

THE MODELING POWER OF THE PE-
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RAILWAY TIMETABLES — AND BEYOND

by

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No. 2004/20

The Modeling Power of the Periodic Event Scheduling Problem: Railway Timetables — and Beyond*

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Summary. In the planning process of railway companies, we propose to integrate important decisions of network planning, line planning, and vehicle scheduling into the task of periodic timetabling. From such an integration, we expect to achieve an additional potential for optimization.

Models for periodic timetabling are commonly based on the Periodic Event Scheduling Problem (PESP). We show that, for our purpose of this integration, the PESP has to be extended by only two features, namely a linear objective function and a symmetry requirement. These extensions of the PESP do not really impose new types of constraints, because practitioners have already required them even when only planning timetables autonomously without interaction with other planning steps.

1 Introduction

Traditionally, the planning process of railway companies is subdivided into several tasks. From the strategic level down to the operational level, the most prominent subtasks are network planning, line planning, timetable generation, vehicle scheduling, crew scheduling, and crew rostering, see Figure 1.

For a detailed description of these planning steps as well as for an overview of solution approaches, we refer to Bussieck, Winter, and Zimmermann [4]. Notice that network planning and line planning are of course part of the strategic planning process of public transportation companies. In contrast, vehicle scheduling and crew scheduling are of operational nature. In other words, timetabling forms the linkage between service and operation. An important reason for the division into at least five subtasks is the high complexity of the overall planning process ([4], [7]).

However, during the last years, a trend towards the integration of several planning steps has emerged. For example, vehicle and crew scheduling were successfully combined by Borndörfer, Löbel, and Weider [3] and by Haase, Desaulniers, and Desrosiers [8]. Similarly, a combination of line planning and network planning is

* Supported by the DFG Research Center “Mathematics for key technologies” in Berlin

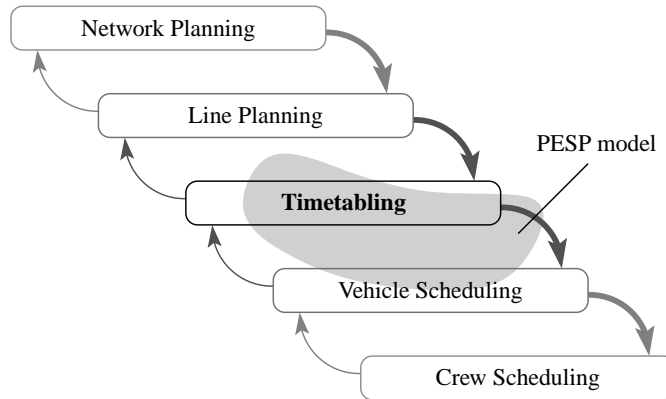


Fig. 1. Planning phases covered by the PESP beforehand

the objective of Borndörfer, Grötschel, and Pfetsch [2]. Periodic timetabling has also served as a starting point for such attempts. Kolonko and Engelhardt-Funke [9] consider investments into infrastructure by using multi-criteria optimization. Liebchen and Peeters [15] add important aspects of vehicle scheduling.

In this paper, we demonstrate how periodic timetable construction can be combined with other planning steps and also incorporate other practical conditions on timetables. This concerns timetable symmetry, line planning, and even infrastructure decisions. We show that this can in fact be achieved with only slight variations of the commonly used model for periodic timetable construction, the PESP model introduced by Serafini and Ukovich [24] in 1989. The variations keep much of the properties of the PESP model and are again mixed integer programs over a feasibility domain with essentially the same structure as the original PESP. In particular, all of the valid inequalities for the PESP stay valid, and some of the new formulations even speed up the solution time of standard MIP solvers.

In the discussion of these modeling features, we will also lay out the map of the borderline between what still fits into the traditional PESP model, and what does require new features, and at which cost. To this end, we also review the traditional PESP modeling issues, thus altogether providing a selfcontained presentation of the PESP modeling capabilities and its extensions to symmetry, line planning, and network planning.

The paper is organized as follows. Section 2 introduces the PESP. It presents its main formulations as a graph theoretic potential problem and as a mixed integer program, and reports on its complexity and a useful characterization of periodic timetables.

Section 3 discusses requirements for cyclic timetables that can be met by the PESP. These include simple requirements such as collision-free traffic on single tracks and headway between successive trains but also more sophisticated ones such

as bundling of lines, train coupling and sharing, fixed events in connection with hierarchical planning, and also disjunctive constraints and soft constraints.

Section 4 is devoted to timetable requirements that leave the traditional PESP, such as balanced reduction of service and symmetry of timetables. We show that, in the case of symmetry, the PESP or its MIP model only needs to be extended slightly in order to accommodate them.

Finally, in Section 5, we consider the integration of aspects of other planning steps into periodic timetable construction, in particular vehicle scheduling (minimization of rolling stock), line planning (simultaneous construction of line plan and timetable), and network planning (making infrastructure decisions). This integration makes essential use of the flexibility of the PESP (in particular disjunctive constraints), uses symmetry and, as a new technique, integrates aspects of graph techniques into the PESP in order to handle line planning.

All model features are illustrated by examples from our practical experience with timetable construction at Deutsche Bahn AG, S-Bahn Berlin GmbH, and BVG (Berlin Underground).

2 The Periodic Event Scheduling Problem (PESP)

In 1989, Serafini and Ukovich [24] introduced the periodic event scheduling problem (PESP), by which periodic timetabling instances may be formulated in a very compact way. Since then, this model has been widely used ([23],[17],[16]). In the Periodic Event Scheduling Problem (PESP), we are given a period time T and a set V of events, where an event models either the arrival or the departure of a directed traffic line at a certain station. Furthermore, we are given a set of constraints A . Every constraint $a = (i, j)$ relates a pair of events i, j by a lower bound ℓ_a and an upper bound u_a .

A solution of a PESP instance is a node assignment $\pi : V \mapsto [0, T)$ that satisfies

$$(\pi_j - \pi_i - \ell_a) \bmod T \leq u_a - \ell_a, \forall a = (i, j) \in A,$$

or $\pi_j - \pi_i \in [\ell_a, u_a]_T$ for short. We call a feasible node potential a feasible *timetable*. Notice that we can scale an instance such that $0 \leq \ell_a < T$, and for the *span* $d_a := u_a - \ell_a$ of a *feasible interval* $[\ell_a, u_a]_T$ we may assume w.l.o.g. $d_a < T$. Furthermore, for every fixed event i_0 , every fixed point of time $t_0 \in [0, T)$, and every feasible timetable π there exists an equivalent timetable π' with $\pi'_{i_0} = t_0$. This is achieved by performing the simple shift $\pi'_i := (\pi_i - (\pi_{i_0} - t_0)) \bmod T$. Let us denote by $D = (V, A, \ell, u)$ the *constraint graph* modeling a PESP instance.

There are several practical aspects of periodic timetabling which profit from the presence of a linear objective function of the form

$$\sum_{a=(i,j) \in A} c_a \cdot (\pi_j - \pi_i - \ell_a) \bmod T,$$

with costs c_a . In our opinion, the most striking one is the integration of central aspects of vehicle scheduling, cf. section 5.1.

Another perspective of periodic scheduling can be obtained by considering tensions instead of potentials. In a straightforward way, define for a given node potential π its tension

$$\hat{x}_a := \pi_j - \pi_i, \forall a = (i, j) \in A.$$

Recall that a vector \hat{x} is a tension, if and only if for some cycle basis \mathcal{C} , and each of its cycles $C \in \mathcal{C}$ with incidence vectors $\gamma_C \in \{-1, 0, 1\}^A$, there holds $\gamma_C \hat{x} = 0$. This yields the following MIP formulation

$$\left. \begin{array}{l} \min c^t(\hat{x} + pT) \\ \text{s.t. } \Gamma \hat{x} = 0 \\ \ell \leq \hat{x} + pT \leq u \\ p \in \mathbb{Z}^A, \end{array} \quad \text{or} \quad \begin{array}{l} \min c^t x \\ \text{s.t. } \Gamma(x - pT) = 0 \\ \ell \leq x \leq u \\ p \in \mathbb{Z}^A, \end{array} \right] \quad (1)$$

where $\Gamma \in \{-1, 0, 1\}^{(|A|-|V|+1) \times |A|}$ denotes the cycle-arc incidence matrix (*cycle matrix*) of some cycle basis of the graph D . Notice that the x variables are in fact a *periodic tension*, which we formally define for a given node potential π to be

$$x_{ij} := (\pi_j - \pi_i - \ell_{ij}) \bmod T + \ell_{ij}.$$

Already Serafini and Ukovich made the following simple but useful observation.

Lemma 1 (Serafini and Ukovich [24]). *If we relax the requirement $\pi \in [0, T]^V$ to $\pi \in \mathbb{Q}^V$, then for every spanning tree H and every feasible timetable π there exists a feasible timetable π' of the same objective value, but which induces $p_a = 0$ for $a \in H$.*

Notice that we may interpret the remaining non-zero integer variables as the representants of the elements of a (strictly) fundamental cycle basis. A generalization to integral cycle bases yields many variants of problem formulation 1, some of which are easier to solve for MIP solvers ([14]).

Periodic tensions can be characterized similarly to classic aperiodic tensions.

Lemma 2 (Cycle Periodicity Property). *A vector $x \in \mathbb{Q}^A$ is a periodic tension, if and only if for every cycle C with incidence vector $\gamma_C \in \{-1, 0, 1\}^A$, there exists some $z_C \in \mathbb{Z}$, such that*

$$\gamma_C x = z_C T. \quad (2)$$

Sometimes, things become more obvious by defining *slack variables* $\tilde{x}_a := x_a - \ell_a$.

The PESP is \mathcal{NP} -complete, since it obviously generalizes Vertex Coloring ([21]). We just orient the edges of a Coloring instance arbitrarily and assign feasible periodic intervals $[1, T - 1]_T$ to each of them. Solution methods for the PESP include Constraint Programming ([23]), Genetic Algorithms ([19]), and of course integer programming techniques. For the latter, a very important ingredient is

Theorem 1 (Odijk [21]). *An integer vector p allows a feasible solution for the MIP (1), if and only if for every oriented cycle C of the constraint graph, the following cycle inequalities hold*

$$\left[\frac{1}{T} \left(\sum_{a \in C^+} \ell_a - \sum_{a \in C^-} u_a \right) \right] \leq \sum_{a \in C^+} p_a - \sum_{a \in C^-} p_a \leq \left[\frac{1}{T} \left(\sum_{a \in C^+} u_a - \sum_{a \in C^-} \ell_a \right) \right], \quad (3)$$

where C^+ and C^- denote the forward and the backward arcs of the cycle C .

We close this section by listing several other totally different practical applications which can be modeled via the PESP ([24]). Rather prominent ones are the scheduling of systems of traffic lights, and periodic job shop scheduling.

3 Timetabling Requirements Covered by the PESP

This section gives a broad overview of the modeling capabilities of the PESP. Contrary to the following sections, practical requirements to be modeled are limited to those arising in periodic timetabling. Nevertheless, there are many facts we have to discuss in order to give a self-contained overview.

However, let us start by naming two facts which are definitely beyond the scope of the PESP: routing of trains through stations or even alternative tracks, and routing of the passenger flow. Hence, throughout this paper we assume fixed routes for both, trains and passengers. A short motivation for these assumptions will be given at the beginning of Section 4

For the vast majority of practical requirements which we are going to model, we are trying to provide examples which are close to practice. Notice, however, that in particular time and track information might not always reflect practice exactly. Depending on the fact to be modeled, we will provide a track map, a line plan, a visualization² of the timetable of a given track, and last but not least the resulting PESP subgraph. For readers not being familiar with the first three types of charts, we refer to any textbook on railway engineering.

Most of our real-world examples will be taken from the surroundings of the station Köln-Deutz (Cologne), which is part of the German ICE/IC-network. Figure 2 displays the general track map of Köln-Deutz. Unless stated otherwise, we assume a period time of $T = 60$ minutes.

3.1 Elementary Requirements

Both, for sake of completeness and in order to introduce the notation we use in the following figures, we start by modeling the three most elementary actions within public transportation networks: trips, stops, and changeovers.

In Figure 3 (a), we highlight the tracks used by two lines which cross at Köln-Deutz. The lines themselves are given in Figure 3 (b). Finally, we provide the constraint graph which models running, stopping, and changeover activities of these lines at Köln-Deutz in Figure 3 (c) as PESP constraints. Notice that we ensure changeover quality by linearly penalizing changeover times which exceed a certain

² In German: “Bildfahrplan.”

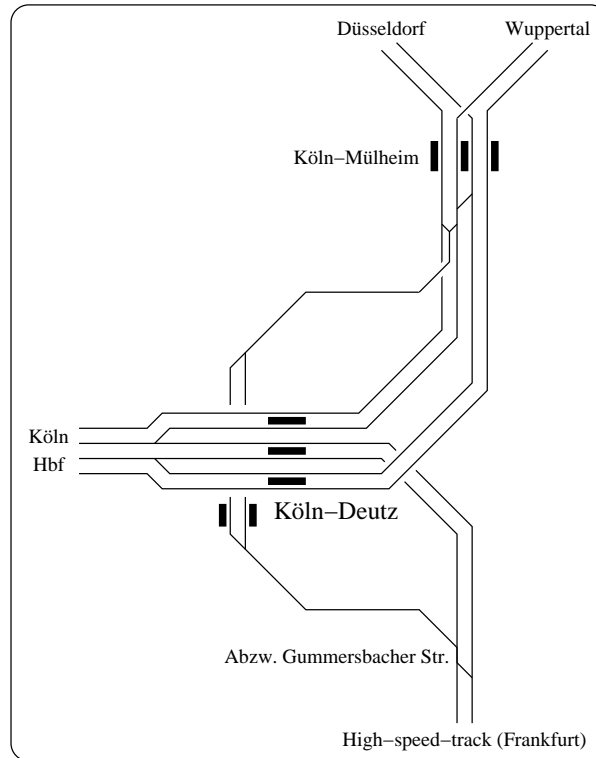


Fig. 2. Track map of Köln-Deutz (Cologne) — based on [12].

minimal changeover time required for changing platforms. In our example, a minimal changeover time of six minutes is assumed when connecting from Dortmund to Frankfurt. Using this approach, changeover arcs typically have a wide span.

An alternative way of modeling changeovers is to require some important ones not to exceed a maximal amount of effective waiting time. Then, we end up with rather small spans for changeover arcs. Schrijver and Steenbeek [23] follow this approach, which seems to be very suitable for constraint programming solvers.

Stopping arcs typically have very small span. In rather unimportant stations, it is a good choice to fix the span to zero, in particular if there is neither a junction of tracks, nor a single track, nor any changeovers.

Just as trip arcs, stopping arcs with span zero constitute redundancies which can be eliminated very efficiently in a preprocessing step. For example, one can contract any *fixed arc*, i.e. having zero span, together with its target node. Doing so, the arcs which were incident with the contracted target node have only to be redirected to the source node of the contracted arc, after having shifted their feasible interval appropriately, of course. Moreover, an arc being (anti-) parallel to another one can obviously be eliminated, if its feasible interval is a superset of the other arc. In addi-

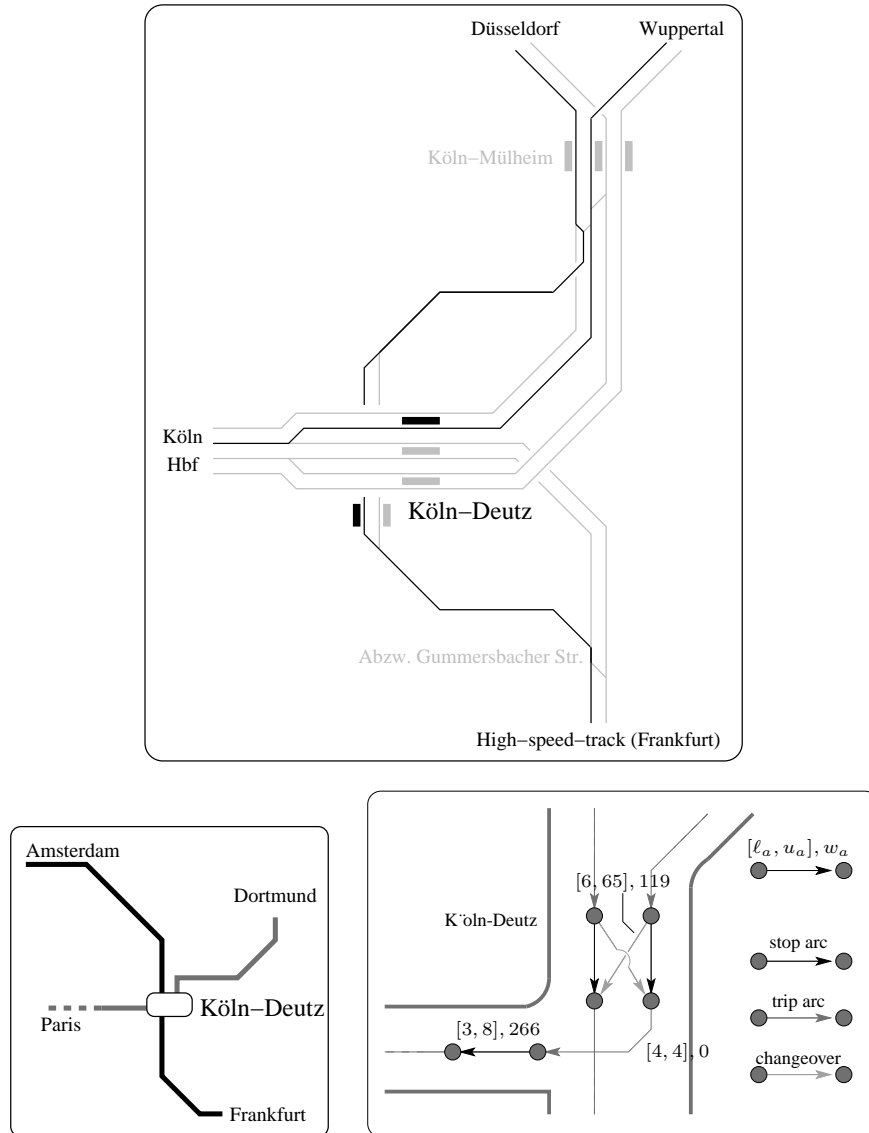


Fig. 3. Modeling elementary requirements: (a) two disjoint routes of lines serving Köln-Deutz; (b) the corresponding line plan; (c) PESP constraints modeling running activities, stopping activities, and changeover activities

tion to nodes with degree at most two, Lindner [16] gives further situations, in which we may simplify the graph.

If there are several lines using the same track into the same direction, sometimes it is required that a balanced service is offered. For n lines, this can easily be achieved by introducing arcs with feasible interval $[\frac{T}{n}, T - \frac{T}{n}]_T$ between any unordered pair of events that represent the departure at the first station of the common track. Certainly, strict balancedness may be relaxed by increasing the feasible interval.

Safety Requirements. If, in contrast to the previous discussion, there is no need for a balanced service, then at least a minimal headway d between any two of them has to be ensured. In the easiest case, the lines are operated with the same type of trains, and their running time is fixed. Then, we can sufficiently separate any two lines by introducing constraints much similar to the above ones, having feasible interval $[d, T - d]_T$. These can be inserted either at the beginning or at the end of their common track. More sophisticated constellations, in particular involving different speeds of the trains, will be discussed in Section 3.2.

But two trains may also use the same track in opposite directions. This is in particular the case for single tracks, see Figure 4 (a). Obviously, a train may not enter the single track until the train of the opposite direction has left it. In Figure 4 (b), we give one of the visualizations of timetables which are extremely useful in particular for single tracks. We assume a fixed local signaling, and the grey boxes visualize the time a train blocks a certain part of the track. Surprisingly, there is only one single constraint needed to prevent two trains of opposite directions from colliding within the single track, as can be seen in Figure 4 (c).

Note that we have not cared about any buffer times and blocking times when setting the feasible interval to $[0, T - ((a_1 - d_1) + (a_2 - d_2))]_T$. If, for example, time d_2 was the earliest possible departure time respecting the above buffers after an arrival at time a_1 , and if we assume the same *minimal crossing time* for Abzw. Gummersbacher Straße, the appropriate constraint would have been

$$[d_2 - a_1, T - ((a_1 - d_1) + (a_2 - d_2) + (d_2 - a_1))]_T = [d_2 - a_1, T - (a_2 - d_1)]_T.$$

Again, if there are several lines that have to be scheduled on a single track, one constraint for every unordered pair of opposite directions is needed.

Some authors ([10]) consider situations at crossings, where trains are shortly using the track of the opposite direction (cf. Figure 5), as another modeling feature. But if the network is blown up such that Abzw. Gummersbacher Straße splits into a northern station and a southern station which are linked by an eastern and a western track, this is just a special case of single tracks.

3.2 More Sophisticated Requirements

Whereas the practical requirements discussed in the previous section might arise in almost every railway network, the following aspects are of a more specialized nature.

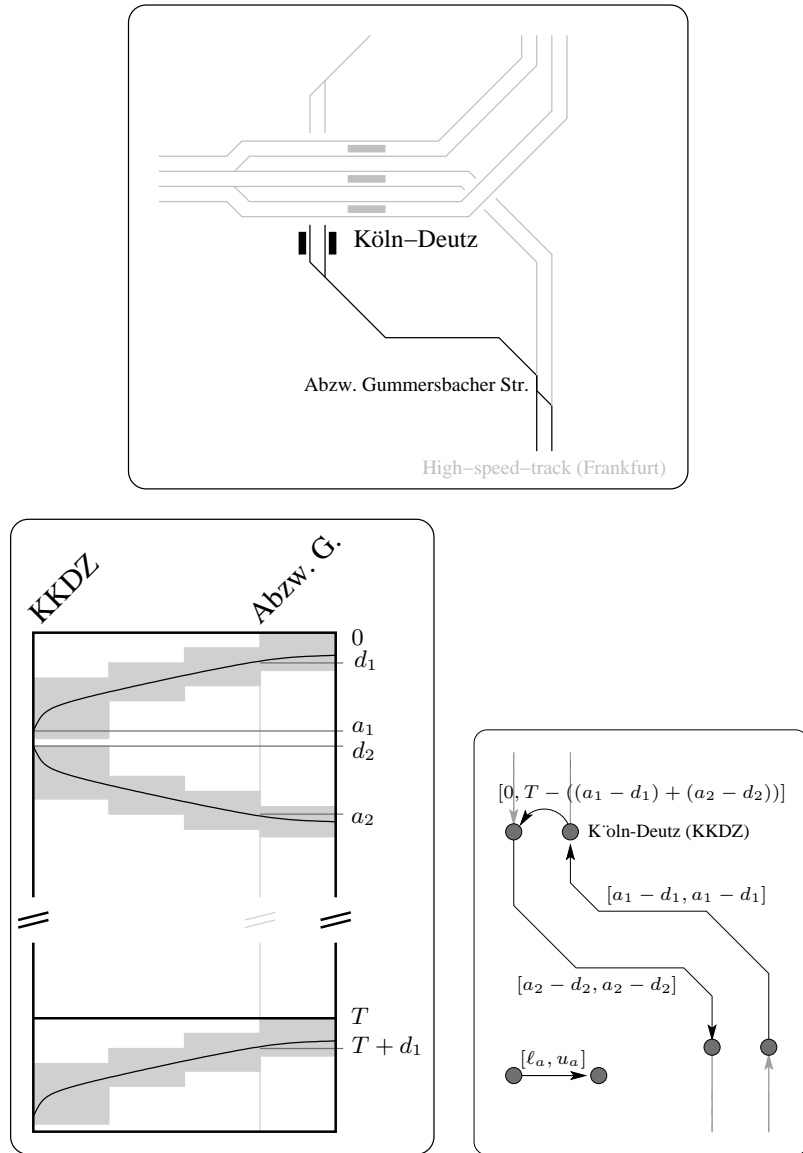


Fig. 4. Modeling single tracks: (a) a single track south of Köln-Deutz; (b) visualization of a feasible timetable for that single track; (c) PESP constraints ensuring safety distance for single track

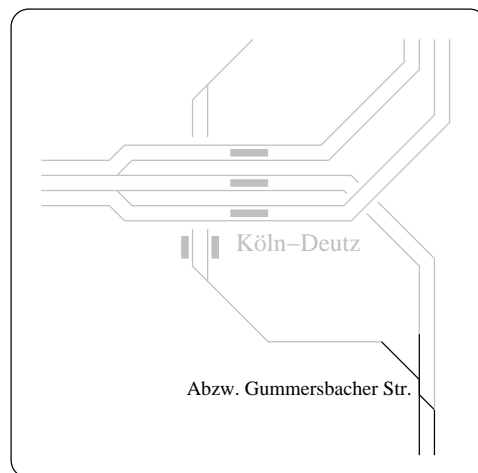


Fig. 5. Crossing of track of the opposite direction south of Köln-Deutz

Fixed Events. When planning a timetable hierarchically, e.g. from international trains down to local trains, one has to consider the fixed settings of previous hierarchies without replanning their times. Hence, the capability to fix an event to a certain point of time is another important modeling feature.

Fortunately, due to the periodic nature of the PESP, we may shift every feasible timetable such that a fixed event i_0 is fixed to a desired point in time $t_0 \in [0, T)$, i.e. $\pi_{i_0} = t_0$, and the objective value remains unchanged. By defining one of the events to be fixed as a kind of “anchor” event, we can easily relate the other events i_j to be fixed to certain points of time t_j by introducing arcs $a_j = (i_0, i_j)$ with $\ell_{a_j} = u_{a_j} = t_j - t_0$.

Bundling of Lines. Hierarchical planning gives rise to an even more challenging aspect of timetabling. Notice that if a track is used by trains of different speeds, the capacity of that track depends significantly on the ordering of the trains. The first two parts of Figure 6 visualize this effect. In the first scenario, slow and fast trains alternate, which implies that only two hourly lines of the two train types can be scheduled. However, if lines are bundled with respect to their speeds, three lines of the same two types of trains can be scheduled on the same infrastructure, cf. Figure 6 (b).

On the one hand, when only planning the high-speed lines in the first step of a hierarchical approach, it may happen that decisions on a higher level result in infeasibility on a lower level. On the other hand, hierarchical decomposition might have been chosen because an overall planning was considered to be too complex.

In order to keep the advantage of decomposition but limit the risk of infeasibility on lower levels, we propose to only bundle the lines of the current level of hierarchy. Figure 6 (c) gives the complete set of lines which should be operated on the track in question. In Figure 6 (d), we provide the PESP graph for the ICE/IC network.

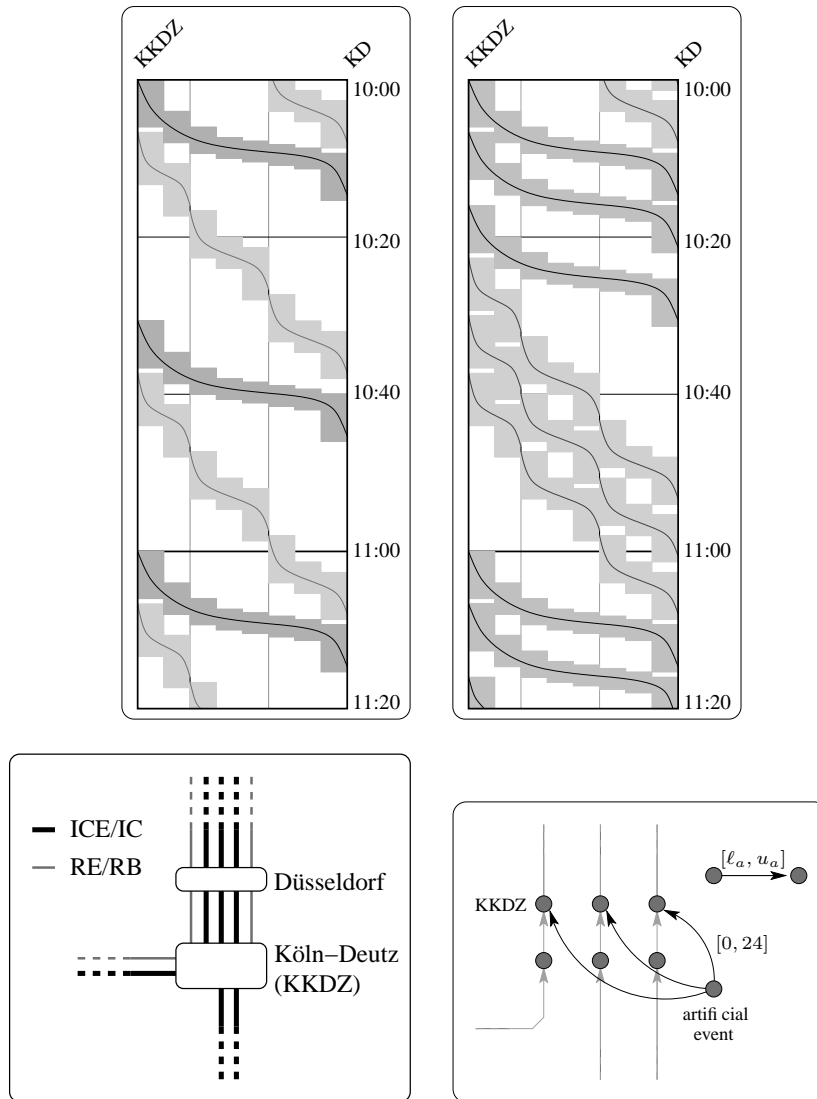


Fig. 6. Bundling of lines: (a) poor capacity if slow and fast trains are alternating; (b) increase of capacity by bundling trains of the same type; (c) complete line plan for all the types of lines; (d) PESP constraints ensuring enough capacity for RE/RB lines already when planning only ICE/IC lines within the first step of a hierarchical planning

To bundle the three active lines, we introduce an artificial event and require each of the departure events to be sufficiently close to that artificial event. Obviously, the departure events will be close to each other as well.

Notice that we must not choose one of the existing events as “anchor”, because this would predict the corresponding line to be the head of the sequence of bundled lines. This must definitely be avoided, because — contrary to assumptions made by Krista [10] — the ordering of lines is indeed a major result of timetabling.

Train Coupling/Train Sharing. During the last decade, in railway passenger traffic a trend emerged towards train units which can easily be coupled and shared. Doing so, more direct connections can be offered without increasing the capacity of some bottleneck tracks.

In Figure 7 (a), we display a line which is operated by two coupled train units between Berlin and Hamm. They split in Hamm to serve the two major routes of the Ruhr area, hereby offering direct connections from Berlin to the most important cities of that region. Still, this line occupies in particular the high-speed track between Berlin and Hannover only once per hour.

In Figure 7 (b), we provide PESP constraints which ensure the time for splitting the two train units in Hamm to be at least five minutes. Furthermore, for the two

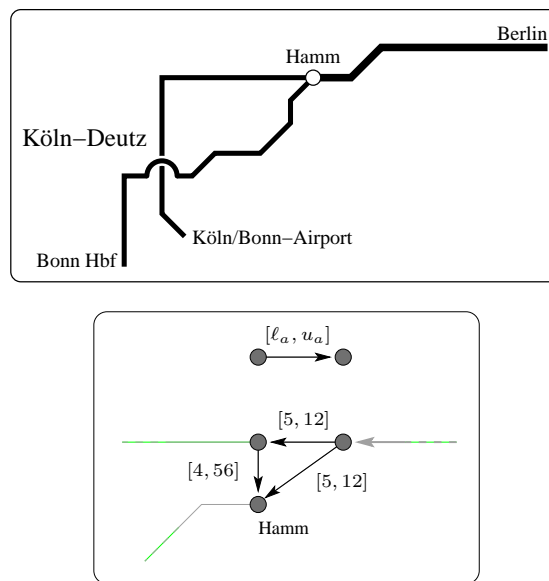


Fig. 7. Modeling train sharing: (a) line plan for line Berlin-Hamm- $\{\text{Bonn Hbf} \mid \text{Köln/Bonn-Airport}\}$; (b) PESP constraints ensuring safety distance and time to split train units, but not specifying the ordering of departures

departing trains, a safety distance of four minutes is guaranteed. Notice that we do

not need to specify which train should leave Hamm first. This decision will be made implicitly, and in an optimized way, by the PESP solver.

Variable Trip Times. As long as trip times are fixed, a usual safety constraint prevents two identical trains from overtaking each other. With d being the minimal headway for the track, we put an arc with feasible interval $[d, T - d]_T$ between the two events of entering the common track. If the line at the tail of the constraints is by f time units faster than the line at the tail of the constraints, overtaking can be prevented by modifying the constraint to $[d + f, T - d]_T$. This can be understood easily by having again a look at the corresponding situation in Figure 6 (a).

But this is no longer guaranteed if the model includes variable trip times. Even ensuring the minimal headway at the end of the track, too, does no longer prevent overtaking (even of trains of the same type) if the span in the trip times is at least twice the safety distance d , i.e. $u_a - \ell_a \geq 2d$. Schrijver and Steenbeek [23] and Kroon and Peeters [11] tackle this phenomenon by adding extra constraints, and hereby leave the PESP model.

In order to stay within the PESP model, we propose to subdivide³ an initial trip arc into new smaller ones such that $u_a - \ell_a < 2d$ for every new trip arc. For an example, we refer to Figure 8, where **bold** arcs represent arcs of the spanning tree for which we set $p_a = 0$, cf. Lemma 1.

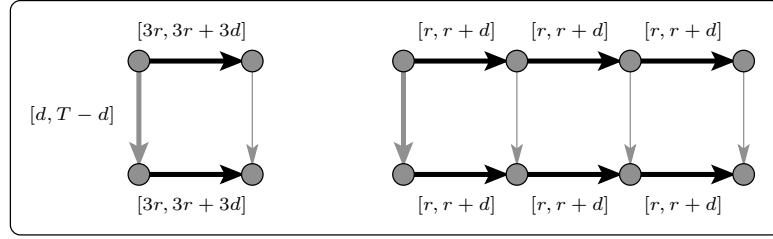


Fig. 8. Overtaking and variable trip times: (a) standard granularity does not prevent overtaking; (b) finer granularity prevents overtaking

Although this might seem to expand the model, the approach behaves rather well. More precisely, in every feasible timetable, the integer variables which we have to introduce for our additional arcs are in fact fixed to zero. This can simply be seen by applying the cycle inequalities (3) to any of the three squares in Figure 8 (b),

$$\underline{p} = \left\lceil \frac{1}{T}(r + d - (T - d) - (r + d)) \right\rceil = \left\lceil \frac{d - T}{T} \right\rceil = 0,$$

$$\bar{p} = \left\lfloor \frac{1}{T}((r + d) + (T - d) - d - r) \right\rfloor = \left\lfloor \frac{T - d}{T} \right\rfloor = 0.$$

³ This approach has also been discussed with Peeters [22] some years ago.

Notice that the corresponding bounds for the initial formulation are only -1 and 1. But this is very natural, because there are three different types of timetables possible, of which we have to cut off two. The value one, for instance, models the fact that the second (lower) train is overtaking the first (upper) train.

Although we showed that the inconveniences caused by flexible running times can be overcome, we will assume fixed running times throughout the remainder of this paper.

3.3 General Modeling Capabilities

There are also important non-timetabling features which can be modeled by the PESP in a very elegant way. The types of such constraints are disjunctive constraints and soft constraints. Although they were originally introduced for their own sake, they turn out to be very useful for even more specialized requirements, which we were asked by practitioners to model.

Disjunctive Constraints. The feasible region of MIPs are commonly given as the intersection of finitely many half-spaces, plus some integrality conditions. If disjunctive constraints have to be modeled, usually artificial integer variables are introduced. However, the PESP offers a much more elegant way.

When introducing the PESP, Serafini and Ukovich [24] already made the important observation that the intersection of two PESP constraints is not always again a single PESP constraint. Rather, the feasible interval for a tension variable can become the *union* of two PESP constraints, e.g.

$$\pi_j - \pi_i \in [\ell_1, u_1]_T \cap [\ell_2, u_2]_T \Leftrightarrow \pi_j - \pi_i \in [\ell_1, u_2]_T \cup [\ell_2, u_1]_T.$$

We illustrate their observation in Figure 9. Nachtigall [18] observed that any union

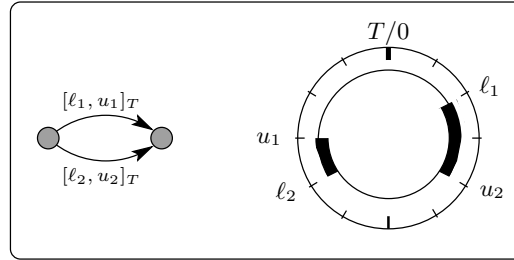


Fig. 9. Disjunctive constraints

of k PESP constraints can be formulated as the intersection of at most k PESP constraints.

As an immediate practical application of disjunctive constraints, we consider optional operational stops. Long single tracks with no stop may cause the timetable of

a line to be fixed within only small tolerances. In such a situation, Deutsche Bahn AG considers the option of letting the ICE/IC trains of one direction stop somewhere, although there is no ICE/IC station. In the current timetable, this takes places on the line between Stuttgart and Zurich, at Epfendorf.

If we want periodic timetable optimization to be competitive, we should enable the PESP to introduce an additional stop as well. We do so by introducing a pair of disjunctive constraints. The first constraint is a usual stop arc a_1 . We set the lower bound ℓ_{a_1} to zero, which models the option of not introducing an additional stop. The upper bound u_{a_1} will be set to the sum of the minimal increase b of travel time occurring from braking and accelerating, plus the maximal amount of stopping time s at the station. Obviously, the effected increase x_a of travel time must fulfill

$$x_a \in \{0\}_T \cup [b, b + s]_T,$$

which, of course, is nothing but a disjunctive constraint. Notice that additional waiting time should be penalized in this situation similarly to an extension of a regular service stop.

Obviously, the introduction of an additional stop can also be due to the construction of a new station. Since such decisions are a part of network planning, we postpone this discussion until section 5.3.

Soft Constraints. Nachtigall [20] investigated the combination of two antiparallel arcs $a_1 = (i, j)$ and $a_2 = (j, i)$. If they have an identical coefficient in the objective function, as well as upper bounds $u_{a_i} = \ell_{a_i} + T - 1$, then they model a *soft constraint*.

Classically, if a certain tension value x_a does not satisfy a given PESP constraint $[\ell_a, u_a]_T$, one would declare the complete timetable as infeasible. But sometimes, it can be an alternative only to produce a significant penalty in the objective function, if a constraint is not satisfied.

To that end, we relax the upper bound of the original constraint to $\ell + T - 1$ and introduce a new antiparallel arc with feasible interval according to Figure 10. Then,

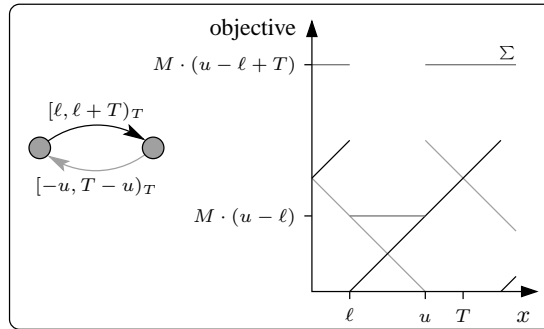


Fig. 10. Soft constraints

these two constraints yield a piecewise constant behavior of the objective function,

which serves as an indicator for the violation of the original constraint, but without guaranteeing feasibility.

In our cooperation with Berlin Underground, we were asked to construct a timetable that, among the top 50 most important connections, maximizes the number connections having a waiting time at most five minutes. Obviously, soft constraints are well-suited for letting MIP solvers produce a timetable being optimal subject to this kind of objective function.

4 Timetabling Requirements *Not* Covered by the PESP

Although the most important practical requirements for a periodic timetable can be modeled within the PESP, we are still aware of some special features for which the PESP fails.

First, one may think of situations, in which it is not fixed which trains will be operated on which track, for example within stations. Consider a station having two tracks in the same direction and three lines serving that direction. Then, we cannot decide a priori, which pair of lines shall be within the station at the same time, hence omitting the sequencing constraint between these two lines. This observation is the motivation for the DONS system to be subdivided into CADANS, covering the timetabling step, and STATIONS, covering the routing aspect ([6]).

Apart from this rather important requirement, which unfortunately is simply out of scope for the PESP, we will analyze a very special situation in more detail. Finally, we will introduce the important notion of *symmetry*. On the one hand, symmetry slightly exceeds the original PESP, but on the other hand, when added explicitly, gives rise to a mechanism to include important aspects of line planning into the very same planning step as periodic timetabling and vehicle scheduling.

4.1 Balanced Reduction of Service

The Berlin fast train company (S-Bahn Berlin GmbH) aims at operating only one timetable for one whole day. The late evening service differs from the rush hour only in that some trains are omitted. Hence, the timetable must respect the available capacity during the rush hour, and it has to offer a balanced service in the late evening as well.

To sidestep an intraday change of the timetable structure could seem strange considering all the information technology available in the 21st century. But it is still the policy of the company. It is given as a motivation that customers really expect to have only one single timetable to be kept in mind for their station.

Consider the approximately 10 km long track from Zoo station to Berlin East station. On it, a minimal headway of 2.5 minutes has to be respected. The period time is 20 minutes and eight lines (having identical train types) per period and direction have to be scheduled, one of them being only a free slot for occasional non-passenger trips. In the late evening service, there are four trains every 20 minutes, two of them being fixed to a 10 minutes time lag. We call these lines *core-lines*.

Of course it would be ideal to have a five minutes time lag between two consecutive trains in the evening. But this is impossible because for one of the evening trains it is required to serve Potsdam every 10 minutes together with a rush hour train. Hence, one should ensure that the maximal time lag between two consecutive trains does not exceed 7.5 minutes.

But this simple requirement cannot be covered by the PESP. Consider the two types of timetables given in Table 1. Timetables of type 1 satisfy our requirement,

Timetable	Departure times ($T = 20$ minutes)								
Type 1	0.0	–	–	7.5	10.0	12.5	–	–	(20.0)
Type 2	0.0	2.5	–	7.5	10.0	–	–	–	(20.0)

Table 1. Timetables

but type 2 does not.

Proposition 1. *For every set of PESP constraints either timetables of both types are feasible, or timetables of both types are infeasible.*

Proof. There are two types of constraints to be analyzed:

- i. one constraint between the two non-core lines,
- ii. four constraints between one of the two core lines and one of the two non-core lines.

Since we must not specify the sequence of the lines in advance, only symmetric constraints $[\ell, T - \ell]_T$ make sense. Moreover, all constraints of type (ii) have to be identical for the same reason.

To guarantee feasibility of type 1 timetables, we deduce $\ell \leq 5$ for the constraint of type (i) and $\ell \leq 2.5$ for the constraints of type (ii). But then, timetables of type 2 stay feasible as well. Hence, in order to cut off timetables of type 2, we have to increment one of the given bounds. But since they are obviously tight, this would immediately cut off timetables of type 1 as well. \square

4.2 Symmetry of a Periodic Timetable

Throughout our discussion of symmetry, we will assume that for every directed line there exists another directed line serving the same stations just in opposite order. Moreover, the concept of symmetry makes only sense, if for every traffic line, the running and stopping of its two opposite directions are the same. Furthermore, the passenger flow is assumed to be symmetric.

A periodic railway timetable is called *symmetric*, if at time 0 every train in the network meets a train of the opposite direction of its line. Large parts of the timetables of central European countries, such as Germany and Switzerland, are symmetric within only small tolerances, see Figure 11 for an example. For instance in

Station/Stop	Date	Time	Platform	Products	Comments
Berlin Zoologischer Garten	05.06.03	dep 09:54	4	ICE 952	InterCityExpress BordRestaurant
Wolfsburg		dep 10:54			
Hannover Hbf		dep 11:31			
Bielefeld Hbf		dep 12:24			
Hamm(Westf)		dep 12:54			
Hagen Hbf		dep 13:25			
Wuppertal Hbf		dep 13:42			
Köln-Deutz		dep 14:11			
Köln Hbf		arr 14:14			
Köln Hbf		05.06.03			
Aachen Hbf	dep 15:52				
Aachen Süd(Gr)					
Liege-Guillemins					
Bruxelles-Midi	05.06.03	arr 17:46			

Duration: 7:52; runs daily

All information is issued without liability. Software/Data: HAFAS 5.00.DB.4.5 - 20.05.03 [5.00.DB.4.5/v4.05.p0.13_data:59e79704]

Station/Stop	Date	Time	Platform	Products	Comments
Bruxelles-Midi	05.06.03	dep 12:16		ICE 15	InterCityExpress Onboard meeting place
Liege-Guillemins		dep 13:28			
Aachen Süd(Gr)	05.06.03	dep 14:10	3		
Aachen Hbf		arr 14:46			
Köln Hbf	05.06.03	dep 15:47	2	ICE 953	InterCityExpress BordRestaurant
Köln-Deutz		dep 15:51			
Wuppertal Hbf		dep 16:17			
Hagen Hbf		dep 16:35			
Hamm(Westf)		dep 17:10			
Bielefeld Hbf		dep 17:37			
Hannover Hbf		dep 18:31			
Wolfsburg		dep 19:05			
Berlin Zoologischer Garten		arr 20:02			

Duration: 7:46; runs Mo - Fr, not 29. May, 9. Jun, 21. Jul, 15. Aug, 11. Nov
Hint: Prolonged stop

All information is issued without liability. Software/Data: HAFAS 5.00.DB.4.5 - 20.05.03 [5.00.DB.4.5/v4.05.p0.13_data:59e79704]

Fig. 11. Symmetric timetables in practice

Köln-Deutz, the train from Berlin arrives at minute 09, which is two minutes before its departure at minute 11. The opposite direction leaves Köln-Deutz at minute 51. Hence, they sum up to zero, modulo the period time of sixty minutes. Notice that this line has one of its meeting points at minute 00 between Köln-Deutz and Wuppertal Hbf.

For the arrival or departure event of a directed line at a certain station, we denote by its *complementary event* the departure or arrival, resp., of the opposite line at the same station. Trivially, under the assumptions made, symmetry is equivalent to the fact that the times $\pi \in [0, T)$ assigned to two complementary events sum up to either 0 or the period time T .

Here, one can think of time 0 as a *symmetry axis*. But of course, other symmetry axes are possible. Since trains of the two opposite directions of the same traffic line meet each other twice within their period time T , for the symmetry axis s we have w.l.o.g $s \in [0, \frac{T}{2})$. In this more general case, symmetry is fulfilled, if

$$\frac{(\pi_a + \pi_d) \bmod T}{2} = s. \quad (4)$$

Anyway, for ease of notation we assume $s = 0$ throughout this article.

To define a counterpart of condition (4) for the tension formulations (1), we define two antiparallel arcs $a = (i, j)$ and $b = (j, i)$ to be *complementary*, if events i and j are complementary and we have $\ell_a = \ell_b$ and $u_a = u_b$. We then consider the following simple condition on their slack variables

$$\tilde{x}_a = \tilde{x}_b. \quad (5)$$

Trivially, if ensured for every complementary pair of nodes or arcs, respectively, the two conditions (4) and (5) are equivalent, except that only the node formulation explicitly fixes the symmetry axis of the system to a specific point of time. Surely, one can introduce a certain tolerance Δ on the symmetry requirement. But notice that in this case, condition (5) has to be blown up by a new integer variable.

If the line plan of a traffic network is connected, then we are able to give a more compact characterization of symmetry. Given that complementary stopping activities show identical extensions of their minimal stopping times, then a periodic timetable is symmetric, if and only if for every pair of complementary connections, these have identical changeover times.

Some practitioners consider this property to be an important advantage of symmetric timetables. Even though this might depend on personal preferences, we do *not* consider this really to be a striking argument for symmetry. Actually, there are examples which prove that symmetric timetables are only suboptimal, even if the input data is symmetric ([13]).

Apparently there are not yet many discussions on symmetric timetables available. But among further motivations for symmetry, as they can be found in Liebchen [13], the most convincing one seems to be that symmetry halves the complexity of an instance. This can in particular be useful if there are complex interfaces to international trains or to regional traffic, and when planning is performed manually. However, this argument should become less important in the future, as PESP solvers hopefully achieve some more progress in performance, and hence find their way into practice.

Let us finally mention, that it is unlikely to integrate symmetry requirements into the PESP framework ([13]). Rather, we will have to add constraints of type 5 to the MIP formulations of a PESP instance. Nevertheless, in particular when designing periodic timetables for national railway companies, the symmetry requirement has to be ensured, because we made the experience that it is really a knockout criterion there.

Hence, besides a linear objective function, symmetry is the second important requirement arising in the practice of periodic railway timetabling, by which the initial PESP model should be extended. Fortunately, MIP solvers are able to profit from the addition of symmetry constraints, in particular in formulation (5) ([13]). Notice that such a generalized MIP model inherits large parts of the structure of a pure PESP model. In particular, the important cycle inequalities (3) remain valid.

5 Further Planning Steps Covered by the PESP

In the following, we will demonstrate that the modeling capabilities of the PESP are not limited only to periodic timetabling. Rather, central aspects of both preceding and succeeding planning steps in the sense of Figure 1 can be integrated.

We start this discussion with the well-established technique of minimizing the number of vehicles required to operate a periodic timetable by penalizing waiting times of vehicles. Hereafter, we provide first ideas for the integration of important decisions of line planning. We close this section by proposing a way to model some specialized decisions as they arise in network planning.

5.1 Aspects of Vehicle Scheduling

Almost all companies in public transportation have in common that they want to minimize the amount of rolling stock required to serve their network. Notice that the quality of the vehicle schedule for a fully periodic timetable, i.e. with no peak trips included, is largely determined by the timetable.

Consider for example the hourly line displayed in Figure 12 (a). Assume the minimal travel times between the two endpoints to be 235 minutes for each direction. Given strict minimal turnover times of 45 and 60 minutes, respectively, the minimal number of vehicles required to operate this line is precisely

$$N := \left\lceil \frac{1}{60}(235 + 235 + 45 + 60) \right\rceil = 10.$$

A timetable which lets the trains leave at the full hour from Frankfurt and Amsterdam can indeed be operated with only 10 trains, at least if the stopping times are extended only moderately. On the contrary, a timetable in which only the trains starting at Frankfurt depart at minute 00, but the trains from Amsterdam leave at minute 30 will require at least 11 vehicles. Hence, the amount of vehicles depends on the timetable.

We will analyze in which special cases pure PESP constraints are able to control the number of trains required. After that, we show that a linear objective function covers many more of the practical cases.

Proposition 2 (Nachtigall [18]). *Consider a fixed traffic line with period time T . If we assume trains always to serve only this line, and if we do not allow to insert additional stopping time, then there exist upper bounds u for the turnover activities, such that the only feasible timetables are those which can be operated with the minimal amount of trains.*

Proof. We present a proof of this simple fact, both in order to provide the notation used in the following paragraphs, and because it avoids modulo-notation.

Denote the endpoints of the line by A and B . Let ℓ_{AB} denote the minimal travel time from A to B , i.e. the sum of the minimal stopping and running times of the

activities of this directed traffic line. Moreover, denote by ℓ_B the minimal amount of time a train has to stay in endpoint B between two consecutive trips.

The minimal number N of trains required to operate this line is precisely

$$N = \left\lceil \frac{\ell_{AB} + \ell_B + \ell_{BA} + \ell_A}{T} \right\rceil.$$

Notice that from the cycle periodicity property (2) we know that every feasible timetable x fulfills

$$x_{AB} + x_B + x_{BA} + x_A = zT, \quad (6)$$

for some $z \in \mathbb{Z}$. Hence, we must ensure $z = N$. To that end, consider the slack

$$s := NT - (\ell_{AB} + \ell_B + \ell_{BA} + \ell_A)$$

of this traffic line, implying $(x_A - \ell_A) + (x_B - \ell_B) = s$. But since $s < T$, by setting

$$u_A := \ell_A + s \quad (7)$$

we even ensure $x_{AB} + x_B + x_{BA} + x_A < (N + 1)T$, q.e.d. \square

Let us now analyze the case in which additional stopping times may be inserted, i.e. $u_{AB} > \ell_{AB}$. We will show that together with the constraints (7), some timetables which require an additional train may become feasible.

On the one hand, consider a timetable for which we have $x \equiv \ell$ for all activities, except for the turnover time in one endpoint. Obviously, this timetable can still be operated with the minimal number of trains, showing that decreasing the value (7) for u_A would cut off timetables we seek for.

On the other hand, assume $x_{AB} = u_{AB}$ and $x_{BA} = u_{BA}$. If

$$(u_{AB} - \ell_{AB}) + (u_{BA} - \ell_{BA}) + s \geq T, \quad (8)$$

then we can extend x to a timetable that still respects (7), but which requires at least one additional train. For instance, if inequality (8) is tight, then $x \equiv u$ yields $x_{AB} + x_B + x_{BA} + x_A = (N + 1)T$.

The above dilemma is our main motivation for the need of a linear objective function. Such a function takes advantage of equation (6): By assigning a value M to the arcs modeling a traffic line, every additional train pays $M \cdot T$ to the objective function value. Of course, it suffices to consider arcs with positive span, cf. Figure 12 (b). If the value for M is chosen relatively large compared to the passenger weights, the objective function essentially models the piecewise constant behavior of the cost of the rolling stock for operating the railway network.

From a more local perspective, we just penalize idle time of trains. But this can even be done without knowing a priori the circulation plan of the trains. Although an exact model involves a quadratic objective function, Liebchen and Peeters [15] report that a simple linear relaxation yields results of high quality.

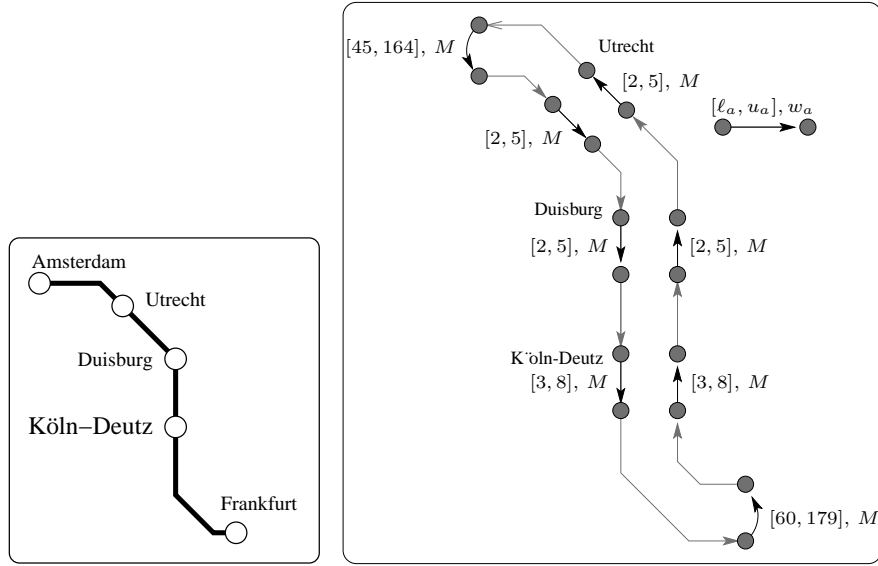


Fig. 12. Modeling aspects of vehicle scheduling: (a) line plan; (b) PESP constraints measuring the number of trains required to operate the line

5.2 Aspects of Line Planning

Our main idea for letting PESP solvers even take decisions of line planning is to combine — or match — pre-defined line-segments. To that end, we will make intensive use of disjunctive constraints. Unfortunately, we will only be able to ensure symmetric line plans if we require symmetry also within the stations where lines are matched.

We are aware of only one other approach for integrating the planning phases of line planning, timetabling and vehicle scheduling ([25]). Whereas that approach is based on the assumption that the line plan contains no cycles, our ideas do not require any restrictive assumptions on the topology of the network. Rather, we are able to keep even very important technical restrictions such as single tracks.

Notice that bad decisions at the level of line planning may cause very bad results also for vehicle scheduling. Consider the four line segments displayed in Figure 13. We assume a period time of $T = 60$ minutes and a minimal turnover time of 30 minutes at each of the four terminal stations. The time for a one-way trip from the matching station to one of the endpoints is indicated at the corresponding edge.

In fact, the vehicle schedule is fixed due to the distinct endpoints. Combining the south-west segment with the north-east segment causes this line to require at least

$$\left\lceil \frac{1}{60}(60 + 95 + 30 + 95 + 60 + 30) \right\rceil = \left\lceil \frac{370}{60} \right\rceil = 7 \text{ trains.}$$

The other line of the same matching requires seven trains, too.

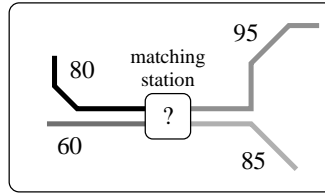


Fig. 13. Line segments where only one matching provides good vehicle schedules

In contrast, the other matching implies seven trains only for the northern line. But the other line can be operated with only six trains. Hence, already the line plan has a major impact on the cost of operation. Zwaneveld et al [5] consider this phenomenon in their approach for constructing cost-optimal line plans.

However, they omit the important intermediate linking step of computing a timetable. Therefore, their approach must also consider possible constellations in which there is no feasible timetable using only six trains for the southern line. This would be the case, if there was a single track with travel time 25 minutes for every direction just at the end of the south-east segment. The same holds if it is required that the two lines together form an exact half-hourly service along the backbone of the network.

We consider a track which has to be served in the same direction by n directed lines which are operated by trains of identical type. We denote the matching station by S . It has to reside in between of the two endpoints of the common track. We consider n line segments L_1^a, \dots, L_n^a which have station S as their common endpoint, and n line segments L_1^d, \dots, L_n^d having station S as their common starting point. Any (bipartite) matching between the arriving and the departing line segments induces a line plan.

But from the perspective of timetabling, there are only n arrival events a_1, \dots, a_n as well as n departure events d_1, \dots, d_n visible. Hence, we must deduce only from their arrival times π_{a_i} and their departure times π_{d_j} which arriving line segment L_i^a should be matched with which departing line segment L_j^d .

This can be done in a canonic way, if we choose the matching station S such that it has only one track in the direction of the line segments we consider. If necessary, we add an artificial station in the middle of some track. Then, at most one train can be in S at the same time. Timetables respecting this constraint can be characterized very easily.

Definition 1 (Alternating timetable). For a fixed station S and a fixed direction, a periodic timetable π with n pairwise different arrival times $0 \leq \pi_{a_1} < \dots < \pi_{a_n} < T$ and n pairwise different departure times $0 \leq \pi_{d_1} < \dots < \pi_{d_n} < T$ is called alternating at S , if either $\pi_{a_i} \leq \pi_{d_i} < \pi_{a_{i+1}}$ for every $i = 1, \dots, n$, or $\pi_{d_i} < \pi_{a_i} \leq \pi_{d_{i+1}}$ for every $i = 1, \dots, n$, where we define $\pi_{\cdot, n+1} := \pi_{\cdot, 1} + T$.

Lemma 3. A timetable π ensures that there is always at most one train at station S if and only if it is alternating at S .

Hence, for an alternating periodic timetable, we will combine the arriving line segment L_i^a with the departing line segment L_j^d , if and only if the latter marks the unique first possible departure. In the sequel, we will give PESP constraints ensuring every feasible timetable to be alternating at S . Thus, every feasible timetable will encode some unique matching and the associated line plan.

The first two sets of constraints will ensure the minimal headway d in front of and behind the matching station S :

$$\forall i, j \in \{1, \dots, n\} : \pi_{a_j} - \pi_{a_i} \in [d, T - d]_T, \quad (9)$$

$$\forall i, j \in \{1, \dots, n\} : \pi_{d_j} - \pi_{d_i} \in [d, T - d]_T. \quad (10)$$

Obviously, (9) and (10) can only be fulfilled, if $0 \leq d \leq \frac{T}{n}$. Moreover, we relate arrival events to departure events by the following disjunctive constraints

$$\forall i, j \in \{1, \dots, n\} : \pi_{d_j} - \pi_{a_i} \in [0, T - d + h]_T, \quad (11)$$

$$\forall i, j \in \{1, \dots, n\} : \pi_{d_j} - \pi_{a_i} \in [d, T + h]_T, \quad (12)$$

where we denote by h the maximal stopping time for a train at station S . Notice that constraints (11) and (12) together yield

$$(\pi_{d_j} - \pi_{a_i}) \bmod T \in [0, h] \dot{\cup} [d, T - d + h]. \quad (13)$$

Trivially, $0 \leq h < d$ is necessary for every feasible timetable π to be alternating at S .

Theorem 2. *Let π be a timetable respecting constraints (9) to (12). Then for every departure event j , there exists a unique arrival event a_i satisfying*

$$\pi_{d_j} - \pi_{a_i} \in [0, h]_T, \quad (14)$$

if and only if $h < (n + 1)d - T$.

Notice that from $h < (n + 1)d - T$ and $d \leq \frac{T}{n+1}$ it would follow that $h < 0$, which makes no sense. Hence, we have $0 \leq h < d$ and $\frac{T}{n+1} < d \leq \frac{T}{n}$.

Proof. “ \Rightarrow ”: We assume $h \geq (n + 1)d - T$. Since $d = \frac{T}{n}$ would imply $h \geq d$, we must only investigate the case that $d < \frac{T}{n}$. We will construct a timetable which respects the constraints (9) to (12), but which contradicts (14).

Define $\pi_{a_i} := (i - 1)d$, for all $i = 1, \dots, n$, and $\pi_{d_j} := j \cdot d$, for all $j = 1, \dots, n$. By construction, all the constraints are satisfied. However, since $\pi_{a_n} + h < n \cdot d = \pi_{d_n}$, for departure π_{d_n} none of the arrival events fulfills (14), q.e.d.

“ \Leftarrow ”: We assume there exists a timetable π having one departure event d_0 such that

$$\forall i = 1, \dots, n : (\pi_{d_0} - \pi_{a_i}) \bmod T > h,$$

but which respects the constraints (9) to (12). We may assume w.l.o.g. that for the cyclic predecessor arrival a_1 of d_0 we have $\pi_{a_1} = 0$. As π is feasible, it satisfies (13). From our assumption, we conclude $d \leq \pi_{d_0}$ and $\pi_{d_0} + (d - h) \leq \pi_{a_2}$, and hence

$\pi_{a_2} - \pi_{a_1} \geq 2d - h$. By defining $\pi_{a_{n+1}} := T$, we have $\pi_{a_{i+1}} - \pi_{a_i} \geq d$, for all $i = 2, \dots, n$. By construction, we know that

$$\sum_{i=1}^n (\pi_{a_{i+1}} - \pi_{a_i}) = \pi_{a_{n+1}} - \pi_{a_1} = T.$$

Summing up the lower bounds yields $T \geq (n+1)d - h$, which contradicts the hypothesis of Theorem 2. \square

Corollary 1. *If $h < (n+1)d - T$, then every timetable which respects constraints (9) to (12) is an alternating timetable.*

In Figure 14, we provide an example for the easiest case, namely matching two lines. As usual, we assume the period time to be 60 minutes.

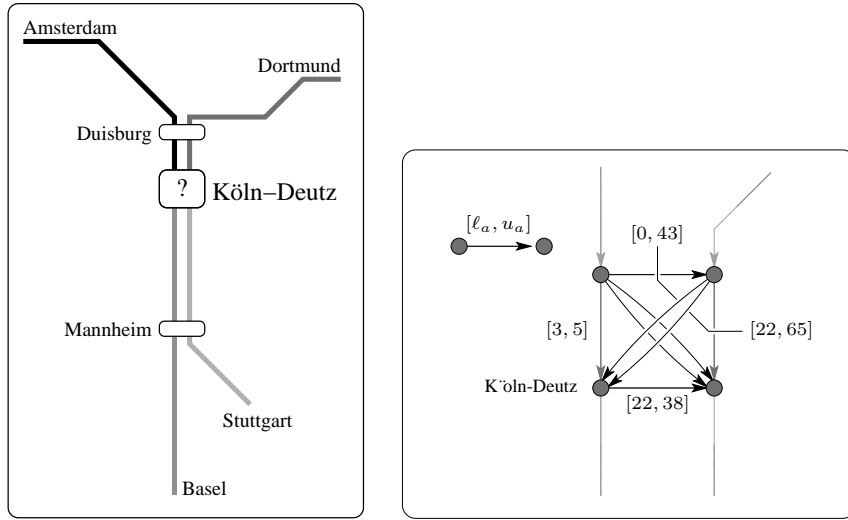


Fig. 14. Modeling aspects of line planning: (a) line segments; (b) PESP constraints ensuring the segments to be matched

Remark 1. There are of course alternating periodic timetables in the case $d \leq \frac{T}{n+1}$. PESP solvers are able to detect even those, if we were able to pre-define sufficiently many empty slots. By an ‘empty slot’ we understand an artificial line which we have to schedule in the same way as the original lines, hereby separating the lines before and after the empty slot.

In more detail, let us assume that $\frac{T}{n^*+1} < d \leq \frac{T}{n^*}$ for some $n^* > n$, and that h satisfies the assumptions of Theorem 2 for n^* . We then introduce $n^* - n$ artificial dummy arrival and departure events a_i and d_i , $i = n+1, \dots, n^*$. To prevent

the original line segments from being matched with an artificial event, we require $\pi_{d_i} - \pi_{a_i} \in [0, h]$ for all $i = n + 1, \dots, n^*$.

By construction, the only feasible timetables will let the original arrivals and departures alternate. However, optimally balanced timetables, i.e. $\pi_{a_i} := (i - 1) \frac{T}{n}$, will be infeasible under these settings if $n^* < 2n$, since they do not provide $n^* - n$ empty slots.

Recall that so far we have considered only one direction. Hence, there is no mechanism yet to bind the matching of one direction to that of the opposite direction. But the matchings of opposite directions must fulfill the symmetry assumption that we gave at the beginning of section 4.2. Otherwise, the trains from direction A could pass the matching station S in order to continue towards B , but the trains from B pass S before continuing in direction C . Thus, it would not be possible to communicate the line plan in the way customers are used to, because it may no more be visualized by an undirected graph. However, limited asymmetries in operation are accepted in practice.

Example 1 (S-Bahn Berlin GmbH). We consider the line S2 serving the route Blankenfelde-Lichtenrade-Buch-Bernau. Between Lichtenrade and Buch, a 10 minutes frequency must be offered, for the remaining parts 20 minutes suffice.

In the current timetable ([1]), this line is served in an asymmetric way. In order to cope with the single tracks (which are present at both endpoints) to limit the total amount of stopping time, and to ensure an efficient employment of the rolling stock, an asymmetric service is offered, and we present it in table 2.

Blankenfelde dep		10:09		arr o		11:14	
Lichtenrade dep ↓		10:15	10:25	arr o		11:05	11:15
Buch arr o		11:06	11:16	dep ↑		10:14	10:24
Bernau arr o		11:21		dep			10:10

Table 2. Asymmetric service of line S2 (Berlin)

In order to ensure symmetric line plans, we have to guarantee the following condition. If we combine the arrival event a_i with the departure event d_j in one direction, then in the opposite direction the complementary arrival event a'_j must be combined with the departure event d'_i . More precisely, when considering the corresponding tension variables $x_{a_i d_j}$ and $x_{a'_j d'_i}$, they must fulfill

$$x_{a_i d_j} \in [0, h] \Leftrightarrow x_{a'_j d'_i} \in [0, h]. \quad (15)$$

Of course, this condition is quite similar to the symmetry constraints (5). What makes things more complicated is the fact that we must not predict in advance for which pairs (i, j) requirement (15) has to hold, and for which pairs it may be violated. Hence, we propose to guarantee property (15) for the matched pairs by imposing symmetry requirements on *every* pair of complementary junctions. But it is clear

that this approach cuts off feasible timetables for symmetric line plans just because such timetables need not to be symmetric, see e.g. example 2.

Example 2 (S-Bahn Berlin GmbH). Consider the current timetable ([1]) of the ring subnetwork of S-Bahn Berlin GmbH, of which we provide an excerpt in table 3. Obviously, the line plan is symmetric. But the timetable is not symmetric. This can

Direction A						
Line	S45	S46	S8	S9	S47	S8
Origin	BFHS	BKW	BGA	BFHS	BSPF	BZN
Schöneeweide dep ↓	xx:01	xx:06	xx:10	xx:13	xx:15	xx:18
Baumschulenweg arr o	xx:03	xx:09	xx:13	xx:16	xx:17	xx:21
Destination	BHMS	BGS	BPKR	BZOO	BWES	BPKR
Direction B						
Line	S8	S46	S9	S47	S8	S45
Origin	BPKR	BGS	BZOO	BWES	BPKR	BHMS
Baumschulenweg dep ↓	xx:02	xx:06	xx:08	xx:13	xx:14	xx:19
Schöneeweide arr o	xx:05	xx:08	xx:10	xx:15	xx:17	xx:21
Destination	BGA	BKW	BFHS	BSPF	BZN	BFHS

Table 3. Symmetric line plan but asymmetric timetable

be seen by calculating the symmetry axes of lines S47 and S9 at station Schöneeweide. Departure and arrival of line S47 sum up to 30, hence the trains of this line meet at times 5 and 15. For line S9 the sum yields 23, providing a symmetry axis of 1.5. An easier argument for asymmetry is that the sequence of the trains in direction B is not the inverse of the one in direction A.

There are two main objectives for the matching approach. First, we want to offer direct trips for as many passengers as possible. Second, the timetable should require only few trains for operation.

For the second criterion, in the case $h = 0$, no additional weight on arcs within the matching node is required in order to minimize the amount of rolling stock required to operate the timetable. In the case $h > 0$, one could put the vehicle weight on the arcs with feasible interval $[0, T - d + h]$. But this would no longer yield the desired exact piecewise-constant behavior of the objective, because some double counting can appear.

For maximizing the number of direct travelers, we consider the number of passengers w_{ij} starting their trip before the common track on a train covering line segment L_i^a , and finishing their trip after the common endpoint on a train covering line segment L_j^d . The value w_{ij} will be added to the weight of the arc $a = (a_i, d_j)$ with $\ell_a = 0$ and $u_a = [0, T - d + h]$. Obviously, the resulting cost coefficients in the objective function make even sense for pairs of line segments which are not matched, because in that case long changeover times of many passengers are penalized.

Notice that the values w_{ij} are only well-defined if the two line segments do not serve a second matching station. This shows that the decisions to be taken within a matching station are of a rather local nature.

Summarizing, there are important scenarios in which the PESP can successfully integrate relevant aspects of line planning into a model suited for timetabling and key issues of vehicle scheduling. This is in particular the case, if symmetric timetables and balanced sequences along the common tracks, i.e. $d > \frac{T}{n+1}$, are requested for their own sake. Moreover, we observed that the larger the distance between two matching stations, the more reliable the passenger cost coefficient obtained with the weights we propose.

We think that in particular fast train networks of European agglomerations, such as Frankfurt, Munich, or Paris (RER), should be interesting candidates for this approach. There, many passengers might have their origin or destination somewhere on the backbone route, and balanced sequences must be ensured due to the large number of lines per period.

5.3 Aspects of Network Planning

We propose to also model two questions which arise in network planning within the PESP: the extension of existing tracks, and thus lines, beyond their current endpoints, and the construction of faster tracks as substitution for existing ones. Taking into account that, in these questions, we have to select one option out of a small number of disjoint options, it is evident that we will make intensive use of disjunctive constraints. We will only discuss the construction of faster tracks in detail. But the reader will have no difficulty to adapt our suggestions to the very similar task of the extension of tracks.

In Figure 15, we provide a constraint graph which offers the option of a new track between Aachen and Köln, being then part of the European high-speed line PBK (Paris-Brussels-Köln). Notice that we have to take into account three different types of objectives: The number of customers c who profit from a new track by shorter travel times, the trains whose shorter travel times may reduce the number of trains required, and the cost M' of the investment. Obviously, it will be an absolutely non-trivial management decision to derive an hourly cost coefficient M' from the total cost of the investment.

Similarly to line planning, investments into infrastructure will only make sense if they are effected for both directions at the same time. Again, we will ensure symmetric investments by requiring the timetable to be symmetric.

Let us now analyze the situation in which several lines have the option of using the same new, faster track. Of course, we want to ensure that infrastructure is only paid once in terms of the objective function. Hence, we have to partition the total cost onto all of the concerned lines. But what if a PESP solver decides to route only one line over the new track?

Obviously, a reasonable allocation of the total costs is only possible, if we know in advance how many lines will have to use the new track. Unfortunately, we are only

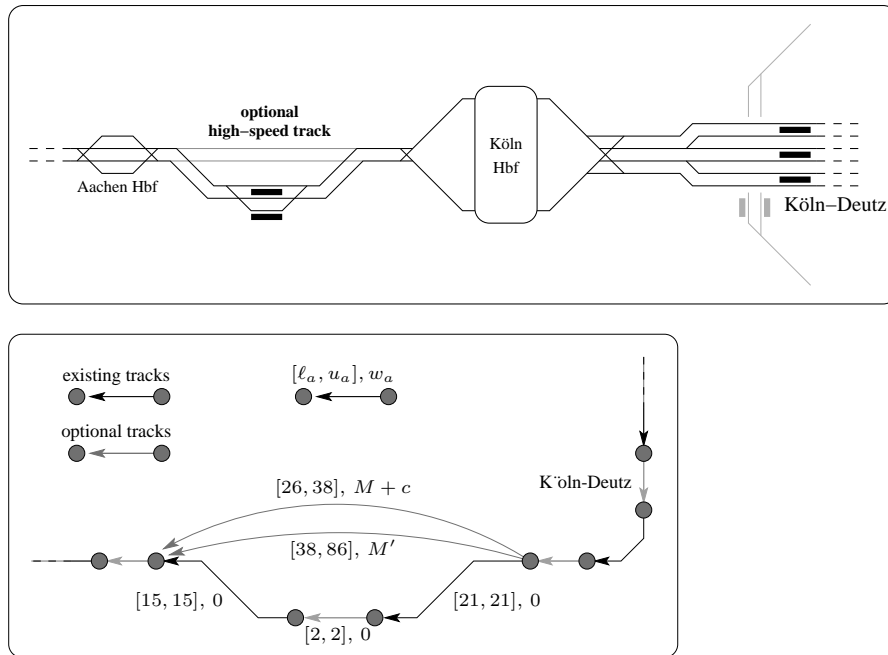


Fig. 15. Modeling aspects of network planning: (a) infrastructure including optional high-speed track; (b) PESP constraints taking into account the two infrastructural alternatives

able to ensure this with constraints of the types already introduced, if *all* the lines must use the same track. This would e.g. be the case when analyzing two mutually exclusive variants of constructing a new track.

We can guarantee that all the lines use the same track simply by enforcing the same running time for each line. This is achieved by introducing constraints of type (5). But notice that in this case we cheat a bit, because those constraints no longer relate only pairs of complementary arcs to each other. . . Anyway, the MIP formulation of this even slightly more extended model incorporates many of the computational aspects of the pure PESP model.

6 Conclusion

Our discussion of the PESP model shows that it has a great modeling power and extendability. We have demonstrated that many non-standard requirements for periodic timetables and also important aspects of other – traditionally separate – planning phases can be integrated into the PESP. Figure 16 displays the gain by this modeling power over the traditional use of the PESP displayed in Figure 1.

Interestingly, this integration into the PESP has been possible without seemingly complicating it too much. In all cases, we obtained mixed integer programs that still

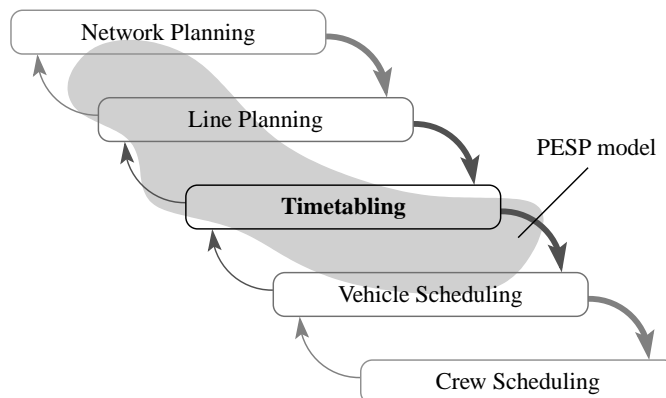


Fig. 16. Planning phases covered by the PESP with our contribution

have the characteristics of a PESP. Hence we believe that these extended models stay computationally tractable also for networks of relevant sizes. So far, our belief is confirmed by a confidential study for S-Bahn Berlin GmbH for two of its three major subnetworks.

We therefore hope that these models, through their integrative approach to vehicle scheduling, timetabling, line planning, and infrastructure planning, will eventually lead to better decision making in practice.

7 Acknowledgments

We want to thank the staff of Deutsche Bahn AG, S-Bahn Berlin GmbH, and Berliner Verkehrsbetriebe (BVG) for providing us with both, real-world data and very detailed requirements of their specific periodic timetabling problems.

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