

Map of the algorithm „Self-organization of Vectors“

This is a map τ from $(\mathbf{R}^n)^k \rightarrow (\mathbf{R}^n)^k$. Vectors are coupled by the dot product $\mathbf{r} \cdot \mathbf{dr}$.

The smallest unit of the algorithm is a pair of points with the transformation $\mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}^n \times \mathbf{R}^n$.

Given vectors: $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}, \mathbf{dr}_{12} \in \mathbf{R}^n$ with $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$; $|\mathbf{dr}_{12}| = 1$.

The map τ depends of two scalars e and s with $e \geq s$.

$\tau(\mathbf{r}_1, \mathbf{r}_2)$:

First calculation of next \mathbf{dr}_{12} .

For $|\mathbf{r}_{12}| \leq e$ then $\mathbf{dr}_{12} = \text{constant}$.

For $|\mathbf{r}_{12}| > e$ then $|\mathbf{r}_{12} \wedge \mathbf{dr}_{12}| = s$ and $\mathbf{r}_{12} \cdot \mathbf{dr}_{12} > 0$.

Then calculation of the new positions of the two points

$$\tau(\mathbf{r}_1) = \mathbf{r}_1 + \mathbf{dr}_{12}, \quad \tau(\mathbf{r}_2) = \mathbf{r}_2 + \mathbf{dr}_{21} = \mathbf{r}_2 - \mathbf{dr}_{12}.$$

The map τ extends to a map $(\mathbf{R}^n)^k \rightarrow (\mathbf{R}^n)^k$, defined for a list of k points, via a principle of superposition.

Given vectors : $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_k, \mathbf{r}_{\{ab\}}, \mathbf{dr}_{\{ab\}} \in \mathbf{R}^n$ with $\mathbf{r}_{\{ab\}} = \mathbf{r}_b - \mathbf{r}_a$; $|\mathbf{dr}_{\{ab\}}| = 1$, where $\{ab\}$ are the chosen pairs of points.

$\tau(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_k)$:

First calculation of all next $\mathbf{dr}_{\{ab\}}$.

For $|\mathbf{r}_{\{ab\}}| \leq e$ then $\mathbf{dr}_{\{ab\}} = \text{constant}$.

For $|\mathbf{r}_{\{ab\}}| > e$ then $|\mathbf{r}_{\{ab\}} \wedge \mathbf{dr}_{\{ab\}}| = s$ and $\mathbf{r}_{\{ab\}} \cdot \mathbf{dr}_{\{ab\}} > 0$.

Then calculation of the new positions of all points by superposition

$$\tau(\mathbf{r}_i) = \mathbf{r}_i + \sum \mathbf{dr}_{\{ij\}} \quad \text{with} \quad \mathbf{dr}_{\{ij\}} = -\mathbf{dr}_{\{ji\}} \quad \text{where } i = 1, 2, \dots, k.$$

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