

# **Dynamic Analysis of the Dortmund University Campus Sky Train**

**Reinhold Meisinger**

Mechanical Engineering Department  
Nuremberg University of Applied Sciences  
Kesslerplatz 12, 90121 Nuremberg, Germany

## **Abstract**

The Sky Train at Dortmund University campus is a driverless transit system. Cabins suspended from the elevated guideway glide along towards to their destination. The vehicles can run on the double track route at one-minute intervalls with a nominal speed of 50 km/h. The guideway consists of box-shaped steel beams with a narrow slot on bottom. With its structure it forms the inside support wheel and guidance wheel tracks for the bogies. The passenger cabins hanging on the bogies are oscillating in lateral direction influenced by the periodic track irregularities. For different system parameters the response of the lateral cabin acceleration at the center of gravity is determined by frequency response analysis and computer simulation. It is shown that by optimizing the design parameters the passenger ride comfort in lateral direction can be improved enormously. Further improvement is possible only by active control.

The original version of the paper was presented on the Second International Conference on Dynamics, Vibration and Control, Beijing, P.R. China 2006.



## 1. Introduction

The Sky Train is an automatic guided transit system for passenger transport used in the university campus of Dortmund, cf. Boehm et al [1]. Depending on the number of customers, up to five vehicles can operate simultaneously on the double-track route in each direction. During off-peak hours they can operate on request: namely, a vehicle automatically starts moving when passengers request it and takes them to their individual destinations, cf. Rahier [2]. As shown in Fig. 1 the design of the guideway beams and the guidance system make a considerable contribution to the availability of the overall system. The box-shaped steel beams are entirely closed except for a narrow slot along the bottom. With its supporting structure it forms the inside tracks for support and guidance wheels on the bogies. The mechanism for the track switches and the conductor rails to supply electric power to the vehicles are also enclosed and therefore optimally protected against inclement weather even in snow and ice and accidental contact, cf. Siemens [2]. Each Sky Train vehicle consists of one or two passenger cabins, each hanging on two bogies. These driverless vehicles are moved by the automatic low-noise propulsion system with four solid-rubber tired support wheels per bogie at speeds up to 50 km/h. In vertical direction comfortable air suspensions on the top of the cabin ensure a pleasant ride comfort at higher speeds. However in lateral direction each cabin connected by joints with the bogies is oscillating influenced by the periodic irregularities of the support tracks. While significant information exists for the dynamic response in the vertical direction very little information exists on the dynamic response of the cabin acceleration in lateral direction, cf. Meisinger [4].

In this paper frequency response analysis is used to compute the resonance diagram of the lateral cabin center of gravity acceleration for the undamped system. For practical cases this is sufficient because the circular damping of the cabin is very small and at nominal speed the excitation frequency is very high compared with the natural frequency of the system. Simulation results with included relative circular damping between passenger cabin and bogie confirm this assumption. Because state space notation is used, for further investigations the computer simulation easily can be extended with an actively controlled passenger cabin.



Fig. 1: Sky Train at Dortmund University Campus, Germany

## 2. Mathematical Model of the Track

As shown in Fig. 2 and Fig. 3, the critical wave length of the track irregularities and the according amplitude are  $l$  and  $A$  respectively. Due to the distance between the two bogies is assumed to be  $3 \cdot l$  and the distance between the axes is assumed to be  $l$  all left support wheels are following the track with  $A \cdot \cos(\Omega \cdot t)$  while all right support wheels are following the track with  $-A \cdot \cos(\Omega \cdot t)$ . Because the bogies with the relevant dimensions  $a$  and  $b$  are assumed to be rigid, the lateral displacement of the bogie in the joint B can be computed as

$$y = A \cdot b/a \cdot \cos(\Omega \cdot t), \quad (1)$$

with the circular excitation frequency  $\Omega = 2 \cdot \pi \cdot v/l$ , where  $v$  is the constant vehicle speed.

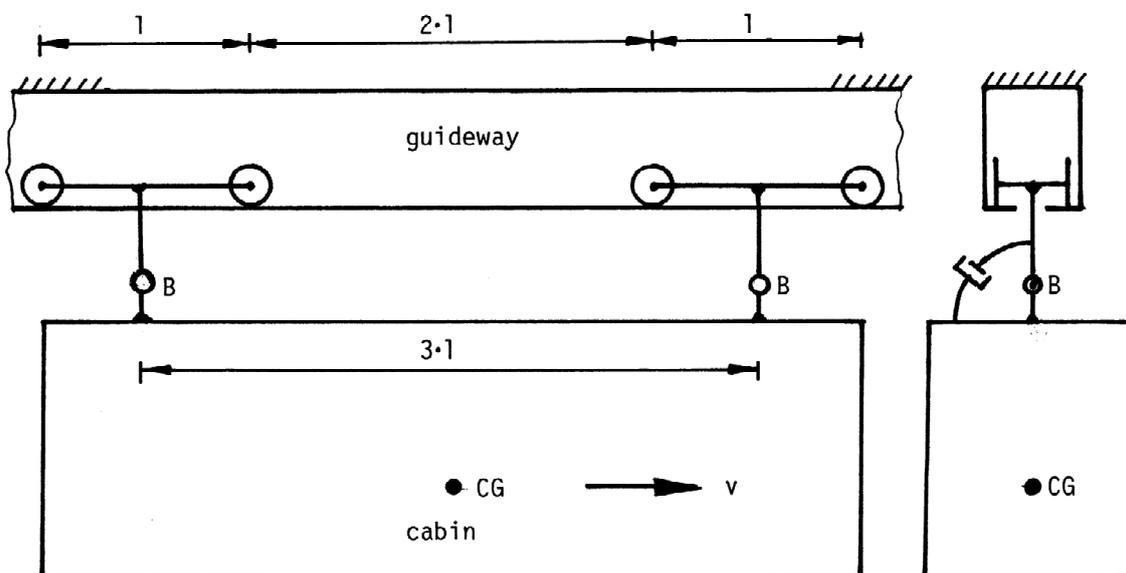


Fig. 2: Mathematical model of the Sky Train and the track

## 3. Mathematical Model of the Cabin

The cabin with mass  $m$ , center of gravity CG and moment of inertia  $J_c$  with respect to CG is connected with the bogie in joint B. The distance between cabin center of gravity CG and joint B is  $l_c$ , cf. Fig. 3. With the two degrees of freedom (DOF)  $\varphi$  and  $y_c$ , which are assumed to be small, the differential equation of motion without circular damping can be written as

$$J_c \cdot d^2\varphi/dt^2 + m \cdot l_c \cdot d^2y_c/dt^2 + m \cdot g \cdot l_c \cdot \varphi = 0. \quad (2)$$

With  $y_c = y + \varphi \cdot l_c$  and with the moment of inertia  $J_B = J_c + m \cdot l_c^2$  with respect to B another form of eq. (2) is

$$J_B \cdot d^2\varphi/dt^2 + m \cdot g \cdot l_c \cdot \varphi = -m \cdot l_c \cdot d^2y/dt^2. \quad (3)$$

Differential equation (3) will be directly used for the frequency response analysis of the undamped system.

With eq. (1) the amplitude of the lateral acceleration  $d^2y/dt^2$  in the joint B is  $\Omega^2 \cdot A \cdot b/a$ . The according differential equation with included circular damping is given in eq. (8).

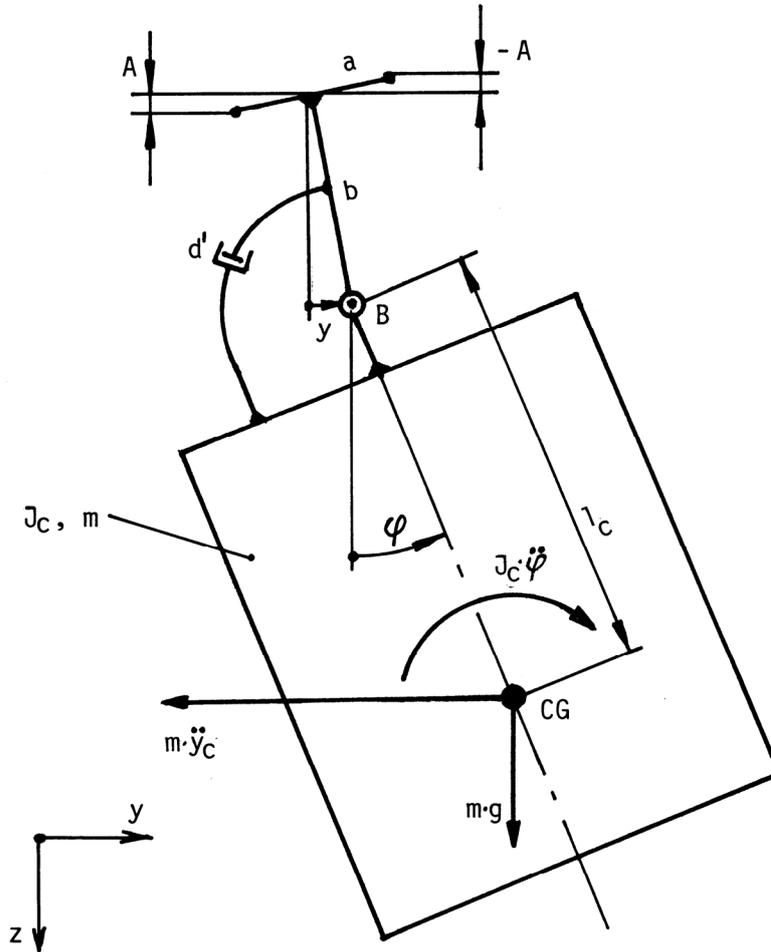


Fig. 3: Mathematical model of the passenger cabin

#### 4. Frequency Response Analysis

With LAPLACE transformation of eq. (3) the complex transfer function

$$\varphi = \frac{-p^2 \cdot m \cdot l_c}{p^2 \cdot J_B + m \cdot g \cdot l_c} \cdot y \tag{4}$$

is obtained, where  $p = j\Omega$ . With  $y_c = y + \varphi \cdot l_c$  the frequency response for the lateral center of gravity displacement  $y_c$  of the passenger cabin can be written as

$$y_c = \frac{m \cdot g \cdot l_c - J_c \cdot \Omega^2}{m \cdot g \cdot l_c - J_B \cdot \Omega^2} \cdot y \tag{5}$$

With the frequency ratio  $\eta = \Omega/\omega_0$ , where  $\omega_0$  is the natural circular frequency ( $\omega_0^2 = m \cdot g \cdot l_c / J_B$ ) of the undamped passenger cabin, with eq.(5) the frequency response for the lateral center of gravity acceleration becomes

$$\frac{d^2 y_c / dt^2}{\Omega^2 \cdot A \cdot b / a} = \frac{1 - J_C / J_B \cdot \eta^2}{1 - \eta^2} \quad (6)$$

If the frequency ratio  $\eta > 10$ , eq. (6) can be simplified as

$$\frac{d^2 y_c / dt^2}{\Omega^2 \cdot A \cdot b / a} = \frac{J_C}{J_B} \quad (7)$$

From eq. (7) it can be seen, that small ratios  $J_C / J_B$  leads to small lateral accelerations of the cabin in CG.

## 5. Computer Simulation

If eq. (3) is extended by the circular damping the following cabin differential equation is obtained

$$J_B \cdot d^2 \varphi / dt^2 + d' \cdot d\varphi / dt + \omega_0^2 \cdot \varphi = d' \cdot d\beta / dt - m \cdot l_c \cdot d^2 y / dt^2, \quad (8)$$

where  $d'$  is the circular damping constant and  $\beta = y/b$ , cf. Fig. 3. The system will be extended by the differential equation for the periodic lateral excitation in joint B with the initial condition  $y_0 = A \cdot b/a$ .

$$d^2 y / dt^2 = -\Omega^2 \cdot y. \quad (9)$$

With  $\omega_0^2 = m \cdot g \cdot l_c / J_B$  and the circular damping factor  $D = d' / (2 \cdot J_B \cdot \omega_0)$  the two differential equations (8) and (9) can be written in state space notation

$$dx/dt = \mathbf{A} \cdot \mathbf{x}, \quad d^2 y_c / dt^2 = \mathbf{C} \cdot \mathbf{x}, \quad \mathbf{x}^T = [\varphi, d\varphi/dt, y, dy/dt], \quad y(0) = y_0, \quad (10)$$

where  $\mathbf{x}$  is the 4×1-state vector including the states of the cabin and the states of the periodic excitation,  $\mathbf{A}$  is the 4×4-system matrix and  $\mathbf{C}$  is the 1×4-output matrix with the following matrix elements

$$a_{12} = a_{34} = 1 \quad a_{21} = -\omega_0^2 \quad a_{22} = -2 \cdot D \cdot \omega_0 \quad a_{23} = \omega_0^2 \cdot \Omega^2 / g \quad a_{24} = 2 \cdot D \cdot \omega_0 / b \quad a_{43} = -\Omega^2$$

$$c_{11} = a_{21} \cdot l_c \quad c_{12} = a_{22} \cdot l_c \quad c_{13} = a_{23} \cdot l_c - \Omega^2 \quad c_{14} = a_{24} \cdot l_c$$

All other matrix elements in  $\mathbf{A}$  and  $\mathbf{C}$  are zero.

## 6. Numerical Results

With eq. (6) the resonance diagram without circular damping shown in Fig. 4 is plotted for different moment of inertia ratios  $J_C / J_B$ . This diagram includes two limiting cases: With  $J_C / J_B = 0$  the cabin is a single mass, that means  $J_C = 0$  and with  $J_C / J_B = 1$  joint B is at the center of gravity CG, that means  $l_c = 0$ .

For the computer simulation the following system parameters are assumed, which are not equal but similar to the parameters of the real Sky Train.

cabin mass	$m = 5000 \text{ kg}$	cabin moment of inertia	$J_C = 10000 \text{ kgm}^2$	speed $v = 50 \text{ km/h}$
wave length	$l = 2 \text{ m}$	damping factor	$D = 0.2$	distance $l_c = 2 \text{ m}$
amplitude	$A = 0.001 \text{ m}$	bogie distances	$a = 0.2 \text{ m}; b = 0.6 \text{ m}$	

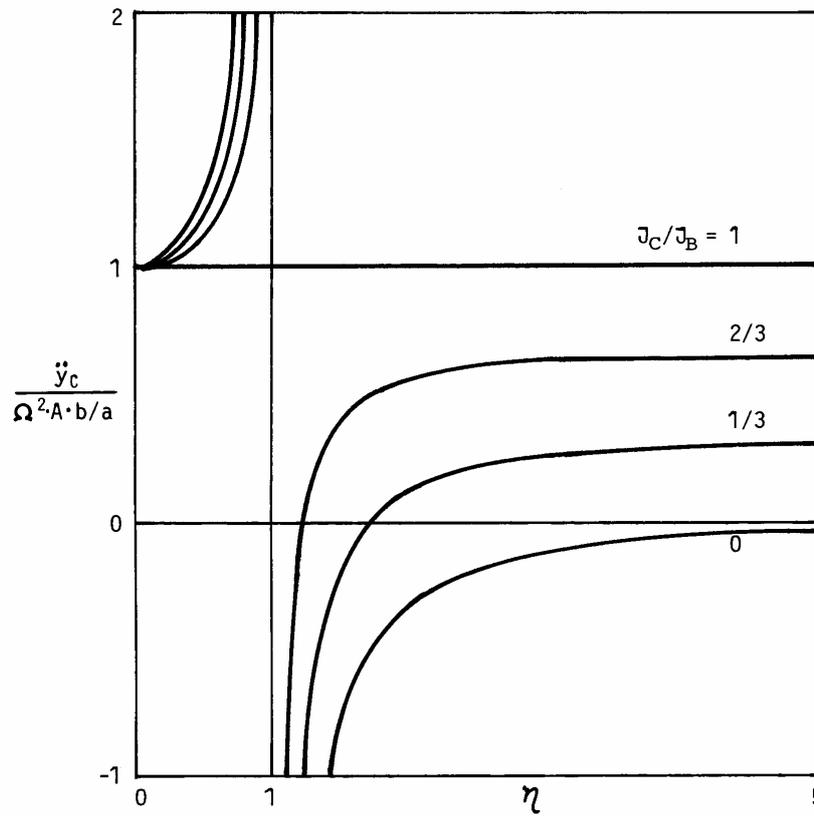


Fig. 4: Resonance diagram of lateral cabin acceleration without circular damping

Then the natural circular frequency  $\omega_0$ , the circular excitation frequency  $\Omega$ , the frequency ratio  $\eta$ , the cabin moment of inertia  $J_B$  with respect to B and the initial condition  $y_0$  are:

$$\omega_0 = 1.80 \text{ 1/s} \quad \Omega = 43.63 \text{ 1/s} \quad \eta = 24.22 \quad J_B = 30000 \text{ kgm}^2 \quad y_0 = 0.003 \text{ m}$$

With this system parameters the lateral acceleration in cabin center of gravity results in  $1.9 \text{ m/s}^2$ , cf. Fig. 5. The same result is received from frequency response analysis in Fig. 4 with a moment of inertia ratio  $J_C/J_B = 1/3$  and a lateral joint acceleration  $\Omega^2 A \cdot b/a = 5.71 \text{ m/s}^2$ .

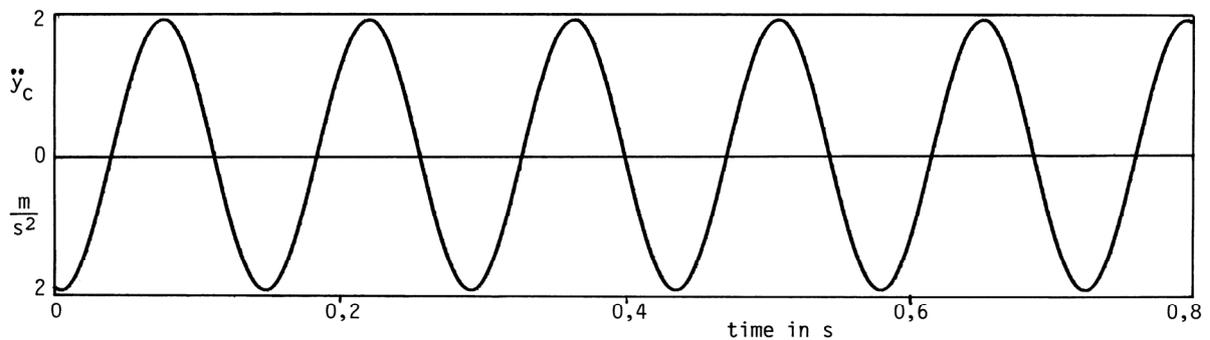


Fig. 5: Time histories of lateral cabin acceleration with circular damping factor  $D = 0.2$

## 7. Conclusion

A basic analysis to determine the response of the lateral center of gravity acceleration of a Sky Train cabin on a guideway with periodic track irregularities has been developed. Studies have shown that the frequency response analysis for the undamped system is in very good agreement with simulation results for the system with circular damping when the exciting frequency is high compared with the natural frequency of the system. For the given system parameters and a travelling speed of 50 km/h the lateral acceleration in center of gravity of the cabin is  $1.9 \text{ m/s}^2$ . If the bogie distance  $b$  is reduced to 0.1 m and the distance  $l_c$  is enlarged to 2.5 m (joint B is moved up 0.5 m) then a moment of inertia ratio  $J_C/J_B = 0.242$  results. This gives a reduction of lateral acceleration to  $0.23 \text{ m/s}^2$ . With this simple change of the design parameters the passenger ride comfort in lateral direction can be improved enormous. In further research an actively controlled cabin will be investigated.

## REFERENCES

- [1] Boehm, E., Frisch, H., 1994, "The New Operating System of the H-Train in Dortmund", Verkehr und Technik, Band 47, Heft 10, S. 465-470.
- [2] Rahier, H.W., Scharf, P., 2002; "Sicherheitstechnische Prüfung der fahrerlosen Kabinenbahn des Flughafens Duesseldorf", Signal und Draht, Band 94, Heft 10, S. 20-22.
- [3] Siemens Transportation Systems, 1996; "The Sky Train at Duesseldorf International Airport" Turnkey Systems, Order No. A19100-V400-B557-X-7600.
- [4] Meisinger, R. , Analysis of the Lateral Dynamics of a Sky Train with Periodic Track Irregularities. Proc. of the Second International Conference on Dynamics, Vibration and Control, Beijing, P.R. China 2006.