

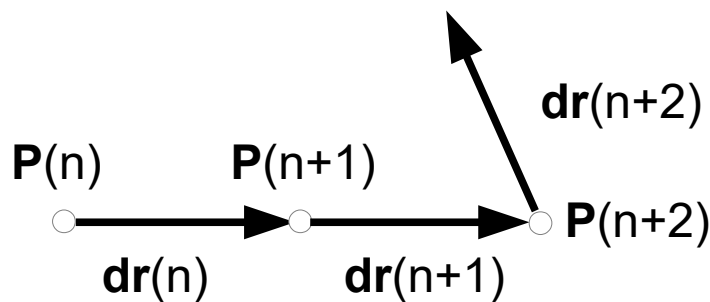
Experiments with vectors

In the Euclidean space some bivectors are created. These can be linked together. As a result, more or less stable particles are generated by self-organization. These particles influence each other in many ways. They can merge, disintegrate or are positioned more or less close to each other. The executable program Particle.jar demonstrates the variety of dynamic behavior in which the particles interact.

It has long been attractive for me to try out mathematical experiments with my computer. I wanted to move through the Euclidean space with the help of simple vector calculation rules. These experiments produced effects that, I think, are quite entertaining.

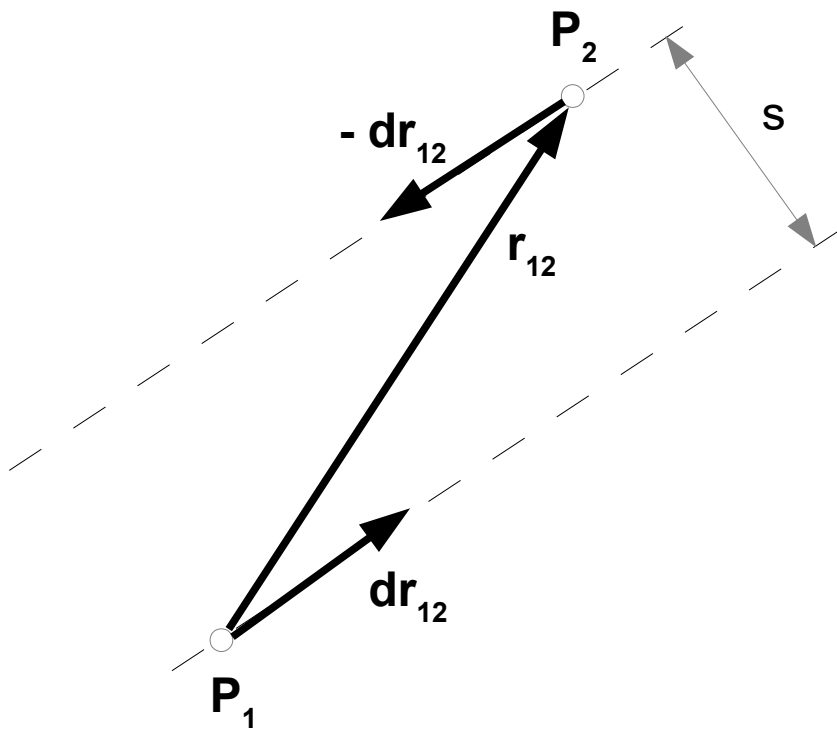
Iteration

Let's start with a point \mathbf{P} in space. This position vector \mathbf{P} should be moved. This can be done step by step with a displacement vector \mathbf{dr} . The magnitude of the displacement vector is equal to one for the sake of simplicity. In the computer program, I insert a loop, which is processed n times. This opens up a discrete dynamic system.



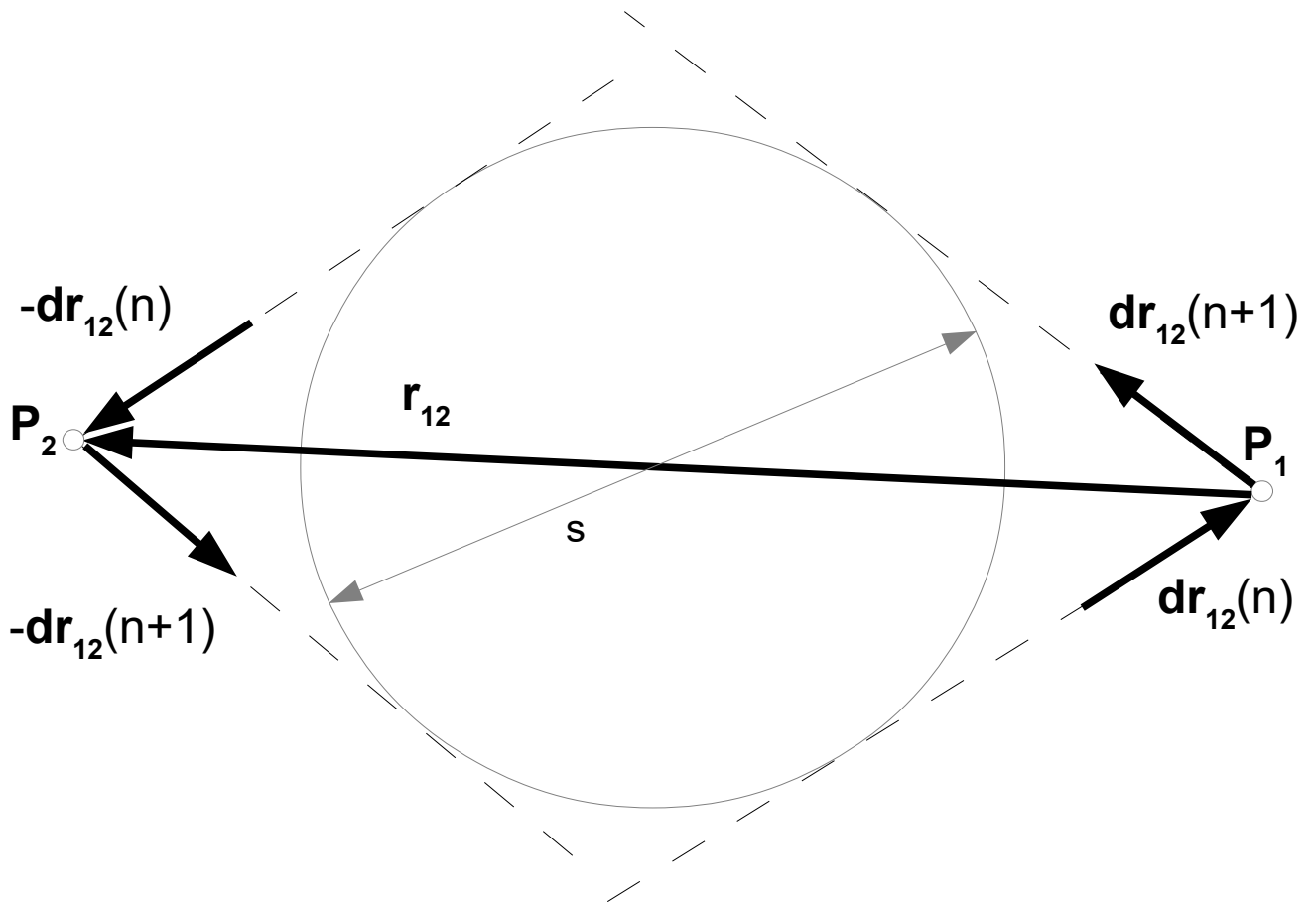
Linking

A point P_1 that moves step by step along a straight line is still pretty boring. Let's give him a partner P_2 and link them together. This is to be done so that the points move in opposite directions on two parallels with the distance s . As a result, the center of gravity of this pair of points remains fixed in space and also this constructed "particle" has a constant torque s . The displacement vector $d\mathbf{r}_{12}$ and the distance vector \mathbf{r}_{12} form a parallelogram whose area is equal to s . Such configurations are known as simple bivectors, which are plane segments with a certain orientation and area in the space.



Change of direction

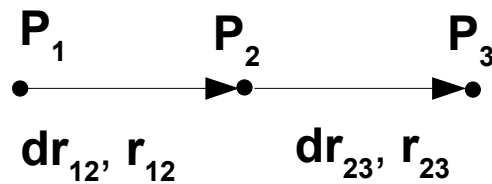
So far, the algorithm is still not very efficient. The two points move antiparallel straight ahead and soon disappear from the screen. So I have to install a change of direction. It would be nice if the bivector did not change, meaning that the torque and orientation in the space remained constant. Well, this task is also known, this is a special form of the mathematical system Outer Billiard. The change of direction takes place when the condition $|\mathbf{r}_{12}| > e$ applies. The parameter e thus determines when the points \mathbf{P}_1 and \mathbf{P}_2 are reflected. On the screen now appears a circular disk, which has any orientation in the space.



Superposition

So far the algorithm still provides some monotonous shapes, but that changes immediately as we place more points into the space and link them together. In order to keep track, it makes sense to introduce directional graphs. The points corresponds to the nodes and the linking of two points corresponds to the edges of the graph.

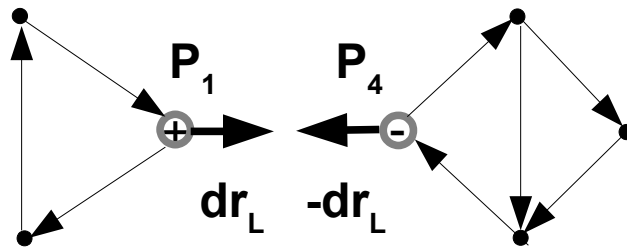
The graph drawn in the example consists of 3 nodes and 2 edges. Point P_1 is moved by the displacement vector \mathbf{dr}_{12} , point P_2 by $(-\mathbf{dr}_{12} + \mathbf{dr}_{23})$ and point P_3 by $-\mathbf{dr}_{23}$. The displacements of the points are simply superpositioned. Even now, the center of gravity remains constant.



Now the situation becomes interesting. Starting with a chaos, the vectors often organize after a while to more or less ordered structures, the variety of possible "particles" is virtually unlimited.

Charges

Now an additional method brings more movement into the process. For this one forms several graphs, which are not linked to each other. These are initially self-organizing completely independently of each other. Then I introduce some points which carry a sort of „charges“. However, this should mean nothing else than that the charged points are moved towards each other radially in the case of unlikely charges by the vector \mathbf{dr}_L . In the case of likely charges, the corresponding points move slightly away from each other. It should be $0 \leq |\mathbf{dr}_L| < 1$ apply. These shifts are also superposed for each point.



Now the particles are coupled with each other and they are performing a quite moving scenario. The Java program Particle.jar makes it possible to create any number of situations by varying the parameters and settings. The intensity of the charge coupling $|\mathbf{dr}_L|$ can be adjusted by "pull".

The program Particle.jar can be downloaded:

<https://drive.google.com/file/d/1bciPNyIINusKPjJIDJb4UFnUAbY8ITLq/view?usp=sharing>

Now the joy of experimenting are no longer limited.

Links

<https://en.wikipedia.org/wiki/Bivector>

https://en.wikipedia.org/wiki/Outer_billiard

<https://opus4.kobv.de/opus4-ohm/frontdoor/index/index/docId/126>