# Simulation of a Single and Double-Span Guideway under Action of Moving MAGLEV Vehicles with Constant Force and Constant Gap

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## Abstract

In 2003 the Shanghai MAGLEV Transportation System Transrapid starts operation on an eleveted single and double-span guideway with speeds over 400 km/h. The general simulation problem of a high speed MAGLEV vehicle on a flexible guideway includes two closely related problems, both of which are, in a sense, limiting cases. In the first case the magnet force is constant and in the second case the magnet gap is constant. To describe the dynamics of the distributed parameter system, modal analysis technique is used. Simulation results are given comparing the solutions for dynamic displacements of a single and a double-span guideway, when the moving vehicle is modelled with a constant force or with a constant gap.

ISSN 1616-0762 Sonderdruck Schriftenreihe der Georg-Simon-Ohm-Fachhochschule Nürnberg Nr.14, Nürnberg 2002

# 1. Introduction

For economic reasons high speed MAGLEV vehicles will operate on elevated periodically supported single or double-span guideways which will be light and flexible [1,2,3]. For qualitative investigations the system can be approximated by a single mass vehicle and an undamped flexible single or double-span beam with rigid piers. The general control and simulation problem of the vehicle/guideway dynamics at high speeds includes two limiting cases, cf.[4]. In the first case, the magnet force is constant (only the weight of the vehicle) and in the second case the magnet gap is constant (the vehicle is following the guideway). In the paper the elastic structure of the guideway is approximated by homogenous elastic single and double-span beams described by partial BERNOULLI-EULER beam equations [3,4]. For typical guideway configurations this approximation is permissible since the lenghts of the beams are large compared with the other dimensions and these also large compared with the deflections. The partial differential equation can be transformed into an infinite number of ordinary differential equations by means of a modal transformation, cf.[5]. The kernels of this transformation consist on the eigenfunctions (modes) of the force-free undamped beam. An approximation is achieved by considering only the first modes. The mathematical description of the linear time variable system with periodic coefficients is given in state space notation which can be directly used for the digital simulation. In the simulation it is assumed, that the static deflection of the beam by its own weight is compensated.

# 2. Single and Double-Span Guideway Model

The guideway is described as a homogenous elastic single and double-span beam mounted with pivots on rigid piers. Equations of motions are derived on the basis of BERNOULLI-EULER beam theory, cf. [3].
If l is the length of one span and $l_b$ the length of the beam, the guideway is determined by its first vibration mode frequency $f_1$ and its span mass $m_l$. As shown in Fig.1 the concentrated magnet force, $F(t) = m_f\,(g-\ddot{z})$, moves along the span with constant velocity v.

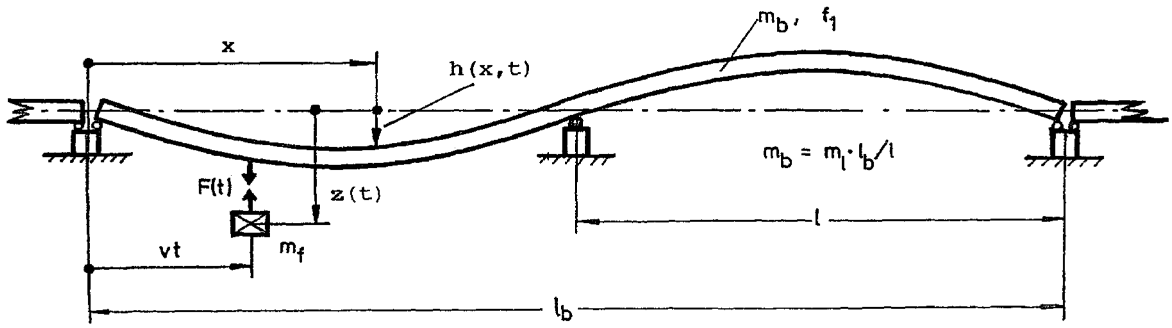


Fig. 1: MAGLEV single mass vehicle on an elevated double-span guideway

For the further investigations the following normalized system parameters are introduced:

$$\kappa = \frac{v/l}{2f_1}, \qquad \mu = m_f / m_l, \qquad \lambda_b = l_b / l,$$

where $\kappa$ is the span crossing frequency ratio, $\lambda_b$ is the beam to span length ratio which indicates a single or double-span guideway and $\mu$ is the vehicle to span mass ratio.

With the nondimensional variables

$$\xi = x/l, \quad \tau = vt/l, \quad \bar{h}(\xi,\tau) = h(x,t)/h_{sm},$$

where $h_{sm}$ is the maximum static span deflection of a single-span caused from the concentrated weight $m_f\,g$ of the vehicle, the nondimensional guideway equation of motion is

$$\frac{\partial^2 \overline{h}(\xi,\tau)}{\partial \tau^2} + \frac{1}{\pi^2 \kappa^2} \frac{\partial^4 \overline{h}(\xi,\tau)}{\partial \xi^4} = \overline{F}(\tau)\, \delta(\xi - \tau), \tag{1}$$

with the 4 boundary and ($\lambda_b$ - 1) intermediate conditions of the beam

$$\frac{\partial^2 \overline{h}(\xi,\tau)}{\partial \xi^2} = 0, \qquad \xi = 0, \lambda_b \qquad \text{and} \qquad \overline{h}(\xi,\tau) = 0, \qquad \xi = 0 \ \ (1)\, \lambda_b \tag{2}$$

In eq. (1) $\delta$ is the DIRAC delta function and $\overline{F}(\tau)$ is the nondimensional magnet force which can be written as

$$\overline{F}(\tau) = \frac{48}{\pi^2 \kappa^2} - \mu\, \ddot{\overline{z}}, \qquad \ddot{\overline{z}} = \frac{(l/v)^2}{h_{sm}}\, \ddot{z}, \tag{3}$$

where $\ddot{z}$ is the vehicle acceleration and $\ddot{\overline{z}}$ is the corresponding nondimensional variable.

**Modal approximation**

Based on the boundary and intermediate conditions of the beam in eq.(2), eigenfunctions $\varphi_j(\xi)$, and eigenvalues $\lambda_j$ can be obtained by solving the eigenvalue problem

$$\frac{1}{\pi^2 \kappa^2} \frac{\partial^4 \varphi_j(\xi)}{\partial \xi^4} = -\lambda_j \cdot \varphi_j(\xi), \qquad j = 1\,(1)\,\infty\,. \tag{4}$$

With the orthonormality relation

$$\lambda_b \int_0^1 \varphi_j(\xi) \cdot \varphi_k(\xi)\, d\xi = \delta_{jk} \qquad \text{and} \qquad \beta_j^4 = -\lambda_j\, \pi^2\, \kappa^2, \qquad \delta_{jk} = \text{ Kronecker–symbol},$$

the following results are obtained:

a) Eigenfunctions of a single-span ($\lambda_b$ = 1) and antimetric eigenfunctions of a double-span ($\lambda_b$ = 2)

$\varphi_j(\xi) = C_j \sin(\beta_j \xi)$, j = 1,2,3,….. for $\lambda_b$ = 1, j = 1,3,5,….. for $\lambda_b$ = 2

parameter $\beta_j$: coefficient $C_j$:

$\beta_j = (\lambda_b + j - 1)\pi / \lambda_b$ , $C_j = \sqrt{2/\lambda_b}$ ,

b) Symmetric eigenfunctions of a double-span, j = 2,4,6,…..

$\varphi_j(\xi) = C_j\,[\cosh(\beta_j) \cdot \sin(\beta_j \eta) - \cos(\beta_j) \cdot \sinh(\beta_j \eta)]$,

where $\eta = \xi$ for $\xi \in [0,1]$, $\eta = (2 - \xi)$ for $\xi \in [1,2]$

| parameter $\beta_j$: | coefficient $C_j$: |
|---|---|
| $\beta_2$ = 3.926602 | $C_2$ = $3.942106 \cdot 10^{-2}$ |
| $\beta_4$ = 7.068582 | $C_4$ = $1.702875 \cdot 10^{-3}$ |
| $\beta_6$ = 10.210176 | $C_8$ = $7.358789 \cdot 10^{-5}$ |
| $\beta_8$ = 13.351768 | $C_8$ = $3.180021 \cdot 10^{-6}$ |
| $\beta_{10}$ = 16.493361 | $C_{10}$ = $1.374213 \cdot 10^{-7}$ |
| . | . |
| . | . |

When only the first n* eigenfunctions $\varphi_j$ are considered, it can be shown that an approximation of the solution of eq.(1) can be written as, cf. [4,5]

$$\bar{h}(\xi,\tau) = \sum_{1}^{n^*} \varphi_j(\xi) \cdot h_j^*(\xi) = \boldsymbol{\varphi}^T(\xi) \cdot \mathbf{h}^*(\tau), \tag{5}$$

where $\varphi$ is the $n^* \times 1$- vector of the eigenfunctions and **h*** is the $n^* \times 1$- vector of the modal coordinates which is obtained as solution from the ordinary differential equation

$$\ddot{\mathbf{h}}^* = \boldsymbol{\Lambda} \cdot \mathbf{h}^* + \boldsymbol{\varphi}(\tau) \cdot \left[ \frac{48}{\pi^2 \kappa^2} - \mu \cdot \ddot{\bar{z}} \right], \tag{6}$$

with the $n^* \times n^*$- diagonal matrix $\boldsymbol{\Lambda}$ = diag($\lambda_j$).

Note: Eq. (5) is exact in the case $n^* \to \infty$. For a finite number n* the approximation converges with $(\lambda_b/n^*)^4$. Investigations have shown [4], that the result is close enough if approximately $n^* = 2\,\lambda_b$ modes are considered.

## 3. Vehicle with Moving Constant Force

Because the magnet force is constant, the vehicle acceleration in eq.(6) is zero. Then the following linear differential equation with constant coefficients is obtained:

$$\ddot{\mathbf{h}}^* = \boldsymbol{\Lambda} \cdot \mathbf{h}^* + \boldsymbol{\varphi}(\tau)\ \frac{48}{\pi^2 \kappa^2}. \tag{7}$$

As shown in [4], eq.(7) can be solved analytically. Though eq.(7) here is solved numerically as a limiting case of the vehicle with moving constant gap ($\mu = 0$).

## 4. Vehicle with Moving Constant Gap

Because the magnet gap is constant, the nondimensional vehicle acceleration $\ddot{\bar{z}}$ in eq.(6) is the same as the nondimensional guideway acceleration under the moving vehicle ( $\xi = \tau$ ).
With eq.(5) the acceleration can be written as:

$$\ddot{\bar{z}} = \ddot{\bar{h}}(\xi = \tau) = \ddot{\boldsymbol{\varphi}}^T(\tau) \cdot \mathbf{h}^* + 2 \cdot \dot{\boldsymbol{\varphi}}^T(\tau) \cdot \dot{\mathbf{h}}^* + \boldsymbol{\varphi}^T(\tau) \cdot \ddot{\mathbf{h}}^*, \tag{8}$$

where $\varphi(\tau) = \varphi(\xi = \tau)$ and $\dot{\varphi}(\tau)$, $\ddot{\varphi}(\tau)$ are the according derivatives with respect to time.

Together with eq.(6) and eq.(8) the following linear time variable differential equation with periodic coefficients is obtained:

$$\mathbf{M}(\tau)\, \ddot{\mathbf{h}}^* = \mathbf{K}(\tau)\, \mathbf{h}^* + \mathbf{D}(\tau)\, \dot{\mathbf{h}}^* + \boldsymbol{\varphi}(\tau) \frac{48}{\pi^2 \kappa^2}, \tag{9}$$

with the symmetric $n^* \times n^*$-matrices

$$\mathbf{M}(\tau) = [\mathbf{E} + \mu \cdot \boldsymbol{\varphi}(\tau) \cdot \boldsymbol{\varphi}^T(\tau)],$$

$$\mathbf{K}(\tau) = [\boldsymbol{\Lambda} - \mu \cdot \boldsymbol{\varphi}(\tau) \cdot \ddot{\boldsymbol{\varphi}}^T(\tau)],$$

$$\mathbf{D}(\tau) = [-2 \cdot \mu \cdot \boldsymbol{\varphi}(\tau) \cdot \dot{\boldsymbol{\varphi}}^T(\tau)].$$

With eq.(5) the nondimensional span deflection under the moving vehicle ($\xi = \tau$) can be written as:

$$\bar{h}(\tau) = \boldsymbol{\varphi}^T(\tau) \cdot \mathbf{h}^*. \tag{10}$$

For the computer simulation eq.(9) and eq.(10) will be written in state space notation:

$$\begin{bmatrix} \dot{\mathbf{h}}^* \\ \ddot{\mathbf{h}}^* \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{E} \\ \mathbf{M}^{-1}(\tau)\cdot\mathbf{K}(\tau) & \mathbf{M}^{-1}(\tau)\cdot\mathbf{D}(\tau) \end{bmatrix} \begin{bmatrix} \mathbf{h}^* \\ \dot{\mathbf{h}}^* \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}(\tau)\cdot\varphi(\tau) \end{bmatrix} \frac{48}{\pi^2\kappa^2}$$

$$\left[\bar{h}\right] = \left[\varphi^T(\tau) \quad \mathbf{0}\right] \begin{bmatrix} \mathbf{h}^* \\ \dot{\mathbf{h}}^* \end{bmatrix}. \tag{11}$$

Eq.(11) has the general form

$$\mathbf{x} = \mathbf{A}(\tau)\cdot\mathbf{x} + \mathbf{B}(\tau)\cdot\mathbf{b}$$

$$\mathbf{y} = \mathbf{C}(\tau)\cdot\mathbf{x}\,, \tag{12}$$

where $\mathbf{A}(\tau)$ is the $2n^* \times 2n^*$-system matrix, $\mathbf{B}(\tau)$ the $2n^* \times 1$-disturbance matrix, $\mathbf{C}(\tau)$ the $1 \times n^*$-measurement matrix and **x**, **y**, **b** are vectors with the according dimensions.

This linear, time variable system with periodic coefficients can be integrated numerically with a 4th order RUNGE-KUTTA algorithm over one period.
To reduce the elapsed computing time, in the numerical integration an analytical expression for $M^{-1}$ is used:

$$\mathbf{M}^{-1}(\tau) = \left[\mathbf{E} - \frac{\mu}{1+\mu\cdot\varphi^T(\tau)\cdot\varphi(\tau)}\,\varphi(\tau)\cdot\varphi^T(\tau)\right], \tag{13}$$

where $\varphi^T(\tau)\cdot\varphi(\tau)$ is a scalar quantity, cf.[6].

Note: For $\mu = 0$ from eq.(11) the state equation for the moving vehicle with constant force is obtained.

## 5. Simulation Results

The normalized system parameters used in the simulation are:

$\kappa$ = 0; 0.25; 0.5; 0.75; 1; 1.25;    $\mu = 0.5$;    $\lambda_b$ = 1; 2.

For the single-span $n^* = 3$ and for the double-span $n^* = 6$ modes are considered in the approximation.
For the limiting case $\kappa = 0$ the steady-state solution for the guideway deflection under the vehicle

$$\bar{h}(\tau) = 48\sum_{1}^{n^*}\frac{\varphi_j^2(\tau)}{\beta_j^4} \tag{14}$$

is used, which can be obtained from eq.(6) and eq.(10).

Fig.2 and Fig.3 illustrate the influence of span crossing frequency ratio $\kappa$ on the nondimensional deflection of a double-span guideway under a moving vehicle with constant force and a moving vehicle with a constant gap.

Fig.4 and Fig.5 show the same results for a single-span guideway.

The results show, that for low values of span crossing frequency ratio $\kappa$ the deflections of the double-span and for high values of $\kappa$ the deflections of the single-span guideway are smaller. Note that the trajectories for the first span of the double-span are closely similar to those of the single-span guideway.
When $\kappa$ approaches zero, the two problems of constant force and constant gap reduce to a static one and the trajectories are all exactly alike.
An important observation is the increase in exitation of higher mode components as the vehicle to span mass ratio $\mu$ is increasing.

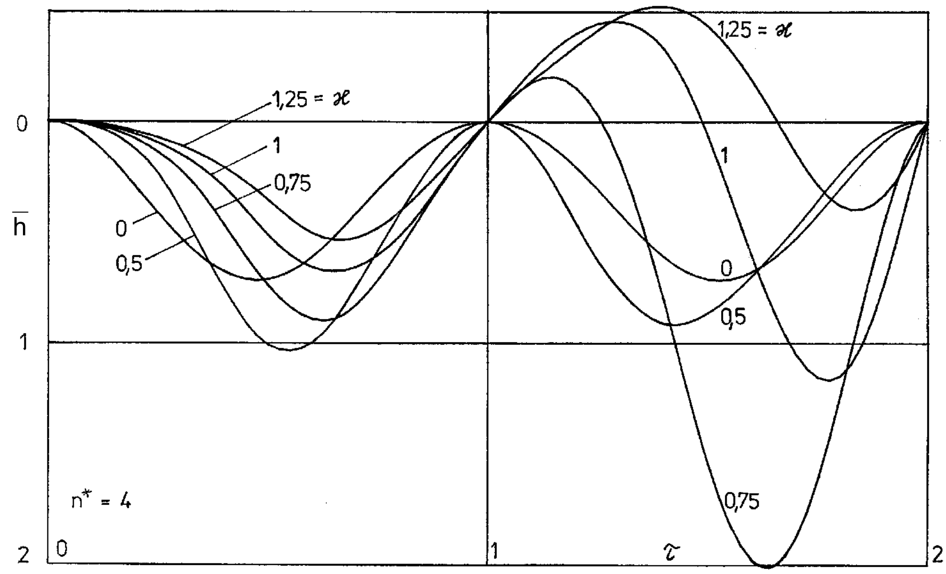


Fig. 2: Histories of double-span deflection under a moving vehicle with constant force

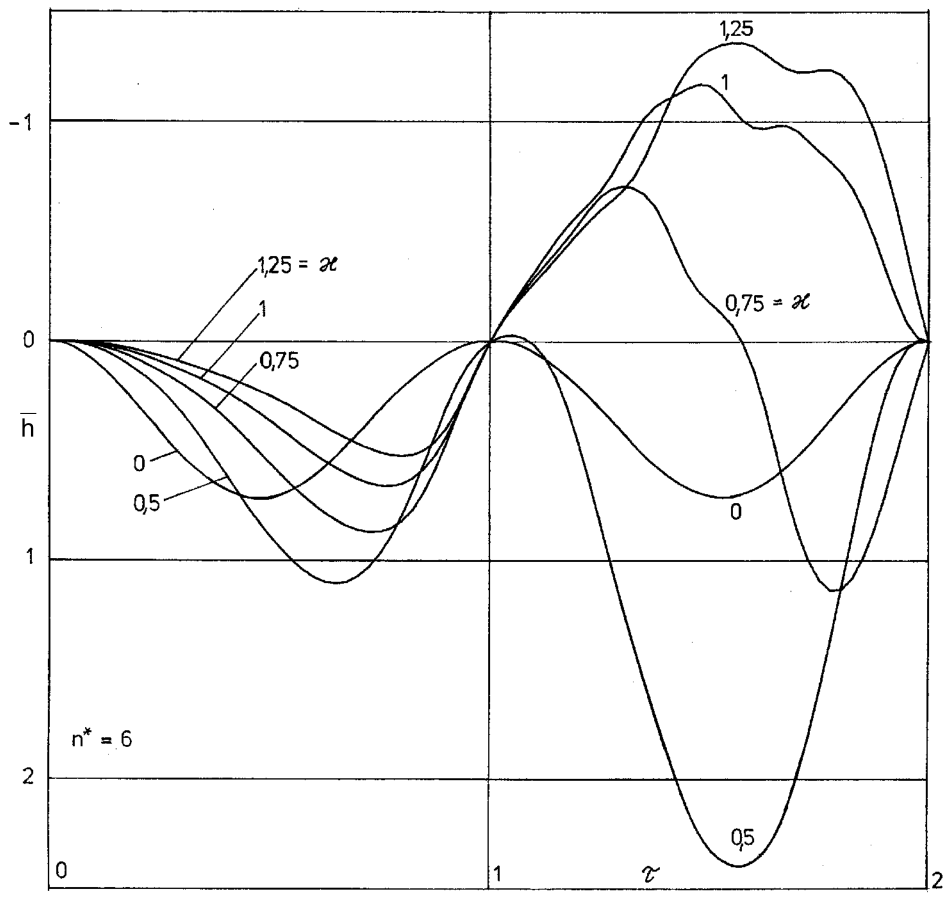


Fig. 3: Histories of double-span deflection under a moving vehicle with constant gap ($\mu$ = 0.5)

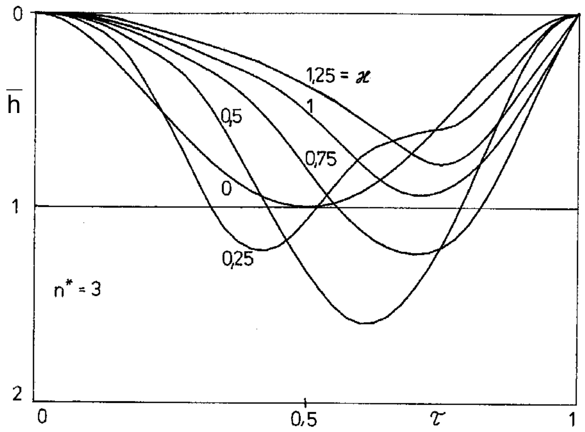


Fig. 4: Histories of single-span deflection under a moving vehicle with constant force

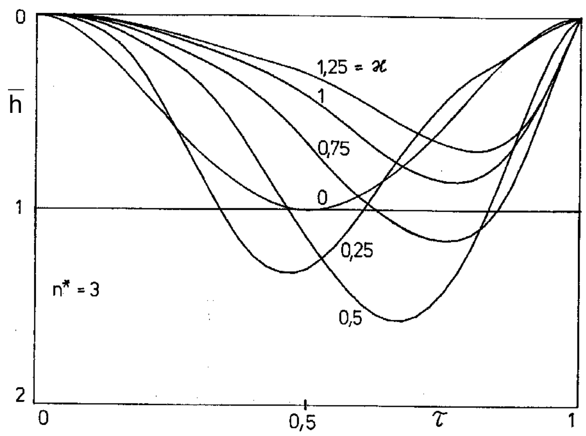


Fig. 5: Histories of single-span deflection under a moving vehicle with constant gap ($\mu$ = 0.5)

## 6. Conclusion

In this paper modal analysis technique has been applied to obtain the state equation of a moving single mass vehicle on an elastic single and double-span guideway. Simulation results for the guideway trajectories were given for the two problems when the MAGLEV vehicle is moving with constant magnet force and moving with constant magnet gap. Comparing the maximum deflections the results show, that for low values of span crossing frequency ratio $\kappa$ (quasistatic region) the double-span guideway is recommended, while for higher values of $\kappa$ the single-span guideway must be favoured. More realistic simulations with an actively controlled magnet force are given in [3,4].

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