Mean-risk optimization models for electricity portfolio management

Andreas Eichhorn and Werner Römisch

Abstract— **The possibility of controlling risk in stochastic power optimization by incorporating special risk functionals, so-called polyhedral risk measures, into the objective is demonstrated. We present an exemplary optimization model for mean-risk optimization of an electricity portfolios of a price-taking retailer. Stochasticity enters the model via uncertain electricity demand, heat demand, spot prices, and future prices. The objective is to maximize the expected overall revenue and, simultaneously, to minimize risk in terms of multiperiod risk measures, i.e., risk measures that take into account intermediate cash values in order to avoid liquidity problems at any time. We compare the effect of different multiperiod polyhedral risk measures that had been suggested in our earlier work.**

Index Terms— **Electricity futures, Electricity portfolio, Optimization methods, Polyhedral risk measures, Power industry, Risk analysis, Scenario tree, Stochastic programming**

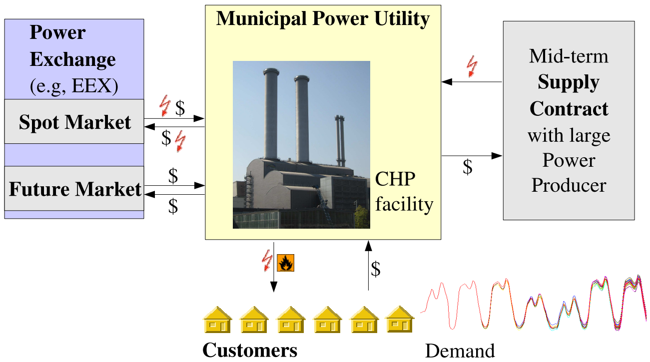

Fig. 1. Schema of the optimization model components

I. Introduction

The deregulation of energy markets has lead to an increased number of uncertainty factors and higher financial risk for electric utilities. Classical (stochastic and deterministic) optimization models for power production and trading are focused on (expected) profit maximization; see, e.g., [1], [2]. Risk management is usually considered as a separate task. Formally, stochastic programming (optimization) models provide the possibility of considering financial risk directly, e.g., by probabilistic constraints or by incorporating risk measures. Carrying out profit maximization and risk aversion simultaneously promises additional overall efficiency for power utilities; see, e.g., [3], [4], [5]. However, most of such risk aversion strategies in stochastic programming make the resulting problems harder to solve, thus, only smaller sized problems can be handled. In this paper we demonstrate a possibility of risk aversion by incorporating special risk functionals, so-called *polyhedral risk measures* [6], into the objectives of stochastic programs. In this case, the increase of complexity is moderate. The case study presented here is rather large-scale, it employs the framework of multistage stochastic programming; cf. [7].

Multistage stochastic programs represent the situation when a decision maker has to make (optimal) decisions at several time stages, in each case based on observations of the past and on statistical information about the futures. An abstract formulation of a typical multistage stochastic program reads

$$\min_x \left\{ \mathbb{F}(z_1, ..., z_T) \;\middle|\; \begin{array}{l} z_t := \sum_{\tau=1}^{t} b_\tau(\xi_\tau)\cdot x_\tau, \\ x_t = x_t(\xi_1, ..., \xi_t), \quad x_t \in X_t, \\ \sum_{\tau=0}^{t-1} A_{t,\tau}(\xi_t) x_{t-\tau} = h_t(\xi_t) \\ (t = 1, ..., T) \end{array} \right\} \qquad (1)$$

Andreas Eichhorn and Werner Römisch are with the Institute of Mathematics, Humboldt-University Berlin, 10099 Berlin, Germany.

This work was supported by DFG Research Center MATHEON *Mathematics for Key Technologies* in Berlin, Germany (http://www.matheon.de).

with a finite (discrete) time horizon $t = 1, ..., T$, a (multivariate) stochastic input process $\xi_1, ..., \xi_T$, given vectors b_t, h_t, matrices A_t (possibly depending on the stochastics), and Euclidian subsets X_t. The x_t-vectors are referred to as the decisions, and the conditions $x_t \in X_t$ and $\sum_{\tau=0}^{t-1} A_{t,\tau}(\xi_t)x_{t-\tau} = h_t(\xi_t)$ are the constraints. Note that x_t may depend on the stochastic input process, but only on the components prior to time t (non-anticipativity). The z_t variables represent the (accumulated) revenues until time t and depend on the stochastics in the same manner as well as on the decisions. Finally, a (mean-risk) functional $\mathbb{F}$ is required that maps the stochastic process of revenues into the real numbers.

The multistage stochastic optimization model presented in this paper is tailored to the requirements of a typical Germnan municipal power utility, which has to serve an electricity demand and a heat demand of customers in a city and its vicinity, cf. Fig. 1. The power utility owns a *combined heat and power* (CHP) production facility that can serve the heat demand completely and the electricity demand partly. Further electricity can be obtained by purchasing volumes for each hour at the (day-ahead) spot market of the European Energy Exchange (EEX), and by signing a supply contract for a medium term horizon with a larger power producer. The latter possibility is suspected to be expensive, but relying on the spot market only is known to be extremely risky. Spot price risk, however, may be reduced (hedged) by obtaining electricity futures at EEX. Futures at EEX are purely financial contracts relating to a specified delivery period in the future. Obtaining a future at a certain market value results, at the end of the corresponding delivery period, in a compensation of the difference between this market value and the average spot price in the delivery period. When a specific supply contract is offered to the power utility by a power producer, the question arises, whether it is beneficial to accept that offer or it is better

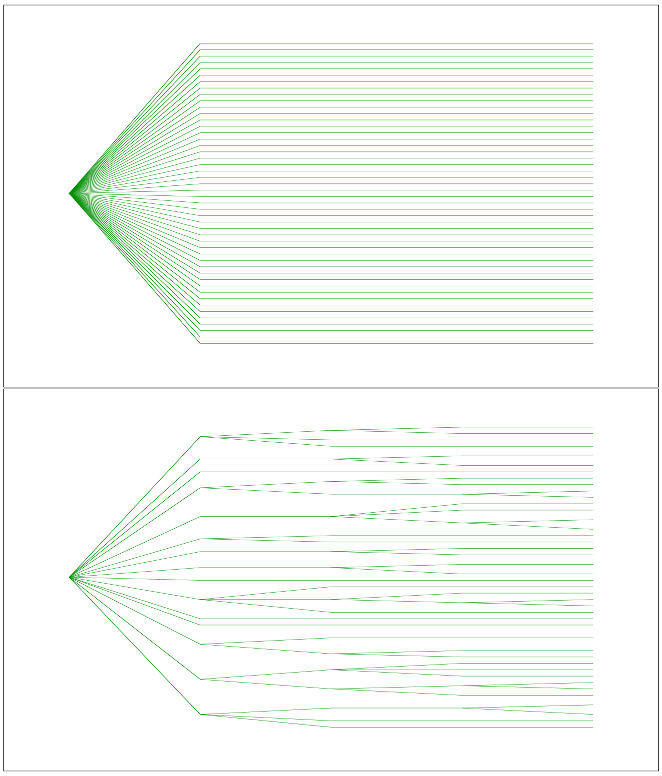

Fig. 2. Top: Schematic illustration of "fan" of scenarios for the future, scenarios are only connected at the beginning (present, $t = 1$). Bottom: Scenario tree shows branching also at intermediate time steps ($t > 1$).

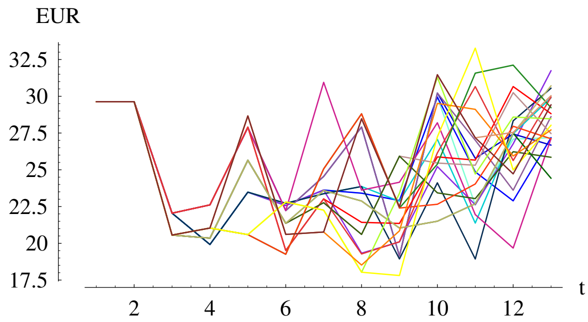

Fig. 3. Scenario tree data for monthly average spot prices

to rely on spot and future market only. That decision will be an output of the optimization which aims to maximize the mean overall revenue and, simultaneously, to minimize a risk measure.

To put this in concrete terms, we take an hourly discretization and an optimization horizon of one year as a basis. We suppose that two types of contracts are available: a fix contract (fix delivery schedule, fix price), and a flexible contract. The latter is based on the same delivery schedule, but, at the end of each month, it is allowed to alter these pre-arranged volumes for each hour of the following month by a certain percentage and, in addition, to realter these new volumes in a day-ahead manner by another percentage. The price of this contract may depend on the overall volume and on the maximum power (demand rate). Other (similar) types of flexible contracts such as swing options would also be possible here, but the type needs to be fixed a priori. Finally, since electricity production together with contract volumes might exceed the demand, we also allow for selling at EEX spot and future market.

Due to the medium term horizon, we slightly simplified the technical restrictions of the CHP facility in the model such that no integer variables appear, we only impose that the heat and the electricity production are within certain interdependent bounds and that the electricity production of two consecutive time-steps must not differ more than a given delta. Furthermore, we assume linear production costs (cf. section IV). For the spot market, we restrict ourselves to price-independent bids. This guarantees full volume safety. We fully incorporate the trading rules of EEX including transaction costs, day-ahead offering, and initial and variation margins[1] for futures. We consider monthly base and peak futures for each month within the optimization horizon, i.e., we neglect futures for quarters and their cascading. We allow for rebalancing the future stock on every trading day at 12 am.

The remaining paper is organized as follows: After describing the employed methodology of risk aversion, we sketch the statistical models and the procedure of generating scenario trees. Then, in section IV, we formalize the above optimization model. Together with a concrete scenario tree, the model can be understood as a linear program. This so-called *deterministic equivalent* is solved with a commercial LP solver and simulation results are presented and interpreted in section V.

II. Multiperiod polyhedral risk measures

The classical choice for the objective functional $\mathbb{F}(z_1, .., z_T)$ in (1) is $\mathbb{E}[-z_T]$, i.e., the expected (mean) overall costs are minimized. A lot is known about this case (cf., e.g., [7]). To achieve risk aversive solutions for (1) it is common to incorporate so-called risk measures ([8], [9]) such as Value-at-Risk (VaR) into the objective. This yields mean-risk objectives such as $\gamma \cdot \mathrm{VaR}(z_T) - (1-\gamma) \cdot \mathbb{E}[z_T]$ with some weighting factor $\gamma \in [0, 1]$ (cf., e.g., [10]). In this paper however, we are dealing with a time horizon of one year and it will be seen below that the application of a one-period risk measure such as VaR is not sufficient here. A better alternative is to include a multiperiod risk measure ρ into $\mathbb{F}$. Such risk measures do not focus on the terminal wealth z_T only, but also take into account the wealth at intermediate time stages $t_1 \leq ... \leq t_{T'}$ in order to avoid liquidity problems at all time [11], [12].

In this paper, we apply the multiperiod risk measures to the cash values at the end of each week within the optimization horizon. We use risk measure taken from the class of *polyhedral risk measures*, that have been shown to be particularly suitable for being optimized in a stochastic program [6]. They are, basically, multiperiod extentions of the Conditional-Value-at-Risk (CVaR), which is, in turn, an improvement of VaR that is known to have certain drawbacks [10]. The key-idea is that such a risk measure can be written as the optimal value of a

[1]When a future is obtained from EEX, a deposit, the initial margin, has to be payed rather than the market value. As long as the future is held, changes of the market value have to be compensated immediately (variation margin).

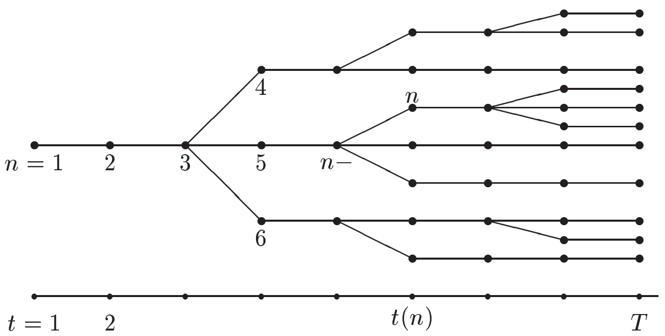

Fig. 4. Scenario tree notations

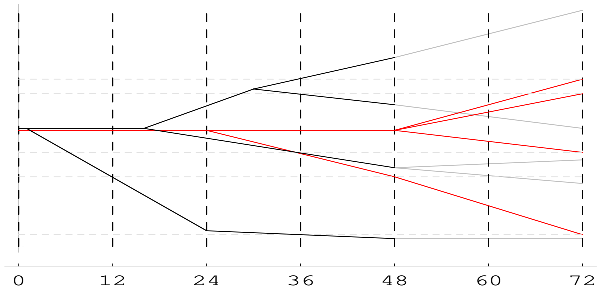

Fig. 5. The (beginning of the) original scenario tree $\mathcal{T}$ (black) and the trading tree $\mathcal{T}^{\text{trade}}$ (red/gray) which is derived by delaying branching in $\mathcal{T}$ until the beginning of the next trading day ($t = 24$, $t = 48$).

specific stochastic minimization problem of the form

$$\rho(z_{t_1}, ..., z_{t_{T'}}) = \min_{y} \left\{ \mathbb{E}\left[\sum_{j=1}^{T'} c_j \cdot y_j\right] \middle| y_j \in \mathcal{Y}_j(z_{t_j}, y_1, ..., y_{j-1}) \right\} \quad (1)$$

with certain dynamic linear constraints $y_j \in \mathcal{Y}_j$. Inserting this definition in (1) with $\mathbb{F}(z_1, .., z_T) = \rho(z_{t_1}, ..., z_{t_{T'}})$ leads to

$$\min_{x,y} \left\{ \mathbb{E}\left[\sum_{j=1}^{T'} c_j \cdot y_j\right] \middle| \begin{array}{l} z_t := \sum_{\tau=1}^{t} b_\tau(\xi_\tau) \cdot x_\tau, \\ x_t = x_t(\xi_1, ..., \xi_t), \quad x_t \in X_t, \\ \sum_{\tau=0}^{t-1} A_{t,\tau}(\xi_t) x_{t-\tau} = h_t(\xi_t), \\ y_j \in \mathcal{Y}_j(z_{t_j}, y_1, ..., y_{j-1}) \end{array} \right\}.$$

Hence, minimizing (1) with $\mathbb{F}(z_1, .., z_T) = \rho(z_{t_1}, ..., z_{t_{T'}})$ is in many respects equivalent to minimizing an expectation. The same is true for the mean-risk objective $\gamma \cdot \rho(z_{t_1}, ..., z_{t_{T'}}) - (1-\gamma) \cdot \mathbb{E}[z_T]$. In mathematical terms this means that the nonlinearity of ρ is transformed into a linear objective and linear constraints. In particular, scenario tree generation can be carried out in the same manner as for purely expectation-based stochastic programs. In [6], five multiperiod instances of polyhedral risk measures have been suggested of which one goes back to [12]. The different effects of two of these functionals (ρ_2 and ρ_4) will be demonstrated in Section V.

III. Stochastic input data and scenario trees

Before setting up the stochastic optimization model, it is necessary to identify the random input data $\xi_1, ..., \xi_T$ and to represent it by suitable statistical models. However, optimization models based on arbitrary statistical models usually cannot be solved in practice. Hence, the statistical models must

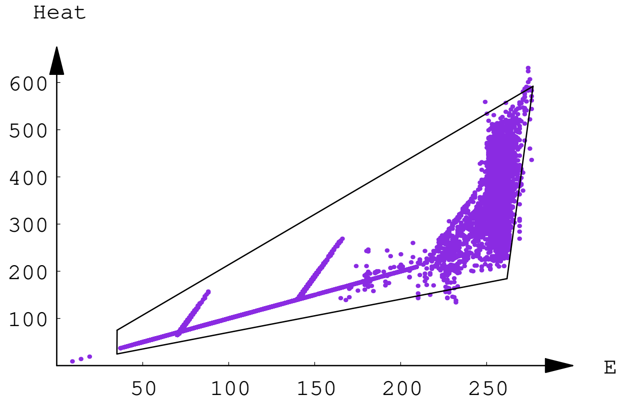

Fig. 6. Feasible region of CHP facility (x-axis: amount of electricity production, y-axis: amount of heat production). The polyhedron is defined through a matrix A^p and a vector b^p by $A^p x^p \leq b^p$ with $x^p = (x^{pe}, x^{ph})'$ representing the amounts of production at a time. The dots within the polyhedron represent true possible states of the facility from historical data.

be approximated by means of appropriate sampling techniques yielding a scenario tree.

For the stochastic input data of the optimization model here (electricity demand, heat demand, and electricity spot prices), a very heterogeneous statistical model is employed. It is adapted to historical data in a rather involved procedure. It consists of a cluster classification for the intra-day (demand and price) profiles and a three dimensional time series model for the daily average values. The latter consists of deterministic trend functions and a trivariate ARMA model for the (stationary) residual time series; see [13] for further details. An arbitrary number of three dimensional sample paths (scenarios) can easily be obtained by simulating white noise processes for the ARMA model and by adding on the trend functions and matched intra-day profiles from the clusters afterwards.

However, such a bunch of sample paths (scenario fan, cf. Fig. 2) does not reflect the information structure in multistage stochastic optimization, i.e., it neglects the fact that information is revealed gradually over time. In other words: for multistage stochastic programming, a sample approximation of a statistical time series model must provide stochasticity (branching) not only at the beginning ($t = 1$) but also at intermediate time steps. Thus, the collection of scenarios is required to have a tree structure (scenario tree, cf. Fig. 2). For this reason, specialized scenario tree construction algorithms for multistage stochastic programs have been developed in [14], [15]. These algorithms are based on stability results for stochastic programs (cf. [16] and [7, Chapter 8]). Beside generating a tree structure from a fan structure, the algorithms can simultaneously reduce the number of scenarios in an optimal way in order to keep the resulting optimization problem tractable; cf. [14], [15] for further details.

Now, consider a given scenario tree (generated as described above) that describes the random input data appropriately. For the optimization model in this paper, the formulation is essentially based on the (input) scenario tree $\mathcal{T}$, which consists of the tree structure (nodes $n \in \mathcal{N}$ and predecessor mapping), node probabilities π_n, and the random data (D_n^e, D_n^h, C_n^s) for $n \in \mathcal{N}$ (electricity demand, heat demand, and electricity

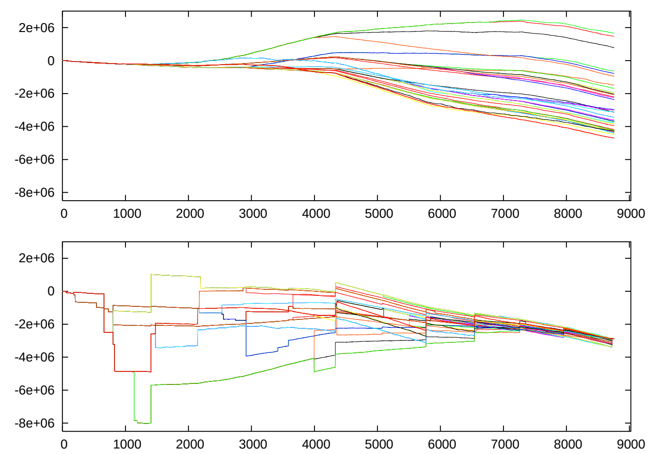

Fig. 7. Resulting optimal cash values over time for each scenario for the case that no contract is included and without incorporating multiperiod risk measures. Top: Only $\mathbb{E}[z_T]$ is optimized. Bottom: $0.9 \cdot \text{CVaR}(z_T) - 0.1 \cdot \mathbb{E}[z_T]$ is minimized. There is *considerably high spreading* and many scenarios reach fairly low cash values at the end or in the meantime, respectively. Of course, the collection of revenue curves in each figure has the same structure as the input scenario tree since the z variables are defined on this tree structure.

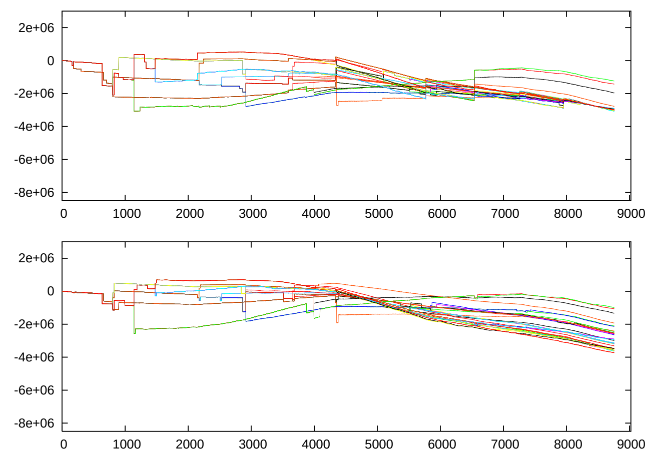

Fig. 8. Resulting optimal cash values over time for each scenario for the case that no contract is included. Two different multiperiod polyhedral risk measures are optimized, ρ_2 (top) and ρ_4 (bottom), cf. [6]. Obviously, multiperiod risk measures aim to reduce spreading at all time.

spot prices, respectively, cf. Fig. 3). The nodes of $\mathcal{T}$ are numbered successively beginning with the root node $n = 1$, cf. Fig. 4. Every node $n \in \mathcal{N} \setminus \{1\}$ has a unique predecessor denoted by $n-$ and a unique corresponding time-step $t(n) \in \{1, ..., 365 \cdot 24\}$. Furthermore, we set $\text{path}(n) = \{n, n-, ..., 1\}$ the set of all nodes between n and the root node. The node probabilities π_n are understood unconditional, i.e., for each time-step t it holds that $\sum_{\{n \in \mathcal{N}: t(n)=t\}} \pi_n = 1$. Beside the random input data, also the decision variables are defined on the scenario tree. This guarantees the non-anticipativity of the decisions.

The prices for the electricity futures are calculated a posteriori from the spot prices in the scenario tree. A future for a month m expires at the end of this month. Then, the final future price is fixed to the average electricity spot price in this month m. (Note that, for peak futures, only the hours between 8 am and 8 pm on trading days contribute to the respective average, whereas, for base futures, every hour of month m is taken into account.) Hence, for the price of a future for month m before the end of this month, it is natural to assume so-called *fair prices*, i.e., the market value of the future at some point in time $t < \text{end}(m)$ is given by the conditional expectation of the (temporal) average of the (stochastic) spot prices with respect to the information that is available at this time t. This approach guarantees the future prices to be arbitrage-free.

IV. Optimization model

A. Parameters

The scenario tree data can be understood as parameters indexed by node numbers. All the other parameters are indexed by time-step or they are not indexed at all:

D_n^e, D_n^h: Demand of electricity, heat at node $n \in \mathcal{N}$ in MW
C_n^s: Spot price costs for electricity in Euro/MWh ($n \in \mathcal{N}$)
$C_n^{fb,m}$, $C_n^{fp,m}$: Prices for base, peak futures in Euro/MWh
$C^{s,\text{trans}} = 0.04\,\text{Euro}/\text{MWh}$: Spot market transaction costs
$C^{f,\text{trans}} = 0.02\,\text{Euro}/\text{MWh}$: Future market transaction costs
$C^{f,\text{imar}} = 2.0\,\text{Euro}/\text{MWh}$: Initial margin for futures
C^{pe}: Cost factor for electricity production in Euro/MWh
C^{ph}: Cost factor for production of heat in Euro/MWh
δ^{pe}: Maximum gradient for electricity production in MW
P^e: Selling price for electricity in Euro/MWh
P^h: Selling price for heat in Euro/MWh
V_t^c: Pre-arranged contract volumes ($t = 1, ..., 365 \cdot 24$)
$C^{c,\text{fix}}$: Energy rate for fix contract in Euro/MWh
$C^{c,\text{flex},p}$: Peak energy rate, flexible contr. in Euro/MWh
$C^{c,\text{flex},o}$: Off-peak energy rate in Euro/MWh
$C^{c,\text{flex},d}$: Maximum demand rate in Euro/MW

B. Derived trees

To formulate the optimization model, it is useful to introduce further (smaller) trees derived from $\mathcal{T}$ by delaying branching points or by eliminating time-steps. These trees reflect further non-anticipativity constraints, e.g. day-ahead requirements for spot market decisions. All decision variables are defined on the nodes of the trees. The nodes of the trees are numbered in the same way as for the original scenario tree:

- Future tree $\mathcal{T}^{\text{fut}}$: based on the original scenario tree, the number of time-steps and, hence, the number of nodes is reduced such that there is one time-step at each trading day at 12 am. In addition, there are time-steps (and nodes) for the final billing of the futures at the end of each month (11 pm). Every node $d \in \mathcal{N}^{\text{fut}}$ has a unique corresponding node $n(d) \in \mathcal{N}$ in $\mathcal{T}$.
- Trading day tree $\mathcal{T}^{\text{trade}}$: based on the original scenario tree. For every day and every scenario, branching between 12 am, previous day, and 12 am, current day, is delayed in time until the beginning of the next trading day (mon-fri and not a holiday), cf. Fig. 5. Each node $n \in \mathcal{N}$ of the original scenario tree has a unique corresponding node $j(n) \in \mathcal{N}^{\text{trade}}$ such that for the time-steps of the nodes it holds that $t(n) = t(j(n))$.

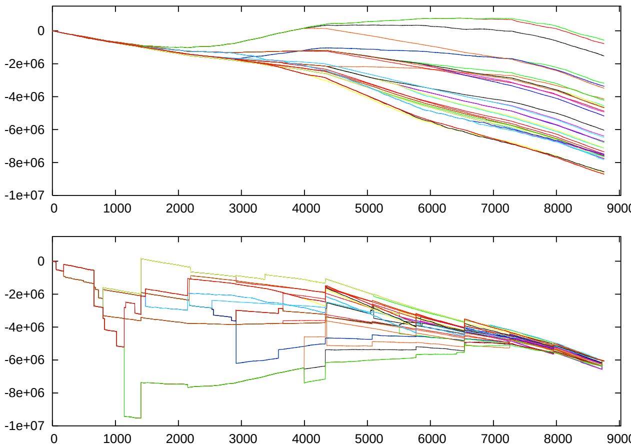

Fig. 9. Resulting optimal cash values over time for each scenario for the case that no contract is included and with **high fuel costs**. Top: Only $\mathbb{E}[z_T]$ is optimized. Bottom: $0.9 \cdot \mathrm{CVaR}(z_T) - 0.1 \cdot \mathbb{E}[z_T]$ is minimized.

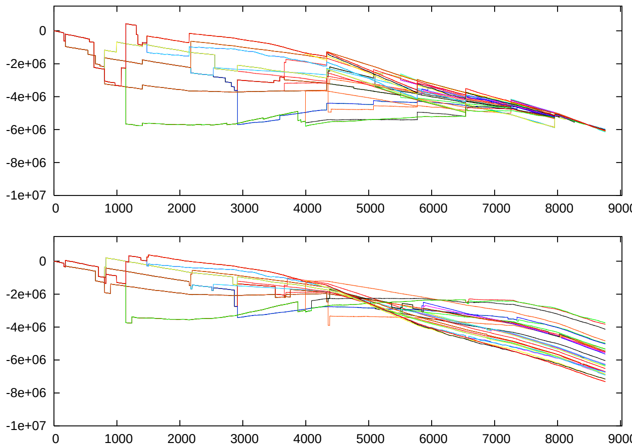

Fig. 10. Resulting optimal cash values over time for each scenario for the case that no contract is included and with **high fuel costs** using ρ_2 (top) and ρ_4 (bottom)

- Contract tree $\mathcal{T}^{\text{contr}}$: based on $\mathcal{T}^{\text{trade}}$, branching is (further) delayed to the 1st day of the following month. For each node j of $\mathcal{T}^{\text{trade}}$ there is a unique corresponding node $l(j) \in \mathcal{N}^{\text{contr}}$.

Note that the decision about the contract alternatives (fix, flexible, or none) has to be made already at the beginning (here-and-now decision), i.e., the respective decision variable would be defined on the root node 1 rather than on one of the above trees.

C. Decision variables

Decision variables will be denoted by the letter x. All of them are defined on one of the trees described in the previous section and, hence, are indexed by the respective node number:
Future stock for month m (base): $x_d^{fb,m} \in \mathbb{R},\ d \in \mathcal{N}^{\text{fut}}$
Future stock for month m (peak): $x_d^{fp,m} \in \mathbb{R},\ d \in \mathcal{N}^{\text{fut}}$
Spot market volumes: $x_j^s \in \mathbb{R},\ j \in \mathcal{N}^{\text{trade}}$
Power production, electricity: $x_n^{pe} \in \mathbb{R}_+,\ n \in \mathcal{N}$
Power production, heat (thermal): $x_n^{ph} \in \mathbb{R}_+,\ n \in \mathcal{N}$
Power production: $x_n^p = (x_n^{pe}, x_n^{ph}) \in \mathbb{R}^2,\ n \in \mathcal{N}$
Monthly declared contr. volumes: $x_l^{c,\text{flex,decl}} \in \mathbb{R}_+,\ l \in \mathcal{N}^{\text{contr}}$
Contract volumes: $x_j^c \in \mathbb{R}_+,\ j \in \mathcal{N}^{\text{trade}}$

D. Constraints

For the future trading variables, we impose that the initial future stock is empty and that, after future for month m has expired, the respective amount of futures is zero:
$x_1^{fb,m} = x_1^{fp,m} = 0$ for $m = 1, ..., 12$,
$x_d^{fb,m} = x_d^{fp,m} = 0$ if $t(d) \geq \text{end}(m)$ for $m = 1, ..., 12$.

For the CHP facility we impose a gradient restriction for the production of electricity, the heat demand satisfaction constraint, and that, for all time-steps, the two-dimensional vector x_n^p lies within some given bounded polyhedron in $\mathbb{R}^2$, cf. Fig. 6, that is given through a matrix A^p and a vector b^p:
$|x_n^{pe} - x_{n-}^{pe}| \leq \delta^{pe}$ for $n \in \mathcal{N} \setminus \{1\}$,
$x_n^{ph} \geq D_n^h$ and $A^p x_n^p \leq b^p$ for $n \in \mathcal{N}$.

For the contract volumes we have that $x_j^c = 0$ if no contract is purchased and, if the fix contract is included, $x_j^c = V_{t(j)}^c$ for $j \in \mathcal{N}^{\text{trade}}$. For the case that the flexible contract is chosen, the monthly declared volumes and the effective volumes, respectively, have to satisfy:

$$\begin{array}{lcl} x_l^{c,\text{flex,decl}} & \in & [(1-\alpha) \cdot V_{t(l)}^c, (1+\alpha) \cdot V_{t(l)}^c] \\ x_j^c & \in & [(1-\beta) \cdot x_{l(j)}^{c,\text{flex,decl}}, (1+\beta) \cdot x_{l(j)}^{c,\text{flex,decl}}] \end{array}$$

for $l \in \mathcal{N}^{\text{contr}}$, $j \in \mathcal{N}^{\text{trade}}$ with some given percentages α, β.

For the spot market, no further constraints are imposed. It remains to require the satisfaction of the electricity demand:

$$x_{j(n)}^s + x_n^{pe} + x_{j(n)}^c \geq D_n^e,\ n \in \mathcal{N} \tag{1}$$

E. Cash values

For formulating the objective, we introduce auxiliary variables z_n $(n \in \mathcal{N})$ that represent the wealth at time $t(n)$ in the respective scenario, i.e., the accumulated revenues. These *cash values* are composed of the revenues from satisfying the demands, the costs of power production and contracts, and the cash flows caused by spot market activity and future trading:

$$\begin{array}{rcl} z_n & = & z_{n-} + P^e \cdot D_n^e + P^h \cdot D_n^h \\ & + & z_n^p + z_n^c + z_n^s \\ & + & \sum_{m=1}^{12} z_n^{fb,m} + \sum_{m=1}^{12} z_n^{fp,m} \end{array} \tag{2}$$

Note that the z variables depend on the decisions. The cash flows for power production and spot market are given by

$$\begin{array}{rcl} z_n^p & = & -C^{pe} \cdot x_n^{pe} - C^{ph} \cdot x_n^{ph} \\ z_n^s & = & -x_{j(n)}^s \cdot C_n^s - |x_{j(n)}^s| \cdot C^{s,\text{trans}}, \end{array}$$

respectively. Because we allow for future trading only on trading days at noon, $z_n^{fb,m} = z_n^{fp,m} = 0$ if $t(n)$ does not correspond to such point in time. If $t(n)$ does correspond to 12 am on a trading day, i,e., if there is a corresponding node $d(n) \in \mathcal{N}^{\text{fut}}$, then

$$\begin{array}{rcl} z_n^{fb,m} & = & x_{d(n)-}^{fb,m} \cdot \left(C_{d(n)}^{fb,m} - C_{d(n)-}^{fb,m}\right) \\ & - & \left(|x_{d(n)}^{fb,m}| - |x_{d(n)-}^{fb,m}|\right) \cdot C^{f,\text{imar}} \\ & - & \left|x_{d(n)}^{fb,m} - x_{d(n)-}^{fb,m}\right| \cdot C^{f,\text{trans}} \cdot \mathbf{1}_{\{t(n) \neq \text{end}(m)\}} \end{array}$$

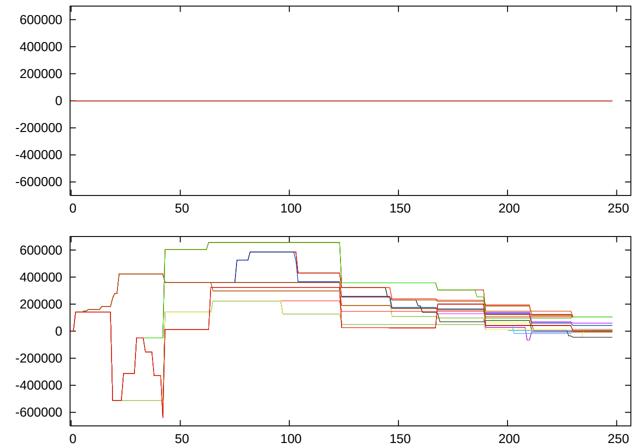

Fig. 11. Overall future stock over time for each scenario for the case that no contract is included and with high fuel costs. Top: Only $\mathbb{E}[z_T]$ is optimized. Bottom: $0.9 \cdot \text{CVaR}(z_T) - 0.1 \cdot \mathbb{E}[z_T]$ is minimized.

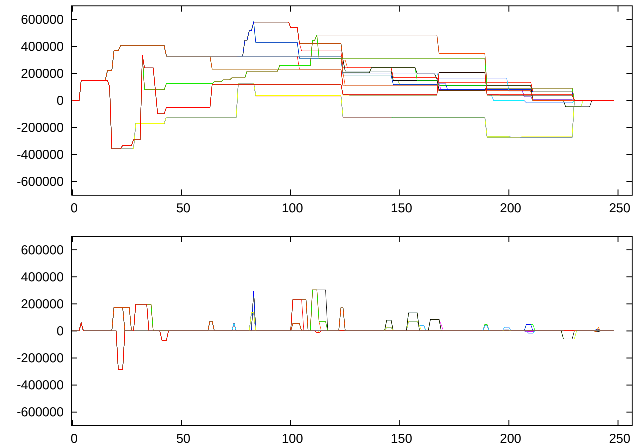

Fig. 12. Overall future stock over time for each scenario for the case that no contract is included and with high fuel costs using ρ_2 (top) and ρ_4 (bottom)

for base futures of month $m = 1, ..., 12$. The first and the second summand in the above equation represent the variation margin and the initial margin, respectively. The indicator function in the third summand reflects the fact, that transaction costs don't need to be payed when the future contract expires. For peak futures, the cost functions $z_n^{fp,m}$ are analogous.

For the contracts cash flow z_n^c, we have to distinguish between the fix and the flexible contract. For both of them, there is a volume dependent price to be payed, but for the latter, there is, in addition, an extrapolated demand rate $z_n^{c,\text{flex},d}$ depending on the maximum demand within the elapsed time, which is to be payed at the end of each month.

$$z_n^c = \begin{cases} -x_{j(n)}^c \cdot C^{c,\text{fix}} & \text{for the fix contract} \\ -x_{j(n)}^c \cdot C_{t(n)}^{c,\text{flex}} - z_n^{c,\text{flex},d} & \text{for the flexible contract} \end{cases}$$

The monthly demand rate is adapted such that, at the end of the term, the overall payment is proportional to the overall maximum power, hence,

$$\sum_{\{\tilde{n} \in \text{path}(n)\}} z_{\tilde{n}}^{c,\text{flex},d} = C^{c,\text{flex},d} \cdot \max_{\{j \in \text{path}(j(n))\}} x_j^{c,\text{flex}}$$

for all leaves n, i.e., for $n \in \mathcal{N}$ such that $t(n) = T$. Note that $z_{\tilde{n}}^{c,\text{flex},d} = 0$ if $t(\tilde{n})$ is not the end of a month.

F. Objective

The above cash values z_n can be understood, together with the node probabilities π_n, as discrete random variables z_t, $t = 1, ..., T$, with $T = 365 \cdot 24$ and $z_t = (z_n)_{\{n \in \mathcal{N}: t(n)=t\}}$. Note that the cash values are defined in a cumulative sense. Thus, the overall expected revenue is given by $\mathbb{E}[z_T]$ and the multiperiod risk measure ρ applied to the time-steps $t_1, ..., t_{T'}$ reads $\rho(z_{t_1}, ..., z_{t_{T'}})$. Hence, the objective can be written as

$$\min \ \gamma \cdot \rho(z_{t_1}, ..., z_{t_{T'}}) - (1 - \gamma) \cdot \mathbb{E}[z_T] \tag{3}$$

with some weighting parameter $\gamma \in [0, 1]$. The minimization is over all the x variables from section IV-C with respect to the constraints from section IV-D. For the simulations, we used $\gamma = 0.9$ and for $t_1, ..., t_{T'}$ we took the end of each week within the optimization horizon, i.e., $T' = 52$ and $t_j = j \cdot 7 \cdot 24$.

V. Simulation Results

The model is implemented and solved with ILOG CPLEX 9.1, the ILOG Concert Technology 21 library, and GNU C++ on a 2 GHz Linux PC with 1 GB memory. We used a scenario tree with 40 scenarios, $T = 365 \cdot 24 = 8760$ time-steps, and approx. $150,000$ nodes.

We ran the simulation successively for the case that the fix contract, the flexible one, or no contract at all is included. We separated this decision from the rest of the optimization model, because all the remaining decision variables are continuous, hence, the three remaining (sub-) problems are purely linear programs. Time for solution is in either case around two hours. We optimized with CVaR, with 2 multiperiod risk measures, and without risk measure and obtained the following values:

	no contract	fix contr.	flex. contr.
opt. $0.9\rho_2 - 0.1\mathbb{E}$	$2.887.700$	$2.348.420$	$3.751.450$
$\rho_2(z_{t_1}, ..., z_{t_{52}})$	$2.886.930$	$2.402.930$	$3.766.220$
$\mathbb{E}[z_T]$	$-2.894.640$	$-1.857.900$	$-3.618.460$
opt. $0.9\rho_2 - 0.1\mathbb{E}$	$1.110.470$	595.918	$1.335.880$
$\rho_4(z_{t_1}, ..., z_{t_{52}})$	911.662	453.436	$1.078.550$
$\mathbb{E}[z_T]$	$-2.899.720$	$-1.878.250$	$-3.651.910$
opt. $0.9\text{CVaR} - 0.1\mathbb{E}$	$2.894.770$	$1.872.500$	$3.629.540$
$\text{CVaR}(z_T)$	$2.894.770$	$1.872.500$	$3.629.540$
$\mathbb{E}[z_T]$	$-2.894.770$	$-1.872.500$	$-3.629.540$
opt. $\mathbb{E}[z_T]$ ($\gamma = 0$)	$-2.877.230$	$-1.846.890$	$-3.601.580$

These values suggest, that going without any contract is the best alternative in terms of expected revenue and, surprisingly, in terms of risk, too. Note that the absolute values of the risk measures may not have a significant meaning, but can be compared for the three contract alternatives.

Beside the (optimal) magnitude of the risk measure and the expected terminal wealth, the shape of the cash values over all time-steps seems to be the most relevant output information. For the case that no contract is considered, the effect of different risk measures can be observed very well, cf. Fig. 7-10. Fig. 7 and 8 result from simulations using the reference parameter. Obviously, optimizing without a risk measure causes high spread for the distribution of the overall revenue z_T. The incorporation of the (one-period) CVaR

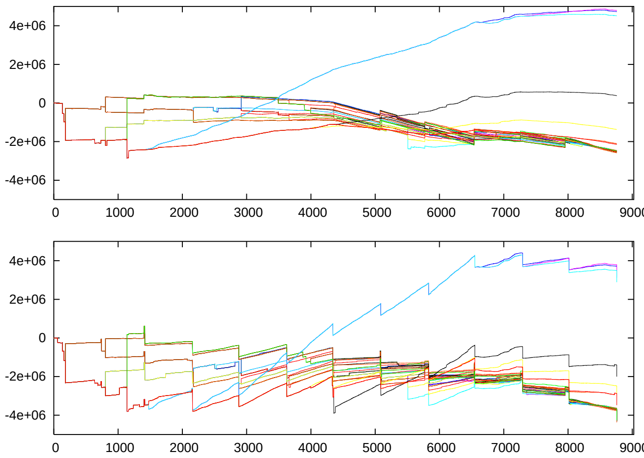

Fig. 13. Resulting cash values over time using the multiperiod risk measure ρ_2 for the case that the fix contract (top) or the flexible contract (bottom) is included. In the latter case, there are jumps due to the monthly demand rate.

applied to z_T reduces this spread considerably for the price of high spread and very low values for z_t at time $t < T$. The multiperiod polyhedral risk measures are effective such that spreading is somehow more equal over time.

Fig. 8 suggests that the effect of ρ_2 and ρ_4 is more or less the same, but Fig. 10 reveals that this is not the case. For the calculation of Fig. 9 and 10, the parameters C^{pe} and C^{ph} have been augmented in order to give the cash value curves a different direction. The difference between ρ_2 and ρ_4 is, roughly speaking, that ρ_4 tries to bring equal spread at all times, whereas ρ_2 tries to find a maximal level that is rarely underrun.

The different shapes of the curves are achieved by different policies of future trading. Future trading is revealed through the jumps in the curves and is explicitely shown in Fig. 11 and 12. If no risk is considered then there is no future trading at all since, due to the fair-price assumption, there is no benefit from futures in terms of the expected revenue. Using CVaR or ρ_2 leads to extensive future trading activity, whereas the application of ρ_4 yields more moderate future trading.

For the case that a delivery contract is considered, future trading activity is reduced, cf. Fig. 13. The curve shapes are basically determined by the special properties of the contracts.

VI. Conclusions and Outlook

Regarding the optimal values, relying on spot and future market appears to be the better choice than purchasing one of the available delivery contracts. However, the situation may be different if the conditions, i.e., the parameters, are changed, or if we no longer assume fair prices for the futures. Due to this fair-price assumption, futures are almost too perfectly capable of reducing spot price risk.

The model could be adapted and improved in numerous directions, e.g. by allowing for price-dependent spot market bids or by introducing integer variables into the CHP production facility model. Moreover, another goal is to enlarge the number of scenarios in order to approximate the uncertainty more accurately. Therefore, one would need more efficient solution methods. Currently, we are working on a decomposition approach based on Lagrangian relaxation of the coupling constraint (1) and the coupling induced by the non-linearity of the polyhedral risk measures.

References

[1] N. Gröwe-Kuska, K. C. Kiwiel, M. P. Nowak, W. Römisch, and I. Wegner, "Power management in a hydro-thermal system under uncertainty by Lagrangian relaxation," in *Decision Making under Uncertainty: Energy and Power*, ser. IMA Volumes in Mathematics and its Applications, C. Greengard and A. Ruszczyński, Eds. New York: Springer, 2002, vol. 128, pp. 39–70.

[2] S. Takriti, B. Krasenbrink, and L. S.-Y. Wu, "Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem," *Operations Research*, vol. 48, pp. 268–280, 2000.

[3] S.-E. Fleten, S. W. Wallace, and W. T. Ziemba, "Hedging electricity portfolios via stochastic programming," in *Decision Making under Uncertainty: Energy and Power*, ser. IMA Volumes in Mathematics and its Applications, C. Greengard and A. Ruszczyński, Eds. New York: Springer, 2002, vol. 128, pp. 71–93.

[4] S. Sen, L. Yu, and T. Genc, "A stochastic programming approach to power portfolio optimization," *Operations Research*, vol. 54, pp. 55–72, 2006.

[5] A. J. Conejo, F. J. Nogales, J. M. Arroyo, and R. García-Bertrand, "Risk-constrained self-scheduling of a thermal power producer," *IEEE Transactions on Power Systems*, vol. 19, pp. 1–6, 2004.

[6] A. Eichhorn and W. Römisch, "Polyhedral risk measures in stochastic programming," *SIAM Journal on Optimization*, vol. 16, pp. 69–95, 2005.

[7] A. Ruszczyński and A. Shapiro, Eds., *Stochastic Programming*, 1st ed., ser. Handbooks in Operations Research and Management Science. Amsterdam: Elsevier, 2003, vol. 10.

[8] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, "Coherent measures of risk," *Mathematical Finance*, vol. 9, pp. 203–228, 1999.

[9] H. Föllmer and A. Schied, *Stochastic Finance: An Introduction in Discrete Time*, 2nd ed., ser. De Gruyter Studies in Mathematics. Berlin: Walter de Gruyter, 2004, vol. 27.

[10] R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *Journal of Banking & Finance*, vol. 26, pp. 1443–1471, 2002.

[11] P. Artzner, F. Delbaen, J.-M. Eber, D. Heath, and H. Ku, "Coherent multiperiod risk adjusted values and Bellman's principle," Department of Mathematics, ETH Zürich, Switzerland," Preprint, 2004.

[12] G. C. Pflug and A. Ruszczyński, "A risk measure for income processes," in *Risk Measures for the 21st Century*, ser. The Wiley Finance Series, G. Szegö, Ed. New York: Wiley, 2004, ch. 13.

[13] A. Eichhorn, W. Römisch, and I. Wegner, "Mean-risk optimization of electricity portfolios using multiperiod polyhedral risk measures," *IEEE St.Petersburg PowerTech Proceedings*, 2004.

[14] H. Heitsch and W. Römisch, "Generation of multivariate scenario trees to model stochasticity in power management," in *IEEE St.Petersburg PowerTech Proceedings*, 2005.

[15] ——, "Scenario tree modelling for multistage stochastic programs," DFG Research Center MATHEON *Mathematics for key technologies*, Berlin, Germany," Preprint 296, 2005.

[16] H. Heitsch, W. Römisch, and C. Strugarek, "Stability of multistage stochastic programs," *SIAM Journal on Optimization*, 2006, to appear.

Biographies

Andreas Eichhorn is a research assistant at the Department of Mathematics, Humboldt-University Berlin. He graduated in technical mathematics in 1999 at the University of Karlsruhe, Germany. Between 1999 and 2002 he worked as a computer programmer for a German software company.
E-Mail: `eichhorn@math.hu-berlin.de`
Web: `http://www.math.hu-berlin.de/~eichhorn/`

Werner Römisch is a Full Professor at the Department of Mathematics, Humboldt-University Berlin, Germany. His current research interests are in theory and solution methods for mixed-integer stochastic programming problems. He is actively working on several applications, especially in the electric power industry. He is co-editor of the *Stochastic Programming E-Print Series* (`http://www.speps.org`).
E-Mail: `romisch@math.hu-berlin.de`
Web: `http://www.math.hu-berlin.de/~romisch/`