

# Systematic assessment of isogeometric sheet metal forming simulations based on trimmed, multi-patch NURBS models in LS-DYNA

Christoph Hollweck<sup>1,2</sup>, Lukas Leidinger<sup>3</sup>, Stefan Hartmann<sup>3</sup>, Liping Li<sup>4</sup>, Marcus Wagner<sup>2</sup>, Roland Wüchner<sup>1</sup>

<sup>1</sup> Technische Universität Braunschweig, Institute of Structural Analysis  
Beethovenstraße 51, D-38106 Braunschweig, Germany

<sup>2</sup> Ostbayerische Technische Hochschule Regensburg, Laboratory Finite-Element-Methods  
Galgenbergstraße 30, D-93053 Regensburg, Germany

<sup>3</sup> DYNAmore GmbH, an ANSYS Company  
Industriestr. 2, D-70565 Stuttgart, Germany

<sup>4</sup> ANSYS  
7374 Las Positas Road, Livermore, CA 95411, USA

## 1 Introduction

Isogeometric sheet metal forming simulation is a promising numerical technique utilized for predicting the behavior of sheet metal parts during the forming process, aiming to establish a stronger connection with Computer Aided Design (CAD) descriptions. This approach combines the well-established framework of traditional finite element analysis (FEA) with the power of non-uniform rational B-splines (NURBS), known as isogeometric analysis (IGA). Unlike the conventional FEA framework, IGA directly employs the ansatz space of the CAD geometry for analysis, thereby enabling analysis on the exact geometry. Additionally, the smoothness of NURBS basis functions offers enhanced simulation accuracy and a larger timestep in explicit dynamics.

From a mechanical and numerical perspective, the requirements for the simulation process are very demanding. This is due to the strong non-linearities caused by contact, large deformation, and elasto-plastic material behavior, as well as highly localized plastification in curved regions.

Adaptive mesh refinement offers a compelling approach to achieve higher accuracy precisely where it is needed by utilizing a fine mesh exclusively in critical areas. This powerful method effectively reduces computational effort in sheet metal forming applications, while maintaining precise control over local errors. However, its application to complex industrial sheet metal forming simulations based on trimmed NURBS models, typically described in Boundary Representation (B-Rep) CAD models, remains a challenge. The efficient implementation of a local adaptive mesh refinement strategy in this context has yet to be fully explored. Initial investigations have been conducted for explicit dynamics, as documented in [1].

In this investigation, we provide a detailed comparison between FEA and IGA in the context of sheet metal forming applications. The current state of the art for both techniques is contrasted, highlighting their respective strengths and limitations. Furthermore, the need for an efficient adaptive mesh refinement strategy is discussed, emphasizing its importance in achieving accurate and efficient simulations. The proposed strategy is expected to accelerate the product development process and facilitate the application of IGA in industrial sheet metal forming simulations.

This paper is organized into five sections to present our study on the S-Rail model using Isogeometric Analysis and Finite Element methods.

Section 2 introduces the general setup of the S-Rail model, providing a comprehensive overview of the initial assumptions and parameters used in our analysis.

In Section 3, we delve into the error norm that serves as a critical metric for assessing the accuracy of our simulations.

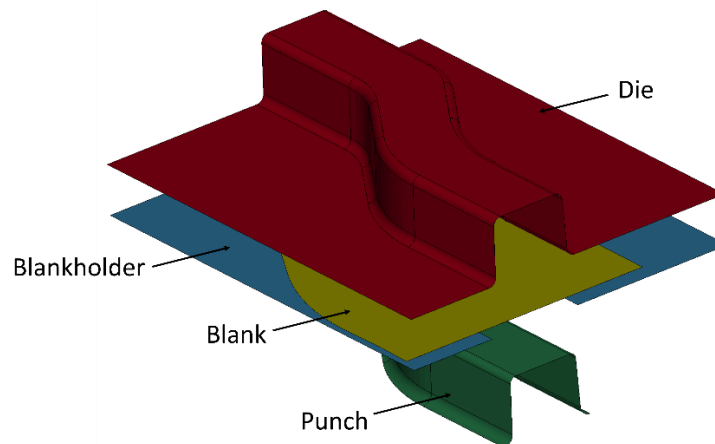
Section 4 is dedicated to exploring the various settings employed in the isogeometric sheet metal forming simulation, focusing on critical aspects such as contact conditions and integration rules. Through systematic investigations, we identify and present the optimal settings that yield the most accurate and efficient results.

The comparative study between IGA and FE is the main focus of Section 5. We evaluate both methods concerning the timestep size, computational costs, accuracy, and overall efficiency. By examining these key aspects, we draw meaningful conclusions and insights regarding the performance of IGA and FE in our specific example. As we will demonstrate, IGA outperforms FE, establishing it as the superior approach for our sheet metal forming simulation.

In Section 6, we provide a concise summary of our research findings, highlighting the key results and conclusions drawn from our investigation on the S-Rail model using Isogeometric Analysis. Additionally, we address the necessity for a local adaptive mesh refinement strategy tailored specifically for isogeometric shells.

## 2 General Setup

In our investigation, we employed the widely recognized S-Rail Benchmark model, which was originally introduced at the Numisheet conference of 1996. This model encompasses all essential components necessary for a comprehensive forming simulation. A conventional approach involves representing the tools as rigid bodies, namely the punch, die, and blankholder. As our work is based on the B-Rep description, we can use the tools with their corresponding B-Rep descriptions directly from CAD and therefore use isogeometric shell elements. Die and punch consist of multiple patches, see figure 1 (right). Those patches are coupled at common edges. We employed the Reissner-Mindlin shell formulation (ELFORM=3 for IGA; ELFORM=2 for FE) to describe its underlying shell structure. For simplicity, we refrain from modelling drawbeads and use the well-known elasto-plastic Material model `*MAT_PIECEWISE_LINEAR_PLASTICITY` for the deformable blank with a thickness of 1mm. To conduct the simulations, we employed explicit time integration, utilizing a lumped mass matrix, which is also a widely adopted scheme in industrial practice.



*Fig 1: Forming setup containing die, punch, blankholder and blank. The tools are modelled purely with isogeometric shells, while for the comparison of FE and IGA the blank is discretized with isogeometric shells and finite element shells, respectively. The die for instance consists of 33 single patches, that are coupled at common edges.*

## 3 Definition of an adequate error measurement

In order to compare the accuracy of Finite Element Analysis and Isogeometric Analysis, it is imperative to establish a suitable error measurement. A well-known and commonly used metric in industry for this purpose is the so called "draw-in". The draw-in refers to the deformation of the blank's contour, and its measurement is straightforward. However, it is essential to acknowledge that the draw-in provides a global assessment of the solution quality, and local phenomena may have minimal or negligible impact

on the draw-in value. Nonetheless, considering that the S-Rail is known to be formable within our specific experimental setup, this tradeoff is deemed acceptable.

For our analysis, we take a reference solution with an element size of 0.5 mm for both IGA and FE, labeled as  $C_R$ , which match very well, see figure 2 . Against this reference solution, we evaluate the performance of the investigated approximation, denoted as  $C_S$ . The error metric is formulated as the mean deviation in the draw-in approximation compared to the reference solution. This error measurement will enable us to quantitatively assess and compare the accuracy of both FEA and IGA.

The error norm  $e$  is defined as

$$e = \frac{1}{L_R} \oint \left\| (C_R(\xi) - C_S(\xi)) \frac{dx}{d\xi} \right\|_2 d\xi \approx \frac{1}{n} \sum_{i=1}^n d_i$$

with  $L_R$  as the full circumference length of the reference draw-in and  $\frac{dx}{d\xi}$  as the Jacobian, as the integration is done in parameter space. For simplicity, this integral is approximated by a sum of distance evaluations, as shown in figure 3.

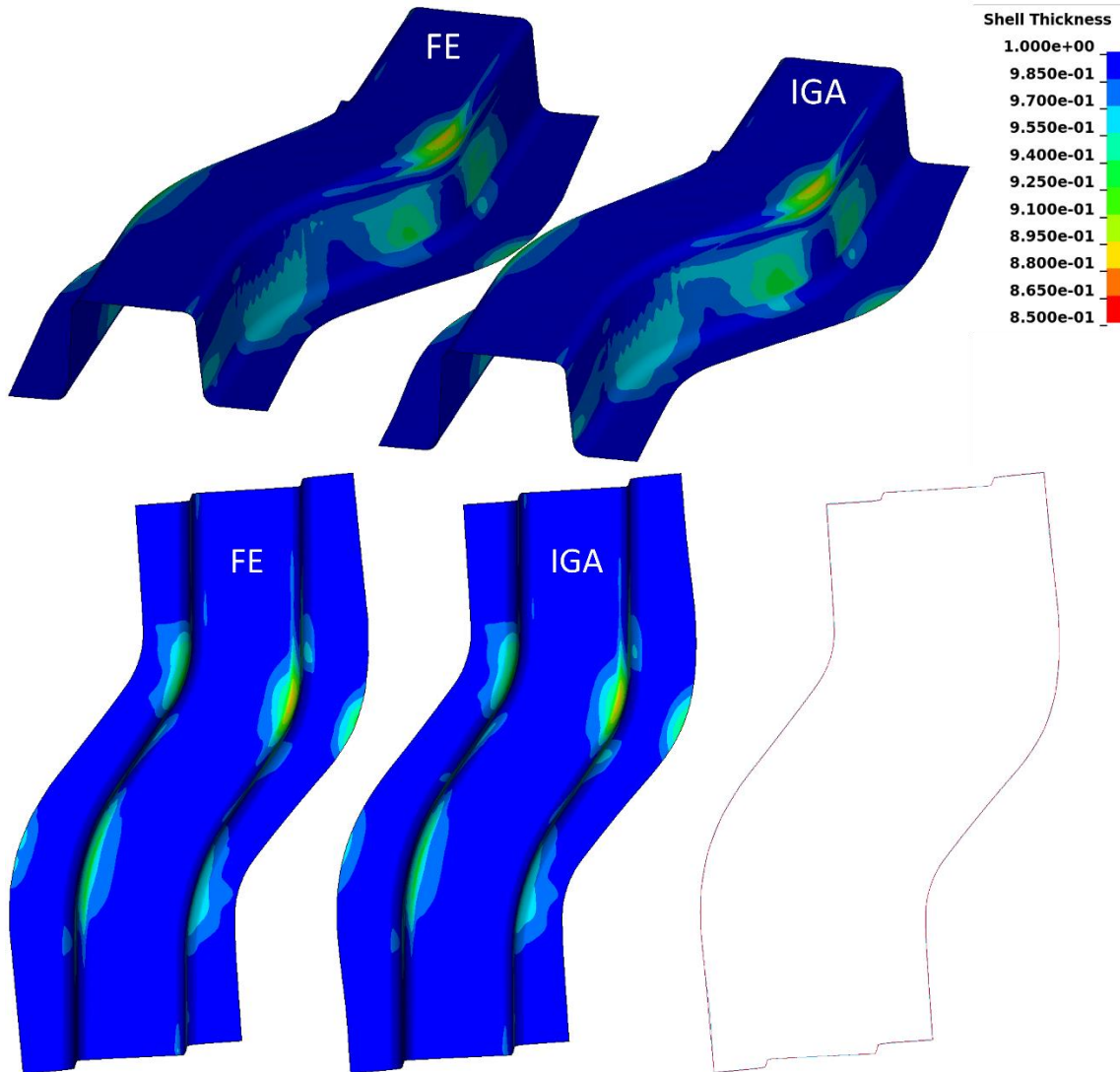


Fig 2: Comparison of the shell thickness of the reference solutions with a mesh size of 0.5 mm for FE and IGA, respectively. The bottom right contour plot shows the draw-in of the FE (red) and IGA (blue) results. Even though there is no visible difference in the draw-in, we always compare the FE results to the FE reference and the IGA results to the IGA reference for a fair comparison.

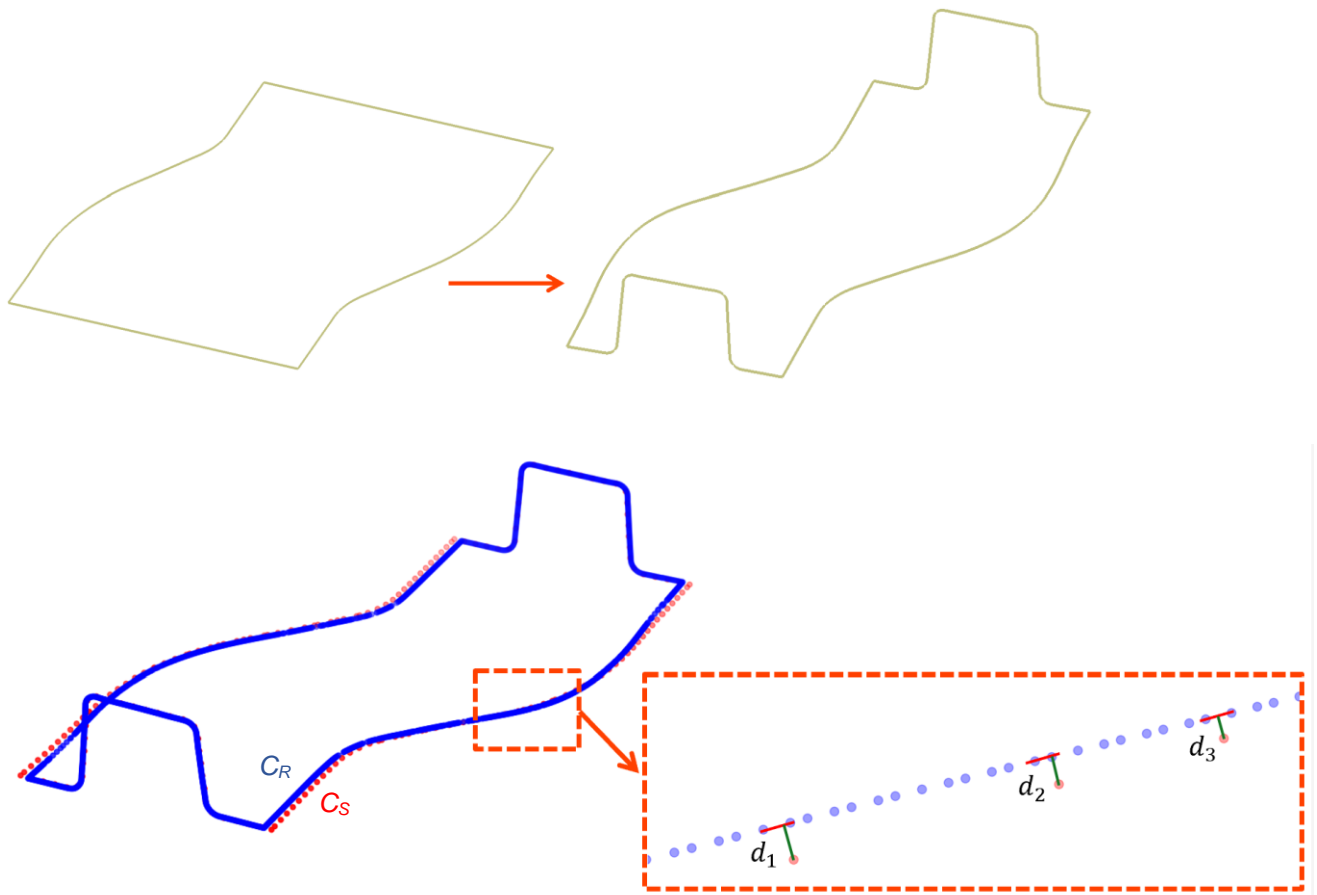


Fig. 3: Deformation of the draw-in (top) and reference solution  $C_R$  (blue, bottom) with solution of investigation  $C_S$  (red, bottom).

## 4 Assessment of the solution quality

This section aims to explore the current state of the art for Isogeometric Analysis simulations for sheet metal forming. Considering the numerous advantages offered by IGA, including enhanced capabilities in LS-Dyna regarding contact, integration procedure, and timestep handling, our objective is to identify optimal simulation settings that yield superior results while minimizing computational costs.

### 4.1 Contact procedure for IGA

The contact is efficiently and reliably captured using a so-called interpolation mesh, which provides a simple and effective method. Each knot span (=IGA element) is approximated using a specific number of interpolation shell elements, which are linear Lagrange elements, commonly used in Finite Elements. These elements interpolate the geometry, enabling the utilization of standard contact formulations in LS-Dyna. We employ the forming contact `*CONTACT_FORMING_ONE_WAY_SURFACE_TO_SURFACE` for all of the investigations. Additionally, the interpolation mesh is utilized for postprocessing tasks such as strain mapping. It is important to note that the interpolation elements do not contribute to mass or stiffness at all.

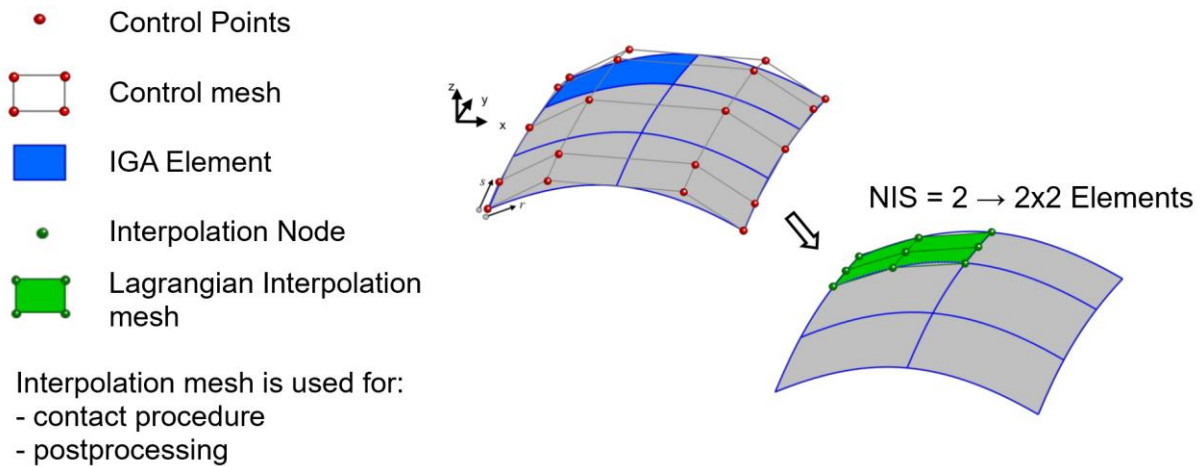


Fig. 4: Interpolation mesh (green) for one IGA element (blue). Adopted from LS-Dyna Compact: Introduction to Isogeometric Analysis with LS-Dyna, 2022

Figure 4 depicts a highlighted knot span (shown in blue) that is approximated using 2x2 interpolation elements (shown in green). As a user, you are interested in determining the number of interpolation elements (NIS) required to represent the contact physics. Increasing NIS improves the accuracy of the contact representation. However, it also increases the computational cost of the contact procedure. Therefore, it is essential to determine the optimal setting for high efficiency.

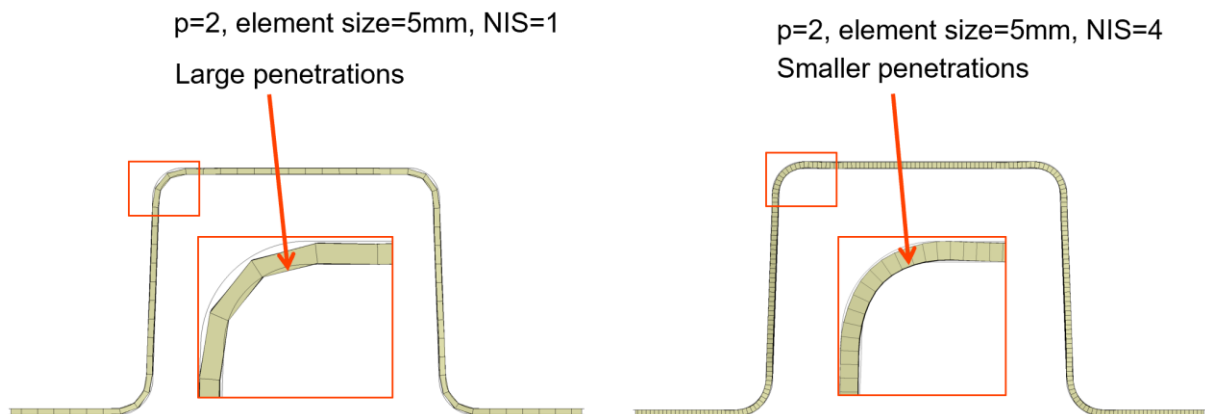


Fig. 5: Tool penetration for NIS=1 (left) and NIS=4 (right)

Figure 5 illustrates a setup with an IGA element size of 5mm. On the left, a coarse representation of the contact with NIS=1 is chosen, resulting in significant tool penetration. On the right, NIS=4 is utilized, providing a more refined representation of the geometry. For all our investigations, we use a degree of  $p=2$  and  $p=3$ , respectively. The NIS is varied between NIS=1 and NIS=4. For the different parameters, the error metric is plotted against the CPU time. The element size, varied between 1mm and 5mm, appears only implicitly as parametric coordinate. Figure 6 shows a good tradeoff for the different degrees for NIS=2. Consequently, it is sufficient to approximate an IGA element with four Interpolation elements.

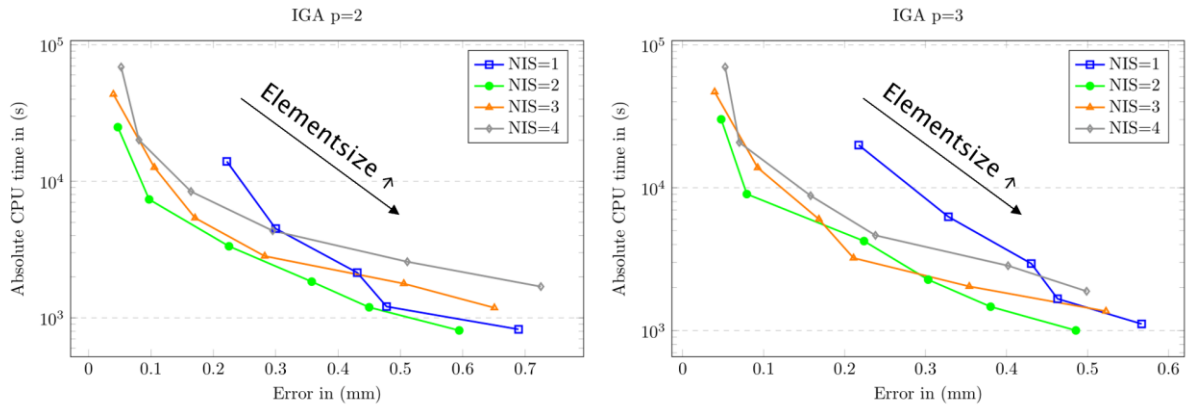


Fig. 6: Absolute CPU time vs. error metric for degree  $p=2$  and  $p=3$

#### 4.2 Integration procedure for IGA

Within the context of our investigation, we employ the standard elementwise Gauss-Legendre-Integration procedure from the Finite Elements framework, which is also applicable to Isogeometric Analysis. We differentiate between full Integration, employing  $(p+1)^2$  Integration points per element, and reduced Integration, which utilizes  $p^2$  Integration points (see figure 7, left).

Given that NURBS basis functions span multiple knot spans, we address the moment fitting equation, which determines the weights and locations of the quadrature points. Remarkably, as the basis functions are defined over a larger domain compared to FE, the moment fitting equation, that is used to calculate the quadrature points and weights, can be satisfied in a macro elementwise manner. Thereby it allows for a reduction in the number of Integration points. Hiemstra et al. [2] introduce the concept of "optimal Gauss quadrature" where they demonstrate that fewer integration points cannot yield the same level of accuracy. This optimal Gauss quadrature approach is only applicable to untrimmed regions (highlighted in figure 7, right). For trimmed elements, a more sophisticated elementwise approach has to be employed (see [3]). Notably, each macro element represents a rectangular knot span domain for which the integration points and weights are computed. Figure 7, right, visually illustrates a possible division of a quarter ring into macro elements according to Messmer et al. [4].

In the context of the current research, the optimal Gauss quadrature also offers the possibility to enhance numerical efficiency by employing a reduced integration scheme, resulting in a reduced number of integration points. These techniques enable us to achieve both efficiency and accuracy in our numerical analysis. In this study, we focus on investigating the integration schemes for degrees  $p=2$  and  $p=3$ , aiming to strike a favorable balance between accuracy and computational cost.

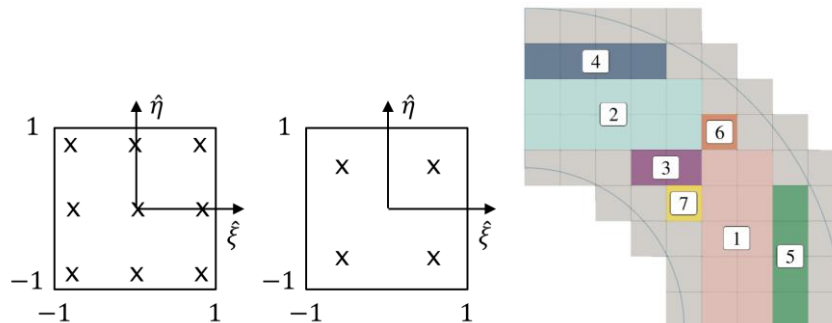


Fig. 7: Integration point locations for full and reduced elementwise Gauss-Legendre-Integration for  $p=2$  (left) and the division of the trimmed region into macro elements for optimal Gauss quadrature according to [4]. The quadrature points are not shown for the macro elements on the right. Illustration from [4]



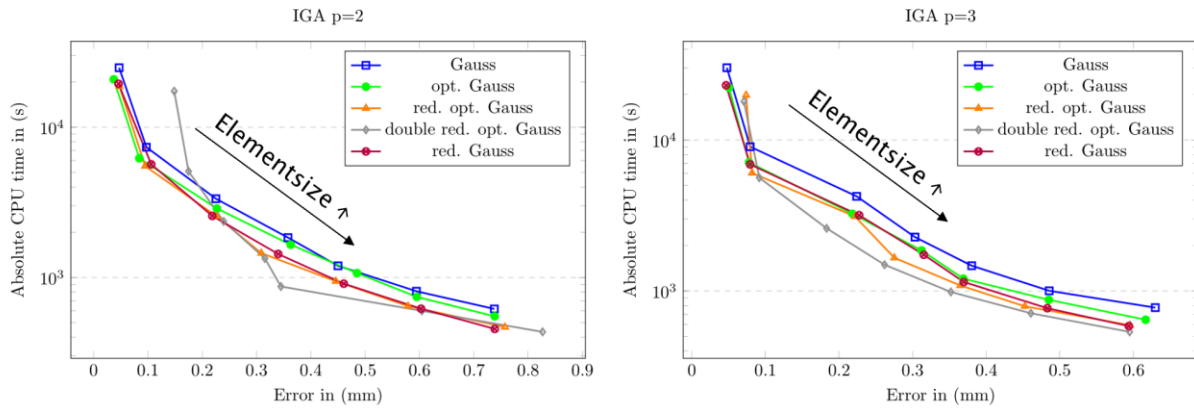


Fig. 8: Comparison of different integration rules for degree  $p=2$  and  $p=3$

Figure 8 illustrates the outcomes of our investigations. For  $p=2$ , both the reduced Gauss-Legendre and reduced optimal Gauss integration schemes exhibit promising results, providing accurate solutions with a notable reduction in the number of integration points. Moving to  $p=3$ , we explore the utilization of a double reduced optimal Gauss integration approach, which not only maintains good accuracy but also significantly reduces the required number of integration points, leading to substantial computational savings. For the following comparison of FE and IGA the red. opt. Gauss for  $p=2$  and the double red. opt. Gauss for  $p=3$  are used.

## 5 Comparison of FE and IGA

The primary objective of this section is to conduct a comprehensive comparison between sheet metal forming simulations employing Isogeometric Analysis and Finite Element methods. We will assess various critical aspects, including timestep considerations, accuracy of results, computational costs, and overall efficiency. By examining these factors, we aim to gain valuable insights into the performance and suitability of both IGA and FE approaches in the context of our sheet metal forming simulation.

### 5.1 Timestep size

Explicit time integration schemes are only conditionally stable. Consequently, a critical time step must be adhered to. The critical timestep is inversely proportional to the largest numerical eigenvalue characterizing the entire structure. Unfortunately, solving the eigenvalue problem for each time step for the whole structure incurs significant computational costs. To circumvent this issue, a practical alternative is employed. It can be shown, that the largest eigenvalue of the entire structure is consistently smaller than that of individual elements. Therefore, the eigenvalue problem is solved for each element separately, which proves more computationally efficient compared to solving it for the entire structure.

Considering this numerical aspect, the largest eigenvalue is heavily influenced by the degree and continuity of the basis functions. Reduced continuity across element boundaries results in an increased largest eigenvalue, affecting the timestep adversely for IGA [5]. In our application, we have the freedom to choose the continuity for the patch. To determine the maximum timestep, we always select the maximal inter-element continuity of  $C^{p-1}$ . However, the boundary basis functions are always of  $C^{-1}$  continuity, determining the critical timestep. To mitigate this negative impact, we employ an extended patch and remove at least  $p-1$  element rows and columns along the boundary, thereby ensuring that they do not influence the timestep. Figure 9 (right) illustrates an extended patch for  $p=2$  with trimmed-off boundary rows and columns. We observe that IGA allows for a significantly larger timestep compared to FE (ELFORM=2 and ELFORM=16), with a ratio of more than two. Consequently, the IGA simulation requires only half the number of timesteps compared to FE. Figure 9 (left) depicts this difference in timestep sizes between IGA and FE simulations.

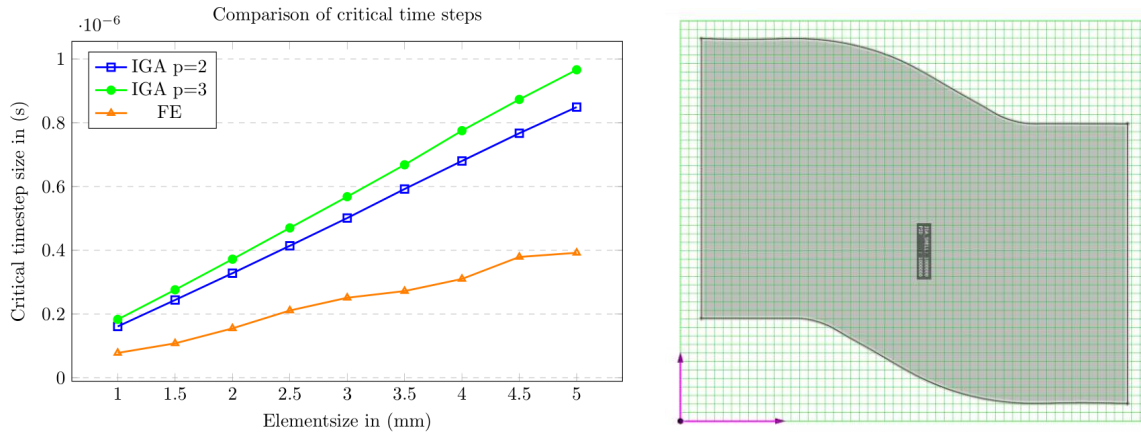


Fig. 9: Comparison of timestep size for IGA and FE (left) and trimmed and extended patch (right)

## 5.2 Accuracy, computational costs and efficiency

In the preceding section, we observed that Isogeometric Analysis permits the use of larger time steps compared to the Finite Element Method. However, it is noteworthy that standard FE still outperforms IGA in terms of computational speed for the same element size. This disparity is expected, given that the element routines for FEM have been optimized over several decades, while IGA is a relatively new approach. Even with the optimal settings we have identified so far, regarding the Number of Interpolation elements and Integration rule for our application, the computational speed of the under-integrated SHELL2 (ELFORM=2) in FE with only one in-plane integration point remains unmatched. To be fair, it is important to note that by using e.g. NIS=2 for IGA, we end up using a total of 4 interpolation elements. This is four times more than for the FE approach. Consequently, the cost for the contact procedure increases correspondingly. Additionally, when we compare IGA with degree  $p=2$  and  $p=3$  to FE with a polynomial degree of  $p=1$ , it becomes clear that IGA requires a higher number of in-plane integration points compared to FE. Figure 10 (left) illustrates the computational costs for different element sizes, highlighting this difference.

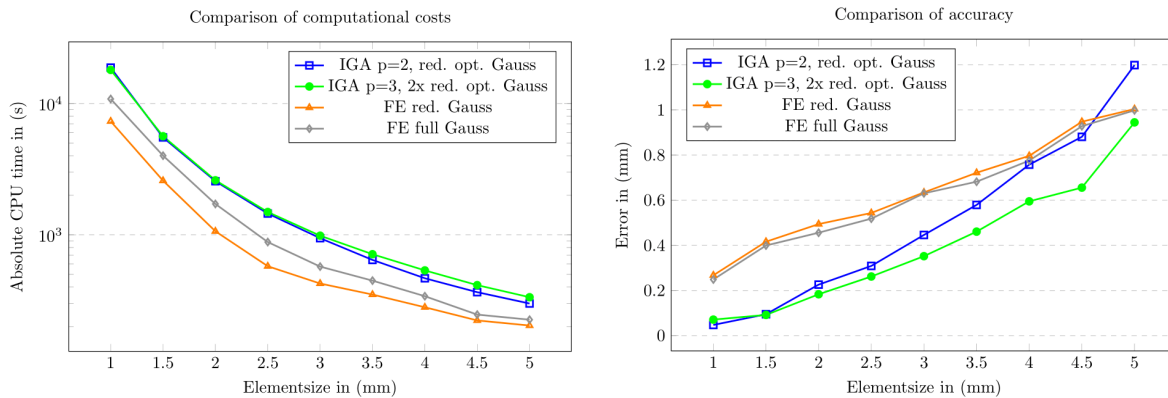


Fig. 10: Computational costs (left) and error norm (right) over element size for IGA and FE

Examining the error norm in figure 10 (right), we find that SHELL2 and SHELL16 (ELFORM=16, FE, full integration) exhibit comparable accuracy for the same element size, while IGA demonstrates superior accuracy within the convergence zone for  $p=2$  (reduced optimal Gauss Integration) and  $p=3$  (double-red. opt. Gauss Integration).

To determine the most efficient numerical approach for our simulation, we combine the insights from the two aforementioned plots. This combination allows us to represent efficiency by plotting the absolute CPU time against the error, see figure 11. Within the converging zone spanning element sizes of 3mm to 1mm, we observe that IGA yields better results for the same computational costs or offers more cost-effective simulations for the same level of accuracy. Based on these findings, we can confidently



conclude that IGA exhibits superior behavior compared to FE, making it a favorable choice for our specific application.

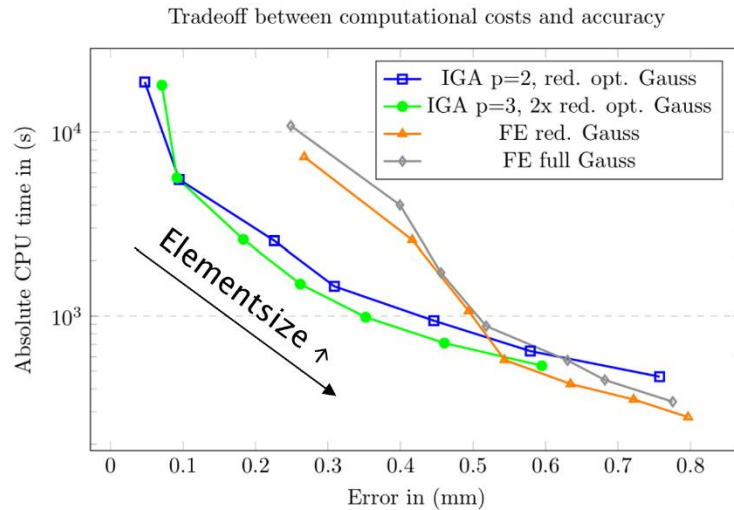


Fig. 11: Absolute CPU time over error for IGA and FE

## 6 Summary and future work

This paper presents a systematic assessment of Isogeometric sheet metal forming simulations using trimmed, multi-patch Non-Uniform Rational B-splines (NURBS) models in LS-Dyna. The study explores various aspects, including contact procedure for IGA, integration schemes, and timestep considerations. It identifies optimal simulation settings for IGA, achieving a good balance between accuracy and computational cost. Comparing FE and IGA, the paper demonstrates that IGA allows for larger timesteps, requiring fewer time steps for simulations. Although standard FE is faster for the same element size, IGA outperforms FE in terms of accuracy and efficiency within the convergence zone.

Overall, the results indicate the potential of IGA for sheet metal forming simulations, offering better accuracy and computational efficiency compared to traditional FEA approaches.

Currently, the absence of an adaptive mesh refinement strategy tailored for isogeometric shells presents a significant limitation, hindering the ability to control errors locally in critical regions and optimize computational resources for large-scale industrial models in comparison to the Finite Element Method. Thus, the development of an efficient mesh refinement approach based on more advanced spline formulations like LR-Splines (Locally Refined Splines) or THB-Splines (Truncated Hierarchical Splines) is imperative, as the standard patch formulation lacks the necessary flexibility for a true local refinement. Consequently, a thorough investigation of these alternative formulations is justified to determine their suitability and effectiveness in explicit simulations for isogeometric shell analysis.

## 7 Literature

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