

Simple Bivectors and Attractors

Wolfhard Hoevel

The suggested model consists of a pair of points \mathbf{r}_1 and \mathbf{r}_2 in the Euclidean space. The one point is moved iteratively by a unit vector $d\mathbf{r}_{12}$ and the second point simultaneously by $-d\mathbf{r}_{12}$. The distance of the parallels is constant $=s$ on which the two points are moved (Figure 1). The consequence is that the center of mass M and the torque of the pair of points remain constant.

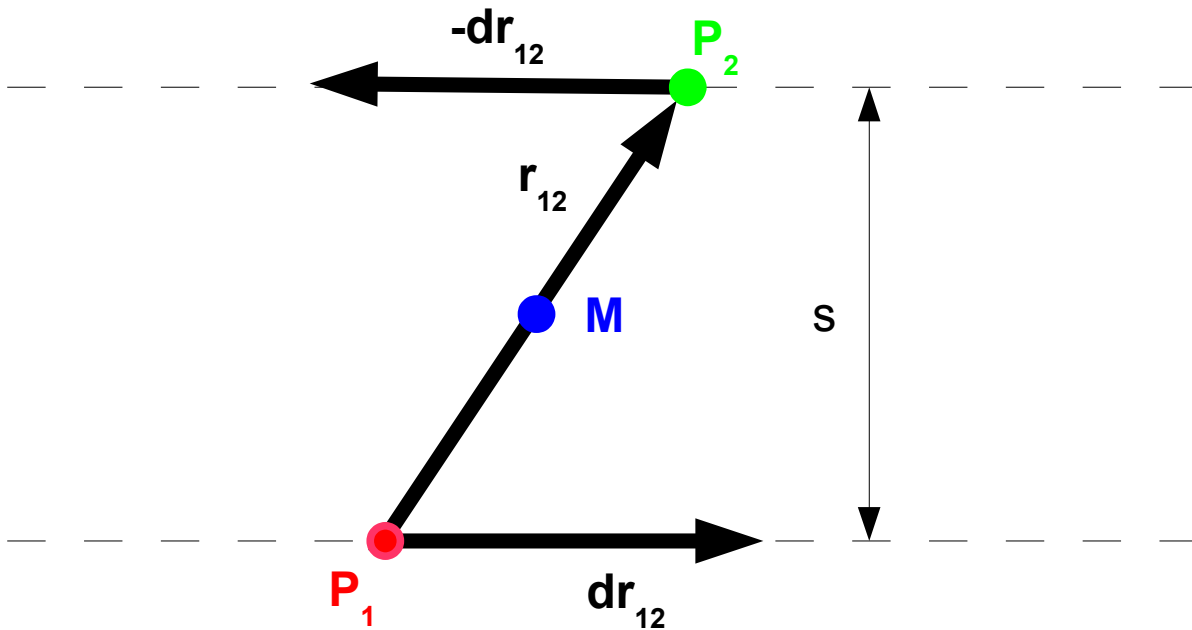


Figure 1.

The distance vector between the two points $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$ and the unit vector $d\mathbf{r}_{12}$ are spanning a simple bivector \mathbf{B}_{12} in the n -dimensional space with a constant area $|\mathbf{B}_{12}|$ (Figure 2).

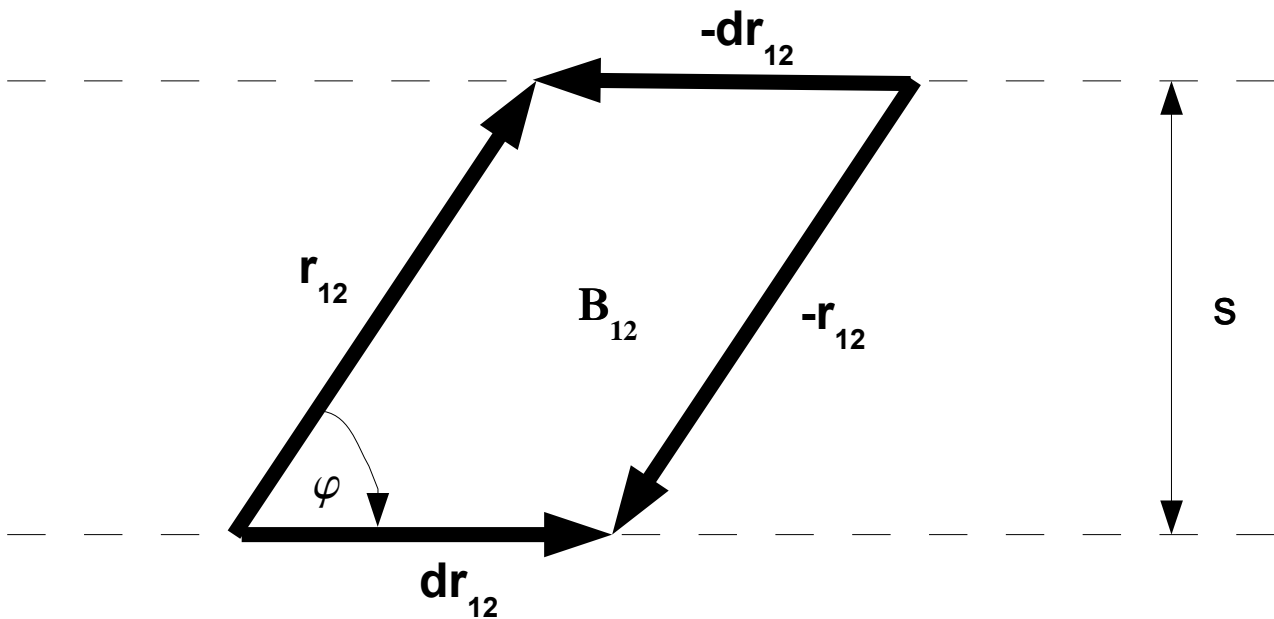


Figure 2.

The simple bivector \mathbf{B}_{12} can be expressed by $\mathbf{r}_{12} \wedge \mathbf{dr}_{12}$ or $-\mathbf{r}_{12} \wedge -\mathbf{dr}_{12}$

$$\mathbf{B}_{12} = \mathbf{r}_{12} \wedge \mathbf{dr}_{12} = (-\mathbf{r}_{12}) \wedge (-\mathbf{dr}_{12}) \quad .$$

The magnitude of the bivector equals to the area of the parallelogram

$$|\mathbf{B}_{12}| = |\mathbf{r}_{12}| |\mathbf{dr}_{12}| \sin \varphi = |\mathbf{r}_{12}| |\mathbf{dr}_{12}| \sqrt{1 - \cos^2 \varphi} \quad .$$

With $|\mathbf{dr}_{12}| = 1$

$$|\mathbf{B}_{12}| = \sqrt{|\mathbf{r}_{12}|^2 - |\mathbf{r}_{12}|^2 \cos^2 \varphi} = \sqrt{|\mathbf{r}_{12}|^2 - (\mathbf{r}_{12} \cdot \mathbf{dr}_{12})^2} \quad .$$

The magnitude of the simple bivector $|\mathbf{B}_{12}|$ represents the area of the parallelogram which is spanned by the vectors \mathbf{r}_{12} and \mathbf{dr}_{12} and in this case shall apply

$$|\mathbf{B}_{12}| = s \quad .$$

The model is defined as a time discrete dynamic system. Every time step the points are moved by the vector \mathbf{dr}_{12} respectively by $-\mathbf{dr}_{12}$ as shown in figure 3.

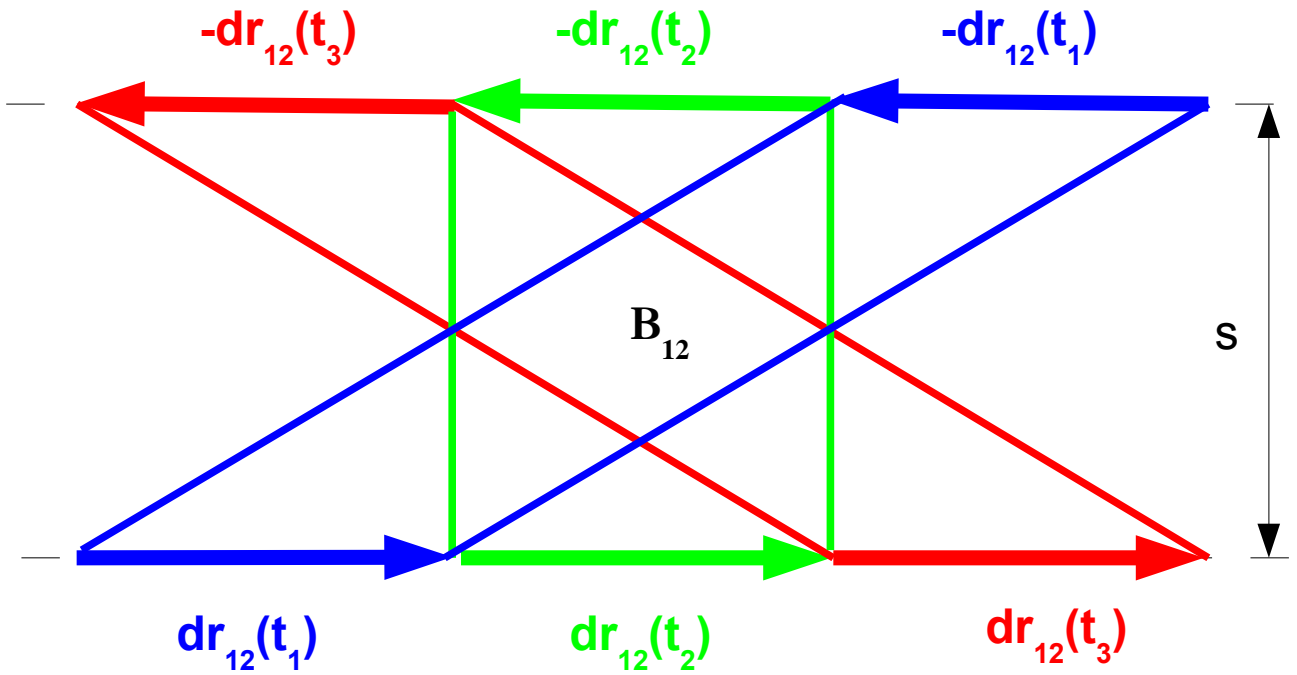


Figure 3.

The bivector \mathbf{B}_{12} and the center of mass and the torque of the pair of points remain constant. The dot product $\mathbf{r}_{12} \cdot \mathbf{dr}_{12}$ changes from positiv to negativ values. When the magnitude of \mathbf{r}_{12} exceeds a given boundary e then a reflection takes place. The bivector \mathbf{B}_{12} remains still constant but the direction of \mathbf{dr}_{12} is changed in such a way that the dot product $\mathbf{r}_{12} \cdot \mathbf{dr}_{12}$ becomes positiv as shown in figure 4.

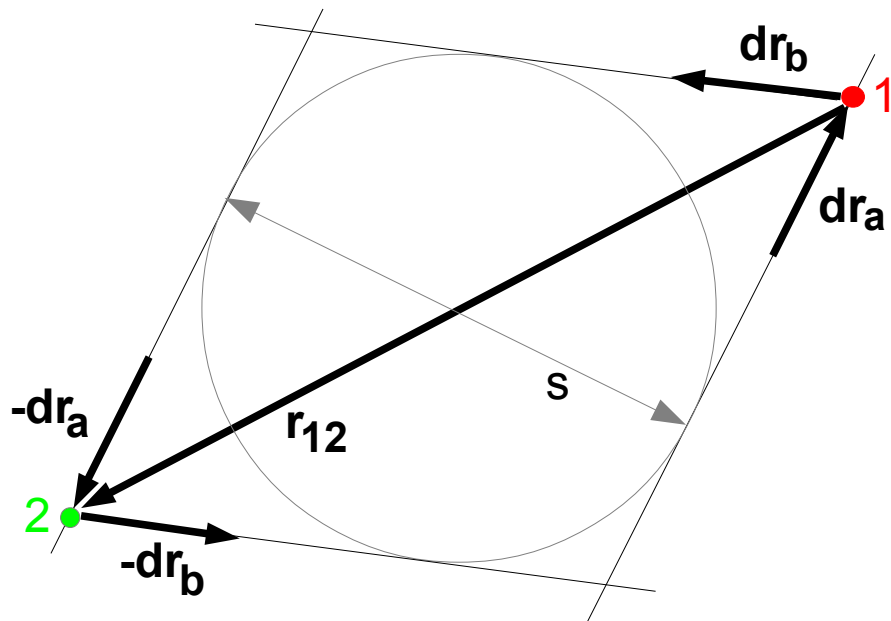


Figure 4.

In the case the condition $|r_{12}| > e$ is met the unit vector $\mathbf{dr}_{12} = \mathbf{dr}_a$ is changed to $\mathbf{dr}_{12} = \mathbf{dr}_b$. Until now only one pair of points was considered but it is very simple to introduce more points. It is possible to choose any number of pair of points between the points. The displacement of the points can be calculated via the principle of superposition.

The situation can be demonstrated by a graph. The points correspond to the nodes and the pair of points correspond to the edges of the graph. Every pair of points can be understood as a separate simple bivector.

The described dynamical system leads to a variety of different attractors. The dynamical behavior of the attractors is demonstrated by the executable Java file „Attractor.jar“.