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# Time-use and Income: A Trivariate Relative Poverty Surface

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## Abstract

Understanding poverty and well-being requires moving beyond income-based measures to account for how work and time are organized within households. While income and unpaid work sustain household living standards, in combination with leisure they determine how these standards are produced and experienced at the individual level, yet the two time dimensions remain largely absent from conventional poverty measurement. To assess living standards and set poverty thresholds we determine a bivariate relative poverty line on the household level and advance this approach to a trivariate poverty surface on the individual level. We base this measure on non-parametric multivariate quantiles. Using the 2018 Mexican National Survey of Household Income and Expenditure reveals substantial deprivation overlooked by income-based measures. At the household level, 17.89% are bidimensional poor but located above the univariate thresholds. At the individual level, 26.9% are trivariate poor but above univariate thresholds, with pronounced intersectional differences. Trivariate deprivation is systematically associated with education, age, gender, and ethnicity. The resulting poverty surface further allows us to identify binding constraints and discuss pathways out of poverty in relation to social and ecological sustainability.

*Keywords:* Multidimensional poverty assessment, income distribution, time-use distribution, intersectional differences, time and income poverty

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# 1 Introduction

To effectively assess poverty in a way that reflects lived well-being, it is essential to consider not only income but also time use – particularly unpaid work and leisure. These dimensions jointly shape individual opportunities, household living standards, and subjective well-being. Approaches that focus exclusively on income risk overlook deprivation arising from time poverty, especially in contexts where unpaid labor substitutes for market services to sustain household consumption. Time-use not only directly affects well-being but also shapes how households meet their needs, often encouraging greater reliance on market-based, resource-intensive forms of consumption when time for unpaid responsibilities is limited. This perspective is particularly important for understanding intersectional inequalities, as time and income constraints are not evenly distributed within or across households. Because indicators guide both diagnosis and policy prioritization, what is measured influences which constraints become visible and actionable (Costanza et al., 2014). Extending poverty measurement to include time-use can therefore improve assessments of well-being and help connect distributional outcomes to broader sustainability concerns.

Women, who continue to shoulder a disproportionate share of unpaid work<sup>1</sup>, often face binding time constraints that limit leisure, reduce labor market participation, and restrict income-generating opportunities. At the same time, this unpaid work is indispensable for maintaining household functioning and living standards. When time for care and non-market activities is scarce, households frequently compensate by increasing monetary spending and outsourcing services, thereby reinforcing both social and material pressures. Conversely, when income is low but time is available, households can compensate by providing more goods and services themselves, reducing reliance on market goods and thereby potentially lowering environmental pressure (Schor, 2005, 2010). Going beyond income and particularly including time use is therefore crucial for a meaningful assessment of well-being and poverty.

Our contribution advances a growing literature that integrates time-use measures into poverty and gender analysis (Vickery, 1977; Zacharias et al., 2012; Merz and Rathjen, 2014; Dorn et al., 2024). Existing approaches differ substantially in how time use is incorporated and how it relates to income, yet none fully integrates leisure, unpaid work, and income simultaneously, nor explicitly models their interdependence. In contrast, most multidimensional poverty frameworks aggregate several indicators into a composite index (Alkire and Santos, 2013). While informative, such aggregation entails a loss of information by imposing linear substitutability and abstracting from the joint distribution of dimensions. However, the distributional structure is central to understanding vulnerability, persistence, and relative deprivation (Pogge, 2002; Mukhopadhyay and Chatterjee, 2011; Ferreira and Lugo, 2013; Decancq, 2014; Dorn et al., 2024).

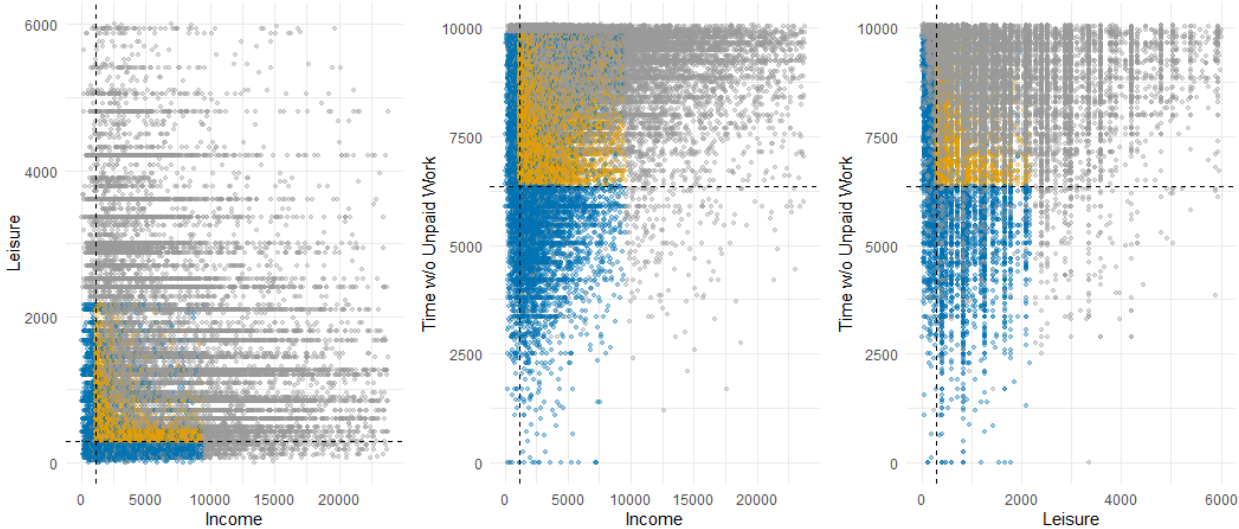
We adopt a two-stage analytical strategy to advance the literature on time and income poverty. This approach enables a comprehensive assessment by distinguishing household living standards from intra-household inequalities. In the first stage, we use a bidimensional approach for unpaid work time and income,

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<sup>1</sup>Unpaid work includes activities that could, in principle, be delegated to a third party, following the third-party criterion introduced by Reid (1934).

as these two dimensions determine household living standards. We apply a non-linear, data-driven approach based on the joint distribution of unpaid work time and income, defining the Bivariate Relative Poverty Line (BRPL) following Dorn et al. (2024). The BRPL enables us to identify the necessary bundles of income and time required to sustain a household. In the second stage, the unit of analysis shifts to the individual to uncover intra-household poverty and inequality. Even when households face similar aggregate constraints, access to monetary resources and the allocation of unpaid work and leisure are typically unequal. We construct individual-level measures of income access, leisure, and time without unpaid work. Defining the unpaid-work dimension as time available net of unpaid work allows deprivation to be interpreted consistently as a deficit in all dimensions. At the household level, poverty arises from too little unpaid work time to sustain living standards given income. At the individual level, poverty reflects the opposite constraint: high unpaid-work burdens reduce leisure and limit opportunities for paid work. The resulting trivariate analysis reveals intersectional disparities that are hidden in household-level analyses. For efficient evaluation of the three-dimensional empirical distribution function, we apply the divide-and-conquer algorithm to construct a trivariate relative poverty surface (TRPS) that is fully data-driven and avoids collapsing multiple dimensions into a single scalar index.

Figure 1: Pairwise plots displaying in which category an individual falls: univariate poor (blue), Below the TRPS but not poor in any univariate dimension (yellow) or not poor (grey).



Both approaches identify households and individuals who are deprived in two and three dimensions, respectively, due to an insufficient combination of time and money, but are hidden in conventional intersectional or union poverty approaches. Ignoring this group risks understating poverty in societies where norms, expectations, and distributions of time and income matter for well-being. Figure 1 illustrates that a non-negligible share of individuals classified as poor by the TRPS measure (yellow) are not poor in any single dimension (blue). These individuals appear above all univariate poverty lines in pairwise projections, yet fall below the multidimensional poverty frontier, revealing a form of hidden deprivation driven by interacting income

and time constraints. We additionally apply logit regressions to analyse which factors are associated with trivariate poverty risk. Finally, we identify the binding constraints of multidimensional poverty by estimating the minimal one-dimensional improvement required for individuals to exit trivariate poverty, distinguishing income, unpaid work, leisure, and mixed bottlenecks.

We apply our framework to data from the 2018 Mexican National Survey of Household Income and Expenditure (ENIGH), which provides detailed information on income, unpaid work, and leisure (INEGI, 2018). At the household level, we find that 17.89% of households are jointly deprived in income and unpaid work despite exceeding univariate thresholds, with marked rural-urban differences. At the individual level, 26.91% of individuals fall below the trivariate relative poverty surface while remaining above all univariate thresholds, revealing substantial multidimensional deprivation that is invisible to conventional measures. This poverty is gendered, intensifying with caregiving responsibilities and across most education levels, and is more pronounced among non-Indigenous individuals and is reversed in high educational levels. Analysing the binding constraints of trivariate poverty provides guidance on potential pathways out of deprivation. Effective poverty reduction requires integrated policy frameworks that address both income and time poverty, with a focus on women’s disproportionate burden of unpaid work, to avoid shifting deprivation across dimensions and to enable synergies with ecological sustainability.

## 2 Literature Review

Time use is widely recognized as a critical component of poverty analysis, particularly for understanding gendered and intersectional forms of deprivation. A substantial body of research shows that unpaid work and limited leisure shape living standards in ways that remain invisible in purely monetary assessments of poverty, especially for women (Vickery, 1977; Bardasi and Wodon, 2010; Zacharias et al., 2012; Dorn et al., 2024). These studies highlight that time constraints are not merely ancillary to income poverty but constitute a distinct and consequential dimension of deprivation.

More broadly, a large literature has emphasized the limitations of unidimensional monetary poverty measures, arguing that non-monetary factors are essential for understanding well-being and deprivation (UN, 2004; UNDP, 2021; OHCHR, 2022; Alkire and Foster, 2011b). Multidimensional poverty approaches respond to this critique by incorporating multiple dimensions, most commonly health, education, and living standards (Alkire and Santos, 2013; Klugman, 2010; Alkire et al., 2021; Decancq and Lugo, 2012). However, despite their relevance for household functioning and gender inequality, time-use dimensions and particularly unpaid work and leisure remain largely absent from these frameworks.

Even where time use is incorporated, existing poverty measures rarely integrate income, unpaid work, and leisure simultaneously in a way that captures their interdependence and resulting multidimensional poverty patterns (Dorn, 2025). Most time-and-income poverty measures rely on composite indices (Bardasi and Wodon, 2010; Zacharias et al., 2012; Aloè, 2023), which tend to obscure distributional structure, assume linear

substitutability across dimensions, and fail to capture how constraints in one domain amplify deprivation in others. Yet the joint distribution and dependence between poverty dimensions are central for understanding vulnerability to poverty and its persistence (Sen, 1976; Pogge, 2002; Kong and Mizera, 2012).

Time-use aspects are particularly important for understanding vulnerability to poverty. Unpaid work contributes significantly to household living standards while simultaneously constraining individuals' labor-market opportunities, particularly for women. Low-income households typically spend more time on unpaid work to maintain basic living standards, as they lack the financial means to outsource household tasks (Antonopoulos and Hirway, 2010; Ferrant et al., 2014). In contrast, higher-income households are more likely to substitute unpaid work with market services (Antonopoulos and Hirway, 2010). At the same time, leisure time constitutes an essential component of well-being. Together, unpaid work and leisure are therefore indispensable for sustaining living standards and individual welfare (Dorn and Folbre, 2022). While several approaches incorporate time-use aspects into poverty analysis (Vickery, 1977; Bardasi and Wodon, 2010; Zacharias et al., 2012; Merz and Rathjen, 2014; Dorn et al., 2024), they vary substantially in scope and methodology and lack a common framework. None fully accounts for both unpaid work and leisure while explicitly modeling their joint interaction with income.

A key distinction in time-inclusive poverty measurement concerns the different economic meanings of unpaid work and leisure across levels of analysis. Unpaid work primarily contributes to household production and can, at least in principle, be pooled and substituted within households to sustain shared living standards, making it a meaningful dimension at the household level (Vickery, 1977; Folbre and Bittman, 2004; Dorn, 2025). Leisure, by contrast, constitutes an inherently individual and non-substitutable dimension of well-being that cannot be transferred or aggregated across household members. Individuals face distinct trade-offs between paid work, unpaid work, and leisure, and insufficient leisure directly reflects individual time poverty and welfare loss (Goodin et al., 2005; Bardasi and Wodon, 2010; Dorn, 2025). As a result, household-level measures based on income and unpaid work may mask substantial individual deprivation when leisure constraints and access to household resources are unevenly distributed within households, a concern emphasized in the literature on individual poverty and intra-household inequality (Klasen and Lahoti, 2021). This conceptual distinction motivates a bivariate framework at the household level and a trivariate framework at the individual level.

A related strand of the literature emphasizes that poverty and well-being assessments are closely linked to questions of environmental sustainability. Research beyond income-based metrics highlights that living standards depend not only on material consumption but also on how work, care, and time are organized under resource constraints (Jackson, 2017; Raworth, 2017). Time allocation is therefore central not only to individual well-being but also to patterns of consumption and environmental pressure. Studies debate that longer working hours and time scarcity are associated with greater reliance on market-based, resource-intensive consumption, whereas reduced working time can enable more time-intensive but materially lighter forms of provisioning, including care, household production, and leisure. When time for care and non-market

activities is scarce, households often compensate by increasing monetary spending and purchasing energy-intensive goods and services, thereby reinforcing both social and ecological pressures (Schor, 2005, 2010). Ignoring unpaid work and leisure in poverty and well-being assessments thus risks overstating the role of consumption while understating the social and temporal foundations of sustainable living standards (Folbre, 2001; Sen, 1987).

Beyond dimensional coverage, distributional aspects play a crucial role in multidimensional poverty assessment. Many index-based approaches obscure correlations between poverty dimensions and thereby underestimate vulnerability arising from simultaneous deprivations (Ravallion, 2011; Chiappero-Martinetti and Von Jacobi, 2012; Kong and Mizera, 2012). Experiencing multiple deprivations concurrently is more severe than facing each in isolation, making it essential to examine interdependencies explicitly (Pogge, 2002; Decancq, 2014). While marginal distributions provide insights into specific aspects of deprivation, analyzing joint distributions reveals additional information on how poverty dimensions interact and reinforce each other (Ferreira and Lugo, 2013; Mukhopadhyay and Chatterjee, 2011; García-Gómez et al., 2021). However, combining multiple dimensions simultaneously poses methodological challenges, which is why many studies consider them sequentially rather than jointly.

Various approaches to modeling dependence structures among multiple poverty dimensions exist. Copula-based models are frequently applied, though they typically rely on parametric assumptions regarding the underlying data-generating process (García-Gómez et al., 2021; Tkach and Gigliarano, 2020; Dorn et al., 2024). An alternative strand of the literature focuses on extending univariate order statistics and quantiles to multiple dimensions, providing distribution-sensitive tools for multivariate analysis (Chaudhuri, 1996; Serfling, 2002; Hallin et al., 2010; Klein and Kneib, 2020; Guggisberg, 2023). Relying on the empirical distributions to generate quantiles is particularly suited for relative poverty assessment, as this avoids strong parametric assumptions. Building on this literature, Dorn et al. (2024) proposes a bidimensional approach to relative poverty based on the joint distribution of leisure time and income. We apply this framework to determine the BRPL of unpaid work and income by a line search algorithm on household level. For individuals, we extend this by introducing leisure as a third dimension, thereby developing a trivariate poverty surface that captures the full interaction between income, unpaid work, and leisure. Rather than performing repeated line searches, we compute the empirical distribution function using a divide-and-conquer algorithm, which allows efficient and exact evaluation of the distribution function and thus direct identification of the poverty surface (Bentley, 1980; Lee and Joe, 2018). This extension affects only the numerical implementation and not the underlying poverty concept. Additionally, this approach minimizes prior assumptions and ensures that the resulting poverty threshold closely reflects the observed data structure (Scott, 2012, 1992).

Finally, Mexico provides a particularly relevant empirical context for this analysis. Persistent gender norms and limited social protection systems contribute to pronounced inequalities in the distribution of paid and unpaid work. Women are typically expected to assume responsibility for household tasks while also contributing to household income, often resulting in a double work burden (Pedrero Nieto, 2005; Segrest

et al., 2003; Dorn, 2021). In response, many women engage in informal employment that offers flexibility but is poorly paid and lacks social protection (Rodin et al., 2012; Lyon et al., 2017; Dorn, 2021). Using data from the 2018 Mexican National Survey of Household Income and Expenditure (ENIGH), this paper contributes to the literature by examining how income, unpaid work, and leisure jointly shape poverty and inequality in this context.

In summary, this paper contributes to the literature by introducing a statistical framework for measuring poverty across three interrelated dimensions: income, unpaid work, and leisure. By relying on the empirical cumulative distribution function, the approach captures the joint distribution of these dimensions without imposing restrictive parametric assumptions. The resulting trivariate relative poverty surface (TRPS) provides a distribution-sensitive representation of time and income deprivation within the population. Applying this framework to data from the 2018 Mexican National Survey of Household Income and Expenditure (ENIGH), we generate new estimates of poverty that extend existing assessments for Mexico.

### 3 Methodology

This section introduces our distributional framework for measuring multidimensional poverty based on the joint empirical distribution of income, leisure, and unpaid work. Building on the bidimensional income–time poverty approach of Dorn et al. (2024), we extend the analysis to three dimensions and define poverty relative to a quantile frontier of the joint distribution. The poverty threshold is set by the quantile level corresponding to the absolute income poverty line in Mexico. This framework accommodates variables with different units and supports both continuous and discrete outcomes, allowing us to combine income (Mexican pesos) with leisure and unpaid work (minutes) in a unified distributional analysis.

#### 3.1 Multidimensional Poverty and Quantiles

We begin by formalizing the distributional objects that underpin our poverty measure, starting with the definition of univariate and multivariate quantiles and their interpretation as multidimensional poverty frontiers.

##### Distribution functions

We consider the real-valued variables  $\mathbf{Y} = (Y_1, \dots, Y_N)' \in \mathbb{R}^{N \times d}$  and the empirical counterpart  $y_1, \dots, y_N$  with  $N$  being the number of observations and  $d$  the number of variables. The joint *cumulative distribution function* (CDF)  $F$  for  $\mathbf{Y} = (Y_1, \dots, Y_d)^T$  satisfies

$$F(y_1, \dots, y_d) = P(Y_1 \leq y_1, \dots, Y_d \leq y_d) \tag{3.1}$$

with  $P$  representing the probability for  $\mathbf{Y} \sim F$ . The multivariate *empirical cumulative distribution function (ECDF)* for  $Y \in \mathbb{R}^d$  takes the form of

$$F_N(\mathbf{y}) = \frac{\#\{\mathbf{y}_i \leq \mathbf{y}\}}{n}, \quad (3.2)$$

where for each datapoint  $\mathbf{y} = (y_1, \dots, y_d)$  the inequality  $\{\mathbf{y}_i \leq \mathbf{y}\}$  is evaluated and counted (Scott, 1992). The multivariate *empirical probability distribution function (EPDF)*  $f_N$  can be derived from Equation (3.2) through the theoretical relationship  $f_N(y) = F_N^{-1}(y)$ . The PDF is defined as

$$\begin{aligned} f_N(y) &= \frac{d}{dy} F_N(y) \\ &= \frac{1}{n} \sum_{i=1}^n \delta(y - y_i) \end{aligned}$$

where the Dirac delta function is represented as  $\delta(t)$  with  $t = y - y_i$ , which is a discrete uniform distribution and has for each data point a probability mass of  $n^{-1}$ . Calculation of the multivariate ECDF is complex due to the lack of ordering in multiple dimensions but often necessary for statistical applications (Scott, 1992). Multivariate ECDF calculation can be decomposed into two subproblems. First, the ECDF calculation based on all points of the data by defining a rank for each observation. The second problem considers ECDF searching, i.e. rank, for a particular observation in the data structure (Bentley, 1980).

### Domination and ranks

For determining the order of the data points in the multivariate setting we need to define when one data point is larger than another one. In this sense, point domination considers the coordinates of points with  $i$ ,  $1 \leq i \leq d$  for dimensions  $d$ . Point  $A_i$  *dominates* point  $B_i$  if  $A_i > B_i$  for all  $i$ . If neither point  $A$  or point  $B$  dominate the other, they are called *incomparable*. Partial ordering in multiple dimensions is achieved with this definition. The *rank* for a point  $x$  in a set  $S$  of  $N$  points can be defined as the number of points dominated by  $x$  in  $S$ . The ECDF of a point  $x$  is  $rank(x)/N$ .

### Univariate and multivariate quantiles

Univariate quantiles are widely used, e.g., as a measure of variability, for assessing the degree of extremity (Klein and Kneib, 2020). Unconditional quantiles do not require any parametric preconditions about the structure of the data. In a two dimensional setting, Dorn et al. (2024) apply directional quantiles for measuring time and income poverty. We extend this approach to a three-dimensional framework and therefore begin by formalizing the notion of quantiles.

Quantiles provide a distributional characterization that goes beyond the mean. In the univariate case, the  $\tau$ -*quantile* of a random variable  $Y$  with CDF  $F$  is defined as

$$Q_Y(\tau) = \inf\{y \in \mathbb{R} : F(y) \geq \tau\}, \quad \tau \in (0, 1),$$

and represents the value below which a fraction  $\tau$  of the population lies (Koenker, 2005). To extend this concept to multiple dimensions, we consider the joint CDF  $F(\mathbf{y}) = P(\mathbf{Y} \leq \mathbf{y})$  and define the *multivariate  $\tau$ -quantile region* as

$$\mathcal{Q}_\tau = \{\mathbf{q} \in \mathbb{R}^3 : F(\mathbf{q}) \leq \tau\}, \quad (3.3)$$

with *quantile surface*  $\partial\mathcal{Q}_\tau = \{\mathbf{q} : F_{\mathbf{Y}}(\mathbf{q}) = \tau\}$ . This lower-orthant quantile set generalizes univariate quantiles to higher dimensions and provides a natural basis for distributional analysis and multidimensional poverty measurement.

### Poverty risk

We define multidimensional poverty risk as the probability that a household lies below a given poverty frontier  $\mathbf{z}$ ,

$$\rho(\mathbf{z}) = \Pr(\mathbf{Y} \leq \mathbf{z}) = F_{\mathbf{Y}}(\mathbf{z}).$$

If  $\mathbf{z} \in \partial\mathcal{Q}_\tau$ , then  $\rho(\mathbf{z}) = \tau$ , so the quantile surface indexes poverty risk. In the empirical analysis, we estimate this risk at the household level using a logit model described in 3.3

## 3.2 Relative poverty lines and associated poverty regions

Defining poverty across multiple dimensions broadens the concept of deprivation but also raises the question of where and how poverty thresholds should be set when dimensions interact. To address this, we adopt a relative approach to poverty measurement that defines thresholds in terms of the joint distribution of the relevant dimensions. This allows for the identification of individuals and households living at the margins of society and avoids reliance on purely marginal cutoffs.

We illustrate the resulting poverty regimes exemplarily in the bivariate case of income and unpaid work, following the framework proposed by Dorn et al. (2024). The different regimes are displayed in Figure 2. Rather than defining poverty solely through one-dimensional thresholds, this approach specifies a joint poverty threshold by fixing a quantile level of the joint distribution. In this way, the Bivariate Relative Poverty Line (BRPL) reflects relative deprivation in the population and captures adverse combinations of dimensions that are not visible in univariate analyses. Formally, the joint poverty threshold is defined through the multivariate quantile in Equation (3.3). In the trivariate case, this quantile unifies the three one-dimensional quantiles of income, unpaid work, and leisure, and identifies observations that cannot be captured by univariate thresholds alone. The resulting quantile surface, defined below Equation (3.3), generalizes conventional relative poverty lines to multiple dimensions and explicitly accounts for their interdependence. Incorporating three dimensions, this specifically allows us to define the Trivariate Relative Poverty Surface (TRPS).

## Bivariate case (Figure 2)

Conventional bidimensional poverty measurement often relies on either a *union* or an *intersection* approach. The union approach classifies households as poor if they fall below the threshold in either dimension, whereas the intersection approach identifies only those below both thresholds simultaneously. Both rules rely on marginal cutoffs and therefore abstract from the joint distribution of income and time, overlooking how deprivations co-occur and interact across dimensions.

In contrast, the bivariate relative poverty line (BRPL) is distribution-sensitive. It identifies households as bidimensional poor in income and unpaid work if they lie below the BRPL within the relevant bounds of the joint distribution. This definition captures households that are not classified as poor by univariate income or unpaid-work thresholds but are relatively disadvantaged due to unfavorable combinations of the two dimensions. These households correspond to Area 6 in Figure 2 and remain unaccounted for under both union and intersection approaches. Table 7 defines the different areas below the bivariate relative poverty line and separates the union and intersection approach from it (as illustrated in Figure 2).

Table 1: Areas in Figure 3 (income  $\times$  unpaid work) and interpretation

Area	Name & Definition	Interpretation
1–5	Union approach $M < M_0$ and/or $U < U_0$	Below the univariate unpaid work threshold or the univariate income threshold
5	Intersection poor $M < M_0$ , and $U < U_0$	Severe joint deprivation under the intersection approach.
6	BRPL poor $M > M_0$ , $U > U_0$ , and ( $M, U$ ) below the BRPL	Relatively poor due to insufficient time–income combinations; missed by union and intersection measures.
1	Univariate unpaid work time poor $M > M_1$ , $U \leq U_0$	Below the univariate minimum threshold of unpaid work time but in the top 10 percentile of income
2	Univariate income poor $U > U_1$ , $M \leq M_0$	Below the univariate minimum threshold of income but in the top 10 percentile of time used for unpaid work

## Trivariate extension (Figure 3)

The logic of the bivariate relative poverty line extends to the trivariate case by incorporating leisure as a third dimension. The resulting trivariate poverty surface defines relative poverty in the joint space of income

( $M$ ), unpaid work ( $U$ ), and leisure ( $L$ ). Individuals are classified as tridimensional poor if their observed combination of these three dimensions lies below the surface at the chosen quantile level.

Figure 3 illustrates this trivariate relative poverty surface. Marginal cutoffs describe shortfalls in individual dimensions, while the poverty surface captures the joint relative deprivation arising from their interaction. In particular, the region below the trivariate surface but above the marginal and the bidimensional cutoffs corresponds to the three-dimensional analogue of Area 6 in the bivariate framework: individuals who are not poor according to univariate or pairwise criteria but are relatively disadvantaged in the joint combination of all three dimensions. Defining unpaid work in inverted form, as time available without unpaid work (time w/o unpaid work), allows poverty to be represented as a deficit in all dimensions, located in the lower-left corner.

Table 2: Additional regions below the TRPS compared to the BRPL

Area	Name & Definition	Interpretation
1_TRPS	Univariate leisure-deprived $L < L_0, M \leq M_1, U \leq U_1$	Individual leisure time poverty but above the univariate income threshold and with insufficient free time.
5_TRPS	Trivariate intersection poor $L < L_0, M < M_0, U < U_0$	Severe multidimensional deprivation across all three dimensions.
6_TRPS	TRPS poor $M > M_0, U > U_0, L > L_0,$ $(M, U, L)$ below the TRPS	Hidden individual poverty arising from adverse combinations of income, unpaid work, and leisure.

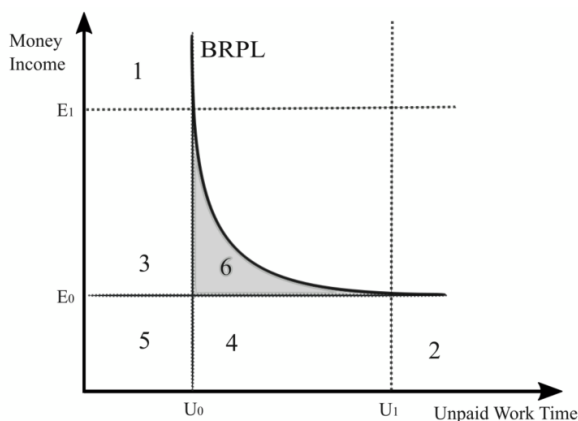


Figure 2: Bivariate Relative Poverty Line (BRPL) splitting the population into different poverty regimes.

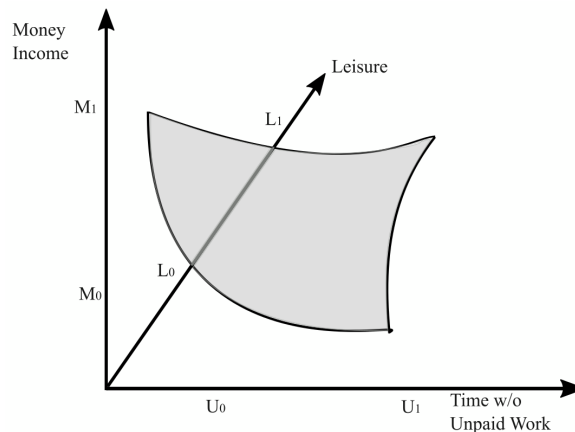


Figure 3: Trivariate relative poverty surface (TRPS).

### 3.3 Conditional Poverty Risk and Logit Model

To relate the three poverty dimensions to potential driving factors, we estimate a logit regression. Specifically, we regress the probability of being below the one-, two-, or three-dimensional poverty line on socioeconomic covariates such as age, gender, and ethnicity. This allows us to assess whether and how these factors are differently associated with multidimensional poverty. We model the probability that household  $i$  is multidimensionally poor using a logit specification. Let

$$D_i(\mathbf{z}) = \mathbf{1}\{\mathbf{Y}_i \leq \mathbf{z}\}$$

indicate whether household  $i$  falls below the poverty frontier  $\mathbf{z}$ . We model the conditional poverty risk as

$$P(D_i(\mathbf{z}) = 1 \mid \mathbf{X}_i) = \Lambda(\mathbf{X}_i^\top \boldsymbol{\beta}), \quad \Lambda(z) = \frac{e^z}{1 + e^z},$$

where  $\mathbf{X}_i$  is a vector of socioeconomic covariates. Identification relies on the assumption that

$$\mathbb{E}[D_i(\mathbf{z}) \mid \mathbf{X}_i] = \Lambda(\mathbf{X}_i^\top \boldsymbol{\beta}),$$

so that the logit model provides a parametric approximation to the conditional poverty risk function. This formulation links multidimensional poverty risk to observable characteristics and allows us to assess how socioeconomic factors shift the probability of lying below a given quantile surface. The logistic link function ensures that predicted probabilities lie in  $[0, 1]$ , while allowing nonlinear and heterogeneous marginal effects of covariates on poverty risk.

### 3.4 Computing the Joint Distribution by the Divide-and-Conquer Algorithm

All objects introduced above (quantile regions, poverty risk, and conditional poverty probabilities) depend on the joint distribution  $F$ . We therefore conclude this section by describing how the joint ECDF is computed efficiently in three dimensions.

As a sorting algorithm, the *divide-and-conquer algorithm* is implemented for various problems such as the calculation of the multivariate ECDF. Similar to the univariate calculation of the ECDF, this approach takes the determination of ranks for the calculation of the ECDF in the multivariate setting (Bentley, 1980). The algorithm by Bentley (1980) solves the multidimensional sorting problem by successively solving several subproblems. The ECDF is computed by sorting the observations and then assigning ranks via recursive sorting functions (Lee and Joe, 2018; Bentley, 1980). This calculation is performed on the ratio of values (i.e., ranks). Therefore, the original unit of the variable is irrelevant, and the included variables may have different scales. In multi-dimensional spaces with points defined by several coordinates, rank computation is a domination problem. The rank for multivariate observations corresponds to the points dominated in every dimension (Lee and Joe, 2018; Bentley, 1980).

The divide-and-conquer dominance counting algorithm allows the joint empirical distribution function

ECDF to be computed efficiently in  $d$  dimensions, with computational complexity  $O(N \log^{d-1} N)$ , making it near-linear for fixed  $d$  and substantially faster than naive  $O(N^2)$  methods. The algorithm works by recursively sorting and splitting the data along successive dimensions, reducing the dominance problem to smaller subproblems whose results are merged. In this paper, the method is implemented for three dimensions: income, leisure, and time w/o unpaid work using income as the primary sorting dimension to handle frequent ties in time-use data, ensuring computational stability and reliable estimation of the joint ECDF. More details and the pseudocode are provided in Appendix A.2.

## 4 Data

Our empirical analysis uses data from the Mexican National Survey of Household Income and Expenditure (Encuesta Nacional de Ingresos y Gastos de los Hogares, ENIGH), collected between August and November 2018 by the Mexican National Institute of Statistics and Geography (INEGI) (INEGI, 2018). The survey is nationally representative and provides detailed information at both the household and individual level on income sources, time use, and socio-demographic characteristics.

To ensure a clear interpretation of income pooling and time allocation, we restrict the sample to households with one or two adults, with or without children. In households with more than two adults, income sharing and time allocation cannot be reliably disentangled empirically (Dorn et al., 2024). In addition, we restrict the analysis to heterosexual households, as the data report expenditures separately for male and female household members. To reduce the influence of extreme values, households in the top 1 percent of the income distribution are excluded.

The construction of the poverty thresholds requires consistent measures of income, unpaid work, and leisure at both the household and the individual level. While income and unpaid work primarily determine household living standards, leisure represents an inherently individual dimension of well-being. We therefore distinguish explicitly between household-level and individual-level variables.

**Income** Household income is defined as the sum of labor income, private and public transfers, capital income, and other income sources reported monthly (INEGI, 2018). To account for household size and economies of scale, household income is equivalized using the square-root equivalence scale.

For the individual-level analysis, household income is allocated using expenditure shares that distinguish between household public goods and gender-specific private consumption. Expenditures on household public goods are assumed to be shared equally among adult household members, while gender-specific private expenditures capture differences in individual consumption. Individual access to household resources is then defined as the sum of the equal share of public goods and individual private expenditures. We use the resulting proportions as proxies for income shares (Dorn et al., 2024).

**Unpaid work** Unpaid work includes all non-market activities that could, in principle, be performed by a third party, following the third-party criterion for defining work introduced by Reid (1934). ENIGH records weekly hours and minutes spent on unpaid activities across four categories relevant for household consumption: housework, caregiving, repair work, and time spent collecting water or fuel. Total unpaid work is computed by summing across these categories. Unpaid work is reported using a short activity list, which does not allow us to distinguish which categories were completed simultaneously. This may lead to an overreporting of time spent on unpaid work. However, existing evidence suggests that aggregate estimates of unpaid work derived from short lists are broadly comparable to diary-based measures Insolera et al. (2019); Gautham and Folbre (2022).

- Household level: Unpaid work is aggregated and adjusted using equivalence scales to account for household size and composition. We apply the unpaid-work equivalence scale proposed by Folbre et al. (2018).<sup>2</sup>
- Individual level: Unpaid work corresponds to each respondent’s reported weekly time. To estimate the trivariate poverty surface, unpaid work is converted into available time, defined as total weekly minutes minus those spent on unpaid work. In the following, we call this variable **Time w/o Unpaid Work**. This transformation places the poverty surface in the lower-left corner of the joint distribution, simplifying interpretation while preserving the dependence structure among income, unpaid work, and leisure.

**Leisure** Leisure captures time available for personal, recreational, and culturally valued activities and reflects an important dimension of individual well-being (Narayan et al., 2000; Ferrant et al., 2014; Merz and Rathjen, 2014; Dorn et al., 2024). To define leisure, we use the ENIGH measure in which respondents report the amount of time remaining during the previous week that they had available for activities they enjoy.<sup>3</sup>

<sup>4</sup> Conceptually, we interpret this category as the counterpart to the third-party criterion: it captures time devoted to activities that cannot be delegated to others and are undertaken primarily for personal enjoyment, recreation, or cultural fulfillment. Leisure is constructed exclusively at the individual level, as it cannot be pooled or transferred across household members.

**Covariates** To analyze which individuals and households fall below the poverty thresholds, we include a set of socio-demographic covariates commonly used in poverty and inequality research: age, gender, education,

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<sup>2</sup>For a household  $h$  with  $A$  adults and  $K$  children, equivalized unpaid work is defined as

$$UW_h^{eq} = UW_h / (A + 2K)^{0.5},$$

where  $UW_h$  denotes total household unpaid work time. This scale reflects that children increase unpaid work requirements more strongly than income needs.

<sup>3</sup>Original survey question: “Durante la semana pasada, ¿cuánto tiempo le quedó para realizar actividades que a usted le gustan?” (INEGI, 2018)

<sup>4</sup>Since the variables leisure and unpaid work rely on self-reporting, they are subject to recall bias and depend on the respondent’s perception. This reliance may result in increased errors or accumulated values in the data for leisure and unpaid work compared to income.

ethnicity, urban residence, partnership status, and the presence of children under 5 and between 6 and 14.

## 4.1 Summary Statistics

Table 3: Weighted summary statistics at the household level.

Variable	Mean	SD	P25	Median	P75
Income	10150	13214	4550	7274	11850
Unpaid Work	2007	1422	891	1680	2800

*Note:* Income refers to monthly household income equivalized using the square-root scale. Unpaid work denotes total weekly minutes of unpaid work at the household level, equivalized using the unpaid-work equivalence scale. All statistics are weighted using survey weights.

Table 4: Weighted summary statistics at the individual level by gender.

Variable	Mean	SD	P25	Median	P75
<i>Men</i>					
Income	6790	10200	2695	4492	7717
Time w/o Unpaid Work	922	867	360	630	1260
Leisure	1105	908	420	840	1440
<i>Women</i>					
Income	5939	7491	2359	4005	6993
Time w/o Unpaid Work	2242	1558	1020	1860	3120
Leisure	1083	878	420	840	1260

*Note:* Income is measured at the individual level using expenditure-based income shares derived from household expenditures. Unpaid work and leisure are reported in minutes per week. All statistics are weighted using survey weights.

Table 3 reports weighted summary statistics for equivalized household income and unpaid work. Median equivalized household income amounts to 7,274 Mexican Pesos per month, while the median household reports 1,680 minutes (approximately 28 hours) of unpaid work per week. This unpaid work reflects the time invested in meeting essential household needs such as cooking, cleaning, and care and makes visible forms of socially necessary labor that are typically excluded from conventional economic accounts.

Interpreting unpaid work at the household level is inherently challenging, particularly in couple households, where many activities are performed jointly or overlap in time. Tasks such as cooking or childcare may be reported by more than one adult, even though household needs would be met if only one person performed the task. Moreover, unpaid work is often undertaken alongside other activities. Most notably, supervisory care, such as monitoring children or dependent relatives while doing housework or engaging in leisure, is blurring the boundary between unpaid work and leisure. Differences in access to informal care support across households further complicate interpretation. As a result, aggregated household unpaid work may overstate the labor time strictly required to sustain household functioning (Dorn, 2026).

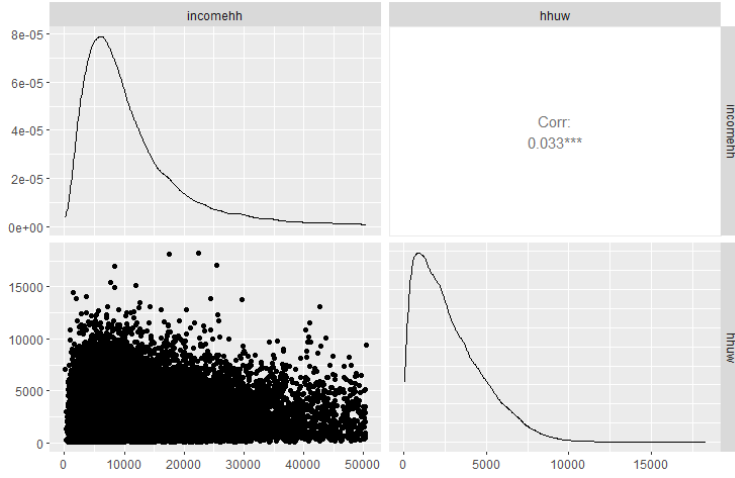
Despite these limitations, incorporating unpaid work is crucial for recognizing the benefits and contributions of care and household production to well-being. Given current data constraints, household-level unpaid work should be interpreted as a first-order indicator of care and production demands rather than a precise measure of necessary labor input. The household-level analysis thus establishes joint income–time constraints, providing a consistent reference point for the paper’s main focus on individual-level deprivation and intra-household inequality.

Table 4 presents weighted summary statistics at the individual level, disaggregated by gender. Clear gender disparities emerge. Women perform substantially more unpaid work than men: mean unpaid work time amounts to 2,242 minutes per week (37.4 hours) for women compared to 922 minutes (15.4 hours) for men – nearly two and a half times higher. At the same time, men have higher average incomes. Mean individual income amounts to 6,790 Mexican Pesos for men and 5,939 Mexican Pesos for women. These patterns reflect the allocation of household income according to gender-specific expenditure shares and indicate systematically lower access to monetary resources for women within households.

Differences in leisure time are comparatively small. Mean leisure time amounts to 1,105 minutes per week (18.4 hours) for men and 1,083 minutes (18.1 hours) for women, while the median leisure time is identical across genders at 840 minutes (14 hours). This suggests that gender inequalities are more pronounced in unpaid work and income than in leisure, reflecting the constraints from paid work for men and unpaid work for women.

As illustrated in Figure 4, the joint distribution of income and unpaid work reveals patterns that are not captured by average values alone. While mean levels of income and unpaid work provide a summary of central tendencies, the concentration of households in the scatterplot underscores the need to analyze their joint distribution. This clustering indicates that time and income constraints intersect for specific groups of households, motivating a multidimensional perspective that goes beyond univariate or mean-based assessments of deprivation.

Figure 4: Pair Plot of Income and Unpaid Work on household level.

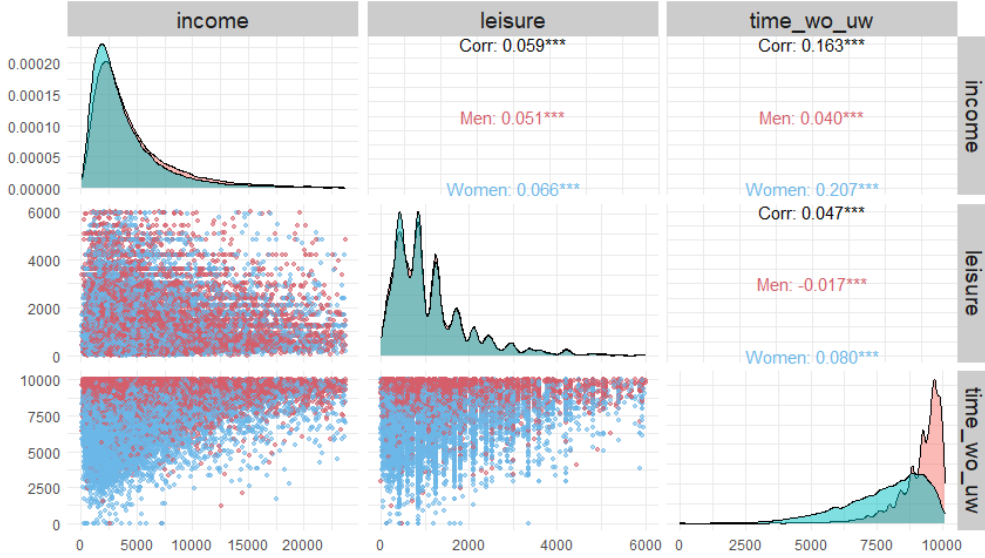


*Note:* Income is reported in Mexican Pesos and unpaid work in minutes per week.

Figure 5 displays the joint distribution of individual income, leisure, and unpaid work. Income is right-skewed for both genders, while unpaid work exhibits a pronounced left tail for women, indicating a substantial share of women with high unpaid work burdens. The scatter plots reveal only weak associations between income and leisure for both men and women, whereas the relationship between unpaid work and leisure differs markedly by gender. In particular, unpaid work and leisure are negatively related for women, while this association is close to zero or slightly positive for men. These patterns highlight pronounced gender differences in individual time allocation that are not visible from mean values alone.

The pairwise nonparametric dependence measures reported in Table 5 confirm these visual impressions. Correlations between income and leisure are negligible, whereas income and unpaid work, as well as leisure and unpaid work, display statistically significant but modest dependence. Overall, the figure underscores the importance of examining the joint distribution of income and time use at the individual level, as marginal associations alone obscure gender-specific trade-offs between unpaid work, income, and leisure.

Figure 5: Pair Plot of Income, Leisure and Time w/o Unpaid Work on individual level.



*Note:* Income is reported in Mexican Pesos, leisure and time w/o unpaid work in minutes per week.

Measuring dependencies beyond normality in multivariate settings can take various forms (Herwartz and Maxand, 2020). Applying the definition of ranks we can go beyond univariate linear dependence measures and define two classical bivariate dependence measures, i.e., Spearman’s rank criterion and Kendall’s tau. Pairwise rank correlations indicate only weak monotonic but significant dependence between income, leisure, and unpaid work (see Table 5). However, weak pairwise associations do not preclude the existence of substantial multivariate dependence, in particular when dependence is nonlinear, non-monotonic, or concentrated in specific regions of the joint distribution. To assess higher-order dependence, we therefore complement bivariate rank correlations with a multivariate test of joint independence by the d-variable Hilbert-Schmidt independence criterion (dHSIC, Pfister et al. (2017)).<sup>5</sup> The results show highly significant trivariate dependence.

Table 5: Pairwise nonparametric dependence measures for the individual level data.

Variable Pair	Spearman’s $\rho$	$p$ -value	Kendall’s $\tau$	dHSIC (3dim)	
(Income, Leisure)	0.0551	0.000	0.0378	test statistic	5.6507
(Income, Time w/o Unpaid Work)	0.1890	0.000	0.1281	crit value	0.7994
(Leisure, Time w/o Unpaid Work)	-0.0239	0.000	-0.0160	p value	0.0010

*Notes:* The table reports pairwise Spearman rank correlation coefficients  $\rho$  with corresponding  $p$ -values, Kendall’s  $\tau$  and dHSIC test for mutual dependence for the individual level data.

The descriptive analysis reveals substantial heterogeneity in both income and time use across households and individuals, as well as pronounced gender differences in unpaid work and access to monetary resources.

<sup>5</sup>Due to the quadratic computational complexity of kernel-based independence tests, the dHSIC statistic is computed on random subsamples of the full dataset. Results are stable across multiple subsamples.

While household-level measures highlight joint income-time constraints and considerable dispersion in unpaid work requirements, individual-level data show that women systematically bear higher unpaid work burdens and have lower income access than men, whereas differences in leisure are comparatively small. These patterns underscore the limitations of unidimensional poverty measures and motivate the need for a multidimensional approach that jointly considers income and time. The next section builds on these insights by deriving relative poverty thresholds and examining how income-time trade-offs translate into household and individual poverty outcomes.

## 5 Results

In this section, we evaluate the bivariate relative poverty line (BRPL) at the household level and the trivariate relative poverty surface (TRPS) for individuals. Household-level poverty thresholds are anchored to Mexico’s official 2018 CONEVAL poverty lines for food and non-food expenditures. Specifically, the urban threshold is set at 2,996 pesos (the 10.51st percentile of urban household incomes) and the rural threshold at 1,938 pesos (the 18.29th percentile of rural household incomes) (CONEVAL, 2026). We use these percentiles as a quantile label,  $\tau$ , to calibrate BRPL points, ensuring that no household classified as non-poor under the BRPL falls below the official income poverty line.

For urban households, this income threshold corresponds to approximately 6 hours of unpaid work, a value that aligns closely with the socially necessary unpaid-work minimum identified for the United States (Dorn, 2026). For rural households, the corresponding minimum is 11 hours, which we interpret as the socially necessary minimum level of unpaid work in rural contexts.

Beyond fixed cutoffs, the BRPL framework identifies the set of income–time bundles sufficient to remain above the poverty line. In this sense, the minimum unpaid-work requirement is linked to the maximum income level needed to sustain a given number of unpaid-work hours, highlighting the trade-offs between time and income.

For individual-level analysis, we use the 10th percentile ( $\tau = 0.10$ ) in each dimension to assess gender differences in resource access. In terms of unpaid work, this implies that more than 60 hours per week constitute the poverty threshold, a range that coincides with very low income levels. Table 6 reports the corresponding univariate threshold values for income and time-use.

Table 6: Poverty thresholds by dimension and area (weekly values).

Dimension	Urban (10.51%)	Rural (18.29%)	Individual (10%)
Income (pesos)	2996.45	1938.06	1185.28
Leisure (hours/week)	–	–	5
Unpaid Work (hours/week)	6	11	-
Time w/o Unpaid Work (hours/week)	-	-	108

*Note:* The first two columns display the numbers below which households are defined as poor; individuals below the values in the third column are defined as poor in the respective dimension.

## 5.1 BRPL on household level

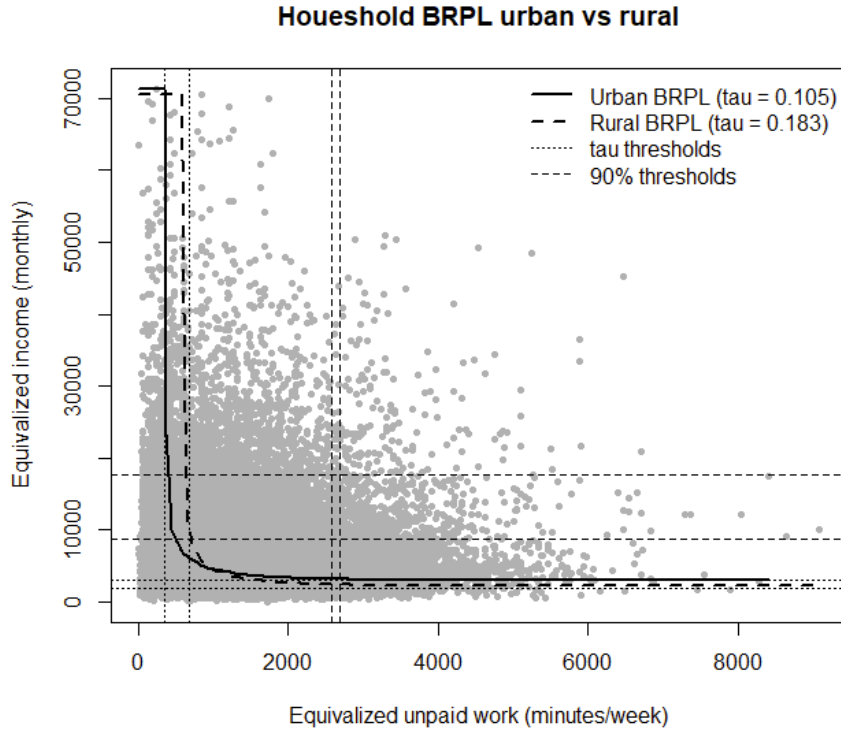
Table 7 and Figure 6 highlight substantial hidden vulnerability in the Mexican income–time distribution. While households that are poor under the intersection approach constitute only a very small share of the population, vulnerability becomes pronounced when the joint allocation of income and unpaid work is considered. As shown in Table 7, the univariate union criterion classifies 24.11% of households as poor. Beyond this group, an additional 17.89% lie above both univariate thresholds yet below the BRPL (and below the 90% boundary). These households are not poor under conventional cutoffs, but their time–income bundles are nevertheless insufficient, indicating relative poverty driven by adverse trade-offs rather than by shortfalls in a single dimension. The table further reveals marked differences between urban and rural areas.

Figure 6 visualizes this mechanism. The scatter plot of equivalized unpaid work and income shows observations clustering around the nonlinear BRPL frontier, where small increases in unpaid work require large compensating income gains to remain non-poor (and vice versa). This nonlinear relationship explains why the BRPL identifies a large group of vulnerable households that univariate and standard multidimensional poverty measures fail to capture. The 90% lines coincide with the region in which the BRPL and the univariate thresholds become asymptotic. This makes the 90th percentile an empirically well-suited cutoff for defining univariate poverty. Households beyond this point are therefore not bidimensional poor, as they lie outside the region where joint income–time trade-offs determine poverty status.

Table 7: Population shares of household poverty regimes in income and unpaid work under the BRPL, as illustrated in Figure 3.

Area	Name & Definition	Share		
		Total	Urban	Rural
1–5	Union approach $M < M_0$ and/or $U < U_0$	24.11	20.39	33.98
5	Intersection poor $M < M_0$ , and $U < U_0$	1.26	0.71	2.70
6	BRPL poor $M > M_0$ , $U > U_0$ , and $(M, U)$ below the BRPL	17.89	16.36	21.96
1	Univariate unpaid work time poor $M > M_1$ , $U \leq U_0$	1.31	1.92	2.51
2	Univariate income poor $U > U_1$ , $M \leq M_0$	1.43	1.31	1.76

Figure 6: Empirical BRPL for household level unpaid work and income, as illustrated in Figure 3.



## 5.2 Individual differences in trivariate poverty

We examine the temporal dimension of poverty in greater detail by defining trivariate poverty for individuals. When considering univariate poverty lines in income, leisure and time w/o unpaid work separately, for instance, individuals at the income poverty line are still likely poor in one of the other dimensions, unpaid work and leisure, indicating that unidimensional poverty measures mask proximity to multidimensional deprivation. Histograms of the distribution around the poverty lines illustrate this and are displayed in Figure 8, 9 and 10 in the Appendix. Figure 7 visualizes the trivariate distribution of individuals in the space of income, leisure, and time w/o unpaid work. The surface represents the empirical trivariate relative poverty threshold, constructed as the level set of the joint empirical distribution function. Observations located inside this surface are classified as trivariate poor.

Table 8 identifies individuals who fall below the TRPS based on income, leisure, and unpaid work. The results show that conventional univariate and union-based measures substantially understate both the extent and the gendered nature of multidimensional poverty. As in the household-level analysis, very few individuals are simultaneously deprived at the absolute minimum in all three dimensions. This confirms that applying absolute univariate thresholds separately, and focusing only on their strict intersection, yields a highly incomplete picture of multidimensional poverty.

The TRPS addresses this limitation by identifying individuals whose joint allocation of income, leisure,

Figure 7: Trivariate Relative Poverty Surface and the data split into trivariate poor (light blue) and non-poor (dark blue).

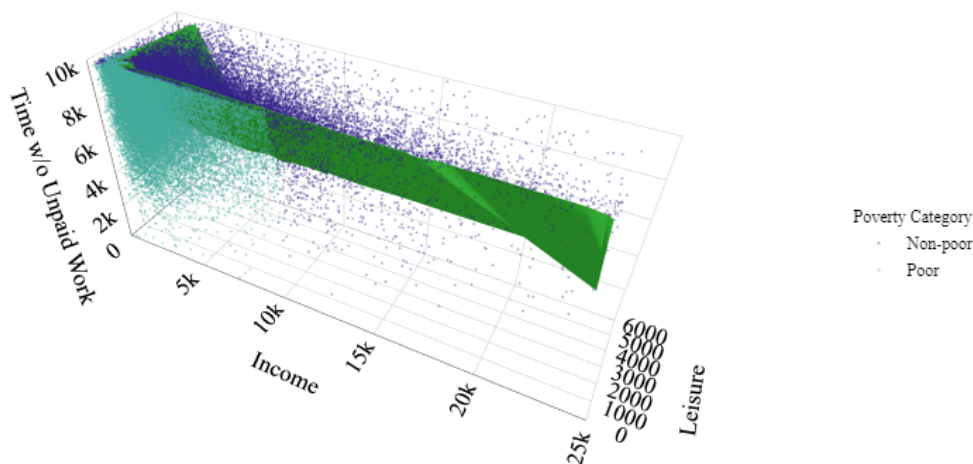


Table 8: Population shares of individual poverty regimes under the trivariate relative poverty surface

	Total	Men	Women
Income	10.00	8.29	11.54
Leisure	9.34	9.70	9.02
Time w/o Unpaid Work	9.82	1.49	17.33
Intersection Income & Leisure & Time w/o Unpaid Work	0.17	0.02	0.30
Union Income & Leisure & Time w/o Unpaid Work	25.77	18.38	32.44
Trivariate Relative Poverty Line	48.80	36.29	60.08
Trivariate without Union of Univariate Quantiles	26.91	22.44	30.95
Non-poor	51.20	63.71	39.92

and unpaid work is unsustainable, even if none of the univariate thresholds is violated. According to this measure, 26.9% of individuals are trivariate poor but would not be classified as poor under conventional poverty regimes. This form of hidden deprivation is highly gendered: 31.0% of women fall below the TRPS compared to 22.4% of men. Univariate differences are most pronounced in unpaid work, where almost no men fall below the threshold, compared with 17.3% of women, whereas gender gaps in leisure are small and income differences are more moderate. Overall, the TRPS reveals a stark gender asymmetry in multidimensional poverty. Men are predominantly classified as non-poor or only relatively deprived, whereas women are disproportionately concentrated among the trivariate poor, often combining relative deprivation with excessive unpaid-work burdens. These results are consistent with Zacharias et al. (2012), who find that in Mexico income poverty without accompanying time poverty affects only a small proportion of the population, while excessive time for unpaid work primarily affects women.

### 5.3 Factors associated with poverty in various dimensions

Table 9 reports average marginal effects from logit models estimating the probability of belonging to different poverty regions defined over income, leisure, and unpaid work. While columns (1)–(3) focus on univariate poverty in each dimension, columns (4) and (5) are central for understanding multidimensional deprivation. Column (4) estimates the probability of falling below the Trivariate Relative Poverty Surface (TRPS), capturing joint deprivation across all three dimensions. Column (5) conditions on individuals below the TRPS and identifies those who are above all univariate thresholds yet still trivariate poor, isolating relative poverty that is not detected by conventional measures. All models control for age, education, ethnicity, gender, urban residence, and partnership status, and Logit (5) additionally includes interactions between gender and education, ethnicity, and children (displayed in Table 10). Table 10 reports discrete interaction effects from the logit model, computed as differences in predicted probabilities between women and men for each education and ethnicity category. Specifically, each entry shows  $P(Y = 1 \mid \text{sexo} = 1, X = k) - P(Y = 1 \mid \text{sexo} = 0, X = k)$ , where  $X$  denotes either education or ethnicity. Positive values indicate a higher predicted probability for women than for men. Standard errors and p-values are obtained using the delta method. Following Ai and Norton (2003), these effects represent the appropriate interpretation of interaction terms in nonlinear models.

The results from Logit (4) show that trivariate poverty is strongly structured by gender, ethnicity, and family responsibilities. Women are 24.3 percentage points more likely than men to fall below the TRPS, and Indigenous individuals face an additional 11.4 percentage point higher probability of trivariate poverty. Care responsibilities further amplify joint deprivation: having children under five increases the probability of falling below the TRPS by 15.8 percentage points, and having children aged 6–14 by 13.2 percentage points. These effects highlight that joint constraints on income, leisure, and unpaid work disproportionately affect women, Indigenous individuals, and caregivers.

Logit (5) refines this picture by distinguishing how trivariate poverty manifests among those below the TRPS. Within this group, women are 12 percentage points less likely than men to be above all univariate thresholds while still falling below the TRPS, indicating that women’s deprivation is more often already captured by at least one univariate dimension, particularly unpaid work. A similar pattern holds for Indigenous individuals, who are 16.8 percentage points less likely than non-Indigenous individuals to be in this relative, trade-off-based poverty region.

The interaction effects reported in Table 10 reveal substantial heterogeneity behind these average patterns. Across ethnic groups, women are significantly less likely than men to experience trivariate poverty exclusively, with the gender gap being particularly pronounced among non-Indigenous individuals (a 12.4 percentage point difference compared to 9.3 percentage points among Indigenous individuals). Education-based interactions further indicate that, across most education levels, women’s deprivation is more often detected by univariate measures. However, this pattern reverses at the top of the education distribution: among individuals with postgraduate education, women are 40.5 percentage points more likely than similarly

educated men to fall below the TRPS while remaining above all univariate thresholds. This indicates that at higher education levels, gender disparities in multidimensional poverty arise less from absolute shortfalls and more from relative income–time trade-offs.

Taken together, these results demonstrate that trivariate poverty is both highly gendered and structurally heterogeneous. While women and Indigenous individuals face a higher overall risk of falling below the TRPS, the mechanisms through which this deprivation manifests differ systematically across education, ethnicity, and family structure.

	(1) Uni. income poor	(2) Uni. leisure poor	(3) Uni. unpaid work poor	(4) Under TRPS	(5) Under TRPS but above Uni
Children under 5	0.051*** (0.004)	0.015*** (0.003)	0.099*** (0.004)	0.158*** (0.005)	-0.102*** (0.007)
Children between 6 and 14	0.021*** (0.003)	0.028*** (0.003)	0.030*** (0.003)	0.132*** (0.004)	-0.030*** (0.007)
Age	-0.0005*** (0.0001)	-0.0005*** (0.0001)	-0.0002+ (0.0001)	-0.001*** (0.0002)	0.001*** (0.0003)
Preschool	-0.079+ (0.041)	0.162** (0.057)	-0.020 (0.027)	0.130* (0.053)	-0.033 (0.072)
Incomplete primary education	-0.064*** (0.008)	0.012* (0.006)	0.019** (0.006)	-0.050*** (0.009)	0.008 (0.014)
Complete primary education	-0.125*** (0.009)	0.019** (0.006)	0.030*** (0.006)	-0.093*** (0.010)	0.028+ (0.014)
Incomplete lower secondary education	-0.156*** (0.009)	0.015* (0.006)	0.024*** (0.007)	-0.154*** (0.010)	0.038* (0.016)
Complete lower secondary education	-0.210*** (0.014)	0.023 (0.020)	0.017 (0.018)	-0.289*** (0.027)	0.051 (0.069)
Upper secondary education (high school)	-0.198*** (0.010)	0.054*** (0.010)	0.024** (0.009)	-0.179*** (0.014)	-0.032 (0.023)
Technical or vocational education	-0.200*** (0.009)	0.036*** (0.007)	-0.002 (0.007)	-0.247*** (0.010)	0.033+ (0.017)
University degree (Bachelor's level)	-0.230*** (0.008)	0.075*** (0.015)	-0.030** (0.010)	-0.350*** (0.019)	0.006 (0.043)
Postgraduate education (Master's or PhD)	-0.230*** (0.008)	0.098** (0.035)	-0.009 (0.024)	-0.428*** (0.037)	-0.225* (0.095)
Indigenous	0.116*** (0.003)	0.012* (0.005)	-0.005 (0.005)	0.114*** (0.008)	-0.168*** (0.010)
Partner	0.097*** (0.003)	0.011*** (0.003)	0.047*** (0.003)	0.178*** (0.005)	-0.088*** (0.008)
Gender (Women)	0.045*** (0.003)	-0.007** (0.003)	0.167*** (0.002)	0.243*** (0.004)	-0.120*** (0.006)
Urban	-0.104*** (0.003)	0.001 (0.003)	-0.010*** (0.003)	-0.098*** (0.005)	0.093*** (0.007)
AIC	24 085.2	30 840.6	25 248.9	56 915.6	31 904.8
BIC	24 342.0	31 097.4	25 505.7	57 172.4	32 141.0
RMSE	0.28	0.28	0.28	0.44	0.48

+ p < 0.1, \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

Table 9: Average Marginal Effects; the last column is based on the observations under the TRPL only, i.e., conditioning on being below the line the entries display the variable's effect to be in Area 6.

Table 10: Gender differences by education, ethnicity, and children for logit (5)

Type	Group		Gender Difference		
	Category	Estimate	Std. Error		
No schooling		-0.024	0.025		
Preschool		0.100	0.151		
Incomplete primary education		-0.100	0.012	***	
Complete primary education		-0.173	0.011	***	
Incomplete lower secondary education		-0.151	0.015	***	
Complete lower secondary education		0.247	0.168		
Upper secondary education (high school)		0.020	0.042		
Technical or vocational education		-0.050	0.021	**	
University degree (Bachelor’s level)		0.060	0.087		
Postgraduate education (Master’s or PhD)		0.405	0.161	**	
Ethnicity	0	-0.124	0.007	***	
Ethnicity	1	-0.093	0.018	***	
Children under 5	0	-0.058	0.008	***	
Children under 5	1	-0.211	0.010	***	
Children age 6–14	0	-0.091	0.010	***	
Children age 6–14	1	-0.143	0.009	***	

#### 5.4 Binding constraints and ways out of the trivariate poverty

To characterize the structure of multidimensional deprivation among individuals classified as poor by the trivariate measure but not poor in any single dimension (Area 6), we decompose poverty into binding-constraint regimes using a minimal-relaxation approach. For each individual, we compute the smallest one-dimensional improvement required in each dimension to exit multidimensional poverty, holding the other dimensions constant. The binding constraint is defined as the dimension for which the minimal standardized improvement is smallest; ties are classified as mixed constraints. This procedure is conceptually related to measures binding constraints in multidimensional welfare analysis (e.g. Alkire and Foster, 2011a), but differs by explicitly identifying the locally binding dimension at the individual level. The bottleneck analysis directly informs policy design by identifying the smallest marginal change needed for individuals to escape multidimensional poverty. The bottleneck is not where deprivation is greatest, but where the least intervention can move individuals above the trivariate poverty surface, allowing policies to target locally binding constraints efficiently.

Table 11: Distribution of binding constraints in trivariate relative poverty (Area 6).

Bottleneck	Overall	Men	Women
income	34.1	45.2	28.7
time w/o unpaid work	21.7	12.4	31.9
leisure	20.3	22.1	18.4
mixed	23.9	20.3	21.0

The decomposition is displayed in Table 11 and reveals substantial heterogeneity in the sources of multidimensional deprivation. Table 11 shows that only about one third (34.1%) of individuals in Area 6 are primarily income-constrained, while nearly half face time-related bottlenecks through excessive unpaid work or insufficient leisure. These constraints are strongly gendered: unpaid work is the dominant bottleneck for women (31.9%), whereas men are more often income-constrained (45.2%). This implies that income-based interventions alone will leave a substantial share of multidimensional poverty unaddressed.

Reducing poverty requires coordinated policies that address income and time constraints simultaneously. Focusing on (female) labor force participation and hours of employment may shift deprivation from income to time constraints rather than alleviating poverty overall (Zacharias et al., 2012). For individuals constrained by unpaid work, effective policies must go beyond formal childcare and be directly accessible to women. Measures such as income support for caregivers, community kitchens, neighborhood-based care networks, and improved access to basic services can reduce time burdens while sustaining household provisioning. Importantly, many such time-saving interventions are compatible with ecological sustainability. Policies that shorten distances to essential services, support local provisioning, reduce commuting, and lower the time intensity of daily reproduction can simultaneously ease time pressure and reduce energy use, emissions, and material throughput (Smetschka et al., 2023; Biesecker and Hofmeister, 2010).

## 6 Conclusion

This paper advances the measurement of poverty by developing and applying a Bivariate Relative Poverty Line (BRPL) at the household level and a Trivariate Relative Poverty Surface (TRPS) at the individual level, jointly accounting for income, leisure, and unpaid work. Together, these approaches show that deprivation is driven less by isolated income shortfalls than by unsustainable income–time bundles, in which excessive unpaid work and constrained leisure prevent individuals and households from maintaining living standards above poverty in a sustainable way. This reframes poverty as a problem of how paid and unpaid work are organized and how time is allocated, with direct implications for policies aimed at sustainable livelihoods and well-being.

At the household level, the BRPL reveals a substantial degree of vulnerability that remains invisible under conventional poverty regimes. While the union of univariate income and unpaid-work thresholds classifies 24.11% of households as poor, an additional 17.89% lie above both univariate thresholds yet below the BRPL. These households are not poor in any single dimension, but their income–time bundles are insufficient when the interaction between unpaid work and income is accounted for. In contrast, only 1.26% of households fall below the strict intersection of univariate thresholds. This stark difference demonstrates that poverty at the household level is rarely characterized by simultaneous extreme shortfalls in both income and unpaid work. Rather it arises from unsustainable combinations of time and money that are more pronounced in rural than in urban areas. The BRPL also highlights that unpaid work plays a crucial role in sustaining household

consumption and living standards, thereby reinforcing the view that poverty depends on the sufficiency of combined time and income rather than on income alone. The nonlinearity of the BRPL implies that policies targeting income alone or time use alone may be insufficient to remain above the poverty threshold; households require coordinated improvements in both income and time conditions.

While unpaid work contributes to household living standards by supporting consumption and daily functioning, it simultaneously restricts individual well-being by limiting leisure and opportunities for paid work. These constraints are particularly relevant at the individual level and are central to understanding gendered vulnerability to poverty. To capture these dynamics, we construct the TRPS. At the individual level, the TRPS extends the household-level insights by incorporating leisure, thereby revealing the full extent of multidimensional deprivation arising from constrained combinations of income, unpaid work, and leisure time. Individual poverty in this paper is interpreted as relative deprivation in access to resources, not needs-based subsistence poverty. Because individuals in multi-person households do not command income independently, individual income is measured as access to household resources scaled by intra-household allocation shares. This perspective foregrounds intra-household inequality. Compared to the union of absolute poverty thresholds, which identifies 26.61% of individuals as poor, the TRPS captures an additional 48.80 percentage points who fall below the trivariate surface while remaining above all absolute univariate thresholds. This region is strongly gendered: 36.3% of men but 60.08% of women fall below the TRPS in this category, underscoring the pronounced gender asymmetry in multidimensional time and money deprivation. Strict trivariate intersections indicate that individuals rarely fall below absolute minima across all dimensions simultaneously. Poverty, therefore, is not a matter of subsistence-level deprivation but of constrained combinations of income and time.

The logit results based on the TRPS reveal a pronounced intersectional asymmetry in multidimensional poverty. Women have a significantly higher probability of falling below the trivariate relative poverty surface than men, even after controlling for age, education, ethnicity, household structure, and place of residence. Indigenous individuals and those with children also face markedly higher risks of trivariate poverty. Conditional on being below the TRPS, women are significantly less likely than men to be above all univariate poverty thresholds, indicating that their deprivation is more often already captured by at least one single dimension, particularly unpaid work. The gender gap is higher for non-Indigenous individuals compared to Indigenous individuals. At the same time, gender differences persist across education levels, but highly educated women are more likely than similarly educated men to experience trivariate poverty that is not detected by conventional univariate measures.

These findings show that poverty and sustainability cannot be assessed solely through monetary indicators, as unpaid work and leisure are fundamental dimensions of well-being and social provisioning. Excessive unpaid-work burdens and constrained leisure signal unsustainable patterns of work organization that undermine individuals' capacity to live above poverty, even in the absence of income poverty. Policies that focus exclusively on income growth or labor market participation risk reinforcing these pressures if they do not

simultaneously address time constraints, care responsibilities, and the unequal distribution of unpaid work.

By explicitly modeling the joint constraints between income and time, the BRPL and TRPS make visible forms of deprivation that are closely linked to the sustainability and resilience of livelihoods. When households and individuals operate near the limits of feasible income–time combinations, their capacity to absorb shocks—such as economic downturns, health crises, or environmental stresses—is reduced. Time scarcity and excessive unpaid work constrain not only individual well-being but also adaptive capacity, community care, and social resilience, which are central to long-term ecological sustainability. Overall, the paper demonstrates that moving beyond isolated thresholds toward a joint, multidimensional understanding of poverty is essential for designing policies that support sustainable living standards, gender equity, to foster social and ecological sustainability.

## References

- Ai, C. and Norton, E. C. (2003). Interaction terms in logit and probit models. *Economics Letters*, 80(1):123–129.
- Alkire, S. and Foster, J. (2011a). Counting and multidimensional poverty measurement. *Journal of public economics*, 95(7-8):476–487.
- Alkire, S. and Foster, J. (2011b). Understandings and misunderstandings of multidimensional poverty measurement. *The Journal of Economic Inequality*, 9(2):289–314.
- Alkire, S., Kanagaratnam, U., and Suppa, N. (2021). The global multidimensional poverty index (mpi) 2021. OPHI MPI Methodological Note 51, Oxford Poverty and Human Development Initiative (OPHI), University of Oxford, Oxford.
- Alkire, S. and Santos, M. E. (2013). A multidimensional approach: poverty measurement & beyond. *Social indicators research*, 112(2):239–257.
- Aloè, E. (2023). Time and income poverty measurement. an ongoing debate on the inclusion of time in poverty assessment. *Social Indicators Research*, 169(1):283–322.
- Antonopoulos, R. and Hirway, I. (2010). Unpaid work and the economy. In Antonopoulos, R. and Hirway, I., editors, *Unpaid Work and the Economy*, pages 1–57. Palgrave Macmillan, London.
- Bardasi, E. and Wodon, Q. (2010). Working long hours and having no choice: Time poverty in guinea. *Feminist Economics*, 16(3):45–78.
- Bentley, J. L. (1980). Multidimensional divide-and-conquer. *Communications of the ACM*, 23(4):214–229.
- Biesecker, A. and Hofmeister, S. (2010). Focus:(re) productivity: Sustainable relations both between society and nature and between the genders. *Ecological Economics*, 69(8):1703–1711.

- Chaudhuri, P. (1996). On a geometric notion of quantiles for multivariate data. *Journal of the American statistical association*, 91(434):862–872.
- Chiappero-Martinetti, E. and Von Jacobi, N. (2012). Light and shade of multidimensional indexes: How methodological choices impact on empirical results. In *Quality of Life in Italy: Research and Reflections*, pages 69–103. Springer, Dordrecht.
- CONEVAL (2026). Consejo nacional de evaluación de la política de desarrollo social. medición de la pobreza: Evolución de las líneas de pobreza por ingresos. [https://www.coneval.org.mx/Medicion/MP/Paginas/Lineas\\_Pobreza\\_Ingresos\\_Serie\\_1992-2021.aspx](https://www.coneval.org.mx/Medicion/MP/Paginas/Lineas_Pobreza_Ingresos_Serie_1992-2021.aspx). Retrieved 17.01.2026.
- Costanza, R., Kubiszewski, I., Giovannini, E., Lovins, H., McGlade, J., Pickett, K. E., Ragnarsdóttir, K. V., Roberts, D., De Vogli, R., and Wilkinson, R. (2014). Development: Time to leave gdp behind. *Nature*, 505(7483):283–285.
- Decancq, K. (2014). Copula-based measurement of dependence between dimensions of well-being. *Oxford Economic Papers*, 66(3):681–701.
- Decancq, K. and Lugo, M. A. (2012). Inequality of wellbeing: A multidimensional approach. *Economica*, 79(316):721–746.
- Dorn, F. (2021). *It's Time: Poverty, Inequality and Sustainability beyond the Mean*. PhD thesis, Georg-August-Universität Göttingen. DOI: <https://doi.org/10.53846/goediss-9107>.
- Dorn, F. (2025). Time and income poverty: Definitions, thresholds and tradeoffs. Technical report, ifso working paper.
- Dorn, F. (2026). Thresholds for time and income poverty in households: Evidence from joint distributions. ifso Working Paper 60, Institute for Socioeconomics, University of Duisburg–Essen.
- Dorn, F. and Folbre, N. (2022). Income and time poverty: Definitions, thresholds and tradeoffs. Paper prepared for the 37th IARIW General Conference.
- Dorn, F., Radice, R., Marra, G., and Kneib, T. (2024). A bivariate relative poverty line for leisure time and income poverty: Detecting intersectional differences using distributional copulas. *Review of Income and Wealth*, 70(2):395–419.
- Ferrant, G., Pesando, L. M., and Nowacka, K. (2014). Unpaid care work: The missing link in the analysis of gender gaps in labour outcomes. *Boulogne Billancourt: OECD Development Center*.
- Ferreira, F. H. and Lugo, M. A. (2013). Multidimensional poverty analysis: Looking for a middle ground. *The World Bank Research Observer*, 28(2):220–235.
- Folbre, N. (2001). *The Invisible Heart*. The New Press, New York.

- Folbre, N. and Bittman, M. (2004). *Family time: The social organization of care*, volume 2. Routledge IAFFE advances in feminist economics, Routledge, London and New York.
- Folbre, N., Murray-Close, M., and Suh, J. (2018). Equivalence scales for extended income in the us. *Review of Economics of the Household*, 16(2):189–227.
- García-Gómez, C., Pérez, A., and Prieto-Alaiz, M. (2021). Copula-based analysis of multivariate dependence patterns between dimensions of poverty in europe. *Review of Income and Wealth*, 67(1):165–195.
- Gautham, L. and Folbre, N. (2022). Parental expenditures of time and money on children in the us. *Review of Income and Wealth*, 70(4):1011–1036.
- Goodin, R. E., Rice, J. M., Bittman, M., and Saunders, P. (2005). The time-pressure illusion: Discretionary time vs. free time. *Social indicators research*, 73(1):43–70.
- Guggisberg, M. (2023). A bayesian approach to multiple-output quantile regression. *Journal of the American Statistical Association*, 118(544):2736–2745.
- Hallin, M., Paindaveine, D., and Šiman, M. (2010). Multivariate quantiles and multiple-output regression quantiles: From  $l_1$  optimization to halfspace depth. *The Annals of Statistics*, 38(2):635–669.
- Herwartz, H. and Maxand, S. (2020). Nonparametric tests for independence: a review and comparative simulation study with an application to malnutrition data in india. *Statistical Papers*, 61:2175–2201.
- INEGI (2018). Encuesta nacional de ingresos y gastos de los hogares (ENIGH). Instituto Nacional de Estadística y Geografía, Website: [www.inegi.org.mx/programas/enigh/nc/2018/](http://www.inegi.org.mx/programas/enigh/nc/2018/) retrieved 17.05.2020.
- Insolera, N. E., Johnson, D. S., and Simmert, B. A. (2019). Evaluation of the time use data in psid with comparisons to atus. Technical Report 19-02, Institute for Social Research, University of Michigan.
- Jackson, T. (2017). *Prosperity without Growth: Foundations for the Economy of Tomorrow*. Routledge, London and New York, 2 edition.
- Klasen, S. and Lahoti, R. (2021). How serious is the neglect of intra-household inequality in multidimensional poverty and inequality analyses? evidence from india. *Review of Income and Wealth*, 67(3):705–731.
- Klein, N. and Kneib, T. (2020). Directional bivariate quantiles: a robust approach based on the cumulative distribution function. *AStA Advances in Statistical Analysis*, 104(2):225–260.
- Klugman, J. (2010). *Human Development Report 2010: 20th Anniversary Edition. The Real Wealth of Nations: Pathways to Human Development*. Palgrave Macmillan, Houndmills, Basingstoke and New York.
- Koenker, R. (2005). *Quantile Regression*, volume 38 of *Econometric Society Monographs*. Cambridge University Press, UK.

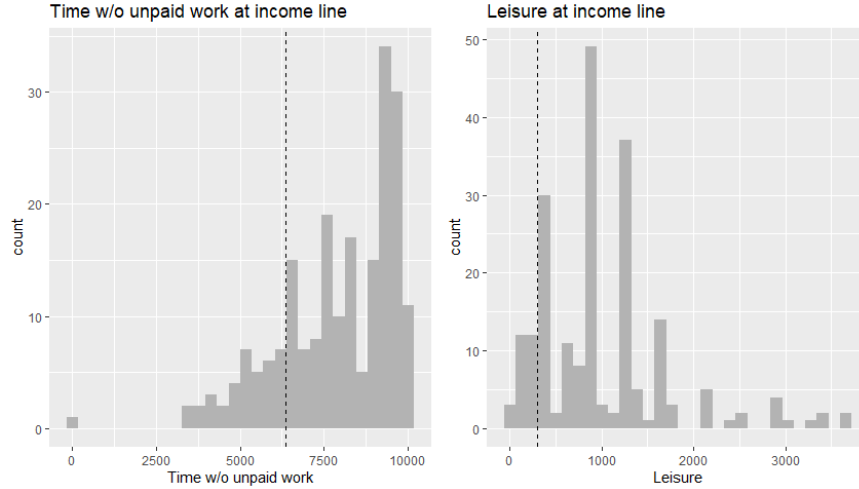
- Kong, L. and Mizera, I. (2012). Quantile tomography: using quantiles with multivariate data. *Statistica Sinica*, 22:1589–1610.
- Langrené, N. and Warin, X. (2021). Fast multivariate empirical cumulative distribution function with connection to kernel density estimation. *Computational Statistics & Data Analysis*, 162:107267.
- Lee, D. and Joe, H. (2018). Efficient computation of multivariate empirical distribution functions at the observed values. *Computational Statistics*, 33(3):1413–1428.
- Lyon, S., Mutersbaugh, T., and Worthen, H. (2017). The triple burden: The impact of time poverty on women’s participation in coffee producer organizational governance in mexico. *Agriculture and Human Values*, 34(2):317–331.
- Merz, J. and Rathjen, T. (2014). Time and income poverty: An interdependent multidimensional poverty approach with german time use diary data. *Review of Income and Wealth*, 60(3):450–479.
- Mukhopadhyay, N. D. and Chatterjee, S. (2011). High dimensional data analysis using multivariate generalized spatial quantiles. *Journal of multivariate analysis*, 102(4):768–780.
- Narayan, D., Chambers, R., Shah, M. K., and Patesch, P. (2000). *Voices of the Poor: Crying out for Change*. New York: Oxford University Press for the World Bank.
- OHCHR (2022). *About Extreme Poverty and human rights*. Office of the United Nations High Commissioner for Human Rights. <https://www.ohchr.org/EN/AboutUs/Pages/ContactUs.aspx>. Retrieved 18.01.2022.
- Pedrero Nieto, M. (2005). *Trabajo doméstico no remunerado en Méxicouna estimación de su valor económico a través de la encuesta nacional sobre uso del tiempo 2002*. Instituto Nacional de las Mujeres, México, D.F.
- Pfister, N., Bühlmann, P., Schölkopf, B., and Peters, J. (2017). Kernel-based tests for joint independence. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 80(1):5–31.
- Pogge, T. W. (2002). Can the capability approach be justified? *Philosophical topics*, 30(2):167–228.
- Ravallion, M. (2011). On multidimensional indices of poverty. *World Bank Policy Research Working Papers no. 5580*.
- Raworth, K. (2017). *Doughnut Economics: Seven Ways to Think Like a 21st-Century Economist*. Random House Business, London.
- Reid, M. G. (1934). *Economics of Household Production*. New York, JohnWiley and Sons.
- Rodin, D. L., McNeill, K., Vite-Leon, N., and Heymann, J. (2012). Determinants of informal employment among working mothers in mexico. *Community, Work & Family*, 15(1):85–99.

- Schor, J. B. (2005). Sustainable consumption and worktime reduction. *Journal of Industrial Ecology*, 9(1–2):37–50.
- Schor, J. B. (2010). *Plenitude: The New Economics of True Wealth*. Penguin Press, New York.
- Scott, D. W. (1992). *Multivariate density estimation: Theory, Practice, and Visualization*. John Wiley & Sons, New York.
- Scott, D. W. (2012). Multivariate density estimation and visualization. In *Handbook of computational statistics*, pages 549–569. Springer.
- Segrest, S. L., Romero, E. J., and Domke-Damonte, D. J. (2003). Exploring the role of machismo in gender discrimination: A comparison of Mexico and the US. *Equal Opportunities International*, 22(1):13–31.
- Sen, A. (1976). Poverty: an ordinal approach to measurement. *Econometrica*, 44(2):219–231.
- Sen, A. (1987). Gender and cooperative conflicts. *WIDER Working Papers 18*.
- Serfling, R. (2002). Quantile functions for multivariate analysis: approaches and applications. *Statistica Neerlandica*, 56(2):214–232.
- Smetschka, B., Gaube, V., and Mader, K. (2023). Time to care—care for time—how spending more time for care than consumption helps to mitigate climate change. *Frontiers in Sustainability*, 4:1070253.
- Tkach, K. and Gigliarano, C. (2020). Multidimensional poverty index with dependence-based weights. *Social Indicators Research*, pages 1–30.
- UN (2004). *Human Rights and Poverty Reduction: A Conceptual Framework*. United Nations. New York and Geneva, <https://www.ohchr.org/Documents/Publications/PovertyReductionen.pdf>. Retrieved 18.01.2022.
- UNDP (2021). *Human Development Report 2020: The Next Frontier—human Development and the Anthropocene*. United Nations Development Programme, New York.
- Vickery, C. (1977). The time-poor: a new look at poverty. *Journal of Human Resources*, 12(1):27–48.
- Zacharias, A., Antonopoulos, R., and Masterson, T. (2012). Why time deficits matter: Implications for the measurement of poverty. *Informe de Proyecto de Investigación. Programa de las Naciones Unidas para el Desarrollo (UNDP)/Levy Economics Institute of Bard College*.

## A Appendix

### A.1 Additional figures

Figure 8: Histograms of poverty dimensions along the univariate income poverty line



*Note:* Individuals are selected within a  $\pm 1\%$  band around the poverty line, and vertical dashed lines indicate the corresponding poverty threshold in the displayed dimension.

## A.2 Implementation of the Divide-and-Conquer Algorithm

The algorithm is applicable to data of arbitrary dimension  $d$ , but its computational cost increases rapidly with dimensionality because natural ordering becomes more complex (Bentley, 1980). Let  $T(N, d)$  denote the running time required to compute the ECDF for  $N$  observations in  $d$  dimensions using the divide-and-conquer dominance counting algorithm. The recursion for the running time is given by

$$T(N, d) = O(N) + 2T(N/2, d) + T(N, d - 1),$$

where the first term corresponds to splitting the data after sorting by the first dimension, the second term reflects the cost of solving the two subproblems of size  $N/2$ , and the last term accounts for recursively solving the dominance problem in the remaining  $d - 1$  dimensions. Solving this recursion yields

$$T(N, d) = O\left(N \log^{d-1} N\right),$$

implying that the algorithm runs in near-linear time for fixed  $d$ . In particular, for three dimensions the complexity is  $T(N, 3) = O(N \log^2 N)$ .

Intuitively, the algorithm proceeds by sorting the observations along the first dimension and splitting them into two halves. Observations in the lower half cannot dominate observations in the upper half in that dimension, so dominance counting is only required across the remaining  $d - 1$  dimensions. Each subproblem is solved recursively and the partial results are then combined, yielding a substantial speed-up relative to naive pairwise comparison, which would require  $O(N^2)$  operations.

In this paper, we implement the algorithm for data of dimension  $d = 3$ . An approach for computing the joint ECDF for three variables is given by Lee and Joe (2018). A generalization to higher dimensions  $d \geq 2$

Figure 9: Histograms of poverty dimensions along the univariate unpaid work poverty line

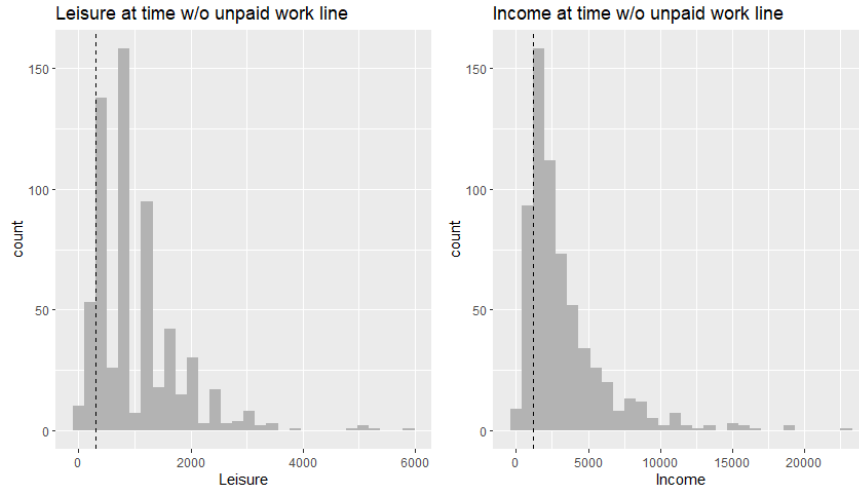
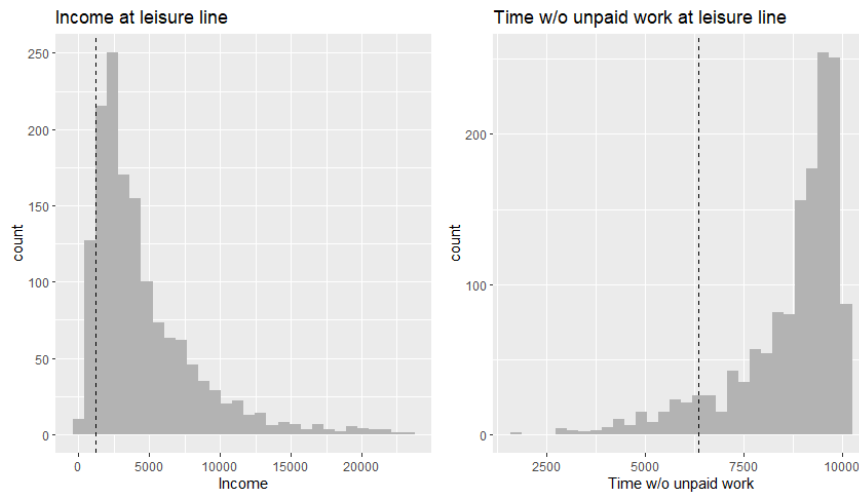


Figure 10: Histograms of poverty dimensions along the univariate leisure poverty line



for the divide-and-conquer algorithm (also called merge sort algorithm) is given by Langrené and Warin (2021). Details are given in their supplementary material. We consider three variables  $Y = (Y_1, Y_2, Y_3)$  with different units, i.e., a continuous monetary variable (Mexican pesos) and two variables measuring time in minutes (leisure  $Y_2$  and unpaid work  $Y_3$ ), which include self-reported rounded values. Because time-use data are not absolutely continuous, ties occur frequently, especially for  $Y_2$  and  $Y_3$ . The algorithm explicitly accounts for such ties. The key idea is that ties only matter within the same dimension. If ties occur in the second or third dimension, the algorithm still works because observations are first strictly ordered by the continuous variable  $Y_1$ . Once two observations differ in  $Y_1$ , they can no longer dominate each other, so ties in later dimensions do not create ambiguity. If ties occur in the first dimension, the second dimension is used as a tie-breaker, following the procedure in Lee and Joe (2018). By using income as the first (continuous) dimension, we avoid computational problems caused by ties in the time-use variables, ensuring that the joint

ECDF can be computed efficiently and reliably.

The algorithm computes the joint ECDF using a sequence of sorting and splitting steps. To reduce runtime, we first preprocess the data by sorting all observations according to the first dimension  $Y_1$  (income). Each observation is assigned a unique index so that its rank can be tracked throughout the algorithm. The remaining dimensions  $Y_2$  and  $Y_3$  are then handled recursively. At each recursion step, the data are split into two subsets according to the current dimension: a *left* set containing smaller values and a *right* set containing larger values. Because all observations in the left set are already smaller in the previous dimension, they cannot dominate observations in the right set. This allows the dominance problem to be reduced to smaller and simpler subproblems. For each observation, the algorithm counts how many other observations dominate it across all dimensions. Auxiliary vectors are used to keep track of which observations are eligible for counting and to store the number of dominating points. These counts are updated recursively and then merged back together. Once the recursion is complete, the algorithm returns a vector of dominance counts. Dividing these counts by the total number of observations yields the ECDF, ensuring that every observation contributes to the final distribution, even if it does not dominate any other observation.

The following Algorithms 1 to 4 are based on the multi-dimensional divide-and-conquer algorithm by Bentley (1980) and the trivariate approach by Lee and Joe (2018). The implementation of the algorithm was conducted in R.

---

**Algorithm 1** Divide-and-Conquer Algorithm - Sort Function

---

**Require:** Variables for sorting  $\mathbf{Y} = (Y_2, Y_3)$  with  $Y_i = (Y_{i1}, \dots, Y_{iN})$  and  $N$  the number of observations, index for observations  $ind$ , counter for left (less) elements  $count\_left$ , variable determining if variable is eligible for counting  $eligible\_count$

**Initialize:** $N \leftarrow \text{Numberofobservations}$  $count\_left \leftarrow \text{vector}(0, \text{length} = N)$  $eligible\_count \leftarrow \text{vector}(0, \text{length} = N)$  $ind \leftarrow 1 : N$ FUNCTION **Sort Function**( $x, y, xind$ ) $N \leftarrow \text{length}(x)$ **IF**  $N = 1$  **THEN****RETURN**  $list(x = x, y = y, xind = xind)$ **ELSE** $m \leftarrow N/2$  $u_1 \leftarrow \text{Sort Function}(x[1 : m], y[1 : m], xind[1 : m])$  $u_2 \leftarrow \text{Sort Function}(x[(m + 1) : N], y[(m + 1) : N], xind[(m + 1) : N])$  $u \leftarrow \text{Merge Function}(u_1\$x, u_2\$x, u_1\$y, u_2\$y, u_1\$xind, u_2\$xind)$ **RETURN**  $list(x = u\$x, xind = u\$xind, y = u\$y, yind = u\$yind)$ **END IF**

---

---

**Algorithm 2** Divide-and-Conquer Algorithm - Merge Function

---

**Require:** Output of **Sort Function**

```
FUNCTION Merge Function( $x_1, x_2, y_1, y_2, xind_1, xind_2$ )
 $N_1 \leftarrow \text{length}(x_1); N_2 \leftarrow \text{length}(x_2); N \leftarrow N_1 + N_2$ 
 $i \leftarrow j \leftarrow k \leftarrow 1; x \leftarrow y \leftarrow xind \leftarrow \text{vector}(0, \text{length} = N)$ 
WHILE  $i \leq N_1$  AND  $j \leq N_2$  DO
    IF  $x_1[i] < x_2[j]$  THEN
         $x[k] \leftarrow x_1[i]; y[k] \leftarrow y_1[i]; xind[k] \leftarrow xind_1[i]$ 
         $\text{eligible\_count}[xind_1[i]] \leftarrow 1$  {LEFT ELEMENT IS ELIGIBLE FOR COUNT (=1)}
         $i \leftarrow i + 1$ 
    ELSE
         $x[k] \leftarrow x_2[j]; y[k] \leftarrow y_2[j]; xind[k] \leftarrow xind_2[j]$ 
         $\text{eligible\_count}[xind_2[j]] \leftarrow 0$  {RIGHT ELEMENT IS NOT ELIGIBLE FOR COUNT (=0)}
         $j \leftarrow j + 1$ 
    END IF
     $k \leftarrow k + 1$ 
END WHILE
IF  $i \leq N_1$  THEN
    {THE ELEMENTS LEFT IN THE LEFT VECTOR ARE USED TO FILL UP THE RESULTING VECTORS  $x$ ,  $y$ 
    AND  $xind$ . THEY ARE ELIGIBLE FOR COUNTING.}
     $x[k : N] \leftarrow x_1[i : N_1]; y[k : N] \leftarrow y_1[i : N_1]; xind[k : N] \leftarrow xind_1[i : N_1]$ 
     $\text{eligible\_count}[xind_1[i : N_1]] \leftarrow 1$ 
END IF
IF  $j \leq N_2$  THEN
    {THE ELEMENTS LEFT IN THE RIGHT VECTOR ARE USED TO FILL UP THE RESULTING VECTORS  $x$ ,
     $y$  AND  $xind$ . THEY ARE NOT ELIGIBLE FOR COUNTING.}
     $x[k : N] \leftarrow x_2[j : N_2]; y[k : N] \leftarrow y_2[j : N_2]; xind[k : N] \leftarrow xind_2[j : N_2]$ 
     $\text{eligible\_count}[xind_2[j : N_2]] \leftarrow 0$ 
END IF
 $y \leftarrow \text{Sort Inner Function}(y, xind)$ 
RETURN  $\text{list}(x = x, y = y\$y, xind = xind, yind = y\$yind)$ 
```

---

---

**Algorithm 3** Divide-and-Conquer Algorithm - Sort Inner Function

---

**Require:** Output of Merge Function

```
FUNCTION Sort Inner Function( $y, yind$ )
 $L \leftarrow \text{length}(y)$ 
IF  $L = 1$  THEN
    RETURN  $\text{list}(y = y, yind = yind)$ 
ELSE
     $m \leftarrow L/2$ 
     $v_1 \leftarrow \text{sort\_inner\_fct}(y[1 : m], yind[1 : m])$ 
     $v_2 \leftarrow \text{sort\_inner\_fct}(y[(m + 1) : L], yind[(m + 1) : L])$ 
     $v \leftarrow \text{merge\_inner\_fct}(v_1\$y, v_2\$y, v_1\$yind, v_2\$yind)$ 
    RETURN  $v$  {RETURNS A LIST OF Y AND YIND.}
END IF
```

---

---

**Algorithm 4** Divide-and-Conquer Algorithm - Merge Inner Function

---

**Require:** Output of Sort Inner Function

```
FUNCTION Merge Inner Function( $y_1, y_2, yind_1, yind_2$ )
 $L_1 \leftarrow \text{length}(y_1); L_2 \leftarrow \text{length}(y_2); L \leftarrow L_1 + L_2$ 
 $i \leftarrow j \leftarrow k \leftarrow 1; \text{count\_left\_helper} \leftarrow 0; y \leftarrow yind \leftarrow \text{vector}(0, \text{length} = L)$ 
WHILE  $i \leq L_1$  AND  $j \leq L_2$  DO
  IF  $y_1[i] < y_2[j]$  THEN
     $y[k] \leftarrow y_1[i]; yind[k] \leftarrow yind_1[i]$ 
    IF  $\text{eligible\_count}[yind_1[i]] = 1$  THEN
      {IF THE LEFT ELEMENT WAS ELIGIBLE FOR COUNTING BEFORE, THAN IT IS ADDED TO THE
      COUNT NOW.}
       $\text{count\_left\_helper} \leftarrow \text{count\_left\_helper} + 1$ 
    END IF
     $i \leftarrow i + 1$ 
  ELSE
     $y[k] \leftarrow y_2[j]; yind[k] \leftarrow yind_2[j]$ 
    IF  $\text{eligible\_count}[yind_2[j]] = 0$  THEN
       $\text{count\_left}[yind_2[j]] \leftarrow \text{count\_left}[yind_2[j]] + \text{count\_left\_helper}$ 
    END IF
     $j \leftarrow j + 1$ 
  END IF
   $k \leftarrow k + 1$ 
END WHILE
IF  $i \leq L_1$  THEN
   $y[k : L] \leftarrow y_1[i : L_1]; yind[k : L] \leftarrow yind_1[i : L_1]$ 
END IF
IF  $j \leq L_2$  THEN
   $y[k : L] \leftarrow y_2[j : L_2]; yind[k : L] \leftarrow yind_2[j : L_2]$ 
  FOR  $ind \in yind_2[j : L_2]$  DO
    IF  $\text{eligible\_count}[ind] = 0$  THEN
       $\text{count\_left}[ind] \leftarrow \text{count\_left}[ind] + \text{count\_left\_helper}$ 
    END IF
  END FOR
END IF
RETURN  $\text{list}(y = y, yind = yind)$ 
```

---