FINANCIAL RESEARCH NOTE 04

Asset Allocation with Black-Litterman in a case study of Robo Advisor Betterment

AUGUST 2018

Financial Research Note 04

Asset Management

Asset Allocation with Black-Litterman

24 Seiten

Deutsche National Bibliothek

Internet: ISSN 2364-9437

The Black-Litterman model extends the framework of modern portfolio theory by allowing portfolio managers to include their views in the optimization process. The study tries to reproduce elements of Betterment's approach to Black-Litterman in asset allocation. The case of Betterment is used to illustrate the strengths and weaknesses of the application of Black-Litterman in the asset allocation process of innovative Robo Advisory platforms as well as to analyze the risk assessment and the ETF selection processes for the investor.

Autoren
Daniel Pirner
Matthias Fischer

Herausgeber
Prof. Dr. Matthias Fischer

Kompetenzzentrum Finanzen
Technische Hochschule Nürnberg
Internet:
www.th-nuernberg.de
Email:
matthias.fischer [at] th-nuernberg.de

Case Study Methodology

In order to reconstruct Betterment's approach to Black-Litterman, both Excel and Excel Solver are used. Asset selection is done using ETF.com's screening tool, applying filters in line with Betterment's asset-selection philosophy. Price quotes are obtained from Yahoo! Finance, returns thereof are converted to steady returns due to their increased compatibility with the normal distribution. The mathematical concepts explained below are executed in Excel by using the matrix functions MMULT, MTRANS, and MINV, as well as nested IF constructs. In order to implement Betterment's model as precisely as possible, the shrinkage estimator of Ledoit and Wolf is built and added to the Excel model and substitutes the basic sample covariance matrix to enhance portfolio robustness. Both the self-constructed portfolios and Betterment's portfolios are then compared against an equally weighted, a market-capitalization-weighted, and a naive portfolio.

Case study company Betterment

Founded in 2008, Betterment is a first-generation robo-advisory start-up that currently has around $ 10 billion of assets under management. Direct competition includes other start-up robo-advisors such as Wealthfront or AssetBuilder, as well as robo-advisory offerings from traditional asset managers such as Charles Schwab Intelligent Portfolios or Vanguard Personal Advisor.[1] While Betterment is the largest of the start-up robo-advisors by assets under management (AUM), solutions from Vanguard or Charles Schwab currently have AUM between $20 and 100 billion.

[1] Credio Investment Advisers.com (2018)

Over time, Betterment expanded its business to also offer 401(k) solutions to B2B clients, and offer back-office management to financial advisors. Total disclosed funding of Betterment sums up to $ 275 million,[2] valuations however only center around $800 – 900 million.[3] This could reflect the market's issues with profitability based on high costs of customer acquisition and low fees due to competitive pressure from traditional asset managers.[4] Retail investments are the core of Betterments business, and ETF-portfolios built with the Black-Litterman model can be tailored towards e.g. tax loss harvesting or retirement.

Theoretical principles

The Black-Litterman model makes use of the mean-variance-optimization (MVO) framework of modern portfolio theory (MPT)[5] as well as the global equilibrium model for asset pricing by CAPM.[6] Although CAPM implies a constant market equilibrium, the Black-Litterman (BL) model partially rejects that assumption. Instead, it is assumed that when risk premiums diverge from equilibrium values, market imbalances have the tendency to reverse these developments in the long run. The model leads to asset weights close to market-capitalization weights rather than abnormally large short positions or corner solutions.[7] Equilibrium market returns required for the model are obtained by reverse-optimization of an MVO problem.[8] The model's foundation rests upon the following three pillars:

The Black-Litterman model extends the framework of modern portfolio theory by allowing portfolio managers to include their views in the optimization process.

Figure 1: The three pillars of the Black-Litterman model[9]

In general, portfolio-building includes collecting both public and private information, that is, market data and analysts' insights exclusive to asset managers. Both processes are costly and time-intensive in practice. If the market equilibrium is now viewed as the aggregation of all individual perspectives on the market, it is implied that publicly accessible information is fully represented in asset prices. This can be expressed by the hypothesis of semi-strong market efficiency,[10] which holds that the generation of α through reliance on public sources of information is impossible, and abnormal returns may only be achieved by the use of private insights or exclusive techniques.[11] This leads to the conclusion that without individual superior insights, one should follow the

[2] Crunchbase.com (2018)
[3] Funderbeam.com (2018); account creation required.
[4] Fastcompany.com (2017)
[5] Markowitz, H. (1952), p. 77ff
[6] Sharpe, W. F. (1990), p. 312ff
[7] Ernst, D.; Schurer, M. (2015), p. 495
[8] Cheung, W. (2009), p. 11
[9] own illustration, based on Cheung, W. (2009), p. 3
[10] Fama, E.F. (1970), p. 415
[11] see Cheung, W. (2009), p. 3ff for a mathematical derivation of the reliance on Fama's hypothesis.

© Prof. Dr. Matthias Fischer

market, represented by holding a passively managed share of the market portfolio based on CAPM. [12]

Asset variances and correlations are used to calculate forward-looking return estimates that are interrelated and dynamic.

With or without superior knowledge – especially with a large number of assets – a derivation of mean-equilibrium returns is not trivial, since different assets are all interrelated via relative volatilities and correlations. For example, if a private view is that the Asian market will outperform the US, this view can be expressed quite easily, but the implications on relative returns of the Asian and other markets are far more difficult to comprehend and factor in. But due to the logic behind MVO, the intended overweight of the Asian market cannot be achieved by simply increasing the expected returns on those stocks. Instead, by using relative volatilities and correlations, the logic of MVO suggests that a more sophisticated portfolio with less risk and more expected return can be constructed.

Within the BL model, this particular view is handled as a positive return expectation for a portfolio comprised of a long position in Asian stocks and a market-weighted short position in the US market. The resulting adjusted returns then suggest the optimization tool the desired overweighting of the Asian stock market.[13] The framework for generating robust and intuitive forward-looking return forecasts is set by the conditional probabilities of the Bayes' theorem.

Unlike with modern portfolio theory an investor does not need to form any views for a set of assets. If no views are included, the investor will simply hold the equilibrium portfolio.

To summarize, the BL-model uses variances and market capitalizations of an array of assets to create an equilibrium portfolio through the use of a reverse-optimized utility function. This equilibrium portfolio is the portfolio of an investor with no specific opinion on capital markets. Equilibrium returns are always positive, since they are calculated from the assets' covariance matrix and not from historical time series. The next step is to factor in personal views on asset returns. The model introduces a new parameter, Tau, which resembles an investor's trust in the equilibrium portfolio and serves as a scaling factor in further calculations. An investor is not required to form any views and can thus focus on a number of assets. Views can be stated in absolute or relative terms. Within a view assets are separated into outperforming and underperforming groups, where every asset's market capitalization is weighted against the sum of market capitalizations in that group. This way, the impact of a view on expected returns more accurately resembles the actual situation in the market.[14]

The Black-Litterman portfolio is based on the equilibrium portfolio. According to personal views, asset weights differ up- or downwards.

After that, potential view errors are recognized, again with the impact in proportion to the relative market weight and the level of trust in the equilibrium portfolio. The final calculation results in a vector of Black-Litterman returns that incorporate information from the equilibrium portfolio and personal views. Asset weights also resemble the equilibrium portfolio, but are "corrected" up- or downwards, depending on views. This essentially means that a Black-Litterman portfolio con-

[12] Idzorek, T.M. (2005), p. 6; p. 14
[13] He, G.; Litterman, R. (1999), p. 5; Black, F; Litterman, R. (1992), pp. 14-15
[14] He, G.; Litterman, R. (1999), pp. 3-6; Idzorek, T.M. (2005), pp. 4-19; Black, F; Litterman, R. (1992), pp. 17-38

© Prof. Dr. Matthias Fischer

sists of two portfolios – the equilibrium portfolio, and a view-portfolio of long and short positions.

To follow Betterment's portfolio strategy more closely, two adjustments are implemented. The above mentioned market-capitalization based view-weighting highly reduces estimation errors compared to an equal weighting scheme. Furthermore, the shrinkage estimator of Ledoit and Wolf[15] is used. The estimator addresses issues regarding estimation error within covariance matrices of financial models. Here, a common theme is the high amount of estimation error when sample sizes are equal to or smaller than the number of individual assets. The shrinkage estimator Ledoit and Wolf propose pulls extreme estimates either up or down, towards the sample's average correlation.

Case Study

Among traditional asset classes, commodities are excluded from Betterment's portfolios due to their insufficient hedging potential against inflation, relatively high risk, and real-term negative returns in the long run. Real estate, which would be assessed by investing in REITs[16] or ETFs, behaves fairly similar to US mid-cap equities[17] and is thus also excluded. Cash is included in the portfolio by utilizing cash-equivalent treasuries, which can enhance diversification in the short run.[18] Equities and fixed income make up the bulge of their portfolios and are categorized further below. ETFs are selected following the criteria shown below:

Betterment employs tight constraints in their asset selection process that exclude up to 97% of ETFs from being considered in their portfolios.

ETF Selection Criteria	Assessment
Low Cost-to-Trade	Narrow Bid/Ask Spreads High Trading Volumes
Low Cost-to-Hold	Low TER Low Tracking Difference
Index Tracking	Passive ETFs
High Liquidity	High AUM
Tax Efficiency	Physical ETFs with earnings distribution
Minimal Market Impact	High AUM
High Diversification	Wide scope of market definition

Table 1: ETF selection criteria at Betterment[19]

The first two rows indicate that Betterment's holistic understanding of costs is somewhat ahead of legislation of the *"markets in financial instruments directive II"* (MIFID II), as the company not only considers the total expense ratio (TER), but also tracking difference and the bid/ask spread,

[15] Ledoit, O.; Wolf, M. (2004), pp. 5-6
[16] REIT stands for Real Estate Investment Trust
[17] REITs are traded like stocks and do not correlate with the Case-Schiller RE-Index according to Betterment.com (2014a)
[18] All from: Betterment.com (2014a); Betterment.com (2014b); Betterment.com (2017); Betterment.com (2015)
[19] own illustration, based on data from Betterment.com (2018b)

© Prof. Dr. Matthias Fischer

thus favoring high liquidity and low costs to arrive at its measure of the total annual cost of ownership (TACO). Although Betterment does not further specify its selection criteria, examining the parameters of the current asset allocation can narrow the number of products down sufficiently. This is demonstrated by following Betterment's selection process using the same tool, the ETF screener from ETF.com. The general selection process for robo-advisors usually narrows the number of investable ETFs down to 3–6% of available products.[20] Betterment eliminates ETFs based on a range of criteria including niche, strategy, size, and TER.[21] Applying the broad market definitions of the ETFs Betterment is currently invested in yields a wide range of investment opportunities that need to be narrowed down. Table 2 depicts this process.

Market definitions of selected ETFs show a strong focus on US equity and fixed income. Cash is included through US Treasury Cash Equivalents (SHV).

Equities Subcategories	Products	AUM > $ 1B	TER < 0.15%	Spread < 0.05%	Avg. Daily Trading Volume > $ 100M	Selection	Results
Emerging Markets - Total Market	51	8	4	3	2	VWO and IEMG; VWO has higher AUM	VWO
Dev. Markets ex. US - Total Market	60	12	4	4	2	VEA and IEFA; VEA has higher AUM & lower TER	VEA
US Small Cap Value	10	4	1	1	1	VBR	VBR
US Mid Cap Value	10	3	1	1	1	VOE	VOE
US Large Cap Value	23	9	5	4	1	VTV	VTV
US Total Market	109	21	6	6	2	VTI and VIG; VTI has lower TER	VTI
Fixed Income Subcategories	**Products**	**AUM > $ 1B**	**TER < 0.15%**	**Spread < 0.05%**	**Avg. Daily Trading Volume > $ 100M**	**Selection**	**Results**
Emerging Markets - Sovereigns	11	4	3*	1	1	EMB	EMB
Broad Market ex. US - IG	2	1	1	1	1	BNDX	BNDX
US Corp. IG	14	2	2	2	1	LQD	LQD
US Broad Market IG	14	4	4	4	2	AGG and BND; Betterment used AGG, now uses BND	BND
Govt. Treasury Cash Equivalents	4	2	2	2	1	SHV	SHV
US Govt. TIPS Short	4	2	2	2	2	VTIP and STIP; VTIP has higher AUM	VTIP

* Emerging Markets Fixed Income have higher avg. TERs, so the TER-filter is widened to "TER < 0.5%"

Table 2: Reproducing ETF selection with an ETF screener[22]

The applied selection criteria are given in columns 2 through 6; in case of multiple suggestions, an explanation is provided in column 7. Tough constraints almost always eliminate all but one product. Table 3 then shows the correlation matrix of Betterment's investment universe.

[20] Orçun, K. (2017), p. 5f
[21] Betterment.com (2018b)
[22] own illustration, based on data from ETF.com (2018)

© Prof. Dr. Matthias Fischer

Due to a tilt towards value, a substantial portion of equity ETFs shows high correlation. This is somewhat countered by negative correlation with the fixed income ETFs.

	VTIP	VWO	VEA	VBR	VOE	VTV	VTI	EMB	BNDX	LQD	BND	SHV
VTIP	1.00											
VWO	-0.07	1.00										
VEA	-0.13	0.77	1.00									
VBR	0.06	0.46	0.59	1.00								
VOE	-0.03	0.59	0.74	0.92	1.00							
VTV	-0.06	0.58	0.71	0.87	0.93	1.00						
VTI	-0.09	0.63	0.79	0.88	0.96	0.96	1.00					
EMB	-0.05	0.68	0.59	0.22	0.33	0.27	0.34	1.00				
BNDX	0.05	-0.12	0.01	-0.21	-0.13	-0.14	-0.15	0.11	1.00			
LQD	0.05	0.42	0.34	0.12	0.19	0.11	0.18	0.79	0.09	1.00		
BND	0.03	0.24	0.16	-0.11	-0.04	-0.13	-0.05	0.73	0.12	0.91	1.00	
SHV	-0.14	0.28	0.12	-0.10	-0.03	-0.01	0.00	0.31	-0.10	0.28	0.30	1.00

Table 3: Sample correlation matrix for Betterment's ETFs[23]

The value ETFs are highly positively correlated with the US Total Stock Market ETF. US Corporate Bonds and the Broad Bond Market ETF are highly correlated to Emerging Markets Bonds. In its whitepaper, Betterment declares that for portfolio selection, it tilts its portfolios based on implications from the 3-factor models of Fama and French.[24] As this could not be reproduced in the financial model, this study assumes that in selecting three different value ETFs, a tilt towards value is sufficiently achieved. The high correlations of value stocks are therefore assumed to be acceptable from Betterment's perspective.

Betterments risk assessment approach is similar to a target-date fund. Individual risk tolerance is substituted by the risk tolerance necessary to reach a certain goal.

Most other robo-advisors assess risk preferences via an array of questions centered around loss aversion, behavior in bearish markets, and sometimes about behavioral biases. Betterment only indirectly assesses risk tolerance, reasoning that standard questionnaires attempt to capture perceived individual risk tolerance at a given moment rather than the risk tolerance that is necessary in order to achieve a certain goal.[25] This means that Betterment's approach to risk-tolerance assessment, detailed in Table 4, is not comparable to most competitors.

[23] own illustration and calculations, based on 5 years of monthly price quotes from Yahoo! Finance; as of 07.03.2018
[24] Fama, E.F.; French, K. (2004), pp. 37-38
[25] Betterment.com (2018c)

Betterment's Questionnaire			
Nr.	Question	Possible answers	Additions
1	Let's get started. Are you retired?	- Yes - No	Age determines stock allocation
2	What's your primary reason for investing?	- Saving for retirement - General investing - Saving for an emergency fund - Saving for a major purchase	Determines stock allocation over time
3	Are you currently investing?	- Yes - No	-
4	How are you currently investing?	- I'm doing it myself - I have an employer plan - I have an investment advisor	-
5	What are your investable assets?	(Amount $ xy)	Deposit determines stock allocation
6	Would you like unlimited access to our team of CFP professionals?	- Yes - No	Selection of payment plan
7	Which plan would you like to start with?	- Digital Plan (0.25% fees) - Premium Plan (0.4% fees)	Selection of payment plan
8	Goal Setting	- Individual (usable for any goal) - 401(k) Rollovers/Transfers - IRAs - Joint Account - Trust Account	Goal, age, deposits, and time horizon determine stock ratios over time **Possible Advice:** - Increase auto-deposits - Add one-time deposits - Adjust time horizon - Adjust stock ratio

Table 4: Betterment's questionnaire[26]

What stands out in Betterment's questionnaire is the lack of questions concerning risk tolerance, time horizon, and other standard questions about age, income, or liabilities. Also, no asset allocation is recommended upon completion of the questionnaire without creating and verifying an account.

Tighter regulation requires German robo advisors to ask more questions on personal liabilities, capital market experience and risk tolerance than their US counterparts.

While some of Betterment's competitors ask questions concerning risk, none of the US-providers analyzed inquire about short- or long-term liabilities of investors – a crucial factor for retail investors. Compared to questionnaires from German robo-advisors following regulatory requirements of §34 sp. 3WpHG-E, American competitors are not required to obtain detailed information on liabilities. Questionnaires of German counterparts are characterized by manifold questions and small examples to assess risk tolerance.[27] Few of the American robo-advisors analyzed inquired about individuals' capital market experiences or assessed risk tolerance in detail. [28] If regulation is made more robust in the future, this could adversely affect American robo-advisors, and in the face of another crisis, this approach could lead to further complications in the form of lawsuits from private investors.

[26] own illustration, based on Betterment (2018d); Betterment (2014c)
[27] Fischer, M. (2017), p. 188
[28] Questionnaires from Charles Schwab IPS.com (2018), AssetBuilder.com (2018a) and Wealthfront.com (2018a) have been completed online.

© Prof. Dr. Matthias Fischer

Betterment assesses risk in a rather generalized way, emphasizing the "goal" in goal-based investing. Stock allocation is adjusted over time in accordance with a determined goal, for example $1 million in retirement savings. Consequently, if a 35-year-old individual[29] wants to retire at age 65, the stock allocation will be relatively high at the point of investment in comparison to other robo-advisors, and adjusted downwards as time progresses. These and other differences in areas including investor onboarding, risk classification, and portfolio recommendation are illustrated in Figure 2, which compares Betterment's approach to a possible standard approach.

Compared to possible standard approaches, Betterments recommended glide path ensures dynamic asset allocations. Risk is modelled through a portfolio's stock allocation.

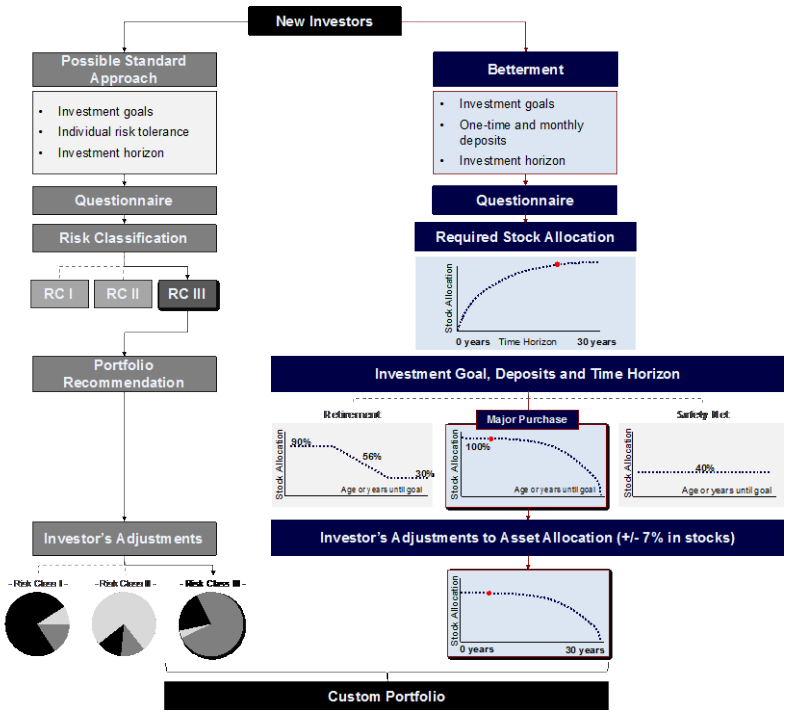


Figure 2: Differences in risk assessment and portfolio recommendation[30]

As Figure 3 illustrates, Betterment makes use of the Black-Litterman model, the CAPM, and MVO, with additions for robustness of asset allocations such as a shrinkage estimator for the covariance matrix and a market-capitalization-based weighting scheme for the view matrix. Views are formulated from historical and empirical observations and in a manner that favors value over growth ETFs. After a vector of portfolio weights is obtained with BL, multiple iterations of Monto Carlo scenarios further increase the robustness of portfolio weights.[31] Thus, weights are a complex average of the implied equilibrium portfolio, the view-weighted BL portfolio, and Monto Carlo averages. For a robo-advisor, even one of the largest, this is a highly sophisticated approach.[32]

[29] According to Betterment.com (2018g), if „General Investing" is selected, Betterment assumes the time horizon is the individual's 65th birthday; the asset allocation will be riskier than with the "Retirement" option.
[30] own illustration, based on Betterment.com (2018g); "other Robo-advisors" based on observations completing questionnaires of German & American providers; Orçun, K. (2017), pp. 2-3
[31] This is not reproduced in this study.
[32] Betterment.com (2018e)

Apart from Betterment, only Wealthfront[33] and FutureAdvisor[34] use their own interpretations of the BL model.

Betterment's use of Black-Litterman produces more robust asset allocations through many iterations of Monte Carlo and the use of shrinkage estimators and market based weighting schemes.

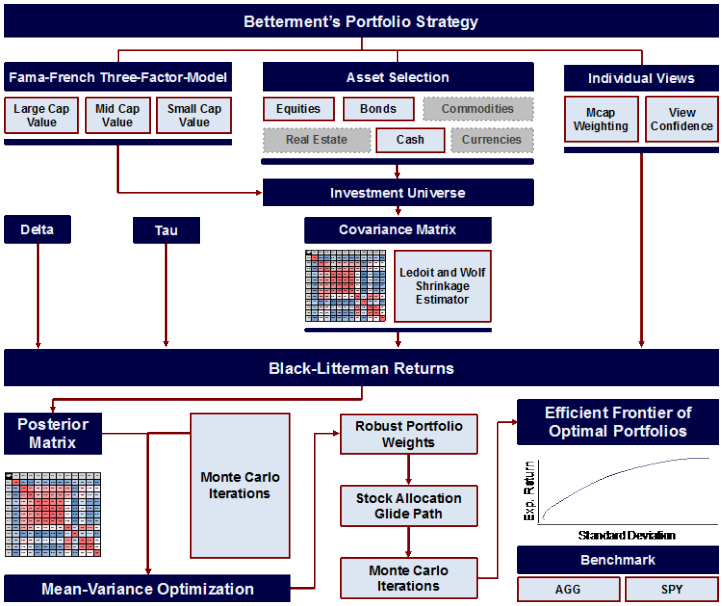


Figure 3: The Betterment portfolio strategy[35]

As a starting point for the case study, five years of historical price quotes on a monthly basis are obtained from Yahoo! Finance to generate historical mean returns and variances. Then, discrete monthly returns are converted to steady returns, since these behave better under the assumption of a normal distribution.[36] From these returns, annualized estimates of sample mean excess returns and variances are obtained assuming a 0% risk-free rate.[37]

Additionally, sample correlation and annualized covariance matrices are generated. These are a necessary input for the shrunk covariance matrix Betterment uses for the Bayesian prior. From the sample correlation matrix, the average correlation is calculated and multiplied with sample variance, volatility, and transposed volatility in order to obtain the constant correlation matrix. The shrinkage constant is assumed to be 0.45.[38] All elements of the sample correlation matrix are now multiplied and weighted by the shrinkage constant to obtain the shrunk covariance matrix. The upgraded matrix is clearly more balanced and centered than the original sample covariance matrix.

This upgraded covariance matrix is the first input required to solve the reverse-optimization problem for the implied equilibrium returns. The risk-aversion coefficient Delta is assumed to be 2.5 in accordance with He and Litterman

[33] Wealthfront.com (2018b)
[34] FutureAdvisor.com (2018)
[35] Own illustration based on Cheung, W. (2009), p. 12; Betterment.com (2018e); Idzorek, T.M. (2005), p. 16
[36] Ernst, D.; Schurer, S. (2015), p. 74
[37] This is done to ease calculations, excess returns close to zero result in extreme asset weights.
[38] Since the equation could not be reproduced, the assumption of Ernst, D.; Schurer, S. (2015), p. 458 is stated here.

(1999).[39] Another necessary input is the market capitalization of all assets. As the weights used by Betterment are unknown, and in the absence of stock-like market capitalizations, this study resorts to using the net assets from each of the 12 ETFs listed in Table 5.

ETFs net assets that are available online are used as a substitute for market capitalizations to obtain the equilibrium weights.

ETF	Net Assets (B$)	Relative Weight
VTIP	$ 24.0	1.70%
VWO	$ 100.9	7.13%
VEA	$ 113.6	8.03%
VBR	$ 30.5	2.16%
VOE	$ 18.1	1.28%
VTV	$ 68.3	4.83%
VTI	$ 701.1	49.56%
EMB	$ 14.1	1.00%
BNDX	$ 102.7	7.26%
LQD	$ 36.9	2.60%
BND	$ 196.0	13.86%
SHV	$ 8.4	0.60%

Table 5: Global net assets and derived relative market weights of Betterment's investable universe (as of 07.03.2018)[40]

As an ETF's AUM is a decisive factor during asset selection, this study deems the use of net assets as capitalization weights justifiable. These relative weights are then used to obtain the equilibrium returns. Black and Litterman reason that cleared markets resemble a state of equilibrium. In the case of investments, cleared markets mean every individual achieves the desired risk-return combination. Investors are considered to be risk-averse, which means that in equilibrium, the minimum return needed to clear the market has to be equal to volatility times global risk aversion. This results in the relatively low but more balanced and always positive equilibrium returns shown in Table 6.

[39] He, G.; Litterman, R. (1999), p. 10; since Betterment does not provide a value on their own and this assumption is quite common in literature.
[40] Own illustration, based on data from Yahoo! Finance; this study accepts the fact that an ETF's net assets might not be representative for the true market weight of an asset class.

Equilibrium returns are the returns implied by a given set of market weights and a covariance matrix. Thus, they are always positive and forward-looking. Compared to historical mean returns, they are also more balanced.

ETF	Implied Equilibrium Returns	Annual Historical Returns (5y mean)
VTIP	0.04%	-0.52%
VWO	1.68%	1.73%
VEA	1.42%	4.25%
VBR	1.39%	10.49%
VOE	1.25%	11.40%
VTV	1.19%	10.68%
VTI	1.63%	11.82%
EMB	0.53%	-1.12%
BNDX	0.05%	1.74%
LQD	0.31%	-0.37%
BND	0.13%	-0.93%
SHV	0.004%	0.01%

Table 6: Equilibrium and historical returns compared (as of 07.03.2018)[41]

The equilibrium portfolio shown in figure 4 below is the portfolio investors should hold in the case of no views. Contrary to historical weight suggestions, there are no short positions in equilibrium market weights. Since the returns and variances of some assets are close to zero, suggested historical weights are extremely high, and thus scaled to 100% to allow for some form of comparison.[42]

Equilibrium weights are also more balanced than historical weight suggestions. Since no views are implemented yet, they are equal to relative market weights.

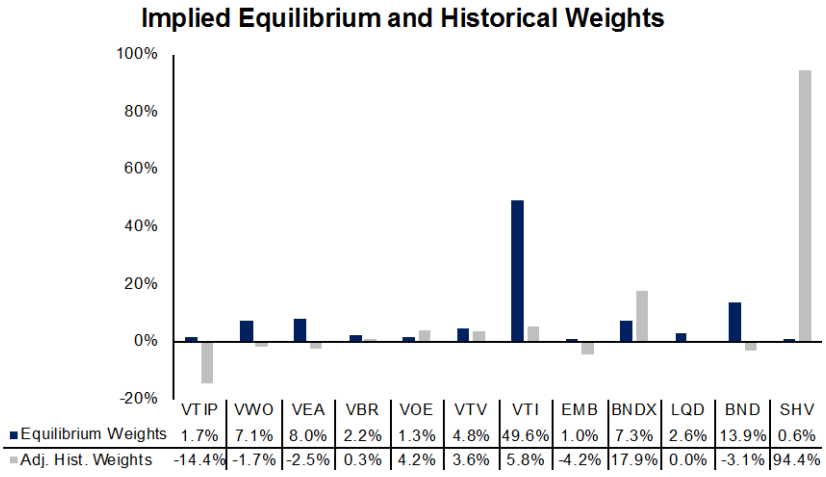


Figure 4: Implied equilibrium and adjusted historical weights compared[43]

The next input in the BL model are the individual views. Since it is based on the Bayesian prior covariance matrix, the expression of a single view ultimately affects every return estimate. Lacking more robust data, this study makes use of what little information Betterment does provide, namely its tilting toward value stocks. Conceptually, through

[41] own illustration and calculations, based on data from Yahoo! Finance (2018)
[42] This is done by dividing each implied historical weight by the sum of all weights.
[43] own illustration and calculations, based on data from Yahoo! Finance (2018); Ernst, D.; Schurer, M. (2015), p. 527

formulating relative and positive views concerning value ETFs, a respective shift in asset allocation should lead to long positions in these ETFs.

	Views for the Matrix *Q*
1	VTIPS and SHV will be outperformed by EMB, LQD and BNDX with 1%
2	VEA will outperform VTI and VOE by 2%
3	VTV and VOE will be outperformed by VBR by 3%
4	VEA and VWO will outperform VTI and VTV by 4%

Table 7: Formulated views for the matrix Q[44]

Result of combining personal views, the equilibrium portfolio, market weights and an error measure is a set of Black-Litterman returns that fluctuates around the equilibrium returns.

View returns are not centered around any real-world observations, and underperforming assets were randomly selected. As explained, the impact of multiple views in a large portfolio is nearly impossible to predict. Factoring in the views yields the vector of Black-Litterman returns, which is compared to historical and equilibrium estimates in table 8 below:

ETF	Black-Litterman Returns	Implied Equilibrium Returns	Historical Returns
VTIP	-0.01%	0.04%	-0.52%
VWO	4.01%	1.68%	1.73%
VEA	3.16%	1.42%	4.25%
VBR	2.64%	1.39%	10.49%
VOE	1.56%	1.25%	11.40%
VTV	1.01%	1.19%	10.68%
VTI	1.41%	1.63%	11.82%
EMB	1.52%	0.53%	-1.12%
BNDX	0.34%	0.05%	1.74%
LQD	1.00%	0.31%	-0.37%
BND	0.47%	0.13%	-0.93%
SHV	0.01%	0.00%	0.01%

Table 8: Black-Litterman returns for Betterment's portfolio, compared with historical and equilibrium returns[45]

Negative views on VTIP and VTI force the returns down compared to equilibrium returns, while returns of VWO and BNDX, for example, are corrected upwards. This shows how views influence return expectations with BL compared to the static and highly diverse historical returns. Betterment does not provide any public guidance in the form of its own estimates or assumptions, so a comparison of any kind at this point is impossible. The newly acquired return vector is then used to calculate the optimal unconstrained BL portfolio:

[44] own illustration, based on the reasoning to weight value stocks higher of Betterment.com (2018e).
[45] own illustration and calculations, based on data from Yahoo! Finance (2018)

The result of combining the Black-Litterman return vector with the posterior covariance matrix is the unconstrained Black-Litterman portfolio. Strong views result in high weight shifts compared to the equilibrium portfolio (BNDX), no views result in no shifts (BND).

ETF	BL Weights	Equilibrium Weights	Adj. Hist. Weights
VTIP	-143.9%	1.7%	-14.4%
VWO	42.6%	7.1%	-1.7%
VEA	57.7%	8.0%	-2.5%
VBR	68.3%	2.2%	0.3%
VOE	-12.9%	1.3%	4.2%
VTV	-54.2%	4.8%	3.6%
VTI	-28.7%	49.6%	5.8%
EMB	19.1%	1.0%	-4.2%
BNDX	138.7%	7.3%	17.9%
LQD	49.7%	2.6%	0.0%
BND	13.9%	13.9%	-3.1%
SHV	-50.5%	0.6%	94.4%

Table 9: The optimal unconstrained Black-Litterman portfolio, compared with adjusted historical and equilibrium weights[46]

Table 9 illuminates the edge BL holds over MPT. Negative views result in short positions of varying magnitude from the equilibrium portfolio; positive views yield higher long positions. No views are formulated for BND, so its weight remains unchanged. This essentially means that the BL portfolio can be separated into the equilibrium portfolio and another portfolio consisting of the suggested long and short positions.[47] In this case, weights with BL sum up to 100% since only relative views are stated, so long and short positions offset each other. Absolute views yield total weights above or below 100%, dependent on the views.

In order to use MVO for construction of the efficient frontier, the posterior covariance matrix must be calculated. Then, the BL returns are adjusted for Betterment's fees of 0.25%.[48] Since Betterment provides no guidance on tax adjustments, taxes are neglected in these calculations. To arrive at sensible asset weights and diversified portfolios, several levels of constraints are applied.

It is important not to employ too many constraints, since portfolios can come to resemble one's own biased perspective, making the use of optimization techniques redundant. Further calculations compare modern portfolio theory (MPT) and Black-Litterman (BL) in their capability to reproduce Betterments asset allocation. Three groups are formed, on which an increasing level of constraints is applied.

[46] own illustration and calculations, based on data from Yahoo! Finance (2018); without adjustments, historical weights will not equal 100%; resulting allocations beyond 500%.
[47] He, G.; Litterman, R. (1999), pp. 12-13
[48] applied fees resemble Betterment's "Digital" payment plan; see Betterment.com (2018f)

Constraints are necessary in an optimization process. Here, three groups with increasing constraints are formed and then compared against self-built benchmarks and Betterment's allocation and frontier.

Constraints Applied on Group Portfolios						
Constraints	**BL I**	**BL II**	**BL III**	**MPT I**	**MPT II**	**MPT III**
Short-Sale Constraint	x	x	x	x	x	x
Budget Constraint	x	x	x	x	x	x
Stock-Ratio Constraint	x	x	x	x	x	x
Maximum-Weight Constraint		x	x		x	x
SHV & VTIP Dynamic-Weight Constraint			x			x

Constraints Applied on Benchmark Portfolios				
Constraints	**MPT MP**	**BL MP**	**Equal Weighted**	**Market Weighted**
Short-Sale Constraint	x	x	x	x
Budget Constraint	x	x	x	x
Stock-Ratio Constraint	x	x	x	x
Maximum-Weight Constraint				
SHV & VTIP Dynamic-Weight Constraint				

Table 10: Groups for portfolio reconstruction by constraints applied[49]

Group BL and MPT I only uses short-sale and budget constraints, stock-ratio constraints are always applied during calculations for all groups. This means that the optimizer is constrained to present portfolios with a certain percentage of stock allocation from 0–100% in intervals of 5%, leading to a total of 20 portfolios per group. This process is justified by the above explained observation that Betterment is modeling portfolio risk through stock allocation, not by setting target returns as in standard MVO. The optimizer is then further constrained by setting maximum asset weights in BL and MPT II, obtained by analyzing Betterment's asset allocation, illustrated in figure 5.

Due to their near-zero parameters, the final level of constraints in BL and MPT III dynamically restricts the optimizer to use only the weights of VTIP and SHV that Betterment assigns to stock allocations along the 5% intervals. This is necessary, as these two assets continuously disturb the optimization process. Benchmarks are generated dynamically with Excel formulas, scaling stock ratios of optimized market portfolios up and down. Respective market portfolios are obtained by maximizing the Sharpe ratio under short-sale and budget constraints. Figure 5 illustrates the asset allocation of Betterment's 101 portfolios, against which all other group portfolios are compared.

[49] own illustration and methodology; see e.g. Stein, M. (2007), pp. 13-15 for possible constraints.

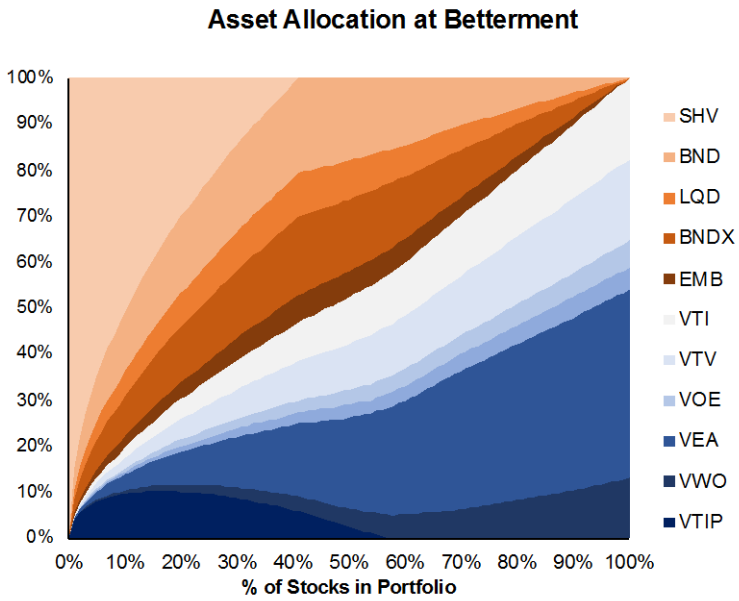


Figure 5: The 101 portfolios of Betterment, by stock ratio[50]

Betterments asset allocation shows smooth entering and exiting of asset classes, as well as an overall high degree of diversification. Using the BL returns calculated above, the efficient frontier of these portfolios and single assets is plotted in figure 6.

Betterment's weight restrictions result in an efficient frontier that does not encompass all assets. The only asset on the frontier is SHV, since their low risk portfolio is allocated 100% in SHV.

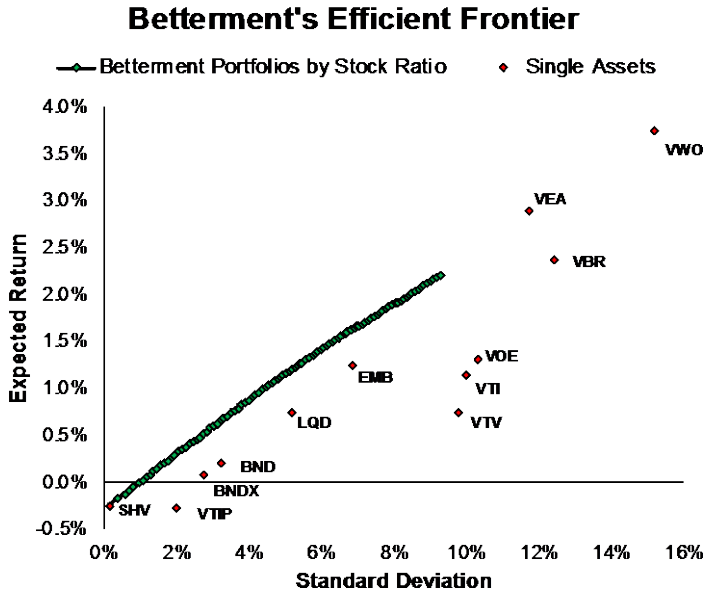


Figure 6: Single-asset portfolios and Betterment's efficient frontier[51]

Due to Betterment's weighting restrictions, some assets are not represented on the frontier. Different levels of constraints with MPT and BL optimization lead to several surprising

[50] own illustration, based on data from Betterment.com (2017)
[51] own illustration and calculations, based on data from Betterment.com (2017) and prices quotes from Yahoo! Finance.

results. In this specific framework of variance minimization subject to a given stock allocation, there is practically no difference in the average allocation in portfolios from BL and MPT I–III. This can be seen by analyzing the correlations between the 20 self-constructed portfolio weights and Betterments weights for each corresponding portfolio by stock ratio:

With increasing constraints, resulting allocations are correlated higher with Betterment's allocations.

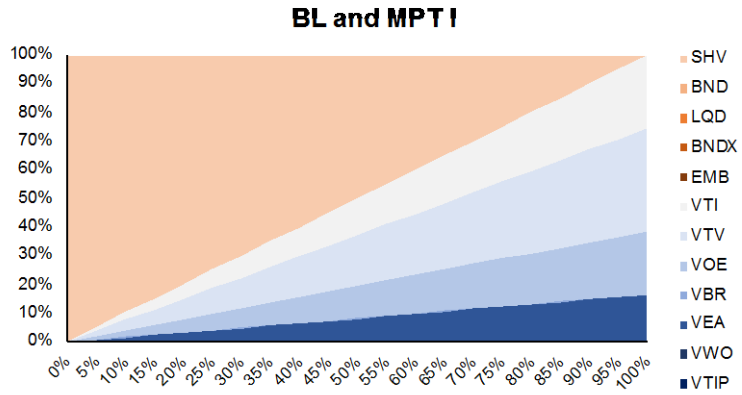
Avg. Correlation with Betterment's Asset Allocation			
BL I	0.5620	0.5620	**MPT I**
BL II	0.6389	0.6388	**MPT II**
BL III	0.9381	0.9380	**MPT III**

Table 11: Correlation of portfolio group asset allocation with Betterment's allocation[52]

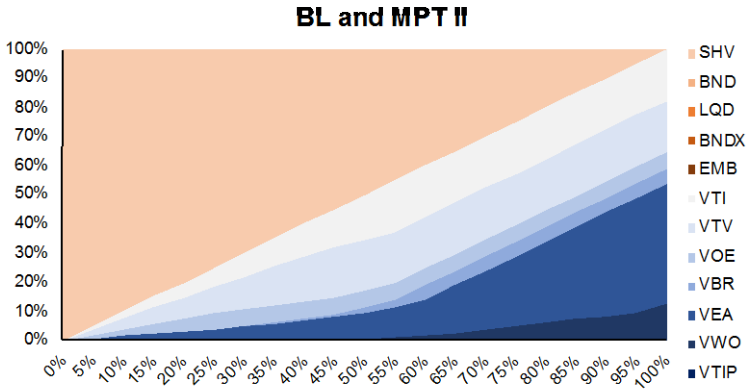
It is clear that the asset allocation resembles Betterment's more closely with each new constraint. But the difference between the two approaches is negligible. Whereas BL uses the posterior covariance matrix, MPT uses the prior covariance matrix since no views are formed within MPT. If both would use the same matrix, the difference should be zero. The following figure 7 treats both approaches from each group as equal.

[52] own illustration and calculations.

Optimization is done by adjusting stock allocation, both approaches eliminate 7 of the 12 ETFs from the portfolios.



Constraining the optimizer to stay equal to or below Betterment's maximum weights per asset results in the inclusion of two additional ETFs. SHV still dominates bond-ETF allocations.



Only by forcing the optimizer to use Betterment's weights for SHV and VTIP for a given stock allocation, portfolios close to Betterment could be built.

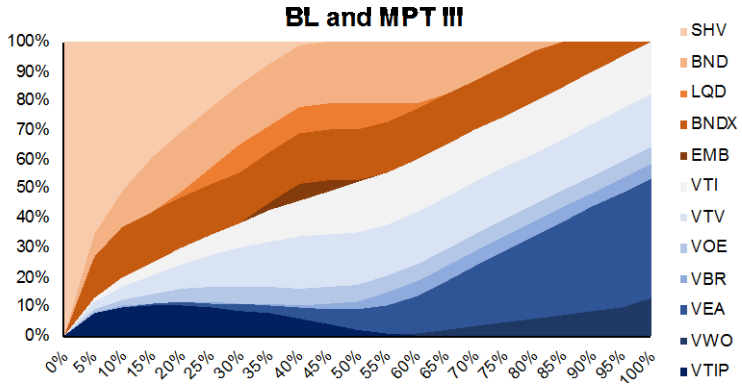


Figure 7: Self-constructed portfolios of groups BL and MPT I–III[53]

Groups I and II are of little representative value since most assets are forced out of the portfolios. Subsequent constraints allow for more diversified and intuitive portfolios heavily dependent on Betterment's maximum-weight constraints. But even with an extensive set of constraints, the continuously high diversification cannot be reproduced precisely. However, correlation suggests that BL and MPT III is a close approximation of Betterment's asset allocation overall. To compare the output above to an array of benchmarks, the market portfolio (MP) from both BL and MPT is scaled by stock ratio from 0 – 100%. The same is done with an equally weighted and a market weighted portfolio. Table 12 shows the average correlation with Betterments asset allocation.

[53] own illustration and calculations.

Although equal in correlation to Betterment's allocations, the market portfolios of BL and MPT show high differences in diversification and the number of assets included.

Avg. Correlation with Betterment's Asset Allocation			
BL MP	0.0978	0.2507	Market Weighted
MPT MP	0.0977	0.4684	Equal Weighted

Table 12: Correlation of benchmark portfolios with Betterment's allocation[54]

Low representative value is obtained in scaling the stock ratio of BL's and MPT's market portfolios. At first glance these are almost equal by correlation, but as Figure 8 demonstrates, their asset allocation differs greatly. Equal weighting scales down weights in accordance with the preset intervals for stock ratios, and the same mechanism is used for the market-weighted benchmark. Market or equal weighted benchmarks are by definition highly diversified, but not based on any form of optimization. While overall asset allocation is constantly equal between BL and MPT I–III, scaling the market portfolios of BL and MPT yields widely different allocations, as figure 8 shows.

The equal weighted portfolios are highly diversified, while the market weighted allocations are heavily invested in the US Total Market ETF.

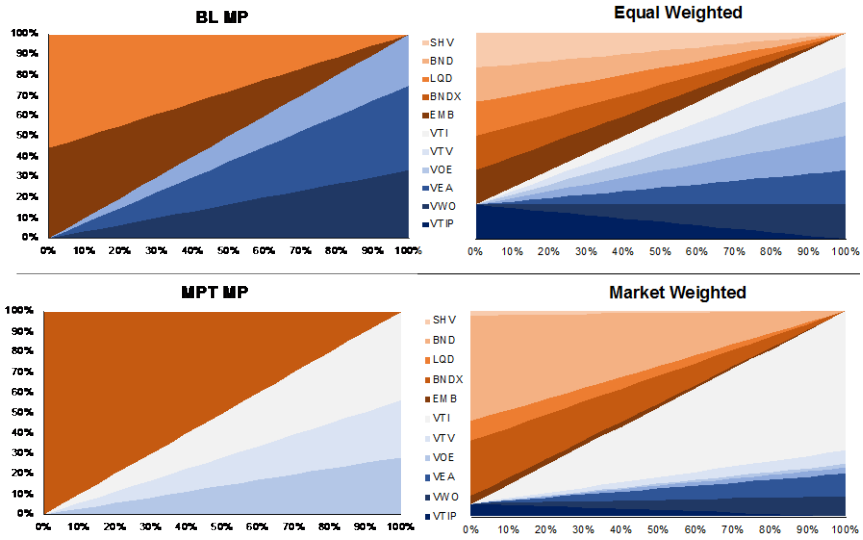


Figure 8: Self-constructed benchmark portfolios scaled by stock ratio[55]

The BL market portfolio continuously contains one more asset and is diversified on asset-class and country levels, whereas MPT's market portfolio consists only of US stocks and US corporate bonds – all highly correlated with each other.

Each self-built efficient frontier consists of 21 portfolios with stock allocations from 0–100%, making a total of 210 self-constructed portfolios. Scaling by stock ratio rather than by risk aversion mimics Betterment's scaling, as individual risk aversion is assumed to be equal to global risk aversion. Plotting expected return and standard deviation leads to the efficient frontiers in Figure 9.

[54] own illustration and calculations.
[55] own calculations and illustrations.

© Prof. Dr. Matthias Fischer

With increasing constraints, group frontiers come closer to Betterment's. The equally weighted benchmark also closely follows Betterments frontier.

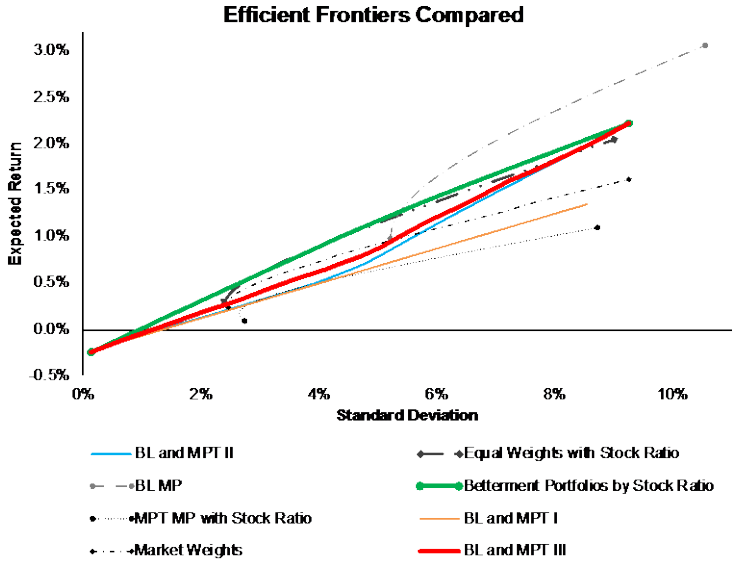


Figure 9: Efficient frontiers of all group and benchmark portfolios, compared with Betterment's frontier[56]

As constraints tighten from BL and MPT I to III, the gap between Betterment and the self-constructed frontier decreases. While the corners of the frontier are reproduced well, allocations in the middle of Betterment's frontier could not be reproduced, presumably due to the expressed views and missing data. An interesting observation is that the equal-weighted benchmark manages to very closely follow the risk-return profile of Betterment's frontier until stock allocations are down to approximately 20%. Another observation compares the two market portfolios. If scaled by stock ratio, the results differ greatly, with MPT recommending the portfolio with the lowest return expectations, where BL's frontier suggests portfolios with the highest expected returns. Difficulties in reproducing the portfolio weights of Betterment also stem from the ETFs SHV and VWO, as Figure 10 shows. This figure compares the divergence from Betterment's asset allocation of all group portfolios for a certain stock allocation.

Across all group portfolios, the ETFs SHV and VWO were responsible for most of the divergence from Betterment's allocation.

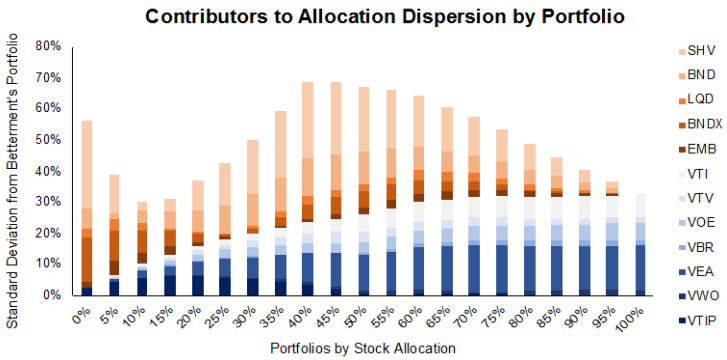


Figure 10: Contributors to allocation dispersion, by stock ratio[57]

[56] own illustration and calculations.
[57] own calculations and illustration.

© Prof. Dr. Matthias Fischer

Across all stock allocations and groups, the optimizer's weight suggestions on average diverge most for these two assets, mainly because the formulated views and return and variance of SHV are near or equal to zero. Low standard deviation of VBR, VOE, and VTV suggests that either the views taken here are stated well with regard to value ETFs or the estimated risk-return profile is close to Betterment's. Further analysis of how well groups I to III and self-defined benchmarks reproduce the allocations of each asset is displayed in Figure 11. By first averaging the per-asset allocations across all different group portfolios and benchmarks, then subtracting the average of Betterment's allocation in that asset, the absolute deviation from the desired outcome is attained.

The lowest deviations from Betterment's portfolios per asset can be found with BL and MPT III and the equal weighted benchmark. The market portfolios expressed the highest deviations across most of the assets.

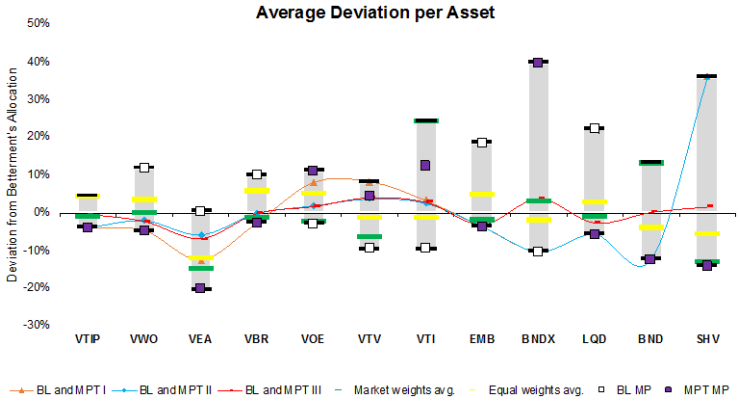


Figure 11: Average per-asset deviation from Betterment's allocation[58]

The data also corresponds to the deviations from Figure 10, now highlighting the positive effects constraints have on reproduction accuracy. Self-constructed BL and MPT market portfolios are represented by the white and purple squares, respectively, and tend to deviate greatly. On average, an equally weighted and stock-ratio-scaled portfolio comprising these 12 assets produces the best results. Reasons for this can be found among the manifold unknown variables like assumptions regarding market weights, risk aversion, covariance, confidence, and scalar parameters. Manually smoothing the allocation curve in an arbitrary way adds to distortion. Overall, the constrained portfolios from BL and MPT III are considered a valid representation of Betterment's asset allocation, given the high degree of arbitrariness from an external point of view.

[58] own calculations and illustration.

Concluding Remarks

Following through with the asset selection process and arriving at an asset allocation using Black-Litterman is highly complex compared to using classical mean-variance optimization. While Black-Litterman circumvents some weaknesses of modern portfolio theory, it has limitations of its own. Many of the model's parameters are often assumed a value due to the inability of a truthful estimation. This includes the confidence scalar Tau, global risk aversion Delta, and view error variances.[59] The views themselves are then prone to behavioral biases,[60] and imbalanced market weights can lead to distorted equilibrium portfolios. As this study showed, when optimizing portfolio risk by adjusting the stock ratio, both approaches produce almost equal results. Other relevant issues encountered during calculations were variances and returns near or equal to zero, leading to large short and long positions reminiscent of portfolios from modern portfolio theory.

Asset allocation at Betterment could not be reproduced exactly, but even without numerous iterations of Monte Carlo at various points and without insights into their assumptions, results are within the range of the desired outcome. The asset allocations obtained from using the Black-Litterman model may be intuitive and *"easy to understand*,"[61] but the way of obtaining them certainly is not. In case of no views, unclear views, or great confidence in the market, an investor should invest in the equilibrium portfolio and avoid the complexity of the BL model. However, in the case of an informed investor with confidence in his views, the Black-Litterman model allows for a comprehensive yet complex solution with a personal touch.

[59] Idzorek, T.M. (2005), pp. 10-11; He, G.; Litterman, R. (1999), p. 11
[60] post.nyssa.org (2011)
[61] He, G.; Litterman, R. (1999), p. 13

References

AssetBuilder.com (2018a): Questionnaire; from: https://assetbuilder.com/start-now; last accessed on 07.03.2018.

Betterment.com (2014a): Investment Selection: Building Portfolios, Fund by Fund; from: https://www.betterment.com/resources/investment-strategy/etfs/good-investment-selection-science-art/; last accessed on 07.03.2018.

Betterment.com (2014b): The Betterment Guide to 'Investment' Cash; from: https://www.betterment.com/resources/investment-strategy/portfolio-management/what-is-the-role-of-cash-in-investing/; last accessed on 07.03.2018.

Betterment.com (2014c): Inside Betterment's Investment Advice; from: https://www.betterment.com/resources/inside-betterment/investment-advice/; last accessed on 07.03.2018.

Betterment.com (2015): The Case Against Commodities; from: https://www.betterment.com/resources/investment-strategy/the-case-against-investing-in-commodities/; last accessed on 07.03.2018.

Betterment.com (2016): The History of Betterment: How we started a Company that changed an Industry; from: https://www.betterment.com/resources/inside-betterment/our-story/the-history-of-betterment/; last accessed on 07.03.2018.

Betterment.com (2017): Where in the World are you Invested?; from: https://www.betterment.com/resources/investment-strategy/portfolio-management/portfolio-diversification/; last accessed on 07.03.2018.

Betterment.com (2018a): Betterment's own and third party portfolios; from: https://www.betterment.com/portfolio/; last accessed on 07.03.2018.

Betterment.com (2018b): Our Investment Selection Methodology, from: https://www.betterment.com/resources/research/etf-portfolio-selection-methodology/; last accessed on 07.03.2018.

Betterment.com (2018c): Our Stock Allocation Advice; from: https://www.betterment.com/resources/research/stock-allocation-advice/; last accessed on 07.03.2018.

Betterment.com (2018d): Questionnaire; from: https://wwws.betterment.com/app/guidance?signupStartSource=BrochureSignupCta; last accessed on 07.03.2018.

Betterment.com (2018e): The Betterment Portfolio Strategy; from: https://www.betterment.com/resources/research/betterment-portfolio-strategy/; last accessed on 07.03.2018.

Betterment.com (2018f): Pricing; from: https://www.betterment.com/pricing/; last accessed on 07.03.2018.

Betterment.com (2018g): Our Goals and Advice Explained; from: https://www.betterment.com/resources/research/goals-advice-explained/; last accessed on 11.03.2018.

Black, Fisher; Litterman, Robert (1992): Global Asset Allocation With Equities, Bonds and Currencies; In: Goldman Sachs Fixed Income Research; Goldman Sachs & Co.

Charles Schwab IPS .com (2018): Questionnaire; from: https://hg.schwab.com/client/#/getStarted?return=%2Fpublic%2Fintelligent%2Fhome.html; last accessed on 07.03.2018.

Cheung, Wing (2009): The Black-Litterman Model Explained; Nomura International PLC; London.

Credio Investment Advisors (2018): Robo-Advisor Company Profiles; from: http://investment-advisors.credio.com/l/31372/Betterment; similar manner for all other named advisors except Schwab IPS and Vanguard; last accessed on 07.03.2018.

Crunchbase.com (2018): Betterment company profile; from: https://www.crunchbase.com/organization/betterment; last accessed on 07.03.2018.

Ernst, Dietmar; Schurer, Marc (2015): Portfolio Management – Theorie und Praxis mit Excel und Matlab; UVK Verlagsgesellschaft mbH; Konstanz und München.

ETF.com (2018): ETF screening tool used for asset selection; from: http://www.etf.com/etfanalytics/etf-finder; last accessed on 07.03.2018.

Fama, Eugene F. (1970): Efficient Capital Markets: A Review of Theory and Empirical Work; In: The Journal of Finance, Vol. 25 Nr. 2; May, 1970; pp. 383-417.

Fama, Eugene F.; French, Kenneth (2004): The Capital Asset Pricing Model: Theory and Evidence; In: Journal of Economic Perspectives; Vol. 18 Nr. 3; Summer 2004; pp. 25-46.

Fastcompany.com (2017): Robo-Advisor Betterment Is On A Personalization Push As It Surpasses $10 Billion In AUM; from: https://www.fastcompany.com/40442080/robo-advisor-betterment-is-on-a-personalization-push-as-it-surpasses-10-billion-in-aum; last accessed on 07.03.2018.

Fischer, Matthias (2017): Robo Advisory und automatisierte Vermögensverwaltung; In: Zeitschrift für das gesamte Genossenschaftswesen; Band 67, Heft 3; pp. 183-193.

Funderbeam.com (2018): Betterment Company Profile; Requires account creation, from: https://funderbeam.com/startups/betterment; last accessed on 07.03.2018.

FutureAdvisor.com (2018): Investment Approach; from: https://www.futureadvisor.com/content/what-we-do/investment-philosophy/nobel-prize-winning-long-term-strategy; last accessed on 08.03.2018.

He, Guangliang; Litterman, Robert (1999): The Intuition Behind Black-Litterman Model Portfolios; Goldman Sachs; New York.

Idzorek, Thomas M. (2005): A Step-by-Step Guide to the Black-Litterman Model – Incorporating User-Specified Confidence Levels; Ibbotson Associates; Chicago.

Ledoit, Olivier; Wolf, Michael (2004): Honey, I Shrunk the Sample Covariance Matrix; In: Journal of Portfolio Management; Vol. 31, Nr. 1; Fall 2004; pp. 110-119.

Markowitz, Harry (1952): Portfolio Selection; In: The Journal of Finance; Vol. 7 Nr.1; pp. 77-91.

Orçun, Kaya (2017): Robo-advice – a true innovation; from: https://www.dbresearch.de/PROD/RPS_DE-PROD/PROD0000000000449125.pdf; last accessed on 07.03.2018.

post.nyssa.org (2011): Black-Litterman's Model: Portfolio's Friend or Foe?; from: http://post.nyssa.org/nyssa-news/2011/03/black-littermans-model-portfolios-friend-or-foe.html; last accessed on: 25.02.2018.

Sharpe, William F. (1990): Capital Asset Prices With and Without Negative Holdings; In: Nobel Lecture, December 7, 1990; pp. 312-332.

Stein, Michael (2007): Mean-Variance Portfolio Selection With Complex Constraints; Karlsruher Institut für Angewandte Informatik und Formale Beschreibungsverfahren; Karlsruhe.

Wealthfront.com (2018a): Questionnaire; from: https://www.wealthfront.com/questions; last accessed on 07.03.2018.

Wealthfront.com (2018b): Investment Methodology Whitepaper; from: https://research.wealthfront.com/whitepapers/investment-methodology/; last accessed on 07.03.2018.

Yahoo! Finance (2018): Historical ETF Price Quotes; from: https://de.finance.yahoo.com; last accessed on 07.03.2018.

© Prof. Dr. Matthias Fischer

Kontakt

FINANCIAL RESEARCH NOTES

Professor Dr. Matthias Fischer
Kompetenzzentrum Finanzen
Technische Hochschule Nürnberg Georg Simon Ohm
Fakultät Betriebswirtschaft
Bahnhofstr. 87
90402 Nürnberg
matthias.fischer [at] th-nuernberg.de
www.th-nuernberg.de

Important Disclosures

© Prof. Dr. Matthias Fischer. All rights reserved.

FINANCIAL RESEARCH NOTES

© Copyright 2018 Matthias Fischer, Nürnberg. Alle Rechte vorbehalten. Bei Zitaten wird um Quellenangabe „Matthias Fischer Financial Research Notes“ gebeten.

Die vorstehenden Ausführungen stellen weder eine Anlage-, Rechts- noch Steuerberatung dar. Sämtliche Meinungsäußerungen geben die aktuelle Einschätzung des Verfassers wieder, die nicht zwingend der Meinung von Matthias Fischer entspricht. Alle Meinungen können ohne vorherige Ankündigung geändert werden. Die Aussagen können von Einschätzungen abweichen, die in anderen von Matthias Fischer veröffentlichten Dokumenten, einschließlich Research-Publikationen, vertreten werden. Die vorstehenden Angaben werden nur zum Zwecke der Information und ohne vertragliche oder sonstige Verpflichtung zur Verfügung gestellt. Für die Richtigkeit, Vollständigkeit oder Angemessenheit der Informationen & Meinungen in dieser Studie wird keine Gewähr übernommen. Soweit wir in den Meinungsäußerungen dieser Studie Prognosen oder Erwartungen äußern oder unsere Aussagen die Zukunft betreffen, können diese Aussagen mit bekannten und unbekannten Risiken und Ungewissheiten verbunden sein. Die tatsächlichen Ergebnisse und Entwicklungen können sich daher deutlich von den geäußerten Erwartungen und Annahmen differenzieren. Neben weiteren, hier nicht erwähnten Gründen, können sich eventuelle Abweichungen aus Veränderungen der allgemeinen wirtschaftlichen Lage, der Wettbewerbssituation und der Rechtslage, vor allem in Kerngeschäftsfeldern und -märkten der Unternehmen, ergeben. Auch die Entwicklung der Finanzmärkte und Wechselkurse sowie nationale und internationale Normen- und Gesetzesänderungen können einen Einfluss haben. Matthias Fischer übernimmt keine Gewähr für Fehler und Aktualität in der Studie. Die Meinungen dieser Studie sind keine Investitionsberatung und ersetzen keine persönliche und individuelle Finanzanalyse und Beratung.

The opinions expressed in this material do not constitute investment advice or individual financial analysis and you should consult your investment or corporate finance advisor before you make any financing or investment decision The value and income of any of the securities or investments and the price of shares and the income derived from them, which are mentioned in this material, may fall as well as rise. Investors may not receive the original amount invested in return. Statements concerning taxation are based on our understanding of the taxation law in force at the time of publication. The levels and bases of taxation may change. You should obtain professional advice on taxation where appropriate before proceeding with any investment. Investors should also be aware that past performance is not necessarily a guide to future performance. No liability is accepted for the information in this material. This material does not constitute a solicitation in any jurisdiction in which such a solicitation is unlawful or to any person to whom it is unlawful. Moreover, this information neither constitutes an offer to enter into an investment agreement with the recipient of this document nor an invitation to respond to the document by making an offer to enter into an investment agreement. This material has been communicated in Germany. Opinions expressed in this material are current opinions as of the date appearing in this material only. Matthias Fischer does not guarantee that the information in this material is correct and up to date. The information in this material can be changed anytime. No part of this material may, without prior written consent of Matthias Fischer, be (i) copied, photocopied or duplicated in any form, by any means, or (ii) distributed to any person that is not an authorised person of the recipient. In case of citation please use “Matthias Fischer Financial Research Notes” as the source.

Internet: ISSN 2364-9437