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The Deferred Acceptance (DA) algorithm is stable and strategy-proof, but can produce matchings that are Pareto-inefficient for students, and thus several alternatives have been proposed to correct this inefficiency that only involve consented priority violations. However, we show that these approaches cannot correct DA's suboptimal rank distribution, because this shortcoming can arise even in cases where DA is Pareto-efficient.

We also examine student segregation in settings with tiered priority structures. We prove that the demographic composition of every school is perfectly preserved under any Pareto-efficient rule that dominates DA, and consequently fully segregated schools under DA maintain their extreme homogeneity.

KEYWORDS. school choice, rank-efficiency, Rawlsian welfare, segregation.

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1. INTRODUCTION

School choice systems shape the educational opportunities of millions of students worldwide. Among these, the student-proposing Deferred Acceptance (DA) algorithm has emerged as the gold standard, adopted by major cities from New York to Paris. Its appeal lies in two compelling properties: it produces the unique student-optimal stable matching (SOSM) while incentivizing parents to truthfully report their preferences. Yet these virtues come at a cost—DA often generates Pareto-inefficient outcomes, leaving some students worse off than necessary. This has been observed in theoretical and empirical studies alike (e.g. [Abdulkadiroğlu and Sönmez, 2003](#), [Abdulkadiroğlu et al., 2009](#), [Kesten, 2010](#)).¹

DA's Pareto-inefficiency has sparked a growing literature in market design, leading to the construction of sophisticated algorithms that improve students' welfare compared to DA's baseline while violating only the priorities of consenting students (starting with the seminal work of [Kesten \(2010\)](#)). The point of this paper is that such efficiency-adjusted alternatives may not correct other problems of the SOSM that have received less attention in the literature: namely their poor rank distribution and segregation. By poor rank distribution, we mean two related but distinct problems: rank-inefficiency (poor average ranks) and poor Rawlsian welfare (poor ranks for the worst-off students). This concern is underscored by the observation that Pareto-efficiency may be “*a very limited kind of success, and in itself may or may not guarantee much*” ([Sen, 1999](#), p.31–32).

A Motivating Example. To illustrate the poor rank distribution problem that we highlighted, consider the school choice problem in [Example 1](#) below with six students (i_1, \dots, i_6) and six schools (s_1, \dots, s_6) , each with unit capacity.

The unique stable allocation appears in squares, with every student assigned to the school with the same index. Three students are assigned to schools in the bottom half of their preferences, with student i_6 assigned to his least-preferred school. Note that this allocation is already Pareto-efficient. Interestingly, an alternative Pareto-efficient allocation, in circles, is remarkably more compelling because of two important reasons:

1. *it is more rank-efficient*: the median student gets assigned to his first choice, rather than to a school in the middle of his preference list, with five out of six students preferring the allocation in circles.

¹Following the standard approach in school choice, our analysis adopts a student-centric welfare perspective, treating priorities as policy constraints rather than preferences. This justifies analyzing efficiency improvements that involve consented priority violations; see [Section 3](#) for a discussion.

EXAMPLE 1. SOSM in squares.

i_1	i_2	i_3	i_4	i_5	i_6	s_1	s_2	s_3	s_4	s_5	s_6
s_1	s_1	s_2	s_3	s_4	s_5	i_1	i_2	i_3	i_4	i_5	\cdot
s_6	s_2	s_1	s_1	s_1	s_1	i_2	i_3	i_4	i_5	i_6	\cdot
\cdot	\cdot	s_3	s_2	s_2	s_2	i_3	i_4	i_5	i_6	\cdot	\cdot
\cdot	\cdot	\cdot	s_4	s_3	s_3	i_4	i_5	i_6	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot	s_5	s_4	i_5	i_6	\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot	\cdot	s_6	i_6	\cdot	\cdot	\cdot	\cdot	\cdot

2. *it has a higher Rawlsian welfare*: the worst-off student is assigned to his second-ranked school, unlike in the SOSM where some student ends up in his worst-ranked school.

Interestingly, because the SOSM was Pareto-efficient in our previous example, any Pareto-efficient rule that weakly Pareto dominates DA—a class we henceforth refer to as *stable-dominating*—also selects the same allocation (and since there is a unique stable matching, this holds for every Pareto-efficient rule that weakly dominates *any* stable rule).² Thus, the previous example illustrates a concern of stable and stable-dominating rules: they may select Pareto-efficient allocations with poor rank distributions when alternative Pareto-efficient allocations are more appealing on both rank-efficiency and Rawlsian welfare dimensions.³ In Theorem 1, we generalize the previous example to provide tight bounds quantifying the worst rank-efficiency and Rawlsian welfare generated by stable and stable-dominating rules.

The Persistency of Segregation. A related concern is DA's tendency to produce segregated allocations, where students with lower priority are concentrated in schools with predominantly low-achieving peers (as in Chile (Kutscher et al., 2023) and the UK (Terrer et al., 2021)). This observation raises a second research question: can the efficiency improvements over DA foster greater school diversity?

²The stable-dominating terminology was introduced by Alva and Manjunath (2019), who proved important results for this large class of rules, though their study goes back to the seminal work of Kesten (2010). Alva and Manjunath's definition only requires that a stable-dominating matching weakly Pareto-dominates some stable matching, not necessarily the student-optimal one, and does not require Pareto-efficiency. We impose the stronger restriction motivated by our focus on rules that improve on student-proposing DA, which is the status quo in many school districts.

³This flaw is not exclusive of stable-dominating rules: the top trading cycle (TTC) algorithm, in which students are endowed with ownership of schools at which they have top priority, also selects the SOSM in our example.

Our analysis in Theorem 2 reveals limitations of stable-dominating rules for reducing segregation. We focus on tiered priority settings—where students are organized into priority tiers with complete separation across all schools. Such tier structures arise in practice: looked-after children in the UK and citizens in Singapore receive universally higher priority at all schools. Under these conditions, we prove that the demographic composition of every school is perfectly preserved under any stable-dominating rule. This invariance occurs because students can only trade positions within their own tier, never across them. Consequently, any school that is fully segregated under DA, admitting exclusively students from one tier only, remains segregated under any Pareto-efficient rule that dominates DA. More strikingly, even mixed schools maintain their exact proportion of students from each tier, as students from better tiers at such schools cannot be displaced through any efficiency-improving trades.

2. RELATED LITERATURE

Stable-Domination. A growing literature has studied and developed algorithms that Pareto-dominate DA from the student perspective. This focus on student preferences in defining Pareto-efficiency reflects a normative stance specific to school choice markets. Unlike other two-sided markets such as marriage or doctor-hospital matching, where both sides' preferences matter for social welfare, school choice involves students as the primary welfare recipients. School priorities are policy instruments chosen by districts to achieve social objectives, not welfare-relevant preferences in themselves. This perspective justifies treating priority violations as acceptable when they benefit students, particularly when the students whose priorities are violated consent to such violations.

The Efficiency-Adjusted Deferred Acceptance (EADA) algorithm (Kesten, 2010) stands as the most prominent mechanism in this literature, allowing controlled priority violations among consenting students who agree to waive their priority rights. EADA satisfies several refined stability concepts including sticky stability (Afacan et al., 2017), essentially stability (Trojan et al., 2020), weak-stability (Tang and Zhang, 2021) and priority-neutrality (Reny, 2022), as well as weak strategy-proof properties such as non-obvious manipulability (Trojan and Morrill, 2020), regret-free truth-telling (Chen and Moller, 2023) and behavioural strategy-proofness (Shirakawa, 2025), demonstrating the theoretical sophistication possible within this framework.

Beyond EADA, this class includes Top Trading Cycles using DA's allocation as endowment, the Maximum Improvement over DA (MIDA) algorithm (Knipe and Ortega, 2025), and various approaches within the Student Exchange under Partial Fairness (SEPF)

framework (Dur et al., 2019). The practical appeal of these algorithms is evidenced by EADA's scheduled implementation in Flanders (Cerrone et al., 2024).

Our analysis examines whether these Pareto improvements can also address other distributional concerns about DA allocations, particularly regarding rank-efficiency, Rawlsian welfare, and segregation. There is a caveat, however. It is well-known that DA is constrained-efficient, meaning that any matching that Pareto-dominates it must necessarily violate the priority of some student. Thus, every stable-dominating rule *must* violate the priority of some student. The key in this literature is to either i) violate the priority of a student who consented to such violation, or ii) violate the priority of students who could not have benefited from a better placement under any matching that harms nobody.

Rank-Efficiency. The distribution of ranks obtained in matching markets is of particular interest to a large literature in random matching markets as a measure of efficiency (e.g. Knuth, 1976), and has gathered particular interest since the work of Featherstone (2020).⁴ The average rank under DA significantly exceeds (i.e. is worse) that obtained by the rank-minimizing algorithm, and this occurs in both random markets and a dataset from the Hungarian high school allocation (Ortega and Klein, 2023). However, DA's average rank is roughly the same as that obtained by other Pareto-efficient rules such as TTC and random serial dictatorship. Pycia (2019) provides a detailed overview of the literature and a theoretical explanation. Regarding stable-dominating rules, Diebold and Bichler (2017) and Ortega and Klein (2023) have examined the rank-efficiency on stable-dominating rules in practice, that while better than DA's, remains far from that attained by the first best.

Rawlsian welfare. DA can produce highly unequal allocations, with the worst-off student receiving significantly worse placements than under Pareto-efficient alternatives (Galichon et al., 2023). Stable-dominating rules cannot improve the placement of poorly matched students, and thus are limited in the extent to which they can improve the placement of the students who need said improvement most (Tang and Yu, 2014, Alva and Manjunath, 2019, Ortega et al., 2025). Efficiency-adjustments over DA such as EADA

⁴Featherstone (2020) defines a matching as rank-efficient if the corresponding rank distribution cannot be stochastically dominated. We use a weaker version of this notion based on the associated average ranks. This allows us to compare the rank-efficiency of different rules that are not rank-efficient in the Featherstone definition, and thus incomparable. See Feizi (2024) for a discussion.

can increase variance in the rank distribution, yet reducing the corresponding Gini index (Cerrone et al., 2024). Nevertheless, our companion paper shows that almost all students could trade their DA placement to be assigned to a more desirable school in large random markets, so that even when Rawlsian welfare does not improve, the welfare of most students improves under any efficient stable-dominating rule (Ortega et al., 2025). The focus on inequality in assignment problems and the search for allocations that are Lorenz dominant goes back to the seminal work of Bogomolnaia and Moulin (2004) and Pycia and Ünver (2017).

Segregation. Empirical evidence from England and Chile shows DA can worsen segregation: in England, switching to DA harmed low-SES students who attended schools with lower value-added and more disadvantaged peers (Terrier et al., 2021); in Chile, DA increased within-school segregation in areas with high residential segregation or many private schools (Kutscher et al., 2023). More broadly, school choice policies can significantly increase segregation, with every 3% increase in choice availability affecting over 564,000 US children (Ukanwa et al., 2022). However, alternative algorithms using minority reserves can reduce segregation without harming efficiency: simulations of Swedish school choice show that reserved seats reduce segregation while maintaining high shares of students assigned to preferred schools (Kessel and Olme, 2018), while in Chile such reserves would reduce segregation while improving rank-efficiency (Escobar and Huerta, 2025). Theoretically, Calsamiglia and Miralles (2023) show how DA can limit disadvantaged students' access to good schools, albeit it being immune to a type of segregation that arises due to heterogeneous risk attitudes (Calsamiglia et al., 2021).

Our Contribution. While the literature establishes that DA suffers from poor rank distribution and high segregation, our contribution is proving that stable-dominating rules cannot eliminate these shortcomings. This highlights limitations of Pareto-efficiency reforms in addressing distributional concerns.

3. MODEL

Following Abdulkadiroğlu and Sönmez (2003), a school choice problem P consists of a finite set of students I and a finite set of schools S . Each student i has a strict preference \succ_i over the schools. Each school s has a quota of available seats q_s and a strict priority over the students \triangleright_s , determined by local educational regulations. For simplicity, we assume that $\sum_{s \in S} q_s = |I|$. We use $m = |I|$ and $n = |S|$ to denote the number of students and schools, respectively. Let $\mathcal{P}_{m,n}$ denote the set of all such school choice problems.

For a given school choice problem P , a *matching* μ is a mapping from I to S such that no school is matched to more students than its quota. We denote by μ_i the school to which student i is assigned and by μ_s^{-1} the set of students assigned to school s . With this notation, a matching needs to satisfy $|\mu_s^{-1}| \leq q_s$ for every $s \in S$.

The function $\text{rk}_i : S \rightarrow \{1, \dots, n\}$ specifies the rank of school s according to the preference profile \succ_i of student i , i.e.

$$\text{rk}_i(s) = |\{s' \in S : s' \succ_i s\}| + 1,$$

so that the most desirable option gets a rank of 1, whereas the least desirable gets a rank of n . With some abuse of notation, we use the same rank function to specify the students' rank as per the priority profile of schools, i.e. $\text{rk}_s(i) = |\{j \in I : j \triangleright_s i\}| + 1$.

A matching μ *weakly Pareto-dominates* matching ν if, for every student $i \in I$, $\text{rk}_i(\mu_i) \leq \text{rk}_i(\nu_i)$. A matching μ *Pareto-dominates* matching ν if μ weakly dominates ν and there exists a student $j \in I$ with $\text{rk}_j(\mu_j) < \text{rk}_j(\nu_j)$. A matching is *Pareto-dominated* if there exists a matching that Pareto-dominates it and is *Pareto-efficient* if it is not Pareto-dominated.

A strictly stronger efficiency notion is that of rank-efficiency. A matching μ is *rank-efficient* if there does not exist an alternative matching ν such that $\sum_{i \in I} \text{rk}_i(\nu_i) < \sum_{i \in I} \text{rk}_i(\mu_i)$. Every rank-efficient matching is Pareto-efficient but the converse is not true.

Student i *desires* school s in matching μ if $\text{rk}_i(s) < \text{rk}_i(\mu_i)$. We say that student j *violates* student i 's priority at school s in matching μ if i desires s , $\mu_j = s$, and $\text{rk}_s(i) < \text{rk}_s(j)$. A matching μ is *non-wasteful* if every school s that is desired by some student in μ satisfies $|\mu_s^{-1}| = q_s$. A matching μ is *stable* if it is non-wasteful and no student's priority at any school is violated in μ .

A *rule* associates a matching to every school choice problem.⁵ For rule r , $r(P)$ denotes the resulting matching for school choice problem P . We are mainly interested in the *student-proposing Deferred Acceptance (DA) rule* (Gale and Shapley, 1962), which works as follows:

- Round 1: Every student applies to her most preferred school. Every school tentatively accepts the best students according to its priority, up to its capacity, and rejects the rest.

⁵Our analysis focuses on the welfare properties of the matchings these rules produce, rather than strategic behavior or incentive compatibility. That is why we use the terms *rule* instead of *mechanism*. Informally, we also used *algorithm* for the same object before.

Round k : Every student rejected in the previous round applies to her next best school. Among both new applicants and previously accepted students, every school accepts the best students according to its priority, up to its capacity, and rejects the rest.

The procedure stops when there is a round without any new rejection. We use $\text{DA}(P)$ to denote the (unique) resulting matching generated by DA in school choice problem P . $\text{DA}_i(P)$ denotes the school to which student i is assigned. $\text{DA}_s^{-1}(P)$ denotes the set of students assigned to school s in $\text{DA}(P)$.

A matching ν is stable-dominating if, for each student $i \in I$, $\text{rk}_i(\nu_i) \leq \text{rk}_i(\text{DA}_i(P))$. Note that the DA is stable-dominating. Stable-dominating matchings were first studied by [Alva and Manjunath \(2019\)](#). A rule r is *stable-dominating* if, for every school choice P , $r(P)$ is a stable-dominating matching. Stable-dominating rules include EADA ([Kesten, 2010](#)), TTC using DA's allocation as endowment, and the Maximum Implementable Improvement over DA rule (MIIDA) ([Knipe and Ortega, 2025](#)).

3.1 Preliminary, Known Results

A student is *unimprovable* if he is assigned to the same school in every stable-dominating rule. The following lemmas will be useful for our analysis.

LEMMA 1 (Lemma 1, [Tang and Yu \(2014\)](#); Lemma 1, [Alva and Manjunath \(2019\)](#)). *If a student i is assigned to a school that did not reject any student during DA's execution, then student i is unimprovable.*

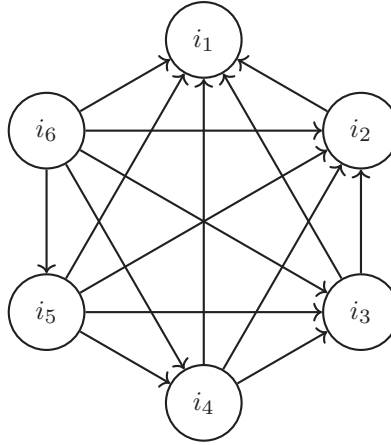
A second known lemma that is useful for us states that the number of students assigned to a school is constant across stable-dominating matchings.

LEMMA 2 (Lemma 1, [Tang and Yu \(2014\)](#); Lemma 1, [Alva and Manjunath \(2019\)](#)). *For any school choice problem P , any stable-dominating rule r and any school $s \in S$, $|\text{DA}_s^{-1}(P)| = |r_s^{-1}(P)|$.*

This result parallels the celebrated rural hospital theorem ([McVitie and Wilson, 1970](#), [Roth, 1986](#)), which shows that each school's enrolment remains constant across all stable matchings. Lemma 2 establishes the same invariance property within the class of stable-dominating rules.⁶

⁶For further extensions of rural hospital-type results to other domains, see [Irving et al. \(2000\)](#), [Hatfield and Kojima \(2010\)](#), [Kojima \(2012b\)](#) and [Klijn and Yazıcı \(2014\)](#).

The third lemma connects unimprovable students with cycles in the envy digraph induced by Deferred Acceptance (DA)—a novel concept introduced in our companion paper (Ortega et al., 2025). In this digraph, denoted $G^{\text{DA}}(P)$, each student is a node with directed edges to other students whose assignments they prefer to their own. The envy digraph for our motivating example appears below; note the absence of cycles, which reflects DA's Pareto-efficiency in this case.



The next lemma relates DA's envy digraph and unimprovable students.⁷

LEMMA 3 (Reshuffling Lemma, Erdil (2014); Proposition 1, Ortega et al. (2025)). *A student is unimprovable if and only if he does not belong to any cycles in $G^{\text{DA}}(P)$.*

4. RESULTS

4.1 Bounds

Rawlsian Welfare. We assess a rule's Rawlsian welfare by comparing the maximum rank assigned to any student (i.e., the rank of the worst-off student) against the minimum possible maximum rank achievable. This minimum is attained by the *Rawlsian rule* (RW), which selects a matching that minimizes the maximum rank for every school choice problem P (Demeulemeester and Pereyra, 2022, Afacan and Dur, 2024, Kuvalekar and Romero-Medina, 2024). This comparison reflects a societal welfare perspective focused on the worst-off individual, following Rawls (1971). The Rawlsian criterion is particularly appropriate for school choice contexts where equity concerns are paramount and policymakers focus on ensuring no student receives extremely poor educational

⁷This Lemma is mentioned in many papers beyond those cited, and is well-known in the literature.

outcomes. Unlike utilitarian approaches that might sacrifice individual students for aggregate gains, the Rawlsian perspective prioritizes protecting the most disadvantaged students—often an important concern in educational policy. Although the Rawlsian matching $\text{RW}(P)$ is not always unique, this potential multiplicity and its resolution are irrelevant for our analysis.⁸

We define the *Rawlsian welfare* (Rw) of a rule r for a school choice problem with n schools as the following ratio:⁹

$$\text{Rw}(r; n) := \sup_{P \in \mathcal{P}_{m,n}} \left[\frac{\max_{i \in I} \text{rk}_i[r_i(P)]}{\max_{i \in I} \text{rk}_i[\text{RW}_i(P)]} \right]. \quad (1)$$

Rank-Inefficiency. Analogously, the *Rank-Minimizing rule* (RM) selects a matching minimizing the average rank for every P (Featherstone, 2020). We define the rank-inefficiency (RkI) of r as

$$\text{RkI}(r; n) := \sup_{P \in \mathcal{P}_{m,n}} \left[\frac{\sum_{i \in I} \text{rk}_i[r_i(P)]}{\sum_{i \in I} \text{rk}_i[\text{RM}_i(P)]} \right]. \quad (2)$$

We note that, unlike DA, neither the Rawlsian nor the rank-minimizing rule satisfy strategy-proofness or stability, and thus may be unattainable in practice. Our analysis employs these first-best benchmarks solely to establish performance bounds and to illuminate the trade-offs inherent to stable-dominating rules, rather than to advocate for their adoption.¹⁰

These definitions enable us to characterize the Rawlsian welfare and rank-inefficiency of stable-dominating rules, demonstrating that they may assign the worst-off student and the average student to schools ranked $n/2$ times worse than the respective first-best rules.

⁸Because every Pareto-efficient matching assigns some student to their top school (due to a fundamental connection between Pareto-efficiency and serial dictatorships (Knuth, 1996, Abdulkadiroğlu and Sönmez, 1998)), the maximum rank directly tells us about the range of ranks achieved.

⁹This concept is related to the *price of anarchy* and *price of stability* concepts in algorithmic game theory (Koutsoupas and Papadimitriou, 1999, Anshelevich et al., 2008). A similar concept was proposed by Boudreau and Knoblauch (2013) to measure the efficiency loss of stable matching rules.

¹⁰Both rules, however, satisfy weaker non-manipulability properties. The rank-minimizing rule is not obviously manipulable (Trojan, 2024), while the Rawlsian rule satisfies the lower invariance axiom (Mennle and Seuken, 2021, Demeulemeester and Pereyra, 2022).

THEOREM 1. For any efficient rule r^* , any stable-dominating rule r and all $n \geq 2$,

$$\text{Rw}(r; n) = \frac{n}{2} \geq \text{Rw}(r^*; n).$$

Assume that $q_s = q$ for all schools $s \in S$. For any efficient rule r^* , any stable-dominating and Pareto-efficient rule r' and all $n \geq 2$,

$$\text{RkI}(r'; n) = \frac{n}{2} \geq \text{RkI}(r^*; n).$$

PROOF. **A worst-case construction.** We generalize Example 1 to arbitrary n to show the $n/2$ bound is achievable with unit capacities. Let students' preferences be:

- Student i_1 : $\text{rk}_{i_1}(s_1) = 1$ and $\text{rk}_{i_1}(s_n) = 2$.
- Student i_k for $k \geq 2$: $\text{rk}_{i_k}(s_{k-1}) = 1$, $\text{rk}_{i_k}(s_k) = k$, and for schools with index $j \in \{1, \dots, k-2\}$, $\text{rk}_{i_k}(s_j) = j+1$.

Schools' priorities satisfy: for any s_k and students i_ℓ, i_f with $k \leq \ell < f \leq n$, we have $\text{rk}_{s_k}(i_\ell) < \text{rk}_{s_k}(i_f)$.¹¹

Under DA, student i_k matches with s_k for all k : stability requires i_1 and s_1 match (mutual top choices), then among remaining agents i_2 and s_2 match, and so on. This allocation is Pareto-efficient, so every stable-dominating rule selects it. Here $\text{rk}_{i_n}[DA_{i_n}(P)] = n$, while the alternative Pareto-efficient matching with $\mu_{i_k} = s_{(k-1) \bmod n}$ achieves $\max_{i \in I} \text{rk}_i(\mu_i) = 2$, yielding Rawlsian ratio $n/2$.

For rank-inefficiency, DA's rank sum is $\sum_{k=1}^n k = n(n+1)/2$. The alternative matching assigns i_1 rank 2 and all others rank 1, for total $n+1$, yielding ratio $n/2$.

For uniform capacity $q \geq 1$, replicating each student q times with identical preferences preserves both ratios.

Upper bound. We now show that $n/2$ is an upper bound for all stable-dominating or Pareto-efficient rules.

Rawlsian welfare. The argument does not depend on capacities. In any matching μ , every student has rank at most n . The Rawlsian benchmark $\text{Rw}(P)$ assigns all students to their top choice only in the trivial case where this is feasible; in such cases every Pareto-efficient or stable-dominating rule must also assign everyone their top choice, so the

¹¹Although preferences and priorities are not fully specified, this partial order suffices because unspecified elements are irrelevant.

ratio equals $1 < n/2$. In every nontrivial instance the Rawlsian minimum is at least 2, so

$$\text{Rw}(\bar{r}; n) \leq \frac{n}{2}$$

for every efficient or stable-dominating rule \bar{r} .

Rank-inefficiency for unit capacities. It is well-known that any Pareto-efficient matching can be generated by a serial dictatorship: the j -th dictator receives rank at most j (Knuth, 1996, Abdulkadiroğlu and Sönmez, 1998). Hence every Pareto-efficient matching satisfies

$$\sum_{i \in I} \text{rk}_i(\mu_i) \leq \frac{n(n+1)}{2}.$$

The rank-minimizing sum is at least $n+1$: otherwise all students would receive rank 1, and the rank-minimizing matching would coincide with every Pareto-efficient matching. Thus

$$\text{RkI}(\bar{r}; n) \leq \frac{n(n+1)/2}{n+1} = \frac{n}{2}.$$

Rank-inefficiency with uniform capacities. Let $q_s = q \geq 1$ for all s , and $m = nq$. Every Pareto-efficient matching can be implemented by a serial dictatorship with quotas: students are processed in some order, and each selects the best school with remaining seats (Knuth, 1996, Abdulkadiroğlu and Sönmez, 1998). By an analogous argument to the unit capacity case, the maximum rank sum in any Pareto-efficient matching is at most $qn(n+1)/2$.

Let μ^* be a rank-minimizing matching. Write

$$\sum_{i \in I} \text{rk}_i(\mu_i^*) = nq + r$$

for some $r \geq 0$. This means exactly r students fail to receive rank 1 in μ^* . First, note that $r \geq 1$ is the interesting case, because if $r = 0$, all students receive rank 1 in μ^* , implying every Pareto-efficient matching assigns all students their top choice, yielding ratio 1. Now, we proceed in cases for $r \geq q$.

Case 1: $r \geq q$. The denominator in RkI satisfies $\text{rk}(\mu^*) = nq + r \geq (n+1)q$. The numerator satisfies $\text{rk}(\mu) \leq qn(n+1)/2$ by the serial dictatorship bound. Therefore,

$$\text{RkI}(\bar{r}; n) \leq \frac{qn(n+1)/2}{(n+1)q} = \frac{n}{2}.$$

Case 2: $1 \leq r < q$. We establish a tighter bound on the numerator using a refined batch argument.

LEMMA 4. *For any set S of schools, the number of students whose top school lies in S is at most $|S|q + r$.*

PROOF. If more than $|S|q + r$ students had their top choice in S , then at least $r + 1$ would be assigned outside S in every feasible matching (since S has capacity $|S|q$). Each such student contributes at least 1 to $\sum_i (\text{rk}_i(\mu_i^*) - 1) = r$, a contradiction. \square

We now bound the rank sum for any Pareto-efficient matching μ . Fix a serial dictatorship order producing μ . Label schools s_1, \dots, s_n in the order they become full during this process. For $t = 1, \dots, n$, let k_t be the number of students who have chosen by the time s_t becomes full (so $k_n = nq$).

Define batch 1 as the first k_1 students (those choosing before s_1 becomes full). For $t = 2, \dots, n$, define batch t as students $k_{t-1} + 1$ to k_t (those choosing after s_{t-1} becomes full and up to when s_t becomes full), with $k_0 := 0$. Batch t contains $k_t - k_{t-1}$ students.

Key property of batches: For any student in batch $t \geq 2$, the schools s_1, \dots, s_{t-1} are full at her turn, while s_t becomes full only at the end of batch t (position k_t). By the fill-order labeling, no other school can be full during batch t : if school u were to become full at some position j with $k_{t-1} < j \leq k_t$, then u would fill before or simultaneously with s_t , contradicting the labeling. Therefore, at the moment of choice for any student in batch t , the set of full schools is exactly $\{s_1, \dots, s_{t-1}\}$.

Bounding the contribution of batch t for $t \geq 2$: Let $S = \{s_1, \dots, s_{t-1}\}$. By Lemma 4, at most $(t-1)q + r$ students have their top choice in S .

Consider the $(t-1)q$ students assigned to schools in S during batches $1, \dots, t-1$ (these schools are full, each with exactly q students). We claim each such student has her top choice in S . Indeed, suppose student i is assigned to school $s \in S$, but her top choice is school $u \notin S$. At i 's turn (which occurs before batch t), school u is not in $\{s_1, \dots, s_{t-1}\}$, so u is not yet full and has at least one available seat. Therefore, i would have chosen u (her top choice) rather than s , contradicting that i chose s in the serial dictatorship.

Thus, all $(t-1)q$ students assigned to S have their top choice in S . By Lemma 4, at most $(t-1)q + r$ students total have top choice in S , so at most r students with top choice in S remain to choose in batch t or later. In particular, at most r students in batch t have their top choice in S .

For students in batch t :

- Those with top choice in S cannot obtain their top (all schools in S are full). In the worst case, they rank all schools in S above their assigned school, receiving rank at most t .
- Those with top choice outside S find their top available (by the key property), receiving rank 1.

Let b_t be the number of students in batch t whose top choice is in S . Then $b_t \leq r$ and $b_t \leq k_t - k_{t-1}$. Batch t contributes at most:

$$b_t \cdot t + ((k_t - k_{t-1}) - b_t) \cdot 1 = (k_t - k_{t-1}) + b_t(t - 1) \leq (k_t - k_{t-1}) + r(t - 1).$$

(The inequality uses $b_t \leq r$.)

For batch 1, no school is full, so all k_1 students receive rank 1. Summing over all batches,

$$\sum_{i \in I} \text{rk}_i(\mu_i) \leq k_1 + \sum_{t=2}^n [(k_t - k_{t-1}) + r(t - 1)] = k_n + r \sum_{t=1}^{n-1} t = nq + \frac{rn(n-1)}{2}.$$

Therefore

$$\text{RkI}(\bar{r}; n) \leq \frac{nq + \frac{rn(n-1)}{2}}{nq + r}.$$

A simple algebraic manipulation shows this is strictly less than $n/2$ for $n \geq 3$ and $0 \leq r < q$.¹² Combining all cases yields

$$\text{RkI}(\bar{r}; n) \leq \frac{n}{2} \quad \text{for all } q \geq 1,$$

with equality achieved by our earlier construction. □

These bounds represent worst-case scenarios that reveal a fundamental limitation of efficiency-based reforms. The $n/2$ ratio establishes that as the number of schools grows, the performance gap between stable-dominating rules and first-best benchmarks grows linearly: the worst-off student can be assigned to a school ranked $n/2$ times worse than necessary, and average ranks can be $n/2$ times higher than achievable. This performance degradation persists for both metrics: the Rawlsian welfare bound holds regardless of school capacity, while the rank-inefficiency bound holds under uniform capacities. This demonstrates that the problem is intrinsic to stable-dominating rules rather

¹²Expanding both sides shows the inequality is equivalent to $(n-2)n(q-r) > 0$.

than a consequence of modelling choices. While allowing for non-constant school capacities may in principle lead to a larger worst-case bound, the $n/2$ ratio from remains attainable. In this sense, extending the analysis to non-uniform capacities would, if anything, strengthen the negative message of our result. Even in very large markets, any stable-dominating or Pareto-efficient rule may assign students an average rank up to a factor $n/2$ worse than the first-best. Thus the difference between the Rawlsian or rank-minimizing benchmark and the best Pareto improvements over DA is of first-order magnitude, whereas capacity-induced refinements affect the bound only at second order.

This result parallels [Kesten \(2010\)](#)'s Proposition 1, which shows that DA may assign all students to their worst- or second-worst school—a finding that motivated the development of stable-dominating rules. Our contribution is to show that while stable-dominating rules successfully address Pareto-inefficiency, they do so in a way that fails to correct DA's poor rank distributions. Efficiency improvements through consented priority violations cannot fix the rank distribution problem.¹³

We note that, since the example we constructed has a unique stable matching, [Theorem 1](#) applies not only to any rule that weakly Pareto dominates student-proposing DA, but in fact to any rule that weakly Pareto dominates *any* stable rule. This shows that our critique extends beyond stable-dominating rules to any efficiency adjustment that improves upon stable allocations.

MAIN IMPLICATION. Stable-dominating rules can assign the worst-off student to a school ranked $n/2$ times worse than necessary (Rawlsian welfare) and can generate average ranks $n/2$ times higher than achievable (rank-efficiency). Both worst-case ratios equal $n/2$, demonstrating that efficiency-based reforms cannot fundamentally address rank distribution problems.

Empirical Relevance. It is natural to ask how informative our welfare bounds are for the performance of stable and stable-dominating mechanisms in practice. There is in fact evidence that the *true, underlying* rank distribution generated by DA can be highly suboptimal. For example, in the Hungarian high-school match in Budapest, the worst student is assigned to their 13th-ranked school (out of 14) under both DA and EADA, according to the *stated* preferences (with RM assigning the worst-off student to his 6th

¹³Our construction also performs poorly on alternative welfare metrics. The rank vector $(1, 2, 3, \dots, n)$ in our worst-case construction has variance $\frac{n^2-1}{12}$, compared to variance $\frac{n-1}{n^2}$ for the best possible Rawlsian allocation with rank vector $(1, 1, \dots, 1, 2)$. We thank an anonymous reviewer for making this point.

choice). More strikingly, when true preferences are estimated—accounting for the well-documented self-selection of students who avoid applying to schools they perceive as unattainable (Chen and Pereyra, 2019)—the maximum rank in DA and EADA becomes 241 out of 244. However, the RM assigns the worst-off student to his 16th choice (Ortega and Klein, 2023).

Thus, although our bounds describe worst-case performance, they help explain why stable and stable-dominating mechanisms can generate poor rank distributions in practice.¹⁴

4.2 Segregation

We now turn to whether stable-dominating rules can improve (or fail to improve) the placement of low-priority students, and in particular whether Pareto improvements over DA can alter patterns of segregation in school compositions. In many applications, low-priority students correspond to groups facing systemic disadvantages that are embedded in admissions criteria—such as students from low-income households, immigrants, ethnic, racial, or religious minorities, or students with disabilities. In other contexts, however, low-priority tiers may reflect entitlement-based policies rather than disadvantage; our analysis is agnostic to the normative interpretation of priorities. Several studies have shown that DA can generate high levels of school segregation (Terrier et al., 2021, Kutscher et al., 2023). It is therefore of policy interest to understand whether any stable-dominating rule can reduce the segregation arising under DA.

We partition the set of students into k non-empty and disjoint tiers T_1, \dots, T_k with $\bigcup_{h=1}^k T_h = I$ and $T_h \cap T_j = \emptyset$ for all $h \neq j$. Students in tier T_h have higher priority than every student in tier T_j whenever $h < j$; that is, for any $i \in T_h$ and $i' \in T_j$ with $h < j$, we have $\text{rk}_s(i) < \text{rk}_s(i')$ at every school s . This priority structure applies at all schools. We refer to students in T_1 and T_k as highest- and lowest-priority students, respectively.¹⁵

¹⁴Additional empirical evidence supports this finding. First, using nineteen datasets on course allocation, Diebold and Bichler (2017) find that in seven of them the rank distributions generated by an efficient, stable-dominating mechanism (EADA) are not statistically different from those produced by DA, further illustrating the limited ability of stable-dominating mechanisms to correct DA's rank distribution. Second, while 83% of students in NYC's 2025 high school match received one of their top three *stated* choices, analysis of NYC's 2009-2010 match (Che and Tercieux, 2018) reveals that even among stated preferences, over 12,000 students (16%) received worse than their 5th stated choice, with over 8,600 (11%) receiving worse than their 10th choice or remaining unassigned. Since students could rank only 12 schools until 2025, true preference ranks over all 700+ programs would be substantially higher, as the Budapest evidence suggests.

¹⁵Tiered preference and priority structures in matching markets have been discussed in a number of papers, including Allman et al. (2025), Ashlagi et al. (2017a,b, 2021), Çelebi and Flynn (2022), Cai (2025),

Our assumption that priorities are fully tiered at every school is admittedly strong, but it allows for a clean analysis. Moreover, stratified priority structures of this kind are observed in practice. For example, Singapore’s primary-school admissions system gives universally higher priority to citizens, followed by permanent residents, and only later to international students; this ordering is primarily entitlement-based.¹⁶ In the UK, looked-after and previously looked-after children have higher priority than any other prospective applicant at every school under the School Admissions Code (2021), paragraph 1.7.¹⁷

Our tier terminology describes position in the priority structure rather than social status. High-priority tiers may correspond to genuinely advantaged groups (as in entitlement-based systems) or to vulnerable populations granted protective priority by policy (as with looked-after children in the UK). Our invariance results, showing that stable-dominating mechanisms preserve demographic composition under tiered priorities, apply regardless of the normative rationale for tiers. Thus the policy implications are context-dependent: concentration of protected vulnerable students may limit peer diversity, whereas concentration of policy-favoured groups may reinforce existing inequalities.

Under the aforementioned priority structure, we obtain the following result.

LEMMA 5. *There is at least one lowest-priority student whose DA assignment does not improve under any stable-dominating rule.*

PROOF. Let t be the last round of DA’s execution where a lowest-priority student applies to a school. Such student i is unimprovable if he is unmatched in DA by Lemma 1. Thus, assume instead that i is assigned to school s . Let $O^t(s)$ denote the set of student proposals that school s receives during round t , and q_s^{t-1} be the number of students accepted by school s at time $t - 1$. If $|O^t(s)| + q_s^{t-1} > q_s$, then school s rejects at least one student at time t . Since i is accepted and is a lowest-priority student, the student who was rejected at time t was himself lowest-priority. This rejected lowest-priority student either (1) applies to some school at a round $t' > t$, contradicting our definition of t , or (2) becomes unmatched, in which case he is unimprovable by Lemma 1. Therefore,

Dur et al. (2016) and, more related to us, Calsamiglia and Miralles (2023). However, none of those papers study stable-dominating rules.

¹⁶Source: Ministry of Education, Singapore. <https://www.moe.gov.sg/international-students/admission/>, accessed on 2025-08-29.

¹⁷Source: School Admissions Code, Department of Education, England. <https://www.gov.uk/government/publications/school-admissions-code--2>, accessed on 2025-08-29.

$|O^t(s)| + q_s^{t-1} \leq q_s$. Then school s does not reject any student at time $t' \leq t$. But then i is unimprovable by Lemma 1. \square

The existence of at least one unimprovable lowest-priority student raises a natural question: can students from low-priority tiers (larger tier numbers) improve their assignments by trading with students from higher-priority tiers (lower tier numbers)? The following result shows this is impossible due to the priority structure.

LEMMA 6. *No trading cycle in G^{DA} involves students from different tiers.*

PROOF. Suppose, for contradiction, that there exists a cycle containing students from two tiers, $T_j < T_h$. Without loss of generality, there exists $i_2 \rightarrow i_1$ as part of such a cycle where $i_1 \in T_h$ has universally worse priority than $i_2 \in T_j$. The edge $i_2 \rightarrow i_1$ means that higher-priority student i_2 envies the lower-priority student i_1 's assignment, i.e., i_2 prefers school $s = \text{DA}_{i_1}(P)$ to their own assignment $\text{DA}_{i_2}(P)$. However, by assumption, i_1 has lower priority than i_2 at every school, including school s . Then, (i_2, s) is a blocking pair of $\text{DA}(P)$, contradicting stability. Therefore, cycles in $G^{\text{DA}}(P)$ cannot contain students from different tiers. \square

Lemma 6 reveals that improvement cycles respect tier boundaries, so that students can only trade within their own tier. This segregation in trading possibilities has immediate implications for school compositions under any stable-dominating matching. To formalize these implications, we introduce the following definition. We say that a school s is *fully segregated* under matching μ if it only accepts students belonging to the same tier, i.e. if $\mu_s^{-1} \subseteq T_j$ for some $1 \leq j \leq k$. While this binary measure captures only the most extreme form of segregation, our results show that stable-dominating rules cannot eliminate even these most severe cases of separation.¹⁸

LEMMA 7. *Every student assigned to a school that also has students from some worse tier is unimprovable.*

PROOF. Let $i \in T_t$ be a student assigned to school s under DA, where school s also admits at least one student $j \in T_{t'}$ with $t' > t$ (i.e., j has universally worse priority than i).

¹⁸Many other measures of segregation are available in the literature, such as the Duncan index (Duncan and Duncan, 1955, Escobar and Huerta, 2025). Our choice of the extreme binary measure was deliberate because we wanted to establish that stable-dominating rules cannot eliminate even the most severe form of segregation. See also Calsamiglia et al. (2021) for a measure of segregation based on differences in reported preferences due to different risk attitudes.

Suppose, for contradiction, that i belongs to a cycle in $G^{\text{DA}}(P)$. By Lemma 6, this cycle contains only students from tier T_t . Therefore, there must exist another student $i' \in T_t$ who envies i 's placement at school s .

However, this is impossible. Since s admits student j from a worse tier $T_{t'}$ (with $t' > t$), and students in tier T_t have universally better priority than students in tier $T_{t'}$, any student from tier T_t who applied to s would have been accepted over j . The fact that j is at s means that no student from tier T_t other than those already at s ever applied there.

In particular, student $i' \in T_t$ never applied to s during DA's execution, which means i' does not prefer s to their current assignment. This contradicts our assumption that i' envies i 's placement at s .

Therefore, no student from tier T_t envies i , so i cannot be part of any cycle and is unimprovable. \square

We have established three useful facts: some lowest-priority students are unimprovable (Lemma 5), no trades occur across tiers (Lemma 6), and students from better tiers at mixed schools cannot improve (Lemma 7). These results combine to yield our main segregation Theorem.

THEOREM 2. *The number of students accepted to each school from each tier is constant under any stable-dominating rule. Thus, if a school is fully segregated under DA, it remains fully segregated under any stable-dominating rule.*

PROOF. By Lemma 6, no improvement cycle contains students from two different tiers. Hence any improvement by a student in tier T_t only displaces students in tier T_t . Consider the following two cases:

- (i) Consider a segregated school s that admits students from a single tier under DA. Since cycles cannot mix tiers, no student from any other tier can be brought into s via an improvement cycle. By Lemma 2, the total number of students at s is the same under any stable-dominating rule as under DA. Therefore, the number from each tier at s is unchanged, and s remains fully segregated.
- (ii) Consider a mixed school s which admits students from at least two tiers under DA. Let \mathcal{T}_s be the set of tiers with students at s under DA, and let $T_{t^*} \in \mathcal{T}_s$ be the lowest-priority tier present at s (i.e., $t^* > t$ for all other $T_t \in \mathcal{T}_s$). By Lemma 7, every student at s from any tier $T_t \in \mathcal{T}_s \setminus \{T_{t^*}\}$ is unimprovable (because s also admits students from the lower-priority tier T_{t^*}). Therefore, the number of students from each of

these higher-priority tiers is fixed. By Lemma 2, the total enrolment at s is also fixed. It follows that the number of students from the lowest-priority tier T_{t^*} must also be fixed as the residual. Thus, for each tier present at s , the number of students admitted is the same as in DA under any stable-dominating matching.

As the previous two cases are exhaustive, we conclude that the demographic composition of every school is perfectly preserved under any stable-dominating rule. \square

Theorem 2 presents an invariance result that complements the rural hospital theorem for stable-dominating rules (Lemma 2). The rural hospital theorem shows that while enrolment numbers are constant across all stable matchings at every school, the exact identities are preserved only at under-subscribed schools. Our result demonstrates a different invariance pattern: under stratified priority structures, the demographic composition at every school—both under-subscribed and over-subscribed—remains constant across stable-dominating rules, even though individual student identities may vary.

Theorem 2 can be understood through an intuitive k -stage perspective. Any stable-dominating rule operates as if it first matches students from the highest-priority tier, T_1 . Then, students from T_2 are matched to the remaining seats, and so on, proceeding sequentially through all k tiers. Due to the tiered priority structure, students from a lower-priority tier T_t can never displace students from a higher-priority tier $T_{t'}$ (where $t' < t$). Applications to seats already filled by higher-priority students would be automatically rejected. The occupancy invariance property (Lemma 2) thus applies at each stage: first preserving the total number of T_1 students at each school, then the total number of T_2 students in the remaining capacity, and so on for all k tiers. This sequential structure reveals why efficiency improvements operate strictly within the tier boundaries (as shown in Lemma 6): students from a given tier T_t can only trade with other students from the *same* tier T_t .

Our analysis reveals important limitations of efficiency-based reforms in school choice settings with priority stratification. Under the specific assumption of complete tier separation, stable-dominating rules like EADA can improve welfare within each group but cannot bridge divides between groups or create more integrated school environments. These theoretical results provide a benchmark for understanding when efficiency-based reforms may be constrained in addressing segregation. Whether such limitations apply in practice depends critically on the specific priority structures in

place. In settings where systematic priority advantages approximate our tiered priority assumption, segregation patterns similar to those documented under DA in England (Terrier et al., 2021) and Chile (Kutscher et al., 2023) may persist under stable-dominating rules. However, our results do not directly imply such persistence in these specific cases, as the priority structures there may differ from our theoretical framework.

A Second Example. To illustrate the preservation of segregation, consider the school choice problem in Example 2 with eight students and four schools, each with two seats.

EXAMPLE 2. SOSM in squares, stable-dominating in bold.

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	s_1	s_2	s_3	s_4
s_2	s_2	s_1	s_1	s_1	s_1	s_3	s_3	i_1	i_3	i_1	\cdot
s_1	s_1	s_2	s_2	s_2	s_2	s_4	s_4	i_2	i_4	i_2	\cdot
s_3	s_3	s_4	s_3	s_3	s_3	s_2	s_1	i_5	i_5	i_3	\cdot
s_4	s_4	s_3	s_4	s_4	s_4	s_1	s_2	i_3	i_1	i_4	\cdot
								i_4	i_2	i_5	\cdot
								i_8	i_6	i_7	\cdot
								i_6	\cdot	i_8	\cdot
								\cdot	\cdot	i_6	\cdot

Here, students i_1, \dots, i_5 are high-priority students, whereas i_6, i_7 , and i_8 are low-priority ones. The DA matching assigns i_1, i_2 to s_1 , i_3, i_4 to s_2 , i_5, i_7 to s_3 , and i_6, i_8 to s_4 . Three of four schools are fully segregated. Yet this DA allocation is not Pareto-efficient: it is dominated by the stable-dominating allocation i_3, i_4 to s_1 , i_1, i_2 to s_2 , i_5, i_7 to s_3 , and i_6, i_8 to s_4 . Segregation persists: the three fully segregated schools remain fully homogeneous.

There is, however, an alternative Pareto-efficient allocation with less segregation: i_4, i_6 to s_1 , i_1, i_2 to s_2 , i_5, i_7 to s_3 , and i_3, i_8 to s_4 . Here, only s_2 is fully segregated, and both rank-efficiency (average rank 1.625 vs. 1.75) and Rawlsian welfare (3 vs. 4) improve relative to the stable-dominating outcome. This comes at the cost of more blocking pairs (4 vs. 2).

4.3 Discussion

Our findings suggest that in highly stratified environments, policies to reduce school segregation may need to rely on interventions beyond Pareto improvements over stable matchings. Pareto-efficient allocations with reduced segregation can be obtained through DA modifications with minority reserves (e.g. [Hafalir et al., 2013](#)).¹⁹ Empirical evidence supports this: Swedish simulations show that reserved seats reduce segregation compared to proximity- or lottery-based priorities while maintaining high shares of students at preferred schools ([Kessel and Olme, 2018](#)), and in Chile such reserves both reduce segregation and improve rank-efficiency ([Escobar and Huerta, 2025](#)).

One can also relax Pareto domination in favor of weaker but still meaningful dominance concepts such as rank-dominance ([Featherstone, 2020](#)), which can escape the constraints we identify and achieve better bounds than those shown for stable-dominating rules in [Theorem 1](#).

In stratified preference settings, “*access to better schools*” is also a valuable outcome measure. While our model does not impose assumptions on school quality rankings, extending our analysis to such environments is a promising direction. More broadly, rules like TTC, which do not Pareto-dominate DA, can escape the segregation and rank-distribution limitations as shown by [Calsamiglia and Miralles \(2023\)](#). Policymakers should therefore consider these alternatives if Pareto-dominance over DA is not a requirement.

5. CONCLUSION

We have shown that Pareto-efficiency improvements that do not harm any student compared to the DA baseline may not be able to correct three potential problems of DA: its rank-inefficiency, its poor Rawlsian welfare, and its associated segregation. Our findings should be interpreted as a cautionary tale about efficiency-adjustments rather than arguing against their need and scope. Indeed, a substantial literature—in particular our companion paper ([Ortega et al., 2025](#))—demonstrates that most students can benefit from rules that Pareto-dominate DA. The contribution here is to precisely characterize what such rules cannot fix, and the need for additional policies orthogonal to Pareto-efficiency to address such shortcomings.

¹⁹The literature on minority reserves in school choice is extensive and typically finds matchings that do not Pareto dominate DA (e.g. [Kojima, 2012a](#), [Aygün and Bó, 2021](#)). Our focus here is on reforms that preserve Pareto-dominance over DA.

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