

Mean-risk optimization of electricity portfolios

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We present a mathematical model with stochastic input data for mean-risk optimization of electricity portfolios containing several physical components and energy derivative products. The model is designed for a medium term optimization horizon of one year in hourly discretization. With the objective of maximization of the mean book value of the portfolio at the end of optimization horizon simultaneously several risk measures are taken into account. We present numerical results for a large-scale realistic problem adapted to a municipal utility and study the effects of varying weighting of risk on the book value of the portfolio during the whole time horizon.

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1 Introduction

The deregulation of energy markets has lead to an increased awareness of the need for profit maximization with simultaneous consideration of risk management, adapted to individual risk aversion of market participants. Mathematical modelling of such problems with uncertain input data results in mixed-integer large-scale stochastic programming models. We refer to a wide range of literature dealing with power management in a hydro-thermal system and simultaneous optimization of power production and electricity trading, e.g. [6] and [8]. We suppose that each historical observation of electrical load and spot price is a realization of certain bivariate random variables. The joint distribution of the stochastic process will be characterized by a time series model. To ensure the numerical tractability of the optimization problem we generate a large number of Monte-Carlo scenarios from this time series model. By means of scenario reduction techniques we generate from this initial approximation of the underlying probability distribution a specific form of an approximation - a scenario tree taking into account the information structure of the optimization problem. For the mathematical description of scenario trees, see figure

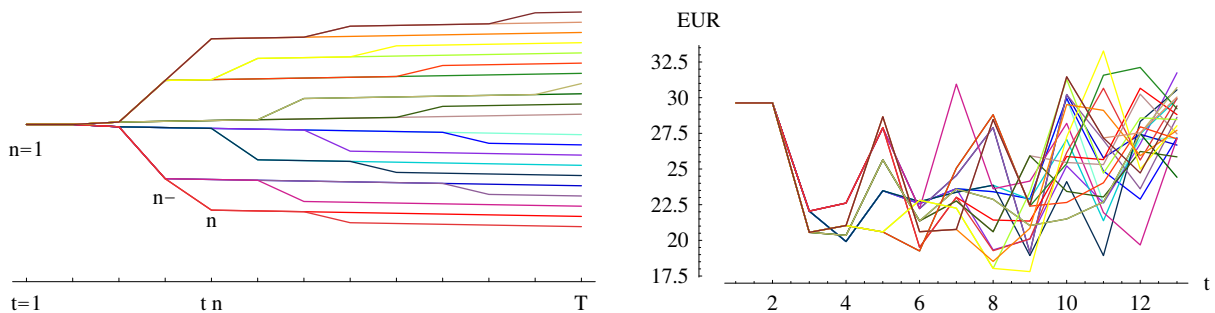


Fig. 1 Scenario tree structure and spot prices - base load

1, we number serially the nodes of the tree. Except for the root node $n = 1$ every node $n \in \mathcal{N}$ has a unique predecessor but possibly several successors forming the set $\mathcal{N}_+(n)$. By \mathcal{N}_T we denote the set of leafs of the scenario tree. Further let $path(n)$ be the set of nodes from the root node to the node n , whereas $t(n)$ denotes the time period related to n . To every node is assigned a unique node probability π_n .

2 Modelling stochastic processes

Based on earlier studies, e.g. [6], and [8] we suggest a decomposition strategy for the original bivariate series $\{(L_t, C_t^{Sp}), t = 1, \dots, T\}$ in hourly discretization and a separate modelling of intra daily behavior and average daily behavior. We generate intra daily scenarios by using a distribution free resampling procedure based on a cluster analysis. For description of modelling the bivariate average daily process let L_k denote the load and C_k^{Sp} the spot price at day k . The model

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comprises a deterministic function $(f^1(k), f^2(k))$ representing a yearly trend with seasonal patterns, a stochastic component $(X^1(k), X^2(k))$ modelled by a bivariate autoregressive moving-average process, whereas extreme spot price outliers are modelled by a discretized jump-diffusion process with time varying jump parameters. For details we refer to [7].

$$\begin{pmatrix} L_k \\ C_k^{Sp} \end{pmatrix} = \begin{pmatrix} f^1(k) \\ f^2(k) \end{pmatrix} + \begin{pmatrix} X_k^1 \\ X_k^2 \end{pmatrix} + \begin{pmatrix} 0 \\ O_k^2 \end{pmatrix}, \quad k = 1, \dots, K = T/24$$

In addition to physical components the electricity portfolio contains energy derivative products, represented by EEX futures of various load and delivery types. Futures are standardized forward transactions. Both buyer and seller agree on the current date to call-off respectively to supply a certain quantity of electricity at a delivery period in the future for a certain price or to effect respective payments. In order to generate arbitrage free future prices as an input for the optimization model we calculate faire prices, adjusted to EEX rules. The settlement price for month futures on the last trading day is the mean spot price for the delivery month and the associated load type. Disregarding transaction fees the faire price for a base load month future at time t with delivery period $\langle T_1, T_2 \rangle$ in a liquid market related to the underlying spot price C_t^{Sp} is defined as:

$$C_{t,T_2}^{f,M} = \mathbb{E} \left(\sum_{j=T_1}^{T_2} C_j^{Sp} \mid C_t^{Sp} \right), \quad T_0 \leq t \leq T_2 \quad (1)$$

3 Risk measure

The risk of losses of a position or a portfolio is assessed by means of risk measures. Risk measures are defined as mappings from a space of random variables to the real numbers. A distinction is drawn between one-period risk measures, i.e., risk measures that have only one scalar random variable as argument, and multi period risk measures that quantify risk of a random process. Well known risk measures are, e.g., Value-at-Risk at level α (VaR_α) and Conditional-Value-at-Risk ($CVaR_\alpha$). The latter turned out to be very suitable for stochastic optimization problems. In addition, it satisfies several axioms that are considered essential from an economic point of view [1].

In our application, however, a random process is to be assessed with respect to its risk, hence, a multi period risk measure is needed. In [3], the class of polyhedral risk measures was introduced and it was shown that risk measures from this class are suitable for stochastic optimization problems. Moreover, multi period extensions of $CVaR_\alpha$ with different emphasis were suggested and analyzed there. We will use these risk measures in our model, see also [4].

4 Optimization model

As the optimization problem is solvable only for a limited number of scenarios it does not seem too restrictive, to narrow the branching structure of the original \mathcal{N} -tree. We branch in the existing model once a month and reduce thereby future trading activities. A portfolio switching is made at the end of a month. Mathematically we model this restricted structure by a second scenario tree with a node set $\mathcal{M} \subset \mathcal{N}$. The \mathcal{M} -tree comprises all nodes from the \mathcal{N} -tree coinciding with the last hour of every month and $m = 1 \leftrightarrow n = 1$.

In order to model a risk-orientated profit maximization the objective function appears as a weighted sum of the mean portfolio book value at the end of the optimization horizon and a risk measure.

$$f(\gamma, z) = (1 - \gamma) \sum_{m \in \mathcal{M}_T} \pi_m z_m - \gamma \rho(z), \quad \gamma \in [0, 1] \quad (2)$$

The weighting factor γ controls the relation between profit maximization and risk aversion. The book value of the portfolio in a node m is composed of all monthly debits and credits for the portfolio elements: power production, power supply contracts, EEX power contracts, supply due to legislation on the priority of renewable energies and futures.

In view of medium-term optimization horizon modelling of the thermal units of the power plant is reduced to upper and lower bounds for the power production and a maximum power variation velocity during one hour. So the model comprises only two integer variables among a large number of real variables. Hence, the problem is for a limited number of scenarios solvable with commercial solvers (CPLEX). Modelling of bid behavior in the auction trade system of EEX spot market is not an essential aim of the existing model. Hence, spot volumes are treated like balance energy with spot prices.

In the first years following the deregulation of energy markets municipal utilities normally placed long- or medium-term supply contracts with large power concerns in addition to customer generation. The form of supply contract considerably differs with respect to flexibility and charges. Therefore we modelled two types of contracts: a fixed and a flexible supply contract. The former is placed with a term of one year for a fixed price. Delivery will be made as agreed, subsequent changes of supply quantities are not possible.

The form of the flexible supply contract allows for an adjustment of agreed quantities within certain bounds during specified time periods. Equations (3) and (4) reflect the readjustment in node n of monthly scheduled quantities $P_n^{O,M}$ based on the fixed

yearly supply schedule $P_{t(n)}^V$ at time $t(n)$ and the correction of daily supply schedules $P_n^{O,T}$ based on the monthly quantities.

$$(1 - \alpha) \cdot u^O \cdot P_{t(n)}^V \leq P_n^{O,M} \leq (1 + \alpha) \cdot u^O \cdot P_{t(n)}^V \quad (3)$$

$$(1 - \beta) \cdot P_n^{O,M} \leq P_n^{O,T} \leq (1 + \beta) \cdot P_n^{O,M} \quad (4)$$

Here u^O denotes the binary decision variable for the flexible supply contract. Note that for the final daily quantities holds $P_n^{O,T} = 0$ if $u^O = 0$. The declaration of monthly supply quantities takes place in the previous month, i.e., the information structure of the associated decision variable is not in accordance with the data scenario tree. Therefore, we add explicit non-anticipativity restrictions for monthly supply quantities to the model.

For the flexible supply contract the customer has to pay an energy rate depending on time and on actual power volumes and a demand rate depending on maximum annual power. In (5) the peak load $P_n^{O,max}$ for every scenario leaf is determined.

$$P_n^{O,max} = \max_{l \in \text{path}(n)} \{P_l^{O,T}\}, \quad \forall n \in \mathcal{N}_T \quad (5)$$

All these physical components of the portfolio are coupled by the load constraint, where P_n^{El} denotes the power production and P_n^{Sp} volumes traded on the spot market at node n as well as $P_{t(n)}^{RE}$ a fixed portion of renewable energies at time $t(n)$.

$$L_n = P_n^{El} + P_n^{O,T} + u^V P_{t(n)}^V + P_n^{Sp} + P_{t(n)}^{RE} \quad (6)$$

Within the optimization model we allow for free future trading without any artificial time restrictions. In interaction with the risk measure a moderate future trading may be expected, hence, we can understand this financial portfolio component as a hedging instrument. At the European Energy Exchange (EEX) futures for various delivery periods, month, quarter and year, are traded. With regard to the optimization horizon of one year we model the former two and in each case the load types base and peak. Tradable are the next six months and the current delivery month and the respectively next seven quarters. Quarter contracts are fulfilled by cascading. Cascading means automatically splitting into three month contracts of the respective quarter contract on the last trading day before the transition to the delivery period takes place. Month contracts are fulfilled by cash settlement. Futures are characterized by a daily profit and loss equation and by the obligation to deposit securities.

For the sake of clarity we exemplarily describe the future balance for a month future taking into account a cascading quarter future of the same load type. The variables $F_{m,k}^{M,cas}$, $F_{m,l}^{M,cas}$ and $F_{m,r}^{M,cas}$ are auxiliary variables. Let $t(k) = t(m) + 1$, $t(l) = t(k) + 1$, $t(r) = t(l) + 1$ correspond to three consecutive months and H_k , H_l , H_r denote the number of delivery hours of the respective month futures. In equation (7) the number $F_{m,k}^M$ of a month future with delivery period k in a node m is updated. The balance contains the number of futures in previous month $F_{m-,k}^M$, the number of purchased futures $F_{m,k}^{M,p}$ and the number of sold futures $F_{m,k}^{M,s}$ in node m as well as month futures $F_{m,k}^{M,cas}$ stemming from a cascading quarter future, where $F_{m-,m}^Q$ denotes the remaining number of quarter futures cascading at time $t(m)$.

$$F_{m,k}^M = F_{m-,k}^M + F_{m,k}^{M,p} - F_{m,k}^{M,s} + F_{m,k}^{M,cas} \quad (7)$$

$$F_{m,k}^{M,cas} = F_{m-,m}^Q \cdot H_k / (H_k + H_l + H_r) \quad (8)$$

$$F_{m-,m}^Q = F_{m--,m}^Q + F_{m-,m}^{Q,p} - F_{m-,m}^{Q,s} \quad (9)$$

These balance equations are formulated for all involved future types, month and quarter, and the load types base and peak. Further we added lower and upper bounds for the future stock, constraints for trading periods and time constraints related to cascading quarter futures to the model. When opening a position, a basic security, the so-called initial margin, must be deposited. The initial margin is bound for the entire duration of the contract. The change in value of the futures position results from the difference between the settlement price of the current day and the settlement price of the previous day. This change in value multiplied with the number of contracts leads to daily credit notes or additional payments which are called variation margin. Neglecting transaction fees we can assign the following financial transactions to the month future considered in (7).

$$\begin{aligned} z_m^{f,M} &= z_{m-}^{f,M} + C^{f,IM,M}(F_{m,k}^{M,s} - F_{m,k}^{M,p}) + F_{m,k}^{M,cas}(C^{f,IM,Q} - C^{f,IM,M}) + \\ &F_{m-,k}^M(C_{m,k}^{f,M} - C_{m-,k}^{f,M}) + F_{m,k}^{M,cas}(C_{m,k}^{f,M} - C_{m-,m}^{f,Q}) + F_{m-,m}^M C_{m-,m}^{f,IM,M} \end{aligned} \quad (10)$$

The future cash flow in (10) is composed of the previous month's cash value, payment and repayment of initial margins, correction of initial margins due to cascading, variation margin of month future, variation margin of cascaded futures and repayment of initial margin for expired futures.

Let now z_m^f denote the sum of all $z_m^{f,M}$ and $z_m^{f,Q}$ for all month and quarter futures, respectively, that enter in the balance sheet at node m . Further let z_m denote the book value of the portfolio at node m including all cash inflow and outflow. In particular, we have to consider total revenue from spot and futures market, payments from consumers of electricity, expenditures for

power production, for supply contracts as well as for renewable energies. Let $m \in \mathcal{M} \leftrightarrow l \in \mathcal{N}$ and $m_- \leftrightarrow r \in \mathcal{N}$, $t(m) > 1$.

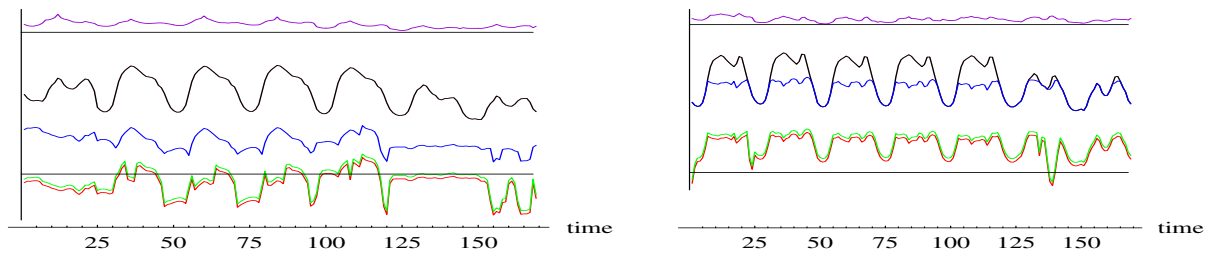
$$z_m = z_{m_-} + z_m^f + \sum_{\substack{n \in \text{path}(l), \\ t(r) < t(n) \leq t(l)}} \left(L_n C^G - C_n^{\text{Sp}} P_n^{\text{Sp}} - C_{t(n)}^{\text{El}} P_n^{\text{El}} - C_{t(n)}^{\text{RE}} P_{t(n)}^{\text{RE}} \right. \\ \left. - u^V C_{t(n)}^V P_{t(n)}^V - C_{t(n)}^{\text{O,T}} P_n^{\text{O,T}} - \mathbf{1}_{(m \in \mathcal{M}_T \leftrightarrow n \in \mathcal{N}_T)} P_n^{\text{O,max}} C^{\text{O,max}} \right) \quad (11)$$

5 Numerical results

The following table gives an overview about the used hard- and software. Computation are made for 21 scenarios leading to 2.4 millions of real variables. For different risk measures and a varying risk weighting parameter a portfolio switching and

Calculations	Intel (R) Celeron (R), CPU 2 GHz RAM 512 MB
Statistic software	Time Series (Mathematica 4.2), Statistica 6.1, SPlus 4.5
Optimization software	GAMS 21.1, CPLEX 8.1.

significant differences in the distribution function of the book value of the portfolio can be observed. For details concerning the risk measure we refer to [3]. In particular, in case of CVaR the book value of the portfolio shows a high spread for the scenarios throughout the whole year except for the time horizon. Independent of γ the portfolio consist of the same physical components: the fixed supply contract, the power production and EEX power contracts. Further extensive future trading activities take place. In contrast to the CVaR the multi period risk measure ρ_4 from [3] reduces the spread of the book value throughout the whole optimization horizon. For small γ the portfolio composition coincides with the former, whereas $\gamma = 0.5$ leads to a portfolio switching: the fixed supply contract is replaced with the more expensive flexible supply contract. Apparently, due to small number of branchings in the scenario tree, future trading activities are not able to control the book value taking into account the constrains for the multi period risk measure. The next figure shows quantitative seasonal



differences for the different physical portfolio components: volumes from the spot market (red), renewable energies (green), power production (blue) and the volumes of the flexible supply contract (black). During summer time power contracts at the spot market are predominantly sold. They are purchased only during daily peak load times. The flexible supply contract is in use the whole day. During winter time power contracts are mainly purchased. The flexible supply contract is in use only during peak load times and almost never at weekends.

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References

- [1] Artzner, P., Delbaen, F., Eber, J.-M., Heath, D., Coherent measures of risk, *Mathematical Finance* 9, 1999.
- [2] Clewlow, L., Strickland, C., *Energy Derivatives, Pricing and Risk Management*, LACIMA Publications, London, 2000.
- [3] Eichhorn, A., Römisch, W., Polyhedral risk measures in stochastic programming, Preprint 04-05, Institut für Mathematik, Humboldt-Universität Berlin, 2004 and submitted.
- [4] Eichhorn, A., Römisch, G., Wegner, I.: Optimizing electricity portfolios using polyhedral risk measures, *Proceedings in Applied Mathematics and Mechanics*, 2004.
- [5] Escribano, Á., Peña, J., Villaplana, P., Modeling electricity prices, Working Paper 02-27, Economics Series 08, 2002.
- [6] Gröwe-Kuska, N., Kiwiel, K.C., Nowak, M.P., Römisch, W., Wegner, I., Power management in a hydro-thermal system under uncertainty by Lagrangian relaxation. In: *Decision Making under Uncertainty: Energy and Power* (C. Greengard, A. Ruszczyński eds.), IMA Volumes in Mathematics and its Applications Vol. 128, Springer, New York 2002, 39–70.
- [7] N. Gröwe-Kuska, A. Liebscher, M. Lucht, W. Römisch, G. Spangardt und I. Wegner, Mittelfristige risikoorientierte Optimierung von Strombeschaffungs-Portfolios kleinerer Marktteilnehmer, Preprint 03-11, Institut für Mathematik, Humboldt-Universität Berlin, 2003.
- [8] Sen, S., Yu, L., Genc, T., A stochastic programming approach to power portfolio optimization, *Stochastic Programming E-Print Series* 02-2003 (<www.speps.info>).