INP</td>
</tr>
<tr>
<td>16 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17 VDD Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18 RxD DIO PA_10 (P31)</td>
<td>FT</td>
<td>USART1_RX, USART1_TX</td>
<td>N</td>
<td>DIO, I2C1_SCL, TIM4_CH1</td>
</tr>
<tr>
<td>19 TxD DIO PA_9 (P30)</td>
<td>FT</td>
<td>USART1_TX</td>
<td>N</td>
<td>DIO</td>
</tr>
<tr>
<td>20 P11 DIO PA_8 (P29)</td>
<td>FT</td>
<td>DIO</td>
<td>N</td>
<td>USART1_CK, MCO</td>
</tr>
<tr>
<td>21 P12 DIO PB_8 (P45)</td>
<td>FT</td>
<td>DIO, I2C1_SCL</td>
<td>N</td>
<td>TIM4_CH3, TIM10_CH1</td>
</tr>
<tr>
<td>22 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>23 P13 DIO PB_9 (P46)</td>
<td>FT</td>
<td>DIO, I2C1_SDA</td>
<td>N</td>
<td>TIM4_CH4, TIM11_CH1</td>
</tr>
<tr>
<td>24 P14 DIO PC_13 (P2)</td>
<td>FT</td>
<td>DIO, WKUP2</td>
<td>N</td>
<td>RTC_TAMP1, RTC_TS, RTC_OUT</td>
</tr>
<tr>
<td>25 P15 DIO, Ain PA_0 (P10)</td>
<td>FT</td>
<td>DIO, WKUP1</td>
<td>Y</td>
<td>USART2_CTS, TIM2_CH1_ETR, COMP1_INP, RTC_TAMP2</td>
</tr>
<tr>
<td>26 BOOT Din BOOT0 (P44)</td>
<td>B</td>
<td>BOOTO</td>
<td>N</td>
<td>-</td>
</tr>
<tr>
<td>27 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28 P16 DIO, Ain PA_2 (P12)</td>
<td>FT</td>
<td>DIO, USART2_TX</td>
<td>Y</td>
<td>TIM2_CH3, TIM9_CH1, COMP1_INP</td>
</tr>
<tr>
<td>29 P17 DIO, Ain PA_3 (P13)</td>
<td>TC</td>
<td>DIO, USART2_RX</td>
<td>Y</td>
<td>TIM2_CH4, TIM9_CH2, COMP1_INP</td>
</tr>
<tr>
<td>30 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>31 RF AIO</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>32 GND Supply</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The MCU’s RTC_OUT pin is available at pad 24. The RTC’s alarm events can be routed to this output pin (STM, 2012, p. 35);

Numerous pins are connected to general-purpose timers. Among their functions (STM, 2016, p. 29) is output PWM, thus partially compensating for not routing any of the MCU’s DACs;

Another interesting feature of the MCU that is made available to the module’s user is MCO – one of the internal clocks is output and thus available for other applications (STM, 2016, p. 21).

Some functions of the pins were specified in Table 2.2, while others were omitted. Also, some were presented but not discussed. Only some of the MCU’s features were talked about and they may or may not be relevant for the user’s application. Users wanting to exploit most of the MCU’s capabilities relevant to their application would have to rigorously consult the literature cited in this Section.

2.1.4. RF-CONSIDERATIONS / USE OF BEST PRACTICES

2.1.4.1. Decoupling capacitors

Modern electronics operate at high-frequencies. 16–32 MHz are not uncommon clock speeds for MCUs, while modern desktop computers have processors running 100 times faster. Since CMOS devices draw current only at transitions of the clock signal (Smith, 1994, p. 101) and the clock signals are almost-square waves, the instantaneous currents are many times larger than the average current consumption of the device. Moreover, the frequency components of these transient currents have a much broader bandwidth than the clock speed. This is made evident by Smith (1994, p. 103, Figure 1): if one were to take the FFT of the shown current curve, the result would have non-negligible frequency components way above the processor’s operating frequency.

Power supplies are slow: they cannot provide the momentary high currents that processors require. While their response is acceptable up to about 100 kHz (Smith, 1994, p. 103), their performance drastically decreases with increasing frequency. Due to this poor response, the load (device to be powered) sees the power supply as high impedance above about 100 kHz. If no measures are taken, the transient currents combined with the high impedance lead to voltage ripple. These voltage swings negatively affect the circuits’ performance, sometimes causing them not to function at all.

The currents at frequencies above the power supply’s capabilities can be supplied by capacitors connected between the positive and negative power supply rails. A capacitor’s impedance \( Z_C \) depends on the frequency \( (f) \) and is expressed as follows:

\[
Z_C = \frac{-j}{2\pi fC},
\]  

\( \textbf{2.1} \)
where $j$ is the imaginary unit, $\pi = 3.14159...$ and $C$ is the capacitance measured in Farads. It can be seen that for any capacitance value the magnitude of the impedance decreases with increasing frequency. According to this formula, the impedance of a capacitor approaches zero as the frequency tends to infinity. Ideally, a large capacitor could be used to supply currents at all the frequencies above $\approx 100$ kHz. The power supply would then have to (slowly) recharge the capacitor.

Unfortunately, real capacitors behave differently from ideal ones. Specifically, their parasitic inductance (ESL\textsuperscript{73}) dominates the impedance after a certain (resonant) frequency. An example is shown in Figure 2.7: while an ideal capacitor could easily deal with any frequency above that where the power supply’s response is still adequate ($100$ kHz), a real one fails at the higher frequencies because it behaves like an inductor, rather than a capacitor. An inductor’s impedance ($Z_L$) depends on frequency and on inductance ($L$):

$$Z_L = 2\pi f L$$  \hspace{1cm} (2.2)

\textbf{Figure 2.7:} Ideal versus real capacitor impedance. log-log graph. The ideal capacitor’s impedance was estimated using Equation 2.1. The real capacitor curve was obtained by simulating a capacitor in series with an inductor and a resistor, the values of which were chosen such that the result closely resembles the curve given by TAIYO YUDEN (2010). The "Target impedance" curve represents the desirable maximum impedance ($200$ mΩ) from $100$ kHz to almost $500$ MHz.

\textsuperscript{73}equivalent series inductance
The target impedance of 200 mΩ corresponds to 100 mV ripple for $I_{\text{max}} = 500$ mA. This maximum current has a safety margin of 4 respective to the maximum currents indicated in MCU's (STM, 2016, p. 55) and TRX's (Semtech Corporation, 2016e, p. 22) datasheets. The 100 mV ripple is assumed to not cause harmful effects to the device’s operation. The target impedance was calculated using the method shown by Smith (1994, p. 103).

Due to the behaviour shown in Figure 2.7, one large capacitor is not enough to properly decouple a power supply. A circuit designer must therefore use a mix of capacitances, each being effective for only a range of frequencies.

![Figure 2.8: Impedance versus frequency for real capacitors. log-log graph. The first five curves represent individual capacitors (1 nF–10 µF). These were modelled as capacitors in series with inductors and resistors. Values taken from (TDK Corporation, 2015; TAIYO YUDEN, 2010). The “Power plane + via” curve assumes a small capacitance due to sandwiching of power planes in series with a via’s small inductance and resistance. “Combined 1” puts one of each capacitors in parallel + the model of power plane + via. “Combined 2” is the same as “Combined 1”, but has two extra capacitors: 100 nF and 10 nF. “Combined 3” is the same as “Combined 2” but ignores the capacitance and inductance associated with the power planes. The “Target impedance” curve represents the desirable maximum impedance (200 mΩ) from 100 kHz to almost 500 MHz.

Figure 2.8 presents an example of a potential decoupling solution involving an array of parallel capacitors\(^{74}\), thus covering a wide range of frequencies: from a power supply's “cut-off” frequency up to the 15\(^{th}\) harmonic of a 32 MHz signal. It clearly shows that none of the individual capacitors can adequately handle all the frequencies of interest.

\(^{74}\) Unless otherwise specified, all the mentioned capacitors from this point on are real ones, having both equivalent series inductance and equivalent series resistance.
Regarding the frequency range, it is chosen somewhat arbitrarily: while the lower bound can be defined by the characteristics of the power supply, the upper bound should be high enough to encompass the frequency components of the clock signal, i.e. its BW. Since the clock signal is not perfectly square, one only needs to know its rise time in order to find its BW (Bogatin, 2013). In this simplistic example, it was considered that the first 15 harmonics of the processor’s operating frequency would be enough, given that a harmonic’s contribution to the composite signal decreases with its increasing order (Weisstein, n.d.).

All the shown capacitors and their parasitic inductance have resonant frequencies where the resulting impedance dips to a minimum. However, there are also anti-resonant frequencies formed by two parallel capacitors. There the impedance suddenly spikes. This happens at frequencies where one capacitor already behaves like an inductor, while the other one’s capacitance still dominates its behaviour. This effect can be observed very well at about 6.5 GHz (Combined 1) and 8 GHz (Combined 2) in Figure 2.8: while all the discrete capacitors are already in their inductive region, the capacitance due to power planes’ sandwiching has not been overcome by vias’ inductance yet.

![Figure 2.9: Resonance and anti-resonance. log-log graph. The first two lines show the impedances of an ideal inductor ($R_L = 0$) and of an ideal capacitor ($R_C = 0$) as a function of frequency. The next two curves show the same capacitor and inductor in series and parallel. Where the capacitor’s and inductor’s reactances ($X$) are equal, their impedances are also equal in magnitude but opposite in sign ($X_C = X_L \implies Z_C = -Z_L$ @ 10 MHz), the series combination (resonance) leads to zero impedance and the parallel arrangement (anti–resonance) - to infinite impedance.](image-url)
Figure 2.9 shows how arranging ideal capacitors and inductors in series and in parallel affects their total impedance. While one case leads to zero impedance at the resonant frequency, the other one - to infinite impedance at the same frequency. Even though not so acute, the same effects can be observed in Figure 2.8, where every trough corresponds to the resonance of a capacitor with its own ESL and every peak to the anti-resonance of a capacitor with the ESL of the capacitor that has its resonance at the next lowest frequency. What is bounding these troughs and peaks is the ESR\textsuperscript{75} of the said capacitors: by having positive real parts the combined impedances cannot "explode" to infinity or sink to zero.

It would be desirable to obtain only troughs and no peaks in the inductance of a decoupling array. However, due to the properties of real capacitors, some of which were discussed here, such a result seems impossible. Nonetheless, a few design guidelines can be outlined. Following these, the circuit designer has better chances to properly decouple the power supply, providing the required high-frequency currents from capacitors. These, in turn, can be recharged by the power supply at a "comfortable" pace.

First, a range of capacitors should be used. Bulk microFarad-range capacitors are required for supplying the lower frequency currents and storing power intermediately between the power supply and the lower-value capacitors. These in turn have to do the most critical decoupling, supplying high-frequency currents and depending on the system’s clock speed, they have to be in the the nano- and even picoFarads-range.

On one hand multiple capacitors of the same value are very useful for reducing their combined ESL and ESR, therefore making the valleys deeper at their resonant frequencies (Smith, Anderson, Forehand, Pelc, & Roy, 1999, p. 288, Figure 11). On the other hand, multiple different values are required to obtain a low impedance over a range of frequencies. Even more, the "gaps" in capacitance should be made as small as possible to avoid large anti-resonant spikes (Smith et al., 1999, p. 288, Figure 12), like e.g. in Figure 2.8: Combined 1 @ 6.5 GHz. The said peak is caused by anti-resonance of power planes’ capacitance and 1 nFarad capacitor’s inductance. Given these two seemingly opposing objectives, one must find a balance between diversity of capacitances used and the number of capacitors of each value.

Next, the use of high quality ceramic capacitors is necessary. While the use of aluminium electrolytic and tantalum capacitors upwards of tens of $\mu$Farads might be needed for smoothing the power supply’s output, their ESL and ESR make them poor choices for higher frequencies (Carter, 2001, p. 8). Moreover, the capacitors should be SMDs\textsuperscript{76} whenever possible, because, compared to through-hole components, they have less parasitic effects and are generally better fitted for high frequencies (TI, 1999, p. 1).

The use of COG(NP0) capacitors is desirable, these having the best thermal characteristics (Novacap, 2017, p. 1) and therefore the most predictable behaviour over a range of temperature. However, NP0’s relative dielectric permittivity and respectively its "volumetric capacitance density" is two orders of magnitude lower than that of other ceramics

\textsuperscript{75}Equivalent series resistance

\textsuperscript{76}Surface-mount devices
(Horowitz & Hill, 2015, p. 18). Therefore, since the ESL increases with size (TDK Corporation, 2015), the use of C0G ceramic capacitors is feasible only up to $1–2$ nFarads in the 0402 package size. Anything above $\approx 2$ nFarads would either be too large, taking up too much board space and/or having too high ESL and/or be too expensive (Digi-Key Corporation, 2017).

Taking into account NP0's limitations, one should use X7R ceramic capacitors for anything above two nanoFarads. They have the next best thermal performance after NP0 (Novacap, 2017, p. 4, Figure J-3). The use of X7R capacitors is feasible up to $\approx 1$ microFarad in the 0402 package size. Since capacitors above $1$ µFarad are not needed next to ICs, their size need not be limited to 0402. Then values up to $10$ and $22$ µFarads become feasible in the 0805 and 1206 package sizes, respectively (Digi-Key Corporation, 2017).

Nonetheless, when the temperature range where the device has to operate is contained between $-55$ and $+85$ °C, the X7Rs can be replaced by X5Rs, since they have the same thermal stability, just in a narrower range. The Y and Z series of ceramic capacitors are to be avoided, due to their small temperature range and extremely poor thermal stability (Novacap, 2017, p. 4, Figure J-3). Smith et al. (1999, p. 289, Figure 13) also recommend using either NP0 or X7R depending on the target frequencies, i.e. capacitances.

Weiler and Pakosta (2006, p. 13) stress the importance of correctly placing the decoupling capacitors: the smallest values must be as close as possible to the ICs’ power pins with the rest also as close as possible in order of increasing capacitance. This is of utmost importance, since the PCB traces between components posses inductance which increases with length and the more inductance, the lower the resonating frequency and respectively the effectiveness of decoupling.

Also, the GND-pads of the capacitors must be directly connected to the GND plane by at least one via per capacitor. Roy, Smith, and Prymak (1998) discuss the capacitors’ via and pad layout and argue that their inductance must be minimized by placing the via as close as possible to the pad. Furthermore, they call for having the GND and VDD vias right next to each other, thus reducing the current loop and the associated inductance.

Last but not least, the datasheets of the ICs will often indicate which values and amounts of capacitors to use for proper decoupling. This section is only a short and simplified overview of the topic. Regarding the iM282A module, the guidelines outlined here were followed while designing it: numerous and varied well-placed high quality capacitors, multiple vias to GND whenever possible and a large area assigned to VDD on a layer next to the internal GND plane. The exact values of the components used are not given, in order to protect IMST’s intellectual property.

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77Values in text always refer to package size in imperial units. 0402 corresponds to four thousands of an inch by two thousands of an inch, i.e. to one millimeter by half a millimeter.
2.1.4.2. Routing

Routing is an essential step of PCB design. When carelessly done, it can lead to all kinds of unwanted effects: crosstalk, reflections and EM radiation to name just a few. Crosstalk refers to the interference between signals on the same board where one can couple into the other and disturb its operation. Reflections are discussed in Section 1.3.3; if not dealt with they disturb signals and can lead to large voltage swings thus damaging ICs. EM Radiation occurs because high-speed traces can act as antennas. It is undesirable, leading to interference with other devices and hindering certification procedures.

Fortunately, these problems can be avoided by following a few simple rules. Weiler and Pakosta (2006) give a theoretical overview of the harmful interactions taking place in high-speed environments and then outline basic PCB design techniques to circumvent these problems. The following is a short outline of some of Weiler and Pakosta’s guidelines (2006) which were followed as best as possible while designing the iM282A module.

The most basic thing to do when designing a PCB is to avoid $90^\circ$-angles on traces. These lead to signal reflections and to increased radiation from the trace. Round bends are the best solution, but $45^\circ$-angles are also acceptable and the most widely used in modern designs.

Next, it was ensured that every signal has a low-impedance path for the return of high-frequency currents either in the GND pour on the same layer or in the GND plane layer directly under the signal trace. This is a fundamental prerequisite in avoiding large current loops, that would otherwise lead to crosstalk and EM-radiation. Regarding the return paths, these are not only essential within the module but also for all the outgoing and incoming signals. On the module’s level they are implemented by assigning eight of the module’s pads to GND. These pads are located all-around the module (Figure 2.2), thus ensuring that with careful design of the carrier board each signal entering or leaving the module has a short low-impedance return path.

Also a measure to reduce radiation, fast signal traces (the communication interfaces) were routed mainly in an internal layer right next to the GND plane. This way, their field lines are terminated immediately on the GND-mass and do not propagate as radiation. Another critical aspect of high-speed design is to keep traces as short as possible. This is done in order to avoid signal delays and corresponding degradation of communication.

Besides these basic requirements, some more subtle aspects had to be tackled during the design of the PCB for the iM282A module. Sometimes there were dilemmas arising from seemingly contradicting design guidelines. Whenever these were encountered, the engineers from IMST’s Wireless Solution department were most kind in explaining how to prioritize the design objectives, giving references for further reading and generally answering any questions.
2.1.5. IMPEDANCE MATCHING

The importance of impedance matching was established in Section 1.3.3. The present Section describes the methods used while matching the TRX RFIO pin's impedance to the 50 Ω antenna pad. While Equations 2.1, 2.2 and a few more would be enough to calculate the impedance of any AC source or load and therefore be able to match them, doing so is less practical than taking advantage of another excellent tool for this task – the Smith chart. Readers unfamiliar with it are encouraged to read Maxim Integrated's application note (2002). For those who wish to see a much broader range of usage examples, Caron (1989) can be a good start.

![Smith Chart](image)

Source: Current and following four figures captured in (Dellsperger & Baud, 1995). Used with author’s permission (Dellsperger, 2017).

Figure 2.10: A basic Smith chart with impedance normalized to 50 Ω. The red circles contained within the chart are lines of constant resistance; the blue ones – lines of constant conductance. The red circles that have their centres outside the chart are lines of constant reactance; the blue ones – lines of constant susceptance. Both real and imaginary quantities are symmetric about the horizontal line going through the middle, but the imaginary ones have opposite signs: reactances are positive in the upper half and susceptances - in the lower one. The point in the center of the chart represents the target impedance: (50 + 0j) Ω.
For understanding the current Section, one has to know what a Smith chart looks like. It is shown in Figure 2.10. For the further discussion, it is useful to refer back to the circuit shown in Figure 1.6, but it has to be said that the TRX’s RFIO pin forms the load impedance ($Z_L$), while the source impedance ($Z_S$) consists of the rest of the components. This perspective is more consistent with the software used (Dellsperger & Baud, 1995). If the point of view is thus shifted, it shall be said that the source impedance is matched to the load impedance, which seems to contradict what was said in Section 1.3.3, but in fact is just another way of stating the problem of impedance matching. In this case, the TRX is a receiver.

The matching process usually starts with plotting the TRX RFIO pin’s impedance at a given frequency as a point on the Smith chart. At least 2–3 frequencies in the spectrum of interest, corresponding to as many points, should be considered. The process then continues with addition of components between the pin and the antenna gradually moving the points until they reach the centre of the chart, where the target impedance lies. Normally, matching the whole spectrum to the target impedance is not possible, so getting reasonably close is enough.

The usual matching networks consist of capacitors and inductors. They can be added in series or parallel. Besides, transmission lines can be added. The Table 2.3 summarizes the effect of each element on the point representing the impedance of the source (containing the RFIO pin, the transmission lines and the matching components).

Table 2.3: Influence of matching components on the source impedance.

<table>
<thead>
<tr>
<th>Component (structure)</th>
<th>Orientation Series</th>
<th>Orientation Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductor</td>
<td>Clockwise along constant resistance</td>
<td>Counter-clockwise along constant conductance</td>
</tr>
<tr>
<td>Capacitor</td>
<td>Counter-clockwise along constant resistance</td>
<td>Clockwise along constant conductance</td>
</tr>
<tr>
<td>Transmission line(a)</td>
<td>Clockwise in a circle centred at the line’s $Z_0$</td>
<td>Open: same as parallel capacitor</td>
</tr>
</tbody>
</table>

(a) While the “movement” caused by inductors and capacitors is constrained between the starting point and the right (left) extremity of the chart for series (parallel) elements, transmission lines rotate the result indefinitely, with $\frac{1}{2}\lambda$ corresponding to a complete rotation.

The Figures 2.11–2.14 show some examples of impedance matching. In all four Figures, the starting point is $(18.84 + 15.90j)$ Ω and the target is $(50 + 0j)$ Ω. The operating frequency is 2440 MHz. The Figures demonstrate that it is possible to arrive at the target impedance by a number of paths, i.e. using different combinations of matching elements.

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78 This is only the case for reactive matching. Resistive matching is also possible, but usually not desirable in RF, due to the associated resistive losses (Silver, 2012).
While the circuits presented in Figures 2.11 and 2.12 accomplish the matching for the given frequency, they may perform poorly for a range of frequencies, e.g. the whole 2.4 GHz SRD radio band: 2400 MHz–2483.5 MHz. It follows from Caron’s examples (1989, Chapter 6) that having more elements is beneficial when matching a range of frequencies, because every element affects each frequency differently and when there are more elements, their values can be adjusted in a way that brings the entire range of frequencies as close as possible to the target impedance. Figures 2.13 and 2.14 show how the same problem is solved with three and respectively four matching elements.

When it comes to implementation there are more variables to be taken into account. For instance there is a limited set of standard capacitor and inductor values that can be used. Moreover, they can substantially deviate from the nominal value. As an example, a high-quality ceramic 0.1 pFarad capacitor has a ±0.1 pFarad tolerance, corresponding to ±100 % variation. Even though the variation in the impedance will not be so large since there are also other components in the circuit, it is easy to see why actual matching should
be done with standard-valued elements with low tolerances. This can be interpreted in two ways regarding the use of more matching elements versus two-element solutions: either the variations of the components add up and make the result completely different from one module to another or they even themselves out. Unfortunately, no reference has been found to support either of the claims, but it was assumed here that this is an argument in favour of using more matching elements: they give the designer more flexibility in how to match, not being constrained to just two elements that have to be perfect in order for the circuit to function properly.

Regarding the impedance matching for the iM282A module, the TRX manufacturer’s reference design (Semtech Corporation, 2016e, p. 123) was used initially. Besides the mentioned components, provisions were made for some additional series and parallel elements. Unfortunately, due to some differences between the module and the manufacturer’s reference board, the performance of the module was inferior to the one advertised. Consequently, a different matching solution was desirable. This redesign however depended on knowing the impedance of the TRX’s RFIO pin - an information which was not made available during the writing of this thesis. One approach attempted the estimation of the said impedance. It is described in the following paragraphs.

Table 2.3 describes how each element in a matching network moves the impedance through the Smith chart. The amount of displacement however depends on the frequency of interest and on the value of the capacitor/inductor considered. It is dictated by the Equations 2.1 and 2.2. By transforming these equations and by taking into account that inductors and capacitors behave exactly like the opposite of each other both in parallel and in series configurations, it is fairly easy to show that the displacement done by a capacitor can be undone by an inductor placed in the same configuration and vice-versa. The values of these "complementary" components are related by the following Equation:

\[
LC = \frac{1}{(2\pi f)^2}
\]  

(2.3)

Using this knowledge, as well as the fact that series transmission lines just rotate the impedance about their characteristic impedance with \(\frac{1}{2}\lambda\) corresponding to a full rotation, it was only needed to measure the impedance at the antenna port with the current matching and solve backwards, element per element, for the TRX’s pin impedance. While the approach is straightforward in theory, in practice it proved rather troublesome, since tiny variations in the considered transmission line length led to completely different results for the pin’s impedance. Consequently, the TRX pin’s impedance was not found and another approach was instead followed.

The next attempt consisted of solving an optimization problem. The elements in the existing network were exchanged one by one for an element with a higher/lower value and the gradient was followed. While also a valid method for either situations with unlimited time resources or experienced RF-designers, it does not guarantee to yield the best results, since local optimal solutions do not always correspond to the overall optimum of the system. The results of this work were not measured directly. Instead, they influence the module’s RF-performance. Therefore, the evaluation of the matching network is part of Section 3.1.
2.2. Measurements

Various tests have been performed at different stages of the design process. Among these were the measurement of current consumption, radiated power, measurement of the crystal oscillators performance, measurement of unwanted emissions and quantification of range performance.

2.2.1. Current Consumption, Transmit Power and Frequency Error

The current consumption and effective transmit power of the module, as well as its frequency error in transmit mode were measured with one set-up. It was done in order to assess the current requirements versus the effective transmit power as well as the efficacy of the matching network and the TRX’s XTAL, i.e. what power is really transmitted and at which frequency versus what is set in software.

LR24™ is a long-range technology directed mainly at low-power gadgets in remote locations, so devices using it should be able to operate a long time relying only on batteries and with no grid connection. As explained in Appendix A, the life-span of a battery depends on the current drawn by the device it powers. Meanwhile, the range of the radio depends, among other things, on the transmitted power. Consequently, it is important for the iM282A module to draw little current, while maximizing the transmitted power.

Regarding the frequency error, what is meant is the TRX’s oscillator frequency accuracy and its impact on the carrier radio frequency. The XTAL is supposed to oscillate at a certain frequency, but a range of factors, like inappropriate loading capacitance, XTAL ageing and temperature variations can lead to an offset in this frequency, which then affects the RF as well. What was measured is the offset in the radio frequency.

In order to perform the said measurements, a module was soldered onto an adapter board (WiMOD_AB_01, V2.1), which was then mounted onto a special testing board, which is a scaled down version of the demo board WiMOD_DB V1.0 (For reference, see IMST GmbH, 2009). The scaled down version provides only a USB-to-UART interface to the module and various options for power supply, excluding all the actuator and sensor capabilities of the WiMOD_DB V1.0.

The test board also features a connector where the module can be supplied with power and its current consumption can be monitored. A bench power supply was used for supplying the module and a multimeter (KEITHLEY 2000) was connected in series to measure the current flow.

Further, the adapter board features an SMA79 connector, which was used for connecting the module’s RF output to a spectrum analyser’s input through a 50-Ω coaxial cable. The module, as well as all the instruments were connected to a controlling PC. The described set-up is presented in Figure 2.15.

79sub-miniature version A
A script was running on the host PC. It set the voltage on the power supply, set the operation mode (CW\textsuperscript{80}), transmit power and central frequency on the module, after which the MCU was put in sleep mode, so that only the TRX's consumption was recorded. That is an important aspect, since the MCU's current requirements are detailed in its datasheet (STM, 2016), while those of the TRX depend on the impedance matching network among other factors. Afterwards, the script set the measurement parameters on the spectrum analyser and read the current measurements from the multimeter and the power measurements on the spectrum analyser.

The script consisted of three nested loops. First, it looped through different voltage levels (2.1 V to 3.6 V in steps of 0.3 V). At the next level, it looped through different central frequencies (2401 MHz to 2481 MHz in steps of 10 MHz) and in the innermost loop – through the different power settings of the module: $-18$ dBm to $+12.5$ dBm in steps of 1.0 dBm, with the last step of 0.5 dBm. The current consumption, transmit power and frequency deviation were recorded at each iteration.

\textsuperscript{80}continuous wave
2.2.2. RANGE MEASUREMENTS

The range performance of the iM282A module is of utmost importance, since one of the motivations to develop it was to achieve long range in the 2.4 GHz frequency band. The range of a pair of radio modules is directly related to their available link budget. The importance of this budget lies in the fact that if the received power is higher than the receiver’s sensitivity, the signal can be demodulated. In its simplest form, the link budget can be expressed as follows:

\[ P_{RX} = P_{TX} + G - L, \]  

where \( P_{RX} \) is the received power, \( P_{TX} \) is the transmitted power; \( G \) stands for gains and \( L \) for losses. All the quantities are expressed in decibels. The gains and some losses are due to the transmitting and receiving antennas. However, a certain part of the \( L \) term occurs because of the distance between the radio devices. This is known as the path loss. All other things being equal, the larger the distance – the larger the path loss, so the higher the transmit power required to satisfy a certain sensitivity requirement.

While radio system designers can influence the output power on the transmitter side and the sensitivity on the receiver side, they have no control over the path loss, which is environment-dependent. The most basic component of the path loss is the FSPL\(^{81}\), which is characterized by the following equation:

\[ FSPL = \left( \frac{4 \pi df}{c} \right)^2, \]  

where \( d \) is the distance between devices, \( f \) is the signal’s frequency and \( c \) is the speed of light. It can be readily observed that the free space path loss increases with both distance and frequency. Figure 2.16 demonstrates how the received power attenuates as a function of distance, for a number of carrier frequencies. The FSPL for each frequency was calculated according to (ITU, 2016a, p. 2, Equation 4).

Figure 2.16 is only representative for a hypothetical scenario, where only FSPL is taking place, i.e. there are no obstacles, no atmospheric influences, no multipath propagation. This scenario cannot be achieved on Earth. Nevertheless, it is a good indication of what the best-case performance of a system might be if path loss were taking place only as a result of free-space propagation. The Figure also specifies other characteristics of the system and indicates the maximum achievable range for the given specification.

Unfortunately, in real life the path loss is more complicated than Figure 2.16. A variety of factors influence it: antenna efficiency and directivity, atmospheric conditions and obstacles among others. A very basic assessment of the range performance of the radio modules can be performed by LOS\(^{82}\) range measurements. This way the influence of obstacles can be minimized and some factors such as antenna performance can be

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\(^{81}\)free space path loss

\(^{82}\)line-of-sight
controlled. Moreover, this establishes an upper limit on the system performance for a given set-up. The user can later use this as a reference value and apply the relevant methods from e.g. (ITU, 2015) and (ITU, 2016b) in order to estimate the expected performance for other real-life conditions.

The range measurements were performed using two modules which were soldered onto WiMOD_AB_01, V2.1 adapter boards. These were mounted on WiMOD_DB V1.0 demo boards. Each demo board was connected to a laptop-PC via an USB interface. This was necessary for interfering with IMST’s software tools.

The measurements were performed on two different occasions at two different sites. The first measurements took place on a 1.2 km straight street with only few trees on the side and almost no buildings. It was during winter, so the trees’ crowns did not greatly impact the measurements. The aforementioned street is Gerade Straße, situated in the fields around Kamp-Lintfort, Germany (Figure 3.7). The day when the measurements took place was sunny, the temperature was about 0 °C.
For the measurement itself, one module had a fixed position in the south end of the street. The other module was being moved along the street and at certain positions a "Radio Link Test" (IMST GmbH, 2013c, p. 11) was performed using LR_Studio (a software tool from IMST). The test consisted of sending 100 packets 15 bytes each from one module, getting the response from the other one and registering the number of received packets in each direction. The target was to get the maximum range possible with zero or almost no PER\(^{83}\) at any given configuration. All the configurations had the central frequency equal to 2440 MHz and a signal BW of 406.25 kHz. The range was measured for SFs 5–8.

The second measurement stage took place on a relatively straight bank of the river Rhine (Figure 3.8). On that day, it was still winter and sunny, the temperature was about 10 \(^{\circ}\)C. It was performed at two distances only for SF 12, at BW = 406.25 and 812.50 kHz. The central frequency was the same as before. Even though LOS measurements were intended also at this site, in the end in reality there were some trees and possibly even some buildings between the modules. The results of both measurements are presented and discussed in Section 3.2.

### 2.2.3. UNWANTED EMISSIONS MEASUREMENT

Among the unwanted emissions originating from the operation of the radio module are harmonics: integer multiples of the desired frequency. The maximum power level of these emissions is regulated and for a device to pass the certification requirements, it must be below a certain level. In the EU, the spurious emissions of short range devices operating in the 1 GHz to 40 GHz range must be below \(-30\) dBm (ETSI, 2010, p. 31, Table 5).

However, the FCC\(^{84}\) in the USA has different regulations. Some of the harmonics of the 2.4 GHz band fall within the so-called "restricted-bands" (FCC, 2017, §15.205), therefore their power shall not exceed \(-41.25\) dBm (FCC, 2017, §15.209).

The cabinet measurements were performed in IMST’s RF anechoic chamber. A module was equipped with an artificial antenna (ETSI, 2010, p. 19, Section 6.2) and set in CW-mode at 2440 MHz and mounted onto a mobile arm in the chamber. During the measurement, the arm rotated in two orthogonal directions, so that a receiving device measured the radiated power over an entire sphere, with a resolution of 5 degrees in each of the two directions. The measurements were performed for the second, third, fourth and fifth harmonics.

The conducted emissions were not measured in an anechoic chamber but rather in a normal lab environment. A module’s RF output was connected to the input of a spectrum analyser. The module was then set in transmit mode at maximum power level at the following frequencies: 2401 MHz, 2441 MHz and 2482.5 MHz – the lower and upper boundaries of the spectrum of interest, as well as the middle point. The power of the 2\(^{nd}\) through 5\(^{th}\) harmonics was recorded.

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\(^{83}\) packet error rate

\(^{84}\) Federal Communications Commission
3. RESULTS AND DISCUSSION

3.1. RF PERFORMANCE

3.1.1. TRANSMIT POWER

![Graph showing measured and set transmit power comparison](image)

**Figure 3.1:** iM282A actual transmit power versus set transmit power. One curve for all the voltage and central frequency ($f_c$) settings. The vertical error bars (almost zero height) represent ±1 standard deviation. Measured at room temperature.

As can be seen from Figure 3.1, the module has good response characteristics regarding the transmitted power. The actual power barely exceeds the set one and it happens for less than half the available range. Usually it falls behind the setting, but by only a small amount: at most $-2.5$ dBm for the $-18$ dBm setting.

While this response is not ideal, the expected difference between the setting and the reality can easily be read from Figure 3.1. This way, if the exact power is of importance for a given application any of the available power settings can be achieved with a ±0.7 dBm difference in the worst-case. Such a level of accuracy would seldom find application in real-world though, where it is desirable to exceed the link budget by a good margin, ensuring that all the packets get through without the need to resend.
Nonetheless, an example comes to mind where the ability to set the transmit power that precisely would be useful: in a predictable environment, where the user is sure the losses are always the same, thus being able to use the system at the edge of the receiver's sensitivity. In such a scenario, the benefit of operating at the lowest possible power would mean transmission of data with the least current consumption possible. The merit of this approach will be revised later, after more information will have been presented.

Something else can be observed from Figure 3.1: the transmitted power is hardly influenced by the VDD or the frequency of operation. This means that no correction for any of these two factors has to be taken into account while transmitting at a certain level.

It must be noted, however, that the transmitted power is measured at the SMA connector of the adapter board (WiMOD_AB_01, V2.1). The results might be different if the module is soldered onto a board whose RF trace is much longer, has a different characteristic impedance, or is in some other way significantly different. Moreover, the actual transmitted power will be a function of the used antenna’s efficiency and directivity.

### 3.1.2. CURRENT CONSUMPTION

![Graph](image)

*Figure 3.2:* iM282A transceiver's current consumption versus set transmit power. The results are averaged over all the tested central frequencies. The vertical error bars represent ±1 standard deviation. Different curves for different voltage settings. Measured at room temperature.
Figure 3.2 demonstrates how the module’s current consumption varies with transmitted power and operating voltage. It can be seen that the current can be estimated for any given voltage and power level with a high degree of confidence. This and the fact that the transmit power can also be set accurately allows for high certainty while calculating the expected battery life of a module given its required range and data throughput.

A number of observations can be made from the graph. First, the current increases exponentially when the power is expressed in dBm. While this can trick one into thinking that operating at lower transmit power saves battery, it is likely not a correct conclusion. This aspect shall be examined later.

Second, the operating voltage has a strong influence on the current: distinct curves can be drawn for each voltage. This is to be expected, since operating at a higher voltage requires less current for obtaining the same power and vice-versa. It must be clarified at this point that all the measurements regarding current consumption and transmit power have been carried out with the TRX’s DC-DC converter enabled. If one were to use the LDO instead, the current would be the same for every set voltage. It would be about twice as large as the current recorded at 3.6 V. This was found out by experimenting with the power management settings: it seems that the TRX operates internally at 1.8 V. Furthermore, the DC-DC converter’s excellent efficiency can be deduced from Figure 3.3: the transmitter’s efficiency (i.e. radiated power over electric power) does not depend on the voltage, as long as it is within the module’s specification.

3.1.3. Transmitter Efficiency

Figure 3.3 is meant to shed some light on the previous allusions that lower transmit power levels might benefit the battery life. It clearly shows that in the case of iM282A the best conversion of electrical power into EM radiation and therefore useful signals is achieved with the highest transmit power levels. This occurs as a result of the power consumed by all the elements of the transmit chain except the PA itself. While the PA’s consumption is proportional to the output power and depends on the PA’s efficiency, the rest of the circuit’s consumption can be assumed to be constant for any power level.

This has implications for the final users’ optimal performance strategy. With the information available so far, it can be said that for achieving the longest battery life possible one should always transmit at the maximal allowed power, but choose the highest data-rate possible at the given range. This way the module has to spend less time on air for transmitting the same amount of data, therefore consuming less energy over a longer period of time. Obviously, at very short ranges when any data-rate can be used the power can be turned down as long as the received power is better than the module’s sensitivity.

At this point, some observations can be made regarding the performance of the module’s matching circuit. Even though not shown in results due to lack of thorough measurements, there were some other variants of the matching network, i.e. the optimization process discussed in Section 2.1.5. Their performance was inferior to the variant used

"power amplifier"
Figure 3.3: iM282A transceiver's transmit power efficiency. log–y scale. The transmitter efficiency is the ratio of measured transmit power in mW to the electrical power consumed by the TRX, also in mW. The results are averaged over all the tested central frequencies. The curves for different voltage levels completely overlap. The “Reference” curve shows the efficiency of the reference design (Semtech Corporation, 2016e, p. 22).

The reference design’s performance is not shown in Figures 3.1 and 3.2, but it can be found in the TRX’s datasheet (Semtech Corporation, 2016e, p. 22). Still, it is summarized in Figure 3.3, where the discrepancy in performance is most evident: while the iM282A’s transmitter efficiency peaks at 15.2 %, the reference design has a maximum efficiency of 22.4 %. The difficulties encountered during the impedance matching were discussed in Section 2.1.5. Here it shall only be stated that with better information future iterations of the impedance matching circuits have the chance to function much better than the current one, hence boosting the module’s RF-performance.
3.1.4. Radio Frequency Error

Figure 3.4: iM282A error in the radio frequency versus set transmit power. The results are averaged over all the tested central frequencies. The vertical error bars represent ±1 standard deviation. Different subplots for different voltage settings. Measured at room temperature.

Figures 3.4 and 3.5 show how the radio frequency is affected by various factors: set transmit power, voltage and central frequency. The following observations can be made: overall the frequency is very stable. Even though the TRX’s XTAL has a specified tolerance of ±10 ppm (Semtech Corporation, 2016e, p. 123), the variations of the RF are within 1 ppm of the set frequency. This is important for proper wireless communication: if the TRXs would have large discrepancy in the RF, they probably would not be able to communicate with each other, but instead interfere with adjacent channels.

Regarding the transmitting power, this is the factor with the largest contribution to the variation in the transmitted RF. While sweeping through the available power levels, a characteristic curve can be observed. It looks like a concave parabola and for any given voltage is just slightly shifted up– or downwards: Figure 3.4.
Figure 3.5: iM282A error in the radio frequency versus set central frequency. The results are averaged over all the tested voltages and transmit power levels. The vertical error bars represent ±1 standard deviation. Measured at room temperature.

Regarding the effect of the set frequency, it is lower in magnitude than that caused by the transmit power. Moreover, its shape is stranger, the RF tending to be ever lower than the one set with increasing frequency, with the exception of the upward “movement” between 2421 and 2431 MHz. No reasonable explanation could be found for any of these observations. In fact, before performing the measurements, it was assumed that the radio frequency would be the largest contributor to the resulting error.

It must however be mentioned that even though present, and apparently a function of transmit power among others, the frequency error is by far not critical. For instance, it was found that in the BW 406.25 kHz, the TRXs can have central frequencies set as much as 90 kHz apart and still be able to send and receive messages from one another. While a good thing for XTAL tolerances, this fact would probably have consequences for the minimum spacing between channels.

All the presented measurement results are based on a single module. It was later found that other modules had larger frequency errors: between 0 and −8 kHz for the highest power level. First, this is an indication of how unreliable results obtained from a single unit might be. Second, since all the detected frequency errors are negative, it can be concluded that the loading capacitance of the XTAL is too high and the problem can be solved in future module revisions by minimizing the stray capacitance at the XTAL’s pads.
3.1.5. Spurious emissions

Figure 3.6: iM282A spurious emissions at different central frequencies. The cabinet emissions were measured with 2 x AAA batteries, so VDD ≈ 3.0 V. Conducted emissions were measured at VDD = 3.3 V. The ETSI EN limit is shown at a continuous line at −30 dBm. The FCC restricted bands - as an interrupted line at −41.25 dBm. All measurements done with transmit power set to maximum; room temperature.

Figure 3.6 presents the iM282A’s unintended radiation. Initial measurements in a broader spectrum have been made, but no emissions distinguishable from the noise level have been detected except the harmonics of the carrier frequency, therefore only these are shown here. It can be seen from the Figure that the iM282A can easily pass the European regulations. Concerning the FCC’s regulations – the module can pass them as well, since all the recorded values that fall into a restricted band are below the limit value.

Nonetheless, as the example with the RF error demonstrated, results obtained from a single module are not representative and should not be taken as an absolute truth. Evidently more measurements will have to be done to fully assess the module’s spurious emissions. Still, the current results are promising. All the measurements so far seem to agree that the second harmonic is the trickiest one. In further revisions of the module special care should be taken that unwanted emissions are completely avoided. This is a fundamental requirement for having a successful certification procedure.
3.2. RANGE PERFORMANCE

The methods used while measuring the range performance of the iM282A module were described in the Section 2.2.2. Figure 3.7 shows the result for the measurements performed at Gerade Straße in Kamp-Lintfort. For the 406.25 kHz BW, the SF5, which corresponds to 50.78 kbps, offered just above 300 m of range. The signal modulated with SF8, corresponding to 10.15 kbps, could be received up to a distance of 660 m for an almost perfect LOS environment. The PERs registered during these tests were between 0 and 2 %. While normally the pass criterion is at most 1 % PER, these results are also accepted and the larger errors are attributed to the operator’s constant movement due to cold and cars on the street.

Source: Satellite image captured in Google Earth software

Figure 3.7: Results for the 1st set of range measurements.

For the range measurements on the bank of river Rhine, only SF12 was tested in the 406.25 kHz (0.95 kbps) and 812.5 kHz (1.90 kbps) BWs. For the 812.5 kHz BW, unsatisfactory results were obtained already for a distance of 1300 m. 27 % of the packets sent by the master module (downlink) were not received by the slave and 8 % of the packets sent by the slave (uplink) were not received by the master.

At the time of the measurements, another matching network for the iM282A module was being developed at IMST. It differs both in the number and placement of the matching elements from the one whose performance is presented in Section 3.1. Unfortunately, no other measurements involving the new matching have been performed, so it is not yet characterized, therefore no other mention about it is made in the present thesis. Nonetheless, with a module that featured the matching circuit acting as a master and no change in the slave device, a reduction of PER to 15 % for the downlink and 7 % for the uplink respectively has been achieved.
For the 406.25 kHz BW, the results were better. At 1300 m, the PER was 0 % downlink and 1 % uplink. However, at 1800 m, 12 and 15 packets out of every hundred were lost for downlink and uplink respectively. This is rather unfortunate, since better range was expected. For instance, Semtech Corporation (2016b, p. 6) reported a range of 3.6 km in non-line-of-sight conditions and speculated that over 4 km are possible in LOS. These results have been obtained by using the SF12 and the BW equal to 203.125 kHz (0.47 kbps). Unfortunately, they could not be replicated, given that for some reason the iM282A does not seem to receive packets with the BW 203.125 kHz.

There is one more impediment to finding out why the results of this thesis are so far away from the expected ones. No sensitivity measurements of the module have been performed to date, so it cannot be known with certainty whether the modules are not as sensitive as in the manufacturer’s reference design (Semtech Corporation, 2016e, p. 37) or whether the cause is external, i.e. antennas and the environment.

Wendt, Volk, and Mackensen (2015, p. 3) managed to transmit data with PER less than 20 % at 9.75 km LOS. They used the SF12 on a CSS-enabled long–range radio module for 2.4 GHz. According to that module’s fact sheet (EM Microelectronic, 2014), the BW is fixed to 812.5 MHz. This is hard to believe, since the whole SRD band at 2.4 GHz is only 83.5 MHz wide. Probably 812.5 kHz is meant. Nonetheless, it is clear that better performance can be obtained. One way to achieve this is likely through the redesign of the impedance matching network of the iM282A module.

86 The transmitted waveform was checked with a network analyser and no problems could be detected.
3.3. Sensitivity Analysis of the GCPWG

The reason to perform a sensitivity analysis of the GCPWG’s characteristic impedance was to study how a change in each of the defining parameters would influence the result. Each of the parameters was varied by $\pm 15\%$ and the corresponding change in the calculated characteristic impedance was noted down. During the variation of each parameter (width of the trace, separation from ground, height of dielectric and relative dielectric permittivity), the other three were kept constant, so only the influence of the parameter of interest was recorded.

![Figure 3.9: GCPWG's $Z_0$ sensitivity analysis. Small variation of parameters. Both axes represent relative change of the depicted quantities.](image)

Figure 3.9 demonstrates each parameter’s influence on the GCPWG’s characteristic impedance. It can be seen that varying the trace width influences the resulting impedance the most and varying the trace’s separation from the GND plane influences it the least.

Nonetheless, a $\pm 15\%$ variation in any parameter leads to at most $\pm 10\%$ variation in the resulting characteristic impedance. This is an important result, showing that the waveguide has an almost constant impedance for most practical variations in material properties and production equipment’s mechanical performance.

The curves in Figure 3.9 are almost straight and can be modelled as linear functions. Consequently, their analysis can be very simple. But this apparent linearity is only a local effect. When larger variations than those in Figure 3.9 are considered, it becomes clear that these functions are not linear, an effect presented in Figure 3.10.
According to the theoretical model used to analyse the GCPWG (Appendix E.1) and to the sensitivity analysis, a correct characteristic impedance was achieved (with some tolerable variability). However, there are some reasons not to trust this result and question whether the achieved variability is indeed acceptable.

For instance, Wadell (1991, p. 80) specifies that the height of the dielectric material must be much larger than the width of the trace plus the separation from GND on both sides in order to avoid microstrip line mode of operation. In the case of the GCPWG used in the current work, this requirement was not taken into account, so it is very possible that the GCPWG functions as a microstrip. While this is not a problem in itself, there is a different model governing the operation of a microstrip, which yields a different characteristic impedance (9% higher) for the parameters used here. Even more, no hybrid models which would correctly deal with the present situation were found.

Another reason to question the correctness of the calculation used here is that the module has a metal shielding which is connected to GND, therefore it is not just a grounded coplanar waveguide that is obtained, it is a shielded grounded coplanar waveguide, for which there is yet another model for impedance calculation (Wadell, 1991, pp. 80–81). Nonetheless, due to the scale of the shielding relative to the GCPWG, the characteristic impedance of the waveguide should deviate by no more than ±1.5% from the calculated one (Wadell, 1991, p. 80).
Finally, even though there are reasons to question the calculated characteristic impedance of the waveguide, the discrepancy between different models is not high enough to lead to serious deviation of reality from the calculated values. This is further confirmed by the adequate RF performance of the iM282A module.

4. SUMMARY AND OUTLOOK

The present thesis described the hardware design process of the iM282A – a long-range low-power radio module for the 2.4 GHz SRD band. Even though specific circuits and component values were often omitted, the principles used in the design phase of the PCB were discussed at length. A strong accent was put on the RF part of the module. Impedance matching and transmission lines were tackled from theoretical and practical perspectives. The performance of the matching circuit was measured indirectly, while the viability of the implemented transmission line was subjected to a sensitivity analysis.

Other PCB design issues discussed in the current paper are proper ICs’ decoupling, adequate XTAL choice guidelines, PCB construction, layer assignment, vias’ usage and general best–practices regarding routing. When appropriate, trade-offs between various design targets were highlighted. Minor sustainability issues were also considered while designing the module’s PCB.

In the RF domain, the iM282A relies on Semtech’s SX1280 transceiver. This, in turn, uses a chirp spread spectrum (CSS) modulation technique (LR24™) to achieve long ranges with little power consumption. Since not a merit of the current thesis, CSS was only discussed at a very basic level and the actual modulation scheme was not explained. Nonetheless, Appendix B discusses CSS signals’ representation and Appendix C provides an in–depth discussion of these signals’ orthogonality. This has implications for channels’ coexistence, which directly affects the throughput capacity and spectral efficiency of LR24™. That discussion can likely be extended to other CSS modulation schemes.

The iM282A also has a time-of-flight (ToF) distance estimation feature, which relies on LR24™ and is implemented in the SX1280 TRX. The principle used and its dissimilarity to a more popular ToF ranging scheme were presented. Nonetheless, no evaluation of this feature’s performance took place.

The present thesis presented the design of the first version of iM282A. Due to its sub-optimal range performance and slightly elevated current consumption, it is likely that another revision of the module will be necessary. Nonetheless, the current implementation served as a good first prototype and learning platform for future iterations. It is considered that the module’s power requirements and attainable range can be greatly improved, reaching at least the capabilities of the manufacturer’s reference design.

The assessment of a module’s "long–range, low–power" performance can be ambiguous. It is therefore desired to implement a metric for its quantification. A good starting point
could be Wendt et al.’s “milliJoules per byte” measure (2015). However, "milliJoules per byte and kilometre" for different environments (urban, forest, field) would be more useful for quantifying also the attainable ranges. This metric could be an indispensable decision-making tool while selecting an appropriate technology for a given application.

Appendix A presents a simple model for evaluation of a radio module’s power consumption and respectively — expected battery life. Such a calculation is a must while choosing a wireless solution and should be used in conjunction with the previously mentioned metric.

The 2.4 GHz radio band offers a number of advantages: broader bandwidths available and lack of duty-cycle limitations compared to sub–GHz SRD bands, resulting in larger data–throughput. Also, the permitted radiated power is larger than in some sub–GHz bands. The present module was developed to take advantage of these benefits. Nonetheless, it must also accept the cost of operating at a higher frequency: increased path loss. Also, since the 2.4 GHz band is so attractive, there are more devices and protocols operating there, so the module has to accept more interference. Both these effects result in lower ranges compared to other CSS schemes in lower frequency bands.

Overall, it can be seen that the developed module has both advantages and disadvantages when compared to existing technologies. Even though there is still work to do, when brought to its full potential the module will present a viable wireless solution for low–power, long–range, medium data–rate applications. Transmission of environmental and industrial monitoring data could well be such an application, given that the combination of data–rate and attainable ranges fit the module’s specification. Combined with its ToF ranging capability, the iM282A becomes attractive to mobile nodes, where both data transmission and localization could be achieved with one chip, even indoors. The range of the module’s feasible applications is yet another aspect that will need further research.
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Appendices
A. Battery Life Estimation

The radio module is expected to be used in battery powered applications. Batteries’ energy storage capacity (the work that they can do) is finite, and therefore a limiting factor on a battery’s useful life. The energy storage capacity is usually expressed in mAh (milliampere-hours) and a capacity of 1000 mAh means that the given battery can supply 1000 mA for 1 hour, 100 mA for 10 hours, or 1 mA for 1000 hours, and so on. It can be seen that a way to optimize the useful life of an application is to lower its power consumption.

The following equations provide a method to estimate the useful life of a battery depending on the application’s current consumption.

Generally, power \( P \) is the rate at which work \( W \) is done (Tipler & Mosca, 2008, p. 186):

\[
P = \frac{dW}{dt} \tag{A.1}
\]

Therefore, work can be calculated as the integral of power over time \( t \):

\[
W = \int_{0}^{t'} P \, dt, \tag{A.2}
\]

where \( t' \) represents the total time considered. When the total time is divided into intervals with constant \( P \), the integral can be expressed as a sum:

\[
W = \sum_{i=1}^{N'} P_i \Delta t_i, \tag{A.3}
\]

where \( N' \) is the number of constant-\( P \) intervals, and \( P_i, \Delta t_i \) are the respective intervals’ power and duration. When these intervals are periodic, e.g. a radio module sleeping for 95% of the time, then receiving for 4% of the time and transmitting for the remaining 1%, while the period of this cycle is much smaller than the total time considered \( (T << t') \), the work can be expressed as the following sum:

\[
W = t' \sum_{i=1}^{N} P_i x_i, \tag{A.4}
\]

where \( N \) is the number of states considered (e.g. transmit, receive, sleep, etc.), and \( P_i, x_i \) are the respective states’ power and relative duration \( (x_i = \frac{\Delta t_i}{T}, \forall i \in \{1, 2, \ldots, N\}, x_i \in [0, 1] \) and \( \sum_{i=1}^{N} x_i = 1 \)). The power \( P \) can now be expressed as the product of current \( I \) and voltage \( V \):
\[ W = t' \sum_{i=1}^{N} V_i I_i x_i \]  

(A.5)

Considering that the voltage provided by a battery is almost constant and that the energy stored in a battery (the work it can do) is its capacity times voltage, the following can be obtained:

\[ \text{Capacity } Y = t' Y \sum_{i=1}^{N} I_i x_i \]  

(A.6)

Since the variable of interest is the battery’s life, the previous equation is rearranged to obtain:

\[ t' = \frac{\text{Capacity}}{N \sum_{i=1}^{N} I_i x_i} \]  

(A.7)

where battery’s duration \( t' \) is measured in hours, the \text{Capacity} in mAh, \( I_i \) in mA and \( x_i \) is a dimensionless quantity.

**Figure A.1:** Potential battery usage diagram.
B. Mathematical representation of CSS signals

Various authors choose to represent chirped signals in different ways. Sometimes it is just a matter of notation, other-times it is about including or excluding terms. For example, three different representations\(^{87}\) are given in the Equations B.1–B.3.

Berni and Gregg (1973, p. 749) chose the following form:

\[
x(t) = \cos \left( \omega_0 t + \frac{1}{2} \alpha t^2 \right),
\]

(B.1)

where \(\omega_0\) is the initial angular frequency and \(\alpha\) is the chirp rate.

Kaminsky and Simanjuntak (2005, p. 896) preferred the following representation\(^{88}\):

\[
x(t) = e^{2\pi j t \left( \left( f_{\text{max}} - f_{\text{min}} \right) t/T_s + f_{\text{min}} \right)},
\]

(B.2)

where \(j\) is the imaginary unit, \(\pi = 3.14159\ldots\), \(f_{\text{max}}\) and \(f_{\text{min}}\) are the frequencies between which the chirping is taking place and \(T_s\) is the period of a chirp – the time it needs to swipe from \(f_{\text{min}}\) to \(f_{\text{max}}\) for an up-chirp and vice-versa for a down–chirp.

Ouyang and Zhao (2016, p. 3948) chose the following alternative form:

\[
x(t) = e^{j \left( \pi \alpha t^2 + \varphi_0 \right)},
\]

(B.3)

where \(\varphi_0\) is the initial phase of the signal.

Berni and Gregg’s representation is simpler than the other two in that it considers only the real part of the signal. On the other hand, Berni and Gregg & Kaminsky and Simanjuntak provide a starting frequency, while Ouyang and Zhao make use of an initial phase.

Another difference is that Kaminsky and Simanjuntak express the chirp rate (\(\alpha\)) and the starting frequency (\(f_0\)) through tangible quantities, specific to their application, while the other two studies keep them general.

While there are obvious differences between the forms presented so far, they are essentially describing the same phenomenon: an oscillating signal with a linearly increasing/decreasing frequency. The choice to use one form of another, to keep things general or be specific and so forth was likely influenced by the application of the study. Probably, the authors also chose the simplest form that still exhibited the properties of interest.

\(^{87}\)In the original sources, the notation was different, but it is harmonized here, in order to avoid confusion.

\(^{88}\)Only the up-chirp is presented, the down-chirp is analogous.
In the context of the present thesis the following form is used:

\[ x(t) = e^{2\pi jt (f_c \pm BW (t/T_s \text{ mod } 1) - 1/2)}, \forall t \geq 0, \quad (B.4) \]

where the "±" is a "+" for an up–chirp and a "−" for a down–chirp. The frequency of this signal can be expressed as follows:

\[ f(t) = f_c \pm BW ((t/T_s \text{ mod } 1) - 1/2), \forall t \geq 0 \quad (B.5) \]

For the purposes of this thesis, the symbol period \( (T_s) \) is defined (Semtech Corporation, 2016e, p. 31) as:

\[ T_s = 2^{SF}/BW, \quad (B.6) \]

where \( SF \) is the signal’s spreading factor and \( BW \) is its bandwidth. The representation given in Equation B.4 has the advantage that the frequency is always contained between \( f_{\text{min}} = f_c - BW/2 \) and \( f_{\text{max}} = f_c + BW/2 \). In the case of an up-chirp, the frequency is increasing from \( f_{\text{min}} \) to \( f_{\text{max}} \) after which it is wrapped back to \( f_{\text{min}} \) and this process continues indefinitely. For a down-chirp, the opposite is true.

This is a crucial difference between the notation used in this thesis and those used by the three studies introduced so far, in that it represents the actual behaviour of the chirps used. Figure B.1 depicts the frequency of an unmodulated\(^{89}\) signal consisting of up-chirps which is correctly represented by Equation B.4.

---

\(^{89}\)Unmodulated chirps carry no useful information and just sweep from one frequency to another. Modulated chirps encode useful information by "jumps" in frequency while sweeping.
C. Orthogonality of CSS Signals

The spreading factor plays a central role in CSS modulation systems. In combination with the bandwidth, it dictates the symbol period \( T_s \), which in turn is a key factor in such a system’s performance: All other things being equal, the larger \( T_s \) – the better the sensitivity and the lower the data-rate.

According to Semtech Corporation (2016e, p. 30), "different spreading factors are orthogonal to each other". In the present section, this claim is examined by first looking at the definition of orthogonality and then seeing how it applies to CSS-systems.

Generally speaking, "orthogonal" means uncorrelated, independent. The property of orthogonality is often discussed for vectors. Two vectors are said to be orthogonal if their inner (dot) product is zero. The same idea can be extended to signals. This is a useful concept, since orthogonal signals can share a channel and not interfere with each other, i.e. they can be correctly demodulated by their intended receivers.

Proakis and Salehi (2008, p. 26) denote the dot product of two complex-valued signals \( x_1(t) \) and \( x_2(t) \) by \( \langle x_1(t), x_2(t) \rangle \), for which they give the following definition:

\[
\langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{+\infty} x_1(t) x_2^*(t) \, dt,
\]

where the \( x_2^*(t) \) denotes the complex conjugate of the \( x_2(t) \). Proakis and Salehi (2008) further explain that in order for the two signals to be orthogonal their inner product has to be zero. However, Cook (1974, pp. 471–472) argues that this definition refers to strict orthogonality between signals, which often is not achieved in practice. Furthermore, Cook (1974, p. 472) provides a looser definition of orthogonality:

\[
\left| \frac{1}{T} \int_{0}^{T} x_1(t) \, x_2^*(t) \, dt \right| \leq \epsilon,
\]

where \( \epsilon \) is a measure of acceptable interference between the signals \( x_1 \) and \( x_2 \). Since the two signals are periodic, it makes more sense to integrate just over a period \( T \), rather than from minus to plus infinity. Dividing the result by the given period and taking its absolute value normalizes it to the interval \([0, 1]\), where a value of 0 represents strict orthogonality between the signals and a value of 1 - strict correlation between the two.

In the context of two LR24\(^{TM}\)-modulated signals operating at the same central frequency \( f_c \), it was found that strict orthogonality is unachievable: the integration over the signal period \( T_s \) and the subsequent normalization results in a value that is larger than zero. This does not disprove Semtech’s claim, but it shows that the orthogonality meant is not strict: a small interference between the different SFs is taking place but is still acceptable.
One issue that is not mentioned by Semtech Corporation (2016e), but discussed at length by Champion and Sornin (2016), is that while different spreading factors lead to orthogonal signals when the bandwidth is constant, this condition is not always satisfied when the bandwidths of the signals are also varying. Specifically, given the eight possible SFs and the five possible BWs, there are 40 possible combinations of the two and this section will try to highlight the combinations which are not mutually orthogonal.

For the further discussion, it is helpful to reintroduce the chirp rate (\(\alpha\)). It measures how quickly the signal changes its frequency, i.e. \(\alpha = \frac{\Delta f}{\Delta t}\). Since only linear chirps are considered, \(\alpha = \frac{\Delta f}{\Delta t}\). Given the notions introduced so far and the Equation B.6, the chirp rate can be expressed as follows:

\[
\alpha = \frac{BW}{T_s} = \frac{BW^2}{2^{SF}} \tag{C.3}
\]

The limited set of valid spreading factors and bandwidths leads to a limited number of possible chirp rates. Table C.1 lists the chirp rates obtained by combining the allowed SFs and BWs. The results are normalized by division to the minimum chirp rate in order to make them more readable.

Table C.1: Possible chirp rates, normalized to the minimum. Cell's background helps distinguishing equal chirp rates.

A couple of observations can be made based on the Table C.1. Firstly, all the possible chirp rates are integer multiples of \(\alpha_{\text{min}}\). Secondly, these integers are all powers of two. Thirdly, for any given BW, the chirp rate for every SF is unique. Fourthly, the chirp rates are not unique when multiple BWs and SFs are considered.

The first two observations follow from these facts: (1) the different BWs are powers-of-two-multiples of \(BW_{\text{min}}\); (2) the signal periods are defined as ratios of powers-of-two of SFs to these BWs and (3) the chirp rates are ratios of the said BWs to symbol periods \(T_s\).

\(^{90}\)Unless otherwise specified, from this point on all the discussion is about non-strict orthogonality, i.e. as explained in Equation C.2.
The third observation is what makes the different SFs orthogonal. This is further demonstrated graphically in Figure C.1. It can be seen that the three signals having different SFs do have the same frequencies only during times which are very short compared to their periods. Therefore, the three presented signals are orthogonal.

![Graphical representation of three mutually orthogonal CSS signals](image1)

**Figure C.1:** Three mutually orthogonal CSS signals. These up-chirps have the same bandwidth \( (BW = \text{kHz}) \) and central frequency, but different spreading factors.

The fourth observation is the one interesting for the research of orthogonality between signals that are different combinations of SFs and BWs. It shows that there are signals with different SFs which will significantly correlate with each other, therefore being far from orthogonal. A particular example is (SF=12, BW=406.25 kHz) v. (SF=10, BW=203.125 kHz), depicted in Figure C.2. Both of these combinations have the same chirp rate:

![Graphical representation of two mutually non-orthogonal CSS signals](image2)

**Figure C.2:** Two mutually non-orthogonal CSS signals. The green (dashed) line is overlapping the red (continuous) one. These signals have the same central frequency, but different spreading factors and bandwidths.

The same observation can be made about most other chirp rates.
In order to calculate the dot product between all the combinations of SFs and BWs, it is necessary to evaluate the following expression:

$$\langle x_1(t), x_2(t) \rangle' = \frac{1}{T} \int_0^T x_1(t)x_2^*(t)dt$$

$$= \frac{1}{T} \int_0^T e^{2\pi j t (f_1(t) - f_2(t))} dt,$$  \hspace{1cm} (C.4)

where \( \langle \rangle' \) notation is used to differentiate from Equation C.1, stressing that the integration limits are different and the result is divided by the difference of these limits. \( f_1(t) \) and \( f_2(t) \) are defined by the following equations:

$$f_1(t) = f_c + BW_1 \left( \left( \frac{t}{T_{s1}} - \frac{1}{2} \right) \right.$$

$$\left( \frac{t}{T_{s1}} - \frac{1}{2} \right) \right) \mod 1 - \frac{1}{2} \right) \right) \right) \right) \right) \right)$$ \hspace{1cm} (C.5)

$$f_2(t) = f_c + BW_2 \left( \left( \frac{t}{T_{s2}} - \frac{1}{2} \right) \right.$$

$$\left( \frac{t}{T_{s2}} - \frac{1}{2} \right) \right) \mod 1 - \frac{1}{2} \right) \right) \right) \right) \right)$$ \hspace{1cm} (C.6)

These are based on Equation B.5, but differ in some aspects: the "±" was replaced by a "+" in order to consider only up-chirps and another \(-\frac{1}{2}\) term was added to ensure that all signals start at \( f_c \). The real and imaginary parts of the Equation C.4 can be expressed as follows:

$$Re \left\{ \langle x_1(t), x_2(t) \rangle' \right\} = \frac{1}{T} \int_0^T \cos \left( 2\pi t \left( f_1(t) - f_2(t) \right) \right) dt$$ \hspace{1cm} (C.7)

$$Im \left\{ \langle x_1(t), x_2(t) \rangle' \right\} = \frac{1}{T} \int_0^T \sin \left( 2\pi t \left( f_1(t) - f_2(t) \right) \right) dt$$ \hspace{1cm} (C.8)

A first brute-force approach was to evaluate the real and imaginary parts of the Equation C.4 numerically using the trapezoidal rule. The results were unsatisfactory, on one hand taking too much time to compute due to the large degree of discretization required and on the other hand resulting in unacceptably large relative errors.

The later, more careful approach consisted of analysing a number of possible cases and devising specialized solutions, rather than generic ones. It is presented in the following paragraphs.

First, it must be noted that the frequencies \( f_1(t) \) and \( f_2(t) \) of both signals are not continuous functions (Figure C.1) even for a single symbol period \( (T_s) \), due to their wrapping and bounding to the BW. Therefore the integral C.4 will be evaluated at smaller intervals \( (T_{int}) \) where both frequencies are continuous. The integration period \( (T_{int}) \) was found to be \( \frac{1}{2} \) of the smallest symbol period of the two signals.

Next, for each integration period there are a number of cases of what \( f_1(t) - f_2(t) \)
evaluates to. First, the easiest case is when the frequencies are equal, so their difference is zero (both signals have the same chirp rate, e.g. Figure C.2: first and fourth quarters of the time). In the second case the frequencies are changing at the same rate, so their difference is a constant (the signals have the same chirp rate, but one of them has a higher BW than the other, e.g. Figure C.2: second and third quarters of the time). The last case is when the frequencies are changing at different rates, so their difference is a function of time (the signals have different chirp rates, e.g. Figure C.1).

The first case is the easiest to calculate:

$$\langle x_1(t), x_2(t) \rangle' = \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} x_1(t)x_2^*(t)dt$$

$$= \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} e^{2\pi jt} (f_1(t) - f_2(t)) \ dt$$

$$= \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} e^{2\pi j(0)} \ dt$$

$$= \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} 1 \ dt$$

$$= \frac{1}{T_{\text{int}}} (T_{\text{int}} - 0)$$

$$= 1 + 0j \quad (C.9)$$

This result makes perfect sense, since it means that the signals are perfectly correlated at these intervals, which is in fact the case. The two signals have the same frequencies at any given time during the considered intervals. For the second case, the constant frequency difference \(a\) is introduced. The integral can then be calculated as follows:

$$\langle x_1(t), x_2(t) \rangle' = \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} x_1(t)x_2^*(t)dt$$

$$= \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} e^{2\pi j t} (f_1(t) - f_2(t)) \ dt$$

$$= \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} e^{2\pi j(0)} \ dt$$

$$= \frac{1}{T_{\text{int}}} (\int_0^{T_{\text{int}}} \cos(2\pi at) dt + \int_0^{T_{\text{int}}} j \sin(2\pi at) dt)$$

$$= \frac{1}{2\pi a T_{\text{int}}} \left( \sin(2\pi a T_{\text{int}}) - \sin(0) - j \cos(2\pi a T_{\text{int}}) - \cos(0) \right)$$

$$= \frac{\sin(2\pi a T_{\text{int}}) + j \cos(2\pi a T_{\text{int}})}{2\pi a T_{\text{int}}}$$

$$= 1 + 0j \quad (C.10)$$
The third and last case is the most challenging. Calculating it requires solving the following integral:

\[
\langle x_1(t), x_2(t) \rangle' = \frac{1}{T_{int}} \int_0^{T_{int}} x_1(t)x_2^*(t)dt
\]

\[
= \frac{1}{T_{int}} \int_0^{T_{int}} e^{2\pi j t(f_1(t) - f_2(t))} dt
\]

\[
= \frac{1}{T_{int}} \int_0^{T_{int}} e^{2\pi j t(a(\omega t + d))} dt ,
\]

(C.11)

where \(a, \omega \& d\) are some constant coefficients. This integral cannot be expressed in terms of elementary functions. However, it can be formulated as a linear combination of Fresnel integrals. The Fresnel Cosine \(C(z)\) and Fresnel Sine \(S(z)\) integrals are defined (Temme, 2016, Equations 7.2.7 & 7.2.8) as:

\[
C(z) = \int_0^z \cos \left( \frac{\pi}{2} t^2 \right) dt
\]

(C.12)

\[
S(z) = \int_0^z \sin \left( \frac{\pi}{2} t^2 \right) dt
\]

(C.13)

A simpler form of the Equation C.11 could be:

\[
\langle x_1(t), x_2(t) \rangle'' = \int_0^b e^{jat^2} dt ,
\]

(C.14)

where \(a \& b\) are general parameters. The real part of this Equation can be expressed as:

\[
Re \left\{ \langle x_1(t), x_2(t) \rangle'' \right\} = \int_0^b \cos(at^2) dt
\]

(C.15)

In order to express it as a \(C(z)\) integral, the following substitution can be applied:

\[
\frac{\pi}{2} u^2 = at^2
\]

\[
\frac{\pi}{2} u^2 = a \frac{\pi}{2} t^2
\]

\[
u = \sqrt{a \frac{\pi}{2} t}
\]

\[
\frac{du}{dt} = \sqrt{2a \frac{\pi}{2a}}
\]

\[
dt = du \sqrt{\frac{\pi}{2a}}
\]

(C.16)
Using the substitution from Equations C.16 & C.17, and calculating the new limits for integration, the integral from Equation C.15 becomes:

\[
\int_0^b \cos(at^2) \, dt = \int_0^{b\sqrt{2a/\pi}} \cos\left(\frac{\pi}{2}u^2\right) \sqrt{\frac{2a}{\pi}} \, du \\
= \sqrt{\frac{\pi}{2a}} \int_0^{b\sqrt{2a/\pi}} \cos\left(\frac{\pi}{2}u^2\right) \, du \\
= \sqrt{\frac{\pi}{2a}} \left( b \sqrt{\frac{2a}{\pi}} \right) \tag{C.18}
\]

The derivation of the imaginary part of the Equation C.14 is analogous and the result is:

\[
\text{Im} \left\{ \langle x_1(t), x_2(t) \rangle'' \right\} = \text{sgn}(a) \sqrt{\frac{\pi}{2a}} S \left( b \sqrt{\frac{2a}{\pi}} \right), \tag{C.19}
\]

where \( \text{sgn}(x) \) is the sign function, which returns \(-1\) when \( x < 0 \), \(0\) when \( x = 0 \) and \(1\) when \( x > 0 \). These results are rather important, since Fresnel integrals can be evaluated very rapidly by computers. Unfortunately, the obtained integrals can only be expressed as shown in Equation C.11, but not in the form presented in Equation C.14, so the result in Equation C.18 cannot be used directly. Nonetheless, expressing the results of this Appendix in the form of Fresnel integrals is very desirable, since it would greatly reduce the computation complexity of the problem at hand. With this in mind, a series of C.11-like Equations were tested in WolframAlpha\(^92\) and the results helped in deducing that:

\[
\begin{align*}
\int_0^b \cos(atwt + d) \, dt &= -ui \left( h \left( C(|d|q) - uC(fq) \right) + g \left( S(|d|q) - uS(fq) \right) \right) \tag{C.20} \\
\int_0^b \sin(atwt + d) \, dt &= -vi \left( g \left( C(fq) - uC(|d|q) \right) + h \left( uS(|d|q) - S(fq) \right) \right), \tag{C.21}
\end{align*}
\]

where:

\[
\begin{align*}
u &= \text{sgn}(wd), \tag{C.22} \\
v &= \text{sgn}(wa), \tag{C.23} \\
i &= \sqrt{\frac{\pi}{2aw}}, \tag{C.24} \\
q &= \sqrt{\frac{a}{2\pi w}}, \tag{C.25} \\
f &= (2b + |d|)|w| - (|w| - u)|d|, \tag{C.26} \\
g &= \sin \left( \frac{ad^2}{4w} \right), \tag{C.27} \\
h &= \cos \left( \frac{ad^2}{4w} \right), \tag{C.28} \\
a, b, d, w &\in \mathbb{R}^* \quad \text{and} \quad b > 0 \tag{C.29}
\end{align*}
\]
\(^92\)url: https://www.wolframalpha.com/
Now that each of the three cases can be evaluated, a short Program (C.1) was written to calculate the inner product (as defined in Equation C.4) between each of the 40 possible combinations of spreading factors and bandwidths. Moreover, the same calculation was performed for all the combinations evaluated by Champion and Sornin (2016, p. 5). This step was necessary in order to make sense of the results: Champion and Sornin have labelled the non-orthogonal combinations, implying that the rest are orthogonal.

In Figure C.3 both the results for LR24™ and Champion and Sornin’s considered combinations are presented. As a way to differentiate orthogonal combinations from non-orthogonal ones, a circle centred at the origin was drawn with the radius equal to Champion and Sornin’s largest absolute value of an orthogonal combination. It is assumed that any two signals with a dot product’s absolute value falling within this circle is orthogonal. Another larger circle with the radius of the smallest absolute value of a non-orthogonal combination originating from Champion and Sornin’s data was drawn. The assumption is that any two signals whose dot product falls outside this circle are not orthogonal.

It can be seen that a number of points belonging to LR24™ combinations fall between these two circles, therefore in the “uncertainty” region. In the current thesis, no attempt was made to verify whether (a) The circles are correct indications of orthogonal versus non–orthogonal regions and (b) Where between the two circles could a more decisive one be drawn, thus eliminating the uncertainty region. Accomplishing (b) would mean finding the $\epsilon$ that would satisfy Cook’s orthogonality principle (Cook, 1974; Equation C.2). Still, for the purposes of this study, it is assumed that combinations of signals having their inner product less than 0.25 are orthogonal.

As expected, all the signals have an inner product of $1 + 0j$ with themselves.
Furthermore, a number of signal pairs have an inner product of \(0.5 + 0j\). These combinations can be found in Table C.1 by taking any cell, then moving two cells to the right and one down. An example of such a pair is \((\text{SF12}, \text{BW}=406,250 \text{ kHz}) \text{ v. } (\text{SF10}, \text{BW}=203,125 \text{ kHz})\). Their frequencies over time are plotted in Figure C.2. Then there are combinations yielding \(0.25 + 0j\). These can be found in Table C.1 by choosing a cell, then moving four cells to the right two cells down. Next, there are a few pairs with a dot product of \(0.125 + 0j\). These can be found by taking any cell, then moving six cells to the right and three cells down.

The present discussion about orthogonality was not applied to any practical aspects yet. As mentioned in the beginning of this section, the orthogonality is important for signals’ coexistence in a channel. Respectively, it influences the maximum data-rate available in this channel. To show the importance of the work presented so far, the maximum data-carrying capacity of a 1625 kHz-wide channel used for LR24\textsuperscript{TM} modulation will be discussed next. Since it was found that unacceptable interference occurs between certain combinations of SFs and BWs, only one combination from each respective set of mutually correlated combinations will have to be chosen.

In order to make a choice, first the data-rate capacity of each combination of SFs and BWs must be calculated. The effective bit rate\textsuperscript{96} of an LR24\textsuperscript{TM}-modulated signal can be calculated using Equation C.30, which is based on the Equations from Semtech Corporation (2016e, p. 31).

\[
R_{\text{eff}}(\text{BW}, \text{SF}, \text{CR}) = \frac{4\text{SF}}{(4 + \text{CR})T_s} \text{ bit/s}, \quad (\text{C.30})
\]

where \(\text{CR}\) represents the coding rate - a characteristic of the forward error correction which is used to improve the robustness of the link. The \(T_s\) is a function of \(\text{SF}\) and \(\text{BW}\) and is calculated according to Equation B.6. For the sake of simplicity, a coding rate of 1 (one) is assumed for all the following calculations. Table C.2 contains the effective bit rate for each combination of SFs and BWs. Out of each set of mutually non-orthogonal combinations of SFs and BWs only one is chosen such that the total effective bit rate of the whole 1625 kHz-wide channel is maximized. The non-orthogonal combinations which are not summed up are struck through on a gray background.

Table C.2 demonstrates the effect of interference between various signals: out of the 40 possible combinations of SFs and BWs, only 20 can be used without causing any perceptible interference among the remaining signals. For the calculation of the total available bandwidth in a 1625 kHz channel, each row in Table C.2 is summed and multiplied by \(\frac{4\text{SF}}{(4 + \text{CR})T_s}\). That means that in the 1625 kHz channel allocated earlier there can coexist

\[
\begin{array}{cccccc}
\hline
\text{SF} & \text{BW} & \text{CR} & R_{\text{eff}} & \text{Available Bandwidth} \\
\hline
\text{SF12} & 406,250 & 1 & 3.33 & 625 kHz \\
\text{SF10} & 203,125 & 1 & 1.67 & 500 kHz \\
\hline
\end{array}
\]

The effective bit rate is introduced in (Semtech Corporation, 2016e, p. 31) and depends, besides BW and SF, on the Coding rate (CR).
The resulting total effective bit rate according to this calculation is 1804.639 kbit/s, which is just short of 75% of the maximum possible if no interference were to occur (2415.283 kbit/s). The current approach assumed no guard band between the 8 sub-channels in the 203.125 kHz bandwidth. In reality, it might be that a certain "distance" is necessary between the adjacent sub-channels, therefore reducing the total available channel capacity. Also, the calculation performed had the objective to maximize the total possible bit rate. One might, however, be interested in maximizing the range on the used systems. Then the decisions of which combinations to use and which not to use would be different, resulting in an overall lower total bit-rate, but in better range performance.
Program C.1: Calculation of two CSS signals’ inner product.

@author: Marcel Dogotari

from time import time
from scipy.special import fresnel
from numpy import pi, cos, sin, sign

def quasi_fresnel(a, b, d, w):
    if a==0 or b==0 or w==0:
        print "a, b and w cannot be zero!"
        return -1
    if d==0:
        S0, C0 = fresnel(b * (abs(2 * a * w) / pi) ** 0.5)
        p = (pi / abs(2 * a * w)) ** 0.5
        return S0 * p * sign(a * w), C0 * p

    """ The following function calculates integral(cos(a*t*(t+w+d))dt) from 0 to b and integral(sin(a*t*(t+w+d))dt) from 0 to b
    I hope to use it to calculate the inner product of chirped signals in their linear range, i.e. even when they are not starting at f=0.
    See https://www.wolframalpha.com/input/?i=integral(sin(5*x→*(3*x%2B7)))+from+0+to+17
    and https://www.wolframalpha.com/input/?i=integral(cos(5*x→*(3*x%2B7)))+from+0+to+17
    for reference.
    """

    """ The following expressions appear often in the calculations, therefore new variables are assigned to calculate their value only once. """
    u = sign(w * d)
    v = sign(w * a)
    b = abs(b)
    e = abs(a / (2 * pi * w)) ** 0.5
    f = (2.0 * b + abs(d)) * abs(w) - (abs(w) - u) * abs(float(d))
    j = abs(float(d))
    g = sin(abs(d ** 2 * a / 4.0 / w))
    h = cos(abs(d ** 2 * a / 4.0 / w))
    i = abs(pi / (2 * a * w)) ** 0.5

    """ The sine and cosine fresnel integrals with the same arguments appear each, so they are calculated once and two variables hold these results"""
    S1, C1 = fresnel(f * e)
    S2, C2 = fresnel(j * e)

    """ Variables to hold the final results """
    COS = - u * i * ((C2 - u * C1) * h + (S2 - u * S1) * g)
\[
\sin = v \cdot i \cdot ((C_1 - u \cdot C_2) \cdot g + (u \cdot S_2 - S_1) \cdot h)
\]

""" Return the result as 2 elements. (sin, cos). each can be accessed by the index 0 or 1 """
return SIN, COS

Bandwidths =
for i in xrange(len(Bandwidths)):
    Bandwidths[i] = Bandwidths[i]

SFactors = [12, 11, 10, 9, 8, 7, 6, 5]

counter = 0

fh = open("orthog_14.txt", "w")
start = time()
for BW_0 in Bandwidths:
    for BW_1 in Bandwidths:
        for SF_0 in SFactors:
            for SF_1 in SFactors:
                # if counter > 0:
                #     break

                Ts_0 = 2 ** SF_0 / BW_0
                Ts_1 = 2 ** SF_1 / BW_1

                SF_max = max(SF_0, SF_1)
                SF_min = min(SF_0, SF_1)

                Ts_max = max(Ts_0, Ts_1)
                Ts_min = min(Ts_0, Ts_1)

                real = 0
                cplx = 0

                T_int = 0.5 * Ts_min

                loops = Ts_max / T_int
                #print "T_int", T_int, "loops", loops

                a = 2 * pi
                b = T_int
                w = (BW_0 / Ts_0 - BW_1 / Ts_1)
                shift0 = 0.5 * Ts_0
                shift1 = 0.5 * Ts_1

                for k in range(int(loops)):
                    d = (BW_0 * shift0 / Ts_0 - BW_1 * shift1 / Ts_1)
                    #True chirp
if w != 0:
    temp = quasi_fresnel(a, b, d, w)
    cplx += temp[0]
    real += temp[1]
    # just linear freq. difference
elif d != 0:
    real += sin(2 * pi * T_int * d) / (2 * pi * d)
    cplx += (1 - cos(2 * pi * T_int * d)) / (2 * pi * d)
    # print real, cplx
    # same freq.
else:
    real += T_int
    cplx += 0
    shift0 += T_int
    if shift0 >= 0.5 * Ts_0: shift0 -= Ts_0
    shift1 += T_int
    if shift1 >= 0.5 * Ts_1: shift1 -= Ts_1
    real /= Ts_max
    cplx /= Ts_max
    magn = (real ** 2 + cplx ** 2) ** 0.5
    # optional, scale with sqrt(magn)
    real /= magn ** 0.5
    cplx /= magn ** 0.5
    magn = (real ** 2 + cplx ** 2) ** 0.5
    string = str(SF_0) +", " +
              str(BW_0) +", " +
              str(SF_1) +", " +
              str(BW_1) +", " +
              str(real) +", " +
              str(cplx) +", " +
              str(magn) +"\n"
    fh.write(string)
    counter +=1
print time()-start
fh.close()
D. Doppler shift resilience of CSS signals

If the transmitter and the receiver of a signal move relative to each other, the frequency received will be different from that which is sent. This phenomenon is known as the Doppler effect (Tipler & Mosca, 2008, p. 518). It can be easily seen that this phenomenon can disrupt communications when the transmitter, the receiver or both are mobile.

It is often claimed that CSS modulation techniques are resilient to the Doppler effect, i.e. they tolerate some mismatch in the sent and received frequency (Cook, 1974, p. 474; Yanjun et al., 2012, p. 621; Semtech Corporation, 2015a, p. 9). Since usually this affirmation is not backed up by numbers, the present Appendix attempts to do a rough estimation of how fast can a pair of iM282A modules move relative to one another and still be able to communicate.

In the case of EM waves, one should normally use the formula for the relativistic Doppler effect in order to achieve correct results. However, when the transmitter and/or the receiver move much slower than light, which is expected to be the case for any device carrying the iM282A, the approximation \( \frac{\Delta f}{f_s} \approx \pm \frac{u}{c} \) works well enough (Tipler & Mosca, 2008, p. 522). \( f_s \) is the transmitter’s frequency, \( \Delta f \) is the frequency difference, \( u \) is the relative speed of the transmitter to the receiver and \( c \) is the speed of light.

Using the previous formula it is easy to find the maximum relative speed at which two modules can move and still communicate with each other, by finding the largest frequency shift at which they can still demodulate each other’s messages. A pair of iM282A modules was placed in the same room, approx 3 m apart. They were set at the central frequency 2440 MHz in the 406.25 kHz bandwidth with the spreading factor 9. One module’s settings stayed constant, while the other module’s central frequency was being varied. It was found that the modules can communicate at frequency shifts up to 90 kHz. This corresponds to \( \approx \) 39 thousand km/h. The resulting speed is too high to even consider in which systems it may be a bottleneck. On one hand, no civilian land or air-borne vehicles move faster than the speed of sound \( (\approx 1.2 \text{ thousand km/h}) \) and on the other hand, if iM282A modules were to move that fast relative to each other, they would be out of each other’s range after less than 100 bytes of data.

Obviously, the simplistic assumptions here are likely wrong, by not considering also time shifts in the signal and how higher and lower frequencies of the same signal are affected differently. In real life, these effects might interrupt the communication already at much lower speeds. Moreover, the 90 kHz figure was obtained for a very specific set of parameters, and is probably different for various central frequencies, spreading factors and bandwidths. Nonetheless, the simple calculations presented in this Appendix help establishing a comparatively high upper bound on the relative velocities of two iM282A modules.
**E. GROUNDED COPLANAR WAVEGUIDE**

**E.1. CHARACTERISTIC IMPEDANCE CALCULATION**

Wadell (1991, pp. 79-80) presents a set of equations that can be used to calculate a grounded coplanar waveguide (GCPWG)’s characteristic impedance ($Z_0$). Equation E.1 gives the characteristic impedance:

$$Z_0 = \frac{60\pi}{\sqrt{\varepsilon_{eff}}\left(\frac{K(k)}{K(k')} + \frac{K(k_1)}{K(k_1')}\right)}$$  \hspace{1cm} (E.1)

It depends, however, on the result of the equations E.2-E.6.

$b$ is defined as the width of the trace plus the separation width on its both sides (see Figure 1.9).

$$b = w + 2s$$ \hspace{1cm} (E.2)

$k$, $k_1$, $k'$ and $k'_1$ are intermediary variables and are defined as follows:

$$k = \frac{w}{b}$$ \hspace{1cm} (E.3)

$$k_1 = \frac{\tanh\left(\frac{\pi w}{4h}\right)}{\tanh\left(\frac{\pi b}{4h}\right)}$$ \hspace{1cm} (E.4)

$$k'_1 = \sqrt{1 - k^2}$$ \hspace{1cm} (E.5)

The importance of the dielectric’s effective relative permittivity ($\varepsilon_{eff}$) is described in (Wadell, 1991, pp. 2-3). For a GCPWG, it is defined as follows:

$$\varepsilon_{eff} = \left(1 + \varepsilon_r \frac{K(k')K(k_1)}{K(k)K(k_1')}\right) / \left(1 + \frac{K(k')K(k_1)}{K(k)K(k_1')}\right)$$ \hspace{1cm} (E.6)

The function $K(k)$ represents the complete elliptic integral of the first kind of $k$. It is defined (Carlson, 2016, Equations 19.2.4 & 19.2.8) as follows:

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2(\theta)}}$$ \hspace{1cm} (E.7)
E.2. PYTHON IMPLEMENTATION

The following short function calculates a GCPWG’s characteristic impedance given its trace’s width, the separation from GND on the same plane, height of the dielectric material and its relative dielectric permittivity (Figure 1.9). The formulas presented in Appendix E.1 were used.

There are numerous approaches to numerically calculate the complete elliptic integral of the first kind \( K(k) \) (Lozier & Olver, 1994, Sec. 5.3.1). In the current work, the functions \( \text{ellipk}(k) \) and \( \text{ellipkm1}(k) \) from Python’s library Scipy were used. Unfortunately, no specification was found for their implementation. However, their results agree in all the presented decimal places with those from MATLAB and WolframAlpha and since the calculations in Python were almost instantaneous it was decided to just use them.

Program E.1: Calculation of a GCPWG’s characteristic impedance given the waveguide’s geometry and board’s material properties.

```python
    @author: Marcel Dogotari

    #Use "true division", rather than "floor division"
    #See https://www.python.org/dev/peps/pep-0238/
    from __future__ import division

    #Necessary functions and constants from the "math" library
    from math import tanh, pi

    #The library containing elliptical integrals
    from scipy.special import ellipk, ellipkm1

    """
    Function to calculate the characteristic impedance of a GCPWG.
    Uses equations from (Wadell, 1991, pp. 79–80)
    Input:
    s – separation between trace and GND
    w – width of trace
    h – height of dielectric material
    \( \varepsilon_r \) – relative permittivity of dielectric material
    s, w & h must be in the same units
    Output:
    \( Z_0 \) – characteristic impedance of GCPWG
    ""
    def imped(s,w,h,eps_r):
        a = w
        b = w + 2 * s

        #k, k_prime, k1 & k1_prime are intermediary variables
        k = a / b
        k_prime = (1 - (k ** 2)) ** 0.5
        k1 = tanh(pi * a / 4 / h) \n```
\[
\tanh(\pi \times b / 4 / h)
\]
\[
k_{1\text{prime}} = (1 - (k1 \times 2)) \times 0.5
\]

""

The following 4 equations all have the same structure:
If the k is not close to 1, use \texttt{ellipk}(k),
Otherwise: use \texttt{ellipkm1}(k-1)
It is done in order to achieve higher accuracy
""

\[
K_k = \text{ellipk}(k \times 2) \\text{ if } k < 0.5 \\text{ else } \text{ellipkm1}(1 - k \times 2)
\]
\[
K_{k\text{prime}} = \text{ellipk}(k_{\text{prime}} \times 2) \\text{ if } k_{\text{prime}} < 0.5 \\text{ else } \text{ellipkm1}(1 - k_{\text{prime}} \times 2)
\]
\[
K_{k1} = \text{ellipk}(k1 \times 2) \\text{ if } k1 < 0.5 \\text{ else } \text{ellipkm1}(1 - k1 \times 2)
\]
\[
K_{k1\text{prime}} = \text{ellipk}(k_{1\text{prime}} \times 2) \\text{ if } k_{1\text{prime}} < 0.5 \\text{ else } \text{ellipkm1}(1 - k_{1\text{prime}} \times 2)
\]

# Calculates the effective relative permittivity
\[
\text{eps\_eff} = (1 + \text{eps\_r} \times K_{k\text{prime}} \times K_{k1} / K_k / K_{k1\text{prime}}) / (1 + K_{k\text{prime}} \times K_{k1} / K_k / K_{k1\text{prime}})
\]

# Calculates the characteristic impedance, the result to be returned
\[
Z_0 = 60 \times \pi / (\text{eps\_eff} \times 0.5) / (K_k / K_{k\text{prime}} + K_{k1} / K_{k1\text{prime}})
\]

return Z_0
DECLARATION OF AUTHENTICITY

I, Marcel Dogotari, hereby declare that the work presented herein is my own work completed without the use of any aids other than those listed. Any material from other sources or works done by others has been given due acknowledgement and listed in the reference section. Sentences or parts of sentences quoted literally are marked as quotations; identification of other references with regard to the statement and scope of the work is quoted. The work presented herein has not been published or submitted elsewhere for assessment in the same or a similar form. I will retain a copy of this assignment until after the Board of Examiners has published the results, which I will make available on request.